Uluslararası Birim Sistemi (SI)

→ temel büyüklük

Fiziksel nicelikler	Birim	Sembol
1) Uzunluk	metre	M
2) Kütle	kilogram	kg
3) Zaman	saniye	S
4) Elektrik akımı	amper	A
5) Termodinamik (Sıcaklık)	Kelvin	K
6) Madde miktarı	Mol	mol
7) Işık şiddeti	kandela	cd

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1) Kütle yoğunluğu	$\frac{\text{kilogram}}{(\text{metre})^3}$	kg/M^3 → türetilmiş büyüklük
2) Sürat / hız	metre / saniye	M/S
3) İvme	$\frac{\text{metre / saniye}}{\text{saniye}}$	M/S^2
4) Kuvvet	$\text{kilogram} \cdot \frac{\text{metre}}{(\text{saniye})^2}$	$\frac{kg.M}{S^2}$ ⇒ Newton (N)

Birimlerin dönüştürülmesi

1 inç = 2,54 cm

1 mph (mil per hour) ⇒ 1h = 3600s

mi/h

1 mi = 1,609 km = 1609 m

$1 mph = \frac{1609}{3600} \frac{m}{s} = 0,447$ M/S

Boyut Analizi

[L] = uzunluk [M] = kütle [T] = zaman

$$x = x_0 + v_0 \cdot t + \frac{1}{2} \cdot a t^2$$

$$[L] = [L] \quad [L] \quad \frac{[L]}{[T]} \cdot [T] \quad \frac{1}{2} \quad \frac{[L]}{[T]^2} \cdot [T]^2$$

$$[L] = [L] \quad [L] \quad [L]$$

Belirsizlik ve Anlamlı Basamaklar

Δ = Mutlak belirsizlik ε = yüzde belirsizlik (bağıntısız)

Örnek: $m = 3,3 \pm 0,2$ kg $= 3,3 + 6,1\,\%$ $\varepsilon = \frac{0,2}{3,3} \cdot 100 = 6,1\%$

Örnek: $6,5 \pm 0,5$ m ve $3,3 \pm 0,1$ m

toplama ve çıkarmada belirsizlik

$A = (6,5 \pm 0,5) + (3,3 \pm 0,1) = (9,8 \pm 0,6)$ m

$B = (6,5 \pm 0,5) - (3,3 \pm 0,1) = (3,2 \pm 0,6)$ m

Örnek: $5,0$ m $\pm\ 4,0\,\%$ ve $(3,05 \pm 3,3\,\%)$

çarpma ve bölmede belirsizlik

$C = (5,0\text{m} \pm 4,0\%) \times (3,05 \pm 3,3\%) = (15,0\ \text{m} \pm 7,3\%)$

$D = \frac{(5,0\ \text{m} \pm 4,0\%)}{(3,0 \pm 3,3\%)} = (1,7\ \text{m/s} \pm 7,3\%)$

Anlamlı basamaklar

Örnek:
$$\begin{array}{r} 54,4 \text{ cm} \\ +\ 3,02 \text{ cm} \\ \hline 57,4 \text{ cm} \end{array}$$

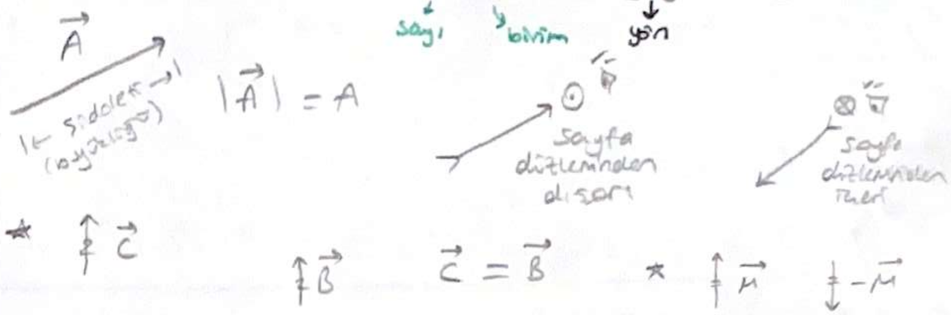
Örnek: $\frac{4,2 \cdot 10^3 \cdot 15,94}{2,255 \cdot 10^{-4}} = 2,968892 \cdot 10^{-8}$ → en az anlamlı basamak

$3,0 \cdot 10^8$

Vektörler

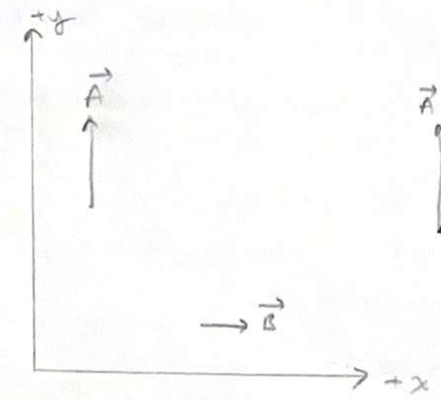
Skaler büyüklük: $m = 93$ kg (sayı → birim)

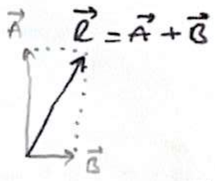
Vektörel büyüklük: $v = 20$ m/s kuzey yönünde (sayı, birim, yön)



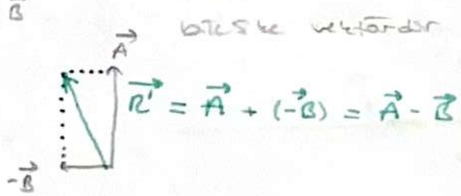
Vektörlerin toplanması

Paralelkenar yöntemi

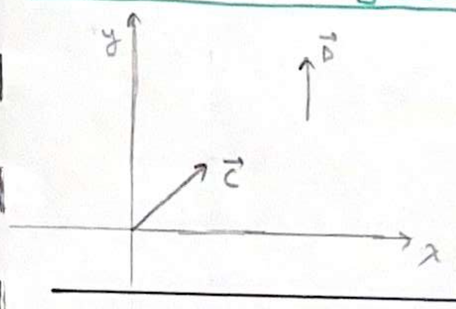


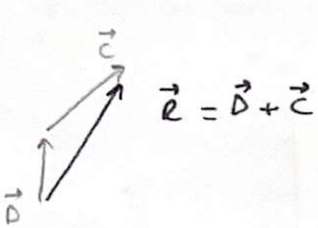


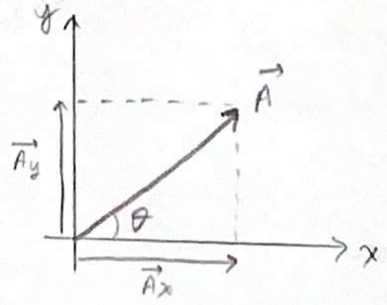
Vektörlerin başlangıç noktaları birleştirilir ve paralelkenara tamamlanır. Oluşan sekiz köşegeni birleştiren vektördür.



Uç uca ekleme yöntemi



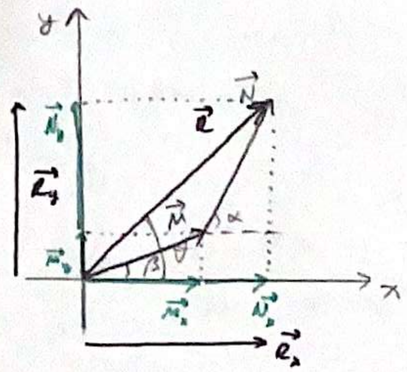




$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A \cdot \cos\theta$

$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A \sin\theta$

$A = \sqrt{(A_x)^2 + (A_y)^2}$ $\tan\theta = \frac{A_y}{A_x}$

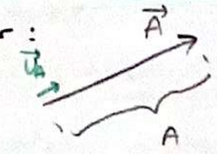


$\vec{R} = \vec{M} + \vec{N}$ → $R_x = M_x + N_x$, $R_y = M_y + N_y$

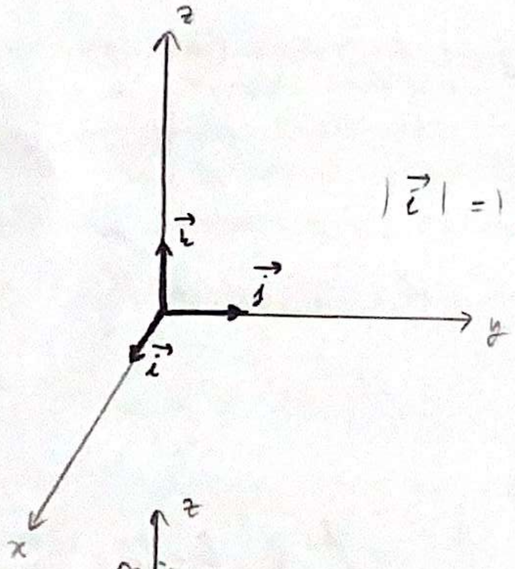
$R = \sqrt{(R_x)^2 + (R_y)^2}$ $\quad \tan\theta = \frac{R_y}{R_x}$

Vektörlerin birim vektörler ile gösterilmesi

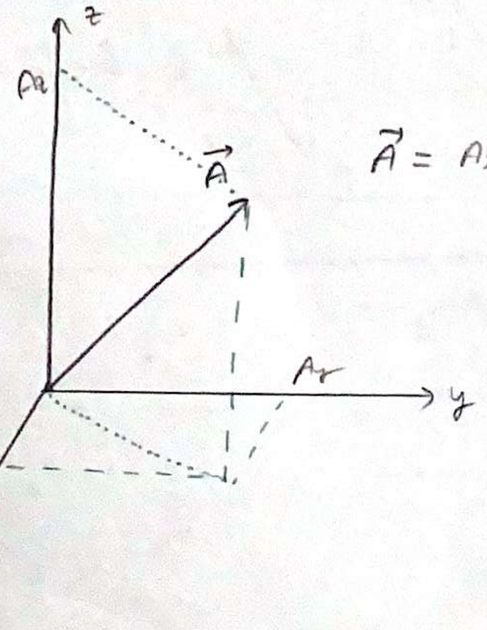
Birim vektör:



$\vec{u_A} = \frac{\vec{A}}{A}$ $\quad |\vec{u_A}| = 1$ (Yönü $\vec{A}$ ile aynıdır)



$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$



$\vec{A} = A_x \cdot \vec{i} + A_y \cdot \vec{j} + A_z \cdot \vec{k}$

Soru: $\vec{A} = 2\vec{i} + 3\vec{j} - 2\vec{k}$ ve $\vec{B} = 4\vec{j} + \vec{k}$

(x: 2 — y: 3 — z: −2 ; $\vec{B}$: y: 4 — z: 1)

a) $\vec{R} = \vec{A} + \vec{B}$

$\vec{R} = (2\vec{i} + 3\vec{j} - 2\vec{k}) + (4\vec{j} + \vec{k})$

$\vec{R} = (2\vec{i} + 7\vec{j} - \vec{k})$ $\quad R = \sqrt{2^2 + 7^2 + (-1)^2}$

$\sqrt{4 + 49 + 1} = \sqrt{54}$

$= 3\sqrt{6}$ br

b) $\vec{R'} = \vec{A} - \vec{B}$

$\vec{R'} = (2\vec{i} + 3\vec{j} - 2\vec{k}) - (4\vec{j} + \vec{k})$

$-4\vec{j} - \vec{k}$

$\vec{R'} = 2\vec{i} - \vec{j} - 3\vec{k}$ $\quad R' = \sqrt{2^2 + (-1)^2 + (-3)^2}$

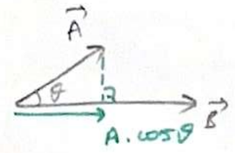
$\sqrt{4 + 1 + 9} = \sqrt{14}$ birim

Vektör Çarpımı

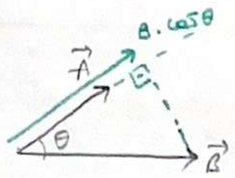
İki vektörün skaler çarpımı

$\vec{A} \cdot \vec{B} = A \cdot B \cdot \cos\theta$

↳ skaler çarpma



$(A \cdot \cos\theta) \cdot B$ → Birbirine paralel bileşenlerin çarpımı

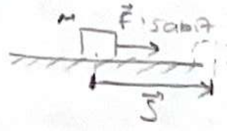


$A \cdot (B \cdot \cos\theta)$ →

i) "+", "−" veya 0 olabilir sonuç skaler bir sonuçtur.

a) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

↳ örnek: "İş"



$W = \vec{F} \cdot \vec{S}$ (W: 1 joule, F: N, S: m) $\quad$ 1 joule = 1 Nm

Birim vektörlerin skaler çarpımı

$\vec{A} \cdot \vec{B} = ?$

$\vec{A} = A_x \cdot \vec{i} + A_y \cdot \vec{j} + A_z \vec{k}$

$\vec{B} = B_x \cdot \vec{i} + B_y \cdot \vec{j} + B_z \cdot \vec{k}$

$\vec{i} \cdot \vec{i} = 1.1.\cos 0° = 1$

$\vec{i} \cdot \vec{j} = 1.1 \cos 90° = 0$

$\vec{A} \cdot \vec{B} = (A_x \cdot \vec{i} + A_y \cdot \vec{j} + A_z \cdot \vec{k}) \cdot (B_x \cdot \vec{i} + B_y \cdot \vec{j} + B_z \cdot \vec{k})$

$\vec{A} \cdot \vec{B} = A_x B_x + 0 + 0 + 0 + A_y B_y + 0 + 0 + A_z B_z$

$$\boxed{\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z}$$

Soru: $\vec{A} = 2\vec{i} + 3\vec{j} - 2\vec{k}$ ve $\vec{B} = 4\vec{j} + \vec{k}$

1)

$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$

$(2.0) + (3.4) + (-2).1 = 0 + 12 + (-2) = 10$

$\boxed{\vec{A} \cdot \vec{B} = 10}$

2)

$\vec{A}$ ile $\vec{B}$ arasındaki açı ne kadardır?

$\vec{A} \cdot \vec{B} = A \cdot B \cdot \cos\theta$ $\quad \cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{A \cdot B}$

$A = \sqrt{2^2 + 3^2 + (-2)^2}$

$\boxed{A = \sqrt{17}}$

$B = \sqrt{4^2 + 1^2}$

$\boxed{B = \sqrt{17}}$

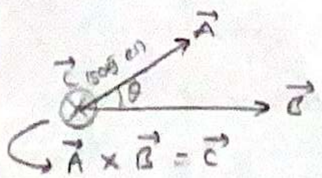
$\cos\theta = \dfrac{10}{\sqrt{17}.\sqrt{17}} = \dfrac{10}{17} = 0{,}588$

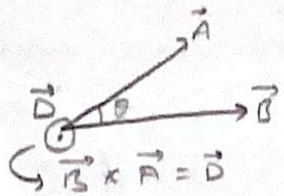
$\hookrightarrow \theta = 54°$

İki vektörün vektörel çarpımı

$\vec{A} \times \vec{B} = \vec{C}$ → vektör

$|\vec{C}| = |\vec{A} \times \vec{B}| = A \cdot B \cdot \sin\theta$ ← aralarındaki açı





$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}|$

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

Not: Bulunan $\vec{C}$ / $\vec{D}$ vektörü her zaman $\vec{A}$ ve $\vec{B}$'e diktir

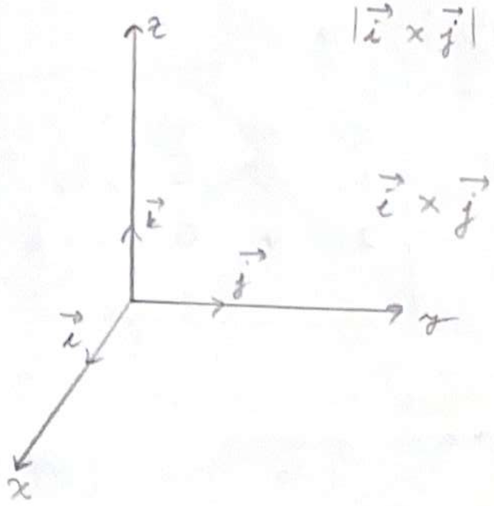
$\vec{A} \perp \vec{C}$ ve $\vec{B} \perp \vec{C}$ / $\vec{A} \perp \vec{D}$ ve $\vec{B} \perp \vec{D}$

Birim vektörlerle vektörel çarpım

$\vec{A} \times \vec{B} = ?$

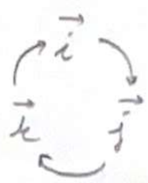
$|\vec{i} \times \vec{i}| = 1.1.\underbrace{\sin 0°}_{0} = 0$ (aynı doğrultuda)

$|\vec{i} \times \vec{j}| = 1.1.\underbrace{\sin 90°}_{1} = 1$



$\vec{i} \times \vec{j} = \vec{k}$ $\quad \vec{j} \times \vec{k} = \vec{i}$ $\quad \vec{k} \times \vec{i} = \vec{j}$

$\vec{j} \times \vec{i} = -\vec{k}$



Soru: $\vec{A} \times \vec{B} = (A_x \vec{i} + A_y \vec{j} + A_z \cdot \vec{k}) \times (B_x \cdot \vec{i} + B_y \cdot \vec{j} + B_z \cdot \vec{k})$

(1.yol)

$[(A_x \cdot \vec{i}) \times (B_x \cdot \vec{i})] + [(A_x \cdot \vec{i}) \times (B_y \cdot \vec{j})] + [(A_x \cdot \vec{i}) \times (B_z \cdot \vec{k})] +$

$(A_x \cdot B_y)(\vec{k})$ $\quad (A_x \cdot B_z)(-\vec{j})$

$[(A_y \cdot \vec{j}) \times (B_x \cdot \vec{i})] + [(A_y \cdot \vec{j}) \times (B_y \cdot \vec{j})] + [(A_y \cdot \vec{j}) \times (B_z \cdot \vec{k})] +$

$(A_y \cdot B_x)(-\vec{k})$ $\quad (A_y \cdot B_z)(\vec{i})$

$[(A_z \cdot \vec{k}) \times (B_x \cdot \vec{i})] + [(A_z \cdot \vec{k}) \times (B_y \cdot \vec{j})] + [(A_z \cdot \vec{k}) \times (B_z \cdot \vec{k})]$

$(A_z \cdot B_x)(\vec{j})$ $\quad (A_z \cdot B_y)(-\vec{i})$

$= [(A_x B_y)(\vec{k})] + [(A_x \cdot B_z)(-\vec{j})] + [(A_y \cdot B_x)(-\vec{k})] + [(A_y \cdot B_z)(\vec{i})] +$

$[(A_z \cdot B_x)(\vec{j})] + [(A_z \cdot B_y)(-\vec{i})]$

$= [(A_y \cdot B_z) - (A_z \cdot B_y)]\vec{i} + [(A_z B_x) - (A_x \cdot B_z)] \cdot \vec{j} + [(A_x \cdot B_y) - (A_y \cdot B_x)]\vec{k}$

(2.yol)

$\vec{A} \times \vec{B} =$

	+	−	+
	$\vec{i}$	$\vec{j}$	$\vec{k}$
	A_x	A_y	A_z
	B_x	B_y	B_z

$\Rightarrow +\vec{i}\,(A_y \cdot B_z - A_z \cdot B_y)$

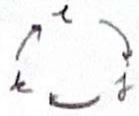
$\Rightarrow -\vec{j}\,(A_x \cdot B_z - A_z \cdot B_x)$

$\Rightarrow +\vec{k}\,(A_x \cdot B_y - A_y \cdot B_x)$

$C_x = A_y \cdot B_z - A_z \cdot B_y \quad C_y = A_z \cdot B_x - A_x \cdot B_z \quad C_z = A_x \cdot B_y - A_y \cdot B_x$

$$\vec{C} = C_x \vec{i} + C_y \vec{j} + C_z \vec{k}$$

Soru: $\vec{A} = 2\vec{i} + 4\vec{j}$ ve $\vec{B} = 2\vec{j} + 3\vec{k}$



① $\vec{C} = \vec{A} \times \vec{B}$

$(2\vec{i} \times 2\vec{j}) + (2\vec{i} \times 3\vec{k}) + (4\vec{j} \times 2\vec{j}) + (4\vec{j} \times 3\vec{k})$

$4\vec{k} + 6(-\vec{j}) + 0 + 12\vec{i}$

$\boxed{\vec{C} = 12\vec{i} - 6\vec{j} + 4\vec{k}}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 0 \\ 0 & 2 & 3 \end{vmatrix} \begin{matrix} +12\vec{i} \\ -6\vec{j} \\ +4\vec{k} \end{matrix}$$

② $\vec{A}$ ile $\vec{C}$ vektörlerinin dik olduklarını gösteriniz

$\vec{A} \cdot \vec{C} = (2\vec{i} + 4\vec{j}) \cdot (12\vec{i} - 6\vec{j} + 4\vec{k})$

$(2\vec{i} \cdot 12\vec{i}) + (2\vec{i} \cdot -6\vec{j}) + (2\vec{i} \cdot 4\vec{k}) + (4\vec{j} \cdot 12\vec{i}) + (4\vec{j} \cdot (-6\vec{j})) + 4\vec{j} \cdot 4\vec{k}$

$(24 \cdot 1) + (-12 \cdot 0) + (8 \cdot 0) + (48 \cdot 0) + ((-24) \cdot 1) + (16 \cdot 0)$

$24 - 24 = 0$

$A = \sqrt{2^2 + 4^2} = \sqrt{20}$

$c = \sqrt{12^2 + (-6)^2 + 4^2} = 14$

$\vec{A} \cdot \vec{C} = A \cdot C \cdot \cos\theta$

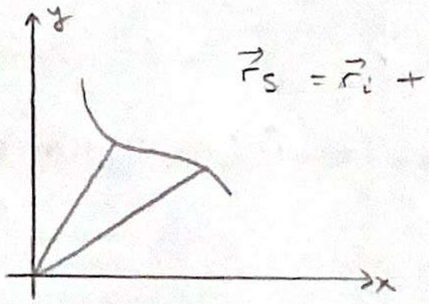
$\frac{\vec{A} \cdot \vec{C}}{A \cdot C} = \cos\theta \qquad \frac{0}{28\sqrt{5}} \Rightarrow 0$

$\boxed{\arccos 0 = 90°}$

13.10.2025 (Pzrts)

Yer değiştirme, hız ve ivme vektörleri

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

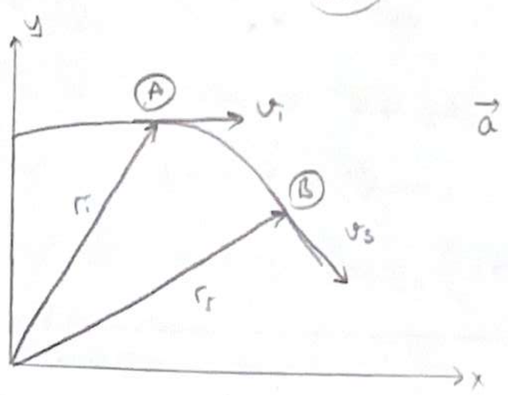


$\vec{r}_s = \vec{r}_i + \Delta\vec{r}$

$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_s - \vec{r}_i}{t_s - t_i}$

ortalama hız

$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$ → anlık hız $\quad |\vec{v}| \Rightarrow$ sürat



$\vec{a} = \frac{\vec{v}_s - \vec{v}_i}{t_s - t_i} = \frac{\Delta\vec{v}}{\Delta t}$ ortalama ivme

$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$ → anlık ivme

→ $\vec{a}$ vektörü $\Delta\vec{v}$ hız değişimi yönündedir

• Büyüklüğü sabit olsa da yön değişirse ivmeli hareket vardır

2 boyutlu

$\vec{r} = x_s\vec{i} + y_s\vec{j} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} \Rightarrow \vec{v} = v_x\vec{i} + v_y\vec{j}$

$v_s = v_i + at \qquad \vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j}$

$v_{xs} = v_{xi} + a_x t \qquad v_{ys} = v_{yi} + a_y t$

$\vec{v}_s = (v_{xi} + a_x t)\vec{i} + (v_{yi} + a_y t)\vec{j}$

$\vec{a} = a_x\vec{i} + a_y\vec{j} \qquad \vec{v}_0 = v_{x0}\vec{i} + v_{y0}\vec{j} + v_{z0}\vec{k}$

$v_x = v_{x0} + a_x t \; ; \; v_y = v_{y0} + a_y t \; ; \; v_z = v_{z0} + a_z t$

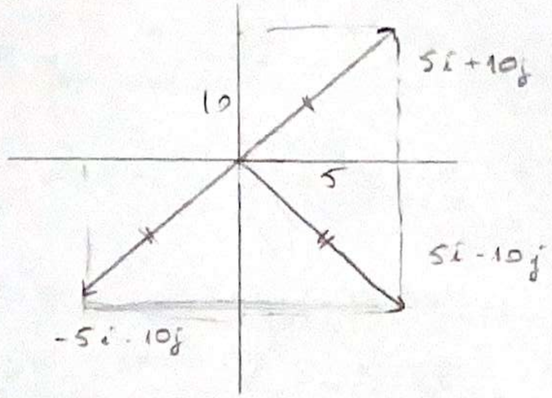
$\vec{v} = (v_{x0} + a_x t)\vec{i} - (v_{y0} + a_y t)\vec{j} + (v_{z0} + a_z t)\vec{k}$

$\vec{v} = (v_{x0}\vec{i} + v_{y0}\vec{j} + v_{z0}\vec{k}) + (a_x\vec{i} + a_y\vec{j} + a_z\vec{k})t$

$\vec{v} = \vec{v}_0 + \vec{a}\,t$

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j}$ $\quad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j}$

$\vec{a} = a_x\vec{i} + a_y\vec{j}$ $\quad x_s = x_i + v_{xi}t + \frac{1}{2}a_x t^2$ (x bileşeni)



$5\sqrt{5}$

$\vec{r} = x\hat{i} + y\hat{j}$ $\quad x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$

$\vec{r} = (x_0 + v_{x0}t + \frac{1}{2}a_x t^2)\hat{i} + (y_0 + v_{y0}t + \frac{1}{2}a_y t^2)\hat{j}$

$\vec{r} = \underbrace{(x_0\hat{i} + y_0\hat{j})}_{\vec{r}_0} + \underbrace{(v_{x0}\hat{i} + v_{y0}\hat{j})}_{\vec{v}_0}t + \underbrace{\frac{1}{2}(a_x\hat{i} + a_y\hat{j})}_{\vec{a}}t^2$

$= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$v_{x0} = 20$ m/s $\quad v_{y0} = -15$ m/s $\quad t = 0$ $\quad x_0 = 0$, $y_0 = 0$

$a_x = 4$ m/s² $\quad a_y = 0$

$v = f(t)$, $\quad \vec{v} = v_x\hat{i} + v_y\hat{j}$ $\quad v_x = v_{x0} + a_x \cdot t$

$v_x = 20 + 4.5$ $\quad \boxed{v_x = 40}$

$v_y = v_{y0} + a_y t$

$v_y = -15 + 0$ $\quad \boxed{v_y = -15}$

$\vec{v} = v_x\hat{i} - v_y\hat{j}$

$\vec{v} = 40\hat{i} - 15\hat{j}$ → formül

$\vec{v} = (20 + 4t)\hat{i} - 15\hat{j}$ m/s

Eğik atış

$\vec{g} = g(-\hat{j}) = -g\hat{j}$

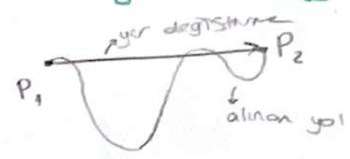
$x = x_0 + v_{x0}\cdot t + \frac{1}{2}a_x t$ $\quad x = v_{x0}t + v_0\cos v_0 t$

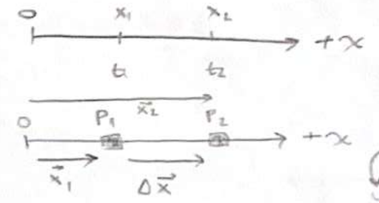
$y = v_{y0}t + \frac{1}{2}a_y t^2 = v_0 \sin v_{0y} t - \frac{1}{2}gt^2$

$v_x = v_{x0} + a_x t = v_0 \cos v_0$ yatay

$v_y = v_{y0} + a_y t = v_0 \sin v_{y0} - gt$ düşey

Bir boyutta hareket

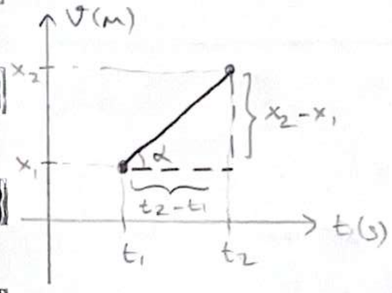




$\boxed{\Delta\vec{x} = \vec{x}_2 - \vec{x}_1}$

Yer değiştirme vektörü

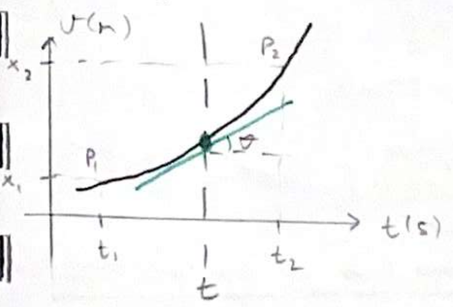
Ortalama hız



$v_{ort} = \vec{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$

$\tan\alpha = \frac{\Delta x}{\Delta t}$ $\quad \Delta\vec{x}$: vektör ⟶ $\vec{v}$: vektör (m/s)

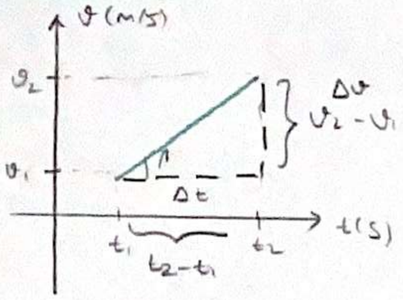
Anlık hız



$\Delta t = t_2 - t_1 \to 0$

$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ $\quad \tan\theta = \frac{dx}{dt}$

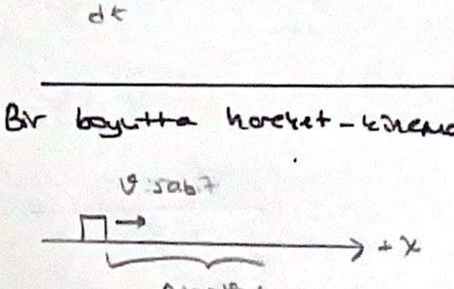
Ortalama ivme



$$a_{ort} = \vec{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \quad (m/s^2)$$

$$\tan\beta = \frac{\Delta v}{\Delta t} \qquad \Delta\vec{v}: \text{vektör} \rightarrow \vec{a}: \text{vektör}$$

Ani ivme



$$\Delta t = t_2 - t_1 \rightarrow 0$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad \tan\theta = \frac{dv}{dt}$$

Örnek: Bir cisim x-ekseni boyunca $x(t) = (10t^2 + 2)$ m denklemine göre hareket etmektedir. (t: saniye)

a) $t = 2s$, $t = 4s$ arasındaki $\Delta x = ?$ $v_{ort} = ?$

$x(2) = 10 \cdot 2^2 + 2 = 42m$ $\quad x(4) = 10 \cdot 4^2 + 2 = 162$ $\quad \Delta x = 162 - 42 = \boxed{120m = \Delta x}$

$$\frac{\Delta v}{\Delta t} = \frac{162 - 42}{4 - 2} = \frac{120}{2} = \boxed{60 m/s = v_{ort}}$$

b) " , " ani hız ve ani ivmeleri nedir?

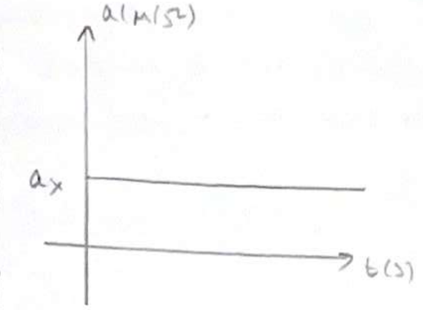
$$v = \frac{dx}{dt} \Rightarrow 20t \qquad v(2) = 40 m/s \qquad v(4) = 80 m/s$$

$$a = \frac{dv}{dt} \overset{20t}{=} 20 m/s^2 \qquad a(2) = 20 m/s^2 \qquad a(4) = 20 m/s^2$$

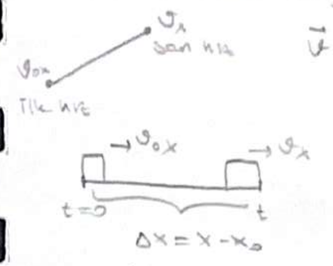
ortalama ivme de eşit çünkü sabit

Bir boyutta hareket - kinematik

Düzgün doğrusal hareket



↳ Sabit ivme olduğu için



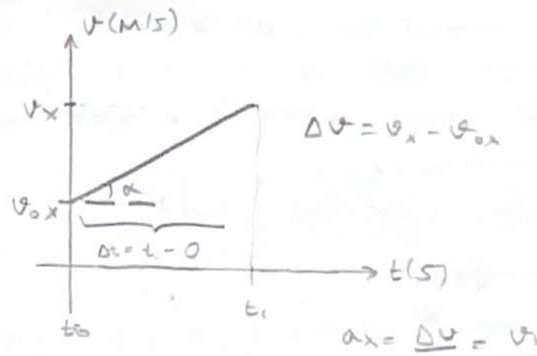
$$\vec{v} = v_{ort} = \frac{v_{0x} + v_x}{2}$$

$$v_{ort} = \frac{\Delta x}{\Delta t} \Rightarrow \frac{x - x_0}{t} = \frac{v_{0x} + v_x}{2}$$

$$\frac{x - x_0}{t} = \frac{v_{0x} + (v_{0x} + a_x t)}{2}$$

$$x - x_0 = \frac{2v_{0x}t + a_x t^2}{2} \Rightarrow \boxed{\Delta x = v_{0x}t + \frac{1}{2}a_x t^2}$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$



$$\Delta v = v_x - v_{0x}$$

$$a_x = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_{0x}}{t - 0}$$

$$a_x = \frac{v_x - v_{0x}}{t}$$

$$\boxed{v_x = v_{0x} + a_x t}$$

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$$

$$v_x = v_{0x} + a_x t$$

$$a_x t = v_x - v_{0x}$$

$$\boxed{t = \frac{v_x - v_{0x}}{a_x}}$$

$$\Delta x = v_{0x}\left(\frac{v_x - v_{0x}}{a_x}\right) + \frac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

$$\Delta x = \frac{v_x v_{0x}}{a_x} - \frac{(v_{0x})^2}{a_x} + \frac{1}{2}a_x \frac{(v_x - v_{0x})^2}{(a_x)^2} \quad \Big| \cdot 2a_x$$

$$2v_x v_{0x} - 2v_{0x}^2 + v_x^2 - 2v_{0x}v_x + v_{0x}^2 = 2\Delta x$$

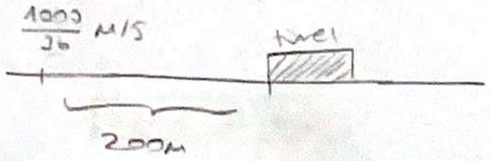
$$\frac{v_x^2 - v_{0x}^2}{a_x} = 2\Delta x \Rightarrow \boxed{v_x^2 = v_{0x}^2 + 2a_x\Delta x}$$

Soru: Hızı 100 km/h olan bir otomobilin sürücüsü 200 m önde bir tünel girişi ve sürücü sabit bir hızla ilerlediğinde tünele 70 km/h hızla girmesi için sabit hızla hareket ederek 12 sn sonra tünelden çıkıp $1{,}5\ m/s^2$ ivme ile hızlanarak 100 km/h hızına ulaşır

a) Tünele girene kadar geçen süre?

b) Tünel uzunluğu ?

c) Tünelden çıkıncaya ne kadar sürede o hıza ulaşır ve ne kadar yol alır?



$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$

$30\ \frac{1000}{3600} - \frac{300}{36}\ m/s \Rightarrow \Delta v$

$200 = \frac{1000}{36}\cdot t - \frac{1}{2}\cdot\frac{300}{36}t^2$

$720 = 100t - 15$

$7{,}35 = t$?

a)

$v_x = v_{0x} + a_x t \qquad \Delta x = v_{0x}t + \frac{1}{2}a_x t^2$

$\frac{v_x - v_{0x}}{a_x} = t \qquad \Delta x = v_{0x}\cdot\left(\frac{v_x - v_{0x}}{a_x}\right) + \frac{1}{2}a_x\cdot\frac{(v_x - v_{0x})^2}{(a_x)^2}$

$2a_x\left(\Delta x = \frac{v_{0x}\cdot v_x}{a_x} - \frac{(v_{0x})^2}{a_x} + \frac{1}{2}\frac{(v_x - v_{0x})^2}{a_x}\right)$

$2a_x\cdot\Delta x = 2v_{0x}v_x - 2v_{0x}^2 + v_x^2 - 2v_x v_{0x} + v_{0x}^2$

$-v_{0x}^2$

$2a_x\Delta x + v_{0x}^2 = v_1^2$

2. $a_x\ 200 + \left(\frac{1000}{36}\right)^2 = \left(\frac{700}{36}\right)^2$

$400\cdot a_x = -\frac{1700\cdot 300}{36\cdot 36}$

$a_x = -\frac{17\cdot 300}{36\cdot 36\cdot 4}$

b) $x = v\cdot t$

$x = \frac{700}{36}\cdot 12 \qquad x = \frac{700}{3}$

c) $v_2^2 = v_1^2 + 2a_2\Delta x \Rightarrow \left(\frac{1000}{36}\right)^2 - \left(\frac{700}{36}\right)^2 = 2\cdot\frac{3}{2}\cdot\Delta x$

1yol

$\frac{1700\ 100}{36\cdot 36} = 3\cdot\Delta x$

$\frac{1700\ 100}{36\ 36} = \Delta x$

$v_2 = v_1 + at \qquad \frac{1000}{36} - \frac{700}{36} = \frac{3}{2}t$

$\frac{300}{36} = \frac{3}{2}t \qquad \frac{100}{18} = t$

2yol

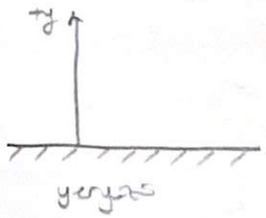
$\Delta x = v_{0x}t + \frac{1}{2}\cdot a_x t^2 \qquad v_2 = v_1 + at$

$\frac{700}{36}\cdot t + \frac{1}{2}\cdot\frac{3}{2}\cdot t^2 \qquad \frac{1000}{36} - \frac{700}{36} = \frac{3}{2}t \qquad \frac{100}{18} = t$

$\Delta x = \frac{100}{18}\left(\frac{700}{36} + \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{25}{18}\cdot\frac{100}{18}\right)$

$\Delta x = \frac{100}{18}\cdot\frac{850}{36}$

y-ekseni boyunca hareket



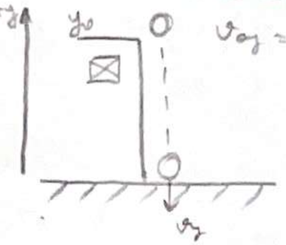
$g = -9{,}8\ m/s^2$

$v_y = v_{0y} - gt$

$\Delta y = v_{0y}t - \frac{1}{2}\cdot g t^2$

$v_y^2 = v_{0y}^2 - 2g\Delta y$

1) Serbest düşme ($v_{0y} = 0$)

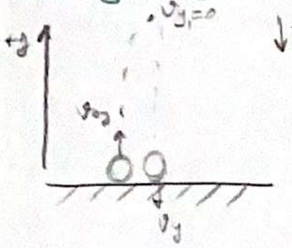


$v_y = v_{0y} - gt \Rightarrow v_y = -gt$

$\Delta y = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow \Delta y = -\frac{1}{2}gt^2$

$v_y^2 = v_{0y}^2 - 2g\Delta y \Rightarrow v_y^2 = -2g\Delta x$

2) Aşağıdan yukarı atış ($v_{oy} > 0$)



$v_y = +v_{oy} - gt$

$\Delta y = +v_{oy}t - \frac{1}{2}gt^2$

$v_y^2 = v_{oy}^2 - 2g\Delta y$

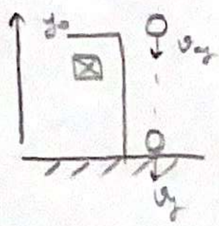
$t_{çıkış} \Rightarrow v_y = 0 \Rightarrow v_y = v_{oy} - gt$

$\frac{v_{oy}}{g} = t_{çıkış}$

$t_{uçuş} \Rightarrow \Delta y = 0 \Rightarrow v_{oy}t - \frac{1}{2}gt^2 \quad \frac{1}{2}gt = v_{oy}$

$t_{uçuş} = \frac{2 \cdot v_{oy}}{g}$

3) Yukarıdan aşağıya atış ($v_{oy} < 0$)

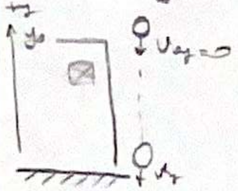


$v_y = -v_{oy} - gt$

$\Delta y = -v_{oy}t - \frac{1}{2}gt^2$

$v_y^2 = v_{oy}^2 - 2g\Delta y$

Örnek: Yüksekliği 60 m olan bir binanın tepesinden serbest bırakılan bir cisim yere ne zaman ve hangi hız ile çarpar? ($g = -9{,}8\ m/s^2$)



$v_y = v_{oy} - gt$

$v_y = 0 + 9{,}8t$

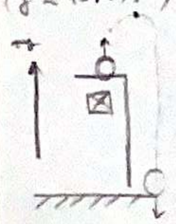
$v_y = \sqrt{9{,}8 \cdot 120}$

$\Delta y = v_{oy}t - \frac{1}{2}gt^2$

$60 = \frac{9{,}8}{2}t^2$

$\sqrt{\frac{120}{9{,}8}} = t$

Örnek: Yüksekliği 60m olan bir binanın tepesinden yere doğru 20 m/s hız ile atılan bir cisim yere ne zaman ve hangi hız ile çarpar? ($g \simeq 10\ m/s^2$)



6 sn sonra -40 m/s hız ile

$0 = v_{oy} - gt$

$0 = 20 - 10t$

$t = 2$

1) $\Delta y = v_{oy}t - \frac{1}{2}gt^2$

$\Delta_2 = 20 \cdot 2 - \frac{1}{2} 10 \cdot 2^2$

$\Delta y = 20$

2) $80 = v_{oy}t_1 - \frac{1}{2}g t_1^2$

$80 = \frac{1}{2}10 t_1^2$

$t_1 = 4$

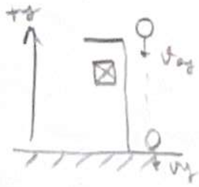
$v_y = 40$ m/s

$v_y = +10 \cdot 4$

$v_y = 0 - gt$

Örnek: Yüksekliği 60 m olan bir binanın tepesinden aşağı doğru 20 m/s hız ile atılan bir cisim yere ne zaman ve hangi hız ile çarpacağını hesaplayınız ($g = 10\ m/s^2$)

2 sn sonra -40 m/s



$v_y = v_{oy} - g \cdot t$

$\Delta y = v_{oy} \cdot t - \frac{1}{2}gt^2$

$60 = 20t + \frac{10}{2}t^2$

$12 = 4t + t^2 \qquad t^2 + 4t - 12 \quad (t+6)(t-2)$

$t = 2$ sn

$v_y = v_{oy} - gt$

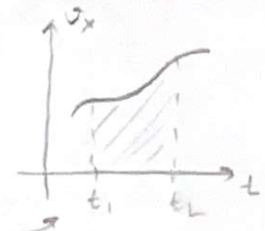
$v_y = 20 - 10 \cdot 2 \Rightarrow v_y = 40$ m/s

Değişken ivmeli hareket

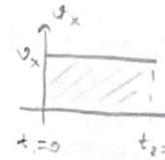
$\Delta x = x_2 - x_1 \qquad v_x = \frac{dx}{dt} \qquad a_x = \frac{dv_x}{dt}$

$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x\, dt$

$\cdot x \Big|_{x_1}^{x_2} = \int_{t_1}^{t_2} v_x\, dt$



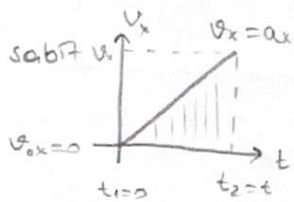
a) v_x sabit



$\Delta x = \int_{t_1}^{t_2} v_x\, dt = \int_0^t v_x\, dt = v_x \int_0^t dt$

$= v_x (t|_0^t) = v_x \cdot t$

b) a_x sabit



$\Delta x = x_2 - x_1 = \int_{t_1}^{t_2} v_x\, dt = \int_0^t (a_x t)\, dt = a_x \int_0^t t\, dt$

$\Delta x = a_x \left(\frac{t^2}{2}\Big|_0^t\right) = \frac{1}{2}a_x t^2 \quad (v_{0x} = 0)$

$a_x = \frac{dv_x}{dt}$

$\int_{v_1}^{v_2} dv_x = \int_{t_1}^{t_2} a_x\, dt \qquad \Delta v = v_2 - v_1 = \int_{t_1}^{t_2} a_x\, dt$

Örnek: Bir cisim $a_x = b + c\,t$ ivmesi ile ilk hızıyla x-ekseninde hareket etmektedir. Cismin ilk dört saniye sonundaki hızı 40 m/s ve ivmesi 18 m/s² ise b ve c nedir?

$a_x = b + c\,t$ $\quad 18 = b + c\cdot 4$ $\qquad v_2 - v_1 = \int_0^4 a_x\,dt$

$40 - 0 = \int_0^4 (b + c\,t)\,dt \Rightarrow 40 = bt\,|_0^4 - \frac{ct^2}{2}\,|_0^4$

$40 = 4b + 8c$

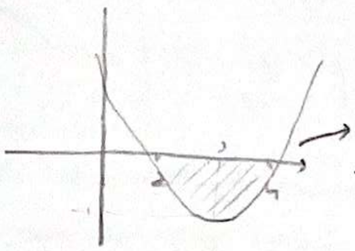
$20 = 2b + 4c$ $\quad b = 2$ $\quad c = 4$

Örnek: Bir cisim $v_x = t^2 - 6t + 8$ hız denklemi ile x-ekseninde hareket etmektedir.

a) Cismin hızı ne zaman sıfır olur? 2 ve 4. sn.

b) $2 \le t \le 4$ aralığındaki yer değiştirme ne kadardır?

$(t-4)(t-2)$



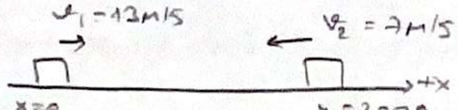
$\int_2^4 (t^2 - 6t + 8)\,dt$

$\left(\frac{t^3}{3} - 3t^2 + 8t\right)\Big|_2^4$

$\left(\frac{64}{3} - 48 + 32\right) - \left(\frac{8}{3} - 12 + 16\right)$

$\frac{64}{3} - 16 - \frac{8}{3} - 4 = \frac{52}{3} - 20 = -\frac{4}{3}\,m$

Örnek:



Ne zaman ve hangi konumda karşılaşırlar?

$v \cdot t = x$

$(13 + 7)\cdot t = 200$ $\quad t = 10$ sn

10 sn sonra (130, 0)

$x_1(t) = v_1 \cdot t$

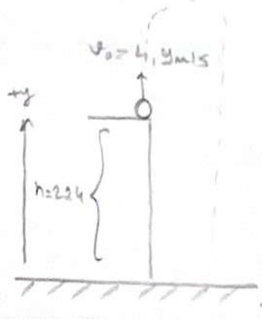
$x_2(t) = x_0 - v_2 t$ $\quad x_1(t) = x_2(t)$ $\quad 13t = 200 - 7t$

Örnek: Bir çocuk t=0 anında elindeki taşı $h = 29.4$ m yüksekteki yatay-dan $v_0 = 4.9$ m/s büyüklüğündeki hızla düşey doğrultuda yukarı doğru fırlatıyor. Buna göre; ($g = -9.8$ m/s²)

a) Taşın hareket denklemlerini integral hesap ile bulunuz.

b) Taş yüksüze sıfır tepe noktasına ne kadar sürede ulaşır ve bu noktada yerden kaç metre yüksektedir?

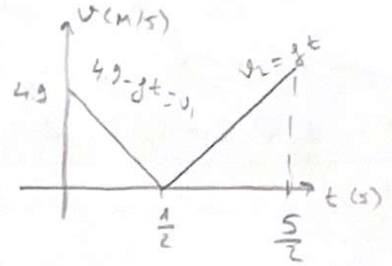
c) Taş fırlatıldıktan kaç saniye sonra yere çarpar ve yere çarpma hızı nedir?



$\downarrow -g = -9.8$ m/s²

I. durum $v_1 = v_0 - gt$ $\quad v_1 = 4.9 - 9.8t$

II. durum $v_2 = 0 + gt$ $\quad v_2 = 9.8t$



I. durum $\Delta x = v_{0x}\,t + \frac{1}{2} g t^2$

$\Delta x = (4.9)\frac{1}{2} + \frac{1}{2}(-9.8)\frac{1}{4}$

$\Delta x = \frac{4.9.4}{8} - \frac{4.8}{8}$ $\quad \Delta x = \frac{9.8}{4}$

II. durum

$\left(\frac{9.8}{4} + 29.4\right) = v_{0x}\,t - \frac{1}{2} g t^2$

$\frac{9.8 + 89.6}{4.2} = \frac{1}{2}(9.8)\cdot t^2$

$\frac{99.4}{19.6} = t^2$ $\quad t = 2$

a) II $\int_{\frac{1}{2}}^{\frac{5}{2}} g\,t\,dt \Rightarrow \frac{(9.8)\cdot t^2}{2}\,|_{\frac{1}{2}}^{\frac{5}{2}} = \frac{4.8\,(\frac{5}{2})^2}{2} - \frac{9.8\,(\frac{1}{2})^2}{2} = \frac{9.8}{2}(x.2)$

$= 3\,9.8 \Rightarrow 29.4$ m

I $\int_0^{\frac{1}{2}} 4.9 - gt\,dt \Rightarrow (4.9)\,t - \frac{gt^2}{2}\,|_0^{\frac{1}{2}} = (4.9)\cdot\frac{1}{2} - \frac{(9.8)\cdot(\frac{1}{2})^2}{2} = \frac{9.8}{8} \Rightarrow$

$\frac{9.8}{4} - \frac{9.8}{8}$

b) $t = 1/2$ sn $\quad x = \frac{9.8}{4}\cdot\frac{1}{4} + 29.4 = 1.225 + 29.4 \simeq 30.6$ m

c) $t = 5/2$ sn $\quad |\vec{v}| = 24.5$ m/s $\quad -24.5$ m/s

a) $t(0) \longrightarrow v_0(0) = 4.9\ m/s \quad y(0) = h = 29.4\ m$

$a = \frac{dv}{dt} \quad a \cdot dt = dv \quad -g \cdot dt \quad v = -g \int dt \Rightarrow -gt + A$

$v_0 = -g \cdot 0 + A \quad \boxed{A = v_0}$

$\boxed{v(t) = v_0 - gt}$

$v = \frac{dy}{dt} \quad dy = v\,dt \quad y(t) = \int dy = \int v \cdot dt$

$= \int (v_0 - gt)\,dt$

$= v_0 t - \frac{1}{2} g t^2 + B$

$t = 0$ anında $y_0 = 29.4\ m$

$y(0) = v_0 \cdot 0 - \frac{1}{2} g \cdot 0 + B = h \quad \boxed{B = h} \quad \boxed{y(t) = v_0 t - \frac{1}{2} g t^2 + 29.4}$

b) $v(t) = v_0 - gt$

$0 = 4.9 - 9.8\,t \quad 9.8\,t = 4.9 \Rightarrow t = \frac{1}{2}$

$y(t) = v_0 t - \frac{1}{2} g t^2 + 29.4 \Rightarrow y(t) = (4.9)\frac{1}{2} - \frac{1}{2}(9.2)\frac{1}{4} + 29.4$

$\frac{9.3}{8} + 29.4 = 1.245 + 29.4$

$= 30.645$

$\approx 30.6\ m$

c) $t_{\text{uçuş}} = ?$

$y(t) = 29.4 + 4.9\,t - 4.9\,t^2 = 0$

$(t^2 - t - 6) \Rightarrow (t-3)(t+2)$

$t = 3 \quad t = -2$

$v(t) = v_0 - gt$

$= 4.9 - 9.8 \cdot 3 = -24.5\ m/s$

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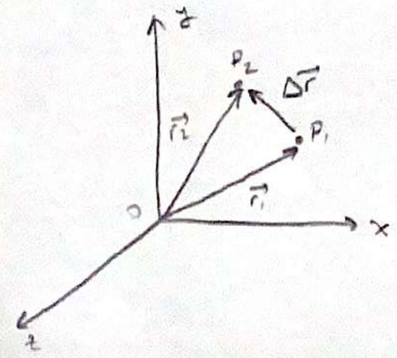
iki veya üç boyutta hareket



$\vec{r}_1 = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$

$\vec{r}_2 = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \underbrace{(x_2 - x_1)}_{\Delta x}\vec{i} + \underbrace{(y_2 - y_1)}_{\Delta y}\vec{j} + \underbrace{(z_2 - z_1)}_{\Delta z}\vec{k}$

ortalama hız

$\vec{v}_{ort} = \vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\vec{i} + \frac{\Delta y}{\Delta t}\cdot\vec{j} + \frac{\Delta z}{\Delta t}\cdot\vec{k}$

Anı hız

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \quad \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

$\vec{v} = \frac{dx}{dt}\cdot\vec{i} + \frac{dy}{dt}\cdot\vec{j} + \frac{dz}{dt}\cdot\vec{k}$

ortalama ivme

$\vec{a}_{ort} = \vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

Anı ivme

$\lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

$\vec{a} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\cdot\vec{j} - \frac{dv_z}{dt}\cdot\vec{k}$

Örnek: Bir cismin konum vektörü $\vec{r} = (10t^2 + 2)\vec{i} + (4t)\vec{j} + (2)\vec{k}$ m olarak veriliyor. Buna göre;

a) $t = 0$ ile $t = 2$ arasındaki yer değiştirmesini, $(40\vec{i} + 8\vec{j})$ m

b) " " " ortalama hızını, $(20\vec{i} + 4\vec{j})$ m/s

c) " "deki anı hızını, $t=0$ için $(4\vec{j})$ m/s, $t=2$ için $(40\vec{i} + 4\vec{j})$ m/s

d) " " arasındaki ortalama ivmesini, $20\vec{i}$ m/s²

e) " "deki anı ivmesini bulunuz. $\vec{a}_0 = \vec{a}_2 \Rightarrow$ sabit

a) $t = 0 \quad \vec{r}_0 = 2\vec{i} + 2\vec{k}$

$t = 2 \quad \vec{r}_2 = 42\vec{i} + 8\vec{j} + 2\vec{k}$

$42\vec{i} + 8\vec{j} + 2\vec{k}$

$-2\vec{i} - 2\vec{k}$

$40\vec{i} + 8\vec{j} = \Delta\vec{r}$

b) $\vec{v}_{ort} = \frac{\Delta\vec{r}}{\Delta t} = \frac{40\vec{i}}{2} + \frac{8\vec{j}}{2} \Rightarrow 20\vec{i} + 4\vec{j}$

c) $\vec{v} = \frac{d\vec{r}}{dt} = (20t)\vec{i} + 4\vec{j}$ $\left\{\begin{array}{l} t=0 \text{ için } (4\vec{j})\ m/s \\ t=2 \text{ için } (40\vec{i} + 4\vec{j})\ m/s \end{array}\right.$

d) $\frac{(40\vec{i} + 4\vec{j}) - (4\vec{j})}{2} = (20\vec{i})\ m/s^2$

e) $20\vec{i} \Rightarrow \vec{a}_0 = \vec{a}_2$

↳ zamandan bağımsız

Örnek: Bir cismin ilk hızı $\vec{v}_0 = 5\vec{j}$ m/s veriliyor ve $\vec{a} = (t+2)\vec{i} + 2\vec{j}$ (m/s²) ivmesiyle hareket ediyor. $t=0$ anında cisim orijinde ise $(x_0=0 ; y_0=0)$ ilk 2 saniye sonundaki konumunu ve hızını bulunuz.

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \int d\vec{v} = \int \vec{a}\,dt \qquad \vec{v} = \left(\frac{t^2}{2} + 2t + C_1\right)\vec{i} + (3t + C_2)\vec{j} \text{ m/s}$$

$t=0$ $\left(\frac{0^2}{2} + 2.0 + C_1\right).\vec{i} + (3.0 + C_2)\vec{j} \Rightarrow C_1\vec{i} + C_2\vec{j} = 5\vec{j}$

$\boxed{C_1 = 0}$ $\boxed{C_2 = 5}$

$t=2$ $\left(\frac{2^2}{2} + 2.2 + 0\right)\vec{i} + (3.2 + 5)\vec{j} \Rightarrow 6\vec{i} + 11\vec{j}$ $\boxed{\vec{v}_2 = 6\vec{i} + 11\vec{j}}$

$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \int d\vec{r} = \int \vec{v}\,dt \qquad \vec{r} = \left(\frac{t^3}{6} + t^2 + C_3\right)\vec{i} + \left(\frac{3t^2}{2} + 5t + C_4\right)\vec{j}$$

$t=0$ $(0 + 0 + C_3)\vec{i} + (0 + 0 + C_4)\vec{j} = C_3\vec{i} + C_4\vec{j} = 0\vec{i} + 0\vec{j}$

$\boxed{C_3 = 0}$ $\boxed{C_4 = 0}$

$t=2$ $\left(\frac{2^3}{6} + 2^2\right)\vec{i} + \left(\frac{3.2^2}{2} + 5.2\right)\vec{j} = \frac{16}{3}\vec{i} + 16\vec{j}$

$\boxed{\vec{r} = \frac{16}{3}\vec{i} + 16\vec{j}}$

Örnek: Bir parçacık 20 m/s'lik x bileşeni ve −15 m/s'lik y bileşeni ile ilk hızla $t=0$ anında başlangıç noktasından hareketle geçmektedir. Parçacık sadece $0x = 4\,m/s^2$ lik sabit ivmenin 2 bileşeniyle xy düzleminde hareket etmektedir.

a) Zamanın fonksiyonu olarak herhangi bir andaki hızın bileşenlerini ve toplam hız vektörünü bulunuz. $\vec{v} = (20 + 4t)\vec{i} + (-15)\vec{j}$

b) $t=5$ anında parçacığın hızının büyüklüğü, yönü ve doğrultusunu hesaplayınız.

43 m/s, −21°

$v_{0x} = 20$ m/s $\quad a_x = 4\,m/s^2$ $\quad \vec{v}_x = v_{0x} + a_x t \Rightarrow \vec{v}_x = (20 + 4t).\vec{i}$

$v_{0y} = -15$ m/s $\quad a_y = 0\,m/s^2$ $\quad \vec{v}_y = v_{0y} + a_y t \Rightarrow \vec{v}_y = -15\vec{j}$

$\vec{v} = (20 + 4t)\vec{i} - 15\vec{j}$

$\vec{v}_5 = (20 + 4.5)\vec{i} - 15\vec{j} \Rightarrow 40\vec{i} - 15\vec{j}$ $\qquad \frac{-15}{40} \simeq -21°$ yön

$|v| = \sqrt{(-15)^2 + (40)^2} = \sqrt{225 + 1600} = 43$ m/s

Örnek: Bir cismin ilk hız bileşenleri $v_{0x} = 12$ m/s ve $v_{0y} = -9$ m/s olacak şekilde $t=0$ anında orijinden hareketle başlıyor. xy-düzleminde hareket eden bu cismin ivme bileşenleri de $a_x = 4\,m/s^2$ ve $a_y = 0\,m/s^2$'dir. Buna göre;

a) Cismin herhangi bir andaki hızını veren denklemi bulunuz.

b) Cismin herhangi bir andaki konumunu veren denklemi bulunuz.

$\vec{v}_x = v_{0x} + a_x t \qquad \vec{v}_x = (12 + 4.t)\vec{i}$

$\vec{v}_y = v_{0y} + a_y.t \qquad \vec{v}_y = (-9 + 0.t) = (-9)\vec{j}$

$\boxed{\vec{v} = (12 + 4t)\vec{i} + (-9)\vec{j}}$

(1.yol) $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \int d\vec{r} = \int \vec{v}.dt \Rightarrow \vec{r} = (12t + 2t^2 + C_1)\vec{i} + (-9t + C_2)\vec{j}$

$\vec{r}_0 = (12.0 + 2.0^2 + C_1)\vec{i} + (-9.0 + C_2)\vec{j}$

$\vec{r}_0 = C_1\vec{i} + C_2\vec{j} = 0\vec{i} + 0\vec{j}$

$\boxed{C_1 = 0}$ $\boxed{C_2 = 0}$

$\boxed{\vec{r} = (12t + 2t^2)\vec{i} + (-9t)\vec{j}}$

(2.yol) $\vec{r}(t) = x(t)\,i - y(t)\,j$ $\quad t=0$ $\quad x(0)=x_0 \Rightarrow x_0 = 0$ $\quad y(0)=y_0 \Rightarrow y_0 = 0$

$x = x_0 + v_{0x}.t + \frac{1}{2}a_x t^2$

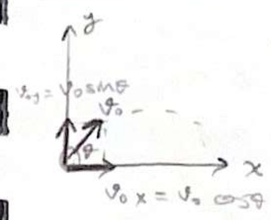
$y = y_0 + v_{0y}.t + \frac{1}{2}a_y t^2$

$x(t) = 0 + 12.t + \frac{1}{2}.4.t^2 \Rightarrow (12t + 2t^2)\vec{i}$

$y(t) = 0 + (-9t) + \frac{1}{2}.0.t^2 \Rightarrow (-9t)\vec{j}$

$\Big\} \; r(t) = (12t + 2t^2)\vec{i} + (-9t)\vec{j}$

Eğik Atış



$\downarrow -g$

x

$a_x = 0$

$v_x = v_{0x} + a_x t$

$\boxed{v_x = v_0.\cos\theta}$

$\boxed{\Delta x = v_x t = v_0.\cos\theta\; t}$

$\boxed{v_x^2 = v_{0x}^2 + 2a_x\Delta x}$

y

$a_y = -g$

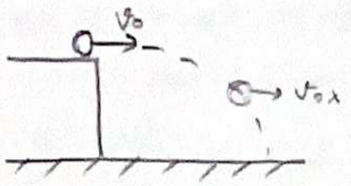
$\boxed{v_y = v_{0y} - g.t}$

$\boxed{\Delta y = v_y t - \frac{1}{2}g t^2}$

$\boxed{v_y^2 = v_{0y}^2 + 2.(-g).\Delta y}$

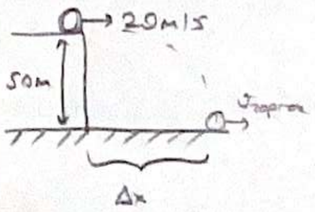
↳ Sabit ivmeli hareket zamansız hız denklemi

Yatay Atış



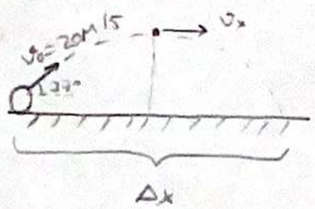
x	y
$v_x = v_{0x} = v_0$	$v_{0y} = 0$ $(\theta = 0°)$
$\Delta x = v_0 t$	$v_y = v_{0y} - g t$
	$\boxed{v_y = -g t}$
	$\boxed{\Delta y = -\frac{1}{2} g t^2}$

Örnek: Bir cisim 50m yükseklikten yere paralel 20m/s hız ile atılıyor. Cismin havada kalma süresi ($t_{uçuş}$), x ekseninde toplam yer değiştirmesi (Δx) ve toprağa çarpma hızını (v_{toprak}) bulunuz. ($g \approx 10 m/s^2$)



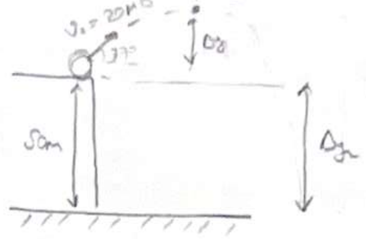
x	y
$v_x = v_0$	$v_y = -g t$
$20 \cdot t = \Delta x$	$\Delta y = -\frac{1}{2} g t^2$
$\boxed{\Delta x = 20\sqrt{10}\ m}$	$-50 = -\frac{1}{2} \cdot 10 \cdot t^2$
	$\boxed{t = \sqrt{10}\ s}$
	$v_y = -10 \cdot \sqrt{10} \Rightarrow \boxed{v_y = -10\sqrt{10}\ m/s}$

Örnek: Bir cisim yerden 37° açı yapacak şekilde 20m/s lik hız ile atılıyor. Cismin en üst noktaya çıkma süresi ($t_{çıkış}$), çıkabileceği maksimum yükseklik (h_{max}), cismin havada kalış süresini ($t_{uçuş}$) ve x ekseni boyunca alacağı yolu (Δx) hesaplayınız. ($\cos 37° = 0,8$; $\sin 37° = 0,6$; $g \approx 10 m/s^2$)



x	y
$v_x = v_{0x} = v_{0x} \cos\theta$	$v_{0y} = v_0 \cdot \sin\theta$
$v_x = 20 \cdot 0,8$	$v_{0y} = 20 \cdot 0,6$
$\boxed{v_x = 16\ m/s}$	$\boxed{v_{0y} = 12\ m/s}$
$\Delta x = v_x \cdot t_{uçuş}$	$v_y = v_{0y} - g t$
$\Delta x = 16 \cdot$	$v_{y2} = v_{0y} - g t_{uçuş}$
	$\Delta y = -\frac{1}{2} g t^2$
	$-12 = 12 - 10 t_{uçuş}$ $\boxed{t_{uçuş} = 2,4\ s}$
	$0 = 12 - 10 \cdot t_{çıkış}$
	$t_{çıkış} = 1,2\ s$
	$\Delta y = \frac{1}{2} \cdot 10 \cdot \frac{144}{100}$ $\Delta y = 7,2\ m$
	$\boxed{h_{max} = 7,2\ m}$

Örnek: Bir cisim yerden 50m yükseklikten x yere göre 37° açı ile 20m/s lik hızla atılıyor. Cismin en üst noktaya çıkma süresini ($t_{çıkış}$), çıkabileceği maksimum yüksekliği (h_{max}), havada kalma süresini ($t_{uçuş}$) ve x-ekseninde alacağı yolu (Δx) hesaplayınız. ($\cos 37° = 0,8$; $\sin 37° = 0,6$; $g \approx 10 m/s^2$)



x	y
$v_x = v_{0x} = v_0 \cdot \cos\theta$	$v_{0y} = 12\ m/s$
$v_x = 20 \cdot 0,8$	$v_y = v_{0y} - g t$
$\boxed{v_x = 16\ m/s}$	$0 = 12 - 10 t$ $\boxed{t_{çıkış} = 1,2\ s}$
	$\Delta y = v_{0y} t - \frac{1}{2} g t^2$
	$\Delta y = 12 \cdot 1,2 - \frac{1}{2} \cdot 10 \cdot \frac{144}{100}$
	$12 \cdot (1,2 - 0,6)$
	$12 \cdot 0,6 = 7,2\ m$
	$h_{max} = y_0 + \Delta y$
	$= 50 + 7,2$
	$\boxed{h_{max} = 57,2\ m}$

$\Delta y_{\downarrow} = v_{0y} \cdot t_u - \frac{1}{2} g t_u^2$

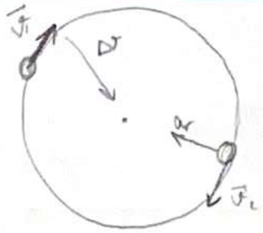
$-50 = 12 \cdot t_u - \frac{1}{2} \cdot 10 \cdot t_u^2$

$5t^2 - 12t - 50$

$\boxed{t_u = 4,58\ s}$

$\Delta x = v_{0x} \cdot t_u \Rightarrow \Delta x = 16 \cdot 4,58 \Rightarrow \boxed{\Delta x = 73,28\ m}$

Düzgün dairesel hareket



$|\vec{v_1}| = |\vec{v_2}|$ $a_{radyan} = a_{merkezcil}$ ivme

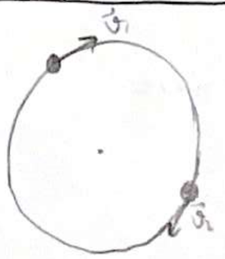
$a_r = \frac{v^2}{r}$ T: periyot

$x = v \cdot t$

$2\pi r = v \cdot T$ $\boxed{T = \frac{2\pi r}{v}}$

$f = \frac{v}{2\pi r}$

Düzgün olmayan dairesel hareket



$|\vec{v_2}| > |\vec{v_1}|$ ise $a_{teğet}$ vardır, yönü de hızla aynıdır

$a = \sqrt{a_r^2 + a_{teğet}^2}$

$|\vec{v_2}| < |\vec{v_1}|$ ise $a_{teğet}$ vardır, yönü hız ile ters tir

Örnek: Bir cisim 40 m yarıçaplı dairesel bir yörünge etrafında 20 m/s sabit sürat ile dönmektedir. Cismin üzerine etki eden radyal ivme ve periyodunu hesaplayınız.

$$a_{radyal} = \frac{v^2}{r} \Rightarrow \frac{20.20}{40} \Rightarrow \boxed{a_{radyal} = 10 m/s^2}$$

$$x = v.T$$

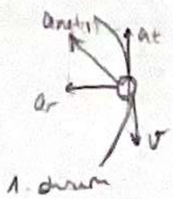
$$2\pi.r = v.T \qquad 2.\pi.40 = 20.T \qquad \boxed{4\pi = T}$$

Örnek: Bir cisim 40 m yarıçaplı bir çember etrafında sayfa düzlemine paralel dönmektedir. Cismin sürati değişken olarak 5 saniyede 20 m/s'den 10 m/s'ye düşmektedir. Her iki anda cisme etki eden net ivmeleri hesaplayınız.

$$a_{radyal} = \frac{v^2}{r} \Rightarrow a_{radyal(1)} = \frac{20.20}{40} \Rightarrow \boxed{a_{radyal(1)} = 10 m/s^2}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{-10}{5} = -2$$

a_t (2) $= 2 m/s^2$



1. durum

$$a_{net(1)} = \sqrt{a_{radyal(1)}^2 + a_{t(1)}^2}$$

$$\sqrt{10^2 + (-2)^2} \Rightarrow a_{net(1)} = \sqrt{104} = 2\sqrt{26}$$

$-2\sqrt{26}$

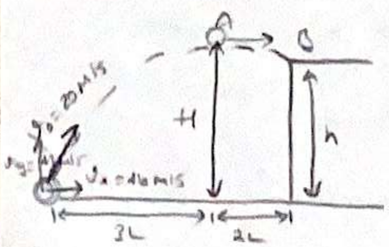
$$a_{radyal(2)} = \frac{10}{40} \frac{10}{} = 2,5 m/s^2$$

$a_{t(2)} = -2 m/s^2$

$$a_{net(2)} = \sqrt{a_{radyal(2)}^2 + a_{t(2)}^2}$$

$$\sqrt{(2,5)^2 + (-2)^2} \Rightarrow a_{net(2)} = \frac{\sqrt{41}}{2} m/s^2$$

Örnek: Yatayla 37° açı yapan bir cisim 20 m/s hız ile fırlatılıyor. Cismin ulaştığı maksimum yükseklik H kadardır. Cisim duvara B noktasında çarpmaktadır. H/h oranını hesaplayınız. ($\sin 37° = 0,6$; $\cos 37° = 0,8$; $g = 9,8 m/s^2$)



$$v_y = v_{oy} - g t$$

$$0 = 12 - 9,8 t \qquad \frac{60}{49} = t \Rightarrow 1,23 s$$

$$\Delta y = v_{oy} t - \frac{1}{2} g t^2$$

$$\Delta y = 12.(1,23) - \frac{1}{2}(9,8).(1,23)^2$$

$$H = 7,35 m$$

$v_x = v_0 \cos 37°$

$20 \cdot 0,8 = 16$

$v_{0y} = v_0 \sin 37° = 20 \cdot 0,6 = 12$

$$\Delta x = 3L = v_0 \cos 37°.(1,23) \qquad \frac{3L = 20.0,8.1,23}{2L = 20.0,8.t \to 0,82 s}$$

$$t_{toplam} = 1,23 + 0,82 = 2,05 s$$

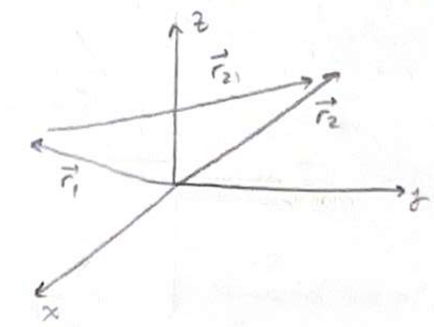
$$\Delta y = v_{oy}.t - \frac{1}{2} g t^2 \Rightarrow \Delta y = 12.(2,05) - \frac{1}{2}.(9,8).(2,05)^2 \Rightarrow \Delta y = 4,01 m$$

$$\frac{H}{h} = \frac{7,35}{4,01} = 1,83$$

$$\Delta y_{(1)} = v_{oy(1)}.t - \frac{1}{2} g t^2 \qquad t = 0,82$$

$$\Delta y_{(1)} = 0 - \frac{1}{2}(9,8).(0,82)^2 \qquad H - \Delta y_{(1)} = h$$

Örnek: Yere göre $v_1 = 200 m/s$ büyüklüğünde hızla ve 53° kuzeydoğu yönünde uçmakta olan bir uçağın pilotu, askeri bir helikopteri $v_{21} = 70 m/s$ hızla güney yönünde uçuyormuş gibi görüyor. Buna göre helikopterin hızının birim-vektör notasyonunda bulunuz. $v_2 = ?$

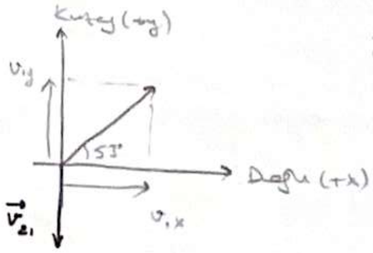


$$\vec{r}_1 + \vec{r}_{21} = \vec{r}_2$$

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

$$\frac{d\vec{r}_{21}}{dt} = \frac{d\vec{r}_2}{dt} - \frac{d\vec{r}_1}{dt} \qquad \vec{v}_{21} = \vec{v}_2 - \vec{v}_1$$

$\vec{v}_{21}$: helikopterin bağıl hızı; $\vec{v}_2$: helikopter; $\vec{v}_1$: uçak



$$\vec{v}_1 = v_1 \cdot \cos 53° \vec{i} + v_1 \cdot \sin 53° \vec{j}$$

$$\boxed{\vec{v}_1 = 120 \vec{i} + 160 \vec{j}}$$

$$\vec{v}_{21} = -70 \vec{j} \qquad \vec{v}_{21} = \vec{v}_2 - \vec{v}_1$$

$$-70 \vec{j} = \vec{v}_2 - (120 \vec{i} + 160 \vec{j})$$

$$\boxed{\vec{v}_2 = (120 \vec{i} - 90 \vec{j}) m/s}$$

$$v_2 = \sqrt{(120)^2 + (90)^2} = 150 m/s$$

$\tan\theta = \frac{4}{3} \Rightarrow \theta = 53°$

Örnek) Üç boyutlu uzayda hareket eden noktasal bir parçacığın konum vektörünün zamana bağlı ifadesi $\vec{r} = 2t^2\vec{i} - 3t\vec{j} + 5\vec{k}$ (m) cinsinden veriliyor. Buna göre:

a) Parçacığın $t=2$ s anındaki hızını, $(8\vec{i} - 3\vec{j})$ m/s

b) $t=0$ ile $t=2$ s zaman aralığındaki ortalama hızını, $(4\vec{i} - 3\vec{j})$ m/s

c) $t=2$ s anındaki ivmesi, $4\vec{i}$ m/s²

d) $t=0$ ile $t=2$ s zaman aralığındaki ortalama ivmesini bulunuz. $4\vec{i}$ m/s²

a) $\frac{d\vec{r}}{dt} = 4t\,\vec{i} - 3\vec{j} \Rightarrow t=2 \quad 8\vec{i} - 3\vec{j}$ m/s

b) $\vec{v}_{ort} = \vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_{son} - \vec{r}_{ilk}}{t_{son} - t_{ilk}} \Rightarrow \vec{r}_2 = 8\vec{i} - 6\vec{j} + 5\vec{k}$

$\vec{r}_0 = 5\vec{k}$

$\frac{8\vec{i} - 6\vec{j}}{2-0} = (4\vec{i} - 3\vec{j})$ m/s

c) $\frac{d}{dt}\vec{v} = 4\vec{i}$ m/s²

d) $\vec{a}_{ort} = \vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_{son} - \vec{v}_{ilk}}{t_{son} - t_{ilk}} = \frac{(8\vec{i} - 3\vec{j}) - (-3\vec{j})}{2-0} = 4\vec{i}$ m/s²

Örnek) $t=0$ anındaki ilk hızı $\vec{v}_0 = -12\vec{i}$ m/s olan hareketli bir parçacığın ivmesinin zamana bağlı olup $\vec{a} = 6t\,\vec{i} + 12t^2\vec{j}$ (m/s²) şeklinde olduğu biliniyor. Parçacık $t=0$ anında harekete orijinden başladığına göre,

a) Parçacığın hız ve konum vektörlerini integral ile elde ediniz.

$\vec{v} = (3t^2 - 12)\,\vec{i} + 4t^3\vec{j}$ (m/s) | $\vec{r} = (t^3 - 12t)\,\vec{i} + t^4\vec{j}$ (m)

b) Parçacık P(−16, 16) noktasından ne zaman geçer?

2 sn sonra ($t=2$ anında)

c) Parçacığın hızının x bileşeni ne zaman yön değiştirir?

$t=2$ anında

a) $\vec{a} = \frac{d\vec{v}}{dt} \qquad \vec{a}\,dt = d\vec{v} \qquad \int \vec{a}\,dt = \int d\vec{v}$

$\int (6t\,\vec{i} + 12t^2\vec{j})\,dt = \int d\vec{v}$

$3t^2\,\vec{i} + 4t^3\vec{j} + \vec{A} = \vec{v} \qquad t=0 \quad \vec{A} = -12\,\vec{i}$

$\boxed{\vec{v} = (3t^2 - 12)\,\vec{i} + 4t^3\vec{j}}$

$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{v}\,dt = d\vec{r} \qquad \int \vec{v}\,dt = \int d\vec{r}$

$\int ((3t^2 - 12)\,\vec{i} + 4t^3\vec{j})\,dt = \int d\vec{r}$

$\left(\frac{3t^3}{3} - 12t\right)\vec{i} + t^4\vec{j} + \vec{B} = \vec{r} \qquad \vec{r}_0 = 0 \qquad \vec{B} = 0$

$\boxed{\vec{r} = (t^3 - 12t)\,\vec{i} + t^4\vec{j}}$

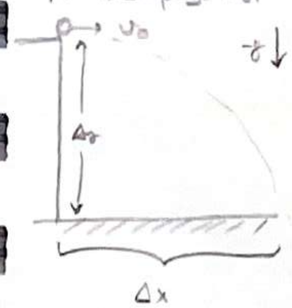
b) $t^3 - 12t = -16 \qquad t^4 = 16$

$\boxed{t = 2}$

c) $\vec{v} = (3t^2 - 12)\,\vec{i} + 4t^3\vec{j}$

$3t^2 - 12 = 0 \qquad 3(t^2 - 4) = 0 \qquad \boxed{t = 2}$

Örnek) $t=0$ anında, yerden h yüksekliğe bırakılan bir nesneden $v_0 = 8$ m/s büyüklüğünde hızla yatay olarak fırlatılan bir taş, yere düşene kadar yatay doğrultuda $d = 24$ m yol aldığına göre taşın fırlatıldığı noktanın yerden yüksekliği h kaç m'dir? ($g = 9.8$ m/s²)



$v_x = v_0$

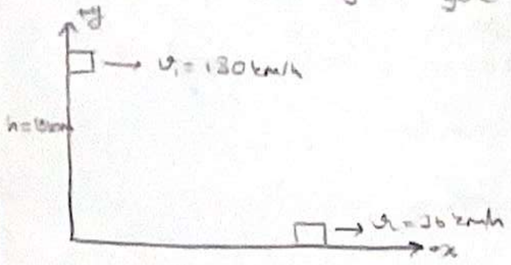
$\Delta x = 24 \qquad 24 = v_x \cdot t \qquad 24 = 8t \qquad \boxed{t = 3\ \text{sn}}$

$\Delta y = v_{0y}\,t - \frac{1}{2}gt^2$

$\Delta y = 0 - \frac{1}{2}(9.8)\cdot 3^2 = \Delta y = 44.1$

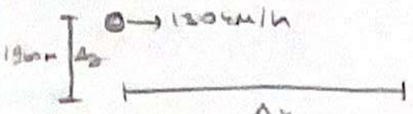
$\frac{49}{10}\cdot 9$

Örnek: Bir savaş uçağı, $v_1 = 180$ km/h büyüklüğündeki hızla yatay doğrultuda uçuş seyir irtifasından x_0 uzaktaki $v_2 = 36$ km/h hızla hareketsiz olan tekneyi görüp bombasını bırakıyor. Pilot bombayı yerden $h = 1960$ m yükseklikte bırakıp tam isabet kaydettiğine göre x_0 uzaklığını hesaplayınız.



$180 \cdot \frac{100}{3600} = 50$ m/s $= v_1$

$36 \cdot \frac{100}{3600} = 10$ m/s $= v_2$



$\Delta y = v_{0y} t - \frac{1}{2} g t^2$

$1960 = 0 \cdot t - \frac{1}{2} (9.8) \cdot t^2$

$1960 = \frac{49}{10} t^2$

$t = 20$

$\Delta x = v_1 \cdot t$

$50 \cdot 20 = 1000$ m

$\Delta x_2 = v_2 \cdot t$

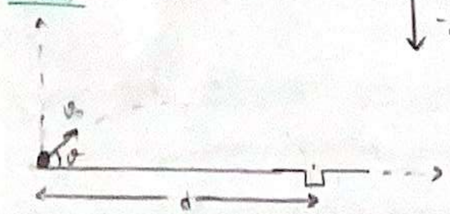
$10 \cdot 20 = 200$

$\Delta x = x_{0x} + \Delta x_2$

$1000 = x_{0x} + 200$

$x_{0x} = 800$ m

Örnek:



a) Taşın atıldığı noktadan d kadar uzaktaki çukura girebilmesi için v_0 ne olmalıdır? (Diğer değişkenler cinsinden)

b) $d = 10$ m, $g = 9.8$ m/s^2 ve $\theta = 15°$ için v_0 sayısal değerini bulunuz.

$(v_0 \cos\theta) \cdot t = d$

$-v_0 \sin\theta = v_0 \sin\theta - g t$

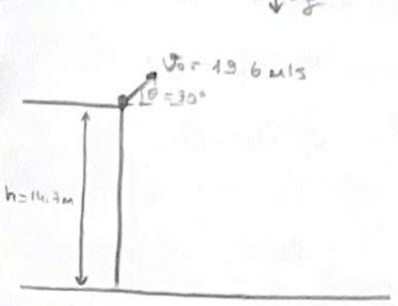
$t = \frac{2 v_0 \sin\theta}{g}$

$v_0 \cdot \cos\theta \cdot \frac{2 v_0 \cdot \sin\theta}{g} = d$

$v_0 = \sqrt{\frac{d g}{\sin(2\theta)}}$

$\sqrt{\frac{10 \cdot \frac{98}{10}}{\frac{1}{2}}} \Rightarrow v_0 = 14$ m/s

Örnek:



a) Taş yörüngenin en üst noktasına kaç saniyede ulaşır? (1 sn)

b) Taş ne kadar süre sonra yere çarpar? (3 sn)

c) Tüm hareket boyunca taşın yatay düzlemde aldığı yol nedir? ($29.4\sqrt{3}$ m)

d) Taşın yere çarpma hızının yatayla yaptığı açı nedir? ($49.11°$)

$v_0 = 19.6$ m/s $\quad v_{0y} = 9.8$ m/s $\quad v_{0x} = 9.8\sqrt{3}$ m/s

$0 = 9.8 - gt = 9.8 t = 9.8 \Rightarrow t = 1$

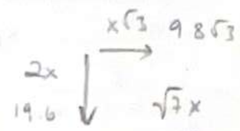
$x = v_{0y} \cdot t - \frac{1}{2} g t^2$

$x = 9.8 \cdot 1 - \frac{1}{2} \cdot 9.8 \cdot 1 \quad x = 4.9$ m

$19.6 = h_{max}$

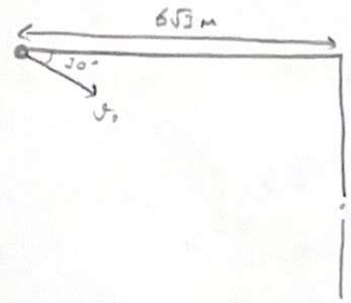
$19.6 = v_0 \cdot t + \frac{1}{2} \cdot 9.8 \cdot t^2$ (t = 2)

$9.8\sqrt{3} \cdot 3 =$ yatay yol $\quad 29.4\sqrt{3}$ m



$x(t) = (v_0 \cdot \cos\theta) \cdot t \qquad y(t) = h + (v_0 \cdot \sin\theta) \cdot t - \frac{1}{2} g t^2$

Örnek:



Top deliden geçtiğine göre ne kadar büyüklüğe sahip olmalıdır? $v_0 = ?$

$9.6 = \frac{v_0}{2} t + \frac{1}{2} \cdot 9.8 \cdot t^2$

$\frac{v_0}{2} \cdot \sqrt{3} \cdot t = 6\sqrt{3}$ $\quad v_0 t = 12$

$19.2 = v_0 t + 9.8 t^2$

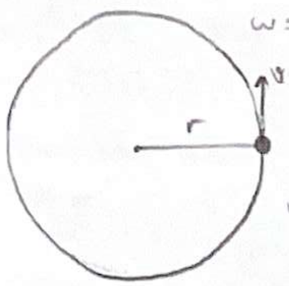
$7.2 = 9.8 t^2 \quad t = \frac{6}{7} \quad v_0 = 14$

Dairesel hareket

Örnek:

$r = 0.5$ m, saniyede $\frac{2}{\pi}$ tur atıyor.

$\omega = ?$, $v = ?$, $a_r = ?$



$x = v \cdot t$

$2\pi r = v \cdot T$

$v = \frac{2 \cdot \pi \cdot \frac{1}{2}}{\frac{\pi}{1}} = 2$ m/s

$\omega = \frac{2\pi}{T} = 2\pi f$

$\omega = \frac{2\pi}{\pi} = 4$ rad/s

$a_r = \frac{v^2}{r} = \omega^2 r$

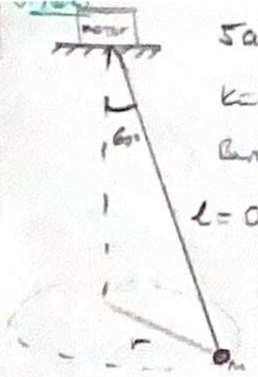
$a_r = 8$ m/s^2

$f = \frac{2}{\pi}$ s^{-1} (Hz)

$T = \frac{\pi}{2}$ s

$s = \theta \cdot r$

$v = \omega \cdot r$



Saniyede 1.115 tur dönen motorun ucundaki M kütleli boncuk düzgün dairesel hareket yapmaktadır. Buna göre boncuğa ait T, ω, v, r, a hesaplayınız.

$\ell = 0{,}4\,m$ $\quad f_h = \frac{1115}{1000}$ $\quad T_h = \frac{1000}{1115}$

$\frac{\sqrt{3}}{2} = \frac{r}{0{,}4}$ $\quad \boxed{\frac{r\sqrt{3}}{5} = r}$ $\quad (2{,}23)^2 \cdot \frac{\sqrt{3}}{5} = a_r$

$2\pi r = v \cdot T$ $\quad 2\pi \cdot \frac{r\sqrt{3}}{5} = v \cdot \frac{1000}{1115}$

$\frac{446\sqrt{3}}{1000}$ $\quad 0{,}446 \cdot \sqrt{3} = v$ m/s

$v = \omega \cdot r$

$\frac{0{,}446 \cdot \sqrt{3}}{\frac{\sqrt{3}}{5}} = 2{,}230$ rad/s

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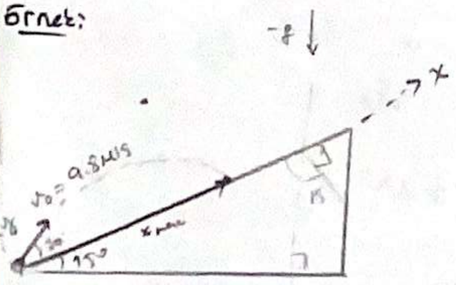
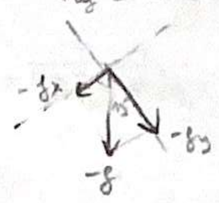
Örnek:



$x_{max} = ?$

$v_{oy} = 4.9$ $\quad v_{ox} = (4.9) \cdot \sqrt{3} = 8.49$



$g_x = -9.8 \cdot 0{,}2588 = -2{,}54$

$-g_y = -9.8 \cdot 0{,}9659 = -9.47$

$x = v_0 \cdot t - \frac{1}{2} g_x t^2$

$x = 8.49t - \frac{1}{2} 2.54 t^2$

$\boxed{x(t) = 8.49t - 1.27t^2}$

$y(t) = v_{oy} \cdot t - \frac{1}{2} g_y t^2$

$y(t) = (4.4)t - \frac{1}{2} 9.47 t^2$

↳ 0 tes y'de yer değiştirmek

$9.8 = 9.47\,t$ $\quad \boxed{t = 1.03\,s}$

$x(1.03) = (8.49)(1.03) - 1.27(1.03)^2$

$= 7.4\,m$ $\quad \boxed{x_{max} = 7.4\,m}$

NEWTON HAREKET YASALARI

1) Eylemsizlik prensibi

• Bir cisim üzerine etki eden net kuvvet 0 ise cisim

↳ $v_0 = 0$ ise durmaya ↳ $v_0 = $ sabit sabit hızla hareketine devam eder

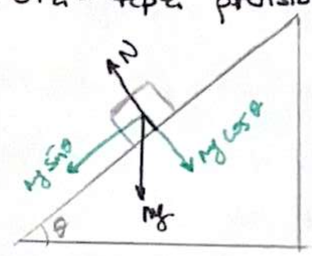
$$\sum \vec{F}_{net} = 0 \Big\} \; v_0 = 0 \;,\; v_0 = \text{sabit} \Big\rangle \; \vec{a} = 0$$

2) Temel prensip

• Bir cisim üzerine etki eden net kuvvet sıfırdan farklı ise cisim ivmelenir.

$$\sum \vec{F}_{net} = M \cdot \vec{a} \qquad 1\,\text{Newton} = 1\,kg\,\frac{m}{s^2}$$

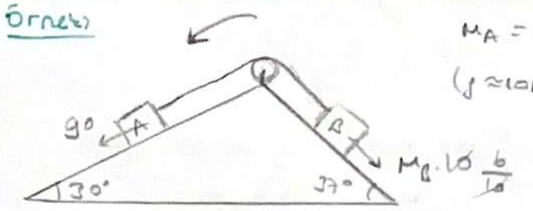
3) Etki - tepki prensibi



$N = mg\cos\theta$

Farklı cisimler üzerindedir

Örnek:



$M_A = 18\,kg$, Şekilde değişik sabit sürtünmeli ($g \approx 10\,m/s^2$) hareket vaziyeti $M_B = ?$

$6M_B = 90$ $\quad \boxed{M_B = 15\,kg}$

Not: Normal kuvvet $\vec{F}_N - \vec{F}_g = M \cdot \vec{a}_y \Rightarrow \vec{F}_N = M(g + a_y)$

İş ve Enerji

$W = \vec{F} \cdot \vec{\Delta x}$ Hareket doğrultusundaki yer değiştirme ile skaler çarpım

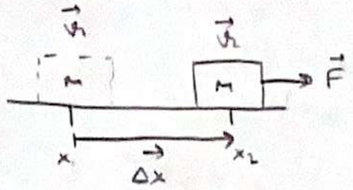
$W = (|\vec{F}| \cdot \cos\theta) \cdot |\vec{\Delta x}|$

İŞ = Newton · Metre $\Rightarrow$ İŞ = $kg\,\frac{m^2}{s^2}$

$kg\,\frac{m}{s^2}$ $\quad m$

$(m \cdot a)$

İŞ skalerdir pozitif, negatif veya 0 olabilir



$$v_2^2 = v_1^2 + 2 a_x \Delta x$$

$$\vec{a_x} = \frac{v_2^2 - v_1^2}{2\vec{\Delta x}}$$

$$\vec{F} = M \cdot \vec{a_x} = M \frac{v_2^2 - v_1^2}{2\vec{\Delta x}}$$

$$\vec{F} \cdot \vec{\Delta x} = \left(M \cdot \frac{v_2^2 - v_1^2}{2\vec{\Delta x}} \right) \cdot \vec{\Delta x} = \frac{1}{2} M v_2^2 - \frac{1}{2} M v_1^2 \Rightarrow W = \Delta E_k$$

Kinetik enerji de skalerdir yani değişim pozitif olması artığını, negatif olması azaldığını 0 olması da değişmediğini gösterir.

Durgun halden harekete geçen cisimler için $W = K_{son} - K_{ilk}$

$$\Rightarrow W = K - 0 = \frac{1}{2} M v^2$$

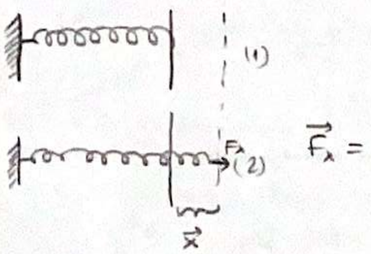
$$W = \int_{x_1}^{x_2} F_x \, dx$$

x_2 → final pozisyon

x_1 → başlangıç pozisyon

} kuvvet ile bileşenin integrali

Yaydaki geriye kuvvet



$$\vec{F_x} = k \cdot \vec{x} \qquad W = \int_0^x \vec{F_x} \, d\vec{x} = \int_0^x k \vec{x} \, d\vec{x} = \frac{1}{2} k x^2$$

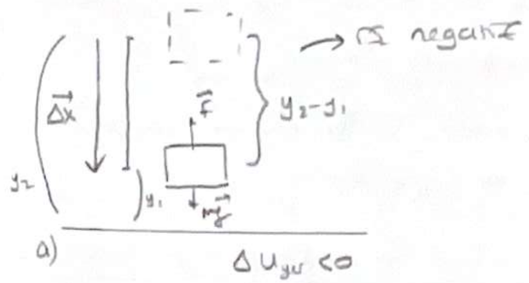
$$W = \int_{x_1}^{x_2} \vec{F_x} \, d\vec{x} = \frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2$$ } yer değiştirme pozitif yönlü (Kuvvet ile aynı yön)

$$W = \int_{x_1}^{x_2} k \cdot (-\vec{x}) \, d\vec{x} = \frac{1}{2} k x_1^2 - \frac{1}{2} k x_2^2$$

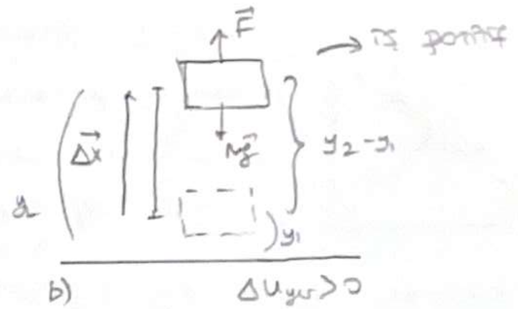
Güç (watt = Joule / s)

$$P_{ort} = \frac{\Delta W}{\Delta t} \qquad P_{ani} = \frac{\Delta W}{\Delta t} \qquad P = \vec{F} \frac{\vec{\Delta x}}{\Delta t} \Rightarrow P = \vec{F} \cdot \vec{v}$$

cisme hızlandıran ve harekete geçiren



a) $\Delta U_{yer} < 0$



b) $\Delta U_{yer} > 0$

$$W_g = \vec{F} \cdot \vec{\Delta x} = m \cdot \vec{g} \, (\vec{y_1} - \vec{y_2})$$

$$W_g = U_{g1} - U_{g2} = -(U_{g2} - U_{g1}) = -\Delta U_g$$

Mekanik enerjinin korunumu

$$\Delta K = K_2 - K_1 = -\Delta U_g = -(U_{g2} - U_{g1})$$

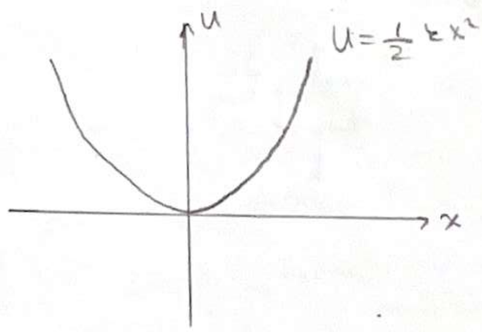
Sistemde yalnızca yer çekimi iş yaparsa mekanik enerji korunacaktır.

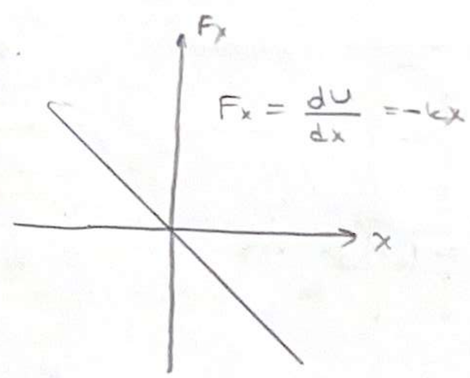
$$K_1 + U_{g1} = K_2 + U_{g2} \qquad \frac{1}{2} m v_1^2 + mgy_1 = \frac{1}{2} m v_2^2 + mgy_2$$

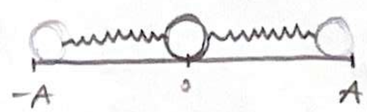
b) $W_g + W_d = K_2 - K_1$ ve $W_g = -\Delta U_g = -(U_{g2} - U_{g1})$

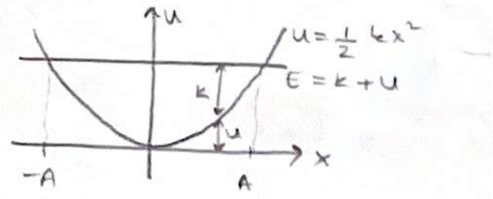
$$W_d + U_{g1} - U_{g2} = K_2 - K_1 \Rightarrow U_{g1} + K_1 + W_d = U_{g2} + K_2$$

W_d pozitif olunca $U_{g2} + K_2 > U_{g1} + K_1 + W_d$

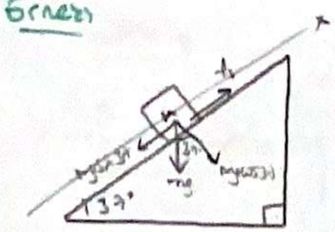








Örnek:



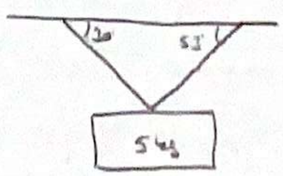
$m = 5\,kg$, statik ve kinetik sürtünme katsayıları, sırasıyla $\mu = 0,2$ değerindedir

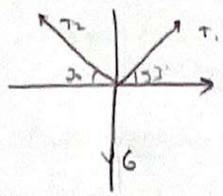
Sürtünme kuvveti kaç N'dur? Cisim hareket geçiyorsa ivmesi ne kadar olur? ($g \approx 10 m/s^2$)

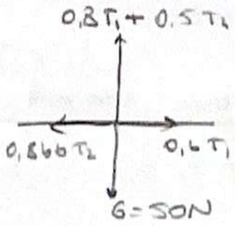
$mg = 5.10 = 50$ $\quad mg\sin37 = 0,6 \cdot 50 = 30\,N$ $\quad F_s = mg\cos37 \cdot \mu = 50 \cdot (0,8) \cdot 0,2 = 8$

$F_{ny} - F_s = F_{net} = m \cdot a \quad 30 - 18 = 5\,a \Rightarrow a = 4,4\,m/s^2$

Örnek:







$\left(\frac{\sqrt{3}}{2} = 0,866\right)$

$0,6\,T_1 = 0,866\,T_2$

$0,8\,T_1 + 0,5\,T_2 = 50N$

$\underbrace{0,8 \cdot (1,44)}T_2 + 0,5\,T_2 = 50N$

$1,155\,T_2$

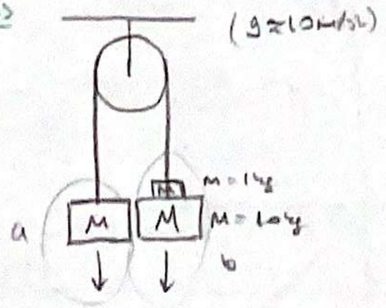
$T_1 = \frac{0,866}{0,6} T_2$

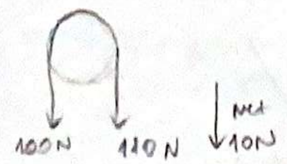
$T_1 = 1,44\,T_2$

$T_2 = \frac{50}{1,655} N \Rightarrow T_2 = 30,2\,N$

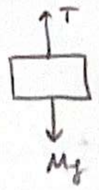
$(1,44)(30,2) \Rightarrow T_1 = 43,5\,N$

Örnek:

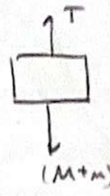




a)



b)



$T - Mg = Ma$ $\quad (M+m) \cdot a = (M+m)g - T$

$M(g+a) = (M+m)(g-a)$

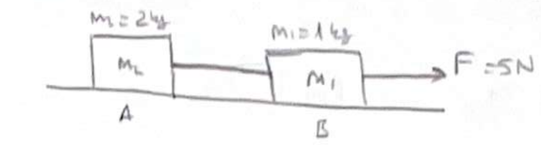
$10 \cdot (10+a) = 11 \cdot (10-a)$

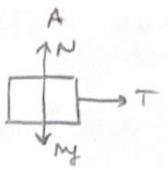
$100 + 10a = 110 - 11a$

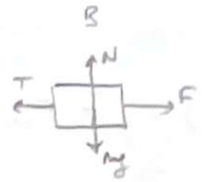
$21\,a = 10$

$a = \frac{10}{21}\,m/s^2$

Örnek:



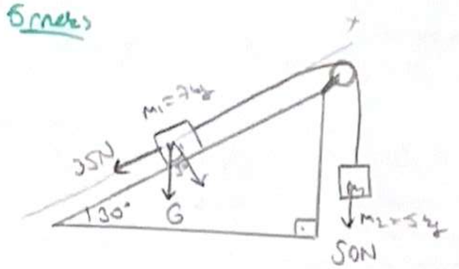


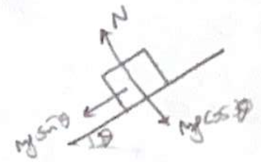


$F = (m_1 + m_2) \cdot a \quad 5 = 3 \cdot a \quad \boxed{\frac{5}{3} = a}$

$\left[T = m_2 \cdot a \quad F - T = m_1 \cdot a\right] \quad T = 2 \cdot \frac{5}{3} \Rightarrow T = \frac{10}{3} N$

Örnek:





$G = m_1 g = 7 \cdot 10 \quad 70 \cdot \sin 30 = mg\cos 30 = 35$

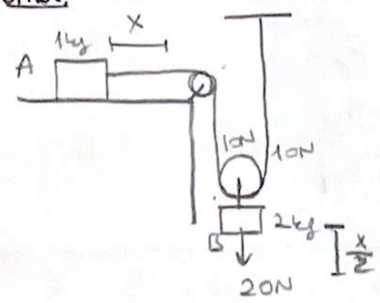
$50 - T = 5 \cdot a$

$T - 35 = 7 \cdot a$

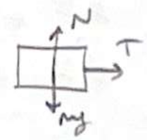
$15 = 12a \Rightarrow a = 1,25\,m/s^2$

$50 - T = 5 \cdot (1,25) \Rightarrow 50 - 6,25 = T \Rightarrow T = 43,75\,N$

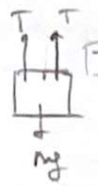
Örnek:



A)



B)



T = 10N

$a_1 = 2a_2$

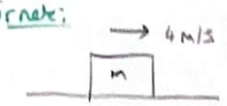
$T = m_1 \cdot 2a_2 \quad 2T - m_2 g = m_2 \cdot a_2$

$T = 1 \cdot 2a_2$

$2T - 20 = -2\,a_2$ (2. yöntem)

$\boxed{T = 6,66\,N}$ $\quad (a_1 = 6,66\,m/s^2\,,\ a_2 = 3,33\,m/s^2)$

Örnek:



$\mu = 0,15$

→ hareket yönü

$f_s = mg \cdot \mu \quad 10m \cdot \frac{1}{4} \Rightarrow f_s = 2,5m$

$f_s = m \cdot a \quad 2,5m = ma \quad \boxed{a = 2,5}$

$X = 6,4 - 3,2$

$\boxed{X = 3,2\,m}$

$x = \cancel{x_0} + v_0 \cdot t - \frac{1}{2} a t^2$

$x = 4 \cdot t - \frac{1}{2} \cdot \frac{5}{2} \cdot t^2$

$v_{son}^{\,0} = v_0 - at$

$\frac{5}{2} t = 4$

$\boxed{t = 1,6}$

Lineer Momentum ve Çarpışmalar

$$\Sigma \vec{F}_1 = F_{21} + F_{31} + F_{41} + \dots + F_{n1} \qquad F_{21} = -F_{12}$$

$$\Sigma \vec{F}_2 = F_{12} + F_{32} + F_{42} + \dots + F_{n2}$$

$$\vdots$$

$$\Sigma \vec{F}_n = F_{1n} + F_{2n} + \dots + F_{(n-1)n}$$

+

$$\Sigma \vec{F}_1 + \Sigma \vec{F}_2 + \dots + \Sigma \vec{F}_n = 0$$

$$m_1 a_1 + m_2 a_2 + \dots + m_n a_n = 0$$

$$m_1 \cdot \frac{d\vec{v}_1}{dt} + m_2 \cdot \frac{d\vec{v}_2}{dt} + \dots + m_n \cdot \frac{d\vec{v}_n}{dt} = 0$$

$$\frac{d}{dt}\left(m_1 \cdot \vec{v}_1 + m_2 \cdot \vec{v}_2 + \dots + m_n \cdot \vec{v}_n\right) = 0$$

$$\frac{d}{dt}\left(\underbrace{\vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_n}\right) = 0$$

$$\frac{d}{dt}\left(\Sigma \vec{P}\right) = 0 \longrightarrow \Sigma \vec{P}_i = \Sigma \vec{P}_s$$

$$\Sigma \vec{P}_{ix} = \Sigma \vec{P}_{sx} \qquad \Sigma \vec{P}_i \hat{i} = \Sigma \vec{P}_s \hat{i}$$

$$\Sigma \vec{P}_{iy} = \Sigma \vec{P}_{sy}$$

$$\Sigma \vec{F} = m \cdot \frac{d\vec{v}}{dt} \Rightarrow \Sigma \vec{F} = \frac{d(m\vec{v})}{dt} \Rightarrow \Sigma \vec{F} = \frac{d\vec{P}}{dt}$$

$$\int_{\vec{P}_i}^{\vec{P}_2} \Sigma \vec{F} \cdot dt = \int_{t_1}^{t_2} d\vec{P} \Rightarrow \vec{P}_2 - \vec{P}_i = \Delta \vec{P} = \int_{t_1}^{t_2} \Sigma \vec{F}\, dt$$

$$F_{ort} = \frac{1}{\Delta t} \int_{t_1}^{t_2} \Sigma \vec{F}\, dt \qquad \Delta P = f_{ort} \cdot \Delta t$$

$m_1 \rightarrow \vec{v}_{1i}$ ..... $m_2 \rightarrow \vec{v}_{2i}$ (esnek çarp.)

$$\vec{P}_i = m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = m_1 \cdot \vec{v}_{1s} + m_2 \cdot \vec{v}_{2s} = \vec{P}_s$$

$$\frac{1}{2} m_1 (v_{1i})^2 + \frac{1}{2} m_2 (v_{2i})^2 = \frac{1}{2} m_1 (v_{1s})^2 + \frac{1}{2} m_2 (v_{2s})^2$$

$$m_1 (v_{1i} - v_{1s})(v_{1i} + v_{1s}) = m_2 (v_{2s} - v_{2i})(v_{2s} + v_{2i})$$

$$m_1 (v_{1i} - v_{1s}) = m_2 (v_{2s} - v_{2i})$$

$$m_2 (v_{2s} - v_{2i}) \cdot (v_{1i} + v_{1s}) = m_2 (v_{2s} - v_{2i}) (v_{2s} + v_{2i})$$

$$\boxed{v_{1i} + v_{1s} = v_{2s} + v_{2i}}$$

$$m_1 \cdot v_{1i} + m_2 \cdot v_{2i} = m_1 \cdot v_{1s} + m_2 (v_{1i} + v_{1s} - v_{2i})$$

$$m_2 v_{1i} + m_2 v_{1s} - m_2 v_{2i}$$

$$(m_1 - m_2) v_{1i} + 2m_2 v_{2i} = (m_1 + m_2) v_{1s}$$

$$\boxed{v_{1s} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2 v_{2i}}{m_1 + m_2}}$$

$$m_1 \cdot v_{1i} + m_2 \cdot v_{2i} = m_1 (v_{2s} + v_{2i} - v_{1i}) + m_2 \cdot v_{2s}$$

$$= m_1 \cdot v_{2s} + m_1 v_{2i} - m_1 v_{1i}$$

$$2m_1 v_{1i} + (m_2 - m_1) \cdot v_{2i} = (m_1 + m_2) v_{2s}$$

$$\boxed{v_{2s} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} \cdot v_{2i}}$$

Kütle merkezi

$$x_{km} = \frac{m_1 \cdot x_1 + m_2 \cdot x_2 + \dots + m_N \cdot x_N}{m_1 + m_2 + \dots + m_N} \qquad M = m_1 + m_2 + m_3 + \dots + m_N$$

$$x_{km} = \frac{1}{M} \sum_{i=0}^{N} m_i \cdot x_i$$

$$y_{km} = \frac{1}{M} \sum_{i=0}^{N} m_i \cdot y_i$$

$$z_{km} = \frac{1}{M} \sum_{i=0}^{N} m_i \cdot z_i$$

$$\vec{r}_{km} = x_{km} \cdot \hat{i} + y_{km} \cdot \hat{j} + z_{km} \hat{k}$$

$$\vec{r}_{km} = \frac{1}{M} \sum_{i=0}^{N} \left[(m_i \cdot x_i)\hat{i} + (m_i \cdot y_i)\hat{j} + (m_i \cdot z_i) \cdot \hat{k} \right]$$

Sürekli kütle dağılımı

$$x_{km} \simeq \frac{1}{M} \sum_{i=0}^{N} \Delta m_i \cdot x_i$$

$$x_{km} = \frac{1}{M} \lim_{\Delta m_i \to 0} \sum_{i=0}^{N} \Delta m_i \cdot x_i \to \int x \, dm$$

$$x_{km} = \frac{1}{M} \int x \, dm$$

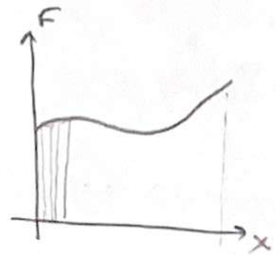
$$y_{km} = \frac{1}{M} \int y \, dm$$

$$z_{km} = \frac{1}{M} \int z \, dm$$

Fizik-1 (2. ara sınav)

$W = \vec{F} \cdot \Delta \vec{r} \Rightarrow W = F \, \Delta r \cdot \cos\theta$ (kuvvet ile yerdeğiştirme arasındaki açı)

↳ sabit kuvvetin yaptığı iş



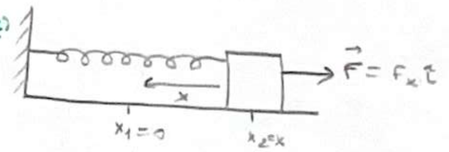
↳ değişken kuvvetin yaptığı iş

$$W_x = \lim_{\Delta x_i \to 0} \sum_{i=1}^{\infty} F_i \cdot \Delta x_i = \int_{x_i}^{x_s} F_x \, dx$$

$$W = \int_{\vec{r}_i}^{\vec{r}_s} \underbrace{F_x \, dx + F_y dy + F_z \, dz}_{\vec{F} \cdot d\vec{r}}$$

$$\boxed{W = \int_{\vec{r}_i}^{\vec{r}_s} \vec{F} \cdot d\vec{r}}$$

Örnek:



$\vec{F} = F_x \hat{i}$ $\quad \vec{F}_{yay} = -kx\hat{i}$

$\Sigma \vec{F} = -kx\hat{i}$ $\quad d\vec{r}_x = dx\hat{i}$

$$W = \int_0^x -kx\hat{i} \cdot dx\hat{i} = -k \cdot \int_0^x x \, dx \Rightarrow \boxed{W = -\frac{1}{2} k x^2}$$

İş - kinetik enerji

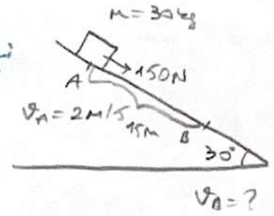
$$W = \int \Sigma \vec{F} \cdot d\vec{r} \qquad \Sigma \vec{F} = m \cdot \vec{a} = m \frac{d\vec{v}}{dt}$$

$$W = \int m \frac{d\vec{v}}{dt} \cdot d\vec{r} \qquad W = m \int \underbrace{d\vec{v} \cdot \vec{v}}_{(\text{aynı yönlü}) \text{ ardalanadki açı } 0 - \text{sıfır çarpım}} \qquad W = m \int_{v_i}^{v_s} v \, dv = \frac{1}{2} m v^2 \Big|_{v_i}^{v_s}$$

$$W = \frac{1}{2} m v_s^2 - \frac{1}{2} m v_i^2$$

$$\boxed{E_k = \frac{1}{2} m v^2} \qquad \boxed{W = E_{ks} - E_{ki} = \Delta E_k}$$

Örnek:

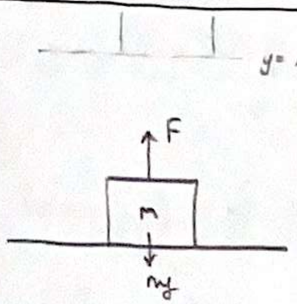


$$150 \cdot 15 = \frac{1}{2} 30 v_B^2 - \frac{1}{2} 30 \cdot 2^2$$

$$\sqrt{154} = \sqrt{v_B^2}$$

$$2\sqrt{39} = v_B^2$$

Kütle çekim potansiyel enerjisi



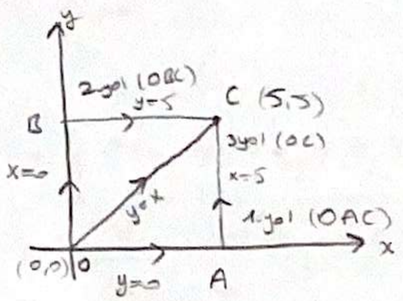
$$W = \int \Sigma \vec{F}\, d\vec{r}$$

$$\Sigma \vec{F} = F\hat{j} = mg\hat{j} \qquad d\vec{r} = dy\,\hat{j}$$

$$W = \int_0^h mg\hat{j}\cdot dy\,\hat{j} \qquad W = mg\int_0^h dy$$

$$\boxed{W = mgh}$$

Korunumlu - korunumsuz kuvvetler



$F = 2y\,\hat{i} + x^2\hat{j}$

1.yol (OAC)

$W_{OAC} = W_{OA} + W_{AC}$

$W_{OA} = \int (2y\,\hat{i} + x^2\hat{j})\, dx\,\hat{i}$

$W_{OA} = 2y\int_0^5 dx\,\hat{i} = 0$

$W_{AC} = \int (2y\,\hat{i} + x^2\hat{j})\, dy\,\hat{j}$

$W_{AC} = x^2\int_0^5 dy\,\hat{j} = 25.5 = 125$

$$\boxed{W_{OAC} = 125 \text{ joule}}$$

2.yol (OBC)

$W_{OBC} = W_{OB} + W_{BC}$

$W_{OB} = \int (2y\,\hat{i} + x^2\hat{j})\, dy\,\hat{j}$

$W_{OB} = x^2\int dy\,\hat{j} = 0$

$W_{BC} = \int (2y\,\hat{i} + x^2\hat{j})\, dx\,\hat{i}$

$2y\int_0^5 dx\,\hat{i} = 50$ joule

$$\boxed{W_{OBC} = 50 \text{ joule}}$$

3.yol (OC)

$W_{OC} = \int (2y\,\hat{i} + x^2\hat{j})\,(dx\,\hat{i} + dy\,\hat{j}) = \int 2y\,dx + \int x^2\,dy$

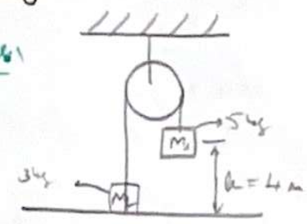
$\int_0^5 2x\,dx + \int_0^5 y^2\,dy$

$\underbrace{x^2|_0^5}_{25} + \underbrace{\frac{y^3}{3}|_0^5}_{125/3} = 66.67$ joule

$$\boxed{W_{OBC} = 66.67 \text{ joule}}$$

Enerji Korunumu

Örnek:



Yerden aynı yüksekliğe gelebilmeleri için hızları ne olur?

$m_1 \Rightarrow 5.10.2 = 100$

$100 = \frac{1}{2}\,5\,v_1^2$ $\qquad \boxed{2\sqrt{10} = v_1}$

$m_2 \Rightarrow 3.10.2 = 60$

$60 = \frac{1}{2}\,3\,v_2^2$ $\qquad \boxed{2\sqrt{10} = v_2}$

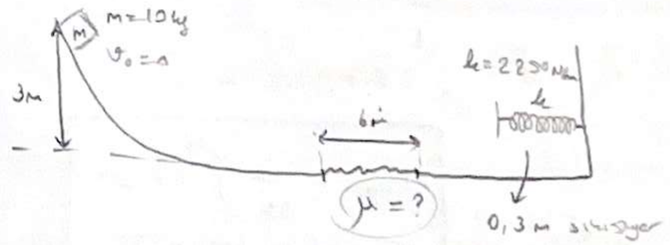
$\Delta U_g + \Delta K = 0$

$(U_{gs} - U_{gi})_1 + (U_{gs} - U_{gi})_2 + (K_s - K_i)_1 + (K_s - K_i)_2 = 0$

$\underbrace{\left(m_1 g\frac{h}{2} - m_1 gh\right)}_{-\frac{1}{2}gh(m_2 - m_1)} + \left(m_2 g\frac{h}{2} - 0\right) + \underbrace{\left(\frac{1}{2}m_1 v_1^2 - 0\right) + \left(\frac{1}{2}m_2 v_2^2 - 0\right)}_{\frac{1}{2}(m_1+m_2)v^2} = 0$

$$v = \sqrt{\frac{(m_1 - m_2)\,gh}{(m_1+m_2)}}$$

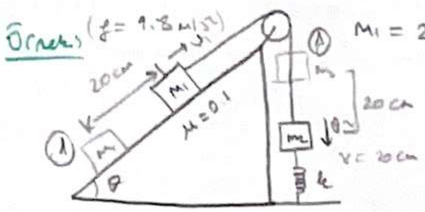
Örnek: ($g = 9.8\ m/s^2$)



$E_{ilk} = E_{son}$

$10.(9.8)\,3 = \frac{1}{2}\,2250.\frac{9}{100} + 10.(9.8).\mu.6$ $\qquad \mu = 0.33$

Örnek: ($g = 9.8\ m/s^2$) $\quad M_1 = 20kg \quad M_2 = 30kg \quad k = 250\ N/m \quad \theta = 40° \quad \mu = 0.1$



(ilk durum ile yeniden geldiğimiz nokta) $\quad (v_1 = v_2$ aynı IP)

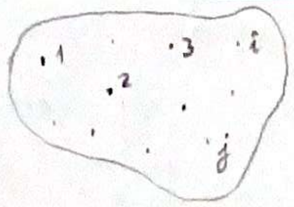
$(U_{gs} - U_{gi})_1 + (U_{gs} - U_{gi})_2 + (K_s - K_i)_1 + (K_s - K_i)_2 + (E_{ys} - E_{yi}) = E_s$

$20.(9.8)\cos 40°.0.2.0.1$

$(20.9.8.(0.2)\underbrace{\sin 40°}_{0.64} - 0) + (0 - 30.(9.8).0.1) + (\tfrac{1}{2}\,20\,v_1^2 - 0) + (\tfrac{1}{2}\,30\,v_2^2 - 0) + \tfrac{1}{2}.250.(0.2)^2 =$

$25.088 - 58.8 + 25v^2 + 5 = 3.028$ $\qquad \boxed{v = 1.11\ m/s}$

Lineer momentum ve Sarpışmalar



$$\Sigma \vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} + \dots + \vec{F}_{i1} + \dots$$

$$\Sigma \vec{F}_2 = \vec{F}_{12} + \vec{F}_{32} + \vec{F}_{42} + \dots$$

$$+ \quad \Sigma \vec{F}_i = \vec{F}_{1i} + \vec{F}_{2i} + \dots$$

$$\Sigma \vec{F}_1 + \Sigma \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_i = 0$$

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 + \dots + m_i \vec{a}_i = 0$$

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + \dots + m_i \frac{d\vec{v}_i}{dt} = 0$$

$$\frac{d}{dt}(m_1 \vec{v}_1) + \frac{d}{dt}(m_2 \vec{v}_2) + \dots + \frac{d}{dt}(m_i \vec{v}_i) = 0$$

$$\frac{d}{dt}\left(m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_i \vec{v}_i\right) = 0$$

$$\left(\vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_i\right)$$

$$\frac{d}{dt}\left(\vec{P}_{toplam}\right) = 0 \quad \rightarrow \text{sabit}$$

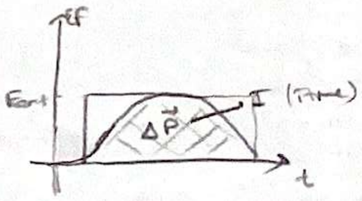
$$\boxed{\Sigma \vec{P}_i = \Sigma \vec{P}_s}$$

$$\Sigma \vec{P}_{ix} = \Sigma \vec{P}_{sx} \quad \dots$$

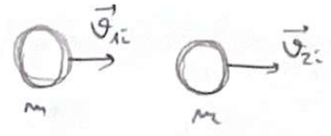
$$\Sigma \vec{F} = m \cdot \frac{d\vec{v}}{dt} \Rightarrow \Sigma \vec{F} = \frac{d}{dt}(m\vec{v}) = \Sigma \vec{F} = \frac{d\vec{P}}{dt}$$

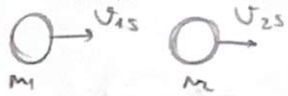
$$\int_{\vec{P}_i}^{\vec{P}} d\vec{P} = \int_0^t \Sigma \vec{F} \cdot dt \Rightarrow \vec{P} - \vec{P}_i = \Delta \vec{P} = \int_0^t \Sigma \vec{F}\, dt$$

$$\boxed{\Delta \vec{P} = \int_0^t \Sigma \vec{F}\, dt}$$



$$\boxed{F_{ort} = \frac{1}{\Delta t} \int_{t_1}^{t_2} \Sigma F\, dt \qquad F_{ort} \cdot \Delta t = \Delta \vec{P}}$$





$$\Sigma \vec{P}_i = \Sigma \vec{P}_s \qquad \Sigma K_i = \Sigma K_s \text{ esnek carpışmalarda}$$

① $m_1 \cdot \vec{v}_{1i} + m_2 \cdot \vec{v}_{2i} = m_1 \cdot \vec{v}_{1s} + m_2 \cdot \vec{v}_{2s}$

② $\frac{1}{2} m_1 (v_{1i})^2 + \frac{1}{2} m_2 (v_{2i})^2 = \frac{1}{2} m_1 (v_{1s})^2 + \frac{1}{2} m_2 (v_{2s})^2$

$m_1 (v_{1i} - v_{1s}) = m_2 (v_{2s} - v_{2i})$

$m_1 ((v_{1i})^2 - (v_{1s})^2) = m_2 ((v_{2s})^2 - (v_{2i})^2)$

② $m_1 (v_{1i} - v_{1s})(v_{1i} + v_{1s}) = m_2 (v_{2s} - v_{2i})(v_{2s} + v_{2i})$

① $m_1 (v_{1i} - v_{1s}) = m_2 (v_{2s} - v_{2i})$

$$\boxed{v_{1i} + v_{1s} = v_{2i} + v_{2s}}$$

$$v_{2s} = v_{1i} + v_{1s} - v_{2i} \rightarrow$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1s} + m_2 (v_{1i} + v_{1s} - v_{2i})$$

$$(m_1 + m_2) v_{1s} = (m_1 - m_2) v_{1i} + 2 m_2 v_{2i}$$

$$\boxed{v_{1s} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}}$$

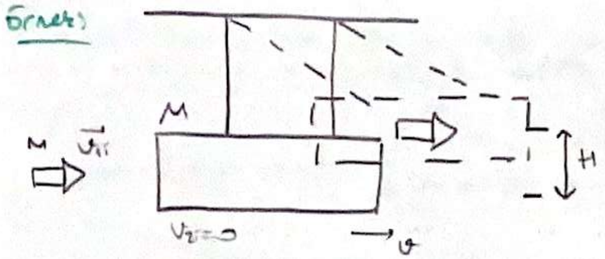
$$v_{1s} = v_{2i} + v_{2s} - v_{1i}$$

$$m_1 \cdot v_{1i} + m_2 v_{2i} = m_1 (v_{2i} + v_{2s} - v_{1i}) + m_2 v_{2s}$$

$$2m_1 \cdot v_{1i} + (m_2 - m_1) v_{2i} = (m_1 + m_2) v_{2s}$$

$$\boxed{v_{2s} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i} + \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}}$$

Örnek:



$\sum \vec{P}_i = \sum \vec{P}_s$

$m \cdot \vec{v}_{i1} = (m+M) v_s$

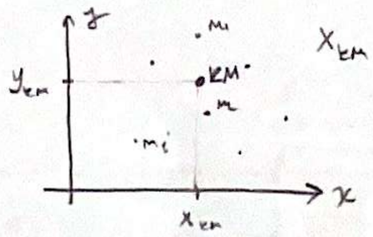
$v_s = \left(\frac{m}{m+M}\right) v_{i1}$

$\frac{1}{2}(m+M) v_s^2 = (m+M) g H$

$\frac{m^2}{2(m+M)^2} v_{i1}^2 = gH$

$$\boxed{v_{i1} = \left(\frac{m+M}{m}\right)\sqrt{2gH}}$$

Kütle merkezi



$$X_{km} = \frac{m_1 x_1 + m_2 x_2 + \dots + m_i x_i}{m_1 + m_2 + \dots + m_i}$$

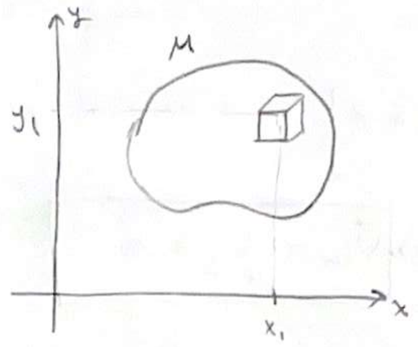
$$\boxed{X_{km} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i} \qquad \boxed{y_{km} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i}$$

$m_1 + m_2 + \dots + m_i = M$

$\vec{r}_{km} = x_{km}\cdot\hat{i} + y_{km}\cdot\hat{j}$

$$\boxed{\vec{r}_{km} = \frac{1}{M}\sum_{i=1}^{n}\left[(m_i x_i)\hat{i} + (m_i y_i)\hat{j}\right]} \rightarrow \text{2 bağıntı}$$

Sürekli kütle dağılımına sahip cisimler

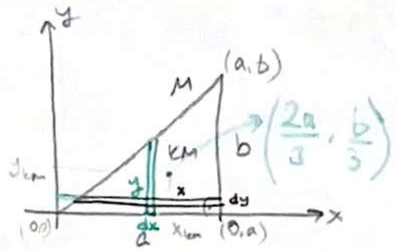


$$X_{km} = \frac{1}{M}\lim_{\Delta m_i \to 0}\sum_{i=1}^{\infty}\Delta m_i x_i$$

$$\boxed{X_{km} = \frac{1}{M}\int x\, dm}$$

$$y_{km} = \frac{1}{M}\int y\, dm$$

Örnek:



$$X_{km} = \frac{1}{M}\int \frac{2My}{ab}\, dx$$

$$X_{km} = \frac{1}{M}\cdot\frac{2M}{ab}\cdot\frac{b}{a}\int_0^a x^2\, dx$$

$$X_{km} = \frac{2}{a^2}\left(\frac{a^3}{3}\right) \Rightarrow \boxed{X_{km} = \frac{2a}{3}}$$

$\frac{ab}{2}$ — M

$y\,dx$ — dm

$dm = \frac{2My}{ab}\,dx$

$\frac{b}{a} = \frac{y}{x} \quad \frac{bx}{a} = y$

$$y_{km} = \frac{1}{M}\int y\frac{2Mx}{ab}\, dy \qquad x = a - \frac{a}{b}y$$

$$y_{km} = \frac{1}{M}\cdot\frac{2M}{ab}\int_0^b \left(a - \frac{a}{b}y\right) y\, dy$$

$\left(ay - \frac{ay^2}{b}\right)$

$$\frac{ay^2}{2} - \frac{ay^3}{3b}\Big|_0^b \Rightarrow \frac{3ab^2}{6b} - \frac{2ab^2}{6b} = \frac{ab^2}{6b} = \frac{ab^2}{6}$$

$$y_{km} = \frac{2}{ab}\cdot\frac{ab^2}{6} \Rightarrow \boxed{y_{km} = \frac{b}{3}}$$

$$\vec{r}_{KM} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\vec{v}_{KM} = \frac{d\vec{r}_{KM}}{dt} = \frac{d}{dt}\left[\frac{1}{M} \sum_i m_i \vec{r}_i\right]$$

$$\vec{v}_{KM} = \frac{1}{M}\left[\sum_i m_i \left(\frac{d\vec{r}_i}{dt}\right)\right] \Rightarrow \boxed{\vec{v}_{KM} = \frac{1}{M}\left(\sum_i m_i \vec{v}_i\right)}$$

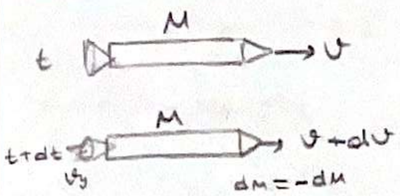
$$\boxed{M \cdot \vec{v}_{KM} = \sum_i m_i \cdot \vec{v}_i} \quad (\vec{p})$$

$$\vec{a} = \frac{d\vec{v}_{KM}}{dt} \qquad \vec{a} = \frac{d}{dt}\left[\frac{1}{M}\left(\sum_i m_i \vec{v}_i\right)\right] \qquad \vec{a} = \frac{1}{M} \sum_i m_i \left(\frac{d\vec{v}_i}{dt}\right)$$

$$\boxed{\vec{a} = \frac{1}{M} \sum_i m_i \vec{a}_i}$$

$$\boxed{M \cdot \vec{a} = \sum_i m_i \cdot \vec{a}_i} \quad (\vec{F})$$

Roket hareketi



$v - v_y$: yakıtın yere göre hızı

$$\vec{P}_i = \vec{P}_s$$

$$(M+dM)v = M(v+dv) + dM\,(v - v_y)$$

$$Mv + v\,dM = Mv + M\,dv + dM\,v - dM\,v_y$$

$$v_y\,dM = M\,dv$$

$$-v_y\,dM = M\,dv$$

$$\boxed{dv = -v_y \cdot \frac{dM}{M}}$$

$$\int_{v_i}^{v_s} dv = -v_y \int_{M_i}^{M_s} \frac{dM}{M}$$

$$v_s - v_i = -v_y \ln\left(\frac{M_s}{M_i}\right) \Rightarrow v_s = v_i + v_y \cdot \ln\left(\frac{M_i}{M_s}\right)$$

$$M\,dv = -v_y\,dM \qquad T = M\frac{dv}{dt}$$

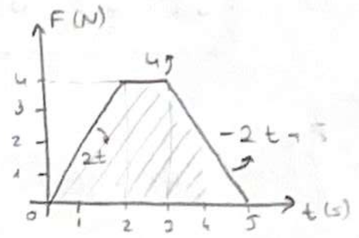
$$\underbrace{M\frac{dv}{dt}}_{T} = -v_y \frac{dM}{dt} \qquad T = -v_y \frac{dM}{dt} \qquad \boxed{T = \left|v_y \frac{dM}{dt}\right|}$$

Örnek: $K = 275$ Joule ve $P = 25$ kg m/s M ? v ?

$$K = \frac{1}{2} \cdot M v^2 = 275 \qquad P = M \cdot v = 25$$

$$\frac{1}{2} M v \cdot v = 275 \qquad v = 22 \qquad \frac{25}{22} = M$$

Örnek:



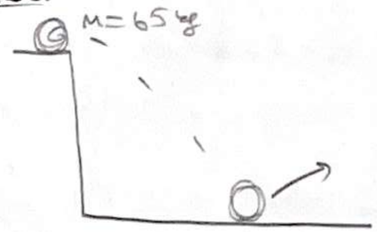
$m = 2$ kg $(I = ?\ \ \Delta t = 5)$ $(v = ?\ \ \Delta t = 5)\ \ v_0 = 0$

$$\sum \vec{F} = \frac{\Delta \vec{P}}{dt} \qquad \int_{P_i}^{P_s} dP = \int_0^t \sum F\,dt$$

$$\Delta P = \int_0^t \sum F\,dt \Rightarrow \boxed{\Delta P = 12 = I}$$

$$\Delta P = I \qquad m \cdot v_s - m v_i = 12 \qquad v_s = \frac{12}{2} \cdot 10 \qquad \boxed{v_s = 4.8 \text{ m/s}}$$

Örnek:

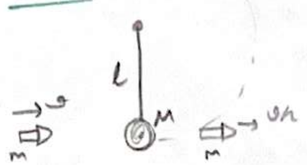


$$F = 9200t - 11500t^2$$

$$\sum \vec{F} = \frac{\Delta \vec{P}}{dt} \qquad \Delta P = \boxed{\int_0^t \sum F\,dt = I}$$

$$\int_0^{0.8} (9200t - 11500t^2)\,dt = 4600t^2 - \frac{11500}{3}t^3 \Big|_0^{0.8} = I$$

Soru:



v kaç olmalı ki M en tepe ulaşsın? (en az)

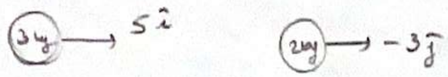
$$P_i = P_s \qquad mv = m\frac{v}{2} + M \cdot v' \qquad \boxed{\frac{mv}{2M} = v'}$$

$$\frac{1}{2} M \left(\frac{m^2 v^2}{4M^2}\right) = M \cdot g \cdot 2l \qquad \left((v')^2\right)$$

$$v = \sqrt{16 g l M^2 \cdot M^2} \qquad \boxed{v = 4M\sqrt{gl}}$$

Örnek: ① $m = 3kg$ $\vec{v}_1 = 5\hat{i}$ m/s ② $m = 2kg$ $\vec{v}_2 = -3\hat{j}$ m/s

yapışıyorlar ve yeni hızları ne olur?



$P_i = P_s \quad 3.5\hat{i} + 2.-3\hat{j} = (3+2).\vec{v}$ $\boxed{3\hat{i} - \frac{6}{5}\hat{j} = \vec{v}}$

Örnek: 30 cm uzunlukta $\lambda = (50 + 20x)$ g/m

a) Kütle kaç gram $\int dm = \int \lambda\, dx \Rightarrow M = \int_0^L (50+20x)\,dx \Rightarrow \boxed{M = 15.9\,g}$

b) KM orijinden ne kadar uzak $dm = \lambda \cdot dx$

$$X_{KM} = \frac{1}{M}\int x\, dM \qquad X_{KM} = \frac{1}{M}\int_0^L x(50+20x)\,dx$$

$$X_{KM} = \frac{10}{159}\left(25x^2 + \frac{20x^3}{3}\right)\Big|_0^L = 15.3\,cm \qquad \boxed{X_{KM} = 15.3\,cm}$$

Örnek: $m_1 = 2kg$ $\vec{v}_1 = (2\hat{i} - 3\hat{j})$ m/s , $m_2 = 3kg$ $\vec{v}_2 = (\hat{i} - 6\hat{j})$ m/s

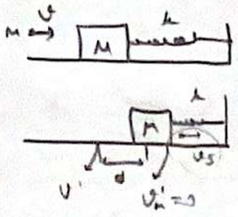
a) $\vec{v}_{KM} = ?$ b) $\Sigma\vec{p} = ?$

$$\vec{v}_{KM} = \frac{d\vec{r}_{KM}}{dt} \qquad \vec{r}_{KM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} \Rightarrow \vec{r}_{KM} = \frac{1}{M}(m_1\vec{r}_1 + m_2\vec{r}_2)$$

$$\vec{v}_{KM} = \frac{d\vec{r}_{KM}}{dt} = \frac{1}{M}\left(m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt}\right) = \frac{1}{M}\underbrace{(m_1\vec{v}_1 + m_2\vec{v}_2)}_{b)\ \Sigma\vec{p}}$$

$$\vec{v}_{KM} = \frac{1}{5}\left[2.(2i-3j) + 3(i+6j)\right] = \underbrace{\left(\frac{7}{5}\hat{i} + \frac{12}{5}\hat{j}\right)}_{a)}\ m/s$$

Örnek: $v = 400$ m/s $m = 0.005$ kg $M = 1$ kg $k = 900$ N/m $d = 0.05$ m



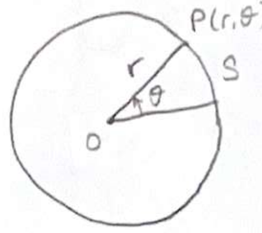
$P_i = P_s \qquad m.v = m.v_s + M.v'$

$$\frac{m(v - v_s)}{M} = v'$$

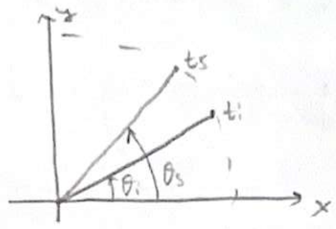
$$\frac{1}{2}M(v')^2 = \frac{1}{2}kx^2 \Rightarrow 1.(v')^2 = 900\cdot\frac{25}{10000} \qquad \boxed{v' = 1.5\ m/s}$$

$$\frac{\frac{0.001}{0.005}(400 - v_s)}{1} = \frac{0.3}{1.5} \qquad \boxed{v_s = 100\ m/s}$$

Dönme Hareketi



$S = r.\theta \qquad 2\pi.r = r.\theta \qquad \theta = 2\pi$



★ Açısal yer değiştirme $\Delta\theta = \theta_s - \theta_i$

★ Ortalama açısal hız $w_{ort} = \frac{\Delta\theta}{\Delta t}$

★ Anlık açısal hız $w = \lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta(t)}{dt}$

★ Anlık açısal ivme $\alpha = \lim_{\Delta t\to 0}\frac{\Delta w}{\Delta t} = \frac{dw}{dt}$

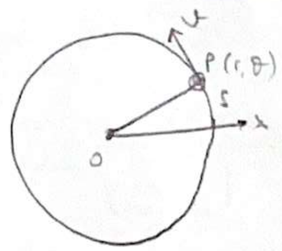
★ Ortalama açısal ivme $\alpha_{ort} = \frac{\Delta w}{\Delta t}$

Sabit açısal ivmeli

$$w_s = w_i + \alpha t$$

$$\theta_s = \theta_i + w_i t + \frac{1}{2}\alpha t^2$$

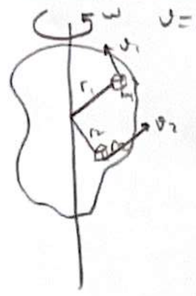
$$w_s^2 = w_i^2 + 2\alpha(\theta_s - \theta_i)$$



$S = r\theta \qquad \frac{dS}{dt} = r.\frac{d\theta}{dt} \qquad \boxed{v = r.w}$

$$\frac{dv}{dt} = r.\frac{dw}{dt} \qquad \boxed{a = r.\alpha}$$

Dönme kinetik enerjisi



$$K \approx \frac{1}{2}\Delta m_1 v_1^2 + \frac{1}{2}\Delta m_2 v_2^2 + \dots + \frac{1}{2}\Delta m_N v_N^2$$

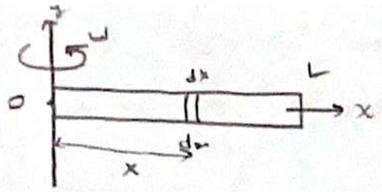
$$K \approx \frac{1}{2}\Delta m_1 r_1^2 w^2 + \frac{1}{2}\Delta m_2 r_2^2 w^2 + \dots + \frac{1}{2}\Delta m_N r_N^2 w^2$$

$$K \approx \frac{1}{2}w^2\left(\sum_{i=1}^{N}\Delta m_i r_i^2\right)$$

$$K = \frac{1}{2}\left(\lim_{\Delta m_i\to 0}\sum_{i=1}^{\infty}\Delta m_i r_i^2\right)w^2$$

$$K = \frac{1}{2}\left(\int r^2\, dm\right)w^2 \qquad \boxed{K = \frac{1}{2}.I w^2}$$

Örnek:



$dm = \lambda . dx$

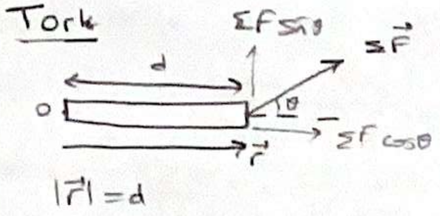
$\lambda = \frac{M}{L}$

$I = \int x^2 dm$

$I = \lambda \int_0^L x^2 dx$

$I = \lambda . \left(\frac{x^3}{3} \Big|_0^L \right)$ $\quad \boxed{I = \frac{1}{3} M L^2}$

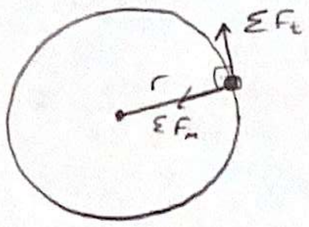
Tork



$|\vec{r}| = d$

$\boxed{\Sigma \tau = (\Sigma F \sin\theta) . d}$

$\boxed{\Sigma \vec{\tau} = \vec{r} \times \Sigma \vec{F}}$



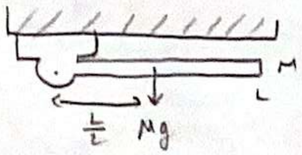
$\Sigma \tau = \Sigma F_t . r . \sin 90$

$\Sigma \tau = \Sigma F_t . r$ $\quad \Sigma F_t = M . a_t$

$\Sigma \tau = M . \alpha . r^2$ $\quad a_t = \alpha r$

$\boxed{\Sigma \tau = I . \alpha}$

Örnek:



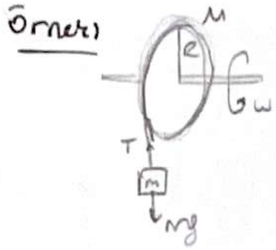
$\Sigma \tau = Mg \frac{L}{2}$ $\quad I = \frac{1}{3} M L^2$

$\Sigma \tau = Mg \frac{L}{2} = I . \alpha$

$Mg \frac{L}{2} = \frac{1}{3} . M . L^2 . \alpha$ $\quad \boxed{\alpha = \frac{3g}{2L}}$

$a = \frac{3g}{2L} . L = \boxed{\frac{3g}{2} = a}$

Örnek:



$\Sigma \tau = \frac{T}{Mg} R = I . \alpha$ $\quad \frac{TR}{I} = \alpha$ $\quad \frac{TR^2}{I} = a$

$F = m . a$

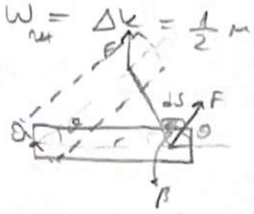
$\frac{mg - T}{m} = a = \frac{TR^2}{I}$ $\quad mgI - TI = mTR^2$

$mgI = T(mR^2 + I)$

$T = \frac{mgI}{I + mR^2}$ $\quad \boxed{a = \frac{mgR^2}{I + mR^2}}$

Dönme hareketinde enerji

$W_{NET} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$



$ds = r . d\theta$ $\quad dW = \vec{F} . d\vec{s}$

$dW = F . ds . \cos\beta$ $\quad \alpha + \beta = 90$

$dW = F . ds . \sin\theta$

$dW = F . r . d\theta . \sin\theta$

$dW = \tau . d\theta$

$\tau = I\alpha = I \frac{dv}{dt}$

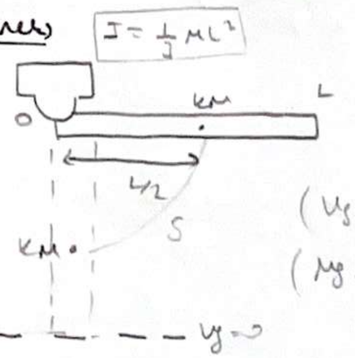
$dW = I \frac{dv}{dt} d\theta$

$\int_{W_i}^{W_f} dW = I \int_{w_i}^{w_f} w \, dw$

$W_{net} = I \left(\frac{w_f^2}{2} - \frac{w_i^2}{2} \right)$

$W_{net} = \frac{1}{2} I w_f^2 - \frac{1}{2} I w_i^2$ $\quad W_{net} = \Delta K_D$

Örnek:



a) Cubuk duşey konuma geldiğinde w = ?

b) KM ve ve uç noktası engebel hızları?

$(U_{gs} - U_{gi}) + (K_s - K_i)_{dönme} = 0$

$\left(Mg \frac{L}{2} - Mg L \right) + \left(\frac{1}{2} I w_s^2 - 0 \right) = 0$

$\frac{1}{2} . \frac{1}{3} M L^2 . w_s^2 = Mg \frac{L}{2}$ $\quad \boxed{w_s = \sqrt{\frac{3g}{L}}}$

$v_{KM} = \frac{L}{2} . \sqrt{\frac{3g}{L}}$ $\quad v_{uç} = L . \sqrt{\frac{3g}{L}}$

Örnek: $\theta = (5 + 10t + 2t^2)$ rad $t = 3$ sn için açısal konum, açısal hız ve açısal ivmeyi bulunuz.

$\theta(3) = 5 + 10 \cdot 3 + 2 \cdot (3)^2 = 59$

$\alpha(3) = 4$

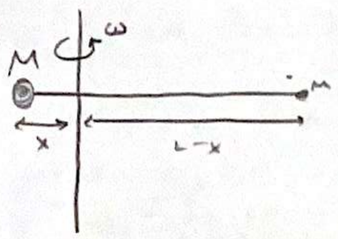
$\omega(3) = 10 + 4 \cdot 3 = 22$

Örnek) Açısal hız $(10 + 6t)$ rad/s² ile hareket ediyor. 4 saniye sonra θ açısından ne kadar açı farkı?

$$\int_0^4 (10 + 6t)\, dt = \overbrace{10t + 3t^2}^{\omega} \qquad \int_0^4 (10t + 3t^2)\, dt = \overbrace{5t^2 + t^3}^{\alpha} \Big|_0^4$$

$5 \cdot 16 + 64 = 144$

Örnek)



a) Cisim dönüş eksen eşit zamanda kendi ekseni etrafında eylemsizlik momentine sahip olduğu gibi ve $I = \mu L^2$ $\left(\mu = \frac{Mm}{M+m}\right)$ olduğu gösterin.

$I = \sum m_i r_i^2$

$I = Mx^2 + m(L-x)^2$

$\left(I = Mx^2 + mL - 2mLx + mx^2\right)' \Rightarrow \frac{I(x)}{dx} = 2Mx - 2mL + 2mx = 0$

$Mx = m(L-x)$

$Mx = mL - mx \qquad x(M+m) = \frac{m}{(M+m)} L$

$$\boxed{x = \frac{mL}{M+m}}$$

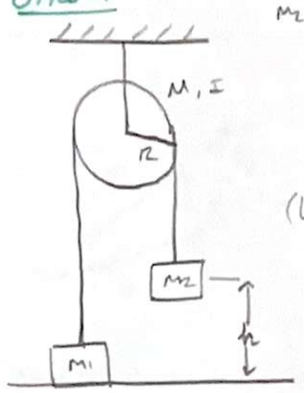
Moment

$Mg \cdot \left(\frac{m}{M+m}\right) \cdot L = mg\left(L - \left(\frac{mL}{M+m}\right)\right) \qquad \frac{Mm}{M+m} = \frac{MmL}{M+M}$ ✓

$$I = M \frac{m^2 L^2}{(M+m)^2} + m\left(L - \frac{mL}{M+m}\right)^2$$

$$= \frac{m^2 \cdot L^2 \cdot M}{(M+M)^2} + \frac{m \cdot M^2 L^2}{(M+m)^2} = \frac{MmL^2}{(M+m)^2} \cdot (M+m) \Rightarrow \underbrace{\frac{MmL^2}{(M+m)}}_{\mu}$$

Örnek)



$m_2 > m_1$ Aynı yüksekliğe geldiklerinde her kütenin momentini bulunuz.

$$\Delta U_g + \Delta K_D + \Delta K_\theta = 0$$

$$(U_{g_3} - U_{g_1})_1 + (U_{g_2} - U_{g_1})_2 + (K_{D_3} - K_{D_1}) + (K_{\theta_3} - K_{\theta_1})_1 + (K_{\theta_2} - K_{\theta_1})_2 = 0$$

$$\left(m_1 \cdot g\frac{h}{2} - m_1 \cdot gh\right) + \left(m_2 g\frac{h}{2} - 0\right) + \left(0 - \frac{1}{2} I\omega^2\right) + \left(\frac{1}{2} m_1 v^2 - 0\right) + \left(\frac{1}{2} m_2 v^2 - 0\right) = 0$$

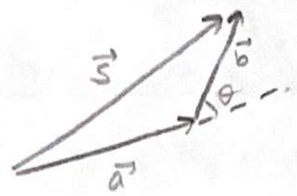
$$g\frac{h}{2}(m_2 - m_1) + \frac{v^2}{2}(m_1 + m_2) - \frac{1}{2} I \omega^2 = 0 \qquad \omega \cdot R = v \qquad \omega = \frac{v}{R}$$

$$g\frac{h}{2}(m_2 - m_1) + \frac{v^2}{2}(m_1 + m_2) - \frac{1}{2} \cdot I \cdot \frac{v^2}{R^2} = 0$$

$$\frac{v^2}{2}\left(m_1 + m_2 - \frac{I}{R^2}\right) = (m_1 - m_2) \cdot g\frac{h}{2}$$

$$\boxed{v = \sqrt{\frac{(m_1 - m_2) g h}{\left(\frac{m_1 + m_2}{2} - \frac{I}{R^2}\right)}}}$$

FİZİK-1 (FİNAL ÖZET)



$$s^2 = a^2 + b^2 + 2ab\cos\theta$$

Skaler çarpma: $\vec{a}\cdot\vec{b} = a.b\cos\phi = a_x b_x + a_y b_y + a_z . b_z$

Vektörel çarpma: $\vec{c} = \vec{a}\times\vec{b} \Rightarrow c = a.b.\sin\phi$

$\vec{c} = -(\vec{b}\times\vec{a})$

Kinematik denklemler:

$v = v_0 + a.t$

$x = x_0 + v_0.t + \frac{1}{2}at^2$ $\quad v^2 = v_0^2 + 2a(x - x_0)$

Düzgün dairesel hareket:

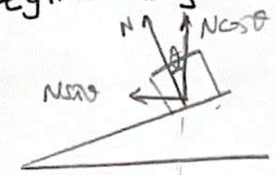
Merkezcil ivme $a = \frac{v^2}{r}$ $\quad T = \frac{2\pi r}{v}$

Asansör sorular

Yukarı ivmeli $\Rightarrow T - mg = ma \Rightarrow T = m(g+a)$

Aşağı ivmeli $\Rightarrow T - mg = -ma \Rightarrow T = m(g-a)$

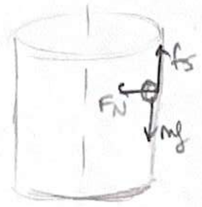
Eğimli viraj



$N\sin\theta = \frac{mv^2}{r}$ $\quad N\cos\theta - mg = 0$

$\tan\theta = \frac{v^2}{gr}$

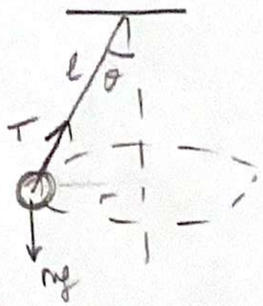
Rotor



$F_N = m.a = \frac{mv^2}{r}$ $\quad f_s = \mu_s . F_N = mg$

$v_{min} = \sqrt{\frac{rg}{\mu_s}}$

Konik sarkaç



$T\cos\theta = mg \qquad T\sin\theta = \frac{mv^2}{r} \qquad \tan\theta = \frac{v^2}{gr}$

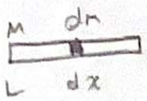
$v = \sqrt{\tan\theta\, g\, r} = \sqrt{\tan\theta\, g\, l\sin\theta}$

Güç

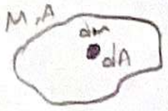
$P_{ort} = \frac{W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F}\cdot\vec{v}$ | $\vec{P} = \vec{\tau}\cdot\omega$

Kütle merkezi

$x_{km} = \frac{1}{M}\int x\, dm$ (homojen)



$\lambda = \frac{dm}{dx} = \frac{M}{L} \qquad x_{km} = \frac{1}{M}\int x\,\lambda\, dx$



$\sigma = \frac{dm}{dA} = \frac{M}{A} \qquad x_{km} = \frac{1}{M}\int x\,\sigma\, dA$

$\rho = \frac{dm}{dV} = \frac{M}{V} \qquad x_{km} = \frac{1}{M}\int x\,\rho\, dV$

Dönme hareketi

$s = \theta\cdot r \qquad a_r = \frac{v^2}{r}$ radyal ivme

$a = \sqrt{a_t^2 + a_r^2}$

Eylemsizlik momenti

$I = \int r^2\, dm \qquad I = I_{km} + Mr^2$

r keder uzakta eylemsizlik nok.

Tork

$\tau = I\alpha$

İş ve dönme kinetik enerjisi

$dW = F_t\, r\, d\theta = \tau\, d\theta$

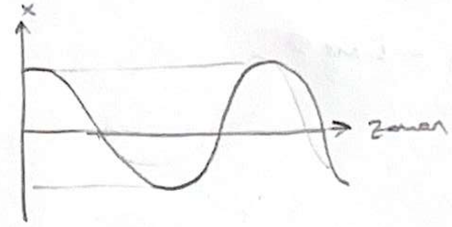
$W = \int F_t\, r\, d\theta = \int_{\theta_i}^{\theta_f} \tau\, d\theta$

Açısal momentum

$\vec{\ell} = \vec{r}\times\vec{p} \qquad \ell = r\cdot m\cdot v\cdot\sin\phi \qquad \vec{\tau}_{net} = \frac{d\vec{\ell}}{dt}$

$L = I\cdot\omega$

BHH



$x(t) = x_M\cos(\omega t + \phi)$

$\omega = 2\pi f = \frac{2\pi}{T}$

$v(t) = \frac{d(x(t))}{dt} = -\omega x_M\sin(\omega t + \phi)$

$a(t) = \frac{d(v(t))}{dt} = -\omega^2 x_M\cos(\omega t + \phi) = -\omega^2 x(t)$

$F = m\cdot a = -m\omega^2 x$

$F = -kx \qquad m\omega^2 = k \longrightarrow T = 2\pi\sqrt{\frac{m}{k}}$

$\omega = \sqrt{\frac{k}{m}}$

$$U = \frac{1}{2} k x^2 = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)$$

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 x_m^2 \sin^2(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow K = \frac{1}{2} k x_m^2 \sin^2(\omega t + \phi)$$

Burdina sarkaria

$\tau = -k\theta$ $\quad F = -Cx$ $\quad C = k$

$$\omega = \sqrt{\frac{C}{I}} = \sqrt{\frac{k}{I}} \qquad T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{I}{k}}$$

Basa sarkaria

$$\tau = r_\perp f_g = -Lmg \underbrace{\sin\theta}_{\approx\theta} \qquad \tau \cong -Lmg\theta$$

$F = -Cx$ $\quad C = Lmg$

$$\omega = \sqrt{\frac{C}{I}} = \sqrt{\frac{mgL}{I}} \qquad T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{ML^2}{Lmg}} = 2\pi\sqrt{\frac{L}{g}}$$