

# Modyfikacje/hybrydyzacje algorytmu PSO w zadaniu optymalizacji globalnej wielowymiarowej funkcji ciaglej

## PSO-DE Hybrid

Jakub Ruszkowski, Mateusz Kaczmarek

### ABSTRACT

Dokumentacja uzyskanych wyników hybrydy PSO-DE

### Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

### General Terms

Algorithms

### Keywords

Benchmarking, PSODE, Optymalizacja wielowymiarowej funkcji ciaglej

of trials that actually reached  $f_t$  [?, ?]. **Statistical significance** is tested with the rank-sum test for a given target  $\Delta f_t$  ( $10^{-8}$  as in Figure 1) using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by  $-1$ ), or, if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

## 1. CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the **PSO-DE Hybrid** on the function  $f_8$  with restarts for at least 30 seconds and until a maximum budget equal to  $400(D + 2)$  is reached. The code was run on a **Mac Intel(R) Core(TM) i5-2400S CPU @ 2.50GHz** with **1** processor and **4** cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20, **40** equals  **$x.x$ ,  $x.x$ ,  $x.x$ ,  $x.x$ ,  $xxx$ , and  $xxx$**  milliseconds respectively.

repeat the above for the second algorithm

## 2. RESULTS

Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2 and 3 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

GECCO'13, July 6-10, 2013, Amsterdam, The Netherlands.

Copyright 2013 ACM TBA ...\$15.00.

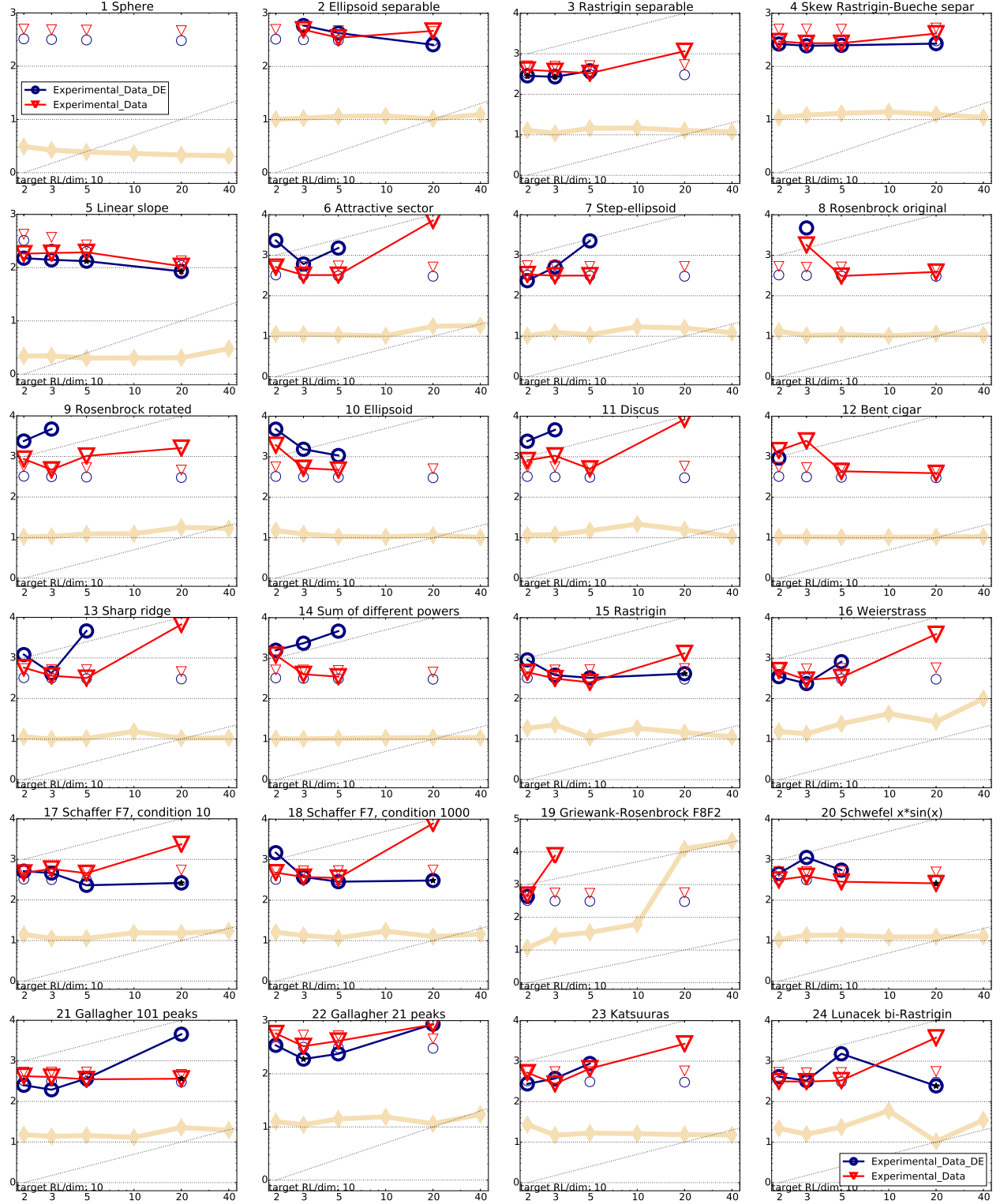


Figure 1: Expected running time (ERT in number of  $f$ -evaluations as  $\log_{10}$  value) divided by dimension versus dimension. The target function value is chosen such that the bestGECCO2009 artificial algorithm just failed to achieve an ERT of  $10 \times \text{DIM}$ . Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with  $p < 0.01$  and Bonferroni correction number of dimensions (six). Legend:  $\circ$ :Experimental Data DE,  $\nabla$ :Experimental Data.

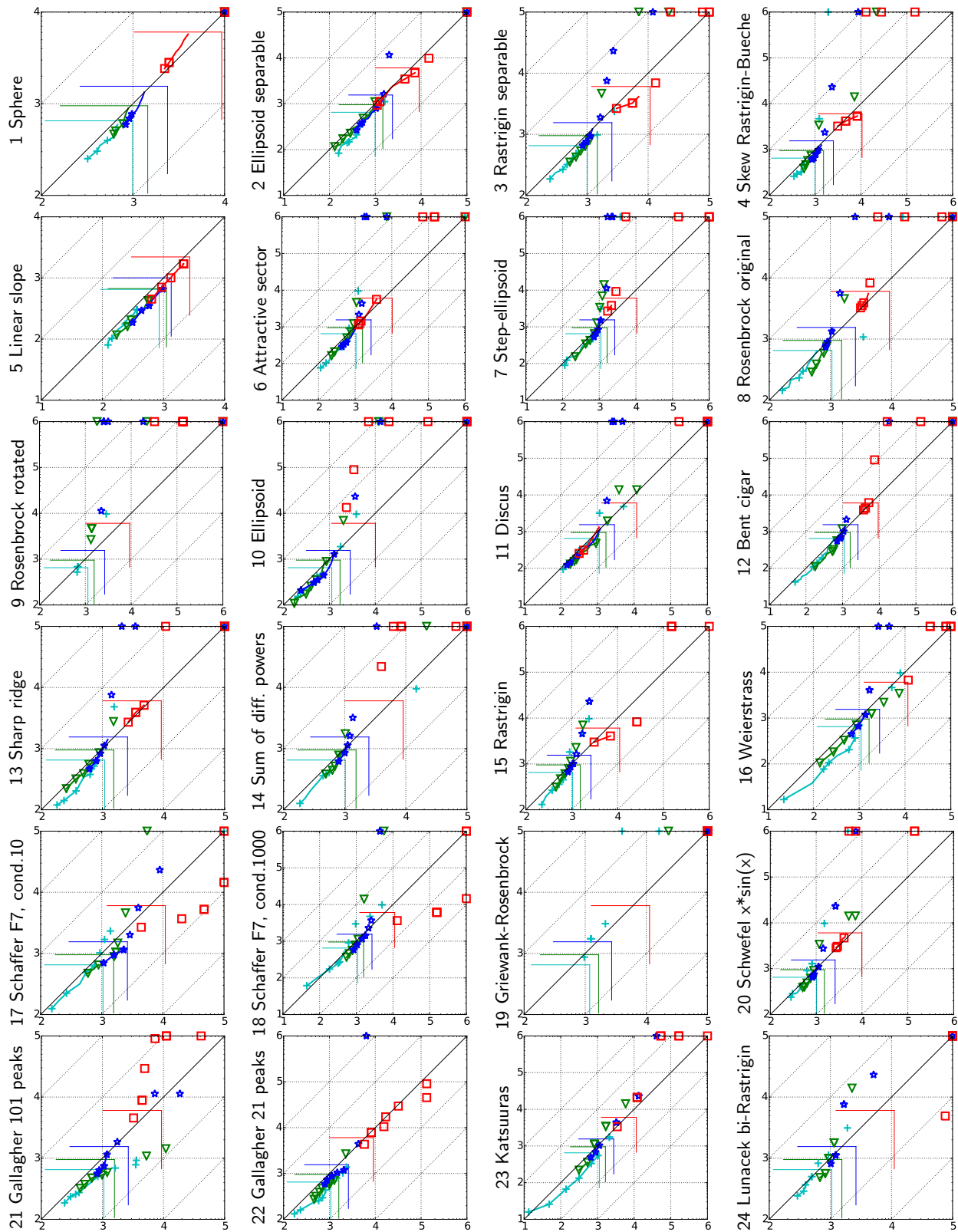
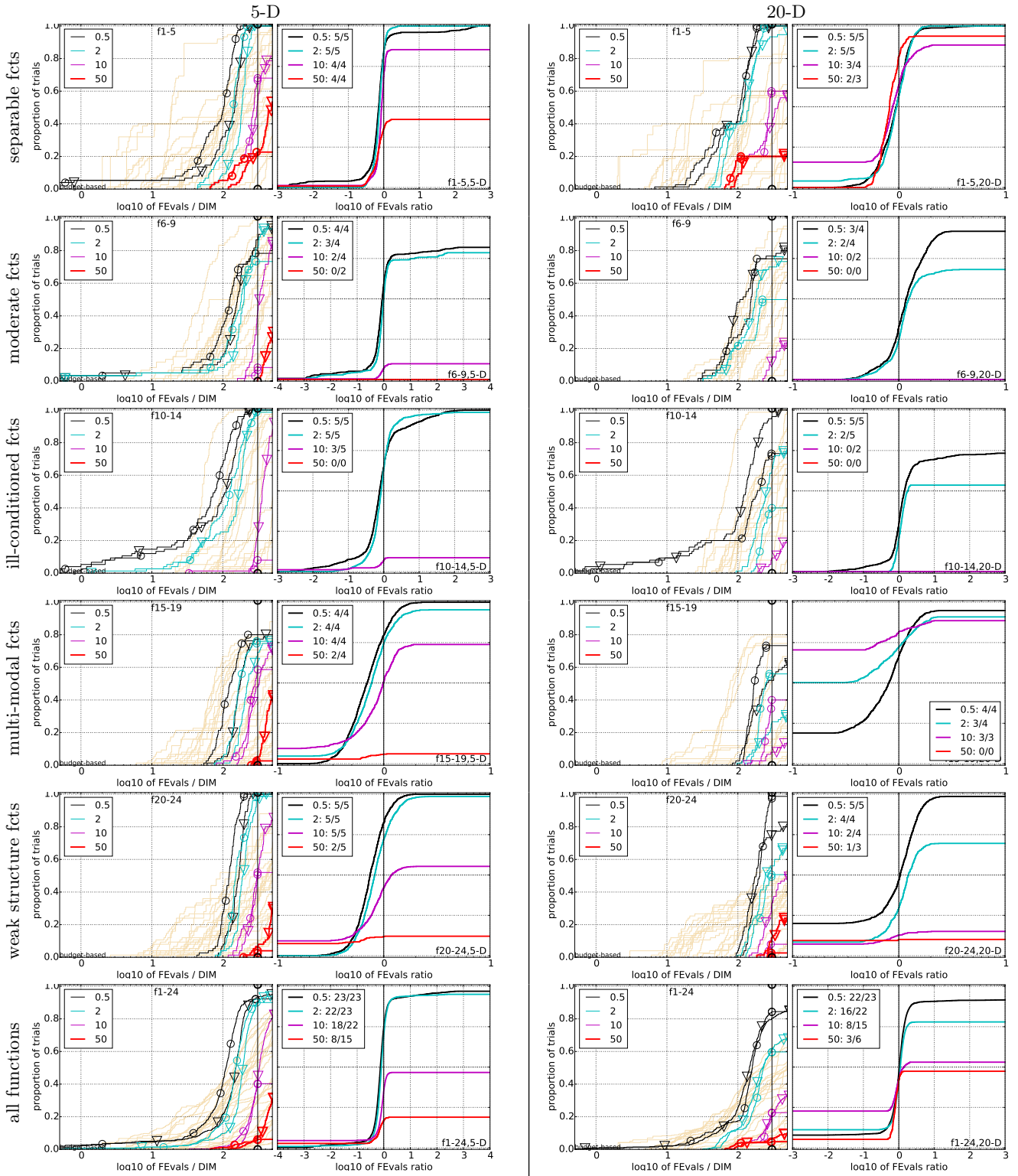


Figure 2: Expected running time (ERT in  $\log_{10}$  of number of function evaluations) of Experimental Data DE ( $y$ -axis) versus Experimental Data ( $x$ -axis) for 8 runlength-based target function values for budgets between  $0.5 \times \text{DIM}$  and  $50 \times \text{DIM}$  evaluations. Each runlength-based target  $f$ -value is chosen such that the ERTs of the bestGECCO2009 artificial algorithm for the given and a slightly easier target bracket the reference budget. Markers on the upper or right edge indicate that the respective target value was never reached. Markers represent dimension: 2:+, 3:▽, 5:\*, 10:○, 20:□, 40:◇.



**Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right).** Left sub-columns: ECDF of the number of function evaluations divided by dimension  $D$  (FEvals/ $D$ ) to fall below  $f_{\text{opt}} + \Delta f$  for Experimental Data DE ( $\circ$ ) and Experimental Data BBOB-2009 ( $\nabla$ ) where  $\Delta f$  is the target just not reached by the GECCO-BBOB-2009 best algorithm within a budget of  $k \times \text{DIM}$  evaluations, with  $k$  being the value in the legend. Right sub-columns: ECDF of FEval ratios of Experimental Data DE divided by Experimental Data BBOB-2009 for run-length-based targets; all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being  $> 0$  or  $< 1$ . The legends indicate the target budget of  $k \times \text{DIM}$  evaluations and, after the colon, the number of functions that were solved in at least one trial (Experimental Data DE first).

## 5-D

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f<sub>1</sub></b>	<i>2.5e+1:4.8</i>	<i>1.6e+1:7.6</i>	<i>1.0e-8:12</i>	<i>1.0e-8:12</i>	<i>1.0e-8:12</i>	15/15
1: Exp	<b>124(24)*<sup>3</sup></b>	<b>91(24)*</b>	$\infty$	$\infty$	$\infty$ 1600	0/15
2: Exp	174(38)	121(16)	$\infty$	$\infty$	$\infty$ 2300	0/15
<b>f<sub>2</sub></b>	<i>1.6e+6:2.9</i>	<i>4.0e+5:11</i>	<i>4.0e+4:15</i>	<i>6.3e+2:58</i>	<i>1.0e-8:95</i>	15/15
1: Exp	93(57)	35(13)	45(12)	37(34)	$\infty$ 1600*	0/15
2: Exp	137(39)	47(26)	60(15)	30(5)	$\infty$ 2300	0/15
<b>f<sub>3</sub></b>	<i>1.6e+2:4.1</i>	<i>1.0e+2:15</i>	<i>6.3e+1:23</i>	<i>2.5e+1:73</i>	<i>1.0e+1:716</i>	15/15
1: Exp	159(33)	52(18)	40(9)	26(17)	32(44)	1/15
2: Exp	206(35)	67(10)	48(7)	23(3)	3.8(2)	12/15
<b>f<sub>4</sub></b>	<i>2.5e+2:2.6</i>	<i>1.6e+2:10</i>	<i>1.0e+2:19</i>	<i>4.0e+1:65</i>	<i>1.6e+1:434</i>	15/15
1: Exp	<b>235(46)*<sup>2</sup></b>	<b>68(11)*<sup>3</sup></b>	<b>42(11)*</b>	19(2)	54(49)	1/15
2: Exp	330(36)	98(14)	55(9)	21(3)	5.4(6)	13/15
<b>f<sub>5</sub></b>	<i>6.3e+1:4.0</i>	<i>4.0e+1:10</i>	<i>1.0e-8:10</i>	<i>1.0e-8:10</i>	<i>1.0e-8:10</i>	15/15
1: Exp	47(46)	30(34)	<b>66(32)*</b>	<b>66(28)*</b>	$\infty$ 1600	0/15
2: Exp	78(77)	44(35)	97(23)	97(26)	97(27)	15/15
<b>f<sub>6</sub></b>	<i>1.0e+5:3.0</i>	<i>6.3e+3:8.4</i>	<i>1.0e+2:16</i>	<i>2.5e+1:54</i>	<i>2.5e-1:254</i>	15/15
1: Exp	96(96)	43(37)	76(55)	139(192)	$\infty$ 1600	0/15
2: Exp	146(135)	60(43)	70(16)	30(10)	29(18)	5/15
<b>f<sub>7</sub></b>	<i>1.6e+2:4.2</i>	<i>1.0e+2:6.2</i>	<i>2.5e+1:20</i>	<i>4.0e+0:54</i>	<i>1.0e+0:324</i>	15/15
1: Exp	127(53)	109(28)	75(15)	209(209)	$\infty$ 1600	0/15
2: Exp	172(39)	132(12)	55(11)	29(11)	7.2(0.5)	13/15
<b>f<sub>8</sub></b>	<i>1.0e+4:4.6</i>	<i>6.3e+3:6.8</i>	<i>1.0e+3:18</i>	<i>6.3e+1:54</i>	<i>1.6e+0:258</i>	15/15
1: Exp	161(46)	117(19)	67(51)	$\infty$	$\infty$ 1600	0/15
2: Exp	181(38)	125(26)	57(15)	28(5)	$\infty$ 2500	0/15
<b>f<sub>9</sub></b>	<i>2.5e+1:20</i>	<i>1.6e+1:26</i>	<i>1.0e+1:35</i>	<i>4.0e+0:62</i>	<i>1.6e-2:256</i>	15/15
1: Exp	573(533)	$\infty$	$\infty$	$\infty$	$\infty$ 1600	0/15
2: Exp	113(72)	98(129)	77(76)	83(33)	$\infty$ 2400	0/15
<b>f<sub>10</sub></b>	<i>2.5e+6:2.9</i>	<i>6.3e+5:7.0</i>	<i>2.5e+5:17</i>	<i>6.3e+3:54</i>	<i>2.5e+1:297</i>	15/15
1: Exp	73(50)	44(50)	23(11)	100(56)	$\infty$ 1600	0/15
2: Exp	84(78)	66(59)	38(42)	44(30)	$\infty$ 2700	0/15
<b>f<sub>11</sub></b>	<i>1.0e+6:3.0</i>	<i>6.3e+4:6.2</i>	<i>6.3e+2:16</i>	<i>6.3e-1:74</i>	<i>6.3e-1:298</i>	15/15
1: Exp	41(47)	29(24)	36(50)	$\infty$	$\infty$ 1600	0/15
2: Exp	49(54)	35(35)	51(32)	34(20)	$\infty$ 2800	0/15
<b>f<sub>12</sub></b>	<i>4.0e+7:3.6</i>	<i>1.6e+7:7.6</i>	<i>4.0e+6:19</i>	<i>1.6e+4:52</i>	<i>1.0e+0:268</i>	15/15
1: Exp	154(55)	86(38)	53(56)	$\infty$	$\infty$ 1600	0/15
2: Exp	197(100)	115(12)	55(11)	42(18)	$\infty$ 2500	0/15
<b>f<sub>13</sub></b>	<i>1.0e+9:2.8</i>	<i>6.3e+8:8.4</i>	<i>4.0e+2:17</i>	<i>6.3e-1:52</i>	<i>6.3e-2:264</i>	15/15
1: Exp	166(72)	75(21)	50(22)	445(224)	$\infty$ 1600	0/15
2: Exp	219(70)	94(8)	55(8)	31(7)	$\infty$ 2500	0/15
<b>f<sub>14</sub></b>	<i>1.0e+1:3.0</i>	<i>1.0e+1:10</i>	<i>6.3e+0:15</i>	<i>2.5e-1:53</i>	<i>1.0e-5:251</i>	15/15
1: Exp	207(104)	87(30)	74(54)	442(450)	$\infty$ 1600	0/15
2: Exp	266(63)	103(22)	73(19)	33(7)	$\infty$ 2400	0/15
<b>f<sub>15</sub></b>	<i>1.6e+2:3.0</i>	<i>1.0e+2:13</i>	<i>6.3e+1:24</i>	<i>4.0e+1:55</i>	<i>1.6e+1:289</i>	5/5
1: Exp	224(115)	66(13)	41(9)	29(30)	80(81)	1/15
2: Exp	277(79)	73(18)	44(6)	23(4)	8.4(9)	12/15
<b>f<sub>16</sub></b>	<i>4.0e+1:4.8</i>	<i>2.5e+1:16</i>	<i>1.6e+1:46</i>	<i>1.0e+1:120</i>	<i>4.0e+0:334</i>	15/15
1: Exp	94(32)	42(20)	26(19)	34(41)	$\infty$ 1600	0/15
2: Exp	142(52)	61(19)	29(11)	14(4)	14(8)	8/15
<b>f<sub>17</sub></b>	<i>1.0e+1:5.2</i>	<i>6.3e+0:26</i>	<i>4.0e+0:57</i>	<i>2.5e+0:110</i>	<i>6.3e-1:412</i>	15/15
1: Exp	135(48)	36(14)	20(5)	18(8)	56(31)	1/15
2: Exp	207(86)	61(64)	40(29)	26(41)	22(28)	4/15
<b>f<sub>18</sub></b>	<i>6.3e+1:3.4</i>	<i>4.0e+1:7.2</i>	<i>2.5e+1:20</i>	<i>1.6e+1:58</i>	<i>1.6e+0:318</i>	15/15
1: Exp	171(48)	110(21)	58(66)	24(23)	$\infty$ 1600	0/15
2: Exp	250(58)	146(52)	72(53)	30(21)	14(13)	8/15
<b>f<sub>19</sub></b>	<i>1.6e-1:172</i>	<i>1.0e-1:242</i>	<i>6.3e-2:675</i>	<i>4.0e-2:3078</i>	<i>2.5e-2:4946</i>	15/15
1: Exp	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 1600	0/15
2: Exp	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 2600	0/15
<b>f<sub>20</sub></b>	<i>6.3e+3:5.1</i>	<i>4.0e+3:8.4</i>	<i>4.0e+1:15</i>	<i>2.5e+0:69</i>	<i>1.0e+0:851</i>	15/15
1: Exp	127(34)	83(21)	64(10)	40(23)	$\infty$ 1600	0/15
2: Exp	161(36)	103(14)	70(10)	21(3)	8.6(7)	5/15
<b>f<sub>21</sub></b>	<i>4.0e+1:3.9</i>	<i>2.5e+1:11</i>	<i>1.6e+1:31</i>	<i>6.3e+0:73</i>	<i>1.6e+0:347</i>	5/5
1: Exp	<b>142(34)*</b>	<b>60(14)*</b>	<b>24(6)*<sup>2</sup></b>	25(8)	33(22)	2/15
2: Exp	209(42)	84(21)	34(6)	24(5)	21(15)	5/15
<b>f<sub>22</sub></b>	<i>6.3e+1:3.6</i>	<i>4.0e+1:15</i>	<i>2.5e+1:32</i>	<i>1.0e+1:71</i>	<i>1.6e+0:341</i>	5/5
1: Exp	164(52)	49(14)	27(9)	17(11)	$\infty$ 1600	0/15
2: Exp	221(73)	60(18)	35(39)	29(37)	19(28)	5/15
<b>f<sub>23</sub></b>	<i>1.0e+1:3.0</i>	<i>6.3e+0:9.0</i>	<i>4.0e+0:33</i>	<i>2.5e+0:84</i>	<i>1.0e+0:518</i>	15/15
1: Exp	164(55)	75(15)	31(9)	52(66)	$\infty$ 1600	0/15
2: Exp	220(62)	99(29)	34(13)	39(27)	79(77)	1/15
<b>f<sub>24</sub></b>	<i>6.3e+1:15</i>	<i>4.0e+1:37</i>	<i>4.0e+1:37</i>	<i>2.5e+1:118</i>	<i>1.6e+1:692</i>	15/15
1: Exp	56(8)	31(5)	31(6)	64(40)	33(21)	1/15
2: Exp	70(16)	33(7)	33(5)	14(3)	7.4(2)	7/15

## 20-D

#Fes/D	0.5	1.2	3	10	50	#succ
<b>f<sub>1</sub></b>	6.3e+1:24	4.0e+1:42	1.0e-8:43	1.0e-8:43	1.0e-8:43	15/15
1: Exp	100(8)	67(8)	∞	∞	∞6100	0/15
2: Exp	92(8)	59(9)*	∞	∞	∞9200	0/15
<b>f<sub>2</sub></b>	4.0e+6:29	2.5e+6:42	1.0e+5:65	1.0e+4:207	1.0e-8:412	15/15
1: Exp	33(8)	27(4)	48(5)	25(2)	∞6100* <sup>3</sup>	0/15
2: Exp	39(12)	30(6)	55(3)	45(25)	∞9000	0/15
<b>f<sub>3</sub></b>	6.3e+2:33	4.0e+2:44	1.6e+2:109	1.0e+2:255	2.5e+1:3277	15/15
1: Exp	80(11)	74(6)	63(27)	∞	∞6100	0/15
2: Exp	92(22)	127(112)	121(79)	91(72)	∞1.0e4	0/15
<b>f<sub>4</sub></b>	6.3e+2:22	4.0e+2:91	2.5e+2:250	1.6e+2:332	6.3e+1:1927	15/15
1: Exp	150(25)	46(3)	21(2)	∞	∞6100	0/15
2: Exp	144(54)	51(75)	33(35)	39(50)	78(114)	1/15
<b>f<sub>5</sub></b>	2.5e+2:19	1.6e+2:34	1.0e-8:41	1.0e-8:41	1.0e-8:41	15/15
1: Exp	24(9)	21(8)	42(5)*	42(9)*	42(5)*	15/15
2: Exp	34(17)	27(9)	52(9)	52(11)	52(8)	15/15
<b>f<sub>6</sub></b>	2.5e+5:16	6.3e+4:43	1.6e+4:62	1.6e+2:353	1.6e+1:1078	15/15
1: Exp	71(25)	33(10)	27(14)	∞	∞6100	0/15
2: Exp	77(33)	33(10)	25(9)	408(549)	∞9900	0/15
<b>f<sub>7</sub></b>	1.0e+3:11	4.0e+2:39	2.5e+2:74	6.3e+1:319	1.0e+1:1351	15/15
1: Exp	254(46)	237(206)	∞	∞	∞6100	0/15
2: Exp	161(25)* <sup>2</sup>	73(24)* <sup>3</sup>	72(25)* <sup>2</sup>	∞	∞1.0e4	0/15
<b>f<sub>8</sub></b>	4.0e+4:19	2.5e+4:35	4.0e+3:67	2.5e+2:231	1.6e+1:1470	15/15
1: Exp	173(12)	100(11)	82(8)	∞	∞6100	0/15
2: Exp	166(31)	94(13)	62(8)* <sup>2</sup>	34(27)	∞9100	0/15
<b>f<sub>9</sub></b>	1.0e+2:357	6.3e+1:560	4.0e+1:684	2.5e+1:756	1.0e+1:1716	15/15
1: Exp	∞	∞	∞	∞	∞6100	0/15
2: Exp	91(115)	241(414)	∞	∞	∞9100	0/15
<b>f<sub>10</sub></b>	1.6e+6:15	1.0e+6:27	4.0e+5:70	6.3e+4:231	4.0e+3:1015	15/15
1: Exp	893(459)	3271(4969)	∞	∞	∞6100	0/15
2: Exp	157(118)* <sup>3</sup>	124(40)* <sup>3</sup>	102(125)* <sup>3</sup>	∞	∞9800	0/15
<b>f<sub>11</sub></b>	4.0e+4:11	2.5e+3:27	1.6e+2:313	1.0e+2:481	1.0e+1:1002	15/15
1: Exp	23(20)	49(43)	∞	∞	∞6100	0/15
2: Exp	28(34)	41(39)	536(815)	∞	∞1.2e4	0/15
<b>f<sub>12</sub></b>	1.0e+8:23	6.3e+7:39	2.5e+7:76	4.0e+6:209	1.0e+1:1042	15/15
1: Exp	169(33)	111(20)	81(9)	∞	∞6100	0/15
2: Exp	161(48)	106(41)	68(18)	38(25)	∞9100	0/15
<b>f<sub>13</sub></b>	1.6e+3:28	1.0e+3:64	6.3e+2:79	4.0e+1:211	2.5e+0:1724	15/15
1: Exp	98(14)	61(6)	65(5)	∞	∞6100	0/15
2: Exp	96(38)	55(30)	61(41)	636(874)	∞9100	0/15
<b>f<sub>14</sub></b>	2.5e+1:15	1.6e+1:42	1.0e+1:75	1.6e+0:219	6.3e+4:1106	15/15
1: Exp	1510(1070)	∞	∞	∞	∞6100	0/15
2: Exp	273(140)* <sup>2</sup>	147(119)* <sup>2</sup>	113(19)	∞	∞9000	0/15
<b>f<sub>15</sub></b>	6.3e+2:15	4.0e+2:67	2.5e+2:292	1.6e+2:846	1.0e+2:1671	15/15
1: Exp	194(59)	60(12)	28(27)*	∞	∞6100*	0/15
2: Exp	200(70)	104(117)	90(176)	178(101)	∞1.0e4	0/15
<b>f<sub>16</sub></b>	4.0e+1:26	2.5e+1:127	1.6e+1:540	1.6e+1:540	1.0e+1:1384	15/15
1: Exp	255(134)	∞	∞	∞	∞6100	0/15
2: Exp	446(467)	280(271)	145(302)	145(102)	∞1.1e4	0/15
<b>f<sub>17</sub></b>	1.6e+1:11	1.0e+1:63	6.3e+0:305	4.0e+0:468	1.0e+0:1030	15/15
1: Exp	253(37)	59(8)*	17(6)* <sup>2</sup>	31(29)* <sup>3</sup>	∞6100* <sup>3</sup>	0/15
2: Exp	418(177)	323(493)	155(189)	∞	∞1.1e4	0/15
<b>f<sub>18</sub></b>	4.0e+1:116	2.5e+1:252	1.6e+1:430	1.0e+1:621	4.0e+0:1090	15/15
1: Exp	31(5)	24(8)* <sup>3</sup>	34(22)* <sup>3</sup>	∞* <sup>3</sup>	∞6100* <sup>3</sup>	0/15
2: Exp	112(98)	621(425)	∞	∞	∞1.1e4	0/15
<b>f<sub>19</sub></b>	1.6e-1:2.5e5	1.0e-1:3.4e5	6.3e-2:3.4e5	4.0e-2:3.4e5	2.5e-2:3.4e5	3/15
1: Exp	∞	∞	∞	∞	∞6100	0/15
2: Exp	∞	∞	∞	∞	∞1.1e4	0/15
<b>f<sub>20</sub></b>	1.6e+1:38	1.0e+1:42	2.5e+2:62	2.5e+0:250	1.6e+0:2536	15/15
1: Exp	75(8)	73(9)	74(6)	∞	∞6100	0/15
2: Exp	71(22)	69(21)	62(20)	21(7)*	2.9(2)	14/15
<b>f<sub>21</sub></b>	6.3e+1:36	4.0e+1:77	4.0e+1:77	1.6e+1:456	4.0e+0:1094	15/15
1: Exp	127(57)	116(101)	116(42)	198(252)	∞6100	0/15
2: Exp	90(21)	58(36)*	58(9)*	16(12)*	38(38)	3/15
<b>f<sub>22</sub></b>	6.3e+1:45	4.0e+1:68	4.0e+1:68	1.6e+1:231	6.3e+0:1219	15/15
1: Exp	96(18)	116(49)	116(55)	74(80)	37(40)	2/15
2: Exp	129(33)	121(67)	121(75)	73(34)	109(78)	1/15
<b>f<sub>23</sub></b>	6.3e+0:29	4.0e+0:118	2.5e+0:306	2.5e+0:306	1.0e+0:1614	15/15
1: Exp	117(30)	179(229)	∞	∞	∞6100	0/15
2: Exp	121(70)	103(15)	176(115)	176(209)	∞1.1e4	0/15
<b>f<sub>24</sub></b>	2.5e+2:208	1.6e+2:918	1.0e+2:6628	6.3e+1:9885	4.0e+1:31629	15/15
1: Exp	24(2)* <sup>2</sup>	∞* <sup>3</sup>	∞* <sup>3</sup>	∞* <sup>3</sup>	∞6100* <sup>3</sup>	0/15
2: Exp	364(302)	∞	∞	∞	∞1.1e4	0/15