# Indefinite Integral

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## Type 1

$$\int \frac{dx}{ax^2+bx+c}$$

### Example 1

$$\int \frac{dx}{x^2-2x-3}$$

$$\frac{1}{x^2-2x-3}=\frac{1}{(x+1)(x-3)}=\frac{A}{x+1}+\frac{B}{x-3}=\frac{-\frac{1}{4}}{x+1}+\frac{\frac{1}{4}}{x-3}$$

$$\Rightarrow \int \frac{dx}{x^2-2x-3}=\int \frac{-\frac{1}{4}}{x+1}dx+\int \frac{\frac{1}{4}}{x-3}dx=\frac{1}{4}\int \frac{dx}{x-3}-\frac{1}{4}\int \frac{dx}{x+1}=$$

$$=\frac{1}{4}\ln|x-3|-\frac{1}{4}\ln|x+1|+c=\frac{1}{4}\ln\left|\frac{x-3}{x+1}\right|+c$$

### Example 2

$$\int \frac{dx}{x^2-4x+4}$$

$$\int \frac{dx}{x^2-4x+4}=\int \frac{dx}{(x-2)^2}=I$$

$$u=x-2\Rightarrow \frac{du}{dx}=1\Rightarrow dx=du$$

$$\Rightarrow I=\int \frac{du}{u^2}=-\frac{1}{u}+c=-\frac{1}{x-2}+c=\frac{1}{2-x}+c$$

### Example 3

$$\int \frac{dx}{x^2-2x+5}$$

$$\int \frac{dx}{x^2-2x+5} = \int \frac{dx}{(x-1)^2+4} = \int \frac{dx}{4+(x-1)^2} = I$$

$$u = x-1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$$

$$\Rightarrow I = \int \frac{du}{4+u^2} = \frac{1}{2}\arctan\frac{u}{2} + c = \frac{1}{2}\arctan\frac{x-1}{2} + c$$

## Type 2

$$\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

### Example 1

$$\int \frac{dx}{\sqrt{x^2-2x-3}}$$

$$\int \frac{dx}{\sqrt{x^2-2x-3}} = \int \frac{dx}{\sqrt{(x-1)^2-4}} = I$$

$$u = x-1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$$

$$\Rightarrow I = \int \frac{du}{\sqrt{u^2-4}} = \ln\left|\sqrt{u^2-4}+u\right| + c = \ln\left|\sqrt{x^2-2x-3}+x-1\right| + c$$

### Example 2

$$\int \frac{dx}{\sqrt{-x^2+2x+3}}$$

$$\int \frac{dx}{\sqrt{-x^2+2x+3}} = \int \frac{dx}{\sqrt{-\left(x-1^2\right)+4}} = \int \frac{dx}{\sqrt{4-(x-1)^2}} = I$$

$$u = x-1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$$

$$\Rightarrow I = \int \frac{du}{\sqrt{4-u^2}} = \arcsin\frac{u}{2} + c = \arcsin\frac{x-1}{2} + c$$

### Example 3

$$\int \frac{dx}{\sqrt{x^2 - 4x + 4}}$$

$$\int \frac{dx}{\sqrt{x^2 - 4x + 4}} = \int \frac{dx}{\sqrt{(x-2)^2}} = \int \frac{dx}{|x-2|} = \ln|x-2| + c$$

**Example 4**

$$\int \frac{dx}{\sqrt{x^2 - 2x + 5}}$$

$$\int \frac{dx}{\sqrt{x^2 - 2x + 5}} = \int \frac{dx}{\sqrt{(x-1)^2 + 4}} = \int \frac{dx}{\sqrt{4 + (x-1)^2}} = I$$

$$u = x - 1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$$

$$\Rightarrow I = \int \frac{du}{\sqrt{4 + u^2}} = \ln\left|\sqrt{4 + u^2} + u\right| + c = \ln\left|\sqrt{x^2 - 2x + 5} + x - 1\right| + c$$

## Type 3

$$\int \sqrt{ax^2 + bx + c}\ dx$$

**Example 1**

$$\int \sqrt{x^2 - 2x - 3}\ dx$$

$$\int \sqrt{x^2 - 2x - 3}\ dx = \int \sqrt{(x-1)^2 - 4}\ dx = I$$

$$u = x - 1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$$

$$\Rightarrow I = \int \sqrt{u^2 - 4}\ du = \frac{1}{2}u\sqrt{u^2 - 4} - \frac{1}{2} \times 4 \times \ln\left|\sqrt{u^2 - 4} + u\right| + c =$$

$$= \frac{1}{2}(x-1)\sqrt{x^2 - 2x - 3} - 2\ln\left|\sqrt{x^2 - 2x - 3} + x - 1\right| + c$$

**Example 2**

$$\int \sqrt{-x^2 + 2x + 3}\ dx$$

$$\int \sqrt{-x^2+2x+3}\ dx = \int \sqrt{-(x-1)^2+4}\ dx = \int \sqrt{4-(x-1)^2}\ dx = I$$

$$u = x-1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$$

$$\Rightarrow I = \int \sqrt{4-u^2}\ du = \frac{1}{2}u\sqrt{4-u^2} + \frac{1}{2}\times 4\times \arcsin\frac{u}{2} + c =$$

$$= \frac{1}{2}(x-1)\sqrt{-x^2+2x+3} + 2\arcsin\frac{x-1}{2} + c$$

**Example 3**

$$\int \sqrt{x^2-4x+4}\ dx$$

$$\int \sqrt{x^2-4x+4}\ dx = \int \sqrt{(x-2)^2}dx = \int |x-2|\ dx = \left|\frac{1}{2}x^2-2x\right| + c$$

**Example 4**

$$\int \sqrt{x^2-2x+5}\ dx$$

$$\int \sqrt{x^2-2x+5}\ dx = \int \sqrt{(x-1)^2+4}\ dx = \int \sqrt{4+(x-1)^2}\ dx = I$$

$$u = x-1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$$

$$\Rightarrow I = \int \sqrt{4+u^2}du = \frac{1}{2}u\sqrt{4+u^2} + \frac{1}{2}\times 4\times \ln\left|\sqrt{4+u^2}+u\right| + c =$$

$$= \frac{1}{2}(x-1)\sqrt{x^2-2x+5} + 2\ln\left|\sqrt{x^2-2x+5}+x-1\right| + c$$