

Figure 1: Expected number of f -evaluations (ERT, lines) to reach $f_{\text{opt}} + \Delta f$; median number of f -evaluations (+) to reach the most difficult target that was reached not always but at least once; maximum number of f -evaluations in any trial (\times); interquartile range with median (notched boxes) of simulated runlengths to reach $f_{\text{opt}} + \Delta f$; all values are divided by dimension and plotted as \log_{10} values versus dimension. Shown is the ERT for targets just not reached by the artificial GECCO-BBOB-2009 best algorithm within the given budget $k \times \text{DIM}$, where k is shown in the legend. Numbers above ERT-symbols indicate the number of trials reaching the respective target. The light thick line with diamonds indicates the respective best result from BBOB-2009 for the most difficult target. Slanted grid lines indicate a scaling with $\mathcal{O}(\text{DIM})$ compared to $\mathcal{O}(1)$ when using the respective 2009 best algorithm.

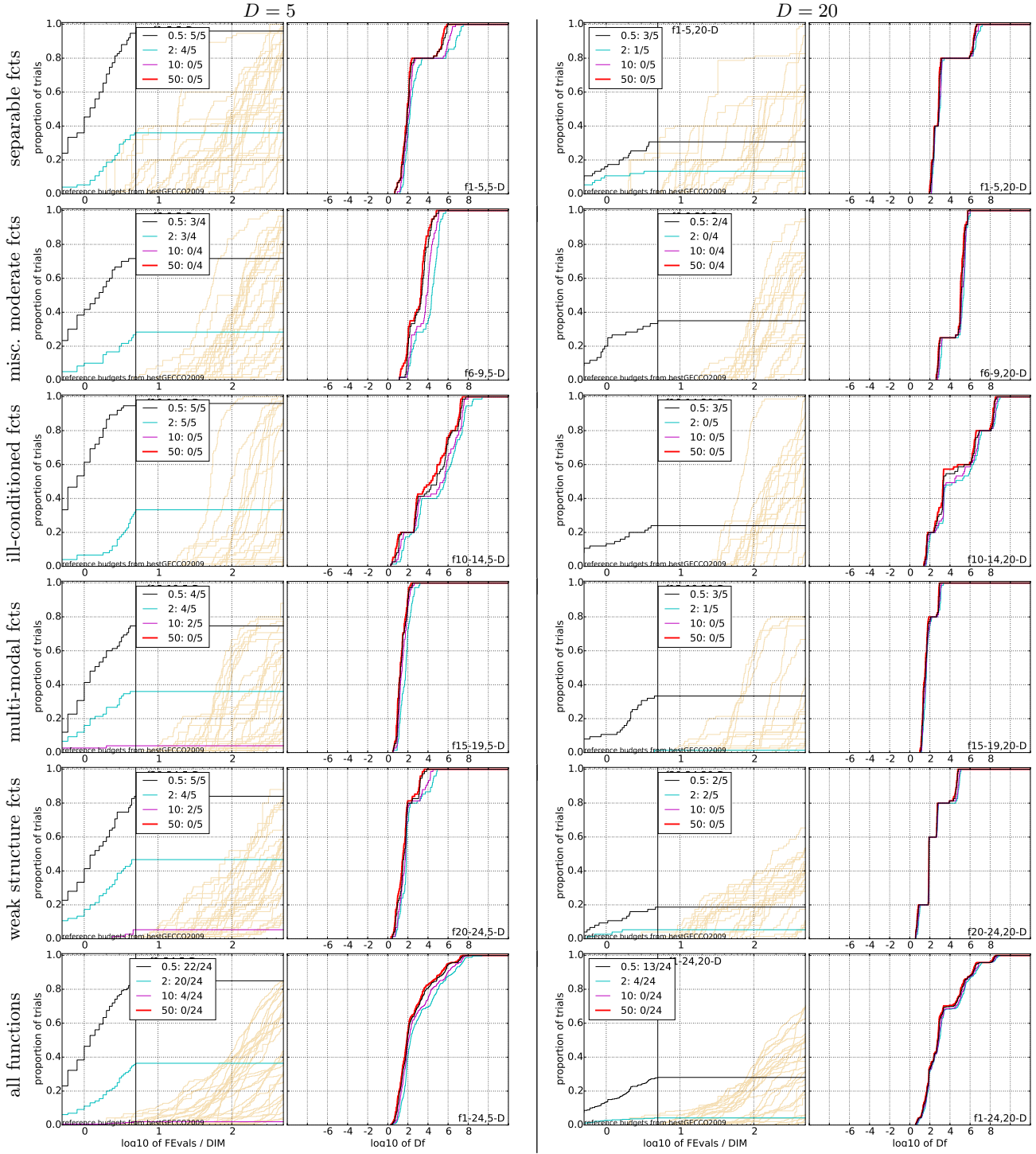


Figure 2: Empirical cumulative distribution functions (ECDF), plotting the fraction of trials with an outcome not larger than the respective value on the x -axis. Left subplots: ECDF of number of function evaluations (FEvals) divided by search space dimension D , to fall below $f_{\text{opt}} + \Delta f$ where Δf is the target just not reached by the GECCO-BBOB-2009 best algorithm within a budget of $k \times \text{DIM}$ evaluations, where k is the first value in the legend. Legends indicate for each target the number of functions that were solved in at least one trial within the displayed budget. Right subplots: ECDF of the best achieved Δf for running times of $0.5D, 1.2D, 3D, 10D, 100D, 1000D, \dots$ function evaluations (from right to left cycling cyan-magenta-black...) and final Δf -value (red), where Δf and Df denote the difference to the optimal function value. Light brown lines in the background show ECDFs for the most difficult target of all algorithms benchmarked during BBOB-2009.

5-D							20-D						
#FEs/D	0.5	1.2	3	10	50	#succ	#FEs/D	0.5	1.2	3	10	50	#succ
f₁	<i>2.5e+1:4.8</i> 2.2(0.9)	<i>1.6e+1:7.6</i> 6.1(8)	<i>1.0e-8:12</i> ∞	<i>1.0e-8:12</i> ∞	<i>1.0e-8:12</i> ∞	15/15 0/15	f₁	<i>6.3e+1:24</i> ∞	<i>4.0e+1:42</i> ∞	<i>1.0e-8:43</i> ∞	<i>1.0e-8:43</i> ∞	<i>1.0e-8:43</i> ∞	15/15 0/15
f₂	<i>1.6e+6:2.9</i> 2.0(2)	<i>4.0e+5:11</i> 1.8(0.8)	<i>4.0e+4:15</i> 24(13)	<i>6.3e+2:58</i> ∞	<i>1.0e-8:95</i> ∞	15/15 0/15	f₂	<i>4.0e+6:29</i> 0.74(1)	<i>2.5e+6:42</i> 1.6(3)	<i>1.0e+5:65</i> ∞	<i>1.0e+4:207</i> ∞	<i>1.0e-8:412</i> ∞	15/15 0/15
f₃	<i>1.6e+2:4.1</i> 2.1(2)	<i>1.0e+2:15</i> 2.2(2)	<i>6.3e+1:23</i> 15(16)	<i>2.5e+1:73</i> ∞	<i>1.0e+1:716</i> ∞	15/15 0/15	f₃	<i>6.3e+2:33</i> 45(23)	<i>4.0e+2:44</i> ∞	<i>1.6e+2:109</i> ∞	<i>1.0e+2:255</i> ∞	<i>2.5e+1:3277</i> ∞	15/15 0/15
f₄	<i>2.5e+2:2.6</i> 2.8(4)	<i>1.6e+2:10</i> 2.5(3)	<i>1.0e+2:19</i> 9.1(10)	<i>4.0e+1:65</i> ∞	<i>1.6e+1:434</i> ∞	15/15 0/15	f₄	<i>6.3e+2:22</i> ∞	<i>4.0e+2:91</i> ∞	<i>2.5e+2:250</i> ∞	<i>1.6e+2:332</i> ∞	<i>6.3e+1:1927</i> ∞	15/15 0/15
f₅	<i>6.3e+1:4.0</i> 2.8(4)	<i>4.0e+1:10</i> 5.1(3)	<i>1.0e-8:10</i> ∞	<i>1.0e-8:10</i> ∞	<i>1.0e-8:10</i> ∞	15/15 0/15	f₅	<i>2.5e+2:19</i> 7.0(6)	<i>1.6e+2:34</i> ∞	<i>1.0e-8:41</i> ∞	<i>1.0e-8:41</i> ∞	<i>1.0e-8:41</i> ∞	15/15 0/15
f₆	<i>1.0e+5:3.0</i> 1.7(3)	<i>2.5e+4:8.4</i> 1.5(1)	<i>1.0e+2:16</i> 23(25)	<i>2.5e+1:54</i> ∞	<i>2.5e-1:254</i> ∞	15/15 0/15	f₆	<i>2.5e+5:16</i> 10(14)	<i>6.3e+4:43</i> ∞	<i>1.6e+4:62</i> ∞	<i>1.6e+2:353</i> ∞	<i>1.6e+1:1078</i> ∞	15/15 0/15
f₇	<i>1.6e+2:4.2</i> 1.5(1)	<i>1.0e+2:6.2</i> 2.8(2)	<i>2.5e+1:20</i> 4.2(6)	<i>4.0e+0:54</i> ∞	<i>1.0e+0:324</i> ∞	15/15 0/15	f₇	<i>1.0e+3:11</i> 2.3(3)	<i>4.0e+2:39</i> ∞	<i>2.5e+2:74</i> ∞	<i>6.3e+1:319</i> ∞	<i>1.0e+1:1351</i> ∞	15/15 0/15
f₈	<i>1.0e+4:4.6</i> 2.3(4)	<i>6.3e+3:6.8</i> 1.7(1)	<i>1.0e+3:18</i> 20(25)	<i>6.3e+1:54</i> ∞	<i>1.6e+0:258</i> ∞	15/15 0/15	f₈	<i>4.0e+4:19</i> ∞	<i>2.5e+4:35</i> ∞	<i>4.0e+3:67</i> ∞	<i>2.5e+2:231</i> ∞	<i>1.6e+1:1470</i> ∞	15/15 0/15
f₉	<i>2.5e+1:20</i> ∞	<i>1.6e+1:26</i> ∞	<i>1.0e+1:35</i> ∞	<i>4.0e+0:62</i> ∞	<i>1.6e-2:256</i> ∞	15/15 0/15	f₉	<i>1.0e+2:357</i> ∞	<i>6.3e+1:560</i> ∞	<i>4.0e+1:684</i> ∞	<i>2.5e+1:756</i> ∞	<i>1.0e+1:1716</i> ∞	15/15 0/15
f₁₀	<i>2.5e+6:2.9</i> 2.2(3)	<i>6.3e+5:7.0</i> 3.3(5)	<i>2.5e+5:17</i> 3.2(4)	<i>6.3e+3:54</i> ∞	<i>2.5e+1:297</i> ∞	15/15 0/15	f₁₀	<i>1.6e+6:15</i> 29(67)	<i>1.0e+6:27</i> 54(53)	<i>4.0e+5:70</i> ∞	<i>6.3e+4:231</i> ∞	<i>4.0e+3:1015</i> ∞	15/15 0/15
f₁₁	<i>1.0e+6:3.0</i> 1.7(1)	<i>6.3e+4:6.2</i> 5.6(3)	<i>6.3e+2:16</i> 11(8)	<i>6.3e+1:74</i> ∞	<i>6.3e-1:298</i> ∞	15/15 0/15	f₁₁	<i>4.0e+4:11</i> 2.6(1)	<i>2.5e+3:27</i> 1.8(2)	<i>1.6e+2:313</i> ∞	<i>1.0e+2:481</i> ∞	<i>1.0e+1:1002</i> ∞	15/15 0/15
f₁₂	<i>4.0e+7:3.6</i> 1.3(0.5)	<i>1.6e+7:7.6</i> 3.7(3)	<i>1.6e+6:19</i> 19(15)	<i>1.6e+4:52</i> ∞	<i>1.0e+0:268</i> ∞	15/15 0/15	f₁₂	<i>1.0e+8:23</i> 64(59)	<i>6.3e+7:39</i> ∞	<i>2.5e+7:76</i> ∞	<i>4.0e+6:209</i> ∞	<i>1.0e+1:1042</i> ∞	15/15 0/15
f₁₃	<i>1.0e+3:2.8</i> 2.2(3)	<i>6.3e+2:8.4</i> 3.3(4)	<i>4.0e+2:17</i> 22(31)	<i>6.3e+1:52</i> ∞	<i>6.3e-2:264</i> ∞	15/15 0/15	f₁₃	<i>1.6e+3:28</i> ∞	<i>1.0e+3:64</i> ∞	<i>6.3e+2:79</i> ∞	<i>4.0e+1:211</i> ∞	<i>2.5e+0:1724</i> ∞	15/15 0/15
f₁₄	<i>1.6e+1:3.0</i> 2.9(2)	<i>1.0e+1:10</i> 2.4(2)	<i>6.3e+0:15</i> 2.2(2)	<i>2.5e-1:53</i> ∞	<i>1.0e-5:251</i> ∞	15/15 0/15	f₁₄	<i>2.5e+1:15</i> ∞	<i>1.6e+1:42</i> ∞	<i>1.0e+1:75</i> ∞	<i>1.6e+0:219</i> ∞	<i>6.3e-4:1106</i> ∞	15/15 0/15
f₁₅	<i>1.6e+2:3.0</i> 3.1(6)	<i>1.0e+2:13</i> 2.4(2)	<i>6.3e+1:24</i> 7.0(4)	<i>4.0e+1:55</i> ∞	<i>1.6e+1:289</i> ∞	5/5 0/15	f₁₅	<i>6.3e+2:15</i> 44(53)	<i>4.0e+2:67</i> ∞	<i>2.5e+2:292</i> ∞	<i>1.6e+2:846</i> ∞	<i>1.0e+2:1671</i> ∞	15/15 0/15
f₁₆	<i>4.0e+1:4.8</i> 1.2(0.9)	<i>2.5e+1:16</i> 1.6(3)	<i>1.6e+1:46</i> 2.3(2)	<i>1.0e+1:120</i> ∞	<i>4.0e+0:334</i> ∞	15/15 0/15	f₁₆	<i>4.0e+1:26</i> 2.0(2)	<i>2.5e+1:127</i> 12(24)	<i>1.6e+1:540</i> ∞	<i>1.6e+1:540</i> ∞	<i>1.0e+1:1384</i> ∞	15/15 0/15
f₁₇	<i>1.0e+1:5.2</i> 2.3(2)	<i>6.3e+0:26</i> 2.9(4)	<i>4.0e+0:57</i> 3.0(5)	<i>2.5e+0:110</i> ∞	<i>6.3e-1:412</i> ∞	15/15 0/15	f₁₇	<i>1.6e+1:11</i> 7.8(10)	<i>1.0e+1:63</i> ∞	<i>6.3e+0:305</i> ∞	<i>4.0e+0:468</i> ∞	<i>1.0e+0:1030</i> ∞	15/15 0/15
f₁₈	<i>6.3e+1:3.4</i> 2.5(6)	<i>4.0e+1:7.2</i> 3.7(6)	<i>2.5e+1:20</i> 2.1(3)	<i>1.6e+1:58</i> 6.0(8)	<i>1.6e+0:318</i> ∞	15/15 0/15	f₁₈	<i>4.0e+1:116</i> ∞	<i>2.5e+1:252</i> ∞	<i>1.6e+1:430</i> ∞	<i>1.0e+1:621</i> ∞	<i>4.0e+0:1090</i> ∞	15/15 0/15
f₁₉	<i>1.6e-1:172</i> ∞	<i>1.0e-1:242</i> ∞	<i>6.3e-2:675</i> ∞	<i>4.0e-2:3078</i> ∞	<i>2.5e-2:4946</i> ∞	15/15 0/15	f₁₉	<i>1.6e-1:2.5e5</i> ∞	<i>1.0e-1:3.4e5</i> ∞	<i>6.3e-2:3.4e5</i> ∞	<i>4.0e-2:3.4e5</i> ∞	<i>2.5e-2:3.4e5</i> ∞	3/15 0/15
f₂₀	<i>6.3e+3:5.1</i> 1.5(2)	<i>4.0e+3:8.4</i> 1.1(1)	<i>4.0e+1:15</i> 24(18)	<i>2.5e+0:69</i> ∞	<i>1.0e+0:851</i> ∞	15/15 0/15	f₂₀	<i>1.6e+4:38</i> 37(36)	<i>1.0e+4:42</i> 33(28)	<i>2.5e+2:62</i> ∞	<i>2.5e+0:250</i> ∞	<i>1.6e+0:2536</i> ∞	15/15 0/15
f₂₁	<i>4.0e+1:3.9</i> 2.0(2)	<i>2.5e+1:11</i> 2.0(3)	<i>1.6e+1:31</i> 3.7(3)	<i>6.3e+0:73</i> ∞	<i>1.6e+0:347</i> ∞	5/5 0/15	f₂₁	<i>6.3e+1:36</i> ∞	<i>4.0e+1:77</i> ∞	<i>4.0e+1:77</i> ∞	<i>1.6e+1:456</i> ∞	<i>4.0e+0:1094</i> ∞	15/15 0/15
f₂₂	<i>6.3e+1:3.6</i> 2.8(0.6)	<i>4.0e+1:15</i> 1.5(2)	<i>2.5e+1:32</i> 1.9(2)	<i>1.0e+1:71</i> 1.7(2)	<i>1.6e+0:341</i> ∞	5/5 0/15	f₂₂	<i>6.3e+1:45</i> ∞	<i>4.0e+1:68</i> ∞	<i>4.0e+1:68</i> ∞	<i>1.6e+1:231</i> ∞	<i>6.3e+0:1219</i> ∞	15/15 0/15
f₂₃	<i>1.0e+1:3.0</i> 2.1(3)	<i>6.3e+0:9.0</i> 1.8(1)	<i>4.0e+0:33</i> 3.2(3)	<i>2.5e+0:84</i> 4.5(4)	<i>1.0e+0:518</i> ∞	15/15 0/15	f₂₃	<i>6.3e+0:29</i> 1.6(2)	<i>4.0e+0:118</i> 3.6(2)	<i>2.5e+0:306</i> ∞	<i>2.5e+0:306</i> ∞	<i>1.0e+0:1614</i> ∞	15/15 0/15
f₂₄	<i>6.3e+1:15</i> 4.3(4)	<i>4.0e+1:37</i> ∞	<i>4.0e+1:37</i> ∞	<i>2.5e+1:118</i> ∞	<i>1.6e+1:692</i> ∞	15/15 0/15	f₂₄	<i>2.5e+2:208</i> ∞	<i>1.6e+2:918</i> ∞	<i>1.0e+2:6628</i> ∞	<i>6.3e+1:9885</i> ∞	<i>4.0e+1:31629</i> ∞	15/15 0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009. The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear in the second row of each cell, the best ERT (preceded by the target Δf -value in *italics*) in the first. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Bold entries are statistically significantly better (according to the rank-sum test) compared to the best algorithm in BBOB-2009, with $p = 0.05$ or $p = 10^{-k}$ when the number $k > 1$ is following the \downarrow symbol, with Bonferroni correction by the number of functions.

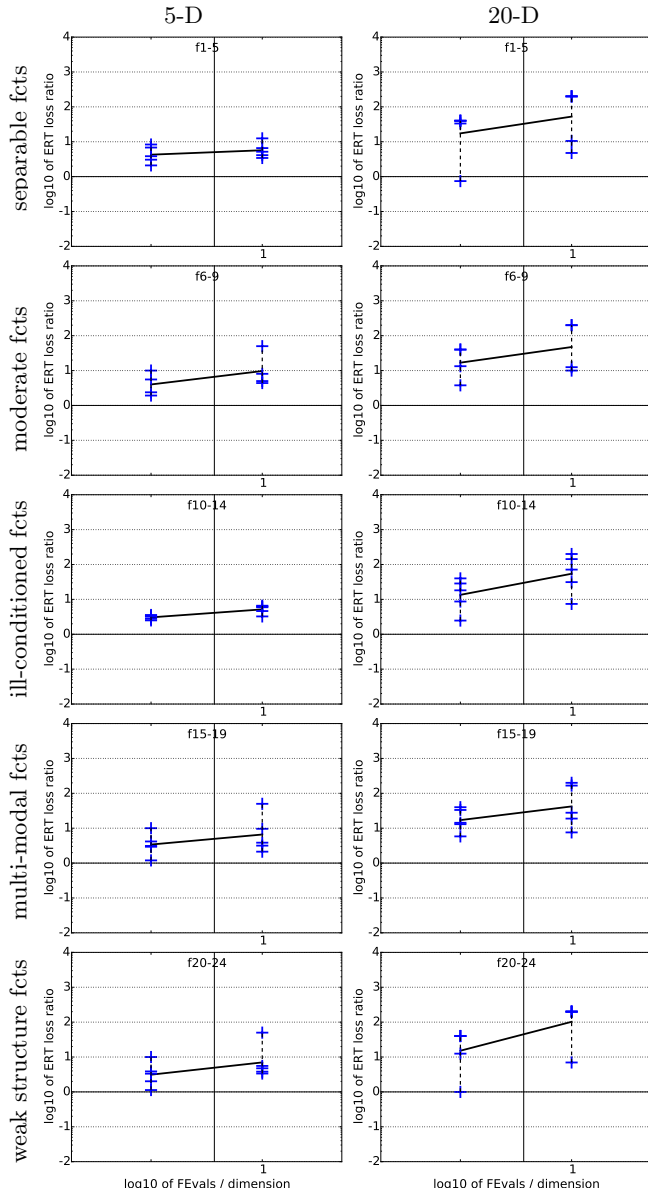


Figure 4: ERT loss ratios (see Figure 3 for details). Each cross (+) represents a single function, the line is the geometric mean.