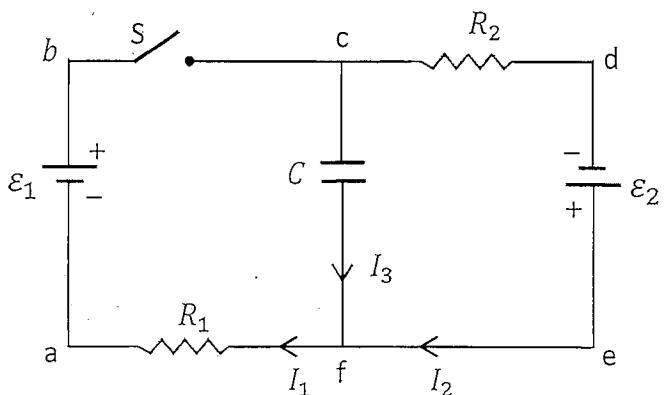




P1	P2	P3	P4	TOTAL
Name Surname				
Registration No				
Department				
Group No	Exam Hall	Signature of the Student		
Lecturer's Name Surname			The 9 th article of Student Disciplinary Regulations of YÖK Law No.2547 states "Cheating or helping to cheat or attempt to cheat in exams" de facto perpetrators takes one or two semesters suspension penalty. Calculators are not allowed. Do not ask any questions about the problems. There will be no explanations. Use the allocated areas for your answers and write legible	

PROBLEM 1

The switch S in the circuit is initially open and the capacitor is uncharged. In here, $\varepsilon_1 = 1\text{ V}$, $\varepsilon_2 = 3\text{ V}$, $R_1 = 0.2\Omega$, $R_2 = 0.3\Omega$ and $C = 5\mu\text{F}$.



a) Find the currents running in the circuit and the charge on the capacitor after a long time while S is still open.

$$I_1 = I_2 = I_3 = 0 \quad (3)$$

$$Q_i = C \Delta V_C \quad (1)$$

$$Q_i = C \cdot \varepsilon_2 = (5\mu\text{F})(3\text{V})$$

$$\boxed{Q_i = 15\mu\text{C}} \quad (2)$$

b) Now the switch is closed. Find the currents and the charge on the capacitor after a long time after S is closed.

$$(1) \quad I_3 = 0 \quad ; \quad I_1 = I_2 = \frac{\varepsilon_1 + \varepsilon_2}{R_1 + R_2} \quad (2)$$

$$I_1 = I_2 = \frac{4}{0.5} = \frac{40}{5}$$

$$\boxed{I_1 = I_2 = 8\text{A}} \quad (2)$$

Charge; Loop abcfa;
or Q_f

$$\varepsilon_1 + \Delta V_C - I_1 R_1 = 0 \quad (3)$$

$$\Delta V_C = I_1 R_1 - \varepsilon_1 = 8(0.2) - 1$$

$$\boxed{\Delta V_C = 0.6\text{V}}$$

$$Q_f = C \cdot \Delta V_C = (5\mu\text{F})(0.6)$$

$$\boxed{Q_f = 3\mu\text{C}} \quad (3)$$

c) Find powers supplied by the batteries and consumed across the resistors after switch S is closed for a long time.

$$P_{E_1} = \varepsilon_1 I_1 = 1 \cdot 8 = 8\text{W} \quad (1)$$

$$P_{E_2} = \varepsilon_2 I_2 = 3 \cdot 8 = 24\text{W} \quad (1)$$

$$P_E = P_{E_1} + P_{E_2} \Rightarrow \boxed{P_E = 32\text{W}}$$

Supplied power

— o —

$$P_{R_1} = I_1^2 R_1 = 8^2(0.2) = 12.8\text{W} \quad (2)$$

$$P_{R_2} = I_2^2 R_2 = 8^2(0.3) = 19.2\text{W} \quad (2)$$

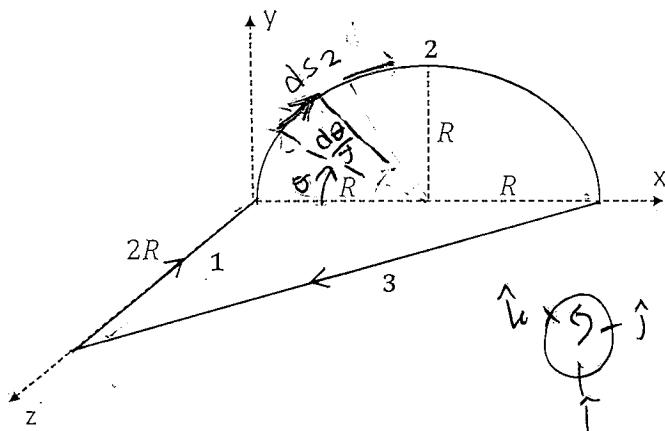
$$P_R = P_{R_1} + P_{R_2} = 12.8 + 19.2$$

$$\boxed{P_R = 32\text{W}}$$

Consumed power

PROBLEM 2

A closed loop carrying a constant current I is in a uniform magnetic field given by $\vec{B} = B_0(\hat{i} + \hat{j})$ (T) as shown in figure ($\pi = 3$).



a) Find the magnetic forces acting on each wire.

$$\text{Wire 1: } \vec{F}_1 = I\vec{l}_1 \times \vec{B}; \quad \vec{l}_1 = 2R(-\hat{k}) \quad (1)$$

$$\vec{F}_1 = 2IRB_0(-\hat{k}) \times (\hat{i} + \hat{j})$$

$$\boxed{\vec{F}_1 = 2IRB_0(\hat{i} - \hat{j})} \quad (3)$$

$$\text{Wire 2: } \vec{F}_2 = I\vec{l}_2 \times \vec{B}; \quad \vec{l}_2 = 2R\hat{i} \quad (2)$$

$$\vec{F}_2 = 2IRB_0\hat{i} \times (\hat{i} + \hat{j})$$

$$\boxed{\vec{F}_2 = 2IRB_0\hat{i}} \quad (3)$$

$$\text{Wire 3: } \vec{F}_3 = I\vec{l}_3 \times \vec{B}, \quad \vec{l}_3 = 2R(-\hat{i} + \hat{k}) \quad (2)$$

$$\vec{F}_3 = 2IRB_0(-\hat{i} + \hat{k}) \times (\hat{i} + \hat{j})$$

$$\boxed{\vec{F}_3 = 2IRB_0(-\hat{k} + \hat{j} - \hat{i})} \quad (3)$$

$$\vec{F}_3 = 2IRB_0(-\hat{i} + \hat{j} + \hat{k})$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 //$$

Alternative method for wire 2:

$$\vec{F}_2 = I \int d\vec{s}_2 \times \vec{B}; \quad d\vec{s}_2 = R d\theta (\sin\theta \hat{i} + \cos\theta \hat{j})$$

$$\vec{F}_2 = IRB_0 \int_0^\pi ((\sin\theta \hat{i} + \cos\theta \hat{j}) \times (\hat{i} + \hat{j})) d\theta$$

$$\vec{F}_2 = IRB_0 \int_0^\pi (\sin\theta - \cos\theta) d\theta \hat{k}$$

$$\vec{F}_2 = IRB_0 \left(-\cos\theta - \sin\theta \right) \Big|_0^\pi \hat{k}$$

$$\boxed{\vec{F}_2 = 2IRB_0(\hat{k})}$$

b) Find the magnetic dipole moment $\vec{\mu}$ of the loop.

$$\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2 \quad (1)$$

$$\vec{\mu}_1 = IA_1 \Rightarrow \vec{\mu}_1 = I \frac{\pi R^2}{2} (-\hat{k}) \quad (2)$$

$$\vec{\mu}_2 = IA_2 \Rightarrow \vec{\mu}_2 = I \frac{4\pi R^2}{2} (-\hat{j}) \quad (2)$$

$$\vec{\mu} = \frac{3\pi R^2}{2} (-\hat{k}) + 2\pi R^2 (-\hat{j})$$

$$\boxed{\vec{\mu} = IR^2(-2\hat{j} - \frac{3}{2}\hat{k})} \quad (2)$$

c) Find the magnetic potential energy U of the loop.

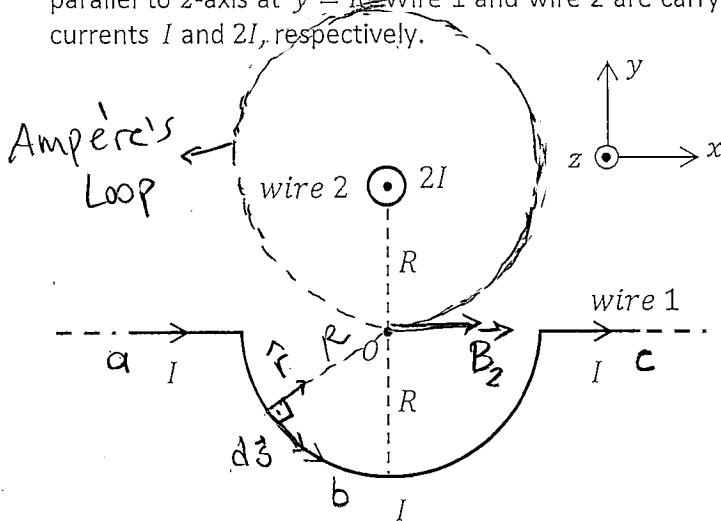
$$U = -\vec{\mu} \cdot \vec{B} \quad (2)$$

$$U = -IR^2(2\hat{j} + \frac{3}{2}\hat{k}) \cdot B_0(\hat{i} + \hat{j})$$

$$\boxed{U = 2IB_0R^2} \quad (2)$$

PROBLEM 3

An infinitely long current wire (wire 1) is bent into a semicircle of radius R and two semi-infinitely long straight wires. Wire 1 is placed on xy -plane as shown in figure. Another infinitely long straight wire (wire 2) is placed parallel to z -axis at $y = R$. Wire 1 and wire 2 are carrying currents I and $2I$, respectively.



- a) Find the total magnetic field vector at point O due to the wire 1 and wire 2.

Wire 1: Since $|d\vec{s} \times \hat{r}| = 0$ there is no contribution from segments a and c ; $B_a = B_c = 0$

(2)

Segment b: Biot-Savart Law or $B = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \sin\theta}{r^2}$

$$B_b = \frac{\mu_0 I}{4\pi} \int \frac{|d\vec{s} \times \hat{r}|}{r^2} ; r = R \quad (1)$$

$$|d\vec{s} \times \hat{r}| = ds \text{ direction } \rightarrow \hat{k}$$

$$B_b = \frac{\mu_0 I}{4\pi} \int \frac{ds}{R^2} = \frac{\mu_0 I}{4\pi R^2} \int \frac{ds}{\pi R} \quad (2)$$

$$\boxed{\vec{B}_b = \frac{\mu_0 I}{4R} \hat{k}} \quad (1) \quad B = \frac{\mu_0 I}{4R} \quad (2)$$

$$\vec{B}_1 = \vec{B}_a + \vec{B}_b + \vec{B}_2$$

$$\boxed{\vec{B}_1 = \frac{\mu_0 I}{4R} \hat{k}} \quad (2)$$

Wire 2: Ampère's Law

$$\oint \vec{B}_2 \cdot d\vec{l} = \mu_0 I_{in} \quad (2)$$

$$B_2 \cdot 2\pi R = \mu_0 \cdot 2I \quad (2)$$

$$\boxed{\vec{B}_2 = \frac{\mu_0 I}{\pi R} \hat{i}} \quad (2)$$

$$\vec{B}_o = \vec{B}_1 + \vec{B}_2$$

$$\boxed{\vec{B}_o = \frac{\mu_0 I}{R} \left(\frac{1}{\pi} \hat{i} + \frac{1}{4} \hat{k} \right)} \quad (3)$$

- b) A charge $+q$ is passing through point O with a velocity of $\vec{v} = v_0 \hat{j}$. Find the magnetic force acting on the charge.

$$\vec{F}_B = q \vec{v} \times \vec{B}_o \quad (2)$$

$$\vec{F}_B = \frac{qv_0 \mu_0 I}{R} \hat{j} \times \left(\frac{1}{\pi} \hat{i} + \frac{1}{4} \hat{k} \right)$$

$$\boxed{\vec{F}_B = \frac{qv_0 \mu_0 I}{R} \left(\frac{1}{4} \hat{i} - \frac{1}{\pi} \hat{k} \right)} \quad (3)$$

PROBLEM 4

A circular conductive loop of radius r is placed in a uniform magnetic field $\vec{B} = B_0 \hat{j}$ (figure 1). Loop rotates about z-axis with a constant angular speed ω (figure 2). ($\theta = \omega t$)

Figure 1

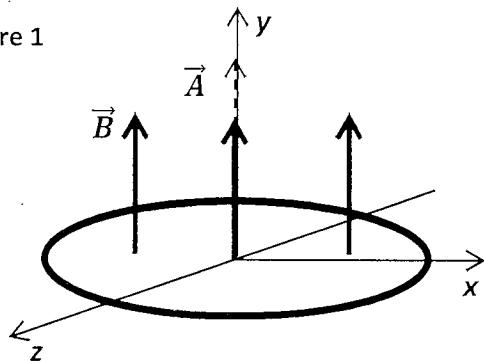
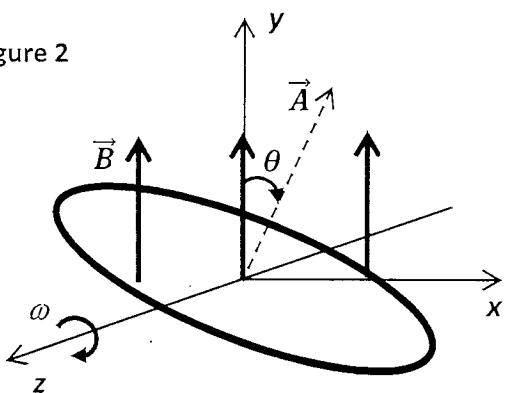


Figure 2



a) Find the electromotive force induced in the loop during the rotation.

$$\mathcal{E} = - \frac{d\Phi_B}{dt} \quad (2)$$

$$\Phi_B = B_0 A \cos \theta \quad (4), \quad \theta = \omega t$$

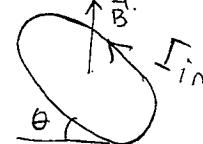
$$\Phi_B = B_0 A \cos \omega t$$

$$\mathcal{E} = - \frac{d}{dt} (B_0 A \cos \omega t) \quad (2)$$

$$\boxed{\mathcal{E} = +B_0 A \omega \sin \omega t} \quad (4)$$

b) If the resistance of the loop is R , find the magnitude of the induced current. Explain the direction of the induced current when $0 < \theta < 90^\circ$ according to the Lenz Law.

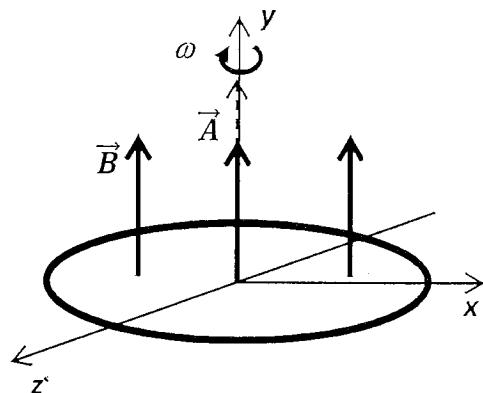
Since magnetic flux decreases when $0 < \theta < 90^\circ$; induced current I_i must be counter clockwise in order to create magnetic field in the same direction 2



$$I_i = \frac{|\mathcal{E}|}{R} \Rightarrow \quad (1)$$

$$\boxed{I_i = \frac{B_0 A \omega \sin \omega t}{R}} \quad (3)$$

c) Find the electromotive force induced in the loop if it rotates about y-axis with a constant angular speed ω .



In this case; $\Phi_B = B_0 A \cos 0^\circ$ 2

and constant all the time.
Therefore;

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = 0 \quad (5)$$