Indefinite Integral

Hossein Rahimzadeh
www.cafeplanck.com
info@cafeplanck.com

Type 1

$$\boxed{\int \arcsin x dx = x \arcsin x + \sqrt{1-x^2} + c}$$

$$\int \underbrace{\arcsin x}_{u} \underbrace{dx}_{dv} = I$$

$$\begin{cases} u = \arcsin x \Rightarrow du = \dfrac{dx}{\sqrt{1-x^2}} \\ dv = dx \Rightarrow v = x \end{cases}$$

$$\Rightarrow I = uv - \int v du = x \arcsin x - \underbrace{\int \frac{xdx}{\sqrt{1-x^2}}}_{I'}$$

$$\int \frac{xdx}{\sqrt{1-x^2}} = I'$$

$$u = 1 - x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$$

$$\Rightarrow I' = \int \frac{x \dfrac{du}{-2x}}{\sqrt{u}} = -\frac{1}{2} \int \frac{du}{\sqrt{u}} = -\frac{1}{2} \int u^{-\frac{1}{2}} du = -\frac{1}{2} \times \frac{1}{\frac{1}{2}} u^{\frac{1}{2}} + c = -u^{\frac{1}{2}} + c = -\sqrt{1-x^2} + c$$

Type 2

$$\boxed{\int \arccos x dx = x \arccos x - \sqrt{1-x^2} + c}$$

$$\int \underbrace{\arccos x}_{u} \underbrace{dx}_{dv} = I$$

$$\begin{cases} u = \arccos x \Rightarrow du = \dfrac{-dx}{\sqrt{1-x^2}} \\ dv = dx \Rightarrow v = x \end{cases}$$

$$\Rightarrow I = uv - \int v du = x \arccos x + \underbrace{\int \frac{xdx}{\sqrt{1-x^2}}}_{I'}$$

$$I' = \int \frac{xdx}{\sqrt{1-x^2}}$$

$$u = 1-x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$$

$$\Rightarrow I' = \int \frac{x \frac{du}{-2x}}{\sqrt{u}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-\frac{1}{2}} du = -\frac{1}{2} \times \frac{1}{\frac{1}{2}} u^{\frac{1}{2}} + c = -u^{\frac{1}{2}} + c = -\sqrt{1-x^2} + c$$

Type 3

$$\boxed{\int \arctan x dx = x\arctan x - \frac{1}{2}\ln\left(1+x^2\right) + c}$$

$$\int \underbrace{\arctan x}_{u} \underbrace{dx}_{dv} = I$$

$$\begin{cases} u = \arctan x \Rightarrow du = \dfrac{dx}{1+x^2} \\ dv = dx \Rightarrow v = x \end{cases}$$

$$\Rightarrow I = uv - \int vdu = x\arctan x - \underbrace{\int \frac{xdx}{1+x^2}}_{I'}$$

$$I' = \int \frac{xdx}{1+x^2}$$

$$u = 1+x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$$

$$\Rightarrow I' = \int \frac{x\dfrac{du}{2x}}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln\left|1+x^2\right| + c$$

Type 4

$$\boxed{\int \operatorname{arccot} x dx = x \operatorname{arccot} x + \frac{1}{2}\ln\left(1+x^2\right) + c}$$

$$\int \underbrace{\operatorname{arccot} x}_{u} \underbrace{dx}_{dv} = I$$

$$\begin{cases} u = \operatorname{arccot} x \Rightarrow du = \dfrac{-dx}{1+x^2} \\ dv = dx \Rightarrow v = x \end{cases}$$

$$\int \operatorname{arccot} x dx = uv - \int v du = x \ \operatorname{arccot} x + \underbrace{\int \frac{x dx}{1+x^2}}_{I'}$$

$$I' = \int \frac{x dx}{1+x^2}$$

$$u = 1 + x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$$

$$\Rightarrow I' = \int \frac{x \frac{du}{2x}}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln\left|1+x^2\right| + c$$