Örn

$4x-3y=9$ ve $x^2+z^2=9$ bölgeleriyle sınırlandırılmış $f(x,y,z)=3x^2+y$ fonksiyonunun max ve min değerleri?

$$L(x,y,z,\lambda_1,\lambda_2)=3x^2+y+\lambda_1(4x-3y-9)+\lambda_2(x^2+z^2-9)$$

$L_x = 6x+4\lambda_1+2\lambda_2 x=0$

$L_y = 1-3\lambda_1=0 \Rightarrow \lambda_1 = 1/3$

$L_z = 2z\lambda_2=0$

$L_{\lambda_1} = 4x-3y-9=0$

$L_{\lambda_2} = x^2+z^2-9=0$

i) $z=0$ veya ii) $\lambda_2=0$

i) $z=0$

$L_{\lambda_2}=x^2+0-9=0$

$x=\mp 3$

* $x=-3 \Rightarrow 4x-3y-9=0$, $y=-7$

$\boxed{(-3,-7,0)}$

* $x=3 \Rightarrow 4.3-3y-9=0$

$y=1$

$\boxed{(3,1,0)}$

ii) $\lambda_2=0$

$6x+4.\underset{1/3}{\lambda_1}=0$

$x=-2/9$

$4.\left(-\frac{2}{9}\right)-3y-9=0$

$y=-\frac{89}{27}$

$\frac{4}{81}+z^2-9=0$

$z=\mp\frac{5\sqrt{29}}{9}$

$\boxed{\left(-\frac{2}{9},-\frac{89}{27},-\frac{5\sqrt{29}}{9}\right)}$

$\boxed{\left(-\frac{2}{9},-\frac{89}{27},\frac{5\sqrt{29}}{9}\right)}$

$f(-3,7,0)=20$

$f(3,1,0)=28$ ~ mutlak max

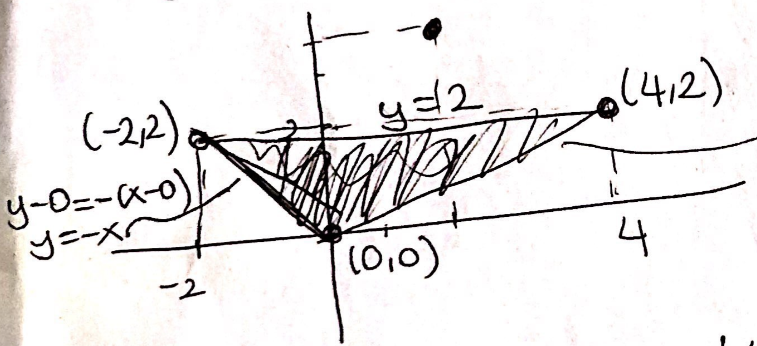
$f\left(-\frac{2}{9}, -\frac{89}{27}, -\frac{5\sqrt{29}}{9}\right)=-\frac{85}{27}$

$f\left(-\frac{2}{9}, -\frac{89}{27}, \frac{5\sqrt{29}}{9}\right)=-\frac{85}{27}$

mutlak min

Örn

$f(x,y)=192x^3+y^2-4xy^2$ fonksiyonunun mutlak extremumları $(0,0)$ $(4,2)$ ve $(-2,2)$ üçgensel bölgede bulunuz



$y-0=\frac{1}{2}(x-0)$

$y=\frac{x}{2}$

1. $(0,0)$ $(-2,2)$ $(4,2)$

2. $f_x=576x^2-4y^2$, $f_y=2y-8xy$

$f_x=576x^2-4y^2=0$

$f_x=0 \Rightarrow 576x^2-4y^2=0$

$f_y=0 \Rightarrow 2y(1-4x)=0$

i) $y=0$

$576x^2=0$

$(0,0)$

$x=1/4$

$576x^2-4y^2=0$

$36-4y^2=0$

$y=\mp 3$

$\left(\frac{1}{4},3\right)$ ve $\left(\frac{1}{4},-3\right)$

3

3 $y=2$: $-2 \le x \le 4$

$f(x,2) = g(x) = 192x^3 - 16x + 4$

$g'(x) = 576x^2 - 16 = 0$

$x = \mp 1/6$

$(-\frac{1}{6}, 2)$ $(\frac{1}{6}, 2)$

$\rightarrow y = x/2$, $0 \le x \le 4$

$f(x, \frac{x}{2}) = g(x) = \frac{x^2}{4} + 191x^3$

$g'(x) = \frac{x}{2} + 573x^2 = x(573x + \frac{1}{2}) = 0$

i) $x=0$ ii) $\cancel{x = -\frac{1}{1146}}$

bölgenin dışın

$(0,0)$

$\rightarrow y = -x$ $-2 \le x \le 0$

$f(x,-x) = g(x) = x^2 + 188x^3$

$g'(x) = 2x + 564x^2 = 2x(1 + 282x)$

$x = 0$ ✓, $x = -\frac{1}{282}$ ✓

$(0,0)$ $(-\frac{1}{282}, \frac{1}{282})$

(x,y)	$f(x,y)$	
$(0,0)$	0	
$(1/6, 2)$	2019	
$(-1/6, 2)$	5219	~ mutlak min.
$(-2,2)$	-1500	
	12.228	~ mutlak max
$(4,2)$		
$(-\frac{1}{282}, \frac{1}{282})$	$1/238.572$	

④

Örn

$f(x,y) = x.\ln(x^2+y^2)$ ise f nin yerel ext?

$f_x = \ln(x^2+y^2) + \frac{2x^2}{x^2+y^2}$

$f_y = \frac{2xy}{x^2+y^2}$

$f_y = 0 \Rightarrow y=0$ veya x=

i) $y=0$ ve

$f_x = 0$

$\ln x^2 + \frac{2x^2}{x^2} = 0$

$\ln x^2 + 2 = 0$

$x = \mp e^{-1}$

ii) $x=0$ ve $f_x = 0$

$\ln y^2 = 0 \Rightarrow y = \mp 1$

$P_1(-e^{-1}, 0)$ $P_2(e^{-1}, 0)$ $Q_1(0,-1)$ $Q_2(0,1)$

kritik noktalar

$f_{xx} = \frac{2x}{x^2+y^2} + \frac{4x(x^2+y^2) - 4x^3}{(x^2+y^2)^2} = \frac{2x(x^2+3y^2)}{(x^2+y^2)^2} = A$

$f_{yy} = \frac{2x(x^2+y^2) - 4xy^2}{(x^2+y^2)^2} = \frac{2x(x^2-y^2)}{(x^2+y^2)^2} = C$

$f_{xy} = \frac{2y}{x^2+y^2} - \frac{4x^2y}{(x^2+y^2)^2} = \frac{2y(y^2-x^2)}{(x^2+y^2)^2} = B$

$\Delta P_{1,2} = 4e^2$ $\quad A|_{P_1} = 2e > 0$

$A|_{P_2} = -2e < 0$

$f(P_1) = -2e^{-1}$ yerel min

$f(P_2) = 2e^{-1}$ " max

$\Delta Q_{1,2} = -2 < 0$ old.

Q_1 ve Q_2 semer noktalardır.