Final 27/09/2023

Ejercicio 2

Filtro no disipativo que se conecta a generador con $R_g = 1\,\Omega$. $V_2 = \emptyset V$.

$$Y(s) = \frac{k \cdot s}{s^3 + 2s^2 + 4s + 1} \quad \Big\} \text{ Admitancia de Transferencia NORMALIZADA.}$$

a) Síntesis gráfica del filtro normalizado para ver topología.

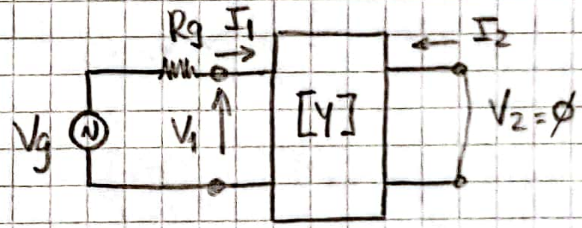
b) Calcular el valor de los componentes de la red.

c) Verificar por MAI o interconexión de cuadripolos para hallar el valor de k.

a) Síntesis gráfica

$$Y(s) = \frac{k \cdot s}{s^3 + 2s^2 + 4s + 1} \rightarrow \text{Admitancia de Transferencia Directa con la Salida en Corto}$$

$$Y = \left.\frac{I_2}{V_g}\right|_{V_2=0} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \cdot \left.\frac{V_1}{V_g}\right|_{V_2=0}$$

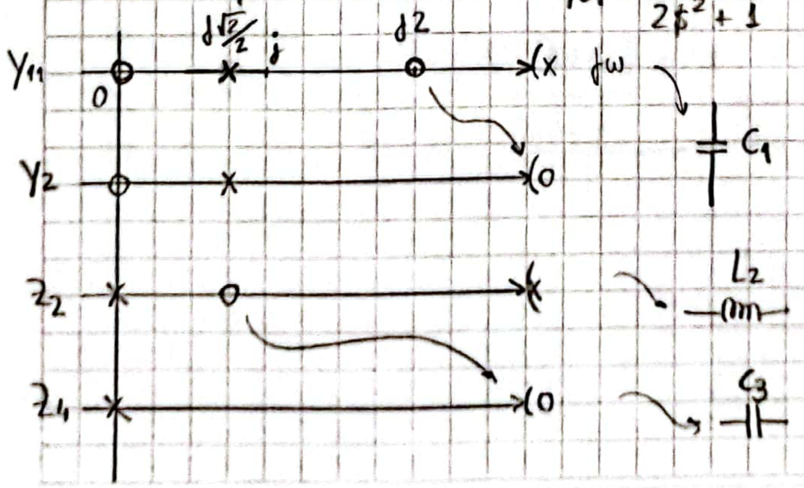


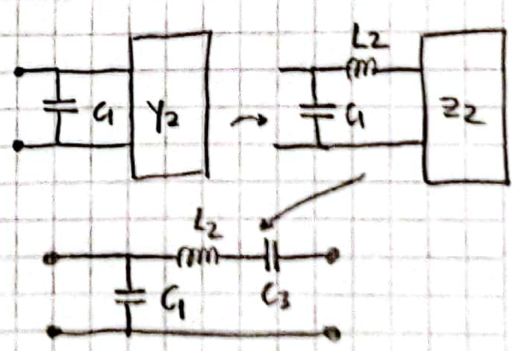
$$Y = \frac{K \cdot s}{s^3 + 2s^2 + 4s + 1}$$

$$Y = k \cdot \frac{\dfrac{s}{2s^2+1}}{1 + \dfrac{s^3 + 4s}{2s^2+1}}$$

$$Y = \frac{Y_{21}}{1 + \dfrac{Y_{11}}{Y_g}} = \frac{Y_{21}}{1 + Y_{11}}$$

Entonces, definimos: $Y_{21} = \dfrac{s}{2s^2+1}$ y $Y_{11} = \dfrac{s^3 + 4s}{2s^2+1}$





b) Calcular componentes de la red.

$$K_\infty = \lim_{s\to\infty} \frac{1}{s}\cdot Y_{11} = \lim_{s\to\infty} \frac{1}{s}\cdot\frac{s(s^2+4)}{2s^2+1} = \lim_{s\to\infty} \frac{1}{2}\cdot\frac{s^2+4}{s^2+1/2}$$

$$\boxed{K_\infty = 1/2} \rightsquigarrow C_1 = 1/2$$

$$Y_2 = Y_{11} - K_\infty \cdot s = \frac{s^3+4s}{2s^2+1} - \frac{1}{2}s = \frac{s^3+4s-s^3-1/2\,s}{2s^2+1} = \frac{7/2\,s}{2s^2+1}$$

$$\boxed{Y_2 = \frac{7/2\,s}{2s^2+1}}$$

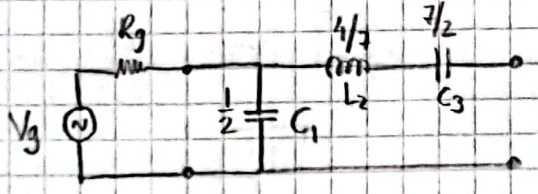
$$Z_4 = \frac{1}{Y_2} - K'_\infty \cdot s$$

$$K'_\infty = \lim_{s\to\infty} \cdot \frac{1}{s}\cdot\frac{1}{Y_2} = \lim_{s\to\infty} \frac{1}{s}\,\frac{2s^2+1}{7/2\,s} = \lim_{s\to\infty} \frac{2}{7/2}\cdot\frac{s^2+1/2}{s^2}$$

$$\boxed{K'_\infty = \frac{4}{7}} \rightsquigarrow L_2 = \frac{4}{7}$$

$$Z_4 = \frac{1}{Y_2} - K'_\infty s = \frac{2s^2+1}{7/2\,s} - \frac{4}{7}s = \frac{2s^2+1-\frac{4}{7}\cdot\frac{7}{2}s}{7/2\,s} = \frac{1}{7/2\,s}$$

$$\boxed{Z_4 = \frac{1}{\frac{7}{2}s}} \rightsquigarrow C_3 = \frac{7}{2}$$



c) Verificar para hallar el valor de k

$$Z = s L_2 + \frac{1}{s C_3} = \frac{s^2 L_2 C_3 + 1}{s C_3}$$

$$T = \begin{bmatrix} 1 & R_g \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ s C_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{s^2 L_2 C_3 + 1}{s C_3} \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 + s C_1 & R_g \\ s C_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{s^2 L_2 C_3 + 1}{s C_3} \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 + \frac{R_g}{s C_1} & (1 + s C_1)\left(\frac{s^2 L_2 C_3 + 1}{s C_3}\right) + R_g \\ s C_1 & s C_1 \cdot \frac{s^2 L_2 C_3 + 1}{s C_3} + 1 \end{bmatrix}$$

De parámetros T sabemos que

$$T \begin{cases} V_1 = A \cdot V_2 + B(-I_2) \\ I_1 = C \cdot V_2 + D(-I_2) \end{cases} \Rightarrow \frac{1}{B} = \left. -\frac{I_2}{V_1} \right|_{V_2 = 0}$$

$$B = (1 + s C_1)\left(\frac{s^2 L_2 C_3 + 1}{s C_3}\right) + R_g = \left(\frac{s^2 L_2 C_3 + 1}{s C_3} + \frac{s^3 C_1 L_2 C_3 + s C_1}{s C_3}\right) + R_g$$

$$B = \frac{s^2 L_2 C_3 + 1 + s^3 C_1 L_2 C_3 + s C_1 + s C_3 R_g}{s C_3}$$

$$B = \frac{\frac{s^3 (C_1 L_2 C_3)}{(C_1 L_2 C_3)} + \frac{s^2 (L_2 C_3)}{C_1 L_2 C_3} + \frac{s (C_1 + C_3 R_g)}{C_1 L_2 C_3} + \frac{1}{C_1 L_2 C_3}}{s C_3} \cdot C_1 . L_2 . C_3$$

$$\frac{1}{B} = \frac{C_3}{C_1 L_2 C_3} \cdot \frac{s}{s^3 + \frac{L_2 C_3}{C_1 L_2 C_3} s + \frac{C_1 + C_3 R_g}{C_1 L_2 C_3} s + \frac{1}{C_1 L_2 C_3}}$$

$$\frac{1}{B} = \frac{C_3}{C_1 L_2 C_3} \cdot \frac{s}{s^3 + s^2 \frac{L_2 C_3}{C_1 L_2 C_3} + s \frac{C_1 + C_3 R_g}{C_1 L_2 C_3} + \frac{1}{C_1 L_2 C_3}}$$

$$\frac{1}{B} = k \cdot \frac{s}{s^3 + 2s^2 + 4s + 1}$$

Entonces igualamos miembro a miembro para corroborar

$$1 = \frac{1}{C_1 L_2 C_3} = \frac{1}{\frac{1}{2} \cdot \frac{4}{7} \cdot \frac{7}{2}} = 1 \quad ✓$$

$$4s = s \cdot \frac{C_1 + C_3 R_g}{C_1 L_2 C_3} = s \frac{\frac{1}{2} + \frac{7}{2} \cdot 1}{\frac{1}{2} \cdot \frac{4}{7} \cdot \frac{7}{2}} = 4s \quad ✓$$

$$2s^2 = s^2 \cdot \frac{L_2 C_3}{C_1 L_2 C_3} = s^2 \cdot \frac{\frac{4}{7} \cdot \frac{7}{2}}{\frac{1}{2} \cdot \frac{4}{7} \cdot \frac{7}{2}} = s^2 \cdot 2 \quad ✓$$

Finalmente, hallamos k:

$$k = \frac{C_3}{C_1 L_2 C_3} = \frac{7/2}{\frac{1}{2} \cdot \frac{4}{7} \cdot \frac{7}{2}} \Rightarrow \boxed{k = 7/2}$$

$$\boxed{Y_{(s)} = \frac{7}{2} \frac{s}{s^3 + 2s^2 + 4s + 1}}$$