Manual Calculation:

① $x[n] = \{1, 2, 3, 4\}$

$N = \text{length}\{x(n)\} = 4$

W.K.T

$$x[k] = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi k n}{N}} \quad - ①$$

for $N = 4 \Rightarrow \frac{2\pi k n}{4} = \frac{\pi k n}{2}$

$$\Rightarrow x[k] = x(0)\, e^{-j\frac{2\pi k(0)}{2}} + x(1)\, e^{-j\frac{\pi k(2)}{2}} + x(2)\, e^{-j\frac{\pi k(2)}{2}}$$

$$+ x(3)\, e^{-j\frac{\pi k(3)}{2}}$$

$$x[k] = 1e^{-j(0)} + 2e^{-j\frac{\pi}{2}k} + 3e^{-j\pi k} + 4e^{-j\frac{3\pi k}{2}} \quad - ②$$

$\Rightarrow$ for $k = 0$ to $N-1$ eq^n ② becomes

$x[0] = 10$

$x[1] = -2 + 2j$

$x[2] = -2$

$x[3] = 2 - 2j$

$\Rightarrow$ Magnitude and phase spectrum

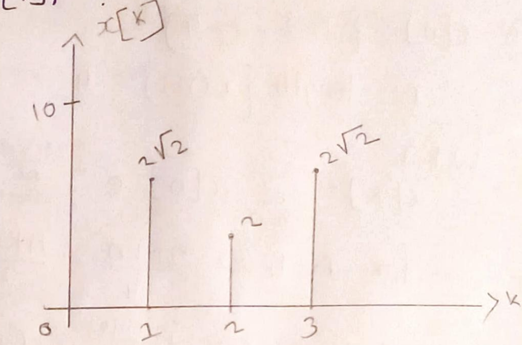
- Magnitude spectra $= |X[k]| = ?$

$|X(0)| = 10$

$|X(1)| = 2\sqrt{2}$

$|X(2)| = 2$

$|X(3)| = 2\sqrt{2}$



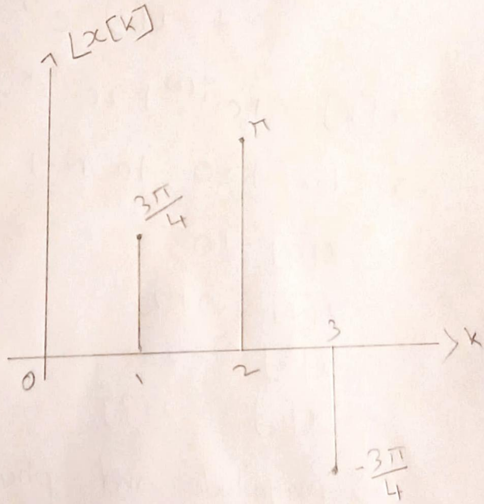
- Phase spectra $= \angle X[k]$

$\angle X(0) = 0$

$\angle X(1) = \frac{3\pi}{4}$

$\angle X(2) = \pi$

$\angle X(3) = -\frac{3\pi}{4}$



2

Manual Calculation (Circular Convolution using DFT-IDFT):

$x[n] = \{1, 2, 3\}$ & $h[n] = \{1, 1\}$

$N = \text{len}\{x(n)\} = 3$, $M = \text{len}\{h(n)\} = 2$

$L_{cin} = \max\{M, N\} = 3$

$h_{pad}(n) = \{1, 1, 0\}$

$N_{DFT} = 3$

$$X[k] = \sum_{n=0}^{2} x(n)\, e^{-j\frac{2\pi kn}{3}}$$

$$X[k] = 1 + 2e^{-j\frac{2\pi k}{3}} + 3e^{-j\frac{4\pi k}{3}} \quad -\ (1)$$

& $$H_{pad}[k] = \sum_{k=0}^{1} h[n] \cdot e^{-j\frac{2\pi kn}{3}}$$

$$H_{pad}[k] = 1 + e^{-j\frac{2\pi}{3}k} + 0$$

$$H_{pad}[k] = 1 + e^{-j\frac{2\pi}{3}k} \quad -\ (2)$$

for $k = 0$ to 2

$$x[k] = \left\{6,\ -\frac{3}{2} + \frac{\sqrt{3}}{2}j,\ -\frac{3}{2} - \frac{\sqrt{3}}{2}j\right\}$$

$$H_{pad}[k] = \left\{ 2,\ \frac{1}{2} - \frac{\sqrt{3}}{2}j,\ \frac{1}{2} + \frac{\sqrt{3}}{2}j \right\}$$

$$Y[k] = \begin{bmatrix} 6 \\ -\frac{3}{2} + \frac{\sqrt{3}}{2}j \\ -\frac{3}{2} - \frac{\sqrt{3}}{2}j \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -\frac{1}{2} - \frac{\sqrt{3}}{2}j \\ \frac{1}{2} + \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 12 \\ \sqrt{3}j \\ -\sqrt{3}j \end{bmatrix}$$

$$y[k] = \begin{bmatrix} 12 \\ \sqrt{3}\angle \pi/2 \\ -\sqrt{3}\angle -\pi/2 \end{bmatrix}$$

By Inv DFT :-

$$y(n) = \frac{1}{N} \sum_{n=0}^{2} Y[k]\, e^{j\frac{2\pi kn}{3}}$$

$$y(n) = \frac{1}{3}\left[12 + \sqrt{3}\, e^{j\frac{\pi}{2}} e^{j\frac{2\pi n}{3}} + \sqrt{3}\, e^{-j\frac{\pi}{2}} e^{j\frac{4\pi n}{3}} \right]$$

$$y(n) = \frac{1}{3}\left[12 + \sqrt{3}\angle\left(\frac{\pi}{2} + \frac{2\pi n}{3}\right) + \sqrt{3}\angle\left(-\frac{\pi}{2} + \frac{4\pi n}{3}\right) \right]$$

$$y(n) = \frac{1}{3}\begin{bmatrix} 12 \\ 9 \\ 15 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

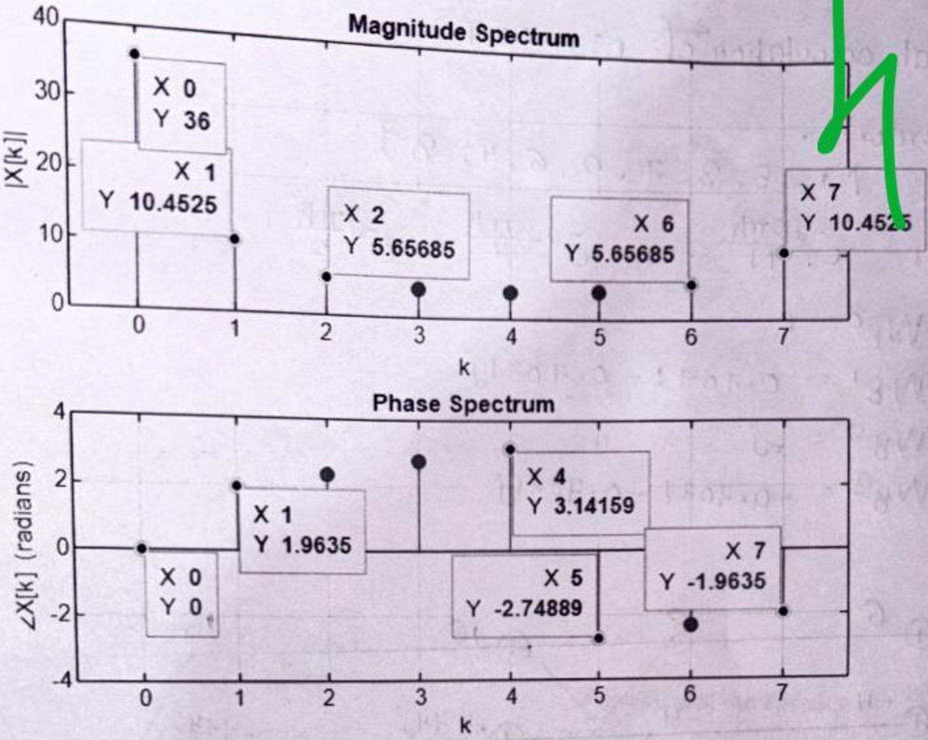


Fig. No.2 Magnitude and Phase Spectrum.

Manual Calculations:

$\therefore x = [1, 2, 3, 4, 5, 6, 7, 8]$

$N = 8$

0 -	000 -	000 -	0
1 -	001 -	100 -	4
2 -	010 -	010 -	2
3 -	011 -	110 -	6
4 -	100 -	001 -	1
5 -	101 -	101 -	5
6 -	110 -	011 -	3
7 -	111 -	111 -	7

Manual calculation of DIT-FFT

order is.

[1, 5, 3, 7, 2, 6, 4, 8]

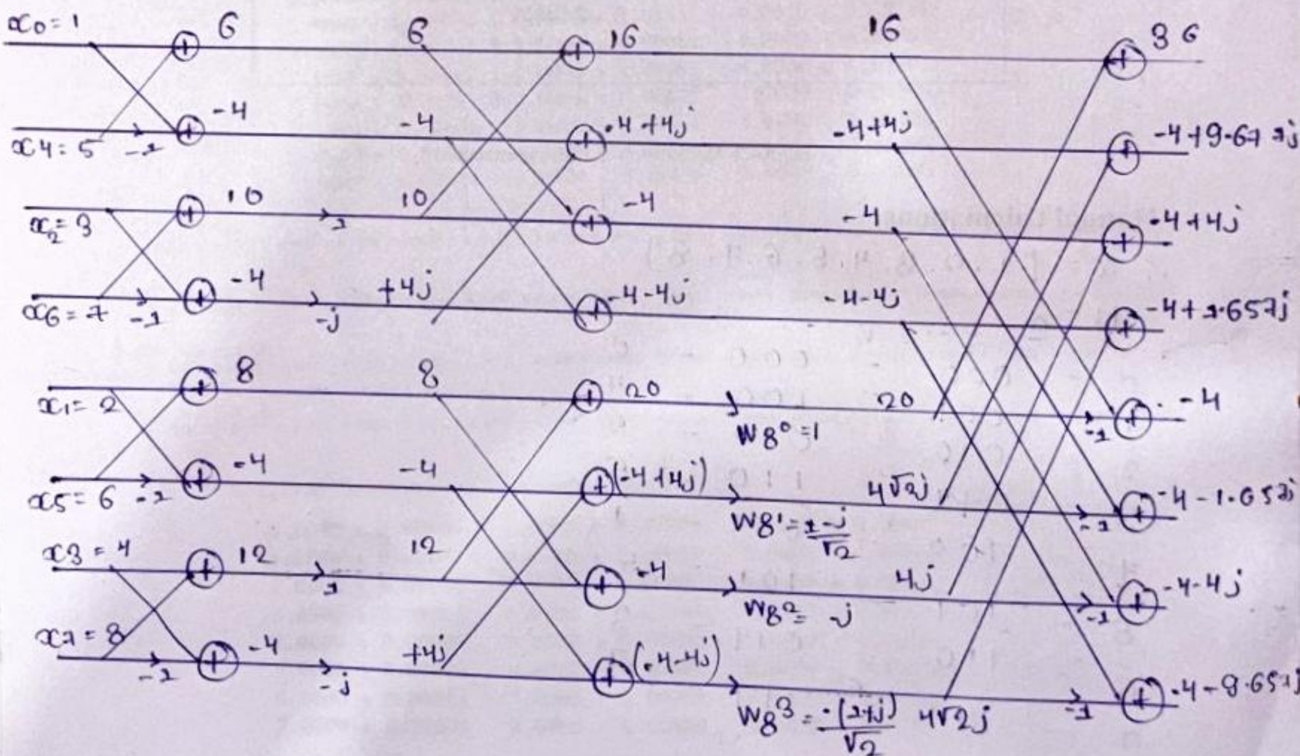
$$W_N = e^{-\frac{j2\pi k}{N}} = e^{-j\frac{2\pi k}{4}} = e^{-j\frac{\pi k}{2}}$$

$W_8^0 = 1$

$W_8^1 = 0.7071 - 0.7071j$

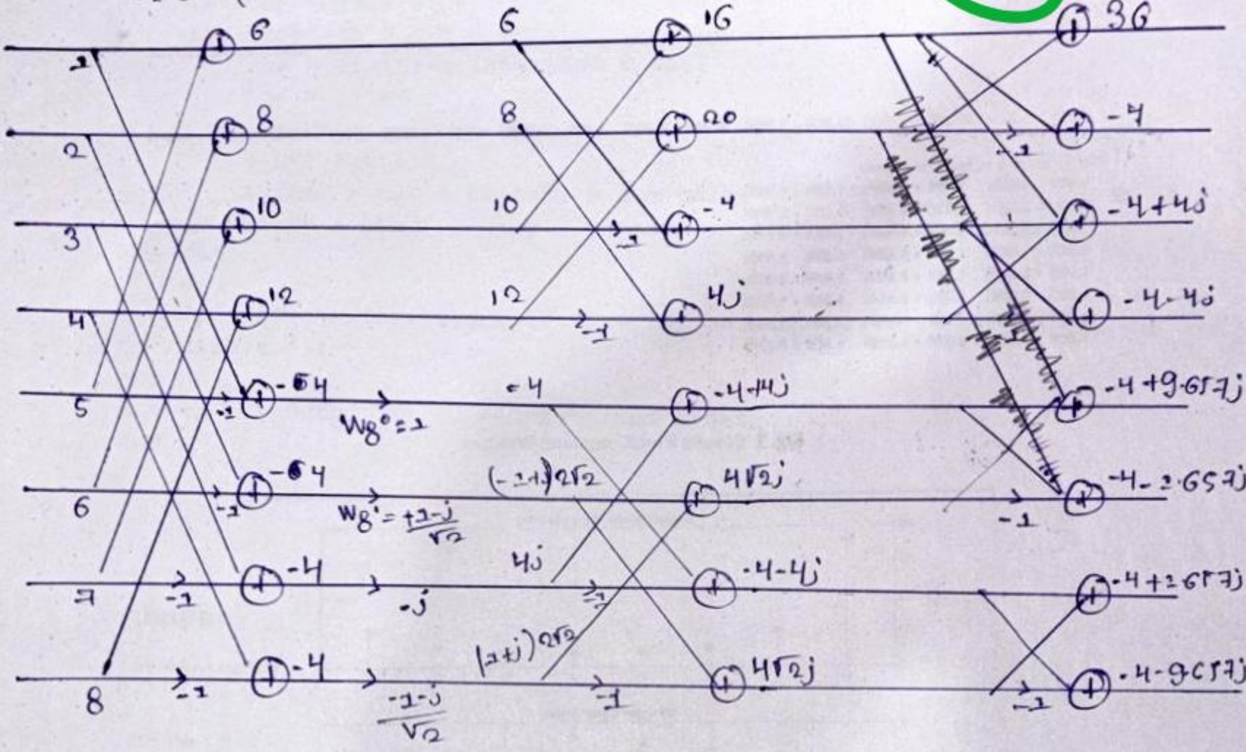
$W_8^2 = -j$

$W_8^3 = -0.7071 - 0.7071j$



Manual Calculations:

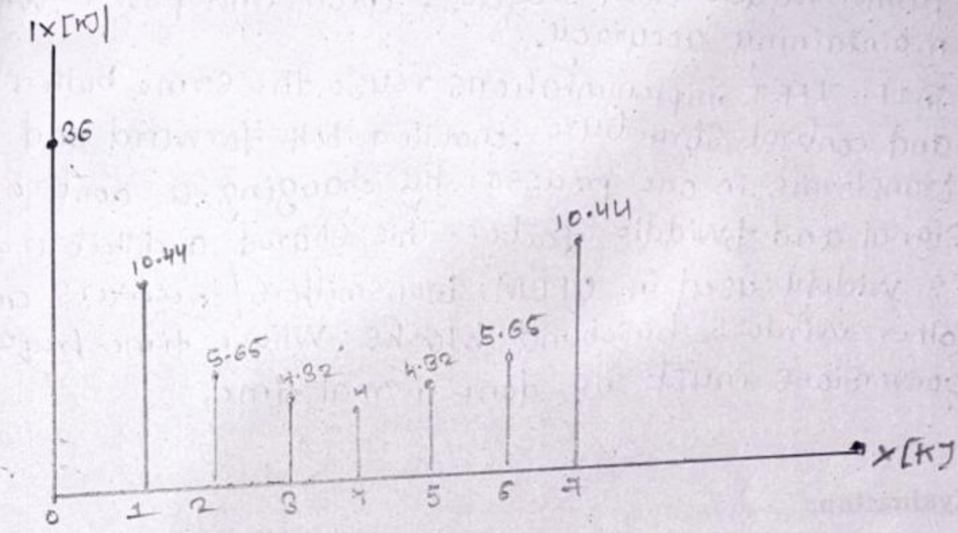
$x[n] = \{1, 2, 3, 4, 5, 6, 7, 8\}$



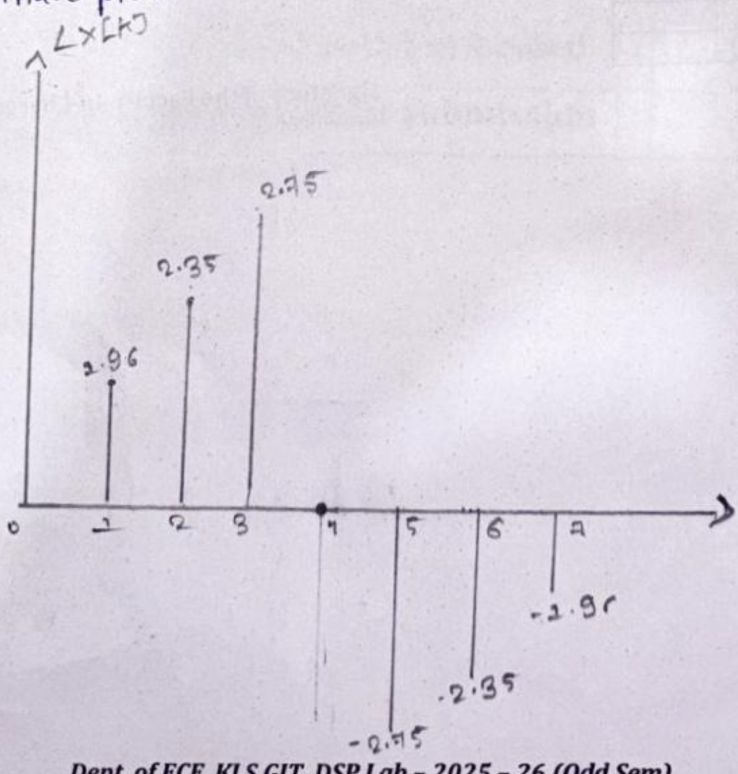
$X[k] = \{36, -4+9.65j, -4+4j, -4+1.65j, -4, -4-1.65j, -4-4j, -4-9.57j\}$

k	X[k]	|X[k]|	∠X[k]
0	36	36	0
1	-4+9.65j	10.44	1.96
2	-4+4j	5.65	2.35
3	-4+1.65j	4.32	2.75
4	-4	~~3.65~~ 4	0
5	-4-1.65j	4.32	-2.75
6	-4-4j	5.65	-2.35
7	-4-9.65j	10.44	-1.96

* Magnitude plot :-



* Phase plot



Manual Calculations:

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Given,

$f_s = 8000\,Hz$

$f_p = 1000\,Hz$

$f_s' = 3000\,Hz$

$A_p = 2\,dB$

$A_s = 20\,dB$

$$\omega_p = \frac{2\pi f_p}{f_s} = 2\pi \frac{\times 1000}{8000} = \frac{\pi}{4} \approx 0.7854\ rad$$

$$\omega_s = \frac{2\pi f_s'}{f_s} = \frac{2\pi \times 3000}{8000} = \frac{3\pi}{4} \approx 2.3562\ rad$$

$$\Omega_p = 2\tan(\omega_p/2) = 2\times\tan\left(\frac{\pi}{8}\right) = 0.8284\ rad/s$$

$$\Omega_s = 2\tan(\omega_s/2) = 2\times\tan\left(\frac{3\pi}{8}\right) = 4.8284\ rad/s$$

$\Omega_s > \Omega_p$ = low pass filter

$$N \geq \frac{\log_{10}\left(\frac{10^{A_s/10} - 1}{10^{A_p/10} - 1}\right)}{2\log_{10}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log_{10}\left(\frac{10^{0.1\times 2} - 1}{10^{0.1\times 20} - 1}\right)}{2\log_{10}\left(\frac{4.8284}{0.8284}\right)}$$

$$N \geq \frac{2.229}{1.53} \approx 1.46$$

$[N = 2]$

$$\Omega c = \frac{\Omega p}{[10^{0.1 Ap} - 1]^{1/2N}} = \frac{0.8284}{[10^{0.1 \times 2} - 1]^{1/4}} = 0.95$$

$$\boxed{\Omega c = 0.95 \text{ rad/s}}$$

For $N = 2$.

$$H_N(s) = \frac{1}{s^2 + \sqrt{2} s + 1}$$

put $s = \frac{s}{\Omega c} = \frac{s}{0.95}$

$$H_2(s) = \frac{1}{\left(\frac{s}{0.95}\right)^2 + \sqrt{2}\left(\frac{s}{0.95}\right) + 1}$$

$$H_2(s) = \frac{0.95^2}{s^2 + \sqrt{2} \times 0.95 + 0.95^2}$$

$$\left[H_2(s) = \frac{0.9025}{s^2 + 1.34 s + 0.9025} \right]$$

$$\Omega_p = 2\tan\left[\frac{\omega_p}{2}\right] = 2\tan\left(\frac{0.7854}{2}\right) \approx 0.8284 \text{ rad/sec}$$

$$\Omega_s = 2\tan\left[\frac{\omega_s}{2}\right] = 2\tan\left(\frac{2.3562}{2}\right) \approx 4.8284 \text{ rad/sec}$$

Since $\Omega_s > \Omega_p$ = low pass filter

$$N = \frac{\cosh^{-1}\left[\sqrt{\frac{10^{0.1A_s}-1}{10^{0.1A_p}-1}}\right]}{\cosh^{-1}\left[\frac{\Omega_s}{\Omega_p}\right]}$$

$$N = \cosh^{-1}\left[\sqrt{\frac{10^{0.1\times 20}-1}{10^{0.1\times 2}-1}}\right]$$

$$N \approx 1.33$$

$$[N = 2]$$

cut off freq^n

$$\Omega_c = \Omega_p = 0.8284 \text{ rad/s}$$

ε (ripple parameter)

$$\varepsilon = \sqrt{10^{0.1A_p}-1} = \sqrt{10^{0.1\times 2}-1} = 0.7647$$

$$H_n(s) = \frac{k}{s^2 + a_1 s + a_0}$$

$$H(s) = H_n\left(\frac{s}{\Omega_c}\right)$$

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Department of Electronics and Communication Engineering

$$H(s) = \frac{K'}{s^2 + \alpha s + \beta}$$

$$H(s) = \frac{0.5648}{s^2 + 0.6659 s + 0.5648}$$

$$\begin{bmatrix} \alpha = 0.6659 \\ \beta = 0.5648 \\ K' = 0.5648 \end{bmatrix}$$

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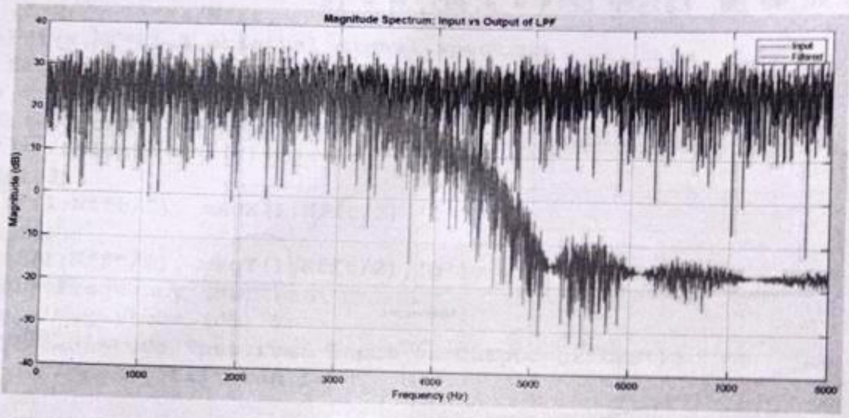
Fig. No.3 Magnitude Spectrum: Input vs Output of LPF



Manual Calculations:

Given $f_S = 16000\,Hz$, $f_P = 2000\,Hz$, $f_S' = 5000\,Hz$

$A_S = 40\,dB$

$\omega = 2\pi f / f_S$

$\omega_P = \dfrac{2\pi \times 2000}{16000} = \dfrac{\pi}{4} = 0.7854\ rad$

$\omega_S = \dfrac{2\pi \times 5000}{16000} = \dfrac{5\pi}{8} \approx 1.9635\ rad$

$\Delta\omega = \omega_S - \omega_P$

$\Delta\omega = 1.9635 - 0.7854 = 1.1781\ rad$

$\therefore$ Kaisen window selection

$A_S = 40\,dB$

since

$21 \leq A_S \leq 50$

$[\beta = 0.5842(As-21)^{0.4} + 0.07886(As-21)]$

Filter length

$$M = \left[\frac{As-8}{2.285\,\Delta\omega}\right]$$

$$= \left[\frac{40-8}{2.285 * 1.1781}\right]$$

$$= [M = 11.88] \cong 12$$

But M should be odd $\boxed{M \approx 13}$

$hd[n]$ ∴ $\left[N = \frac{M-1}{\alpha} = \frac{12}{2} = 6\right]$

$$hd[n] = \frac{\sin(\omega c(n-\alpha))}{\pi(n-\alpha)}, \quad n \neq \alpha$$

$$\frac{\omega c}{\pi}, \quad n = \alpha$$

$$\omega c = \frac{\omega p + \Delta\omega}{2} = \frac{0.7854 + 1.1781}{2} = 0.9817$$

$$\omega[n] = 0.5\left(1 - \cos\left(\frac{2\pi n}{M-1}\right)\right)$$

$H[n] = hd[n] + \omega[n]$

n	hd[n]	ω[n]	H[n]
0	-0.020	0	0
1	-0.062	0.066	-4.092×10^{-3}
2	-0.056	0.250	-0.014
3	0.020	0.500	0.01
4	0.147	0.750	0.11025
5	0.264	0.933	0.2463
6	0.312	1	0.312
7	0.264	0.933	0.2463
8	0.147	0.750	0.01
9	0.020	0.500	-0.014
10	-0.056	0.250	-9.092×10^{-3}
11	-0.062	0.066	0
12	-0.020	0	

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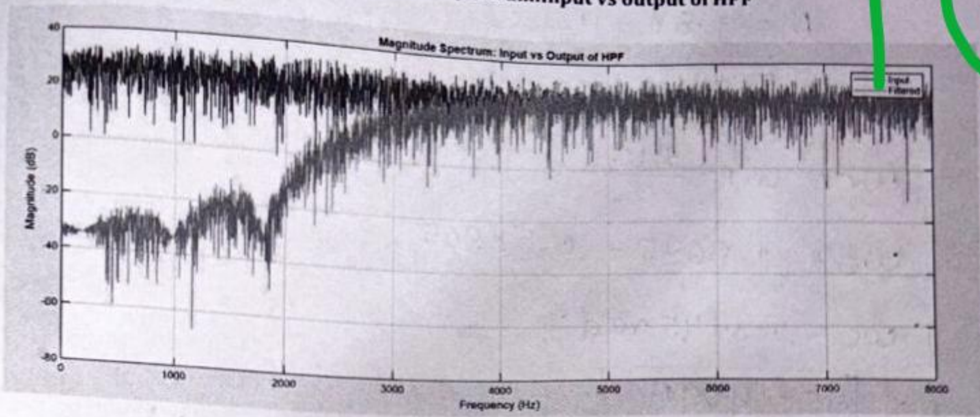
Fig No. 3 : Magnitude Spectrum:Input vs output of HPF



Manual Calculations:

Given Specifications

$f_s = 16000\ Hz$

$f_p = 5000\ Hz$

$f_s' = 2000\ Hz$

$A_s = 40dB$

Since $f_p > f_s \rightarrow$ high pass FIR

$$\omega_p = 2\pi \times \frac{5000}{16000} = \frac{5\pi}{8} = 1.9635\ rad$$

$$\omega_s = 2\pi \times \frac{2000}{16000} = \frac{\pi}{4} = 0.7854\ rad$$

$$\Delta\omega = \omega_p - \omega_s$$

$$\Delta\omega = 1.8635 - 0.7854 = 1.1781\ rad$$

$$M = \left[\frac{A_s - 8}{2.285\Delta\omega}\right] = \left[\frac{32-8}{2.285 - 2.692}\right]$$

$M = 11.88 = 12$

$\boxed{M = 13}$

$\omega c = \omega p - \frac{\Delta \omega}{2}$

$\omega c = 1.9635 - 0.58905$

$[\omega c = 1.3745 \text{ rad}]$

As, $\boxed{As = 40dB.}$

Hanning window, $\omega[n] = 0.5\left(1 - \cos\left(\frac{2\pi n}{M-1}\right)\right)$

$$hd[n] = \begin{cases} \dfrac{-\sin(\omega c(n-\alpha))}{\pi(n-\alpha)} & n \neq \alpha \\ 1 - \dfrac{\omega c}{\pi} & n = \alpha \end{cases}$$

n	k=n-α	hd[n]	ω[n]	H[n]
0	-6	-0.049013	0	0
1	-5	-0.035869	0.058253	-0.002969
2	-4	0.056270	0.250000	0.014067
3	-3	0.088222	0.50000	0.044111
4	-2	-0.060906	0.691342	-0.045679
5	-1	-0.31014	0.941544	-0.291281
6	0	0.562500	1.00000	0.562500
7	1	0.312194	0.841544	-0.291281
8	2	0.060906	0.691342	-0.045679
9	3	-0.088222	0.50000	0.044111
10	4	0.056270	0.25000	0.014067
11	5	-0.035369	0.058253	-0.002969
12	6	-0.049013	0	-0.0000.