# Advanced Calculus 1, Midterm Exam      24.11.2020

**1)** Show that each of the following limits does not exit ( **a)** and **b)** ):

**a)** $\displaystyle\lim_{(x,y)\to(2,0)} \frac{xy-2y}{\sqrt{(x-2)^4+y^4}}$      **b)** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x(-1+\cos y)}{x^3+y^3}$

**c)** Find $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x-\sqrt{xy}}{2x^2-xy-y^2}$ .

**2.** If $\displaystyle f(x,y)=\frac{x^4+y^4}{\left(x^2+y^2\right)^{3/2}}$ for $(x,y)\neq(0,0)$, then make the function $f(x,y)$ continuous at $(x,y)=(0,0)$

(Express the function $f(x,y)$ as a piecewise function).

**3.** If $\begin{cases} x=u\cos\theta-v\sin\theta \\ y=u\sin\theta+v\cos\theta \end{cases}$ , express the expression $\left(F_x\right)^2+\left(F_y\right)^2$ in terms of new coordinates $u$ and $v$.

**4.** Show that the equations $\begin{cases} ye^x+vz-\cos u=2 \\ v\cos x+y^2u-xz^2=1 \end{cases}$ define $u$ and $v$ as differentiable functions of $x$, $y$ and $z$

near the point $(x,y,z,u,v)=(0,2,1,0,1)$ and find $\left(\dfrac{\partial v}{\partial z}\right)_{x,y}$ at $(x,y,z,u,v)=(0,2,1,0,1)$.

**Duration:** 90 Min.