Fizik 1 Tekrar

3 temel büyüklük vardır.

1- Kütle (kg)

2- Uzunluk (m)

3- Zaman (s)

Boyut Analizi

Büyüklük	Boyut	Birim
Kütle	Mass	kg
Uzunluk	Length	metre
Zaman	Time	saniye
Alan	L^2	m^2
Hacim	L^3	m^3
Hız	L/T	m/s
İvme	L/T^2	m/s^2
Kuvvet	$M.L/T^2$	$kg.m/s^2$

* Bir eşitliğin her iki tarafının boyutları aynı olmalıdır.
* Aynı boyuttaki nicelikler toplanıp çıkarılabilir.

Ör/

$$F = G.\frac{m_1.m_2}{r^2}$$

"G" boyut analizini yapınız.

$$\frac{M.L}{T^2} = "G".\frac{M^*}{L^2}$$

$$\frac{L^3}{M.T^2} = "G"$$

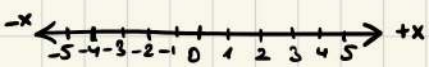
Birim Çevirme

1. $\frac{1km}{1000m} \cdot \frac{3600s}{1h}$

$1\ m/s = ?\ km/h$

$$1\ \frac{m}{s}\left(\frac{3600s}{1h}\right)\cdot\left(\frac{1\ km}{1000m}\right) = 3{,}6\ km/h$$

Bir Boyutlu Hareket



1) Yer değiştirme: $\Delta x = x_s - x_i$ (metre)

x_i = ilk konum | t_i = ilk zaman

x_s = son konum | t_s = son zaman

2) Ortalama Hız: $v_{ort} \equiv \frac{\Delta x}{\Delta t}\ (m/s)$

3) Ani Hız: $\lim_{\Delta t \to 0} v_{ort} \equiv v$

$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

Türev Operatörü

$\lim_{\Delta t \to 0} \frac{\Delta}{\Delta t} \equiv \frac{d}{dt}$

4) Ortalama ivme:

$a_{ort} \equiv \frac{\Delta v}{\Delta t} = \frac{v_s - v_i}{t_s - t_i}\ (m/s^2)$

5) Ani ivme

$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv(t)}{dt}$

Ör/ $-4t + 2t^2 = x$

a) 1sn ile 3sn arasında yer değiştirme

$4t - 4 = v$

$4 = a$

$(t=1)\ x_i = -2m$

$(t=3)\ x_s = 6m$

8m

b) ort hız:

$8/2 = 4\ m/s$

$\frac{8-0}{2} = 4 m/s$

c) $t = 2,5$ sn'de cismin hızı

$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{x_s - x_i}{\Delta t}$

$t_s = t + \Delta t$

$x_s = -4(t+\Delta t) + 2(t+\Delta t)^2$

$\lim_{\Delta t \to 0} \frac{-4t - 4\Delta t + 2t^2 + 4t\Delta t + 2\Delta t^2 + 4t - 2t^2}{\Delta t}$

$\frac{\Delta t(-4 + 4t + 2\Delta t)}{\Delta t} = -4 + 4t$

$(t=2,5) = 6$

$v = \frac{dx}{dt}\Big|_{t=2,5} \rightarrow -4 + 4t$

$6m$

Sabit İvmeli Hareket ($a = sbt$)

- Ortalama ivme, ani ivmeye eşittir.

$$v(t) = v_i + at$$

$+$ anında v hızı
ilk hızı + ivme x zaman

$$v_{ort} = \frac{x(t) - x_i}{t} = \frac{v_i + v(t)}{2}$$

Düzgün ivmede geçerlidir.

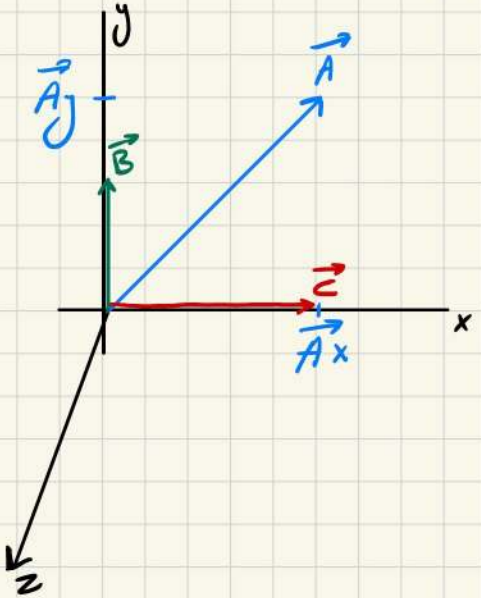
$$x(t) = x_i + v_i t + \frac{1}{2} a t^2$$

$$x_s = x_i + v_0 t + \frac{at^2}{2}$$

$$v^2 = v_i^2 + 2a(x - x_i)$$

zamansız hız formülü

Vektörler



Birim Vektör: $|\hat{i}| = 1$ br

$|\hat{j}| = 1$ br

$|\hat{k}| = 1$ br

$\vec{B} = 3\hat{i}$

$\vec{C} = 4\hat{j}$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

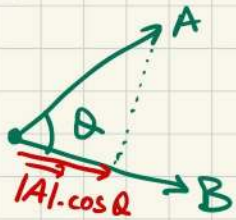
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$\vec{A}$'nın uzunluğu

Vektörlerde Çarpım

1) Skaler Çarpım (iç çarpım)

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos\theta$$



$\hat{i} \cdot \hat{i} = |\hat{i}| \cdot |\hat{i}| \cdot \cos 0^\circ = 1$

$\hat{j} \cdot \hat{j} = |\hat{j}| \cdot |\hat{j}| \cdot \cos 0^\circ = 1$

$\hat{k} \cdot \hat{k} = 1$

$\hat{i} \cdot \hat{j} = |\hat{i}| \cdot |\hat{j}| \cdot \cos 90 = 0$

$\hat{i} \cdot \hat{k} = 0$

$\hat{j} \cdot \hat{k} = 0$

$$\vec{A} \cdot \vec{B} = A_x B_x \hat{i} \cdot \hat{i} + A_x B_y \hat{i} \hat{j} + A_x B_z \hat{i} \cdot \hat{k} + A_y B_x \ldots$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Ör/

$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ (4, 9, 1)

$\vec{B} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ (9, 16, 25)

a) $6 - 12 - 5 = -11$

$6 - 12 - 5 = -11$

$-11 = \sqrt{14} \cdot \sqrt{50} \cos\theta$

$\cos^{-1}\left(\frac{-11}{\sqrt{14}\cdot\sqrt{50}}\right) = \theta$

a) A ve B skaler çarpım

b) A, B arası açı kaç derecedir?

b) $\sqrt{3^2 + (-4)^2 + 5^2} = 7.07$ ($\vec{B}$)

$\sqrt{2^2 + 3^2 + (-1)^2} = 3.74$ ($\vec{A}$)

$-11 = (7.07)(3.74) \cdot \cos\theta$

$\cos\theta = -0,38$

$\theta = 112.3^\circ$

2) Vektörel Çarpım

$\vec{A} \times \vec{B}$

$|\hat{i} \times \hat{i}| = |\hat{i}| \cdot |\hat{i}| \cdot \sin 0^\circ = 0$

$|\hat{j} \times \hat{j}| = |\hat{j}| \cdot |\hat{j}| \cdot \sin 0^\circ = 0$

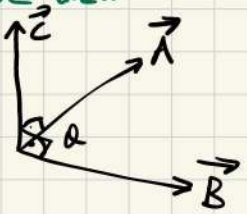
Vektörel Çarpımın Özellikleri

1) İki vektör çarpımı da bir vektördür.

2) $|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin\theta$

3) İki vektör çarpımı diğer vektörlere diktir.

Yani aynı düzlemde değildir.



4) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$\hat{i} \times \hat{j} = \vec{c}$

$\hat{i} \cdot \hat{j} = \hat{k}$

$\vec{c} = |\hat{i}| \cdot |\hat{j}| \cdot \sin 90° = 1$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

$\vec{A} = 2\hat{i} + 3\hat{j}$

$\vec{B} = \hat{i} - 5\hat{j}$

$\vec{A} \times \vec{B} = (2\hat{i} + 3\hat{j}) \times (\hat{i} - 5\hat{j})$

$-3\hat{k}$ $\quad 0$ $\quad 0$ $\quad -10\hat{k}$

$\vec{A} \times \vec{B} = -13\hat{k}$

$|\vec{A} \times \vec{B}| = 13$

$(\vec{A} \times \vec{B}) \cdot \vec{A} = ?$

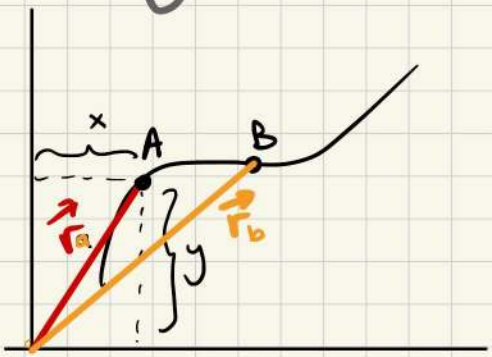
$\vec{C}$

0

$\vec{C} \perp \vec{A}$

$\vec{C} \perp \vec{B}$

İki Boyutlu Hareket



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{r}| = \sqrt{x^2+y^2+z^2}$$

1) Yer değiştirme

$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A = (x_B\hat{i} + y_B\hat{j}) - (x_A\hat{i} + y_A\hat{j})$$

2) Ortalama Hız

$$\vec{v}_{ort} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$\vec{v}_{ort} = v_{ort_x}\hat{i} + v_{ort_y}\hat{j} + v_{ort_z}\hat{k}$$

3) Anlık Hız

$$\vec{v} = \lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

4) Ortalama İvme

$$\vec{a}_{ort} = \frac{\Delta\vec{v}}{\Delta t} = a_{ort_x}\hat{i} + a_{ort_y}\hat{j}$$

5) Ani İvme

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{dt} = \frac{d\vec{v}}{dt}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

Sabit ivmeli iki Boyutlu Hareket

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

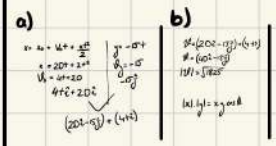
$$v_x = v_{0x} + a_x t$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_y = v_{0y} + a_y t$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{r} = \vec{r_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2$$



$a_x = 4\ m/s^2$ $t=0$ anında orijinde başlayan

$a_y = 0$ $V_{0x} = 20\ m/s$

$V_{0y} = -15\ m/s$

a) Herhangi bir andaki hız vek. yazınız.

b) $(t=5)$ için hız, sürat x ekseni ile açısı

c) Konum vek?

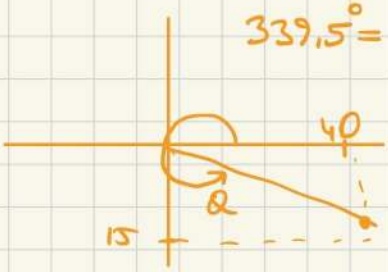
$$\vec{a} = 4\hat{i}$$

$$\vec{v} = \vec{v_0} + \vec{a}t$$

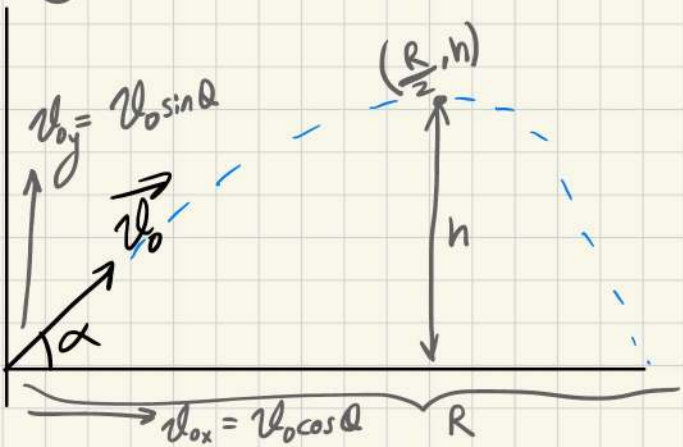
$$\vec{v} = (20\hat{i} - 15\hat{j}) + (4\hat{i}t)$$

c) $r = r_0 + \vec{v_0}t + \frac{1}{2}\vec{a}t^2$

$20i - 15j$



Yatay Atış Hareketi



$a = -g = -9.8\ m/s^2$

1) Cismin ivmesi "g" ve sbt
2) Hava Direnci ihmal

$x = x_o + v_o t + \frac{1}{2} a t^2$ ($x_o = 0$)

$x = v_o \cos\theta \cdot t$

$t = \frac{v_o \sin\theta \cdot 2}{g}$

$$x = \frac{v_o^2 \cdot \sin(2\theta)}{g}$$

Maksimum uzaklık

$v_y = v_{oy} + a_y t$

$$v_y = v_o \sin\theta - g t$$

+ anında hızın y eksenindeki değeri

tepe noktasında $v_y = 0$

$0 = v_o \sin\theta - g t_c$

$$t_c = \frac{v_o \sin\theta}{g} = t_i$$

çıkış = iniş süresi

$y = y_o + v_{oy} t + \frac{1}{2} a_y t^2$ ($a_y = -g$)

$t = t_c$ olursa $y = h$

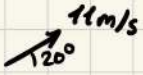
$h = v_o \sin\theta \cdot \frac{v_o \sin\theta}{g} - \frac{1}{2} g \frac{v_o^2 \cdot \sin^2\theta}{g^2}$

$$h = \frac{v_o^2 \cdot \sin^2\theta}{2g}$$

Maksimum yükseklik

Bir taş yerle 20° atılarak ($V_0 = 11\ m/s$)

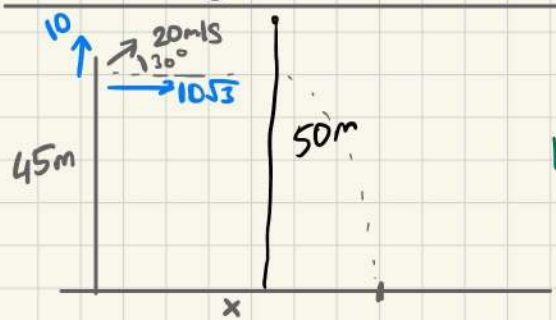
a) Yatayda ne kadar uzağa düşer?



$$x = \frac{V_0^2 \sin 40^\circ}{g} = \frac{121 \cdot \sin 40^\circ}{9.8} = 7.93 \text{ metre}$$

b) Taşın maks yükseklik

$$h = \frac{V_0^2 \cdot \sin^2 \theta}{2g} = \frac{121 \cdot \sin^2 20^\circ}{9.8} = 0,72 m$$



a) taş ne zaman yere çapar

$t_ç = 1sn$

$h_{max} = 45 + 10 - 5 = 50$ | $3.16 + 1 = 4.16$

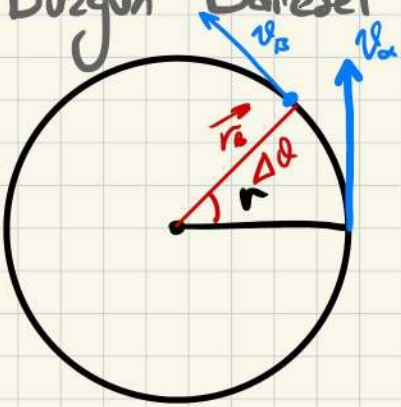
b) taşın yere çarpma hızı nedir?

31.35 m/s

$1 = t_ç$

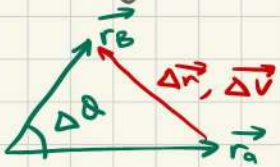
$10 \cdot 1 - \frac{1}{2} 10 \cdot 1 = 5m$

Düzgün Dairesel Hareket



$$\vec{a} = \frac{d\vec{v}}{dt}$$

merkezcil ivme: dairenin merkezine doğru olan ivme



$$|\vec{v_A}| = |\vec{v_B}| = v$$

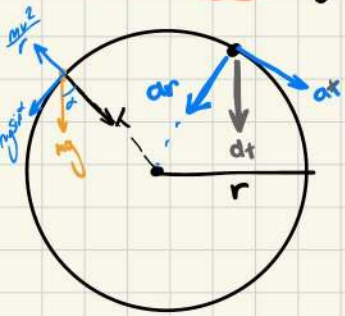
$$|\vec{r_A}| = |\vec{r_B}| = r$$

$$|\vec{a_{ort}}| = \frac{|\Delta\vec{v}|}{\Delta t}$$

$$\frac{|\Delta\vec{r}|}{r} = \frac{|\Delta\vec{v}|}{v}$$

$$v = \lim_{\Delta t \to 0} \frac{|\Delta\vec{r}|}{\Delta t} = \frac{|d\vec{r}|}{dt}$$ } anlık hız

$$|\vec{a}_{anlık}| = \frac{v^2}{r}$$ } merkezcil ivme



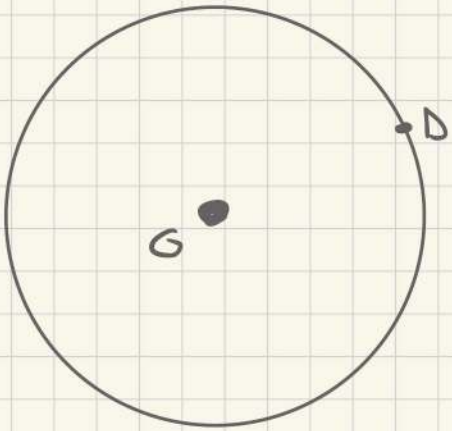
$$\vec{a_r} + \vec{a_x} = \vec{a_t}$$

Düzgün olmayan dairesel hareket

1- Hızın büyüklüğünün değişmesinden ivme $a_1 = \frac{dv}{dt}$

2- Hızın yönünden dolayı ivme $a_2 = \frac{v^2}{r}$

Dünyanın, güneş etrafında dönmesi sonucu merkezcil ivmesi yaklaşık nedir?

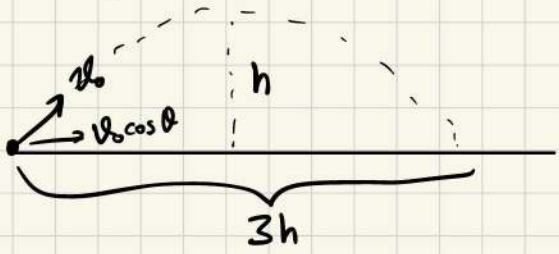


$$a = \frac{v^2}{r} \rightarrow a = \frac{4\pi^2 r^2}{t^2 \cdot r}$$

$$a = 5.93 \cdot 10^{-3}\ m/s^2$$

$$\underbrace{2\pi r}_{\text{yol}} = v \cdot t \quad \downarrow \quad \frac{2\pi r}{t} = v$$

Bir taş yerden belli bir açıyla fırlatıldığında menzil uzaklığı yüksekliğin 3 katı oluyor. Açı=?



$$\frac{v_0^2 \cdot \sin 2\theta}{g} = 3h$$

$$\frac{3 v_0^2 \cdot \sin^2\theta}{2g} = 3h$$

$$\frac{3 v_0^2 \cdot \sin^2\theta}{2g} = \frac{v_0^2 \cdot \sin 2\theta}{g}$$

$$3\sin^2\theta = 2\sin 2\theta$$

$$3s^2 = 2 - 4s^2$$

$$7s^2 = 2$$

$$s = \pm\sqrt{\frac{2}{7}}$$

Orijinde

$\vec{a} = 3\hat{j}\ m/s^2$

$\vec{v_0} = 5\hat{i}\ m/s$

ise

a) herhangi anda konum vek

$x = v_0 t + \frac{1}{2} a t^2$

$r = 5\hat{i} t + \frac{3\hat{j}}{2} t^2$

b) hız vek

$v = 5\hat{i} + 3\hat{j} t = \frac{d\vec{r}}{dt}$

c) $t=2$ için koordinat, sürat

$(10\hat{i}, 6\hat{j})$ / $5\hat{i} + 6\hat{j} = \vec{v}$

$\sqrt{25+36} = |\vec{v}|$

$m/s\ 7.81 = |\vec{v}|$

Hareket Yasaları

Kuvvet: $\vec{F}$

1) Temas Kuvvetleri
2) Alan Kuvvetleri

1- Eylemsizlik

$$\Sigma \vec{F} = 0$$
$$\vec{a} = 0$$

Kütle: Cismin hızının değişimine karşı gösterdiği direnç

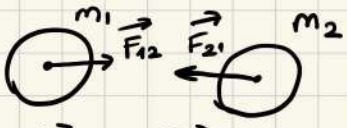
2- $F = m.a$ $(kg/m.s^2)$

$m_1 \rightarrow a_1$

$m_2 \rightarrow a_2$

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

3- Etki/Tepki Prensibi



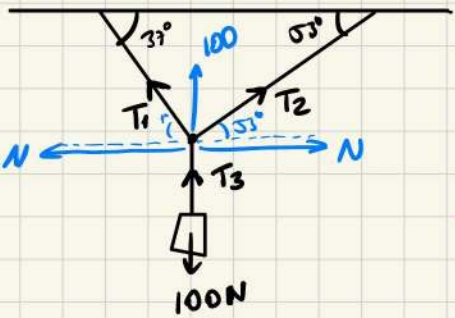
$$\vec{F}_{12} = -\vec{F}_{21}$$

4- Kütle Çekim Kuvveti

$\vec{F}_g$ $\vec{a} = \vec{g}$

$$\Sigma \vec{F} = m.\vec{a}$$
$$\vec{F}_g = m\vec{g}$$

Ağırlık = m.g



$T_{2x} = T_2 \cos 53°$

$T_{2y} = T_2 \cdot \sin 53°$

$T_{1x} = T_1 \cdot \cos 37°$

$T_{1y} = T_1 \cdot \cos 37°$

$$T_1 = \frac{T_2 \sin 37°}{\cos 37°} = \frac{3T_2}{4}$$

$$\frac{4}{5}(T_1 + T_2) = 100$$

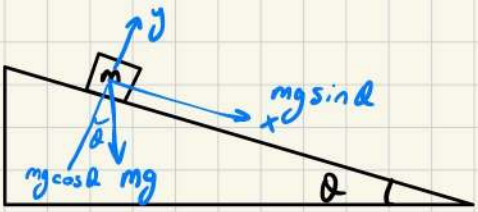
$1 + \frac{4}{3}$

$\frac{3T_2}{4} + T_2$

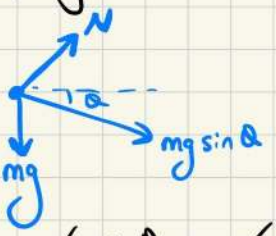
$$\frac{4}{5} \cdot \frac{7T_2}{4} = 100$$

$\frac{500}{7} = T_2$

$\frac{1500}{28} = T_1$



a) bloğun ivmesi



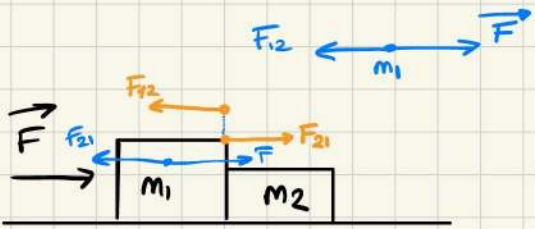
$mg\sin\alpha = m.a_x$

$g\sin\alpha = a_x$

b) eğik düzlemde "d" kadar yol katederse hızı ne olur.

$v^2 = v_0^2 + 2a_x.(x_s - x_0)$

$v = \sqrt{2gsin\alpha d}$

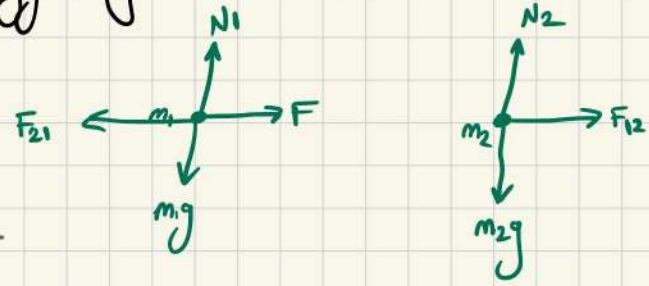


a) sistem ivme

$\sum F_x = m.a_x$

$F = (m_1 + m_2)\,a_x$

b) 1.'nin 2.'ye uyguladığı kuvvet?



$\frac{F - F_{21}}{m_1} = a_1 = a_x$

$\frac{F}{m_1+m_2} = \frac{F - F_{21}}{m_1}$

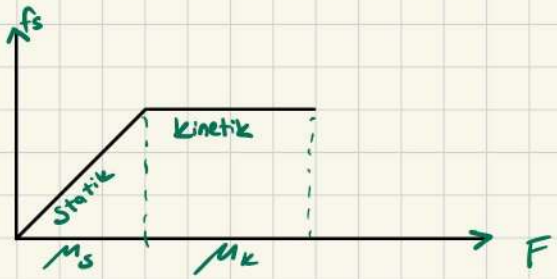
$F.m_1 = F.m_1 + F.m_2 - F_{21}.m_1 - F_{21}.m_2$

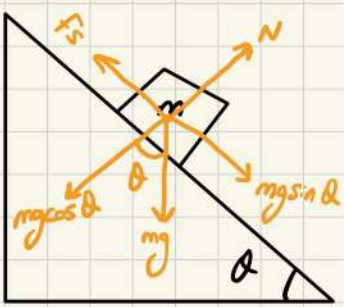
$F_{21} = \left(\frac{m_2}{m_1+m_2}\right).F \longrightarrow$

$\frac{F_{12}}{m_2} = \frac{F}{m_1+m_2}$

$F_{12} = -F\left(\frac{m_2}{m_1+m_2}\right)$

Sürtünme Kuvveti





Ör/ $N = mg\cos\theta$

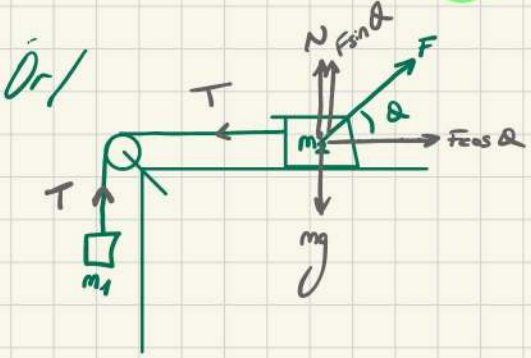
$$f_s = \mu_s N = \mu_s mg\cos\theta$$

Eğer $\Sigma F_x = 0$ ise hareket yoktur.

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

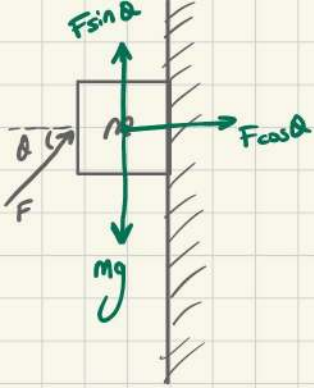
$$\mu_s = \tan\theta_c$$

Ör/



ivmeleri gösteriniz.

$$\frac{T - m_1 g}{m_1} = \frac{F\cos\theta - T - \mu(m_2 g - F\sin\theta)}{m_2}$$



$m = 3kg$ $\theta = 50°$

Statik sürtünme $k = 0,25$ ise
cismi değecek tutacak muhtemel F nedir?

$N = F\cos\theta$

$f_s = \mu F\cos\theta$

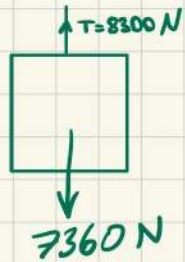
$f_s = F\sin\theta + mg = \mu F\cos\theta$

$F = \frac{mg}{\mu\cos\theta + \sin\theta} = 31.72\ N$ } 1. Durum

$F\sin\theta = mg + \mu F\cos\theta$

$F = 48.56\ N$

Ör/ 75 kilo insan tartılmak isteniyor.

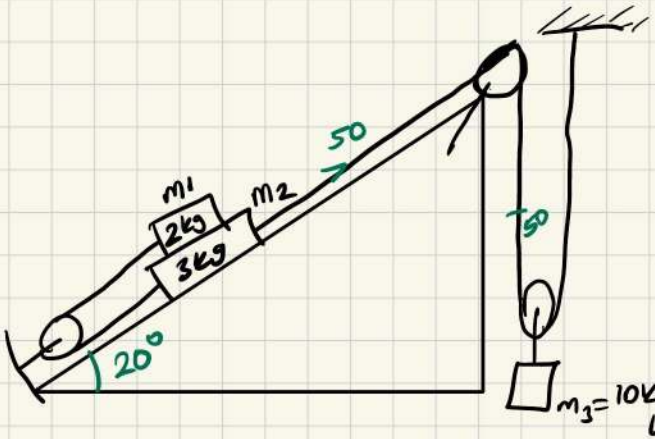


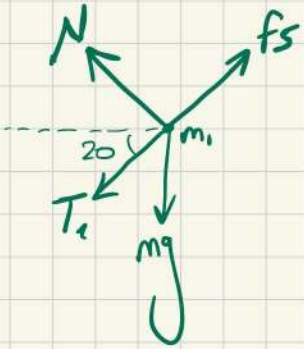
$\frac{940}{750} = 1,25\ m/s^2$

$+\ 9,8$

$11,05\ m/s^2$

$11.05 \cdot 75 = 82.8\ N$





$$m_1 a = T_1 + m_1 g \sin 20° - \mu_1 m_1 g \cos 20°$$

$$2a - 2 = T_1$$

$$m_2 a = T_2 - m_2 g \sin 20° - F_{s_2} - F_{s_1} - T_1$$

$$3a = T_2 - 75 - T_1$$

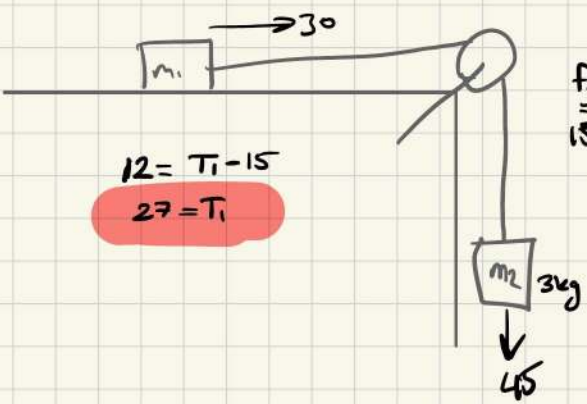
$$F_{s_2} = \mu_2 N_2$$

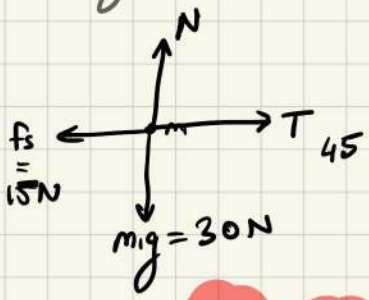
$$m_3 g - 2T_2 = m_3 \cdot \frac{a}{2}$$

Şekilde yukarıya doğru $A = 5\,m/s^2$'lik ivmeyle hareket etmekte olan asansörün içinde serbest bırakılan sistemde yatay doğrultudaki kinetik sürtünme katsayısı 0,5

$m_1 = 2kg$

$m_2 = 3kg$ old. göre blokların ivmeyi bulunuz.





$12 = T_1 - 15$

$27 = T_1$

$\frac{30}{5} = 6$

$45 - T_1 = 18$

$T_1 = 27$

2 koşu sporcusu durgun halden aynı anda 100 metre koşuya başlıyor. Her iki sporcu 10.2 sn de koşuyu tamamlıyor. 1. koşucu ivmesi = 2sn
2. koşucunun ivme = 3sn
ve sonra sbt hızda ilerliyorlar. Koşucuların hızları nedir?

$$x_{s_1} = \frac{1}{2} a_1 4 = 2a_1$$

$$x_{s_2} = \frac{1}{2} a_2 9 = 4{,}5 a_2$$

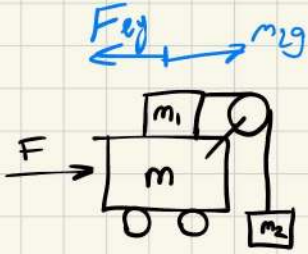
$$x_1 = v_{0x}(8.2) + 2a_1 = 100$$

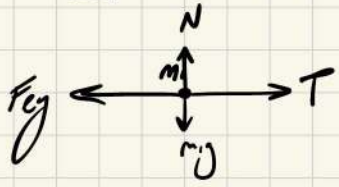
$$x_2 = v_{0y}(7.2) + 4{,}5 a_2 = 100$$

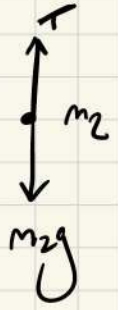
Blokların arabaya göre hareketsiz kalabilmesi için uygulanması gereken F = ?

(Tüm yüzeyler, tekerlekler, makara sürtünmesizdir.)

(ipteki gerilmeyi m_1 kütlesi meydana getirir.)







$$F = (m + m_1 + m_2)\, a$$

$$a = \frac{F}{m + m_1 + m_2}$$

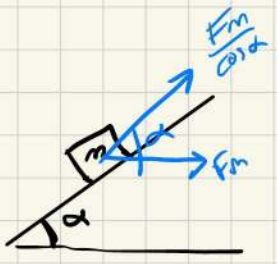
$$F_{eş} = \frac{F \cdot m_1}{m + m_1 + m_2} = T$$

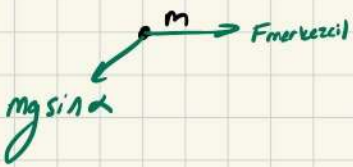
$$m_2 g =$$

Bir mühendis arabaların sürtünmeye güvenmeksizin savrulmadan dönebileceği eğimli bir otoyol istiyor.

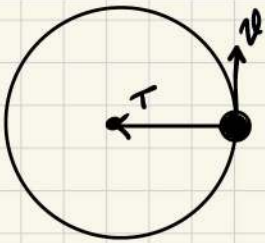
$v = 134\ m/s$'lik hız

virajın yarıçapı = 50m



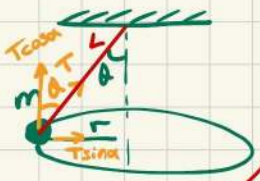


Dairesel Hareket



$$T = \frac{m.v^2}{r}$$

merkezcil kuvvet $(F_r) = m.a_r = \frac{m.v^2}{r}$



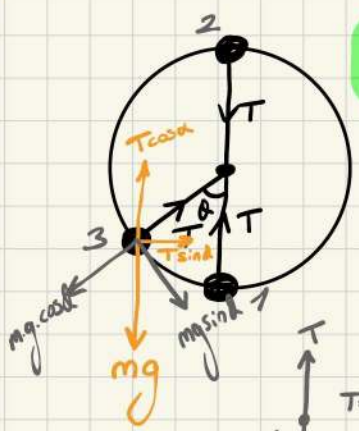
$T\sin\theta = \frac{mv^2}{r}$

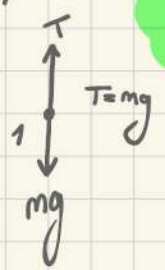
$T\cos\theta = mg$

$\tan\theta = \frac{v^2}{rg}$

$v = \sqrt{rg.\tan\alpha}$

$v\sqrt{L.g.\sin\theta.\tan\theta}$





$T = mg$

$$T_2 = mg\left(\frac{v^2}{rg} - 1\right)$$

$$T_1 = mg.\left(1 + \frac{v^2}{rg}\right)$$

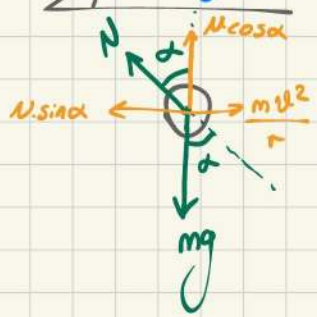
$$T - mg\cos\theta = \frac{m.v^2}{r}$$

$$T_3 = mg\left(\cos\theta + \frac{v^2}{rg}\right)$$

$$v = \sqrt{r.g}$$

Tekerlekle yol arasında sürtünmesiz ortamda ilerlemesi için açı = ?

$v = 13.4\ m/s \quad r = 35\ m$



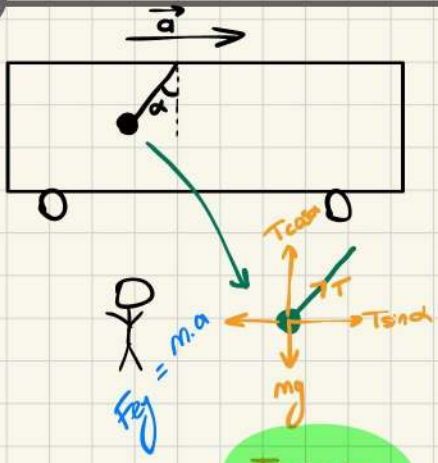
$$\frac{N\sin\alpha = \frac{mv^2}{r}}{N\cos\alpha = mg} = \tan\alpha = \frac{v^2}{r \cdot g}$$

$$v = \sqrt{r \cdot g \tan\alpha}$$

$$13.4 = \sqrt{35 \cdot (9.8) \cdot \tan\alpha}$$

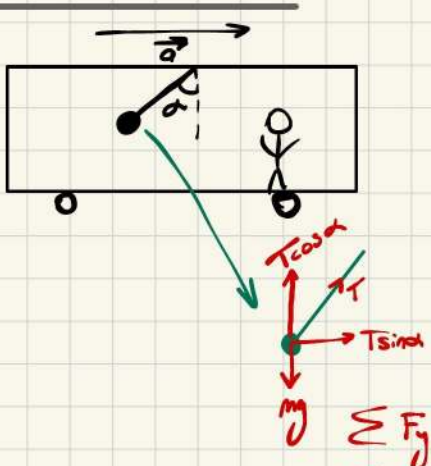
$$\alpha = 27.6^\circ$$

Eylemli Referans Sisteminde Hareket



$T\cos\alpha = mg$

$T\sin\alpha = m.a$



$\sum F_y = 0 \Rightarrow T\cos\alpha = mg$

$\sum F_x = 0$

$F_y = T\sin\alpha$

Direnç Ortamında Hareket

R = direnç kuvveti $\sim v, v^2$

$R = -k.v$

$v_0 = 0$

mg

$\sum F = m.a$

$mg - kv = m.\frac{dv}{dt}$

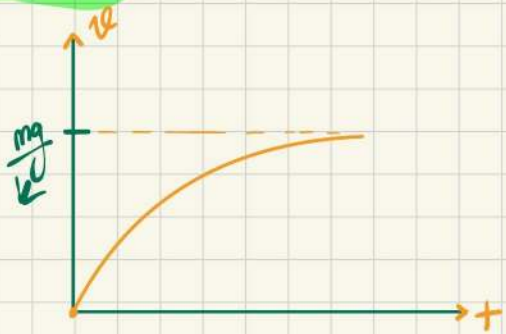
$\frac{dv}{dt} = g - \frac{kv}{m}$

$\int_0^t \frac{dv}{g - \frac{kv}{m}} = \int_0^t dt$

$v(t) = \frac{mg}{k}\left(1 - e^{-\frac{k.t}{m}}\right)$

$v(t) = \frac{mg}{k}\left(1 - \frac{1}{e^{k.t/m}}\right)$

$v_{limit} = \frac{m.g}{k}$



Cismin hızının **limit** hızın %63.2 değerine ulaşması için gereken süreye **zaman sabiti** denir.

2gr kütleli cisim yağ içerisinde serbest bırakılıyor. Cisim yağdan bir direnç hissediyor.

$v_{li} = 5 cm/s$

a) zaman sabiti

b) hızın, limit hızın %90'a ulaşması için geçen süre

$\frac{m.g}{k} = v_{li}$

$v = v_l(1 - e^{-t/\tau})$

$t = -5.1 \times 10^{-3} \ln(0.1) = 0.01$ sn

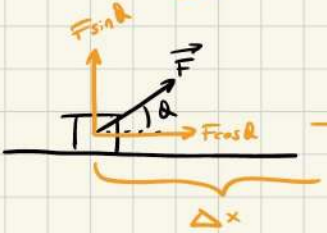
$\frac{2.(980)}{5} = 392 \, gram/saniye = k$

$k = \frac{m}{\tau(s)} \rightarrow g/s$ $\quad \tau = \frac{2}{392} \simeq 5.1 \, m/s$

İş ve Enerji

Sabit Kuvvetin Yaptığı İş

$$W \equiv F_x \cdot \Delta x$$



$W = F\cos\theta \cdot \Delta x$

Conclusion

* Kuvvet, yer değiştirme ile aynı yönde iş yapmalıdır.

Skaler çarpım gereğidir.

$W = F \cdot \Delta r \cdot \cos\theta$

$\vec{A} \cdot \vec{B} = |A|\,|B| \cdot \cos\theta$

$$W = \vec{F} \cdot \Delta\vec{r}$$

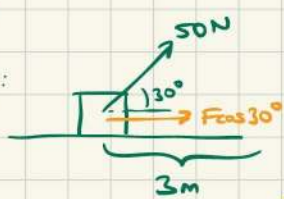
Skaler çarpım

$W = \vec{F} \cdot \Delta\vec{r}$

$N \cdot m = kg \cdot \frac{m^2}{s^2}$

1 Joule= Bir cisme 1 Newton kuvvet uygulanması ile 1 metre yer değiştirmesi sonucu kazandığı enerjiye denir.

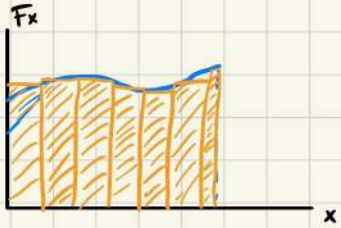
Soru:



W=?

$50 \cdot \cos 30^\circ \cdot 3 = \frac{150\sqrt{3}}{2} = 75\sqrt{3}$

Değişen Kuvvetin Yaptığı İş



$$W \simeq \Delta x_1 . F_1 + \Delta x_2 . F_2 + \dots\dots$$

$$W \simeq \sum_{i=1}^{6} F_i \, \Delta x_i$$

$$W = \lim_{\Delta x_i \to 0} \sum_{i=1}^{\infty} F_i \, \Delta x_i$$

$$\vec{F} = F_x . \hat{i} + F_y \hat{j} + F_z . \hat{k}$$

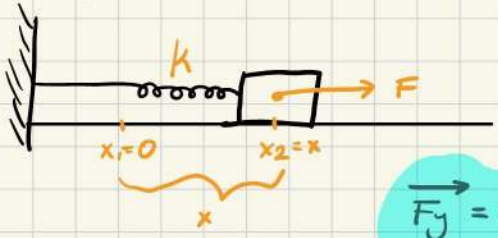
$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\vec{F} . d\vec{r} = F_x . dx + F_y . dy + F_z . dy$$

Vektörel Çarpım

$$W = \int_{x_i}^{x_s} F_x . dx$$

Yay Kuvvetinin Yaptığı İş



$$\vec{F} = F_x . \hat{i}$$

$$F \sim x$$

$$\vec{F_y} = -k . x . \hat{i}$$

$$W = \int \Sigma \vec{F} . d\vec{r}$$

$$W = -k \int_0^x x \, dx$$

$$W = -\frac{1}{2} k x^2$$

İş - Kinetik Enerji Teoremi

$$W = \int \Sigma \vec{F} \cdot d\vec{r}$$

$$\Sigma \vec{F} = m\vec{a} = m \cdot \frac{d\vec{v}}{dt}$$

$$W = \int m \cdot \frac{d\vec{v}}{dt} \cdot d\vec{r}$$

$$W = m \int d\vec{v} \cdot \frac{d\vec{r}}{dt}$$

$$W = m \int_{v_i}^{v_s} \vec{v} \cdot d\vec{v}$$

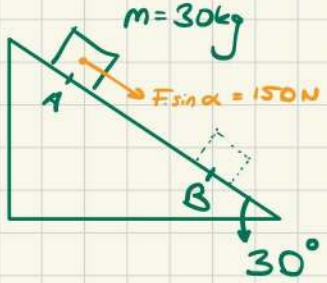
$$W = \frac{m \cdot v^2}{2} \Big|_{V_i}^{V_s}$$

$$W = \frac{1}{2} m \cdot v_s^2 - \frac{1}{2} m \cdot v_i^2$$

$$K_{(\text{kinetik enerji})} = \frac{1}{2} m v^2$$

$$W = \Delta K = K_s - K_i$$

Soru:



$\Delta x = 15m$

$v_A = 2 m/s$

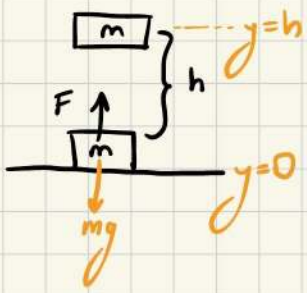
$v_B = ?$

$$150 \cdot 15 = \frac{30 \cdot 2^2}{2} - \frac{30 \cdot v_B^2}{2}$$

$$150 = \frac{30(4 - v_B^2)}{2}$$

$$12.08 = \sqrt{146} = v_B$$

Kütle Çekim Potansiyel Enerjisi



$$W = \int \Sigma \vec{F} \cdot d\vec{r}$$

$$\Sigma \vec{F} = F\hat{j} = mg\hat{j}$$

$$d\vec{r} = dy\hat{j}$$

$$W = mgh$$

$$U = mgh$$

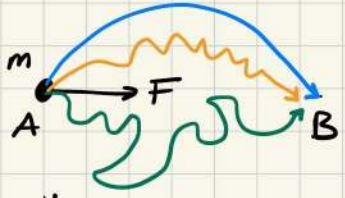
(potenj)

Elastik Potansiyel Enerji

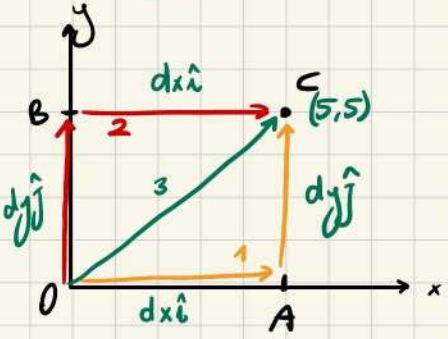
$$W = k\int_0^x x\,dx = \frac{1}{2}kx^2$$

$$U_y = \frac{1}{2}kx^2$$

Korunumlu Kuvvetler



Yapılan iş, yola bağlı değilse, yani gidilen yoldan bağımsızsa **Korunumlu kuvvet** denir.



$$\vec{F} = 2y\hat{i} + x^2\hat{j}$$

Bu cisim üç farklı yolla gidiyor. Korunumlu kuvveti araştırınız.

$$W_{OA} = \int \vec{F} \cdot d\vec{r}$$

$$W_{OA} = \int_0^5 (2y\hat{i} + x^2\hat{j}) \cdot (dx\,\hat{i})$$

$$W_{OA} = 2\int_0^5 y\,dx = 0$$

$0 \rightarrow$ A'ya giderken iş yapılmadı.

$$W_{AC} = \int_0^5 (2y\hat{i} + x^2\hat{j}) \cdot dy\,\hat{i} = x^2 \int_0^5 dy = x^2 \left(y\Big|_0^5\right)$$

$$W_{AC} = 5^2 \cdot 5 = 125 \text{ Joule}$$

$$W_{OB} = \int (2y\hat{i} + x^2\hat{j}) \cdot (dy\,\hat{j})^{0}$$

$$\int x^2\,dy = 0$$

$$W_{BC} = \int (2y\hat{i} + x^2\hat{j}) \cdot dx\,\hat{i}$$

$$2y\int_0^5 dx = 2 \cdot 5 \cdot 5 = 50 \text{ Joule}$$

Korunumlu Kuvvetler bir sistemin potansiyel enerji fonksiyonlarından türetilebilirler.

$$\Delta K + \Delta U = 0$$

$$W = \int \vec{F} \cdot d\vec{r} = \Delta K = -\Delta U(x,y,z)$$

$$\vec{F} \cdot d\vec{r} = -dU(x,y,z)$$

$$\Downarrow$$

$$F_x dx + F_y dy + F_z dz = -dU(x,y,z)$$

$$\boxed{\vec{F} = -\vec{\nabla} U}$$

Soru: $U(x,y) = 3x^3y - 7x$ pot. enj fonksiyonu sisteme etki eden kuvveti bulunuz.

$$\vec{F} = -(9x^2y - 7)\,\hat{i} - (3x^3)\,\hat{j}$$

$$\vec{F_x} = -9x^2y + 7$$

$$\vec{F_y} = -3x^3$$

Soru: Bir cisme $F = 4x\,\hat{i} + 3y\,\hat{j}$ etki ediyor. $x = 5$ noktasına hareket ediyor. İşi bulunuz.

$$W = \int \Sigma \vec{F} \cdot d\vec{r}$$

$$\Sigma \vec{F} = 4x\hat{i} + 3y\hat{j}$$

$$d\vec{r} = dx\,\hat{i}$$

$$W = \int_0^5 4x\,dx \qquad 2x^2 \Big|_0^5 = 50 \text{ Joule}$$

Soru: 3 kg kütleli cismin hız vektörü $\vec{v} = (6\hat{i} - 1\hat{j})$ m/s ile hareket ediyor.

555 J

a) Cismin K=?

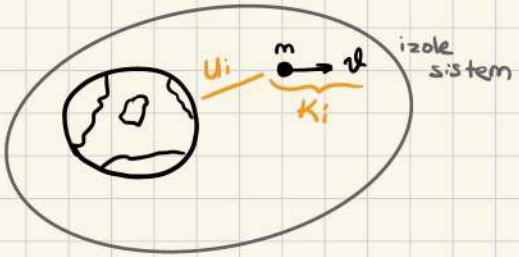
$$\frac{mv^2}{2} \longrightarrow \frac{1}{2} \cdot 3 \cdot \underbrace{[(6i - j) \cdot (6\hat{i} - j)]}_{37} = \frac{101}{2} = 55.5 \text{ Joule}$$

b) Cismin hızı $(8\hat{i} + 4\hat{j})$ m/s değerine değişirse ne kadar iş yapılmış dur?

$$\frac{3}{2} \cdot (8\hat{i} + 4j)^2 = 120 \text{ Joule} \qquad 120 - 55 = 64 \text{ J}$$

Enerji Korunumu

İzole Olmuş Sistem

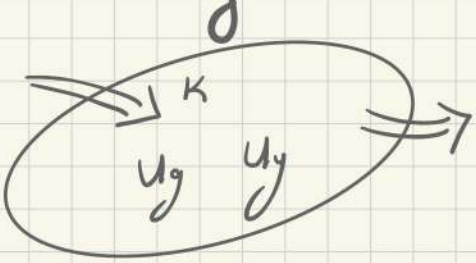


$$\Sigma E_i = \Sigma E_s$$

$$K_i + U_i = K_s + U_s$$

$$\Delta K + \Delta U_g + \Delta U_y = 0$$

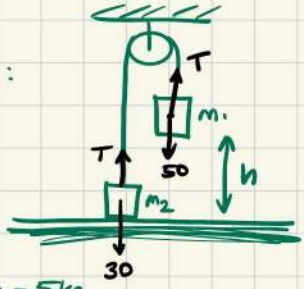
İzole Olmayan Sistem



$$W = \int \Sigma \vec{F} \cdot d\vec{r}$$

$$W_s = -\int f\,ds$$

Soru:



$m_1 = 5kg$

$m_2 = 3kg$

$h = 4m$

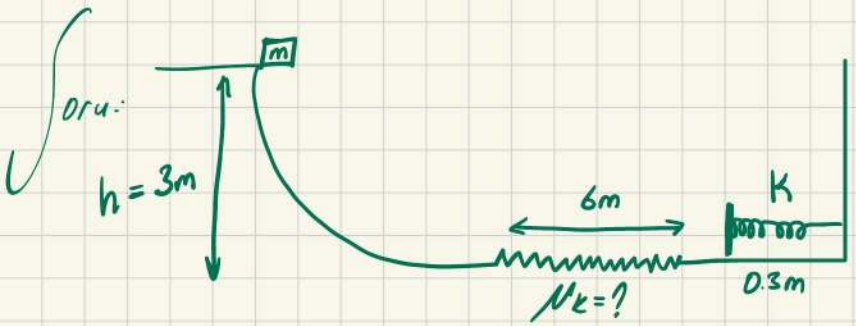
m_1, m_2 aynı yükseklikte hızları ne olur?

$E_1 = E_s = 50 \cdot 4 = 200$ $\quad V_1 = V_2 = V$

$$\cancel{200} = \cancel{80 \cdot 2} + \frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2}$$

$$40 = 4V^2$$

$$\sqrt{10} = V$$



$m = 10kg$

$k = 2250\ N/m$

$E = 300J$

$E_{sürtünme} = 600 . \mu_k$

$$(U_{gs} - U_{gi}) + (U_{ys} - U_{yi}) + (K_s - K_i) = 0$$

$$=$$

$$300 - 600\mu_k = \underbrace{\frac{1}{2} k . x^2}_{101.25}$$

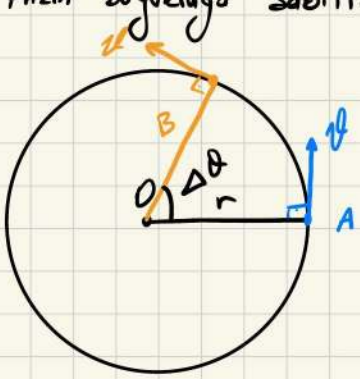
$\mu_k = 0.33$

$$300 - 600\mu_k = \frac{1}{2} 225 . \frac{9}{10}$$

$\mu_k = 0.33$

Düzgün Dairesel Hareket

- Hızın büyüklüğü sabittir.



$$|\vec{v_A}| = |\vec{v_B}| = v$$

$$|\vec{r_A}| = |\vec{r_B}| = r$$

$$\frac{|\Delta\vec{r}|}{r} = \frac{|\Delta\vec{v}|}{v}$$

$$|\Delta\vec{v}| = \frac{v \cdot |\Delta\vec{r}|}{r}$$

$$|\vec{a_{ort}}| = \frac{\Delta\vec{v}}{\Delta t}$$

$$|\vec{a_{ort}}| = \frac{v}{r} \cdot \frac{|\Delta\vec{r}|}{\Delta t}$$

$$a = \frac{v^2}{r}$$

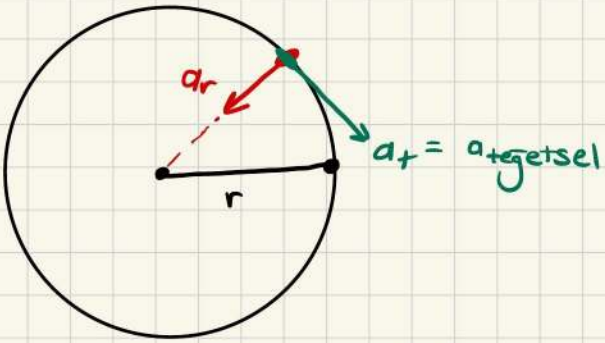
merkezcil ivme büyüklüğü

1) Hızın büyüklüğünün değişiminden dolayı sahip olduğu ivme $a_1 = \frac{dv}{dt}$

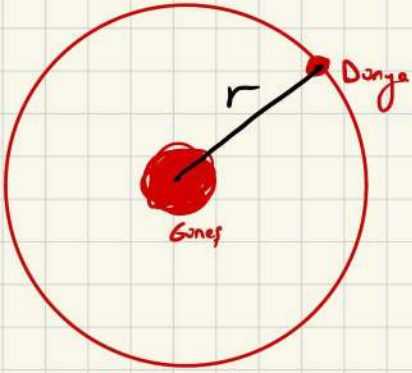
2) Hızın yönünün değişmesinden dolayı sahip olduğu ivme $a_2 = \frac{v^2}{r}$

$a_{toplam} = \vec{a_1} + \vec{a_2}$

Dairesel Hareket



Dünyanın Güneş etrafında dönmesi sonucu sahip olduğu merkezcil ivme nedir?

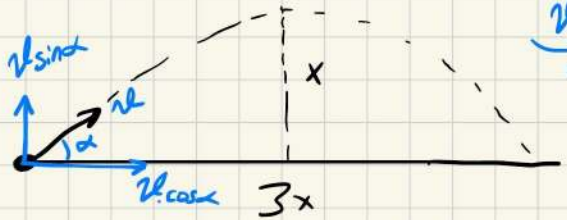


$$a = \frac{v^2}{r}$$

$$2\pi r = v_t$$

$$\frac{\left(\frac{2\pi r}{t}\right)^2}{r} = \frac{4\pi^2 r}{t^2} = a$$

Bir taş yerden belli bir açıyla fırlatılınca



$\underbrace{v\sin\alpha}_{10}\cdot t - \frac{gt^2}{2} = x$

$v\sin\alpha = 10$

$\underbrace{10^2 - 5^2}_{5^2} = x$

$\tan\alpha = \frac{10}{2}\cdot\frac{2}{3}$

$\tan\alpha = \frac{4}{3}$

$\alpha = 53^\circ$

$v\cos x \cdot 2 = 15$

$v\cos x = \frac{15}{2}$

Başlangıçta bir parçacığın $\vec{a} = 3\hat{j}\ m/s^2$

$v_0 = 5\hat{i}$ m/s olarak veriliyor.

x konum vektörü = ?

hız vektörü = ?

t=2 için koordinat = ?

t=2 için sürat = ?

$x = 0 + 5\hat{i}t + \frac{3\hat{j}\cdot t^2}{2}$

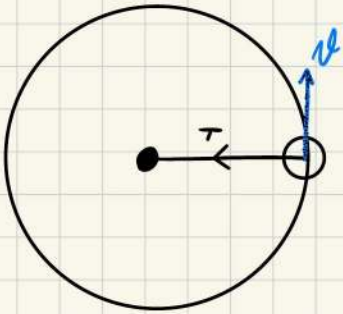
$x = 5t\,\hat{i} + \frac{3t^2}{2}\hat{j}$

$v = 5\hat{i} + 3t\hat{j}$

$(10, 6)$

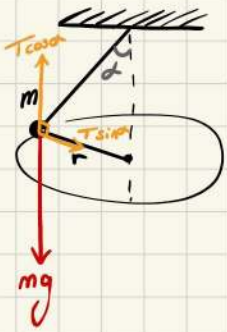
$(5\hat{i} + 6\hat{j}) = $ hız

$25 + 36 = \sqrt{71} = $ sürat

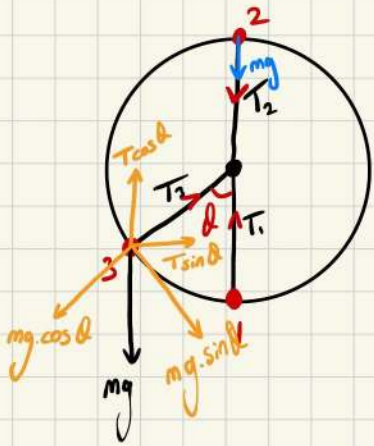


$$T = \frac{m \cdot v^2}{r}$$

$$F_r = m \cdot a_r = \frac{m \cdot v^2}{r}$$



$$\left.\begin{array}{l} T\sin\alpha = \dfrac{m \cdot v^2}{r} \\ \\ T\cos\alpha = mg \end{array}\right\} \Rightarrow v = \sqrt{r \cdot g \cdot \tan\alpha}$$

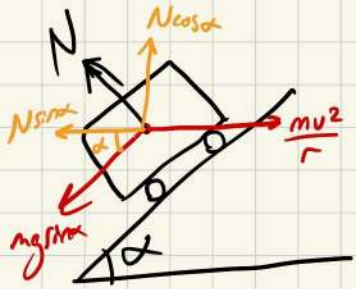


3-) $T - mg\cos\theta = \dfrac{mv^2}{r}$

$$T_3 = mg\left(\cos\theta + \frac{v^2}{rg}\right)$$

$$T_1 = mg\left(1 + \frac{v^2}{rg}\right) = mg + \frac{mv^2}{r}$$

$$T_2 = \left(\frac{mv^2}{r} - mg\right)$$



$v = 13.4\ m/s$

$r = 35m$

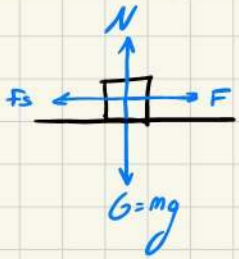
$$N\sin\alpha = \frac{mv^2}{r}$$

$$N\cos\alpha = mg$$

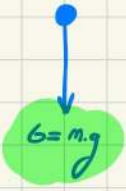
$$\tan\alpha = \frac{v^2}{r \cdot g}$$

Serbest Cisim Diyagramı

Üstünde cisme etki eden bütün kuvvetler ölçeksiz gösterilir.

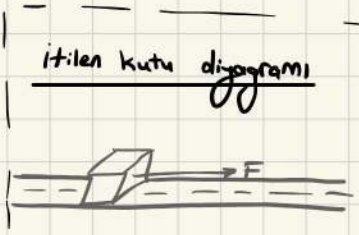


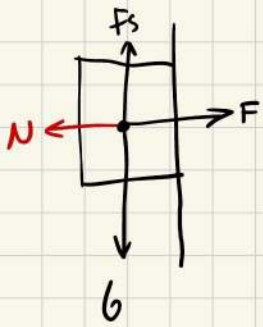
Taş binanın tepesinden bırakılıyor. Taşa etkiyen kuvvet diyagramı=?

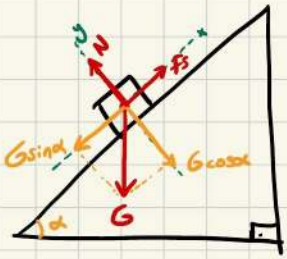


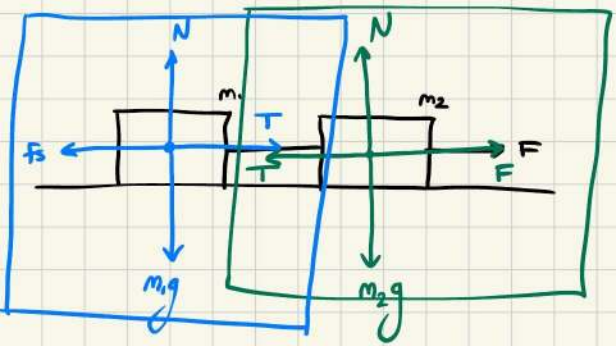
Tavandaki avize serbest cisim diyagramı

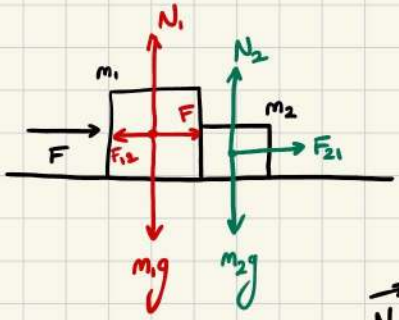
İtilen kutu diyagramı

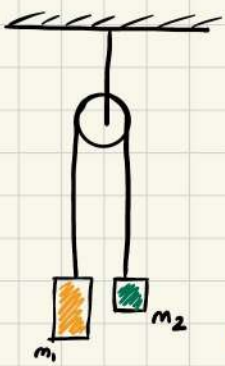


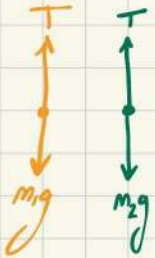


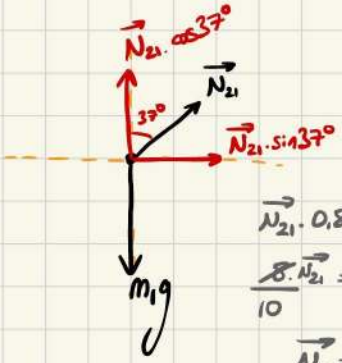


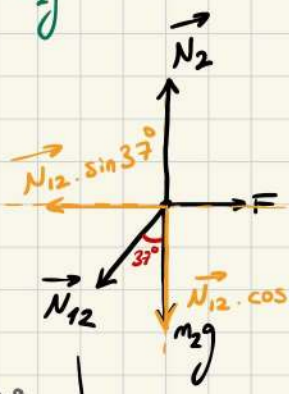












$\vec{N_{21}} \cdot 0{,}8 = m_1 \cdot g$

$\frac{8 \cdot \vec{N_{21}}}{10} = \frac{24}{3}$

$\vec{N_{21}} = 30\,N$

$\frac{30}{10} \quad \frac{\vec{N_{21}} \cdot 6}{10} = 2{,}4 \cdot a$

$F - \vec{N_{12}} \cdot \frac{6}{10} = \frac{30}{4}$

$-\frac{30 \cdot 6}{10} = 30$

-18

$F = 48$

$\frac{30}{4} = \frac{3}{0{,}4} = \frac{18}{2{,}4} = a$

Lineer Momentum ve Korunumu

$$\sum \vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$
$$\sum \vec{F}_2 = \vec{F}_{32} + \vec{F}_{12} + \vec{F}_{42}$$

+

$$\sum \vec{F}_1 + \sum \vec{F}_2 + \sum \vec{F}_3 = 0$$

$$m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 = 0$$

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} = 0$$

$$\frac{d}{dt}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3\right) = 0$$

$$m.\vec{v} = \vec{P} \rightarrow \vec{P}_{top} = sbt$$

* Bir kuvvetsiz düzlemde kütlelerin birbirine uyguladığı toplam kuvvet "0" dır.

* $a = \frac{dv}{dt}$, $\alpha = \frac{d\omega}{dt}$

Kuvvetsiz sistemde $P_i = P_s$

* Dışarıdan kuvvet uygulanmıyorsa lineer momentum muhakkak eşittir.

Soru: Sürtünmesiz düzlemde 50kg okçu 0,5kg kütleli bir ok fırlatıyor. Okun hızı 50m/s ise okçunun hızı = ?

$50.x \longleftrightarrow 0,5.50 = 25\ kg.m/s$

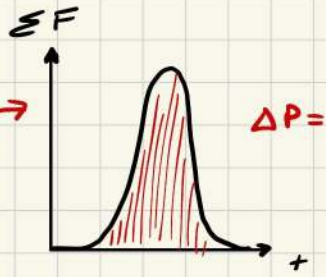
$kg.m/s$

$x = -0,5\ m/s$

$$\sum \vec{F} = m.\frac{d\vec{v}}{dt}$$

$$\sum \vec{F} = \frac{d\vec{P}}{dt}$$

$$\int_{\vec{P}_i}^{\vec{P}_s} d\vec{P} = \int_0^{+} \sum F.\, dt$$



ΔP = İtme

$$\Delta P = F_{ort} \cdot \Delta t$$

$$F_{ort} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \sum F\, dt$$

İtme (İmpuls) =

anlık momentum değişimi

1500kg otomobil duvara çarptırılıyor. Otomobilin çarpışmadan önceki ve sonraki hızları

$\vec{v}_i = -15\hat{i}$ m/s

$\vec{v}_s = 2.6\hat{i}$ m/s

Eğer çarpışma 0,15 sn sürüyorsa çarpışma itmeyi ve otomobile etki eden ortalama kuvveti bulunuz.

2) $26500 = \frac{15}{100}.\, F_{ort}$ 1)

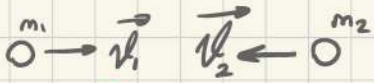
$176000\ N = F_{ort}$

$-22500\hat{i}$ kg.m/s $\longleftrightarrow$ $3900\hat{i}$ kg.m/s

1500. − 15 =

$\Delta P = 26400\hat{i}$

Çarpışmalar



$$\sum \vec{P}_i = \sum \vec{P}_s$$

* Dışarıdan kuvvet bulunmayan tüm lineer durumlarda

$$m_1 v_{1i} \hat{i} + m_2 v_{2i} \hat{i} \ldots = m_1 v_{1s} + m_2 v_{2s} \ldots$$

eğer esnekse kinetik enerji korunacağından:

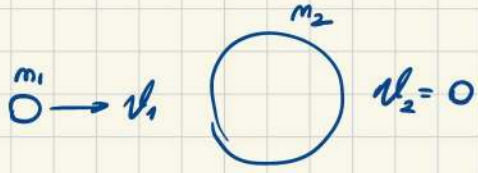
$$\frac{m_1 v_{1i}^2}{2} + \frac{m_2 v_{2i}^2}{2} + \ldots = \frac{m_1 v_{1s}^2}{2} + \frac{m_2 v_{2s}^2}{2} + \ldots$$

$$\vec{v}_{1i} + \vec{v}_{1s} = \vec{v}_{2i} + \vec{v}_{2s} = \ldots$$

$$v_{1s} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2s} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) v_{2i} + \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

1. Durum



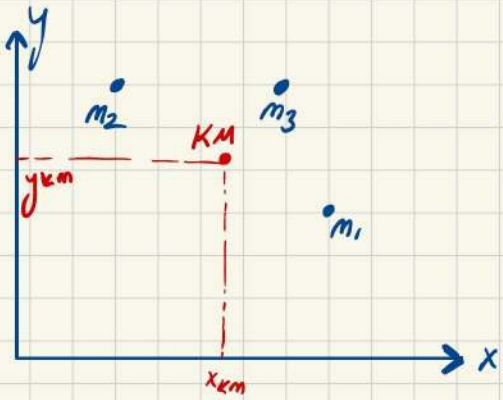
* m_1 ihmal edilebilecek kadar küçük

$$v_{1s} = -v_{1i}$$

$$v_{1s} = -v_{1i}$$

$$v_{2s} \simeq 0$$

Kütle Merkezi



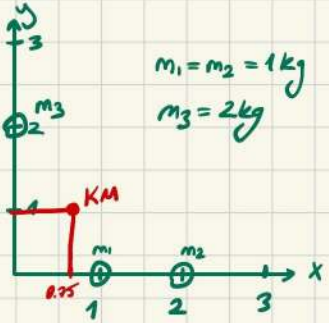
$$x_{KM} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + m_3 \dots}$$

$$x_{KM} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i$$

$$y_{KM} = \frac{1}{M} \cdot \sum_{i=1}^{n} m_i y_i$$

$$\overrightarrow{r_{KM}} = x_{KM}\hat{i} + y_{KM}\hat{j} + z_{KM}k$$

Soru:



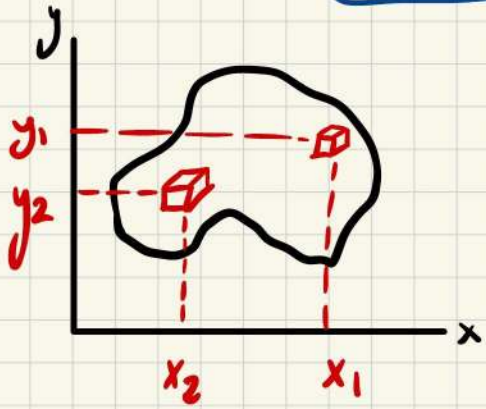
$m_1 = m_2 = 1\,kg$

$m_3 = 2\,kg$

$$x_{KM} = \frac{3}{4} = 0{,}75\,m$$

$$y_{KM} = \frac{4}{4} = 1\,m$$

Sürekli Kütle Dağılımına Sahip Cisimlerin Kütle Merkezleri



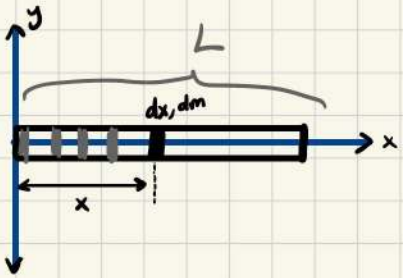
$$x_{KM} \simeq \frac{1}{M} \sum_{i=1}^{n} \Delta m_i x_i$$

$$\downarrow$$

$$x_{KM} \simeq \frac{1}{M} \cdot \lim_{\Delta m_i \to 0} \sum_{i=1}^{\infty} \Delta m_i x_i$$

$$\downarrow$$

$$x_{KM} = \frac{1}{M} \int y \, dm$$



$$\frac{L}{dx} \times \frac{M}{dm}$$

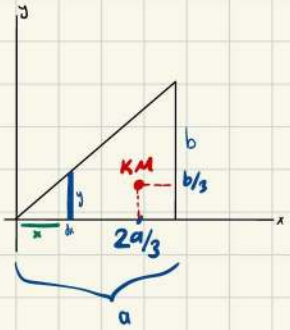
$$dm = \frac{M}{L}\,dx \quad \Big| \quad dm = \lambda\,dx$$

λ: çizgisel kütle yoğunluğu

$$X_{KM} = \frac{1}{M}\int x\,dm$$

$$X_{KM} = \frac{1}{M}\int_0^L x\,\lambda\,dx$$

$$X_{KM} = \frac{\lambda}{M}\cdot\frac{L^2}{2} = \frac{1}{2}L$$



$$dm = \sigma\,dA$$

$$\sigma = \frac{M}{A};\quad dA = y\,dx$$

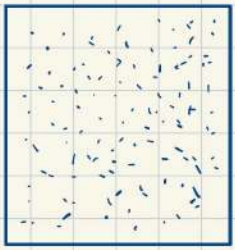
$$dm = \sigma y\,dx$$

$$X_{KM} = \frac{1}{M}\int x\,dm$$

$$X_{KM} = \frac{\sigma}{M}\int_0^a x\,y\,dx$$

$$X_{KM} = \frac{2a}{3}$$

$$y_{KM} = \frac{b}{3}$$



$\vec{v}$

$\vec{a}$

$$\vec{r}_{km} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i$$

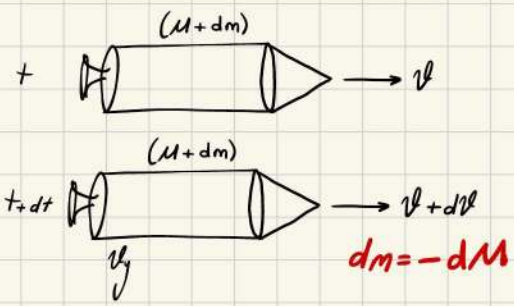
$$\vec{v}_{KM} = \frac{d\vec{r}_{KM}}{dt} = \frac{d}{dt}\left[\frac{1}{M} \sum_i m_i \vec{r}_i\right]$$

$$M \vec{v}_{km} = \sum_i m_i . \vec{v}_i$$

$$\vec{a}_{KM} = \frac{d\vec{v}_{KM}}{dt} = \frac{d}{dt}\left[\frac{1}{M} \sum_i m_i \vec{v}_i\right]$$

$$M.\vec{a} = \sum_i m_i \vec{a}_i \rightarrow \vec{F} = \sum_i \vec{F}_i$$

Roket Hareketi



$dm = -dM$

+ anında

$$(M+dm)v = M(v+dv) + dm(v - v_y)$$

$$v_s = v_i + v_y \ln\left(\frac{M_i}{M_s}\right)$$

$$T = M.\frac{dv}{dt} \rightarrow T = \left| v_y \frac{dM}{dt} \right|$$

Roket hızı $3.10^3 m/s$

Bu anda roket ateşleniyor.

Roketin yakıtı $5.10^3 m/s$'lik hızla terk ediyor.

Roketin kütlesi yarıya düşerse yere göre hız = ?

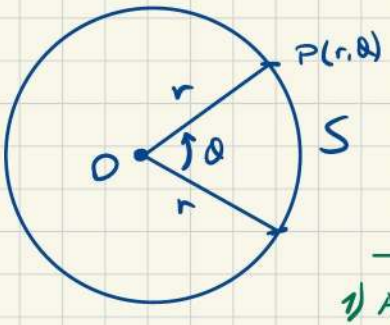
$$v_s = v_i + v_y \ln\left(\frac{M_i}{0.5M_i}\right)$$

$$(3\times10^3) + (5\times10^3).\ln 2 = 6.5\times10^3 m/s$$

Roket yakıtı 50 kg/s oranında yanıyorsa itme kuvveti = ?

$$\left| v_y \frac{dm}{dt} \right| = T = 50.5.10^3 = 25.10^4 N$$

Katı Bir Cismin Sabit Eksende Dönmesi



Tam bir tur için

$2\pi r = r\,\theta$

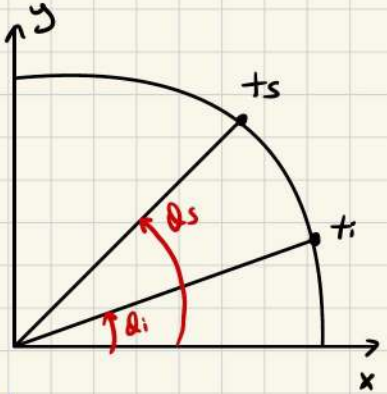
$2\pi = \theta$ (radyan)

$S = r \cdot \theta$

θ birimi = radyan

1) Açısal Yer Değiştirme

$$\Delta\theta = \theta_s - \theta_i$$



2) Ortalama Açısal Hız

$$w_{ort} = \frac{\Delta\theta}{\Delta t} \quad \left(\frac{rad}{s}\right) \quad (s^{-1})$$

3) Ani Açısal Hız

$$w = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta(t)}{dt}$$

türev operatörü

4) Ortalama Açısal İvme

$$\alpha_{ort} = \frac{\Delta w}{\Delta t} \quad \left(\frac{rad}{s^2}\right)$$

5) Ani Açısal İvme

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw(t)}{dt}$$

Sabit Açısal İvmeli Dönme Hareketi

Dönme Hareketi	Öteleme Hareketi
$\omega_s = \omega_0 + \alpha t$	$v_s = v_i + a t$
$\theta_s = \theta_0 + \omega_0 t + \frac{\alpha t^2}{2}$	$x_s = x_i + v_i t + \frac{a t^2}{2}$
$\omega_s^2 = \omega_0^2 + 2\alpha(\theta_s - \theta_i)$	$v_s^2 = v_0^2 + 2a(x - x_0)$
(α = sbt)	(a = sbt)
değilse;	değilse;
$\omega = \frac{d\theta}{dt}$	$v = \frac{dx}{dt}$
$\alpha = \frac{d\omega}{dt}$	$a = \frac{dv}{dt}$

Soru: $\alpha = 3{,}5\ rad/s^2$

$t_0 = 0 \Rightarrow \omega_0 = 2\ rad/s$

a) $t = 2sn$ 'deki $\Delta\theta = ?$

$\theta_s = \theta_i + \omega_0 t + \frac{\alpha t^2}{2}$

$\Delta\theta = 4 + \frac{3{,}5 \cdot 4}{2} = 11\ rad$

b) $t = 2sn$, kaç tur atar?

$\frac{11}{2\pi}$ derece yani $\frac{180 \cdot (11)}{\pi} = 630°$

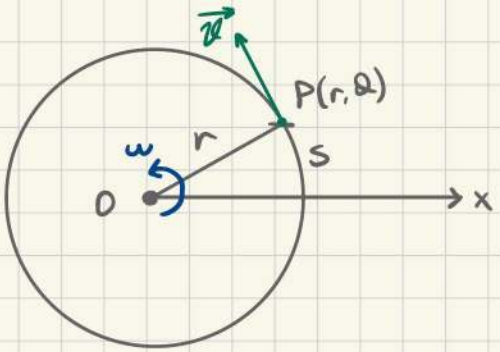
1.75 tur

c) $t = 2sn$ açısal hız = ?

$\omega_s = \omega_0 + \alpha t$

$\omega_s = 2 + 7$

$\omega_s = 9\ rad/s$



$$S = r\,\theta$$

$$\frac{ds}{dt} = r \cdot \frac{d\theta}{dt}$$

$$v = r \cdot \omega$$

$$\frac{dv}{dt} = r \cdot \frac{d\omega}{dt}$$

$$a = \alpha r$$

Dönme Kinetik Enerjisi

$$K \simeq \frac{1}{2}\Delta m_1 v_1^2 + \frac{1}{2}\Delta m_2 v_2^2 + \dots$$

$$K \simeq \sum_i^N \frac{1}{2}\Delta m_i v_i^2$$

$$K \simeq \frac{1}{2}\left(\sum_i^N \Delta m_i r_i^2\right)\omega^2$$

- Sadece "ω" hepsinde aynı.

$$K = \frac{1}{2}\left(\lim_{\Delta m_i \to 0} \sum_{i=1}^{\infty} \Delta m_i r_i^2\right)\cdot \omega^2$$

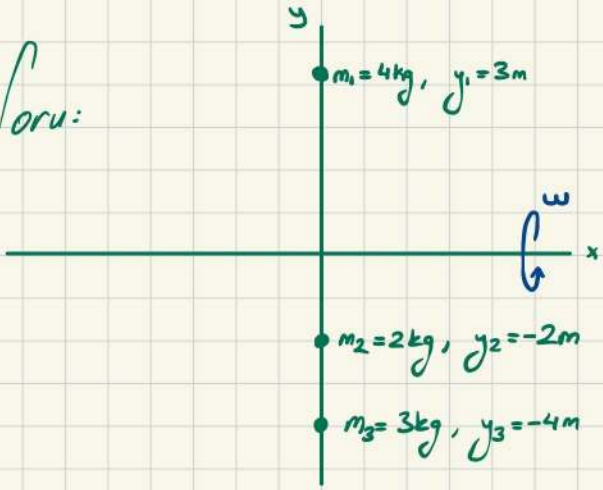
$$\int r^2\,dm = I$$

- eylemsizlik momenti

$$\sum_i m_i r_i^2 = I$$

$$K = \frac{1}{2} I \omega^2$$

Soru:



$\omega = 2\ rad/sn$

Sistemin kinetik enerjisi kaç J = ?

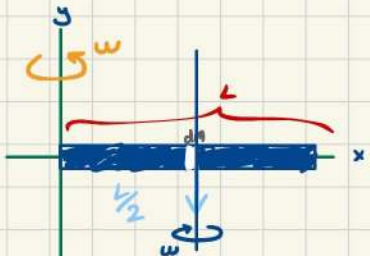
$$K = \frac{1}{2} I \omega^2$$

$$I = \sum_i m_i r_i^2 \text{ ise,}$$

$$\underbrace{\frac{4.9}{36}} + \underbrace{\frac{2.4}{8}} + \underbrace{\frac{3.16}{48}} = 92\ kg.m^2$$

$$\frac{1}{2} 92.2^2 = 184\ J$$

Soru:



$$I = \int x^2\, dm$$

$$dm = \frac{M}{L} \cdot dx$$

$$I = \int_0^L x^2 \frac{M}{L}\, dx$$

$$\frac{x^3}{3}\Big|_0^L = \frac{L^3}{3} \cdot \frac{M}{L} = \frac{ML^2}{3}$$

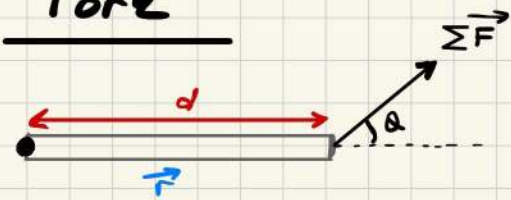
eylemsizlik momenti nedir?

$$I = \int x^2\, dm$$

$$I = \lambda \cdot \frac{L^3}{3}$$

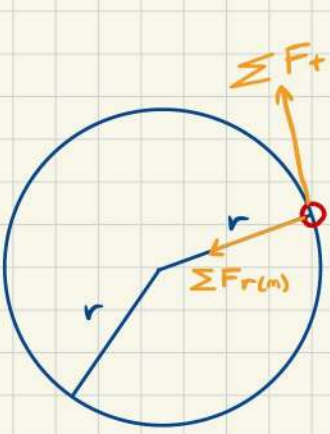
$$I = \frac{1}{3} ML^2$$

Tork



$$\tau = (\Sigma F . \sin\theta) . d$$

$$\tau = \vec{r} \times \Sigma \vec{F}$$



merkezcil kuvvet tork üretmez.

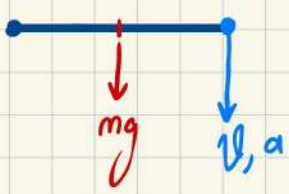
$$\Sigma F_t = m . \alpha_t$$

$$r\alpha = a_t$$

$$\Sigma \tau = (m . r^2)\,\alpha$$

$$\Sigma \tau = I\,\alpha$$

Soru: m kütleli çubuk yatayla paralelken serbest bırakılıyor. Anlık açısal ivmeyi ve çubuğun en ucunun çizgisel ivmesini bulunuz.



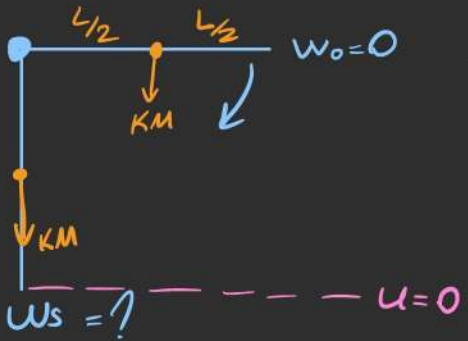
$$I = \frac{1}{3} mL^2$$

$$\tau = I . \alpha = mg . \frac{L}{2}$$

$$\frac{1}{3} mL^2 . \alpha = mg . \frac{L}{2}$$

$$\alpha = \frac{3g}{2L}$$

$$\frac{3g}{2L} . L = \frac{3g}{2}$$



$\Delta E = 0$

$$mgL = K + mg\frac{L}{2}$$

$$\frac{mgL}{2} = K$$

$$\frac{mgL}{2} = \frac{I}{2} w_s^2$$

$$\frac{3\,mgL}{mL^2} = w_s^2$$

$$\sqrt{\frac{3g}{L}} = w_s$$

$r \cdot w = v$

$$\frac{L}{2} \cdot \sqrt{\frac{3g}{L}} = v_{KM}$$

$$\sqrt{3gL} = v_p$$

Dönen katı cismin üzerindeki herhangi bir noktanın bir sabit eksenle yaptığı açının zamanla değişimi

$$\theta = (5 + 10t + 2t^2)\ rad$$

$t = 3$ sn için

açısal konum, hız, ivme

$w = 10 + 4t \longrightarrow 22$

$\alpha = 4 \longrightarrow \alpha = 4$

$\theta = 53\ rad$

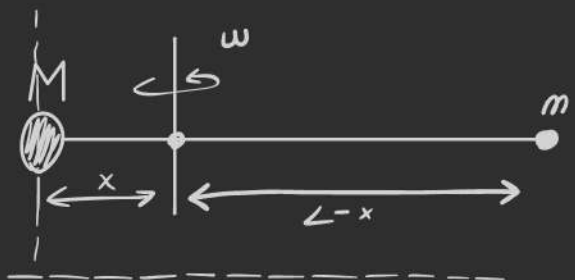
$$\alpha = (10 + 6t)\ rad/s^2$$

$$10 + 6t = \frac{dw}{dt}$$

$$w(t) = (10t + 3t^2)\ rad/s$$

$$\theta(t) = (5 + t^2 + t^3)\ rad$$

$$\theta(4) = 144\ rad$$



$$I = \sum_i m_i r_i^2$$

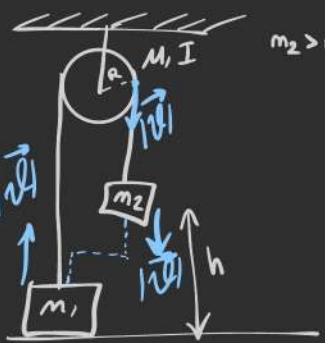
$$I = Mx^2 + m(L-x)^2$$

$$\frac{dI}{dx} = 2Mx + 2m(L-x)(-1)$$

$$I = \mu L^2 \qquad \mu = \frac{Mm}{m+M}$$

$$Mx = mL - mx$$

$$x = \left(\frac{m}{m+M}\right) L$$



$m_2 > m_1$

$$m_2 g h = \frac{m_2 g h}{2} + \frac{m_1 g h}{2} + \frac{m_1 v^2}{2} + \frac{m_2 v^2}{2} + \frac{I\omega^2}{2}$$

$$r \cdot \omega = v$$

$$\sqrt{\frac{(m_2 - m_1) g h}{m_1 + m_2 + I/R^2}} = 1k$$

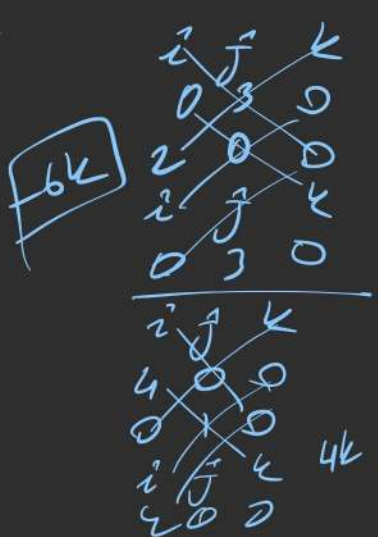
$8 \cdot 40 = 320\ J \qquad \frac{20 \cdot 10 \cdot 9}{2}$

$320 + 180 = \frac{500}{2} J = \frac{20}{40} \cdot \frac{v^2}{2}$

$1k = 5$

$\frac{5}{40} \cdot \frac{6}{8} = \frac{6}{48} \cdot 1k$

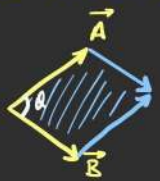
$\vec{F_1} \times \vec{r_1} =$



Açısal Momentum

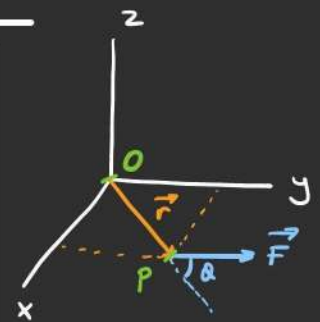
$$\vec{C} = \vec{A} \times \vec{B}$$

$$C = A.B.\sin\theta$$



$$*\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$$

$$*(\vec{A}\times\vec{B})\frac{d}{dt} = \frac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\frac{d\vec{B}}{dt}$$



$$\sum \vec{\tau} = \vec{r} \times \vec{F}$$

$$\sum \vec{\tau} = \frac{d}{dt}(\vec{r}\times\vec{P})$$

$$\Downarrow$$

$$\vec{L} \equiv \vec{r} \times \vec{P}$$

$$\Downarrow$$

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$$

$$L_x = (yP_z - zP_y)$$

$$L_y = (zP_x - xP_z)$$

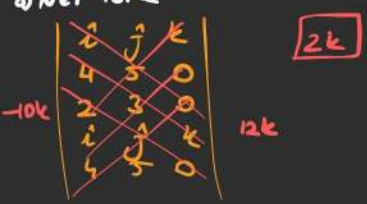
$$L_z = (xP_y - yP_x)$$

Soru

$\vec{F} = (2\hat{i} + 3\hat{j})$ N

$\vec{r} = (4\hat{i} + 5\hat{j})$ m

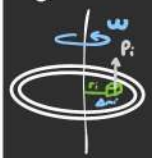
a) Net tork



Noktasal Parçacıklar ise

$$\vec{L}_{top} = \sum_i \vec{L}_i$$

Rijit Cisim Açısal Momentumu



$$\vec{L}_i = \vec{r}_i \times \vec{P}_i$$

$$L_i = r_i.\Delta m_i.v_i = r_i\,\Delta m_i.r_i.\omega$$

$$L_{top} \simeq \left(\sum_i r_i \Delta m_i\right)\omega$$

$$L_{top} = \left(\lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i\right)\omega$$

$$L_{top} = \left(\int r^2 dm\right).\omega \Rightarrow L = I.\omega$$

$\omega = 10$ devir/sn $\longrightarrow$ $10\left(\frac{\text{devir}}{s}\right)\cdot\left(\frac{2\pi\ rad}{1\ devir}\right) = \frac{20\pi\ rad}{s}$

$m = 7kg$

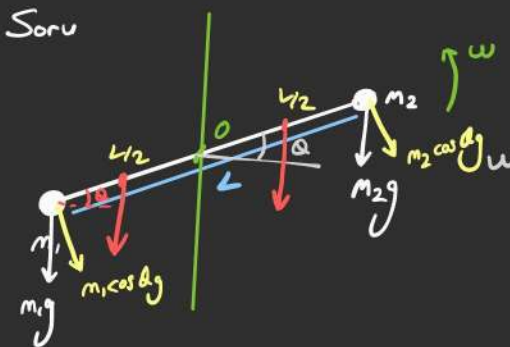
$R = 0.12\ m$

$L = I.\omega$

$\frac{2}{5}.7.(0,12)^2.\ 20\pi =$

$2.53\ kg.m^2/s =$

Soru



Açısal mom korunur.

$L = I.\omega$

$\omega.\left(m_1\frac{L^2}{4} + m_2\frac{L^2}{4} + \frac{ML^2}{12}\right) = \frac{3(m_1+m_2)L^2 + ML^2}{12}.\omega = L$

θ açısı anında sistem açısal ivmeyi bul.

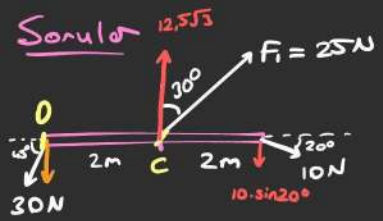
$\tau = I.\alpha$

$|m_1 \cos\theta g - m_2 \cos\theta g| = \frac{3(m_1+m_2)L^2 + ML^2}{12}.\alpha$

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

$\sum \vec{\tau} = 0$ ise açısal momentum sabit

Sorular



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xy düzleminde hareket eden
1,5 kg kütleli cismin hız vektörü
$(4,2\hat{i} - 3,6\hat{j}) = \vec{v}$

Orijinden çizilen konum vek = $(1,3\hat{i} + 2,2\hat{j})$

- Açısal Momentum

$L = I.w$

$L = \vec{r} \times \vec{p}$

$\vec{p} = (6,3\hat{i} - 5,4\hat{j})$

$\vec{r} = (1,3\hat{i} + 2,2\hat{j})$

$\vec{L} = -20,43\hat{k}$

$L = I.w = \vec{r} \times \vec{p}$

$mv = (m+M)\, v_{kop}$

$\frac{mv}{m+M} = v_{kop}$

$(m+M)\, v_{kop} . l$

$r.w = v$

$w = \frac{v}{r}$

Statik Denge ve Esneklik

* Tüm kuvvetlerin bileşkesi sıfır ise cisim dengededir.

$\sum F = 0$ → durağan / sabit hızlı

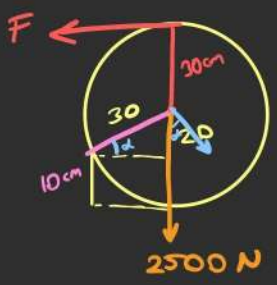
*** Statik = Durgun**

Cisim → noktasal → $\sum F = 0$ yeterlidir.

Cisim → rijit (gerçek) → $\sum F = 0$ ve cismin herhangi bir noktaya göre torku sıfır olmalıdır.

$\vec{\tau} = \vec{F} \cdot d \ (F \perp d) = \vec{F} \times \vec{d}$ — öteleme dengesi

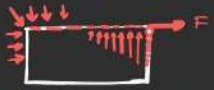
$\vec{\tau} = \vec{F} \cdot d \cdot \sin\alpha$ — dönme dengesi



$\sum \tau =$

Esneklik

* Gerçekte bütün cisimler bozunmaya uğrar.



* Her bir rijit cisim, dış kuvvetlere karşı belirli bir direnme ve rijitliğini koruma sınırı vardır. Direnme noktaları dayanamaz ise kırılma bölgesine uğrar.

Esneklik Modülü

* Young modülü: Katının uzunluğundaki bir değişime karşı gösterdiği direnç oranı
* Kesme sabiti: Katının, atomik düzeylerde birbiri üzerinde kayması -enine kesme- ortaya çıkan direnç
* Hacim modülü: Hacim değişikliklerinde gelebilecek direnç ölçütü

$F_s = -k \cdot x$

$x = A \cdot \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega}$

$a_{max} = A\omega^2$ → maks. genlik

$v_{max} = A\omega$

$v(t) = -A \cdot \omega \cdot \sin(\omega t + \phi)$

$a(t) = -A \cdot \omega^2 \cdot \cos(\omega t + \phi)$

---

yay-kütle

$2\pi\sqrt{\frac{m}{k}} = T = \frac{2\pi}{\omega}$

anlık kuvvet uygulanırsa açısal momentum korunur.

ilerleyen cismin hayaldeğ yarıçapı

$f = 0$

Çizgisel

$mv_0 = mv + mwr$

$= mwd + Mwr$

$= \frac{(\frac{1}{3}ML^2 + md^2)w}{d} = w(md + Mr)$

$r = \frac{L}{2}$

Açısal

$L_i = L_s \quad L = r \times P$

$mv_0 d = I_n \cdot w$

$mv_0 = \frac{(\frac{1}{3}ML^2 + md^2) \cdot w}{d}$

$\frac{1}{3d}ML^2 + md = md + \frac{ML}{2}$

$\frac{1M}{3d} = \frac{ML}{2}$

$d = \frac{2L}{3}$