

# Modyfikacje/hybrydyzacje algorytmu PSO w zadaniu optymalizacji globalnej wielowymiarowej funkcji ciaglej

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## ABSTRACT

Dokumentacja uzyskanych wyników hybrydy PSO-DE

## Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

## General Terms

Algorithms

## Keywords

Benchmarking, PSODE, Optymalizacja wielowymiarowej funkcji ciaglej

## 1. CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the **PSO-DE Hybrid** on the function  $f_8$  with restarts for at least 30 seconds and until a maximum budget equal to  $400(D + 2)$  is reached. The code was run on a **Mac Intel(R) Core(TM) i5-2400S CPU @ 2.50GHz** with 1 processor and 4 cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20, 40 equals  $8,3e^{-11}$ ,  $1,0e^{-10}$ ,  $1,4e^{-10}$ ,  $2,2e^{-10}$ ,  $3,8e^{-10}$ , and  $7,2e^{-10}$  seconds respectively.

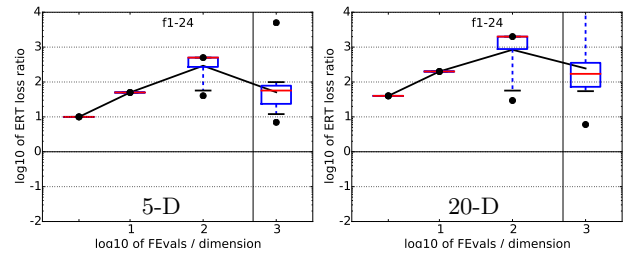
## 2. RESULTS

Results of PSO DE from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2, 3, and 4 and in Tables 1.

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$f_1-f_{24}$ in 5-D, maxFE/D=482						
#FEs/D	best	10%	25%	med	75%	90%
2	10	10	10	10	10	10
10	50	50	50	50	50	50
100	40	50	2.6e2	5.0e2	5.0e2	5.0e2
1e3	7.0	12	22	57	79	1.3e2
RLUS/D	4e2	4e2	4e2	4e2	4e2	5e2
$f_1-f_{24}$ in 20-D, maxFE/D=491						
#FEs/D	best	10%	25%	med	75%	90%
2	40	40	40	40	40	40
10	2.0e2	2.0e2	2.0e2	2.0e2	2.0e2	2.0e2
100	29	49	6.5e2	2.0e3	2.0e3	2.0e3
1e3	6.0	50	71	1.7e2	3.9e2	2.0e4
RLUS/D	4e2	4e2	4e2	4e2	4e2	5e2

**Figure 3:** ERT loss ratio versus the budget in number of  $f$ -evaluations divided by dimension. For each given budget FEvals, the target value  $f_t$  is computed as the best target  $f$ -value reached within the budget by the given algorithm. Shown is then the ERT to reach  $f_t$  for the given algorithm or the budget, if the GECCO-BBOB-2009 best algorithm reached a better target within the budget, divided by the best ERT seen in GECCO-BBOB-2009 to reach  $f_t$ . Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in a single trial in this function subset. See also Figure 4 for results on each function subgroup.

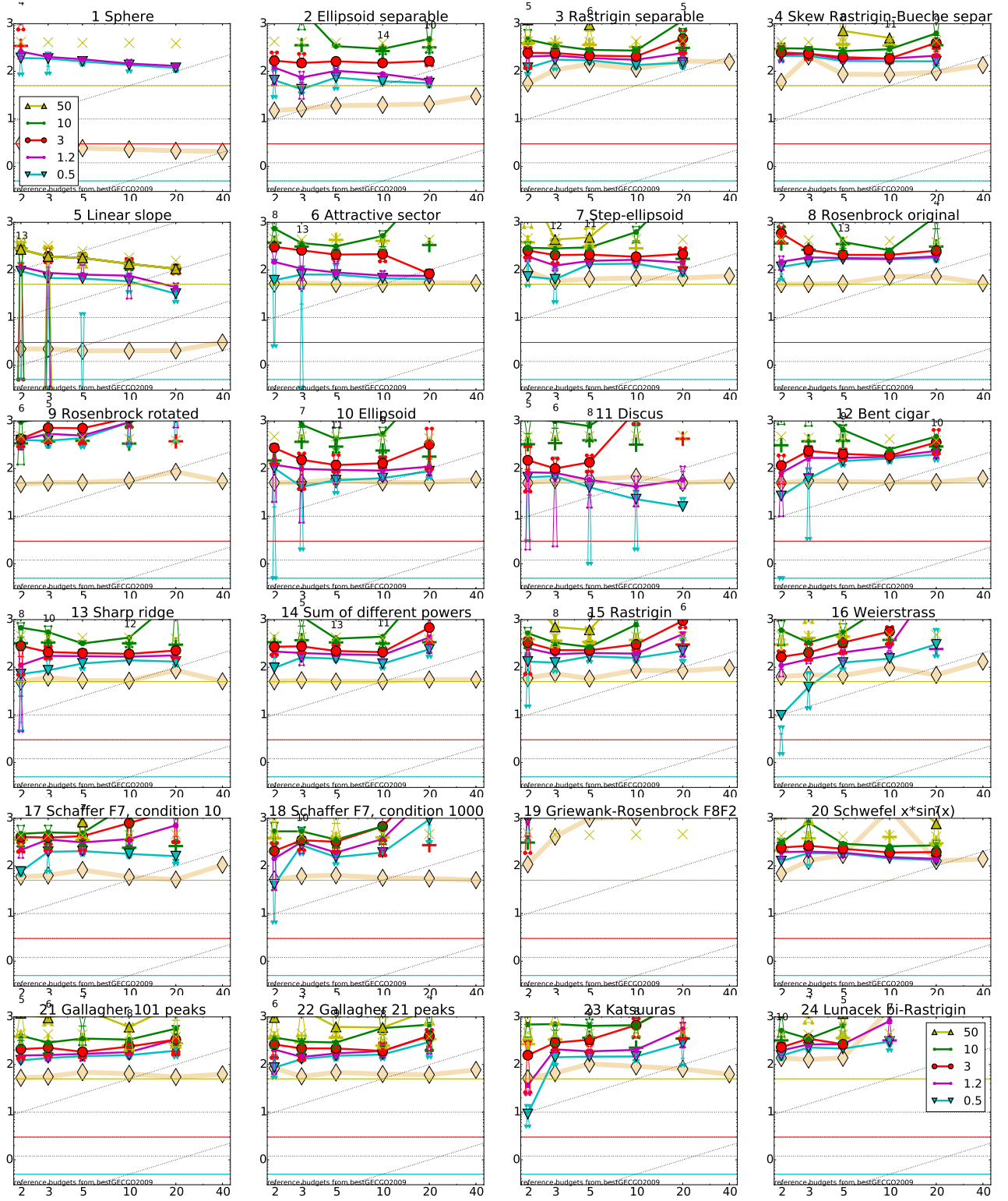
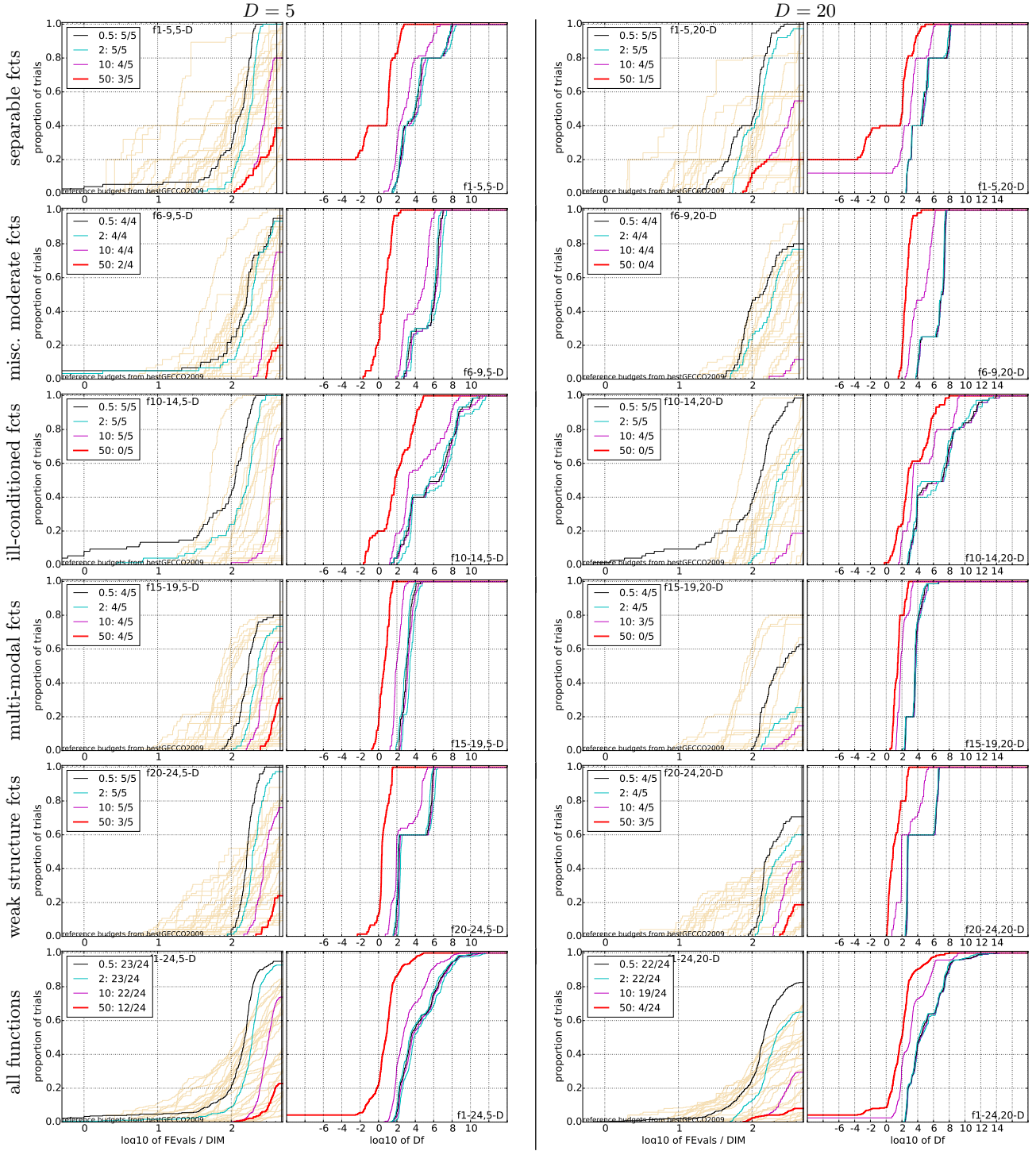


Figure 1: Expected number of  $f$ -evaluations (ERT, lines) to reach  $f_{\text{opt}} + \Delta f$ ; median number of  $f$ -evaluations (+) to reach the most difficult target that was reached not always but at least once; maximum number of  $f$ -evaluations in any trial ( $\times$ ); interquartile range with median (notched boxes) of simulated runlengths to reach  $f_{\text{opt}} + \Delta f$ ; all values are divided by dimension and plotted as  $\log_{10}$  values versus dimension. Shown is the ERT for targets just not reached by the artificial GECCO-BBOB-2009 best algorithm within the given budget  $k \times \text{DIM}$ , where  $k$  is shown in the legend. Numbers above ERT-symbols indicate the number of trials reaching the respective target. The light thick line with diamonds indicates the respective best result from BBOB-2009 for the most difficult target. Slanted grid lines indicate a scaling with  $\mathcal{O}(\text{DIM})$  compared to  $\mathcal{O}(1)$  when using the respective 2009 best algorithm.



**Figure 2: Empirical cumulative distribution functions (ECDF), plotting the fraction of trials with an outcome not larger than the respective value on the  $x$ -axis. Left subplots: ECDF of number of function evaluations (FEvals) divided by search space dimension  $D$ , to fall below  $f_{\text{opt}} + \Delta f$  where  $\Delta f$  is the target just not reached by the GECCO-BBOB-2009 best algorithm within a budget of  $k \times \text{DIM}$  evaluations, where  $k$  is the first value in the legend. Legends indicate for each target the number of functions that were solved in at least one trial within the displayed budget. Right subplots: ECDF of the best achieved  $\Delta f$  for running times of  $0.5D, 1.2D, 3D, 10D, 100D, 1000D, \dots$  function evaluations (from right to left cycling cyan-magenta-black...) and final  $\Delta f$ -value (red), where  $\Delta f$  and  $Df$  denote the difference to the optimal function value. Light brown lines in the background show ECDFs for the most difficult target of all algorithms benchmarked during BBOB-2009.**

5-D							20-D						
#FEs/D	0.5	1.2	3	10	50	#succ	#FEs/D	0.5	1.2	3	10	50	#succ
<b>f<sub>1</sub></b>	<i>2.5e+1:4.8</i>	<i>1.6e+1:7.6</i>	<i>1.0e-8:12</i>	<i>1.0e-8:12</i>	<i>1.0e-8:12</i>	15/15	<b>f<sub>1</sub></b>	<i>6.3e+1:24</i>	<i>4.0e+1:42</i>	<i>1.0e-8:43</i>	<i>1.0e-8:43</i>	<i>1.0e-8:43</i>	15/15
	168(50)	118(26)	∞	∞	∞1900	0/15		97(15)	62(9)	∞	∞	∞7700	0/15
<b>f<sub>2</sub></b>	<i>1.6e+6:2.9</i>	<i>4.0e+5:11</i>	<i>4.0e+4:15</i>	<i>6.3e+2:58</i>	<i>1.0e-8:95</i>	15/15	<b>f<sub>2</sub></b>	<i>4.0e+6:29</i>	<i>2.5e+6:42</i>	<i>1.0e+5:65</i>	<i>1.0e+4:207</i>	<i>1.0e-8:412</i>	15/15
	129(60)	46(22)	53(9)	29(3)	∞2000	0/15		40(6)	31(5)	51(23)	46(34)	∞7600	0/15
<b>f<sub>3</sub></b>	<i>1.6e+2:4.1</i>	<i>1.0e+2:15</i>	<i>6.3e+1:23</i>	<i>2.5e+1:73</i>	<i>1.0e+1:716</i>	15/15	<b>f<sub>3</sub></b>	<i>6.3e+2:33</i>	<i>4.0e+2:44</i>	<i>1.6e+2:109</i>	<i>1.0e+2:255</i>	<i>2.5e+1:3277</i>	15/15
	205(46)	65(12)	46(6)	19(3)	6.6(6)	6/15		92(17)	110(200)	89(80)	91(78)	∞8500	0/15
<b>f<sub>4</sub></b>	<i>2.5e+2:2.6</i>	<i>1.6e+2:10</i>	<i>1.0e+2:19</i>	<i>4.0e+1:65</i>	<i>1.6e+1:434</i>	15/15	<b>f<sub>4</sub></b>	<i>6.3e+2:22</i>	<i>4.0e+2:91</i>	<i>2.5e+2:250</i>	<i>1.6e+2:332</i>	<i>6.3e+1:1927</i>	15/15
	316(46)	93(24)	53(10)	20(4)	8.2(4)	8/15		150(66)	47(13)	32(19)	38(14)	∞8300	0/15
<b>f<sub>5</sub></b>	<i>6.3e+1:4.0</i>	<i>4.0e+1:10</i>	<i>1.0e-8:10</i>	<i>1.0e-8:10</i>	<i>1.0e-8:10</i>	15/15	<b>f<sub>5</sub></b>	<i>2.5e+2:19</i>	<i>1.6e+2:34</i>	<i>1.0e-8:41</i>	<i>1.0e-8:41</i>	<i>1.0e-8:41</i>	15/15
	82(75)	40(16)	90(29)	90(30)	90(31)	15/15		33(9)	25(7)	52(17)	52(13)	52(9)	15/15
<b>f<sub>6</sub></b>	<i>1.0e+5:3.0</i>	<i>2.5e+4:8.4</i>	<i>1.0e+2:16</i>	<i>2.5e+1:54</i>	<i>2.5e-1:254</i>	15/15	<b>f<sub>6</sub></b>	<i>2.5e+5:16</i>	<i>6.3e+4:43</i>	<i>1.6e+4:62</i>	<i>1.6e+2:353</i>	<i>1.6e+1:1078</i>	15/15
	133(143)	53(53)	66(25)	29(14)	123(192)	1/15		81(15)	35(11)	27(5)	349(265)	∞8400	0/15
<b>f<sub>7</sub></b>	<i>1.6e+2:4.2</i>	<i>1.0e+2:6.2</i>	<i>2.5e+1:20</i>	<i>4.0e+0:54</i>	<i>1.0e+0:324</i>	15/15	<b>f<sub>7</sub></b>	<i>1.0e+3:11</i>	<i>4.0e+2:39</i>	<i>2.5e+2:74</i>	<i>6.3e+1:319</i>	<i>1.0e+1:1351</i>	15/15
	157(61)	125(39)	52(9)	27(4)	7.4(7)	11/15		174(50)	73(22)	59(52)	377(693)	∞8400	0/15
<b>f<sub>8</sub></b>	<i>1.0e+4:4.6</i>	<i>6.3e+3:6.8</i>	<i>1.0e+1:18</i>	<i>6.3e+1:54</i>	<i>1.6e+0:258</i>	15/15	<b>f<sub>8</sub></b>	<i>4.0e+4:19</i>	<i>2.5e+4:35</i>	<i>4.0e+3:67</i>	<i>2.5e+2:231</i>	<i>1.6e+1:1470</i>	15/15
	183(28)	131(19)	58(12)	36(15)	∞2000	0/15		191(27)	110(26)	75(19)	117(88)	∞7600	0/15
<b>f<sub>9</sub></b>	<i>2.5e+1:20</i>	<i>1.6e+1:26</i>	<i>1.0e+1:35</i>	<i>4.0e+0:62</i>	<i>1.6e-2:256</i>	15/15	<b>f<sub>9</sub></b>	<i>1.0e+2:357</i>	<i>6.3e+1:560</i>	<i>4.0e+1:684</i>	<i>2.5e+1:756</i>	<i>1.0e+1:1716</i>	15/15
	111(54)	96(49)	102(65)	166(158)	∞2000	0/15		104(113)	101(76)	169(166)	∞	∞7700	0/15
<b>f<sub>10</sub></b>	<i>2.5e+6:2.9</i>	<i>6.3e+5:7.0</i>	<i>2.5e+5:17</i>	<i>6.3e+3:54</i>	<i>2.5e+1:297</i>	15/15	<b>f<sub>10</sub></b>	<i>1.6e+6:15</i>	<i>1.0e+6:27</i>	<i>4.0e+5:70</i>	<i>6.3e+1:231</i>	<i>4.0e+3:1015</i>	15/15
	99(97)	67(66)	35(28)	39(53)	∞2200	0/15		117(24)	80(15)	92(100)	517(785)	∞8300	0/15
<b>f<sub>11</sub></b>	<i>1.0e+6:3.0</i>	<i>6.3e+4:6.2</i>	<i>6.3e+2:16</i>	<i>6.3e+1:74</i>	<i>6.3e-1:298</i>	15/15	<b>f<sub>11</sub></b>	<i>4.0e+4:11</i>	<i>2.5e+3:27</i>	<i>1.6e+2:313</i>	<i>1.0e+2:481</i>	<i>1.0e+1:1002</i>	15/15
	68(90)	47(43)	42(30)	53(31)	∞2300	0/15		28(23)	43(28)	461(466)	∞	∞9700	0/15
<b>f<sub>12</sub></b>	<i>4.0e+7:3.6</i>	<i>1.6e+7:7.6</i>	<i>1.0e+6:19</i>	<i>1.6e+4:52</i>	<i>1.0e+0:268</i>	15/15	<b>f<sub>12</sub></b>	<i>1.0e+8:23</i>	<i>6.3e+7:39</i>	<i>2.5e+7:76</i>	<i>4.0e+6:209</i>	<i>1.0e+1:1042</i>	15/15
	199(48)	110(23)	53(12)	63(13)	∞2100	0/15		171(97)	119(126)	94(57)	45(53)	∞7600	0/15
<b>f<sub>13</sub></b>	<i>1.0e+3:2.8</i>	<i>6.3e+2:8.4</i>	<i>2.5e+2:17</i>	<i>6.3e-1:52</i>	<i>6.3e-2:264</i>	15/15	<b>f<sub>13</sub></b>	<i>1.6e+3:28</i>	<i>1.0e+3:64</i>	<i>6.3e+2:79</i>	<i>4.0e+1:211</i>	<i>2.5e+0:1724</i>	15/15
	214(90)	101(27)	59(5)	30(6)	∞2000	0/15		95(20)	55(17)	57(26)	262(270)	∞8600	0/15
<b>f<sub>14</sub></b>	<i>1.6e+1:3.0</i>	<i>1.0e+1:10</i>	<i>6.3e+0:15</i>	<i>2.5e-1:53</i>	<i>1.0e-5:251</i>	15/15	<b>f<sub>14</sub></b>	<i>2.5e+1:15</i>	<i>1.6e+1:42</i>	<i>1.0e+1:75</i>	<i>1.6e+0:219</i>	<i>6.3e-4:1106</i>	15/15
	252(53)	96(22)	71(18)	38(2)	∞2000	0/15		326(188)	200(92)	182(208)	515(562)	∞7600	0/15
<b>f<sub>15</sub></b>	<i>1.6e+2:3.0</i>	<i>1.0e+2:13</i>	<i>6.3e+1:24</i>	<i>4.0e+1:55</i>	<i>1.6e+1:289</i>	5/5	<b>f<sub>15</sub></b>	<i>6.3e+2:15</i>	<i>4.0e+2:67</i>	<i>2.5e+2:292</i>	<i>1.6e+2:846</i>	<i>1.0e+2:1671</i>	15/15
	282(67)	77(19)	47(11)	24(8)	11(12)	9/15		286(45)	147(173)	65(82)	∞	∞	0/15
<b>f<sub>16</sub></b>	<i>4.0e+1:4.8</i>	<i>2.5e+1:16</i>	<i>1.6e+1:46</i>	<i>1.0e+1:120</i>	<i>4.0e+0:334</i>	15/15	<b>f<sub>16</sub></b>	<i>4.0e+1:26</i>	<i>2.5e+1:127</i>	<i>1.6e+1:540</i>	<i>1.6e+1:540</i>	<i>1.0e+1:1384</i>	15/15
	130(29)	65(28)	36(33)	22(2)	33(37)	3/15		229(284)	1063(748)	∞	∞	∞9300	0/15
<b>f<sub>17</sub></b>	<i>1.0e+1:5.2</i>	<i>6.3e+0:26</i>	<i>4.0e+0:57</i>	<i>2.5e+0:110</i>	<i>6.3e-1:412</i>	15/15	<b>f<sub>17</sub></b>	<i>1.6e+1:11</i>	<i>1.0e+1:63</i>	<i>6.3e+0:305</i>	<i>4.0e+0:468</i>	<i>1.0e+0:1030</i>	15/15
	196(68)	60(17)	37(25)	22(29)	10(5)	7/15		302(74)	226(190)	100(166)	275(263)	∞8800	0/15
<b>f<sub>18</sub></b>	<i>6.3e+1:3.4</i>	<i>4.0e+1:7.2</i>	<i>2.5e+1:20</i>	<i>1.6e+1:58</i>	<i>1.6e+0:318</i>	15/15	<b>f<sub>18</sub></b>	<i>4.0e+1:116</i>	<i>2.5e+1:252</i>	<i>1.6e+1:430</i>	<i>1.0e+1:621</i>	<i>4.0e+0:1090</i>	15/15
	225(83)	132(61)	80(106)	31(8)	24(17)	4/15		162(153)	256(252)	309(253)	∞	∞9100	0/15
<b>f<sub>19</sub></b>	<i>1.6e-1:172</i>	<i>1.0e-1:242</i>	<i>6.3e-2:675</i>	<i>4.0e-2:3078</i>	<i>2.5e-2:4946</i>	15/15	<b>f<sub>19</sub></b>	<i>1.6e-1:2.5e5</i>	<i>1.0e-1:3.4e5</i>	<i>6.3e-2:3.4e5</i>	<i>4.0e-2:3.4e5</i>	<i>2.5e-2:3.4e5</i>	3/15
	∞	∞	∞	∞	∞2100	0/15		∞	∞	∞	∞	∞9200	0/15
<b>f<sub>20</sub></b>	<i>6.3e+3:5.1</i>	<i>4.0e+3:8.4</i>	<i>4.0e+1:15</i>	<i>2.5e+0:69</i>	<i>1.0e+0:851</i>	15/15	<b>f<sub>20</sub></b>	<i>1.6e+4:38</i>	<i>1.0e+4:42</i>	<i>2.5e+2:62</i>	<i>2.5e+0:250</i>	<i>1.6e+0:2536</i>	15/15
	177(36)	113(11)	75(9)	21(3)	∞2000	0/15		69(10)	67(15)	62(19)	22(4)	6.0(5)	7/15
<b>f<sub>21</sub></b>	<i>4.0e+1:3.9</i>	<i>2.5e+1:11</i>	<i>1.6e+1:31</i>	<i>6.3e+0:73</i>	<i>1.6e+0:347</i>	5/5	<b>f<sub>21</sub></b>	<i>6.3e+1:36</i>	<i>4.0e+1:77</i>	<i>4.0e+1:77</i>	<i>1.6e+1:456</i>	<i>4.0e+0:1094</i>	15/15
	191(76)	79(30)	30(9)	24(5)	21(24)	4/15		109(28)	88(100)	88(53)	25(13)	33(32)	3/15
<b>f<sub>22</sub></b>	<i>6.3e+1:3.6</i>	<i>4.0e+1:15</i>	<i>2.5e+1:32</i>	<i>1.0e+1:71</i>	<i>1.6e+0:341</i>	5/5	<b>f<sub>22</sub></b>	<i>6.3e+1:45</i>	<i>4.0e+1:68</i>	<i>4.0e+1:68</i>	<i>1.6e+1:231</i>	<i>6.3e+0:1219</i>	15/15
	212(48)	58(18)	34(34)	20(11)	9.1(7)	9/15		133(231)	119(28)	119(57)	60(51)	23(20)	4/15
<b>f<sub>23</sub></b>	<i>1.0e+1:3.0</i>	<i>6.3e+0:9.0</i>	<i>4.0e+0:33</i>	<i>2.5e+0:84</i>	<i>1.0e+0:518</i>	15/15	<b>f<sub>23</sub></b>	<i>6.3e+0:29</i>	<i>4.0e+0:118</i>	<i>2.5e+0:306</i>	<i>2.5e+0:306</i>	<i>1.0e+0:1614</i>	15/15
	244(58)	105(27)	50(43)	39(30)	∞2200	0/15		196(427)	94(90)	146(246)	146(147)	∞9400	0/15
<b>f<sub>24</sub></b>	<i>6.3e+1:15</i>	<i>4.0e+1:37</i>	<i>4.0e+1:37</i>	<i>2.5e+1:118</i>	<i>1.6e+1:692</i>	15/15	<b>f<sub>24</sub></b>	<i>2.5e+2:208</i>	<i>1.6e+2:918</i>	<i>1.0e+2:6628</i>	<i>6.3e+1:9885</i>	<i>4.0e+1:31629</i>	15/15
	74(21)	37(5)	37(11)	28(22)	8.5(8)	5/15		∞	∞	∞	∞	∞8800	0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009. The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear in the second row of each cell, the best ERT (preceded by the target  $\Delta f$ -value in *italics*) in the first. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Bold entries are statistically significantly better (according to the rank-sum test) compared to the best algorithm in BBOB-2009, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k > 1$  is following the  $\downarrow$  symbol, with Bonferroni correction by the number of functions.

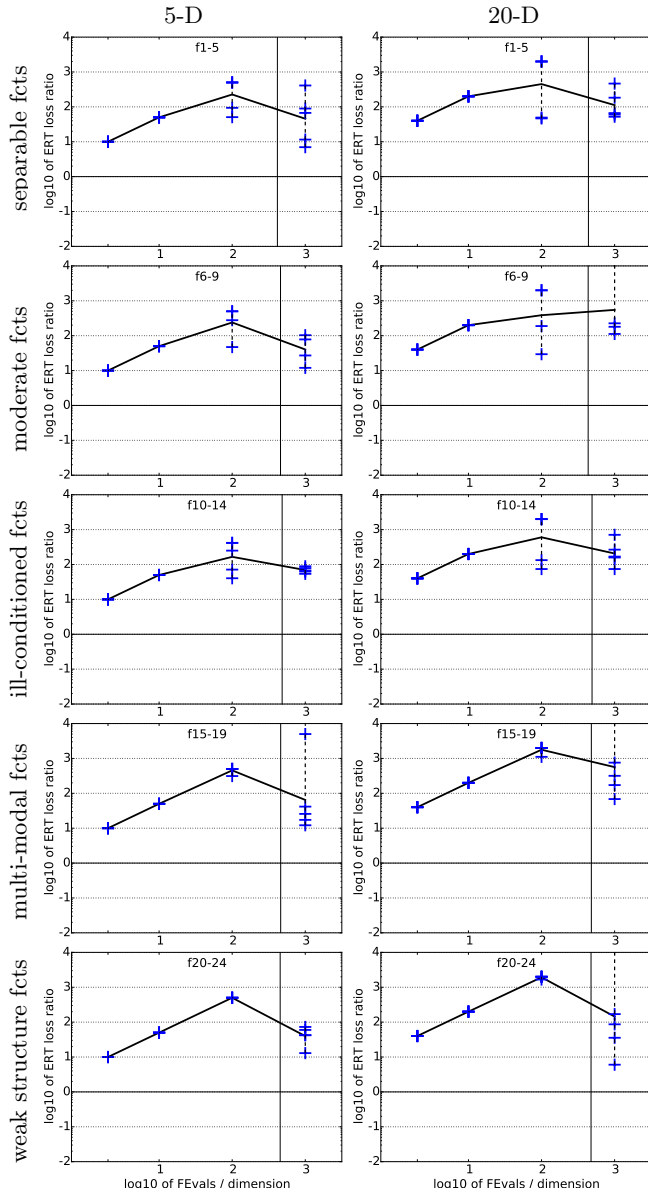


Figure 4: ERT loss ratios (see Figure 3 for details). Each cross (+) represents a single function, the line is the geometric mean.