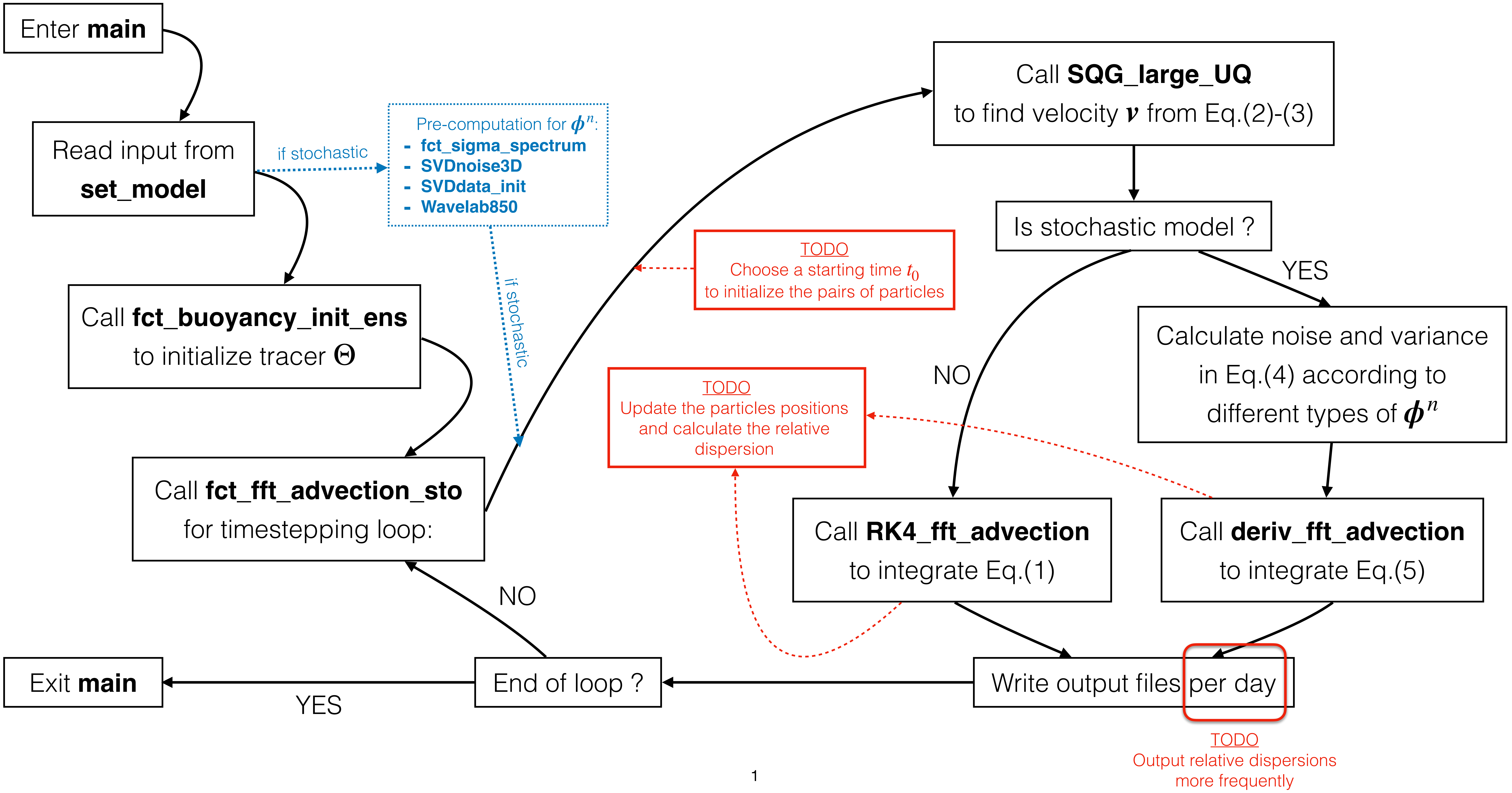
Flow chart of SQGMU-3.0



Deterministic equations

Physical space

$$\partial_t \Theta + \boldsymbol{v} \cdot \boldsymbol{\nabla}\, \Theta = -\mu \Delta^4 \Theta$$

$$\boldsymbol{v} = (-\partial_y \psi, \partial_x \psi)$$

$$\psi = c(\sqrt{-\Delta})^{-\alpha}\Theta$$

Spectral space

$$\partial_t \widehat{\Theta} + \widehat{\boldsymbol{v} \cdot \boldsymbol{\nabla}\, \Theta} = -\mu |\boldsymbol{k}|^8 \widehat{\Theta} \qquad (1)$$

$$\boxed{\widehat{\boldsymbol{v}}} = (-\mathrm{i}k_y \widehat{\psi}, \mathrm{i}k_x \widehat{\psi}) \qquad (2)$$

Fourier Transform

$$\widehat{\psi} = c|\boldsymbol{k}|^{-\alpha}\widehat{\Theta} \qquad (3)$$

$$\boxed{\boldsymbol{k} = (k_x, k_y)}$$

2D wave vector

$$\boldsymbol{v} \cdot \boldsymbol{\nabla}\, \Theta = \boxed{\mathcal{F}^{-1}}\left[\widehat{G}\widehat{\boldsymbol{v}}\right] \cdot \mathcal{F}^{-1}\left[\mathrm{i}(\boxed{\widehat{G}\boldsymbol{k}})\widehat{\Theta}\right]$$

Inverse Fourier Transform

Low-pass filtering

Stochastic equations

$$\partial_t \widehat{\Theta} + \widehat{\boldsymbol{v}^\star \cdot \boldsymbol{\nabla}}\, \Theta - \frac{\mathrm{i}}{2}\boldsymbol{k} \cdot \widehat{\boldsymbol{a}\boldsymbol{\nabla}\Theta} = -\mu|\boldsymbol{k}|^8 \widehat{\Theta} \tag{4}$$

$$\boldsymbol{v}^\star = \boldsymbol{v} - \frac{1}{2}\, \boldsymbol{\nabla} \cdot \boxed{\boldsymbol{a}} + \sum_n \boldsymbol{\phi}^n \boxed{\xi^n}, \qquad \boldsymbol{a} = \delta t \sum_n \boxed{\boldsymbol{\phi}^n} (\boldsymbol{\phi}^n)^T \tag{5}$$

Matrix variance

Scalar Gaussian variables

Vector eigenfunctions

Particle trajectory

Deterministic

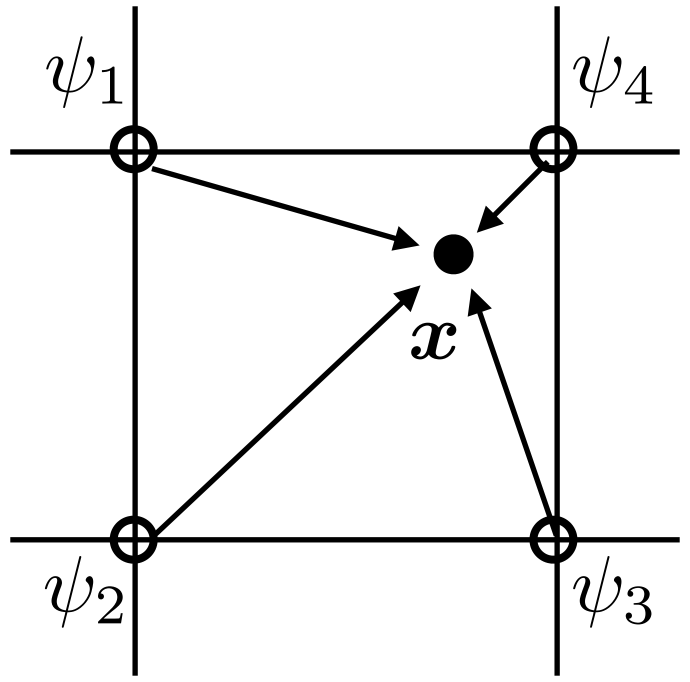
$$\boxed{\partial_t}\boldsymbol{x} = \boldsymbol{v}(\boldsymbol{x}, t)$$

RK4

Stochastic

$$\boxed{\partial_t}\boldsymbol{x} = \boldsymbol{v}^{\star}(\boldsymbol{x}, t)$$

Euler
Maruyama



$$\boldsymbol{v}(\boldsymbol{x}) = \,?$$

$$\{\psi_1, \psi_2, \psi_3, \psi_4\}$$

$\downarrow$ eg. Bicubic interpolation

$$\overline{\psi}(\boldsymbol{x})$$

$\downarrow$ Eq.(2)

$$\boldsymbol{v}(\boldsymbol{x})$$

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NOTES AND CORRESPONDENCE

The Conservation of Potential Vorticity along Lagrangian Trajectories in Simulations of Eddy-Driven Flows

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2 August 1992 and 19 May 1993