

### Homework 3

**Exercise 3.1.** Let  $X$  be a topological space and  $e \in X$  a point. Let  $\mu: X \times X \rightarrow X$  be a map with  $\mu(e, x) = \mu(x, e) = e$  for every point  $x \in X$ . Prove that  $\pi_1(X, e)$  is abelian.

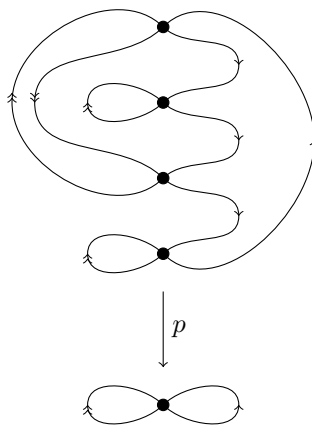
**Exercise 3.2.** Let  $Z$  be a path-connected space, and  $p: Y \rightarrow X$  a cover. Let  $z_0 \in Z$  be a point and  $f: Z \rightarrow X$  a map. Let  $y_0 \in Y$  be a point with  $p(y_0) = x_0 = f(z_0)$ . Let  $z \in Z$  be another point, and  $\gamma, \gamma' \in \Omega(Z, z_0, z)$  paths from  $z_0$  to  $z$ . Let  $\tilde{\gamma}, \tilde{\gamma}': I \rightarrow Y$  be lifts of these paths that start at  $y_0 \in Y$ . Show that if the image of  $\pi_1(f): \pi_1(Z, z) \rightarrow \pi_1(X, f(z))$  is contained in the image of  $\pi_1(p): \pi_1(Y, y) \rightarrow \pi_1(X, f(z))$ , then  $\tilde{\gamma}(1) = \tilde{\gamma}'(1)$ .

**Exercise 3.3.** (1) Let  $X$  be the following graph:



Find all covers of degree 3 up to homeomorphism over  $X$ .

(2) Compute the group of deck transformations  $G(Y)$  of the cover  $p: Y \rightarrow X$ :



**Exercise 3.4.** Let  $X$  be path-connected and  $x \in X$ . Show that  $\pi_1(X, x)$  is trivial if and only if every map  $S^1 \rightarrow X$  extends to a map  $D^2 \rightarrow X$ .