MATH5315M Seminar 1: Question Sheet

Question 1

Consider the two following datasets:

$$1, 3, 4, 5, 6, 6, 6, 7, 10, 12$$

and

$$1, 3, 4, 4, 4, 8, 8, 8, 9, 11$$

Calculate the mean and variance for each data set.
Produce box-plots for each data set.
What is the point of this question?

Question 2

The Bank of England comes up with probabilities of economic growth over some time period (G) given information about controlled inflation (I):

$$P(G|\ I) = 0.7,$$
$$P(G^c|\ I^c) = 0.9.$$

If they judge the probability of controlled inflation to be 0.2, what is their probability for economic growth?

Question 3

In a court case, a prosecution lawyer has asserted that, if the defendant is guilty, there is 0.9 probability of his fingerprints being on the murder weapon and, if the defendant if not guilty, there is 0.01 probability of fingerprints being there. If there is no other evidence in the case, the prior probability of his innocence was 0.9999.
Given that the defendants fingerprints were on the weapon, what is the probability of his guilt?

Question 4

You go to see the doctor about an ingrowing toenail. The doctor selects you at random to have a blood test for swine flu, which lets say is currently suspected to affect 1 in 10,000 people in the UK. The test is 99% accurate, in the sense that the probability of a false positive is 1% (that is, the probability of testing positive without having the flu). The probability of a false negative is zero. You test positive. What is the new probability that you have swine flu?

Now imagine that you recently visited Mexico, and suppose that it is know that 1 in 200 people who visited Mexico recently come back with swine flu. Given the same test result as above, what should your revised estimate be for the probability you have the disease?

Extra Question 1

Let X be a random variable with:

$$\begin{aligned} P(X=-1) &= 0.2, \\ P(X=0) &= 0.4, \\ P(X=1) &= 0.1, \\ P(X=2) &= 0.1, \\ P(X=3) &= 0.2. \end{aligned}$$

Let $Y = X^2 + 1$.
Write out the probabilities for each possible value of Y, and calculate E[Y].

Extra Question 2

Let

$$f_X(x) = \begin{cases} x, & \text{for } 0 \leq x < 1, \\ 2 - x, & \text{for } 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Show that $f_X(x)$ is a density function.
Sketch the function, and, without making any calculations, find the mean of the random variable X.

Extra Question 3

For the density function in **Extra Question 2**, and the probability mass function in **Extra Question 1**, calculate the cumulative distribution function.