GaussianMethod

Software Documentation

Author: matiwa

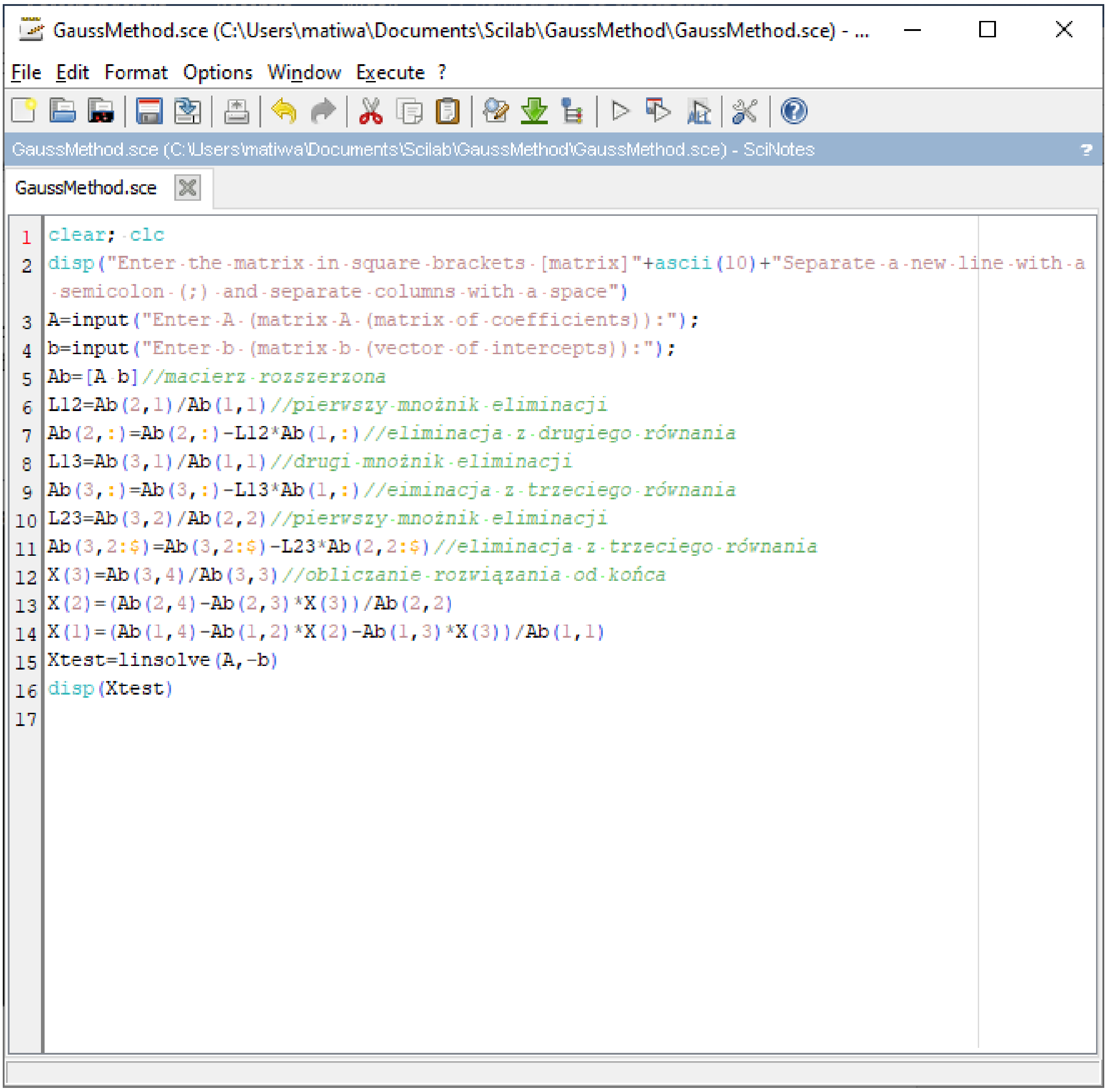
Table of contents

Introduction

This Scilab script file documentation includes: description of the script operation, what is needed for use, used algorithms, description of the source code. This script is used to calculate a system of n equations with n unknowns Ax = b.

Describing of the script's operation

If the user wants to calculate the solutions of the equation system by Gaussian elimination method, he runs the Scilab script in the Scilab Console window in menu -> Applications -> SciNotes. Then, in Scinotes, he opens the file and loads it from the menu -> Execute -> Save and Execute (F5 shortcut).



Drawing 1: Script content [own study]

```
 Enter the matrix in square brackets [matrix]
Separate a new line with a semicolon (;) and separate
 columns with a space
Enter A (matrix A (matrix of coefficients)):[4 -2 4 -2;3 1 4 2; 2 4 2 1; 2 -2 4 2]

Enter b (matrix b (vector of intercepts)):[8;7;10;2]


  -1.
   2.
   3.
  -2.

-->
```

Drawing 2: The contents of the Scilab console window [own study]

After this operation, enter the following input data on the Scilab Console screen: matrix A in square brackets [matrix], separate elements of the same row with a space, and new rows with a semicolon (;). After confirming with Enter by pressing the button, enter the matrix of intercepts b according to the same rules and confirm with Enter by pressing. The console will display solutions starting with x_1 to x_n.

What is needed for use?

The application does not require installation. It only needs the Windows operating system.

Algorithms used

The algorithm used in the Scilab script is based on the Gaussian elimination method.

The Gaussian elimination method allows us to calculate a system of n equations with n unknowns Ax = b. We transform the matrix of coefficients A into the upper triangular matrix R, from which we then calculate the final solution - i.e. the vector x. The upper triangular matrix R is obtained as follows: intercepts b)

1. In the matrix (A, b) we look for an element ar1 other than zero and we go to the next point. If, however, there is no such element, it means that the matrix is singular and we cannot solve this system.
2. We replace the r-th and the first row.
3. We subtract from the i-th row of the matrix l the multiplicity of the i-th and first rows. This can be represented by the following formulas: (i = 2,3, ..., n j = 2,3, ..., n)

$$a_{ij} = a_{ij} - l_i \cdot a$$

$$b_i = b_i - l_i \cdot b_i$$

$$\text{gdzie } \; l_i = \frac{a_{i1}}{a_{11}}$$

Drawing 3: Formula [1]

4. Then we call this procedure from the first point recursively for the matrix (A ', b') - that is, the matrix (A, b) minus the first column and the first row.

$$(A, b) = \left| \begin{array}{cc} a_{11} & \\ a_{21} & \left| \begin{array}{cccc} a_{12} & \dots & a_{1n} & b_n \\ a_{22} & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots \\ a_{n2} & \dots & a_{nn} & b_n \end{array} \right| \\ \dots & \\ a_{n1} & \end{array} \right|$$

$$(A', b') = \left| \begin{array}{cccc} a_{22} & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots \\ a_{n2} & \dots & a_{nn} & b_n \end{array} \right|$$

Drawing 4: Formula [1]

We can obtain a partial selection of the basic element by selecting the element, and in the first point, | ar1 | = max | ai1 | i = 1,2, ..., n

Full selection of the base element with | ars | = max | aij | i = 1,2, ..., n j = 1,2, ..., n We have to rearrange the r-th and first row and the s-th and first column (changes in columns should be remembered because it causes the replacement of unknowns!).

Once we transform the matrix of coefficients A to the upper triangular matrix R, and vector b to vector c, we can find the final solution from the formula: (i = n, n-1, ..., 1)

$$x_i = \frac{c_i - \sum_{k=i+1}^{n} r_{ik} x_k}{r_{ii}}$$

Drawing 5: Formula [1]

For example, let the matrix A and the vector b be given, from which we create the matrix (A, b)

$$(A, b) = \begin{vmatrix} 3 & 1 & 0 & -3 \\ 1 & 2 & -1 & 0 \\ 0 & -1 & 3 & 0 \end{vmatrix}$$

Drawing 6: Matrix (A,b) [1]

after transformations:

$$\begin{vmatrix} 3 & 1 & 0 & -3 \\ 0 & \frac{5}{3} & -1 & 1 \\ 0 & -1 & 3 & 0 \end{vmatrix}$$

Drawing 7: Matrix A [1]

then we call the procedure recursively for:

$$\begin{vmatrix} \frac{5}{3} & -1 & 1 \\ -1 & 3 & 0 \end{vmatrix}$$

Drawing 7: Example solution [1]

after transformations:

$$\begin{vmatrix} \frac{5}{3} & -1 & 1 \\ 0 & \frac{12}{5} & \frac{3}{5} \end{vmatrix}$$

Drawing 8: Example solution [1]

Since the last matrix can no longer be recursively called (after cutting off the top row and the right column, the vector will remain), so we have obtained the matrix R and vector c we are looking for, from which we determine the final solution

$$(R, c) = \begin{vmatrix} 3 & 1 & 0 & -3 \\ 0 & \frac{5}{3} & -1 & 1 \\ 0 & 0 & \frac{12}{5} & \frac{3}{5} \end{vmatrix}$$

Drawing 9: Matrix (R,c) [1]

x [3] = 0.25 x [2] = 0.75 x [1] = - 1.25 [1]

An example from Mother.txt shows the following solution:

Let us solve the above given Gaussian elimination method the following system of equations:

$$\begin{aligned} 4x_1 - 2x_2 + 4x_3 - 2x_4 &= 8 \\ 3x_1 + 1x_2 + 4x_3 + 2x_4 &= 7 \\ 2x_1 + 4x_2 + 2x_3 + 1x_4 &= 10 \\ 2x_1 - 2x_2 + 4x_3 + 2x_4 &= 2 \end{aligned}$$

Drawing 10: System of equations [2]

We begin the variable elimination stage:

- from equation 2 we subtract equation 1 multiplied by 3/4 = 0.75

- from equation 3 we subtract equation 1 multiplied by 2/4 = 0.5

- from equation 4 we subtract equation 1 multiplied by 2/4 = 0.5.

$$3x_1 + x_2 + 4x_3 + 2x_4 - 0{,}75 \cdot (4x_1 - 2x_2 + 4x_3 - 2x_4) = 7 - 0{,}75 \cdot 8$$
$$2x_1 + 4x_2 + 2x_3 + x_4 - 0{,}5 \cdot (4x_1 - 2x_2 + 4x_3 - 2x_4) = 10 - 0{,}5 \cdot 8$$
$$2x_1 - 2x_2 + 4x_3 + 2x_4 - 0{,}5 \cdot (4x_1 - 2x_2 + 4x_3 - 2x_4) = 2 - 0{,}5 \cdot 8$$

$$4x_1 - 2x_2 + 4x_3 - 2x_4 = 8$$
$$3x_1 + x_2 + 4x_3 + 2x_4 - 3x_1 + 1{,}5x_2 - 3x_3 + 1{,}5x_4 = 1$$
$$2x_1 + 4x_2 + 2x_3 + x_4 - 2x_1 + x_2 - 2x_3 + x_4 = 6$$
$$2x_1 - 2x_2 + 4x_3 + 2x_4 - 2x_1 + x_2 - 2x_3 + x_4 = -2$$

$$4x_1 - 2x_2 + 4x_3 - 2x_4 = 8$$
$$3x_1 - 3x_1 + x_2 + 1{,}5x_2 + 4x_3 - 3x_3 + 2x_4 = 1{,}5x_4 = 1$$
$$2x_1 - 2x_1 + 4x_2 + x_2 + 2x_3 - 2x_3 + x_4 + x_4 = 6$$
$$2x_1 - 2x_1 - 2x_2 + x_2 + 4x_3 - 2x_3 + 2x_4 + x_4 = -2$$

$$4x_1 - 2x_2 + 4x_3 - 2x_4 = 8$$

$$\begin{aligned}
4{,}0x_1 - 2{,}0x_2 + 4{,}0x_3 - 2{,}0x_4 &= 8 \\
2{,}5x_2 + x_3 + 3{,}5x_4 &= 1 \\
5{,}0x_2 + 2{,}0x_4 &= 6 \\
-x_2 + 2{,}0x_3 + 3{,}0x_4 &= -2
\end{aligned}$$

Drawing 11: Variable elimination stage [2]

Note that from Equations 2, 3, and 4, the unknown x 1 has disappeared because the coefficient for it has been reduced to zero. We eliminate the next unknowns:

- from equation 3 we subtract equation 2 multiplied by 5 / 2.5 = 2

- from equation 4 we subtract equation 2 multiplied by -1 / 2.5 = -0.4.

$$
\begin{aligned}
&4x_1 - 2x_2 + 4x_3 - 2x_4 = 8\\
&2{,}5x_2 + x_3 + 3{,}5x_4 = 1\\
&5x_2 + 2x_4 - 2 \cdot (2{,}5x_2 + x_3 + 3{,}5x_4) = 6 - 2\\
&-x_2 + 2x_3 + 3x_4 + 0{,}4 \cdot (2{,}5x_2 + x_3 + 3{,}5x_4) = -2 + 0{,}4
\end{aligned}
$$

$$
\begin{aligned}
&4x_1 - 2x_2 + 4x_3 - 2x_4 = 8\\
&2{,}5x_2 + x_3 + 3{,}5x_4 = 1\\
&5x_2 + 2x_4 - 5x_2 - 2x_3 - 7x_4 = 4\\
&-x_2 + 2x_3 + 3x_4 + x_2 + 0{,}4x_3 + 1{,}4x_4 = -1{,}6
\end{aligned}
$$

$$
\begin{aligned}
&4x_1 - 2x_2 + 4x_3 - 2x_4 = 8\\
&2{,}5x_2 + x_3 + 3{,}5x_4 = 1\\
&5x_2 - 5x_2 - 2x_3 + 2x_4 - 7x_4 = 4\\
&-x_2 + x_2 + 2x_3 + 0{,}4x_3 + 3x_4 + 1{,}4x_4 = -1{,}6
\end{aligned}
$$

$$
\begin{aligned}
4{,}0x_1 - 2{,}0x_2 + 4{,}0x_3 - 2{,}0x_4 &= 8{,}0\\
2{,}5x_2 + 1{,}0x_3 + 3{,}5x_4 &= 1{,}0\\
-2{,}0x_3 - 5{,}0x_4 &= 4{,}0\\
2{,}4x_3 + 4{,}4x_4 &= -1{,}6
\end{aligned}
$$

Drawing 12: Elimination of further unknowns [2]

Final Elimination:

- from equation 4 we subtract equation 3 multiplied by 2.4 / -2 = -1.2 with both sides:

$$
\begin{aligned}
&4x_1 - 2x_2 + 4x_3 - 2x_4 = 8\\
&2{,}5x_2 + x_3 + 3{,}5x_4 = 1\\
&-2x_3 - 5x_4 = 4\\
&2{,}4x_3 + 4{,}4x_4 + 1{,}2 \cdot (-2x_3 - 5x_4) = -1{,}6 + 1{,}2 \cdot 4
\end{aligned}
$$

$$
\begin{aligned}
&4x_1 - 2x_2 + 4x_3 - 2x_4 = 8\\
&2{,}5x_2 + x_3 + 3{,}5x_4 = 1\\
&-2x_3 - 5x_4 = 4\\
&2{,}4x_3 + 4{,}4x_4 - 2{,}4x_3 - 6x_4 = 3{,}2
\end{aligned}
$$

$$
\begin{aligned}
&4x_1 - 2x_2 + 4x_3 - 2x_4 = 8\\
&2{,}5x_2 + x_3 + 3{,}5x_4 = 1\\
&-2x_3 - 5x_4 = 4\\
&2{,}4x_3 - 2{,}4x_3 + 4{,}4x_4 - 6x_4 = 3{,}2
\end{aligned}
$$

$$
\begin{array}{rrrrr}
4{,}0x_1 & -2{,}0x_2 + & 4{,}0x_3 - & 2{,}0x_4 & = 8{,}0\\
 & 2{,}5x_2 + & 1{,}0x_3 + & 3{,}5x_4 & = 1{,}0\\
 & & -2{,}0x_3 - & 5{,}0x_4 & = 4{,}0\\
 & & & -1{,}6x_4 & = 3{,}2
\end{array}
$$

Drawing 13: Final elimination [2]

We start the reverse procedure. Going from the end, we set the next unknowns:

$$x_4 = \frac{3{,}2}{-1{,}6} = \mathbf{-2}$$

Drawing 14: Determining the unknowns [2]

Given x 4 we can calculate x 3 from equation 3:

$$x_3 = \frac{4 + 5x_4}{-2} = \frac{4 + 5 \cdot (-2)}{-2} = \frac{4 - 10}{-2} = \frac{-6}{-2} = \mathbf{3}$$

Drawing 15: Determining the unknowns [2]

We calculate x 2 from equation 2:

$$x_2 = \frac{1 - 3{,}5x_4 - x_3}{2{,}5} = \frac{1 - 3{,}5 \cdot (-2) - 3}{2{,}5} = \frac{1 + 7 - 3}{2{,}5} = \frac{5}{2{,}5} = 2$$

Drawing 16: Determining the unknowns [2]

And the last unknown x 1 we get from equation 1:

$$x_1 = \frac{8 + 2x_4 - 4x_3 + 2x_2}{4} = \frac{8 + 2 \cdot (-2) - 4 \cdot 3 + 2 \cdot 2}{4} = \frac{8 - 4 - 12 + 4}{4} = \frac{-4}{4} = -1$$

Drawing 17: Determining the unknowns [2]

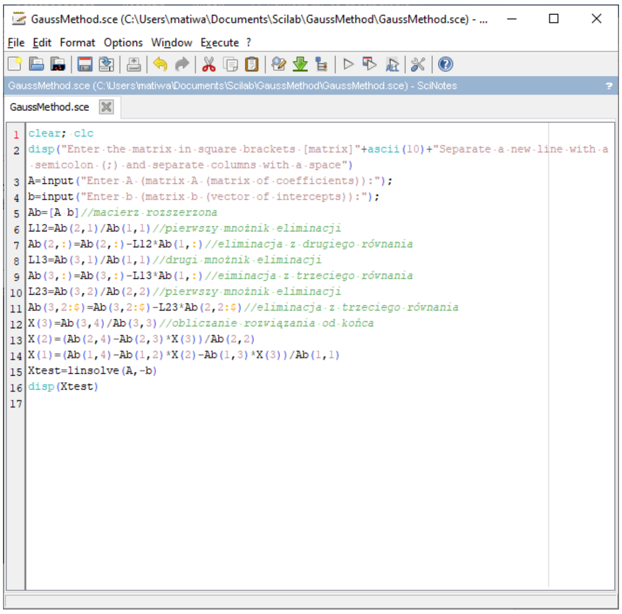
and finally we can save the solution: [2]

$$\begin{aligned} x_1 &= -1 \\ x_2 &= 2 \\ x_3 &= 3 \\ x_4 &= -2 \end{aligned}$$

Drawing 18: Solution [2]

Interface description

Running the code requires Scilab.



Drawing 19: Scilab graphical interface [own study]

Source code description

The project was made in the C++ programming language, in the Dev-C++ programming environment. All work was done on the Windows 10 operating system. The application's source code looks like this.

```
clear; clc
disp("Enter the matrix in square brackets [matrix]"+ascii(10)+"Separate a new line with a semicolon (;) and separate columns with a space")
A=input("Enter A (matrix A (matrix of coefficients)):");
b=input("Enter b (matrix b (vector of intercepts)):");
Ab=[A b]//macierz rozszerzona
L12=Ab(2,1)/Ab(1,1)//pierwszy mnożnik eliminacji
Ab(2,:)=Ab(2,:)-L12*Ab(1,:)//eliminacja z drugiego równania
L13=Ab(3,1)/Ab(1,1)//drugi mnożnik eliminacji
Ab(3,:)=Ab(3,:)-L13*Ab(1,:)//eiminacja z trzeciego równania
L23=Ab(3,2)/Ab(2,2)//pierwszy mnożnik eliminacji
Ab(3,2:$)=Ab(3,2:$)-L23*Ab(2,2:$)//eliminacja z trzeciego równania
X(3)=Ab(3,4)/Ab(3,3)//obliczanie rozwiązania od końca
X(2)=(Ab(2,4)-Ab(2,3)*X(3))/Ab(2,2)
X(1)=(Ab(1,4)-Ab(1,2)*X(2)-Ab(1,3)*X(3))/Ab(1,1)
Xtest=linsolve(A,-b)
disp(Xtest)
```

Listing 1: Source code [own study]

List of drawings

List of listings

Bibliography

[1] http://www.algorytm.org/procedury-numeryczne/metoda-eliminacji-gaussa.html
[2] https://eduinf.waw.pl/inf/alg/001_search/0076.php