SORU-1:

$$\int \frac{x^2+5}{x\left(x^2+2x+5\right)}\,dx$$ integralini hesaplayınız.

$$\frac{x^2+5}{x(x^2+2x+5)} = \underset{(x^2+2x+5)}{\frac{A}{x}} + \underset{(x)}{\frac{Bx+C}{x^2+2x+5}}$$

$$x^2+5 = (A+B)x^2 + (2A+C)x + 5A$$

$$\left.\begin{array}{r} A+B=1 \\ 2A+C=0 \\ 5A=5 \end{array}\right] \begin{array}{l} A=1 \\ B=0 \\ C=-2 \end{array}$$

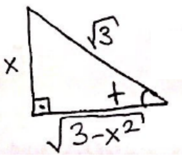
$$\int \frac{x^2+5}{x(x^2+2x+5)} = \int \left[\frac{1}{x} + \frac{-2}{(x+1)^2+4}\right] dx$$

$$= \ln|x| - 2\cdot\frac{1}{2}\arctan\frac{(x+1)}{2} + C$$

SORU-2:

$$\int \frac{x^2}{\left(3-x^2\right)^{3/2}}\,dx$$ integralini hesaplayınız.

$$x=\sqrt{3}\sin t$$
$$dx=\sqrt{3}\cos t\,dt$$



$$= \int \frac{\cancel{3}\sin^2 t}{\cancel{3}\sqrt{3}\,(1-\sin^2 t)^{3/2}}\cdot\sqrt{3}\cos t\,dt$$

$$= \int \frac{\sin^2 t\cdot\cos t}{\cos^3 t}\,dt = \int \tan^2 t\,dt$$

$$= \int (\sec^2 t - 1)\,dt = \tan t - t + C$$

$$= \frac{x}{\sqrt{3-x^2}} - \arcsin\frac{x}{\sqrt{3}} + C$$

SORU-3:

$$\int \frac{\cos^3 t}{\sin^3 t - \sin^2 t - 6\sin t}\,dt$$ integralini hesaplayınız.

$$I = \int \frac{\cos^3 t}{\sin^3 t - \sin^2 t - 6\sin t}\,dt$$

$$\sin t = u$$
$$\cos t\,dt = du$$
$$\cos^2 t = 1-u^2$$

$$= \int \frac{(1-u^2)\,du}{u^3-u^2-6u} = \int \frac{1-u^2}{u(u^2-u-6)}\,du$$

$$I = \int \left(-\frac{1}{6}\cdot\frac{1}{u} - \frac{8}{15}\cdot\frac{1}{u-3} - \frac{3}{10}\cdot\frac{1}{u+2}\right)du$$

$$= -\frac{1}{6}\ln|u| - \frac{8}{15}\ln|u-3| - \frac{3}{10}\ln|u+2| + C$$

$$= -\frac{1}{6}\ln|\sin t| - \frac{8}{15}\ln|\sin t-3| - \frac{3}{10}\ln|\sin t+2| + C$$

$$\frac{1-u^2}{u(u-3)(u+2)} = \frac{A}{u} + \frac{B}{u-3} + \frac{C}{u+2}$$

$$A = \left.\frac{1-u^2}{(u-3)(u+2)}\right|_{u=0} = \frac{-1}{6}$$

$$B = \left.\frac{1-u^2}{u(u+2)}\right|_{u=3} = \frac{-8}{3.5} = \frac{-8}{15}$$

$$C = \left.\frac{1-u^2}{u(u-3)}\right|_{u=-2} = \frac{1-4}{-2.(-5)} = \frac{-3}{10}$$

SORU-4:

$\int \frac{e^t}{e^{3t}-e^{2t}+2e^t-2}\,dt$ integralini hesaplayınız.

$$I=\int \frac{e^t}{e^{3t}-e^{2t}+2e^t-2}\,dt = \int \frac{du}{u^3-u^2+2u-2} \quad , \left\{ \begin{array}{l} e^t=u \\ e^t dt = du \end{array}\right\}$$

$$I=\int \frac{du}{(u-1)(u^2+2)} \Rightarrow \frac{1}{(u-1)(u^2+2)} \equiv \frac{A}{u-1}+\frac{Bu+C}{u^2+2}$$

$$1 \equiv Au^2+2A+Bu^2-Bu+Cu-C$$

$$A+B=0,\ A+C=0,\ -B+C=0,\ 2A-C=1$$

$$A=1/3,\ B=-1/3,\ C=-1/3$$

$$I=\frac{1}{3}\int \frac{du}{u-1}-\frac{1}{3}\int \frac{u+1}{u^2+2}du$$

$$I=\frac{1}{3}\int \frac{du}{u-1}-\frac{1}{3}\int \frac{u\,du}{u^2+2}-\frac{1}{3}\int \frac{du}{u^2+2}$$

$$I=\frac{1}{3}\ln|u-1|-\frac{1}{6}\ln|u^2+2|-\frac{1}{3}\cdot\frac{1}{\sqrt{2}}\cdot\arctan\frac{u}{\sqrt{2}}+C$$

$$I=\frac{1}{3}\ln|e^t-1|-\frac{1}{6}\ln|e^{2t}+2|-\frac{1}{3\sqrt{2}}\arctan\frac{e^t}{\sqrt{2}}+C$$

SORU-5:

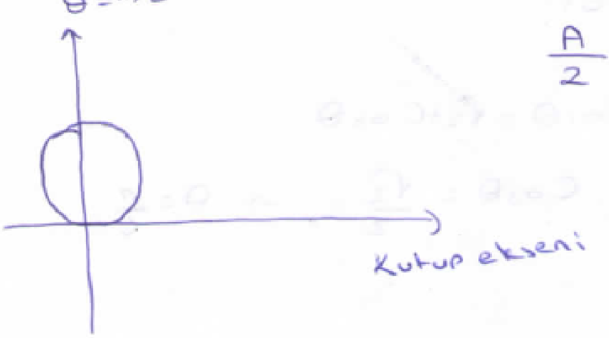
$t, [-1,0]$ aralığında değişken, $x(t)=t^2, y(t)=1-t^2$ ile çizilmiş yolun uzunluğunu bulunuz.

$$L=\int_{-1}^{0}\sqrt{[x'(t)]^2+[y'(t)]^2}\,dt=\int_{-1}^{0}\sqrt{(2t)^2+(-2t)^2}\,dt$$

$$=2\sqrt{2}\int_{-1}^{0}|t|\,dt=-2\sqrt{2}\int_{-1}^{0}t\,dt=\sqrt{2}\text{ br}$$

SORU-6:

$r=\sqrt{2}\,\mathrm{Sin}\,\theta$ ile sınırlı bölgenin alanı?



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/2}(\sqrt{2}\,\mathrm{Sin}\,\theta)^2\,d\theta$$

$$=\frac{1}{2}\int_0^{\pi/2}2\,\mathrm{Sin}^2\theta\,d\theta$$

$$=\int_0^{\pi/2}\frac{1-\mathrm{Cos}\,2\theta}{2}\,d\theta=\frac{\pi}{4}$$

$$\boxed{A=\frac{\pi}{2}}$$

SORU-7:

$r = a\sin^2\frac{\theta}{2}$ eğrisinin $0 \le \theta \le \pi$ aralığındaki uzunluğu? $(a>0)$

$$S = \int_0^{\pi} \sqrt{r^2 + (r')^2}\, d\theta$$

$$r^2 = a^2 \sin^4\frac{\theta}{2}$$

$$r' = a \cdot 2 \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} \cdot \frac{1}{2} = a \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$$

$$(r')^2 = a^2 \sin^2\frac{\theta}{2} \cdot \cos^2\frac{\theta}{2}$$

$$r^2 + (r')^2 = a^2 \sin^4\frac{\theta}{2} + a^2 \sin^2\frac{\theta}{2} \cdot \overbrace{\cos^2\frac{\theta}{2}}^{1-\sin^2\frac{\theta}{2}} = a^2 \cancel{\sin^4\frac{\theta}{2}} + a^2\sin^2\frac{\theta}{2} - a^2 \cancel{\sin^4\frac{\theta}{2}}$$

$$= a^2 \sin^2\frac{\theta}{2}$$

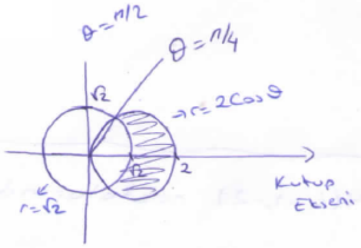
$$\sqrt{r^2 + (r')^2} = \sqrt{a^2 \cdot \sin^2\frac{\theta}{2}} = a \cdot \left|\sin\frac{\theta}{2}\right|$$

$$S = \int_0^{\pi} a \cdot \left|\sin\frac{\theta}{2}\right| d\theta = \int_0^{\pi} a \cdot \sin\frac{\theta}{2}\, d\theta = -2a\cos\frac{\theta}{2}\Big|_0^{\pi}$$

$$= \boxed{2a}$$

SORU-8:

$r = 2\cos\theta$ eğrisinin içinde $r = \sqrt{2}$ nin dışında kalan alan?



$$2\cos\theta = \sqrt{2} \rightarrow \boxed{\theta = \frac{\pi}{4}}$$

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/4} (2\cos\theta)^2\, d\theta - \frac{1}{2}\int_0^{\pi/4} (\sqrt{2})^2\, d\theta$$

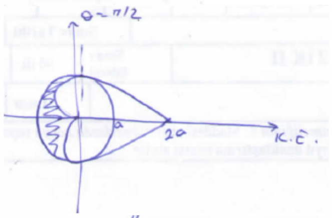
$$= \frac{1}{2}\left[\int_0^{\pi/4} (4\cos^2\theta - 2)\, d\theta\right]$$

$$= \frac{1}{2}\int_0^{\pi/4} 2\cos 2\theta\, d\theta = \frac{\sin 2\theta}{2}\Big|_0^{\pi/4} = \frac{1}{2}$$

$$\boxed{A = 1}$$

SORU-9:

$a>0$ olmak üzere $r=a(1+\cos\theta)$ kardiyoidinin dışında, $r=a$ çemberinin içinde kalan bölgenin alanını hesaplayınız.



$$\frac{A}{2}=\int_{\pi/2}^{\pi} a^2-(a+a\cos\theta)^2\,d\theta=\int_{\pi/2}^{\pi}\Big(-2a^2\cos\theta-a^2\underbrace{\cos^2\theta}_{\frac{1+\cos 2\theta}{2}}\Big)\,d\theta$$

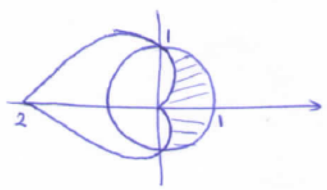
$$=-2a^2\sin\theta-a^2\frac{\theta}{2}-\frac{a^2}{4}\sin 2\theta\,\Big|_{\pi/2}^{\pi}$$

$$=-a^2\frac{\pi}{2}-\left(-2a^2-\frac{a^2\pi}{4}\right)=-\frac{a^2\pi}{2}+2a^2+\frac{a^2\pi}{4}$$

$$=2a^2-\frac{\pi}{4}a^2$$

SORU-10:

$r=1$ çemberinin içinde, $r=1-\cos\theta$ kardiyoidinin dışında kalan bölgenin alanını veren integral?



$$\frac{A}{2}=\int_0^{\pi/2}\frac{1}{2}\,d\theta-\frac{1}{2}\int_0^{\pi/2}(1-\cos\theta)^2\,d\theta$$

$$A=\int_0^{\pi/2}\left(1-(1-\cos\theta)^2\right)d\theta$$

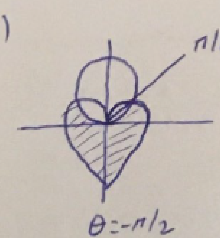
SORU-11:

a) $r=1-\sin\theta$ içi $r=\sin\theta$ dışı alan?

b) $r=1-\sin\theta$ dışı $r=\sin\theta$ içi alan? $1-\sin\theta=\sin\theta$

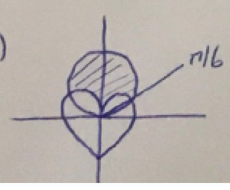
$\sin\theta=\frac{1}{2}$ $\theta=\frac{\pi}{6}$

a)



$$\frac{A}{2}=\frac{1}{2}\int_{-\pi/2}^{\pi/6}(1-\sin\theta)^2\,d\theta-\frac{1}{2}\int_0^{\pi/6}(\sin\theta)^2\,d\theta$$

b)



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/6}^{\pi/2}\left((\sin\theta)^2-(1-\sin\theta)^2\right)d\theta$$