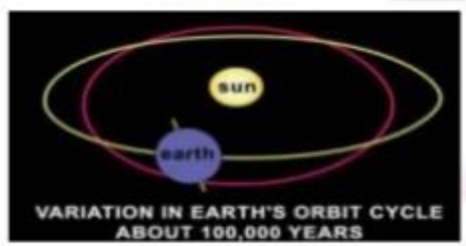
ELİPS

Application of Ellipse:

- Ellipse are contributed to the real world because of Oval shape.
- Tilt a glass of water and the surface of the liquid acquires an elliptical outline.
- The Tycho Brahe plantarium is located in Denmark.this building takes the form of an ellipse and it is clearly shown. Any cylinder sliced at an angle will reveal an ellipse.
- Footballs are elliptic.

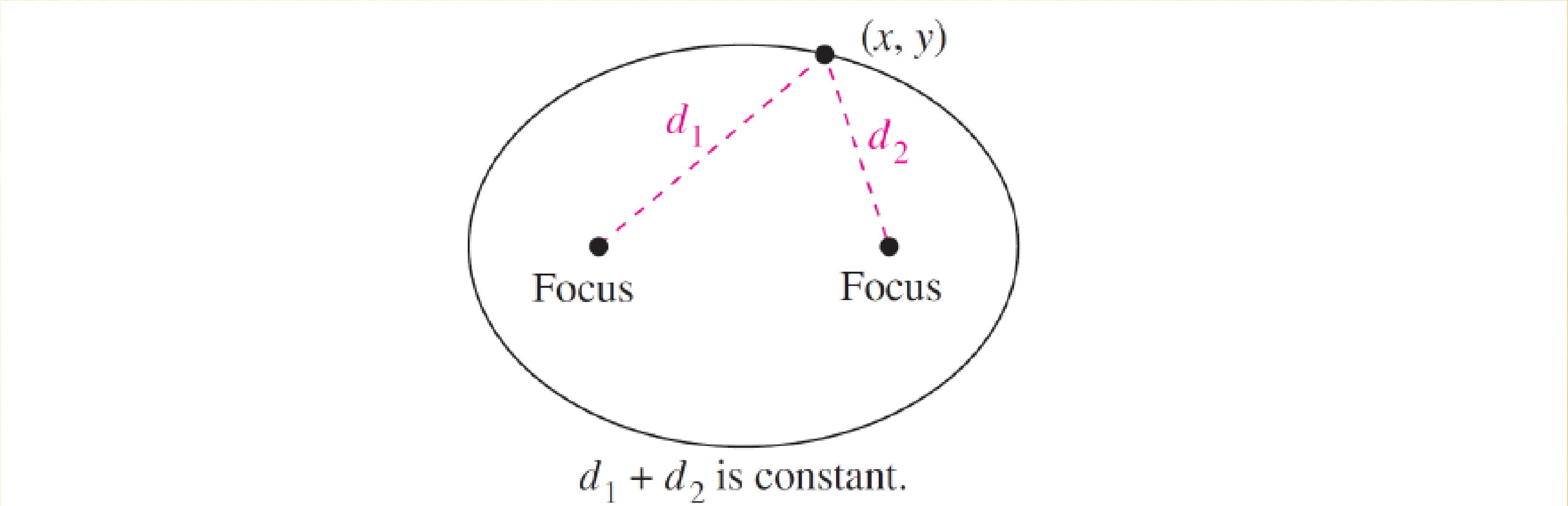
- Your eye is an ellipse! It is a horizontal ellipse, the eye ball can be considered the center and the surrounding shape forms an ellipse, the minor axis is vertical and the major axis is horizontal across the eye. The two ends of the eye can be considered as vertex.
- Bicycle chain is an example of ellipse.
- Earth orbit around the sun is an ellipse. Without the orbit we all .



- The ellipse is found in the rotation of planets in solar system. All planets orbit around the sun creates an ellipse.

ELİPSİN TANIMI

Bir düzlemde iki belirli sabit noktaya (odak noktaları) uzaklıkları toplamı sabit olan tüm (x,y) noktaları kümesine elips denir.

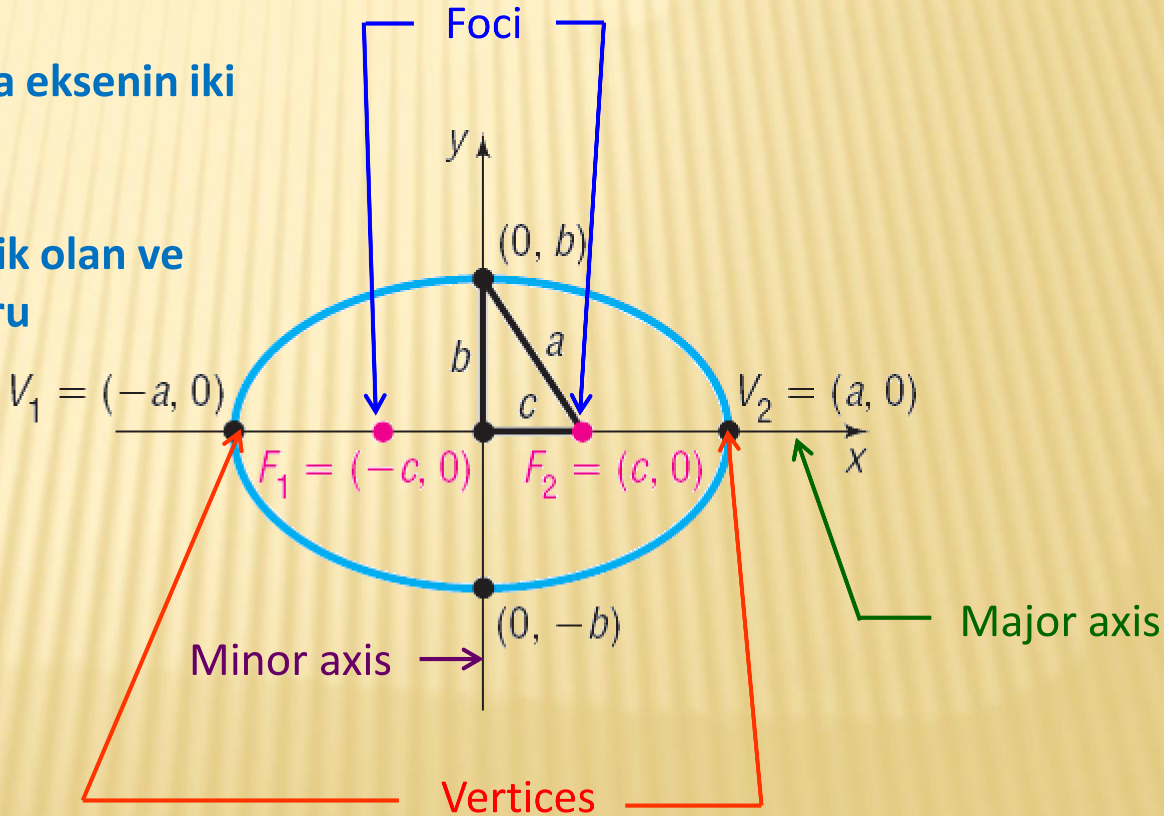


ODAK NOKTALARI: Elips üzerindeki tek bir noktaya uzaklıkları sabit olan F_1 ve F_2 iki sabit noktaları

MAJÖR EKSEN: Odakları içeren ve elipsin merkezinden geçen doğru

KÖŞE NOKTALARI: Elipsin ve ana eksenin iki kesişme noktası V_1 ve V_2

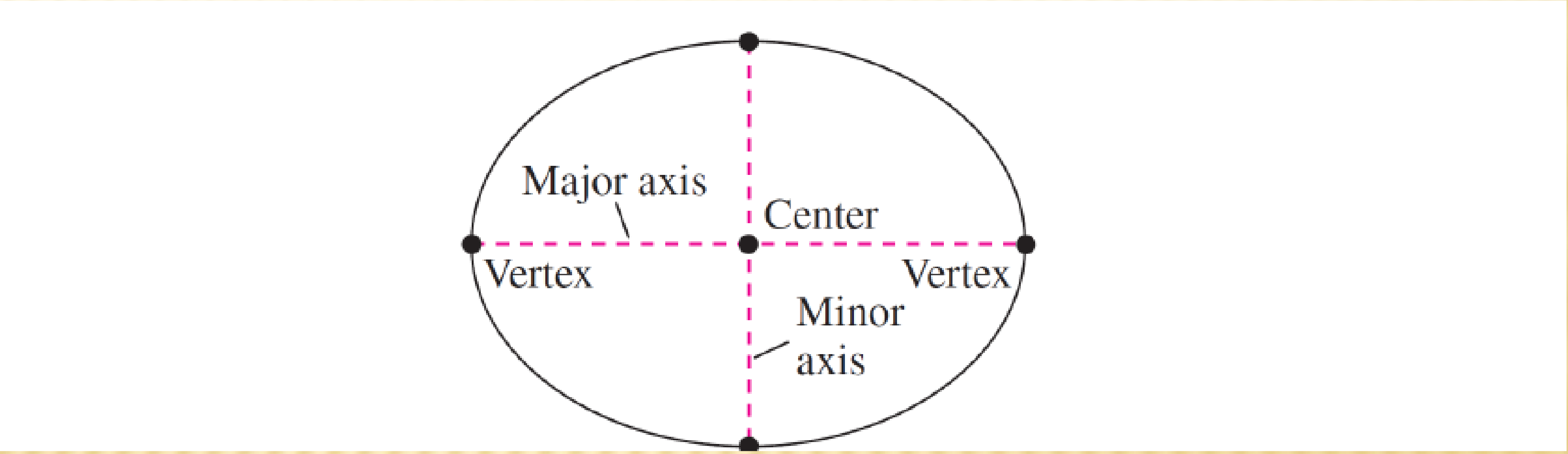
MİNÖR EKSEN: Majör eksene dik olan ve elipsin merkezinden geçen doğru



Odaklardan geçen doğru, elipsi köşe noktaları adı verilen iki noktada keser.

Köşeleri birleştiren kiriş majör eksendir ve orta noktası elipsin merkezidir.

Merkezdeki majör eksene dik olan kiriş, minör eksendir.



BİR ELİPSİN STANDART DENKLEMİ

(h,k) merkezli ve majör ve minör eksenlerinin uzunluğu 0<b<a olmak üzere 2a ve 2b olan bir elips denkleminin standart formu:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad \text{majör eksen yataydır}$$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1 \quad \text{majör eksen dikeydir}$$

Odak noktaları majör eksen üzerinde merkezden c birim uzaktadır.

$$c^2=a^2-b^2$$

Eğer merkez orijin yani (0,0) ise denklem aşağıdaki gibi olur:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{majör eksen yataydır}$$

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \quad \text{majör eksen dikeydir}$$

a>b

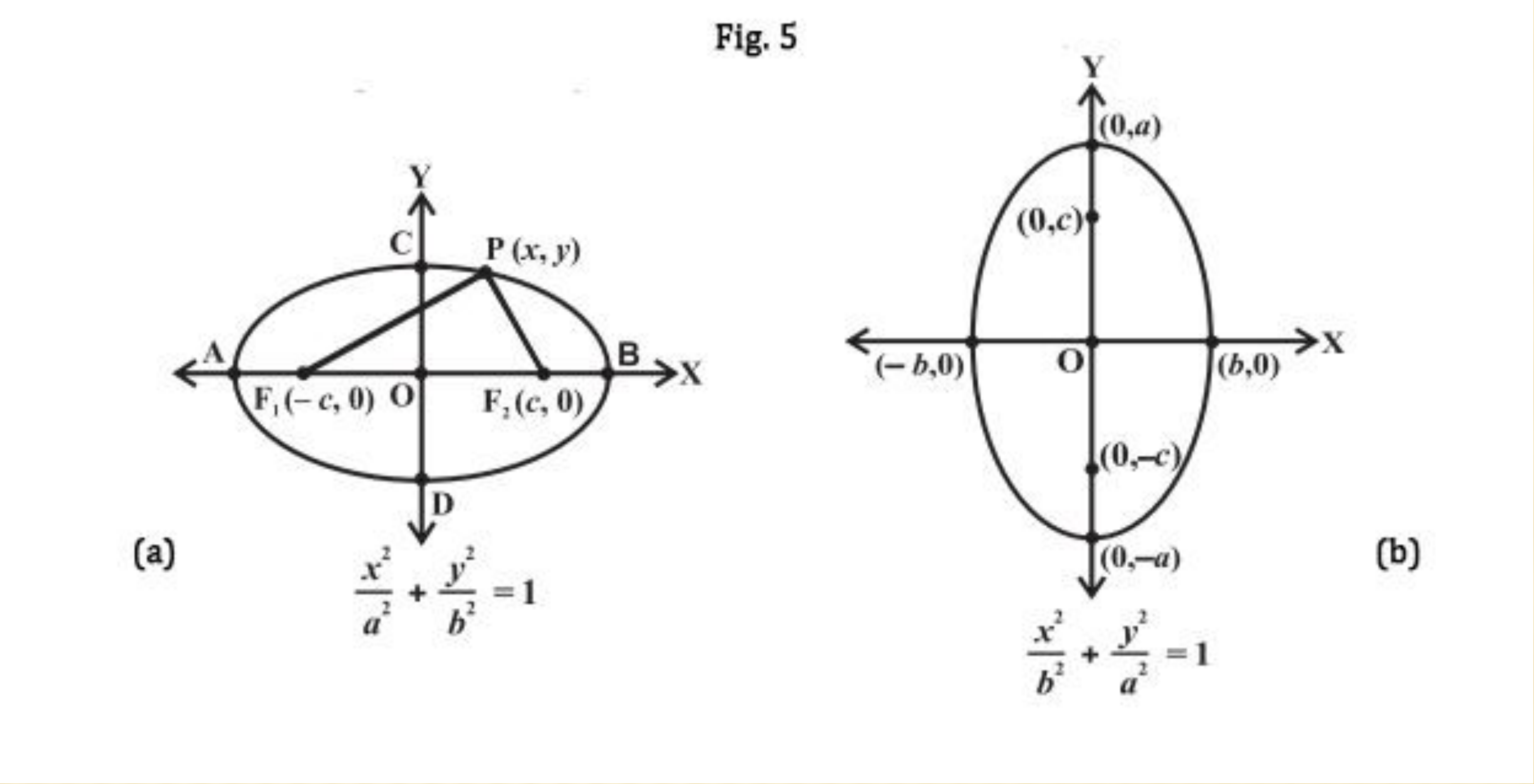
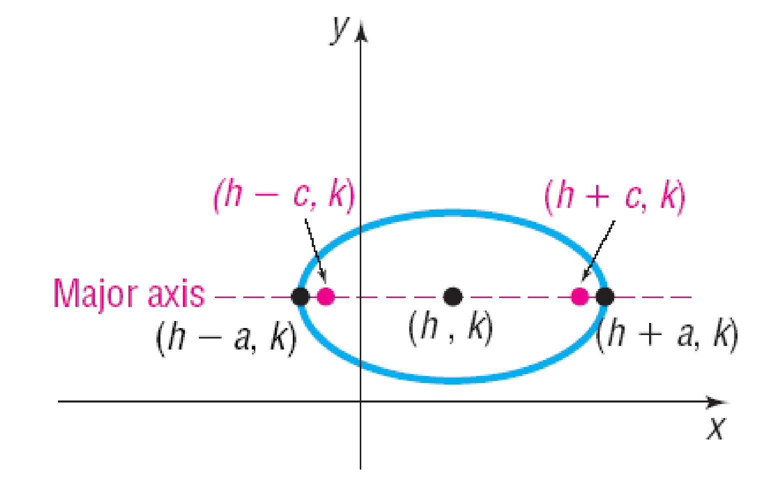


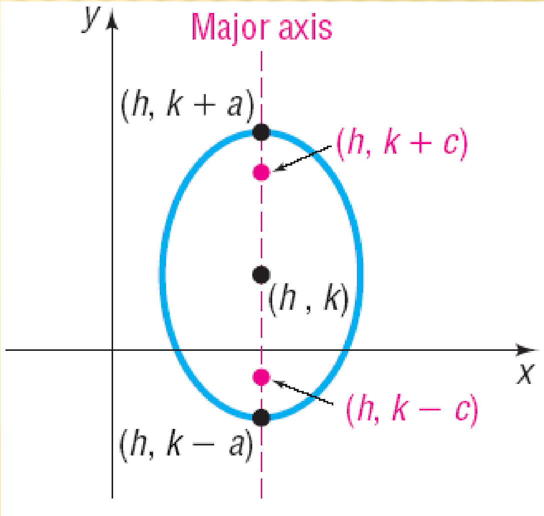
Fig. 5
Y
C
P (x, y)
A
B
X
F₁(− c, 0)
O
F₂(c, 0)
D
(a)
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Y
(0,a)
(0,c)
X
(− b,0)
O
(b,0)
(0,−c)
(0,−a)
(b)
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Equations of an Ellipse: Center at (h, k); Major Axis Parallel to a Coordinate Axis

Center	Major Axis	Foci	Vertices	Equation
(h, k)	Parallel to the x-axis	$(h + c, k)$ $(h - c, k)$	$(h + a, k)$ $(h - a, k)$	$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$, $a > b > 0$ and $b^2 = a^2 - c^2$
(h, k)	Parallel to the y-axis	$(h, k + c)$ $(h, k - c)$	$(h, k + a)$ $(h, k - a)$	$\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$, $a > b > 0$ and $b^2 = a^2 - c^2$



(a) $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$



(b) $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

The first artificial satellite to orbit Earth was Sputnik I (launched by the former Soviet Union in 1957). Its highest point above the Earth's surface was 947 km, and its lowest point was 228 km. The center of the Earth was at one focus of the elliptical orbit and the radius of the Earth is 6378 km. Find the equation of the orbit.

- Imagine one focus (the center of the Earth) is at $(0, 0)$. The major axis of the orbit is

$$\begin{aligned} 2a &= (6378 + 228) + (6378 + 947) \\ a &= 6965.5 \text{ km.} \end{aligned}$$

- If we think of the major axis as horizontal, the vertices of the orbit are $(-7325, 0)$ and $(6606, 0)$.
- The center of the elliptical orbit is the point $((6605 - 7325)/2, 0) = (-359.5, 0)$, thus $c = 359.5$.
- Therefore $b = \sqrt{6965.5^2 - 359.5^2} = 6956.2$ km.

Equation of the elliptical orbit:

$$\frac{(x + 359.5)^2}{(6965.5)^2} + \frac{y^2}{(6956.2)^2} = 1$$

Find the center, vertices, and foci given the following equation of an ellipse.

$$\frac{(x-3)^2}{25}+\frac{(y-9)^2}{9}=1$$

Center: (3,9)

Major axis is along the x-axis

Vertices: $a^2 = 25 \quad a = \pm 5$

$(3-5,9)\ and\ (3+5,9)$

$(-2,9)\ and\ (8,9)$

Vertices of the minor axis

$b^2 = 9 \quad b = \pm 3$

$(3,9-3)\ and\ (3,9+3)$

$(3,6)\ and\ (3,12)$

Foci

$c^2 = a^2 - b^2$

$c^2 = 25 - 9$

$c^2 = 16$

$c = \pm 4$

$(3-4,9)\ and\ (3+4,9)$

$(-1,9)\ and\ (7,9)$

Find the center, vertices, and foci given the following equation of an ellipse.

$$\frac{(x-3)^2}{25}+\frac{(y-9)^2}{9}=1$$

Center:

$(3,9)$

Vertices:

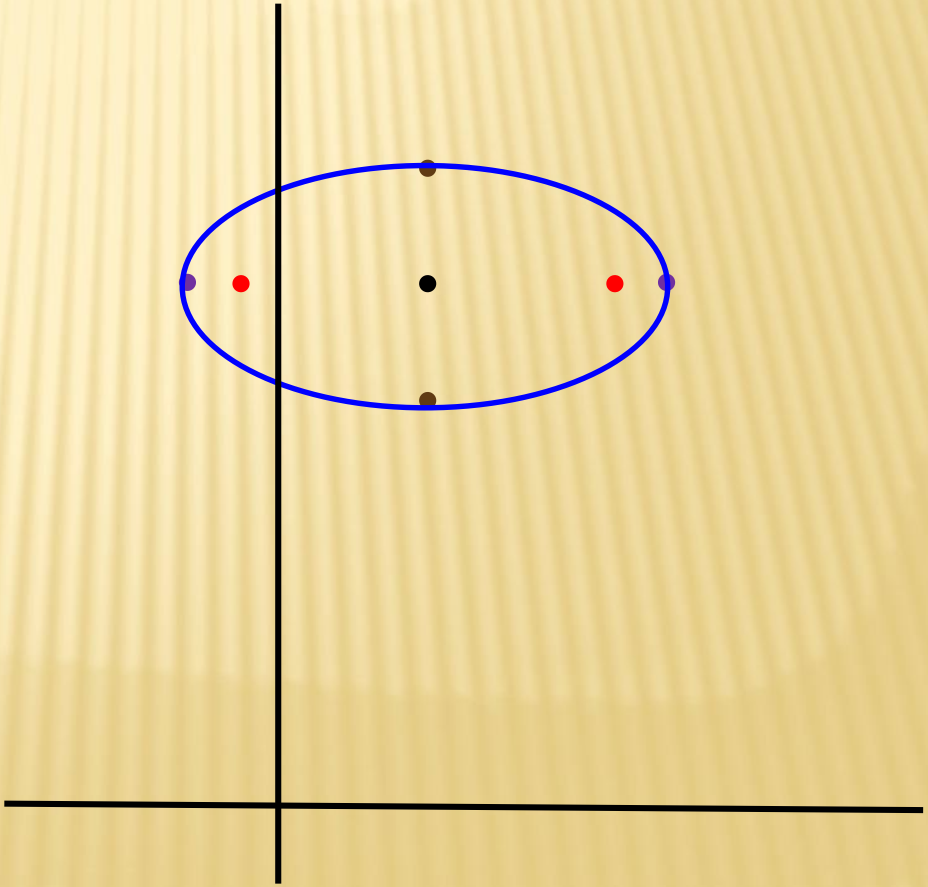
$(-2,9)\ and\ (8,9)$

Vertices of the minor axis

$(3,6)\ and\ (3,12)$

Foci

$(-1,9)\ and\ (7,9)$



Find the center, the vertices of the major and minor axes, and the foci using the following equation of an ellipse.

$$16x^2 + 4y^2 + 96x - 8y + 84 = 0$$

$$16x^2 + 96x + 4y^2 - 8y = -84$$

$$16(x^2 + 6x) + 4(y^2 - 2y) = -84$$

$$\frac{6}{2} = 3 \quad 3^2 = 9 \qquad \frac{-2}{2} = -1 \quad (-1)^2 = 1$$

$$16(x^2 + 6x + 9) + 4(y^2 - 2y + 1) = -84 + 144 + 4$$

$$16(x+3)^2 + 4(y-1)^2 = 64$$

$$\frac{16(x+3)^2}{64} + \frac{4(y-1)^2}{64} = 1$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$$

Center:

$(-3,1)$

Major axis: $x = -3\ (vertical)$

Vertices: $a^2 = 16 \qquad a = \pm 4$

$(-3,1-4)\ and\ (-3,1+4)$

$(-3,-3)\ and\ (-3,5)$

Minor axis: $y = 1\ (horizontal)$

Vertices of the minor axis

$b^2 = 4 \qquad b = \pm 2$

$(-3-2,1)\ and\ (-3+2,1)$

$(-5,1)\ and\ (-1,1)$

Foci

$c^2 = a^2 - b^2$

$c^2 = 16 - 4$

$c^2 = 12$

$c = \pm 2\sqrt{3}$

$\left(-3,1-2\sqrt{3}\right)\ and\ \left(-3,1+2\sqrt{3}\right)$

$(-3,-2.464)\ and\ (-3,4.464)$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$$

Center:

$(-3,1)$

Major axis vertices:

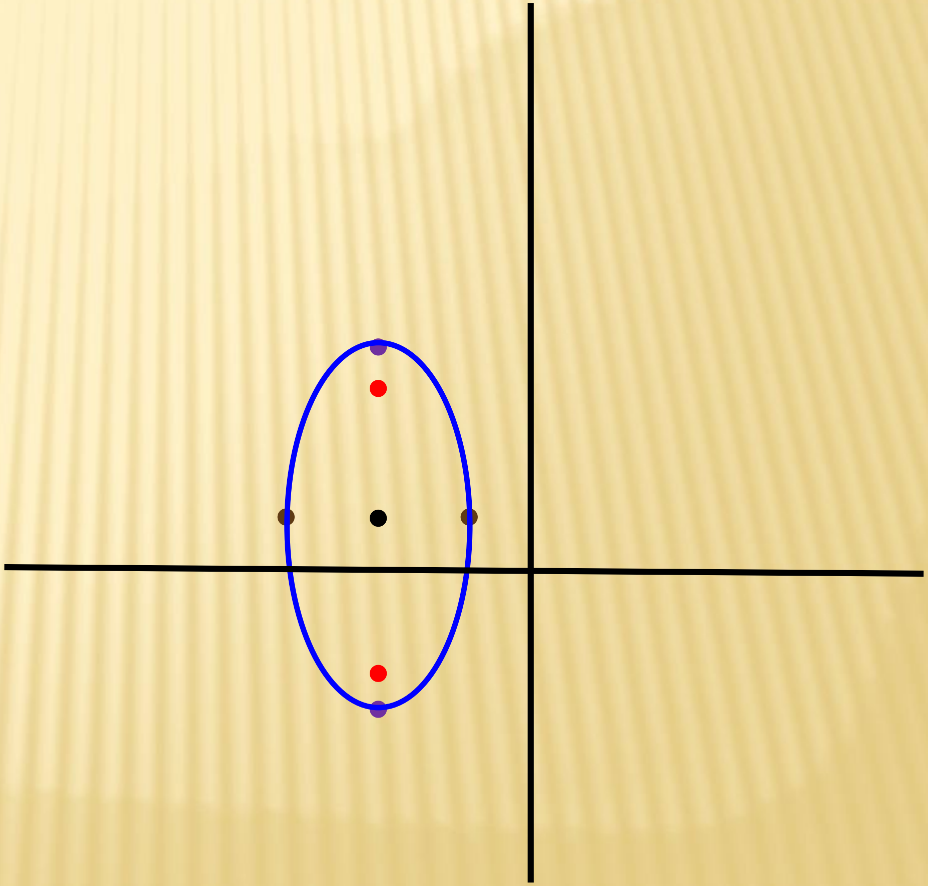
$(-3,-3)\ and\ (-3,5)$

Minor axis vertices:

$(-5,1)\ and\ (-1,1)$

Foci

$(-3,-2.464)\ and\ (-3,4.464)$

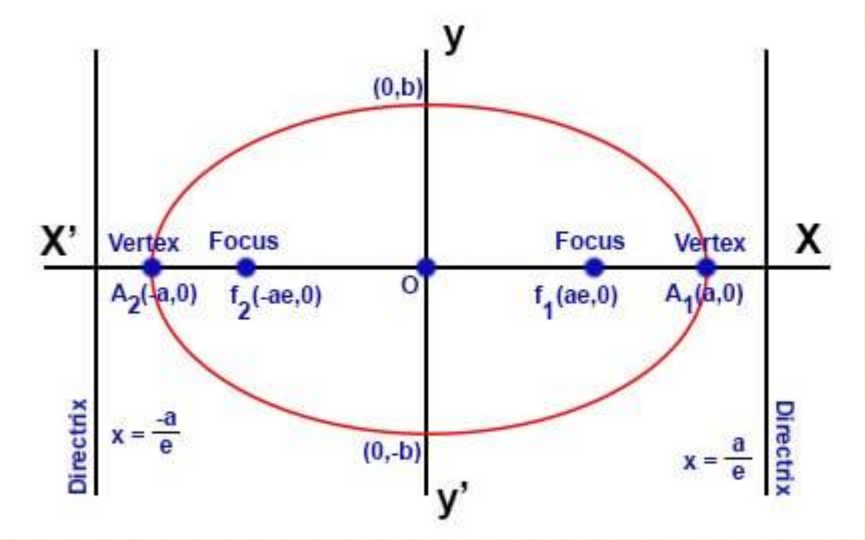


İlk astronomların gezegenlerin yörüngelerinin elips olduğunu saptamasının zor olmasının nedenlerinden biri, gezegenlerin yörüngelerinin odaklarının nispeten merkezlerine yakın olması ve dolayısıyla yörüngelerin neredeyse dairesel olmasıdır.

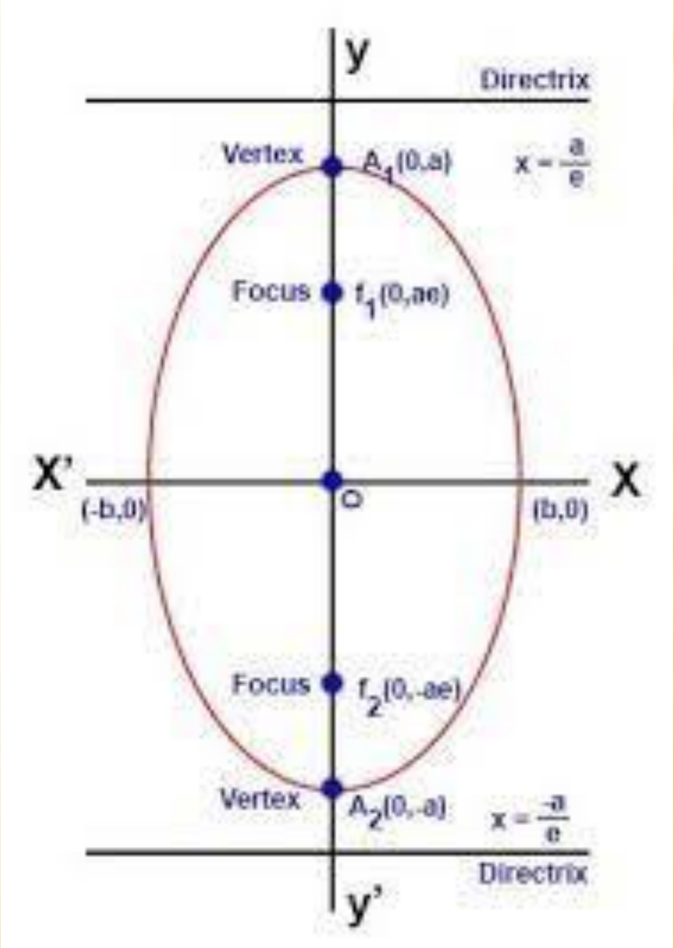
Bir elipsin ovalliğini ölçmek için eksantriklik kavramını kullanabilirsiniz.

Eksantrikliğin tanımı:
Bir elipsin e eksantrikliği $e = \frac{c}{a}$ oranıyla verilir.
Her elips için $0<e<1$ olduğuna dikkat edelim.

Eksantriklik 0'a yakın olduğu zaman elips, ovalden daha daireseldir.



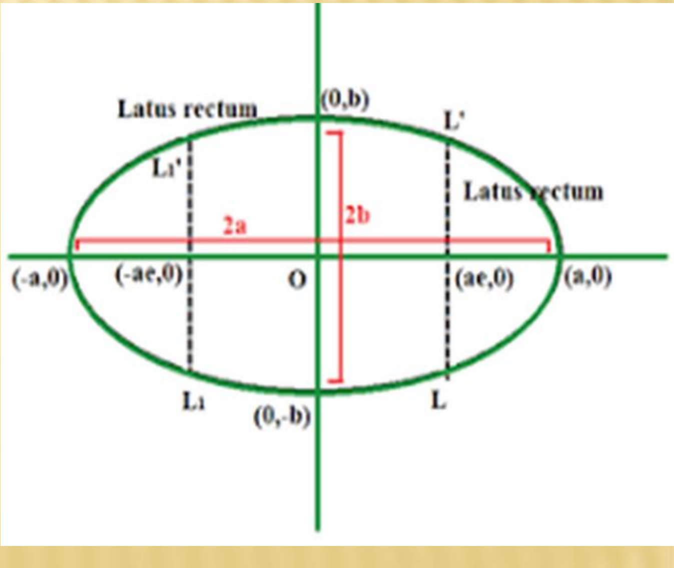
y
(0,b)
X'
Vertex
Focus
Focus
Vertex
X
$A_2(-a,0)$
$f_2(-ae,0)$
O
$f_1(ae,0)$
$A_1(a,0)$
Directrix
$x = \frac{-a}{e}$
(0,-b)
$x = \frac{a}{e}$
Directrix
y'



y
Directrix
Vertex
$A_1(0,a)$
$x = \frac{a}{e}$
Focus
$f_1(0,ae)$
X'
X
O
(-b,0)
(b,0)
Focus
$f_2(0,-ae)$
Vertex
$A_2(0,-a)$
$x = \frac{-a}{e}$
Directrix
y'

Standard equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a>b>0)$	$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1 \quad (a>b>0)$
vertices	(-a,0),(a,0),(0,b),(0,-b)	(-b,0),(b,0),(0,-a),(0,a)
foci	(-ae,0), (ae,0) (-c,0),(c,0)	(0,-ae), (0,ae) (0,-c),(0,c)
directrices	$x \pm \frac{a}{e} = 0$	$y \pm \frac{a}{e} = 0$
Symmetry axes	x-axis, y-axis	x-axis, y-axis
Length of axes	semi major axis=a (x) semi minor axis=b (y)	semi major axis=a (y) semi minor axis=b (x)

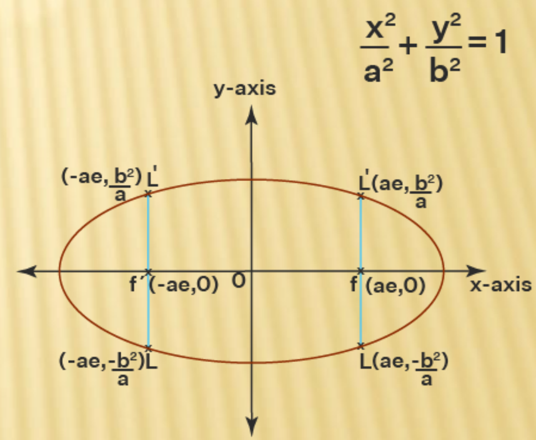
Majör eksen: S’ ve S odaklarının 2a & uzunluğunda olduğu A’A doğru parçasına elipsin majör ekseni (a>b) denir. Ana eksenin doğrultman ile kesiştiği nokta, doğrultmanın ayağı (Z) olarak adlandırılır.
Minör eksen: y ekseni, elipsi B‘(0,-b) ve B(0,b) noktalarında keser. B'B uzunluğundaki 2b (b<a) doğru parçasına elipsin minör ekseni denir.
Ana eksen: Majör ve minör eksenler birlikte elipsin ana ekseni olarak adlandırılır.
Köşe noktaları: Elipsin majör eksen ile kesişim noktaları A’(-a,0) ve A(a,0)
Odak kirişi: Odak noktasından geçen kiriş odak kirişidir.
Çift ordinat: Majör eksene dik olan kiriş çift ordinat olarak adlandırılır.
Özkiriş: Majör eksene dik olan odak kirişi özkiriş olarak adlandırılır.
Özkirişin uzunluğu: $LL' = \frac{2b^2}{a} = \frac{(Minör\ eksen)^2}{(Majör\ eksen)} = 2a(1-e^2)$



Forms of the ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
	$a > b$	$a > b$
Equation of major axis	$y = 0$	$x = 0$
Length of major axis	$2a$	$2a$
Equation of Minor axis	$x = 0$	$y = 0$
Length of Minor axis	$2b$	$2b$
Directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{a}{e}$
Equation of latus rectum	$x = \pm ae$	$y = \pm ae$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Centre	$(0, 0)$	$(0, 0)$

Latus Rectum Of Ellipse

cuemath THE MATH EXPERT



Example 1. Given the ellipse with equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

determine the principal axis, vertices, endpoints of the minor axis, lengths of the major and minor axes, foci, eccentricity and equations of directrices. Draw also a sketch of the ellipse.

SOLUTION

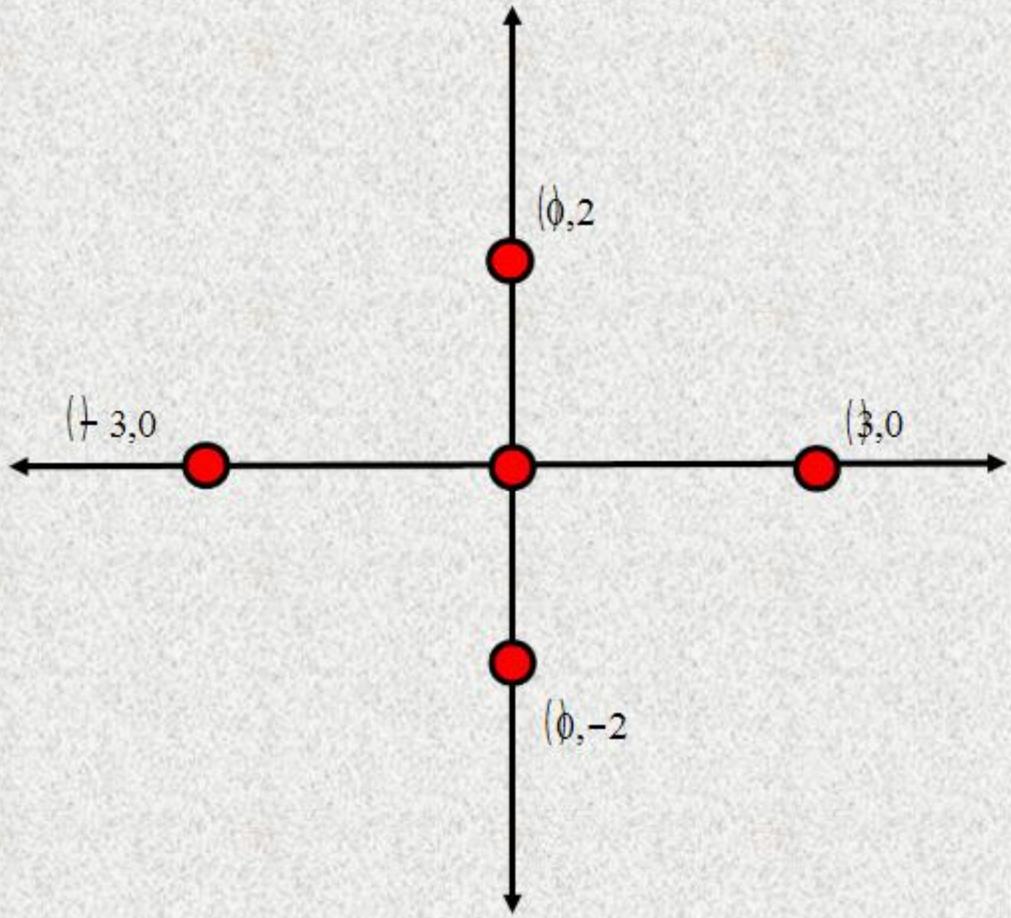
$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$a = 3 \quad b = 2$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{9 - 4} = \sqrt{5}$

- center: (0,0)
- principal axis: x-axis
- vertices:
  (3,0) and (-3,0)
- endpoints of minor axis:
  (0,2) and (0,-2)

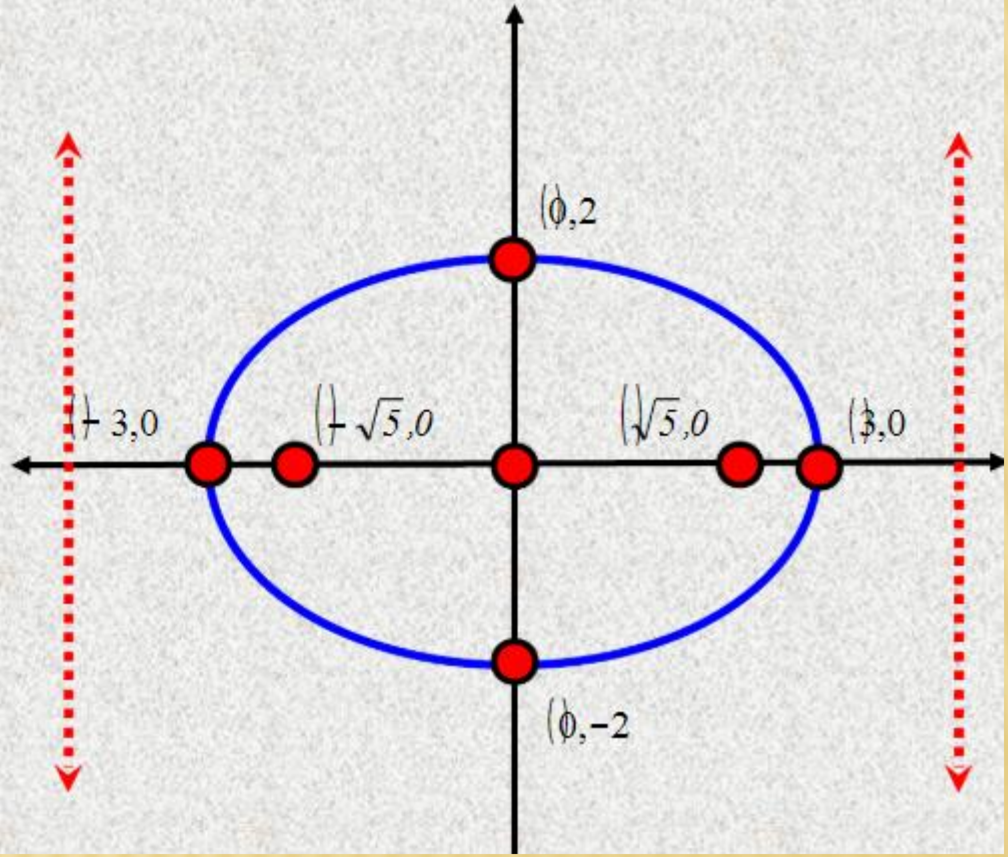


SOLUTION

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

- foci: $\left(\pm\sqrt{5}, 0\right)$
- eccentricity: $\frac{\sqrt{5}}{3}$
- equation of the directrices:

  $x = \pm \frac{9}{\sqrt{5}}$



MUSEUMS In an ellipse, sound or light coming from one focus is reflected to the other focus. In a whispering gallery, a person can hear another person whisper from across the room if the two people are standing at the foci. The whispering gallery at the Museum of Science and Industry in Chicago has an elliptical cross section that is 13 feet 6 inches by 47 feet 4 inches.

a. Write an equation to model this ellipse. Assume that the center is at the origin and the major axis is horizontal.

The length of the major axis is $47\frac{1}{3}$ or $\frac{142}{3}$ feet.

$2a = \frac{142}{3}$ Length of major axis $= \frac{142}{3}$

$a = \frac{71}{3}$ Divide each side by 2.

The length of the minor axis is $13\frac{1}{2}$ or $\frac{27}{2}$ feet.

$2b = \frac{27}{2}$ Length of minor axis $= \frac{27}{2}$

$b = \frac{27}{4}$ Divide each side by 2.

Substitute $a = \frac{71}{3}$ and $b = \frac{27}{4}$ into the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. An equation of the ellipse is $\frac{x^2}{\left(\frac{71}{3}\right)^2} + \frac{y^2}{\left(\frac{27}{4}\right)^2} = 1$.

b. How far apart are the points at which two people should stand to hear each other whisper?

People should stand at the two foci of the ellipse. The distance between the foci is $2c$ units.

$$c^2 = a^2 - b^2 \quad \text{Equation relating } a, b, \text{ and } c$$

$$c = \sqrt{a^2 - b^2} \quad \text{Take the square root of each side.}$$

$$2c = 2\sqrt{a^2 - b^2} \quad \text{Multiply each side by 2.}$$

$$2c = 2\sqrt{\left(\frac{71}{3}\right)^2 - \left(\frac{27}{4}\right)^2} \quad \text{Substitute } a = \frac{71}{3} \text{ and } b = \frac{27}{4}.$$

$$2c \approx 45.37 \quad \text{Use a calculator.}$$

The points where two people should stand to hear each other whisper are about 45.37 feet or 45 feet 4 inches apart.

Bir noktanın bir elipse göre pozisyonu

$P(x_1, y_1)$ noktası $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ elipsinin

dışında ise $\dfrac{x_1^{\,2}}{a^2} + \dfrac{y_1^{\,2}}{b^2} - 1 > 0$

üzerinde ise $\dfrac{x_1^{\,2}}{a^2} + \dfrac{y_1^{\,2}}{b^2} - 1 = 0$

içinde ise $\dfrac{x_1^{\,2}}{a^2} + \dfrac{y_1^{\,2}}{b^2} - 1 < 0$

i) $P(x_1, y_1)$ noktası $PM > QM$ ise $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ elipsinin dışındadır.

yani, $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-1>0$

ii) $P(x_1, y_1)$ noktası $PM = QM$ ise $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ elipsinin üzerindedir.

yani, $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-1=0$

iii) $P(x_1, y_1)$ noktası $PM < QM$ ise $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ elipsinin içindedir.

yani, $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-1<0$

Determine the position of the point (3, - 4) with respect to the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

Solution:

We know that the point (x_1, y_1) lies outside, on or inside the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ according as

$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 >, =$ or < 0.

For the given problem we have,

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 = \frac{3^2}{9} + \frac{(-4)^2}{16} - 1 = \frac{9}{9} + \frac{16}{16} - 1 = 1 + 1 - 1 = 1 > 0.$$

Therefore, the point (3, - 4) lies outside the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

Bir doğru ile bir elipsin kesişimi

$y = mx + c$ doğrusu $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ elipsi ile $c^2 > a^2m^2 + b^2$ koşulu sağlanıyorsa KESİŞMEZ.

$$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$

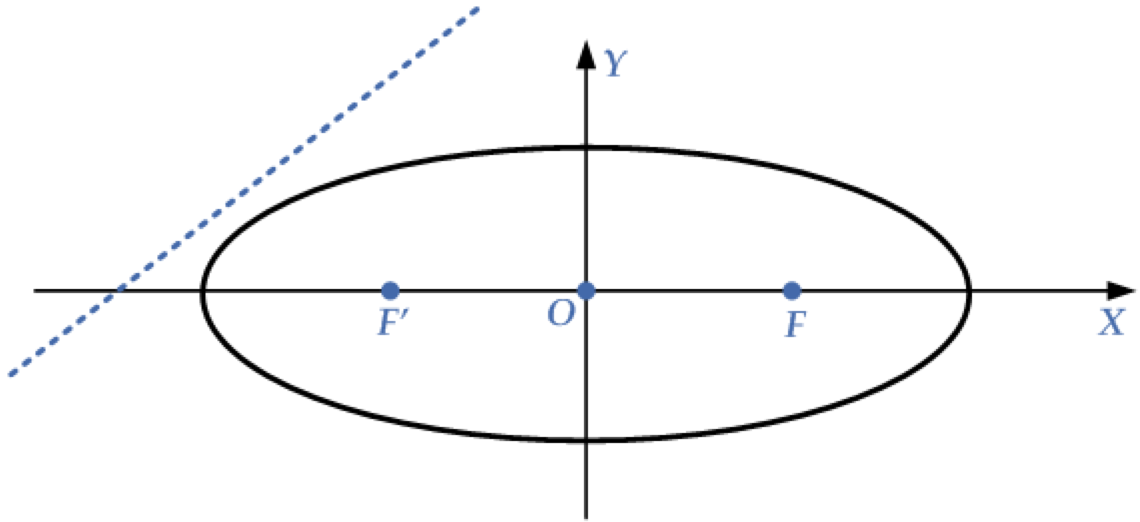
$$\Rightarrow \frac{b^2x^2 + a^2(mx+c)^2}{a^2b^2} = 1$$

$$\Rightarrow b^2x^2 + a^2(mx+c)^2 = a^2b^2$$

$$\Rightarrow b^2x^2 + a^2\left(m^2x^2 + 2mcx + c^2\right) = a^2b^2$$

$$\Rightarrow b^2x^2 + a^2m^2x^2 + 2a^2mcx + a^2c^2 - a^2b^2 = 0$$

$$\Rightarrow \left(a^2m^2 + b^2\right)x^2 + 2a^2mcx + a^2\left(c^2 - b^2\right) = 0 \quad --- \text{(iii)}$$



$$\text{Discriminant} < 0$$

$$\Rightarrow \left(2a^2mc\right)^2 - 4\left(a^2m^2 + b^2\right)a^2\left(c^2 - b^2\right) < 0$$

$$\Rightarrow 4a^4m^2c^2 - 4a^2\left(a^2m^2 + b^2\right)\left(c^2 - b^2\right) < 0$$

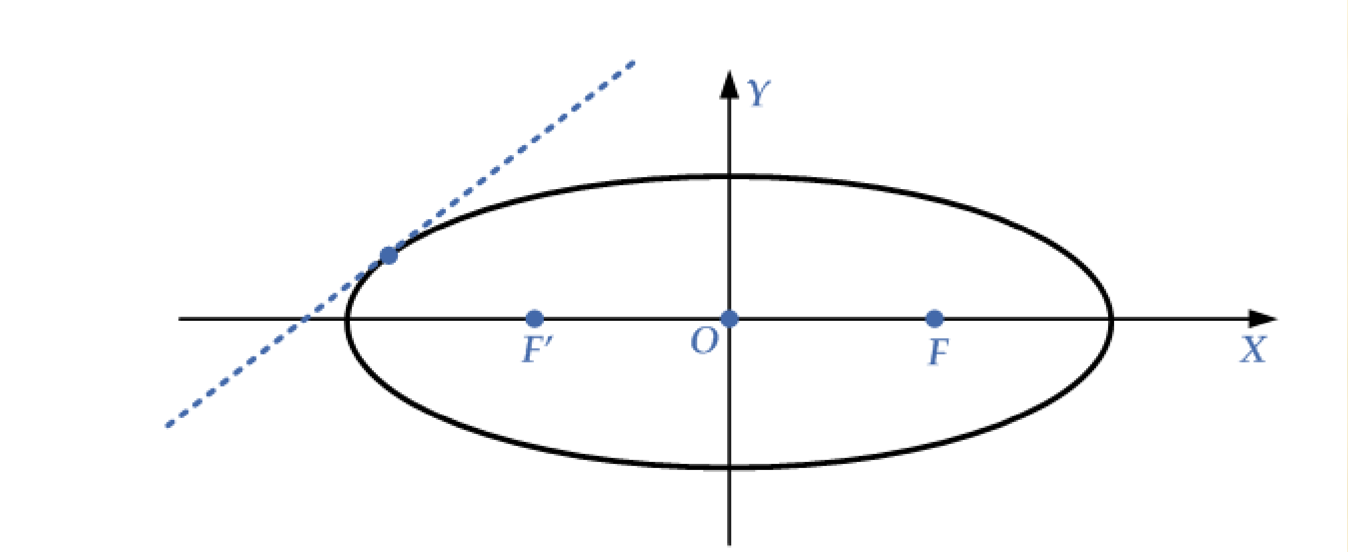
$$\Rightarrow a^2m^2c^2 - \left(a^2m^2 + b^2\right)\left(c^2 - b^2\right) < 0$$

$$\Rightarrow -b^2c^2 + a^2m^2b^2 + b^4 < 0$$

$$\Rightarrow c^2 - a^2m^2 - b^2 > 0$$

$$\Rightarrow \boxed{c^2 > a^2m^2 + b^2}$$

Bir Elipsin Teğeti



Discriminant $=0$

$$\Rightarrow \left(2a^2mc\right)^2 - 4\left(a^2m^2 + b^2\right)a^2\left(c^2 - b^2\right) = 0$$

$$\Rightarrow 4a^4m^2c^2 - 4a^2\left(a^2m^2 + b^2\right)\left(c^2 - b^2\right) = 0$$

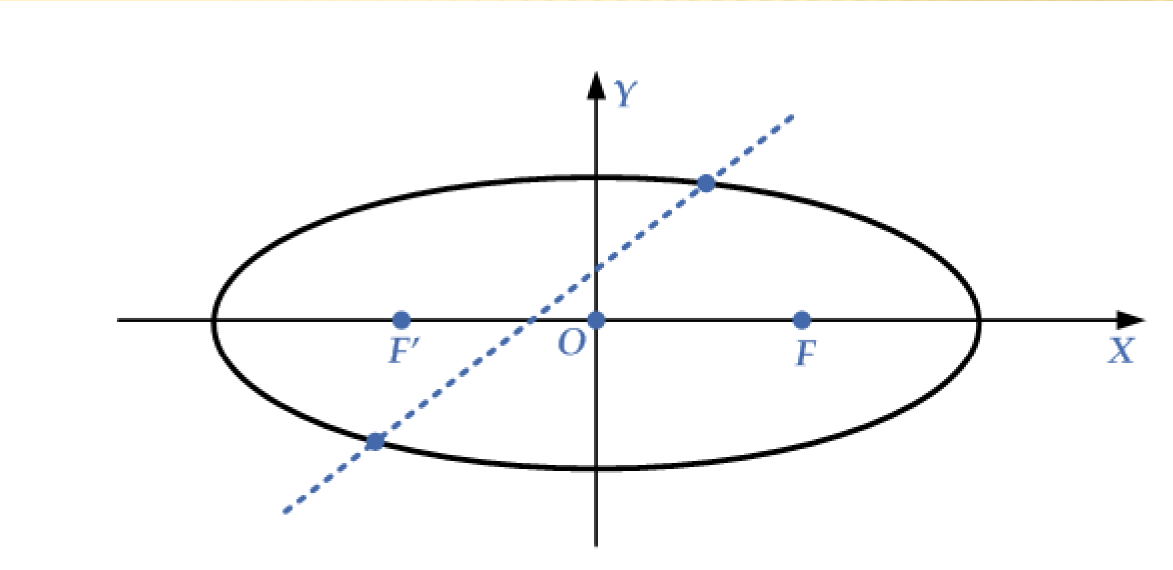
$$\Rightarrow a^2m^2c^2 - \left(a^2m^2 + b^2\right)\left(c^2 - b^2\right) = 0$$

$$\Rightarrow -b^2c^2 + a^2m^2b^2 + b^4 = 0$$

$$\Rightarrow c^2 - a^2m^2 - b^2 = 0$$

$$\Rightarrow \boxed{c^2 = a^2m^2 + b^2}$$

$y = mx + c$ doğrusu $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ elipsi ile $c^2 < a^2 m^2 + b^2$ koşulu sağlanıyorsa maksimum iki noktada kesişir.



$$\begin{gathered}
\text{Discriminant} > 0 \\
\Rightarrow \left(2a^2mc\right)^2 - 4\left(a^2m^2 + b^2\right)a^2\left(c^2 - b^2\right) > 0 \\
\Rightarrow 4a^4m^2c^2 - 4a^2\left(a^2m^2 + b^2\right)\left(c^2 - b^2\right) > 0 \\
\Rightarrow a^2m^2c^2 - \left(a^2m^2 + b^2\right)\left(c^2 - b^2\right) > 0 \\
\Rightarrow -b^2c^2 + a^2m^2b^2 + b^4 > 0 \\
\Rightarrow c^2 - a^2m^2 - b^2 < 0 \\
\Rightarrow \boxed{c^2 < a^2m^2 + b^2}
\end{gathered}$$

Bir elipsin teğet ve normali

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

elipsinin $P(x_1, y_1)$ noktasındaki teğetinin denklemi

$$\frac{x_1 x}{a^2}+\frac{y_1 y}{b^2}=1$$

ile tanımlıdır.

e.g. 3 Find the equation of the tangent at the point (2,3) to the ellipse $3x^2 + 4y^2 = 48$.

Soln:

$$\frac{3x^2}{48} + \frac{4y^2}{48} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$\therefore \ equation\ of\ tangent\ is\ \frac{2x}{16} + \frac{3y}{12} = 1$$

$$x + 2y = 8$$

e.g. 4 **Write down the equation of the tangent at the point (-2,-1) to the ellipse** $9x^2 + 4y^2 = 40$ **.**

Soln:

$$\frac{9x^2}{40} + \frac{4y^2}{40} = 1$$

$$\frac{9x^2}{40} + \frac{y^2}{10} = 1$$

Eqn of tangent at (-2,-1) is $\frac{(-2)9x}{40} + \frac{(-1)y}{10} = 1$

$$-18x - 4y = 40$$

$$9x + 2y + 20 = 0$$

Teğetin denklemi:

i) Nokta formu: $x^2/a^2 + y^2/b^2 = 1$ elipsine (x_1, y_1) noktasından çizilen teğetin denklemi

$$x_1 x/a^2 + y_1 y/b^2 = 1$$

ii) Parametrik form: Bir elipse $(a\cos\theta, b\sin\theta)$ noktasından çizilen teğetin denklemi

$$(x/a)\cos\theta + (y/b)\sin\theta = 1$$

iii) Eğim formu: $x^2/a^2 + y^2/b^2 = 1$ elipsine çizilen eğimi m olan teğetin denklemi

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

ve temas noktasının koordinatları: $\left(\pm \dfrac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp \dfrac{b^2}{\sqrt{a^2m^2+b^2}} \right)$

iv) İki Teğetin Kesişim Noktası: Bir elipse $P(a\cos\theta_1, b\sin\theta_1)$ ve $Q(a\cos\theta_2, b\sin\theta_2)$ noktalarından çizilen teğetlerin denklemleri

$$(x/a)\cos\theta_1 + (y/b)\sin\theta_1 = 1 \text{ ve } (x/a)\cos\theta_2 + (y/b)\sin\theta_2 = 1$$

ve bunlar $\left(\dfrac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}, \dfrac{b\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \right)$ noktasında kesişirler.

Normalin denklemi:

i) Nokta formu: $x^2/a^2+y^2/b^2=1$ elipsine (x_1,y_1) noktasından çizilen normalin denklemi

$$a^2x/x_1+b^2y/y_1=a^2-b^2$$

ii) Parametrik form: Bir elipse $(a\cos\theta, b\sin\theta)$ noktasından çizilen normalin denklemi

$$ax\sec\theta-by\operatorname{cosec}\theta=a^2-b^2$$

iii) Eğim formu: $x^2/a^2+y^2/b^2=1$ elipsine çizilen eğimi m olan normalin denklemi

$$y=mx-m(a^2-b^2)/\sqrt{a^2+b^2m^2}$$

ve temas noktasının koordinatları: $\left(\pm\dfrac{a^2}{\sqrt{a^2+b^2m^2}},\pm\dfrac{b^2m}{\sqrt{a^2+b^2m^2}}\right)$

iv) İki Normalin Kesişim Noktası: Bir elipse $(a\cos\theta_1, b\sin\theta_1)$ ve $(a\cos\theta_2, b\sin\theta_2)$ noktalarından çizilen normallerin kesişim noktası

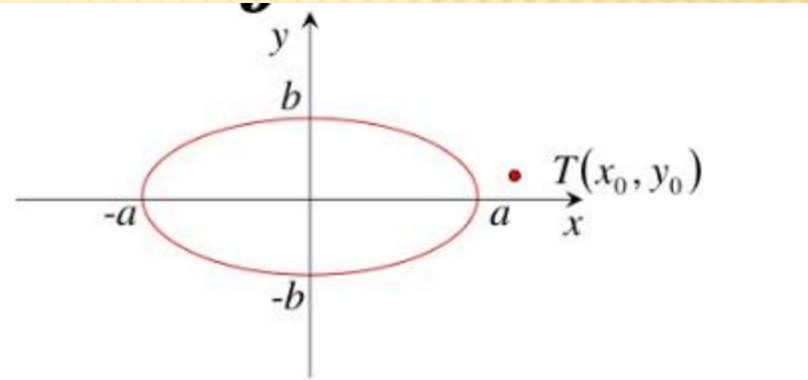
$$\left(\frac{a^2-b^2}{a}\cos\theta_1\cos\theta_2\frac{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}, \frac{-(a^2-b^2)}{b}\sin\theta_1\sin\theta_2\frac{\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}\right)$$

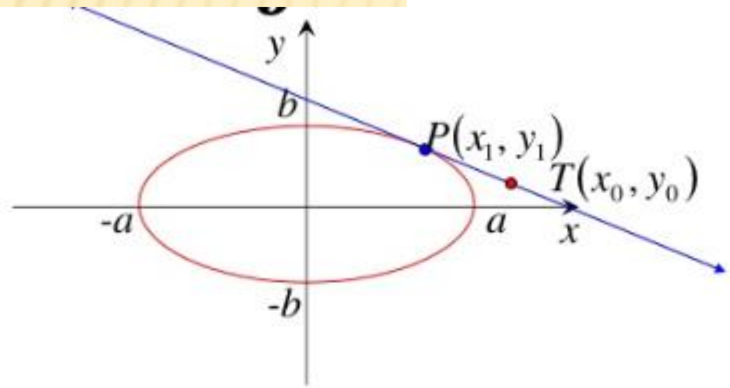
v) $y=mx+c$ doğrusu $x^2/a^2+y^2/b^2=1$ elipsinin bir normali ise

$$c^2=m^2(a^2-b^2)^2/a^2+b^2m^2$$

Temas Kirişi

Bir T dış noktasından iki teğet çizilebilir.



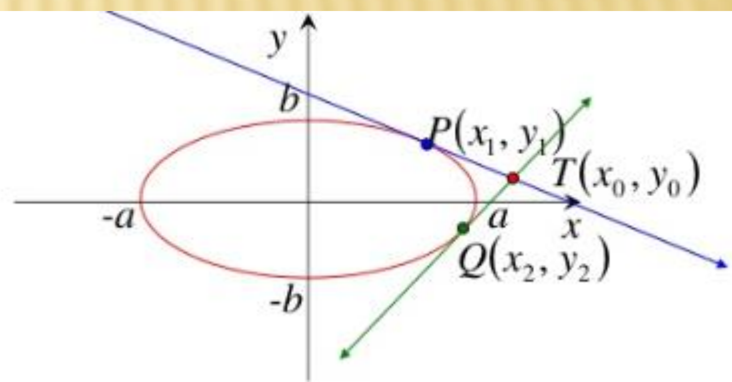


P noktasındaki teğetin denklemi;

$$\frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1$$

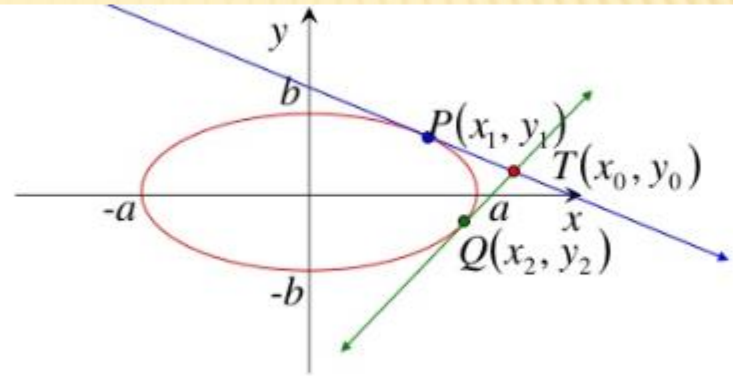
Q noktasındaki teğetin denklemi;

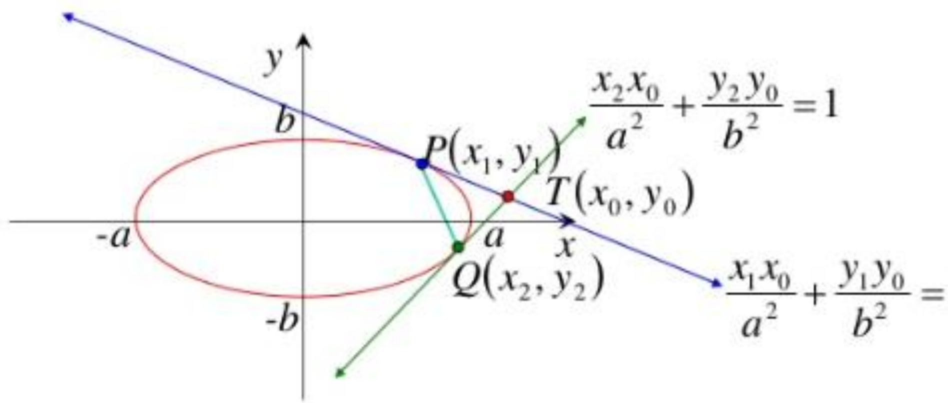
$$\frac{x_2 x}{a^2} + \frac{y_2 y}{b^2} = 1$$



T bu iki doğrunun da üstündedir:

$$\frac{x_1 x_0}{a^2}+\frac{y_1 y_0}{b^2}=1 \text{ ve } \frac{x_2 x_0}{a^2}+\frac{y_2 y_0}{b^2}=1$$





Thus P and Q both must lie on a line with equation;

$$\boxed{\frac{x_0 x}{a^2}+\frac{y_0 y}{b^2}=1}$$

which must be the line PQ i.e. chord of contact

P ve Q aşağıdaki doğru üzerindedir:

$$\frac{x_0 x}{a^2}+\frac{y_0 y}{b^2}=1$$

TEMAS KİRİŞİ

Bir elipsin parametrik denklemleri

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

elipsi aşağıdaki değerleri her zaman sağlar:

$$x = a\cos\theta$$

Majör eksen

$$y = b\sin\theta$$

Minör eksen

θ bir parametredir.

Eğri üzerindeki herhangi bir noktanın *parametrik koordinatları:*

$$(a\cos\theta, b\sin\theta)$$

e.g. 6

Find the parametric coordinates of any point on each of the following ellipses:

$$(i) \quad 4x^2 + 9y^2 = 16$$

$$(ii) \quad (x-2)^2 + 4y^2 = 4$$

Soln: $(i)\ \ 4x^2 + 9y^2 = 16$

$$\frac{4x^2}{16} + \frac{9y^2}{16} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{\frac{16}{9}} = 1$$

a=2 ; b=4/3

$$\therefore \quad x = 2cos\theta\,, \qquad y = \frac{4}{3}sin\theta$$

$$(ii) \quad (x-2)^2 + 4y^2 = 4$$

$$\frac{(x-2)^2}{4} + \frac{4y^2}{4} = 1$$

$$\frac{(x-2)^2}{4} + y^2 = 1$$

$$\therefore \quad x - 2 = 2cos\theta \,, \qquad y = sin\theta$$

$$\therefore \quad x = 2 + 2cos\theta \,, \qquad y = sin\theta$$

ÇAP

Bir elipsin paralel kirişleri sisteminin orta noktasının yeri, denklemi

$$y = -(b^2 / a^2 m)x$$

olan çap olarak adlandırılır.

Her biri kirişleri birbirine paralel olarak ikiye bölecek şekilde bir elipsin iki çapı, eşlenik çaplar olarak adlandırılır.

$$y = m_1 x \text{ ve } y = m_2 x,\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ elipsinin iki çapı}$$

$$\text{eğer } m_1 m_2 = -\frac{b^2}{a^2} \text{ ise eşleniktir.}$$

2. Determine the equation of the ellipse whose directrices along y = ± 9 and foci at (0, ± 4). Also find the length of its latus rectum.

Solution:

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ... (i)

The co-ordinate of the foci are (0, ± 4). This means that the major axes of the ellipse is along y axes and the minor axes of the ellipse is along x axes.

We know that the co-ordinates of the foci are (0, ± be) and the equations of directrices are $y = \pm \frac{b}{e}$

Therefore, $\frac{b}{e} = 9$ .... (ii)

and be = 4 .... (iii)

Now, from (ii) and (iii) we get,

$b^2 = 36$

$\Rightarrow b = 6$

Now, $a^2 = b^2(1 - e^2)$

$\Rightarrow a^2 = b^2 - b^2e^2$

$\Rightarrow a^2 = b^2 - (be)^2$

$\Rightarrow a^2 = 6^2 - 4^2$, [Putting the value of be = 4]

$\Rightarrow a^2 = 36 - 16$

$\Rightarrow a^2 = 20$

Therefore, the required equation of the ellipse is $\frac{x^2}{20} + \frac{y^2}{36} = 1$.

The required length of latus rectum $= 2 \cdot \frac{a^2}{b} = 2 \cdot \frac{20}{6} = \frac{20}{3}$ units.

Find the length of the latus rectum and equation of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 16y + 13 = 0$.

Solution:

The given equation of the ellipse $x^2 + 4y^2 + 2x + 16y + 13 = 0$

Now form the above equation we get,

$(x^2 + 2x + 1) + 4(y^2 + 4y + 4) = 4$

$\Rightarrow (x + 1)^2 + 4(y + 2)^2 = 4.$

Now dividing both sides by 4

$\Rightarrow \frac{(x+1)^2}{4} + (y + 2)^2 = 1.$

$\Rightarrow \frac{(x+1)^2}{2^2} + \frac{(y+2)^2}{1^2}$ .... (i)

Shifting the origin at (-1, -2) without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$x = X - 1$ and $y = Y - 2$ .... (ii)

Using these relations, equation (i) reduces to $\frac{X^2}{2^2} + \frac{Y^2}{1^2} = 1$ .... (iii)

This is of the form $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$, where $a = 2$ and $b = 1$.

Thus, the given equation represents an ellipse.

Clearly, a > b. So, the given equation represents an ellipse whose major and minor axes are along X and Y axes respectively.

Now fine the eccentricity of the ellipse:

We know that e = $\sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{1^2}{2^2}} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$.

Therefore, the length of the latus rectum = $\frac{2b^2}{a} = \frac{2\bullet(1)^2}{2} = \frac{2}{2}$ = 1.

The equations of the latus recta with respect to the new axes are X= ±ae

X = ± 2 · $\frac{\sqrt{3}}{2}$

⇒ X = ± √3

Hence, the equations of the latus recta with respect to the old axes are

x = ±√3 – 1, [Putting X = ± √3 in (ii)]

i.e., x = √3 - 1 and x = -√3 – 1.

Find the focal distance of a point on the ellipse $25x^2 + 9y^2 -150x - 90y + 225 = 0$

Solution:

The given equation of the ellipse is $25x^2 + 9y^2 - 150x - 90y + 225 = 0$.

From the above equation we get,

$25x^2 - 150x + 9y^2 - 90y = - 225$

$\Rightarrow 25(x^2 - 6x) + 9(y^2 - 10y) = -225$

$\Rightarrow 25(x^2 - 6x + 9) + 9(y^2 - 10y + 25) = 225$

$\Rightarrow 25(x - 3)^2 + 9(y - 5)^2 = 225$

$\Rightarrow \frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$ .... (i)

Now transfering the origin at (3, 5) without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by x and y, we have

$x = X + 3$ and $y = Y + 5$ .... (ii)

Using these relations, equation (i) reduces to

$\frac{X^2}{3^2} + \frac{Y^2}{5^2} = 1$ .... (iii)

This is the form of $\frac{X^2}{b^2} + \frac{Y^2}{a^2} = 1$ ($a^2 < b^2$) where $a = 5$ and $b = 3$

Now, we get that $a > b$.

Hence, the equation $\frac{X^2}{3^2} + \frac{Y^2}{5^2} = 1$ represents an ellipse whose major axes along X and minor axes along Y axes.

Therefore, the focal distance of a point on the ellipse $25x^2 + 9y^2 - 150x - 90y + 225 = 0$ is major axis $= 2a = 2 \cdot 5 = 10$ units.