# Modyfikacje/hybrydyzacje algorytmu PSO w zadaniu optymalizacji globalnej wielowymiarowej funkcji ciągłej

## **PSO-DE Hybrid**

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#### **ABSTRACT**

Dokumentacja uzyskanych wyników hybrydy PSO-DE

### **Categories and Subject Descriptors**

G.1.6 [Numerical Analysis]: Optimization—global optimization, unconstrained optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

#### **General Terms**

Algorithms

#### **Keywords**

Benchmarking, PSODE, Optymalizacja wielowymiarowej funkcji ciaglej

#### 1. CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the PSO-DE Hybrid on the function  $f_8$  with restarts for at least 30 seconds and until a maximum budget equal to 400(D+2) is reached. The code was run on a Windows 8 Intel(R) Core(TM) i7-4500U CPU @ 2.39GHz with 1 processor and 2 cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20 equals  $1, 9e^{-10}$ ,  $2, 2e^{-10}$ ,  $2, 4e^{-10}$ ,  $3, 5e^{-10}$  and  $6, 1e^{-10}$  seconds respectively.

#### 2. RESULTS

Results of PSO-DE nowe from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2, 3, and 4 and in Tables 1.

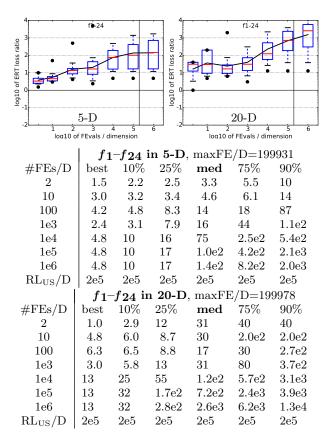


Figure 3: ERT loss ratio versus the budget in number of f-evaluations divided by dimension. For each given budget FEvals, the target value  $f_t$  is computed as the best target f-value reached within the budget by the given algorithm. Shown is then the ERT to reach  $f_t$  for the given algorithm or the budget, if the GECCO-BBOB-2009 best algorithm reached a better target within the budget, divided by the best ERT seen in GECCO-BBOB-2009 to reach  $f_t$ . Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in a single trial in this function subset. See also Figure 4 for results on each function subgroup.

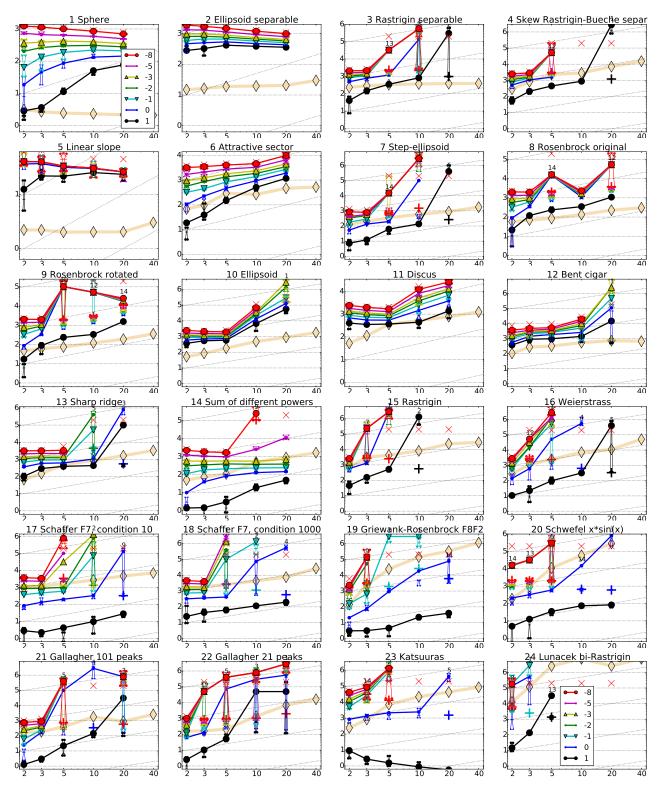


Figure 1: Expected number of f-evaluations (ERT, lines) to reach  $f_{\rm opt} + \Delta f$ ; median number of f-evaluations (+) to reach the most difficult target that was reached not always but at least once; maximum number of f-evaluations in any trial (×); interquartile range with median (notched boxes) of simulated runlengths to reach  $f_{\rm opt} + \Delta f$ ; all values are divided by dimension and plotted as  $\log_{10}$  values versus dimension. Shown are  $\Delta f = 10^{\{1,0,-1,-2,-3,-5,-8\}}$ . Numbers above ERT-symbols (if appearing) indicate the number of trials reaching the respective target. The light thick line with diamonds indicates the respective best result from BBOB-2009 for  $\Delta f = 10^{-8}$ . Horizontal lines mean linear scaling, slanted grid lines depict quadratic scaling.

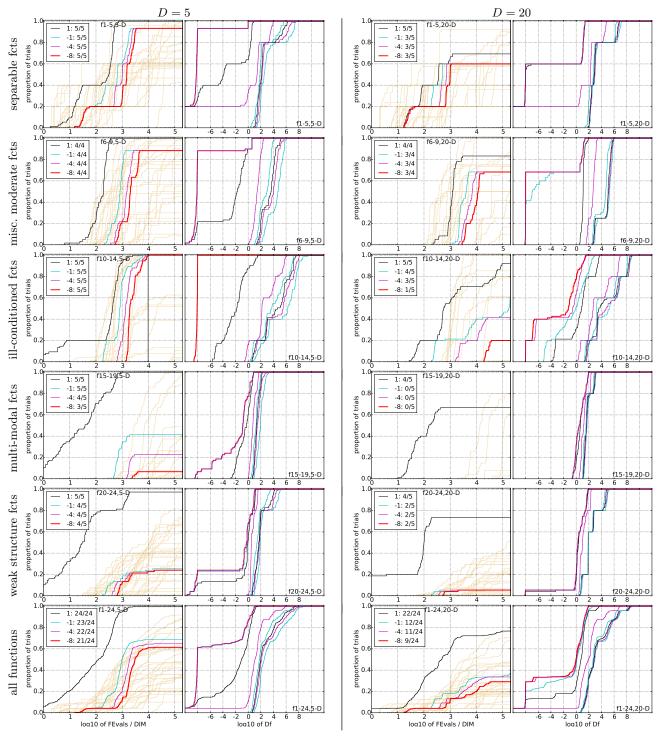


Figure 2: Empirical cumulative distribution functions (ECDF), plotting the fraction of trials with an outcome not larger than the respective value on the x-axis. Left subplots: ECDF of the number of function evaluations (FEvals) divided by search space dimension D, to fall below  $f_{\rm opt} + \Delta f$  with  $\Delta f = 10^k$ , where k is the first value in the legend. The thick red line represents the most difficult target value  $f_{\rm opt} + 10^{-8}$ . Legends indicate for each target the number of functions that were solved in at least one trial within the displayed budget. Right subplots: ECDF of the best achieved  $\Delta f$  for running times of  $0.5D, 1.2D, 3D, 10D, 100D, 1000D, \ldots$  function evaluations (from right to left cycling cyan-magenta-black...) and final  $\Delta f$ -value (red), where  $\Delta f$  and Df denote the difference to the optimal function value. Light brown lines in the background show ECDFs for the most difficult target of all algorithms benchmarked during BBOB-2009.

|                              |                 |                  |                   | 5-D              |                  |                  |                         |                             |                     |                    | 4                 | 20-D                |                           |                  |                         |               |
|------------------------------|-----------------|------------------|-------------------|------------------|------------------|------------------|-------------------------|-----------------------------|---------------------|--------------------|-------------------|---------------------|---------------------------|------------------|-------------------------|---------------|
| $\Delta f$                   | 1e+1            | 1e+0             | 1e-1              | 1e-2             | 1e-3             | 1e-5             | 1e-7                    | #s <b>∆</b> gd              | 1e+1                | 1e+0               | 1e-1              | 1e-2                | 1e-3                      | 1e-5             | 1e-7                    | #succ         |
| f <sub>1</sub>               | 11              | 12               | 12                | 12               | 12               | 12               | 12                      | 15/fl <sub>1</sub> 5        | 43                  | 43                 | 43                | 43                  | 43                        | 43               | 43                      | 15/15         |
|                              | 5.1(5)          | 35(21)           | 77(17)            | 122(3)           | 171(13)          | 261(24)          | 363(13)                 | 15/15                       | 36(4)               | 67(5)              | 97(8)             | 129(11)             | 163(9)                    | 230(10)          | 295(23)                 | 15/15         |
| $\mathbf{f_2}$               | 83              | 87               | 88                | 89               | 90               | 92               | 94                      | 15/fb5                      | 385                 | 386                | 387               | 388                 | 390                       | 391              | 393                     | 15/15         |
|                              | 25(3)           | 31(2)            | 37(2)             | 43(2)            | 49(3)            | 62(3)            | 72(3)                   | 15/15                       | 19(0.7)             | 22(1)              | 26(1)             | 30(1)               | 33(1)                     | 40(2)            | 47(2)                   | 15/15         |
| $f_3$                        | 716             | 1622             | 1637              | 1642             | 1646             | 1650             | 1654                    | 15/ <b>f5</b> 5             | 5066                | 7626               | 7635              | 7637                | 7643                      | 7646             | 7651                    | 15/15         |
|                              | 2.4(0.9         |                  | 98(305)           | 98(1.0)          | 98(303)          | 99(604)          | 99(302)                 | 13/15                       | 1186(1577)          | ∞                  | ∞                 | ∞                   | ∞                         |                  | $\infty 4.0e6$          | 0/15          |
| $f_4$                        | 809<br>2.8(1)   | 1633<br>4.8(1)   | 1688<br>153(149)  | 1758<br>147(425) | 1817<br>143(274) | 1886<br>139(133) | 1903<br>138(262)        | 15/fl <sub>4</sub> 5        | 4722<br>11847(18397 | 7628<br>) ∞        | 7666<br>∞         | 7686<br>∞           | 7700<br>∞                 | 7758<br>∞        | 1.4e5<br>$\infty 4.0e6$ | 9/15<br>0/15  |
| f <sub>5</sub>               | 10              | 10               | 10                | 10               | 10               | 10               | 10                      | 15/fk5                      | 41                  | 41                 | 41                | 41                  | 41                        | 41               | 41                      | 15/15         |
| -5                           | 11(2)           | 16(3)            | 17(7)             | 17(4)            | 17(8)            | 17(9)            | 17(4)                   | 15/15                       | 11(3)               | 13(5)              | 13(2)             | 13(3)               | 13(3)                     | 13(5)            | 13(4)                   | 15/15         |
| f <sub>6</sub>               | 114             | 214              | 281               | 404              | 580              | 1038             | 1332                    | 15/ <b>f</b> <sub>6</sub> 5 | 1296                | 2343               | 3413              | 4255                | 5220                      | 6728             | 8409                    | 15/15         |
|                              | 6.5(2)          | 11(4)            | 15(6)             | 17(5)            | 15(3)            | 13(2)            | 13(1)                   | 15/15                       | 18(12)              | 16(5)              | 16(8)             | 16(6)               | 17(8)                     | 20(10)           | 21(9)                   | 15/15         |
| f <sub>7</sub>               | 24              | 324              | 1171              | 1451             | 1572             | 1572             | 1597                    | 15/fl/5                     | 1351                | 4274               | 9503              | 16523               | 16524                     | 16524            | 16969                   | 15/15         |
|                              | 13(8)           | 3.1(0.5)         | 62(214)           | 51(173)          | 47(159)          | 47(161)          | 47(0.4)                 | 14/15                       | 5925(4441)          | $\infty$           | $\infty$          | $\infty$            | $\infty$                  | $\infty$         | $\infty$ 4.0e6          | 0/15          |
| $f_8$                        | 73              | 273              | 336               | 372              | 391              | 410              | 422                     | 15/fb5                      | 2039                | 3871               | 4040              | 4148                | 4219                      | 4371             | 4484                    | 15/15         |
|                              | 16(4)           | 273(4)           | 226(5)            | 206(675)         | 199(3)           | 193(2)           | 191(595)                | 14/15                       | 11(2)               | 267(2)             | 257(741)          | 252(242)            | 249(711)                  | 242(458)         |                         |               |
| $f_9$                        | 35              | 127              | 214               | 263              | 300              | 335              | 369                     | 15/fb5                      | 1716                | 3102               | 3277              | 3379                | 3455                      | 3594             | 3727                    | 15/15         |
| -                            | 34(8)           |                  | 2356(7004)<br>574 |                  |                  |                  |                         | 10/15                       | 18(7)<br>7413       | 113(3)             | 112(5)            | 114(297)            | 118(6)                    | 126(6)           | 131(5)                  | 14/15         |
| f <sub>10</sub>              | 349<br>8.3(2)   | $500 \\ 7.4(2)$  | 7.8(0.6)          | 607<br>8.7(2)    | 626<br>10(2)     | 829<br>9.0(2)    | 880<br>10(2)            | 15 <b>/16</b><br>15/15      | 7413<br>151(56)     | 8661<br>294(208)   | 10735<br>537(343) | 13641<br>2053(2476) | 14920<br>3834(3040)       | $17073$ $\infty$ | 17476<br>$\infty 3.9e6$ | 15/15<br>0/15 |
| f <sub>11</sub>              | 143             | 202              | 7.8(0.0)          | 977              | 1177             | 1467             | 1673                    | 15#15                       | 1002                | 2228               | 6278              | 8586                | 9762                      | 12285            | 14831                   | 15/15         |
| 111                          | 13(3)           | 13(3)            | 4.6(0.7)          | 4.3(0.5)         | 4.2(0.5)         | 4.4(0.5)         |                         |                             | 28(13)              | 35(18)             | 22(15)            | 24(8)               | 27(7)                     | 30(9)            | 33(7)                   | 15/15         |
| $\overline{\mathbf{f_{12}}}$ |                 | 268              | 371               | 413              | 461              | 1303             | 1494                    | 15/15                       | 1042                | 1938               | 2740              | 3156                | 4140                      | 12407            | 13827                   | 15/15         |
| -12                          | 46(7)           | 35(37)           | 36(32)            | 38(32)           | 38(27)           | 17(7)            | 17(9)                   | 15/15                       |                     |                    |                   |                     | 12509(20751)              |                  | $\infty 3.7e6$          | 0/15          |
| $\overline{\mathbf{f_{13}}}$ |                 | 195              | 250               | 319              | 1310             | 1752             | 2255                    | 15 <b>f15</b>               | 652                 | 2021               | 2751              | 3507                | 18749                     | 24455            | 30201                   | 15/15         |
| 13                           | 15(2)           | 16(3)            | 19(5)             | 19(3)            | 5.8(0.9)         | 6.2(1)           | 6.4(0.8                 | 15/15                       | 3074(7656)          | 7904(13823)        |                   | $\infty$            | $\infty$                  | $\infty$         | $\infty 4.0e6$          | 0/15          |
| f <sub>1.4</sub>             | 10              | 41               | 58                | 90               | 139              | 251              | 476                     | 15 <b>f</b> -1.5            | 75                  | 239                | 304               | 451                 | 932                       | 1648             | 15661                   | 15/15         |
|                              | 1.6(2)          | 10(2)            | 18(1)             | 22(2)            | 21(2)            | 19(2)            | 15(2)                   | 15/15                       | 13(4)               | 12(2)              | 16(1)             | 17(2)               | 15(1)                     | 136(55)          | $\infty 4.0e6$          | 0/15          |
| f <sub>15</sub>              |                 | 9310             | 19369             | 19743            | 20073            | 20769            | 21359                   | 14715                       | 30378               | 1.5e5              | 3.1e5             | 3.2e5               | 3.2e5                     | 4.5e5            | 4.6e5                   | 15/15         |
|                              | 5.1(2)          | 1502(1260)       | 722(812)          | 708(1024)        | 697(1330)        | 673(1130)        |                         | 1/15                        | $\infty$            | $\infty$           | $\infty$          | $\infty$            | $\infty$                  | $\infty$         | $\infty$ 4.0e6          | 0/15          |
| f <sub>16</sub>              |                 | 612              | 2662              | 10163            | 10449            | 11644            | 12095                   | 15 <b>f16</b>               | 1384                | 27265              | 77015             | 1.4e5               | 1.9e5                     | 2.0e5            | 2.2e5                   | 15/15         |
| -                            | 4.1(4)          | 418(826)         | 1033(1032)        | 639(688)         | 622(741)         |                  | 1157(1012)              | 1/15                        | 5776(5772)          | ∞                  | ∞                 | ∞                   | ∞                         | ∞                | ∞4.0e6                  | 0/15          |
| f <sub>17</sub>              | 5.2<br>4.0(2)   | 215<br>4.7(1)    | 899<br>3.3(0.7)   | 2861<br>1.8(0.4) | 3669<br>44(136)  | 6351<br>80(118)  | 7934<br>254(377)        | 15 <b>715</b><br>3/15       | 63<br>8.8(3)        | 1030<br>2549(3864) | 4005<br>∞         | 12242<br>∞          | 30677<br>∞                | $56288$ $\infty$ | 80472<br>$\infty 4.0e6$ | 15/15<br>0/15 |
| f <sub>18</sub>              | 103             | 378              | 3968              | 8451             | 9280             | 10905            | 12469                   | 15#15                       | 621                 | 3972               | 19561             | 28555               | 67569                     | 1.3e5            | 1.5e5                   | 15/15         |
| 118                          | 3.0(2)          | 5.5(1)           | 127(63)           | 237(266)         |                  | 1284(1191)       |                         | 0/15                        | 6.4(2)              | 2748(3272)         | ∞                 | ∞                   | ∞                         | ∞                | $\infty 4.0e6$          | 0/15          |
| f <sub>19</sub>              | 1               | 1                | 242               | 1.0e5            | 1.2e5            | 1.2e5            | 1.2e5                   | 15 <b>f 15</b>              | 1                   | 1                  | 3.4e5             | 4.7e6               | 6.2e6                     | 6.7e6            | 6.7e6                   | 15/15         |
| 10                           | 23(33)          | 4629(2925)       | 57595(47288)      | ) ∞              | $\infty$         | $\infty$         | $\infty 1.0e6$          | 0/15                        | 798(332)            | 1.5e6(3e6)         | $\infty$          | $\infty$            | $\infty$                  | $\infty$         | $\infty 3.8e6$          | 0/15          |
| f <sub>20</sub>              | 16              | 851              | 38111             | 51362            | 54470            | 54861            | 55313                   | 14726                       | 82                  | 46150              | 3.1e6             | 5.5e6               | 5.5e6                     | 5.6e6            | 5.6e6                   | 14/15         |
|                              | 11(6)           | 3.5(2)           | 39(26)            | 29(39)           | 28(37)           | 27(32)           | 27(63)                  | 6/15                        | 20(8)               | 347(390)           | $\infty$          | $\infty$            | $\infty$                  | $\infty$         | $\infty$ 4.0e6          | 0/15          |
| $f_{21}$                     | 41              | 1157             | 1674              | 1692             | 1705             | 1729             | 1757                    | 14725                       | 561                 | 6541               | 14103             | 14318               | 14643                     | 15567            | 17589                   | 15/15         |
|                              | 2.7(4)          | 433(864)         | 1194(2087)        |                  |                  |                  | 1139(1705)              | 5/15                        |                     | 2444(5803)         |                   |                     | 1092(546)                 | 1027(2310        |                         |               |
| f22                          |                 | 386              | 938               | 980              | 1008             | 1040             | 1068                    | 14722                       | 467                 | 5580               | 23491             | 24163               | 24948                     | 26847            | 1.3e5                   | 12/15         |
| -                            | 3.7(3)          |                  | 2131(3726)        |                  | /                |                  |                         | 5/15                        |                     | 1970(2327)         |                   |                     | 2243(2402)                |                  |                         |               |
| $f_{23}$                     |                 | 518              | 14249             | 27890            | 31654            | 33030            | 34256                   | 15#25                       | 3.2                 | 1614               | 67457             | 3.7e5               | 4.9e5                     | 8.1e5            | 8.4e5                   | 15/15         |
| -                            | 2.6(2)          | 21(17)           | 283(281)          | 235(269)         | 207(268)         | 198(280)         | 191(175)                | 2/15                        | 3.5(2)              | 5001(7431)         | ∞                 | ∞ = 0-7             | ∞                         | ∞<br>5.2e7       | $\infty 4.0e6$ $5.2e7$  | 0/15          |
| $f_{24}$                     | 1622<br>99(155) | $2.2e5$ $\infty$ | $6.4e6$ $\infty$  | 9.6e6<br>∞       | 9.6e6<br>∞       | 1.3e7<br>∞       | 1.3e7<br>$\infty 1.0e6$ | 3 <b>#24</b><br>0/15        | 1.3e6<br>∞          | 7.5e6<br>∞         | 5.2e7<br>∞        | 5.2e7<br>∞          | $5.2\mathrm{e}7$ $\infty$ | 5.2e7<br>∞       | $5.2e7$ $\infty 4.0e6$  | 0/15          |
|                              | 99(100)         | 30               | 30                |                  | ×                |                  | ∞1.0e0                  | 0/13                        | 30                  | 00                 | 30                | 30                  | 000                       | 000              | ∞4.0e0                  | 1 0/13        |

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009. The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear in the second row of each cell, the best ERT in the first. The different target  $\Delta f$ -values are shown in the top row. #succ is the number of trials that reached the (final) target  $f_{\rm opt} + 10^{-8}$ . The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Bold entries are statistically significantly better (according to the rank-sum test) compared to the best algorithm in BBOB-2009, with p = 0.05 or  $p = 10^{-k}$  when the number k > 1 is following the  $\downarrow$  symbol, with Bonferroni correction by the number of functions.

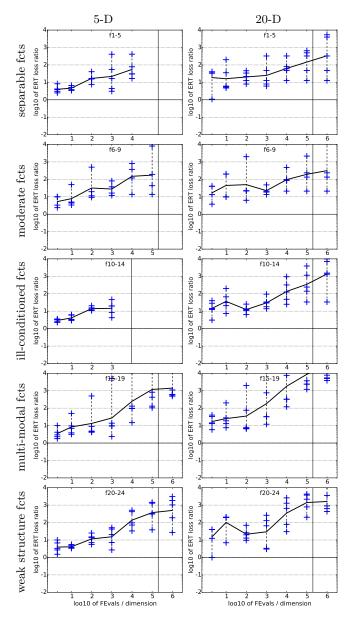


Figure 4: ERT loss ratios (see Figure 3 for details). Each cross (+) represents a single function, the line is the geometric mean.