Math 731 – Algebraic Topology

Instructor: Matthias Nagel   due in class 9:30, October 27

Homework 3

Exercise 3.1. Let X be a topological space and $e \in X$ a point. Let $\mu\colon X \times X \to X$ be a map with $\mu(e, x) = \mu(x, e) = x$ for every point $x \in X$. Prove that $\pi_1(X, e)$ is abelian.

Exercise 3.2. Let Z be a path-connected space, and $p\colon Y \to X$ a cover. Let $z_0 \in Z$ be a point and $f\colon Z \to X$ a map. Let $y_0 \in Y$ be a point with $p(y_0) = x_0 = f(z_0)$. Let $z \in Z$ be another point, and $\gamma, \gamma' \in \Omega(Z, z_0, z)$ paths from z_0 to z. Let $\widetilde{\gamma}, \widetilde{\gamma}'\colon I \to Y$ be lifts of these paths that start at $y_0 \in Y$. Show that if the image of $\pi_1(f)\colon \pi_1(Z, z) \to \pi_1(X, f(z))$ is contained in the image of $\pi_1(p)\colon \pi_1(Y, y) \to \pi_1(X, f(z))$, then $\widetilde{\gamma}(1) = \widetilde{\gamma}'(1)$.

Exercise 3.3.   (1) Let X be the following graph:

   Find all covers of degree 3 up to homeomorphism over X.

   (2) Compute the group of deck transformations $G(Y)$ of the cover $p\colon Y \to X$:
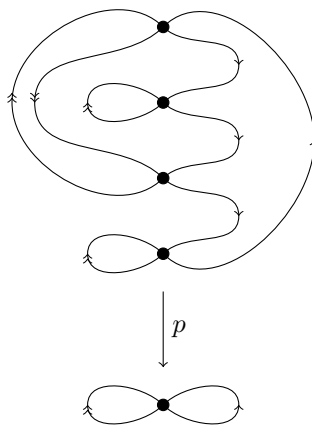
Exercise 3.4. Let X be path-connected and $x \in X$. Show that $\pi_1(X, x)$ is trivial if and only if every map $S^1 \to X$ extends to a map $D^2 \to X$.