1 Four bands

The interlayer coupling in the small angle limit ($\theta \lesssim 10°$) is given by

$$T_{12}^{\alpha\beta}(\mathbf{q}_1, \mathbf{q}_2) = w\, T_{\mathbf{q}_b}^{\alpha\beta} \delta_{\mathbf{q}_1 - \mathbf{q}_2, \mathbf{q}_b} + w\, T_{\mathbf{q}_{tr}}^{\alpha\beta} \delta_{\mathbf{q}_1 - \mathbf{q}_2, -\mathbf{q}_{tr}} + w\, T_{\mathbf{q}_{tl}}^{\alpha\beta} \delta_{\mathbf{q}_1 - \mathbf{q}_2, -\mathbf{q}_{tl}}, \tag{1}$$

where the hopping matrices are

$$T_{\mathbf{q}_b} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

$$T_{\mathbf{q}_{tr}} = e^{-i\mathbf{g}_{12}\cdot\boldsymbol{\tau}_0} \begin{pmatrix} e^{i\phi} & 1 \\ e^{-i\phi} & e^{i\phi} \end{pmatrix},$$

$$T_{\mathbf{q}_{tl}} = e^{-i\mathbf{g}_{13}\cdot\boldsymbol{\tau}_0} \begin{pmatrix} e^{-i\phi} & 1 \\ e^{i\phi} & e^{-i\phi} \end{pmatrix},$$

where $w = t_{k_D}/\Omega$ is the hopping energy, being t_{k_D} the interlayer coupling function at the Dirac point of a monolayer and Ω the monolayer unit cell area.

We consider a momentum state close to the Dirac point of layer 1, with crystal-momentum $\mathbf{K}_1 + \mathbf{q}$, where $|\mathbf{q}| \sim |\Delta\mathbf{K}| = 2\,k_D \sin\theta/2$. According to equation 1, this layer 1 state will couple to three layer 2 states with momenta $\mathbf{q} + \mathbf{q}_b$, $\mathbf{q} + \mathbf{q}_{tr}$, and $\mathbf{q} + \mathbf{q}_{tl}$. Working with a truncated Hamiltonian matrix with only these 4 states, we have

$$H_{4,\text{TBG}}^K(\mathbf{q}) = \begin{pmatrix} H_1^K(\mathbf{q}) & T_{\mathbf{q}_b} & T_{\mathbf{q}_{tr}} & T_{\mathbf{q}_{tl}} \\ T_{\mathbf{q}_b}^\dagger & H_2^K(\mathbf{q} + \mathbf{q}_b) & 0 & 0 \\ T_{\mathbf{q}_{tr}}^\dagger & 0 & H_2^K(\mathbf{q} + \mathbf{q}_{tr}) & 0 \\ T_{\mathbf{q}_{tl}}^\dagger & 0 & 0 & H_2^K(\mathbf{q} + \mathbf{q}_{tl}) \end{pmatrix},$$

in the basis $|1, \mathbf{K}_1 + \mathbf{q}, \alpha\rangle$, $|2, \mathbf{K}_2 + \mathbf{q} + \mathbf{q}_b, \alpha\rangle$, $|2, \mathbf{K}_2 + \mathbf{q} + \mathbf{q}_{tr}, \alpha\rangle$, $|2, \mathbf{K}_2 + \mathbf{q} + \mathbf{q}_{tl}, \alpha\rangle$.