

Question Sheet

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Department

Exam Hall

Instructor's Name Surname

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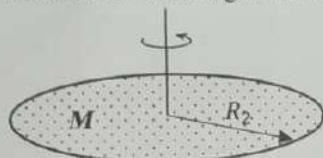
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Student Signature:

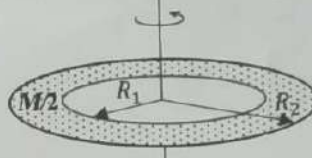
$g = 10 \text{ (m/s}^2\text{)}$

$\vec{F}_{korunumlu} = -\frac{dU}{dr} \hat{r}; W_{korunumlu} = -\Delta U; U = mgy; U = \frac{1}{2}kx^2; \vec{F}_{net} = \frac{d\vec{p}}{dt}; \vec{p} = m\vec{v}; \vec{l} = \Delta\vec{p} = \vec{F}\Delta t; f_s \leq \mu_s N; f_k = \mu_k N$
 $\vec{\omega} = \frac{\Delta\theta}{\Delta t}; \vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}; \vec{\omega} = \frac{d\vec{\theta}}{dt}; \vec{\alpha} = \frac{d\vec{\omega}}{dt}; \vec{\omega} = \vec{\omega}_0 + \vec{\alpha}t; \vec{\theta} = \vec{\theta}_0 + \vec{\omega}_0t + \frac{1}{2}\vec{\alpha}t^2; \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0); v = r\omega; a_t = r\alpha$
 $F = -kx; \vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}; \vec{r}_{cm} = \frac{\int \vec{r} dm}{\int dm}; \vec{\tau} = \vec{r} \times \vec{F}; \vec{\tau}_0 = I_0 \vec{\alpha}; l = \int r^2 dm; P = \vec{\tau} \cdot \vec{\omega}; W = \int \vec{\tau} \cdot d\vec{\theta}; \vec{P} = \frac{\Delta W}{\Delta t}; W = \Delta U + \Delta K$

Question 1) (LABORATORY QUESTION) In a moment of inertia experiment, it was determined that the moment of inertia of a disk with mass M and radius R_2 corresponds to $I_d = \frac{1}{2}MR_2^2$ with respect to the axis passing through the center of mass. A disk with mass $M/2$ and radius R_1 is removed from this disk as shown in the figure. As a result of the same experiment, which of the following is the expression that gives the moment of inertia of the evacuated disk?



$I_d = \frac{1}{2}MR_2^2$



$I = ?$

$\frac{1}{2}MR_2^2 - \frac{1}{2}\frac{M}{2}R_1^2 = \frac{1}{2}\frac{M}{2}(R_1^2 + R_2^2)$
 $2MR_2^2 - MR_1^2 = MR_1^2 + MR_2^2$
 $R_2^2 = 2R_1^2$

$\frac{1}{2}\frac{M}{2}(R_1^2 + R_2^2)$
 $\frac{3}{4}MR_1^2$

A) $\frac{3}{4}MR_1^2$

B) $\frac{1}{4}MR_1^2$

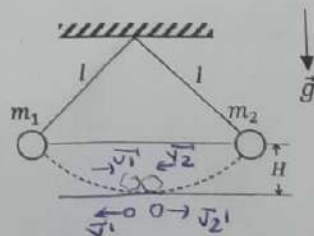
C) $\frac{5}{4}MR_1^2$

D) $\frac{1}{2}MR_1^2$

E) $\frac{3}{2}MR_1^2$

Question 2) Two pendulum balls of masses $m_1 = m$ and $m_2 = 2m$ are released from a height H above the lowest point and collide elastically head-on at the lowest point of their motion. Find the height to which the mass m_1 can rise for the first time after the collision?

$m_1gH = \frac{1}{2}m_1v_1^2$
 $v_1 = \sqrt{2gH} = v_2 = v$



$m_1v + 2m(-v) = m_1(v_1') + 2m(v_2')$
 $-v = -v_1' + 2v_2'$
 $v_1' + v_1' = v_2' + v_2'$
 $v - v_1' = -v + v_2'$
 $2v = v_1' + v_2'$
 $-4v = -2v_2' - 2v_1'$
 $-v = -v_1' + 2v_2'$
 $-5v = -3v_1'$
 $v_1' = \frac{5}{3}v$

$\frac{1}{2}m_1v_1'^2 = m_1gh$

$\frac{1}{2}m \frac{25v^2}{9} = mgh$

$\frac{1}{2} \frac{25}{9} 2gH = gh$

$\frac{25H}{9} = h$

A) $\frac{25H}{9}$

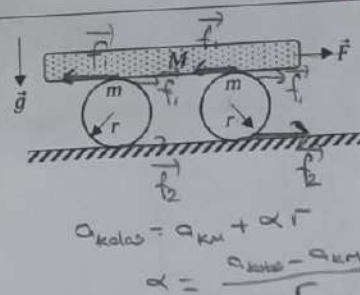
B) $\frac{H}{9}$

C) $\frac{4H}{9}$

D) $\frac{16H}{9}$

E) $\frac{12H}{9}$

Question 3) A long plank of mass M is placed on two identical cylinders of radius r and mass m . The plank begins to be pulled from one end with a force \vec{F} parallel to the ground. If it is assumed that the cylinders roll on the ground without slipping, find the acceleration of the plank.
(For a cylinder of mass m and radius r ; $I = \frac{1}{2}mr^2$.)



$$M: \Sigma F_x = F - 2f_1 = Ma_{\text{plank}} \quad (1)$$

$$m: \Sigma F_x = f_1 + f_2 = ma_{\text{cm}} \quad (2)$$

$$\Sigma \tau = (f_1 - f_2)r = I\alpha$$

$$f_1 - f_2 = \frac{1}{2} \frac{mr^2}{r} \left(\frac{a_{\text{plank}} - a_{\text{cm}}}{r} \right) \quad (3)$$

$$a_{\text{plank}} = a_{\text{cm}} + \alpha r$$

$$\alpha = \frac{a_{\text{plank}} - a_{\text{cm}}}{r}$$

$$a_{\text{plank}} = 2a_{\text{cm}}$$

$$a_{\text{cm}} = \frac{a_{\text{plank}}}{2}$$

$$F = Ma_{\text{plank}} + m a_{\text{cm}} + \frac{1}{2} m (a_{\text{plank}} - a_{\text{cm}})$$

$$F = Ma_{\text{plank}} + m \frac{a_{\text{plank}}}{2} + \frac{1}{2} m \left(a_{\text{plank}} - \frac{a_{\text{plank}}}{2} \right)$$

$$= a_{\text{plank}} \left(M + \frac{m}{2} + \frac{m}{4} \right) = a_{\text{plank}} \left(\frac{4M + 2m + m}{4} \right)$$

$$a_{\text{plank}} = \frac{4F}{4M + 3m}$$

A) $\frac{F}{4M - 3m}$

B) $\frac{3F}{4M + 3m}$

C) $\frac{F}{4M + 3m}$

D) $\frac{F}{M + 3m}$

E) $\frac{4F}{4M + 3m}$

Question 4) A circle that is at rest at $t = 0$ and has a moment of inertia $I = 20 \text{ (kg} \cdot \text{m}^2\text{)}$ moves with an angular acceleration $\alpha = 4t \text{ (rad/s}^2\text{)}$. Here t is in seconds. What is the power on the circle in Watts at $t = 2 \text{ (s)}$?

$$\alpha = 4t$$

$$\tau = I\alpha = 20 \cdot 4t = 80t$$

$$\tau_{t=2} = 160$$

$$P = \tau \omega$$

$$= 160 \cdot 8$$

$$= 1280 \text{ watt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\int d\omega = \int \alpha dt$$

$$\omega = 2t^2 = 8$$

A) 1280

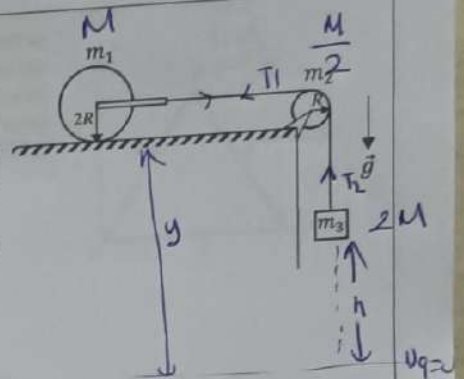
B) 2560

C) 1920

D) 1080

E) 640

Questions 5-6-7) A solid, uniform cylinder of mass $m_1 = M$ and radius $2R$ rests on a horizontal table. It is attached to a frictionless shaft passing through the center of the cylinder, and the cylinder can rotate around this shaft. The rope passes over a pulley with mass $m_2 = \frac{M}{2}$, radius R , and attached to a frictionless shaft passing through its center. A block of mass $m_3 = 2M$ is hung on the free end of the rope, as shown in the figure. The rope does not slip on the pulley surface and the cylinder rolls on the table without slipping. For a cylinder and pulley of mass m and radius r ; $I = \frac{1}{2}mr^2$. The system is released from rest, when the mass m_3 is displaced by h ;



5) Find the speed of the center of mass of the cylinder.

$$m_3: T_2 - m_3g = m_3(-a) \Rightarrow T_2 - 2Mg = 2M(-a)$$

$$T_2 - 2Mg = -2Ma \quad (1)$$

cylinder; $\sum \tau = I \alpha$

$$T_1 - f_s = Ma \quad (2)$$

$$m_1 = Mg \quad (3)$$

$$Z_0 = I_0 \alpha$$

$$f_s 2R = \frac{1}{2} M (2R)^2 \frac{a}{2R}$$

$$f_s = \frac{Ma}{2} \quad (4)$$

pulley; $\sum \tau = I_2 \alpha_2$

$$\sum \tau = I_2 \alpha_2$$

$$(T_2 - T_1)R = \frac{1}{2} \left(\frac{M}{2}\right) R^2 \frac{a}{R}$$

$$T_2 - T_1 = \frac{Ma}{4} \quad (5)$$

$$2Mg - T_2 = 2Ma$$

$$T_1 - f_s = Ma$$

$$T_2 - T_1 = \frac{Ma}{4}$$

$$2Mg - f_s = \frac{13Ma}{4}$$

$$2Mg = \frac{15Ma}{4}$$

$$a = \frac{8g}{15}$$

$$T_1 = \frac{3Mg}{2}$$

$$T_2 = \frac{7Mg}{4}$$

A) $\sqrt{\frac{16gh}{11}}$

B) $\sqrt{\frac{2gh}{15}}$

C) $\sqrt{\frac{4gh}{3}}$

D) $\sqrt{\frac{16gh}{15}}$ $\omega 2R = v$

E) $\sqrt{\frac{7gh}{9}}$

6) What is the tension force in the rope between masses m_2 and m_3 ? $(F = F_1)$

$$T_2 = \frac{7Mg}{4} = \frac{7m}{4} \frac{8g}{15}$$

$$T_2 = \frac{14Mg}{15}$$

$$m_1g + m_3gh = \frac{1}{2}m_1v^2 + \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}m_3v^2 + m_2gh + m_2\frac{1}{2}v^2$$

$$\frac{2Mg}{2}h = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}M\right)\frac{v^2}{R^2} + \frac{1}{2}(2M)v^2 + \frac{1}{2}\left(\frac{1}{2}M\right)\frac{v^2}{R^2}$$

$$4Mgh = Mv^2 + \frac{Mv^2}{2} + 2Mv^2 + \frac{Mv^2}{4}$$

$$4gh = \frac{15v^2}{4} \Rightarrow 16gh = 15v^2$$

$$v = \sqrt{\frac{16gh}{15}}$$

A) $\frac{16Mg}{11}$

B) $\frac{8Mg}{3}$

C) $\frac{14Mg}{15}$

D) $\frac{2Mg}{15}$

E) $\frac{16Mg}{9}$

7) What is the magnitude of the friction force?

$$f_s = \frac{Ma}{2} = \frac{M}{2} \frac{8g}{15}$$

$$f_s = \frac{4Mg}{15}$$

A) $\frac{7Mg}{9}$

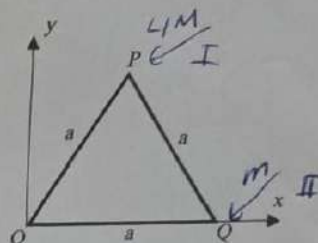
B) $\frac{10Mg}{15}$

C) $\frac{16Mg}{3}$

D) $\frac{8Mg}{11}$

E) $\frac{4Mg}{15}$

Question 8) An equilateral triangle plate with sides a and mass $4m$ is placed as shown in the figure. First, a point object with mass $4m$ is placed in vertex P of the plate, and then a point object with mass $4m$ is removed and a point object with mass m is placed in vertex Q . In both cases, the x -coordinate of the center of mass of the mass system is the same. Find the center of mass x_{CM} of the triangular plate?



$$(x_{cm})_i = (x_{cm})_f$$

$$\frac{4m \cdot \frac{a}{2} + 4m \cdot x_1}{8m} = \frac{4m \cdot x_1 + m \cdot a}{5m}$$

$$32m^2 x_1 + 8m^2 a = 10m^2 a + 20m^2 x_1$$

$$12x_1 = 2a \Rightarrow \boxed{x_1 = \frac{a}{6}}$$

A) $\frac{a}{3}$

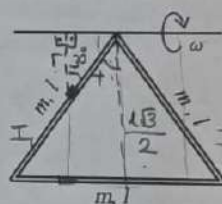
B) $\frac{a}{6}$

C) $\frac{6a}{3}$

D) $\frac{2a}{3}$

E) $\frac{3a}{2}$

Question 9) Three identical thin rods, each of mass m and length l are joint to form an equilateral triangle. As shown in the figure, it rotates with a constant angular velocity ω relative to the axis passing through the vertex of the triangle. Find the rotational kinetic energy in terms of the given ones. The moment of inertia about the axis passing through the center of a rod of mass m and length l is, $I = \frac{1}{12}ml^2$.



$$I_1 = \int r^2 dm = \int_0^l \frac{3}{4} x^2 \lambda dx$$

$$I_1 = \frac{3}{4} \frac{m}{l} \frac{x^3}{3} \Big|_0^l = \frac{m}{4l} l^3 = \frac{1}{4} ml^2 = I_2$$

$$r = x \cos 30^\circ$$

$$r = \frac{\sqrt{3}}{2} x$$

$$I_3 = m \left(\frac{\sqrt{3}l}{2} \right)^2 = \frac{3}{4} ml^2$$

$$I = \frac{1}{4} ml^2 + \frac{1}{4} ml^2 + \frac{3}{4} ml^2 = \frac{5}{4} ml^2$$

$$K = \frac{1}{2} I \omega^2 = \frac{5}{8} ml^2 \omega^2$$

A) $\frac{1}{4} ml^2 \omega^2$

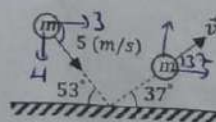
B) $\frac{7}{6} ml^2 \omega^2$

C) $\frac{5}{8} ml^2 \omega^2$

D) $\frac{5}{2} ml^2 \omega^2$

E) $ml^2 \omega^2$

Question 10) An object of mass $m = 1$ kg strikes a smooth horizontal surface as shown in the figure. Find the magnitude of the impulse exerted by the ball on the surface. ($\cos 37^\circ = \sin 53^\circ = 0.8$, $\cos 53^\circ = \sin 37^\circ = 0.6$)



$$\vec{J} = \Delta \vec{p} = \int \vec{F} \cdot dt$$

$$p_{xi} = p_{xs}$$

$$m u \cos 53^\circ = m v \cos 37^\circ$$

$$3 = v / 0.8 \Rightarrow \boxed{v = \frac{30}{8}}$$

$$\vec{v}_i = 5 \cos 53^\circ \hat{i} - 5 \sin 53^\circ \hat{j}$$

$$= 5 \cdot 0.6 \hat{i} - 5 \cdot 0.8 \hat{j}$$

$$\boxed{\vec{v}_i = 3\hat{i} - 4\hat{j}}$$

$$\vec{p}_i = 3\hat{i} - 4\hat{j}$$

$$p_s = \frac{30}{8} \cos 37^\circ \hat{i} + \frac{30}{8} \sin 37^\circ \hat{j}$$

$$p_s = \frac{30}{8} (0.8 \hat{i} + 0.6 \hat{j}) = 3\hat{i} + \frac{9}{4}\hat{j}$$

$$\Delta \vec{p} = \vec{p}_s - \vec{p}_i = \frac{9}{4} + 4 = \frac{25}{4} = 6.25 \text{ Ns}$$

A) 6.25 Ns

B) 1.76 Ns

C) 7.8 Ns

D) 5.25 Ns

E) 2.2 Ns