EL9343 Homework 1 (Due September 20th, 2018)

All problem/exercise numbers are for the third edition of CLRS text book

- **1.** Prove the Symmetry property of $\Theta(\cdot)$, i.e. $f(n) = \Theta(g(n))$ if and only if $g(n) = \Theta(f(n))$.
- 2. Problem 3-2 in CLRS Text book;
- 3. Problem 3-3 (a) in CLRS Text book;
- 4. Problem 3-4 (c) (d) (e) (f) in CLRS Text book;
- 5. Use the substitution method to show that the solution to $T(n) = T(\alpha n) + T((1 \alpha)n) + 10$, with $0 < \alpha < 1$, is $\Theta(n)$
- **6**. First use the iteration method to solve the recurrence:

$$T(n) = 2T\left(\frac{2n}{3}\right) + n$$

then use the substitution method to verify your solution.

7. First use the iteration method to solve the recurrence

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n^2$$

then use the substitution method to verify your solution.

8. Solving the recurrence:

$$T(n) = 9T(n^{\frac{1}{6}}) + \log^2(n).$$

(Hint: Making change of variable)

9. Give asymptotic upper and lower bounds for T(n) in each of the following recurrences. Make your bounds as tight as possible, and justify your answers.

(a)
$$T(n) = 2T(n/3) + n^{\frac{1}{2}} \log n$$

(b)
$$T(n) = 25T(n/5) + n^2$$

(c)
$$T(n) = 4T(n/2) + n^2\sqrt{n}$$

(d)
$$T(n) = T(n-2) + 1/n$$