# Modyfikacje/hybrydyzacje algorytmu PSO w zadaniu optymalizacji globalnej wielowymiarowej funkcji ciaglej

# **PSO-DE Hybrid**

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#### **ABSTRACT**

Dokumentacja uzyskanych wynikow hybrydy PSO-DE

## **Categories and Subject Descriptors**

G.1.6 [Numerical Analysis]: Optimization—global optimization, unconstrained optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

#### **General Terms**

Algorithms

### **Keywords**

Benchmarking, PSODE, Optymalizacja wielowymiarowej funkcji ciaglej

#### 1. CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the PSO-DE Hybrid on the function  $f_8$  with restarts for at least 30 seconds and until a maximum budget equal to 400(D+2) is reached. The code was run on a Mac Intel(R) Core(TM) i5-2400S CPU @ 2.50GHz with 1 processor and 4 cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20, 40 equals x.x, x.x, x.x, x.x, x.x, x.x, and xxx milliseconds respectively.

repeat the above for the second algorithm

#### 2. RESULTS

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Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2 and 3 and in Table 1. The **expected running time** (**ERT**), used in the figures and table, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number

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of trials that actually reached  $f_t$  [?, ?]. Statistical significance is tested with the rank-sum test for a given target  $\Delta f_t$  (10<sup>-8</sup> as in Figure 1) using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by -1), or, if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

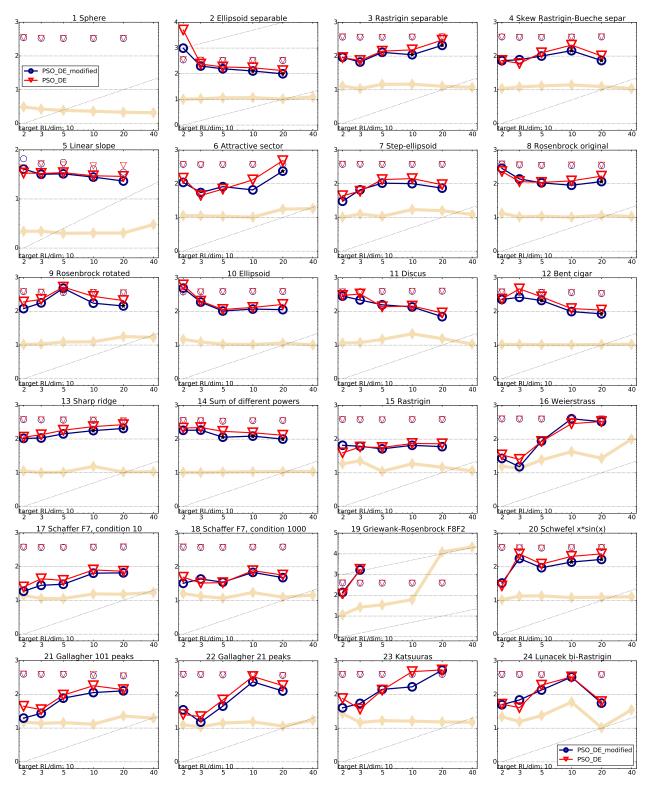


Figure 1: Expected running time (ERT in number of f-evaluations as  $\log_{10}$  value) divided by dimension versus dimension. The target function value is chosen such that the bestGECCO2009 artificial algorithm just failed to achieve an ERT of  $10 \times \text{DIM}$ . Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with p < 0.01 and Bonferroni correction number of dimensions (six). Legend:  $\circ$ :PSO DE modified,  $\nabla$ :PSO DE.

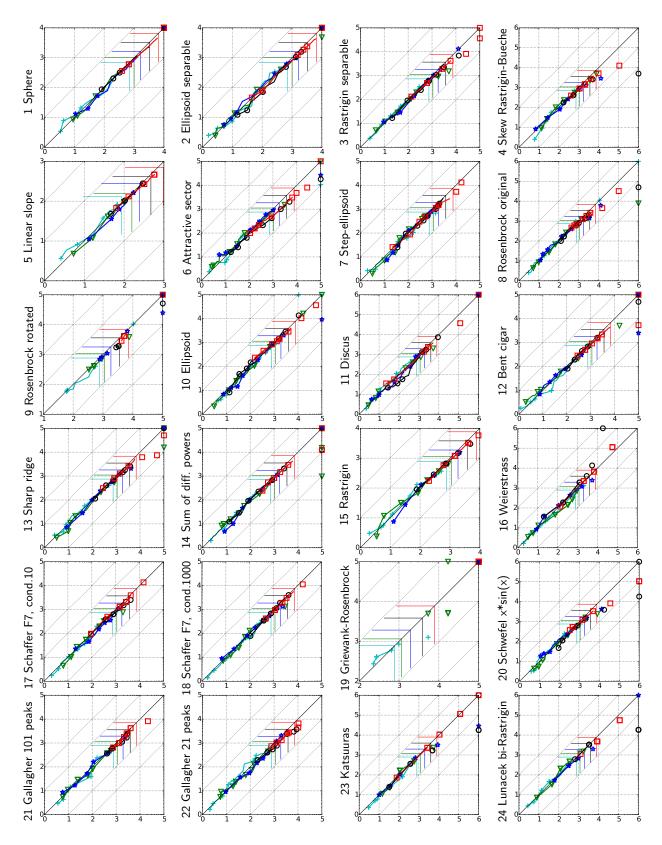


Figure 2: Expected running time (ERT in  $\log_{10}$  of number of function evaluations) of PSO DE modified (y-axis) versus PSO DE (x-axis) for 8 runlength-based target function values for budgets between  $0.5 \times \text{DIM}$  and  $50 \times \text{DIM}$  evaluations. Each runlength-based target f-value is chosen such that the ERTs of the bestGECCO2009 artificial algorithm for the given and a slightly easier target bracket the reference budget. Markers on the upper or right edge indicate that the respective target value was never reached. Markers represent dimension: 2:+, 3: , 5:\*, 10:0, 20: , 40:.

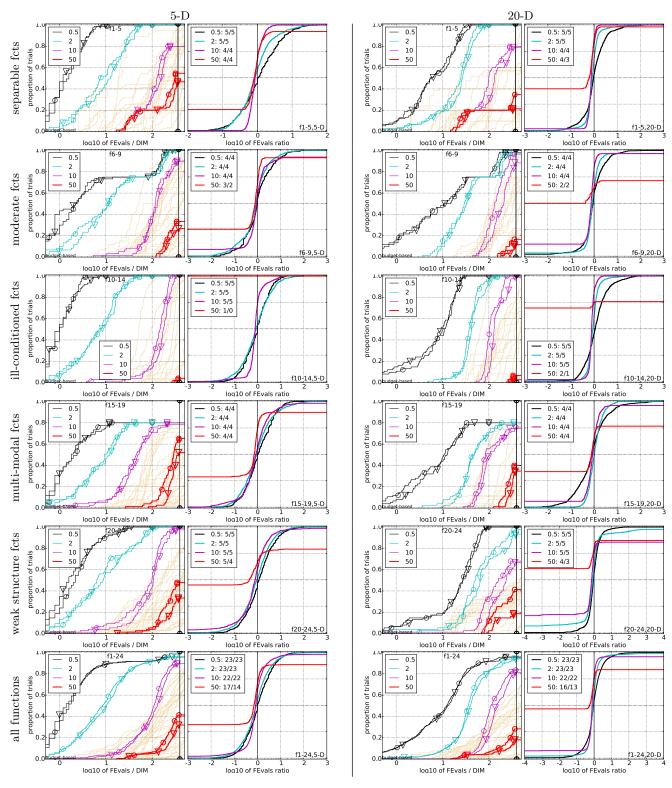


Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to fall below  $f_{\rm opt} + \Delta f$  for PSO DE modified ( $\circ$ ) and PSO DE ( $\bigtriangledown$ ) where  $\Delta f$  is the target just not reached by the GECCO-BBOB-2009 best algorithm within a budget of  $k \times {\rm DIM}$  evaluations, with k being the value in the legend. Right sub-columns: ECDF of FEval ratios of PSO DE modified divided by PSO DE for runlength-based targets; all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being >0 or <1. The legends indicate the target budget of  $k \times {\rm DIM}$  evaluations and, after the colon, the number of functions that were solved in at least one trial (PSO DE modified first).

5-D 20-D

	5-D						20-D						
#FEs/D	0.5	1.2	3	10	50	#succ	#FEs/D	0.5	1.2	3	10	50	#succ
f <sub>1</sub>	2.5e+1:4.8		1 0e=8·12	1 0e=8·12		15/15	$\mathbf{f_1}$	6.3e+1:24	4.0e+1:42	1.0e-8:43	1.0e-8:43	1.0e-8:43	15/15
1: PSO	2.8(2)	2.6(3)	∞*3	∞*3	$\infty 1600^{*3}$	0/15	1: PSO	15(3)	14(2)	∞*3	∞*3	$\infty 6500^{\star 3}$	0/15
2: PSO	2.3(3)	3.8(5)	∞	∞	∞ 1600	0/15	2: PSO	16(7) 4.0e+6:29	16(3)	∞ 1.0.15.05	∞ 1.0e+4:207	∞6600	0/15 15/15
$\mathbf{f_2}$	1.6e+6:2.9	4.0e+5:11	4.0e+4:15	6.3e+2:58	1.0e-8:95	15/15	f <sub>2</sub>		2.5e+6:42	1.0e+5:65	1.0e+4:207 10(0.9)*3	1.0e-8:412 $\infty 6500*3$	1 '
1: PSO	2.0(1)	1.3(1)	11(4)	14(2)	$\infty$ 1600 $\times$ 3	0/15	1: PSO 2: PSO	1.6(2) 1.6(1)	1.9(2) 2.4(3)	17(2)* 21(4)	13(1)	∞ 6500 ∞ 6500	$0/15 \\ 0/15$
2: PSO	1.9(2)	1.2(1)	9.3(5)	16(3)	∞1600	0/15	f <sub>3</sub>	6.3e+2:33	4.0e+2:44	1.6e+2:109	1.0e+2:255	2.5e+1:3277	15/15
1: PSO	1.6e+2:4.1 2.8(3)	1.0e+2:15 2.0(1)	6.3e+1:23 6.1(5)	2.5e+1:73 8.9(2)	1.0e+1:716 2.5(2)	$\frac{15/15}{12/15}$	1: PSO	4.0(4)	11(3)	18(3)*2	16(4)*	11(20)	3/15
2: PSO	2.3(2)	3.2(2)	7.7(3)	10(2)	3.2(2)	10/15	2: PSO	4.2(3)	14(4)	25(4)	23(3)	∞ 7400	0/15
$_{ m f_4}$	2.5e+2:2.6	1.6e+2:10	1.0e+2:19	4.0e+1:65	1.6e+1:434		$f_4$	6.3e+2:22	4.0e+2:91	2.5e+2:250	1.6e+2:332	6.3e+1:1927	15/15
1: PSO	3.2(2)	3.2(3)	4.3(3)	7.7(2)	2.9(2)	14/15	1: PSO	18(7)	10(2)	5.9(1.0)*	8.4(2)*2	6.6(4)*	8/15
2: PSO	2.6(2)	2.9(4)	5.9(2)	10(3)	5.4(4)	10/15	2: PSO	19(14) 2.5e+2:19	12(6) 1.6e+2:34	8.1(1) 1.0e-8:41	13(2) 1.0e-8:41	57(29) 1.0e-8:41	1/15
<b>f</b> 5 1: PSO	6.3e+1:4.0 2.8(2)	4.0e+1:10 3.6(2)	1.0e-8:10 16(5)	1.0e-8:10 16(6)	1.0e-8:10 16(5)	15/15 15/15	1: PSO	3.5(3)	3.8(2)	11(4)	11(2)	11(3)	15/15 $15/15$
2: PSO	3.3(3)	4.9(3)	17(5)	17(5)	17(3)	15/15	2: PSO	3.3(2)	3.8(2)	14(5)	14(2)	14(7)	15/15
<sub>f6</sub>		2.5e+4:8.4		2.5e+1:54	2.5e-1:254	15/15	<sub>f6</sub>	2.5e+5:16	6.3e+4:43	1.6e+4:62	1.6e+2:353		
1: PSO	4.2(4)	2.3(1)	4.3(4)	7.6(3)	104(102)	1/15	1: PŠO	6.2(6)	5.0(3)	4.8(3)	13(4)	∞ 7600	0/15
2: PSO	1.9(0.5)	2.0(1)	5.1(5)	6.4(4)	∞ 1800	0/15	2: PSO	7.0(5)	6.0(1)	6.1(1)	27(18) 6.3e+1:319	∞7500 1.0e+1:1351	0/15 15/15
1: PSO	1.8(1)	2.4(5)	2.5e+1:20 6.2(8)	4.0e+0:54 10(4)	1.0e+0:324 3.0(1)	15/15 = 15/15	1: PSO	1.0e+3:11 2.5(4)	4.0e+2:39 7.6(6)	2.5e+2:74 6.6(5)	4.7(2)	10(8)	7/15
2: PSO	3.1(3)	3.7(4)	5.3(3)	12(4)	3.8(1)	14/15	2: PSO	2.1(3)	7.6(6)	7.1(5)	5.9(2)	13(11)	6/15
f <sub>8</sub>	1.0e+4:4.6	6.3e+3:6.8	1.0e+3:18	6.3e+1:54	1.6e+0:258		f <sub>8</sub>	4.0e+4:19	2.5e+4:35	4.0e+3:67	2.5e+2:231	1.6e+1:1470	15/15
1: PSO	2.4(2)	3.3(3)	6.3(6)	10(3)	24(18)	4/15	1: PSO	25(8)	17(4)	16(4)*	<b>10</b> (1)*2	<b>22</b> (15)*2	3/15
2: PSO	2.6(1.0) 2.5e+1:20	2.3(2) 1.6e+1:26	6.3(5) 1.0e+1:35	10(3) 4.0e+0:62	49(52) 1.6e-2:256	$\frac{2/15}{15/15}$	2: PSO	23(10)	17(6)	21(3)	15(4)	69(114)	1/15
1: PSO	36(12)	32(7)	27(6)	39(36)	2.0e-z:230 ∞1700	0/15	1: PSO	1.0e+2:357 8.1(3)*2	6.3e+1:560	4.0e+1:684	2.5e+1:756 6.6(0.9)*	1.0e+1:1716	15/15 0/15
2: PSO		35(12)	32(23)	42(27)	∞ 1700	0/15	2: PSO	12(11)	7.5(1) 9.2(5)	6.6(8) 8.2(3)	8.4(0.9)	$\infty 6700$ $\infty 6800$	0/15
f <sub>10</sub>		6.3e+5:7.0		6.3e+3:54	2.5e+1:297		f <sub>10</sub>	1.6e+6:15	1.0e+6:27	4.0e+5:70	6.3e+4:231	4.0e+3:1015	
1: PSO 2: PSO	2.4(2)	1.9(2)	1.4(2) 2.2(3)	10(5)	31(28)	3/15	1: PSO	16(9)	17(8)	12(6)	10(5)	36(47)	3/15
11 f <sub>11</sub>	2.6(2) 1.0e+6:3.0	2.3(3) 6.3e+4:6.2		11(8) 6.3e+1:74	∞1900 6.3e-1:298	$0/15 \\ 15/15$	2: PSO	10(9)	13(10)	12(8)	14(5)	56(80)	2/15
1: PSO	1.9(2)	3.1(5)	8.0(8)	11(5)	∞ 1900	0/15	f <sub>11</sub> 1: PSO	4.0e+4:11 3.1(4)	2.5e+3:27 3.6(3)	1.6e+2:313 4.5(3)	1.0e+2:481 4.9(2)	1.0e+1:1002 ∞7900	15/15 0/15
2: PSO	1.4(0.7)	4.8(6)	11(11)	9.3(7)	∞ 1900	0/15	2: PSO	2.0(2)	3.9(3)	5.8(3)	6.0(2)	∞ 8000	0/15
f <sub>12</sub>		1.6e+7:7.6		1.6e+4:52	1.0e+0:268	15/15	f <sub>12</sub>	1.0e+8:23	6.3e+7:39	2.5e+7:76	4.0e+6:209	1.0e+1:1042	15/15
1: PSO	1.9(2)	3.0(3)	4.1(3)	21(4)*2	∞1700 <sup>*3</sup>	0/15	1: PSO	12(7)	13(4)	12(2)	8.1(1)*2	$\infty 6600*3$	0/15
2: PSO f <sub>13</sub>	2.1(1) 1.0e+3:2.8	2.6(1.0) 6.3e+2:8.4	4.5(3) 4.0e+2:17	26(4) 6.3e+1:52	∞1800 6.3e-2:264	$0/15 \\ 15/15$	2: PSO	12(8)	15(4)	15(2)	11(2)	∞6700	0/15
1: PSO	2.6(2)	3.4(2)	5.9(7)	14(3)	∞ 1800	0/15	f <sub>13</sub>	1.6e+3:28	1.0e+3:64	6.3e+2:79	4.0e+1:211	2.5e+0:1724	15/15
2: PSO	3.1(2)	4.7(3)	5.8(5)	17(2)	∞ 1800	0/15	1: PSO 2: PSO	11(4) 10(6)	12(3) 13(4)	16(2) 18(3)	19(3)*3 25(2)	29(16) <sup>★2</sup> ∞6800	2/15 0/15
f <sub>14</sub>	1.6e+1:3.0	1.0e+1:10		2.5e-1:53	1.0e-5:251	15/15	f <sub>14</sub>	2.5e+1:15	1.6e+1:42	1.0e+1:75	1.6e+0:219	6.3e-4:1106	15/15
1: PSO	1.6(0.7)	1.0(1)	3.0(2)	11(3)*2	∞1700	0/15	1: PSO	17(10)	13(6)	11(4)	$9.2(0.7)^{*3}$	∞ 7000	0/15
2: PSO	2.8(4) 1.6e+2:3.0	2.0(1) 1.0e+2:13	3.1(3) 6.3e+1:24	16(4) 4.0e+1:55	∞1700 1.6e+1:289	0/15 5/5	2: PSO	22(11)	13(8)	13(4)	12(2)	∞ 7000	0/15
f <sub>15</sub> 1: PSO	2.5(4)	2.1(3)	5.2(5)	4.6(3)	5.4(7)	11/15	f <sub>15</sub>	6.3e+2:15	4.0e+2:67	2.5e+2:292	1.6e+2:846	1.0e+2:1671	15/15
2: PSO	4.3(5)	2.5(3)	4.5(5)	5.1(4)	7.4(6)	10/15	1: PSO 2: PSO	11(5) 13(11)	8.6(3) 9.0(4)	4.1(1.0) $5.0(1)$	3.8(0.7) 4.8(1)	$3.5(0.7) \\ 5.4(5)$	$\frac{14}{15}$ $\frac{11}{15}$
f <sub>16</sub>	4.0e+1:4.8						f <sub>16</sub>	4.0e+1:26	2.5e+1:127	1.6e+1:540	1.6e+1:540		
1: PSO 2: PSO	1.8(2) 1.8(1)	2.4(3) 1.4(2)	2.9(2) 2.4(1)	3.6(8) 3.5(3)	7.4(5) $16(13)$	9/15 5/15	1: PSO	4.5(8)	18(9)	12(7)	12(7)	81(69)	1/15
f <sub>17</sub>	1.0e+1:5.2		4.0e+0:57				2: PSO	5.1(5)	16(10)	12(7)	12(16)	41(57)	2/15
1: PSO	4.4(5)	1.8(2)	2.7(2)	2.5(2)	2.7(2)	14/15	f <sub>17</sub> 1: PSO	1.6e+1:11 9.2(20)	1.0e+1:63 8.0(5)	6.3e+0:305 4.3(1)	4.0e+0:468 4.5(2)	1.0e+0:1030 $13(13)$	15/15 7/15
2: PSO	4.0(5)	2.9(2)	3.5(3)	3.2(1)	3.1(1)	14/15	2: PSO	8.6(5)	7.1(5)	4.8(1)	5.8(3)	14(13)	7/15
f <sub>18</sub> 1: PSO	6.3e+1:3.4 2.6(2)	4.0e+1:7.2 3.1(6)	2.5e+1:20 $4.0(4)$	1.6e+1:58 3.0(2)	1.6e+0:318 4.2(1)	$\frac{15}{15}$	f <sub>18</sub>	4.0e+1:116	2.5e+1:252	1.6e+1:430	1.0e+1:621	4.0e+0:1090	
2: PSO	2.0(2)	3.1(5)	3.9(3)	2.9(2)	7.6(5)	10/15	1: PSO	3.7(2)	3.8(1)	4.4(1)	4.9(1.0)	10(4)	8/15
f <sub>19</sub>				4.0e-2:3078			2: PSO	4.1(2) 1.6e-1:2.5e5	4.5(2)	5.4(3) 6.3e-2:3.4e5	5.5(2)	15(17) 2.5e-2:3.4e5	6/15 3/15
1: PSO	∞	∞	∞	∞	∞ 1900	0/15	f <sub>19</sub> 1: PSO	1.0e-1:2.5e5 ∞	1.0e-1:3.4e5 ∞	0.5e-z:5.4e5 ∞	4.0e-2:3.4e5 ∞	2.5e-2:3.4e3 ∞7700	0/15
2: PSO	∞ 0.0 + 0.5 1	00	∞ / 0 / / 15	∞ 0.5.10.00	∞1900	0/15	2: PSO	∞	∞	∞	∞	∞ 7700	0/15
f <b>20</b> 1: PSO	3.7(5)	4.0e+3:8.4 2.9(4)	4.0e+1:15 11(4)	2.5e+0:69 6.8(3)	1.0e+0.851 4.9(4)	$\frac{15/15}{6/15}$	f <sub>20</sub>	1.6e+4:38	1.0e+4:42	2.5e+2:62	2.5e+0:250	1.6e+0:2536	
2: PSO	2.2(3)	2.0(1)	8.9(7)	8.8(4)	15(18)	2/15	1: PSO 2: PSO	10(6)	13(2)	17(3)	13(3)	3.2(3)	11/15
f <sub>21</sub>			1.6e+1:31	6.3e+0:73	1.6e+0:347	5/5	2: PSO f <sub>21</sub>	8.6(3) 6.3e+1:36	11(3) 4.0e+1:77	21(6) 4.0e+1:77	19(11) 1.6e+1:456	14(19) 4.0e+0:1094	$\frac{3/15}{15/15}$
1: PSO	2.2(1)	1.6(1)	1.7(3)	5.3(3)	4.5(5)	10/15	1: PSO	18(6)	13(6)	13(7)	5.6(8)	7.5(16)	8/15
2: PSO	1.5(2) 6.3e+1:3.6	1.8(1)	2.0(2) 2.5e+1:32	6.5(4) 1.0e+1:71	6.8(8) 1.6e+0:341	8/15 5/5	2: PSO	21(8)	17(5)	17(8)	6.0(10)	20(14)	4/15
f <b>22</b> 1: PSO	2.5(3)	2.5(1.0)	1.7(2)	3.2(3)	6.0(3)	9/15	f <sub>22</sub>	6.3e+1:45	4.0e+1:68	4.0e+1:68	1.6e+1:231	6.3e+0:1219	
2: PSO	2.6(2)	2.6(3)	2.9(5)	4.8(4)	5.9(1)	10/15	1: PSO 2: PSO	16(5) 17(8)	15(4) 19(7)	15(7) 19(6)	11(9) 16(2)	3.7(2) 8.4(7)	$\frac{11/15}{7/15}$
f <sub>23</sub>		6.3e+0:9.0		2.5e+0:84	1.0e+0:518	15/15	f <sub>23</sub>	6.3e+0:29	4.0e+0:118	2.5e+0:306	2.5e+0:306	1.0e+0:1614	
1: PSO	3.4(3)	3.1(2)	3.2(5)	8.5(3)	55(36)	1/15	1: PSO	2.6(1)	20(10)	34(21)	34(35)	∞7900	0/15
2: PSO f <sub>24</sub>	3.3(3) 6.3e+1:15	4.1(6) 4.0e+1:37	3.4(2) 4.0e+1:37	8.1(4) 2.5e+1:118	∞2000 1.6e+1:692	0/15	2: PSO	2.7(4)	22(16)	35(62)	35(15)	∞ 7900	0/15
1: PSO	3.7(4)	7.8(4)	7.8(4)	5.7(3)	2.9(2)	10/15	1: PSO	2.5e+2:208				4.0e+1:31629 ∞7700	
2: PSO	4.2(5)	8.6(4)	8.6(5)	8.0(8)	7.5(13)	5/15	1: PSO 2: PSO	5.3(1.0) 5.9(2)	5.2(2) 9.2(5)	8.4(8) 17(20)	12(13) ∞	∞7700 ∞7700	0/15 0/15
								1 (=)	/	(==)			1 -/

Table 1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 in dimensions 5 (left) and 20 (right). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding best ERT (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. 1:PSO is PSO DE modified and 2:PSO is PSO DE. Bold entries are statistically significantly better compared to the other algorithm, with p = 0.05 or  $p = 10^{-k}$  where  $k \in \{2, 3, 4, ...\}$  is the number following the \* symbol, with Bonferroni correction of 48. A \$\pi\$ indicates the same tested against the best algorithm of BBOB-2009.