

# Modyfikacje/hybrydyzacje algorytmu PSO w zadaniu optymalizacji globalnej wielowymiarowej funkcji ciaglej

## PSO-DE Hybrid

Jakub Ruszkowski, Mateusz Kaczmarek

### ABSTRACT

Dokumentacja uzyskanych wyników hybrydy PSO-DE

### Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

### General Terms

Algorithms

### Keywords

Benchmarking, PSODE, Optymalizacja wielowymiarowej funkcji ciaglej

of trials that actually reached  $f_t$  [?, ?]. **Statistical significance** is tested with the rank-sum test for a given target  $\Delta f_t$  ( $10^{-8}$  as in Figure 1) using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by  $-1$ ), or, if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

## 1. CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the **PSO-DE Hybrid** on the function  $f_8$  with restarts for at least 30 seconds and until a maximum budget equal to  $400(D + 2)$  is reached. The code was run on a **Mac Intel(R) Core(TM) i5-2400S CPU @ 2.50GHz** with **1** processor and **4** cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20, **40** equals  **$x.x$ ,  $x.x$ ,  $x.x$ ,  $x.x$ ,  $xxx$ , and  $xxx$**  milliseconds respectively.

repeat the above for the second algorithm

## 2. RESULTS

Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2 and 3 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

GECCO'13, July 6-10, 2013, Amsterdam, The Netherlands.

Copyright 2013 ACM TBA ...\$15.00.

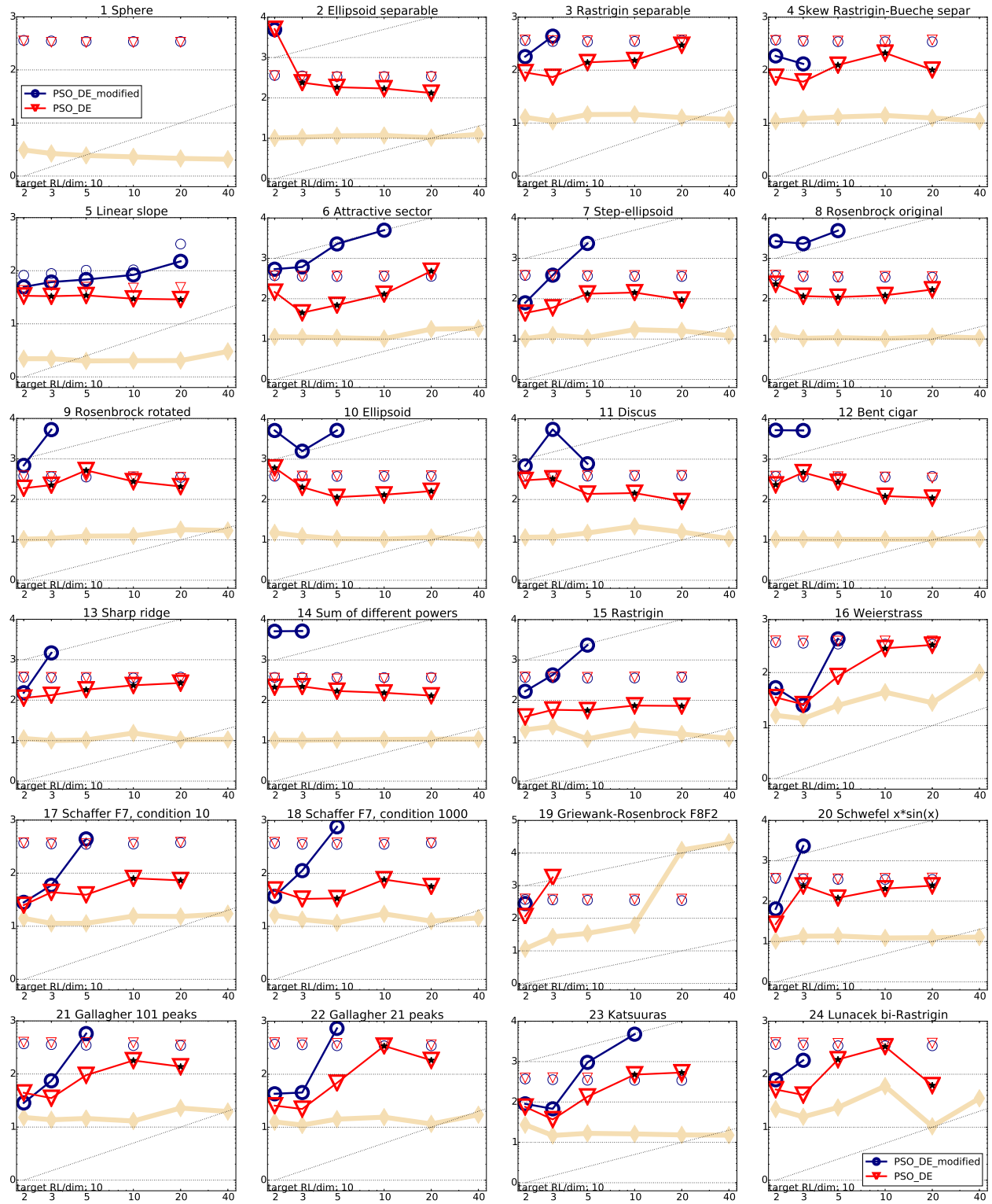


Figure 1: Expected running time (ERT in number of  $f$ -evaluations as  $\log_{10}$  value) divided by dimension versus dimension. The target function value is chosen such that the bestGECCO2009 artificial algorithm just failed to achieve an ERT of  $10 \times \text{DIM}$ . Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with  $p < 0.01$  and Bonferroni correction number of dimensions (six). Legend:  $\circ$ :PSO DE modified,  $\nabla$ :PSO DE.

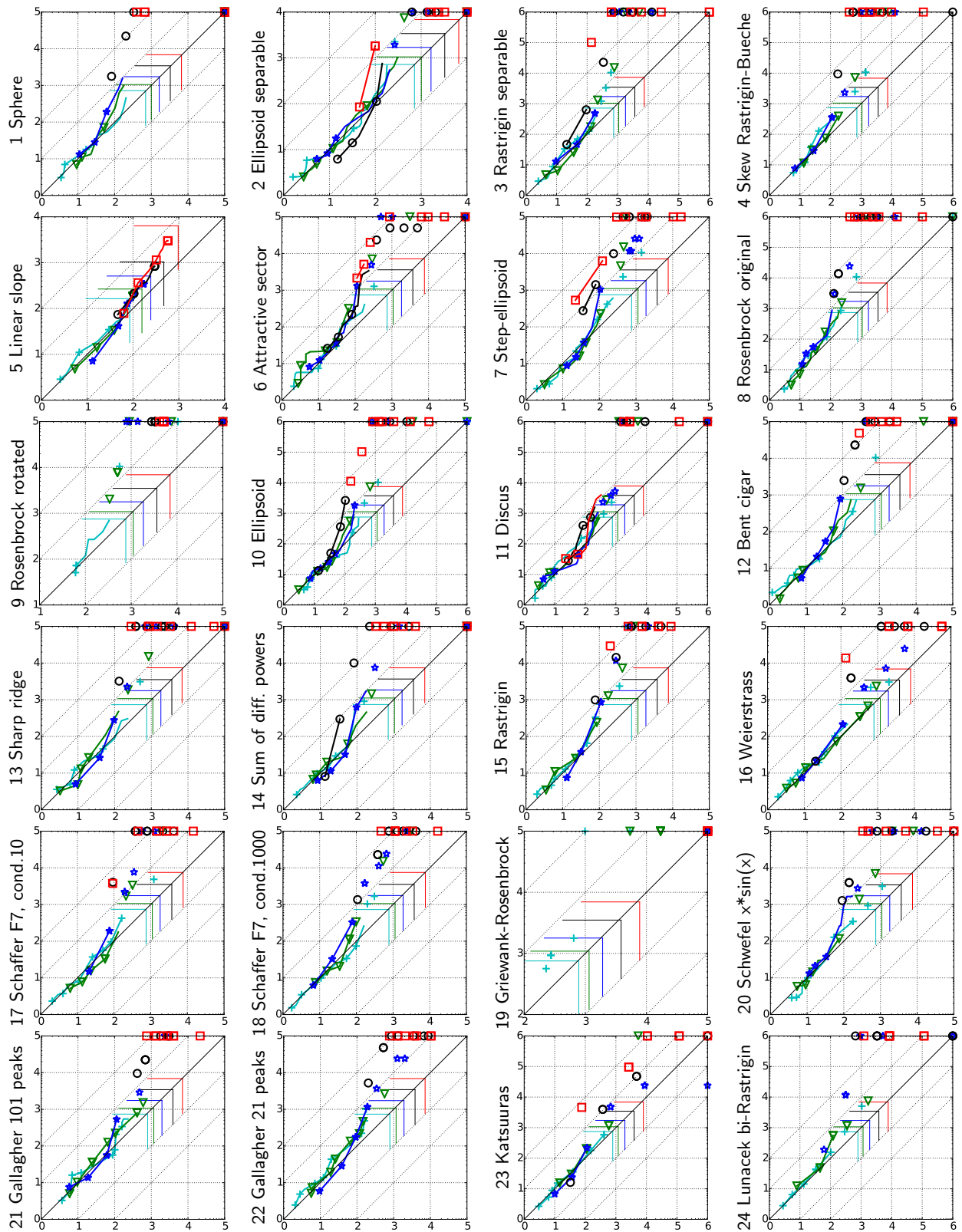
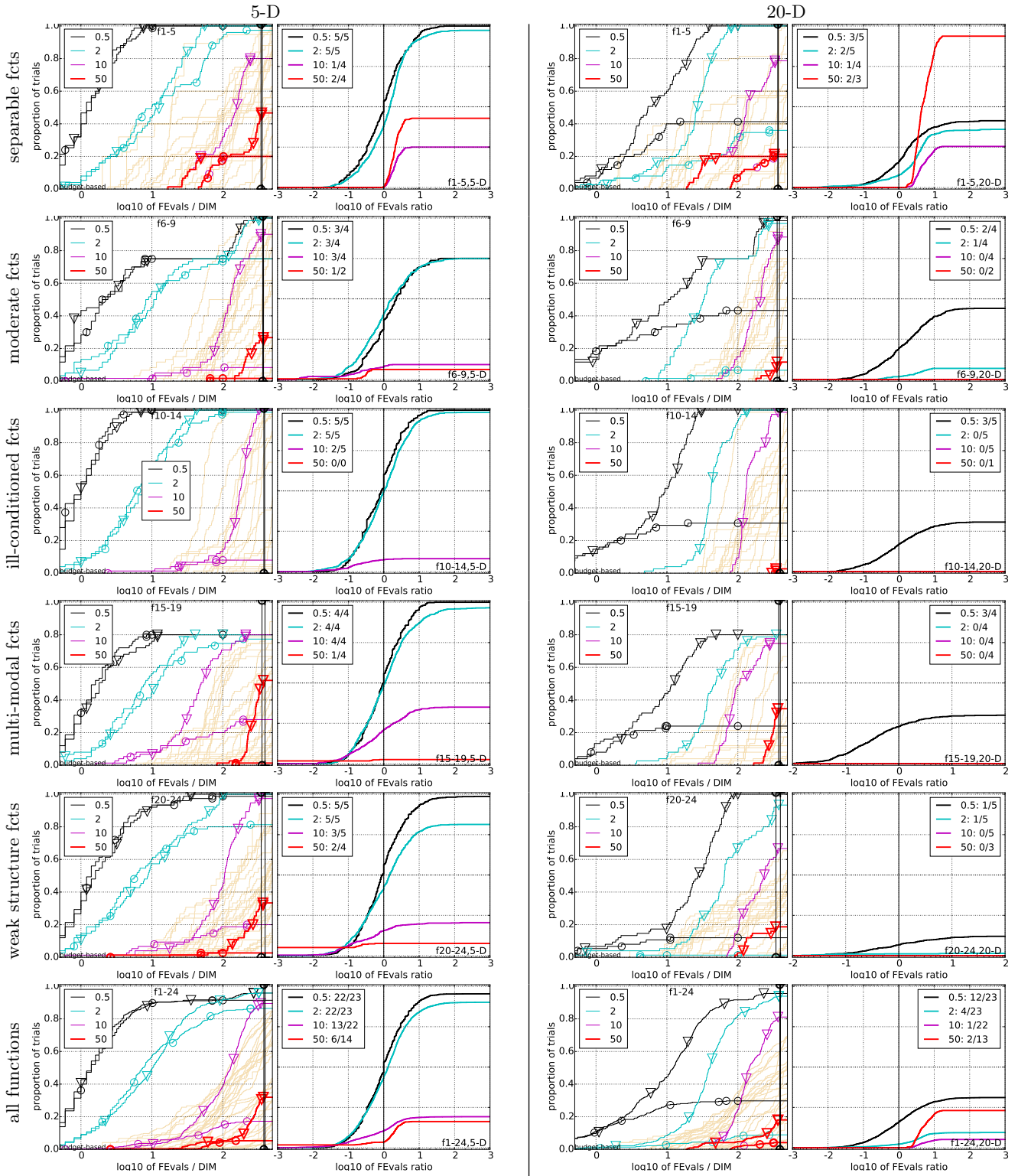


Figure 2: Expected running time (ERT in  $\log_{10}$  of number of function evaluations) of PSO DE modified ( $y$ -axis) versus PSO DE ( $x$ -axis) for 8 runlength-based target function values for budgets between  $0.5 \times \text{DIM}$  and  $50 \times \text{DIM}$  evaluations. Each runlength-based target  $f$ -value is chosen such that the ERTs of the bestGECCO2009 artificial algorithm for the given and a slightly easier target bracket the reference budget. Markers on the upper or right edge indicate that the respective target value was never reached. Markers represent dimension: 2:  $+$ , 3:  $\nabla$ , 5:  $\star$ , 10:  $\circ$ , 20:  $\square$ , 40:  $\diamond$ .



**Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right).** Left sub-columns: ECDF of the number of function evaluations divided by dimension  $D$  (FEvals/ $D$ ) to fall below  $f_{\text{opt}} + \Delta f$  for PSO DE modified ( $\circ$ ) and PSO DE ( $\nabla$ ) where  $\Delta f$  is the target just not reached by the GECCO-BBOB-2009 best algorithm within a budget of  $k \times \text{DIM}$  evaluations, with  $k$  being the value in the legend. Right sub-columns: ECDF of FEval ratios of PSO DE modified divided by PSO DE for run-length-based targets; all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being  $> 0$  or  $< 1$ . The legends indicate the target budget of  $k \times \text{DIM}$  evaluations and, after the colon, the number of functions that were solved in at least one trial (PSO DE modified first).

5-D							20-D						
#FEs/D	0.5	1.2	3	10	50	#succ	#FEs/D	0.5	1.2	3	10	50	#succ
<b>f<sub>1</sub></b>	2.5e+1:4.8	1.6e+1:7.6	1.0e-8:12	1.0e-8:12	1.0e-8:12	15/15	<b>f<sub>1</sub></b>	6.3e+1:24	4.0e+1:42	3.0e+4:43	1.0e-8:43	1.0e-8:43	15/15
1: PSO	2.8(1)	3.7(2)	$\infty$	$\infty$	$\infty$ 1700	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 6800	0/15
2: PSO	2.3(2)	3.8(3)	$\infty$ *3	$\infty$ *3	$\infty$ 1600*3	0/15	2: PSO	<b>16(7)*3</b>	<b>16(4)*3</b>	$\infty$ *3	$\infty$ *3	$\infty$ 6600*3	0/15
<b>f<sub>2</sub></b>	1.6e+6:2.9	4.0e+5:11	4.0e+4:15	6.3e+2:58	1.0e-8:95	15/15	<b>f<sub>2</sub></b>	4.0e+6:29	2.5e+6:42	1.0e+5:65	1.0e+4:207	1.0e-8:412	15/15
1: PSO	2.2(3)	1.6(2)	11(14)	$\infty$	$\infty$ 1700	0/15	1: PSO	3.0(3)	43(83)	$\infty$	$\infty$	$\infty$ 6800	0/15
2: PSO	1.9(2)	1.2(1)	9.3(7)	<b>16(5)*3</b>	$\infty$ 1600*3	0/15	2: PSO	1.6(1)	2.4(2)	<b>21(7)*3</b>	<b>13(0.9)*3</b>	$\infty$ 6500*3	0/15
<b>f<sub>3</sub></b>	1.6e+2:4.1	1.0e+2:15	6.3e+1:23	2.5e+1:73	1.0e+1:716	15/15	<b>f<sub>3</sub></b>	6.3e+2:33	4.0e+2:44	1.6e+2:109	1.0e+2:255	2.5e+1:3277	15/15
1: PSO	3.1(3)	3.2(3)	21(38)	$\infty$	$\infty$ 1700	0/15	1: PSO	3074(3451)	$\infty$	$\infty$	$\infty$	$\infty$ 7200	0/15
2: PSO	2.3(2)	3.2(2)	7.7(4)	<b>10(4)*3</b>	<b>3.2(3)*3</b>	10/15	2: PSO	<b>4.2(2)*3</b>	<b>14(3)*3</b>	<b>25(5)*3</b>	<b>23(3)*3</b>	$\infty$ 7400	0/15
<b>f<sub>4</sub></b>	2.5e+2:2.6	1.6e+2:10	1.0e+2:19	4.0e+1:65	1.6e+1:434	15/15	<b>f<sub>4</sub></b>	6.3e+2:22	4.0e+2:91	2.5e+2:250	1.6e+2:332	6.3e+1:1927	15/15
1: PSO	2.9(3)	3.0(5)	19(7)	$\infty$	$\infty$ 1700	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 6800	0/15
2: PSO	2.6(2)	2.9(6)	5.9(5)	<b>10(3)*3</b>	<b>5.4(5)*3</b>	10/15	2: PSO	<b>19(6)*3</b>	<b>12(3)*3</b>	<b>8.1(2)*3</b>	<b>13(2)*3</b>	57(42)	1/15
<b>f<sub>5</sub></b>	6.3e+1:4.0	4.0e+1:10	1.0e-8:10	1.0e-8:10	1.0e-8:10	15/15	<b>f<sub>5</sub></b>	2.5e+2:19	1.6e+2:34	1.0e-8:41	1.0e-8:41	1.0e-8:41	15/15
1: PSO	1.8(2)	4.2(3)	34(13)	34(12)	34(4)	15/15	1: PSO	4.1(4)	11(4)	74(59)	74(34)	74(54)	15/15
2: PSO	3.3(2)	4.9(2)	<b>17(6)*3</b>	<b>17(4)*3</b>	<b>17(6)*3</b>	15/15	2: PSO	3.3(2)	<b>3.8(2)*3</b>	<b>14(4)*3</b>	<b>14(4)*3</b>	<b>14(3)*3</b>	15/15
<b>f<sub>6</sub></b>	1.0e+5:3.0	2.5e+4:8.4	1.0e+2:16	2.5e+1:54	2.5e-1:254	15/15	<b>f<sub>6</sub></b>	2.5e+5:16	6.3e+4:43	1.6e+4:62	1.6e+2:353	1.6e+1:1078	15/15
1: PSO	2.7(4)	2.2(2)	11(16)	211(73)	$\infty$ 1700	0/15	1: PSO	134(33)	467(1070)	$\infty$	$\infty$	$\infty$ 7100	0/15
2: PSO	1.9(0.5)	2.0(3)	5.1(3)	<b>6.4(4)*</b>	$\infty$ 1800	0/15	2: PSO	7.0(5)	<b>6.0(3)*2</b>	<b>6.1(3)*3</b>	<b>27(28)*3</b>	$\infty$ 7500	0/15
<b>f<sub>7</sub></b>	1.6e+2:4.2	1.0e+2:6.2	2.5e+1:20	4.0e+0:54	1.0e+0:324	15/15	<b>f<sub>7</sub></b>	1.0e+3:11	4.0e+2:39	2.5e+2:74	6.3e+1:319	1.0e+1:1351	15/15
1: PSO	2.1(0.5)	2.4(2)	52(53)	216(256)	78(169)	1/15	1: PSO	50(171)	2525(2433)	$\infty$	$\infty$	$\infty$ 7000	0/15
2: PSO	3.1(4)	3.7(3)	5.3(3)	<b>12(3)*</b>	<b>3.8(3)*2</b>	14/15	2: PSO	2.1(3)	<b>7.6(6)*2</b>	<b>7.1(3)*3</b>	<b>5.9(2)*3</b>	<b>13(8)*3</b>	6/15
<b>f<sub>8</sub></b>	1.0e+4:4.6	6.3e+3:6.8	1.0e+3:18	6.3e+1:54	1.6e+0:258	15/15	<b>f<sub>8</sub></b>	4.0e+4:19	2.5e+4:35	4.0e+3:67	2.5e+2:231	1.6e+1:1470	15/15
1: PSO	3.3(3)	4.9(4)	49(50)	446(561)	$\infty$ 1700	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 6800	0/15
2: PSO	2.6(2)	2.3(2)	6.3(4)	<b>10(4)*3</b>	<b>49(35)*3</b>	2/15	2: PSO	<b>23(9)*3</b>	<b>17(4)*3</b>	<b>21(4)*3</b>	<b>15(3)*3</b>	<b>69(39)*3</b>	1/15
<b>f<sub>9</sub></b>	2.5e+1:20	1.6e+1:26	1.0e+1:35	4.0e+0:62	1.6e-2:256	15/15	<b>f<sub>9</sub></b>	1.0e+2:357	6.3e+1:560	4.0e+1:684	2.5e+1:756	1.0e+1:1716	15/15
1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 1800	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 6900	0/15
2: PSO	<b>39(16)*3</b>	<b>35(15)*3</b>	<b>32(9)*3</b>	<b>42(37)*3</b>	$\infty$ 1700*3	0/15	2: PSO	<b>12(6)*3</b>	<b>9.2(8)*3</b>	<b>8.2(5)*3</b>	<b>8.4(3)*3</b>	$\infty$ 6800*3	0/15
<b>f<sub>10</sub></b>	2.5e+6:2.9	6.3e+5:7.0	2.5e+5:17	6.3e+3:54	2.5e+1:297	15/15	<b>f<sub>10</sub></b>	1.6e+6:15	1.0e+6:27	4.0e+5:70	6.3e+4:231	4.0e+3:1015	15/15
1: PSO	2.6(1)	2.3(0.6)	2.0(2)	482(557)	$\infty$ 1800	0/15	1: PSO	747(1233)	3774(3431)	$\infty$	$\infty$	$\infty$ 7400	0/15
2: PSO	2.6(4)	2.3(2)	2.2(2)	<b>11(6)*2</b>	$\infty$ 1900	0/15	2: PSO	10(9)	<b>13(9)*2</b>	<b>12(7)*3</b>	<b>14(20)*3</b>	56(64)	2/15
<b>f<sub>11</sub></b>	1.0e+6:3.0	6.3e+4:6.2	6.3e+2:16	6.3e+1:74	6.3e-1:298	15/15	<b>f<sub>11</sub></b>	4.0e+4:11	2.5e+3:27	1.6e+2:313	1.0e+2:481	1.0e+1:1002	15/15
1: PSO	2.3(5)	2.9(3)	20(13)	52(57)	$\infty$ 1900	0/15	1: PSO	2.9(2)	2.6(2)	$\infty$	$\infty$	$\infty$ 7800	0/15
2: PSO	1.4(1)	4.8(4)	11(8)	9.3(5)	$\infty$ 1900	0/15	2: PSO	2.0(1)	3.9(5)	<b>5.8(1)*3</b>	<b>6.0(4)*3</b>	$\infty$ 8000	0/15
<b>f<sub>12</sub></b>	4.0e+7:3.6	1.6e+7:7.6	4.0e+6:19	1.6e+4:52	1.0e+0:268	15/15	<b>f<sub>12</sub></b>	1.0e+8:23	6.3e+7:39	2.5e+7:76	4.0e+6:209	1.0e+1:1042	15/15
1: PSO	1.5(3)	2.8(3)	41(28)	$\infty$	$\infty$ 1700	0/15	1: PSO	2092(2411)	$\infty$	$\infty$	$\infty$	$\infty$ 7500	0/15
2: PSO	2.1(1)	2.6(1)	4.5(3)	<b>26(4)*3</b>	$\infty$ 1800	0/15	2: PSO	<b>12(8)*</b>	<b>15(5)*3</b>	<b>15(2)*3</b>	<b>11(1)*3</b>	$\infty$ 6700*3	0/15
<b>f<sub>13</sub></b>	1.0e+3:2.8	6.3e+2:8.4	4.0e+2:17	6.3e+1:52	6.3e-2:264	15/15	<b>f<sub>13</sub></b>	1.6e+3:28	1.0e+3:64	6.3e+2:79	4.0e+1:211	2.5e+0:1724	15/15
1: PSO	1.8(2)	3.1(5)	17(4)	$\infty$	$\infty$ 1700	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 7400	0/15
2: PSO	3.1(3)	4.7(5)	5.8(5)	<b>17(3)*3</b>	$\infty$ 1800	0/15	2: PSO	<b>10(6)*3</b>	<b>13(4)*3</b>	<b>18(4)*3</b>	<b>25(2)*3</b>	$\infty$ 6800*3	0/15
<b>f<sub>14</sub></b>	1.6e+1:3.0	1.0e+1:10	6.3e+0:15	2.5e-1:53	1.0e-5:251	15/15	<b>f<sub>14</sub></b>	2.5e+1:15	1.6e+1:42	1.0e+1:75	1.6e+0:219	6.3e-4:1106	15/15
1: PSO	2.1(2)	2.1(3)	$\infty$	$\infty$	$\infty$ 1800	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 7400	0/15
2: PSO	2.8(4)	2.0(2)	3.1(3)	<b>16(4)*3</b>	$\infty$ 1700*3	0/15	2: PSO	<b>22(12)*3</b>	<b>13(8)*3</b>	<b>13(3)*3</b>	<b>12(1)*3</b>	$\infty$ 7000*3	0/15
<b>f<sub>15</sub></b>	1.6e+2:3.0	1.0e+2:13	6.3e+1:24	4.0e+1:55	1.6e+1:289	5/5	<b>f<sub>15</sub></b>	6.3e+2:15	4.0e+2:67	2.5e+2:292	1.6e+2:846	1.0e+2:1671	15/15
1: PSO	2.5(1)	2.9(8)	35(147)	211(121)	$\infty$ 1700	0/15	1: PSO	1899(4152)	$\infty$	$\infty$	$\infty$	$\infty$ 7300	0/15
2: PSO	4.3(9)	2.5(3)	4.5(2)	<b>5.1(3)*2</b>	<b>7.4(0.9)*3</b>	10/15	2: PSO	<b>13(11)</b>	<b>9.0(2)*3</b>	<b>5.0(0.9)*3</b>	<b>4.8(0.9)*3</b>	<b>5.4(3)*3</b>	11/15
<b>f<sub>16</sub></b>	4.0e+1:4.8	2.5e+1:16	1.6e+1:46	1.0e+1:120	4.0e+0:334	15/15	<b>f<sub>16</sub></b>	4.0e+1:26	2.5e+1:127	1.6e+1:540	1.6e+1:540	1.0e+1:1384	15/15
1: PSO	1.5(1)	1.4(0.7)	4.6(1)	18(18)	74(107)	1/15	1: PSO	520(1682)	$\infty$	$\infty$	$\infty$	$\infty$ 6800	0/15
2: PSO	1.8(2)	1.4(2)	2.4(2)	3.5(4)	16(21)	5/15	2: PSO	5.1(9)	<b>16(25)*3</b>	<b>12(2)*3</b>	<b>12(7)*3</b>	41(28)	2/15
<b>f<sub>17</sub></b>	1.0e+1:5.2	6.3e+0:26	4.0e+0:57	2.5e+0:110	6.3e-1:412	15/15	<b>f<sub>17</sub></b>	1.6e+1:11	1.0e+1:63	6.3e+0:305	4.0e+0:468	1.0e+0:1030	15/15
1: PSO	2.9(2)	7.3(2)	39(31)	69(76)	$\infty$ 1700	0/15	1: PSO	355(881)	$\infty$	$\infty$	$\infty$	$\infty$ 7500	0/15
2: PSO	4.0(3)	2.9(2)	3.5(2)	<b>3.2(1)*2</b>	<b>3.1(0.6)*3</b>	14/15	2: PSO	8.6(6)	<b>7.1(5)*3</b>	<b>4.8(1)*3</b>	<b>5.8(2)*3</b>	<b>14(15)*3</b>	7/15
<b>f<sub>18</sub></b>	6.3e+1:3.4	4.0e+1:7.2	2.5e+1:20	1.6e+1:58	1.6e+0:318	15/15	<b>f<sub>18</sub></b>	4.0e-1:116	2.5e+1:252	1.6e+1:430	1.0e+1:621	4.0e+0:1090	15/15
1: PSO	1.8(2)	4.5(5)	16(17)	65(52)	$\infty$ 1700	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 7400	0/15
2: PSO	2.0(2)	3.1(3)	3.9(2)	2.9(5)	<b>7.6(5)*3</b>	10/15	2: PSO	<b>4.1(2)*3</b>	<b>4.5(3)*3</b>	<b>5.4(2)*3</b>	<b>5.5(2)*3</b>	<b>15(13)*3</b>	6/15
<b>f<sub>19</sub></b>	1.6e-1:172	1.0e-1:242	6.3e-2:675	4.0e-2:3078	2.5e-2:4946	15/15	<b>f<sub>19</sub></b>	1.6e-1:2.5e5	1.0e-1:3.4e5	6.3e-2:3.4e5	4.0e-2:3.4e5	2.5e-2:3.4e5	3/15
1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 1800	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 6900	0/15
2: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 1900	0/15	2: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 7700	0/15
<b>f<sub>20</sub></b>	6.3e+3:5.1	4.0e+3:8.4	4.0e+1:15	2.5e+0:69	1.0e+0:851	15/15	<b>f<sub>20</sub></b>	1.6e+4:38	1.0e+4:42	2.5e+2:62	2.5e+0:250	1.6e+0:2536	15/15
1: PSO	2.6(3)	2.5(3)	108(121)	$\infty$	$\infty$ 1700	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 6900	0/15
2: PSO	2.2(1)	2.0(1)	8.9(6)	<b>8.8(4)*3</b>	15(28)	2/15	2: PSO	<b>8.6(4)*3</b>	<b>11(3)*3</b>	<b>21(6)*3</b>	<b>19(4)*3</b>	14(6)	3/15
<b>f<sub>21</sub></b>	4.0e+1:3.9	2.5e+1:11	1.6e+1:31	6.3e+0:73	1.6e+0:347	5/5	<b>f<sub>21</sub></b>	6.3e+1:36	4.0e+1:77	4.0e+1:77	1.6e+1:456	4.0e+0:1094	15/15
1: PSO	1.9(2)	1.3(1.0)	1.8(2)	40(34)	$\infty$ 1700	0/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 6800	0/15
2: PSO	1.5(1)	1.8(5)	2.0(1)	6.5(2)	<b>6.8(4)*3</b>	8/15	2: PSO	<b>21(11)*3</b>	<b>17(10)*3</b>	<b>17(13)*3</b>	<b>6.0(2)*3</b>	<b>20(22)*3</b>	4/15
<b>f<sub>22</sub></b>	6.3e+1:3.6	4.0e+1:15	2.5e+1:32	1.0e+1:71	1.6e+0:341	5/5	<b>f<sub>22</sub></b>	6.3e+1:45	4.0e+1:68	4.0e+1:68	1.6e+1:231	6.3e+0:1219	15/15
1: PSO	1.6(2)	1.9(4)	5.4(15)	52(68)	71(68)	1/15	1: PSO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ 6900	0/15
2: PSO	2.6(3)	2.6(5)	2.9(4)	4.8(5)	<b>5.9(6)*2</b>	10/15	2: PSO	<b>17(7)*3</b>	<b>19(10)*3</b>	<b>19(4)*3</b>	<b>16(23)*3</b>	<b>8.4(9)*3</b>	7/15
<b>f<sub>23</sub></b>	1.0e+1:3.0	6.3e+0:9.0	4.0e+0:33	2.5e+0:84	1.0e+0:518	15/15	<b>f<sub>23</sub></b>	6.3e+0:29	4.0e+0:118	2.5e+0:306	2.5e+0:306	1.0e+0:1614	15/15
1: PSO	2.3(4)	2.6(4)	6.6(14)	57(46)	46(35)	1/15	1: PSO	160(180)	812(594)	$\infty$	$\infty$		