

# Modyfikacje/hybrydyzacje algorytmu PSO w zadaniu optymalizacji globalnej wielowymiarowej funkcji ciągłej

PSO-DE

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## ABSTRACT

Porównanie algorytmów PSO-DE, jego modyfikacji oraz algorytmu Ewolucji Różnicowej.

if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

## Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

## General Terms

Algorithms

## Keywords

Benchmarking, Black-box optimization

## 1. CPU TIMING

Algorytm był uruchamiany na komputerze z systemem Windows 8 Intel(R) Core(TM) i7-4500U CPU @ 2.39GHz. Czasy ewaluacji funkcji o wymiarach 2, 3, 5, 10, 20 wynosiły odpowiednio  $1,9e^{-10}$ ,  $2,2e^{-10}$ ,  $2,4e^{-10}$ ,  $3,5e^{-10}$  and  $6,1e^{-10}$  sekund.

## 2. RESULTS

Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2 and 3 and in Tables 1 and 2. The **expected running time (ERT)**, used in the figures and tables, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number of trials that actually reached  $f_t$  [?, ?]. **Statistical significance** is tested with the rank-sum test for a given target  $\Delta f_t$  using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by  $-1$ ), or,

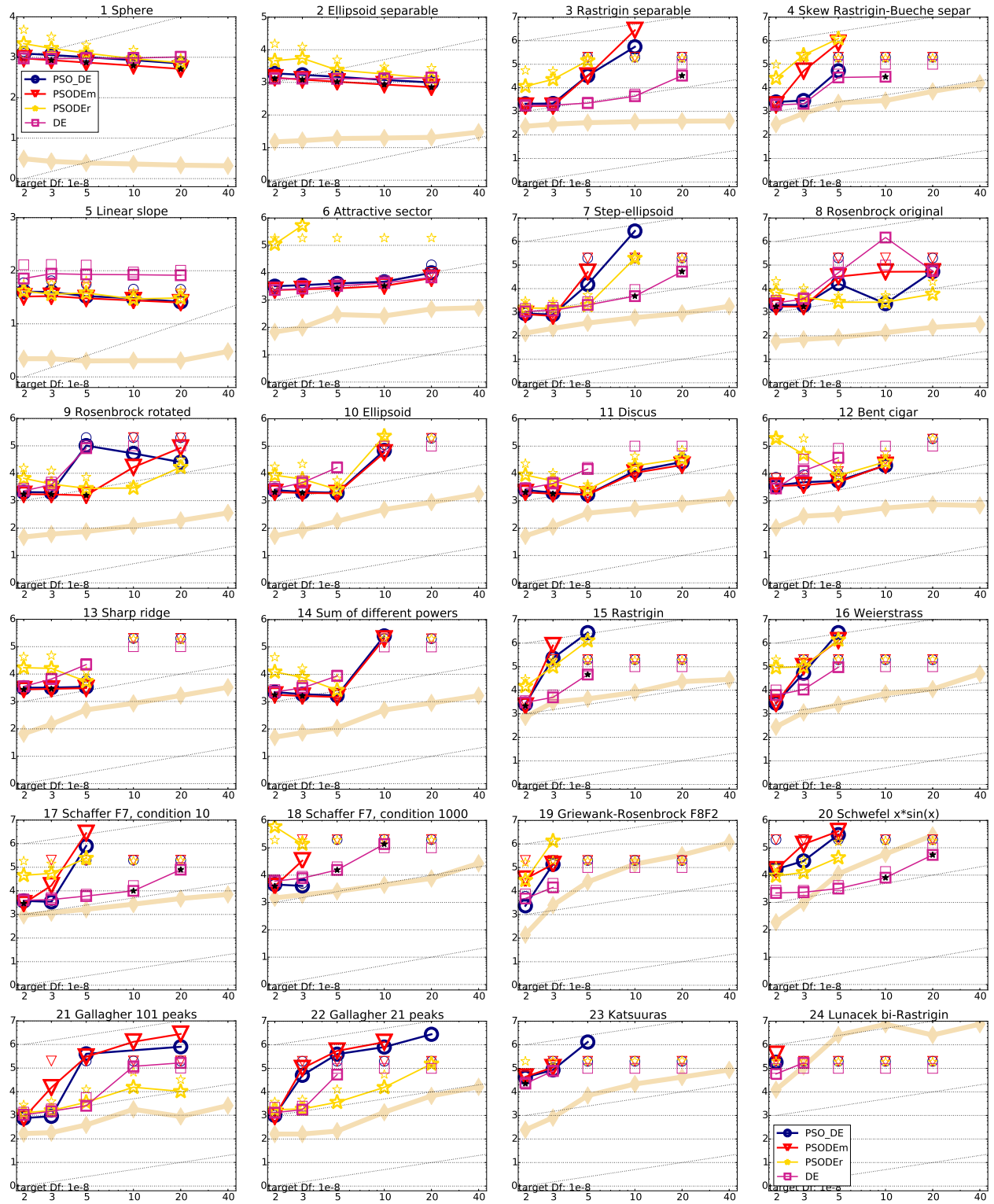


Figure 1: Expected running time (ERT in number of  $f$ -evaluations as  $\log_{10}$  value), divided by dimension for target function value  $10^{-8}$  versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with  $p < 0.01$  and Bonferroni correction number of dimensions (six). Legend:  $\circ$ :PSO DE,  $\nabla$ :PSODEm,  $\star$ :PSODer,  $\square$ :DE

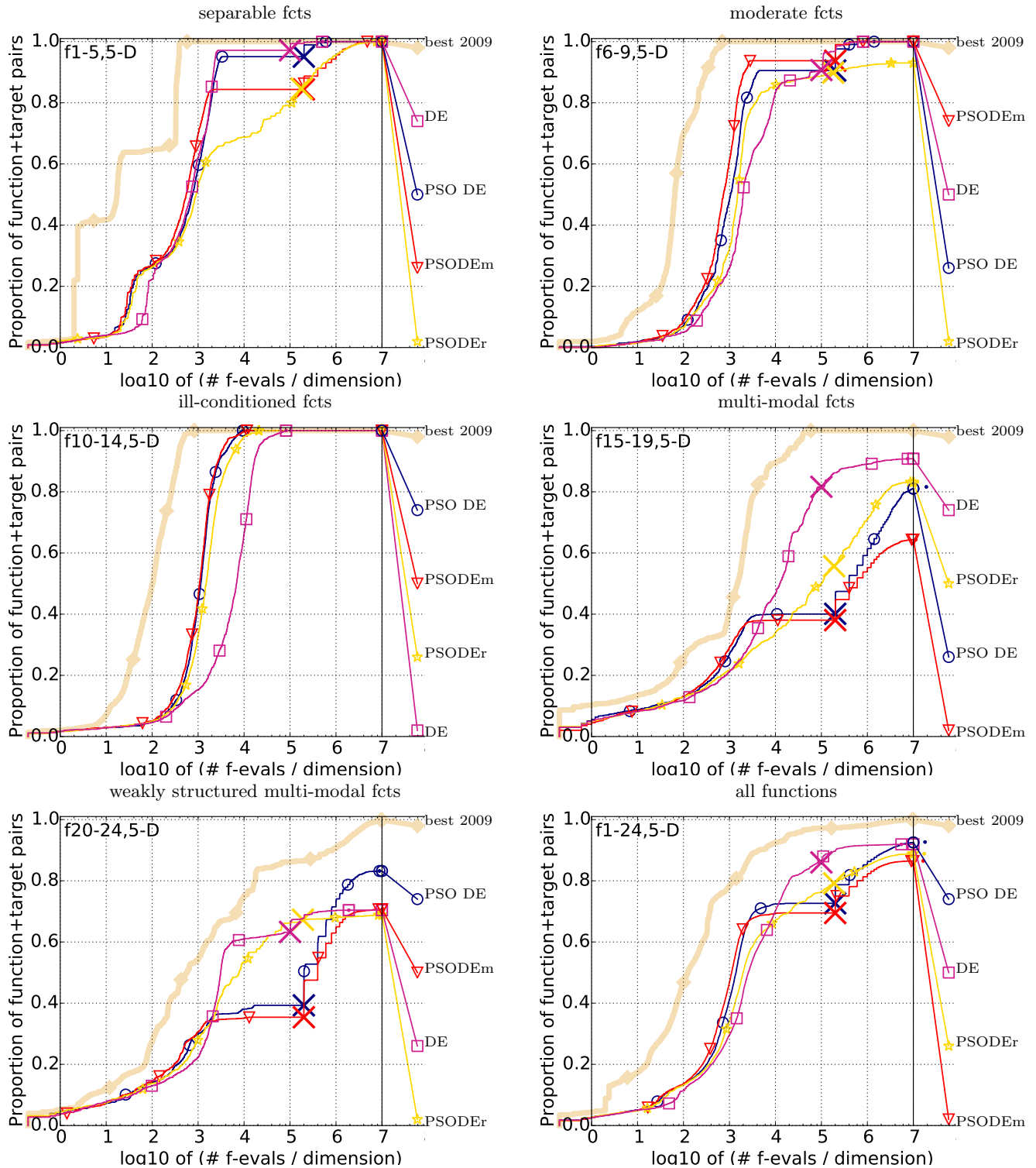


Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in  $10^{[-8..2]}$  for all functions and subgroups in 5-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

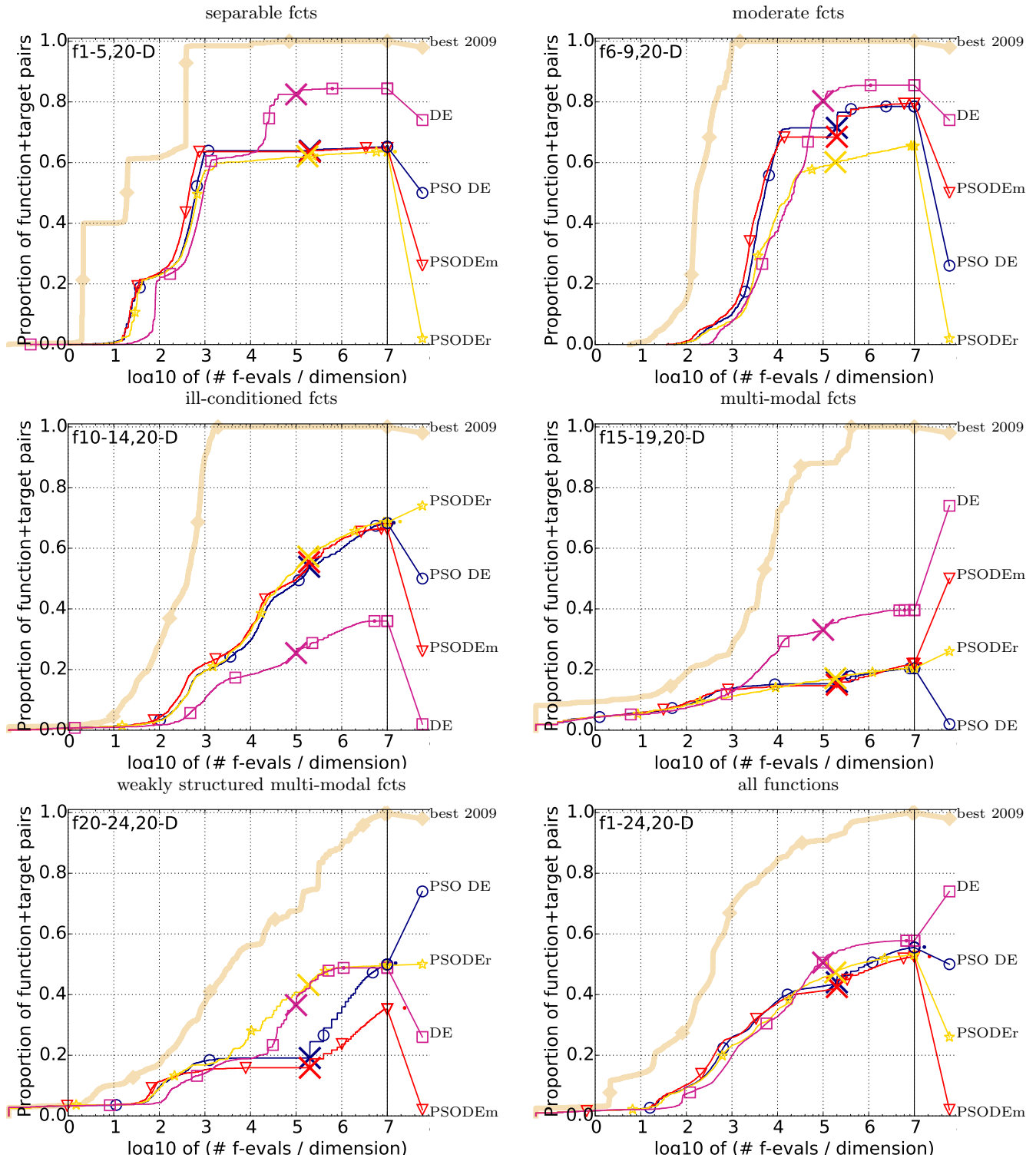


Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in  $10^{[-8..2]}$  for all functions and subgroups in 20-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f1</b>	11	12	12	12	12	12	12	15/15
PSO DE	5.1(2)	35(22)	77(26)	122(11)	171(16)	261(23)	363(21)	15/15
PSODEm	5.1(4)	39(8)	68(15)	103(10)*3	134(10)*4	199(20)*4	271(14)*4	15/15
PSODer	6.0(4)	43(9)	85(11)	126(11)	196(10)	303(66)	416(94)	15/15
DE	5.5(5)	45(4)	90(8)	129(14)	174(16)	259(10)	343(10)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f2</b>	83	87	88	89	90	92	94	15/15
PSO DE	25(3)	31(2)	37(2)	43(2)	49(2)	62(3)	72(2)	15/15
PSODEm	18(2)	22(2)*2	26(2)*3	31(1)*3	35(2)*4	43(2)*4	51(3)*4	15/15
PSODer	31(18)	38(14)	47(33)	53(28)	69(3)	88(39)	102(28)	15/15
DE	20(1)	26(2)	31(1)	37(1.0)	42(2)	53(2)	62(3)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f3</b>	716	1622	1637	1642	1646	1650	1654	15/15
PSO DE	2.4(1)	3.7(0.7)	98(0.6)	98(456)	98(303)	99(302)	99(151)	13/15
PSODEm	2.1(0.8)	2.7(0.8)*97(1)	97(0.5)	97(304)	97(454)	97(754)		13/15
PSODer	4.5(6)	107(210)	192(305)	219(256)	306(554)	428(1096)	428(905)	10/15
DE	2.1(0.6)	3.9(0.4)	4.6(0.5)	4.9(0.6)	5.2(0.4)	5.8(0.4)	6.4(0.4)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f4</b>	809	1633	1688	1758	1817	1886	1903	15/15
PSO DE	2.8(1)	4.8(0.6)	153(296)	147(142)	143(138)	139(132)	138(524)	12/15
PSODEm	1.9(0.8309)	611	2368(4138)	2274(1845)	2200(3569)	2121(2646)	2102(2097)	3/15
PSODer	10(23)	558(360)	1338(817)	1289(1030)	2218(1447)	3247(3706)	3219(3045)	2/15
DE	2.6(0.7)	4.4(0.6)	79(148)	76(427)	74(69)	72(0.2)	72(132)	12/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f5</b>	10	10	10	10	10	10	10	15/15
PSO DE	11(3)	16(5)	17(8)	17(8)	17(7)	17(6)	17(5)	15/15
PSODEm	11(4)	15(4)	15(7)	15(4)	15(6)	15(4)	15(6)	15/15
PSODer	13(6)	19(8)	19(7)	19(5)	19(6)	19(6)	19(9)	15/15
DE	25(6)	39(15)	43(15)	43(13)	43(13)	43(10)	43(11)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f6</b>	114	214	281	404	580	1038	1332	15/15
PSO DE	6.5(3)	11(6)	15(6)	17(3)	15(3)	13(2)	13(1)	15/15
PSODEm	5.6(2)	7.8(2)	10(2)*2	11(2)*3	10(2)*3	8.4(0.5)*8.9(1)*2		15/15
PSODer	7.3(2)	21(11)	75(60)	181(160)	610(2272)	5950(4999)	$\infty$ 9e5	0/15
DE	13(4)	14(2)	17(3)	16(3)	14(3)	11(2)	11(1)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f7</b>	24	324	1171	1451	1572	1572	1597	15/15
PSO DE	13(4)	3.1(0.5)	62(214)	51(345)	47(1)	47(0.9)	47(1)	14/15
PSODEm	10(7)	2.5(0.6)	133(427)	108(0.5)	160(477)	160(318)	158(313)	12/15
PSODer	12(8)	4.1(3)	2.6(1)	4.0(2)	4.5(2)	4.5(2)	4.7(2)	15/15
DE	20(11)	6.6(3)	4.0(1.0)	5.1(0.7)	5.4(1)	5.4(0.8)	5.7(2)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f8</b>	73	273	336	372	391	410	422	15/15
PSO DE	16(6)	273(1830)	226(3)	206(672)	199(3)	193(612)	191(594)	14/15
PSODEm	12(2)	572(1830)	467(3)	424(1342)	405(1279)	389(611)	381(593)	13/15
PSODer	20(12)	21(17)	22(5)	22(18)	23(10)	27(19)	30(14)	15/15
DE	21(5)	23(3)	115(110)	312(468)	557(343)	550(615)	554(595)	11/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f9</b>	35	127	214	263	300	335	369	15/15
PSO DE	34(10)	3959(3939)	2356(3506)	1916(4743)	683(1663)	512(3726)	378(2)	10/15
PSODEm	23(4)*2	18(5)*2	15(5)*3	14(2)*3	15(3)*3	17(3)*3	19(2)*3	15/15
PSODer	36(23)	32(14)	26(7)	24(4)	24(4)	32(25)	35(36)	15/15
DE	45(10)	57(14)	266(251)	1239(1908)	307(2690)	195(2990)	105(2874)	9/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f10</b>	349	500	574	607	626	829	880	15/15
PSO DE	8.3(2)	7.4(1.0)	7.8(0.6)	8.7(1)	10(2)	9.0(2)	10(0.6)	15/15
PSODEm	8.5(3)	7.4(1)	7.6(2)	8.2(1)	9.3(1)	8.8(1)	10(0.7)	15/15
PSODer	11(9)	9.1(6)	9.4(4)	10(4)	12(5)	11(4)	13(4)	15/15
DE	47(22)	48(8)	54(7)	63(5)	73(8)	73(7)	85(5)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f11</b>	143	202	763	977	1177	1467	1673	15/15
PSO DE	13(3)	13(2)	4.6(1)	4.3(0.4)	4.2(0.3)	4.4(0.5)	4.7(0.5)	15/15
PSODEm	11(5)	12(2)	4.2(0.5)	4.0(0.5)	4.0(0.8)	4.2(0.4)	4.5(0.8)	15/15
PSODer	25(9)	28(14)	8.4(4)	8.1(5)	7.3(3)	6.9(3)	7.4(3)	15/15
DE	55(32)	74(12)	28(9)	29(6)	30(5)	34(2)	39(7)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f12</b>	108	268	371	413	461	1303	1494	15/15
PSO DE	46(17)	35(37)	36(34)	38(23)	38(20)	17(12)	17(10)	15/15
PSODEm	55(25)	35(3)	31(26)	32(25)	32(18)	15(11)	15(4)	15/15
PSODer	65(44)	50(31)	51(33)	55(31)	53(32)	27(9)	26(14)	15/15
DE	131(31)	93(15)	100(26)	129(29)	163(55)	101(76)	115(76)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f13</b>	132	195	250	319	1310	1752	2255	15/15
PSO DE	15(6)	16(4)	19(5)	19(2)	5.8(0.7)	6.2(0.9)	6.4(2)	15/15
PSODEm	12(2)	13(1)*	16(3)	17(1)*2	5.1(0.3)*5.5(0.9)	6.0(1)		15/15
PSODer	16(1)	18(2)	19(4)	21(2)	6.8(2)	7.8(2)	10(6)	15/15
DE	26(6)	48(11)	70(16)	90(20)	32(5)	39(5)	43(3)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f14</b>	10	41	58	90	139	251	476	15/15
PSO DE	1.6(2)	10(3)	18(3)	22(4)	21(3)	19(1)	15(3)	15/15
PSODEm	2.2(2)	10(3)	16(3)	16(2)*3	16(3)*2	16(1)*3	13(0.7)	15/15
PSODer	1.6(1)	10(5)	19(6)	24(5)	28(14)	29(11)	24(27)	15/15
DE	2.5(4)	15(4)	23(4)	25(4)	30(9)	69(8)	72(3)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f15</b>	511	9310	19369	19743	20073	20769	21359	14/15
PSO DE	5.1(2)	1502(885)	722(863)	708(771)	697(796)	673(577)	655(292)	1/15
PSODEm	144(4)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 1e6	0/15
PSODer	18(11)	59(36)	213(170)	209(194)	318(386)	307(162)	299(290)	2/15
DE	9.0(5)	9.4(4)*12(28)*2	12(8)*2	11(25)*2	11(7)*2	11(13)*2		12/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f16</b>	120	612	2662	10163	10449	11644	12095	15/15
PSO DE	4.1(3)	418(816)	1033(938)	639(614)	622(884)	558(579)	1157(3345)	1/15
PSODEm	5.6(4)	602(412)	1034(2813)	271(615)	383(717)	558(408)	538(722)	2/15
PSODer	5.3(4)	128(131)	117(151)	51(71)	76(164)	248(424)	571(309)	2/15
DE	3.7(3)	90(39)	91(23)	39(40)	44(26)	40(23)	39(36)	11/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f17</b>	5.2	215	899	2861	3669	6351	7934	15/15
PSO DE	4.0(9)	4.7(1)	3.3(0.7)	1.8(0.3)	44(204)	80(79)	254(346)	5/15
PSODEm	3.3(2)	3.5(1)	82(0.5)	26(0.2)	100(272)	181(197)	820(850)	2/15
PSODer	4.0(4)	4.9(1)	4.4(3)	4.5(2)	7.7(9)	31(52)	109(113)	10/15
DE	3.7(6)	6.1(1)	4.5(0.5)	2.6(0.5)	3.1(0.7)	3.1(0.5)	3.5(0.6)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f18</b>	103	378	3968	8451	9280	10905	12469	15/15
PSO DE	3.0(2)	5.5(1)	127(189)	237(148)	701(1023)	1284(3000)	$\infty$ 1e6	0/15
PSODEm	4.3(2)	4.6(1)	64(126)	104(177)	216(188)	$\infty$	$\infty$ 1e6	0/15
PSODer	2.5(2)	8.0(5)	3.7(5)	7.0(3)	21(18)	612(475)	$\infty$ 9e5	0/15
DE	5.2(2)	9.1(3)	2.6(0.5)	2.3(0.4)	3.3(1.0)	5.0(1)*5.7(1)*4		15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f19</b>	1	1	242	1.0e5	1.2e5	1.2e5	1.2e5	15/15
PSO DE	23(24)	4629(3285)	5.8e4(1e5)	$\infty$	$\infty$	$\infty$	$\infty$ 1e6	0/15
PSODEm	35(36)	3883(2328)	8329(1e4)	$\infty$	$\infty$	$\infty$	$\infty$ 1e6	0/15
PSODer	46(44)	8915(9142)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 9e5	0/15
DE	37(34)	6135(1207)	1603(1983)	69(46)	62(87)	$\infty$	$\infty$ 5e5	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f20</b>	16	851	38111	51362	54470	54861	55313	14/15
PSO DE	11(4)	3.5(2)	39(26)	29(34)	28(28)	27(14)	27(36)	6/15
PSODEm	8.9(6)	2.9(1)	52(52)	39(29)	37(37)	36(23)	36(45)	5/15
PSODer	13(9)	10(11)	3.8(3)	3.0(3)	3.3(2)	3.5(5)	3.9(2)	15/15
DE	12(7)	4.3(2)	0.27(0.1)	0.24(0.1)	0.24(0.1)	0.26(0.1)	0.28(0.1)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f21</b>	41	1157	1674	1692	1705	1729	1757	14/15
PSO DE	2.7(4)	433(432)	1194(1491)	1181(2213)	1172(1611)	1157(2022)	1139(1705)	5/15
PSODEm	2.8(3)	756(1080)	895(745)	886(1475)	879(879)	868(1444)	854(853)	6/15
PSODer	3.0(3)	4.8(5)	6.2(11)	8.4(9)	8.5(14)	10(13)	10(12)	15/15
DE	2.1(2)	3.5(4)	4.2(5)	5.4(6)	5.7(2)	6.4(6)	6.8(5)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f22</b>	71	386	938	980	1008	1040	1068	14/15
PSO DE	3.7(2)	1939(2131)	2129(2040)	2293(1983)	2228(1923)	1680(1874)	1871	5/15
PSODEm	3.9(3)	2957(3879)	2931(4260)	2804(3567)	2726(8173)	2643(4320)	2576(2105)	4/15
PSODer	3.8(4)	15(18)						

$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f1</b>	43	43	43	43	43	43	43	15/15
PSO DE	36(4)	67(6)	97(8)	129(6)	163(9)	230(19)	295(22)	15/15
PSODEm	<b>28(3)*</b>	<b>52(5)*3</b>	<b>76(6)*4</b>	<b>98(10)*4</b>	<b>122(6)*4</b>	<b>168(7)*4</b>	<b>217(6)*4</b>	15/15
PSODer	34(4)	68(6)	100(7)	140(7)	172(33)	240(11)	306(30)	15/15
DE	53(7)	101(8)	149(7)	196(8)	243(7)	335(9)	428(11)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f2</b>	385	386	387	388	390	391	393	15/15
PSO DE	19(0.8)	22(1)	26(0.7)	30(0.8)	33(0.6)	40(1)	47(2)	15/15
PSODEm	<b>13(0.9)*4</b>	<b>16(1)*4</b>	<b>18(1)*4</b>	<b>21(1)*4</b>	<b>23(1)*4</b>	<b>29(2)*4</b>	<b>34(2)*4</b>	15/15
PSODer	20(3)	24(3)	27(1)	33(2)	39(19)	48(4)	64(33)	15/15
DE	25(1)	30(2)	35(2)	41(2)	45(1.0)	56(1)	66(2)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f3</b>	5066	7626	7635	7637	7643	7646	7651	15/15
PSO DE	1186(1380)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	5128(4929)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	<b>89(29)</b>	<b>64(21)*4</b>	<b>84(136)*4</b>	<b>84(9)*4</b>	<b>85(126)*4</b>	<b>85(134)*4</b>	<b>86(25)*4</b>	14/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f4</b>	4722	7628	7666	7686	7700	7758	1.4e5	9/15
PSO DE	1.2e4(8247)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	<b>74(22)*2</b>	<b>455(340)*4</b>	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f5</b>	41	41	41	41	41	41	41	15/15
PSO DE	11(5)	13(5)	13(6)	13(3)	13(6)	13(3)	13(3)	15/15
PSODEm	<b>11(3)</b>	<b>12(4)</b>	<b>12(4)</b>	<b>12(3)</b>	<b>12(4)</b>	<b>12(5)</b>	<b>12(4)</b>	15/15
PSODer	13(4)	15(4)	15(5)	15(3)	15(5)	15(4)	15(4)	15/15
DE	30(2)	38(6)	40(4)	40(5)	40(5)	40(2)	40(3)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f6</b>	1296	2343	3413	4255	5220	6728	8409	15/15
PSO DE	18(10)	16(7)	16(8)	16(5)	17(8)	20(12)	21(11)	15/15
PSODEm	<b>12(4)*</b>	<b>12(4)</b>	<b>11(2)</b>	<b>12(5)</b>	<b>13(3)</b>	<b>13(3)</b>	<b>14(4)</b>	15/15
PSODer	612(286)	2761(2612)	1.6e4(1e4)	1.3e4(2e4)	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	19(3)	15(2)	14(1)	14(1)	14(1)	14(1)	14(1)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f7</b>	1351	4274	9503	16523	16524	16524	16969	15/15
PSO DE	5925(1e4)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	1.2e4(1e4)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	19(17)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	29(10)	<b>37(10)*4</b>	<b>81(225)*4</b>	<b>63(97)*4</b>	<b>63(93)*4</b>	<b>63(92)*4</b>	<b>62(96)*4</b>	11/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f8</b>	2039	3871	4040	4148	4219	4371	4484	15/15
PSO DE	11(1)	267(257)	257(496)	252(242)	249(473)	242(687)	238(1)	12/15
PSODEm	<b>7.5(2)</b>	265(774)	255(249)	249(1443)	246(474)	239(686)	234(668)	12/15
PSODer	10(5)	<b>21(37)</b>	<b>22(35)</b>	<b>22(2)</b>	<b>23(34)</b>	<b>24(11)</b>	<b>25(17)</b>	15/15
DE	94(12)	129(4)	167(6)	205(12)	223(23)	239(39)	244(42)	15/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f9</b>	1716	3102	3277	3379	3455	3594	3727	15/15
PSO DE	18(8)	113(2)	112(5)	114(9)	118(579)	126(282)	131(541)	14/15
PSODEm	<b>17(6)</b>	483(1595)	462(907)	454(293)	450(863)	447(1104)	446(800)	11/15
PSODer	24(8)	<b>57(44)</b>	<b>59(54)</b>	<b>63(42)</b>	<b>67(51)</b>	<b>76(48)</b>	<b>84(45)</b>	15/15
DE	143(8)	258(34)	390(30)	2197(1190)	$\infty$	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f10</b>	7413	8661	10735	13641	14920	17073	17476	15/15
PSO DE	151(60)	294(164)	537(372)	2053(1980)	<b>3834(3493)</b>	$\infty$	$\infty$ 4e6	0/15
PSODEm	<b>131(96)</b>	<b>241(98)</b>	<b>518(550)</b>	<b>810(1227)</b>	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	211(119)	486(338)	5252(2336)	4202(4952)	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f11</b>	1002	2228	6278	8586	9762	12285	14831	15/15
PSO DE	28(17)	35(11)	22(14)	24(15)	27(10)	30(10)	33(9)	15/15
PSODEm	28(12)	<b>32(31)</b>	<b>18(7)</b>	<b>18(7)</b>	<b>20(8)</b>	<b>23(7)</b>	<b>24(7)</b>	15/15
PSODer	<b>27(8)</b>	37(46)	24(11)	22(8)	25(10)	32(12)	41(27)	15/15
DE	964(120)	728(49)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f12</b>	1042	1938	2740	3156	4140	12407	13827	15/15
PSO DE	286(1767)	1145(608)	3496(8122)	1.6e4(4e4)	<b>1.3e4(2e4)</b>	$\infty$	$\infty$ 4e6	0/15
PSODEm	612(1964)	1664(926)	<b>1679(1198)</b>	<b>3316(3408)</b>	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	<b>225(159)</b>	<b>899(2107)</b>	2216(2782)	8442(6955)	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	343(1956)	1217(1811)	2102(2269)	8892(1e4)	$\infty$	$\infty$	$\infty$ 2e6	0/15

$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f13</b>	652	2021	2751	3507	18749	24455	30201	15/15
PSO DE	3074(3063)	7904(2e4)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
PSODEm	1540(1)	3955(4939)	9439(1e4)	1.6e4(3e4)	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	42(50)	<b>70(79)</b>	<b>190(170)*</b>	<b>553(1014)*275</b>	(169)	$\infty$	$\infty$ 3e6	0/15
DE	<b>32(7)</b>	394(1021)	3006(8042)	8016(7130)	$\infty$	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f14</b>	75	239	304	451	932	1648	15661	15/15
PSO DE	13(3)	12(1)	16(3)	17(2)	15(1)	136(46)	$\infty$ 4e6	0/15
PSODEm	<b>11(3)</b>	<b>9.2(1)*</b>	<b>11(0.6)*</b>	<b>12(1)*4</b>	<b>12(4)*</b>	<b>113(38)</b>	<b>1236(1332)</b>	3/15
PSODer	14(6)	12(2)	16(1)	18(7)	17(4)	136(65)	$\infty$ 4e6	0/15
DE	45(13)	31(5)	37(1)	86(22)	1249(483)	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f15</b>	30378	1.5e5	3.1e5	3.2e5	3.2e5	4.5e5	4.6e5	15/15
PSO DE	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f16</b>	1384	27265	77015	1.4e5	1.9e5	2.0e5	2.2e5	15/15
PSO DE	5776(7216)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	5778(7938)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	<b>290(187)</b>	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f17</b>	63	1030	4005	12242	30677	56288	80472	15/15
PSO DE	8.8(4)	2549(3868)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	<b>8.6(5)</b>	2570(1942)	1.4e4(9123)	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	10(11)	364(223)	6291(7330)	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	16(7)	11(2)	<b>7.3(0.8)*</b>	<b>7.4(0.7)*</b>	<b>7.7(0.1)*</b>	<b>14(23)*4</b>	<b>20(19)*4</b>	9/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f18</b>	621	3972	19561	28555	67569	1.3e5	1.5e5	15/15
PSO DE	6.4(2)	2748(2254)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	<b>4.8(1)</b>	1.4e4(2e4)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	17(12)	1728(2618)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	13(5)	11(3)	<b>8.3(4)*4</b>	<b>32(35)*4</b>	<b>54(49)*4</b>	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f19</b>	1	1	3.4e5	4.7e6	6.2e6	6.7e6	6.7e6	15/15
PSO DE	798(342)	<b>1.5e6(1e5)*</b>	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	<b>628(160)</b>	4.5e6(9e6)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	961(432)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	1411(242)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 2e6	0/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f20</b>	82	46150	3.1e6	5.5e6	5.5e6	5.6e6	5.6e6	14/15
PSO DE	20(5)	347(238)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODEm	<b>16(6)</b>	563(1277)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
PSODer	21(4)	11(11)	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 4e6	0/15
DE	37(8)	18(9)	<b>0.30(0.2)*</b>	<b>0.20(0.2)*</b>	<b>0.20(0.1)*</b>	<b>0.19(0.1)*</b>	<b>0.19(0.1)*</b>	14/15
$\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f21</b>	561	6541	14103	14318	14643	15567</		