Split into two cases (①, ②)

$$\frac{\partial C^n}{\partial z_k} = -\frac{y_k^n}{\hat{y}_k^n}\hat{y}_k^n(1-\hat{y}_k^n) - \sum_{k\neq k'}^{K}\frac{y_k^n}{\hat{y}_k^n}(-\hat{y}_k^n\hat{y}_{k'}^n)$$

$$= -y_k^n(1-\hat{y}_k^n) + \sum_{k\neq k'}^{K} y_k^n\hat{y}_{k'}^n$$

$$= y_k^n\hat{y}_k^n - y_k^n + \sum_{k\neq k'}^{K} y_k^n\hat{y}_{k'}^n$$

$$= -y_k^n + \underbrace{\sum_{k=1}^{K} y_k^n\hat{y}_k^n}_{=1} = \hat{y}_k^n - y_k^n$$

$$\frac{\partial z_k}{\partial w_{kj}} = x_j$$

$$\frac{\partial C^n(w)}{\partial w_{kj}} = \frac{\partial z_k}{\partial w_{kj}}\cdot\frac{\partial C^n}{\partial z_k} = -x_j^n\left(y_k^n - \hat{y}_k^n\right)$$