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The uncapacitated hub location problem in networks under decentralized management

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ABSTRACT

We propose a new hub location model defined by the minimization of costs. The main contribution of this work is to permit the analysis of a hub-and-spoke network operated under "decentralized management". In this type of network, various transport companies act independently, and each makes its route choices according to its own criteria, which can include cost, time, frequency, security and other factors, including subjective ones. Therefore, due to the diversity of the various companies' criteria, one can expect that between each origin–destination pair, a fraction of the flow will be carried through hubs and a fraction will be carried by the direct route. to resolve this problem, it becomes necessary to determine the probability that any network user will choose the hub route for each trip to be made (or for each load to be carried). We present an integer programming formulation, subject the new model to experiments with an intermodal general cargo network in Brazil and address questions regarding the solution of the problem in practice.

1. Introduction

In a hub-and-spoke network, the hub vertices are connected to common nodes by routes called spokes. The connections between the hubs themselves are made by shuttle services (Fig. 1), which provide the greatest transport capacities, economies of scale and pollution reduction. The latter two advantages are the main reasons for establishing such a transportation system.

A hub has three principal functions: to aggregate flows arriving from any vertices of the network, to redistribute flows toward each destination point and to send aggregated flow to another hub for further redistribution.

The location of hubs in networks is a nondeterministic polynomial time complete (NP-complete) combinatorial optimization problem with a two-part solution: determination of the network's vertices, which should function as hubs, and attribution of the origin and destination nodes of each flow to their respective hubs. The solution aims to provide the lowest total network cost for routing the flows between all O–D (origin–destination) pairs. The uncapacitated multiple allocation hub location problem (UMHLP) is the specific case that does not include flow capacity limits, neither in the net links nor at the vertices. Its multiple allocation feature allows a vertex to be connected to every hub, contrarily to the single allocation problems that restrict each vertex to send or receive flows by only one hub.

The aim of this work is to resolve an uncapacitated multiple allocation hub location problem (UMHLP) in a hub-and-spoke network operated under decentralized management. To the best of our knowledge this case has not yet been examined in the published models of this problem, but it is a very common case in the analysis of networks of regional or greater scope.

The existing models are only applicable to networks managed by a central entity, which establishes a uniform criterion for choosing the best route such that all vehicles belonging to the network must follow that criterion. For example, an express delivery firm that has its own transport network can establish the deterministic criterion of lowest cost for choosing the route to be followed by its vehicles. With this uniform criterion established throughout the network, the location of the hubs and the allocation of the flows can be determined through use of published hub location models, since, in these models, the flows between any two points i and j occur "totally" over a single route that is considered optimal.

However, in a system under decentralized management, the flow between an O–D pair does not go by a single route because such a system does not have a single criterion for determining the best route. In this type of system, in which different transport firms act independently, each of them makes its own route choices based on its own particular criterion. Hence, with a diversity of criteria from

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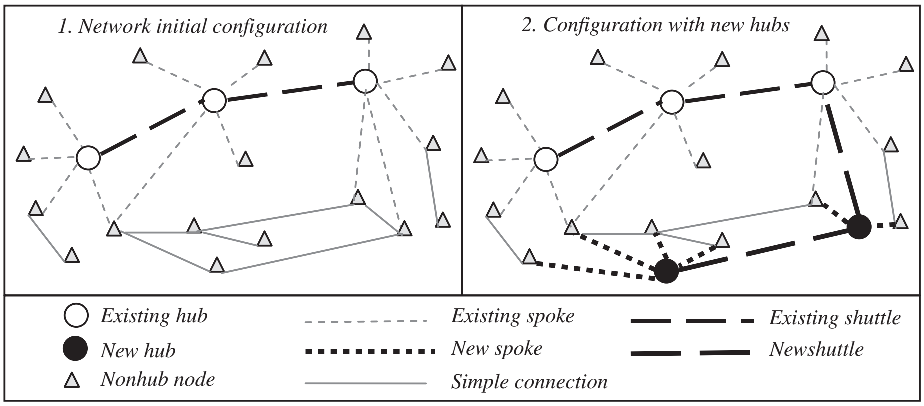

Fig. 1. Example of the establishment of a set of new hubs in a network.

multiple companies, one can expect each O–D pair to account for a fraction of the flow, carried through hubs, and another fraction to be carried by direct routes.

An example of a decentralized management network is the domestic transport of containers in a country, where each company can choose the route and transport mode (or modes) to be used based on a particular combination of parameters (cost, transport time, frequency of shipments, security and other subjective parameters).

Even if this free market does not appear to be an organized system, it is a transport network with flows that can be probabilistically modeled. In this case the hub location problem (HLP) becomes more complex.

It can be intuitively argued that even in a transport system without central management, the conventional hub location model could be applied if a generalized cost function[1] were used by computing all criteria for choosing the best route, including subjective criteria. So, if the generalized costs were computed for all existing routes between an O–D pair, it would be possible to compare them and choose the optimal route (the route with the lowest generalized cost). However, this argument fails because the objective function and constraints of a conventional UMHLP would determine that ''all'' flows between an O–D pair should take a single route with the lowest generalized cost, which cannot be expected in practice.

As stated before, in real-world decentrally managed networks, an O–D flow is splitted into the O–D available routes (direct or via hubs). As seen in Domencich and McFadden [6], this division is better represented by discrete choice models—such as the Logit Model. These models estimate the probability that a user will choose an available route. Hence, this probability also represents the proportion of users that is expected to choose a route type in a determined period (e.g., a year). And, according to these kinds of models, if a new lower-cost route is created due to the establishment of a new hub, the existing direct flow (with higher cost) cannot be expected to migrate ''totally'' to this new route. This means that the probabilities of using both types of routes are non-zero. And, in transportation modeling, they are usually significant. Therefore, a conventional hub location model based on generalized cost cannot be applied as described in the previous paragraph.

Even if one could assume in a specific case that ''all'' of the flow between an O–D pair would migrate from a route with higher generalized cost to a new one with lower generalized cost, it would still not be correct to use the same generalized cost function for all of the network's vertices. It would instead be necessary to divide the network into homogeneous areas and to determine a single generalized cost function for each of these areas because each variable of this function has a different value for each region (e.g., time spent in a highly industrialized region might be more costly than that spent in an agricultural region). Due to the specific probabilistic process utilized to generate generalized cost functions, cost values calculated by different generalized cost functions cannot be compared. Therefore, in many cases these figures cannot be used as absolute values for an objective function that computes the costs of all of the parties of a network.

Due to these limitations on application of the conventional UMHLP model in systems with decentralized management, we believe that development of a new hub location model would be useful. This aim is the most important innovation of this work.

Such a model is of interest to both public policymakers (concerned, for example, with allocating investments in new public terminals) and private operators of large transport systems, such as railroads, cabotage, etc. (interested in attracting customers from competing systems, mainly those that only use trucking).

For background on the hub location problem, we urge readers to consult Alumur and Kara [1], who provide a complete and detailed discussion of the various aspects of the problem and the state of the art of its different formulations and solution methods. Some new solution models for the UMHLP were not mentioned in that survey or have been published since then, so they are commented as follows.

The model of Cánovas et al. [4] included a heuristic based on the dual-ascent technique and an original algorithm to resolve the UMHLP. In this model, a preprocessing step is carried out before performing iterations of the base algorithm. According to the authors, this approach substantially improves the method's efficiency. The algorithm that coordinates all of the functions is of the branch-and-bound type. At each node of the implicit enumeration, a dual-ascent heuristic routine is executed as a fundamental tool of the process. Those authors noted that the most difficult problems are those in which there is symmetry in transport costs or in which values of the discount coefficient for interhub costs (α—described in Section 2.1) are low.

Among all of the solution methods published to date, the only one that outperforms that of Cánovas et al. [4] in solving this problem is that presented in Camargo et al. [3].

Camargo et al. [3] created an algorithm that currently performs best in solving the UMHLP. In addition, their algorithm is an exact optimization method. The procedure is based on the classic Benders decomposition and is able to resolve cases of up to 200 vertices in less than 10,000 s of processing time (using a Sun

[1] Sum of the financial cost and other monetary amounts representing the other transport impedances.

Blade 100 with a 500-MHz Ultra-SPARC processor and 1-gigabyte RAM memory).

Yang [15] makes an important contribution to formulation of the UMHLP. The formulation more precisely models the cases where transport demand is variable during the period considered. To solve this new model of the problem, a two-stage stochastic network design model is proposed.

Limbourg and Jourquin [11], through an iterative process, introduce the multi-modal assignment problem in the solution of the single allocation *p*-hub median problem to consider the variation in transshipment costs according to the amount of transshipped cargoes. Although this type of HLP is different than those addressed in the present work (multiple allocation), the case study presented by those authors is very similar to that presented in Section 3 because it seeks to establish intermodal hubs to attract cargo flows from long-distance trucking to combined transport.

Ishfaq and Sox [9] develop a multiple allocation *p*-hub median model (a problem type very similar to the UMHLP) whose formulation is so detailed in variables and parameters that it provides a more thorough and robust modeling framework for an intermodal logistics hub network (when compared to the already published formulations). It considers different types of shipments, different modes of transportation, fixed hub operating costs, modal connectivity costs, non-linear function of economies-of-scale and service time limit requirements. The work presents a meta-heuristic solution approach (tabu search based) with assessment of its performance, and an empirical study that gives interesting conclusions about the design and management of an intermodal logistics hub network.

Eiselt and Marianov [7] propose the competitive "*p*-hub" location method, which instead of trying to minimize the total network cost, aims to maximize the "demand" of users attracted by a new airline company entering a determined market through the justification that maximizing profits (and/or minimizing costs) is relegated to a second phase after a satisfactory number of customers have been achieved. These authors use the following three principles (which we also have used in our work):

- flows from one node *i* to another *j* do not necessarily have to follow the lowest cost route;
- proportions of these flows over the available routes between *i* and *j* can be defined probabilistically according to the relative utility of each route (perceived by the user); and
- in choosing among these routes, users can consider various factors (including subjective ones) besides those commonly employed (e.g., cost, distance and time).

Although Marín et al. [12] is discussed in the referred state of the art of Alumur and Kara [1], we present a comment on it because its UMHLP formulation is the basis of our proposed model. We took those authors' formulation as a starting point because it generalizes the UMHLP with fixed costs. In addition, it is not subject to the limitations of obedience to triangular inequality and it defines the facets of the problem's solution space. Additional references concerning this formulation are provided in Section 2.1.

Finally, Hamacher and Meyer [8] present an interesting summary of the latest developments in hub location.

The remainder of the paper is structured as follows. Section 2 states the model's formulation and its main features. Section 2.1.1 discusses a reduction in the number of variables, while the next subsection elucidates the practical meaning of an unusual parameter included in the proposed formulation. Section 3 describes the experiments, explains how the necessary input data are obtained and reports the computational results. The last section presents concluding remarks and suggestions for further research.

2. Proposed model

We propose a new model for the UMHLP with fixed costs, which we call the uncapacitated hub location problem with fixed costs in networks under decentralized management (UHLP-DM). We do not include "multiple allocation" in the name because a network under decentralized management necessarily functions according to a multiple allocation system (where each vertex of the network can be linked to more than one hub). Hence, it is not necessary to highlight the difference from the "single allocation" case because a decentralized management network cannot work in the single allocation case given that its various operators have total freedom and flexibility to choose their routes.

Due to this level of operator freedom, the model includes the so-called non-restrictive policy (i.e., permission for direct flows between each origin–destination pair).

As in conventional models, the basic objective of the model proposed here (Fig. 1) is establishment of new hubs to create more economical routes, which capture flows from existing costly routes, thereby generating resource savings. Therefore, optimization seeks to maximize these savings to obtain the lowest total network cost.

In reviewing the literature, we found that the large majority of published articles on hub location present models that start from an initial situation where no node of the network is defined as a hub. Besides being able to perform this way, the model proposed here presents a structure prepared for situations where the aim is to locate "additional" concentration terminals in an existing hub-and-spoke network. Furthermore, in the proposed model a lower number of feasible new hubs in relation to the number of existing hubs causes a reduction in the number of flow variables (x_{ijkm}), as explained in Section 2.1.1.

2.1. Model formulation

A hub-and-spoke network can be represented by a graph $G=(N, A)$, in which each vertex of the set N corresponds to a point of origin and destination of flows and can be chosen for establishment of a hub. The arcs of set A are the elements that constitute the routes.

$W=|w_{ij}|$ is defined as the flow demand matrix between pairs of nodes ij (with $i,j\in N$), such that each flow can pass through one or more intermediate hub node ($k,m\in N$), composing a path from i to j that goes through hubs k and m (more often represented by the sequence $ijkm$). We assume at most two intermediate hubs because it is rare in transportation engineering for more than two hubs to be utilized in the route between an origin–destina tion pair.

The transport cost of demand w_{ij} over a route $ijkm$ is given by $C_{ijkm}=w_{ij}\ (\chi c_{ik}+\alpha c_{km}+\delta c_{mj})$, where c_{ik}, c_{km} and c_{mj} are unit transport costs for each route subsegment and χ, α and δ are discount factors due to economies of scale. Each of these three factors ranges from 0 to 1, and α is expected to be smaller than the other two since the most significant cost reductions are achieved over the interhub links, where the flows are higher than on the spokes.

In contrast, there are no scale economies over the direct route, so $C_{ij}=w_{ij}.c_{ij}$. C_h is the cost of establishing a new hub terminal at vertex $h\in N$.

P_{ijkm} is the probability that a user of the pair ij will opt for the $ijkm$ hub route when it is offered the choice only between this route and the direct route. This probability can be defined by choice models (as proposed by Domencich and McFadden [6] and discussed in Ortúzar and Willumsen [13]). In the case of a route that passes only through existing hubs, this probability can be estimated by means of surveys of revealed or declared preference; however, in the case of a route that includes at least one not yet established hub, the estimation normally has to be done by means of the second type of preference study. This probability estimation is covered in more detail in Section 3.4.

Here, careful attention must be paid to the parameter that introduces the biggest difference between the proposed model and conventional models. We define C^o_{ijkm} as the sum of the transport costs of the direct flows and the flows via hubs of a node i to another node j (according to the following equation):

$$C^o_{ijkm} = P_{ijkm}C_{ijkm} + (1-P_{ijkm})C_{ij} \tag{1}$$

This parameter (C^o_{ijkm}) guarantees that the proposed model more faithfully represents the reality of networks operated under decentralized management, in which the flows from vertex i to vertex j are distributed between two route types (a direct one and another through hubs). We assume that the probability P_{ijkm} represents the fraction of users that will choose route $ijkm$ if this is the best one via hubs. In addition, the second portion of the parameter associates a fraction of the ij transport demand being carried by the direct route ($1-P_{ijkm}$) to the hub route $ijkm$. The adequacy of the parameter P_{ijkm} can be better understood in Section 2.1.2, which describes the practical meaning of the proposed objective function.

As variables, we have: y_h—a binary variable that defines whether there will be a hub established at vertex h; x_{ij}—a binary variable that defines the exclusive use of the direct path from i to j ($x_{ij}=1$ when no hub is used for the transportation of all the flow from i to j); and x_{ijkm}—a binary variable that defines whether the $ijkm$ route will be utilized. $H^* \subseteq N$ is the subset consisting of points where hubs are already installed.

With the defined variables and parameters, a first version of the model is presented by formulation (2a)–(2m) (with more general indices and based on the referred formulation of Marín et al. [12]):

$$\min_{x,y} F(x,y) = \left[\sum_{i\in N}\sum_{j\in N}\left(C_{ij}x_{ij} + \sum_{k\in N}\sum_{m\in N} C^o_{ijkm}x_{ijkm}\right)\right] + \left(\sum_{h\in N} C_h y_h\right) \tag{2a}$$

s. t.

$$x_{ij} + \sum_{k\in N}\sum_{m\in N} x_{ijkm} = 1 \quad \forall i,\ j \in N; \tag{2b}$$

$$y_h = 1 \quad \forall h \in H^*; \tag{2c}$$

$$x_{ij} \in \{0,1\} \quad \forall i,\ j \in N; \tag{2d}$$

$$x_{ijkm} \in \{0,\ 1\} \quad \forall i,j,k,m \in N; \tag{2e}$$

$$x_{ijkm} = 0 \quad \forall i,j,k,m \in N \mid P_{ijkm} = 0; \tag{2f}$$

$$\sum_{m\in N} x_{ijkm} + \sum_{\substack{m\in N\\ m\neq k}} x_{ijmk} \le y_k \quad \forall i,j,k \in N; \tag{2g}$$

$$\sum_{\substack{k\in N\\ \wedge k\neq i}} \sum_{m\in N} x_{ijkm} + y_i \le 1 \quad \forall i,j \in N, i \neq j; \tag{2h}$$

$$\sum_{k\in N}\sum_{\substack{m\in N\\ \wedge m\neq j}} x_{ijkm} + y_j \le 1 \quad \forall i,j \in N, i \neq j; \tag{2i}$$

$$y_i + \sum_{\substack{(k,m)\in A:\\ (k,m)\neq(i,i)}} x_{iikm} \le 1 \quad \forall i \in N; \tag{2j}$$

$$x_{kkkm} = x_{kkmk} = 0 \quad \forall k,m \in N, m \neq k; \tag{2k}$$

$$x_{kjmk} = x_{jkkm} = 0 \quad \forall k,m,j \in N, m \neq k \wedge j \neq k; \tag{2l}$$

$$y_K \in \{0,1\} \quad \forall k \in N. \tag{2m}$$

The objective function (2a) seeks to minimize the total network cost.

Although for each pair ij, the flow over the direct route is considered in terms of C^o_{ijkm} (Eq. (1)), there are normally origin–destination pairs for which there is no route through hubs that presents a lower cost than the direct path. For this reason, the objective function contains a portion with the variable x_{ij}. Hence, when the route via hubs is not used from i to j, the direct flow is computed by its specific variable (x_{ij}); however, when one of the hub routes is used, the associated direct flow is computed by the term C^o_{ijkm}.

The constraints in (2b) guarantee that each flow ij is totally routed. Those of (2c) type define the vertices h of the network that are existing hubs, such that their respective C_h values are nil. Constraints in (2f) guarantee that x_{ijkm} is null and x_{ij} equals 1 when P_{ijkm} vanishes. Note that, if $P_{ijkm}=0$, then C^o_{ijkm} is equal to C_{ij} and the absence of (2f) would allow either x_{ijkm} or x_{ij} to be 1.

Constraints (2g)–(2m) are equal to the respective constraints formulated in Marín et al. [12] and are explained next. As already justified by the end of Section 1, we based our model in those authors' formulation. It is important to observe that the formulation's property of being the facets of the solution space has not been lost with our model's new constraints added. This is justified below.

Marín et al. [12] employed the process of lifting facets of set-packing type, which is based on the following premise: "Any set of set-packing type constraints can be optimally written by grouping in a single constraint all maximal subsets of pairwise incompatible variables. Two 0–1 variables a and b are incompatible if $a+b \le 1$." Because the referred modifications in relation to the original formulation of Marín et al. [12] do not change the compatibility between variables, the constraints continue being facets of the problem's solution polyhedron.

The constraints in (2g) guarantee that a new route that goes through a hub k, only can be used if this concentrator terminal is established. The constraints in (2h) determine that a flow starting from one hub must go directly to a distribution hub without passing through a collection hub because the origin is already considered to be a collection hub. Analogously, the constraints in (2i) determine that a flow headed to one hub must go directly from the collection hub to its destination without passing through a distribution hub because the destination is already considered to be a distribution hub. The constraint in (2j) represents the case where hub i is both the origin and destination of a flow, but this flow may not pass through any other vertex because hub i itself is the collection and distribution hub. The constraints in (2k) and (2l) follow the same logic as those in (2h), (2i) and (2j), which impede flows over redundant routes.

In many practical hub location problems, the number of feasible points to establish new hubs and the number of existing hubs are considerably lower than the cardinality of N. Therefore, we can rewrite the formulation in a second version (that utilized in the experiments), taking into consideration the observation that the number of possible hub pairs will only be $|H \cup H^*|^2$, where $H \subseteq N$ is the subset consisting of feasible points at which new hubs may be installed, and H^* as it was defined in Section 2.1. Constraints (3b)–(3m) are, respectively, analogous to (2b)–(2m), and they are for i, j in N and k, m in $H\cup H^*$

$$\min_{x,y} F(x,y)=\left[\sum_{i\in N}\sum_{j\in N}\left(C_{ij}x_{ij}+\sum_{k\in H\cup H^*}\sum_{m\in H\cup H^*}C^o_{ijkm}x_{ijkm}\right)\right]+\left(\sum_{h\in N}C_h y_h\right), \tag{3a}$$

subject to

$$x_{ij}+\sum_{k\in H\cup H^*}\sum_{m\in H\cup H^*}x_{ijkm}=1 \quad \forall i,j\in N; \tag{3b}$$

$$y_k=1 \quad \forall k\in H^*; \tag{3c}$$

$$x_{ij}\in\{0,1\} \quad \forall i,\ j\in N; \tag{3d}$$

$$x_{ijkm}\in\{0,1\} \quad \forall i,j\in N,\forall k,m\in H\cup H^*; \tag{3e}$$

$$x_{ijkm}=0 \quad \forall i,\ j\in N,\ \forall k,\ m\in H\cup H^*\,|\,P_{ijkm}=0; \tag{3f}$$

$$\sum_{m\in H\cup H^*}x_{ijkm}+\sum_{m\in H\cup H^*,m\neq k}x_{ijmk}-y_k\leq 0 \quad \forall i,\ j\in N,\ \forall k\in H\cup H^*; \tag{3g}$$

$$\sum_{\substack{k\in HUH^*\\ \wedge k\neq i}}\sum_{m\in HUH^*}x_{ijkm}+y_i\leq 1 \quad \forall i,\ j\in N,\ i\neq j; \tag{3h}$$

$$\sum_{k\in HUH^*}\sum_{\substack{m\in HUH^*\\ \wedge m\neq j}}x_{ijkm}+y_j\leq 1 \quad \forall i,\ j\in N,\ i\neq j; \tag{3i}$$

$$y_i+\sum_{\substack{(k,m)\in A:\\ (k,m)\neq(i,.i)}}x_{iikm}\leq 1 \quad \forall i\in N; \tag{3j}$$

$$x_{kkkm}=x_{kkmk}=0 \quad \forall k,\ m\in H\cup H^*,\ m\neq k; \tag{3k}$$

$$x_{kjmk}=x_{jkkm}=0 \quad \forall k,\ m\in H\cup H^*,\ \forall j\in N,\ m\neq k\wedge j\neq k; \tag{3l}$$

$$y_k\in\{0,1\} \quad \forall k\in H. \tag{3m}$$

In certain application cases of the proposed model, it may be necessary to keep a new hub from being established at a place that will compete with an existing hub to the point where the new one renders the old one unfeasible. This task can be accomplished by means of restrictions on the set of vertices in which new hubs may be established ($H\subseteq N$). Thus for example, the nodes very close to an existing hub would not belong to H. The mathematical definition of this proximity depends on the characteristics and parameters for each instance, besides the numeric determination must be done with support of the analyst's experience.

The total number of constraints in the formulation defined by (2a)–(2m) is given by $R_o=|N|^3+3|N|^2-|N|$. In formulations (3a)–(3m), this number of constraints and also the number of variables can be substantially reduced—being the former according to Eq. (4) and the latter as described in the next section

$$R_n=|N|^2|H\cup H^*|+3|N|^2-|N|=(H\cup H^*|+3)|N|^2-|N| \tag{4}$$

2.1.1. Reduction of the number of variables

As observed in the introduction of Section 2, the proposed model structure is prepared for problems that require the location of additional hubs in a pre-existent hub-and-spoke network (HSN). For this kind of problem, the number of variables can be reduced by the preprocessing presented in the following letters a and b.

For each origin–destination pair ij, let $ijpq$ be a path in the pre-existent HSN such that $i,j\in N\wedge p,\ q\in H^*\wedge C^o_{ijpq}\leq C^o_{ijkm}\forall k,\ m\in H^*$ (β/α), i.e., among the paths that go through a pair of pre-established hubs, $ijpq$ has a minimal value for the cost parameter (C^o_{ijpq}):

a. Then $x_{ijkm}=0 \ \ \forall k,\ m\in H^*: C^o_{ijkm}>C^o_{ijpq}$.
b. If $C_{ij}>C^o_{ijpq}\Rightarrow x_{ij}=0$.

Thus, for all pre-existent paths from i to j, it is sufficient to consider in the model only the variables (x_{ij} or x_{ijpq}:$p,q\in H^*$) that present the minimal value for cost parameter (C_{ij} or C^o_{ijpq}).

On the other hand, let $ijk'm'$ be a path where "at least" one of the two later vertices (k', m') is not a pre-established hub such that

a. $k'\in H\cup H^*\wedge m'\in H$ or
b. $k'\in H\wedge m'\in H^*$.

Therefore, each route $ijk'm'$ must be represented in the model's formulation by a variable $x_{ijk'm'}$. The amount of these variables (due to non-pre-existent paths) totals to $|HUH^*|^2-|H^*|^2$ for each O–D pair ij. So the number of variables in a problem of "additional" hub location may be reduced down to the value given by Eq. (5), where number 1, the last term in the parenthesis, represents the minimum quantity of the variables related to pre-existent paths (x_{ij} and x_{ijpq}:$p,q\in H^*$).

$$V=|N|^2(|H\cup H^*|^2-|H^*|^2+1) \tag{5}$$

In conventional formulations, the number of flow variables (x_{ijkm}) is equal to $|N|^2|HUH^*|^2$ because it is necessary to consider both the new flows and all those already going through previously established hubs.

Hence, in cases where the analyst intends to locate additional hubs in an existing HSN, the reduction of the number of variables can be very significant. To give an example of the potential magnitude of this diminution, Table 1 compares the numbers of flow variables (multiplied by $1/|N|^2$) for a case in which the total number of hubs is 12—as in the experiments of Section 3, where there are nine existing hubs and three potential hubs (generating a reduction from $144|N|^2$ to $64|N|^2$ flow variables).

Analogous to the flow variables, the number of decision variables for the establishment of a new hub (y_h) is reduced from $|H\cup H^*|$ to $|H|$.

2.1.2. Practical meaning of the formulation with parameters C^o_{ijkm}

In the hub location models found in the literature, the installation of a new hub allows some flows to change their paths. This implies only a simple cost reduction equal to $C_{ij}-C_{ijkm}$ for each pair ij. However, due to the greater complexity of the costs term in the proposed model (C^o_{ijkm}), we believe that it is advisable to analyze the practical meaning of the effect (on the

Table 1
Numbers (multiplied by $1/|N|^2$) of flow variables (x_{ijkm}).

$\|H^*\|$	$\|H\|$	Conventional $\|H\cup H^*\|^2$	Proposed model $\|H\cup H^*\|^2-\|H^*\|^2+1$
1	11	144	144
2	10	144	141
3	9	144	136
4	8	144	129
5	7	144	120
6	6	144	109
7	5	144	96
8	4	144	81
9	3	144	64
10	2	144	45
11	1	144	24

total network cost) caused by a new solution found by the optimization process.

Consider an existing hub-and-spoke network and an O–D pair ij (such that $i,j \in N$). Suppose the hub path $ijpq$ (such that $p,q \in H^*$) presents the minimal parameter C^o_{ijpq} among all paths from i to j. Then the ij transport flow (due to w_{ij}) goes through $ijpq$ and through the direct path, as explained at the beginning of Section 2.1. When applying the model to this HSN, the portion of the transport costs of the pair ij in the objective function can be determined from the terms of the right hand side of Eq. (6), which include the variable related to the pre-established path $ijpq$ and all variables $x_{ijk'm'}$ that contain at least one non-pre-established hub (as defined by the indices of the summations in the right hand side of the following equation:

$$C_{ij}x_{ij}+\sum_{k\,\in\,H\cup H^*}\sum_{m\,\in\,H\cup H^*}C^o_{ijkm}x_{ijkm}=\sum_{k'\,\in\,H\cup H^*}\sum_{m'\,\in\,H}C^o_{ijk'm'}x_{ijk'm'}+\sum_{k'\,\in\,H}\sum_{m'\,\in\,H^*}C^o_{ijk'm'}x_{ijk'm'}+C^o_{ijpq}x_{ijpq} \tag{6}$$

In Eq. (7), let $x_{ijk^*m^*}$ be the variable with the lowest cost parameter ($C^o_{ijk'm'}$) among all variables that contain at least one non-pre-established hub. Therefore,

$$C_{ij}x_{ij}+\sum_{k\,\in\,H\cup H^*}\sum_{m\,\in\,H\cup H^*}C^o_{ijkm}x_{ijkm}=C^o_{ijk^*m^*}x_{ijk^*m^*}+C^o_{ijpq}x_{ijpq}. \tag{7}$$

In this form, if $C^o_{ijk^*m^*}<C^o_{ijpq}$, then $x_{ijk^*m^*}$ becomes equal to 1 and the former path $ijpq$ stops being used following application of the model. In turn, the following network transport cost reduction will occur:

$$\begin{aligned}C^o_{ijpq}-C^o_{ijk^*m^*}&=[P_{ijpq}C_{ijpq}+(1-P_{ijpq})C_{ij}]-[P_{ijk^*m^*}C_{ijk^*m^*}+(1-P_{ijk^*m^*})C_{ij}]\\&=[P_{ijpq}C_{ijpq}-P_{ijk^*m^*}C_{ijk^*m^*}]+[(1-P_{ijpq})-(1-P_{ijk^*m^*})]C_{ij}\\&=[P_{ijpq}C_{ijpq}-P_{ijk^*m^*}C_{ijk^*m^*}]+[P_{ijk^*m^*}-P_{ijpq}]C_{ij}\end{aligned} \tag{8}$$

Being $P_{ijk^*m^*}=P_{ijpq}+(P_{ijk^*m^*}-P_{ijpq})$, the substitution of $P_{ijk^*m^*}$ in the last expression gives

$$\begin{aligned}&[P_{ijpq}C_{ijpq}-P_{ijpq}C_{ijk^*m^*}-(P_{ijk^*m^*}-P_{ijpq})C_{ijk^*m^*}]+[P_{ijk^*m^*}-P_{ijpq}]C_{ij}\\&=[P_{ijpq}(C_{ijpq}-C_{ijk^*m^*})-(P_{ijk^*m^*}-P_{ijpq})C_{ijk^*m^*}]+[P_{ijk^*m^*}-P_{ijpq}]C_{ij}\\&=[P_{ijpq}(C_{ijpq}-C_{ijk^*m^*})]+[(P_{ijk^*m^*}-P_{ijpq})(C_{ij}-C_{ijk^*m^*})]\end{aligned} \tag{9}$$

The first term of the final expression represents the savings provided by transferring flows from $ijpq$, to the path by hubs k^* and m^*.

Note that in the second term there is a difference between the probabilities of using hub routes ($P_{ijk^*m^*}$–P_{ijpq}). This disparity is due to the difference between the characteristics of the two routes; one route is more attractive than the other, causing greater traffic flow through hubs and less use of direct transport. Therefore, this difference represents the fraction of users that did not utilize a path via hubs and will start to do so. For this reason, the cost reduction associated with these users will be the difference between the cost of the direct route and that of the new route via hubs.

Intuition might lead to the error of thinking that if $C_{ijk^*m^*}>C_{ijpq}$, the probability $P_{ijk^*m^*}$ will be lower than P_{ijpq} (because the route ijk^*m^* could appear to be less attractive than $ijpq$) and hence $C^o_{ijk^*m^*}$ will be higher than C^o_{ijpq}. Analogously, one might also expect that if $P_{ijk^*m^*}<P_{ijpq}$, the cost $C_{ijk^*m^*}$ will probably be higher than C_{ijpq}, causing $C^o_{ijk^*m^*}$ to be higher than C^o_{ijpq}. However, in the practice of transportation modeling, these conclusions are not necessarily true because not only the cost influences the user's choice (and the associated probabilities) but other factors beyond the costs are relevant (time, frequency, security and other subjective factors).

A capacitated model (in which capacity limits can be considered in both existing and new routes) requires a different type of formulation. It would be extremely complex to consider more than one route of the $ijk'm'$ type (which contain at least one non-pre-established hub) simultaneously absorbing the respective parts of the flows between ij. It would be necessary to model the user's options among the direct alternative and all the other alternatives though hubs, generating a specific probability of utilization for each new hub path in the same O–D pair. This approach would make each $P_{ijk'm'}$ depend on the probabilities of using the other hub routes between ij instead of only the probability of using the direct route.

3. Experiments

One of the main strategies of Brazil's Transportation Ministry is to rebalance the country's cargo transport modal mix. Currently, 58% of cargo nationwide is carried by trucking, 26% by rail, 15% by cabotage (on coastal and river shipping) and 1% by air. To achieve this objective, some basic actions have been planned—among them construction of infrastructure for intermodal terminals. The problem thus arises of determining the most propitious sites for these investments, which was one of the main motivations for the present study.

In the experiments carried out, 305 instances of the proposed model were solved. Each instance was modeled as recommended in Section 2 and simulates one case in which the Brazilian government would decide where to apply resources to open general cargo new cabotage terminals in the country—seeking the greatest logistical cost reduction by migration of cargo flows from trucks to the referred larger capacity modal.

Section 3.1 defines and characterizes the network adopted for the experiments. The next three sections explain how input data were obtained: the origin–destination demand matrix values (w_{ij}), the transportation costs (C_{ij}, C_{ijkm}) and the probabilities of the option for transport through hubs (P_{ijkm}). Section 3.5 presents the computational results.

3.1. The network adopted for the experiments and its discount factors

A hub-and-spoke network under decentralized management is very common for cargo transport. Any trunk line (such as a waterway, railway, etc.) effectively linked to a roadway network can constitute infrastructure for this type of system—where the intermodal terminals play the role of hubs. However, the availability of data on networks of this nature is normally very restricted. In the first place, the trunk axes (or shuttle cargo lines) of the network are normally dominated by one or only a few large companies, which generally have no interest in disclosing detailed information about demand, cost, duration of service, cargo losses, etc. (as discussed by Jong [10]). Second, the data on complementary transport on the spokes (almost always by road) depend on very expensive field surveys.

For these reasons, there is no network in Brazil with all the necessary data available for the model. Thus, for the experiments presented here, we chose the network for which the existing set of data on demand and costs was least difficult to complement through alternative processes. This network is the complex of routes formed by the cabotage lines carrying general cargo and the system of federal and state highways (Fig. 2). In this network, the hubs are the ports along the shipping lines, while these lines and the highways make up, respectively, the shuttle services and spokes. The origin and destination vertices are the centroids of 289 homogenous micro-regions that compose the hinterlands of the ports considered in the study. The digital model of this network was supplied by CENTRAN (Center for Excellence in Transport Engineering, the government entity responsible for

Fig. 2. Hub-and-spoke network chosen for the study.

strategic logistics planning, linked to the Transportation and Defense Ministries). The three localities marked with a single geometric form in Fig. 2 are potential hubs considered in all instances (see Section 3.5).

Due to the specific scenario of these experiments, it can be assumed that the interhub flows always travel by cabotage while the direct ones go by road.

The scale economies of the interhub routes were considered according to the model of constant discount factor (represented in the formula for C_{ijkm} in Section 2.1).

Because the experiments examine competition between cabotage and trucking, we adopted the definition of general cargo that is utilized in the Brazilian Port System, which includes practically all products handled at ports that are not bulk liquids or solids.

We considered the management of the network to be decentralized because the demand matrix utilized in the experiments ($W=|w_{ij}|$) includes all of the general cargo carried in the domestic Brazilian market. Because there is free competition in this market, transport is shared by all companies that are active in it.

3.2. Estimation of the O–D matrix

In our research at institutions concerned with transportation, we did not find a general cargo O–D matrix for Brazil that could be used in this study. Therefore, we generated an O–D matrix ($W=|w_{ij}|$) from the available data. We used two methods recommended by Ortúzar and Willumsen [13]: the entropy maximization and the Furness algorithm of successive multi-proportional corrections. The first of which failed and the second of which succeeded in our experiments.

The later method is based on an iterative process involving successive corrections of the values of w_{ij} in the set of equations (10), where each parameter V_a is the yearly flow of general cargo that goes through a determined link a of the Brazilian transport network (obtained in the actual traffic counts of CENTRAN [5]) and Q_{aij} reports whether the flow between the pair i and j contributed to that link a (being 1 or 0 if it contributed or not, respectively). According to Eq. (10), 240 links were considered

$$\sum_{i,j} w_{ij} Q_{aij} = V_a \quad \forall a = 1,\ldots,240 \tag{10}$$

At each iteration, a correction factor is redefined in an attempt to satisfy the equations within a predetermined tolerance. It is a simple process that is sufficiently precise for the analysis in question. It also proved to be very efficient in the cases of very large and sparse matrices.

3.3. Calculation of C_{ij} and C_{ijkm}

Because the optimization problem is modeled from the perspective of a user of the system (i.e., from the standpoint of cost savings of the modal choice), we considered that the transport value is equal to the freight rate charged by the operators.

For highway transport we created (by regression analysis) two curves to model the average freight rate per ton-kilometer (US\$/t km) as a function of the distance traveled (*dist*), which are $C^1_{ij}=1.1364(dist)^{-0.481}$ and $C^2_{ij}=3.0675(dist)^{-0.562}$. In this statistical process, the database of market rates from SIFRECA [14] was used.

The first function (C^1_{ij}) models the freight rate for hauling services to collect or distribute cargoes to or from hubs (i.e., the traffic going through the spokes of the network), while the second function (C^2_{ij}) refers to direct highway transport between an O–D pair.

To establish an average waterway freight rate, we surveyed the three main companies that operate in the Brazilian cabotage market (Hamburg Süd, Log-In and Maersk). We analyzed these data and created an O–D matrix of average freight charges considering all of the studied ports as origins and destinations. These values included transshipment of cargo at the origin and the destination of the interhub transport. Hence, by adding the values from this matrix to those of the referred C^1_{ij}, we composed the parameters C_{ijkm}.

3.4. Calculation of P_{ijkm}

In the experiments we used the Logit choice model (Domencich and McFadden [6]) to determine the proportion into which a flow between a pair ij is routed by a path through hubs instead of by the direct path alternative. This proportion is equivalent to the probability P_{ijkm} (represented by Eq. (11)).

$$P_{ijkm} = \frac{1}{1+\exp(\overline{a}_k(\overline{d}_{ij}-\overline{h}_{ijkm})+a_k^o)}, \tag{11}$$

where, $\overline{a}_k$ is the vector of parameters of the hinterland of the origin hub (k); a_k^o is a constant of the hinterland of the origin hub (k); $\overline{h}_{ijkm}$ is the vector of variables related to the route from i to j that goes through hubs k and m; and $\overline{d}_{ij}$ is the vector of variables related to the direct route.

The parameters used in calculating P_{ijkm} refer to the area of influence (hinterland) of the "origin" hub k because (as stated by Jong [10]) "interviews in the transport market have indicated that for mode choice the shipping firm is the most important decision-maker." Therefore, to obtain P_{ijkm}, it is necessary to estimate the constant a_k^o and the parameters of the vector $\overline{a}_k$ for the hinterland of each port $k \in |H \cup H^*|$. These elements can be estimated by methods such as multiple linear regression, maximum likelihood estimation and others [13]; however, these methods depend on lengthy field surveys, which require considerably more funding than was available for this study. Thus, due to these budgetary limitations, we created an alternative estimation process for this experiment, based on the solution of a system of nonlinear equations.

This system is composed of two equations for each port k: one modeling the annual quantity of outbound cargo handled in k and another modeling the annual quantity of inbound cargo in port k (as Eqs. (13) and (14)). Therefore, due to the number of equations, it is possible to estimate at most two parameters per port (as will be explained throughout this section).

Before describing the alternative estimation procedure, it's necessary to approach the variables of Eq. (11) to which the parameters of vector $\overline{a}_k = (a_{k1}, a_{k2})$ refer. Various characteristics of the transport process can be considered as variables of vectors $\overline{h}_{ijkm}$ and $\overline{d}_{ij}$ (e.g., cost, time, variance of time, etc.). However, because of the limitation of the number of parameters that can be estimated, we had to consider only two kind of variables, which were the average freight rate and the average transport time. Thus, for each O–D pair ij, we assumed the following vectors of variables: $\overline{d}_{ij} = (d1_{ij}, d2_{ij})$ and $\overline{h}_{ijkm} = (h1_{ijkm}, h2_{ijkm})$, where $d1_{ij}$ and $h1_{ijkm}$ represent the average freight rates over the direct and hub routes, respectively (i.e., C_{ij} and C_{ijkm}), while $d2_{ij}$ and $h2_{ijkm}$ represent the average transport times over these routes.

Due to the referred limitation of the alternative process (two parameters per port), we assumed the constants a_k^o as nil because this strategy was better than having only one dimension in each of the vectors of Eq. (11). The vanishing of constant a_k^o was accepted because this alternative process intends to provide experimental input data that only approximate reality, in order to avoid costly survey on transport modal preference. Besides, the objective of this research does not include creating a precise method to estimate modal division functions.

The first step to estimate the parameters a_{k1} and a_{k2} is to formulate the system of nonlinear equations, which is explained below.

Considering a full year of transport activity, it can be assumed that the amount of cargo transported from i to j that goes through ports (hubs) is the product of the annual demand from i to j and the probability P_{ijkm}—such that $ijkm$ is the best route between i and j that goes through hubs. This is represented by Eq. (12) (where k is not an exponent, only an index)

$$Q_{ij}^k = w_{ij} P_{ijkm} = \frac{w_{ij}}{1+\exp(\overline{a}_k(\overline{d}_{ij}-\overline{h}_{ijkm})+a_k^o)} \tag{12}$$

Therefore, it can be said that the annual quantity of outbound cargo handled at a determined port k (Eq. (13)) will be equal to the sum of the annual quantities carried (via ports) from all of the centroids of the area of influence (hinterland) of k:

$$Out_k = \sum_{i \in AI(k)} \sum_{j \in UAI \setminus AI(k)} Q_{ij}^k \tag{13}$$

where $AI(k)$ is the set of centroids of the area of influence of k and UAI is the union of all of the areas of influence.

The total quantity of inbound cargo handled at port k can be calculated analogously. However, in this case the flows come from other areas of influence and the probability of using transport through hubs must be calculated according to each area g of the respective origin

$$In_k = \sum_{i \in UAI \setminus AI(k)} \sum_{j \in AI(k)} Q_{ij}^g \tag{14}$$

These two Eqs. (13) and (14) per port compose the system of nonlinear equations where the terms a_{k1} and a_{k2} are the unknowns. The other terms (that can be obtained) are the following:

- O–D matrix for the general cargo flows between the centroids of the hinterlands (w_{ij});
- the two O–D matrices of transport cost values ($d1_{ij}$ and $h1_{ijkm}$);
- the two O–D matrices of transport times ($d2_{ij}$ and $h2_{ijkm}$); and
- the general cargo cabotage flows arriving at and departing from each port (In_k and Out_k).

In the experiments, we formulated this system of equations for the nine actually existing cabotage Brazilian ports (i.e., the ports $k \in H^*$): Rio Grande, São Francisco do Sul, Santos, Itaguaí, Salvador, Maceió, Suape, Pecém and Manaus. The respective actual flows In_k and Out_k are provided by ANTAQ [2] (Brazilian Waterway Transport Agency). In the case of the ports $k \in H$ (potential hubs), there is no actual values for In_k and Out_k, then these ports were not included in the system and their parameters a_{k1} and a_{k2} were simply determined as explained at the end of this section.

To solve the system, we created a program in MatLab 7.2 (which includes various routines to solve systems of nonlinear equations). We assumed an error in each equation of up to 10% of the annual quantity of outbound or inbound cargo. Nevertheless, even with that tolerance, the solution routines did not converge and the errors remained very large for ports with very small cabotage volumes—such as Port of São Francisco do Sul and Port of Maceió.

Although these solution routines did not converge, we did not discard this alternative method. Instead, we slightly modified the actual data on flows In_k and Out_k and made other assumptions (less significant) until the solution routines errors could be accepted. We believe this approach is justified because, as stated before, our intent is to avoid costly modal preference studies, even by using data that only approximate reality.

Therefore, the parameters $\overline{a}_k = (a_{k1}, a_{k2})$ for the previously established ports were attained (Table 2).

To calculate the flows that go through new ports, it is necessary to create parameters (a_{k1}, a_{k2}) for the areas of influence of the potential hub locations, but the estimation of these parameters depends on declared preference data. Once again, there is a need for substantial human and financial resources for field surveys, which are not possible within the scope of this study.

Hence, to carry out this experiment, we used hypothetical parameters for these new areas of influence, attributing to them the parameter vector of the nearest neighboring port: the parameters

Table 2
Parameters of the probability functions.

Port	a_{k1} (Cost)	a_{k2} (Time)
Rio Grande	0.01634607	−0.05127318
S. Franc. Sul	−0.28701589	−1.73957621
Santos	−0.16534645	−1.81760973
Itaguaí	0.13974524	0.49768476
Salvador	−0.03830048	−0.76618573
Maceió	0.14323009	1.61877310
Suape	0.01023603	−0.29228317
Pecém	0.09369596	0.88607857
Manaus	0.04450863	0.14695807

of Port of Pecém for the areas of São Luís and Belém and that of Port of Itaguaí for the area of Barra do Riacho (due to their respective similarity).

3.5. Computational results

The proposed model was implemented using Xpress-MP optimization software (v.2008) and it was run on a Windows Server 2003 R2 SP2 platform with an Intel Core 2 Duo 2.66 GHz processor and 7.93 Gb of RAM.

As was mentioned previously, we created and solved 305 instances of the problem. In this set, nine instances were very near to a real world case because they include as pre-established hubs the general cargo Brazilian cabotage ports "actually" working,[2] and they consider as not installed hubs three places where public or private institutions have expressed significant interest in making investments of this nature—the cities of Barra do Riacho, São Luís and Belém, represented in Fig. 2 by a circle, triangle and square, respectively. The results of these particular instances are discussed in Section 3.5.2.

The other instances were not created to resemble the actual Brazilian cabotage situation. They were generated to enhance the number of tests of model efficiency. Thus, in these instances, the subsets of not installed hubs were not in accordance with reality because at each instance, it was assumed that some existing Brazilian ports were not yet installed.

In all experimental instances, the number of hubs concerned (summing pre-established and not pre-established hubs) and the number of origin and destination vertices were the same, respectively, 12 and 289 (the latter is explained in Section 3.1).

The instances were distributed into 10 classes according to the number of not installed hubs considered in the instance (from 3 not installed hubs to 12). For each class, at least 30 examples of the problem were run and the average running time and standard deviation of each class were calculated (Table 3). It can be noted that the execution time grows with the number of not installed hubs, which was expected, because the number of variables increases. The growing number of variables is discussed in Section 2.1.1 and shown in Table 1.

We obtained very rough estimates for the fixed costs of terminals because neither detailed proposals nor budget need estimations had been obtained for any of the considered ports. Therefore, the fixed costs were parametrically analyzed as reported in Table 4.

3.5.1. Preprocessing

The maximum number of origin–destination (O–D) pairs is the square of the number of nodes less the number of nodes ($|N|^2-|N|$). Nevertheless, the intermodal transport by cabotage is not feasible for all of them. In each instance of these experiments, a portion of the O–D pairs was disregarded because cabotage in Brazil is not feasible for an O–D pair with a hauling distance of less than 1000 km nor for an O–D pair where the distance from its origin or from its destination to the respective closer port is greater than 400 km. It was possible to eliminate this type of O–D pair because the aim of the study was to minimize the total cost by migration of flows from direct to intermodal transport and not to calculate the total cost of the entire network. Thus, consideration of O–D pairs with direct flows that cannot migrate to an intermodal route is useless in this study.

Some specific polyhedral properties of these experiments instances were applied as additional constraints. These properties are as follows:

- $x_{ijkm}=0 \quad \forall i,\ j \in N,\ \forall k,\ m \in H \cup H^* | k=m$, because no savings occur due to the large capacity mode between k and m (on the intermodal transport);
- $x_{ijkm}=0 \quad \forall i,j \in N, \forall k,m \in H \cup H^* | i=j$, because intermodal transport is not worthwhile from one node of the network to itself;
- $x_{ijkm}=0 \quad \forall i,j \in N, \forall k,m \in H \cup H^* | C_{ij} \le C_{ijkm}$, because we assumed cabotage is not worthwhile if its cost is more than or equal to the direct transport cost.

3.5.2. Real case instances

A deeper analysis of the referred nine real world instances, with potential hubs in the cities of Barra do Riacho, São Luís and Belém, was performed (Fig. 2).

It is worthwhile to note that São Luís and Belém are strategically important cities in Brazil and are along one large segment of the Brazilian coast line that does not yet present a general cargo cabotage significant port. The proximity of these two cities creates a case of competition conflict between hubs because the market in their total area of influence is not large enough to generate transport demand that would require two new cabotage hubs.

We considered a hypothetical situation (but likely in the future) in which cabotage in the region of Barra do Riacho is strongly motivated. We disregarded the existence of its neighbor Port of Vitoria, which has limited capacity and thus may lose its general cargo handling role in the future.

To solve a real instance of the problem with a single application of the proposed model, it would be necessary to obtain fair estimates of hub fixed costs (C_h) at each of the three candidate sites to allow for an optimal combination of hubs and spokes to be used in the domestic market. As explained above, we could not acquire this information.

Hence, to carry out our experiments, we consider that, in governmental decisions, it is often more suitable to carry out a broader analysis with identification of the cost intervals within which specific combinations of projects would be viable. Thus, we conducted nine applications of the model (A–I, Table 4) with several values for the annual fixed costs C_h. The following comments on the results illustrate this kind of parametric analysis and show how useful it can be.

In Case A, only Barra do Riacho was selected for establishment of a new hub. The transport cost savings is US$ 112,424,000 per year, generating a positive project balance of US$ 76,004,000 per year. Therefore, projects in that city with fixed costs of up to US$ 112×10^6 present a positive financial return, even without the installation of another new hub.

In Cases B and C, with the reduction of costs at locations 2 and 3, the establishment of a hub in Belém (3) is included in the optimal solution. Note that for equal fixed costs in Belém and São Luís ($C_2=C_3$), there is competition with only one winner for values in the following intervals: $8.77 \times 10^6 \le C_2 \le 36.41 \times 10^6$ when a hub is

[2] The Brazilian general cargo cabotage hubs actually installed are the ports of Rio Grande, São Francisco do Sul, Santos, Itaguaí, Salvador, Maceió, Suape, Pecém and Manaus—which compose the subset $H^* \subset N$.

Table 3
Statistical information about the classes of the instances run.

Classes of instances	No of potential hubs $\|H\|$	No of variables			Running time (s)	
		Conventional models[a]	Maximum in the proposed model[b]	Average after preprocessing[c]	Average	Standard deviation
Class 1	3	12,027,024	5,345,344	72,704	4.89	1.09
Class 2	4	12,027,024	6,765,201	87,967	11.42	4.29
Class 3	5	12,027,024	8,018,016	101,343	22.93	6.35
Class 4	6	12,027,024	9,103,789	119,912	35.41	9.82
Class 5	7	12,027,024	10,022,520	128,774	45.16	14.27
Class 6	8	12,027,024	10,774,209	132,038	107.84	47.25
Class 7	9	12,027,024	11,358,856	135,791	171.94	66.74
Class 8	10	12,027,024	11,776,461	140,047	259.26	71.57
Class 9	11	12,027,024	12,027,024	141,948	319.71	80.71
Class 10	12	12,027,024	12,110,545	141,948	596.66	143.77

[a] In conventional models, the number of variables is $|N|^2|H \cup H^*|^2$. Here $|N|=289$ and $|H \cup H^*|=12$.
[b] In the proposed model, the maximum number of variables is given by $|N|^2(|H \cup H^*|^2 - |H^*|^2+1)$.
[c] After the preprocessing (3.5.1), each instance included a much lower number of variables.

Table 4
Applications of the new UHLP-DM model.

	C_h—fixed costs (10^6 US$/year)[a]			Obj. function (10^6 US$/year)	Selected hubs	Transport savings (10^6 US$/year)
	1. B. Riacho	**2.** São Luís	**3.** Belém			
A	36.42	36.42	36.42	76.004	1	112.424
B	36.42	36.41	36.41	76.010	1, 3	148.840
C	36.42	8.77	8.77	103.644	1, 3	148.840
D	36.42	8.76	8.76	103.664	1, 2, 3	157.602
E	148.84	36.42	10.00	12.701	3	22.701
F	148.84	10.00	36.42	4.682	2	14.682
G	148.84	3.81	3.81	18.891	3	22.702
H	148.84	3.80	3.80	18.902	2, 3	26.502
I	36.42	32.21	36.42	76.008	1, 2	144.638

[a] US$ 1.00=R$ 2.20 (Brazilian currency).

established in Barra do Riacho ($y_1=1$); and $3.81\times10^6 \le C_2 \le 22.70\times10^6$ when no hub is established in Barra do Riacho ($y_1=0$, for this later interval see cases G and E). In this competition, the optimization decision always includes Belém ($y_3=1$), in detriment to São Luís ($y_2=0$).

Cases D and H show that the competing locations Belém and São Luís can be included together in the optimal solution ($y_2=y_3=1$) if the fixed costs of these both hubs are in the following intervals: $C_2=C_3 \le 3.80\times10^6$ when no hub is established in Barra do Riacho ($y_1=0$); and $C_2=C_3 \le 8.76\times10^6$ when a hub is established in Barra do Riacho ($y_1=1$). In this later interval, the optimization solution includes all three places even for higher fixed costs for the hub of Barra do Riacho.

In Case E, the fixed costs of locations 1 and 2 (C_1 and C_2) were raised while that of C_3 was reduced to verify the transport savings generated by only a hub established in Belém, which totals US$ 22,701,000. Note that the sum of the transport savings separately generated by hubs 1 and 3 (US$ 135,125,000) is less than that generated with the establishment of both hubs (US$ 148,840,000). This difference is to be expected due to the flows that would be carried between these two hubs.

Case F is analogous to E, the only difference being that the objective is to verify the savings generated by establishing only a hub in São Luís (US$ 14,682,000).

Cases I and B reveal a combination of costs in which the optimal solution favors São Luís over Belém: if the fixed cost in Belém (C_3) cannot be reduced below US$ 36.42×10^6, then establishing a hub in São Luís is included when $C_2 \le 32.21\times10^6$ and a hub is established at Barra do Riacho ($y_1=1$).

Various other situations can be simulated to support a decision process in which various decision makers propose scenarios with different cost combinations.

4. Conclusions

We created a model of the uncapacitated hub location problem with fixed costs on networks under decentralized management (UHLP-DM) including an integer programming formulation. We applied the model to intermodal transport of general cargo in Brazil. The results reveal that it is feasible to use the new model as an instrument to support decision making on investments in new cargo terminals.

In the model, we seek to depict the reality of decentralized operations, assuming that between an origin–destination pair *ij* the direct route and the route through hubs can be used at the same time. With this feature we overcome a limitation of conventional hub location models employing cost minimization. This innovation expands the field of combinatorial optimization because its use for locating hubs is no longer restricted to networks in which a uniform criterion prevails for choosing the best path. Therefore, mathematical programming by cost minimization can be utilized for determination of the location of hubs in any network under decentralized management where flows can be probabilistically modeled.

This modeling is carried out by means of behavioral theory applied to transportation [6]. It is the most laborious part of the process because it depends on costly studies of demand and modal preference. However, this problem is not unique to the model in question; it is common to any procedure intended to analyze (through mathematical modeling) transport systems in free markets.

The method permits consideration of subjective criteria on in transport type choice process (direct or via hubs). Given the nature of the objective function, the values of the variables must be numerical. Therefore, the costs must be represented in an objective form—either by their size (monetary value, time interval, etc.) or by another method (e.g., fuzzy logic).

Future work should be aimed at generalizing the model by considering the capacity limits of the arcs and vertices.

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