Indefinite Integral

Hossein Rahimzadeh
www.cafeplanck.com
info@cafeplanck.com

Type 1

$$\boxed{\int \frac{dx}{1+\sin x} = \tan x - \sec x + c = \tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + c}$$

$$\int \frac{dx}{1+\sin x} = \int \frac{1-\sin x}{(1+\sin x)(1-\sin x)}\,dx = \int \frac{1-\sin x}{1-\sin^2 x}dx = \int \frac{1-\sin x}{\cos^2 x}dx =$$

$$\int \frac{1}{\cos^2 x}\,dx - \int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\,dx - \int \sec x \tan x dx =$$

$$= \tan x - \sec x + c = \tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + c$$

Type 2

$$\boxed{\int \frac{dx}{1+\cos x} = -\cot x + \csc x + c = \tan\frac{x}{2} + c}$$

$$\int \frac{dx}{1+\cos x} = \int \frac{1-\cos x}{(1+\cos x)(1-\cos x)}dx = \int \frac{1-\cos x}{1-\cos^2 x}dx$$

$$= \int \frac{1-\cos x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx - \int \frac{\cos x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx - \int \cos x \cot x dx =$$

$$= -\cot x + \csc x + c = \tan\frac{x}{2} + c$$

Type 3

$$\boxed{\int x \sin x dx = -x \cos x + \sin x + c}$$

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = \sin x dx \Rightarrow v = -\cos x \end{cases}$$

$$\Rightarrow \int x \sin x dx = uv - \int v du = -x\cos x - \int -\cos x dx = -x\cos x + \sin x + c$$

Type 4

$$\boxed{\int x \cos x dx = x \sin x + \cos x + c}$$

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = \cos x dx \Rightarrow v = \sin x \end{cases}$$

$$\Rightarrow \int x \cos x dx = uv - \int v du = x \sin x - \int \sin x dx = x \sin x + \cos x + c$$

Type 5

$$\boxed{\int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2\cos x + c}$$

$$\begin{cases} u = x^2 \Rightarrow du = 2x dx \\ dv = \sin x dx \Rightarrow v = -\cos x \end{cases}$$

$$\Rightarrow \int x^2 \sin x dx = uv - \int v du = -x^2 \cos x - \int -2x\cos x dx =$$

$$= -x^2 \cos x + 2\int x \cos x dx = -x^2 \cos x + 2(x \sin x + \cos x + c) =$$

$$= -x^2 \cos x + 2x \sin x + 2\cos x + c$$

Type 6

$$\boxed{\int x^2 \cos x dx = x^2 \sin x + 2x\cos x - 2\sin x + c}$$

$$\begin{cases} u = x^2 \Rightarrow du = 2x dx \\ dv = \cos x dx \Rightarrow v = \sin x \end{cases}$$

$$\Rightarrow \int x^2 \cos x dx = uv - \int v du = x^2 \sin x - \int 2x \sin x dx =$$

$$= x^2 \sin x - 2\int x \sin x dx = x^2 \sin x - 2(-x\cos x + \sin x + c) =$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + c$$

Type 7

$$\boxed{\int x^3 \sin x dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6\sin x + c}$$

$$\begin{cases} u = x^3 \Rightarrow du = 3x^2 dx \\ dv = \sin x dx \Rightarrow v = -\cos x \end{cases}$$

$$\Rightarrow \int x^3 \sin x dx = uv - \int v du = -x^3 \cos x - \int -3x^2 \cos x dx =$$

$$= -x^3 \cos x + 3\int x^2 \cos x dx = -x^3 \cos x + 3\left(x^2 \sin x + 2x \cos x - 2\sin x + c\right) =$$

$$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6\sin x + \underbrace{3c}_{c'} = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6\sin x + c'$$

Type 8

$$\boxed{\int x^3 \cos x dx = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x + c}$$

$$\begin{cases} u = x^3 \Rightarrow du = 3x^2 dx \\ dv = \cos x dx \Rightarrow v = \sin x \end{cases}$$

$$\Rightarrow \int x^3 \cos x dx = uv - \int v du = x^3 \sin x - \int 3x^2 \sin x dx =$$

$$= x^3 \sin - 3\int x^2 \sin x dx = x^3 \sin x - 3\left(-x^2 \cos x + 2x \sin x + 2\cos x + c\right) =$$

$$= x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x + \underbrace{3c}_{c'} = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x + c'$$