# Anderson

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## 1 Modelo

Nós tínhamos exatamente

$$\big[\omega^+ - \epsilon_0 - \Sigma^{(0)}(\omega^+)\big] G_{d\sigma}(\omega^+) = 1 + U \cdot D_{d\sigma}(\omega^+),$$

onde, pela aproximação de *mean-field*

$$\Sigma^{(0)}(\omega^+) = \sum_{\mathbf{k}} \frac{|t_{\mathbf{k}}|^2}{\omega^+ - \epsilon_{\mathbf{k}}} \quad \text{e} \quad D_{d\sigma}(\omega^+) = \Big\langle\Big\langle \hat{n}_{d\bar{\sigma}}\, \hat{c}_{d\sigma} : \hat{c}^\dagger_{d\sigma} \Big\rangle\Big\rangle \approx \langle \hat{n}_{d\bar{\sigma}} \rangle\, G_{d\sigma}(\omega^+).$$

Portanto

$$G_{d\sigma}(\omega^+) = \frac{1}{\omega^+ - \epsilon_0 - \Sigma^{(0)}(\omega^+) - U\,\langle \hat{n}_{d\bar{\sigma}} \rangle} =$$

$$\frac{1}{(\omega - \epsilon_0 - U\,\langle \hat{n}_{d\bar{\sigma}} \rangle - \mathrm{Re}\{\Sigma^{(0)}(\omega^+)\}) + i\,(\eta - \mathrm{Im}\{\Sigma^{(0)}(\omega^+)\})}.$$

e

$$\boxed{\mathrm{Im}\big\{G_{d\sigma}(\omega^+)\big\} = \frac{-\big[\eta - \mathrm{Im}\big\{\Sigma^{(0)}(\omega^+)\big\}\big]}{(\omega - \epsilon_0 - U\,\langle \hat{n}_{d\bar{\sigma}} \rangle - \mathrm{Re}\{\Sigma^{(0)}(\omega^+)\})^2 + (\eta - \mathrm{Im}\{\Sigma^{(0)}(\omega^+)\})^2}.}$$

Podemos então calcular $\Sigma^{(0)}(\omega^+)$ por

$$\Sigma^{(0)}(\omega^+) = \sum_{\mathbf{k}} \frac{|t_{\mathbf{k}}|^2}{\omega^+ - \epsilon_{\mathbf{k}}} = \mathrm{Vol} \cdot \int \frac{\mathrm{d}^d\mathbf{k}}{(2\pi)^d} \frac{|t(\mathbf{k})|^2}{\omega^+ - \epsilon(\mathbf{k})}.$$

Supondo que $t(k)$ e $\epsilon(k)$ só dependem do módulo $k = |\mathbf{k}|$, temos

$$\Sigma^{(0)}(\omega^+) = \frac{\mathrm{Vol}}{(2\pi)^d} \int \mathrm{d}\Omega \int \frac{|t(k)|^2\,\mathrm{d}k}{\omega^+ - \epsilon(k)}.$$

Sendo então que $\mathrm{d}k = d(\epsilon)\,\mathrm{d}\epsilon$ e $\omega^+ = \omega + i\eta$, temos

$$\Sigma^{(0)}(\omega^+) = \frac{\Omega \cdot \mathrm{Vol}}{(2\pi)^d} \int \frac{|t(\epsilon)|^2}{\omega - \epsilon + i\eta}\, d(\epsilon)\,\mathrm{d}\epsilon$$

$$= \frac{\Omega \cdot \mathrm{Vol}}{(2\pi)^d} \left\{ P.V. \int \frac{|t(\epsilon)|^2}{\omega - \epsilon}\, d(\epsilon)\,\mathrm{d}\epsilon - i\pi \int \delta(\omega - \epsilon)\,|t(\epsilon)|^2 d(\epsilon)\,\mathrm{d}\epsilon \right\}$$

Definindo então $\Delta(\epsilon) = \pi\,\frac{\Omega\cdot\mathrm{Vol}}{(2\pi)^d}\,|t(\epsilon)|^2 d(\epsilon)$, temos que

$$\boxed{\Sigma^{(0)}(\omega^+) = P.V. \int \frac{\Delta(\epsilon)}{\omega - \epsilon}\,\mathrm{d}\epsilon - i\Delta(\omega).}$$

Chamando $\boxed{\Lambda(\omega) = \mathrm{Re}\big\{\Sigma^{(0)}(\omega^+)\big\} = P.V. \int \frac{\Delta(\epsilon)}{\omega - \epsilon}\, \mathrm{d}\epsilon}$ e tomando $\eta \to 0^+$, temos

$$A_{d\sigma}(\omega) = -\frac{1}{\pi}\, \mathrm{Im}\big\{G_{d\sigma}(\omega^+)\big\} \;\Rightarrow$$

$$\boxed{A_{d\sigma}(\omega) = \frac{\Delta(\omega)/\pi}{\Big[\omega - \epsilon_0 - U\,\langle \hat{n}_{d\bar{\sigma}}\rangle - \Lambda(\omega)\Big]^2 + \Big[\Delta(\omega)\Big]^2}.}$$

Para temperatura $T = 1/(k_B\beta)$, temos $n_F(\omega) = (e^{\beta\omega} + 1)^{-1}$ e então

$$\boxed{\langle \hat{n}_{d\sigma}\rangle = \int_{-\infty}^{\infty} \frac{A_{d\sigma}(\omega)}{e^{\beta\omega} + 1}\, \mathrm{d}\omega\,.}$$

No caso especial onde $T = 0$ ($\beta = \infty$), temos que $e^{\beta\omega} = +\infty \cdot \theta(\omega)$, o que nos dá

$$\boxed{\langle \hat{n}_{d\sigma}\rangle = \int_{-\infty}^{0} A_{d\sigma}(\omega)\, \mathrm{d}\omega\,, \quad T = 0.}$$

Como $A_{d\sigma}$ depende de $\langle \hat{n}_{d\bar{\sigma}}\rangle$, as equações acima são de **ponto fixo**

$$\boxed{\langle \hat{n}_{d\sigma}\rangle = \mathcal{F}\{\langle \hat{n}_{d\bar{\sigma}}\rangle\}}\ , \text{ onde } \mathcal{F}\{\langle \hat{n}_{d\sigma}\rangle\} = \int_{-\infty}^{\infty} \frac{A_{d\sigma}(\omega, \langle \hat{n}_{d\bar{\sigma}}\rangle)}{e^{\beta\omega} + 1}\, \mathrm{d}\omega\,.$$

Por enquanto escolheremos

$$\Delta(\omega) = \Delta_0 \left[1 - \left(\frac{\omega}{D}\right)^2\right].$$