Indefinite Integral

Hossein Rahimzadeh
www.cafeplanck.com
info@cafeplanck.com

Type1

$$\boxed{\int xe^x dx = xe^x - e^x + c}$$

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = e^x dx \Rightarrow v = e^x \end{cases}$$

$$\Rightarrow \int xe^x dx = uv - \int vdu = xe^x - \int e^x dx = xe^x - e^x + c$$

Type 2

$$\boxed{\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + c}$$

$$\begin{cases} u = x^2 \Rightarrow du = 2xdx \\ dv = e^x dx \Rightarrow v = e^x \end{cases}$$

$$\Rightarrow \int x^2 e^x dx = uv - \int vdu = x^2 e^x - \int 2xe^x dx = x^2 e^x - 2\int xe^x dx =$$

$$= x^2 e^x - 2\left(xe^x - e^x + c\right) = x^2 e^x - 2xe^x + 2e^x \underbrace{-2c}_{c'} = x^2 e^x - 2xe^x + 2e^x + c'$$

Type 3

$$\boxed{\int x^3 e^x = x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + c}$$

$$\begin{cases} u = x^3 \Rightarrow du = 3x^2 dx \\ dv = e^x dx \Rightarrow v = e^x \end{cases}$$

$$\Rightarrow \int x^3 e^x dx = uv - \int vdu = x^3 e^x - \int 3x^2 e^x dx = x^3 e^x - 3\int x^2 e^x dx =$$

$$= x^3 e^x - 3\left(x^2 e^x - 2xe^x + 2e^x + c\right) = x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x \underbrace{-3c}_{c'} =$$

$$= x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + c'$$

Type 4

$$\boxed{\int x^n e^x dx = x^n e^x - n\int x^{n-1} e^x dx}$$

$$\begin{cases} u = x^n \Rightarrow du = nx^{n-1}dx \\ dv = e^x dx \Rightarrow v = e^x \end{cases}$$

$$\Rightarrow \int x^n e^x dx = uv - \int vdu = x^n e^x - \int nx^{n-1} e^x dx = x^n e^x - n\int x^{n-1} e^x dx$$

Examples

$$\int xe^x dx = xe^x - \int x^0 e^x dx = xe^x - e^x + c$$

$$\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx = x^2 e^x - 2xe^x + 2e^x + c$$

$$\int x^3 e^x dx = x^3 e^x - 3\int x^2 e^x dx = x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + c$$

Type 5

$$\boxed{\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) + c}$$

$$\begin{cases} u = e^x \Rightarrow du = e^x dx \\ dv = \sin x dx \Rightarrow v = -\cos x \end{cases}$$

$$I = \int e^x \sin x dx = uv - \int vdu = -e^x \cos x + \int e^x \cos x dx = -e^x \cos x + I'$$

$$I' = \int e^x \cos x dx$$

$$\begin{cases} u = e^x \Rightarrow du = e^x dx \\ dv = \cos x dx \Rightarrow v = \sin x \end{cases}$$

$$\Rightarrow I' = \int e^x \cos x dx = uv - \int vdu = e^x \sin x - \int e^x \sin x dx = e^x \sin x - I$$

$$I = -e^x \cos x + I' = -e^x \cos x + e^x \sin x - I$$

$$\Rightarrow 2I = -e^x \cos x + e^x \sin x \Rightarrow I = \frac{1}{2} e^x (\sin x - \cos x) + c$$

Type 6

$$\boxed{\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x) + c}$$

$$\begin{cases} u = e^x \Rightarrow du = e^x dx \\ dv = \cos x dx \Rightarrow v = \sin x \end{cases}$$

$$I = \int e^x \cos x dx = uv - \int v du = e^x \sin x - \int e^x \sin x dx = e^x \sin x - I'$$

$$I' = \int e^x \sin x dx$$

$$\begin{cases} u = e^x \Rightarrow du = e^x dx \\ dv = \sin x dx \Rightarrow v = -\cos x \end{cases}$$

$$\Rightarrow I' = \int e^x \sin x dx = uv - \int v du = -e^x \cos x + \int e^x \cos x dx = -e^x \cos x + I$$

$$I = e^x \sin x - I' = e^x \sin x + e^x \cos x - I$$

$$\Rightarrow 2I = e^x \sin x + e^x \cos x \Rightarrow I = \frac{1}{2} e^x (\sin x + \cos x) + c$$

Type 7

$$\boxed{\int x e^x \sin x dx = \frac{1}{2} x e^x (\sin x - \cos x) + \frac{1}{2} e^x \cos x + c}$$

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = e^x \sin x dx \Rightarrow v = \frac{1}{2} e^x (\sin x - \cos x) \end{cases}$$

$$\Rightarrow \int x e^x \sin x dx = uv - \int v du = \frac{1}{2} x e^x (\sin x - \cos x) - \int \frac{1}{2} e^x (\sin x - \cos x) dx =$$

$$= \frac{1}{2} x e^x (\sin x - \cos x) - \frac{1}{2} \int e^x \sin x dx + \frac{1}{2} \int e^x \cos x dx =$$

$$= \frac{1}{2} x e^x (\sin x - \cos x) - \frac{1}{2}\left(\frac{1}{2} e^x (\sin x - \cos x)\right) + \frac{1}{2}\left(\frac{1}{2} e^x (\sin x + \cos x)\right) =$$

$$= \frac{1}{2} x e^x (\sin x - \cos x) - \frac{1}{4} e^x (\sin x - \cos x) + \frac{1}{4} e^x (\sin x + \cos x) + c =$$

$$= \frac{1}{2} x e^x (\sin x - \cos x) + \frac{1}{2} e^x \cos x + c$$

Type 8

$$\boxed{\int xe^x \cos x dx = \frac{1}{2} xe^x(\sin x + \cos x) - \frac{1}{2} e^x \sin x + c}$$

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = e^x \cos x dx \Rightarrow v = \frac{1}{2} e^x(\sin x + \cos x) \end{cases}$$

$$\Rightarrow \int xe^x \cos x dx = uv - \int v du = \frac{1}{2} xe^x(\sin x + \cos x) - \int \frac{1}{2} e^x(\sin x + \cos x) dx =$$

$$= \frac{1}{2} xe^x(\sin x + \cos x) - \frac{1}{2}\int e^x \sin x dx - \frac{1}{2}\int e^x \cos x dx =$$

$$= \frac{1}{2} xe^x(\sin x + \cos x) - \frac{1}{2}\left(\frac{1}{2} e^x(\sin x - \cos x)\right) - \frac{1}{2}\left(\frac{1}{2} e^x(\sin x + \cos x)\right) =$$

$$= \frac{1}{2} xe^x(\sin x + \cos x) - \frac{1}{4} e^x(\sin x - \cos x) - \frac{1}{4} e^x(\sin x + \cos x) + c =$$

$$= \frac{1}{2} xe^x(\sin x + \cos x) - \frac{1}{2} e^x \sin x + c$$