Formularium

1 Probability theory

Formula	**Description**
$F_X(x) = Pr[x \leq x]$	Cumulative distribution function of a random variable, this is non-decreasing and right-continues
$\lim_{x->\infty} F_X(x) = 1$	The normalization condition of this CDF
$\mathrm{f}_X(x) = \frac{dF_X(x)}{dx}; \quad \mathrm{f}_X(x)dx = Pr[x < X \leq x + dx]$	The density function, the derivative of the CDF
$\int_0^\infty \mathrm{f}_X(x)dx = 1$	The normalization condition of the density function
$p_X(n) = Pr[X = n], n \in \mathbb{N}$	The probability mass function (discrete RV)
$\sum_{n=0}^\infty p_X(n) = 1$	The normalization function (discrete RV)
$F_{XY}(x, y) = Pr[X \leq x, Y \leq y] \quad x, y \in \mathbb{R}_{\geq 0}$	joint cumulative distribution function of two random variables
$Pr[X \leq x, Y \leq y] \neq Pr[X \leq x]Pr[Y \leq y]$, if they are independent RV it is equal, thus $F_{XY}(x, y) = F_X(x)F_Y(y); \quad F_X(x) = F_{XY}(x, \infty); \quad F_Y(y) = F_{XY}(\infty, y)$	properties of the joint CDF
$f_{XY}(x, y) = \frac{\partial^2 F(x,y)}{\partial x \partial y}$ and have that $f_{XY}(x, y)dxdy = Pr[x < X \leq x + dx, y < Y \leq y + dy]$	Joint density function
$p_{XY}(n, m) = Pr[X = n, Y = m]$	joint mass function (discrete RV)
$f_{X\|Y}(x\|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$ and have that $f_{X\|Y}(x\|y)dx = Pr[x < X \leq x + dx\|Y = y]$	Conditional density function
$p_{X\|Y}(n\|m) = Pr[x = n\|Y = m] = \frac{p_{XY}(n,m)}{p_Y(m)}$	Conditional mass function (discrete RV)
$E[X] = \int_0^\infty x dF_X(x) = \int_0^\infty x f_X(x)dx$	Mean or expected value of a RV, it is the summary of a complete probability distribution
$E[X] = \sum_{n=0}^\infty n p_X(n)$	Mean or expected value of a discrete RV, it is the summary of a complete probability distribution
$E[aX + bY] = aE[X] + bE[Y]$	$E[.]$ is a linear operator
$E[XY] = E[X]E[Y]$	If RV X and Y are independent

Formula	Description
$Var[X] = E[(X - E[X])^2] = E[X^2] - E[X]^2$	The variance of a RV
$E[X] = E[E[X\|Y]]$	The conditional expectation of X given Y

2 Terminology

Name	Description
Arrival instant	The time at which a customer arrives at the queue.
Service instant	The time at which a customer leaves the system after being served completely.
Queue content	The number of customers in the queue waiting for service.
System content	The number of customers in the total system.
Queue capacity	The maximum number of customers in the queue.
System capacity	The maximum number of customers in the system.
Service time	The amount of time that the customer occupies a server.
Waiting time	The amount of time a customer waits in the queue before starting service.
Delay or sojourn time	The amount of time a customer resides in the system.

3 Distributions

Name	Density Function	Mean	Variance
Binomial	$Pr[X = k] = \binom{n}{k} p^k (1-p)^{n-k}$ with $\binom{n}{k} = \frac{n!}{k!(n-k)!}$	np	$np(1-p)$
Geometric	$Pr[X = k] = (1-p)^{k-1} p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Normal	$f(x\|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2
Uniform	$f(x\|a,b) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b, \\ 0 & \text{for } x < a \text{ or } x > b. \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$f(x\|\lambda) = \lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Poisson	$f(k\|\lambda) = \frac{e^{-\lambda} \lambda^k}{k!}$	λ	λ

3.1 Memoryless property

Formula	**Description**
$r(t) = \frac{Pr[X \leq t+dt \mid X>t]}{dt} = \frac{x(t)}{1-X(t)}$	The hazard rate function of a RV
$r(t) = \lambda$	The hazard rate function of an exponential function