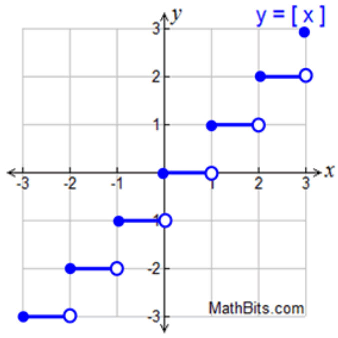
~~~~~~ FUNCTIONS AND PROPERTIES ~~~~~~

GREATEST INTEGER FUNCTION

$\boldsymbol{f(x) = [\![g(x)]\!]}$ ***and*** $\boldsymbol{n} \in \mathbb{Z}$;

$\boldsymbol{f(a) = n + 1}$, ***if*** $\boldsymbol{n < g(a) \leq n + 1}$



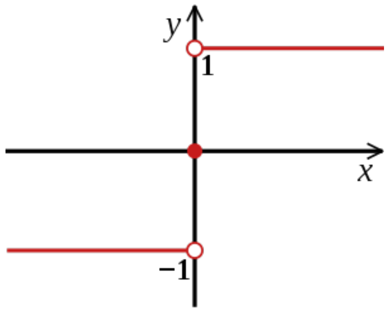
SIGNUM FUNCTION

$$\boldsymbol{sgn(f(x)) = \begin{cases} -1, & if\ f(x) < 0 \\ 0, & if\ f(x) = 0 \\ 1, & if\ f(x) > 0 \end{cases}}$$

$\longrightarrow \quad sgn\big(f(x)\big) = \dfrac{d}{dx}|f(x)| \quad for\ x \neq 0$

$\longrightarrow \quad f(x) = sgn\big(f(x)\big) \cdot |f(x)|$

$\longrightarrow \quad sgn(x^n) = sgn(x)^n$



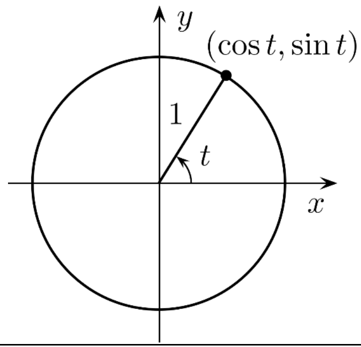
PERIODIC FUNCTIONS

$f(x+T) = f(x)$. *Then f is periodic function and T is called period of f.*

TRIGONOMETRIC FUNCTIONS

Unit Circle

Perimeter of Unit Circle:	$2\pi r = 2\pi, \quad since\ r = 1$
Arc length in Unit Circle: **(t is central angle of arc)**	$l = 2\pi r \cdot \dfrac{t}{2\pi} = t$ So, arc length is equal to dimension of angel in a unit circle



Periods of Trigonometric Functions

$n \in \mathbb{N}$	$\sin^{2n+1}(ax+b)$, $\cos^{2n+1}(ax+b)$, $\sec^{2n+1}(ax+b)$, $\csc^{2n+1}(ax+b)$	$\dfrac{2\pi}{\lvert a \rvert}$
	$\sin^{2n}(ax+b)$, $\cos^{2n}(ax+b)$, $\sec^{2n}(ax+b)$, $\csc^{2n}(ax+b)$, $\tan^{n}(ax+b)$, $\cot^{n}(ax+b)$	$\dfrac{\pi}{\lvert a \rvert}$

LOGARITMIC FUNCTIONS AND PROPERTIES

1. $\log_a x\,,\quad x \in \mathbb{R}^+,\quad a \in \mathbb{R}^+ - \{1\}$	**2.** $f(x) = a^x \quad \Longrightarrow f^{-1}(x) = \log_a x$
3. $\log_a a = 1$	**4.** $\log_a 1 = 0$
5. $\log_a a^n = n$	**6.** $\log_a x^n = n.\log_a x = n$
7. $\log_{a^m} x = \frac{1}{m}.\log_a x = \frac{1}{m}$	**8.** $\log_{a^m} x^n = \frac{n}{m}.\log_a x = \frac{n}{m}$
9. $\log x = \log_{10} x$	**10.** $\ln x = \log_e x$
11. $\log_a x \cdot y = \log_a x + \log_a y$	**12.** $\log_a \frac{x}{y} = \log_a x - \log_a y$
13. $\log_a x = \frac{\log x}{\log a} = \frac{\log_c x}{\log_c a}$	**14.** $\log_a x = \frac{1}{\log_x a}$
15. $\log_a y \cdot \log_y x = \log_a x$	**16.** $a^{\log_c x} = x^{\log_c a}$
17. $a^{\log_a x} = x^{\log_a a} = x$	**18.** $e^{\ln x} = x$

HYPERBOLIC AND INVERSE HYPERBOLIC FUNCTIONS

Hyperbolic Functions	Inverse Hyperbolic Functions	
$\sinh x = \dfrac{e^x - e^{-x}}{2}$	$\operatorname{arcsinh} x = \ln\left(x + \sqrt{x^2+1}\right)$	$x \in \mathbb{R}$
$\cosh x = \dfrac{e^x + e^{-x}}{2}$	$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2-1}\right)$	$x \geq 1$
$\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	$\operatorname{arctanh} x = \dfrac{1}{2}\ln\dfrac{1+x}{1-x}$	$\lvert x\rvert < 1$
$\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$	$\operatorname{arccoth} x = \operatorname{arctanh}\dfrac{1}{x} = \dfrac{1}{2}\ln\dfrac{x+1}{x-1}$	$\lvert x\rvert > 1$
$\operatorname{sech} x = \dfrac{2}{e^x - e^{-x}}$	$\operatorname{arcsech} x = \operatorname{arccosh}\dfrac{1}{x} = \ln\left(\dfrac{1+\sqrt{1-x^2}}{x}\right)$	$0 < x \leq 1$
$\operatorname{csch} x = \dfrac{2}{e^x + e^{-x}}$	$\operatorname{arccsch} x = \operatorname{arcsinh}\dfrac{1}{x} = \ln\left(\dfrac{1}{x} + \dfrac{1+\sqrt{1+x^2}}{\lvert x\rvert}\right)$	$x \neq 0$

Properties of Hyperbolic Functions:

$\longrightarrow \cosh^2 x - \sinh^2 x = 1$
$\longrightarrow \cosh(-x) = \cosh x$
$\longrightarrow \sinh(-x) = -\sinh x$

$\longrightarrow \sinh(x+y) = \sinh x \cosh y + \sinh x \cosh y$
$\longrightarrow \cosh x + y = \cosh x \cosh y + \sinh x \sinh y$
$\longrightarrow \cosh 2x = 2\cosh^2 x - 1 = \sinh^2 x + \cosh^2 x = 2\sinh^2 x + 1$

POLYNOMIAL FUNCTIONS				
$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$				
Polynomial	**Degree**	**Example**	**Number of Terms**	**Name Using Number of Terms**
Constant or Zero Polynomial	0	6	1	Monomial
Linear Polynomial	1	3x+1	2	Binomial
Quadratic Polynomial (Parabola)	2	$3x^2$	1	Monomial
Cubic Polynomial	3	$6x^3 + 3x^2 - 5x + 7$	4	Polynomial of 4 terms
Quartic Polynomial	4	$2x^4 + 4x^3 - 1$	3	Trinomial
Quintic Polynomial	5	$x^5 + 16$	2	Binomial

~~~~~~ LIMIT & DERIVATIVE ~~~~~~

DEFINITION OF LIMIT			
M, N, $\varepsilon, \delta > 0$	$x \to -\infty$	$x \to a$	$x \to \infty$
$f(x) \to -\infty$	$f(x) < -M\ ise;$ $x < -N(M)$	$f(x) < -M\ ise;$ $\lvert x-a \rvert < \delta(M)$	$f(x) < -M\ ise;$ $x > N(M)$
$f(x) \to L$	$\lvert f(x) - L \rvert < \varepsilon\ ise;$ $x < -N(\varepsilon)$	$\lvert f(x) - L \rvert < \varepsilon\ ise;$ $\lvert x-a \rvert < \delta(\varepsilon)$	$\lvert f(x) - L \rvert < \varepsilon\ ise;$ $x > M(\varepsilon)$
$f(x) \to \infty$	$f(x) > M\ ise;$ $x < -N(M)$	$f(x) > M\ ise;$ $\lvert x-a \rvert < \delta(M)$	$f(x) > M\ ise;$ $x > N(M)$

CONVERGENCE AND DIVERGENCE		
If a limit of a thing is equal to a real number; (that is to say, the limit exist) **"the thing is convergent"**		If a limit of a thing is equal to infinity; (that is to say, the limit does not exist) **"the thing is divergent"**
Types of Thing	**Condition of Convergent Thing**	**Condition of Divergent Thing**
Sequence	$\lim_{n\to\infty} a_n = a \quad where\ a \in \mathbb{R}$ This means that the sequence converges to "a"	$\lim_{n\to\infty} a_n = \pm\infty$ This means that the sequence diverges to $\pm\infty$
Function	$\lim_{n\to\pm\infty} f(x) = L \quad where\ L \in \mathbb{R}$ This means that the function converges to "L"	$\lim_{n\to\pm\infty} f(x) = \pm\infty$ This means that the function diverges to $\pm\infty$
Infinite Series (Sum of Sequence)	$\lim_{n\to\infty} \sum_{i=0}^{n} a_n = \lim_{n\to\infty} S_n = S \quad where\ S \in \mathbb{R}$ This means that the series converges to "S"	$\lim_{n\to\infty} \sum_{i=0}^{n} a_n = \lim_{n\to\infty} S_n = \pm\infty$ This means that the series diverges to $\pm\infty$
Improper Integtals (Sum of function)	$\lim_{n\to\infty} \int_m^n f(x)dx = S \quad where\ S \in \mathbb{R}$ This means the integral converges to "S"	$\lim_{n\to\infty} \int_m^n f(x)dx = \pm\infty$ This means that the integral diverges to $\pm\infty$

SOME LIMIT CALCULATIONS

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 \quad and \quad Q(x) = b_m x^m + b_{m-1}x^{m-1} + \cdots + b_2x^2 + b_1x + b_0$$

$$\lim_{x\to\infty}\frac{P(x)}{Q(x)} = \begin{cases} sgn\left(\frac{a_n}{b_m}\right)\cdot\infty, & n > m \\ \frac{a_n}{b_m}, & n = m \\ 0, & n < m \end{cases}$$

$$\lim_{x\to-\infty}\frac{P(x)}{Q(x)} = \begin{cases} sgn\left(\frac{a_n x^n}{b_m x^m}\right)\cdot\infty, & n > m \\ \frac{a_n}{b_m}, & n = m \\ 0, & n < m \end{cases}$$

$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to\infty}\frac{\sin(1/x)}{1/x} = \lim_{x\to\infty} \mathrm{x}\cdot\sin(1/x) = 1$$

$$\lim_{x\to 0}\frac{\sin ax}{bx} = \lim_{x\to\infty}\frac{\sin(a/x)}{b/x} = \frac{a}{b}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to 0}(1+x)^{\frac{1}{x}} = \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{\frac{x}{a}} = e$$

$$\lim_{x\to\infty}\left(1+\frac{a}{cx+d}\right)^{bx} = e^{\frac{a\cdot b}{c}}$$

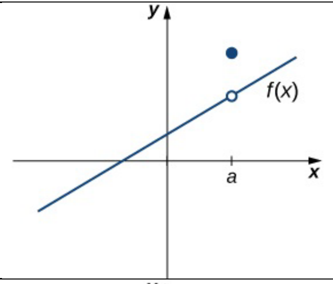
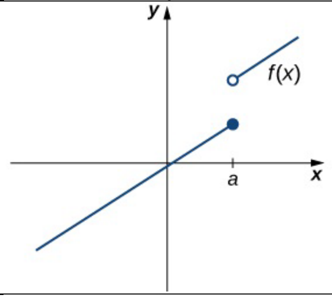
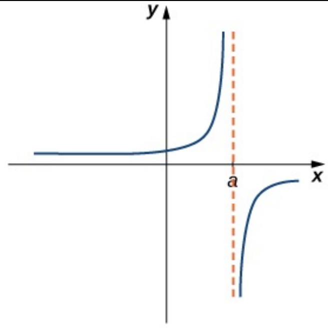
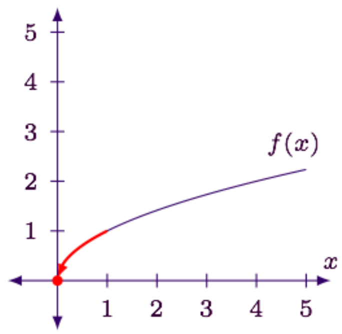
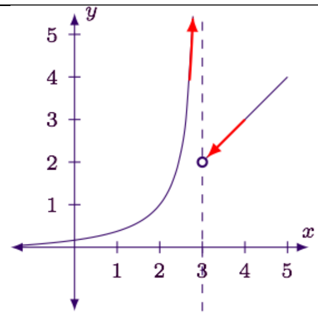
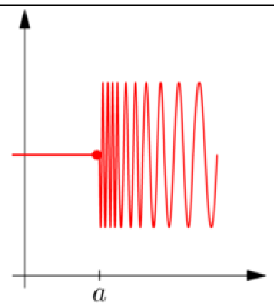
$$\lim_{x\to\infty}\left(1+\frac{a}{Q(x)}\right)^{P(x)} = \begin{cases} \infty, & n > m \text{ and } sgn\left(\frac{a_n}{b_m}\right) = 1 \\ 0, & n > m \text{ and } sgn\left(\frac{a_n}{b_m}\right) = -1 \\ e^{a\cdot\frac{a_n}{b_m}}, & n = m \\ 1, & n < m \end{cases}$$

$$\lim_{x\to 0}\frac{x}{\log_a(1+x)} = \lim_{x\to 0}\frac{a^x-1}{x} = \ln a \qquad (a \neq 1, \quad a > 0)$$

CONDITION of DERIVATIVE, CONTINUITY AND LIMITS

<table>
<tr><td rowspan="7">Conditions of Derivative at Point " a"</td><td rowspan="5">Conditions of Continuity at Point " a"</td><td rowspan="3">Conditions of Limit at Point " a"</td><td>$\lim_{x\to a^-} f(x)$ exists</td></tr>
<tr><td>$\lim_{x\to a^+} f(x)$ exists</td></tr>
<tr><td>$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$
Then $\lim_{x\to a} f(x) = L$</td></tr>
<tr><td colspan="2">$f(a)$ exists (f must be defined at "a")</td></tr>
<tr><td colspan="2">$\lim_{x\to a} f(x) = L = f(a)$</td></tr>
<tr><td colspan="3">$f'_-(a) = \lim_{h\to 0^-}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0^+}\frac{f(a+h)-f(a)}{h} = f'_+(a)$</td></tr>
<tr><td colspan="3">$f'(a)$ exists (f' must be defined at a)
(For example, $y = \sqrt[3]{x}$ then y' does not exist at "0")</td></tr>
</table>

TYPES OF DISCONTINUITIES

Point/Removable Discontinuity	$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ *and* $\lim_{x\to a} f(x) \neq f(a)$ *or* f *is undefined at "a"*	
Jump Discontinuity	$\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$	
Asymptotic/Infinite Discontinuity	At least one of the one-sided limits is(are) infinite; $\lim_{x\to a} f(x) = \pm\infty$, $\lim_{x\to a^\pm} f(x) = -\infty$ *and* $\lim_{x\to a^\mp} f(x) = \infty$, $\lim_{x\to a^\pm} f(x) = \pm\infty$ *and* $\lim_{x\to a^\mp} f(x) = L$, $\lim_{x\to a^\pm} f(x) = \pm\infty$ *and* $\lim_{x\to a^\mp} f(x)$ *does not exist*	
Endpoint Discontinuity	$\lim_{x\to a^\pm} f(x) = L$ $\lim_{x\to a^\mp} f(x)$ *does not exist* *(This means that a is endpoint of f)*	
Mixed Discontinuity	$\lim_{x\to a^\pm} f(x) = \pm\infty$ $\lim_{x\to a^\mp} f(x) = L$	
Oscillating Discontinuity	An oscillating discontinuity exists when the values of the function appear to be approaching two or more values simultaneously. A standard example of this situation is the function $f(x) = \sin\frac{1}{x}$	

CONDITIONS OF CONTINUITY AT INTERVAL

- A function f(x) is **continuous on the open interval** (a,b) if it is continuous;
  - ✓ at every point $x=c$ contained in that interval.
- A function f(x) is **continuous on the closed interval** [a,b] if it is continuous;
  - ✓ on the open interval (a,b),
  - ✓ from the right at $x=a$, and
  - ✓ from the left at $x=b$.
- A function f(x) is **continuous everywhere** if it is continuous;
  - ✓ at every point on the interval $(-\infty,\infty)$.

IMPORTANT THEOREMS AND FORMULAS FOR DERIVATIVE & LIMITS

Squeeze:	$f(x) \leq g(x) \leq h(x)$ and $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} h(x) = c$, then $\lim\limits_{x\to a} g(x) = c$
Bolzano Intermadiate Value:	Let $f: I \to \mathbb{R}$ be a continuous function. If $[a,b] \subset I$ and if $k \in \mathbb{R}$ is such that $f(a) < k < f(b)$ $\big(or\ f(b) < k < f(a)\big)$, then there exists a point $c \in [a,b]$ such that $f(c) = k$
Bolzano Root Value:	Let $f: I \to \mathbb{R}$ be a continuous function. If $[a,b] \subset I$ and $f(a) < 0 < f(b)$ $\big(or\ f(b) < 0 < f(a)\big)$, then there exists a point $c \in [a,b]$ such that $f(c) = 0$
Fermat Extreme Value:	Suppose that $a < c < b$. If a function f is defined on the interval (a,b), and it has a maximum or a minimum at c, then either f' doesn' exist at c or $f'(c) = 0$.
If f is a continuous function on [a,b] and differentiable on (a,b), then there exists a point c in (a,b) such that: (Let A(a,f(a)) and B(b,f(b)))	
Mean Value:	$f'(c) = \dfrac{f(b) - f(a)}{b - a}$ ([AB] ye paralel en az bir tane teğet vardır.)
Rolle:	If $f(a) = f(b)$, then $f'(c) = 0$ ([AB] ye ve aynı zamanda x eksenine paralel en az bir tane teğet vardır.)
L' Hospital's Rules:	
Linear Approximation:	$f(x) = f(x_0 + \Delta x) \cong L(x) = f(x_0) + f'(x_0) \cdot \Delta x = f(x_0) + f'(x_0) \cdot (x - x_0)$
	If $f'(x) > 0$ on interval I_1, then f is increasing on the interval I_1 If $f'(x) < 0$ on interval I_2, then f is decreasing on the interval I_2
	On interval I_1; if $f''(x) > 0$, then f' is increasing. Thus f is concave up On interval I_2; if $f''(x) < 0$, then f' is decreasing. Thus f is concave down
Newton's Method	If x_n is the n^{th} guess for the $\frac{root}{solution}$ of $f(x) = 0$ then $(n+1)^{st}$ guess is $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ provided $f'(x_n)$ exists.

DERIVATIVE AND NOTATIONS

If $y = f(x)$ *then all of the following are equivalent notations for the derivative:*

$f'(x) = y' = \frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx}(f(x)) = Df(x)$	$f''(x) = y'' = \frac{d^2f}{dx^2} = \frac{d^2y}{dx^2} = D^2f(x)$

If $y = f(x)$ *all of the following are equivalent notations for the derivative evaluated at* $x = a$:

$$f'(a) = y'\Big|_{x=a} = \frac{df}{dx}\Big|_{x=a} = \frac{dy}{dx}\Big|_{x=a} = Df(a)$$

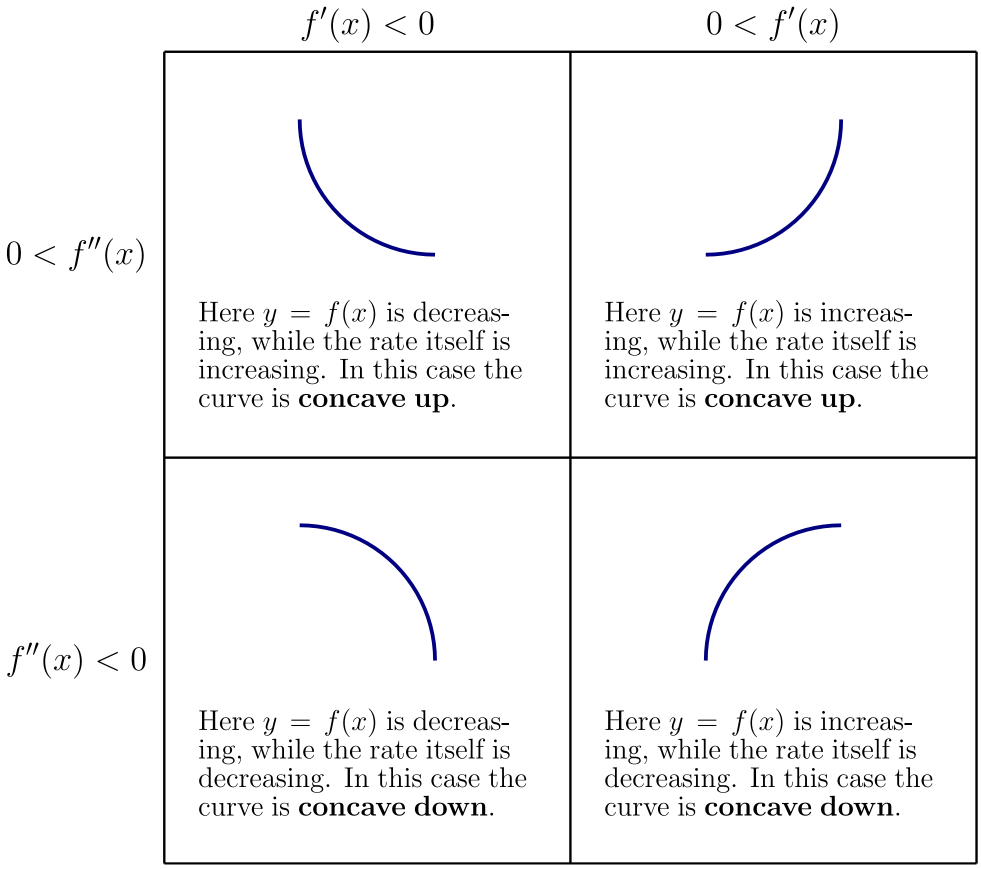
DERIVATION RULES

y	y'	**Description/Comments**
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$1 + \tan^2 x = {}^{1}/_{\cos^2 x} = \sec^2 x$	
$\cot x$	$-(1 + \cot^2 x) = {}^{-1}/_{\sin^2 x} = -\csc^2 x$	
$\sec x$	$\sec x \tan x$	
$\csc x$	$-\csc x \cot x$	
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$	
$\arctan x$	$\frac{1}{1+x^2}$	
$\mathrm{arccot}\, x$	$\frac{-1}{1+x^2}$	
$\mathrm{arcsec}\, x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	
$\mathrm{arccsc}\, x$	$\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	
$\tanh x$	$1 + \tanh^2 x = {}^{1}/_{\cosh^2 x} = \mathrm{sech}^2 x$	
$\coth x$	$-(1 + \coth^2 x) = {}^{-1}/_{\sinh^2 x} = -\mathrm{csch}^2 x$	
$\mathrm{sech}\, x$	$\mathrm{sech}\, x \tanh x$	
$\mathrm{csch}\, x$	$-\mathrm{csch}\, x \coth x$	
$\mathrm{arcsinh}\, x$	$\frac{1}{\sqrt{x^2+1}}$	$for\ \forall x$

$\operatorname{arccosh} x$	$\dfrac{1}{\sqrt{x^2-1}}$	$for\ x > 1$
$\operatorname{arctanh} x$	$\dfrac{1}{x^2-1}$	$for\ \lvert x\rvert < 1$
$\operatorname{arccoth} x$	$\dfrac{1}{1-x^2}$	$for\ x > 1$
$\operatorname{arcsech} x$	$\dfrac{-1}{\lvert x\rvert\sqrt{1-x^2}}$	$for\ 0 < x < 1$
$\operatorname{arccsch} x$	$\dfrac{-1}{\lvert x\rvert\sqrt{x^2+1}}$	$for\ x \neq 0$
$\sqrt{x}$	${}^{1}/_{2\sqrt{x}}$	
e^x	e^x	
a^x	$a^x \cdot \ln a$	
$\ln x$	${}^{1}/_{x}$	
$\log_a x$	$\dfrac{1}{x \cdot \ln a}$	
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$	
$fog(x) = f(g(x))$	$f'(g(x)) \cdot g'(x)$	
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'^{(x)} \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$	$g(x) \neq 0$
$f^{-1}(x)$	$\dfrac{1}{f'(y)} = \dfrac{1}{f'(f^{-1}(x))} = \dfrac{1}{f'of^{-1}(x)}$	(Derivative of Inverse Function)
$[f(x)]^{g(x)}$	$[f(x)]^{g(x)} \cdot \left[g'(x) \cdot \ln f(x) + g(x) \cdot \dfrac{f'(x)}{f(x)}\right]$	$\ln y = g(x) \cdot \ln f(x)$
$y = f(u),$ $u = g(v),$ $v = h(t),$ $t = k(x),$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dv} \cdot \dfrac{dv}{dt} \cdot \dfrac{dt}{dx} = f'(u) \cdot g'(v) \cdot\ h'(t) \cdot k'(x)$	Chain Rule

ASYMPTOTES

Horizontal Asymptote:	$The\ line\ y = L\ is\ called\ a\ horizontal\ asymtote\ for\ y = f(x)\ if\ and\ only\ if;$ $\lim\limits_{x\to\infty} f(x) = L, \quad or \quad \lim\limits_{x\to-\infty} f(x) = L$
Vertical Asymptote:	$The\ line\ x = a\ is\ called\ a\ vertical\ asymptote\ of\ \boldsymbol{f}\ if\ at\ least\ one\ of\ the\ these\ exists:$ $\lim\limits_{x\to a} f(x) = \infty;\quad \lim\limits_{x\to a} f(x) = -\infty;\quad \lim\limits_{x\to a^-} f(x) = \infty$ $\lim\limits_{x\to a^-} f(x) = -\infty;\quad \lim\limits_{x\to a^+} f(x) = \infty;\quad \lim\limits_{x\to a^+} f(x) = -\infty$
Oblique Asymptote:	$The\ line\ y = kx + l\ is\ called\ oblique\ asymptote\ for\ y = f(x) if\ and\ only\ if;$ $\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = k \quad and \quad \lim\limits_{x\to\infty} [f(x) - kx] = l$



SKETCHING GRAPH of CURVES

Cartesien Coordinates:

Step 1: Determine the Domain and Range
Step 2: Find the y-Intercept and x-Intercept(s)
Step 3: Look for Symmetry and Periods
Step 4: Find Asymptote(s)
Step 5: First Derivative: Determine the Intervals of Increase and Decrease and Locate the Relative Extrema
Step 6: Second Derivative: Determine the Intervals of Concavity and Locate the Inflection Points
Step 7: Transfer these informations to the Table
Step 8: Sketch the Graph

Polar Coordinates:

Step 1: Determine the Domain and Range
Step 2: Look for Periods
Step 3: Look for Symmetry:

Cases:	Symmetry:
$f(\theta) = f(-\theta)$ $f(\pi-\theta) = -f(\theta)$	Polar axis ($\theta = 0$)
$f(\theta) = -f(-\theta)$ $f(\pi-\theta) = f(\theta)$	$\theta = {}^{\pi}/_{2}$
$f(\pi+\theta) = f(\theta)$	Origin
$f(\pi+\theta) = -f(\theta)$	T/2 kadar aralıkta çizim yeterlidir. (T: period)

$f(\theta - a) = f(\theta)$ *ise* $P(r,\theta), Q(r,\theta + a)$ *aynı çember üzerinde. Q noktasını kutup noktası etrafında a kadar döndür.*
$f(\theta + a) = f(\theta)$ *ise* $P(r,\theta), Q(-r,\theta + a) = Q(r,\theta + a - \pi)$. *P noktasını Q açısının ters yönünde* $\pi - a$ *kadar döndür.*

Step 4: First Derivative: Determine the Intervals of Increase and Decrease and Locate the Relative Extrema
Step 5: Find the particular values of the function
Step 6: Transfer these informations to the Table
Step 7: Sketch the Graph of periodic part according to table
Step 8: Sketch the Graph completely

~~~~~~ ANTI-DERIVATIVE AND INTEGRALS ~~~~~~

INTEGRATION RULES

y	$\int y\,dx$	Description
$\sin x$	$-\cos x + C$	
$\cos x$	$\sin x + C$	
$\sinh x$	$\cosh x + C$	
$\cosh x$	$\sinh x + C$	
$f'(g(x)) \cdot g'(x)$	$f\big(g(x)\big) + C$	$Substitution\ Rule: u = g(x)$
$f(x) \cdot g'(x)$	$f(x) \cdot g(x) - \int f'(x) \cdot g(x) dx$	$Integration\ by\ Parts\ Rule\ (LAPTU)$: $u = f(x)\ and\ dv = g'(x)dx$
$\sec^2 x = \dfrac{1}{\cos^2 x}$	$\tan x + C$	
$\csc^2 x = \dfrac{1}{\sin^2 x}$	$-\cot x + C$	
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\arcsin \dfrac{x}{a} + C$	
$\dfrac{1}{1 + x^2}$	$\arctan x + C$	
$\dfrac{1}{x}$	$\ln\lvert x\rvert + C$	
$\dfrac{1}{x + a}$	$\ln\lvert x + a\rvert + C$	
$\dfrac{1}{ax + b}$	$\dfrac{1}{a} \ln\lvert ax + b\rvert + C$	
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + C$	
$\dfrac{1}{\sqrt{a^2 + x^2}}$	$\operatorname{arcsinh} \dfrac{x}{a} + C = \ln\left(\dfrac{x + \sqrt{a^2 + x^2}}{a}\right) + C$	
$\dfrac{1}{\sqrt{x^2 - a^2}}$	$\operatorname{arccosh} \dfrac{x}{a} + C = \ln\left(\dfrac{x + \sqrt{x^2 - a^2}}{a}\right) + C$	
e^x	$e^x + C$	
a^x	$\dfrac{a^x}{\ln a} + C$	$a > 0$
$\tan x$	$-\ln\lvert\cos x\rvert + C = \ln\lvert\sec x\rvert + C$	
$\cot x$	$\ln\lvert\sin x\rvert + C$	
$\sec x$	$\ln\lvert\sec x + \tan x\rvert + C$	

$\csc x$	$\ln\lvert\csc x - \cot x\rvert + C$	
$\sin^2 x$	$\frac{1}{2}(x - \sin x \cos x) + C$	
$\cos^2 x$	$\frac{1}{2}(x + \sin x \cos x) + C$	
$\sec^3 x$	$\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln\lvert\sec x + \tan x\rvert + C$	
$\arcsin x$	$x \cdot \arcsin x + \sqrt{1 - x^2} + C$	
$\arccos x$	$x \cdot \arccos x - \sqrt{1 - x^2} + C$	
$\arctan x$	$x \cdot \arctan x - \ln\sqrt{1 + x^2} + C$	
$\operatorname{arcsec} x$		
$\operatorname{arccsc} x$		
$\tanh x$		
$\coth x$		
$\operatorname{sech} x$		
$\operatorname{csch} x$		
$\operatorname{arcsinh} x$		$for\ \forall x$
$\operatorname{arccosh} x$		$for\ x > 1$
$\operatorname{arctanh} x$		$for\ \lvert x\rvert < 1$
$\operatorname{arccoth} x$		$for\ x > 1$
$\operatorname{arcsech} x$		$for\ 0 < x < 1$
$\operatorname{arccsch} x$		$for\ x \neq 0$
$\ln x$	$x \ln x - x + C$	
$\log_a x$	$\frac{1}{\ln a}(x \ln x - x) + C$	

INTEGRATION METHODS

1) *Basit kesirlere ayırma yöntemi:* $\int \frac{P(x)}{Q(x)} dx$ *şeklinde bir integral verildiğinde eğer* $P(x)$*in derecesi* $Q(x)$ *in derecesinden büyük ise ilk olarak polinom bölmesi yapılır:* $\int \frac{P(x)}{Q(x)} dx = \int A(x)\, dx + \int \frac{K(x)}{Q(x)} dx$

$$\int \frac{K(x)}{Q(x)} dx = \int \frac{K(x)}{(x-c_1)(x-c_2)\dots(x-c_n)} dx = \int \frac{A_1}{x-c_1} dx + \int \frac{A_2}{x-c_2} dx + \dots + \int \frac{A_3}{x-c_3} dx \qquad If\ \Delta > 0$$

$$\int \frac{K(x)}{Q(x)} dx = \int \frac{K(x)}{(x-c)^n} dx = \int \frac{A_1}{x-c} dx + \int \frac{A_2}{(x-c)^2} dx + \dots + \int \frac{A_n}{(x-c)^n} dx \qquad If\ \Delta = 0$$

$$\int \frac{K(x)}{Q(x)} dx = \int \frac{K(x)}{(a_1x^2+b_1x-c_1)(a_2+b_2x-c_2)\dots(a_nx^2+b_nx-c_n)} dx$$

$$= \int \frac{A_1x+B_1}{a_1x^2+b_1x-c_1} dx + \int \frac{A_2x+B_2}{a_2x^2+b_2x-c_2} dx + \dots + \int \frac{A_nx+B_n}{a_nx^2+b_nx-c_n} dx$$

$$\int \frac{K(x)}{Q(x)} dx = \int \frac{K(x)}{(ax^2+bx-c)^n} dx = \int \frac{A_1x+B_1}{ax^2+bx-c} dx + \int \frac{A_2x+B_2}{(ax^2+bx-c)^2} dx + \dots + \int \frac{A_nx+B_n}{(ax^2+bx-c)^n} dx$$

2) $\int \sqrt[n]{(ax+b)^m}\, dx$ *şeklinde bir integral verildiğinde* $ax+b=u^n$ *dönüşümüyle* $adx = nu^{n-1}du$ *olur.*

$$\int \sqrt[n]{(ax+b)^m}\, dx = \frac{n}{a}\int u^m u^{n-1}\, du = \frac{nu^{m+n}}{a(m+n)} + C = \frac{n\sqrt[n]{(ax+b)^{m+n}}}{a(m+n)} + C$$

3) $\sqrt[n_i]{(ax+b)^m}$ *şeklindeki ifadeleri içeren fonksiyonların intergalleri alınırken* n_i *kök kuvvetlerinin en küçük ortak katı* p *olmak üzere* $ax+b=u^p$ *değişken dönüşümüyle* $adx = pu^{p-1}du$ *elde edilir.*

4) $\sqrt{a^2-x^2}$ *den başka köklü ifade içermeyen fonksiyonların intergalleri alınırken* $x = a\sin t$ *değişken dönüşümüyle* $\sqrt{a^2-x^2} = a\cos t$ *ve* $dx = a\cos t\, dt$ *elde edilir.* $(0 < t < \pi)$

5) $\sqrt{x^2-a^2}$ *den başka köklü ifade içermeyen fonksiyonların intergalleri alınırken* $x = a\sec t$ *değişken dönüşümüyle* $\sqrt{a^2-x^2} = a\tan t$ *ve* $dx = a\sec t\tan t\, dt$ *elde edilir.* $(0 < t < \frac{\pi}{2})$

(Not: $x = a\cosh t$ *dönüşümü de uygulanabilir.*)

6) $\sqrt{a^2+x^2}$ *den başka köklü ifade içermeyen fonksiyonların intergalleri alınırken* $x = a\tan t$ *değişken dönüşümüyle* $\sqrt{a^2+x^2} = a\sec t$ *ve* $dx = a\sec^2 t\, dt$ *elde edilir.* $(\frac{-\pi}{2} < t < \frac{\pi}{2})$

(Not: $x = a\sinh t$ *dönüşümü de uygulanabilir.*)

7) *Trigonometrik fonksiyonlar cinsinden rasyonel olarak ifade edilen fonksiyonların integrasyonu için, yarım açı metodu denilen* $\tan\frac{x}{2} = t$ *değişken dönüşümüyle* $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$ *ve* $dx = \frac{2dt}{1+t^2}$ *ifadeleri elde edilir.*

TRIGONOMETRIC INTEGRALS AND SUBSTITUTION RULES

$for\ all\ m, n \in \mathbb{N}$

Trigonometric Integral	***Substitution Rule***
$\int \sin^{2n+1} x \cos^{2m} x\,dx = \int (1-t^2)^n t^{2m} dt$	$\cos x = t$
$\int \sin^{2n} x \cos^{2m+1} x\,dx = \int t^{2n}(1-t^2)^m dt$	$\sin x = t$
$\int \sin^{2n+1} x \cos^{2m+1} x\,dx = \int (1-t^2)^n t^{2m+1} dt$ $\int \sin^{2n+1} x \cos^{2m+1} x\,dx = \int t^{2n+1}(1-t^2)^m dt$	$\cos x = t$ or $\sin x = t$
$\int \sin^{2n} x \cos^{2m} x\,dx = \int \sin^{2n} x\,(1-\sin^2 x)^m dx$ $\int \sin^{2n} x \cos^{2m} x\,dx = \int (1-\cos^2 x\,)^n \cos^{2m} x\,dx$	$\cos^2 x = 1-\sin^2 x,\ then\ \sin^2 x = \frac{1-\cos 2x}{2}$ or $\sin^2 x = 1-\cos^2 x,\ then\ \cos^2 x = \frac{1+\cos 2x}{2}$
$\int \tan^{2n} x \sec^{2m} x\,dx = \int t^{2n}(1+t^2)^{m-1} dt$ $\int \tan^{2n+1} x \sec^{2m} x\,dx = \int t^{2n+1}(1+t^2)^{m-1} dt$	$\tan x = t\ (or\ \tan^2 x = t)$
$\int \tan^{2n+1} x \sec^{2m+1} x\,dx = \int (t^2-1)^n t^m dt$	$\sec x = t$
$\int \tan^{2n} x \sec^{2m+1} x\,dx$	Each integral will be dealt with differently.
$\int \cot^{2n} x \csc^{2m} x\,dx$	$\cot x = t$

RECURRENCE FORMULAS
$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
$\int tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$
$\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx =$
$\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx$
$\int \frac{dx}{(x^2+a^2)^n} = \frac{1}{a^2(2n-2)}\left[\frac{x}{(x^2+a^2)^{n-1}} - (2n-3)\int \frac{dx}{(x^2+a^2)^{n-1}}\right]$

PROPERTIES OF DEFINITE INTEGRALS

$\longrightarrow \int_a^b c\,dx = c(b-a)$ $\longrightarrow \int_a^a f(x)dx = 0$	$\longrightarrow \int_a^b f(x)dx = -\int_b^a f(x)dx$ $\longrightarrow \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad c \in (a,b)$ $\longrightarrow \left\lvert\int_a^b f(x)dx\right\rvert \le \int_a^b \lvert f(x)\rvert dx$

IMPORTANT THEOREMS AND FORMULAS FOR INTEGRALS

Fundamental Therom of Calculus Part 1	$F(x) = \int_a^x f(t)\,dt, \quad then\ F'(x) = f(x)$	
Fundamental Therom of Calculus Part 2	$F'(x) = f(x), \quad then \quad \int_a^b f(x)\,dx = F(b) - F(a)$	
	$If\ a \le b\ and\ m \le f(x) \le M\ for\ a \le x \le b, then;$ $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$	
Mean Value of Integral:	$The\ average\ value\ of\ f(x)\ on\ a \le x \le b\ is;$ $f_{avg} = \overline{f} = \frac{1}{b-a}\int_a^b f(x)dx \le$	
Upper and Lower Riemann:	$U(f,P) = \lim_{n\to\infty}\sum_{i=1}^{n} f(u_i)\,\Delta x \qquad L(f,P) = \lim_{n\to\infty}\sum_{i=1}^{n} f(l_i)\,\Delta x$	
Riemann Sum:	$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\,\Delta x, \quad x_i = a + i\Delta x, \quad \Delta x = \frac{b-a}{n}$	
Area:	$A = \int_a^b f(x)dx$	Between two curves: (If $f \ge g$) $A = \int_a^b [f(x) - g(x)]dx$
Volume by Disks for Rotation About the x-axis:	$V = \pi\int_a^b [f(x)]^2 dx$	
Volume by Washers for Rotation About the x-axis:	$V = \pi\int_a^b ([R(x)]^2 - [r(x)]^2)dx$	
Arc Length:	$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$	for parametric curves: $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}dt$
Surface Area:	$SA = 2\pi\int_a^b \lvert y\rvert ds = 2\pi\int_a^b \lvert y\rvert\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx, \quad (rotate\ about\ x-axis)$ $SA = 2\pi\int_c^d \lvert x\rvert ds = 2\pi\int_c^d \lvert x\rvert\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy, \quad (rotate\ about\ y-axis)$	

APPROXIMATING DEFINITE INTEGRALS

Tropezodial Rule:	$\int_a^b f(x)dx \approx T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$ $where\ \Delta x = \frac{b-a}{n}$
Simpson's Rule:	$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{2}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)]$ $where\ \Delta x = \frac{b-a}{n}$
Midpoint Rule:	$\int_a^b f(x)dx \approx \Delta x\left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-2}+x_{n-1}}{2}\right) + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$ $where\ \Delta x = \frac{b-a}{n}$

TYPES OF IMPROPER INTEGRALS

Type 1	**I.** $If\ f\ is\ continuous\ on\ [a,\infty), then$ $\int_a^\infty f(x)\,dx = \lim_{R\to\infty}\int_a^R f(x)\,dx$ **II.** $If\ f\ is\ continuous\ on\ (-\infty,b], then$ $\int_{-\infty}^b f(x)\,dx = \lim_{R\to-\infty}\int_R^b f(x)\,dx$ **III.** $If\ f\ is\ continuous\ on\ (-\infty,\infty), then$ $\int_{-\infty}^\infty f(x)\,dx = \int_0^\infty f(x)\,dx + \int_{-\infty}^0 f(x)\,dx = \lim_{R\to\infty}\int_0^R f(x)\,dx + \lim_{R\to-\infty}\int_R^0 f(x)\,dx$
Type 2	**I.** $If\ f\ is\ continous\ on\ [a,b)\ and\ discontinuous\ at\ b, then$ $\int_a^b f(x)\,dx = \lim_{R\to b^-}\int_a^R f(x)\,dx$ **II.** $If\ f\ is\ continuous\ on\ (a,b]\ and\ discontinuous\ at\ a, then$ $\int_a^b f(x)\,dx = \lim_{R\to a^+}\int_R^b f(x)\,dx$ **III.** $If\ f\ is\ continuous\ on\ [a,b]\ and\ discontinuous\ at\ c \in (a,b), then$ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \lim_{R\to c^-}\int_a^R f(x)\,dx + \lim_{R\to c^+}\int_R^b f(x)\,dx$
Type 3	**(Mixed Type)**

SUMMARY OF CONVERGENCE TESTS FOR IMPROPER INTEGRALS

Tests	For which Integral & When to Use	Conclusions	Comments
$p-test$	$\int_a^\infty \frac{dx}{x^p}$ or $\int_a^\infty x^{-p}\,dx$	$\begin{cases} converges\ to\ \frac{a^{1-p}}{p-1}, & if\ p>1 \\ diverges\ to\ \infty, & if\ p\le 1 \end{cases}$	$Useful\ for\ comparison\ tests\ if\ general\ term\ is\ similar\ to\ \frac{1}{x^p}.$
$Direct\ Comparison$	$\int_a^\infty f(x)dx$ and $\int_a^\infty g(x)dx$ $0\le f(x)\le g(x)$ for all $x\ge a$	$\int_a^\infty g(x)dx\ converges \Rightarrow \int_a^\infty f(x)dx\ converges$ $\int_a^\infty f(x)dx\ diverges \Rightarrow \int_a^\infty g(x)dx\ diverges$	$*\ f, g$ must be continuous on $[a,\infty)$. $*$ If we prove divergence of $\int g(x)dx$, then $f(x)$ is chosen generally as $\frac{1}{x}$. $*$ If we prove convergence of $\int f(x)dx$, then $g(x)$ is chosen generally as $\frac{1}{x^p}$ but p must be greater than 1.
$Limit\ Comparison$	$\int_a^\infty f(x)dx$ and $\int_a^\infty g(x)dx$ $f(x), g(x)>0$ for all x $\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}=L$	If $L\ne 0 \Rightarrow$ both are same character If $L=0$ and $\int_a^\infty g(x)dx$ converges $\Rightarrow \int_a^\infty f(x)dx$ converges If $L=\infty$ and $\int_a^\infty g(x)dx$ diverges $\Rightarrow \int_a^\infty f(x)dx$ diverges	$\int_a^\infty g(x)dx$ is chosen generally as type of $\frac{1}{x^p}$.

~~~~~~ SERIES ~~~~~~

SOME PARTICULAR FINITE SERIES' SUM

$\sum_{i=1}^{n} i=\frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i(i+1)=\frac{n(n+1)(n+2)}{3}$
$\sum_{i=1}^{n}(2i-1)=n^2$	$\sum_{i=1}^{n} i(i+1)(i+2)=\frac{n(n+1)(n+2)(n+3)}{4}$
$\sum_{i=1}^{n} i^2=\frac{n(n+1)(2n+1)}{6}$	$\sum_{i=1}^{n}\frac{1}{i(i+1)}=\frac{n}{n+1}$
$\sum_{i=1}^{n} i^3=\frac{n^2(n+1)^2}{2}$	$\sum_{i=1}^{n}\frac{1}{i(i+1)(i+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$

SUMMARY OF CONVERGENCE TESTS FOR SERIES

Tests	For which Series & When to Use	Conclusions	Comments
n^{th} term test (or the zero test) (Divergence Test)	for any series $\sum a_n$	Diverges if $\lim_{n\to\infty} \lvert a_n\rvert \neq 0$	Inconclusive if $\lim_{n\to\infty} \lvert a_n\rvert = 0$ (Limit 0 ise yakınsaktır diyemeyiz)
Geometric Series	$\sum_{n=0}^{\infty} ax^n$ or $(\sum_{n=1}^{\infty} ax^{n-1})$	Converges to $\frac{a}{1-x}$ only if $\lvert x\rvert < 1$ Diverges if $\lvert x\rvert \geq 1$	Useful for comparison tests if general term is similar to ax^n
$p-series$ ($p-test$)	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ Diverges if $p \leq 1$	Useful for comparison tests if general term is similar to $\frac{1}{n^p}$
Integral Test	$\sum_{n=c}^{\infty} a_n,\ c \geq 0$ $a_n = f(n)$ for all n	Converges if $\int_c^{\infty} f(x)dx$ converges Diverges if $\int_c^{\infty} f(x)dx$ diverges	The function $f(n)$ must be; ✓ continuous ✓ positive, ✓ decreasing ✓ readily integrable for $x \geq c$
Direct Comparison	$\sum a_n$ and $\sum b_n$ $0 \leq a_n \leq b_n$ for all n	$\sum b_n$ converges $\Rightarrow \sum a_n$ converges $\sum a_n$ diverges $\Rightarrow \sum b_n$ diverges	$\sum b_n$ is chosen generally as a geometric or a p series
Limit Comparison	$\sum a_n$ and $\sum b_n$ $a_n, b_n > 0$ for all n $\lim_{n\to\infty} \frac{a_n}{b_n} = L$	If $L \neq 0 \Rightarrow$ both are same character If $L = 0$ and $\sum b_n$ converges $\Rightarrow \sum a_n$ converges If $L = \infty$ and $\sum b_n$ diverges $\Rightarrow \sum a_n$ diverges	$\sum b_n$ is chosen generally as a geometric or a p series
Ratio (D'Alembert Test)	for any series $\sum a_n$ $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L$	Converges (absolutely) if $L < 1$ Diverges if $L > 1$	Inconclusive if $L = 1$ The test useful if a_n involves factorials or n^{th} powers.
Root (Cauchy Test)	for any series $\sum a_n$ $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L$	Converges (absolutely) if $L < 1$ Diverges if $L > 1$	Inconclusive if $L = 1$ The test useful if a_n involves n^{th} powers.
Alterneting Series (Leibnitz Test)	$\sum_{n=0}^{\infty} (-1)^n a_n \quad (a_n > 0)$	Converges if; ✓ $0 < a_{n+1} \leq a_n$ (a_n is decreasing) ✓ $\lim_{n\to\infty} a_n = 0$	Applicable only to series with alternateing terms.
Absolute Convergence	for any series $\sum a_n$	Converges (abs.) if $\sum \lvert a_n\rvert$ converges	The test useful for series containing both positive and negative terms
Conditional Convergence	for any series $\sum a_n$	Converges while $\sum \lvert a_n\rvert$ diverges	

SOME PARTICULAR INFINITE AND MACLAURIN SERIES' SUM

Function	Expansion of the Series	Conditions	Radius of Convergence
$\frac{1}{1-x}$	$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n=1+x+x^2+x^3+\cdots$	$-1<x<1$	$R=1\Longrightarrow \lvert x\rvert<1$
$\frac{a}{1-x}$	$\frac{1^{st}\,term}{1-ratio}=\frac{a}{1-x}=\sum_{n=0}^{\infty}ax^n=a+ax+ax^2+\cdots$	$-1<x<1$	$R=1\Longrightarrow \lvert x\rvert<1$
$\frac{1}{1+x}$	$\frac{1}{1+x}=\sum_{n=0}^{\infty}(-x)^n=1-x+x^2-x^3+\cdots$	$-1<x<1$	$R=1\Longrightarrow \lvert x\rvert<1$
$\frac{1}{(1-x)^2}$	$\frac{1}{(1-x)^2}=\sum_{n=0}^{\infty}(n+1)x^n=1+2x+3x^2+4x^3+\cdots$	$-1<x<1$	$R=1\Longrightarrow \lvert x\rvert<1$
$\frac{1}{(1+x)^2}$	$\frac{1}{(1+x)^2}=\sum_{n=0}^{\infty}(n+1)(-x)^n=1-2x+3x^2-\cdots$	$-1<x<1$	$R=1\Longrightarrow \lvert x\rvert<1$
$(1+x)^k$ Binomial series	$(1+x)^k=\sum_{n=0}^{\infty}\binom{k}{n}x^n=1+kx+\frac{k(k-1)}{2!}x^2+\cdots$	$-1<x<1$	$R=1\Longrightarrow \lvert x\rvert<1$
$\ln(1+x)$	$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$	$-1<x\le 1$	$R=1\Longrightarrow \lvert x\rvert<1$
$-\ln(1-x)$	$-\ln(1-x)=\sum_{n=0}^{\infty}\frac{x^n}{n}=x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\cdots$	$-1\le x<1$	$R=1\Longrightarrow \lvert x\rvert<1$
$\sin x$	$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$	$-\infty<x<\infty$	$R=\infty\Longrightarrow \lvert x\rvert<\infty$
$\cos x$	$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$	$-\infty<x<\infty$	$R=\infty\Longrightarrow \lvert x\rvert<\infty$
e^x	$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots,$	$-\infty<x<\infty$	$R=\infty\Longrightarrow \lvert x\rvert<\infty$
$\tan^{-1}x$	$\tan^{-1}x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$	$-1\le x\le 1$	$R=1\Longrightarrow \lvert x\rvert<1$
$\sinh x$	$\sinh x=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$	$-\infty<x<\infty$	$R=\infty\Longrightarrow \lvert x\rvert<\infty$
$\cosh x$	$\cosh x=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$	$-\infty<x<\infty$	$R=\infty\Longrightarrow \lvert x\rvert<\infty$

POWER, TAYLOR AND MACLAURIN SERIES

Power Series

$$f(x)=\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$$

Cauchy – Hadamard Theorem: $\text{Let } L=\lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right| \quad \text{or} \quad L=\lim_{n\to\infty}\sqrt[n]{|c_n|}\text{, then;}$

I. $L\neq 0 \quad \Longrightarrow \quad R=\frac{1}{L}$ .That is to say $R=\frac{1}{\lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right|}$ or $R=\frac{1}{\lim_{n\to\infty}\sqrt[n]{|c_n|}}$

II. $L=0 \quad \Longrightarrow \quad R=\infty$

III. $L=\infty \quad \Longrightarrow \quad R=0$ (The means that f can be convergent only for $x=0$)

If "radius of convergence" of $f(x)$ is R;

$|x-a|<R \quad \Longrightarrow \quad a-R<x<a+R$

In this interval $f(x)$ is already convergent. But on endpoints of this interval ($a-R$ and $a+R$), they should be checked whether f is convergent or not there

If radius of $f(x)$ is R, radius of $f'^{(x)}$ and raidus of $\int f(x)$ are also R. (f must be differentiable)

If radius of $\sum_{n=0}^{\infty} a_n x^n$ is R_a and radius of $\sum_{n=0}^{\infty} b_n x^n$ is R_b, then;

radius of $\sum_{n=0}^{\infty}(a_n+b_n)x^n$ is $R\geq \min\{R_a,R_b\}$

Taylor and Maclaurin Series

If f can be presented by a power series, then f can be represented by a Taylor series form;

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n = f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots$$

For the special case $a=0$ the Taylor series becomes Maclaurin series:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n = f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3\ \ldots$$

Partial sum of Taylor series is $T_n(x)$, then $f(x)=\lim_{n\to\infty}T_n(x)$

Remainder of Taylor series is $R_n(x)$, then $\lim_{n\to\infty}R_n(x)=0$

Taylor's Inequality: If $\left|f^{(n+1)}(x)\right|\leq M$ for $|x-a|\leq d$, then $R_n(x)$ satisfied the inequality;

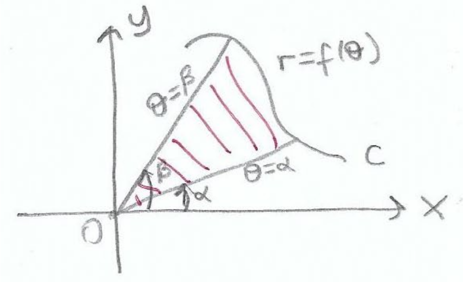
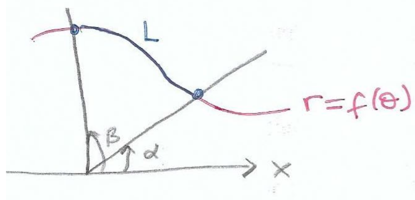
$$|R_n(x)|\leq\frac{M}{(n+1)!}|x-a|^{n+1} \quad \text{for } |x-a|\leq d$$

ERROR ESTIMATE FOR ALTERNATING SERIES

$$Error = |s-s_n|\leq|s_{n+1}-s_n|\leq|a_{n+1}|$$

----------------POLAR COORDINATES----------------

$r = 0$	Orijin noktası
$r = a$	Merkezi (0,0) olan yarıçapı a olan çember
$r = a\sin\theta$	Merkezi (0,a/2) olan yarıçapı a/2 olan çember
$r = a\cos\theta$	Merkezi (a/2,0) olan yarıçapı a/2 olan çember
$r = a(1 \mp \sin\theta)$	Simetri ekseni y ekseni olan **Kardiyoid**
$r = a(1 \mp \cos\theta)$	Simetri ekseni x ekseni olan **Kardiyoid**
$r = a\sin 2\theta$	Simetri eksenleri $y = x$ ve $y = -x$ doğruları olan **4 yapraklı gül**
$r = a\cos 2\theta$	Simetri ekseni x ve y eksenleri olan **4 yapraklı gül**
$r = a\sin 3\theta$	Simetri ekseni y ekseni olan **3 yapraklı gül**
$r = a\cos 2\theta$	Simetri ekseni x ekseni olan **3 yapraklı gül**
$r = a \mp b\sin\theta$	Simetri ekseni y ekseni olan **Limaçon (Snail)**
$r = a \mp b\cos\theta$	Simetri ekseni x ekseni olan **Limaçon (Snail)**
$r = a\theta$	**Spiral**
$r = ae^{b\theta}$	**Logaritmik Spiral**

Tangent	$m = \left.\dfrac{dy}{dx}\right\|_{(r,\theta)} = \dfrac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$	
	$\dfrac{dy}{d\theta} = 0 \; and \; \dfrac{dx}{d\theta} \neq 0, \qquad then \; m = 0$	
	$\dfrac{dx}{d\theta} = 0 \; and \; \dfrac{dy}{d\theta} \neq 0, \qquad then \; m = \infty$	
Area	$A = \dfrac{1}{2}\displaystyle\int_{\alpha}^{\beta} [f(\theta)]^2 d\theta = \dfrac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$	
Arc Length	$L = \displaystyle\int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$	

VEKTÖR DEĞERLİ FONKSİYONLAR			
$\vec{r} = f(t)\vec{\imath} + g(t)\vec{\jmath} + h(t)\vec{k}, \qquad a \le t \le b$			
Hız: $\vec{v} = \frac{d\vec{r}}{dt} = \frac{df}{dt}\vec{\imath} + \frac{dg}{dt}\vec{\jmath} + \frac{dh}{dt}\vec{k}$	Sürat: $\lvert\vec{v}\rvert = \sqrt{\left(\frac{df}{dt}\right)^2 + \left(\frac{dg}{dt}\right)^2 + \left(\frac{dh}{dt}\right)^2}$	İvme: $\vec{a} = \frac{d\vec{v}}{dt}$	Yön: $\frac{\vec{v}}{\lvert\vec{v}\rvert}$

Birim Teğet Vektör	$\vec{T} = \frac{\vec{v}}{\lvert\vec{v}\rvert}$	
Arc Length	$L = \int_a^b \lvert\vec{v}\rvert dt$	$S(t) = \int_{t_0}^{t} \lvert\vec{v}\rvert dt$

~~~~~~ MULTI-VARIABLE FUNCTIONS ~~~~~~

PARTIAL DERIVATIVE AND NOTATIONS		
$F_x(x,y) = \frac{\partial F}{\partial x} = \frac{\partial}{\partial x}(F(x,y))$	$F_y(x,y) = \frac{\partial F}{\partial y} = \frac{\partial}{\partial y}(F(x,y))$	
$F_{xx}(x,y) = \frac{\partial^2 F}{\partial x^2} = \frac{\partial^2}{\partial x^2}(F(x,y))$	$F_{yy}(x,y) = \frac{\partial^2 F}{\partial y^2} = \frac{\partial^2}{\partial y^2}(F(x,y))$	$F_{xy}(x,y) = \frac{\partial^2 F}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x}\right)$
$F_x(a,b) = F_x(x,y)\Big\rvert_{(x,y)\,=\,(a,b)} = \frac{dF}{dx}\Big\rvert_{(x,y)\,=\,(a,b)}$		

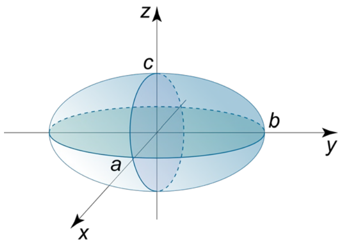
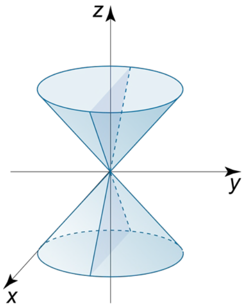
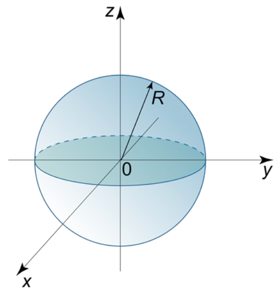
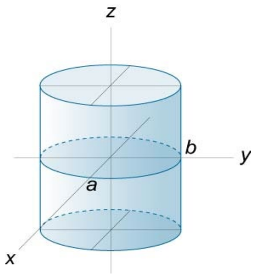
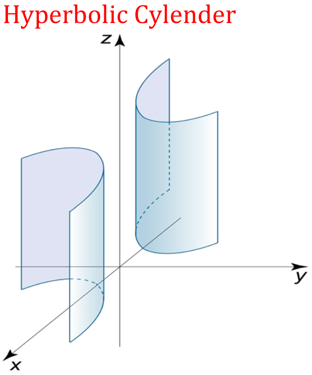
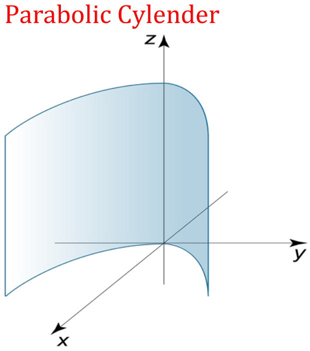
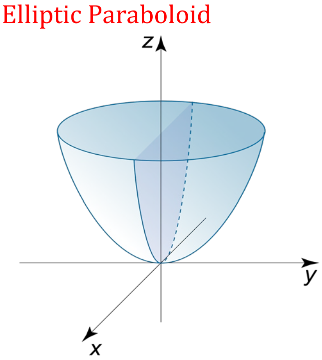
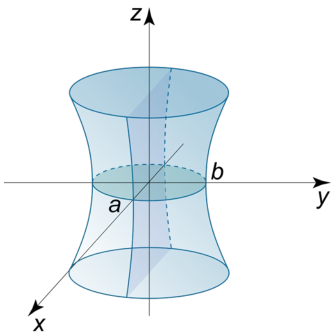
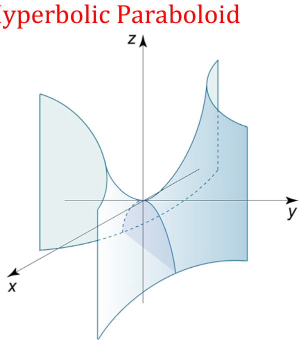
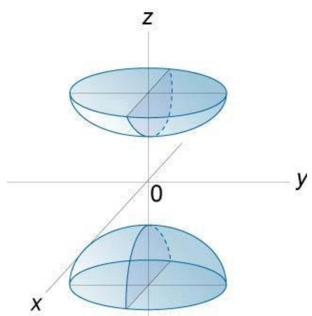
IMPORTANT THEOREMS AND FORMULAS FOR M-V FUNCTIONS		
Divergence, Curl, and Laplacian		
(Fonksiyonel) Jakobian Determinant	$F_1(x_1,x_2,\dots,x_n),\ F_2(x_1,x_2,\dots,x_n),\ F_2(x_1,x_2,\dots,x_n)$ $J = \frac{\partial(F_1,F_2,\dots,F_n)}{\partial(x_1,x_2,\dots,x_n)} = \begin{vmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \cdots & \frac{\partial F_n}{\partial x_n} \end{vmatrix}$	
Laplace Equation	$F_{xx} + F_{yy} = 0$	$F_{xx} + F_{yy} + F_{zz} = 0$
Tam Diferensiyel	$df = f_x dx + f_y dy$	$df = f_x dx + f_y dy + f_z dz$

Gradient Vektörü	$\nabla_f = \frac{df}{dx}\vec{\imath} + \frac{df}{dy}\vec{\jmath}$	$\nabla_f = \frac{df}{dx}\vec{\imath} + \frac{df}{dy}\vec{\jmath} + \frac{df}{dz}\vec{k}$
Doğrultu Türevi	$\vec{u} = u_1\vec{\imath} + u_2\vec{\jmath} + u_3\vec{k}, \quad P_0(x_0, y_0, z_0)$ $(D_u f)_{P_0} = \frac{dF}{ds}\Big\|_{\vec{u}, P_0} = \lim_{s\to 0}\frac{f(x_0 + su_1,\ y_0 + su_2,\ z_0 + su_3) - f(x_0, y_0, z_0)}{s}$	
	$(D_u f)_{P_0} = \frac{dF}{ds}\Big\|_{\vec{u}, P_0} = \nabla_f \Big\|_{P_0} \cdot \vec{u}$	
	$(D_u f)_{P_0} = \|\nabla_f\| \cos\theta$	$\cos\theta = 1$ ise f hızlı artar $\cos\theta = -1$ ise f hızlı azalır $\cos\theta = 0$ ise f in değişimi sıfırdır. (∇_f *ile* $\vec{u}$ *dik ise*)
Leibnitz Formula: (İntegral Altında Türev Alma)	$F(x) = \int_{u(x)}^{v(x)} f(x,y)dy \Rightarrow F'(x) = \int_{u(x)}^{v(x)} \frac{\partial f}{\partial x}dy + v'(x)f[x, v(x)] - u'(x)f[x, u(x)]$	

GRADIENT

Gradient Vector	$\nabla_f = \frac{df}{dx}\vec{\imath} + \frac{df}{dy}\vec{\jmath}$	$\nabla_f = \frac{df}{dx}\vec{\imath} + \frac{df}{dy}\vec{\jmath} + \frac{df}{dz}\vec{k}$
Gradient Rules	$\nabla(kf) = k\nabla f$	$\nabla(f \mp g) = \nabla f \mp \nabla g$
	$\nabla(fg) = f\nabla g + g\nabla f$	$\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$
Seviye Eğrilerinin Granyenti	$\vec{r} = g(t)\vec{\imath} + h(t)\vec{\jmath}$ *and* $F(x = g(t), y = h(t)) = c$	$\nabla_f \cdot \frac{d\vec{r}}{dt} = 0$
Tangent Equaiton	$\nabla_f \Big\|_{P_0} \cdot \overrightarrow{P_0P} = 0$	$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) = 0$
	$f_x(x_0, y_0, z_0)(x - x_0) + f_y(x_0, y_0, z_0)(y - y_0) + f_z(x_0, y_0, z_0)(z - z_0) = 0$	
Tangent Plane	$\nabla_f \cdot \frac{d\vec{r}}{dt} = 0$	
Normal line at P_0 $t \in \mathbb{R}$	$x = x_0 + f_x(P_0)t$ $y = y_0 + f_y(P_0)t$ $z = z_0 + f_z(P_0)t$	
Tangent Plane of $z = f(x, y)$ at P_0	$f_x(x_0, y_0, z_0)(x - x_0) + f_y(x_0, y_0, z_0)(y - y_0) - (z - z_0) = 0$	

QUADRATIC SURFACES AND EQUATIONS

Surface	Equation	Surface	Equation
Ellipsoid 	$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ All traces are ellipses. If $a=b=c=R$, the ellipsoid is a sphere whose radius is R.	Cone 	$$\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$ Horizontal traces are ellipses. Vertical traces in planes $x=k\ and\ y=k$ are hyperbolas if $k\neq 0$ but are pairs of lines if $k=0$
Sphere 	$$x^2+y^2+z^2=R^2$$ All traces are circles. If centre is (a,b,c), then $(x-a)^2+(y-b)^2+(z-c)^2=R^2$	Elliptic Cylinder 	$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$ Horizontal traces are ellipses. Vertical traces are two parallel lines.
Hyperbolic Cylender 	$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$ Horizontal traces are hyperbolas. Vertical traces are two parallel lines.	Parabolic Cylender 	$$\frac{x^2}{a^2}-y=0$$ Horizontal traces are parabolas. Vertical traces are two parallel lines.
Elliptic Paraboloid 	$$\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid	Hyperboloid of One Sheet 	$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coef.ficent is negative.
Hyperbolic Paraboloid 	$$\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c<0$ is illustrated	Hyperboloid of Two Sheets 	$$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ Horizontal traces in z = k are ellipses if $\|k\|>c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets

MAXIMUM AND MINIMUM

Critical Points	1. $f_x = f_y = 0$ 2. $f_x \; or \; f_y \; does \; not \; exist$ 3. $The \; boundary \; points \; of \; the \; function \; f$
Hessian Matrix and Discriminant of f	$\Delta = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - {f_{xy}}^2$

$$f_x(a,b) = f_y(a,b) = 0, \quad then;$$

	$\Delta\Big\vert_{(a,b)} > 0$	$\Delta\Big\vert_{(a,b)} = 0$	$\Delta\Big\vert_{(a,b)} < 0$
$f_{xx}(a,b) > 0$	(a,b) is a local minimum point	We know nothing about the point (a,b) The test is inconclusive at (a,b)	(a,b) is a saddle point (Eyer / Semer noktası)
$f_{xx}(a,b) < 0$	(a,b) is a local maximum point		

Kapalı Sınırlı Bölgede	1. Fonksiyonun bölgenin iç noktalarında kritik noktası varsa bu noktalarda aldığı değerler bulunur. 2. Fonksiyonun bölgenin sınır noktalarında kritik noktası varsa bu noktalarda aldığı değerler bulunur. 3. Tüm bu değerler içindeki en küçük değer fonksiyonun o kapalı bölgedeki en küçük değeri, en büyük değer ise fonksiyonun o kapalı bölgedeki en büyük değeri olur.

LAGRANGE MULTIPLIERS

	Equations	**The Values What Must Be Find**
One Constraint	$\nabla f_x = \lambda \nabla g_y$ $\nabla f_y = \lambda \nabla g_y$ $\nabla f_z = \lambda \nabla g_z$ $g(x,y,z) = 0$	x, y, z, λ
Two Constraints	$\nabla f_x = \lambda \nabla g_x + \mu \nabla h_x$ $\nabla f_y = \lambda \nabla g_y + \mu \nabla h_y$ $\nabla f_z = \lambda \nabla g_z + \mu \nabla h_z$ $g(x,y,z) = 0$ $h(x,y,z) = 0$	x, y, z, λ, μ

~~~~~~~ MULTIPLE INTEGRALS ~~~~~~~

<table>
<tr><th colspan="3">DOUBLE INTEGRALS</th></tr>
<tr><td>Volume</td><td>Between f(x,y) and R on Oxy plane:
$V=\iint\limits_R f(x,y)dA=\iint\limits_R f(x,y)dxdy$</td><td>Between f(x,y) and g(x,y): (If $f\geq g$)
$V=\iint\limits_R [f(x,y)-g(x,y)]dA$</td></tr>
<tr><td>Area</td><td colspan="2">$A=\iint\limits_R dA=\iint\limits_R dxdy$</td></tr>
<tr><td rowspan="3">Fubini Theorem</td><td colspan="2">If f(x,y) is continuous on R whose bounds are $a\leq x\leq b$ and $c\leq y\leq d$, the integral can be written as;
$\iint\limits_R f(x,y)dA=\int_a^b\left(\int_c^d f(x,y)dy\right)dx=\int_c^d\left(\int_a^b f(x,y)dx\right)dy$</td></tr>
<tr><td colspan="2">If f(x,y) is continuous on R whose bounds are $a\leq x\leq b$ and $g_1(x)\leq y\leq g_2(x)$, the integral can be written as;
$\iint\limits_R f(x,y)dA=\int_a^b\left(\int_{g_1(x)}^{g_2(x)} f(x,y)dy\right)dx$</td></tr>
<tr><td colspan="2">If f(x,y) is continuous on R whose bounds are $h_1(y)\leq x\leq h_2(y)$ and $c\leq y\leq d$, the integral can be written as;
$\iint\limits_R f(x,y)dA=\int_c^d\left(\int_{h_1(y)}^{h_2(y)} f(x,y)dx\right)dy$</td></tr>
<tr><td rowspan="2">Parametric Transform</td><td>$J(u,v)=\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}x_u & x_v\\ y_u & y_v\end{vmatrix}=x_uy_v-x_vy_u$</td><td>$dxdy=|J(u,v)|dudv=|x_uy_v-x_vy_u|dudv$</td></tr>
<tr><td colspan="2">$\iint\limits_R f(x,y)dxdy=\iint\limits_{R\prime} f(g(u,v),h(u,v))|x_uy_v-x_vy_u|dudv$</td></tr>
<tr><td rowspan="3">Polar Transform</td><td>$x=r\cos\theta$
$y=r\sin\theta$</td><td>$J(r,\theta)=\frac{\partial(x,y)}{\partial(r,\theta)}=\begin{vmatrix}x_r & x_\theta\\ y_r & y_\theta\end{vmatrix}=\begin{vmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{vmatrix}=r$</td></tr>
<tr><td>$dxdy=rdrd\theta$</td><td>$\iint\limits_R f(x,y)dxdy=\int_{\theta=\alpha}^{\beta}\left(\int_{r=g_1(\theta)}^{g_2(\theta)} f(r\cos\theta,r\sin\theta)rdr\right)d\theta$</td></tr>
<tr><td colspan="2">$Area=A=\iint\limits_R f(x,y)dxdy=\int_{\theta=\alpha}^{\beta}\left(\int_{r=g_1(\theta)}^{g_2(\theta)} f(r\cos\theta,r\sin\theta)rdr\right)d\theta$</td></tr>
</table>

TRIPLE INTEGRALS

Volume

$$V = \iiint_D dV = \iiint_D dxdydz$$

Fubini Theorem

If F(x,y,z) is continuous on D whose bounds are $a \le x \le b,\ c \le y \le d$, *and* $e \le z \le f$, *the integral can be written as;*

$$\iiint_D dV = \int_e^f \int_c^d \int_a^b F(x,y,z)dxdydz$$

If F(x,y,z) is continuous on D whose bounds are $a \le x \le b,\ g_1(x) \le y \le g_2(x)$, *and* $f_1(x,y) \le z \le f_2(x,y)$, *the integral can be written as;*

$$\iiint_D dV = \int_a^b \int_{g_1(x)}^{g_2(x)} \int_{f_1(x,y)}^{f_2(x,y)} F(x,y,z)dzdydx$$

If F(x,y,z) is continuous on D whose bounds are $h_1(y) \le x \le\ h_2(y),\ c \le y \le d$, *and* $f_1(x,y) \le z \le f_2(x,y)$, *the integral can be written as;*

$$\iiint_D dV = \int_c^d \int_{h_1(y)}^{h_2(y)} \int_{f_1(x,y)}^{f_2(x,y)} F(x,y,z)dzdydx$$

Cylindrical Coordinates

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$$J(r,\theta,z) = \frac{\partial(x,y,z)}{\partial(r,\theta,z)} = \begin{vmatrix} x_r & x_\theta & x_z \\ y_r & y_\theta & y_z \\ z_r & z_\theta & z_z \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

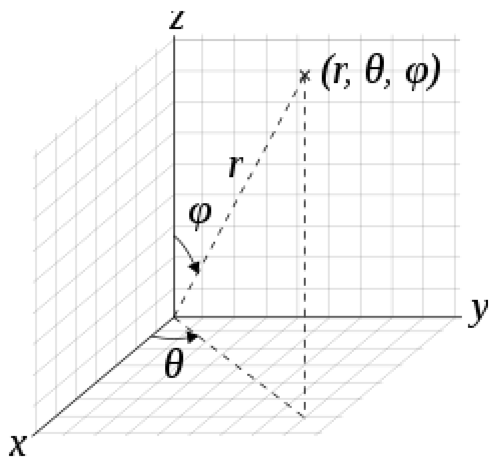
dV=rdzdrdθ

$$\iiint_D dV = \int_{\theta=\alpha}^{\beta} \int_{g_1(\theta)}^{g_2(\theta)} \int_{z=f_1(r,\theta)}^{f_2(r,\theta)} F(r\cos\theta, r\sin\theta, z)rdzdrd\theta$$

Spherical Coordinates

$x = \rho\sin\varphi\cos\theta$
$y = \rho\sin\varphi\sin\theta$
$z = \rho\cos\varphi$

$0 \le \varphi \le \pi$
$0 \le \theta \le 2\pi$
$x^2 + y^2 + z^2 = \rho^2$



$$J(\rho,\varphi,\theta) = \frac{\partial(x,y,z)}{\partial(\rho,\varphi,\theta)} = \begin{vmatrix} x_\rho & x_\varphi & x_\theta \\ y_\rho & y_\varphi & y_\theta \\ z_\rho & z_\varphi & z_\theta \end{vmatrix}$$

$$= \begin{vmatrix} \sin\varphi\cos\theta & \rho\cos\varphi & -\rho\sin\varphi\sin\theta \\ \sin\varphi\sin\theta & \rho\cos\varphi & \rho\sin\varphi\cos\theta \\ \cos\varphi & -\rho\sin\varphi & 0 \end{vmatrix} = \rho^2\sin\varphi$$

$$dV = |J|d\rho d\varphi d\theta = (\rho^2\ sin\varphi)d\rho d\varphi d\theta$$

$$\iiint_D dV = \int_{\theta=\alpha}^{\beta} \int_{\varphi_{min}}^{\varphi_{max}} \int_{\rho=f_1(\theta,\varphi)}^{f_2(\theta,\varphi)} (\rho^2\ sin\varphi)d\rho d\varphi d\theta$$

$z = \rho\cos\phi$

kartezyen	silindirik	küresel
$z=3$ düzlem	$z=3$	$\rho\cos\phi=3 \rightarrow \rho=\frac{3}{\cos\phi}$
$z=x^2+y^2$ parabol	$z=r^2$	$\rho\cos\phi=\rho^2\sin^2\phi \Rightarrow \rho=\frac{\cos\phi}{\sin^2\phi}$ $\rho(\cos\phi-\rho\sin^2\phi)=0$
$z=\sqrt{x^2+y^2}$ koni	$z=r$	$\rho\cos\phi=\rho\sin\phi$ $\rho(\cos\phi-\sin\phi)=0$ $\tan\phi=1 \Rightarrow \phi=\frac{\pi}{4}$
$z^2=x^2+y^2$ koni 	$z^2=r^2$ $z=\mp r$	$\rho^2\cos^2\phi=\rho^2\sin^2\phi$ $\rho^2(\sin^2\phi-\cos^2\phi)=0$ $\sin^2\phi=\cos^2\phi$ $\tan^2\phi=1 \rightarrow \phi=\frac{\pi}{4}$, $\phi=\frac{3\pi}{4}$ $\tan\phi=\mp 1$
$x^2+y^2+z^2=4$ küre	$r^2+z^2=4$ $z=\mp\sqrt{4-r^2}$	$\rho^2=4 \Rightarrow \boxed{\rho=2}$ küre
$z=0$	$z=0$	$\rho\cos\phi=0 \Rightarrow \phi=\frac{\pi}{2}$

$\uparrow z$ $\cdot P(r,\theta,z)$

FONKSİYON DİZİLERİ

Noktasal Yakınsaklık	$\forall \varepsilon > 0$ *ve her* $x \in E$ *için öyle bir* $N \in \mathbb{N}$ *vardır ki;* $n \geq N(\varepsilon)$ *için* $\lvert f_n(x) - f(x) \rvert < \varepsilon$ *sağlanır.*
Düzgün Yakınsaklık	$\forall \varepsilon > 0$ *için öyle bir* $N \in \mathbb{N}$ *vardır ki; her* $x \in E$ *ve* $n \geq N(\varepsilon)$ *için* $\lvert f_n(x) - f(x) \rvert < \varepsilon$ *sağlanır.*
	E *kümesi üzerinde* $f_n \xrightarrow{noktasal} f$ *olsun ve* $M_n = \sup\limits_{x \in E} \lvert f_n(x) - f(x) \rvert$ *şeklinde tanımlansın.* $f_n \xrightarrow{düzgün} f \iff \lim\limits_{n \to \infty} M_n = 0$