Math 731 – Algebraic Topology

Instructor: Matthias Nagel   due in class 9:30, November 17

Homework 4

Exercise 4.1. Let G be a group with a free generating system $S \subset G$, and G' be a group with a free generating system $S' \subset G'$. Suppose S and S' have the same cardinality, prove that G is isomorphic to G'. Hint: Use the universeral map, and go back and forth to create an isomorphism.

Exercise 4.2.   (1) Show that $\mathbb{Z}^2$ is not a free group.
    (2) Show that the coproduct (free product) $\mathbb{Z}/2 * \mathbb{Z}/3$ of two cyclic groups of order 2 and 3 is isomorphic not their product $\mathbb{Z}/2 \times \mathbb{Z}/3$.

Exercise 4.3. Can there be a continous action $\mathbb{Z}$ on S^2 such that for all $x \in S^2$ there exists an open subset $U_x \ni$ with

$$U_x \cap gU_x \neq \quad \text{for every } g \neq e.$$

Exercise 4.4. Let $f \colon (X, x_0) \to (Y, y_0)$ be a based map of path-connected spaces.
    (1) if f is surjective, does $\pi_1(f)$ have to be surjective?
    (2) if f is injective, does $\pi_1(f)$ have to be injective?