- $\lim\limits_{n\to\infty}\left(\frac{1+n^2}{3+n^2}\right)^n = \lim\limits_{n\to\infty}\left(\frac{3+n^2}{3+n^2}+\frac{-2}{3+n^2}\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{-2}{3+n^2}\right)^n$

$\lim\limits_{n\to\infty} n = \lim\limits_{n\to\infty}(n^2+3)\frac{1}{n} = \lim\limits_{n\to\infty}\left(n+\frac{3}{n}\right) = n \Rightarrow \lim\limits_{n\to\infty}\Big[\underbrace{\left(1+\frac{-2}{3+n^2}\right)^{n^2+3}}_{e^{-2}}\Big]^{\frac{1}{n}} = e^{\lim\limits_{n\to\infty}\frac{-2}{n}} = e^0 = 1$

$\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ | $\lim\limits_{n\to\infty} n\left(1+\frac{1}{n^2}\right) = n$ | $\lim\limits_{n\to\infty} n\left(1-\frac{1}{n^2}\right) = n$ | $\lim\limits_{n\to\infty}(1+n^2)\frac{1}{n} = n$ | $\lim\limits_{n\to\infty}(n+n^2)\frac{1}{n} =$

- $\lim\limits_{x\to 1} x^{\frac{1}{x-1}}$ $\quad x=(x-1)+1 \quad \lim\limits_{x\to 1}\left((x-1)+1\right)^{\frac{1}{x-1}} \Rightarrow \lim\limits_{x\to 1}\left(1+\frac{1}{\frac{1}{x-1}}\right)^{\frac{1}{x-1}} \quad \frac{1}{x-1}=u \quad x\to 1 \Rightarrow u\to\infty$

$\lim\limits_{u\to\infty}\left(1+\frac{1}{u}\right)^u = e \qquad \lim\limits_{x\to 1} x^{\frac{1}{x-1}} = e$

- $\lim\limits_{x\to 2}(x-1)^{\frac{1}{x-2}}$ $\quad x-1=(x-2)+1 \quad \lim\limits_{x\to 2}\left(1+\frac{1}{\frac{1}{x-2}}\right)^{\frac{1}{x-2}} \quad \frac{1}{x-2}=u \quad x\to 2 \Rightarrow u\to\infty$

$\Rightarrow \lim\limits_{u\to\infty}\left(1+\frac{1}{u}\right)^u = e \qquad \lim\limits_{x\to 2}(x-1)^{\frac{1}{x-2}} = e$

- $\lim\limits_{x\to 3}(2x-5)^{\frac{1}{3x-9}}$ $\quad 2x-5 = 2x-6+1 = 2(x-3)+1$, $2x-6 = 2(x-3)$ $\quad \lim\limits_{x\to 3}\left[\left(1+\frac{1}{\frac{1}{2(x-3)}}\right)^{\frac{1}{2(x-3)}}\right]^{\frac{2}{3}} \quad \frac{1}{2(x-3)}=u \quad x\to 3 \Rightarrow u\to\infty$

$= \lim\limits_{u\to\infty}\left[\left(1+\frac{1}{u}\right)^u\right]^{\frac{2}{3}} = (e^2)^{\frac{1}{3}} = \sqrt[3]{e^2}$

- $\lim\limits_{x\to 1}\left(\frac{3x^2-2x-1}{x^2-x}\right)$ $\quad \frac{(3x+1)\cdot\cancel{(x-1)}}{x\cancel{(x-1)}} \Rightarrow \lim\limits_{x\to 1}\left(\frac{3x+1}{x}\right) = 4$

- $y = x\cdot\arccos x - \sqrt{1-x^2}$, $y' = ?$ $\quad \frac{d}{dx}\arccos(x) = \frac{-1}{\sqrt{1-x^2}}$

$y' = \arccos x + x\cdot\frac{-1}{\sqrt{1-x^2}} - \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} \Rightarrow y' = \arccos x + \cancel{\frac{-x}{\sqrt{1-x^2}}} + \cancel{\frac{x}{\sqrt{1-x^2}}} \Rightarrow y' = \arccos x$

- $y = \tanh(\ln x)$, $y' = ?$ $\quad \frac{d}{dx}\tanh(x) = \frac{1}{1-x^2} = \operatorname{sech}^2(x)$

$\operatorname{sech}^2(\ln x)(\ln x)' \Rightarrow \frac{\operatorname{sech}^2(\ln x)}{x}$

- $f(x) = x^5$, $(f^{-1})'(32) =$

$f(f^{-1}(x)) = x$

$\frac{d}{dx} f(f^{-1}(x)) = 1$

$f'(f^{-1}(x))\,\frac{d}{dx} f^{-1}(x) = 1$

$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$

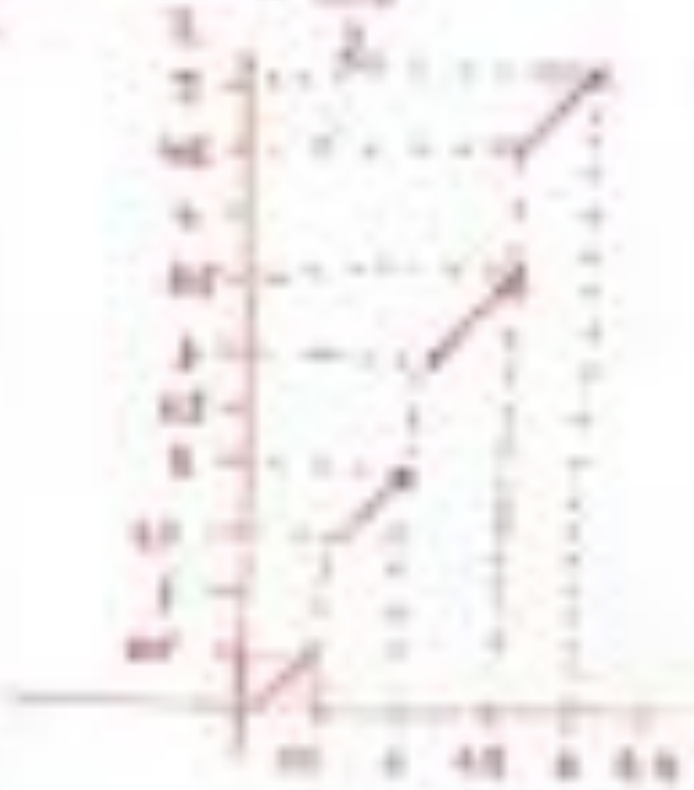
$\frac{d}{dx} f^{-1}(32) = \frac{1}{f'(f^{-1}(32))} \qquad f^{-1}(32) = 2, \quad f'(2) = 80$

$= \frac{1}{80}$

- $f(x) = x + [\![2x]\!]$, examine the discontinuous points of the given function on the interval $[0,2)$ and define types of discontinuities.

$n \le 2x < n+1 \Rightarrow \frac{n}{2} \le x < \frac{n+1}{2} \Rightarrow \frac{n+1}{2} - \frac{n}{2} = \frac{1}{2}$

$$f(x) = x + [\![2x]\!] = \begin{cases} x & ; [0, \frac{1}{2}) \\ x+1 & ; [\frac{1}{2}, 1) \\ x+2 & ; [1, \frac{3}{2}) \\ x+3 & ; [\frac{3}{2}, 2) \end{cases}$$



$\left(\lim_{x \to \frac{1}{2}^+} f(x) = \frac{3}{2}\right) \neq \left(\lim_{x \to \frac{1}{2}^-} f(x) = \frac{1}{2}\right)$; $\left(\lim_{x \to 1^+} f(x) = 3\right) \neq \left(\lim_{x \to 1^-} f(x) = 2\right)$; $\left(\lim_{x \to \frac{3}{2}^+} f(x) = \frac{9}{2}\right) \neq \left(\lim_{x \to \frac{3}{2}^-} f(x) = \frac{7}{2}\right)$

- $\lim_{x \to -2} \left(-\frac{1}{x^3}\right) = \frac{1}{8}$; $\varepsilon - \delta$ technique

for $\lim_{x \to a} f(x) = L$

$\forall \varepsilon \in \mathbb{R}^+ , \exists \delta(\varepsilon) \in \mathbb{R}^+ \ni |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$|x - (-2)| < \delta \Rightarrow \left| -\frac{1}{x^3} - \frac{1}{8} \right| < \varepsilon$ | $|x+2| < \delta \Rightarrow \left| \frac{8 + x^3}{8x^3} \right| < \varepsilon$ $\quad x^3 + 8 = (x+2)(x^2 - 2x + 4)$

$\left| \frac{1}{x^3} + \frac{1}{8} \right| < \varepsilon$

★

---

① $|x+2| < \delta < 1$ $\quad -3 < x < -1$ $\quad |x| < 3$ $\quad \boxed{8|x|^3 < 8}$

② $x \in (-3, -1)$ $\quad x^2 - 2x + 4$ $\quad (-1) \to 7$ $\quad (-3) \to 19$

$7 < x^2 - 2x + 4 < 19$ $\quad \boxed{|x^2 - 2x + 4| < 19}$

$\frac{|x+2| \cdot 19}{8} < \varepsilon$ $\quad |x+2| < \frac{8\varepsilon}{19}$ $\quad |x+2| < \delta < 1$ $\quad \delta = \min\left\{1, \frac{8}{19}\varepsilon\right\}$

- Use the mean value theorem to show that $\sqrt{y} - \sqrt{x} < \frac{y-x}{2\sqrt{x}}$ if $0 < x < y$

Let $f(x) = \sqrt{x}$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$ $\quad 0 < x < c < y \Rightarrow f'(c) = \frac{\sqrt{y} - \sqrt{x}}{y - x}$

$\sqrt{x} < \sqrt{c} < \sqrt{y}$

$\frac{1}{2\sqrt{c}} = \frac{\sqrt{y} - \sqrt{x}}{y - x} < \frac{1}{2\sqrt{x}} \Rightarrow \sqrt{y} - \sqrt{x} < \frac{y - x}{2\sqrt{x}}$

• For the given curve by parametric equations $\begin{cases} x(t) = 6t\cos(t) \\ y(t) = 6\sqrt{3}\,t\sin(t) \end{cases}$

a) Find the equation of the tangent line at $\frac{\pi}{6}$

b) " " " normal line at $\frac{\pi}{6}$

$x\left(\frac{\pi}{6}\right) = 6\cdot\frac{\pi}{6}\cdot\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\pi \qquad y\left(\frac{\pi}{6}\right) = 6\sqrt{3}\cdot\frac{\pi}{6}\cdot\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}\pi}{2}$

$\frac{d}{dt}x(t) = 6\left[\cos(t) + t(-\sin(t))\right] = 6\cos t - 6t\sin t \Rightarrow 6\cos\left(\frac{\pi}{6}\right) - 6\cdot\frac{\pi}{6}\cdot\sin\left(\frac{\pi}{6}\right)$

$\Rightarrow 3\sqrt{3} - \frac{\pi}{2}$

$\frac{d}{dt}y(t) = 6\sqrt{3}\left(\sin(t) + t\cos(t)\right) = 6\sqrt{3}\sin t + 6\sqrt{3}\,t\cos t \Rightarrow 6\sqrt{3}\sin\left(\frac{\pi}{6}\right) + 6\sqrt{3}\cdot\frac{\pi}{6}\cdot\cos\left(\frac{\pi}{6}\right)$

$\Rightarrow 3\sqrt{3} + \frac{3\pi}{2}$

$y'\left(\frac{\pi}{6}\right) = \frac{\dot{y}}{\dot{x}}\Big|_{\pi/6} = \frac{3\sqrt{3} + \frac{3\pi}{2}}{3\sqrt{3} - \frac{\pi}{2}} = \frac{6\sqrt{3} + 3\pi}{6\sqrt{3} - \pi} \qquad m_N \cdot m_T = -1 \qquad m_T = \frac{6\sqrt{3} + 3\pi}{6\sqrt{3} - \pi}$

$m_N = -\left(\frac{6\sqrt{3} - \pi}{6\sqrt{3} + 3\pi}\right)$

a) d_T: $y - y_0 = m_T(x - x_0) \Rightarrow y - \frac{\sqrt{3}\pi}{2} = \frac{6\sqrt{3} + 3\pi}{6\sqrt{3} - \pi}\left(x - \frac{\sqrt{3}}{2}\pi\right)$

b) d_N: $y - y_0 = m_N(x - x_0) \Rightarrow y - \frac{\sqrt{3}}{2}\pi = -\left(\frac{6\sqrt{3} - \pi}{6\sqrt{3} + 3\pi}\right)\cdot\left(x - \frac{\sqrt{3}}{2}\pi\right)$

• A right triangle with hypotenuse of $\sqrt{2}$ is rotated about one of its legs to generate a right circular cone. Find the greatest possible volume of such a cone by determining the lengths of the legs of right triangle. ($V = \frac{1}{3}\pi r^2 h$ volume of cone, r: radius of base, h: height of cone)

because of square of r we write r^2 as h eq.

$V = \frac{1}{3}\pi r^2 h \quad (r^2 = 2 - h^2) \quad V = \frac{1}{3}\pi(2 - h^2)h$

$\frac{d}{dh}V(h) = \frac{1}{3}\pi\left(2 - 3h^2\right) \Rightarrow 2 = 3h^2 \Rightarrow h = \pm 1 \quad \boxed{h = 1}$

$V'' = -2\pi h$

$V''(h = -1) = -2\pi(-1) = 2\pi > 0$ local min for $h = -1$

$V''(h = 1) = -2\pi(1) = -2\pi < 0$ local max for $h = 1$ $\boxed{r = \sqrt{2}}$

• $\lim_{n\to\infty} (\sqrt{n^2+1} - n) = 0$ (ε-δ technique)

$\forall \varepsilon>0,\ 0<|x-x_0|<\delta \Rightarrow |f(x)-L|<\varepsilon$ so, $x_0 > \delta(\varepsilon)$

$$|(\sqrt{n^2+1}-n)-0|<\varepsilon$$

$$\left[(\sqrt{n^2+1}-n)\cdot\frac{(\sqrt{n^2+1}+n)}{(\sqrt{n^2+1}+n)}\right] = \frac{(n^2+1)-n^2}{\sqrt{n^2+1}+n} = \frac{1}{\sqrt{n^2+1}+n}$$

$$\left|\frac{1}{\sqrt{n^2+1}+n}\right| < \frac{1}{|\sqrt{n^2}+n|} < \frac{1}{2n} < \varepsilon \Rightarrow n > \frac{1}{2\varepsilon}$$

As there exists $\delta(\varepsilon) = \frac{1}{2\varepsilon} > 0$ for $\forall\varepsilon>0$, the existence of limit is true.

• $$\lim_{x\to1}\frac{\tan\pi x}{1-x^2} = \lim_{x\to1}\frac{\frac{\sin(\pi x)}{\cos(\pi x)}}{(1+x)(1-x)} = \underbrace{\lim_{x\to1}\frac{\sin(\pi x)}{(1-x)}}_{\pi}\cdot\underbrace{\lim_{x\to1}\frac{1}{(1+x)\cos(\pi x)}}_{-\frac{1}{2}}$$

Let $1-x=u$ $\quad \lim_{u\to0}\frac{\sin(\pi(1-u))}{u} = \lim_{u\to0}\frac{\sin(\pi-\pi u)}{u}$

$$= \lim_{u\to0}\frac{\sin(\pi u)}{u}\cdot\frac{\pi}{\pi} = \pi\cdot\underbrace{\lim_{u\to0}\frac{\sin(\pi u)}{(\pi u)}}_{1} = \pi$$

$$\lim_{x\to1}\frac{\tan(\pi x)}{1-x^2} = -\frac{\pi}{2}$$

• $\lim_{x\to 119}(2x+1) = 239$ (ε-δ technique)

$\forall\varepsilon>0,\ 0<|x-x_0|<\delta \Rightarrow |f(x)-L|<\varepsilon$ so, there exists $x_0>\delta(\varepsilon)$

$0<|x-119|<\delta \Rightarrow |(2x+1)-239|<\varepsilon \rightarrow |2(x-119)|<\varepsilon \quad |x-119|<\frac{\varepsilon}{2}$

For the function $\delta(\varepsilon)=\frac{\varepsilon}{2}$ (under the condition $\forall\varepsilon>0$) there exists limit value in the case $\delta>0$

• $f(x)=\frac{1}{\sqrt{x+1}}$ $\quad f'(x)=?=\lim_{h\to0}\frac{\frac{1}{\sqrt{x+h+1}}-\frac{1}{\sqrt{x+1}}}{h} = \lim_{h\to0}\frac{\sqrt{x+1}-\sqrt{x+h+1}}{h\cdot(\sqrt{x+h+1}\cdot\sqrt{x+1})}\cdot\frac{(\sqrt{x+1}+\sqrt{x+h+1})}{(\sqrt{x+1}+\sqrt{x+h+1})}$

$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

where $f(x)=\frac{1}{\sqrt{x+1}}$,

$f'(x) = -\frac{1}{2}(x+1)^{-3/2}$

$$=\lim_{h\to0}\frac{x+1-(x+h+1)}{h(\sqrt{x+h+1}\cdot\sqrt{x+1})\cdot(\sqrt{x+1}+\sqrt{x+h+1})}$$

$$=\lim_{h\to0}\frac{-\cancel{h}}{\cancel{h}\cdot(\sqrt{x+h+1}\cdot\sqrt{x+1})(\sqrt{x+1}+\sqrt{x+h+1})}$$

$$=-\lim_{h\to0}\frac{1}{(\sqrt{x+h+1}\cdot\sqrt{x+1})(\sqrt{x+1}+\sqrt{x+h+1})}$$

$$\Longrightarrow -\frac{1}{(x+1)\cdot2\sqrt{x+1}}$$

[illegible]

$\lim_{x \to 2} \frac{f(x) - f(2)}{x^2 - 4}$ ([illegible]) $\quad \lim_{x \to 2} \frac{f(x)-f(2)}{x-2} \cdot \lim_{x \to 2} \frac{1}{x+2} = \frac{f'(2)}{4}$

$\frac{d}{dx} [\tan^4(x)]$ [illegible]

$y = \sqrt{u}$ [illegible] $\frac{dy}{dx}$ at $x=0$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dt} \cdot \frac{dt}{dx}$ [illegible]

$f(x) = \frac{x^3}{3} - x^2$ [illegible]

$f'(x) = x^2 - 2x \quad f''(x) = 2x - 2$ [illegible]

$\sqrt{x} + \sqrt{y} = x^2 + y^2$ [illegible] $\frac{d}{dx}(\sqrt{x} + \sqrt{y}) = \frac{d}{dx}(x^2 + y^2)$

[illegible]

$\Rightarrow f'(1) = -1$

$\lim_{x \to \infty}$ [illegible]

$\Rightarrow \lim_{x \to \infty} \left[\left(1 + \ldots\right) \right]$ [illegible] $= e^{\ldots}$

- $f(x) = \dfrac{\sin 2x}{(2 - 2e^{3x})\, 2\cos x}$ is continuous at $x=0$ $\lim_{x\to 0} f(x) = f(0)$

$f(0) = ?$

$f(x) = \dfrac{\sin 2x}{(2-2e^{3x})\,2\cos x} \Rightarrow \dfrac{0}{0}$ $\quad f(x) = \dfrac{2\sin x \cos x}{(2-2e^{3x})\,\cancel{2\cos x}} \Rightarrow f(x) = \dfrac{\sin x}{x} \cdot \dfrac{x}{(2-2e^{3x})}$

$\left(x \to 0 \quad \dfrac{\sin x \le x \le \frac{\sin x}{\cos x}}{\sin x} \quad 1 \le \dfrac{x}{\sin x} \le \dfrac{1}{\cos x} \quad \cos x \le \dfrac{\sin x}{x} \le 1 \right.$

$\left. \underbrace{\lim_{x\to 0} \cos x}_{1} \le \lim_{x\to 0} \dfrac{\sin x}{x} \le \underbrace{\lim_{x\to 0} 1}_{1} \right)$

$\lim_{x\to 0} f(x) = \underbrace{\lim_{x\to 0} \dfrac{\sin x}{x}}_{1} \cdot \underbrace{\lim_{x\to 0} \dfrac{x}{2-2e^{3x}}}_{\frac{1}{2}} \Rightarrow \lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{1}{2}\left(\dfrac{x}{1-e^{3x}}\right)$

Let u: $1 - e^{x} = u \quad 1-u = e^{x} \quad \ln(1-u) = x$

$x \to 0 \Rightarrow u \to 0$

$\lim_{u\to 0} \dfrac{1}{2}\left(\dfrac{\frac{\ln(1-u)}{3}}{u}\right) = \dfrac{1}{6}\lim_{u\to 0} \left(\ln(1-u)\right)^{\frac{1}{u}} = \dfrac{1}{6}\ln\left(\underbrace{\lim_{u\to 0}(1-u)^{\frac{1}{u}}}_{e^{-1}}\right) = \dfrac{1}{6}\ln(e^{-1}) = -\dfrac{1}{6}$

$f(0) = -\frac{1}{6}$ (continuous for that value)

- $\lim_{x\to\infty} \left(\dfrac{x^2-3}{x^2+5}\right)^{x} \quad \lim_{x\to\infty}\left(1 + \dfrac{-8}{x^2+5}\right)^{x} \qquad \lim_{x\to\infty}\left(1+\dfrac{m}{x}\right)^{x} = e^{m}$

$\lim_{x\to\infty}\left[\left(1 + \dfrac{-8}{x^2+5}\right)^{x^2+5}\right]^{\frac{x}{x^2+5}} = \left[\underbrace{\lim_{x\to\infty}\left(1+\dfrac{-8}{x^2+5}\right)^{x^2+5}}_{e^{-8}}\right]^{\frac{x}{x^2+5}} = \left(e^{-8}\right)^{\underbrace{\lim_{x\to\infty}\frac{x}{x^2+5}}_{0}} = \left(e^{-8}\right)^{0} = 1$

$\lim_{x\to\infty} \dfrac{x}{x^2+5} = \dfrac{\frac{1}{x}}{1+\frac{5}{x^2}} = \dfrac{0}{1} = 0$

- $\lim_{x\to\frac{1}{2}} (x-\tfrac{1}{2})^{2} \cdot \sin\left(\dfrac{1}{x-\frac{1}{2}}\right) \Rightarrow \lim_{x\to\frac{1}{2}} (x-\tfrac{1}{2})\, \dfrac{\sin\left(\frac{1}{x-\frac{1}{2}}\right)}{\left(\frac{1}{x-\frac{1}{2}}\right)} = \underbrace{\lim_{x\to\frac{1}{2}}(x-\tfrac{1}{2})}_{0}\ \underbrace{\lim_{x\to\frac{1}{2}} \dfrac{\sin\left(\frac{1}{x-\frac{1}{2}}\right)}{\left(\frac{1}{x-\frac{1}{2}}\right)}}_{0}$

$\lim_{x\to\frac{1}{2}} (x-\tfrac{1}{2})^{2} \sin\left(\dfrac{1}{x-\frac{1}{2}}\right) = 0$

• Let $f(x)$ be a function has inverse function. If the normal line to curve $y=f(x)$ at point $P(x_0,-1)$ is $\underset{=2x+1=y}{y+2x-1=0}$, find $(f^{-1})'(-1)$

$y-y_1 = m\cdot(x-x_0)$

$y-(-1) = -2(x-x_0)$

$y+1 = -2x+2x_0 \quad y = -2x+2x_0-1$

$-2x+2x_0-1 = -2x+1 \Rightarrow x_0=1$

$m_T\cdot m_N = -1 \quad \boxed{m_T = \frac{1}{2}} \quad \boxed{f'(1) = m_T = \frac{1}{2}}$

$m_T = f'(1)$

$f^{-1}(-1)=1 \Rightarrow f(1)=-1$

$f(f^{-1}(x)) = x$

$\frac{d}{dx} f(f^{-1}(x)) = 1$

$f'(f^{-1}(x))\cdot\frac{d}{dx}(f^{-1}(x)) = 1$

[illegible]

$\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))} \Rightarrow \frac{1}{f'(f^{-1}(-1))}$

$\Rightarrow \frac{1}{f'(\underbrace{f^{-1}(-1)}_{1})} = \frac{1}{f'(1)} = \frac{1}{\frac{1}{2}} = 2$

• Check if it's differentiable at $x=1$

$f(x) = \begin{cases} (x-1)\cdot\sin\left(\frac{1}{x-1}\right) & ; x\neq 1 \\ 0 & ; x=1 \end{cases}$

$f'(x) = \lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow \lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0^-}\frac{f(1-h)-f(1)}{(1-h)-1} = \frac{(1-h-1)\cdot\sin\left(\frac{1}{(1-h)-1}\right)-0}{-h} = \lim_{h\to 0^-} -\sin\left(\frac{1}{h}\right)$

$\Rightarrow \lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0^+}\frac{f(1+h)-f(1)}{(1+h)-1} = \frac{((1+h)-1)\cdot\sin\left(\frac{1}{(1+h)-1}\right)-0}{h} = \lim_{h\to 0^+}\sin\frac{1}{h}$

The given function is not differentiable at point $x=1$ because right-hand and left-hand limits are not equal.

• For the function $f(x) = \frac{x^2-x+1}{x}$

i) domain $x\neq 0$, $D = \mathbb{R}-\{0\}$

ii) asymptotes
vertical: $\lim_{x\to 0^-} f(x) = -\infty$, $\lim_{x\to 0^+} f(x) = +\infty$ } $x=0$ vertical asymptote   oblique: [illegible]
horizontal: $\lim_{x\to -\infty} f(x) = -\infty$, $\lim_{x\to +\infty} f(x) = +\infty$ } there is no horizontal asymptote

iii) Intervals on which f is increasing, decreasing, and local extreme values $f'(x) = 1-\frac{1}{x^2}$ $x=1$ $x=-1$

iv) " " concave up and down, and inflection points (if any)

v) sketch the graph

iii)

x		-1		0		1	
f'	$+$		$-$	‖	$-$		$+$
f	↗		↘	‖	↘		↗

$f(x)$ is increasing on $(-\infty,-1)\cup(1,\infty)$

$f(x)$ is decreasing on $(-1,0)\cup(0,1)$

iv) $f''(x) = \frac{2}{x^3}$

x		0	
f''	$-$		$+$
f	∩		∪

But, there is no inflection point ($0\notin D$)

$f(x)$ is concave up on $(0,\infty)$

$f(x)$ is concave down on $(-\infty,0)$

v)