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- BELİRLİ İNTEGRAL -

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^a f(x)\,dx = ?$$

$$\underbrace{\int_a^b f(x)\,dx + \int_b^c f(x)\,dx}_{\int_a^c f(x)\,dx} + \int_c^a f(x)\,dx =$$

$$\int_a^c f(x)\,dx + \int_c^a f(x)\,dx = \int_a^a f(x)\,dx = 0$$

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$$\int_0^2 3f(x)\,dx + \int_1^3 3f(x)\,dx - \int_0^3 2f(x)\,dx - \int_1^2 3f(x)\,dx$$

$$\int_1^3 3f(x)\,dx = \int_1^2 3f(x)\,dx + \int_2^3 3f(x)\,dx$$

$$\int_0^3 2f(x)\,dx = \int_0^2 2f(x)\,dx + \int_2^3 2f(x)\,dx \quad \text{olarak}$$

$$= \int_0^2 3f(x)\,dx + \int_1^2 3f(x)\,dx + \int_2^3 3f(x)\,dx - \int_0^2 2f(x)\,dx - \int_2^3 2f(x)\,dx - \int_1^2 3f(x)\,dx$$

$$= \int_0^2 (3-2)f(x)\,dx + \int_1^2 \underbrace{(3-3)}_{0} f(x)\,dx + \int_2^3 (3-2)f(x)\,dx$$

$$= \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx = \int_0^3 f(x)\,dx$$

27) $\int_{-2}^{2} (x+2)\,dx = ?$

$$\int_{-2}^{2} (x+2)\,dx = \frac{x^2}{2} + 2x \Big|_{-2}^{2} = \left(\frac{2^2}{2} + 2.2\right) - \left(\frac{(-2)^2}{2} + 2(-2)\right)$$
$$= (2+4) - (2-4)$$
$$= 6 + 2$$
$$= 8$$

2. YOL

$$\int_{-2}^{2} (x+2)\,dx = \int_{-2}^{2} x\,dx + \int_{-2}^{2} 2\,dx \quad ; \quad f(-x) = -f(x) \Rightarrow \int_{-a}^{a} f(x)\,dx = 0$$
$$= 0 + 2.\left(x \Big|_{-2}^{2}\right)$$
$$= 2.[2-(-2)]$$
$$= 8$$

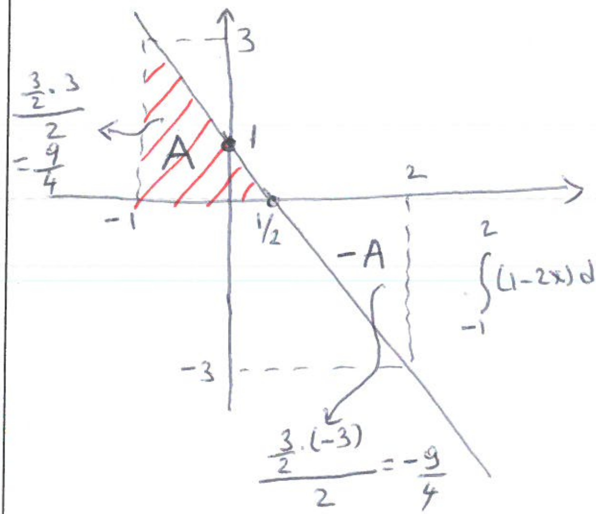
28) $\int_{0}^{2} (3x+1)\,dx = ?$

$$= 3.\frac{x^2}{2} + x \Big|_{0}^{2} = \left(3.\frac{2^2}{2} + 2\right) - \left(3.\frac{0^2}{2} + 0\right)$$
$$= 8$$

29) $\int_{-1}^{2} (1-2x)\,dx = ?$

$$= x - 2\cdot\frac{x^2}{2}\Big|_{-1}^{2} = x - x^2\Big|_{-1}^{2} = (2-2^2) - (-1-(-1)^2)$$
$$= (-2) - (-1-1)$$
$$= 0$$

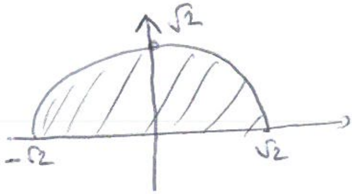
Grafik olarak



$$\int_{-1}^{2}(1-2x)\,dx = \int_{-1}^{1/2}(1-2x)\,dx + \int_{1/2}^{2}(1-2x)\,dx$$
$$= A - A$$
$$= 0$$

30) $\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{2-x^2}\,dx = ?$

$$\int_{-\sqrt{2}}^{\sqrt{2}} \underbrace{\sqrt{2-x^2}}_{y\geq 0}\,dx$$



$f(-x) = f(x)$ olarak

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

olarak

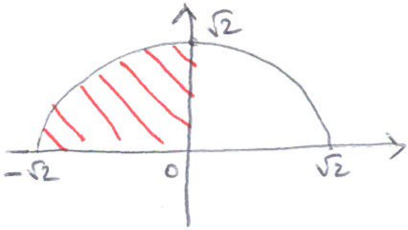
$$\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{2-x^2}\,dx = 2\int_{0}^{\sqrt{2}} \sqrt{2-x^2}\,dx = 2\cdot\left(\frac{\text{Daire Alanı}}{4}\right)$$
$$= 2\cdot\frac{\pi\cdot\sqrt{2}^2}{4} = \pi \text{ br}^2$$

$y = \sqrt{2-x^2}$

$y^2 = 2-x^2$

$x^2+y^2 = 2 = \sqrt{2}^2$

31) $$\int_{-\sqrt{2}}^{0}\sqrt{2-x^2}\,dx = ?$$



$y=\sqrt{2-x^2}$ olarak

$y^2 = 2-x^2$

$x^2+y^2 = 2 = \sqrt{2}^2$

$y \geq 0$ olup $-\sqrt{2} \leq x \leq 0$ alınacaktır.

Dolayısıyla;

$$\int_{-\sqrt{2}}^{0}\sqrt{2-x^2}\,dx = \frac{\text{Alan Daire}}{4} = \frac{\pi(\sqrt{2})^2}{4} = \frac{\pi}{2}\ \text{br}^2.$$

32) $$\int_{-\pi}^{\pi}\sin(x^3)\,dx = ?$$

$f(-x) = -f(x) \Rightarrow f(x)$ Tek.

$f(-x) = f(x) \Rightarrow f(x)$ Çift.

$\sin((-x)^3) = \sin(-x^3) = -\sin x^3$

Dolayısıyla $\sin x^3$ Tek fonksiyondur.

$$\int_{-\pi}^{\pi}\sin x^3\,dx = 0 \quad \text{olur.}$$

$f(x)$ tek $\Rightarrow \int_{-a}^{a} f(x)\,dx = 0$; $f(x)$ çift ise $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

Doç. Dr. Müslüm Özışık

33) $\int_{-a}^{a} (a-|s|)\,ds = ?$

İntegral s üzerinden alınacaktır. (ds)

$f(s) = a-|s|$
olursa
$f(-s) = a-|-s|$
$= a-|s| = f(s)$
$f(s)$ çift fonk.

$$\int_{-a}^{a} (a-|s|)\,ds = 2\int_{0}^{a} (a-|s|)\,ds$$

$$= 2\left[\int_0^a a\,ds - \int_0^a |s|\,ds\right]$$

$$\int_0^a a\,ds = a\int_0^a ds = a.s\Big|_0^a = a.(a-0) = a^2$$

$$\int_0^a |s|\,ds \xrightarrow{s\geq 0} \int_0^a s\,ds = \frac{s^2}{2}\Big|_0^a = \frac{1}{2}(a^2-0) = \frac{a^2}{2}$$

$$\int_0^a |s|\,ds \xrightarrow{s<0} \int_0^a -s\,ds = -\frac{s^2}{2}\Big|_0^a = -\frac{1}{2}(a^2-0) = -\frac{a^2}{2}$$

olarak

$$2\int_0^a (a-|s|)\,ds = 2\Big[\underbrace{\int_0^a a\,ds}_{a^2} - \underbrace{\int_0^a |s|\,ds}_{s\geq 0:\ \frac{a^2}{2} \qquad s<0:\ -\frac{a^2}{2}}\Big]$$

olarak

$$s\geq 0 \Rightarrow 2\left[a^2-\frac{a^2}{2}\right] = a^2$$

$$s<0 \Rightarrow 2\left[a^2-\left(-\frac{a^2}{2}\right)\right] = 3a^2$$

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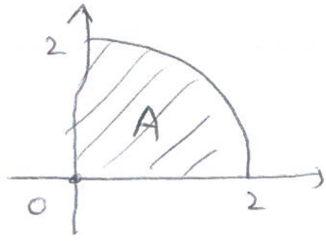
34) $\int_0^2 \underbrace{\sqrt{4-x^2}}_{y}\, dx = ?$



$y \geq 0$ olarak $y=\sqrt{4-x^2}$

$y^2 = 4-x^2$

$x^2+y^2 = 4 = 2^2$

$$\int_0^2 \sqrt{4-x^2}\, dx = \frac{\text{Alan daire}}{4} = \frac{\pi \cdot 2^2}{4} = \pi \ br^2$$

35) $\int_0^1 \left(x^2+\sqrt{1-x^2}\right) dx = ?$

$$= \int_0^1 x^2 dx + \int_0^1 \sqrt{1-x^2}\, dx$$

$$\int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}(1^3-0^3)$$

$$= \frac{1}{3}$$

$$\int_0^1 \sqrt{1-x^2}\, dx = \frac{\text{Alan daire}}{4} = \frac{\pi \cdot 1^2}{4} = \frac{\pi}{4}$$

$y=\sqrt{1-x^2}$

$y^2 = 1-x^2$

$x^2+y^2=1$

olarak

$$= \int_0^1 x^2 dx + \int_0^1 \sqrt{1-x^2}\, dx$$

$$= \frac{1}{3} + \frac{\pi}{4}$$

36) $y=f(x)=x+2$ fonksiyonunun $[0,4]$ için ortalama değerini bulunuz.

$$\bar{f}=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$=\frac{1}{4-0}\int_0^4 (x+2)\,dx$$

$$=\frac{1}{4}\left[x^2+2x\right]_0^4=\frac{1}{4}\left[(16+8)-0\right]=\frac{24}{4}=6$$

37) $f(u)=1+\sin u$ için $[-\pi,\pi]$'de ortalama değerini bulunuz

$$\bar{f}=\frac{1}{b-a}\int_a^b f(u)\,du$$

$$=\frac{1}{\pi-(-\pi)}\int_{-\pi}^{\pi}(1+\sin u)\,du=\frac{1}{2\pi}\left[u-\cos u\right]_{-\pi}^{\pi}$$

$$=\frac{1}{2\pi}\left[(\pi-\cos\pi)-(-\pi-\cos(-\pi))\right]$$

$$=\frac{1}{2\pi}\left[(\pi-1)-(-\pi-1)\right]$$

$$=\frac{1}{2\pi}\left[\pi-1+\pi+1\right]$$

$$\bar{f}=1$$

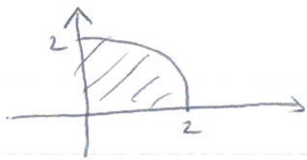
38 $f(x)=\sqrt{4-x^2}$ için $[0,2]$-deki ortalama değerini bulunuz

$$\bar{f}=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$=\frac{1}{2-0}\int_0^2 \sqrt{4-x^2}\,dx$$

$$=\frac{1}{2}\underbrace{\int_0^2 \sqrt{4-x^2}\,dx}_{\frac{\text{Alan daire}}{4}} \quad ; \quad y=\sqrt{4-x^2} \; ; \; y\geq 0 \; : \; [0,2]$$

$$x^2+y^2=2^2$$



$$=\frac{1}{2}\cdot\frac{\pi\cdot 2^2}{4}$$

$$\bar{f}=\frac{\pi}{2}$$

39 $f(t)=\frac{1}{t}$ için $[\frac{1}{2},2]$-deki ortalama değerini bulunuz

$$\bar{f}=\frac{1}{b-a}\int_a^b f(t)\,dt$$

$$=\frac{1}{2-\frac{1}{2}}\int_{1/2}^{2}\frac{1}{t}\cdot dt$$

$$=\frac{2}{3}\ln t\Big|_{1/2}^{2}$$

$$=\frac{2}{3}\left[\ln 2-\ln\left(\frac{1}{2}\right)\right]=\frac{2}{3}\left[\ln 2-\ln 2^{-1}\right]=\frac{2}{3}\left[\ln 2+\ln 2\right]$$

$$=\frac{2}{3}(2\ln 2)=\frac{4}{3}\ln 2$$

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(40) $f(x)=\begin{cases} x+1 & ; x<0 \\ 2 & ; x\geq 0 \end{cases}$ ise $\int_{-3}^{2} f(x)\,dx=?$

Fonksiyon parçalı sürekli olup

$$\int_{-3}^{2} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$= \int_{-3}^{0} (x+1)\,dx + \int_{0}^{2} 2\,dx$$

$$= \left[\frac{x^2}{2}+x\right]_{-3}^{0} + 2.x\Big|_{0}^{2}$$

$$= \left[(0)-\left(\frac{(-3)^2}{2}+(-3)\right)\right] + 2.(2-0)$$

$$= -\frac{9}{2}-3+4$$

$$= \frac{-9-6+8}{2}$$

$$= -\frac{7}{2}$$

(41) $g(x)=\begin{cases} x^2 & ; 0\leq x\leq 1 \\ x & ; 1< x\leq 2 \end{cases}$ ise $\int_{0}^{2} g(x)\,dx=?$

$$\int_{0}^{2} g(x)\,dx = \int_{0}^{1} g(x)\,dx + \int_{1}^{2} g(x)\,dx = \int_{0}^{1} x^2\,dx + \int_{1}^{2} x\,dx$$

$$= \frac{x^3}{3}\Big|_{0}^{1} + \frac{x^2}{2}\Big|_{1}^{2} = \frac{1}{3}[1-0]+\frac{1}{2}[4-1] = \frac{1}{3}-2$$

$$= -5/3$$

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42) $\int \frac{dx}{\cos x} = ?$

$$\int \frac{dx}{\cos x} = \int \frac{\cos x \cdot dx}{\cos x \cdot \cos x} = \int \frac{\cos x}{1-\sin^2 x} dx$$

$$\boxed{\begin{array}{l} \int \frac{dx}{a^2-x^2} = -\frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + c \\ \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + c \end{array}}$$

$u = \sin x$

$du = \cos x\, dx$

$$= \int \frac{du}{1-u^2} = -\frac{1}{2} \ln\left(\frac{1-u}{1+u}\right) + c = \frac{1}{2} \ln\left(\frac{1+u}{1-u}\right) + c$$

$$= \frac{1}{2} \ln\left(\frac{1+u}{1-u}\right) + c = \frac{1}{2} \ln\left(\frac{(1+u)^2}{1-u^2}\right) + c \quad ; \quad u = \sin x$$

$(1+u)$

$$= \frac{1}{2} \ln\left(\frac{(1+u)^2}{\cos^2 x}\right) + c = \ln\left(\frac{1+u}{\cos x}\right) + c = \ln\left(\frac{1+\sin x}{\cos x}\right) + c$$

$$= \frac{1}{2} \ln\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) + c$$

$$= \frac{1}{2} \ln(\sec x + \tan x) + c$$

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(*) $\int \cos^n(ax+b)\,dx$ İNTEGRALİ : $(n>2)$

n çift ise;

$$\int \cos^n(ax+b)\,dx = \int \left[\cos^2(ax+b)\right]^{\frac{n}{2}} dx = \int \left[\frac{1+\cos(2.(ax+b))}{2}\right]^{\frac{n}{2}} dx$$

şeklinde yazılır ve benzer işlemler gerekirse tekrarlanır.

n tek ise;

$$\int \cos^n(ax+b)\,dx = \int \cos(ax+b).\cos^{n-1}(ax+b).dx$$

$$= \int \cos(ax+b).\left[\cos^2(ax+b)\right]^{\frac{n-1}{2}} dx$$

$$= \int \cos(ax+b)\left[1-\sin^2(ax+b)\right]^{\frac{n-1}{2}} dx$$

$u=\sin(ax+b)$ dönüşümü ile integral çözülür.

Not:

① $\int \sin^n(ax+b)\,dx$ integrali de benzer şekilde çözülür.

② $\cos^2 x = \dfrac{1+\cos 2x}{2}$; $\sin^2 x = \dfrac{1-\cos 2x}{2}$ 'dir.

44) $\int \cos^2 x \, dx = ?$

$$\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2} dx = \frac{1}{2}\int (1+\cos 2x) dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c$$

45) $\int \cos^4 x \, dx = ?$

$$\int \cos^4 x \cdot dx = \int (\cos^2 x)^2 dx = \int \left[\frac{1+\cos 2x}{2}\right]^2 dx$$

$$= \frac{1}{4}\int \left[1 + 2\cdot\cos 2x + \cos^2 2x\right] dx$$

$$= \frac{1}{4}\int \left[1 + 2\cdot\cos 2x + \frac{1+\cos 4x}{2}\right] dx$$

$$= \frac{1}{4}\left[x + 2\cdot\frac{\sin 2x}{2} + \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right)\right] + c$$

$$= \frac{\sin 4x}{32} + \frac{\sin 2x}{4} + \frac{3x}{8} + c$$

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$$I=\int \cos^6 x\, dx = ?$$

$$\int \cos^6 x\, dx = \int (\cos^2 x)^3 dx = \int \left(\frac{1+\cos 2x}{2}\right)^3 \cdot dx$$

$$= \frac{1}{8}\int (1+\cos 2x)^3 dx = \frac{1}{8}\int [1+3.\cos 2x + 3.\cos^2 2x + \cos^3 2x] dx$$

$$= \underbrace{\frac{1}{8}\int 1 \cdot dx}_{I_1} + \underbrace{\frac{1}{8}\int 3.\cos 2x\, dx}_{I_2} + \underbrace{\frac{1}{8}\int 3.\cos^2 2x\, dx}_{I_3} + \underbrace{\frac{1}{8}\int \cos^3 2x\, dx}_{I_4}$$

$$I_1 = \frac{1}{8}\int 1 \cdot dx = \frac{x}{8} + C_1$$

$$I_2 = \frac{1}{8}\int 3.\cos 2x\, dx = \frac{3}{8}\, \frac{\sin 2x}{2} + C_2 = \frac{3}{16}\sin 2x + C_2$$

$$I_3 = \frac{1}{8}\int 3.\cos^2 2x\, dx = \frac{3}{8}\int \left[\frac{1+\cos 2x}{2}\right] dx = \frac{3}{16}\left[x + \frac{\sin 2x}{2}\right] + C_3$$

$$= \frac{3.\sin 2x}{32} + \frac{3x}{16} + C_3$$

$$I_4 = \frac{1}{8}\int \cos^3 2x\, dx = \frac{1}{8}\int \cos 2x.\cos^2 2x\, dx = \frac{1}{8}\int \cos 2x.(1-\sin^2 2x) dx$$

$$u = \sin 2x \Rightarrow du = 2.\cos 2x\, dx$$

$$= \frac{1}{8}\cdot\frac{1}{2}\int (1-u^2).du$$

$$= \frac{1}{16}\left[\sin 2x - \frac{\sin^3 2x}{3}\right] + C_4$$

$I = I_1 + I_2 + I_3 + I_4$ olarak düzenlenir.

Not: Bu toplamdaki $C_1+C_2+C_3+C_4 = C$ olarak alınır.

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$\int \sin^3 x . \cos^8 x \, dx = ?$

$\int \sin^2 x . \sin x . \cos^8 x \, dx = \int (1-\cos^2 x) . \cos^8 x . \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$= -\int (1-u^2) . u^8 . du = -\int (u^8 - u^{10}) \, du = -\left(\frac{u^9}{9} - \frac{u^{11}}{11}\right) + c$

$= \frac{u^{11}}{11} - \frac{u^9}{9} + c$

$= \frac{\cos^{11} x}{11} - \frac{\cos^9 x}{9} + c$

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$\int \cos^3 x . \sin^8 x . dx = ?$

$\int \cos^2 x . \cos x . \sin^8 x \, dx = \int (1-\sin^2 x) . \cos x . \sin^8 x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int (1-u^2) . u^8 . du$

$= \int (u^8 - u^{10}) du$

$= \frac{u^9}{9} - \frac{u^{11}}{11} + c$

$= \frac{\sin^9 x}{9} - \frac{\sin^{11} x}{11} + c$

49

$$\int \frac{dx}{\sqrt{4+2x-x^2}} = ?$$

$$4+2x-x^2 = 5-(x-1)^2$$

$$\int \frac{dx}{\sqrt{5-(x-1)^2}} = \int \frac{dx}{\sqrt{\sqrt{5}^2-(x-1)^2}} = \int \frac{dx}{\sqrt{5}\cdot\sqrt{1-\left(\frac{x-1}{\sqrt{5}}\right)^2}} = \int \frac{dx}{\sqrt{5}\cdot\sqrt{1-\left(\frac{x-1}{\sqrt{5}}\right)^2}}$$

$$= \frac{1}{\sqrt{5}} \int \frac{dx}{\sqrt{1-\left(\frac{x-1}{\sqrt{5}}\right)^2}}$$

$$\boxed{\int \frac{dx}{\sqrt{a^2-x^2}} = \text{Arcsin}\left(\frac{x}{a}\right)+c}$$

$$u = \frac{x-1}{\sqrt{5}}$$

$$du = \frac{dx}{\sqrt{5}}$$

$$= \int \frac{du}{\sqrt{1-u^2}} = \text{Arcsin}\,u + c = \text{Arcsin}\left(\frac{x-1}{\sqrt{5}}\right)+c$$

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$$\int \frac{x+1}{\sqrt{1-x^2}}\,dx = \int \frac{x}{\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{-1}{2}\int \frac{2x\,dx}{\sqrt{1-x^2}} + \text{Arcsin}\,x + c$$

$$= -\sqrt{1-x^2} + \text{Arcsin}\,x + c$$

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$$\int \frac{dx}{e^x+e^{-x}} = ?$$

$$\int \frac{dx}{e^x+e^{-x}} = \int \frac{e^x . dx}{e^{2x}+1} = \int \frac{e^x dx}{(e^x)^2+1}$$

$$\boxed{\int \frac{dx}{a^2+x^2} = \frac{1}{a} \text{Arctg}\left(\frac{x}{a}\right)+c}$$

$u = e^x$

$du = e^x . dx$

$$\int \frac{du}{u^2+1} = \text{Arctg}\, u + c = \text{Arctg}\, e^x + c$$

52

$$\int_1^{\sqrt{e}} \frac{\sin(\pi \ln x)}{x} dx = ?$$

$u = \ln x$ için $x=1 \Rightarrow u = \ln 1 = 0$

$x = \sqrt{e} \Rightarrow u = \ln \sqrt{e} = \frac{1}{2}$

$du = \frac{dx}{x}$

$$\int_0^{1/2} \sin(\pi . u) . du = -\frac{1}{\pi} \cos(\pi u) \Big|_0^{1/2}$$

$$= -\frac{1}{\pi}\left[\cos\left(\pi . \frac{1}{2}\right) - \cos(\pi . 0)\right]$$

$$= -\frac{1}{\pi}\left[\cos\frac{\pi}{2} - \cos 0\right]$$

$$= -\frac{1}{\pi}\left[0 - 1\right]$$

$$= \frac{1}{\pi}$$

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$$y=f(x)=\begin{cases}-1 \;; & -1\leq x\leq 0\\ 1 \;; & 0< x\leq 1\end{cases} \Rightarrow \int_{-1}^{1} f(x)\,dx = ?$$

f(x) parçalı sürekli olup;

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

$$= \int_{-1}^{0} (-1)\,dx + \int_{0}^{1} (1)\cdot dx$$

$$= -x\Big|_{-1}^{0} + x\Big|_{0}^{1}$$

$$= -[(0)-(-1)] + [(1)-(0)]$$

$$= -[1] + [1]$$

$$= -1+1$$

$$= 0$$

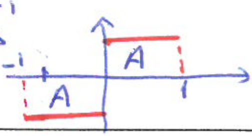
Çözüm-2-

f(x); $x\leq 0$ için -1

$x>0$ için $+1$ olup

$f(-x) = -f(x)$ yani $\int_{-a}^{a} f(x)\,dx = 0$ olarak

$$\int_{-1}^{1} f(x)\,dx = 0$$

Çözüm-3 

olarak $\int_{-1}^{1} f(x)\,dx = 0$

54

$$\int_0^{\pi/2} \sqrt{1+\cos x}\,.\,dx = ?$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= \cos^2 x - (1-\cos^2 x)$$
$$= 2\cos^2 x - 1$$
$$1+\cos 2x = 2.\cos^2 x$$

olarak

$$1+\cos x = 2.\cos^2\left(\frac{x}{2}\right)$$

$$\int_0^{\pi/2} \sqrt{2.\cos^2\frac{x}{2}}\,.\,dx$$

$$\sqrt{2}\int_0^{\pi/2} \cos\left(\frac{x}{2}\right)dx$$

$$\sqrt{2}\,.\,\frac{1}{\left(\frac{1}{2}\right)}\,.\,\sin\left(\frac{x}{2}\right)\Big|_0^{\pi/2}$$

$$= 2\sqrt{2}\sin\left(\frac{x}{2}\right)\Big|_0^{\pi/2} = 2\sqrt{2}\left[\left(\sin\frac{\pi/2}{2}\right)-\left(\sin\frac{0}{2}\right)\right]$$

$$= 2\sqrt{2}\left[\sin\frac{\pi}{4} - \sin 0\right]$$

$$= 2\sqrt{2}\left[\frac{\sqrt{2}}{2} - 0\right]$$

$$= 2$$

55

$$\int\frac{dx}{e^x+1} = ?$$

$$\int\frac{e^x dx}{e^x(e^x+1)} = \int\frac{e^x dx}{e^{2x}+e^x} = \int\frac{e^x dx}{(e^x)^2+e^x} \quad ; \quad e^x = u, \quad e^x dx = du$$

$$= \int\frac{du}{u^2+u} = \int\left[\frac{1}{u} - \frac{1}{u+1}\right]du = \int\frac{du}{u} - \int\frac{du}{u+1}$$

$$= \ln u - \ln(u+1) + c = \ln\left(\frac{u}{u+1}\right) + c$$

$$= \ln\left(\frac{e^x}{e^x+1}\right) + c$$

56 $\int \frac{\cos x}{4+\sin^2 x}\,dx = ?$

$$\boxed{\int \frac{dx}{a^2+x^2} = \frac{1}{a}\,\mathrm{Arctg}\,\frac{x}{a} + c}$$

$u = \sin x$

$du = \cos x\,dx$

$$\int \frac{du}{4+u^2} = \int \frac{du}{2^2+u^2} = \frac{1}{2}\,\mathrm{Arctg}\left(\frac{u}{2}\right) + c$$

$$= \frac{1}{2}\,\mathrm{Arctg}\left(\frac{\sin x}{2}\right) + c$$

57 $\int \frac{x^2\,dx}{2+x^6} = ?$

$\int \frac{x^2\,dx}{2+(x^3)^2}$; $u = x^3$

$du = 3x^2\,dx$

$x^2\,dx = \frac{du}{3}$

$$\int \frac{\left(\frac{du}{3}\right)}{2+u^2} = \frac{1}{3}\int \frac{du}{2+u^2} = \frac{1}{3}\int \frac{du}{\sqrt{2}^2+u^2}$$

$$= \frac{1}{3}\cdot\frac{1}{\sqrt{2}}\,\mathrm{Arctg}\left(\frac{u}{\sqrt{2}}\right) + c$$

$$= \frac{1}{3\sqrt{2}}\,\mathrm{Arctg}\left(\frac{x^3}{\sqrt{2}}\right) + c$$