

# Modyfikacje/hybrydyzacje algorytmu PSO w zadaniu optymalizacji globalnej wielowymiarowej funkcji ciągłej

## PSO-DE Hybrid

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### ABSTRACT

Dokumentacja uzyskanych wyników hybrydy PSO-DE

### Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

### General Terms

Algorithms

### Keywords

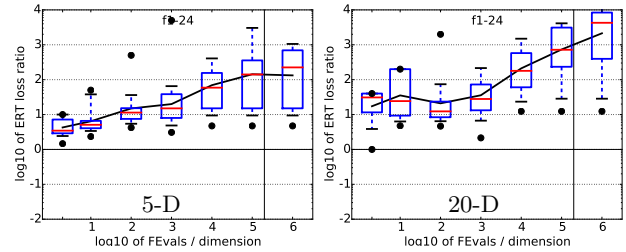
Benchmarking, PSODE, Optymalizacja wielowymiarowej funkcji ciągłej

## 1. CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the PSO-DE Hybrid on the function  $f_8$  with restarts for at least 30 seconds and until a maximum budget equal to  $400(D+2)$  is reached. The code was run on a Windows 8 Intel(R) Core(TM) i7-4500U CPU @ 2.39GHz with 1 processor and 2 cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20 equals  $1,9e^{-10}$ ,  $2,2e^{-10}$ ,  $2,4e^{-10}$ ,  $3,5e^{-10}$  and  $6,1e^{-10}$  seconds respectively.

## 2. RESULTS

Results of PSO DE modified from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2, 3, and 4 and in Tables 1.



$f_1-f_{24}$ in 5-D, maxFE/D=199951						
#FEs/D	best	10%	25%	med	75%	90%
2	1.5	2.4	2.9	3.5	7.6	10
10	2.3	3.3	4.0	5.1	6.6	50
100	4.2	4.8	7.3	11	15	87
1e3	3.1	4.4	7.7	15	40	94
1e4	4.7	8.7	15	59	1.6e2	4.5e2
1e5	4.7	8.7	15	1.4e2	3.6e2	4.0e3
1e6	4.7	8.7	15	2.2e2	7.0e2	1.3e3
RL <sub>US</sub> /D	2e5	2e5	2e5	2e5	2e5	2e5
$f_1-f_{24}$ in 20-D, maxFE/D=199976						
#FEs/D	best	10%	25%	med	75%	90%
2	1.0	3.8	11	31	40	40
10	4.8	6.0	8.7	24	2.0e2	2.0e2
100	4.7	6.4	8.2	12	24	2.7e2
1e3	2.1	5.8	13	28	80	2.4e2
1e4	12	20	60	1.8e2	6.0e2	1.7e3
1e5	12	25	2.3e2	7.2e2	3.3e3	4.3e3
1e6	12	25	3.4e2	4.3e3	8.7e3	1.4e4
RL <sub>US</sub> /D	2e5	2e5	2e5	2e5	2e5	2e5

**Figure 3: ERT loss ratio versus the budget in number of  $f$ -evaluations divided by dimension. For each given budget FEvals, the target value  $f_t$  is computed as the best target  $f$ -value reached within the budget by the given algorithm. Shown is then the ERT to reach  $f_t$  for the given algorithm or the budget, if the GECCO-BBOB-2009 best algorithm reached a better target within the budget, divided by the best ERT seen in GECCO-BBOB-2009 to reach  $f_t$ . Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in a single trial in this function subset. See also Figure 4 for results on each function subgroup.**

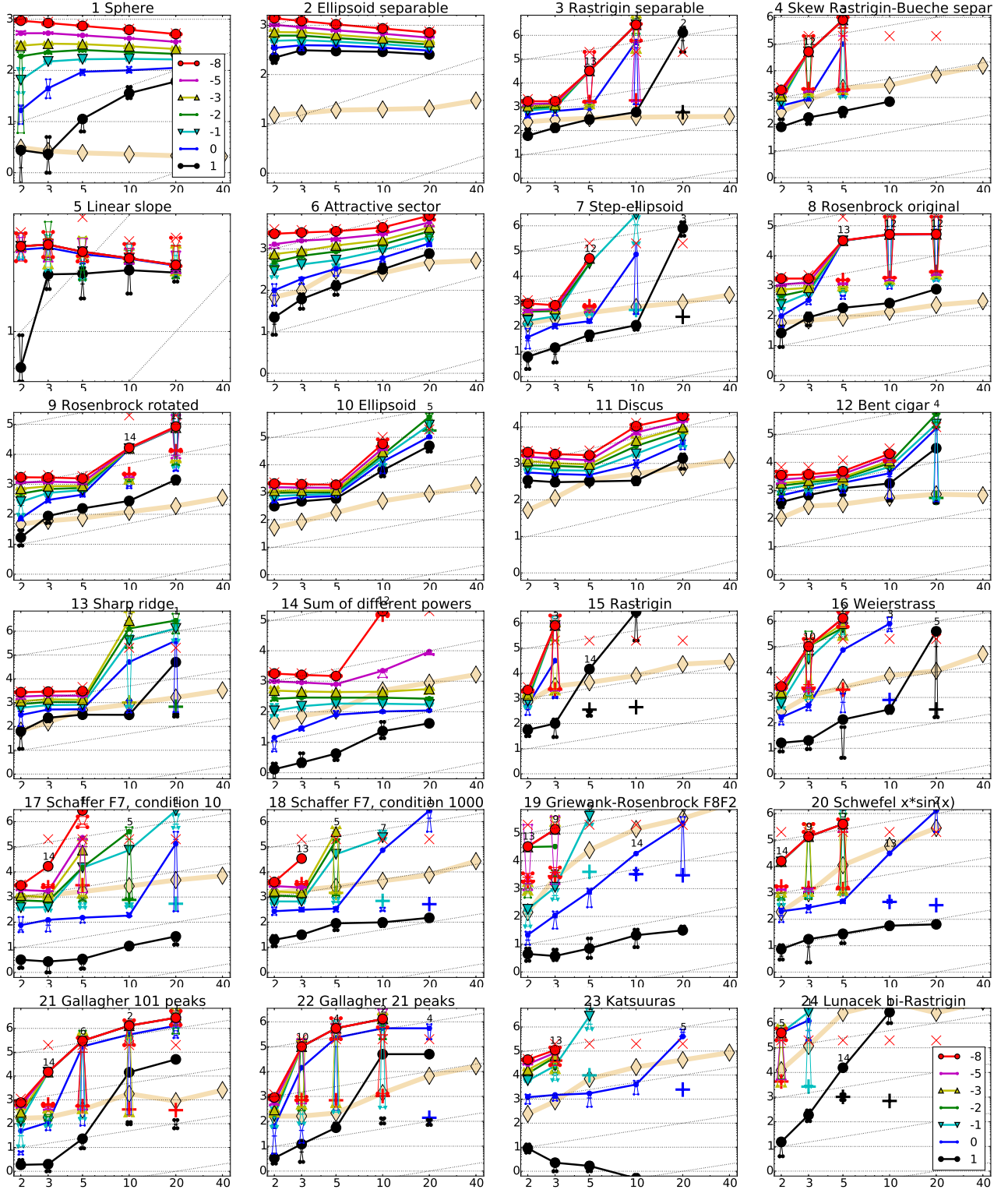
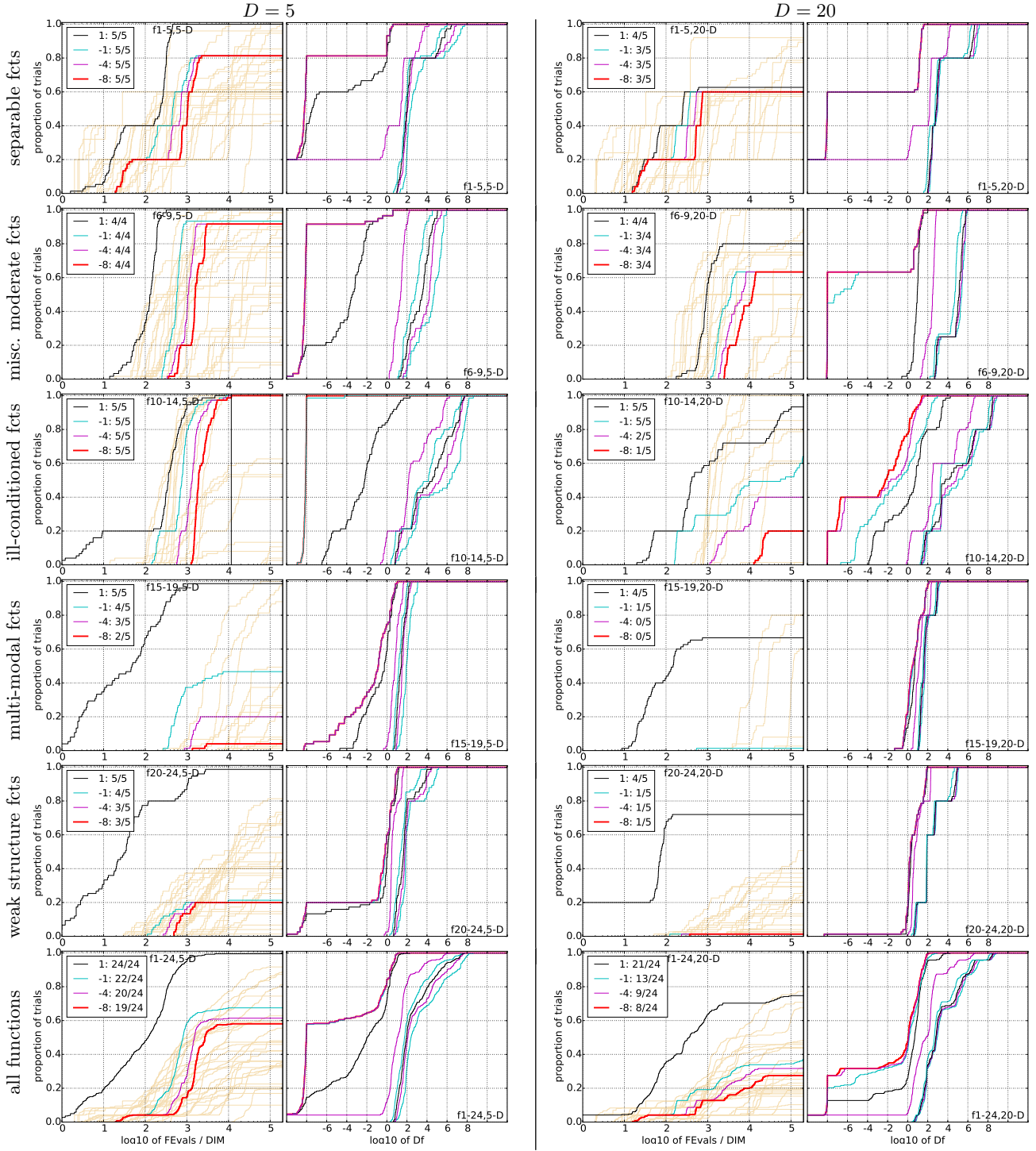


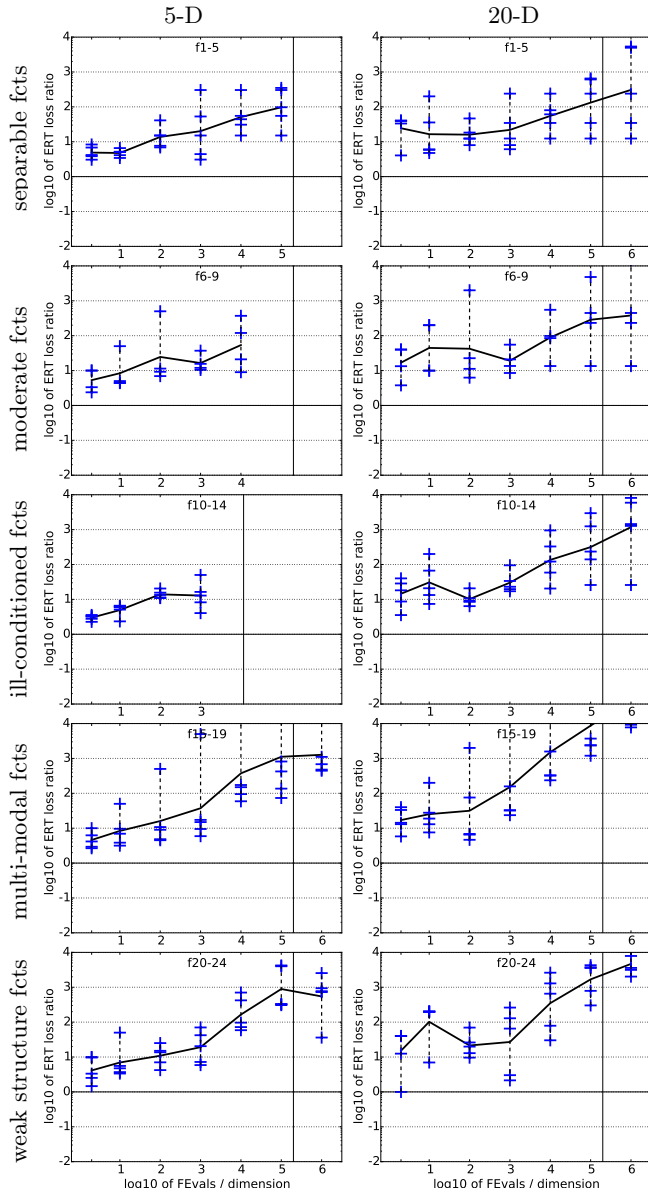
Figure 1: Expected number of  $f$ -evaluations (ERT, lines) to reach  $f_{\text{opt}} + \Delta f$ ; median number of  $f$ -evaluations (+) to reach the most difficult target that was reached not always but at least once; maximum number of  $f$ -evaluations in any trial ( $\times$ ); interquartile range with median (notched boxes) of simulated runlengths to reach  $f_{\text{opt}} + \Delta f$ ; all values are divided by dimension and plotted as  $\log_{10}$  values versus dimension. Shown are  $\Delta f = 10^{\{-8, -5, -3, -2, -1, 0, 1\}}$ . Numbers above ERT-symbols (if appearing) indicate the number of trials reaching the respective target. The light thin line with diamonds indicates the respective best result from BBOB-2009 for  $\Delta f = 10^{-8}$ . Horizontal lines mean linear scaling, slanted grid lines depict quadratic scaling.



**Figure 2: Empirical cumulative distribution functions (ECDF), plotting the fraction of trials with an outcome not larger than the respective value on the  $x$ -axis. Left subplots: ECDF of the number of function evaluations (FEvals) divided by search space dimension  $D$ , to fall below  $f_{\text{opt}} + \Delta f$  with  $\Delta f = 10^k$ , where  $k$  is the first value in the legend. The thick red line represents the most difficult target value  $f_{\text{opt}} + 10^{-8}$ . Legends indicate for each target the number of functions that were solved in at least one trial within the displayed budget. Right subplots: ECDF of the best achieved  $\Delta f$  for running times of  $0.5D, 1.2D, 3D, 10D, 100D, 1000D, \dots$  function evaluations (from right to left cycling cyan-magenta-black...) and final  $\Delta f$ -value (red), where  $\Delta f$  and  $Df$  denote the difference to the optimal function value. Light brown lines in the background show ECDFs for the most difficult target of all algorithms benchmarked during BBOB-2009.**

5-D										20-D									
$\Delta f$	1e+1	1e+0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ		1e+1	1e+0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ		
$f_1$	11 5.1(2)	12 39(7)	12 68(15)	12 103(8)	12 134(10)	12 199(16)	12 271(8)	15/ 15/15	$f_1$	43 28(3)	43 52(1)	43 76(3)	43 98(10)	43 122(5)	43 168(12)	43 217(6)	15/15	15/15	
$f_2$	83 18(2)	87 22(2)	88 26(2)	89 31(2)	90 35(2)	92 43(2)	94 51(2)	15/ 15/15	$f_2$	385 13(0.6)	386 16(0.8)	387 18(1)	388 21(1)	390 23(1)	391 29(1)	393 34(1)	15/15	15/15	
$f_3$	716 2.1(0.8)	1622 2.7(0.6)	1637 97(153)	1642 97(0.9)	1646 97(152)	1650 97(303)	1654 97(302)	15/ 13/15	$f_3$	5066 5128(4929)	7626 $\infty$	7635 $\infty$	7637 $\infty$	7643 $\infty$	7646 $\infty$	7651 $\infty$	15/15	0/15	
$f_4$	809 1.9(0.8)	1633 309(306)	1688 2368(1921)	1758 2274(3690)	1817 2200(2196)	1886 2121(1588)	1903 2102(2884)	15/ 3/15	$f_4$	4722 $\infty$	7628 $\infty$	7666 $\infty$	7686 $\infty$	7700 $\infty$	7758 $\infty$	1.4e5 $\infty$	9/15	0/15	
$f_5$	10 11(3)	10 15(6)	10 15(2)	10 15(6)	10 15(6)	10 15(5)	10 15(6)	15/ 15/15	$f_5$	41 11(3)	41 12(4)	41 12(5)	41 12(5)	41 12(5)	41 12(3)	41 12(4)	15/15	15/15	
$f_6$	114 5.6(2)	214 7.8(2)	281 10(3)	404 11(2)	580 10(2)	1038 8.4(1)	1332 8.9(1)	15/ 15/15	$f_6$	1296 12(2)	2343 12(4)	3413 11(4)	4255 12(3)	5220 13(3)	6728 13(2)	8409 14(5)	15/15	15/15	
$f_7$	24 10(3)	324 2.5(0.7)	1171 133(214)	1451 108(0.6)	1572 160(159)	1572 160(477)	1597 158(157)	15/ 12/15	$f_7$	1351 11844(9620)	4274 $\infty$	9503 $\infty$	16523 $\infty$	16524 $\infty$	16524 $\infty$	16969 $\infty$	15/15	0/15	
$f_8$	73 12(2)	273 572(1832)	336 467(1)	372 424(672)	391 405(640)	410 389(1220)	422 381(2)	15/ 13/15	$f_8$	2039 7.5(2)	3871 265(774)	4040 255(741)	4148 249(481)	4219 246(0.6)	4371 239(458)	4484 234(1)	15/15	12/15	
$f_9$	35 23(5)	127 18(5)	214 15(2)	263 14(3)	300 15(3)	335 17(3)	369 19(3)	15/ 15/15	$f_9$	1716 17(4)	3102 483(639)	3277 462(1208)	3379 454(587)	3455 450(574)	3594 447(829)	3727 446(534)	15/15	11/15	
$f_{10}$	349 8.5(3)	500 7.4(1)	574 7.6(2)	607 8.2(2)	626 9.3(1)	829 8.8(1)	880 10(0.8)	15/ 15/15	$f_{10}$	7413 131(93)	8661 241(106)	10735 518(237)	13641 810(357)	14920 $\infty$	17073 $\infty$	17476 $\infty$	15/15	0/15	
$f_{11}$	143 11(4)	202 12(1)	763 4.2(0.7)	977 4.0(0.6)	1177 4.0(0.5)	1467 4.2(0.7)	1673 4.5(0.3)	15/ 15/15	$f_{11}$	1002 28(16)	2228 32(27)	6278 18(8)	8586 18(5)	9762 20(6)	12285 23(7)	14831 24(10)	15/15	15/15	
$f_{12}$	108 55(32)	268 35(15)	371 31(52)	413 32(12)	461 32(43)	1303 15(7)	1494 15(10)	15/ 15/15	$f_{12}$	1042 612(0.7)	1938 1664(1932)	2740 1679(1637)	3156 3316(4634)	4140 $\infty$	12407 $\infty$	13827 $\infty$	15/15	0/15	
$f_{13}$	132 12(2)	195 13(2)	250 16(2)	319 17(2)	1310 5.1(0.3)	1752 5.5(0.7)	2255 6(0.2)	15/ 15/15	$f_{13}$	652 1540(1532)	2021 3955(3457)	2751 9439(11613)	3507 15945(20780)	18749 $\infty$	24455 $\infty$	30201 $\infty$	15/15	0/15	
$f_{14}$	10 2.2(2)	41 10(2)	58 16(3)	90 16(2)	139 16(3)	251 16(1)	476 13(1)	15/ 15/15	$f_{14}$	75 11(2)	239 9.2(0.7)	304 11(0.7)	451 12(1)	932 12(3)	1648 113(34)	15661 1236(259)	15/15	0/15	
$f_{15}$	511 144(979)	9310 $\infty$	19369 $\infty$	19743 $\infty$	20073 $\infty$	20769 $\infty$	21359 $\infty$	14/ 0/15	$f_{15}$	30378 $\infty$	1.5e5 $\infty$	3.1e5 $\infty$	3.2e5 $\infty$	3.2e5 $\infty$	4.5e5 $\infty$	4.6e5 $\infty$	15/15	0/15	
$f_{16}$	120 5.6(6)	612 602(1632)	2662 1034(1312)	10163 271(221)	10449 383(669)	11644 558(879)	12095 538(702)	15/ 2/15	$f_{16}$	1384 5778(9382)	27265 $\infty$	77015 $\infty$	1.4e5 $\infty$	1.9e5 $\infty$	2.0e5 $\infty$	2.2e5 $\infty$	15/15	0/15	
$f_{17}$	5.2 3.3(5)	215 3.5(1)	899 82(0.8)	2861 26(0.2)	3669 100(68)	6351 181(315)	7934 820(1353)	15/ 1/15	$f_{17}$	63 8.6(5)	1030 2570(3853)	4005 13805(17811)	12242 $\infty$	30677 $\infty$	56288 $\infty$	80472 $\infty$	15/15	0/15	
$f_{18}$	103 4.3(2)	378 4.6(1)	3968 64(126)	8451 104(326)	9280 216(242)	10905 $\infty$	12469 $\infty$	15/ 0/15	$f_{18}$	621 4.8(2)	3972 13838(12880)	19561 $\infty$	28555 $\infty$	67569 $\infty$	1.3e5 $\infty$	1.5e5 $\infty$	15/15	0/15	
$f_{19}$	1 35(39)	1 3883(1944)	242 8329(7226)	1.0e5 $\infty$	1.2e5 $\infty$	1.2e5 $\infty$	1.2e5 $\infty$	15/ 0/15	$f_{19}$	1 628(284)	1 4.5e6 (7e6)	3.4e5 $\infty$	4.7e6 $\infty$	6.2e6 $\infty$	6.7e6 $\infty$	6.7e6 $\infty$	15/15	0/15	
$f_{20}$	16 8.9(6)	851 2.9(1)	38111 52(79)	51362 39(53)	54470 37(92)	54861 36(68)	55313 36(32)	14/ 5/15	$f_{20}$	82 16(3)	46150 563(628)	3.1e6 $\infty$	5.5e6 $\infty$	5.5e6 $\infty$	5.6e6 $\infty$	5.6e6 $\infty$	14/15	0/15	
$f_{21}$	41 2.8(3)	1157 756(1511)	1674 895(894)	1692 886(1033)	1705 879(732)	1729 868(1733)	1757 854(569)	14/ 6/15	$f_{21}$	561 1782(3560)	6541 3972(4277)	14103 3967(3188)	14318 3908(2442)	14643 3821(5936)	15567 3594(2824)	17589 3181(3238)	15/15	1/15	
$f_{22}$	71 3.9(3)	386 2957(6463)	938 2931(3462)	980 2804(2802)	1008 2726(4706)	1040 2643(2881)	1068 2576(3976)	14/ 4/15	$f_{22}$	467 2144(4281)	5580 1970(2328)	23491 $\infty$	24163 $\infty$	24948 $\infty$	26847 $\infty$	1.3e5 $\infty$	12/15	0/15	
$f_{23}$	3.0 2.8(2)	518 17(10)	14249 985(1192)	27890 $\infty$	31654 $\infty$	33030 $\infty$	34256 $\infty$	15/ 0/15	$f_{23}$	3.2 1.7(1)	1614 4993(4956)	67457 $\infty$	3.7e5 $\infty$	4.9e5 $\infty$	8.1e5 $\infty$	8.4e5 $\infty$	15/15	0/15	
$f_{24}$	1622 48(309)	2.2e5 $\infty$	6.4e6 $\infty$	9.6e6 $\infty$	9.6e6 $\infty$	1.3e7 $\infty$	1.3e7 $\infty$	3/ 0/15	$f_{24}$	1.3e6 $\infty$	7.5e6 $\infty$	5.2e7 $\infty$	5.2e7 $\infty$	5.2e7 $\infty$	5.2e7 $\infty$	5.2e7 $\infty$	3/15	0/15	

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009. The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear in the second row of each cell, the best ERT in the first. The different target  $\Delta f$ -values are shown in the top row. #succ is the number of trials that reached the (final) target  $f_{\text{opt}} + 10^{-8}$ . The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Bold entries are statistically significantly better (according to the rank-sum test) compared to the best algorithm in BBOB-2009, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k > 1$  is following the  $\downarrow$  symbol, with Bonferroni correction by the number of functions.



**Figure 4: ERT loss ratios (see Figure 3 for details). Each cross (+) represents a single function, the line is the geometric mean.**