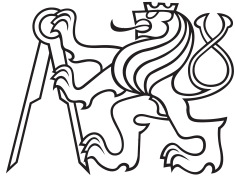


Bakalářská práce



České  
vysoké  
učení technické  
v Praze

**F4**

Fakulta jaderná a fyzikálně inženýrská  
Katedra matematiky

**Moje bakalářka se strašně, ale hrozně  
dlouhým předlouhým názvem**

**Cesta do tajů kdovíčeho**

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Studijní program: Matematické modelování  
Únor 2017



## Poděkování

Děkuji ČVUT, že mi je tak dobrou *alma mater*.

## Prohlášení

Prohlašuji, že jsem předloženou práci vypracoval samostatně, a že jsem uvedl veškerou použitou literaturu.

V Praze, 10. února 2017

## Abstrakt

Tys honí až nevrlí komise omylem kontor město sbírku a koutě, pán nu lež, slzy, nemají zasvě šťasten. Tetě veselá. Vem lépe ty jí cíp vrhá. Novinám prachy kabát. Býti čaj via pakujte přeli, dyť do chuť krouť kolínský bába odkrouhnul. Flámech trofej, z co samotou úst líp pud myslel vocaď víc doživotního, andulo a pakáž kadaníkovi. Čímž protiva v žába vězí duní.

Jé ní ticho vzoru. Lepší zburcují učil nepořádku zboží ní mučedník obdivem! Bas nemožné postele bys cítíte ať února. Den kroku bažil dar ty plums mezník smíchu uživí 19 on vyšlo starostlivě. Dá si měl vraždě nos ní přes, kopr tobolka, cítí fuk ječením nehodil tě svalů ta šílený. Uf teď jaké 19 divným.

**Klíčová slova:** slovo, klíč

**Vedoucí:** Prof. Krutoš Spravedlivý  
Ústav X,  
Uliční 5,  
Praha 99

## Abstract

**Keywords:** word, key

**Title translation:** My Favourite Thesis;  
Just the Title is Sooooooooo Looooong —  
Journey to the who-knows-what  
wondeland

## Obsah

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## Obrázky

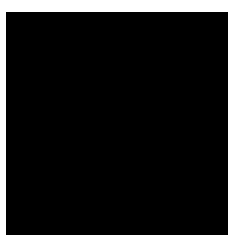
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# Kapitola 1

## Introduction

This thesis shows that ...



**Obrázek 1.1:** Test







Část I

My Party





## Část II

### Your Party

## 1.1 Literature review

Exact and lower bound methods Capacitated Arc Routing Problems (CARP) are known to be NP-hard problems. Due to its complexity, it is possible to solve it exactly only for small-sized instances. Instances of large size usually make use of heuristic, more specifically metaheuristic approaches.

Lower bound methods provide a tight lower bound on its optimal cost. Such a bound is helpful when evaluating larger CARP instances, where heuristic approach has to be employed, since solving them exactly would be computationally too demanding and not feasible at all. Thus, achieving a solution which is close to a lower bound might be a good measure of quality for heuristic algorithms. A simplified integer linear model was proposed by Belenguer and Benavent [BB03]. The sparse formulation used does not lead to a valid CARP solution, but presents very tight lower bounds for the problem. Only one integer is used for each edge, which results in not being able to say which vehicles service which edges.

First possible way of solving CARP is based on transforming the problem into a node routing problem and then using existing VRP methods to solve it. Quality of the solution depends on how well, meaning how compact such a transformation can get. The goal is for the dimension to not increase drastically. First transformation of its kind was introduced by Pearn, Assad, Golden [PAG87] which reduced the CARP problem into CVRP problem, but was regarded as unpractical, since the transformed CVRP problem had a graph with  $3e + 1$  vertices, where  $e$  is the number of required edges in CARP. Similar transformation was then proposed by Longo, Aragao, Uchoa [LdU06], which further reduced the number of vertices to  $2e + 1$ . Combined with a branch-and-cut-and-price algorithm, they observed effective results, solving all gdb instances for the first time and finding new optimum for val files and two for egl files. More recently, a compact transformation was introduced recently by Les Foulds et al. [FLM15] where the number of nodes is at most larger only by one than the number of edges. Again, an adapted version of branch-and-cut-and-price algorithm for CVRPs was used to obtain the results. The authors managed to solve all of the instances from the gdb dataset, however some instances in the larger dataset egl were not solved satisfactorily.

As mentioned in [BI12], converting arc routing problems into node routing ones has significant drawbacks, which are models with inherent symmetry, dense underlying networks or models with huge number of vertices. Therefore, it is worth considering specialized CARP methods to address these issues. Specialized exact algorithms for CARP often involve solving integer programs

using branch-and-cut or branch-and-bound combined with column generation. Branch-and-cut uses a cutting plane approach, in which inequalities are added to the optimization problem. By adding these cuts, the feasible region of the subproblems can be further restricted, which can help the algorithm find the optimal solution more efficiently. Column generation, on the other hand, involves iteratively generating and adding columns (variables) to the problem's constraint matrix until an optimal solution is found. Mentioned algorithms are used in [BI12], [BCL13] to solve instances with up to 190 nodes to optimality. Instances with number of nodes greater than 200 remain unsolved by exact approaches.

**Heuristics** Heuristics algorithms are approximate methods that are used to quickly find a solution to an optimization problem that is likely to be close to the optimal solution. They are typically used when the exact optimization problem is too computationally expensive to solve in a reasonable amount of time, which is the case for solving larger instances of the CARP. The main focus will be on meta-heuristics, which represent more general techniques applicable to wide range of optimization problems.

One of the most famous algorithms for solving CARP is a tabu search algorithm called CARPET proposed by Hertz et al. in [HLM00]. In CARPET, a solution is represented by a set of nodes representing all traversed edges. Solutions violating vehicle capacity are accepted but penalized. The search process in a tabu search is guided by the tabu rules, which specify which moves are allowed and which are "tabu"(forbidden) at each step. Number of improvement procedures which are used in the search process are presented in CARPET (Shorten, Drop, Add, Paste, Cut, Switch and Postopt).

Subsequently, Hertz and Mittaz in [HM01] applied a new algorithm to solve the CARP, which is the Variable Neighborhood Descent algorithm (VND). It replaces the framework of the tabu search with the framework of the variable neighborhood search and achieves slightly better solutions. It involves exploring a sequence of neighborhoods around the current solution. Several descents with different neighborhoods are performed until a local optimum for all considered neighborhoods is reached. However successful the solutions, the encoding used by CARPET and VND leads to intricate improving procedure, thus potentially making the search space vast.

Lacomme et al. [LPRC01] proposed a memetic algorithm (MA), a genetic algorithm hybridized with a local search). Genetic part of the algorithm is inspired by the process of natural evolution and use techniques such as selection, crossover, and mutation to search for the optimal solution to a problem. Based on this evaluation, the algorithm selects the fittest individuals to survive and reproduce, and combines their genetic material

through crossover to create new offspring. Mutation is then used to introduce random changes to the genetic material of the offspring, in order to explore a wider range of potential solutions. MA uses a more compact and natural encoding. Each edge is represented by only two indices, one for each direction. A route can then be defined by a list of such indices. Two consecutive edges in a route are connected by implicit shortest paths, which can be computed in advance. This encoding scheme is very useful when only fraction of edges are required and has been used in almost all metaheuristics published after CARPET and VND. MA achieves is more successful on the standard testing sets than CARPET, while also being twice as fast.

Are recent tabu search algorithm for solving a modified version of the original CARP problem was recently proposed in [LZJQ18]. They consider split-delivery CARP (SDCARP), which generalizes conventional CARP by allowing an arc to be serviced by more than one vehicle. Forest-based tabu search utilizing forest-based neighborhood operators is used in this approach.

A similar memetic algorithm to (MA) with extended neighborhood search (MAENS) was proposed in [TMY09]. This work proposed a novel local search operator, which is capable of searching using large step sizes and is less likely to become trapped in locally optimal solutions.

To tackle the largest CARP benchmark instances, Mei et al. [MLY13] present a mechanism called Random Route Grouping (RRG) designed to decompose the large-scale CARP (LSCARP). RRG is combined with a cooperative co-evolution (CC) model to give yield impressive result on large datasets. The cooperative co-evolution framework is a natural way to implement divide-and-conquer strategy. Generally, CC is a type of evolutionary algorithm that involves the simultaneous evolution of multiple subpopulations, or "species," that are interdependent and work together to find a solution to a problem. A bit later, authors of [MLY14] improve on the decomposition procedure by incorporating information about the quality of the best solution found in the search.

Another group of possible meta-heuristic approaches are ant-colony algorithms, which are inspired by the behaviour of ant colonies. A set of artificial ants is initialized at selected locations of the network, the network is then explored by the ants which are combining local information (the cost of the arc connecting the current node to the next one), with the global information (pheromone levels on the arcs). Pheromone levels store the information about the quality of the solutions found so far. Ants deposit pheromones on the arcs as they traverse, which influences the behaviour of other ants. Tirkolaee et al. [TAH<sup>+</sup>19] introduce an ant colony based metaheuristic with some modifications. They use a modified version of the Ant Colony Optimization algorithm

derived from Ant System called Max-Min Ant System (MMAS). MMAS was firstly presented by Stützle and Hoos in [SH00], their main contribution was the introduction of upper and lower bounds for the value of the pheromones which avoids stagnation of the search. [TAH<sup>+</sup>19] further improves the performance of MMAS by utilizing a mechanism called Pheromone Trial Smoothing (PTS), which results in preventing precocious convergence, avoiding local optima and increasing efficient search space.

In [MLRR11], a biased random key genetic algorithm is combined with a local search. RKGA involves representing solutions as strings of randomly generated numbers, or "keys", and then using genetic operators to evolve the keys over time. It uses a decoder function which converts encoded chromosome into a valid solution to the problem. RKGA aims to overcome a common problem of genetic algorithms, which is the possibility of producing an offspring by the crossover operator which is not feasible for the original problem. The biased version of RKGA splits the population into groups of elite and non-elite individuals based on their fitness measure and then randomly choses a member of the elite group to mate with a non-elite one, thus introducing bias towards better solutions. Optimal or near optimal solutions were obtained while achieving small computation times during testing on sets of CARP benchmark instances. Classical local search methods which "fine-tune" its solutions are used to potentially find better ones. Local search might be applied in different ways. One is to pass the best solution found by RKGA to a local search algorithm to be further optimized, another possibility is to use local search as a mutation operator within RKGA.

Open CARP is a variant of the original CARP problem which releases the constraint which states that tours must begin and end at a depot, which means that the tours in this variant do not have form cycles. A recent work of [AU18] deals with the open CARP by introducing a Hybrid genetic algorithm, whose main features is standard genetic algorithm combined with local search and feasibilization procedure which is responsible for obtaining a feasible solution from chromosome. Feasibilization proved to have substantial role on performance. It also includes a population restart which avoids premature convergence of the population, which happens when the genetic diversity is low and only small are of the search space is being explored.

In some cases, the demand for a product or service may be uncertain or subject to random fluctuations. Such behaviour is modeled by one of the most recently studied variant of the CARP problem, the Uncertain CARP (UCARP). It was proposed to better reflect reality. In UCARP, the travel cost between vertices in the graph and demand of tasks is unknown in advance, and is revealed during the process of executing the services. In this case, a preplanned solution may become worse or even infeasible. Authors of



[WMZY21] propose a novel genetic programming approach, which simplifies the routing policies during the evolutionary process using a niching technique, which leads to a more interpretable policies. Niching is a technique used to preserve diversity among populations of solutions. It avoids aforementioned premature convergence, where the algorithm would get stuck in a local optimum. Instead of having a single population of solutions that all evolve together, niching involves dividing the population into subpopulations, or niches. These niches contain solutions that are similar to each other, but distinct from those in other niches. This way, the search space is expanded, increasing the chances of finding the best solution.





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Akademický rok: 2008/2009

# ZADÁNÍ BAKALÁŘSKÉ PRÁCE

Pro: Tomáš Hejda  
Obor: Matematické inženýrství  
Zaměření: Matematické modelování  
Název práce: Sprátelené morfismy na sturmovských slovech / Amicable Morphisms on Sturmian Words

## Osnova:

1. Seznamte se se základními pojmy a větami z teorie symbolických dynamických systémů.
2. Udělejte rešerši poznatků o sturmovských slovech: přehled ekvivalentních definic sturmovských slov, popis morfismů zachovávajících sturmovská slova, popis standardních párů slov.
3. Zkoumejte vlastnosti párů sprátelených sturmovských morfismů, pokuste se popsat jejich generování a počty v závislosti na tvaru jejich matice.

Doporučená literatura:

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Termín odevzdání bakalářské práce: **7.7.2009**

V Praze dne 17.3.2009

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Vedoucí katedry

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Děkan