	SUGGESTED QUESTIONS (B. Ali İBRAHİMOĞLU) MTM1502 CALCULUS 2 MIDTERM EXAM (25/04/2023)		

QUESTIONS AND SOLUTIONS

Q1)

Assume a certain substitution gives

$$\int e^{\sec x+\ln(\sec x)+\ln(\tan x)}\,dx=\int e^{u}\,du.$$

Which of the following is equal to u?

A) $u=\ln(\tan x)$
B) $u=\ln(\sec x)$
C) $u=\tan x$
D) $u=\sec x$
E) $u=e^{\sec x}$

Solution of Q1) [Subs. Met.] Answer: D) $u=\sec x$

$$\int e^{\sec x+\ln(\sec x)+\ln(\tan x)}\,dx=\int e^{\sec x}e^{\ln(\sec x.\tan x)}\,dx$$
$$=\int e^{\sec x}(\sec x.\tan x)\,dx,$$
$$(u=\sec x\Rightarrow du=\sec x\tan x\,dx)$$
$$=\int e^{u}\,du$$

Q2)

A particle moves along a line so that its velocity at time t is $v(t)=2\cos(2t)$ (measured in meters per second). Find the total distance traveled during the time period $0\le t\le \pi/2$.

A) 0 **B)** $\frac{1}{2}$ **C)** 1 **D)** $\frac{3}{2}$ **E)** 2

Solution of Q2) [Subs. Met.] Answer: E) 2

The total distance traveled: $\int_0^{\pi/2}|v(t)|\,dt$

$$\int_0^{\pi/2}|v(t)|\,dt=\int_0^{\pi/2}|2\cos(2t)|dt$$
$$=\int_0^{\pi/4}2\cos(2t)dt-\int_{\pi/4}^{\pi/2}2\cos(2t)dt$$
$$=2$$
$$\left(\int 2\cos(2t)\,dt=\int\cos(u)\,du=\sin(u)+C\right)$$

Q3)

Which of the following definite integrals are equal to $\int_0^1 \frac{x^3}{\sqrt{x^2+1}}dx$?

I. $x^2\sqrt{x^2+1}\Big]_{x=0}^{x=1} - \int_0^1 2x\sqrt{x^2+1}\,dx$

II. $\int_1^{\sqrt{2}} (u^2-1)\,du$

III. $\int_1^{\sqrt{2}} (u^2+1)\,du$

A) None **B)** I only **C)** I and II only

D) II only **E)** I and III only

Solution of Q3) [Subs. Met.& int. by parts]

Answer:C) I and II only

$\int_0^1 \frac{x^3}{\sqrt{x^2+1}}dx$?

Using integration by parts:

$u = x^2 \Rightarrow du = 2xdx$

$dv = \frac{x}{\sqrt{x^2+1}}dx \Rightarrow v = \sqrt{x^2+1}$

$$\int_0^1 \frac{x^3}{\sqrt{x^2+1}}dx = x^2\sqrt{x^2+1}\Big]_{x=0}^{x=1} - \int_0^1 2x\sqrt{x^2+1}\,dx$$

Using the substitution: $u = \sqrt{x^2+1}$

$du = \frac{x}{\sqrt{x^2+1}}dx$

$$\int_0^1 \frac{x^3}{\sqrt{x^2+1}}dx = \int_1^{\sqrt{2}} (u^2-1)\,du$$

Q4)

Which of the following is an appropriate choice of u and dv for integration by parts of $\int x\,\mathrm{arccot}(x^2)\,dx$?

A) $u = x\,\mathrm{arccot}(x^2),\ dv = dx$

B) $u = \mathrm{arccot}(x^2),\ dv = x\,dx$

C) $u = x,\ dv = \mathrm{arccot}(x^2)dx$

D) $u = 1,\ dv = x\,\mathrm{arccot}(x^2)dx$

E) $u = x^2,\ dv = \mathrm{arccot}(x^2)dx$

Solution of Q4)[int. by parts]

Answer: B) $u = \mathrm{arccot}(x^2),\ dv = x\,dx$

Using integration by parts for $\int \underbrace{x}_{\text{pol}}\ \underbrace{\mathrm{arccot}(x^2)}_{\text{Arc}}\,dx$:

Apply LAPTE rule

$u = \mathrm{arccot}(x^2) \Rightarrow du = -\frac{2x}{x^4+1}$

$dv = x\,dx \Rightarrow v = \frac{x^2}{2}$

$$\int x\,\mathrm{arccot}(x^2)\,dx$$

$$= \frac{x^2}{2}\mathrm{arccot}(x^2) + \frac{1}{4}\ln(x^4+1) + C$$

Q5)

Assume $f(x)$ is a differentiable function. Which of the following expressions is equal to

$\int x^2 f'(x)\,dx$?

A) $x^2 f(x) - \frac{1}{2}\int x f(x)\,dx$

B) $x^2 f(x) - 4\int x f(x)\,dx$

C) $x^2 f(x) - 2\int x f(x)\,dx$

D) $2x^2 f(x) - 2\int x f(x)\,dx$

E) $2x^2 f(x) - \frac{1}{2}\int x f(x)\,dx$

Solution of Q5) [int. by parts]

Answer: C) $x^2 f(x) - 2\int x f(x)\,dx$

Using integration by parts for $\int x^2 f'(x)\,dx$:

$$u = x^2 \Rightarrow du = 2x dx$$
$$dv = f'(x)dx \Rightarrow v = f(x)$$
$$uv - \int v\,du = x^2 f(x) - \int f(x) 2x\,dx$$

Q6)

Which of the following is a reduction formula to evaluate $I_n = \int_0^{\pi/2} x^n \sin x = ?$ (n is an integer)

A) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}, (n \geq 2)$

B) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} + n(n-1)I_{n-2}, (n \geq 2)$

C) $I_n = n\left(-\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}, (n \geq 2)$

D) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - nI_{n-2}, (n \geq 2)$

E) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} + nI_{n-2}, (n \geq 2)$

Solution of Q6) [int. by parts]

Answer: A) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2},\ (n \geq 2)$

$$I_n = \int_0^{\pi/2} x^n \sin x\,dx = uv - \int v\,du$$

$$\begin{pmatrix} u = x^n, & dv = \sin x \\ du = nx^{n-1}dx, & v = -\cos x \end{pmatrix}$$

$$= x^n(-\cos x)\Big|_0^{\pi/2} - \int_0^{\pi/2} (-\cos x) n x^{n-1}\,dx$$

$$= n\int_0^{\pi/2} x^{n-1}\cos x\,dx$$

$$= n\left(uv - \int v\,du\right) \to u = x^{n-1},\ dv = \cos x$$

$$= n\left(x^{n-1}(\sin x)\Big|_0^{\pi/2} - \int_0^{\pi/2} (\sin x)(n-1)x^{n-2}\,dx \right)$$

$$= n\left(\frac{\pi}{2}\right)^{n-1} - \underbrace{n(n-1)\int_0^{\pi/2} x^{n-2}(\sin x)\,dx}_{n(n-1)I_{n-2}}$$

$$\boxed{I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2},\ (n \geq 2).}$$

Q5)

Assume $f(x)$ is a differentiable function. Which of the following expressions is equal to

$\int x^2 f'(x)\,dx$?

A) $x^2 f(x) - \frac{1}{2}\int x\,f(x)\,dx$

B) $x^2 f(x) - 4\int x\,f(x)\,dx$

C) $x^2 f(x) - 2\int x\,f(x)\,dx$

D) $2x^2 f(x) - 2\int x\,f(x)\,dx$

E) $2x^2 f(x) - \frac{1}{2}\int x\,f(x)\,dx$

Solution of Q5) [int. by parts]

Answer: C) $x^2 f(x) - 2\int x\,f(x)\,dx$

Using integration by parts for $\int x^2 f'(x)\,dx$:

$$u = x^2 \Rightarrow du = 2x\,dx$$
$$dv = f'(x)dx \Rightarrow v = f(x)$$
$$uv - \int v\,du = x^2 f(x) - \int f(x)2x\,dx$$

Q6)

Which of the following is a reduction formula to evaluate $I_n = \int_0^{\pi/2} x^n \sin x = ?$ (n is an integer)

A) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}, (n \geq 2)$

B) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} + n(n-1)I_{n-2}, (n \geq 2)$

C) $I_n = n\left(-\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}, (n \geq 2)$

D) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - nI_{n-2}, (n \geq 2)$

E) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} + nI_{n-2}, (n \geq 2)$

Solution of Q6) [int. by parts]

Answer: A) $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2},\ (n \geq 2)$

$$I_n = \int_0^{\pi/2} x^n \sin x\,dx = uv - \int v\,du$$

$$\begin{pmatrix} u = x^n, & dv = \sin x \\ du = nx^{n-1}dx, & v = -\cos x \end{pmatrix}$$

$$= x^n(-\cos x)\Big|_0^{\pi/2} - \int_0^{\pi/2} (-\cos x)nx^{n-1}\,dx$$

$$= n\int_0^{\pi/2} x^{n-1}\cos x\,dx$$

$$= n\left(uv - \int v\,du\right) \to u = x^{n-1},\ dv = \cos x$$

$$= n\left(x^{n-1}(\sin x)\Big|_0^{\pi/2} - \int_0^{\pi/2} (\sin x)(n-1)x^{n-2}\,dx\right)$$

$$= n\left(\frac{\pi}{2}\right)^{n-1} - \underbrace{n(n-1)\int_0^{\pi/2} x^{n-2}(\sin x)\,dx}_{n(n-1)I_{n-2}}$$

$$\boxed{I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2},\ (n \geq 2).}$$

S-1) $\int_0^2 \frac{dx}{\sqrt{1+x^3}}$ integrali hangi aralıkta değer alır? A) $[-2,2]$ B) $\left[-2,\frac{-2}{3}\right]$ C) $\left[-2,\frac{3}{2}\right]$ D) $\left[\frac{2}{3},2\right]$ E) $\left[\frac{3}{2},2\right]$	C-1) $0 \le x \le 2 \;\Rightarrow\; 1 \le 1+x^3 \le 9 \;\Rightarrow$ $1 \le \sqrt{1+x^3} \le 3 \;\Rightarrow 1 \ge \frac{1}{\sqrt{1+x^3}} \ge \frac{1}{3} \;\Rightarrow$ $\int_0^2 dx \ge \int_0^2 \frac{dx}{\sqrt{1+x^3}} \ge \int_0^2 \frac{dx}{3} \;\Rightarrow$ $2 \ge \int_0^2 \frac{dx}{\sqrt{1+x^3}} \ge \frac{2}{3}$ Cevap D şıkkı
S-2) $y=f(x)$ fonksiyonu $\int_0^{2x} Cos(t)dt + \int_0^{y} e^{2t}dt = 0$ şeklinde verilmiş kapalı bir fonksiyon olduğuna göre $\frac{dy}{dx}$ türevi aşağıdakilerden hangisidir. A) $-\frac{Sin(x)}{2e^y}$ B) $\frac{2Cos(2x)}{e^{2x}}$ C) $-\frac{2Cos(2x)}{e^{2y}}$ D) $-\frac{Sin(2x)}{2e^x}$ E) $\frac{2Cos(x)}{e^{2y}}$	C-2) $\frac{d}{dx}\left(\int_0^{2x} Cos(t)dt\right) + \frac{d}{dy}\left(\int_0^{y} e^{2t}dt\right).\frac{dy}{dx} = 0$ $2Cos(2x) + e^{2y}.\frac{dy}{dx} = 0 \;\Rightarrow$ $\frac{dy}{dx} = -\frac{2Cos(2x)}{e^{2y}}$ Cevap C şıkkı
S-3) $f(x)$ sürekli bir fonksiyon olmak üzere $(x+1)^{1/2} = \int_2^{x^2} t.f(t)dt$ olsun. $f(1)$ değerini hesaplayınız. A) $2^{1/2}$ B) $2^{3/2}$ C) $2^{-3/2}$ D) $2^{5/2}$ E) $2^{-5/2}$	C-3) $\left((x+1)^{1/2}\right)' = \left(\int_2^{x^2} t.f(t)dt\right)'$ $\frac{1}{2(x+1)^{1/2}} = x^2.f(x^2).2x$ $x=1$ için $\frac{1}{2(2)^{1/2}} = 1^2.f(1^2).2 \;\Rightarrow$ $f(1) = \frac{1}{4\sqrt{2}} = 2^{-5/2}$ Cevap E şıkkı
S-4) $\int_{-1}^{3} \sqrt{1+x}\,dx$ integraline Ortalama Değer Teoremini uygulayıp c sabitini hesaplayınız. A) $\frac{7}{9}$ B) $\frac{61}{64}$ C) $-\frac{81}{84}$ D) $\frac{81}{84}$ E) $-\frac{7}{9}$	C-4) $a \le c \le b$ olmak üzere $\int_a^b f(x)\,dx = (b-a)f(c)$ $\int_{-1}^{3} \sqrt{1+x}\,dx = (3+1)\sqrt{1+c} \;\Rightarrow$ $c = \frac{7}{9}$ Cevap A şıkkı

S-5) $I=\int_0^{\pi}\left(\sqrt{1+Cos2x}\right)dx$ integralinin eşdeğeri aşağıdakilerden hangisidir. A) $I=\int_{-\pi}^{0}\left(\sqrt{1+Cos2x}\right)dx+\int_0^{\pi}\left(\sqrt{1+Cos2x}\right)dx$ B) $I=\int_0^{\frac{\pi}{2}}\left(\sqrt{Cos2x}\right)dx+\int_{\frac{\pi}{2}}^{\pi}\left(\sqrt{Cos2x}\right)dx$ C) $I=\sqrt{2}\int_0^{\frac{\pi}{2}}Cosx\,dx-\sqrt{2}\int_{\frac{\pi}{2}}^{\pi}Cosx\,dx$ D) $I=\int_0^{\frac{\pi}{2}}2Sin^2x\,dx+\int_{\frac{\pi}{2}}^{\pi}2Sin^2x\,dx$ E) $I=\int_{-\pi}^{0}\sqrt{Sin^2x}\,dx+\int_0^{\pi}\sqrt{Sin^2x}\,dx$	C-5) $\sqrt{1+Cos2x}=\sqrt{2Cos^2x}=\sqrt{2}\lvert Cosx\rvert$ $=\begin{cases}0\le x\le\dfrac{\pi}{2} & \text{ise} \quad \sqrt{2}Cosx\\ \dfrac{\pi}{2}\le x\le\pi & \text{ise} \quad -\sqrt{2}Cosx\end{cases}$ $I=\sqrt{2}\int_0^{\frac{\pi}{2}}Cosx\,dx-\sqrt{2}\int_{\frac{\pi}{2}}^{\pi}Cosx\,dx$ $=\sqrt{2}Sinx\Big\rvert_0^{\frac{\pi}{2}}-\sqrt{2}Sinx\Big\rvert_{\frac{\pi}{2}}^{\pi}=2\sqrt{2}$ Cevap C şıkkı
S-6) $[0,2]$ aralığında $y=x+1$ fonksiyonunun alt Riemann toplamını 5 eşit aralığa $(n=5)$ bölerek hesaplayınız. A) $\frac{17}{4}$ B) $\frac{18}{5}$ C) $\frac{19}{3}$ D) $\frac{20}{3}$ E) $\frac{21}{2}$	C-6) $L(f,\Delta)=\sum_{i=0}^{4}\left(\frac{2i}{5}+1\right)\frac{2}{5}$ $=\frac{2}{5}\left(1+\frac{7}{5}+\frac{9}{5}+\frac{11}{5}+\frac{13}{5}\right)=\frac{18}{5}$ Cevap B şıkkı
S-7) $[0,1]$ aralığında $y=x^2$ fonksiyonunun üst Riemann toplamını 6 eşit aralığa $(n=6)$ bölerek hesaplayınız. A) $\frac{125}{36}$ B) $\frac{30}{216}$ C) $\frac{91}{216}$ D) $\frac{91}{36}$ E) $\frac{14}{36}$	C-7) $U(f,\Delta)=\sum_{i=1}^{6}\left(\frac{i}{6}\right)^2\frac{1}{6}$ $=\frac{1}{6}\left(\frac{1}{36}+\frac{4}{36}+\frac{9}{36}+\frac{16}{36}+\frac{25}{36}+\frac{36}{36}\right)=\frac{91}{216}$ Cevap C şıkkı

MTM1502-Matematik Analiz II

25.04.2023 Tarihli Vize Sınavı Taslak Soru ve Çözümleri

Dr. **Müslüm ÖZIŞIK**

SORU-1) $\displaystyle\int \frac{\mathbf{dx}}{\mathbf{x}\sqrt{\mathbf{x}^2-\mathbf{a}^2}};\ (\mathbf{x}>\mathbf{a})$

integrali için $x = a\cosh(u)$ **dönüşümü uygulanırsa, aşağıdakilerden hangisi çözüm aşamasına oluşabilecek formlardandır?**

A-) $\displaystyle\frac{1}{a}\int du$

B-) $\displaystyle\frac{1}{a}\int \frac{du}{\sec(u)}$

C-) $\displaystyle\frac{1}{a}\int \frac{\cosh(u)du}{1+\sinh^2(u)}$

D-) $\displaystyle\int \frac{du}{\cosh(u)}$

E-) $\displaystyle\frac{1}{a}\int \frac{du}{\cos(u)}$

CEVAP-1)

$$\int \frac{\mathbf{dx}}{\mathbf{x}\sqrt{\mathbf{x}^2-\mathbf{a}^2}};\ (\mathbf{x}>\mathbf{a})$$

(Cevap-C)

$$x = a\cosh(u) \ \Rightarrow dx = a\sinh(u)du$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \int \frac{a\sinh(u)du}{a\cosh(u)\sqrt{(a\cosh(u))^2-a^2}}$$

$$= \int \frac{a\sinh(u)du}{a\cosh(u)\sqrt{a^2\underbrace{\left(\cosh^2(u)-1\right)}_{\sinh^2(u)}}} = \int \frac{a\sinh(u)du}{a\cosh(u)a\sinh(u)} = \frac{1}{a}\int \frac{du}{\cosh(u)}$$

$$= \frac{1}{a}\int \frac{du}{\cosh(u)}\left(\frac{\cosh(u)}{\cosh(u)}\right) = \frac{1}{a}\int \frac{\cosh(u)du}{1+\sinh^2(u)}$$

$$= \frac{1}{a}\arctan(\sinh(u)) + C = \frac{1}{a}\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right) + C$$

SORU-2)

$$\int \mathbf{\sin}(1-\mathbf{x})\mathbf{\cos}(2+5\mathbf{x})\mathbf{dx}$$

integralinin çözüm aşamasına ait formlardan biri hangi seçenekte verilmiştir?

A-) $\displaystyle\int 2\sin\left(\frac{3+4x}{2}\right)\cos\left(\frac{-1-5x}{2}\right)dx$

B-) $\displaystyle\int 2\sin\left(\frac{-1-5x}{2}\right)\cos\left(\frac{3+4x}{2}\right)dx$

C-) $\displaystyle\frac{1}{2}\int[\sin(-3-4x)+\sin(1+6x)]dx$

D-) $\displaystyle=\frac{1}{2}\int[\sin(3+4x)-\sin(-1-6x)]dx$

E-) $\displaystyle=\frac{1}{2}\int[\sin(3+4x)-\sin(1+6x)]dx$

CEVAP-2)

(Cevap-E)

$$\int \sin(1-x)\cos(2+5x)dx \quad ; \quad \sin p\cos q = \frac{1}{2}[\sin(p+q)+\sin(p-q)]$$

$$= \int \sin(1-x)\cos(2+5x)dx$$

$$= \frac{1}{2}\int\left[\sin((1-x)+(2+5x))+\sin((1-x)-(2+5x))\right]dx$$

$$= \frac{1}{2}\int[\sin(3+4x)+\sin(-1-6x)]dx$$

$$= \frac{1}{2}\int \sin(3+4x)dx + \frac{1}{2}\int \sin(-1-6x)dx \ ; \ \sin(-f(x)) = -\sin(f(x))$$

$$= -\frac{}{2}\int \sin(3+4x)dx - \frac{1}{2}\int \sin(1+6x)dx \qquad 1$$

$$= -\frac{1}{8}\cos(3+4x) + \frac{1}{12}\cos(1+6x) + C$$

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SORU-3)

$$\int \frac{\sin x}{\sin x - 1 + \cos x}dx$$

integralinin çözümü için en uygun trigonometrik dönüşüm ve dönüşüm sonrası integralin alacağı form hangi seçenekte verilmiştir.

A-) $u=\tan x; \int \frac{\frac{u}{1+u^2}}{\frac{u}{1+u^2}-1+\frac{1}{1+u^2}}\left(\frac{du}{1+u^2}\right)$

B-) $u=\tan\left(\frac{x}{2}\right)\;;\int \frac{\frac{2u}{1+u^2}}{\frac{2u}{1+u^2}-1+\frac{1-u^2}{1+u^2}}dx$

C-) $u=\tan\left(\frac{x}{2}\right)\;;\;\int \frac{du}{2(u-1)(u^2+1)}$

D-) $u=\tan\left(\frac{x}{2}\right)\;;\;\int \frac{2du}{(1-u)(1+u^2)}$

E-) $u=\tan\left(\frac{x}{2}\right)\;;\;\int \frac{2du}{(1+u)(1-u^2)}$

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CEVAP-3) (Cevap-D)

$$\int \frac{\sin x}{\sin x - 1 + \cos x}dx$$

$$u=\tan\left(\frac{x}{2}\right) \Rightarrow dx=\frac{2du}{1+u^2},\ \sin x=\frac{2u}{1+u^2},\ \cos x=\frac{1-u^2}{1+u^2}$$

$$=\int \frac{\frac{2u}{1+u^2}}{\frac{2u}{1+u^2}-1+\frac{1-u^2}{1+u^2}}\left(\frac{2du}{1+u^2}\right)=\int \frac{2u}{2u-(1+u^2)+(1-u^2)}\left(\frac{2du}{1+u^2}\right)$$

$$=\int \frac{2u}{2u(1-u)}\left(\frac{2du}{1+u^2}\right)=2\int \frac{du}{(1-u)(1+u^2)}=2\left(\int \frac{Adu}{1-u}+\int \frac{(Bu+C)du}{1+u^2}\right)$$

$$\frac{1}{(1-u)(1+u^2)} \equiv \frac{A}{1-u}+\frac{Bu+C}{1+u^2} \Rightarrow A=B=C=1/2$$

$$=\int \frac{du}{1-u}+\int \frac{u+1}{1+u^2}du=\int \frac{du}{1-u}+\frac{1}{2}\int \frac{2u}{1+u^2}du+\int \frac{1}{1+u^2}du$$

$$=-\ln|1-u|+\frac{1}{2}\ln|1+u^2|+\arctan(u)+C$$

$$=-\ln\left(1-\tan\left(\frac{x}{2}\right)\right)+\frac{\ln\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{2}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)+C$$

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SORU-4)

$$\int \frac{2}{1-x+x^2-x^3}dx$$

İntegralinin çözüm aşamasına ait formlardan biri hangi seçenekte verilmiştir? $(A,B,C \in R)$

A-) $\int \frac{A}{x-1}dx-\int \frac{Bx+C}{1+x^2}dx$

B-) $\int \frac{A}{x-1}dx+\int \frac{B}{x}dx+\int \frac{C}{x+1}dx$

C-) $-\int \frac{2A}{x-1}dx+\int \frac{2(Bx+C)}{x^2+1}dx$

D-) $\int \frac{1}{x-1}dx-\int \frac{x+1}{x^2+1}dx$

E-) $\int \frac{1}{(x-1)(1+x^2)}dx$

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CEVAP-4) (Cevap-C)

$$1-x+x^2-x^3=1-x+x^2(1-x)=(1-x)(1+x^2)$$

$$=\int \frac{2dx}{(1-x)(1+x^2)}=2\left(\int \frac{A}{1-x}dx+\int \frac{Bx+C}{1+x^2}dx\right)$$

$$\frac{1}{(1-x)(1+x^2)} \equiv \frac{A}{1-x}+\frac{Bx+C}{1+x^2} \Rightarrow A=B=C=1/2$$

$$=\int \frac{dx}{1-x}+\int \frac{x+1}{1+x^2}dx=\int \frac{dx}{1-x}+\frac{1}{2}\int \frac{2x}{1+x^2}dx+\int \frac{1}{1+x^2}dx$$

$$=-\ln|1-x|+\frac{1}{2}\ln|1+x^2|+\arctan(x)+C$$

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SORU-5) **Aşağıdakilerden hangisi** $\int \mathbf{sin^5(x)sin(2x)dx}$ **integralinin çözüm aşamasına aittir?** A-) $\int \sin^6(x)\cos(x)dx$ B-) $\int\left(\frac{1-\cos 2x}{2}\right)^3 \sin(x)dx$ C-) $\int\left(\frac{\cos 2x-1}{2}\right)^3 \sin(2x)dx$ D-) $\int\left(\sqrt{1-\sin^2(2x)}\right)^3 \sin(2x)dx$ E-) $2\int d\left(\frac{\sin^7 x}{7}\right)$	**CEVAP-5)** (Cevap-E) $\int \sin^5(x)\sin(2x)dx$ $=\int \sin^5(x)\left(2\sin(x)\cos(x)\right)dx$ $=2\int \sin^6(x)\cos(x)dx$ $=2\int d\left(\frac{\sin^7 x}{7}\right)$ $=\frac{2}{7}\sin^7 x + C$
SORU-6) $\int\left[\mathbf{tan^2(1+x)-cot^2(1-x)}\right]\mathbf{dx}$ **integralinin sonucu aşağıdaki seçeneklerden hangisinde doğru olarak verilmiştir? (x, türevlenebilen noktaları ifade etmektedir)** A-) $\ln\lvert\cos(1+x)\rvert-\ln\lvert\sin(1-x)\rvert+C$ B-) $\tan(1+x)-\cot(1-x)+C$ C-) $\ln\lvert\cos(1-x)\rvert+\ln\lvert\sin(1+x)\rvert+C$ D-) $\tan(1+x)+\cot(1-x)+C$ E-) $x+C$	**CEVAP-6)** (Cevap-B) $\int\left[\mathbf{tan^2(1+x)-cot^2(1-x)}\right]\mathbf{dx=?}$ $=\int\left[\tan^2(1+x)-\cot^2(1-x)+1-1\right]dx$ $=\int\left[1+\tan^2(1+x)\right]dx+\int\left[-1-\cot^2(1-x)\right]dx$ $=\int \sec^2(1+x)dx-\int \operatorname{cosec}^2(1-x)dx$ $=\tan(1+x)-\cot(1-x)+C$

SORU-7) Aşağıda verilen integraller ile dönüşümleri en uygun şekilde eşleştiriniz.

İntegraller

$$I_1 = \int \frac{dx}{\left(3 - x^2 - 2x\right)^{3/2}}$$

$$I_2 = \int \frac{dx}{x\sqrt{3 + x^2 - 2x}}$$

$$I_3 = \int \frac{dx}{x\sqrt{-3 + x^2 - 2x}}$$

$$I_4 = \int \frac{dx}{1 - \sin(2x) + \cos(2x)}$$

Dönüşümler

$T_1 : (x+1) = 2\sin t$; $T_2 : (x+1) = 4\sin t$

$T_3 : (x+1) = \sqrt{2}\sin t$; $T_4 : (x-1) = 2\sin t$

$T_5 : (x-1) = \sqrt{2}\sin t$; $T_6 : (x-1) = \sqrt{2}\tan t$

$T_7 : (x-1) = \sqrt{2}\sec t$; $T_8 : (x-1) = 2\tan t$

$T_9 : \tan\left(\frac{x}{2}\right) = t$; $T_{10} : \tan x = t$

$T_{11} : (x+1) = 2\tan t$; $T_{12} : (x-1) = 2\sec t$

A-) $I_1 \to T_1$; $I_2 \to T_6$; $I_3 \to T_{12}$; $I_4 \to T_{10}$

B-) $I_1 \to T_1$; $I_2 \to T_{11}$; $I_3 \to T_8$; $I_4 \to T_9$

C-) $I_1 \to T_3$; $I_2 \to T_6$; $I_3 \to T_7$; $I_4 \to T_{10}$

D-) $I_1 \to T_1$; $I_2 \to T_9$; $I_3 \to T_{12}$; $I_4 \to T_9$

E-) $I_1 \to T_4$; $I_2 \to T_8$; $I_3 \to T_7$; $I_4 \to T_9$

CEVAP-7) (Cevap-A)

$$I_1 = \int \frac{dx}{\left(3 - x^2 - 2x\right)^{3/2}} = \int \frac{dx}{\left(3 - \left(x^2 + 2x\right)\right)^{3/2}} = \int \frac{dx}{\left(3 - (x+1)^2 + 1\right)^{3/2}}$$

$$I_1 = \int \frac{dx}{\left(4 - (x+1)^2\right)^{3/2}};$$

$x + 1 = 2\sin t$ Cevap : T1

$$I_2 = \int \frac{dx}{x\sqrt{3 + x^2 - 2x}} = \int \frac{dx}{x\sqrt{3 + (x-1)^2 - 1}} = \int \frac{dx}{x\sqrt{2 + (x-1)^2}}$$

$x - 1 = \sqrt{2}\tan t$ Cevap : T6

$$I_3 = \int \frac{dx}{x\sqrt{-3 + x^2 - 2x}} = \int \frac{dx}{x\sqrt{-3 + (x-1)^2 - 1}} = \int \frac{dx}{x\sqrt{(x-1)^2 - 4}}$$

$x - 1 = 2\sec t$ Cevap : T12

$$I_4 = \int \frac{dx}{1 - \sin(2x) + \cos(2x)}$$

$t = \tan\left(\frac{2x}{2}\right) = \tan x$ Cevap : T10

NOT: I_1, I_2, I_3 integralleri hiperbolik dönüşümler ile çözülebilir.

$I_1 : (x+1) = 2\tanh t$

$I_2 : (x-1) = \sqrt{2}\sinh t$

$I_3 : (x-1) = 2\cosh t$