

# Modyfikacje/hybrydyzacje algorytmu PSO w zadaniu optymalizacji globalnej wielowymiarowej funkcji ciaglej

## PSO-DE Hybrid

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### ABSTRACT

Dokumentacja uzyskanych wyników hybrydy PSO-DE

### Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

### General Terms

Algorithms

### Keywords

Benchmarking, PSODE, Optymalizacja wielowymiarowej funkcji ciaglej

of trials that actually reached  $f_t$  [?, ?]. **Statistical significance** is tested with the rank-sum test for a given target  $\Delta f_t$  ( $10^{-8}$  as in Figure 1) using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by  $-1$ ), or, if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

## 1. CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the **PSO-DE Hybrid** on the function  $f_8$  with restarts for at least 30 seconds and until a maximum budget equal to  $400(D + 2)$  is reached. The code was run on a **Mac Intel(R) Core(TM) i5-2400S CPU @ 2.50GHz** with **1** processor and **4** cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20, **40** equals  **$x.x$ ,  $x.x$ ,  $x.x$ ,  $x.x$ ,  $xxx$ , and  $xxx$**  milliseconds respectively.

repeat the above for the second algorithm

## 2. RESULTS

Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2 and 3 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number

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GECCO'13, July 6-10, 2013, Amsterdam, The Netherlands.

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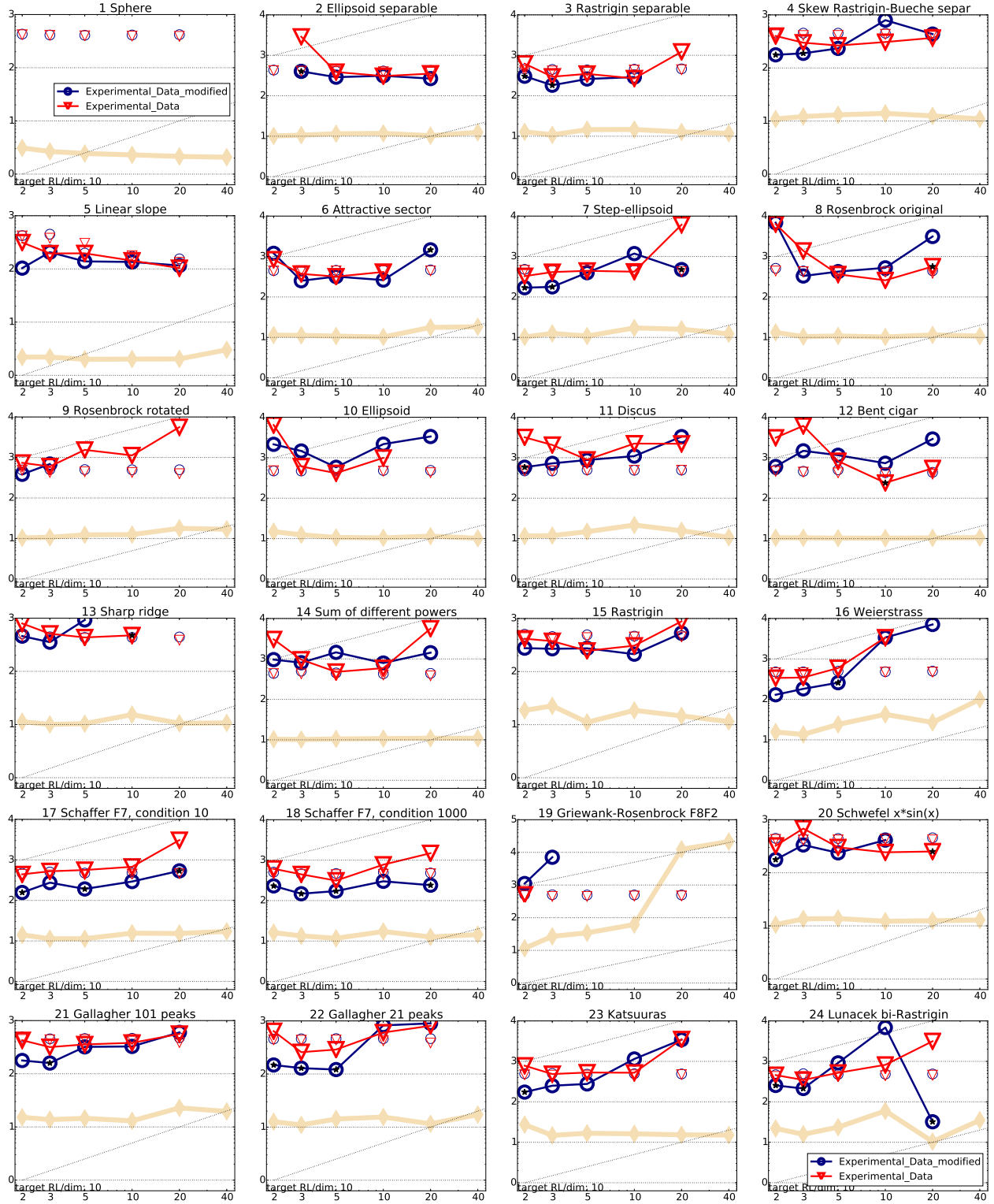
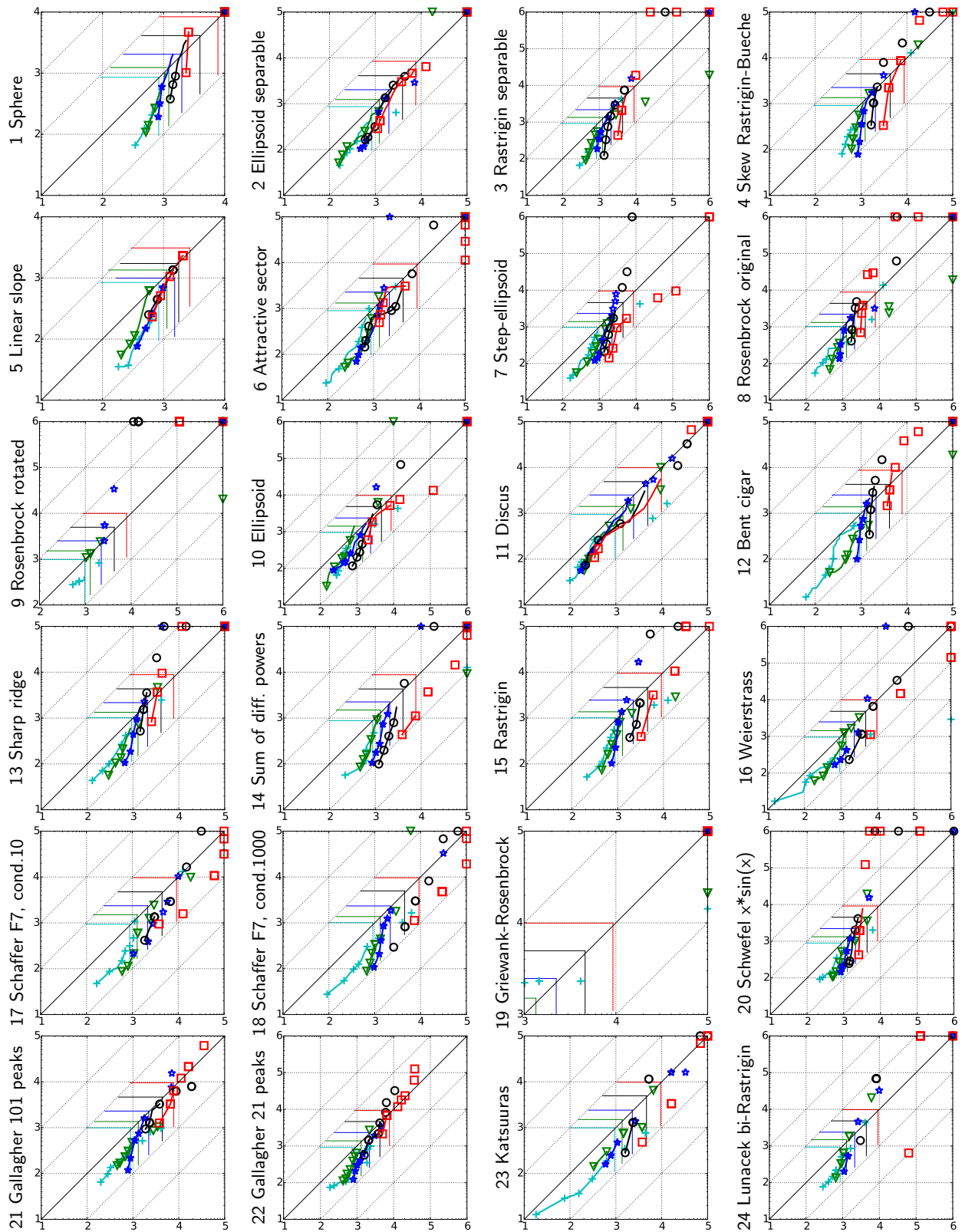
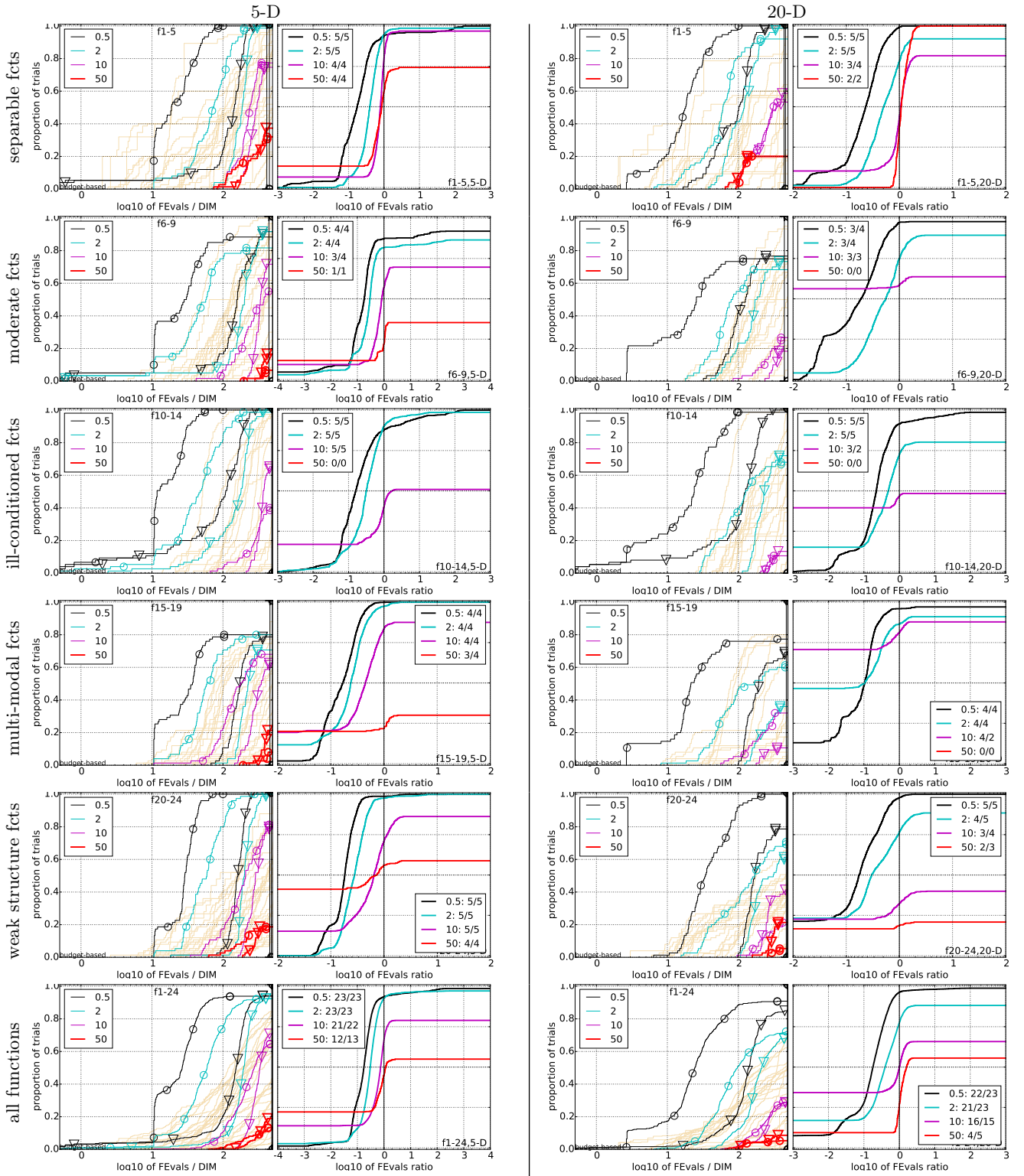


Figure 1: Expected running time (ERT in number of  $f$ -evaluations as  $\log_{10}$  value) divided by dimension versus dimension. The target function value is chosen such that the bestGECCO2009 artificial algorithm just failed to achieve an ERT of  $10 \times \text{DIM}$ . Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with  $p < 0.01$  and Bonferroni correction number of dimensions (six). Legend:  $\circ$ :Experimental Data modified,  $\nabla$ :Experimental Data.



**Figure 2:** Expected running time (ERT in  $\log_{10}$  of number of function evaluations) of Experimental Data modified ( $y$ -axis) versus Experimental Data ( $x$ -axis) for 8 runlength-based target function values for budgets between  $0.5 \times \text{DIM}$  and  $50 \times \text{DIM}$  evaluations. Each runlength-based target  $f$ -value is chosen such that the ERTs of the bestGECCO2009 artificial algorithm for the given and a slightly easier target bracket the reference budget. Markers on the upper or right edge indicate that the respective target value was never reached. Markers represent dimension: 2: +, 3:  $\nabla$ , 5: \*, 10:  $\circ$ , 20:  $\square$ , 40:  $\diamond$ .



**Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right).** Left sub-columns: ECDF of the number of function evaluations divided by dimension  $D$  (FEvals/ $D$ ) to fall below  $f_{\text{opt}} + \Delta f$  for Experimental Data modified ( $\circ$ ) and Experimental Data ( $\nabla$ ) where  $\Delta f$  is the target just not reached by the GECCO-BBOB-2009 best algorithm within a budget of  $k \times \text{DIM}$  evaluations, with  $k$  being the value in the legend. Right sub-columns: ECDF of FEval ratios of Experimental Data modified divided by Experimental Data for run-length-based targets; all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being  $> 0$  or  $< 1$ . The legends indicate the target budget of  $k \times \text{DIM}$  evaluations and, after the colon, the number of functions that were solved in at least one trial (Experimental Data modified first).

## 5-D

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f<sub>1</sub></b>	<i>2.5e+1:4.8</i>	<i>1.6e+1:7.6</i>	<i>1.0e-8:12</i>	<i>1.0e-8:12</i>	<i>1.0e-8:12</i>	15/15
1: Exp	<b>40(12)*<sup>3</sup></b>	<b>42(21)*<sup>3</sup></b>	$\infty^*$	$\infty$	$\infty 2000$	0/15
2: Exp	171(35)	115(27)	$\infty^*$	$\infty^*$	$\infty 2000^*$	0/15
<b>f<sub>2</sub></b>	<i>1.6e+6:2.9</i>	<i>4.0e+5:11</i>	<i>4.0e+4:15</i>	<i>6.3e+2:58</i>	<i>1.0e-8:95</i>	15/15
1: Exp	<b>37(18)*<sup>2</sup></b>	<b>14(7)*<sup>3</sup></b>	<b>22(11)*<sup>3</sup></b>	25(5)	$\infty 2000$	0/15
2: Exp	166(129)	58(24)	63(21)	34(4)	$\infty 1900$	0/15
<b>f<sub>3</sub></b>	<i>1.6e+2:4.1</i>	<i>1.0e+2:15</i>	<i>6.3e+1:23</i>	<i>2.5e+1:73</i>	<i>1.0e+1:716</i>	15/15
1: Exp	<b>45(45)*<sup>3</sup></b>	<b>24(12)*<sup>3</sup></b>	<b>23(10)*<sup>3</sup></b>	18(4)	22(26)	2/15
2: Exp	211(51)	66(14)	47(14)	24(10)	10(9)	4/15
<b>f<sub>4</sub></b>	<i>2.5e+2:2.6</i>	<i>1.6e+2:10</i>	<i>1.0e+2:19</i>	<i>4.0e+1:65</i>	<i>1.6e+1:434</i>	15/15
1: Exp	<b>30(18)*<sup>3</sup></b>	<b>15(8)*<sup>3</sup></b>	<b>19(14)*<sup>3</sup></b>	18(13)	10(9)	7/15
2: Exp	328(85)	96(27)	55(10)	20(5)	7.1(5)	9/15
<b>f<sub>5</sub></b>	<i>6.3e+1:4.0</i>	<i>4.0e+1:10</i>	<i>1.0e-8:10</i>	<i>1.0e-8:10</i>	<i>1.0e-8:10</i>	15/15
1: Exp	19(18)	15(13)	69(23)	69(24)	69(28)	15/15
2: Exp	93(79)	52(31)	98(36)	98(27)	98(30)	15/15
<b>f<sub>6</sub></b>	<i>1.0e+5:3.0</i>	<i>2.5e+4:8.4</i>	<i>1.0e+2:16</i>	<i>2.5e+1:54</i>	<i>2.5e-1:254</i>	15/15
1: Exp	23(21)	13(12)	<b>27(12)*<sup>2</sup></b>	30(46)	$\infty 2100$	0/15
2: Exp	140(138)	58(38)	68(17)	30(14)	$\infty 2100$	0/15
<b>f<sub>7</sub></b>	<i>1.6e+2:4.2</i>	<i>1.0e+2:6.2</i>	<i>2.5e+1:20</i>	<i>4.0e+0:54</i>	<i>1.0e+0:324</i>	15/15
1: Exp	<b>29(20)*<sup>3</sup></b>	<b>23(17)*<sup>3</sup></b>	<b>22(14)*<sup>3</sup></b>	37(17)	24(24)	4/15
2: Exp	179(47)	147(45)	62(15)	41(49)	8.4(8)	10/15
<b>f<sub>8</sub></b>	<i>1.0e+4:4.6</i>	<i>6.3e+3:6.8</i>	<i>1.0e+3:18</i>	<i>6.3e+1:54</i>	<i>1.6e+0:258</i>	15/15
1: Exp	<b>30(19)*<sup>3</sup></b>	<b>26(17)*<sup>3</sup></b>	40(63)	39(38)	$\infty 2100$	0/15
2: Exp	174(45)	124(22)	59(10)	33(13)	$\infty 2000$	0/15
<b>f<sub>9</sub></b>	<i>2.5e+1:20</i>	<i>1.6e+1:26</i>	<i>1.0e+1:35</i>	<i>4.0e+0:62</i>	<i>1.6e-2:256</i>	15/15
1: Exp	127(188)	212(214)	302(321)	$\infty$	$\infty 2200$	0/15
2: Exp	131(139)	102(153)	89(62)	125(138)	$\infty 2000$	0/15
<b>f<sub>10</sub></b>	<i>2.5e+6:2.9</i>	<i>6.3e+5:7.0</i>	<i>2.5e+5:17</i>	<i>6.3e+3:54</i>	<i>2.5e+1:297</i>	15/15
1: Exp	31(25)	20(11)	<b>10(9)*</b>	54(59)	$\infty 2300$	0/15
2: Exp	81(58)	62(57)	34(30)	39(45)	$\infty 2300$	0/15
<b>f<sub>11</sub></b>	<i>1.0e+6:3.0</i>	<i>6.3e+4:6.2</i>	<i>6.3e+2:16</i>	<i>6.3e+1:74</i>	<i>6.3e-1:298</i>	15/15
1: Exp	19(9)	19(12)	44(68)	59(51)	$\infty 2300$	0/15
2: Exp	57(67)	39(25)	56(30)	60(59)	$\infty 2300$	0/15
<b>f<sub>12</sub></b>	<i>4.0e+7:3.6</i>	<i>1.6e+7:7.6</i>	<i>4.0e+6:19</i>	<i>1.6e+4:52</i>	<i>1.0e+0:268</i>	15/15
1: Exp	<b>28(22)*<sup>3</sup></b>	<b>35(15)*<sup>3</sup></b>	39(61)	112(215)	$\infty 2100$	0/15
2: Exp	226(65)	121(33)	57(13)	80(49)	$\infty 2000$	0/15
<b>f<sub>13</sub></b>	<i>1.0e+3:2.8</i>	<i>6.3e+2:8.4</i>	<i>4.0e+2:17</i>	<i>6.3e+1:52</i>	<i>6.3e-2:264</i>	15/15
1: Exp	<b>38(20)*<sup>3</sup></b>	<b>22(13)*<sup>3</sup></b>	<b>26(12)*<sup>3</sup></b>	89(56)	$\infty 2100$	0/15
2: Exp	236(63)	105(21)	61(8)	42(13)	$\infty 2100$	0/15
<b>f<sub>14</sub></b>	<i>1.6e+1:3.0</i>	<i>1.0e+1:10</i>	<i>6.3e+0:15</i>	<i>2.5e-1:53</i>	<i>1.0e-5:251</i>	15/15
1: Exp	<b>35(26)*<sup>3</sup></b>	<b>18(15)*<sup>3</sup></b>	<b>18(6)*<sup>3</sup></b>	138(102)	$\infty 2100$	0/15
2: Exp	284(129)	105(47)	81(32)	45(23)	$\infty 2000$	0/15
<b>f<sub>15</sub></b>	<i>1.6e+2:3.0</i>	<i>1.0e+2:13</i>	<i>6.3e+1:24</i>	<i>4.0e+1:55</i>	<i>1.6e+1:289</i>	5/5
1: Exp	<b>34(33)*<sup>3</sup></b>	<b>17(14)*<sup>3</sup></b>	<b>33(34)*</b>	25(22)	58(81)	2/15
2: Exp	247(66)	69(27)	42(11)	22(6)	<b>10(5)*<sup>2</sup></b>	10/15
<b>f<sub>16</sub></b>	<i>4.0e+1:4.8</i>	<i>2.5e+1:16</i>	<i>1.6e+1:46</i>	<i>1.0e+1:120</i>	<i>4.0e+0:334</i>	15/15
1: Exp	<b>35(17)*<sup>3</sup></b>	<b>15(5)*<sup>3</sup></b>	<b>9(2)*<sup>3</sup></b>	<b>11(3)*<sup>2</sup></b>	$\infty 2300$	0/15
2: Exp	140(48)	62(12)	30(11)	25(12)	50(63)	2/15
<b>f<sub>17</sub></b>	<i>1.0e+1:5.2</i>	<i>6.3e+0:26</i>	<i>4.0e+0:57</i>	<i>2.5e+0:110</i>	<i>6.3e-1:412</i>	15/15
1: Exp	<b>41(37)*<sup>3</sup></b>	<b>15(12)*<sup>3</sup></b>	<b>17(22)*<sup>2</sup></b>	16(14)	25(15)	3/15
2: Exp	202(109)	85(63)	50(60)	43(61)	24(39)	3/15
<b>f<sub>18</sub></b>	<i>6.3e+1:3.4</i>	<i>4.0e+1:7.2</i>	<i>2.5e+1:20</i>	<i>1.6e+1:58</i>	<i>1.6e+0:318</i>	15/15
1: Exp	<b>32(22)*<sup>3</sup></b>	<b>29(9)*<sup>3</sup></b>	<b>21(13)*<sup>2</sup></b>	<b>15(15)*</b>	104(81)	1/15
2: Exp	282(51)	174(99)	69(59)	27(6)	99(85)	1/15
<b>f<sub>19</sub></b>	<i>1.6e-1:172</i>	<i>1.0e-1:242</i>	<i>6.3e-2:675</i>	<i>4.0e-2:3078</i>	<i>2.5e-2:4946</i>	15/15
1: Exp	$\infty$	$\infty$	$\infty$	$\infty$	$\infty 2500$	0/15
2: Exp	$\infty$	$\infty$	$\infty$	$\infty$	$\infty 2100$	0/15
<b>f<sub>20</sub></b>	<i>6.3e+3:5.1</i>	<i>4.0e+3:8.4</i>	<i>4.0e+1:15</i>	<i>2.5e+0:69</i>	<i>1.0e+0:851</i>	15/15
1: Exp	<b>28(7)*<sup>3</sup></b>	<b>22(7)*<sup>3</sup></b>	<b>27(8)*<sup>3</sup></b>	17(9)	$\infty 2200$	0/15
2: Exp	162(36)	107(19)	75(10)	22(3)	$\infty 2000$	0/15
<b>f<sub>21</sub></b>	<i>4.0e+1:3.9</i>	<i>2.5e+1:11</i>	<i>1.6e+1:31</i>	<i>6.3e+0:73</i>	<i>1.6e+0:347</i>	5/5
1: Exp	<b>30(19)*<sup>3</sup></b>	<b>21(6)*<sup>3</sup></b>	<b>17(21)*<sup>2</sup></b>	22(7)	22(19)	4/15
2: Exp	198(43)	83(9)	35(16)	25(17)	20(22)	4/15
<b>f<sub>22</sub></b>	<i>6.3e+1:3.6</i>	<i>4.0e+1:15</i>	<i>2.5e+1:32</i>	<i>1.0e+1:71</i>	<i>1.6e+0:341</i>	5/5
1: Exp	<b>34(18)*<sup>3</sup></b>	<b>14(4)*<sup>3</sup></b>	<b>9(4)*<sup>3</sup></b>	<b>8(5)*<sup>8</sup></b>	10(10)	7/15
2: Exp	220(39)	61(14)	31(11)	20(11)	13(13)	6/15
<b>f<sub>23</sub></b>	<i>1.0e+1:3.0</i>	<i>6.3e+0:9.0</i>	<i>4.0e+0:33</i>	<i>2.5e+0:84</i>	<i>1.0e+0:518</i>	15/15
1: Exp	<b>53(18)*<sup>2</sup></b>	<b>28(7)*<sup>2</sup></b>	<b>14(5)*</b>	17(14)	31(21)	2/15
2: Exp	198(55)	86(50)	32(10)	32(24)	64(69)	1/15
<b>f<sub>24</sub></b>	<i>6.3e+1:15</i>	<i>4.0e+1:37</i>	<i>4.0e+1:37</i>	<i>2.5e+1:118</i>	<i>1.6e+1:692</i>	15/15
1: Exp	<b>14(9)*<sup>3</sup></b>	<b>15(6)*<sup>3</sup></b>	<b>15(5)*<sup>3</sup></b>	39(69)	47(48)	1/15
2: Exp	75(9)	38(11)	38(10)	22(13)	14(11)	3/15

## 20-D

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f<sub>1</sub></b>	6.3e+1:24	4.0e+1:42	1.0e-8:43	1.0e-8:43	1.0e-8:43	15/15
1: Exp	<b>42(13)*<sup>3</sup></b>	112(209)	$\infty$	$\infty$	$\infty 8200$	0/15
2: Exp	97(9)	61(16)	$\infty^*$	$\infty^*$	$\infty 7700^*$	0/15
<b>f<sub>2</sub></b>	4.0e+6:29	2.5e+6:42	1.0e+5:65	1.0e+4:207	1.0e-8:412	15/15
1: Exp	<b>10(4)*<sup>3</sup></b>	<b>10(9)*<sup>3</sup></b>	39(4)	26(5)	$\infty 8400$	0/15
2: Exp	39(10)	30(7)	50(13)	34(29)	$\infty 7600$	0/15
<b>f<sub>3</sub></b>	6.3e+2:33	4.0e+2:44	1.6e+2:109	1.0e+2:255	2.5e+1:3277	15/15
1: Exp	<b>13(10)*<sup>3</sup></b>	<b>48(127)*</b>	173(206)	$\infty$	$\infty 9100$	0/15
2: Exp	97(131)	93(75)	90(15)	96(99)	$\infty 8600$	0/15
<b>f<sub>4</sub></b>	6.3e+2:22	4.0e+2:91	2.5e+2:250	1.6e+2:332	6.3e+1:1927	15/15
1: Exp	<b>16(9)*<sup>3</sup></b>	<b>25(11)*</b>	35(21)	199(193)	$\infty 9100$	0/15
2: Exp	142(51)	44(17)	29(22)	57(62)	$\infty 8400$	0/15
<b>f<sub>5</sub></b>	2.5e+2:19	1.6e+2:34	1.0e-8:41	1.0e-8:41	1.0e-8:41	15/15
1: Exp	<b>12(7)*<sup>3</sup></b>	<b>15(5)*<sup>2</sup></b>	57(8)	57(13)	57(6)	15/15
2: Exp	35(14)	26(5)	51(17)	51(11)	51(12)	15/15
<b>f<sub>6</sub></b>	2.5e+5:16	6.3e+4:43	1.6e+4:62	1.6e+2:353	1.6e+1:1078	15/15
1: Exp	<b>31(17)*<sup>3</sup></b>	31(7)	36(14)	<b>83(52)*<sup>2</sup></b>	$\infty 9100$	0/15
2: Exp	83(46)	38(23)	29(10)	$\infty$	$\infty 8400$	0/15
<b>f<sub>7</sub></b>	1.0e+3:11	4.0e+2:39	2.5e+2:74	6.3e+1:319	1.0e+1:1351	15/15
1: Exp	<b>13(18)*<sup>3</sup></b>	<b>24(9)*<sup>2</sup></b>	<b>23(8)*<sup>2</sup></b>	<b>30(29)*</b>	$\infty 9300$	0/15
2: Exp	173(27)	78(20)	74(24)	384(431)	$\infty 8300$	0/15
<b>f<sub>8</sub></b>	4.0e+4:19	2.5e+4:35	4.0e+3:67	2.5e+2:231	1.6e+1:1470	15/15
1: Exp	<b>37(43)*<sup>3</sup></b>	<b>67(125)*</b>	243(126)	271(301)	$\infty 8500$	0/15
2: Exp	163(25)	93(18)	65(21)	<b>49(20)*<sup>2</sup></b>	$\infty 7600^*$	0/15
<b>f<sub>9</sub></b>	1.0e+2:357	6.3e+1:560	4.0e+1:684	2.5e+1:756	1.0e+1:1716	15/15
1: Exp	$\infty$	$\infty$	$\infty$	$\infty$	$\infty 9300$	0/15
2: Exp	314(144)	202(147)	167(154)	$\infty$	$\infty 7600$	0/15
<b>f<sub>10</sub></b>	1.6e+6:15	1.0e+6:27	4.0e+5:70	6.3e+4:231	4.0e+3:1015	15/15
1: Exp	<b>40(31)*<sup>3</sup></b>	<b>67(169)*</b>	73(86)	290(235)	$\infty 9500$	0/15
2: Exp	135(30)	95(42)	113(69)	$\infty$	$\infty 8300$	0/15
<b>f<sub>11</sub></b>	4.0e+4:11	2.5e+3:27	1.6e+2:313	1.0e+2:481	1.0e+1:1002	15/15
1: Exp	10(9)	20(10)	213(168)	$\infty$	$\infty 9800$	0/15
2: Exp	30(32)	52(32)	142(234)	$\infty$	$\infty 9600$	0/15
<b>f<sub>12</sub></b>	1.0e+8:23	6.3e+7:39	2.5e+7:76	4.0e+6:209	1.0e+1:1042	15/15
1: Exp	<b>63(112)*<sup>2</sup></b>	82(87)	132(155)	279(345)	$\infty 8300$	0/15
2: Exp	161(32)	107(22)	73(17)	53(36)	$\infty 7600$	0/15
<b>f<sub>13</sub></b>	1.6e+3:28	1.0e+3:64	6.3e+2:79	4.0e+1:211	2.5e+0:1724	15/15
1: Exp	<b>30(22)*<sup>3</sup></b>	57(98)	121(133)	$\infty$	$\infty 8600$	0/15
2: Exp	92(21)	53(11)	55(13)	$\infty^*$	$\infty 7600^*$	0/15
<b>f<sub>14</sub></b>	2.5e+1:15	1.6e+1:42	1.0e+1:75	1.6e+0:219	6.3e+4:1106	15/15
1: Exp	<b>30(27)*<sup>3</sup></b>	<b>26(20)*<sup>3</sup></b>	<b>50(116)*<sup>2</sup></b>	130(148)	$\infty 8600$	0/15
2: Exp	264(92)	177(179)	187(85)	512(614)	$\infty 7600$	0/15
<b>f<sub>15</sub></b>	6.3e+2:15	4.0e+2:67	2.5e+2:292	1.6e+2:846	1.0e+2:1671	15/15
1: Exp	<b>26(25)*<sup>3</sup></b>	47(68)	36(102)	$\infty$	$\infty 9400$	0/15
2: Exp	210(71)	88(166)	61(96)	37(38)	$\infty 8600$	0/15
<b>f<sub>16</sub></b>	4.0e+1:26	2.5e+1:127	1.6e+1:540	1.6e+1:540	1.0e+1:1384	15/15
1: Exp	<b>42(99)*<sup>2</sup></b>	117(74)	263(499)	263(342)	$\infty 9700$	0/15
2: Exp	242(235)	332(275)	$\infty$	$\infty$	$\infty 9200$	0/15
<b>f<sub>17</sub></b>	1.6e+1:11	1.0e+1:63	6.3e+0:305	4.0e+0:468	1.0e+0:1030	15/15
1: Exp	<b>89(16)*<sup>2</sup></b>	<b>25(77)*<sup>2</sup></b>	<b>35(35)*</b>	<b>68(83)*</b>	$\infty 9500$	0/15
2: Exp	362(33)	202(202)	201(332)	$\infty$	$\infty 8800$	0/15
<b>f<sub>18</sub></b>	4.0e+1:116	2.5e+1:252	1.6e+1:430	1.0e+1:621	4.0e+0:1090	15/15
1: Exp	<b>10(3)*<sup>2</sup></b>	<b>19(13)*</b>	<b>45(71)*</b>	111(54)	$\infty 9500$	0/15
2: Exp	63(37)	118(125)	$\infty$	$\infty$	$\infty 9000$	0/15
<b>f<sub>19</sub></b>	1.6e-1:2.5e5	1.0e-1:3.4e5	6.3e-2:3.4e5	4.0e-2:3.4e5	2.5e-2:3.4e5	3/15
1: Exp	$\infty$	$\infty$	$\infty$	$\infty$	$\infty 9900$	0/15
2: Exp	$\infty$	$\infty$	$\infty$	$\infty$	$\infty 9200$	0/15
<b>f<sub>20</sub></b>	1.6e+4:38	1.0e+4:42	2.5e+2:62	2.5e+0:250	1.6e+0:2536	15/15
1: Exp	<b>11(3)*<sup>3</sup></b>	<b>46(4)*</b>	607(610)	$\infty$	$\infty 8600$	0/15
2: Exp	66(10)	65(9)	<b>59(10)*</b>	<b>20(4)*<sup>3</sup></b>	<b>3.9(2)*<sup>3</sup></b>	10/15
<b>f<sub>21</sub></b>	6.3e+1:36	4.0e+1:77	2.5e+1:77	1.6e+1:456	4.0e+0:1094	15/15
1: Exp	<b>35(16)*<sup>3</sup></b>	43(16)	43(23)	26(33)	56(86)	2/15
2: Exp	106(67)	86(58)	86(104)	25(18)	32(40)	3/15
<b>f<sub>22</sub></b>	6.3e+1:45	4.0e+1:68	4.0e+1:68	1.6e+1:231	6.3e+0:1219	15/15
1: Exp	<b>47(13)*<sup>2</sup></b>	100(127)	100(60)	78(183)	51(42)	2/15
2: Exp	110(65)	97(85)	97(102)	70(65)	30(41)	3/15
<b>f<sub>23</sub></b>	6.3e+0:29	4.0e+0:118	2.5e+0:306	2.5e+0:306	1.0e+0:1614	15/15
1: Exp	<b>17(10)*<sup>3</sup></b>	<b>28(36)*</b>	226(207)	226(219)	$\infty 9700$	0/15
2: Exp	131(59)	138(89)	229(207)	229(170)	$\infty 9400$	0/15
<b>f<sub>24</sub></b>	2.5e+2:208	1.6e+2:918	1.0e+2:6628	6.3e+1:9885	4.0e+1:31629	15/15
1: Exp	<b>3.1(1)*<sup>3</sup></b>	$\infty$	$\infty$	$\infty$	$\infty 9700$	0/15
2: Exp	302(408)	144(83)	$\infty$	$\infty$	$\infty 9000$	0/15