HW02 for ECE 9343

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1 Question 1: 3-divide maximum subarray

MAXFROMLEFT(A, p, r)

$max = -\infty$
for $i = p$ **to** r
    $max = Sum(A, p, i) > max?Sum(A, p, i) : max$
return max

MAXFROMRIGHT(A, p, r)

$max = -\infty$
for $i = r$ **downto** p
    $max = Sum(A, i, r) > max?Sum(A, i, r) : max$
return max

THREE-FOLD-MAXSUB(A, p, r)

$s = \lfloor (p + r)/3 \rfloor$
$t = \lfloor (p + r)2/3 \rfloor$
if $Sum(A, s, t - 1) > 0$
    return $max(maxFromLeft(A, p, s - 1), maxFromRight(A, t, r)) + Sum(A, s, t - 1)$
else return $max(maxFromLeft(A, p, s - 1), maxFromRight(A, t, r))$

The time complexity is $\Theta(n)$

2 Question 2: Intermediate Sequence

BUBBLE SORT(A)

$A = [11, 8, 7, 5, 3, 1]$
$\rightarrow [8, 11, 7, 5, 3, 1] \rightarrow [8, 7, 11, 5, 3, 1] \rightarrow [8, 7, 5, 11, 3, 1] \rightarrow [8, 7, 5, 3, 11, 1] \rightarrow [8, 7, 5, 3, 1, 11]$
$\rightarrow [7, 8, 5, 3, 1, 11] \rightarrow [7, 5, 8, 3, 1, 11] \rightarrow [7, 5, 3, 8, 1, 11] \rightarrow [7, 5, 3, 1, 8, 11]$
$\rightarrow [5, 7, 3, 1, 8, 11] \rightarrow [5, 3, 7, 1, 8, 11] \rightarrow [5, 3, 1, 7, 8, 11]$
$\rightarrow [3, 5, 1, 7, 8, 11] \rightarrow [3, 1, 5, 7, 8, 11]$
$\rightarrow [1, 3, 5, 7, 8, 11]$

Insertion sort(A)

$A = [11, 8, 7, 5, 3, 1]$
$\rightarrow [8, 11, 7, 5, 3, 1]$
$\rightarrow [8, 7, 11, 5, 3, 1] \rightarrow [7, 8, 11, 5, 3, 1]$
$\rightarrow [7, 8, 5, 11, 3, 1] \rightarrow [7, 5, 8, 11, 3, 1] \rightarrow [5, 7, 8, 11, 3, 1]$
$\rightarrow [5, 7, 8, 3, 11, 1] \rightarrow [5, 7, 3, 8, 11, 1] \rightarrow [5, 3, 7, 8, 11, 1] \rightarrow [3, 5, 7, 8, 11, 1]$
$\rightarrow [3, 5, 7, 8, 1, 11] \rightarrow [3, 5, 7, 1, 8, 11] \rightarrow [3, 5, 1, 7, 8, 11] \rightarrow [3, 1, 5, 7, 8, 11] \rightarrow [1, 3, 5, 7, 8, 11]$

3 Question 3: Illustrate Merge Sort

Merge sort(A)

$15, 16, 25, 29, 30, 40, 48$
$15, 29, 48||16, 25, 30, 40$
$29||15, 48||25, 40||16, 30$
$-||48||15||40||25||16||30$

4 Question 4: CLRS Problem 2-1

4.1 a. show time complexity

$\Theta(T) = \frac{n}{k}\Theta(n^2) = \Theta(nk)$

4.2 b. show merge, c. show whole

There should not be anything special about Merge function, just use the original interface and implement of Merge in CLRS pp 31.

$$T(n) = \begin{cases} n & n \leq k \\ 2T(\frac{1}{2}n) + n & n > k \end{cases}$$

Regarding the iterative tree, it is easy to notice that: For branch (Merge), the complexity is $\Theta(nlg\frac{n}{k})$, For leaf (Insertion sort), is $\Theta(nk)$

Merge-sort(A, p, r, k)

if $r - p + 1 \leq k$
    Insertion-Sort(A, p, r)
    return
elseif $p < r$
    $q = \lfloor (p + r)/2 \rfloor$
    Merge-Sort(A, p, q)
    Merge-Sort$(A, q + 1, r)$
    Merge (A, p, q, r)
    return
else return

5 Question 5: verify

Proof: $T(n) = O(n)$
Suppose $\forall k < n, \exists c_2, T(k) \leq c_2 k - 10$
$\rightarrow T(n) = c_2 \alpha n + c_2(1-\alpha)n - 20 + 10 \leq c_2 n - 10$
$\rightarrow T(n) = O(c_2 n - 10)$
$\rightarrow T(n) = O(n)$

Proof: $T(n) = \Omega(n)$
Suppose $\forall k < n, \exists c_1, T(k) \geq c_1 k$
$\rightarrow T(n) = c_1 \aleph n + c_1(1-\alpha)n + 10 \geq c_1 n$
$\rightarrow T(n) = \Omega(n)$

$T(n) = O(n), T(n) = \Omega(n) \rightarrow T(n) = \Theta(n)$

6 Question 6: solve and verify

Notice that $TreeHeight = h = log_{\frac{3}{2}} n$
For branch $\Theta(n) = \sum_1^{h+1} n(\frac{4}{3})^h = n\frac{(\frac{4}{3})^h - 1}{\frac{4}{3} - 1} = \Theta(n^{\frac{ln2}{ln3 - ln2}})$
For leaf $\Theta(n) = 2^h = \Theta(n^{\frac{ln2}{ln3-ln2}})$
$\rightarrow T(n) = \Theta(n^{\frac{ln2}{ln3-ln2}}) = \Theta(n^{log_{\frac{3}{2}}2}) = \Theta(2^{log_{\frac{3}{2}}n})$

Proof: $T(n) = O(2^{log_{\frac{3}{2}}n}) - 3n$
Suppose $\forall k < n, \exists c_2, T(k) \leq c_2 2^{log_{\frac{3}{2}}n} - 3n$
$\rightarrow T(n) = c_2 2 * 2^{log_{\frac{3}{2}}\frac{2}{3}n} - 4n + n \leq c_2 2^{log_{\frac{3}{2}}n} - 3n$
$\rightarrow T(n) = O(2^{log_{\frac{3}{2}}n} - 3n)$
$\rightarrow T(n) = O(2^{log_{\frac{3}{2}}n})$

Proof: $T(n) = \Omega(n^{log_{\frac{3}{2}}2})$
Suppose $\forall k < n, \exists c_1, T(k) \geq c_1 n^{log_{\frac{3}{2}}2}$
$\rightarrow T(n) = c_1 2(\frac{2}{3}n)^{log_{\frac{3}{2}}2} + \frac{4}{3}n = c_1 2 * (\frac{3}{2})^{log_{\frac{3}{2}}2} * n^{log_{\frac{3}{2}}2} + \frac{4}{3}n = c_1 n^{log_{\frac{3}{2}}2} + \frac{4}{3}n \geq$
$c_1 n^{log_{\frac{3}{2}}2} + n$
$\rightarrow T(n) = \Omega(n^{log_{\frac{3}{2}}2})$

$T(n) = O(2^{log_{\frac{3}{2}}n}), T(n) = \Omega(n^{log_{\frac{3}{2}}2}) \rightarrow T(n) = \Theta(n^{log_{\frac{3}{2}}2})$

7 Question 7: solve and verify

Notice that for iterative tree:
$\Theta(n^2) = 2T(\frac{1}{4}n) + n^2 \leq T(n) \leq 2T(\frac{1}{2}n) + n^2 = \Theta(n^2)$

Proof: $T(n) = O(n^2)$, Suppose $\forall k < n, T(k) = O(k^2)$
$\rightarrow \exists c_2 > \frac{16}{11}, T(k) \leq c_2 n^2, T(n) \leq (\frac{5}{16}c_2 + 1)n^2 \leq c_2 n^2, c_2 > \frac{16}{11}$

Proof: $T(n) = \Omega(n^2)$, Suppose $\forall k < n, T(k) = \Omega(k^2)$
$\rightarrow \exists c_1 < \frac{16}{11}, T(k) \geq c_1 n^2, T(n) \geq (\frac{5}{16}c_1 + 1)n^2 \geq c_1 n^2, c_1 < \frac{16}{11}$
$\rightarrow T(n) = \Theta(n^2)$

8 Question 8: solve

Let $n = 2^m$, Then $T(2^m) = 9T(2^{\frac{m}{6}}) + m^2$
$\rightarrow S(m) = 9S(\frac{1}{6}m) + m^2$
From Branch: $S(m) = m^{log_6 \frac{3}{2} + 2}$
From Leave: $S(m) = m^{log_6 9}$
So, $S(m) = \Theta(m^{log_6 \frac{3}{2} + 2})$
$\rightarrow T(n) = T(2^m) = \Theta((lg(n))^{log_6 \frac{3}{2} + 2})$

9 Question 9: solve and justify

9.1 a

For leaf $\Theta(n) = n^{log_3 2}$
For branch, Notice that $n^{\frac{1}{2}} < n^{\frac{1}{2}} lgn < n^{\frac{1}{2} + \epsilon}$
$\rightarrow 2S(\frac{1}{3}n) + n^{\frac{1}{2}} < T(n) < S(n) = 2S(\frac{1}{3}n) + n^{\frac{1}{2} + \epsilon}$
Notice that the branch complexity of $\Theta(n^{log_3 2}) \leq S(n) = n^{\frac{1}{2} + \epsilon} \leq \Theta(n^{log_3 2})$
$\rightarrow T(n)$ is equally dominated by branch and leaf, $T(n) = \Theta(n^{log_3 2})$

9.2 b

For branch $T(n) = \Theta(hn^2) = \Theta(lognn^2)$
For leaf $T(n) = \Theta(n^2)$
$\rightarrow T(n)$ is dominated by branch, $T(n) = \Theta(lognn^2)$

9.3 c

For leaf $T(n) = \Theta(4^{log_2 n}) = \Theta(n^2)$
For branch, notice that $4 * (\frac{1}{2})^{\frac{5}{2}} = 2^{-\frac{1}{2}} < 1, T(n) = \Theta(n^{\frac{5}{2}})$
$\rightarrow T(n)$ is dominated by branch, $T(n) = \Theta(n^{\frac{5}{2}})$

9.4 d

For branch $TreeHeight = h = \frac{n}{2}, T(n) = \frac{1}{2}\sum_{1}^{h+1} \frac{1}{n} = \frac{1}{2}(lnn - ln2) = \Theta(lnn)$
For leaf $T(n) = \Theta(c)$
$\rightarrow T(n)$ is dominated by branch, $T(n) = \Theta(lnn)$