Challenge 5

Numerical experimentation for Stochastic Models of SQG

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Deterministic SQG model

$$\partial_t \Theta + \boldsymbol{v} \cdot \nabla \Theta = -\mu \Delta^4 \Theta$$

$$\boldsymbol{v} = (v_1, v_2) = (-\partial_{x_2}\psi, \ \partial_{x_1}\psi), \qquad \psi = c \left(\sqrt{-\Delta}\right)^{-\alpha} \Theta$$

$(\alpha = 1)$ Θ — Buoyancy; $\qquad (\alpha = 2)$ Θ — Vorticity

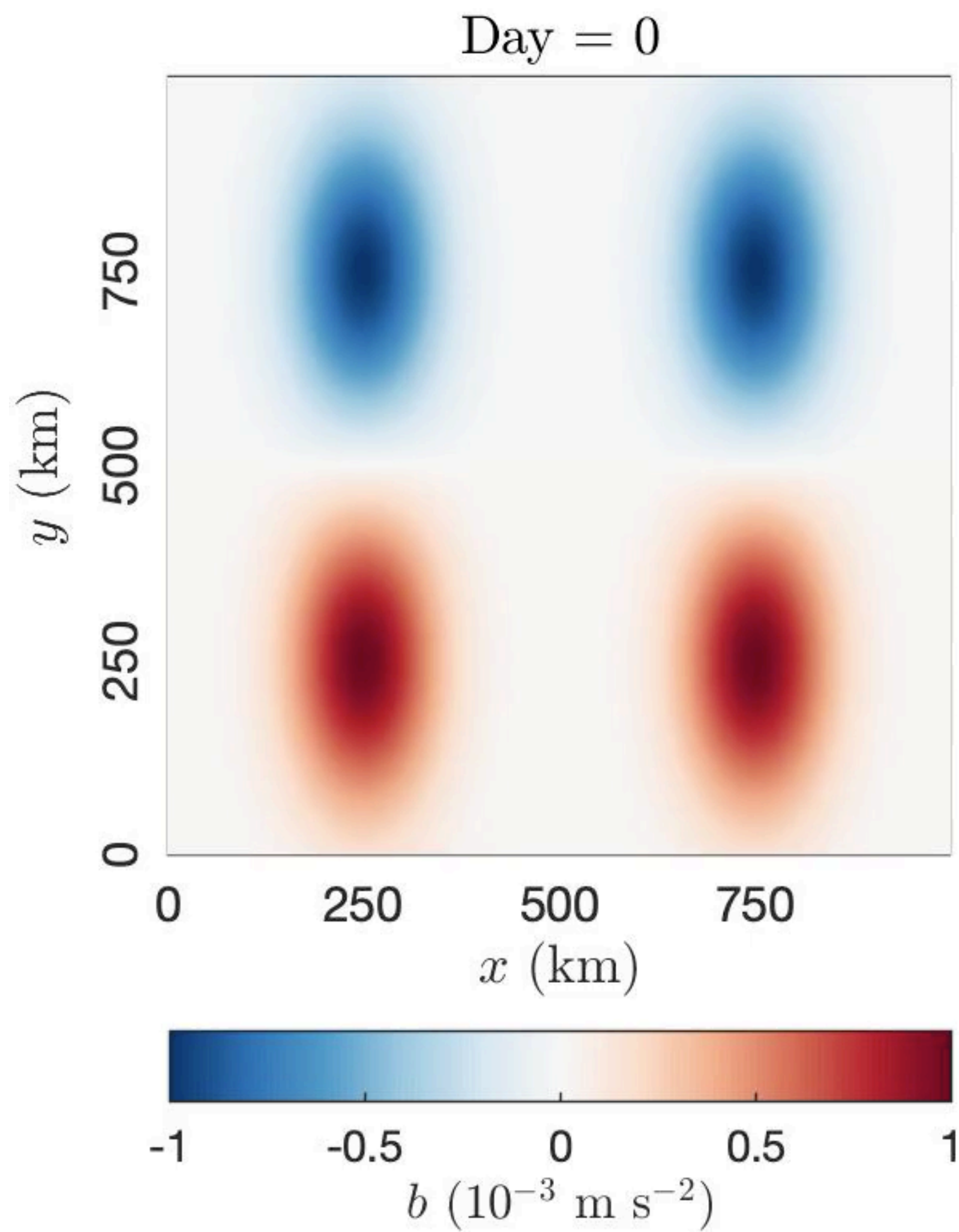# Stochastic SQG model

$= 0$ for SALT

$$\partial_t \Theta + \left(\boldsymbol{v} - \boxed{\frac{1}{2} \nabla \cdot \boldsymbol{a}} + \sum_n \boldsymbol{\phi}^n \xi^n \right) \cdot \nabla \Theta - \frac{1}{2} \nabla \cdot \left(\boldsymbol{a} \nabla \Theta \right) = - \mu \Delta^4 \Theta$$

$$\boldsymbol{\phi}^n = (\phi_1^n, \phi_2^n), \qquad \xi_n \sim \mathcal{N}(0,1)$$
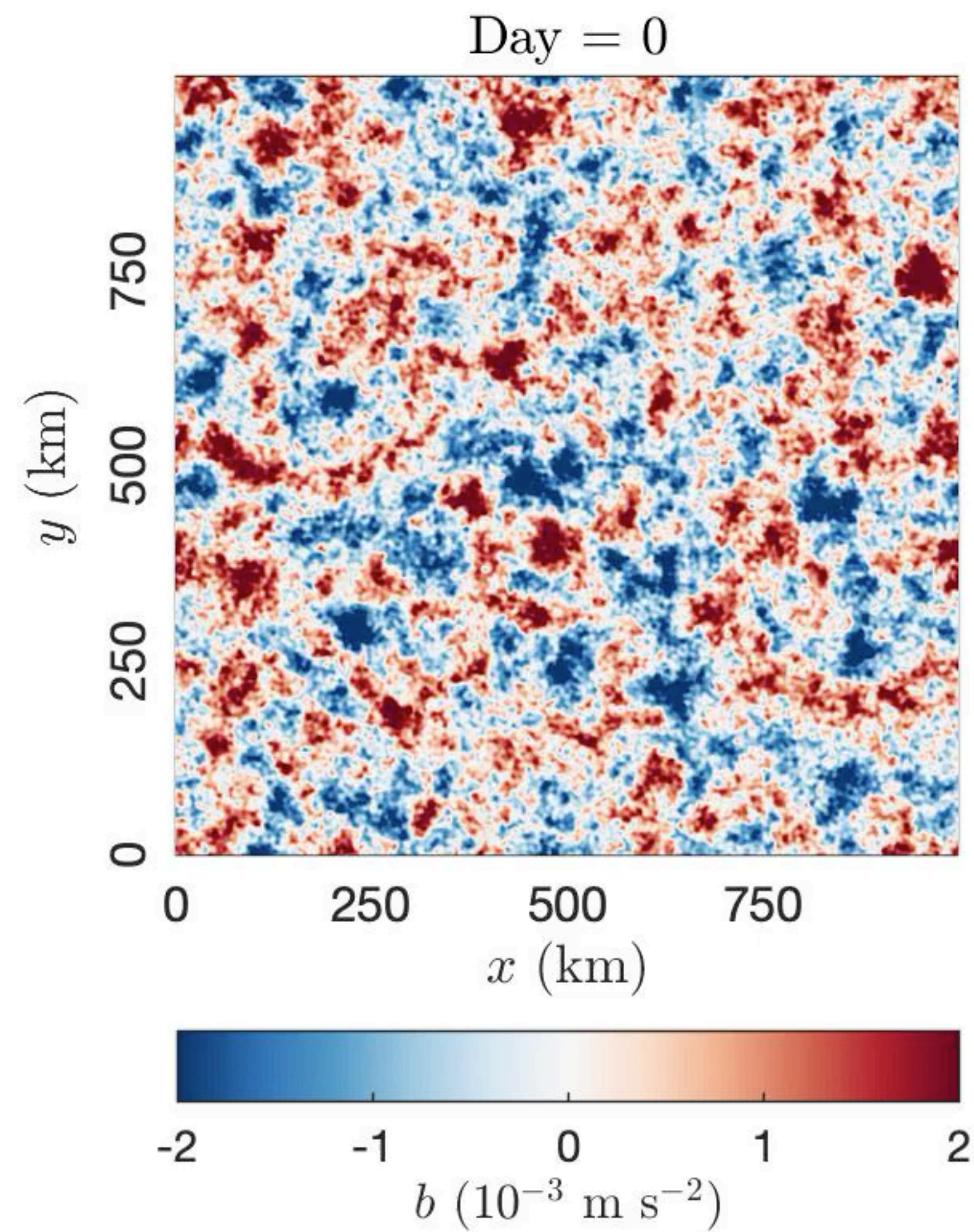
$$\boldsymbol{a} = (a_{ij})_{i,j=1,2}, \qquad a_{ij} = \delta t \sum_n \phi_i^n \phi_j^n$$

$$\boldsymbol{v} = (v_1, v_2) = (-\partial_{x_2} \psi, \, \partial_{x_1} \psi), \qquad \psi = c \left(\sqrt{-\Delta} \right)^{-\alpha} \Theta$$
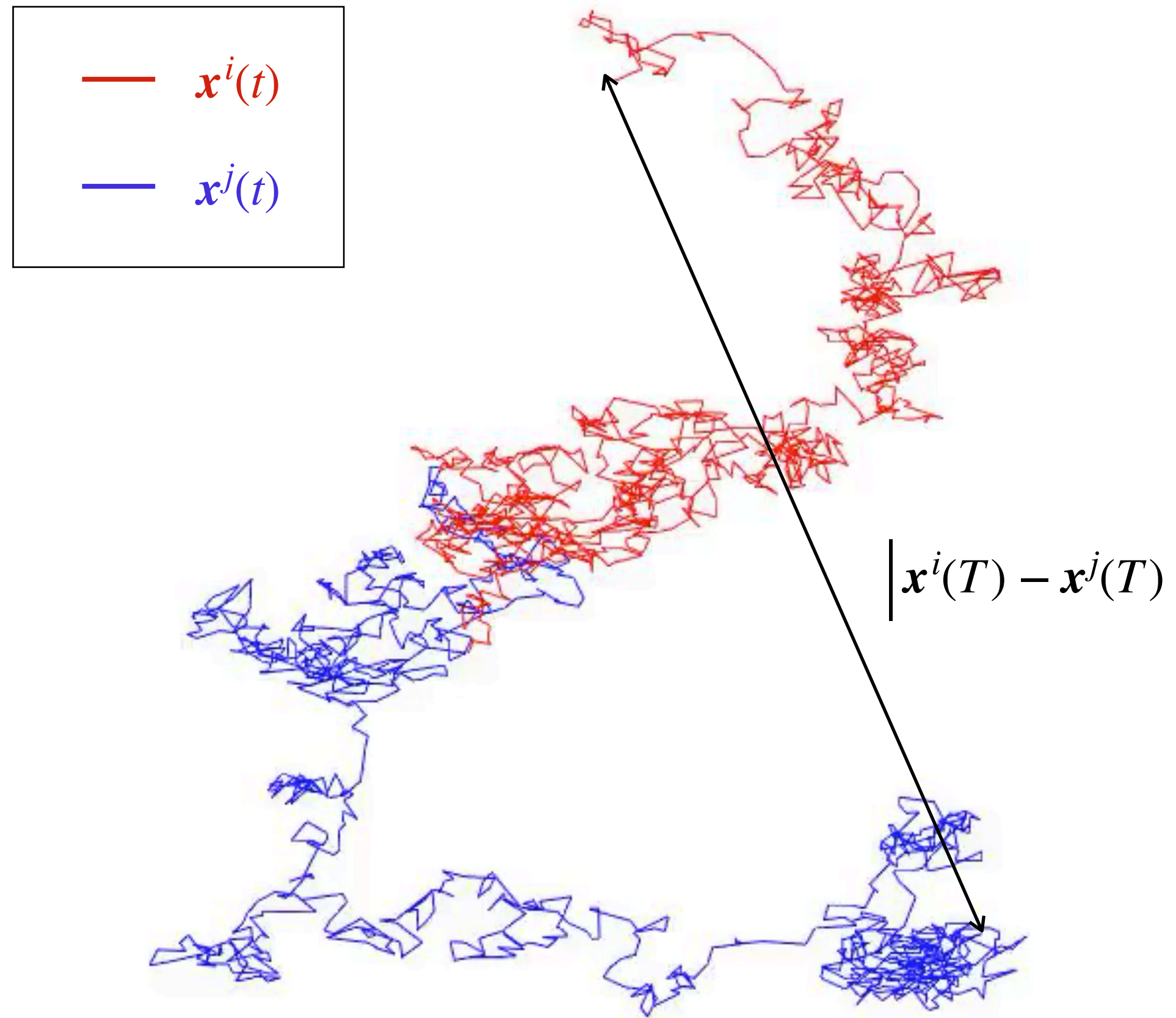
Test case 1

Test case 2

Day = 0

Day = 0

y (km)

x (km)

b (10^{-3} m s^{-2})

b (10^{-3} m s^{-2})

Deterministic simulations (512 x 512) for $\alpha = 1$

Relative dispersion

For N pairs of particles (i, j), do:

$$\partial_t \boldsymbol{x}^i = \boldsymbol{v}(\boldsymbol{x}^i), \qquad \partial_t \boldsymbol{x}^j = \boldsymbol{v}(\boldsymbol{x}^j)$$

$$y^2(t) = \frac{1}{N} \sum_{i,j=1}^{N} \left| \boldsymbol{x}^i(t) - \boldsymbol{x}^j(t) \right|^2$$

Legend:
- $\boldsymbol{x}^i(t)$
- $\boldsymbol{x}^j(t)$

$\left| \boldsymbol{x}^i(T) - \boldsymbol{x}^j(T) \right|$

Example of two particles (i, j)

Plot axes: $y^2(t) - y^2(t_0)$ versus t

Legend:
- $\alpha = 2$
- $\alpha = 1.75$
- $\alpha = 1.5$
- $\alpha = 1.25$
- $\alpha = 1$
- $\alpha = 2$ (viscous)
- $\alpha = 1$ (viscous)

t^3

t^2
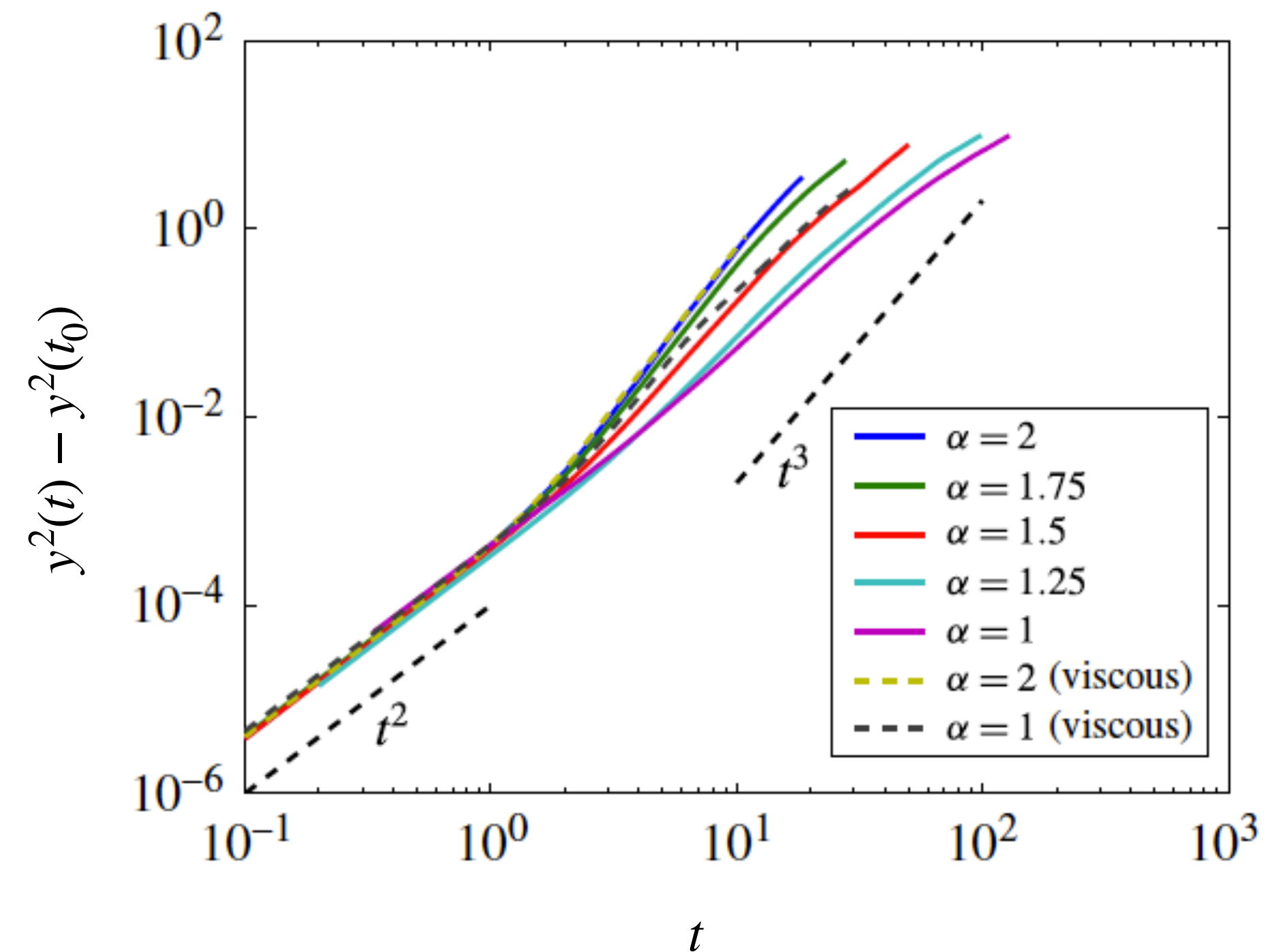
Source

Foussard et al. 2017

Welcome to Challenge 5 !