ν-sim

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1 Numerical description

In general:

$$i\frac{\mathrm{d}\psi}{\mathrm{d}t} = \mathcal{H}\psi \ \Rightarrow \ \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\Re \\ \Im\end{bmatrix} = \begin{bmatrix}\mathcal{H}_\Im & \mathcal{H}_\Re \\ -\mathcal{H}_\Re & \mathcal{H}_\Im\end{bmatrix}\begin{bmatrix}\Re \\ \Im\end{bmatrix},$$

where $\psi = \Re + i\Im = (\Re_1 + i\Im_1, \ldots, \Re_n + i\Im_n)$, $\mathcal{H} = \mathcal{H}_\Re + i\mathcal{H}_\Im$ being $\Re, \Im, \mathcal{H}_\Re, \mathcal{H}_\Im$ real.

The original complex system of n equations becomes a real system with $2n$ equations.

In the case of Neutrino Oscillations in matter, our hamiltonian is always of the form:

$$\mathcal{H} = \mathcal{H}^0 + \mathrm{diag}\big(V(L), 0, \ldots, 0\big),$$

where $\mathcal{H}^0$ is constant and $V(L) = \sqrt{2}\,G_F N_e(L)$ is the only parameter that depends on the traveled distance L (time also, because $L = ct = t$). G_F is the Fermi constant and $N_e(L)$ is the solar electron density. The hamiltonian $\mathcal{H}$ is so simple because the only neutrino that interacts with solar matter is the neutrino ν_e of the electron.

The matrix $\mathcal{H}^0$ is simply given by the sandwich:

$$\mathcal{H}^0 = U\,M\,U^\dagger,$$

where U is the neutrino mixing matrix (it's only a unitary matrix, and it has standard parametrization), and M is the diagonal matrix corresponding to the mass eigenvalues of the neutrinos in vacuum (it can be simplified, making one of its entries zero).

Now, the algorithm is very simple:

1. Assume U, M and a table $L \times N_e(L)$ of data as input.

2. Using the structures `gsl_complex, gsl_matrix_complex, gsl_matrix` of the GSL Library, we easily calculate $\mathcal{H}^0 = \mathcal{H}^0_\Re + i\mathcal{H}^0_\Im$ by complex matrix operations and then obtain $\mathcal{H}^0_\Re$, $\mathcal{H}^0_\Im$ by taking the real and imaginary parts with `GSL_REAL, GSL_IMAG`.

3. Interpolate $N_e(L)$ using Bahcall's data and compute $V(L) = \sqrt{2}\,G_F N_e(L)$ for $-R_\odot \leq L \leq R_\odot$, where $R_\odot$ is the solar radius. Here we use the following header and type from GSL:

   ```
   #include <gsl/gsl_spline.h>       /* gsl_interp_steffen */
   ```

   which by the last 1D Interpolation Example seems to be the best spline for the case.

4. Now we simply solve the following ODE numerically and print the results.

   $$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\Re \\ \Im\end{bmatrix} = \begin{bmatrix}\mathcal{H}_\Im & \mathcal{H}_\Re \\ -\mathcal{H}_\Re & \mathcal{H}_\Im\end{bmatrix}\begin{bmatrix}\Re \\ \Im\end{bmatrix},$$

   where $\mathcal{H}_\Re = \mathcal{H}^0_\Re + \mathrm{diag}\big(V(L), 0, \ldots, 0\big)$ and $\mathcal{H}_\Im = \mathcal{H}^0_\Im$. For this we use the header

   ```
   #include <gsl/gsl_odeiv2.h>.
   ```

2 Neutrinos de massa

Seja $\mathcal{E}$ a base dos neutrinos de interação e, λ, τ e $\mathcal{B}(t)$ a base instantânea dos autoestados de massa $|\nu_1(t)\rangle, |\nu_2(t)\rangle, |\nu_3(t)\rangle$. Seja então $W(t) = [I]_{\mathcal{E}\to\mathcal{B}(t)}$ a matriz mudança de base de $\mathcal{E}$ para $\mathcal{B}(t)$. Isto significa que $|\nu_i(t)\rangle = [I]_{\mathcal{E}\to\mathcal{B}(t)} |\nu_\alpha\rangle$, para $\alpha = e, \lambda, \tau$ e $i = 1, 2, 3$.

Seja ainda

$$\phi(t) = \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \end{bmatrix},$$

onde $|\nu(t)\rangle = \psi_1(t)|\nu_1(t)\rangle + \psi_2(t)|\nu_2(t)\rangle + \psi_3(t)|\nu_3(t)\rangle$ está escrito na base $\mathcal{B}(t)$.

Denotando por $\Lambda(t) = \mathrm{diag}\Big(\lambda_1(t), \lambda_2(t), \lambda_3(t)\Big)$ a matriz diagonal que representa a hamiltoniana na base $\mathcal{B}(t)$, temos então que:

$$i\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = \left[-iW(t)\frac{\mathrm{d}W^\dagger(t)}{\mathrm{d}t} + \Lambda(t)\right]\phi(t).$$

Esta é a mesma equação (9) que o Boechat obteve, antes de substituir os termos.

O problema com ela são os termos $W(t), \frac{\mathrm{d}W^\dagger(t)}{\mathrm{d}t}$, que dificultam numericamente. A dificuldade que eu acho que torna essa EDO intratável é que a diagonalização do GSL **não é necessariamente contínua**. Com isso, quero dizer que os autovetores podem diferir por uma constante multiplicativa complexa. Veja os exemplos do GSL.

Caminhos:

- Buscar algo mais analítico? A mixing matrix é muito feia.
- O que eu tentei fazer? Resolver a EDO na base de interação e mudar para a base $\mathcal{B}(t)$ a todo instante.

Outra ideia é uma equação diferencial para os autovetores:

$$\mathcal{H}|\nu_i\rangle = \lambda_i|\nu_i\rangle \Rightarrow \frac{\mathrm{d}|\nu_i\rangle}{\mathrm{d}t} = \frac{1}{\lambda_i - 1}\frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{H} - \lambda_i I)|\nu_i\rangle.$$

Isso nos dá uma equação para o $W(t)$ mais ou menos assim (provavelmente tá errado, pq tem uma matriz 9x9):

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \begin{pmatrix} \frac{1}{\lambda_1-1} & & \\ & \frac{1}{\lambda_2-1} & \\ & & \frac{1}{\lambda_3-1} \end{pmatrix} \begin{pmatrix} \frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{H}-\lambda_1 I) & & \\ & \frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{H}-\lambda_2 I) & \\ & & \frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{H}-\lambda_2 I) \end{pmatrix} W(t).$$

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