

Homework 4

Exercise 4.1. Let G be a group with a free generating system $S \subset G$, and G' be a group with a free generating system $S' \subset G'$. Suppose S and S' have the same cardinality, prove that G is isomorphic to G' . Hint: Use the universal property, and go back and forth to create an isomorphism.

Exercise 4.2. (1) Show that \mathbb{Z}^2 is not a free group.

(2) Show that the coproduct (free product) $\mathbb{Z}/2 * \mathbb{Z}/3$ of two cyclic groups of order 2 and 3 is isomorphic not their product $\mathbb{Z}/2 \times \mathbb{Z}/3$.

Exercise 4.3. Can there be a continuous action \mathbb{Z} on S^2 such that for all $x \in S^2$ there exists an open subset $U_x \ni x$ with

$$U_x \cap gU_x = \emptyset \text{ for every } g \neq e.$$

Exercise 4.4. Let $f: (X, x_0) \rightarrow (Y, y_0)$ be a based map of path-connected spaces.

(1) if f is surjective, does $\pi_1(f)$ have to be surjective?

(2) if f is injective, does $\pi_1(f)$ have to be injective?