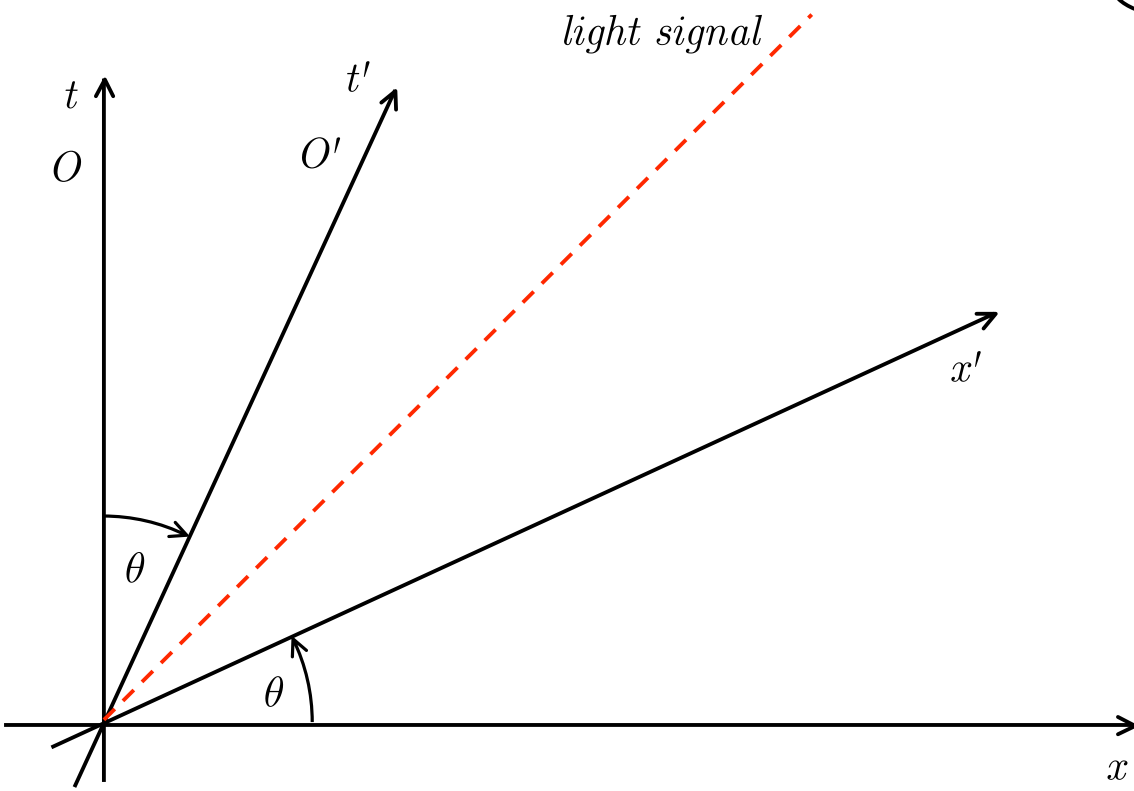
15. Einstein and Minkowski Spacetime

- Light Postulate of Special Relativity entails:

The speed of light c is the same in all inertial reference frames.



$$c = \frac{\Delta x}{\Delta t} = \frac{\Delta x'}{\Delta t'}$$

value of c for O $=$ *value of c for O′*

- O' is moving at constant velocity with respect to O.
- O and O' must measure same speed c for light signal.
- <u>*So*</u>: O and O' must disagree on spatial and temporal measurements!

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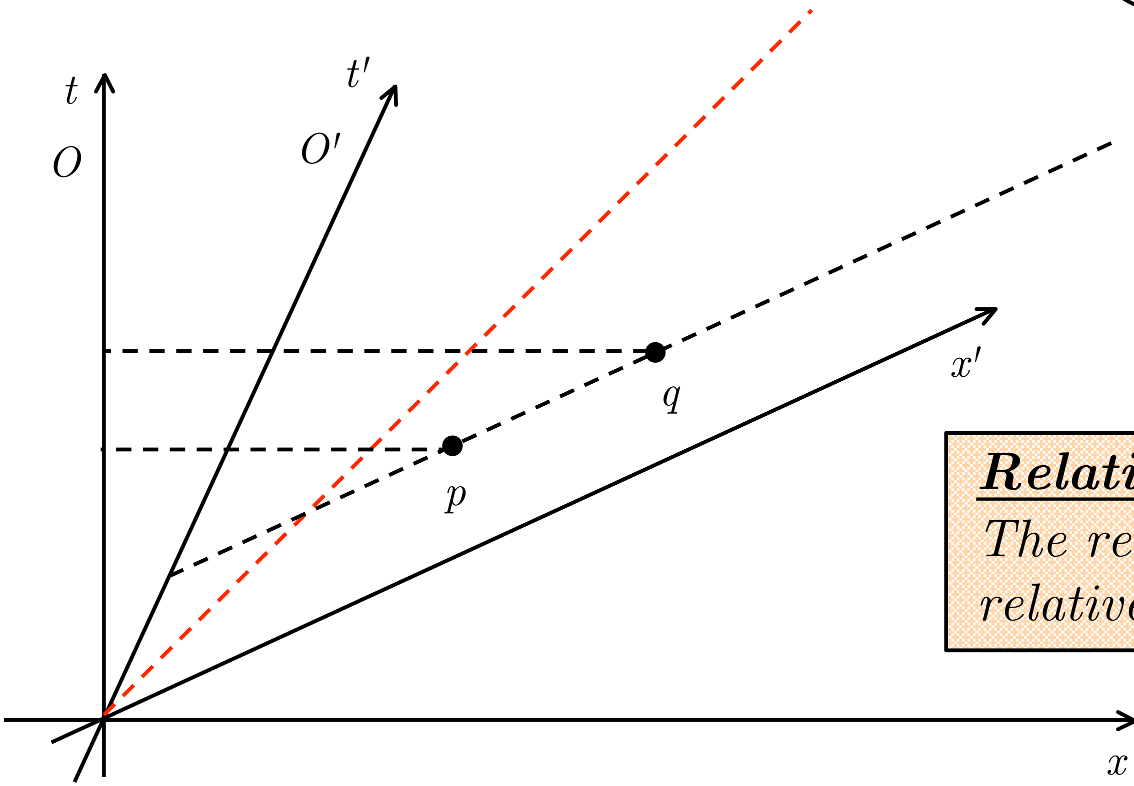


Relativity of Simultaneity

The relation of simultaneity is relative to inertial reference frames.

- O and O' make different judgements of simultaneity.
- p and q are simultaneous according to O'.
- p happens before q according to O.

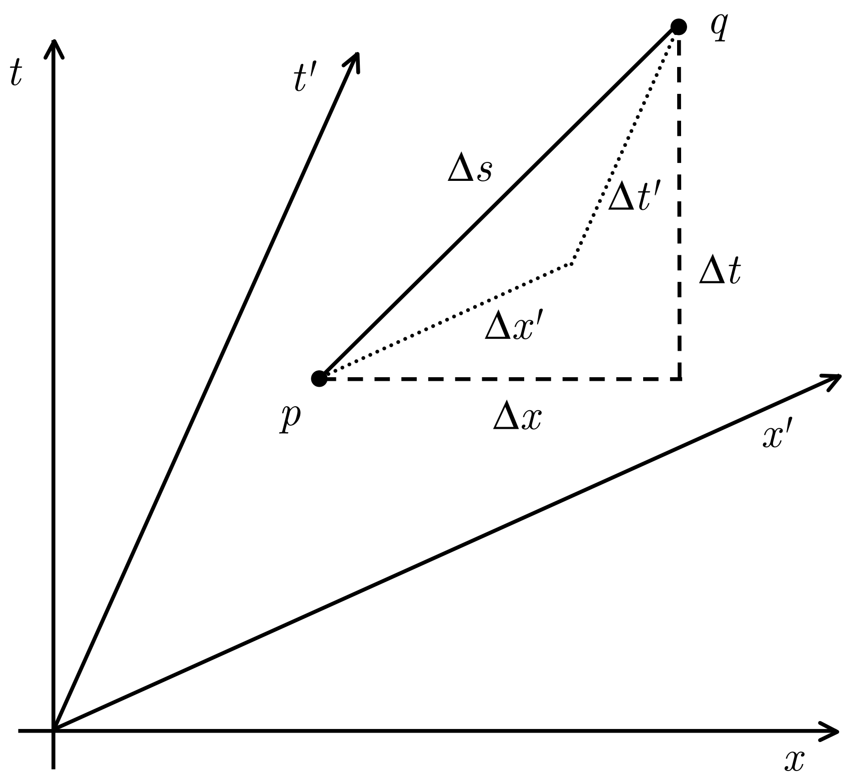
- Spacetime of Special Relativity = Minkowski spacetime

Hermann Minkowski

Minkowski spacetime = 4-dim collection of points such that between any two points $p(t, x, y, x)$, $q(t + \Delta t, x + \Delta x, y + \Delta y, z + \Delta z)$ there is a definite spacetime interval given by

$$\Delta s = \sqrt{-(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}.$$

- Similar to Euclidean *spatial* interval $\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$.
- *But*: Includes the time coordinate difference, too! And it's *negative*!
- *Idea*: All inertial frames will agree on the spatiotemporal distance Δs between any points p and q.
- But they will disagree on how Δs gets split into a temporal part and a spatial part: they will disagree on measurements of time and measurements of space.



$$\Delta s = \sqrt{-(c\Delta t)^2 + (\Delta x)^2}$$

$$= \sqrt{-(c\Delta t')^2 + (\Delta x')^2}$$

- All inertial frames agree on the *spatiotemporal* distance between any two points p and q.
- They will disagree on the *temporal* distance between p and q (time dilation) and on the *spatial* distance (length contraction).
- They will *disagree* on *how* they split the spacetime distance into temporal and spatial parts.

- The Minkowski spacetime interval is encoded in the *Minkowski metric* $\eta_{\mu\nu}$.

$$(\Delta s)^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu \qquad \mu,\, \nu = 0,\, 1,\, 2,\, 3$$

where $\Delta x^0 = c\Delta t$, $\Delta x^1 = \Delta x$, $\Delta x^2 = \Delta y$, $\Delta x^3 = \Delta z$, and $\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- Infinitesimally: $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu$.

- The Minkowski spacetime interval $(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ has three general forms:

(a) *Timelike interval*: $\Delta s < 0$, *or* $(c\Delta t)^2 > (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$.

$$\text{Speed} = \frac{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}{\Delta t} < c.$$

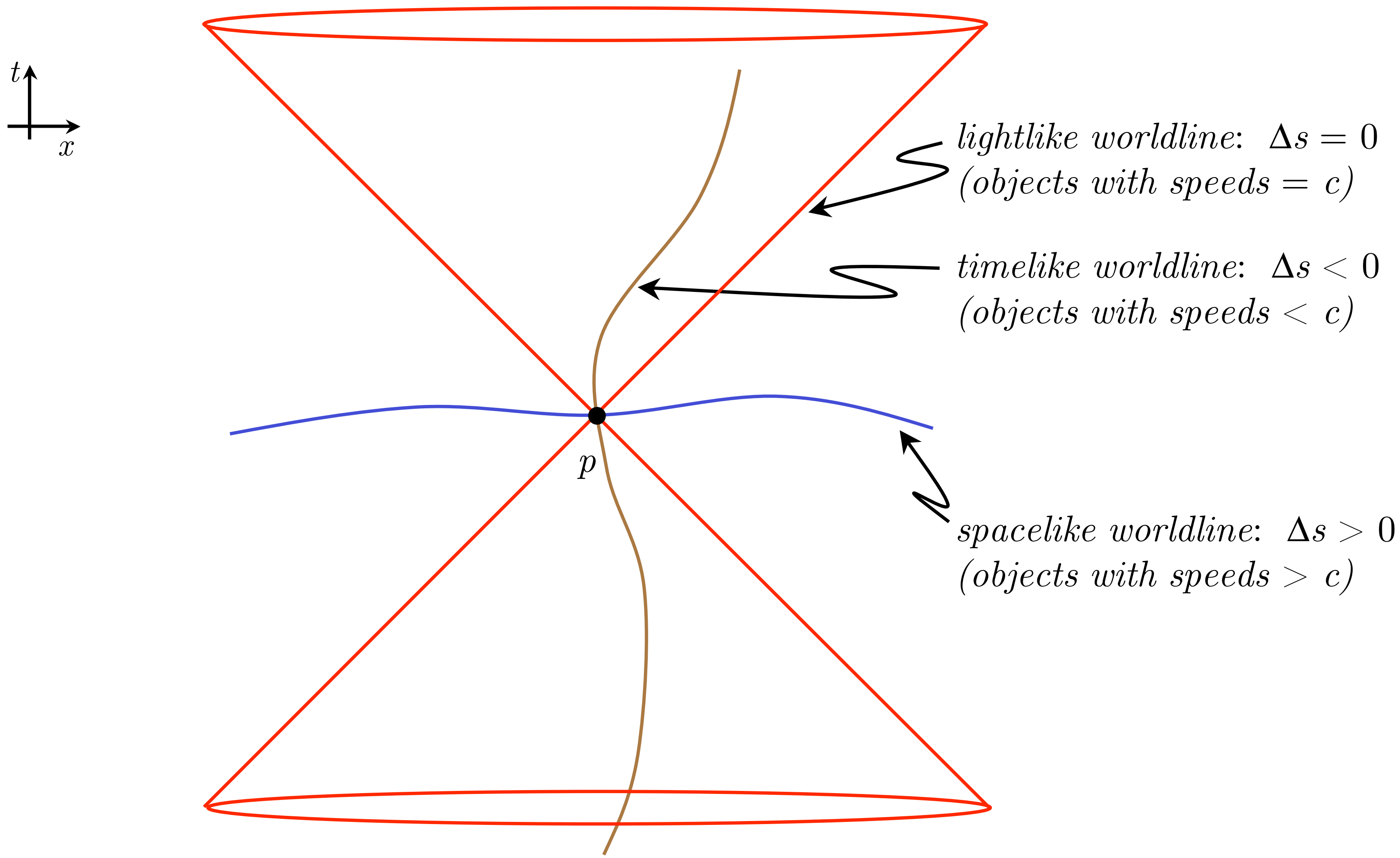
(b) *Lightlike interval*: $\Delta s = 0$, *or* $(c\Delta t)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$.

Speed $= c$.

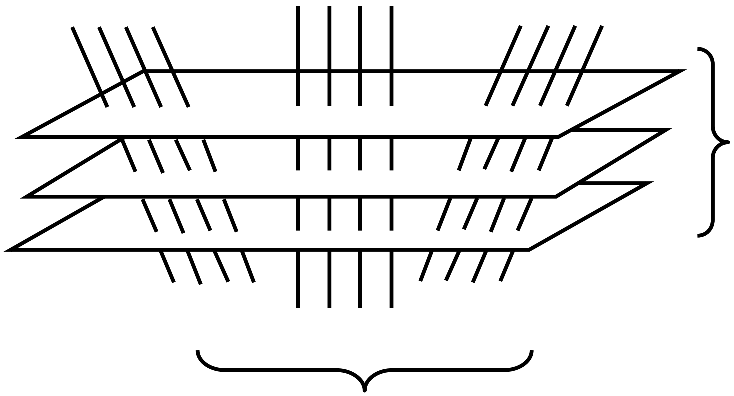
(c) *Spacelike interval*: $\Delta s > 0$, *or* $(c\Delta t)^2 < (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$.

Speed $> c$.

- *Hence*: The Minkowski metric defines a *lightcone* at any point p:

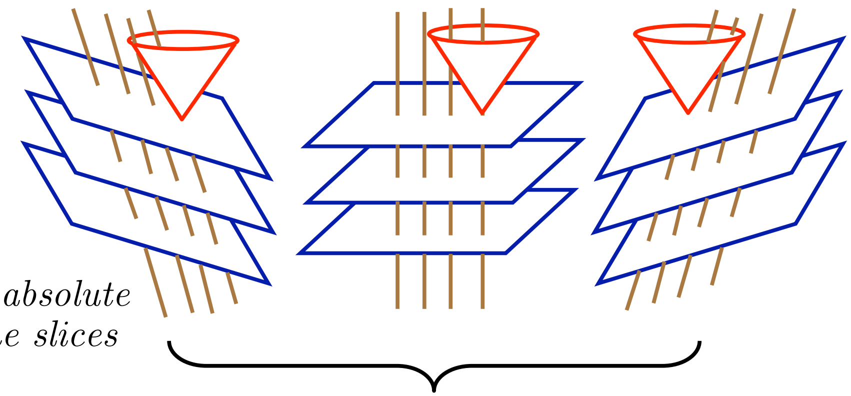


Neo-Newtonian Spacetime



1. Many inertial frames; none privileged.
2. Velocity is relative.
3. Acceleration is absolute.
4. Simultaneity is absolute.

Minkowski Spacetime



1. Many inertial frames; none privileged.
2. Velocity is relative.
3. Acceleration is absolute.
4. Simultaneity is relative.
5. Invariant light-cone structure at each point.

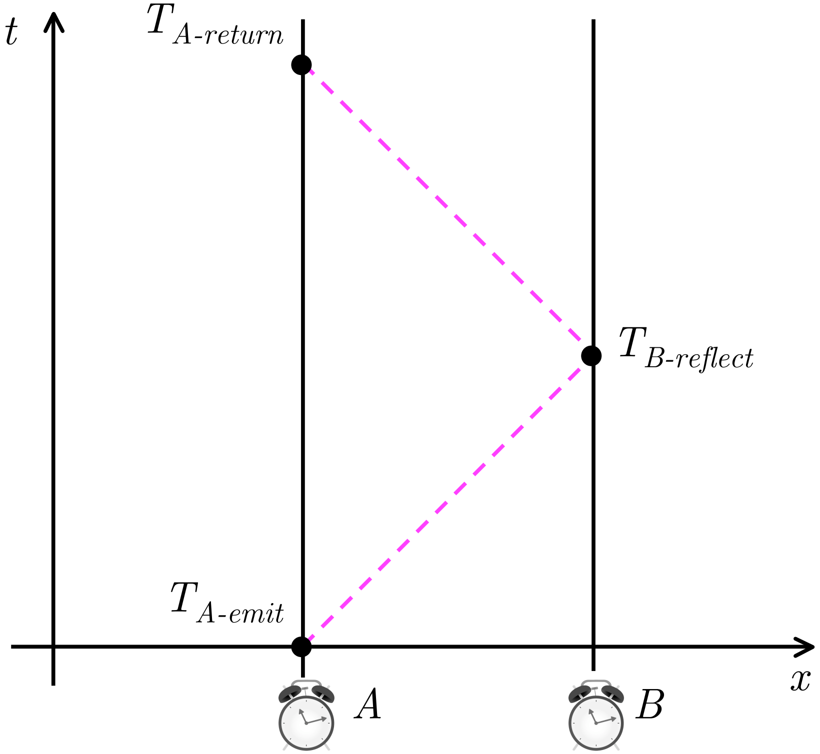
The Conventionality of Simultaneity

Claim: Given an event A, there is no objective fact of the matter as to what *distant* events at rest with respect to A are simultaneous with A. The choice is a matter of convention.

Hans Reichenbach (1891-1953)

- *Relativity of simultaneity* = Different inertial frames judge the simultaneity of events in different ways. (Entailed by the 2 Postulates.)
- *Conventionality of simultaneity* = Within a *single* inertial frame, the simultaneity of *distant* events is not fixed and can be judged in different ways. (*Not* entailed by the 2 Postulates.)

- How can the simultaneity of distant events in the same inertial frame be established?
  - Einstein (1905): By setting up synchronized clocks at these events.
- How can distant clocks in the same inertial frame be synchronized?
  - Einstein (1905): Use light signals.

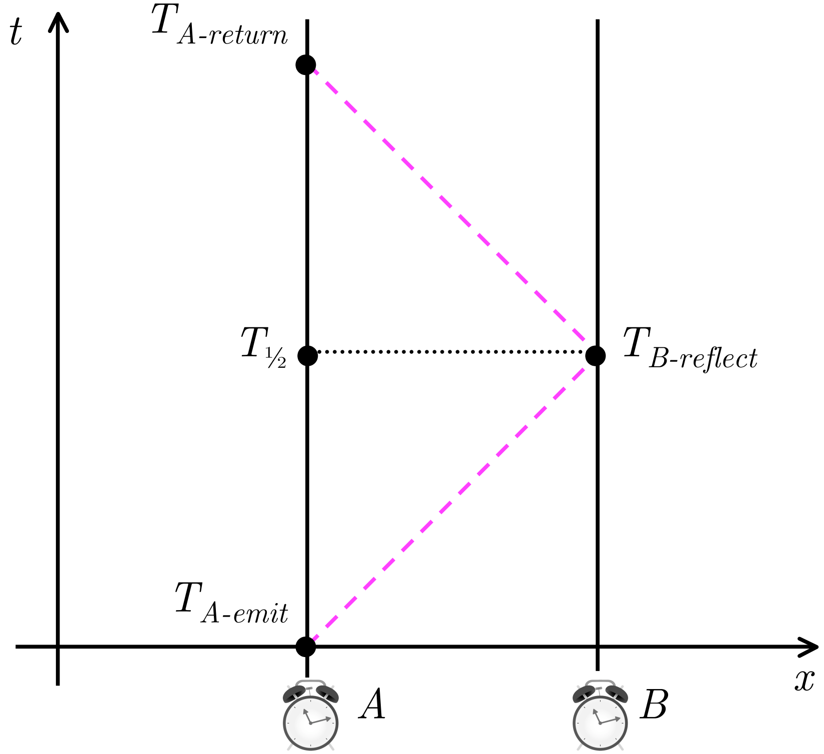


- To synchronize Clock B a given distant from Clock A,

  (1) Emit a light signal from A to B and record the time $T_{A\text{-}emit}$ on A.

  (2) Have B reflect the signal back to A. Record the time on B, $T_{B\text{-}reflect}$.

  (3) Record the A time $T_{A\text{-}return}$ when the light signal returns.

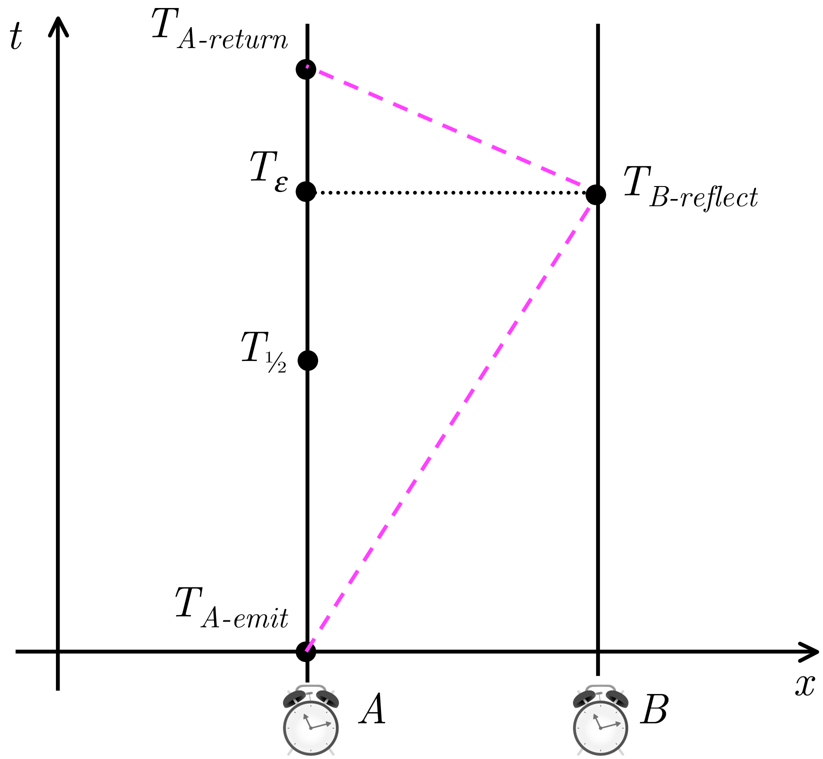


> ***Standard Simultaneity***
>
> The event at $T_{B\text{-}reflect}$ is simultaneous with the event at $T_{½}$.

- *Einstein's Stipulation*: A and B are in synchrony just when

  $T_{B\text{-}reflect} = T_{½} \equiv T_{A\text{-}emit} + ½(T_{A\text{-}return} - T_{A\text{-}emit})$.

  - *Assumption*: Light travels at the same speed c in all directions.



Standard Simultaneity

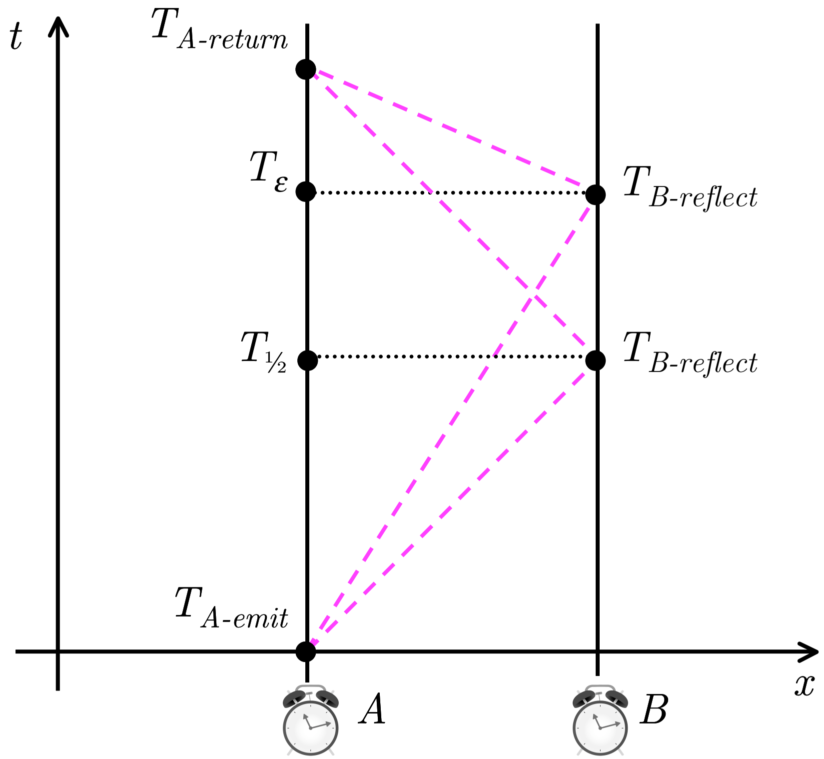
The event at $T_{B\text{-}reflect}$ is simultaneous with the event at $T_{1/2}$.

Non-Standard Simultaneity

The event at $T_{B\text{-}reflect}$ is simultaneous with the event at T_{ε}.

- *Einstein's Stipulation*: A and B are in synchrony just when
  $T_{B\text{-}reflect} = T_{1/2} \equiv T_{A\text{-}emit} + 1/2(T_{A\text{-}return} - T_{A\text{-}emit})$.
  - *Assumption*: Light travels at the same speed c in all directions.

- *Reichenbach's Conventionalism*: A and B may be said to be in synchrony when
  $T_{B\text{-}reflect} = T_{\varepsilon} \equiv T_{A\text{-}emit} + \varepsilon(T_{A\text{-}return} - T_{A\text{-}emit})$, for *any* value of ε, where $0 < \varepsilon < 1$.
  - *Assumption*: Light does *not* necessarily travel at the same speed c in all directions.



Standard Simultaneity

The event at $T_{B\text{-}reflect}$ is simultaneous with the event at $T_{½}$.

Non-Standard Simultaneity

The event at $T_{B\text{-}reflect}$ is simultaneous with the event at T_ε.

- Who's right: Einstein or Reichenbach?
- Does light travel at the same speed in all directions or not?
- How can the "one-way" speed of light be measured?
- *Reichenbach's Claim:*

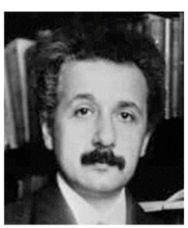
(a) To measure the one-way speed of light, we need synchronized clocks.

(b) But we can only synchronize our clocks if we have prior knowledge of distant simultaneity, which requires prior knowledge of the one-way speed of light.

Realist Response:

- Agree that there is no observational difference between the standard simultaneity relation and any non-standard simultaneity relation.
- *So*: If empirical adquacy (*i.e.*, agreement with observation) is the criterion for how one chooses between competing theories, then there's no reason to prefer the standard relation to any non-standard relation.
- *But*: Why think empirical adquacy is the only criterion of theory choice?

> ◦ Suppose *simplicity* is a criterion of theory choice.
>
> ◦ *Then*: We should prefer the standard simultaneity relation, since it assumes light travels at the same speed in all directions.
>
> ◦ *However*: Simplicity is a highly subjective concept...



Einstein

General relativity is much more simple than Newton's theory of gravity!

???

Average Joe

Realist Response:

- Agree that there is no observational difference between the standard simultaneity relation and any non-standard simultaneity relation.
- *So*: If empirical adquacy (*i.e.*, agreement with observation) is the criterion for how one chooses between competing theories, then there's no reason to prefer the standard relation to any non-standard relation.
- *But*: Why think empirical adquacy is the only criterion of theory choice?

> - Suppose *unifying power* is a criterion of theory choice (*i.e.*, we should choose that theory that fits better with other theories).
> - *Then*: We should prefer the standard simultaneity relation, since Friedman-Robertson-Walker spacetimes in general relativity (*i.e.*, "Big Bang" spacetimes) are isotropic in a way that singles out the standard definition.
> - *But*: Adopting such spacetimes as descriptions of our universe requires many assumptions, one of which *just* is isotropy.