



Ficha de Exercícios 2 - Parte I

Primitivas. Integrais indefinidos

1. Determine os seguintes integrais indefinidos:

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| (a) $\int (3x^2 + 5x + 7) dx$ | (b) $\int \sqrt[3]{x} dx$ | (c) $\int (x^3 + 1)^2 dx$ |
| (d) $\int \frac{\arctg x}{1 + x^2} dx$ | (e) $\int \frac{3x^2}{1 + x^3} dx$ | (f) $\int \frac{1}{x^7} dx$ |
| (g) $\int \frac{x + 1}{2 + 4x^2} dx$ | (h) $\int 4x^3 \cos x^4 dx$ | (i) $\int \frac{x}{\sqrt{1 - x^2}} dx$ |
| (j) $\int \sin x \cos^5 x dx$ | (k) $\int \operatorname{tg} x dx$ | (l) $\int \frac{\ln x}{x} dx$ |
| (m) $\int e^{\operatorname{tg} x} \sec^2 x dx$ | (n) $\int x 7^{x^2} dx$ | (o) $\int \sin(\sqrt{2}x) dx$ |
| (p) $\int \frac{x^2 + 1}{x} dx$ | (q) $\int \frac{x}{(7 + 5x^2)^{\frac{3}{2}}} dx$ | (r) $\int \frac{x^3}{1 + x^8} dx$ |
| (s) $\int \frac{5x^2}{\sqrt{1 - x^6}} dx$ | (t) $\int \frac{1}{x^2 + 7} dx$ | (u) $\int \frac{1}{x^2 + 2x + 5} dx$ |
| (v) $\int \frac{x}{1 + x^4} dx$ | (w) $\int \frac{x}{\sqrt{1 - x^4}} dx$ | (x) $\int \frac{x^3}{\sqrt{1 - x^4}} dx$ |

2. Determine os seguintes integrais indefinidos:

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|--|--|---|
| (a) $\int \frac{e^{\arcsen x}}{\sqrt{1 - x^2}} dx$ | (b) $\int \operatorname{tg}^2 x dx$ | (c) $\int \frac{1}{x} \cos(\ln x) dx$ |
| (d) $\int \frac{6}{x \ln^3(4x)} dx$ | (e) $\int \frac{e^{3x}}{(e^{3x} - 2)^6} dx$ | (f) $\int \operatorname{tg}^3 x dx$ |
| (g) $\int \frac{1}{x \sqrt{1 - \ln^2 x}} dx$ | (h) $\int e^x \sqrt{1 + e^x} dx$ | (i) $\int \frac{1}{x \ln x} dx$ |
| (j) $\int \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$ | (k) $\int \frac{1 + \cos x}{x + \sin x} dx$ | (l) $\int \frac{e^{2x+1}}{e^{2x} + 3} dx$ |
| (m) $\int x^5 \sin(x^6) dx$ | (n) $\int \frac{\arccos x - x}{\sqrt{1 - x^2}} dx$ | (o) $\int \frac{\cos(\ln(x^2))}{x} dx$ |
| (p) $\int \frac{1}{e^x + 9e^{-x}} dx$ | (q) $\int \frac{\sin(\arctg x)}{1 + x^2} dx$ | |

3. Considere a função g definida em \mathbb{R}^+ por $g(x) = \frac{(\ln x)^2}{x}$.

- (a) Determine a família de todas as primitivas de g .
- (b) Indique a primitiva da função g que se anula para $x = e$.

Resolução:

(a) $\int \frac{(\ln x)^2}{x} dx = \frac{(\ln x)^3}{3} + c, \quad c \in \mathbb{R}.$

(b) Para cada $c \in \mathbb{R}$, $G(x) = \frac{(\ln x)^3}{3} + c$ é uma primitiva de g .
Pretendemos então determinar $c \in \mathbb{R}$ tal que $G(e) = 0$.

$$G(e) = 0 \Leftrightarrow \frac{1}{3} + c = 0 \Leftrightarrow c = -\frac{1}{3}$$

Assim, $G(x) = \frac{(\ln x)^3}{3} - \frac{1}{3}$ é a primitiva de g que se anula para $x = e$.

4. Determine a primitiva F para a função $f(x) = \frac{2}{x} + \frac{3}{x^2}$ tal que $F(-1) = 1$.

5. Sabendo que a função f satisfaz a igualdade $\int f(x) dx = \sin x - x \cos x - \frac{1}{2}x^2 + c$, com $c \in \mathbb{R}$, determinar $f(\frac{\pi}{4})$.

6. Determine a primitiva da função $f(x) = \frac{1}{x^2} + 1$ que se anula no ponto $x = 2$.

7. Determine a primitiva da função f definida por $f(x) = \frac{3 \cos(\ln x)}{x}$ que toma o valor 2 em $x = 1$.

8. Determine a função g que verifica as seguintes condições:

$$g'(x) = \frac{1}{(1 + \operatorname{arctg}^2(x))(1 + x^2)} \quad \text{e} \quad \lim_{x \rightarrow +\infty} g(x) = 0.$$

9. Determine, usando a técnica de integração por partes, os seguintes integrais indefinidos:

(a) $\int (x+1) \sin x dx$

Resolução: Fazendo

$$\begin{aligned} f'(x) = \sin x & \quad \text{temos} \quad f(x) = -\cos x \\ g(x) = x+1 & \quad \text{temos} \quad g'(x) = 1 \end{aligned}$$

Assim,

$$\begin{aligned} \int (x+1) \sin x dx &= -(x+1) \cos x + \int \cos x dx \\ &= -(x+1) \cos x + \sin x + c, \quad c \in \mathbb{R} \end{aligned}$$

(b) $\int x \cos x dx$ (c) $\int x^2 \cos x dx$ (d) $\int e^{-3x}(2x+3) dx$ (e) $\int \ln^2 x dx$

(f) $\int \ln x dx$ (g) $\int \ln(x^2+1) dx$ (h) $\int x \operatorname{arctg} x dx$ (i) $\int \cos(\ln x) dx$

(j) $\int e^{2x} \sin(x) dx$ (k) $\int \sin(\ln x) dx$ (l) $\int \operatorname{arcsen} x dx$ (m) $\int x \operatorname{arcsen} x^2 dx$

(n) $\int x^3 e^{x^2} dx$ (o) $\int \operatorname{arctg} x dx$ (p) $\int \operatorname{arctg} \frac{1}{x} dx$ (q) $\int \sqrt{x} \ln x dx$

(r) $\int \sin x \cos(3x) dx$ (s) $\int \cos^2 x dx$ (t) $\int \sec^3 x dx$ (u) $\int \frac{x^2}{\sqrt{(1-x^2)^3}} dx$

10. Determine, usando a técnica de integração por substituição, os seguintes integrais indefinidos:

(a) $\int x\sqrt{x+1} dx$

Resolução:

Consideremos a substituição $x+1 = t^2$, com $t \geq 0$. Definindo $\varphi(t) = t^2 - 1$, $t \geq 0$, temos que φ é invertível, diferenciável e $\varphi'(t) = 2t$. Então

$$\begin{aligned} \int x\sqrt{x+1} dx &= \int (t^2 - 1) \cdot t \cdot 2t dt \\ &= \frac{2t^5}{5} - \frac{2t^3}{3} + c. \end{aligned}$$

Atendendo a que $x+1 = t^2$, com $t \geq 0$, vem que $t = \sqrt{x+1}$. Assim,

$$\int x\sqrt{x+1} dx = \frac{2(x+1)^2\sqrt{x+1}}{5} - \frac{2(x+1)\sqrt{x+1}}{3} + c, \text{ com } c \in \mathbb{R}.$$

(b) $\int \frac{x}{1+\sqrt{x}} dx$ (c) $\int \frac{1}{x^2\sqrt{1-x^2}} dx$ (d) $\int \frac{1}{x^2\sqrt{x^2+4}} dx$ (e) $\int \frac{1}{x\sqrt{x^2-5}} dx$
(f) $\int x^2\sqrt{1-x} dx$ (g) $\int x^2\sqrt{4-x^2} dx$ (h) $\int \frac{1}{x\sqrt{x^2-1}} dx$ (i) $\int \frac{1}{x\sqrt{x^2+4}} dx$
(j) $\int \frac{1}{x^2\sqrt{9-x^2}} dx$ (k) $\int \frac{x^2}{\sqrt{1-2x-x^2}} dx$ (l) $\int \frac{1}{x^2\sqrt{x^2-7}} dx$ (m) $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx$
(n) $\int x(2x+5)^{10} dx$ (o) $\int e^{\sqrt{x}} dx$ (p) $\int \frac{\ln x}{x \cdot \sqrt{1+\ln x}} dx$ (q) $\int \frac{1+\operatorname{tg}^2 x}{\sqrt{\operatorname{tg} x - 1}} dx$

11. Determine os seguintes integrais indefinidos:

(a) $\int \frac{x+2}{(x-1)^2(x^2+4)} dx$

Resolução:

A determinação deste integral indefinido passa por decompor em frações simples a fração

$$\frac{x+2}{(x-1)^2(x^2+4)}.$$

Isto é, passa por escrever a dita fração na seguinte forma

$$\frac{x+2}{(x-1)^2(x^2+4)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+4} \quad (*)$$

com A , B , C e D constantes reais a determinar.

Temos então que

$$\frac{x+2}{(x-1)^2(x^2+4)} = \frac{(A+C)x^3 + (-A+B-2C+D)x^2 + (4A+C-2D)x - 4A+4B+D}{(x-1)^2(x^2+4)}$$

donde resulta a igualdade de polinómios

$$x+2 = (A+C)x^3 + (-A+B-2C+D)x^2 + (4A+C-2D)x - 4A+4B+D.$$

Atendendo à condição de igualdade de polinômios resulta que

$$\begin{cases} A + C = 0 \\ -A + B - 2C + D = 0 \\ 4A + C - 2D = 1 \\ -4A + 4B + D = 2 \end{cases} \Leftrightarrow \begin{cases} A = -\frac{1}{25} \\ B = \frac{15}{25} \\ C = \frac{1}{25} \\ D = -\frac{14}{25} \end{cases}$$

Voltando a (*), podemos escrever

$$\frac{x+2}{(x-1)^2(x^2+4)} = \frac{-\frac{1}{25}}{x-1} + \frac{\frac{15}{25}}{(x-1)^2} + \frac{\frac{1}{25}x - \frac{14}{25}}{x^2+4}.$$

Assim

$$\begin{aligned} \int \frac{x+2}{(x-1)^2(x^2+4)} dx &= -\frac{1}{25} \int \frac{1}{x-1} dx + \frac{15}{25} \int (x-1)^{-2} dx + \frac{1}{25} \int \frac{x-14}{x^2+4} dx \\ &= -\frac{1}{25} \ln|x-1| - \frac{3}{5(x-1)} + \frac{1}{25} \int \frac{x}{x^2+4} dx - \frac{14}{25} \int \frac{1}{x^2+4} dx \\ &= -\frac{1}{25} \ln|x-1| - \frac{3}{5(x-1)} + \frac{1}{50} \ln(x^2+4) - \frac{7}{25} \operatorname{arctg} \frac{x}{2} + c, \quad c \in \mathbb{R} \end{aligned}$$

(b) $\int \frac{2x-1}{(x-2)(x-3)(x+1)} dx$	(c) $\int \frac{1}{(x-1)(x+1)^3} dx$	(d) $\int \frac{1}{x^3+8} dx$
(e) $\int \frac{x^8}{1+x^2} dx$	(f) $\int \frac{1}{x^3(1+x^2)} dx$	(g) $\int \frac{8}{x^4+4x^2} dx$
(h) $\int \frac{x^5+x^4-8}{x^3-4x} dx$	(i) $\int \frac{x^2}{(x-1)^3} dx$	(j) $\int \frac{x^3+3x-1}{x^4-4x^2} dx$
(k) $\int \frac{x+1}{x^3-1} dx$	(l) $\int \frac{x^4}{x^4-1} dx$	(m) $\int \frac{1}{x(x^2+1)^2} dx$
(n) $\int \frac{x+1}{x^2+4x+5} dx$		

12. Determine

(a) $\int \sin^2 \theta d\theta$	(b) $\int \sin^4 x dx$	(c) $\int \sin x \cos^2 x dx$
(d) $\int \sin^3 x dx$	(e) $\int \sin^5 x \cos^2 x dx$	(f) $\int \cos^3 x dx$
(g) $\int \frac{1}{\sqrt{2+x^2}} dx$	(h) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$	(i) $\int \frac{x}{x^2-5x+6} dx$
(j) $\int \frac{1}{\sqrt{2x-x^2}} dx$	(k) $\int x\sqrt{(1+x^2)^3} dx$	(l) $\int \frac{\sqrt{x}}{1+\sqrt{x}} dx$
(m) $\int x \ln x dx$	(n) $\int \frac{1+e^x}{e^{2x}+4} dx$	(o) $\int \frac{x}{\cos^2 x} dx$
(p) $\int \frac{\sin x}{(1-\cos x)^3} dx$	(q) $\int (2x^2+3)\operatorname{arctg} x dx$	(r) $\int \frac{1}{\sqrt{x^2+2x-3}} dx$
(s) $\int \sqrt{1+e^x} dx$	(t) $\int \frac{1}{\sqrt{e^x-1}} dx$	(u) $\int \cos x \cos(5x) dx$
(v) $\int \frac{\sin^3 x}{\sqrt{\cos x}} dx$	(w) $\int \sin^5 x dx$	(x) $\int \frac{\ln x}{x(\ln^2 x+1)} dx$

Exercícios de revisão

13. Determine a primitiva da função $f(x) = \operatorname{tg} x$ cujo gráfico passa pelo ponto de coordenadas $(\pi, 3)$.

14. Determine a função $f : \mathbb{R} \rightarrow \mathbb{R}$ tal que

$$f'(x) = \frac{2e^x}{3 + e^x} \quad \text{e} \quad f(0) = \ln 4.$$

15. Determine a função $f : \mathbb{R} \rightarrow \mathbb{R}$ tal que $f(0) = 1$, $f'(0) = 2$ e $f''(x) = 12x$, para todo o $x \in \mathbb{R}$.

16. Determine os seguintes integrais indefinidos:

(a) $\int \operatorname{sen}(2x)e^{\cos(2x)} dx$

(b) $\int \frac{1}{\sqrt{x} - \sqrt[4]{x}} dx$

(c) $\int \frac{1}{x^2\sqrt{x^2 - 9}} dx$

(d) $\int x^2 \operatorname{arctg} x dx$

(e) $\int \frac{x+2}{x(x^2+4)} dx$

(f) $\int \frac{x^2}{\sqrt{1+x^3}} dx$

(g) $\int \frac{1}{x^2\sqrt{1+x^2}} dx$

(h) $\int \frac{3x-1}{x^3+x} dx$

(i) $\int \frac{-\cos x}{(1+\operatorname{sen} x)^2} dx$

(j) $\int x \cdot \ln(1+x^2) dx$

(k) $\int \cos x \cdot \ln(\operatorname{sen} x) dx$

(l) $\int \frac{1}{x\sqrt{x^2-4}} dx$

(m) $\int x^3 \cos(x^2) dx$

(n) $\int x(\ln x)^2 dx$

(o) $\int \cos(\sqrt{x}) dx$

(p) $\int \operatorname{tg}^3 x dx$

17. Determine a função f tal que

$$f'(x) = \frac{5x-4}{x(x^2-2x+2)} \quad \text{e} \quad \lim_{x \rightarrow +\infty} f(x) = 0.$$

Soluções:

1. (a) $x^3 + \frac{5}{2}x^2 + 7x + c, \quad c \in \mathbb{R}$
(b) $\frac{3}{4}\sqrt[3]{x^4} + c, \quad c \in \mathbb{R}$
(c) $\frac{x^7}{7} + \frac{x^4}{2} + x + c, \quad c \in \mathbb{R}$
(d) $\frac{(\operatorname{arctg} x)^2}{2} + c, \quad c \in \mathbb{R}$
(e) $\ln |1 + x^3| + c, \quad c \in \mathbb{R}$
(f) $-\frac{1}{6x^6} + c, \quad c \in \mathbb{R}$
(g) $\frac{1}{8} \ln(2 + 4x^2) + \frac{\sqrt{2}}{4} \operatorname{arctg}(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
(h) $\operatorname{sen} x^4 + c, \quad c \in \mathbb{R}$
(i) $-\sqrt{1 - x^2} + c, \quad c \in \mathbb{R}$
(j) $-\frac{\cos^6 x}{6} + c, \quad c \in \mathbb{R}$
(k) $-\ln |\cos x| + c, \quad c \in \mathbb{R}$
(l) $\frac{(\ln x)^2}{2} + c, \quad c \in \mathbb{R}$
(m) $e^{\operatorname{tg} x} + c, \quad c \in \mathbb{R}$
(n) $\frac{1}{2 \ln 7} 7^{x^2} + c, \quad c \in \mathbb{R}$
(o) $-\frac{\sqrt{2}}{2} \cos(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
(p) $\frac{x^2}{2} + \ln |x| + c, \quad c \in \mathbb{R}$
(q) $-\frac{1}{5\sqrt{7+5x^2}} + c, \quad c \in \mathbb{R}$
(r) $\frac{1}{4} \operatorname{arctg}(x^4) + c, \quad c \in \mathbb{R}$
(s) $\frac{5}{3} \operatorname{arcsen}(x^3) + c, \quad c \in \mathbb{R}$
(t) $\frac{\sqrt{7}}{7} \operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right) + c, \quad c \in \mathbb{R}$
(u) $\frac{1}{2} \operatorname{arctg}\left(\frac{x+1}{2}\right) + c, \quad c \in \mathbb{R}$
(v) $\frac{1}{2} \operatorname{arctg}(x^2) + c, \quad c \in \mathbb{R}$
(w) $\frac{1}{2} \operatorname{arcsen}(x^2) + c, \quad c \in \mathbb{R}$
(x) $-\frac{1}{2} \sqrt{1 - x^4} + c, \quad c \in \mathbb{R}$
2. (a) $e^{\operatorname{arcsen} x} + c, \quad c \in \mathbb{R}$
(b) $\operatorname{tg} x - x + c, \quad c \in \mathbb{R}$
(c) $\operatorname{sen}(\ln x) + c, \quad c \in \mathbb{R}$
(d) $-\frac{3}{\ln^2(4x)} + c, \quad c \in \mathbb{R}$
(e) $-\frac{1}{15(e^{3x}-2)^5} + c, \quad c \in \mathbb{R}$
(f) $\frac{\operatorname{tg}^2 x}{2} + \ln |\cos x| + c, \quad c \in \mathbb{R}$
(g) $\operatorname{arcsen}(\ln x) + c, \quad c \in \mathbb{R}$
(h) $\frac{2}{3} \sqrt{(1 + e^x)^3} + c, \quad c \in \mathbb{R}$
(i) $\ln |\ln x| + c, \quad c \in \mathbb{R}$
(j) $2e^{\sqrt{x}} + c, \quad c \in \mathbb{R}$
(k) $\ln |x + \operatorname{sen} x| + c, \quad c \in \mathbb{R}$
(l) $\frac{e}{2} \ln(e^{2x} + 3) + c, \quad c \in \mathbb{R}$
(m) $-\frac{\cos(x^6)}{6} + c, \quad c \in \mathbb{R}$

- (n) $-\frac{1}{2}(\arccos x)^2 + \sqrt{1-x^2} + c, \quad c \in \mathbb{R}$
- (o) $\frac{1}{2}\text{sen}(\ln(x^2)) + c, \quad c \in \mathbb{R}$
- (p) $\frac{1}{3}\text{arctg}\left(\frac{e^x}{3}\right) + c, \quad c \in \mathbb{R}$
- (q) $-\cos(\text{arctg } x) + c, \quad c \in \mathbb{R}$

3. Resolvido

4. $F(x) = 2 \ln |x| - \frac{3}{x} - 2$

5. $\frac{\pi}{8}(\sqrt{2} - 2)$

6. $F(x) = -\frac{1}{x} + x - \frac{3}{2}$

7. $F(x) = 3\text{sen}(\ln x) + 2$

8. $g(x) = \text{arctg}(\text{arctg } x) - \text{arctg}(\pi/2)$

9. (a) Resolvido

- (b) $x\text{sen } x + \cos x + c, \quad c \in \mathbb{R}$
- (c) $x^2\text{sen } x + 2x \cos x - 2\text{sen } x + c, \quad c \in \mathbb{R}$
- (d) $-\frac{2x+3}{3}e^{-3x} - \frac{2}{9}e^{-3x} + c, \quad c \in \mathbb{R}$
- (e) $x(\ln^2 x - 2 \ln x + 2) + c, \quad c \in \mathbb{R}$
- (f) $x \ln x - x + c, \quad c \in \mathbb{R}$
- (g) $x \ln(x^2 + 1) - 2(x - \text{arctg } x) + c, \quad c \in \mathbb{R}$
- (h) $\frac{x^2}{2}\text{arctg } x - \frac{1}{2}(x - \text{arctg } x) + c, \quad c \in \mathbb{R}$
- (i) $\frac{x}{2} \cos(\ln x) + \frac{x}{2}\text{sen}(\ln x) + c, \quad c \in \mathbb{R}$
- (j) $\frac{-e^{2x} \cos x + 2e^{2x}\text{sen } x}{5} + c, \quad c \in \mathbb{R}$
- (k) $\frac{x\text{sen}(\ln x) - x \cos(\ln x)}{2} + c, \quad c \in \mathbb{R}$
- (l) $x \arcsen x + \sqrt{1-x^2} + c, \quad c \in \mathbb{R}$
- (m) $\frac{x^2}{2} \arcsen(x^2) + \frac{1}{2}\sqrt{1-x^4} + c, \quad c \in \mathbb{R}$
- (n) $\frac{1}{2}e^{x^2}(x^2 - 1) + c, \quad c \in \mathbb{R}$
- (o) $x \text{arctg } x - \frac{1}{2} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (p) $x \text{arctg } \frac{1}{x} + \frac{1}{2} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (q) $\frac{2}{3}\sqrt{x^3} \ln x - \frac{4}{9}\sqrt{x^3} + c, \quad c \in \mathbb{R}$
- (r) $\frac{\cos x \cos(3x) + 3\text{sen } x \text{sen}(3x)}{8} + c, \quad c \in \mathbb{R}$
- (s) $\frac{\cos x \text{sen } x + x}{2} + c, \quad c \in \mathbb{R}$
- (t) $\frac{\sec x \text{tg } x + \ln|\sec x + \text{tg } x|}{2} + c, \quad c \in \mathbb{R}$
- (u) $\frac{x}{\sqrt{1-x^2}} - \arcsen x + c, \quad c \in \mathbb{R}$

10. (a) Resolvido

- (b) $\frac{2}{3}\sqrt{x^3} - x + 2\sqrt{x} - 2 \ln |\sqrt{x} + 1| + c, \quad c \in \mathbb{R}$
- (c) $-\frac{\sqrt{1-x^2}}{x} + c, \quad c \in \mathbb{R}$
- (d) $-\frac{\sqrt{4+x^2}}{4x} + c, \quad c \in \mathbb{R}$
- (e) $\frac{1}{\sqrt{5}} \arccos\left(\frac{\sqrt{5}}{x}\right) + c, \quad c \in \mathbb{R}$
- (f) $-\frac{2}{3}(1-x)\sqrt{1-x} - \frac{2}{7}(1-x)^3\sqrt{1-x} + \frac{4}{5}(1-x)^2\sqrt{1-x} + c, \quad c \in \mathbb{R}$

- (g) $2\arcsen \frac{x}{2} - \frac{x(2-x^2)\sqrt{4-x^2}}{4} + c, \quad c \in \mathbb{R}$
- (h) $\arccos \frac{1}{x} + c, \quad c \in \mathbb{R}$
- (i) $-\frac{1}{2} \ln \left| \frac{\sqrt{x^2+4}}{x} + \frac{2}{x} \right| + c, \quad c \in \mathbb{R}$
- (j) $-\frac{\sqrt{9-x^2}}{9x} + c, \quad c \in \mathbb{R}$
- (k) $2\arcsen \frac{x+1}{\sqrt{2}} - \frac{(x+1)\sqrt{2-(x+1)^2}}{2} + 2\sqrt{2-(x+1)^2} + c, \quad c \in \mathbb{R}$
- (l) $\frac{\sqrt{x^2-7}}{7x} + c, \quad c \in \mathbb{R}$
- (m) $\frac{6}{7}x\sqrt[6]{x} - \frac{6}{5}\sqrt[6]{x^5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6\operatorname{arctg} \sqrt[6]{x} + c, \quad c \in \mathbb{R}$
- (n) $\frac{1}{48}(2x+5)^{12} - \frac{5}{44}(2x+5)^{11} + c, \quad c \in \mathbb{R}$
- (o) $2e^{\sqrt{x}}(\sqrt{x}-1) + c, \quad c \in \mathbb{R}$
- (p) $\frac{2}{3}(\sqrt{1+\ln x})^3 - 2\sqrt{1+\ln x} + c, \quad c \in \mathbb{R}$
- (q) $2\sqrt{\operatorname{tg} x - 1} + c, \quad c \in \mathbb{R}$
11. (a) Resolvido
- (b) $-\ln |x-2| + \frac{5}{4} \ln |x-3| - \frac{1}{4} \ln |x+1| + c, \quad c \in \mathbb{R}$
- (c) $\frac{1}{8} \ln |x-1| - \frac{1}{8} \ln |x+1| + \frac{1}{4(x+1)} + \frac{1}{4(x+1)^2} + c, \quad c \in \mathbb{R}$
- (d) $\frac{1}{12} \ln |x+2| - \frac{1}{24} \ln(x^2-2x+4) + \frac{\sqrt{3}}{12} \operatorname{arctg} \left(\frac{x-1}{\sqrt{3}} \right) + c, \quad c \in \mathbb{R}$
- (e) $\frac{x^7}{7} - \frac{x^5}{5} + \frac{x^3}{3} - x + \operatorname{arctg} x + c, \quad c \in \mathbb{R}$
- (f) $-\ln |x| - \frac{1}{2x^2} + \frac{1}{2} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
- (g) $-\frac{2}{x} - \operatorname{arctg} \left(\frac{x}{2} \right) + c, \quad c \in \mathbb{R}$
- (h) $\frac{x^3}{3} + \frac{x^2}{2} + 4x + 2 \ln |x| + 5 \ln |x-2| - 3 \ln |x+2| + c, \quad c \in \mathbb{R}$
- (i) $\ln |x-1| - \frac{2}{x-1} - \frac{1}{2(x-1)^2} + c, \quad c \in \mathbb{R}$
- (j) $-\frac{3}{4} \ln |x| - \frac{1}{4x} + \frac{13}{16} \ln |x-2| + \frac{15}{16} \ln |x+2| + c, \quad c \in \mathbb{R}$
- (k) $\frac{1}{3}(2 \ln |x-1| - \ln(x^2+x+1)) + c, \quad c \in \mathbb{R}$
- (l) $\frac{1}{4}(4x + \ln |x-1| - \ln |x+1| - 2\operatorname{arctg} x) + c, \quad c \in \mathbb{R}$
- (m) $\ln |x| - \frac{1}{2} \ln(1+x^2) + \frac{1}{2(x^2+1)} + c, \quad c \in \mathbb{R}$
- (n) $\frac{1}{2} \ln(x^2+4x+5) - \operatorname{arctg}(x+2) + c, \quad c \in \mathbb{R}$
12. (a) $\frac{1}{2}\theta - \frac{1}{4}\operatorname{sen}(2\theta) + c, \quad c \in \mathbb{R}$
- (b) $\frac{3}{8}x - \frac{1}{4}\operatorname{sen}(2x) + \frac{1}{32}\operatorname{sen}(4x) + c, \quad c \in \mathbb{R}$
- (c) $-\frac{\cos^3 x}{3} + c, \quad c \in \mathbb{R}$
- (d) $-\cos x + \frac{1}{3} \cos^3 x + c, \quad c \in \mathbb{R}$
- (e) $-\frac{\cos^3 x}{3} + \frac{2}{5} \cos^5 x - \frac{\cos^7 x}{7} + c, \quad c \in \mathbb{R}$
- (f) $\operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + c, \quad c \in \mathbb{R}$
- (g) $\ln \left| \sqrt{\frac{2+x^2}{2}} + \frac{x}{\sqrt{2}} \right| + c, \quad c \in \mathbb{R}$
- (h) $-2 \cos \sqrt{x} + c, \quad c \in \mathbb{R}$
- (i) $3 \ln |x-3| - 2 \ln |x-2| + c, \quad c \in \mathbb{R}$
- (j) $\arcsen(x-1) + c, \quad c \in \mathbb{R}$
- (k) $\frac{(1+x^2)^2 \sqrt{1+x^2}}{5} + c, \quad c \in \mathbb{R}$
- (l) $x - 2\sqrt{x} + 2 \ln(1+\sqrt{x}) + c, \quad c \in \mathbb{R}$

- (m) $\frac{x^2}{2} \ln x - \frac{x^2}{4} + c, \quad c \in \mathbb{R}$
 (n) $\frac{1}{4}x - \frac{1}{8} \ln(e^{2x} + 4) + \frac{1}{2} \operatorname{arctg} \frac{e^x}{2} + c, \quad c \in \mathbb{R}$
 (o) $x \operatorname{tg} x + \ln |\cos x| + c, \quad c \in \mathbb{R}$
 (p) $-\frac{1}{2(1-\cos x)^2} + c, \quad c \in \mathbb{R}$
 (q) $(\frac{2}{3}x^3 + 3x) \operatorname{arctg} x - \frac{1}{3}x^2 - \frac{7}{6} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
 (r) $\ln \left| \frac{x+1+\sqrt{(x+1)^2-4}}{2} \right| + c, \quad c \in \mathbb{R}$
 (s) $2\sqrt{1+e^x} + \ln |\sqrt{1+e^x} - 1| - \ln(\sqrt{1+e^x} + 1) + c, \quad c \in \mathbb{R}$
 (t) $2 \operatorname{arctg} \sqrt{e^x - 1} + c, \quad c \in \mathbb{R}$
 (u) $\frac{1}{12} \operatorname{sen}(6x) + \frac{1}{8} \operatorname{sen}(4x) + c, \quad c \in \mathbb{R}$
 (v) $-2\sqrt{\cos x} + \frac{2}{5} \sqrt{\cos^5 x} + c, \quad c \in \mathbb{R}$
 (w) $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + c, \quad c \in \mathbb{R}$
 (x) $\frac{1}{2} \ln(\ln^2 x + 1) + c, \quad c \in \mathbb{R}$
13. $F(x) = -\ln |\cos x| + 3$
14. $f(x) = 2 \ln(e^x + 3) - \ln 4$
15. $f(x) = 2x^3 + 2x + 1$
16. (a) $-\frac{1}{2}e^{\cos(2x)} + c, \quad c \in \mathbb{R}$
 (b) $4 \left(\frac{\sqrt{x}}{2} + \sqrt[4]{x} + \ln(\sqrt[4]{x} - 1) \right) + c, \quad c \in \mathbb{R}$
 (c) $\frac{\sqrt{x^2-9}}{9x} + c, \quad c \in \mathbb{R}$
 (d) $\frac{x^3}{3} \operatorname{arctg} x - \frac{x^2}{6} - \frac{1}{6} \ln(1+x^2) + c, \quad c \in \mathbb{R}$
 (e) $\frac{1}{2} \ln |x| - \frac{1}{4} \ln(4+x^2) + \frac{1}{2} \operatorname{arctg} \left(\frac{x}{2} \right) + c, \quad c \in \mathbb{R}$
 (f) $\frac{2}{3} \sqrt{1+x^3} + c, \quad c \in \mathbb{R}$
 (g) $-\frac{\sqrt{1+x^2}}{x} + c, \quad c \in \mathbb{R}$
 (h) $\frac{1}{2} (\ln(x^2 + 1) - 2 \ln |x| + 6 \operatorname{arctg} x) + c, \quad c \in \mathbb{R}$
 (i) $\frac{1}{1+\operatorname{sen} x} + c, \quad c \in \mathbb{R}$
 (j) $\frac{x^2+1}{2} \ln(1+x^2) - \frac{x^2}{2} + c, \quad c \in \mathbb{R}$
 (k) $\operatorname{sen} x \cdot \ln(\operatorname{sen} x) - \operatorname{sen} x + c, \quad c \in \mathbb{R}$
 (l) $\frac{1}{2} \arccos \left(\frac{2}{x} \right) + c, \quad c \in \mathbb{R}$
 (m) $\frac{x^2}{2} \operatorname{sen}(x^2) + \frac{1}{2} \cos(x^2) + c, \quad c \in \mathbb{R}$
 (n) $\frac{x^2}{2} \left((\ln x)^2 - \ln x + \frac{1}{2} \right) + c, \quad c \in \mathbb{R}$
 (o) $2(\sqrt{x} \cdot \operatorname{sen}(\sqrt{x}) + \cos(\sqrt{x})) + c, \quad c \in \mathbb{R}$
 (p) $\frac{\operatorname{tg}^2 x}{2} + \ln |\cos x| + c, \quad c \in \mathbb{R}$
17. $f(x) = \ln \left(\frac{x^2-2x+2}{x^2} \right) + 3 \operatorname{arctg}(x-1) - \frac{3\pi}{2}$