Le potentiel en forme de marche

Mécanique Quantique

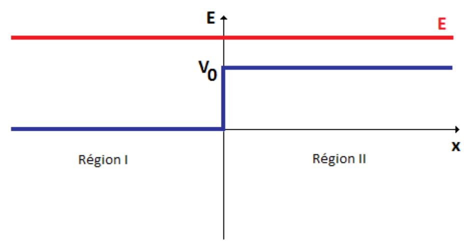
Hossein Rahimzadeh

8/19/2008

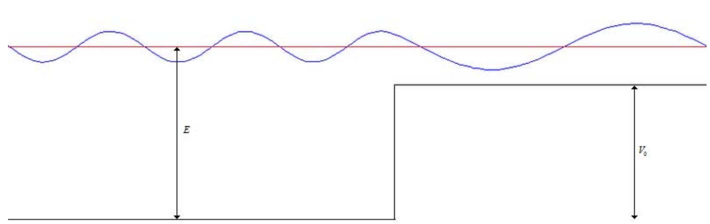
Le potentiel en forme de marche

$$V(x)=\begin{cases}0 & , \quad x<0\\ V_0 & , \quad x>0\end{cases}$$

Le cas $E>V_0$



Région I	Frontière X=0	Région II
$\hat{H}u(x)=Eu(x)$		$\hat{H}u(x)=Eu(x)$
$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right]u(x)=Eu(x)$		$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right]u(x)=Eu(x)$
$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u(x)+V(x)u(x)=Eu(x)$		$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u(x)+V(x)u(x)=Eu(x)$
$V(x)=0$		$V(x)=V_0$
$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u(x)=Eu(x)$		$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u(x)+V_0u(x)=Eu(x)$
$\frac{d^2}{dx^2}u(x)+\frac{2mE}{\hbar^2}u(x)=0$		$\frac{d^2}{dx^2}u(x)+\frac{2m(E-V_0)}{\hbar^2}u(x)=0$
$\frac{d^2}{dx^2}u(x)+k^2u(x)=0$ où $k^2=\frac{2mE}{\hbar^2}$		$\frac{d^2}{dx^2}u(x)+q^2u(x)=0$ où $q^2=\frac{2m(E-V_0)}{\hbar^2}$
$u_I(x)=e^{ikx}+Re^{-ikx}$ où $k^2=\frac{2mE}{\hbar^2}$		$u_{II}(x)=Te^{iqx}$ où $q^2=\frac{2m(E-V_0)}{\hbar^2}$



Les conditions de continuités :

À $x=0$:

1. $u_I(0)=u_{II}(0)\Rightarrow 1+R=T$
2. $u_I'(0)=u_{II}'(0)\Rightarrow ik-ikR=iqT\Rightarrow k\left(1-R\right)=qT$
3. $j_I=j_{II}\Rightarrow j_{In}+J_{Réf}=j_{Tr}\Rightarrow \frac{\hbar k}{m}-\frac{\hbar k}{m}\left|R\right|^2=\frac{\hbar q}{m}\left|T\right|^2\Rightarrow k\left(1-\left|R\right|^2\right)=q\left|T\right|^2$

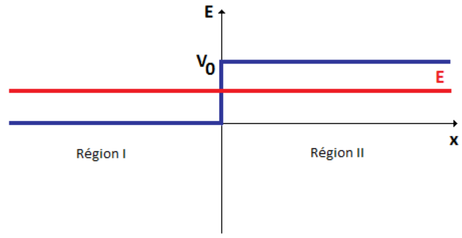
Donc,

Coefficient de réflexion	$R=\frac{k-q}{k+q}$
Coefficient de transmission	$T=\frac{2k}{k+q}$
Le flux d'incidente	$j_{In}=\frac{\hbar k}{m}$
Le flux de réflexion	$j_{Réf}=\frac{\hbar k}{m}\left\|R\right\|^2=\frac{\hbar k}{m}\left(\frac{k-q}{k+q}\right)^2$
Le flux de transmission	$j_{Tr}=\frac{\hbar k}{m}\left\|T\right\|^2=\frac{\hbar k}{m}\left(\frac{2k}{k+q}\right)^2$

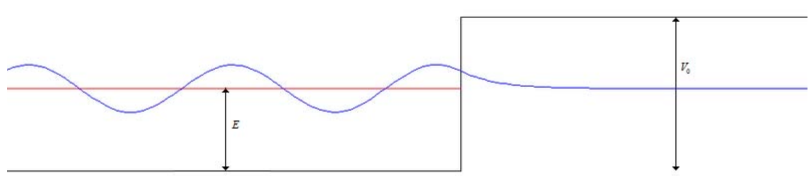
Les cas particuliers :

Réflexion nul et transmission total	$V_0=0\Rightarrow\frac{q}{k}=\sqrt{\frac{E-V_0}{E}}=1\Rightarrow\begin{cases}R=0\\T=1\end{cases}$
Réflexion presque nul et transmission presque total.	$E\gg V_0\Rightarrow\frac{q}{k}=\sqrt{\frac{E-V_0}{E}}\approx 1\Rightarrow\begin{cases}R\approx 0\\T\approx 1\end{cases}$

Le cas $E < V_0$



Région I	Frontière X=0	Région II
$\hat{H}u(x) = Eu(x)$		$\hat{H}u(x) = Eu(x)$
$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]u(x) = Eu(x)$		$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]u(x) = Eu(x)$
$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u(x) + V(x)u(x) = Eu(x)$		$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u(x) + V(x)u(x) = Eu(x)$
$V(x) = 0$		$V(x) = V_0$
$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u(x) = Eu(x)$		$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}u(x) + V_0 u(x) = Eu(x)$
$\frac{d^2}{dx^2}u(x) + \frac{2mE}{\hbar^2}u(x) = 0$		$\frac{d^2}{dx^2}u(x) - \frac{2m(V_0 - E)}{\hbar^2}u(x) = 0$
$\frac{d^2}{dx^2}u(x) + k^2 u(x) = 0$ où $k^2 = \frac{2mE}{\hbar^2}$		$\frac{d^2}{dx^2}u(x) - q^2 u(x) = 0$ où $q^2 = \frac{2m(V_0 - E)}{\hbar^2}$
$u_I(x) = e^{ikx} + Re^{-ikx}$ où $k^2 = \frac{2mE}{\hbar^2}$		$u_{II}(x) = Te^{-qx}$ où $q^2 = \frac{2m(V_0 - E)}{\hbar^2}$



Les conditions de continuités :

À $x = 0$:

1. $u_I(0) = u_{II}(0) \Rightarrow 1 + R = T$
2. $u_I'(0) = u_{II}'(0) \Rightarrow ik - ikR = -qT \Rightarrow ik\left(1 - R\right) = -qT$
3. $j_I = j_{II} \Rightarrow j_{In} + J_{Réf} = j_{Tr} \Rightarrow \frac{\hbar k}{m} - \frac{\hbar k}{m}\left|R\right|^2 = 0 \Rightarrow 1 - \left|R\right|^2 = 0 \Rightarrow \left|R\right|^2 = 1$

Donc,

Coefficient de réflexion	$R = \frac{k - iq}{k + iq}$
Coefficient de transmission	$T = \frac{2k}{k + iq}$
Flux d'incident	$j_{In} = \frac{\hbar k}{m}$
Flux de réfléchi	$j_{Réf} = \frac{\hbar k}{m}\left|R\right|^2 = \frac{\hbar k}{m}$
Flux de transmis	$j_{Tr} = \frac{\hbar k}{m}\left|T\right|^2 = \frac{\hbar k}{m}\left(\frac{2k}{k + iq}\right)^2$

Les cas particuliers :

	$E = V_0 \Rightarrow q = 0 \Rightarrow \begin{cases} R = 1 \\ T = 2 \end{cases}$