Plan d'étude et représentation graphique de $y = f(x) = \dfrac{x^3}{x^2 - 1}$

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Le domaine de définition de f

$$y = f(x) = \frac{x^3}{x^2 - 1} \Rightarrow D_f = R - \{-1,1\} = \underbrace{(-\infty,-1)}_{I_1} \cup \underbrace{(-1,1)}_{I_2} \cup \underbrace{(1,+\infty)}_{I_3}$$

Etudier la fonction au bornes de D_f

A la borne gauche

$$\lim_{x \to -\infty} y = -\infty$$

$$a = \lim_{x \to -\infty} \frac{y}{x} = \lim_{x \to -\infty} \frac{x^3}{x^3 - x} = 1$$

$$b = \lim_{x \to -\infty} (y - ax) = \lim_{x \to -\infty} \left(\frac{x^3}{x^2 - 1} - x \right) = 0$$

$$\lim_{x \to -1^-} y = -\infty$$

$$\lim_{x \to -1^+} y = +\infty$$

$$\lim_{x \to 1^-} y = -\infty$$

$$\lim_{x \to 1^+} y = +\infty$$

$$\lim_{x \to +\infty} y = +\infty$$

$$a = \lim_{x \to +\infty} \frac{y}{x} = \lim_{x \to +\infty} \frac{x^3}{x^3 - x} = 1$$

$$b = \lim_{x \to +\infty}\left(y - ax\right) = \lim_{x \to +\infty}\left(\frac{x^3}{x^2 - 1} - x\right) = 0$$

$$y' = f'(x) = \frac{x^2\left(x^2 - 3\right)}{(x^2 - 1)^2}$$

$$x^2\left(x^2 - 3\right) = 0 \Rightarrow \begin{cases} x = 0 \Rightarrow y = 0 \Rightarrow \left| \begin{matrix} 0 \\ 0 \end{matrix} \right. \\ x = -1.73 \Rightarrow y = -2.59 \Rightarrow \left| \begin{matrix} -1.73 \\ -2.59 \end{matrix} \right. \\ x = 1.73 \Rightarrow y = 2.59 \Rightarrow \left| \begin{matrix} 1.73 \\ 2.59 \end{matrix} \right. \end{cases}$$

$$(x^2 - 1)^2 = 0 \Rightarrow \begin{cases} x = -1 \\ x = 1 \end{cases}$$

$$y'' = f''(x) = \frac{2x(x^2 + 3)}{\left(x^2 - 1\right)^3}$$

$$2x(x^2 + 3) = 0 \Rightarrow x = 0 \Rightarrow y = 0 \Rightarrow \left| \begin{matrix} 0 \\ 0 \end{matrix} \right.$$

$$\left(x^2 - 1\right)^3 = 0 \Rightarrow \begin{cases} x = -1 \\ x = 1 \end{cases}$$

$$m_{x=-1.73} = f'(-1.73) = 0$$
$$m_{x=0} = f'(0) = 0$$
$$m_{x=1.73} = f'(1.73) = 0$$