Población límite

(Población simple)

$$\frac{dP}{dt} = k.P \ , \quad P(t_o) = P_0$$

$$k = A - B.P \quad \Rightarrow \quad \frac{dP}{dt} = \left(A - B.P\right).P \quad \Rightarrow \quad \frac{dP}{dt} = A.P - BP^2$$

$$P(t) = \frac{A/B}{1 + \left(\frac{A/B}{P_0} - 1\right).e^{-A.t}}; \qquad P_{\max} = \lim_{t \to \infty} P(t) = \frac{A}{B}$$

Tomando un cambio de escala:

$$P(0) = P_0; \quad \mathrm{P}(1) = P_1; \quad \mathrm{P}(2) = P_2$$

resulta:

$$e^{-A.t} = \frac{P_0\left(P_2 - P_1\right)}{P_2\left(P_1 - P_0\right)}$$

$$\frac{A}{B} = \frac{P_1\left(P_0 P_1 - 2P_0 P_2 + P_1 P_2\right)}{P_1^2 - P_0 P_2}$$

Ejemplo de

Tabla de datos

$t_1 - t_0 = t_2 - t_1$

t (Nva escala)	T (años)	Población ($x10^3$)	
$t_0 = 0$	1890	62.5	P_0
	1900	70.2	
	1910	88.6	
$t_1 = 1$	1920	106.8	P_1
	1930	113.1	
$t_2 = 2$	1950	150.4	P_2

$$e^{-A} = \frac{62,5(150,4-106,8)}{150,4(106,8-62,5)} = \frac{2725}{6662,72} = 0,40899$$

$$-\ln(0,409) = \boxed{0,894 = A}$$

$$\frac{A}{B} = \frac{106,8(62,5.106,8-2.62,5.150,4+106,8.150,4)}{(106,8)^2-62,5.150,4} = \frac{420548,496}{2006,24} =$$

$$\boxed{\frac{A}{B} = 209,6202}$$

$$P(t) = \frac{209,6202}{1+\left(\frac{209,6202}{62,5}-1\right).(0,409)^t}; \qquad e^{-A.t} = \left(e^{-A}\right)^t$$

$$P(T=1880)=P(-0,3\widehat{3})=\frac{209,6202}{1+\left(2,3539\right).\left(0,409\right)^{-0,\widehat{3}}}=50,2558$$

$$P(T=2010)=P(4)=\frac{209,6202}{1+\left(2,3539\right).\left(0,409\right)^{4}}=196,6658$$

$$P(T=1940)=P(1,6\widehat{6})=\frac{209,6202}{1+\left(2,3539\right).\left(0,409\right)^{1,6\widehat{6}}}=136,96$$

$$P(T=1930)=P(1,3\widehat{3})=\frac{209,6202}{1+\left(2,3539\right).\left(0,409\right)^{1,3\widehat{3}}}=122,25$$