Plan d'étude et représentation graphique de $y = f(x) = \cos x$

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Le domaine de définition de f

$$y = f(x) = \cos x \Rightarrow D_f = {}^\circ \ = \left(-\infty, +\infty\right)$$

$$y = \cos x \Rightarrow T = 2\pi \Rightarrow I = \left[0, 2\pi\right]$$

Etudier la fonction au bornes de I

A la borne gauche

$$x = 0 \Rightarrow y = 1 \Rightarrow \left| \begin{array}{l} 0 \\ 1 \end{array} \right.$$

Alors le point $\left| \begin{array}{l} 0 \\ 1 \end{array} \right.$ est un point d'arrêt.

A la borne droite

$$x = 2\pi \Rightarrow y = 1 \Rightarrow \left| \begin{array}{l} 2\pi \\ 1 \end{array} \right.$$

Alors le point $\left| \begin{array}{l} 2\pi \\ 1 \end{array} \right.$ est un point d'arrêt.

Le sens de variation de f

$$y' = -\sin x$$

$$-\sin x = 0 \Rightarrow x = k\pi$$

$$k = 0 \Rightarrow x = 0 \Rightarrow y = 0 \Rightarrow \left| \begin{array}{l} 0 \\ 0 \end{array} \right.$$

$$k = 1 \Rightarrow x = \pi \Rightarrow y = 0 \Rightarrow \left| \begin{array}{l} \pi \\ 0 \end{array} \right.$$

$$k = 2 \Rightarrow x = 2\pi \Rightarrow y = 0 \Rightarrow \left| \begin{array}{l} 2\pi \\ 0 \end{array} \right.$$

Convexité de f

$$y'' = -\cos x$$

$$-\cos x = 0 \Rightarrow x = k\pi + \frac{\pi}{2}$$

$$k = 0 \Rightarrow x = \frac{\pi}{2} \Rightarrow y = 1 \Rightarrow \left| \begin{array}{l} \frac{\pi}{2} \\ 1 \end{array} \right.$$

$$k = 1 \Rightarrow x = \frac{3\pi}{2} \Rightarrow y = -1 \Rightarrow \left| \begin{array}{l} \frac{3\pi}{2} \\ -1 \end{array} \right.$$

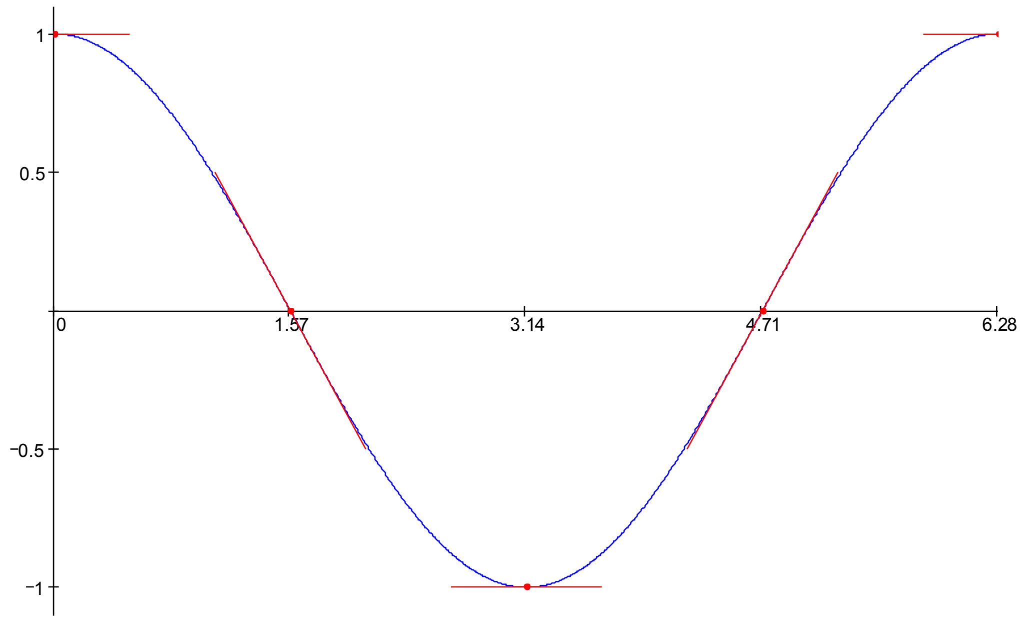
$$m_{\frac{\pi}{2}} = f'(\frac{\pi}{2}) = -1$$

$$m_{\frac{3\pi}{2}} = f'(\frac{3\pi}{2}) = 1$$

Le tableau de variation

x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
y'	0	$-$	-1	$-$	0	$+$	1	$+$	0
y''		$-$	0	$+$		$+$		$-$	
y	1	↘ $\frown$	0 Inf	↘ $\smile$	-1 Min	↗ $\smile$	0 Inf	↗ $\frown$	1

La courbe


1
0.5
−0.5
−1
0
1.57
3.14
4.71
6.28