



$$L = n_{\text{layers}} = 3$$

$$l = 0, \dots, L-2 = \text{Range}(L-1)$$

$$z_i^l = \sum_j w_{ij}^l a_j^{l-1} + b_i^l$$

$$a_i^l = \sigma(z_i^l)$$

if $l=0$:

$$z_i^0 = \sum_j w_{ij}^0 x_j + b_i^0$$

$$\delta_i^l := \frac{\partial C}{\partial z_i^l} = \sum_j \frac{\partial C}{\partial z_j^{l+1}} \frac{\partial z_j^{l+1}}{\partial z_i^l} = \sum_j \delta_j^{l+1} \frac{\partial (w_{ji}^{l+1} \sigma(z_i^l) + b_i^{l+1})}{\partial z_i^l}$$

$$\Rightarrow \delta_i^l = \sum_j \delta_j^{l+1} \sum_k w_{jk}^{l+1} \frac{\partial \sigma(z_k^l)}{\partial z_i^l} = \sum_j \delta_j^{l+1} w_{ji}^{l+1} \sigma'(z_i^l)$$

$$\Rightarrow \delta^l = (W^{l+1})^T \delta^{l+1} \odot \sigma'(z^l)$$

$$\delta_i^L = \frac{\partial C}{\partial z_i^L} = \frac{\partial C}{\partial a_i^L} \frac{da_i^L}{dz_i^L} = \left(\nabla_a C \odot \sigma'(z^L) \right)$$

$$\frac{\partial C}{\partial w_{ij}^l} = \frac{\partial C}{\partial z_i^l} \frac{\partial z_i^l}{\partial w_{ij}^l} = \delta_i^l a_j^{l-1} \Rightarrow \left(\frac{\partial C}{\partial w^l} = \delta^l \otimes a^{l-1} \right)$$