CLRS Exercise

Tongda Xu

October 11, 2018

1 7

1.1 7.3

1.1.1 a

This is certain concerning the *Randomized* procedure, the probability of any index i is chosen from $[0, n-1]$is:
$Pr(pivot = i) = \frac{1}{n}$
$E(X_i) = 1 * Pr(pivot = i) + 0 * Pr(pivot \neq i) = \frac{1}{n}$

1.1.2 b

It is certain that if ith element is chosen as pivot, $Random - Parition$ cost $\Theta(n)$ time, and it will call $QuickSort[1, q-1], QuickSort[q+1, n]$ recursively.
Concerning only the first $Parition$, this would be the result:
$E(T(n)) = \Sigma_{i=1}^{n} Pr(pivot = i)(T(i-1) + T(n-i) + \Theta(n))$
$= \Sigma_{i=1}^{n} X_i(T(i-1) + T(n-i) + \Theta(n))$

1.1.3 c

Concerning $X_i = \frac{1}{n}$
$E(T(n)) = \Sigma_{i=1}^{n} \frac{1}{n}(T(i-1) + T(n-i) + \Theta(n))$
$= \Sigma_{i=1}^{n} \frac{1}{n} T(i-1) + \Sigma_{i=1}^{n} \frac{1}{n} T(n-i) + \Sigma_{i=1}^{n} \frac{1}{n} \Theta(n)$
$= \frac{2}{n} \Sigma_{i=1}^{n-1} T(i) + \Theta(n)$

1.1.4 d

$\Sigma_{k=2}^{n-1} klgk$
$\leq lg\frac{n}{2} \Sigma_{k=2}^{\frac{n}{2}} k + lgn \Sigma_{k=\frac{n}{2}}^{n-1} k$
$= lgn \Sigma_{k=2}^{n-1} k - lg2 \Sigma_{k=2}^{\frac{n}{2}} k$
$= lgn \frac{(n+1)(n-2)}{2} - \frac{(\frac{n}{2}+2)(\frac{n}{2}-1)}{2}$
$\leq lgn \frac{n^2}{2} - \frac{n^2}{8}$
by Calculus, we have:
$(\frac{1}{2}x^2 lgx - \frac{1}{4}x^2)|_1^{n-1} \leq E(T(n)) \leq (\frac{1}{2}x^2 lgx - \frac{1}{4}x^2)|_2^n$

1.1.5 e

Proof of $E(T(n)) = O(nlgn)$:
Assume that $\forall k \in [1, n-1], \exists c, E(T(k)) \leq cklgk - \Theta(k)$
For $k = n, E(T(n)) \leq \frac{n}{2}c(lgn\frac{n^2}{2} - \frac{n^2}{4} - \Theta(n^2)) + \Theta(n) \leq cnlgn - \Theta(n)$
Proof of $E(T(n)) = \Omega(nlgn)$:
Assume that $\forall k \in [1, n-1], \exists c, E(T(k)) \geq cklgk + \Theta(k)$
For $k = n, E(T(n)) \geq \frac{n}{2}c(lgn\frac{(n-1)^2}{2} - \frac{(n-1)^2}{4} + \Theta(n^2)) + \Theta(n) \geq cnlgn + \Theta(n)$
$\rightarrow E(T(n)) = \Theta(nlgn)$

2 15

2.1 15.1-1

$2^n - 1 = \Sigma_{j=0}^{n-1} 2^j$

2.2 15.1-2

Do not know how!

2.3 15.1-3

See Code

2.4 15.1-4

See Code

2.5 15.1-5

See Code

2.6 15.2-1

See Code

2.7 15.2-2

See Code

2.8 15.2-3

Assume that $\forall k \leq n-1, T(k) \geq c2^k$
Then $T(n) = \Sigma_{k=1}^{n-1} T(k)T(n-k) = (n-1)c^2 2^n > c2^n$
So $T(n) = \Omega(n), \omega(n)$

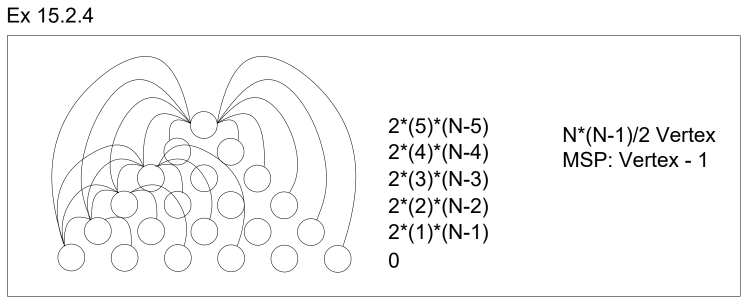

Figure 1: 15.2-4

2.9 15.2-4

See Figure 1

2.10 15.2-5

For each level $h(i) = i(n-i)$
For tree $T(n) = 2\Sigma_{i=1}^{n-1} i(n-i)$
$= \frac{3n^3+3n^2}{3} - \frac{2n^3+3n^2+n}{3}$
$= \frac{n^3-n}{3}$

2.11 15.2-6

Assume that $\forall k \leq n-1, N(k) = k-1$
Then $N(n) = N(n-1) + 1$
So $N(n) = n - 1$

2.12 15.3-1

running through: $T(n) = n * P_n^n = n * n! > 4^n$
running recursion: $T(n) = 2\Sigma_{i=1}^{n-1} 4^i + n = \frac{8}{3}4^{n-1} + n \leq 4^n$
running through takes longer

2.13 15.3-2

no overlapping subproblem call

2.14 15.3-3

Yes

2.15 15.3-4

Do not know how!

2.16 15.4-1

See code

2.17 15.4-2

See code

2.18 15.4-3

See code