



Name Surname

Registration No

Department

Group No

Exam Hall

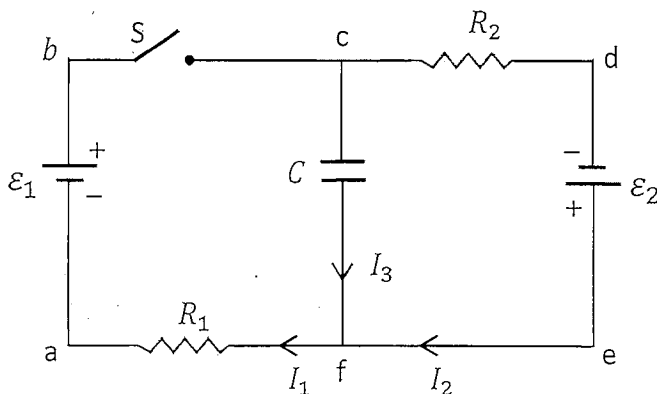
Signature of the Student

Lecturer's
Name Surname

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PROBLEM 1

The switch S in the circuit is initially open and the capacitor is uncharged. In here, $\mathcal{E}_1 = 1\text{ V}$, $\mathcal{E}_2 = 3\text{ V}$, $R_1 = 0.2\ \Omega$, $R_2 = 0.3\ \Omega$ and $C = 5\ \mu\text{F}$.



a) Find the currents running in the circuit and the charge on the capacitor after a long time while S is still open.

$$I_1 = I_2 = I_3 = 0 \quad (3)$$

$$Q_i = C \Delta V_c \quad (1)$$

$$Q_i = C \cdot \mathcal{E}_2 = (5\text{MF})(3\text{V})$$

$$Q_i = 15\text{MC} \quad (2)$$

b) Now the switch is closed. Find the currents and the charge on the capacitor after a long time after S is closed.

$$(1) \quad I_3 = 0 \quad ; \quad I_1 = I_2 = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R_1 + R_2} \quad (2)$$

$$I_1 = I_2 = \frac{4}{0.5} = \frac{40}{5}$$

$$I_1 = I_2 = 8\text{A} \quad (2)$$

Charge; Loop abcfa:

$$\mathcal{E}_1 + \Delta V_c - I_1 R_1 = 0 \quad (3)$$

$$\Delta V_c = I_1 R_1 - \mathcal{E}_1 = 8(0.2) - 1$$

$$\Delta V_c = 0.6\text{V}$$

$$Q_f = C \cdot \Delta V_c = (5\text{MF})(0.6)$$

$$Q_f = 3\text{MC} \quad (3)$$

c) Find powers supplied by the batteries and consumed across the resistors after switch S is closed for a long time.

$$P_{\mathcal{E}_1} = \mathcal{E}_1 I_1 = 1 \cdot 8 = 8\text{W} \quad (2)$$

$$P_{\mathcal{E}_2} = \mathcal{E}_2 I_2 = 3 \cdot 8 = 24\text{W} \quad (2)$$

$$P_E = P_{\mathcal{E}_1} + P_{\mathcal{E}_2} \Rightarrow P_E = 32\text{W}$$

Supplied power

$$P_{R_1} = I_1^2 R_1 = 8^2 (0.2) = 12.8\text{W} \quad (2)$$

$$P_{R_2} = I_2^2 R_2 = 8^2 (0.3) = 19.2\text{W} \quad (2)$$

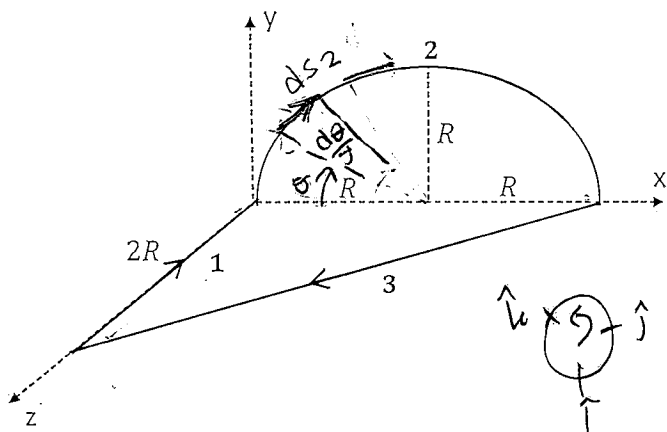
$$P_R = P_{R_1} + P_{R_2} = 12.8 + 19.2$$

$$P_R = 32\text{W}$$

Consumed power

PROBLEM 2

A closed loop carrying a constant current I is in a uniform magnetic field given by $\vec{B} = B_0(\hat{i} + \hat{j})$ (T) as shown in figure ($\pi = 3$).



a) Find the magnetic forces acting on each wire.

Wire 1: $\vec{F}_1 = I \vec{l}_1 \times \vec{B}$; $\vec{l}_1 = 2R(-\hat{k})$ ①

$$\vec{F}_1 = 2IRB_0(-\hat{k}) \times (\hat{i} + \hat{j})$$

$$\boxed{\vec{F}_1 = 2IRB_0(\hat{i} - \hat{j})} \quad ③$$

Wire 2: $\vec{F}_2 = I \vec{l}_2 \times \vec{B}$; $\vec{l}_2 = 2R\hat{i}$ ②

$$\vec{F}_2 = 2IRB_0\hat{i} \times (\hat{i} + \hat{j})$$

$$\boxed{\vec{F}_2 = 2IRB_0\hat{k}} \quad ③$$

Wire 3: $\vec{F}_3 = I \vec{l}_3 \times \vec{B}$, $\vec{l}_3 = 2R(\hat{i} + \hat{k})$ ②

$$\vec{F}_3 = 2IRB_0(-\hat{i} + \hat{k}) \times (\hat{i} + \hat{j})$$

$$\vec{F}_3 = 2IRB_0(-\hat{k} + \hat{j} - \hat{i}) \quad ③$$

$$\vec{F}_3 = 2IRB_0(-\hat{i} + \hat{j} + \hat{k})$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 //$$

Alternative method for wire 2:

$$\vec{F}_2 = I \int d\vec{s}_2 \times \vec{B}; d\vec{s}_2 = R d\theta (\sin\theta \hat{i} + \cos\theta \hat{j})$$

$$\vec{F}_2 = IRB_0 \int_0^\pi (\sin\theta \hat{i} + \cos\theta \hat{j}) \times (\hat{i} + \hat{j}) d\theta$$

$$\vec{F}_2 = IRB_0 \int_0^\pi (\sin\theta - \cos\theta) d\theta \hat{k}$$

$$\vec{F}_2 = IRB_0 (-\cos\theta + \sin\theta) \Big|_0^\pi (\hat{k})$$

$$\boxed{\vec{F}_2 = 2IRB_0(\hat{k})}$$

b) Find the magnetic dipole moment $\vec{\mu}$ of the loop.

$$\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2 \quad ①$$

$$\vec{\mu}_1 = I \vec{A}_1 \Rightarrow \vec{\mu}_1 = I \frac{\pi R^2}{2} (-\hat{k}) \quad ②$$

$$\vec{\mu}_2 = I \vec{A}_2 \Rightarrow \vec{\mu}_2 = I \frac{4R^2}{2} (-\hat{j}) \quad ②$$

$$\vec{\mu} = \frac{3\pi R^2}{2} (-\hat{k}) + 2IR^2 (-\hat{j})$$

$$\boxed{\vec{\mu} = IR^2 (-2\hat{j} - \frac{3}{2}\hat{k})} \quad ②$$

c) Find the magnetic potential energy U of the loop.

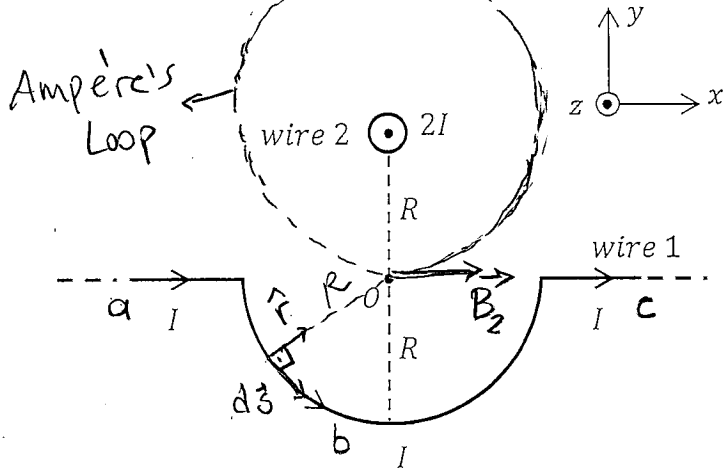
$$U = -\vec{\mu} \cdot \vec{B} \quad ②$$

$$U = -IR^2 (2\hat{j} + \frac{3}{2}\hat{k}) \cdot B_0(\hat{i} + \hat{j})$$

$$\boxed{U = 2IB_0R^2} \quad ②$$

PROBLEM 3

An infinitely long current wire (wire 1) is bent into a semicircle of radius R and two semi-infinitely long straight wires. Wire 1 is placed on xy -plane as shown in figure. Another infinitely long straight wire (wire 2) is placed parallel to z -axis at $y = R$. Wire 1 and wire 2 are carrying currents I and $2I$, respectively.



- a) Find the total magnetic field vector at point O due to the wire 1 and wire 2.

Wire 1: Since $|d\vec{s} \times \vec{r}| = 0$ there is no contribution from segments a and c ; $B_a = B_c = 0$ (2)

Segment b: Biot-Savart Law (1) or $B = \frac{\mu_0 I}{4\pi} \int \frac{ds \sin \theta}{r^2}$
 $B_b = \frac{\mu_0 I}{4\pi} \int \frac{|d\vec{s} \times \vec{r}|}{r^2}$; $r = R$ (1)
 $|d\vec{s} \times \vec{r}| = ds$ direction $\rightarrow \hat{k}$

$$B_b = \frac{\mu_0 I}{4\pi} \int \frac{ds}{R^2} = \frac{\mu_0 I}{4\pi R^2} \int ds$$

$$\vec{B}_b = \frac{\mu_0 I}{4R} \hat{k} \quad (1)$$

$$B = \frac{\mu_0 I}{4R} \quad (2)$$

$$\vec{B}_1 = \vec{B}_a + \vec{B}_b + \vec{B}_c$$

$$\vec{B}_1 = \frac{\mu_0 I}{4R} \hat{k} \quad (2)$$

Wire 2: Ampère's Law

$$\oint \vec{B}_2 \cdot d\vec{s} = \mu_0 I_{in} \quad (2)$$

$$B_2 2\pi R = \mu_0 \cdot 2I \quad (2)$$

$$\vec{B}_2 = \frac{\mu_0 I}{\pi R} \hat{i} \quad (2)$$

$$\vec{B}_0 = \vec{B}_1 + \vec{B}_2$$

$$\vec{B}_0 = \frac{\mu_0 I}{R} \left(\frac{1}{\pi} \hat{i} + \frac{1}{4} \hat{k} \right) \quad (3)$$

- b) A charge $+q$ is passing through point O with a velocity of $\vec{v} = v_0 \hat{j}$. Find the magnetic force acting on the charge.

$$\vec{F}_B = q \vec{v} \times \vec{B}_0 \quad (2)$$

$$\vec{F}_B = \frac{qv_0 \mu_0 I}{R} \hat{j} \times \left(\frac{1}{\pi} \hat{i} + \frac{1}{4} \hat{k} \right)$$

$$\vec{F}_B = \frac{qv_0 \mu_0 I}{R} \left(\frac{1}{4} \hat{i} - \frac{1}{\pi} \hat{k} \right) \quad (3)$$

PROBLEM 4

A circular conductive loop of radius r is placed in a uniform magnetic field $\vec{B} = B_0 \hat{y}$ (figure 1). Loop rotates about z -axis with a constant angular speed ω (figure 2). ($\theta = \omega t$)

Figure 1

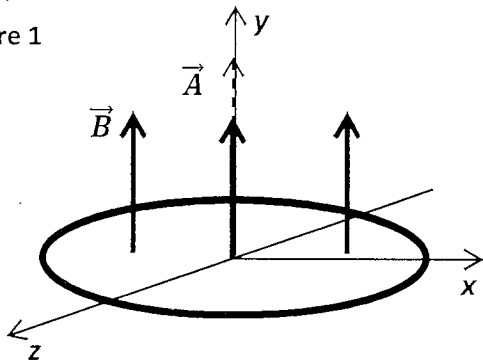
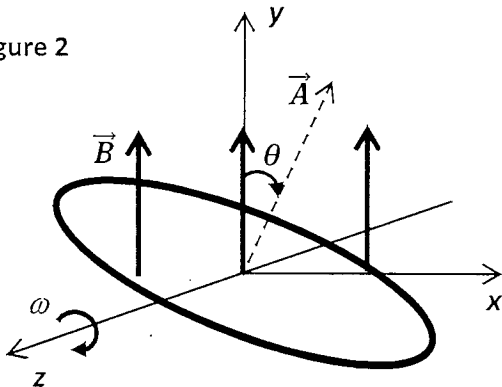


Figure 2



a) Find the electromotive force induced in the loop during the rotation.

$$\mathcal{E} = - \frac{d\Phi_B}{dt} \quad (2)$$

$$\Phi_B = B_0 A \cos \theta \quad (4), \quad \theta = \omega t$$

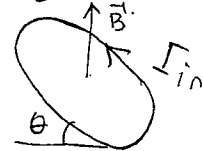
$$\Phi_B = B_0 A \cos \omega t$$

$$\mathcal{E} = - \frac{d}{dt} (B_0 A \cos \omega t) \quad (2)$$

$$\boxed{\mathcal{E} = + B_0 A \omega \sin \omega t} \quad (4)$$

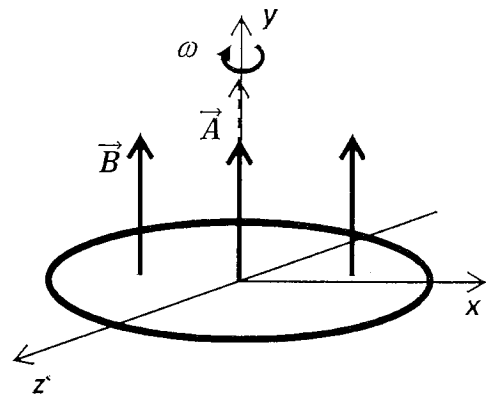
b) If the resistance of the loop is R , find the magnitude of the induced current. Explain the direction of the induced current when $0 < \theta < 90^\circ$ according to the Lenz Law.

Since magnetic flux decreases when $0 < \theta < 90^\circ$; induced current I_i must be counter clockwise in order to create magnetic field in the same direction (2)



$$\frac{I_i}{1} = \frac{|\mathcal{E}|}{R} \Rightarrow \boxed{I_i = \frac{B_0 A \omega \sin \omega t}{R}} \quad (3)$$

c) Find the electromotive force induced in the loop if it rotates about y -axis with a constant angular speed ω .



In this case; $\Phi_B = B_0 A \cos 0^\circ$ (2)
and constant all the time.
Therefore;

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = 0 \quad (5)$$