ECES 301-Chapter 1

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Attendance Policy

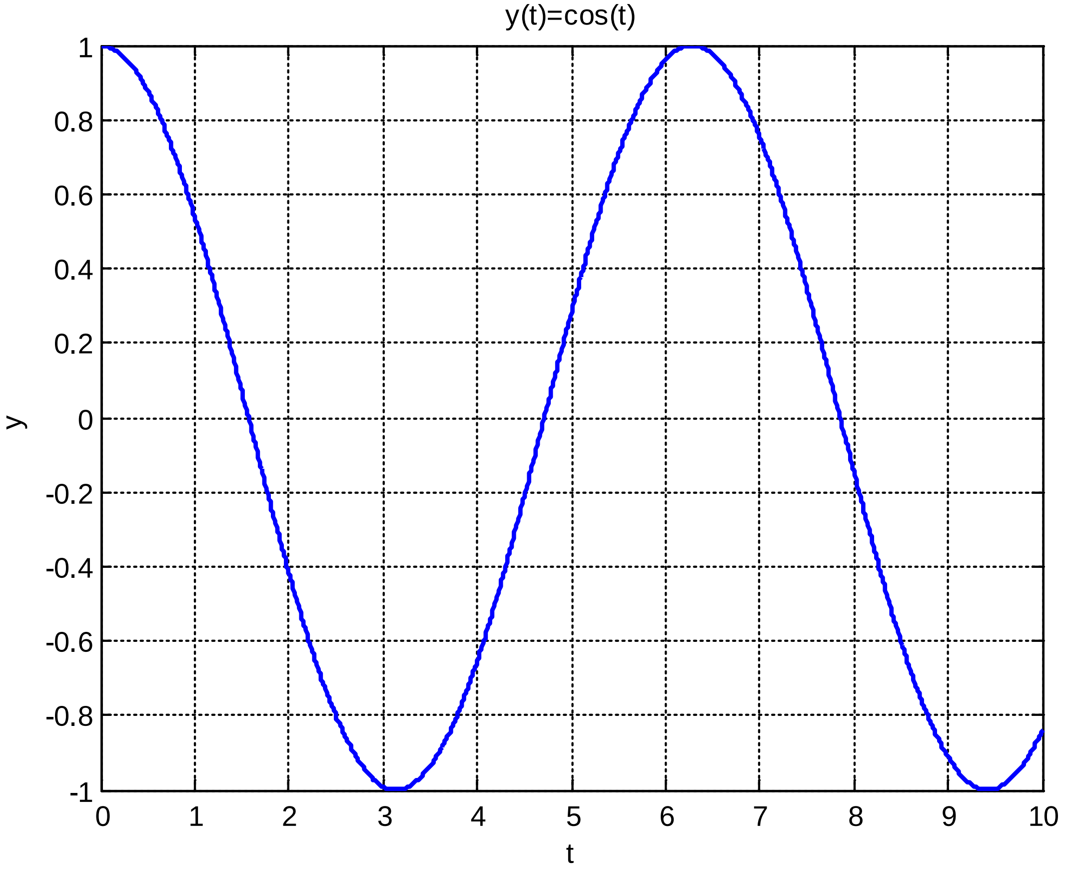
Attendance is mandatory. For every unexcused absence past the first, the student will receive a full 10% reduction of their final grade.

Examples of excused absences are: sickness(with a note), surgeries (with a note), illness/death in family(must have documentation), and military committments.

Missing class due to co-op interviews is NOT an excuse. This actually violates Drexel's co-op interview policy:

http://www.drexel.edu/scdc/co-op/undergraduate/policies-procedures/job-search/

Continuous Signals




```
%% Continuous Signal Example
close all;clear all;clc
t=0:.01:10; % Time
y=cos(t); % y(t)
plot(t,y,'Linewidth',2)
grid on
xlabel('t')
ylabel('y')
title('y(t)=cos(t)')
```

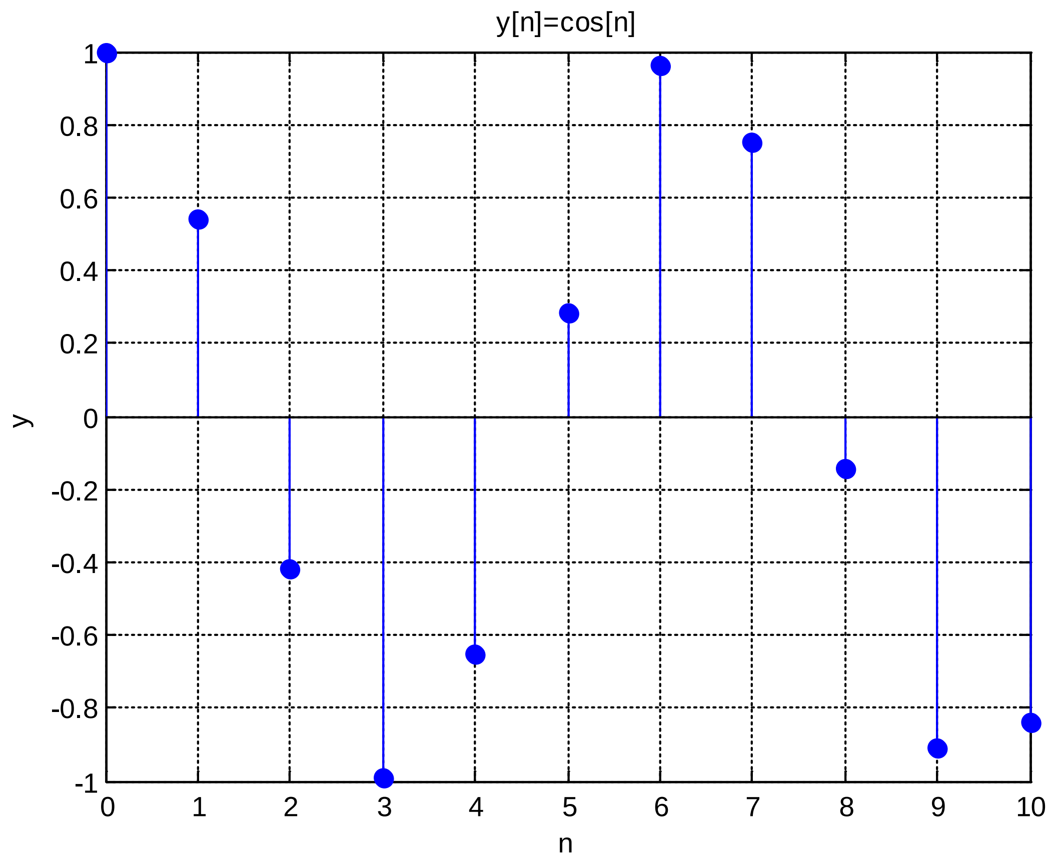
Continuous signals are represented by parenthesis ( )

Why Do We Care About Continuous Signals?

- RF/microwave transmission/generation
- Our bodies generate continuous signals for use (auditory, vocal)
- Musical Instruments
- Power generation/transmission
- Controls
- Circuits

Just about every kind of real signal starts off as a continuous function

Discrete-Time Signals




```
%% Discrete-Time Signal Example
close all;clear all;clc
n=0:10; % Time
y=cos(n); % y[n]
stem(n,y,'filled')
grid on
xlabel('n')
ylabel('y')
title('y[n]=cos[n]')
```

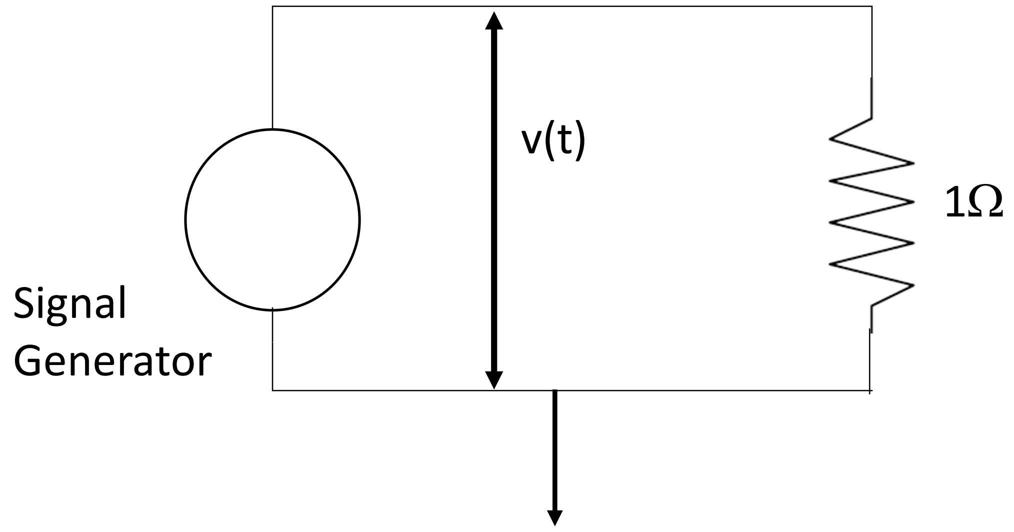
Discrete-time signals are represented by square brackets [ ]

Why Do We Care About Discrete Signals?

- Can’t store continuous signals on media
- Adaptive modification for processing
- Image/Signal processing
- Rapid processing
- Biological signal processing (ocular)
- Low power controls

Signal Energy and Power

Physical Representation of Signal Energy and Power



The instantaneous power consumed in the resistor is $\frac{(v(t)\ Volts)^2}{1\Omega} = v^2(t)$ Watts

The average power consumed in the resistor is $\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v^2(t)(1\Omega)\ dt =$
$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v^2(t)\ dt$ Watts

Energy Signals

The energy of a continuous signal is $$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

If $E_x < \infty$, then x(t) is an energy signal

The energy of a discrete signal is $$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

If $E_x < \infty$, then x[n] is an energy signal

Continuous Example

Determine if the signal $x(t) = 5e^{-3t}u(t)$ is an energy signal

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} 25e^{-6t}u(t)\, dt = 25\int_{0}^{\infty} e^{-6t}u(t)\, dt = 25\left[-\frac{1}{6}e^{-6t}\right]_0^{\infty} = 25\left[0 + \frac{1}{6}\right] = \frac{25}{6}$$

Because E_x is finite, this is an energy signal

Discrete Example

Determine if the signal x[n]=cos(2n), $-5 \leq n \leq 5$ is an energy signal

n	x[n]	$\|x[n]\|^2$
-5	-0.83907	0.704041
-4	-0.1455	0.02117
-3	0.96017	0.921927
-2	-0.65364	0.42725
-1	-0.41615	0.173178
0	1	1
1	-0.41615	0.173178
2	-0.65364	0.42725
3	0.96017	0.921927
4	-0.1455	0.02117
5	-0.83907	0.704041
	Sum	5.495133

Because x[n] has a finite energy, it is an energy signal

Power Signals

The power of continuous, aperiodic signals is $$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

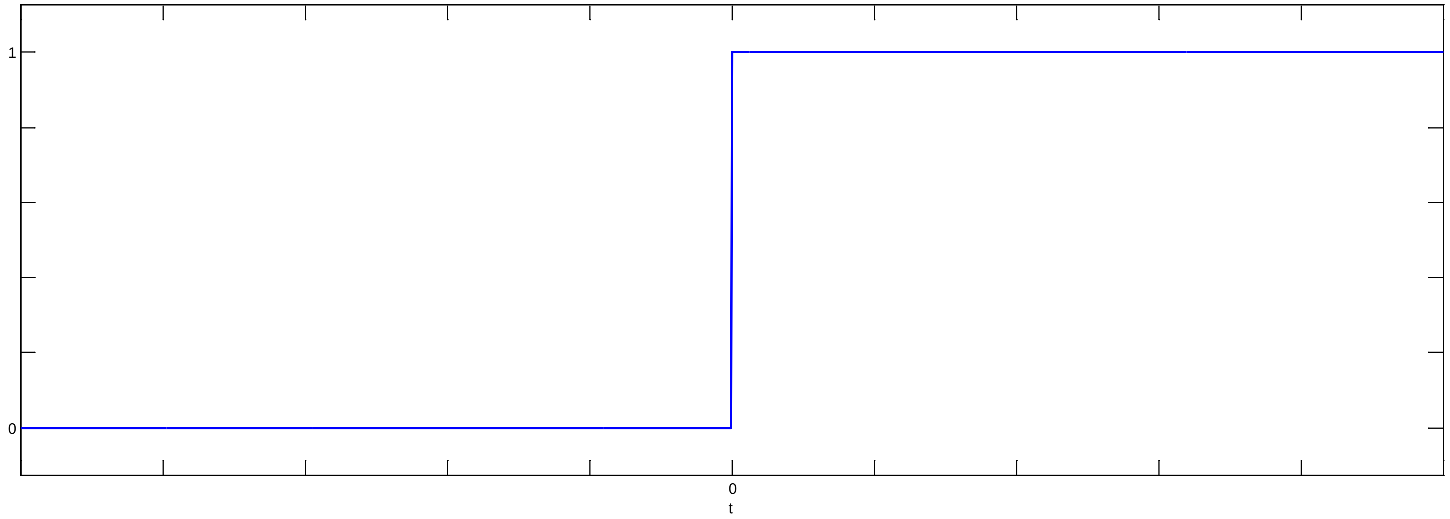
The power of continuous, periodic signals is $$P_x = \frac{1}{T} \int_{t_0}^{t_0+T} |x(t)|^2 \, dt$$

The power of a discrete, aperiodic signals is $$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

The power of a discrete, periodic signals is $$P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

Continuous, Aperiodic Example

Calculate the power of $x(t) = u(t)$ $\qquad u(t) = \begin{cases} 0, t < 0 \\ 1, otherwise \end{cases}$



$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |u(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} 1 \, dt = \lim_{T\to\infty} \frac{T}{2T} = \frac{1}{2}$$

Continuous, Periodic Example

Calculate the power of $x(t) = 5sin(t)$

$$P_x = \frac{1}{T}\int_{t_0}^{t_0+T} |x(t)|^2\,dt = \frac{25}{T}\int_{t_0}^{t_0+T} sin^2(t)\,dt = \frac{25}{T}\left(\frac{1}{2}\right)[t - sin(t)cos(t)]_{t_0}^{t_0+T}$$

$$P_x = \frac{25}{2T}\big(t_0 + T - sin(t_0 + T)cos(t_0 + T) - t_0 + sin(t_0)cos(t_0)\big)$$

Due to periodicity, $sin(t_0)cos(t_0) = sin(t_0 + T)cos(t_0 + T)$

$$P_x = \frac{25}{2}$$

Does this make sense?

Continuous, Aperiodic Example

Calculate the power of $x(t) = u(t)$

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |u(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} 1 \, dt = \lim_{T\to\infty} \frac{T}{2T} = \frac{1}{2}$$

Continuous, Periodic Example

Calculate the power of $x(t) = 5sin(t)$

$$P_x = \frac{1}{T}\int_{t_0}^{t_0+T} |x(t)|^2\,dt = \frac{25}{T}\int_{t_0}^{t_0+T} sin^2(t)\,dt = \frac{25}{T}\left(\frac{1}{2}\right)[t - sin(t)cos(t)]_{t_0}^{t_0+T}$$

$$P_x = \frac{25}{2T}\left(t_0 + T - sin(t_0+T)cos(t_0+T) - t_0 + sin(t_0)cos(t_0)\right)$$

Due to periodicity, $sin(t_0)cos(t_0) = sin(t_0+T)cos(t_0+T)$

$$P_x = \frac{25}{2}$$

Discrete, Aperiodic Example

Calculate the power of $x[n] = u[n]$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |1|^2 = \lim_{N\to\infty} \frac{N+1}{2N+1} = \frac{1}{2}$$

Discrete, Periodic Example

Calculate the power for the periodic sequence

$$x[n] = \{\dots, 1,3,2,5,6,0,1,3,2,5,6,0,1,3,2,5,6, \dots\}$$

$$P_x = \frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2 = \frac{1}{6}(1 + 9 + 4 + 25 + 36 + 0) = \frac{75}{6}$$

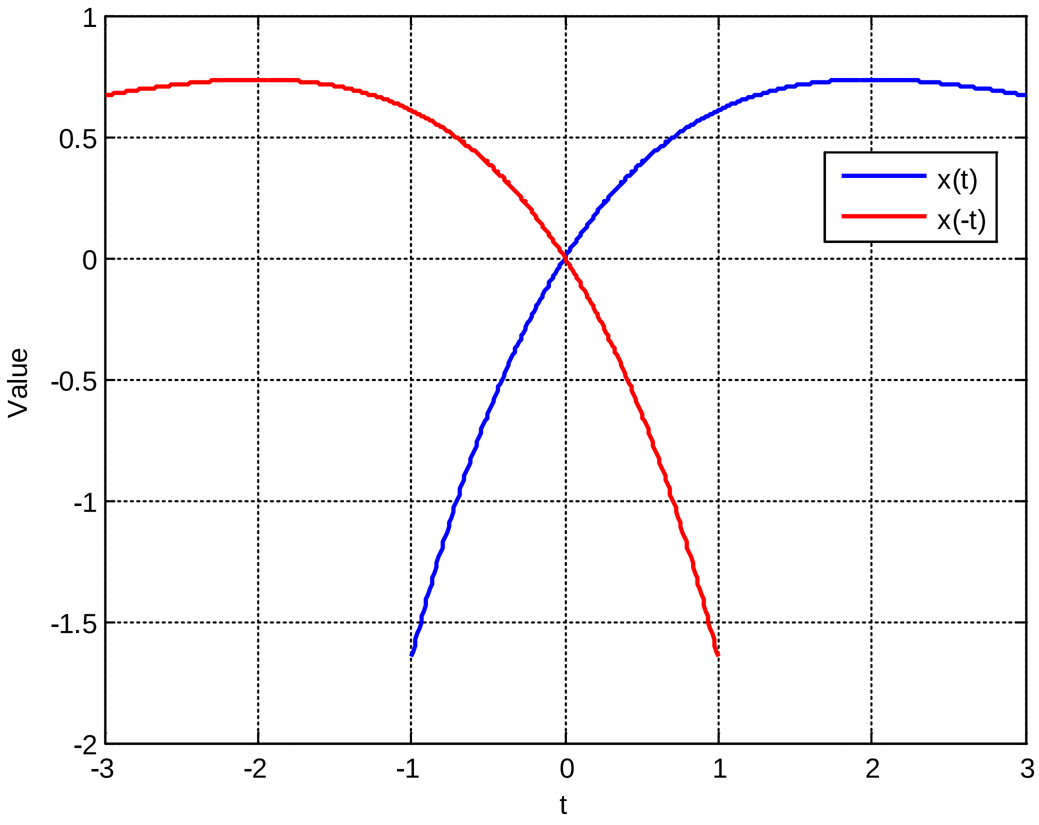
Remarks

- Signals with finite energy have a signal power of zero
- Power signals have infinite energy
- There are signals that are neither energy nor power signals

Fundamental Transformations of Signals

Time Reversal

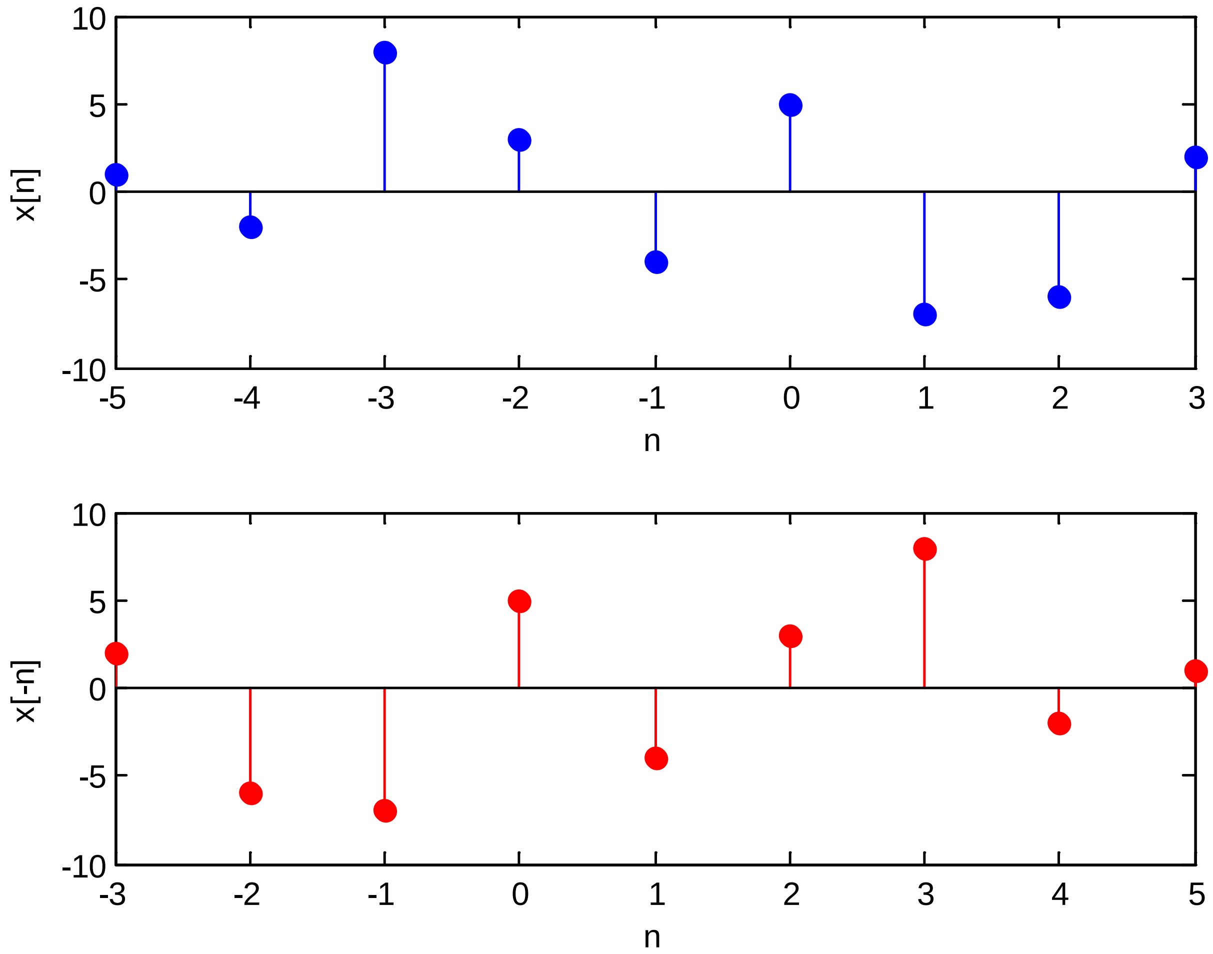
A signal y(t) is a reflection of x(t) (about the vertical axis) if y(t)=x(-t)




```
close all;clear all;clc
t=-1:.01:3;
x=t.*exp(-0.5*t);
figure
plot(t,x,'Linewidth',2)
hold on
plot(-t,x,'r','Linewidth',2)
grid on
xlabel('t')
ylabel('Value')
legend('x(t)','x(-t)',0)
```

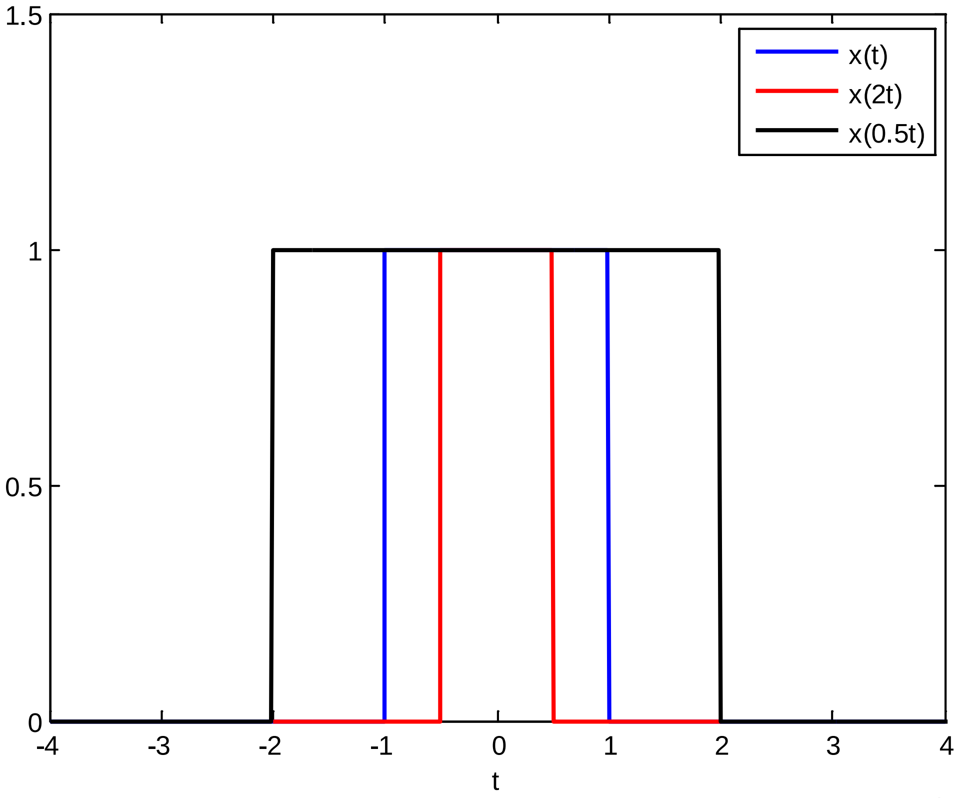
Discrete Example

```
close all;clear all;clc
n=-5:3;
x=[1 -2 8 3 -4 5 -7 -6 2];
figure
subplot(2,1,1)
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
subplot(2,1,2)
stem(-n,x,'r','filled')
xlabel('n')
ylabel('x[-n]')
```



Time Scaling

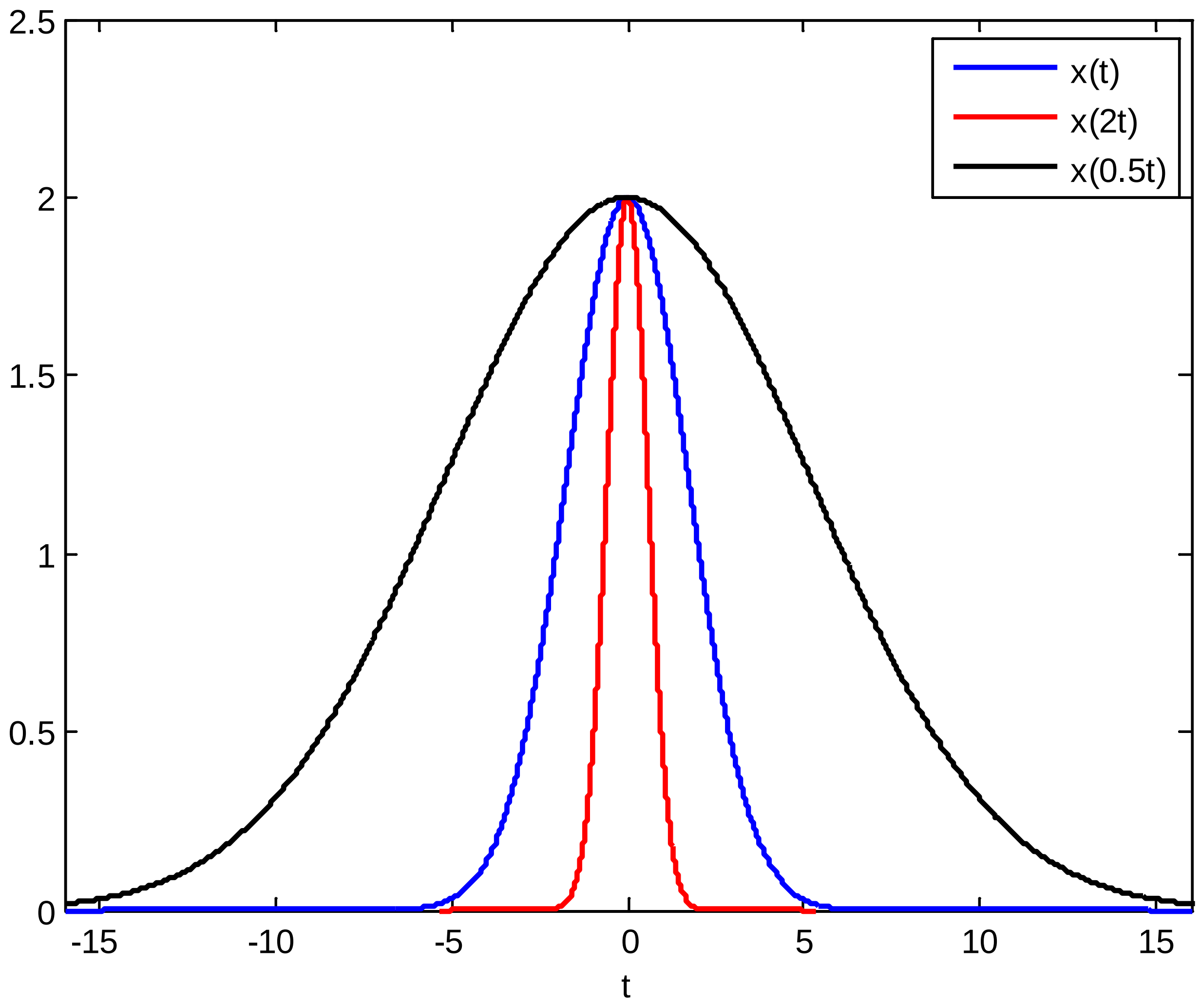
Consider the equation $y(t) = x(at)$, where a is a positive, real value. If a<1, y(t) becomes an expanded version of x(t). If a>1, y(t) becomes a compressed version of x(t).




```
t=-4:.005:4;
y=rectpuls(t,2);
figure
plot(t,y,'Linewidth',2);
hold on
plot(t/2,y,'r','Linewidth',2);
plot(t*2,y,'k','Linewidth',2);
xlabel('t')
legend('x(t)','x(2t)','x(0.5t)')
ylim([0 1.5])
xlim([-4 4])
```

Continuous Example

```
close all;clear all;clc
t=-16:.005:16;
y=2*exp(-t.^2/6);
a=3;
figure
plot(t,y,'Linewidth',2);
hold on
plot(t/a,y,'r','Linewidth',2);
plot(t*a,y,'k','Linewidth',2);
xlabel('t')
legend('x(t)','x(3t)','x(t/3)')
ylim([0 2.5])
xlim([t(1) t(end)])
```



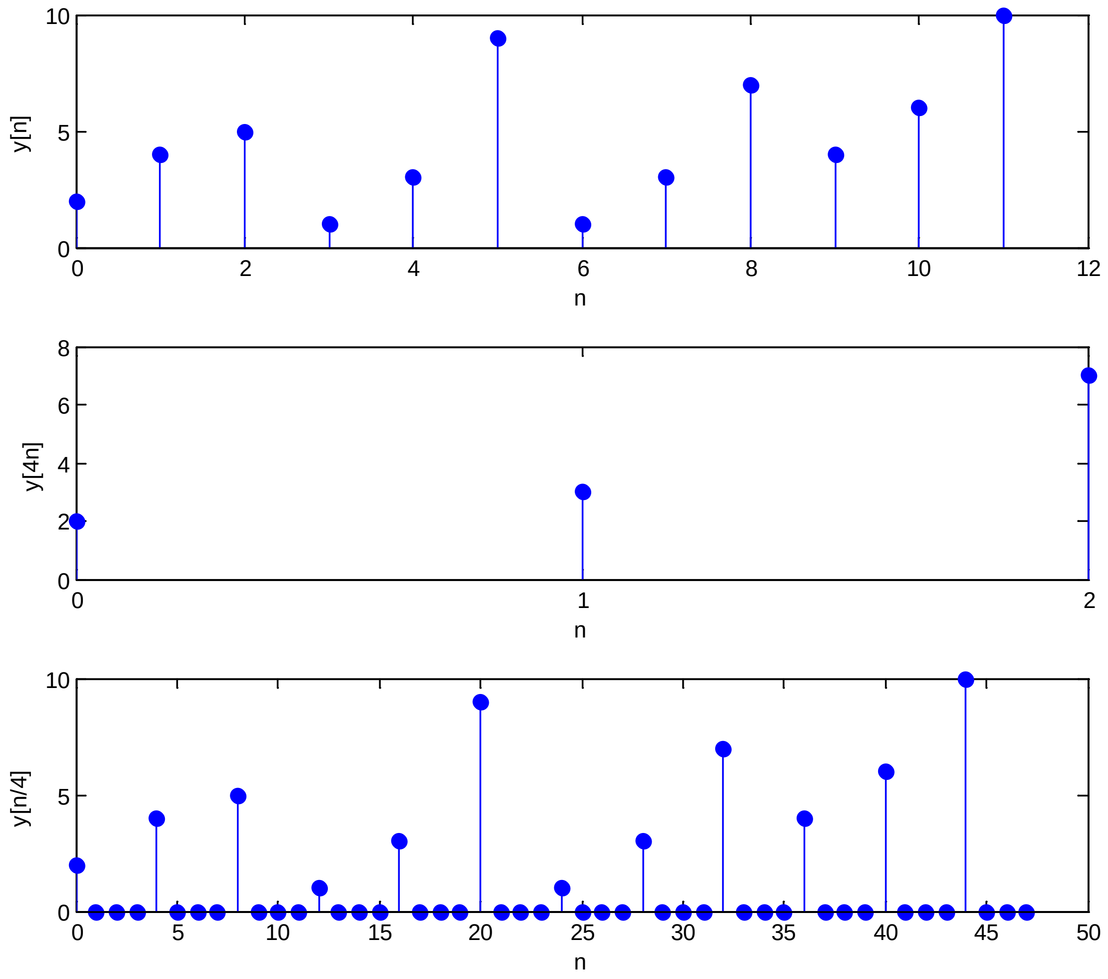
Time Scaling For Discrete Signals

Consider the signal $y[n] = x[an]$, where a is positive integer

- If a>1, the time-shifting operation is called *downsampling*
  - The downsignaled signal is time-compressed
  - Samples are lost (y[n] has less samples than x[n])
- If a<1, the time-shifted operation is called *upsampling*
  - The upsampled signal is time-expanded
  - 1/a-1 zeros are inserted between two consecutive examples of x[n]
  - 1/a must also be a positive integer

Discrete Example

```
close all;clear all;clc
x=[2 4 5 1 3 9 1 3 7 4 6 10];
a=4; % Scale Factor
xdown=downsample(x,a);
xup=upsample(x,a);
figure
subplot(3,1,1)
stem(0:(length(x)-1),x,'filled')
xlabel('n')
ylabel('y[n]')
subplot(3,1,2)
stem(0:(length(xdown)-1),xdown,'filled')
set(gca,'XTick',0:2,'XTickLabel',0:2)
xlabel('n')
ylabel('y[4n]')
subplot(3,1,3)
stem(0:(length(xup)-1),xup,'filled')
xlabel('n')
ylabel('y[n/4]')
```



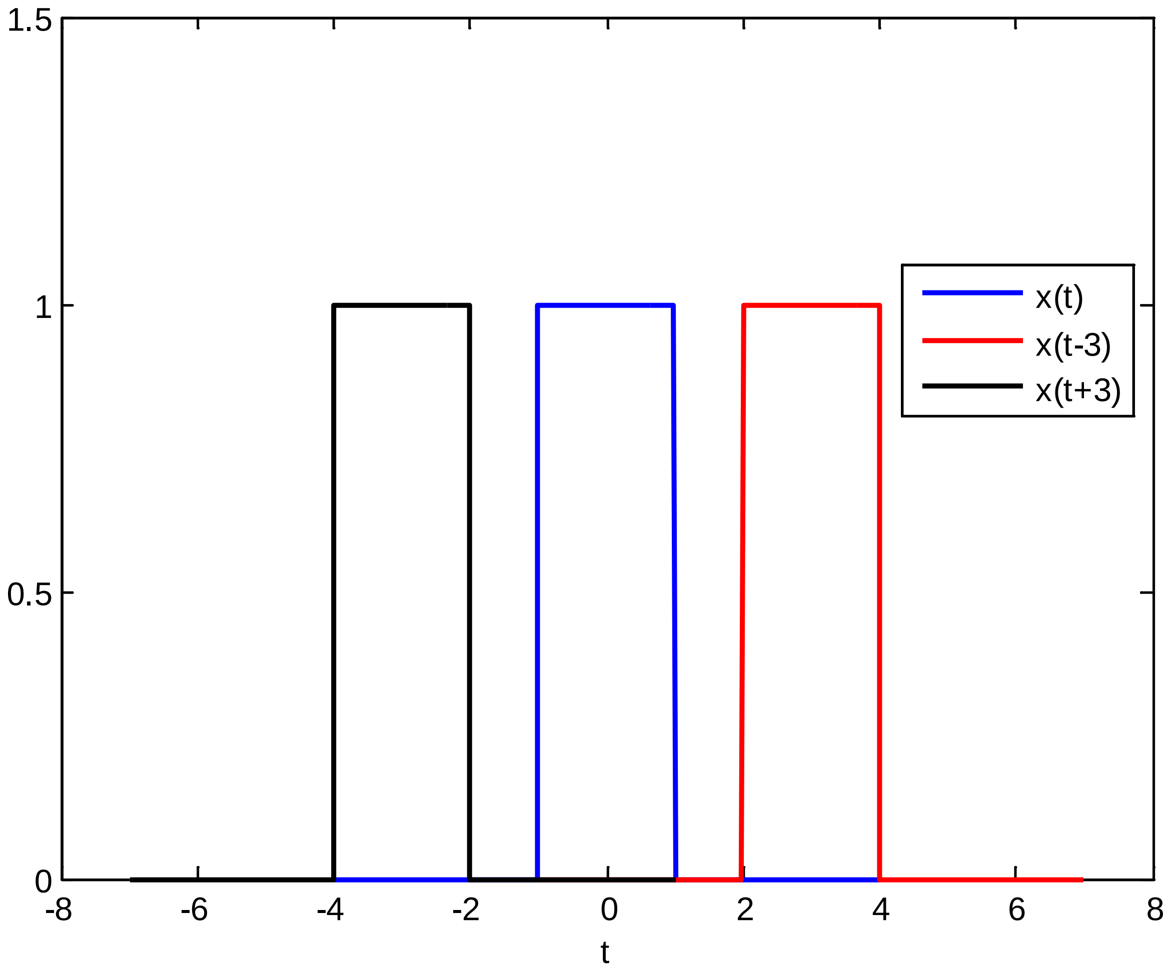
Time Shifting

Consider the function x(t). If $y(t) = x(t - t_0)$, then y(t) is a time-shifted version of x(t)

- If t0 >0, then y(t) is a *delayed* version of x(t)
- If t0<0, then y(t) is an *advanced* version of x(t)

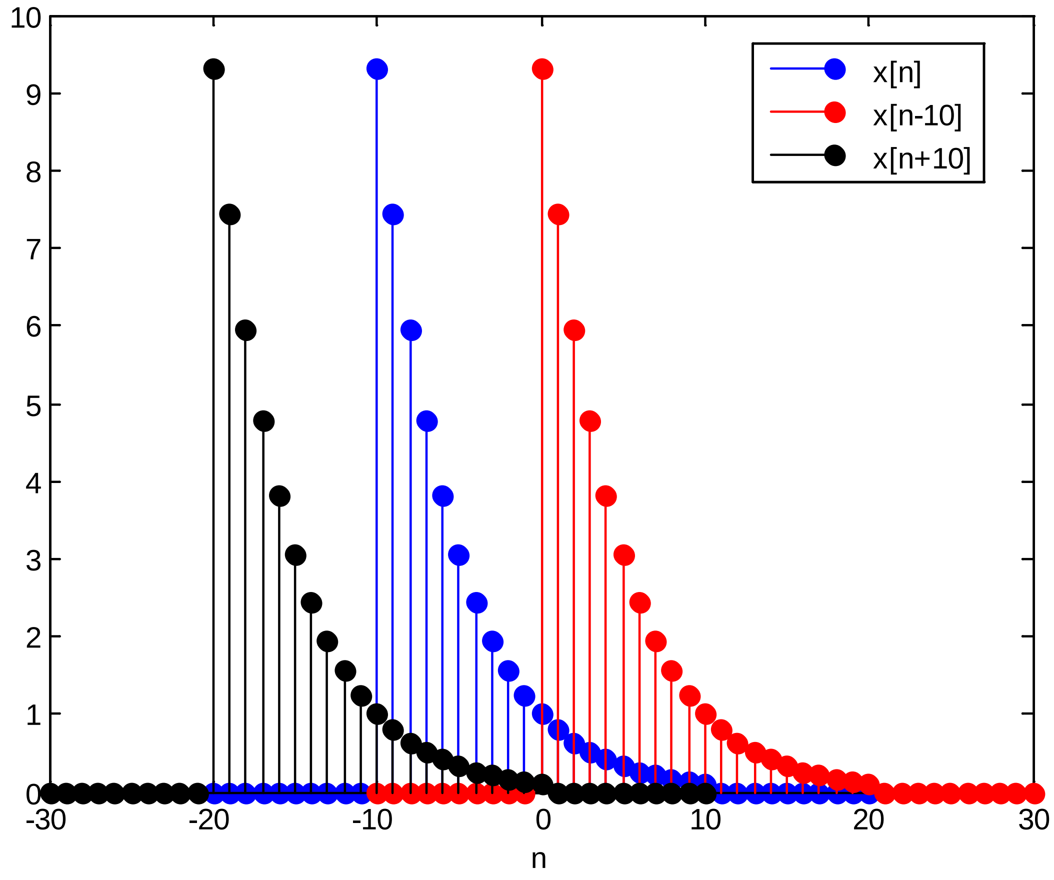
Continuous Example

```
close all;clear all;clc
t=-4:.01:4;
x=rectpuls(t,2);
t0=3;
figure
plot(t,x,'Linewidth',2);
hold on
plot(t+t0,x,'r','Linewidth',2)
plot(t-t0,x,'k','Linewidth',2)
xlabel('t')
legend('x(t)','x(t-3)','x(t+3)',0)
ylim([0 1.5])
```



Discrete Example

```
close all;clear all;clc
n=-20:20;
p=((n>=-10)&(n<=10));
x=(0.8.^n).*p;
n0=10;
figure
stem(n,x,'filled')
hold on
stem(n+n0,x,'r','filled')
stem(n-n0,x,'k','filled')
xlabel('n')
legend('x[n]','x[n-10]','x[n+10]',0)
```



Basic Continuous-Time Signals

Periodic Signals

A continuous-time signal x(t) is periodic if there is a positive number T such that

$$x(t) = x(t + T), \forall t \in \mathbb{R}$$

If T is the smallest positive value that holds in the above equation, then T is called the *fundamental period*.

A discrete-time signal x[n] is periodic if there is a positive number N such that

$$x[n] = x[n + N], \forall N \in \mathbb{Z}$$

If N is the smallest positive value that holds in the above equation, then N is called the *fundamental period*.

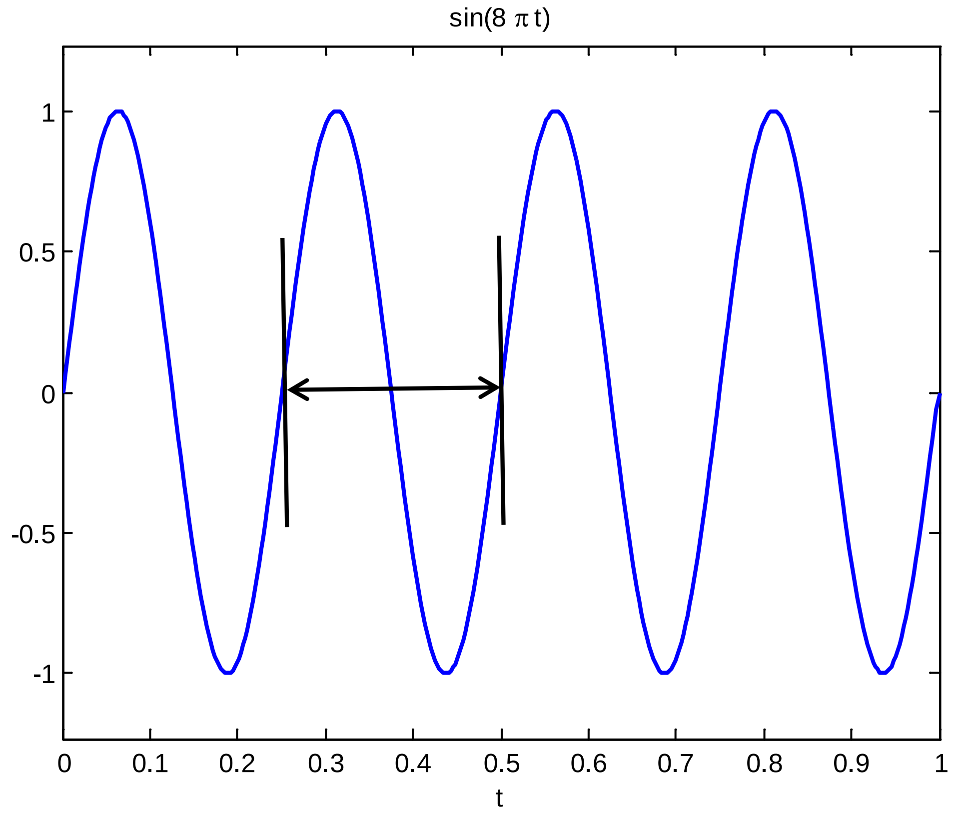
Continuous Sinusoid Example

$$y(t) = sin\left(8\pi t + \frac{\pi}{4}\right)$$

1. The phase offset does NOT affect the periodicity

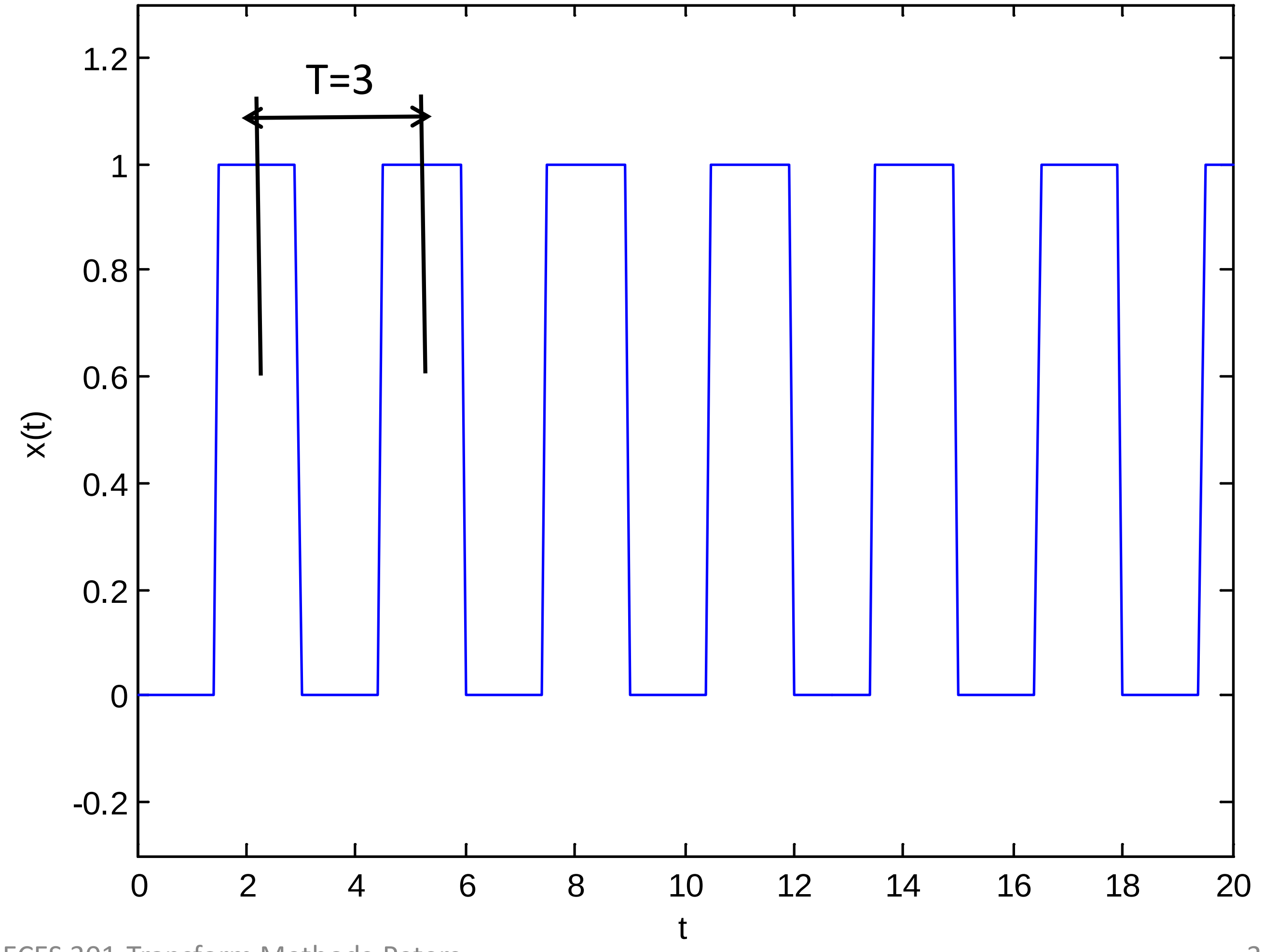
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8\pi} = \frac{1}{4}s$$

```
close all;clear all;clc
syms y t
y=sin(8*pi*t);
ezplot(y,[0 1])
```



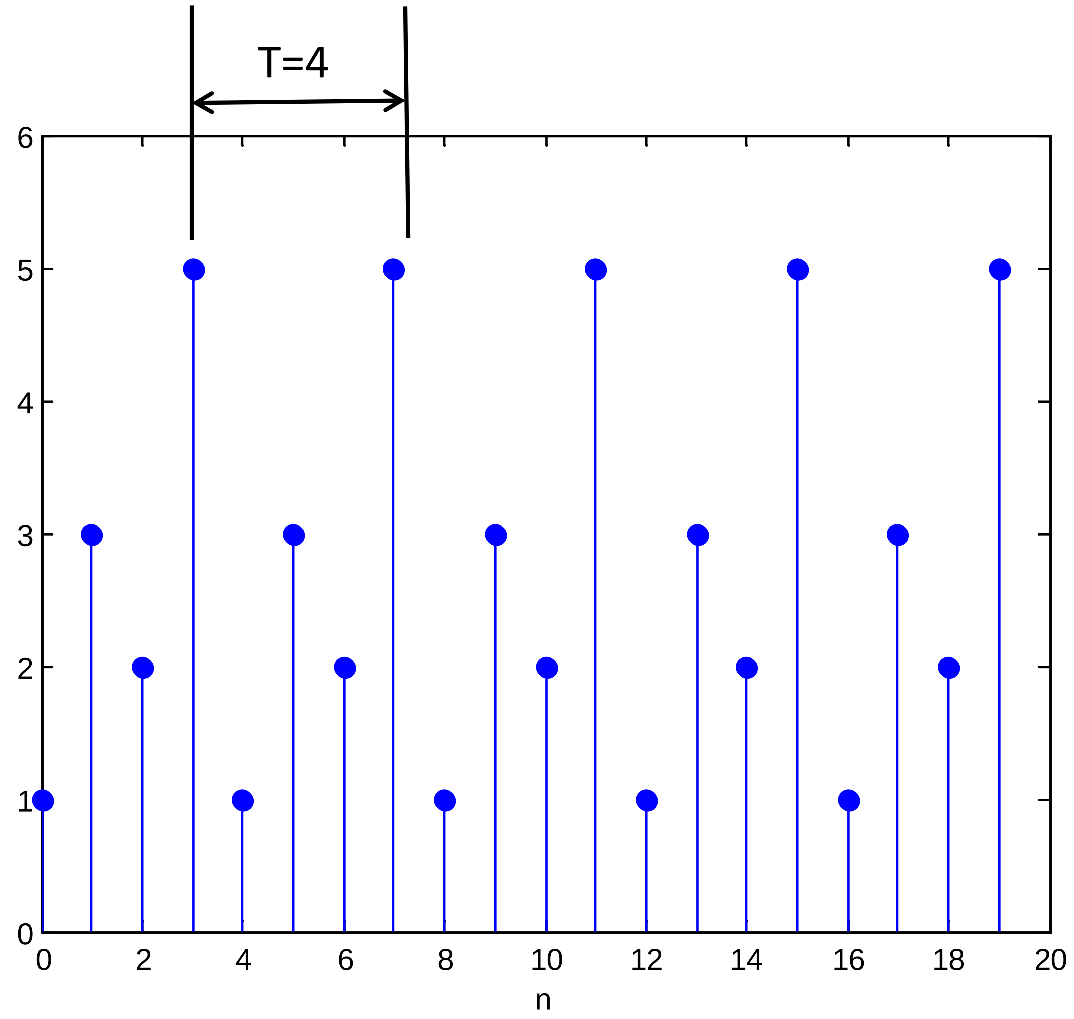
Continuous Periodic Signal Example

```
close all;clear all;clc
[s,t]=gensig('square',3,20,0.1);
plot(t,s);
ylim([-0.3 1.3])
xlabel('t')
ylabel('x(t)')
```



Discrete Periodic Signal Example

```
close all;clear all;clc
x1=[1 3 2 5];
y=repmat(x1,1,5);
n=0:(length(y)-1);
stem(n,y,'filled')
xlabel('n')
ylim([0 6])
```



Sum of Periodic Continuous-Time Signals

Let $x_1(t)$ and $x_2(t)$ be periodic signals:

$x_1(t) = x_1(t + mT_1), m\ is\ an\ integer$

$x_2(t) = x_2(t + nT_2), n\ is\ an\ integer$

The sum of these two signals is

$$x(t) = x_1(t) + x_2(t) = x_1(t + mT_1) + x_2(t + nT_2)$$

The combined signal must be period with period T. Thus,

$x(t) = x(t + T) = x_1(t + T) + x_2(t + T)$

Therefore,

$mT_1 = nT_2 = T, \qquad m, n\ are\ integers$

$$\frac{m}{n} = \frac{T_2}{T_1}$$

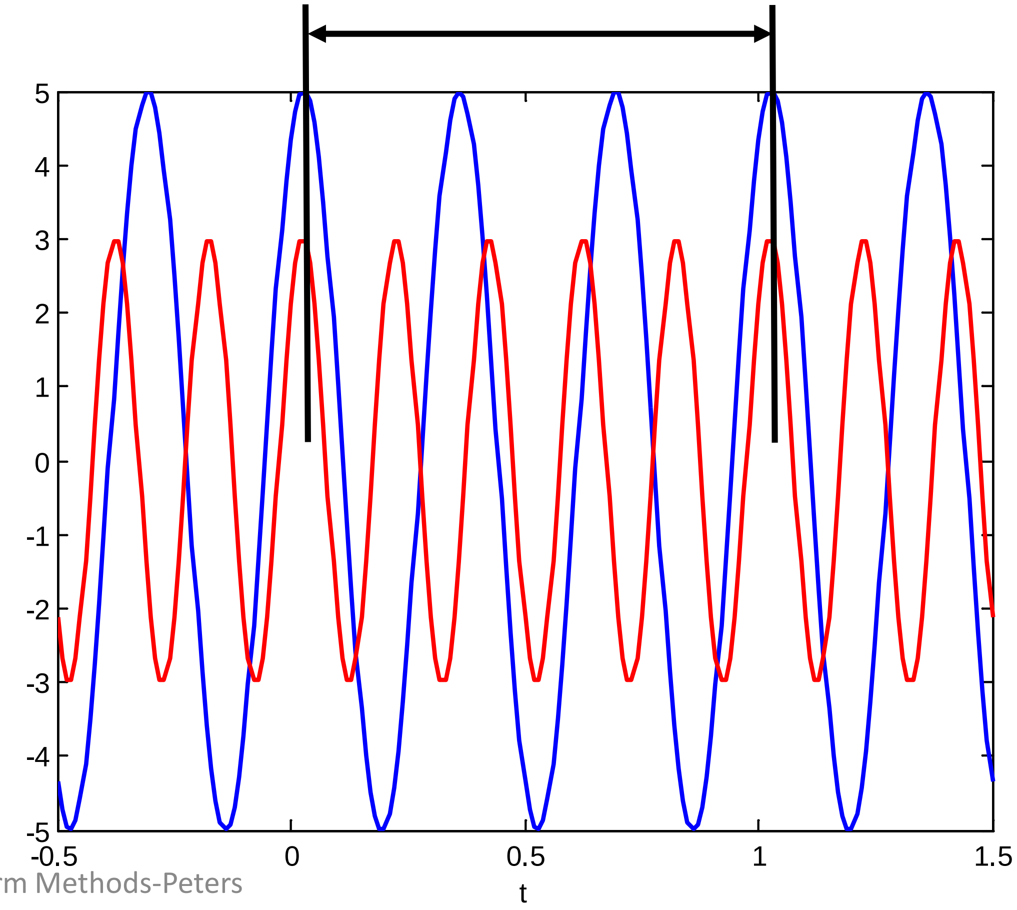
Example of Sum of Two Periodic Signals

What is the periodicity of the signal $x = 5sin\left(6\pi t + \frac{\pi}{3}\right) + 3cos\left(10\pi t - \frac{\pi}{4}\right)$?

$$T_1 = \frac{2\pi}{6\pi} = \frac{1}{3} \qquad T_2 = \frac{2\pi}{10\pi} = \frac{1}{5}$$

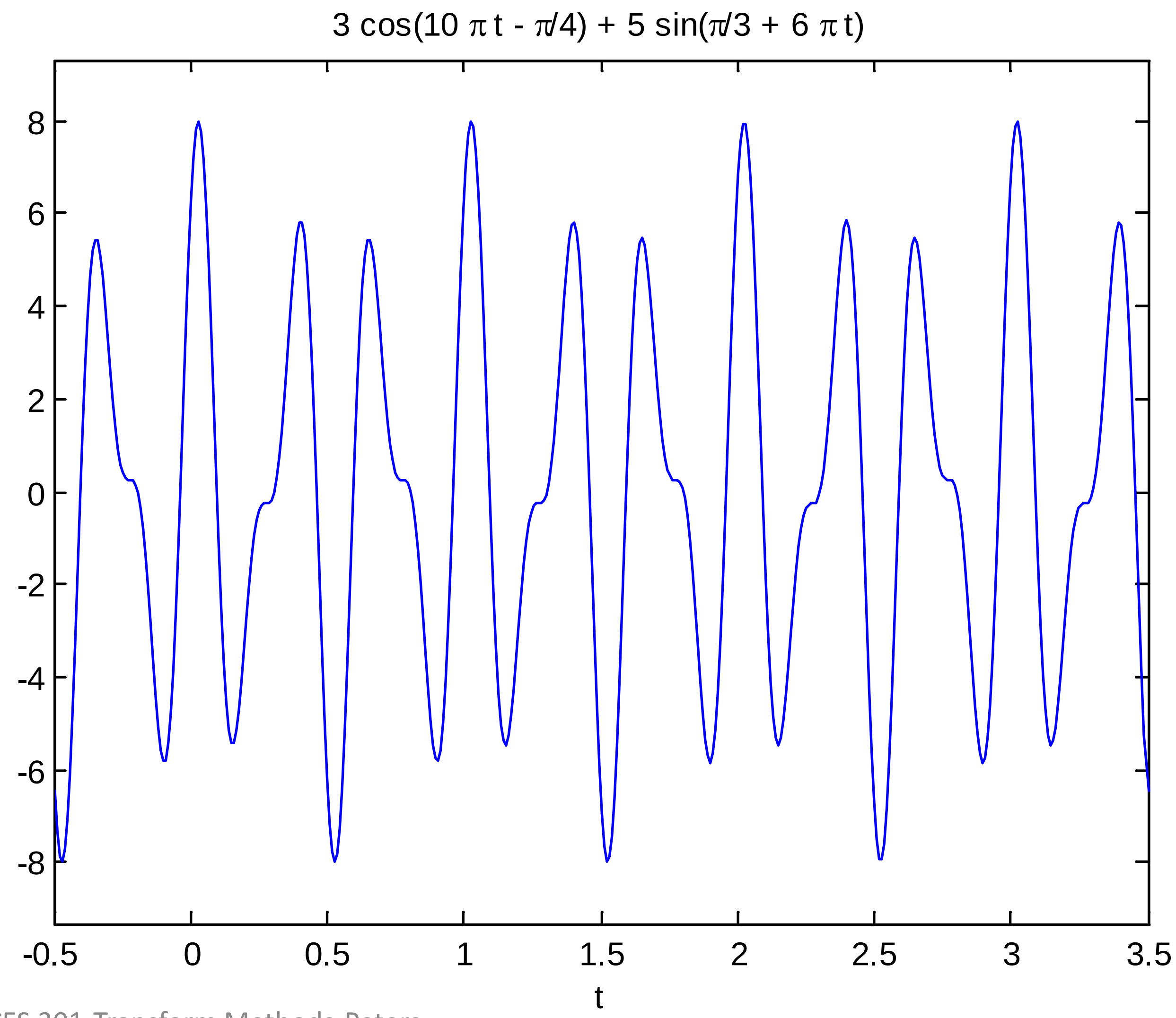
$$m\left(\frac{1}{3}\right) = n\left(\frac{1}{5}\right) = T$$

Here, we can see m=3, and n=5 in order for the two periodic signals to make the basic periodic part of the sum of both signals (here, with a periodicity of T=1)



Construction of Fundamental Periodic Signals in MATLAB

```
close all;clear all;clc
syms y t
y=5*sin(6*pi*t+pi/3)+3*cos(10*pi*t-pi/4);
figure
ezplot(y,[-0.5 3.5])
xlabel('t')
```



Sum of Periodic Continuous-Time Signals

Let $x_1[n]$and $x_2[n]$ be periodic signals:

$x_1[n] = x_1[n + mN_1]\ , m\ is\ an\ integer$

$x_2[n] = x_2[n + kN_2]\ , k\ is\ an\ integer$

The sum of these two signals is

$$x[n] = x_1[n] + x_2[n] = x_1[n + mN_1] + x_2[n + kN_2]$$

The combined signal must be period with period T. Thus,

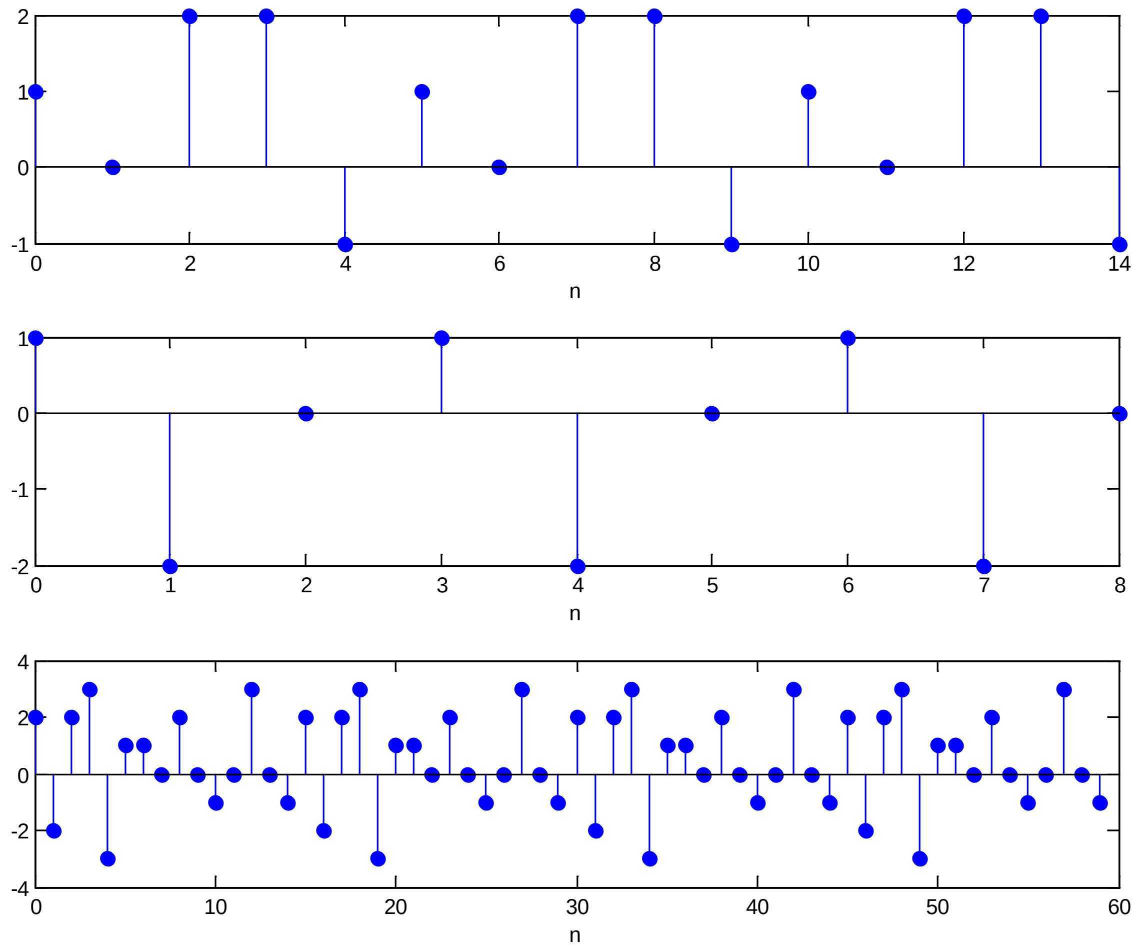
$$x[n] = x[n + N] = x_1[n + mN_1] + x_2[n + kN_2]$$

Therefore,

$$mN_1 = kN_2 = N, \qquad m, k\ are\ integers$$

$$\frac{m}{k} = \frac{N_2}{N_1}$$

Discrete Periodic Example



How do we solve this one?

Even Signals

A signal is even if $x(t) = x(-t)$ (continuous)

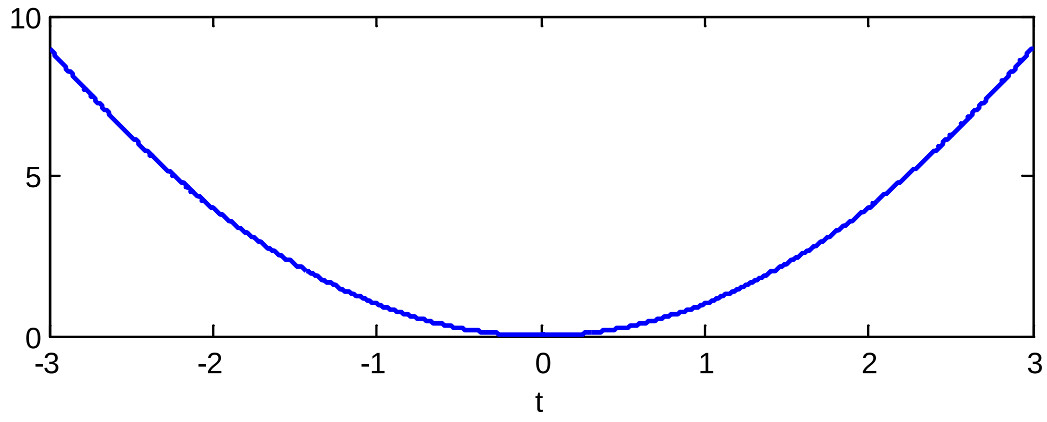
$x[n] = x[-n]$ (discrete)

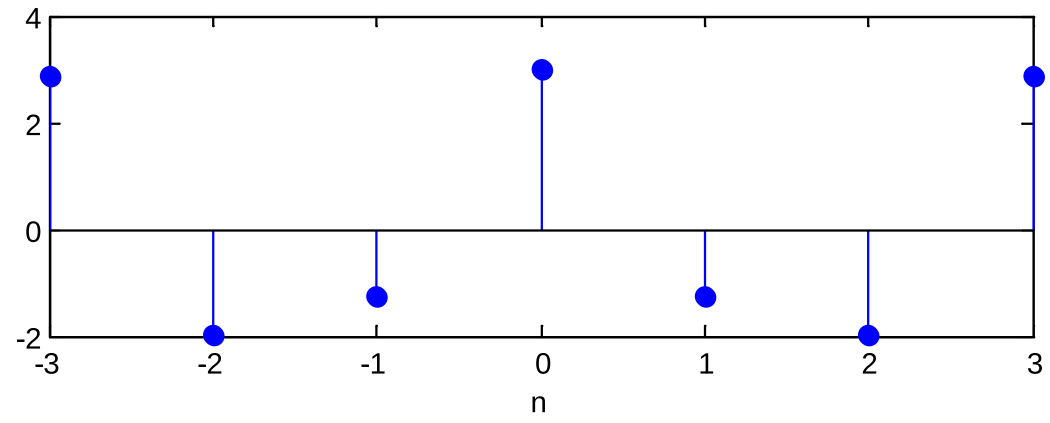
Examples: $x(t) = 5cos(3t)$

$x(t) = t^2$

$x[n] = 3cos[2n]$

$x[n] = 8n^2$

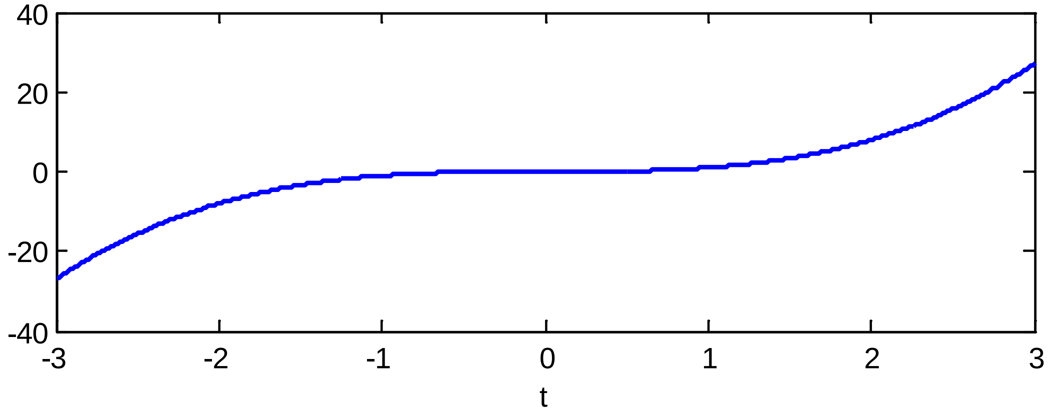




Odd Signals

A signal is odd if $x(t) = -x(-t)$(continuous)

$x[n] = -x[-n]$(discrete)

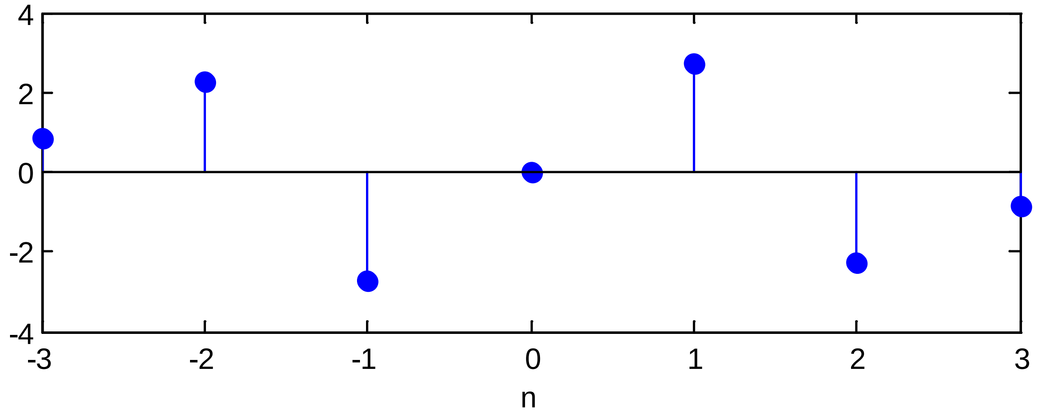


Examples: $x(t) = 5sin(3t)$

$x(t) = t^3$

$x[n] = 3sin[2n]$

$x[n] = 8n^3$



Decomposing Signals Into Even and Odd Parts

Signals, whether continuous or discrete, can be decomposed into even and odd components:

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$x[n] = x_e[n] + x_o[n]$$

$$x_e[n] = \frac{1}{2}(x[n] + x[-n])$$

$$x_o[n] = \frac{1}{2}(x[n] - x[-n])$$

Why would this be so important?

Continuous Example

What are the even and odd components of the signal $x(t) = 3 + 2t + 4t^2 + t^3 + 7t^4$?

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[3 + 2t + 4t^2 + t^3 + 7t^4 + 3 + 2(-t) + 4(-t)^2 + (-t)^3 + 7(-t)^4]$$

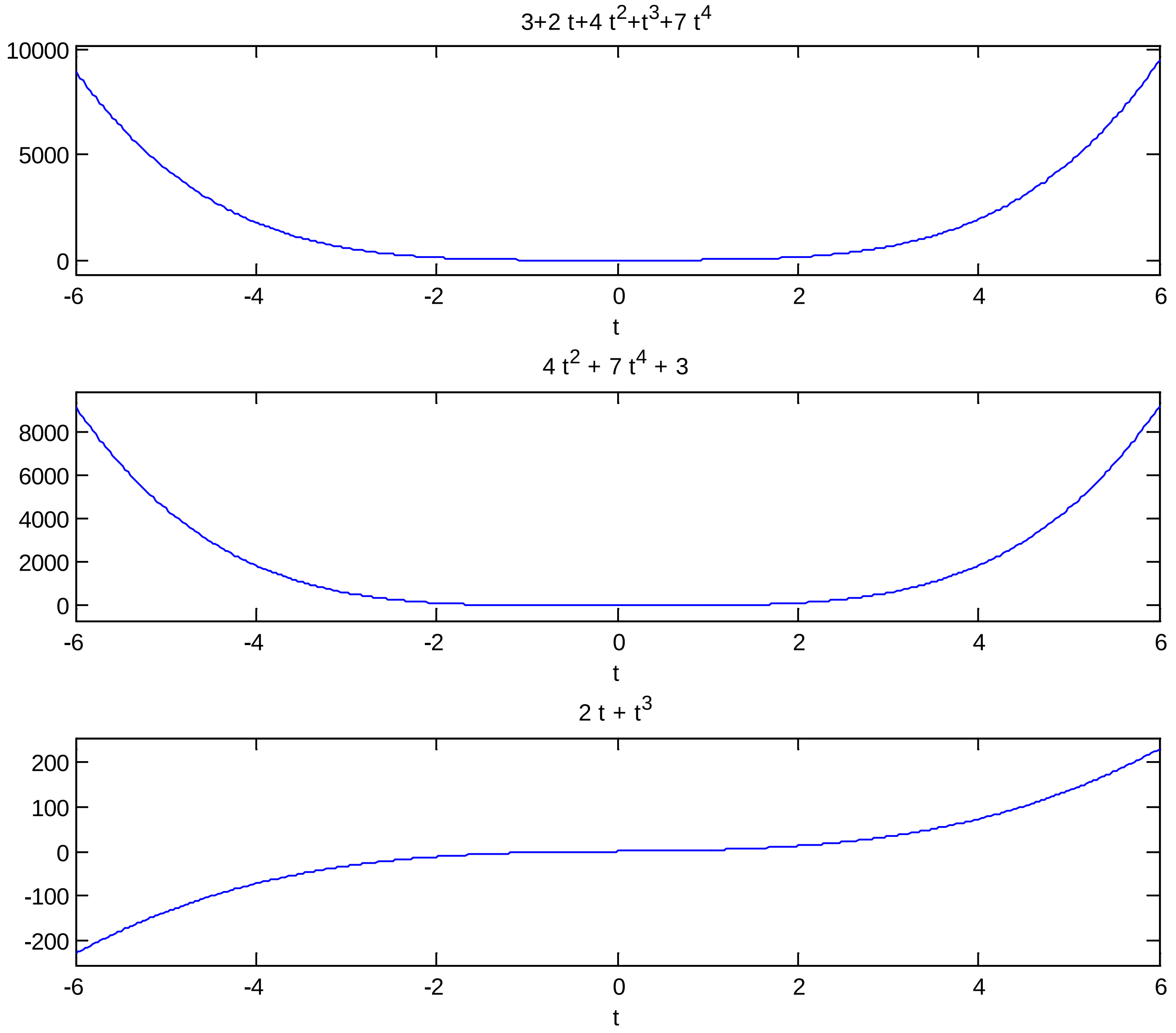
$$x_e(t) = \frac{1}{2}[6 + 8t^2 + 14t^4] = 3 + 4t^2 + 7t^4$$

$$x_0(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[3 + 2t + 4t^2 + t^3 + 7t^4 - 3 - 2(-t) - 4(-t)^2 - (-t)^3 - 7(-t)^4]$$

$$x_0(t) = \frac{1}{2}[4t + 2t^3] = 2t + t^3$$

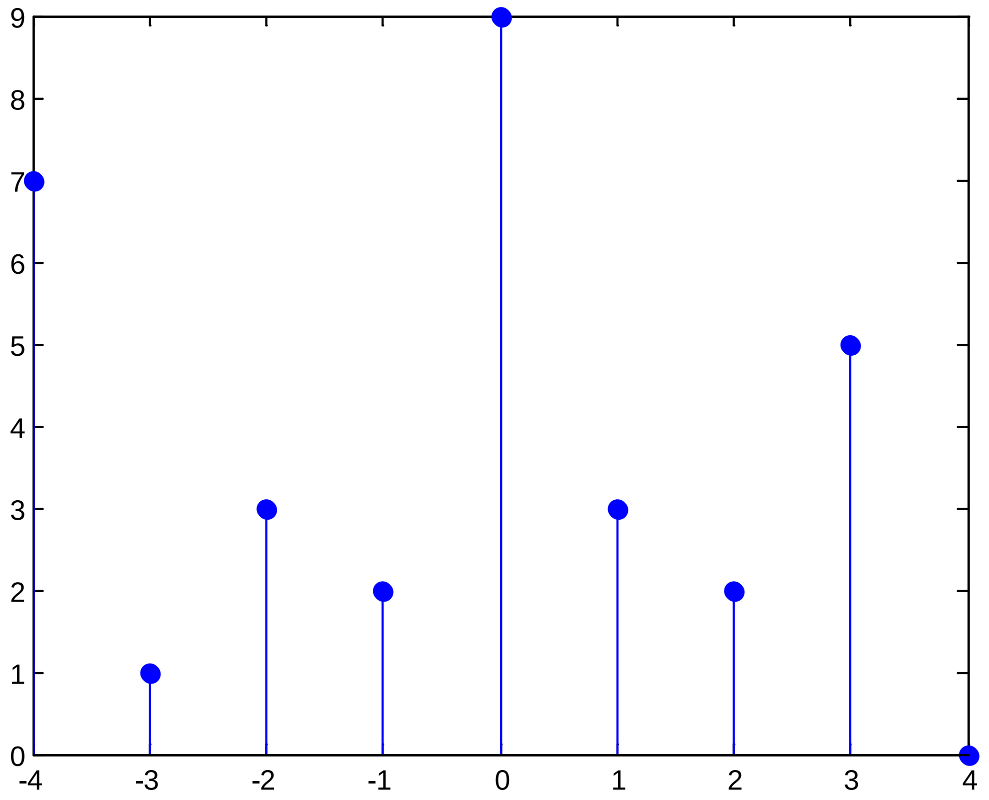
DOES THIS MAKE SENSE?

```
close all;clear all;clc
syms x t x_even x_odd
x=@(t)3+2*t+4*t.^2+t.^3+7*t.^4;
x_even=0.5*(x+x(-t));
x_odd=0.5*(x-x(-t));
figure
subplot(3,1,1)
ezplot(x,[-6 6])
xlabel('t')
subplot(3,1,2)
ezplot(x_even,[-6 6])
xlabel('t')
subplot(3,1,3)
ezplot(x_odd,[-6 6])
xlabel('t')
```

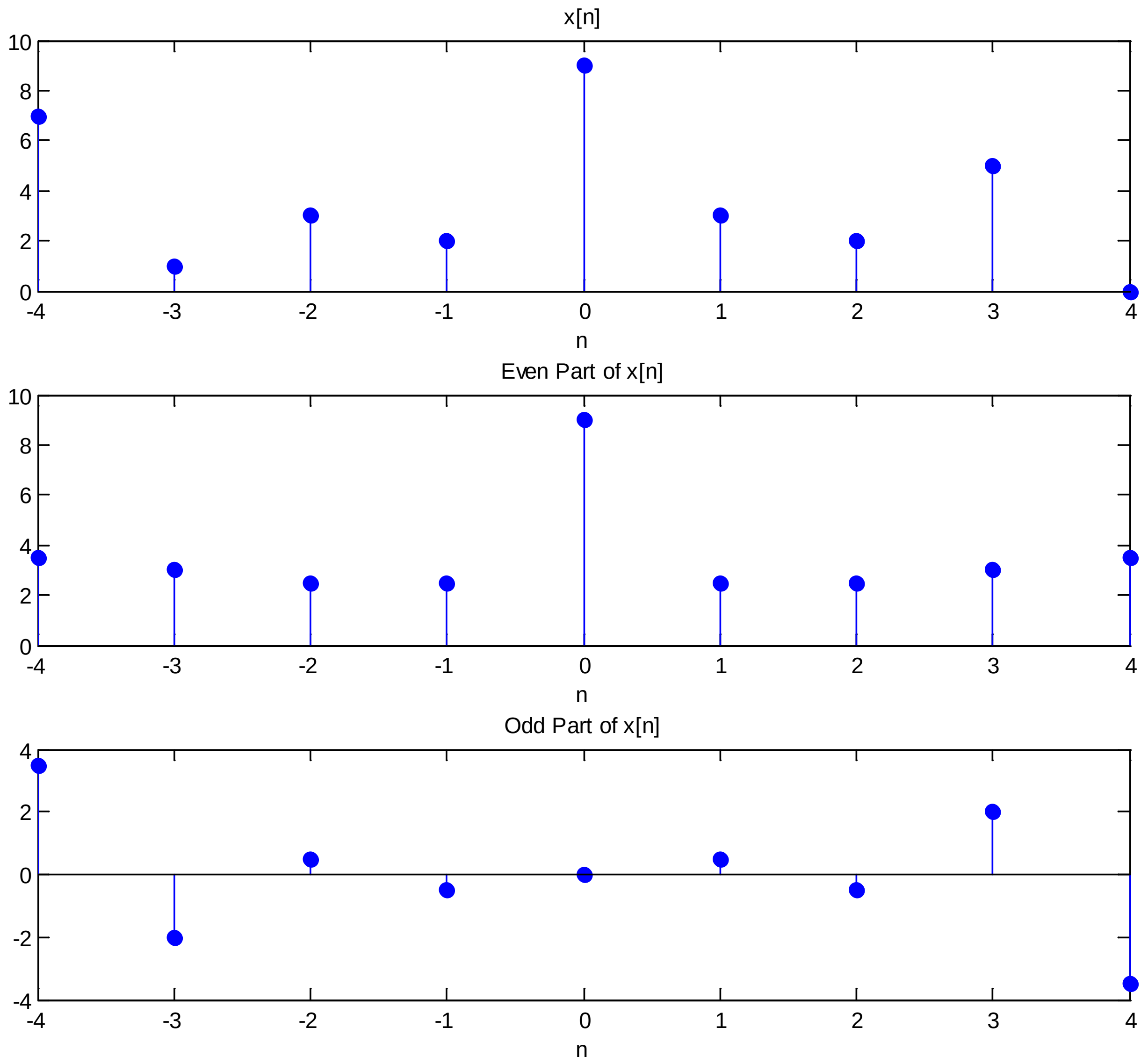


Discrete Example

n	-4	-3	-2	-1	0	1	2	3	4
x[n]	7	1	3	2	9	3	2	5	0
x[-n]	0	5	2	3	9	2	3	1	7
$X_e[n]$	3.5	3	2.5	2.5	9	2.5	2.5	3	3.5
$X_o[n]$	3.5	-2	0.5	-0.5	0	0.5	-0.5	2	-3.5

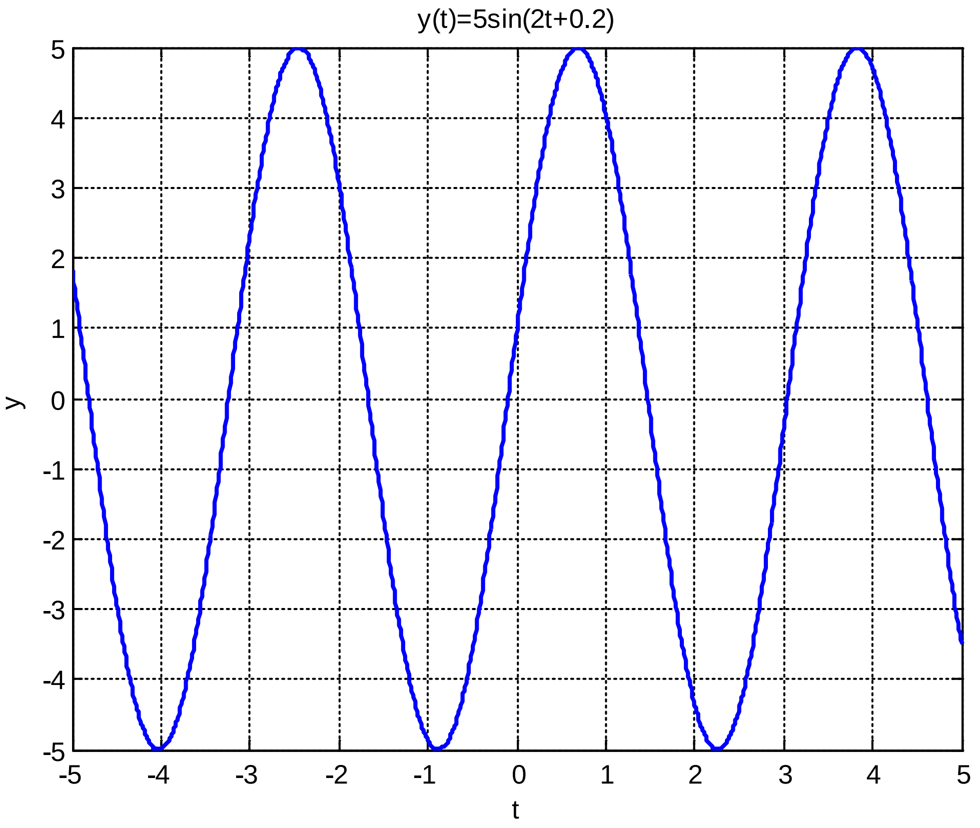


```
close all;clear all;clc
n=-4:4;
x=[7 1 3 2 9 3 2 5 0];
x_even=0.5*(x+fliplr(x));
x_odd=0.5*(x-fliplr(x));
figure
subplot(3,1,1)
stem(n,x,'filled')
title('x[n]')
xlabel('n')
subplot(3,1,2)
stem(n,x_even,'filled')
xlabel('n')
title('Even Part of x[n]')
subplot(3,1,3)
stem(n,x_odd,'filled')
xlabel('n')
title('Odd Part of x[n]')
```



Exponential and Sinusoidal Signals

Sinusoids



$$y(t) = A sin(\omega t + \phi)$$

Amplitude

Radian frequency (rad/s)

Phase offset (radians)

$$\omega = 2\pi f$$

f=frequency (cycles/second)

$$\mathrm{T} = \frac{2\pi}{\omega}$$

T=period (time between periods)

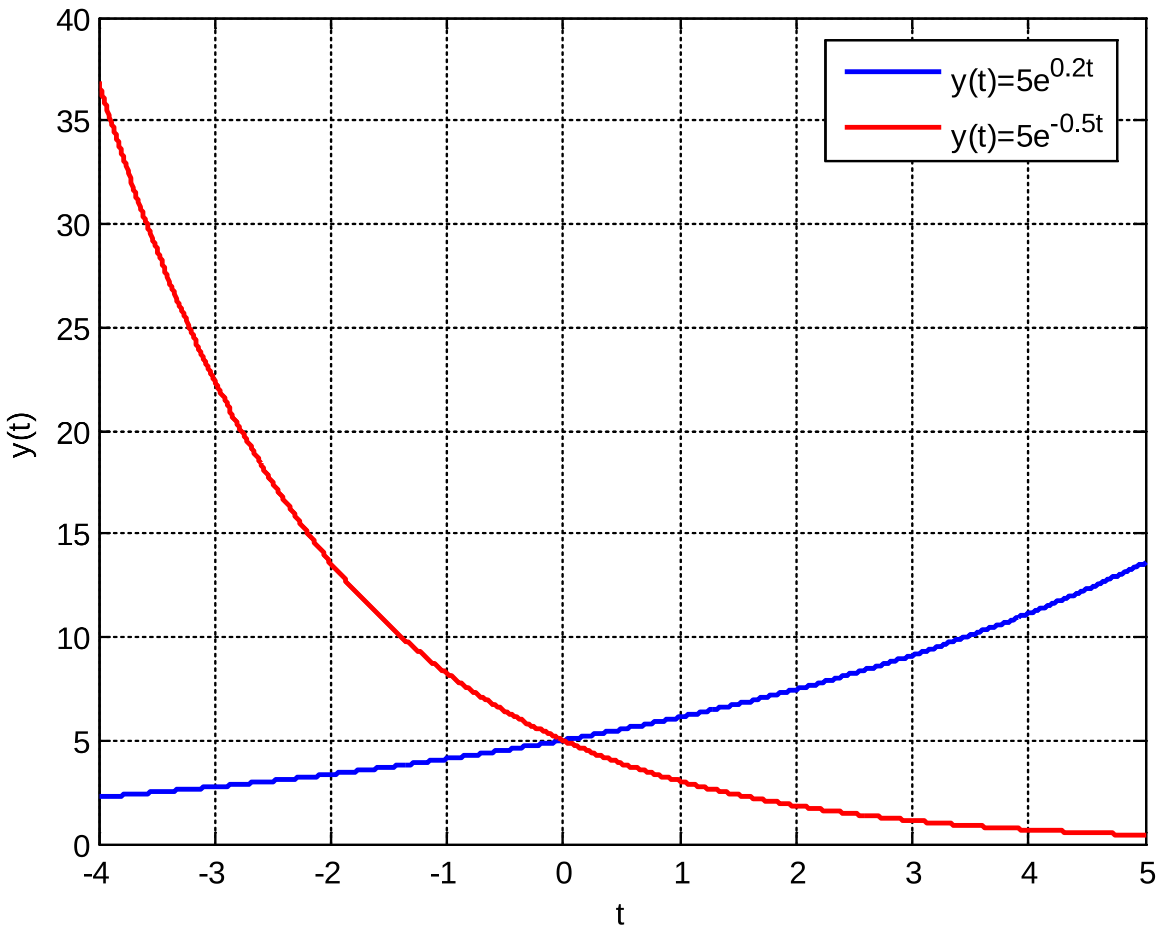
Where Do We Use Sinusoids?

- Acoustics
- Electromagnetics
- Geological effects (e.g., earthquakes)
- Vibrational analysis
- Heat transfer
- Quantum mechanics
- Nuclear reactor neutron flux measurements
- Wave mechanics/modeling
- Plasma instability assessment
- Circuits

Exponential Signals

$$y(t) = Ae^{bt}$$

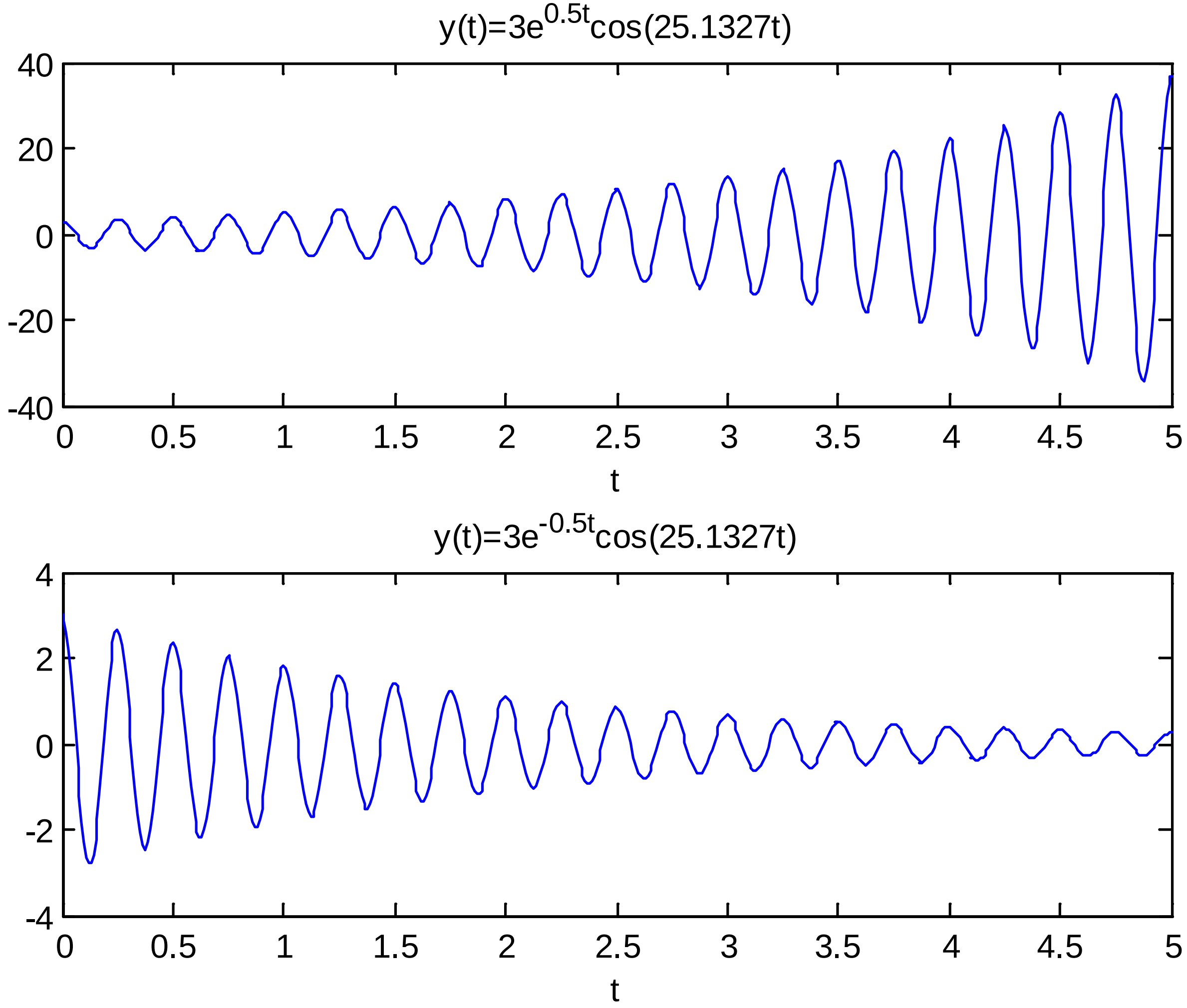
```
%% Exponential Signal Example
close all;clear all;clc
A=5; % Amplitude
b1=0.2;
b2=-0.5;
t=-4:.01:5;
y1=A*exp(b1*t);
y2=A*exp(b2*t);
plot(t,y1,'Linewidth',2)
hold on
plot(t,y2,'r','Linewidth',2)
grid on
xlabel('t')
ylabel('y')
legend('y(t)=5e^{0.2t}','y(t)=5e^{-0.5t}',0)
```



Where Do We Use Exponential Signals?

- Circuits (RC and RL circuits)
- Drug absorption/buildup
- Radioactive Decay
- Radiation Shielding
- RF Attenuation
- Nuclear Reactor Power Growth/Decay
- Controls (with use in eigenvectors)

Exponential Sinusoids $y(t) = ae^{-bt}sin(\omega t + \phi)$



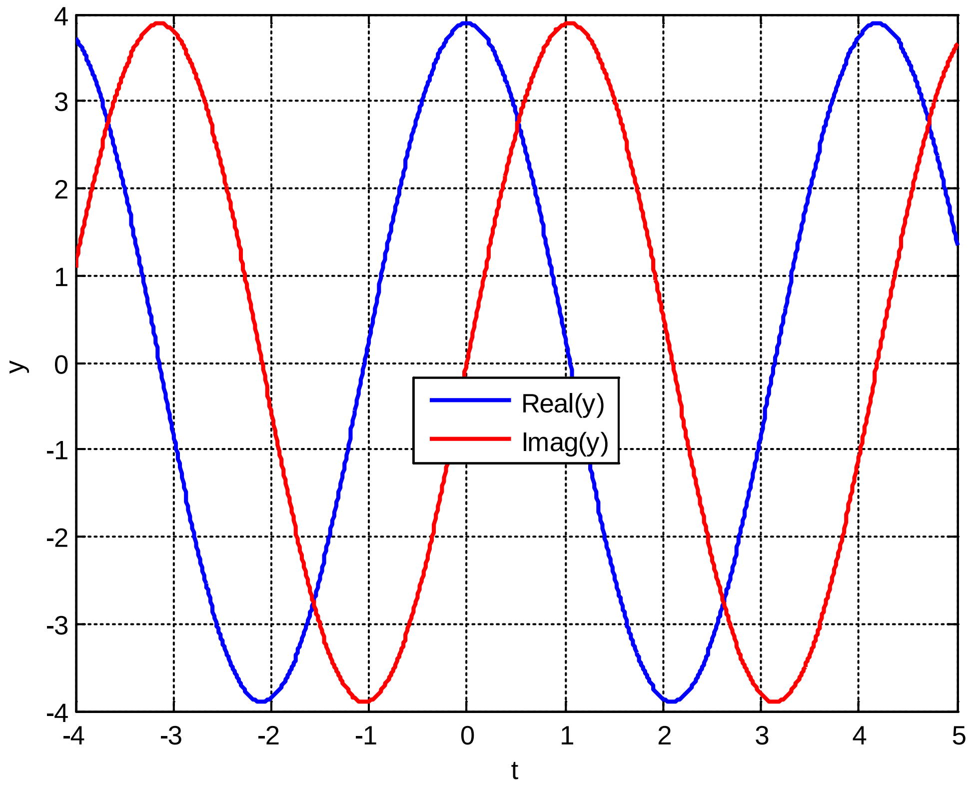
Where Do We Use Exponential Sinusoidal Signals?

- Circuits (RLC circuits)
- Impact analysis/modeling
- Oscillator design/analysis/modeling
- Controls
- Bridge analysis
- Plasma instability analysis

Complex Exponential Signal

$$y(t) = Ae^{j(\omega t+\phi)} = Acos(\omega t + \phi) + jAsin(\omega t + \phi)$$

```
%% Complex Expoenential Signal Example
close all;clear all;clc
A=2; % Amplitude
w=1.5;
t=-4:.01:5;
phi=2/3;
y=A*exp(j*w*t+phi);
plot(t,real(y),'Linewidth',2)
hold on
plot(t,imag(y),'r','Linewidth',2)
grid on
xlabel('t')
ylabel('y')
legend('Real(y)','Imag(y)',0)
```



Where Do We Use Complex Exponential Signals?

- Everywhere sinusoidal and damped sinusoids are used

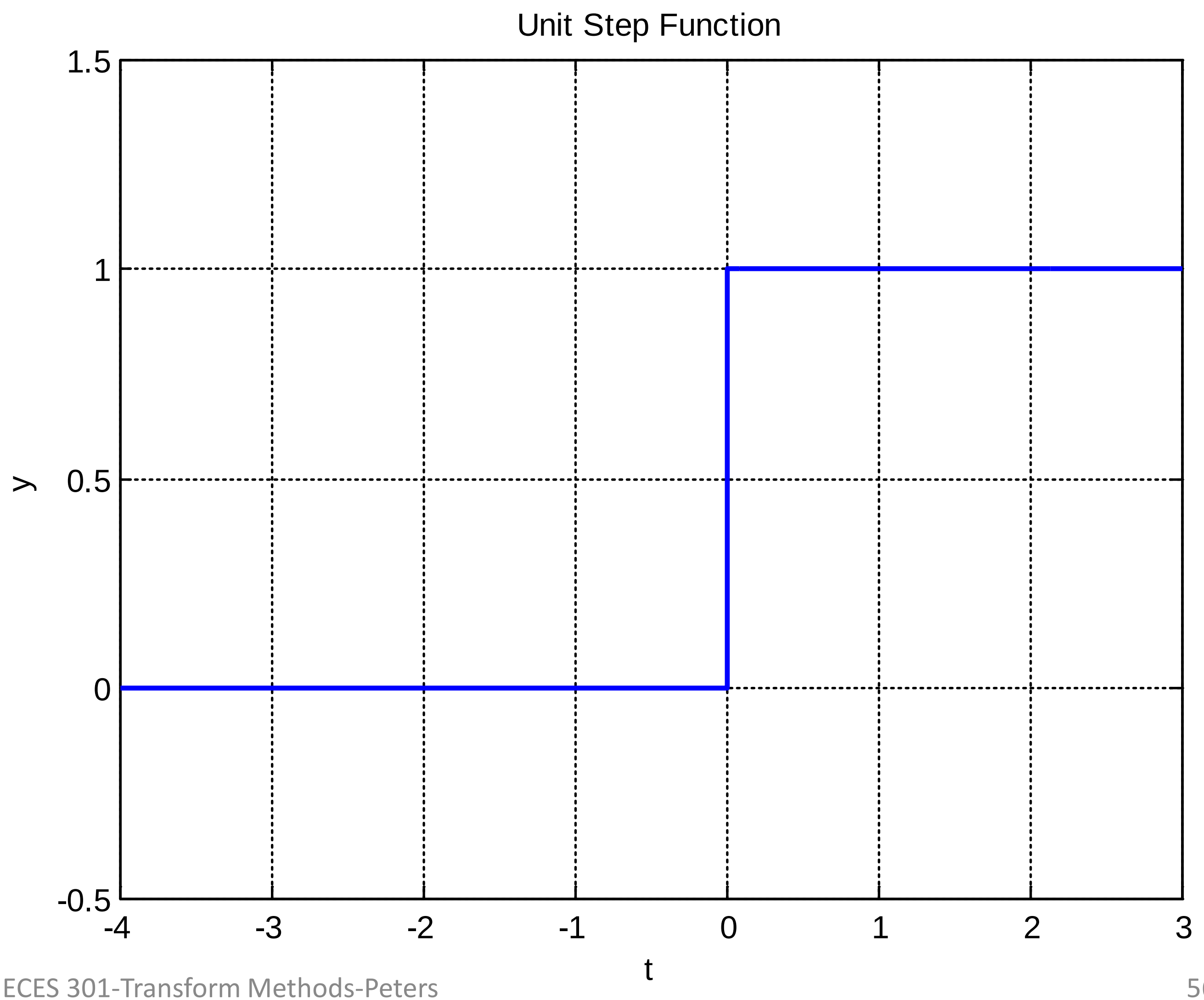
Transforms 2 will be using complex exponential signals for analysis immensely (Laplace and Z transforms)

This class will be using complex, undamped exponential signals for analysis immensely (Fourier Series/Transforms)

Unit Step Function

$$u(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}$$

```
%% Unit Step Signal Example
close all;clear all;clc
t1=-4:.01:0;
t2=0:.01:3;
y1=zeros(size(t1));
y2=ones(size(t2));
t=[t1 t2];
y=[y1 y2];
plot(t,y,'Linewidth',2)
grid on
xlabel('t')
ylabel('y')
title('Unit Step Function')
ylim([-.5 1.5])
```



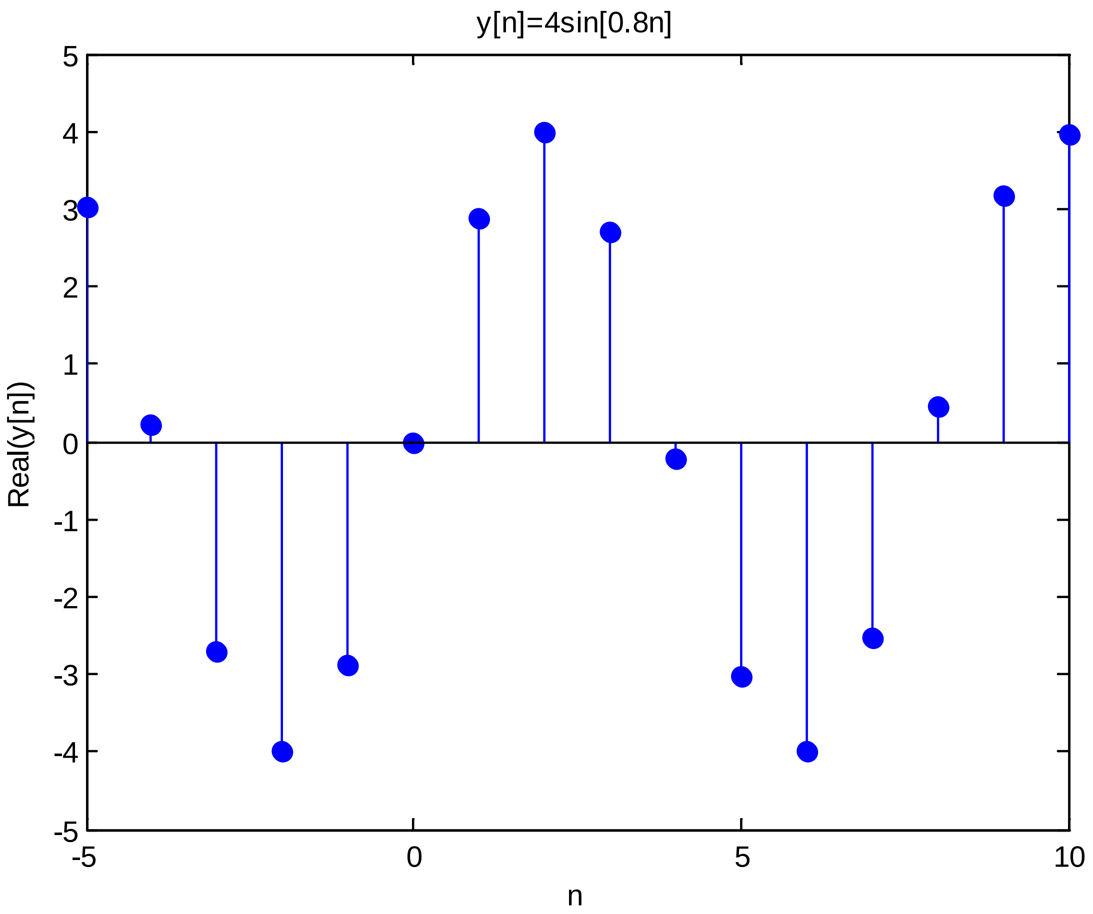
Where Do We Use the Step Function?

- Controls
- Switching
- Real signal modeling/analysis
- Turn on/Turn off behavior

Discrete Sinusoidal Sequence

$y[n] = Asin(\omega n + \phi)$

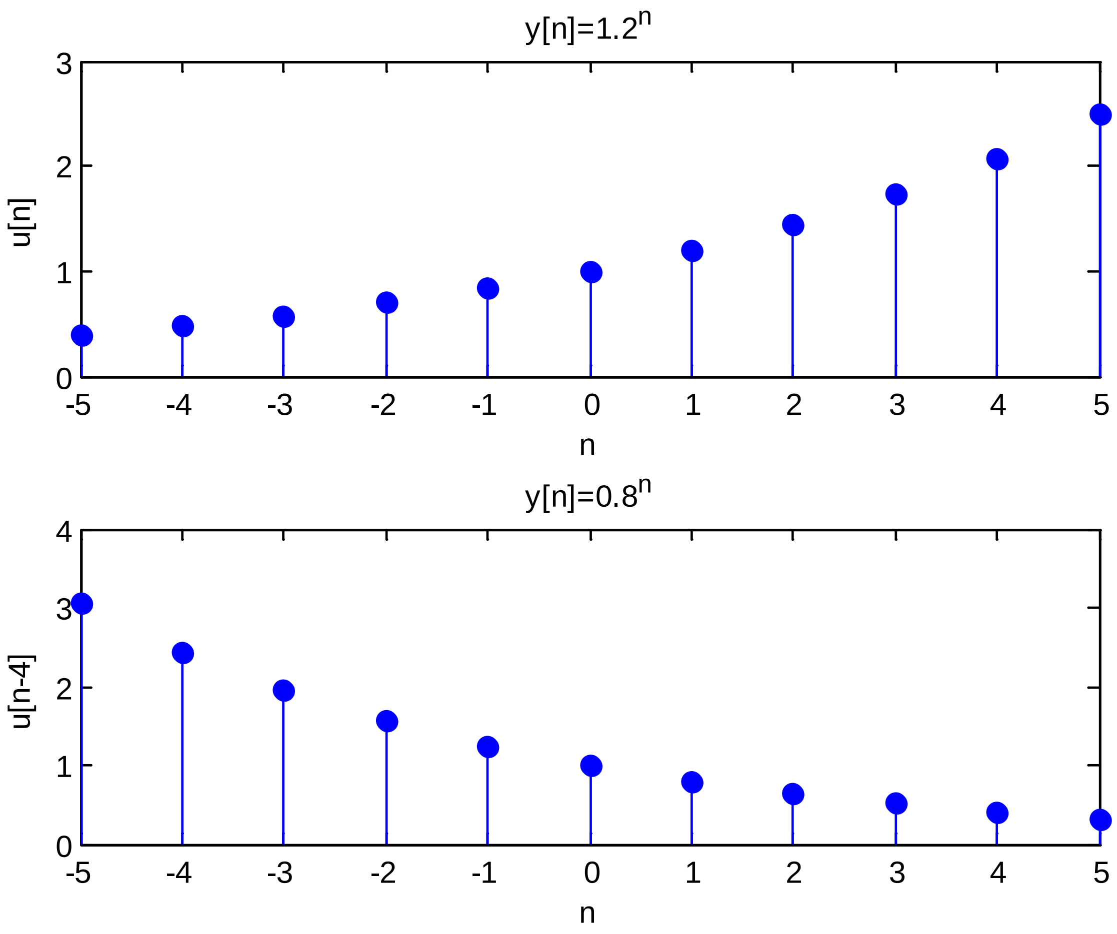
```
%% Discrete Sinusoid Example
close all;clear all;clc
n=-5:10; % Discrete sample number
w=0.8;
y=4*sin(w*n);
figure
stem(n,y,'filled');
xlabel('n')
ylabel('Real(y[n])')
title('y[n]=4sin[0.8n]')
ylim([-5 5])
```



Discrete Real Exponential Sequence

$y[n] = a^n$

```
%% Discrete Unit Exponential Example
close all;clear all;clc
n=-5:5; % Discrete sample number
u1=1.2.^n;
u2=0.8.^n;
figure
subplot(2,1,1)
stem(n,u1,'filled');
xlabel('n')
ylabel('u[n]')
title('y[n]=1.2^n')
subplot(2,1,2)
stem(n,u2,'filled')
xlabel('n')
ylabel('u[n-4]')
title('y[n]=0.8^n')
```

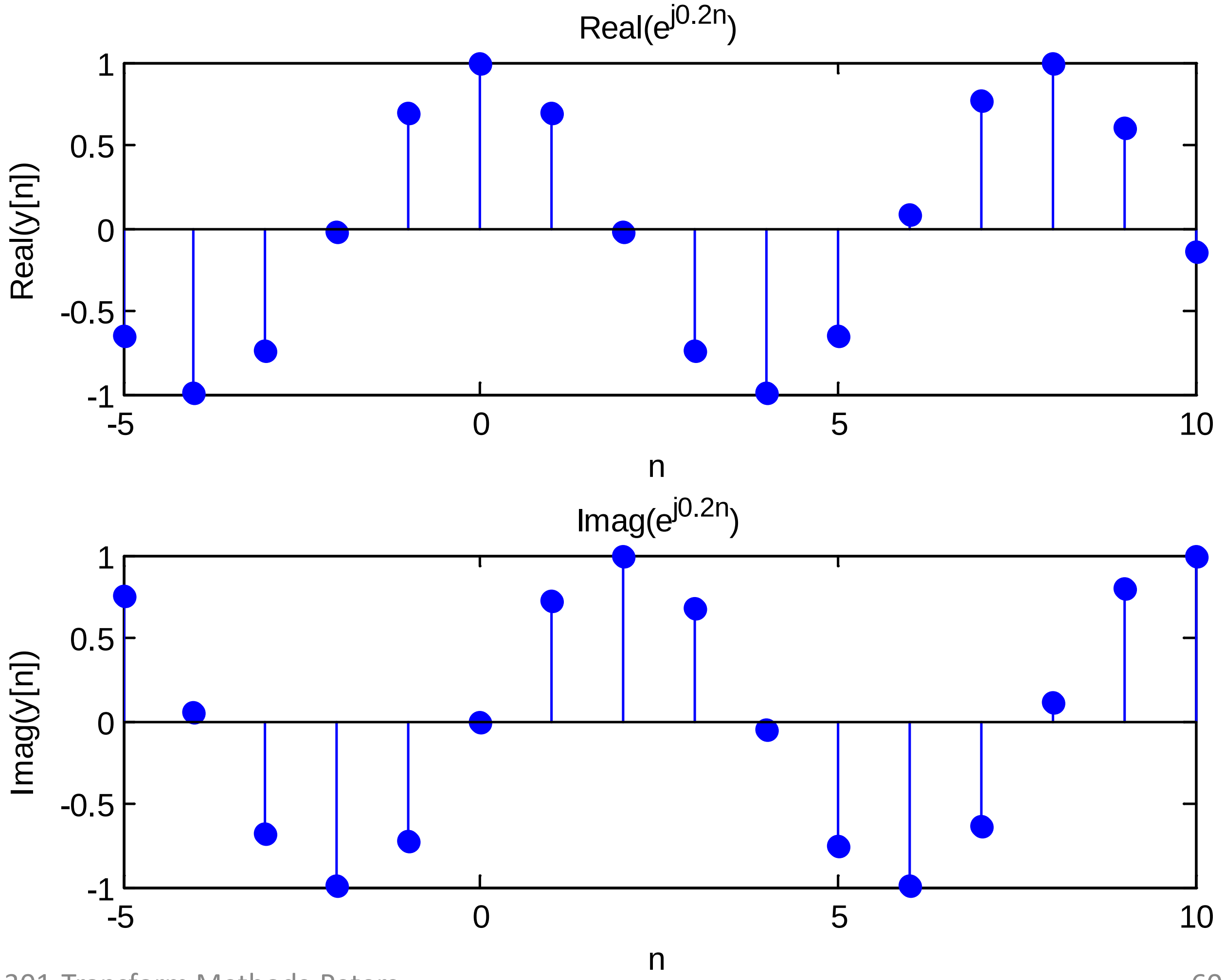


Discrete Complex Exponential Sequence

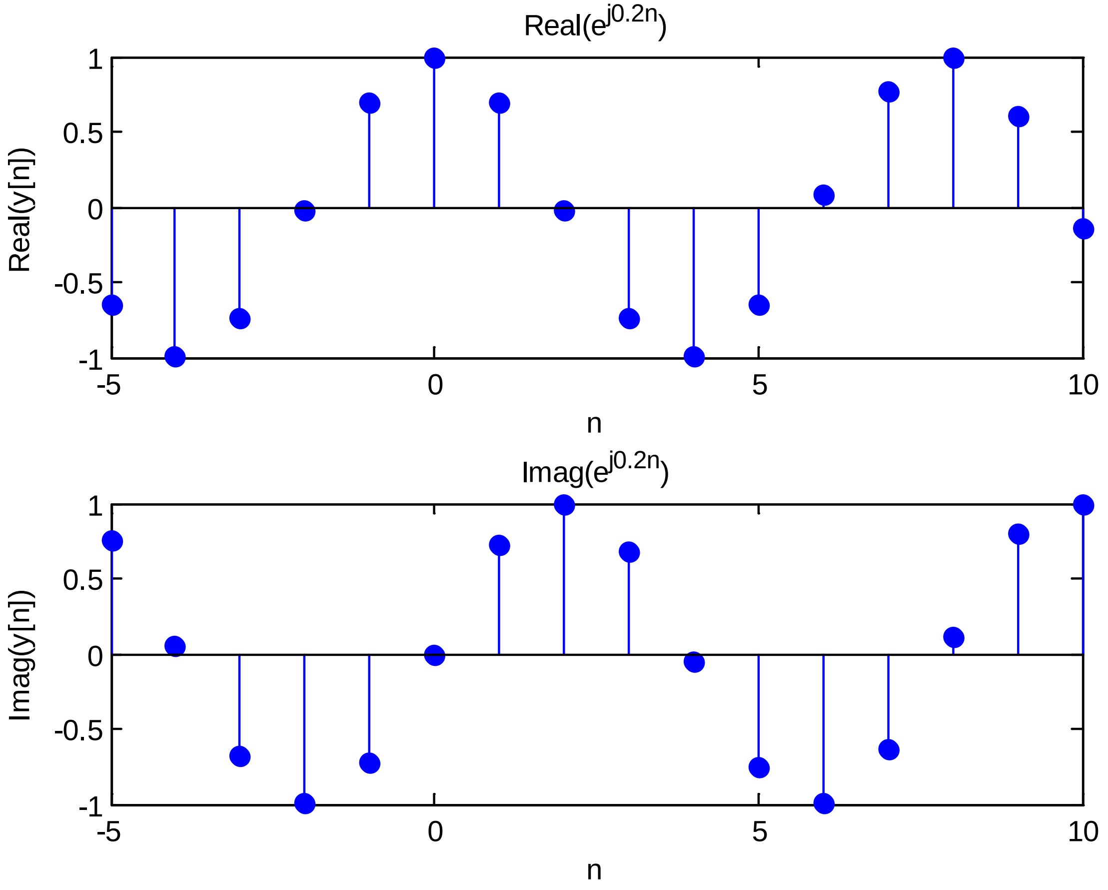
$$y[n] = Ae^{j\omega n} = A\cos(\omega n) + jA\sin(\omega n)$$

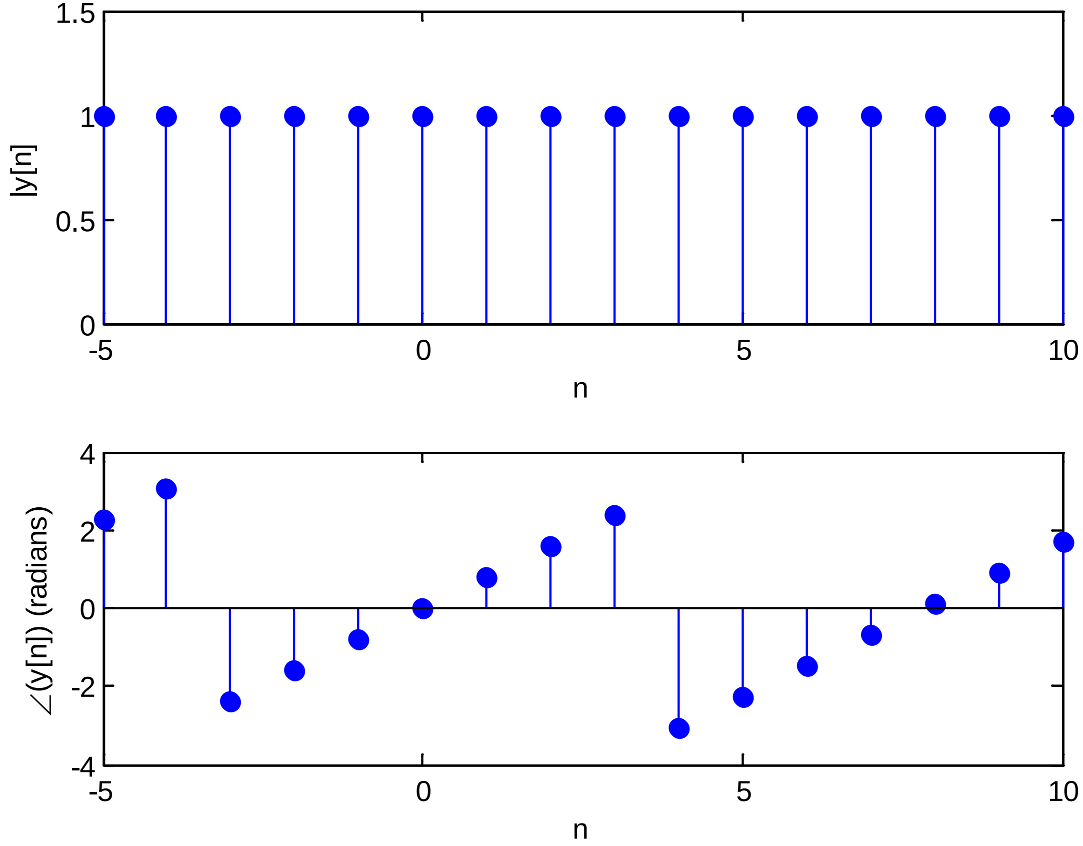
$$|y[n]| = A$$

$$\angle y[n] = \omega n$$



Discrete Complex Exponential Sequence



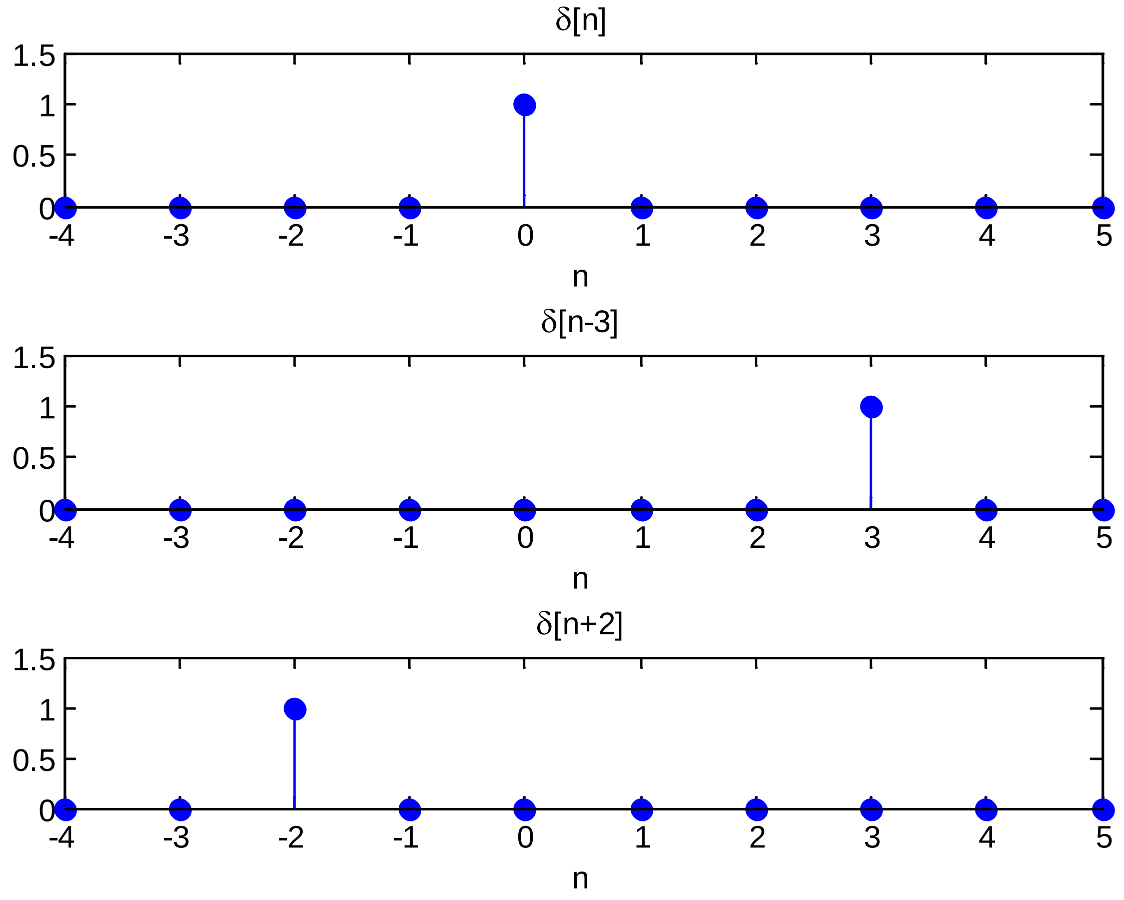


The Unit Impulse and Unit Step Functions

Discrete-Time Unit Impulse

$$\delta[n]\begin{cases}0, n \neq 0\\ 1, n = 0\end{cases}$$

$$\delta[n-k]\begin{cases}0, n \neq k\\ 1, n = k\end{cases}$$

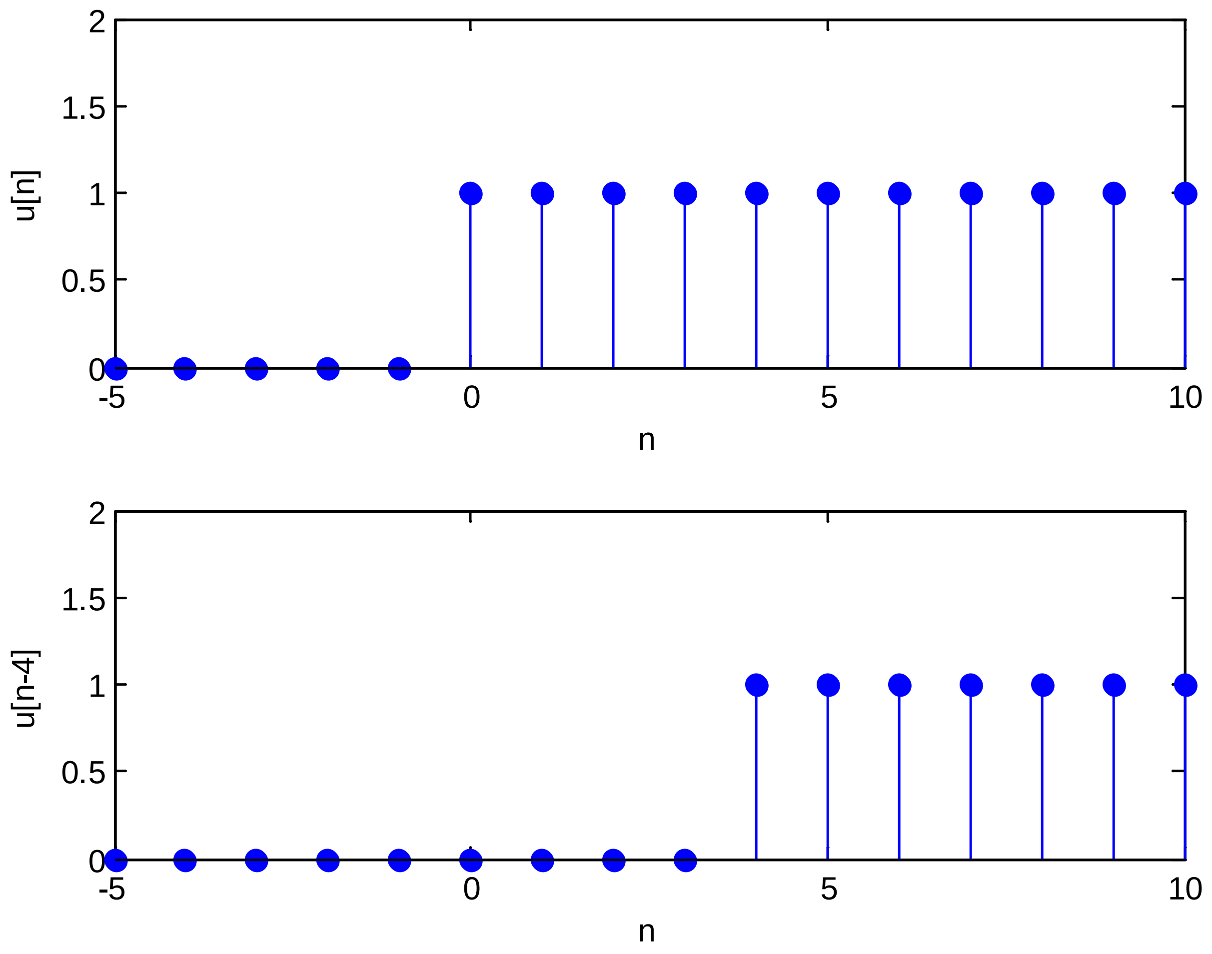


Discrete-Time Unit Step Sequence

$$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$$

$$u[n - n_0] = \begin{cases} 1, n \geq n_0 \\ 0, n < n_0 \end{cases}$$

```
%% Discrete Unit Step Example
n=-5:10; % Discrete sample number
U=zeros(size(n));
u(find(n>=0))=1;
n0=4;
u1=zeros(size(n));
u1(find(n>=n0))=1;
figure
subplot(2,1,1)
stem(n,u,'filled');
xlabel('n')
ylabel('u[n]')
ylim([0 2])
subplot(2,1,2)
stem(n,u1,'filled')
xlabel('n')
ylabel('u[n-4]')
ylim([0 2])
```

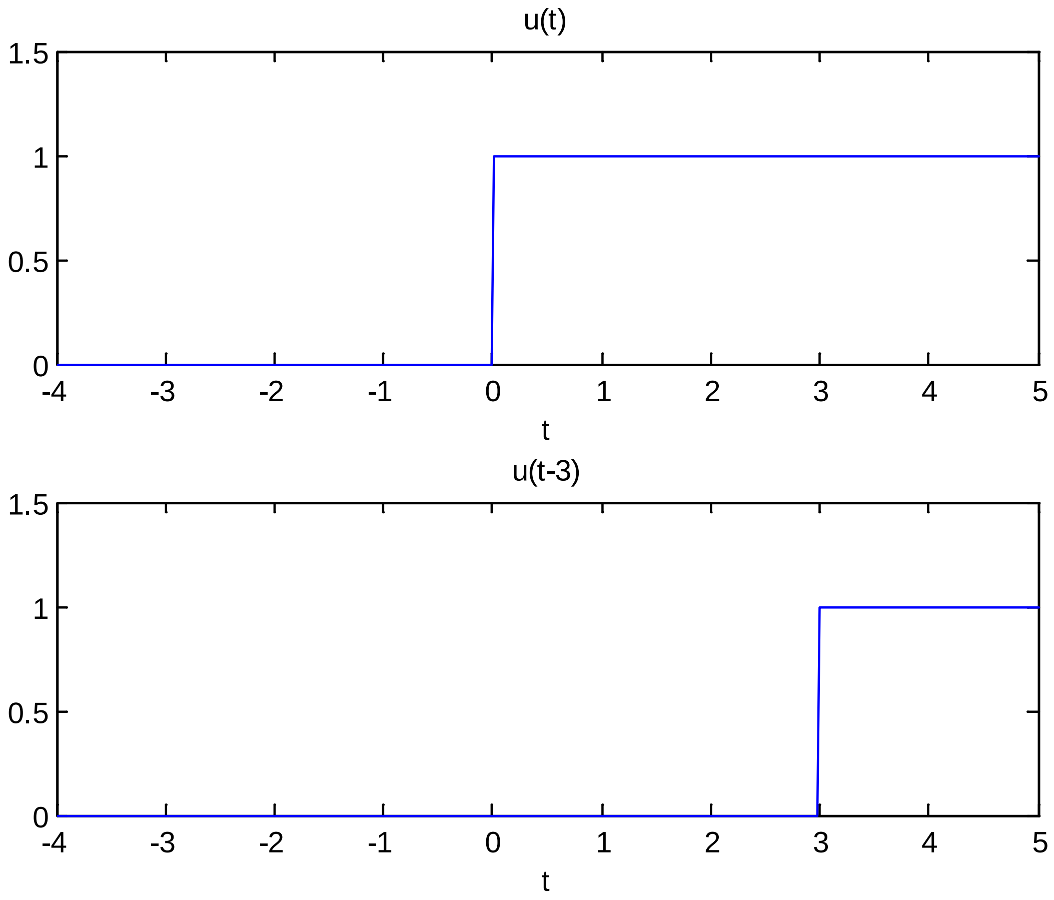


Continuous Time Unit Step Function

$$u(t) = \begin{cases} 1, t \geq 0 \\ 0, n < 0 \end{cases}$$

$$u(t - t_0) = \begin{cases} 1, t \geq t_0 \\ 0, t < t_0 \end{cases}$$

```
close all;clear all;clc
u=@(t) t>=0;
u3=@(t) u(t-3);
figure
subplot(2,1,1)
fplot(u,[-4 5])
xlabel('t')
title('u(t)')
ylim([0 1.5])
subplot(2,1,2)
fplot(u3,[-4 5])
xlabel('t')
title('u(t-3)')
ylim([0 1.5])
```

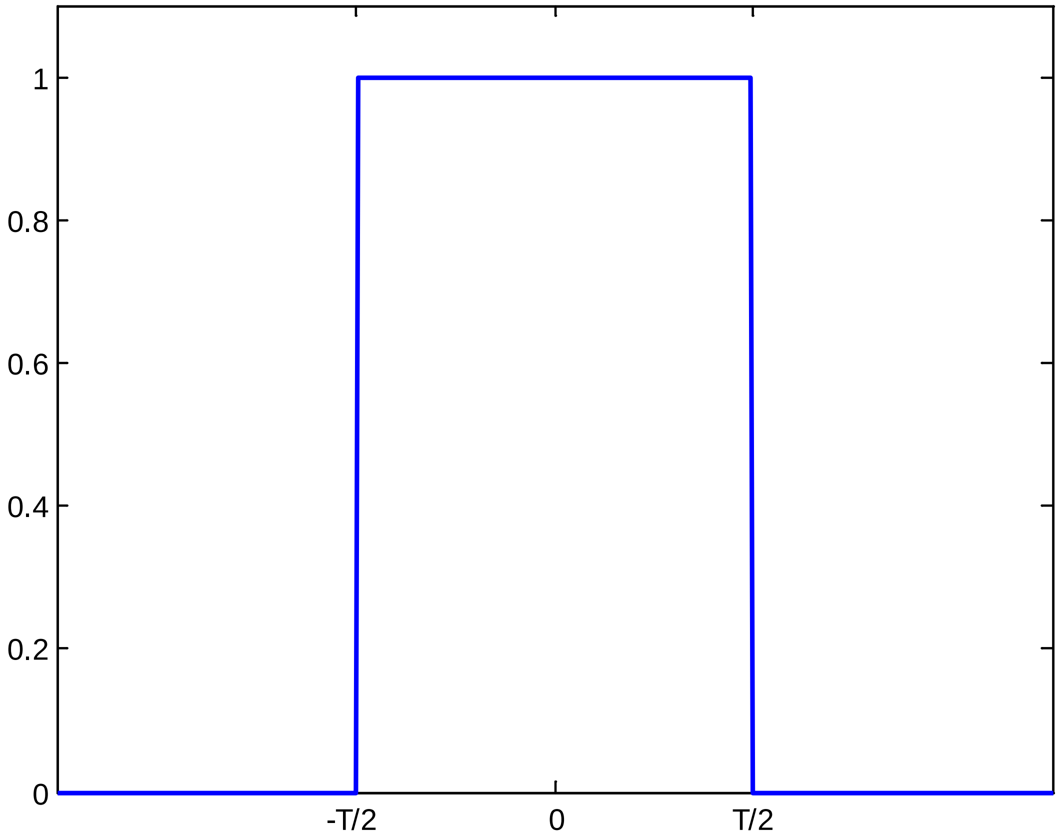


The Rect Function

$$rect(t/T) = \begin{cases} 1, -\frac{T}{2} < t < \frac{T}{2} \\ 0, otherwise \end{cases}$$

$$rect(t/T) = \begin{cases} u(t + T/2) - u(t - \frac{T}{2}) \\ u\left(\frac{T}{2} - t\right) - u(-\frac{T}{2} - t) \\ u\left(t + \frac{T}{2}\right) u(\frac{T}{2} - t) \end{cases}$$

$$rect\left((t - t_0)/T\right) = \begin{cases} 1, t_0 - \frac{T}{2} < t < t_0 + \frac{T}{2} \\ 0, otherwise \end{cases}$$



The Continuous-Time Delta Function

The continuous time delta function $\delta(t)$ is related to the unit step function u(t):

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

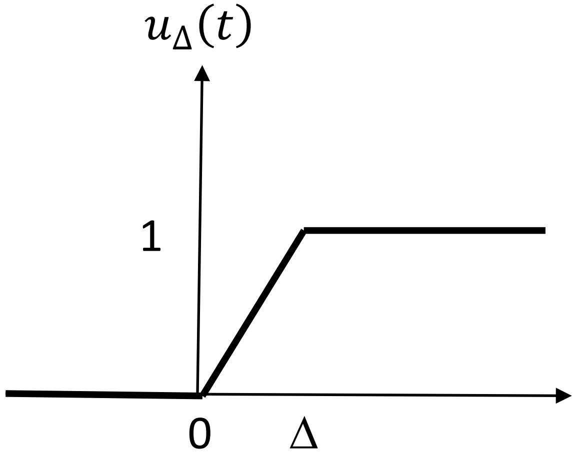
Thus,

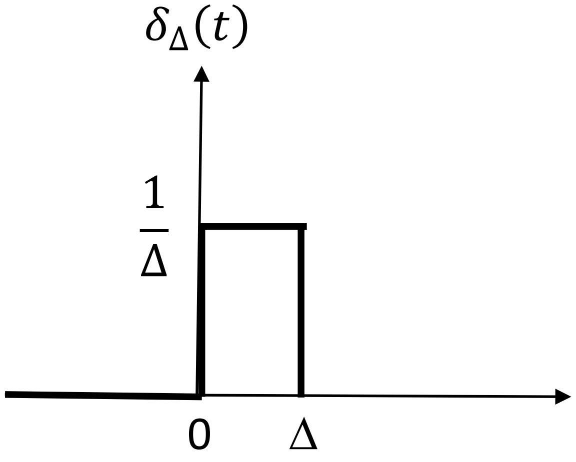
$$\delta(t) = \frac{du(t)}{dt}$$

How do we approach this?

The Continuous-Time Delta Function

We approach the understanding of the delta function as unit step modified with a ramp of width Δ



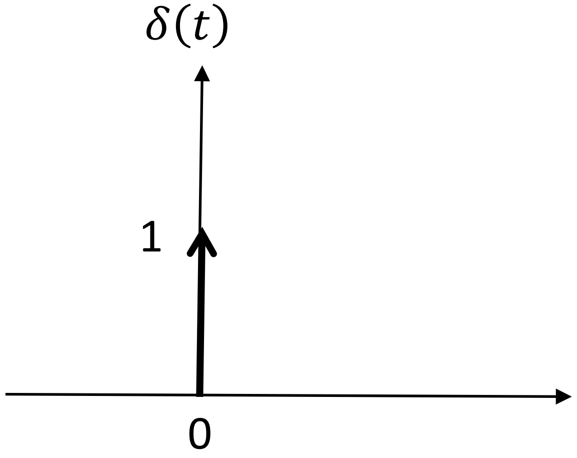


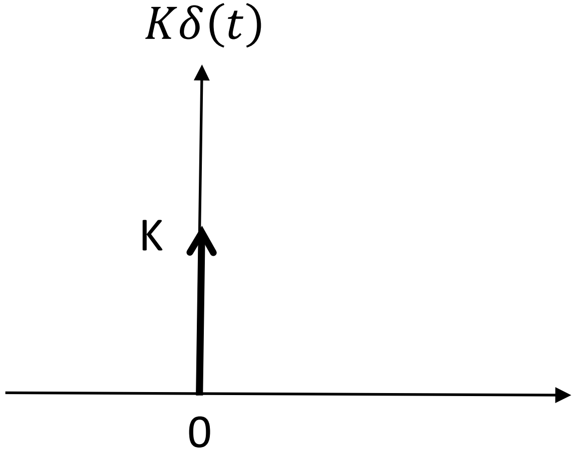
$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$

The area underneath $\delta_\Delta(t)$ is always 1

The Continuous-Time Delta Function

As $\Delta \rightarrow 0$, $\delta_\Delta(t) \rightarrow \delta(t)$





Properties of The Continuous-Time Delta Function

$$x(t)\delta(t) = x(0)\delta(t)$$

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

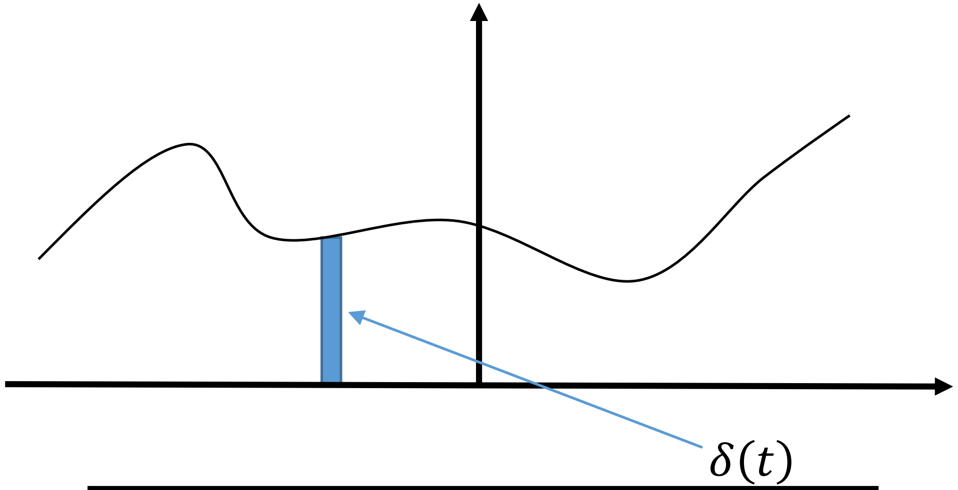
$$\int_{-\infty}^{\infty} x(t - t_0)\delta(t)\, dt = \int_{-\infty}^{\infty} x(t)\delta(t - t_0)\, dt = x(t_0)$$

The Delta Function for Continuous Functions

Consider a function x(t). We would like to create a function that can be used to represent any point in the function. Call it the delta function, $\delta(t)$

We can envision a rectangular pulse of width Δ, and height $1/\Delta$. The area of that pulse is one. Now, if the limit of Δ approaches zero, the height must increase, but the total area stays the same.

If Δ is small, and multiply x(t) by $\delta(t)$, we get a piecewise approximation of the function. Thus, integration will yield the value of x(t) where the delta function is located.



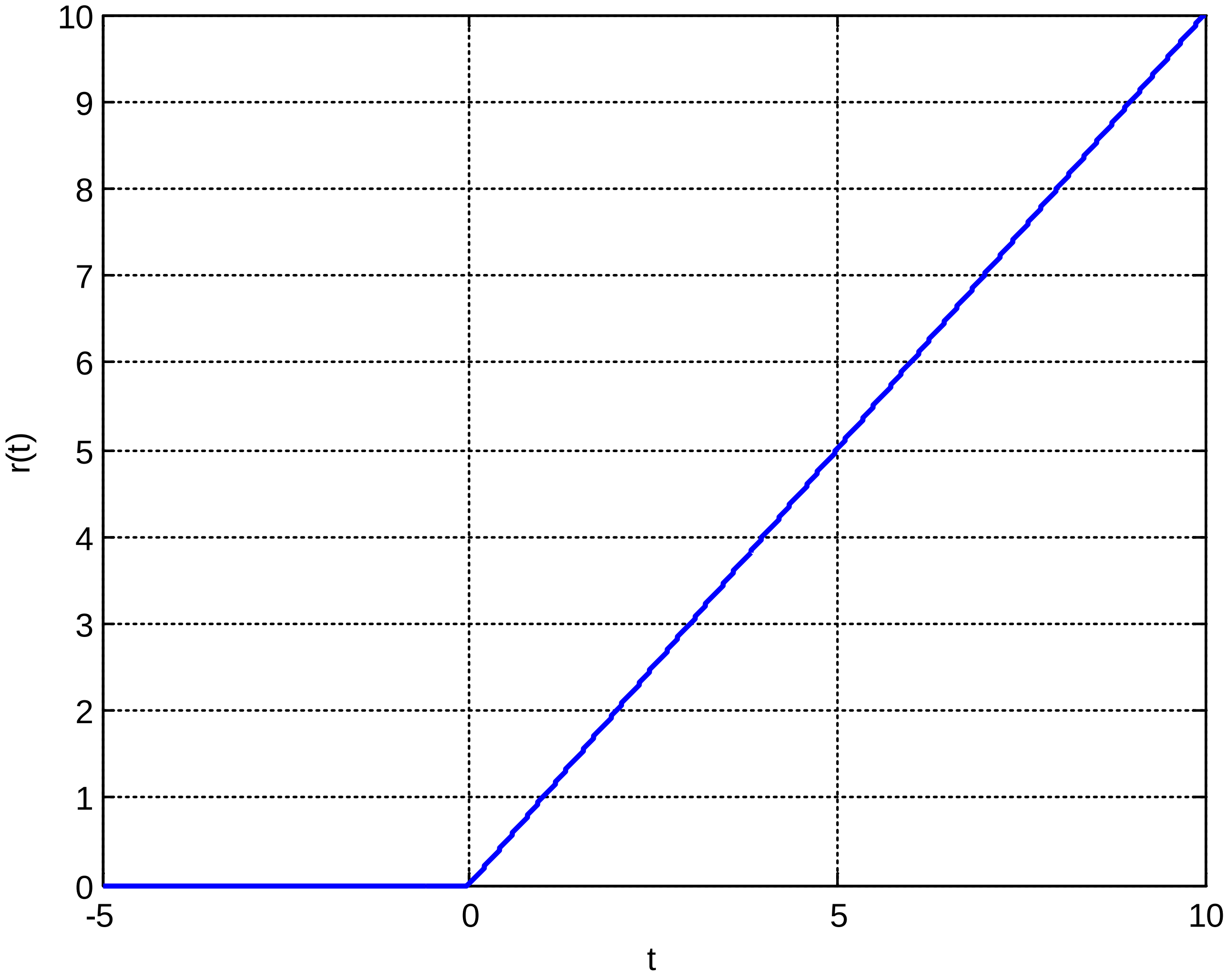
$$\delta(t) = \begin{cases} \frac{1}{\Delta}, 0 \le t < \Delta \\ 0, otherwise \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Ramp Function

$$r(t) = t \cdot u(t) = \begin{cases} t, t \geq 0 \\ 0, t < 0 \end{cases}$$

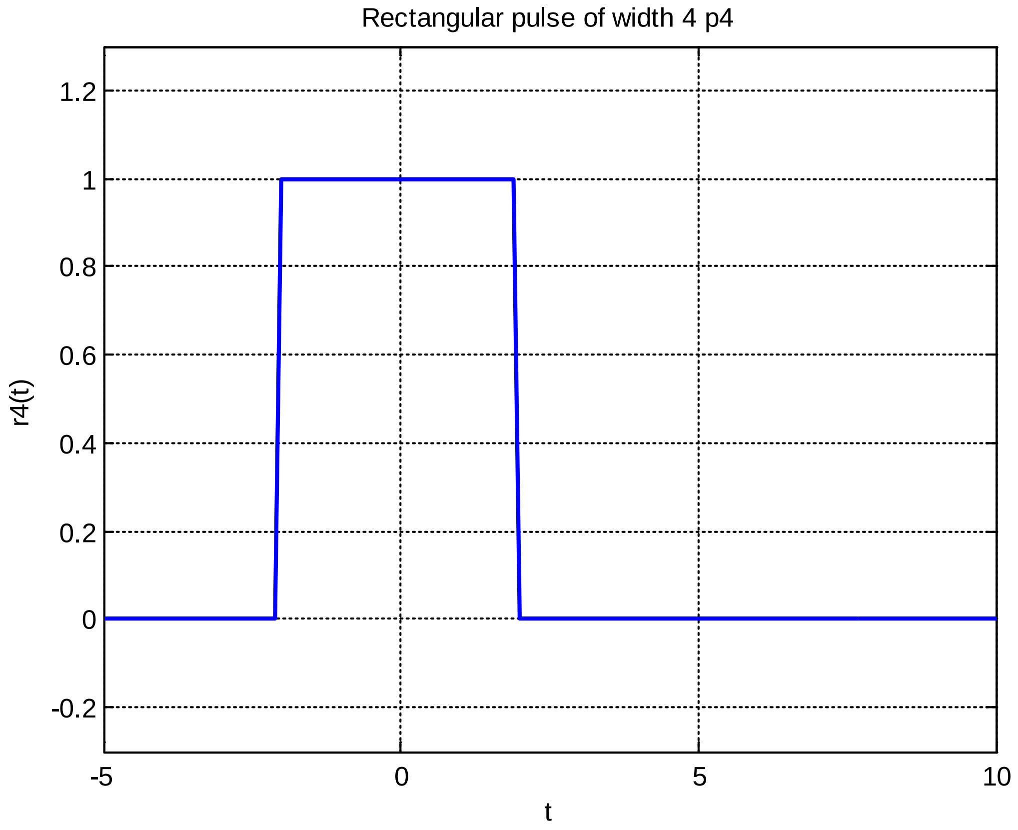
$$r(t - t_0) = (t - t_0)\mathrm{u}(t - t_0) = \begin{cases} t - t_0, t \geq t_0 \\ 0, t < t_0 \end{cases}$$



Rectangular Pulse

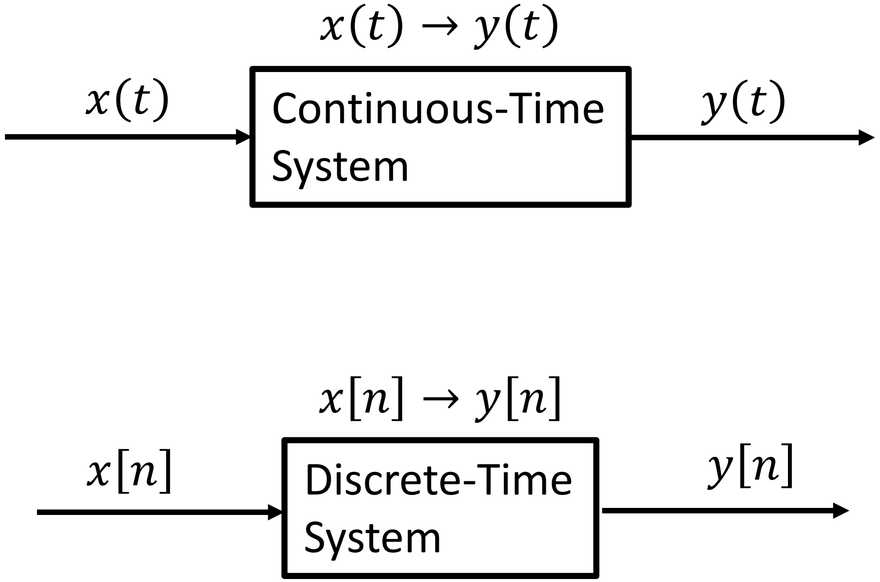
$$pT = u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right) = \begin{cases} 1, -T/2 \leq t \leq T/2 \\ 0, otherwise \end{cases}$$

```
%% Unit Step Signal Example
t=-5:.1:10;
s=rectpuls(t,4);
plot(t,s,'linewidth',2);
ylim([-0.3 1.3])
grid on
xlabel('t')
ylabel('r4(t)')
title('Rectangular pulse of width 4 p4')
```



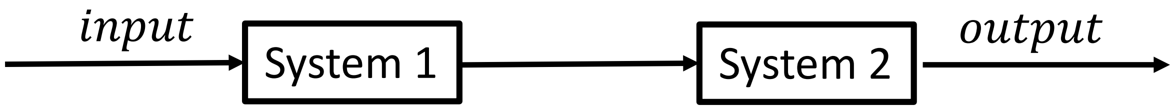
Properties of Systems

Input/Output Relationship of Systems

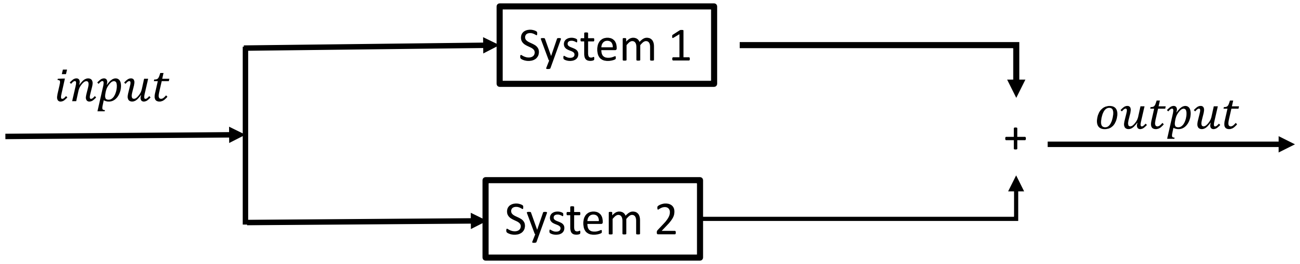


Input/Output Relationship of Systems

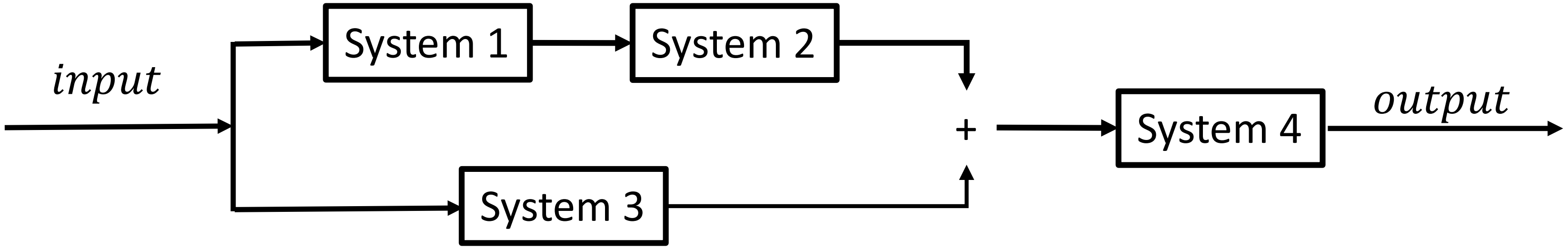
Series/cascaded connection



Parallel connection



Series-parallel connection



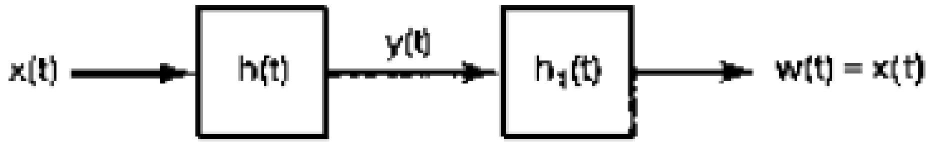
Memory

A system has memory if it relies on past and present values of the input signal x(t)

A system is *memoryless* if it relies only on present values of the input signal x(t)

Invertibility of Systems

A system is invertible if and only if there exists a system such that when the two systems are cascaded, the output is the same as the input. Thus, the overall impulse response of the cascaded system must be $\delta[n]$ for discrete systems and $\delta(t)$ for continuous systems



(a)

Example of Invertible System

Consider the following system:

$$y[n] = 3x[n]$$

Recall that the cascaded system must be able to return the original signal. It should be obvious here the inverse of this system is $y_{inv}[n] = \frac{1}{3}x[n]$

In other words, for a system to be invertible, there must be a one-to-one mapping of the input/output relationship. Distinct inputs lead to distinct outputs

Example of Non-Invertible System

Consider the following system:

$$y[n] = x^2[n]$$

This system is not invertible. Let's try $y_{inv}[n] = \sqrt{x[n]}$. The result of this could have either a positive or negative value (assuming the input is positive). Since there is no a one-to-one mapping of the input/ouput relationship, the system is non-invertible

Causal Systems

A system is *causal* IF and ONLY IF the system uses only past and present values of the input signal

Example: The sound of a doorbell cannot start until someone pushes the door bell itself.

This is perhaps the most important type of signals in this class. Why?

Non-Causal Systems

A system is *noncausal* if and only if there is at least one nonzero value in the input signal before time t=0 (continuous) and n=0 (discrete). NOTE: in this case, there are nonzero values after t=0 (or n=0).

Example of where used: determining the distance a target is when using radar

Can a non-causal signal exist in the real world?

Anti-Causal Systems

A system is anti-*causal* IF and ONLY IF the input signal is nonzero for t>0 (continuous) or n>0 (discrete)

Time-Invariance

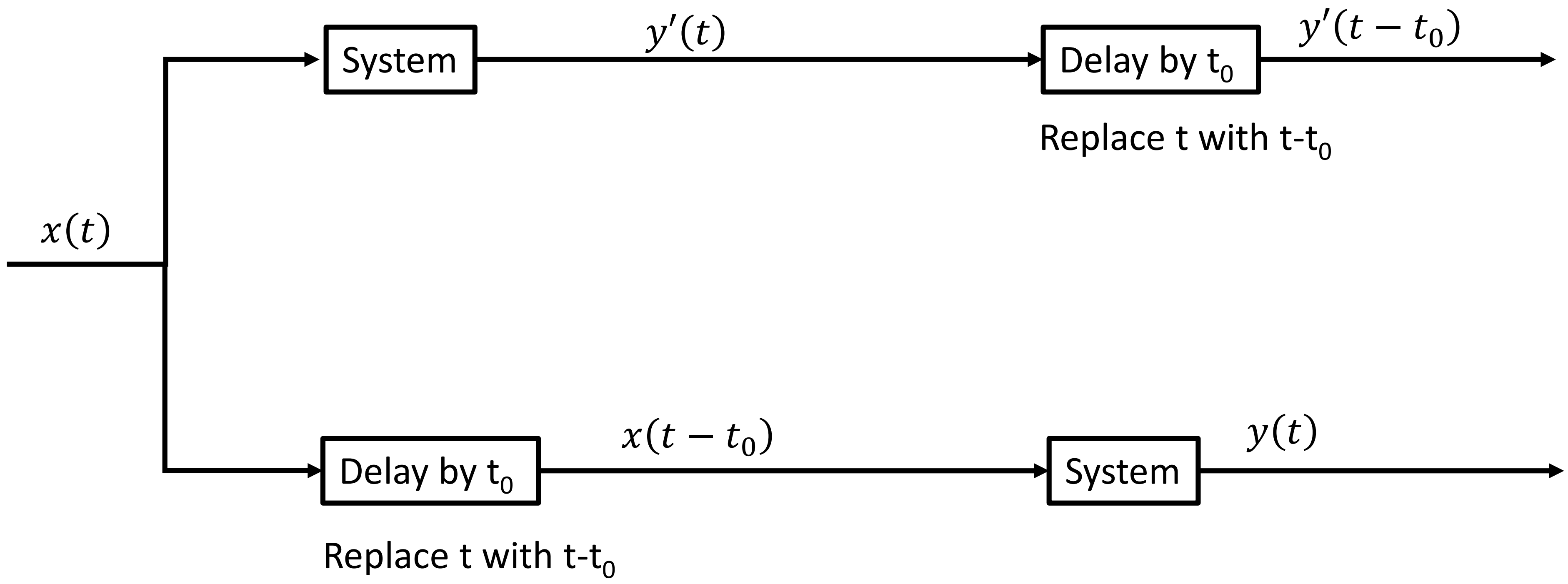
A system is time-invariant if, when excited with a delayed input signal $x(t-t_0)$ or $x[n-n_0]$, produces a delayed version of the unshifted output $y(t-t_0)$ or $x[n-n_0]$. In other words,

$$x(t-t_0) \to y(t-t_0)$$

$$x[n-n_0] \to y[n-n_0]$$

How do we test for time-invariance/time-variance?

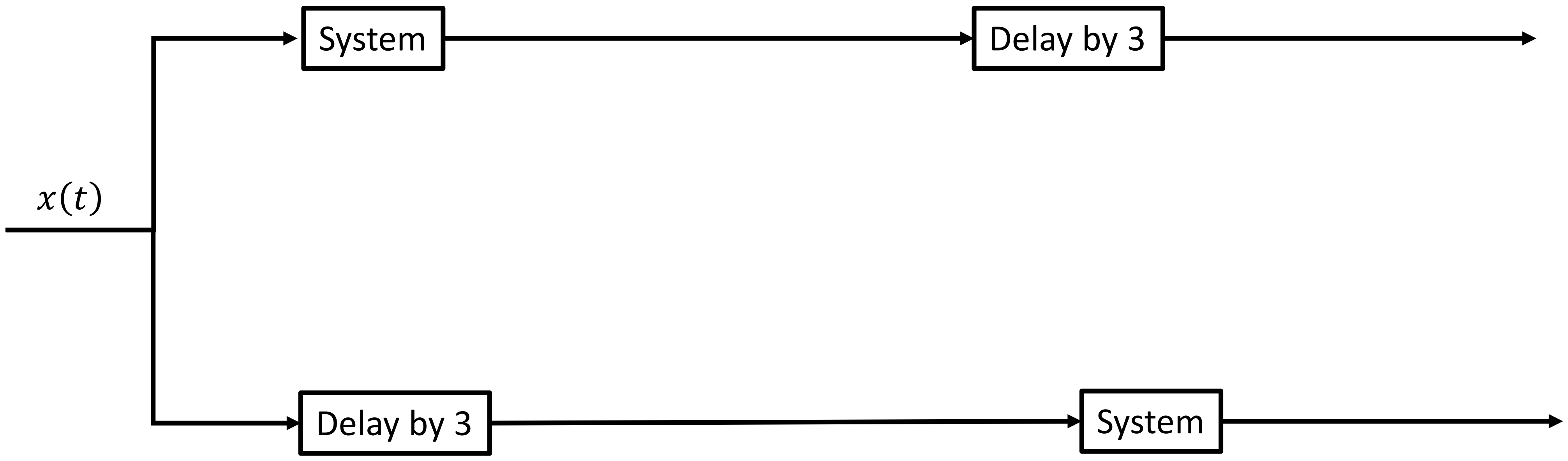
Time-Invariance Test



If the results of both legs are equal, then the system is time-invariant to the input signal

Time-Invariance Test Example 1

Suppose the response of a system to an input signal is $y(t) = te^{-t}x(t)$. Determine if the system is time-invariant by using the input signal $x(t) = u(t) - u(t-5)$



Time-Invariance Test Example 1

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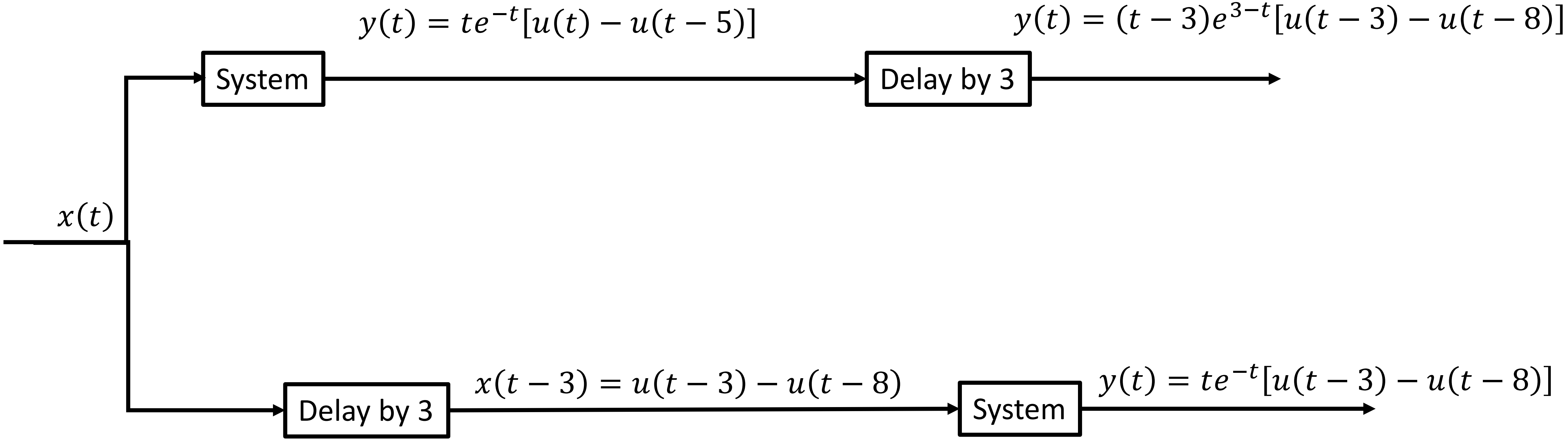
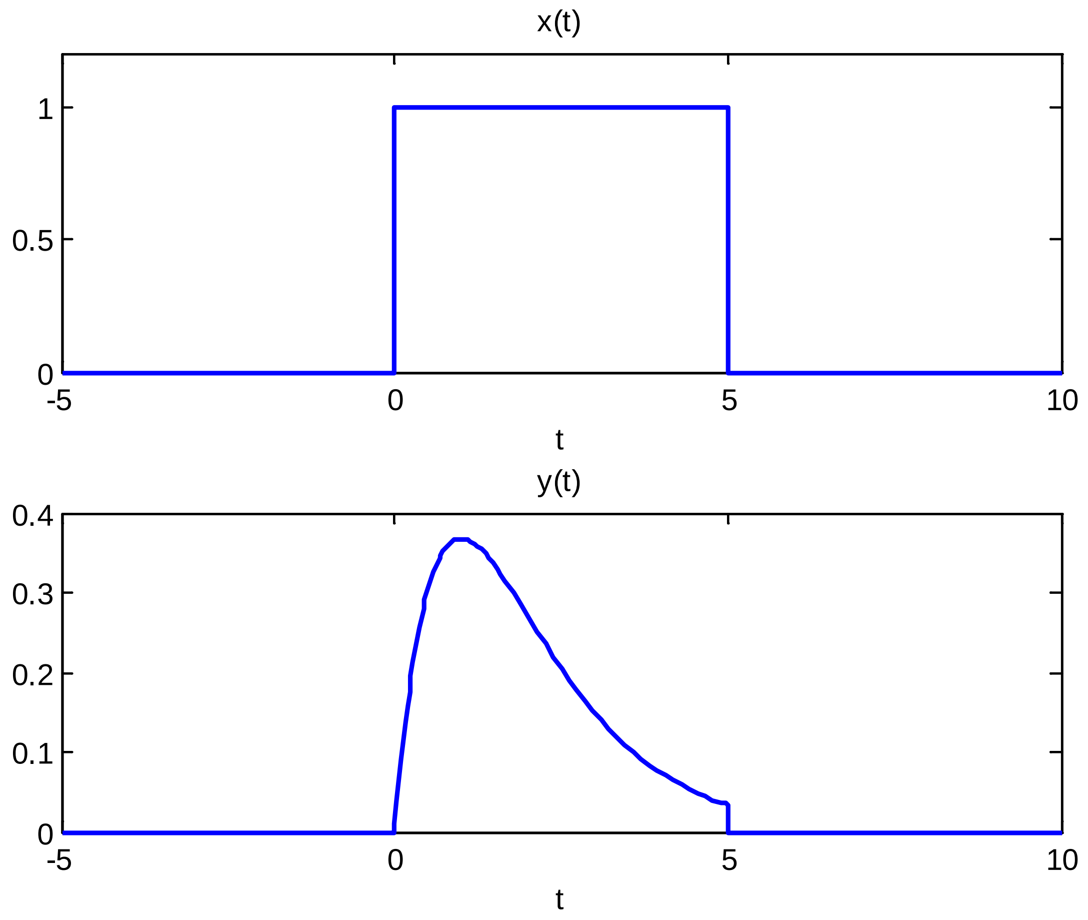


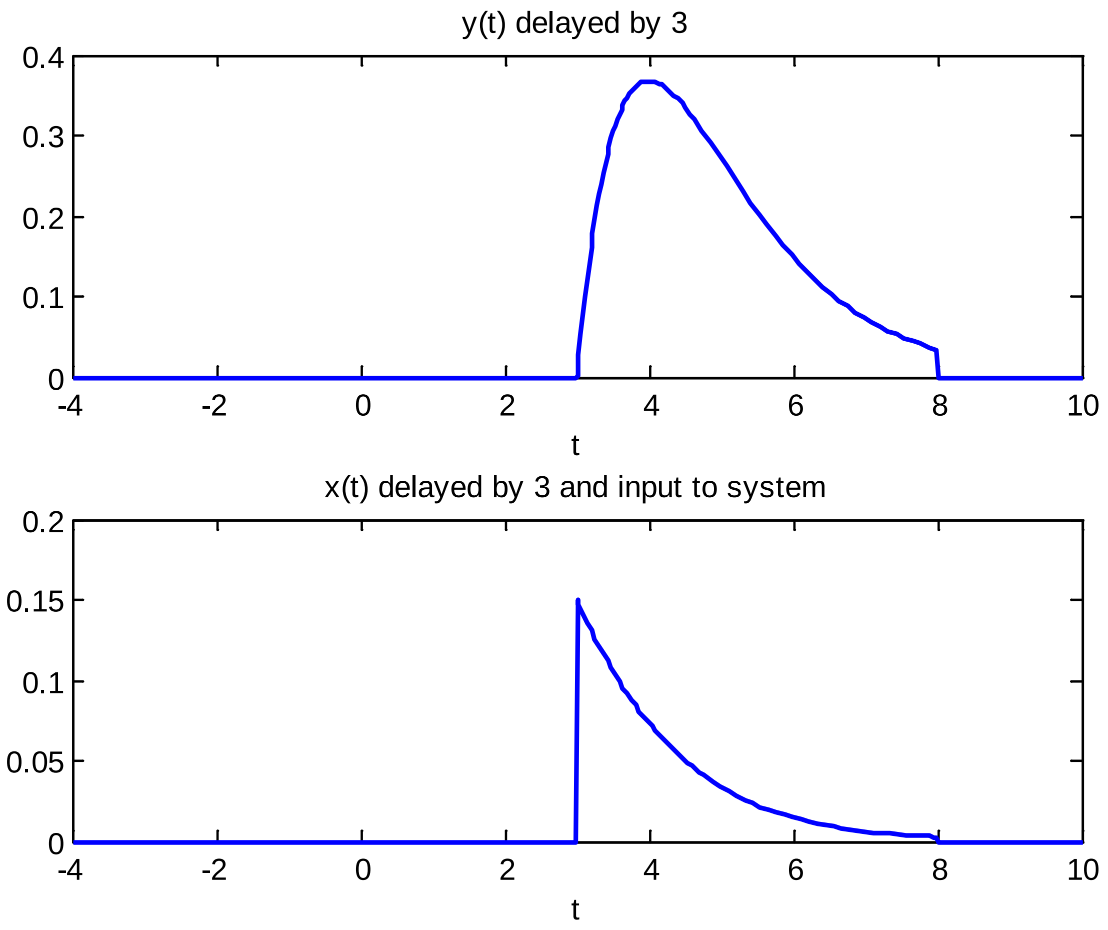
Time-Invariance Test Example 1

```
close all;clear all;clc
u=@(t) t>0;
x=@(t) u(t)-u(t-5);
y=@(t) t*exp(-t)*x(t);
figure
subplot(2,1,1)
fplot(x,[-5 10])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('x(t)')
ylim([0 1.2])
subplot(2,1,2)
fplot(y,[-5 10])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('y(t)')
```



Time-Invariance Test Example 1

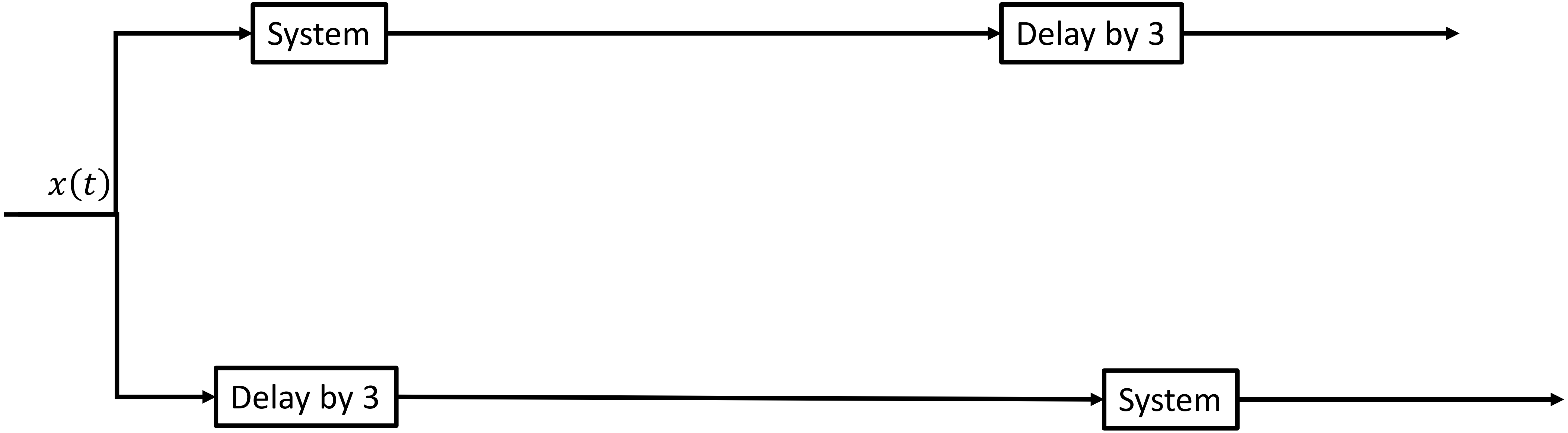
```
%% Delay y by 3
y_delayed=@(t) y(t-3);
%% Delay input by 3 and then put through system
x_delayed=@(t) x(t-3);
y_delayed1=@(t) t*exp(-t)*x_delayed(t);
%% Plot Results
figure
subplot(2,1,1)
fplot(y_delayed,[-4 10])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('y(t) delayed by 3')
subplot(2,1,2)
fplot(y_delayed1,[-4 10])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('x(t) delayed by 3 and input to system')
```



Because the two results are not the same, the system is time-variant

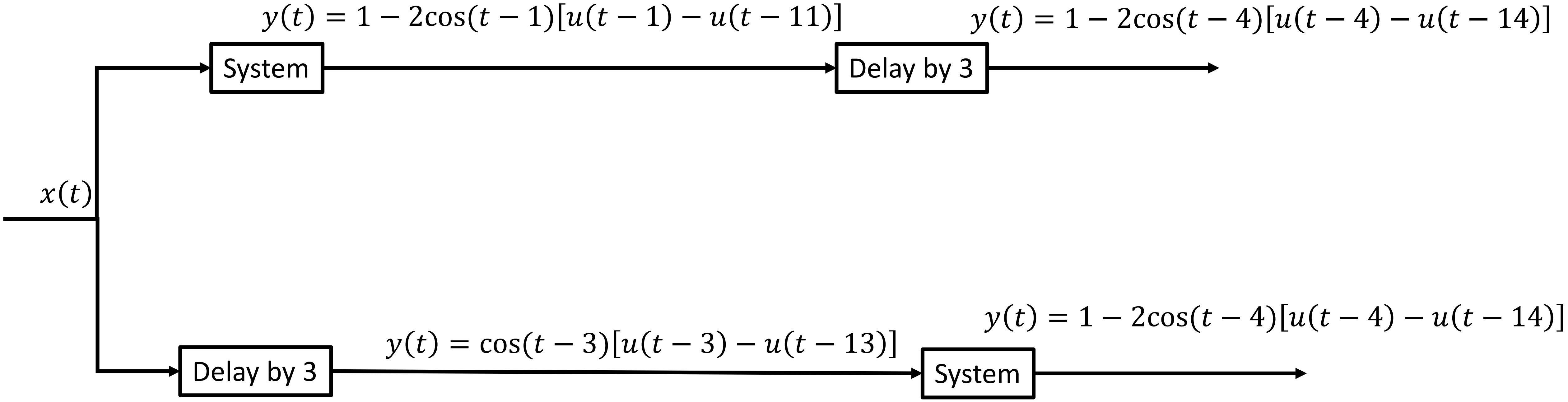
Time-Invariance Test Example 2

Suppose the response of a system to an input signal is $y(t) = 1 - 2x(t-1)$. Determine if the system is time-invariant by using the input signal $x(t) = cos(t)[u(t) - u(t-10)]$



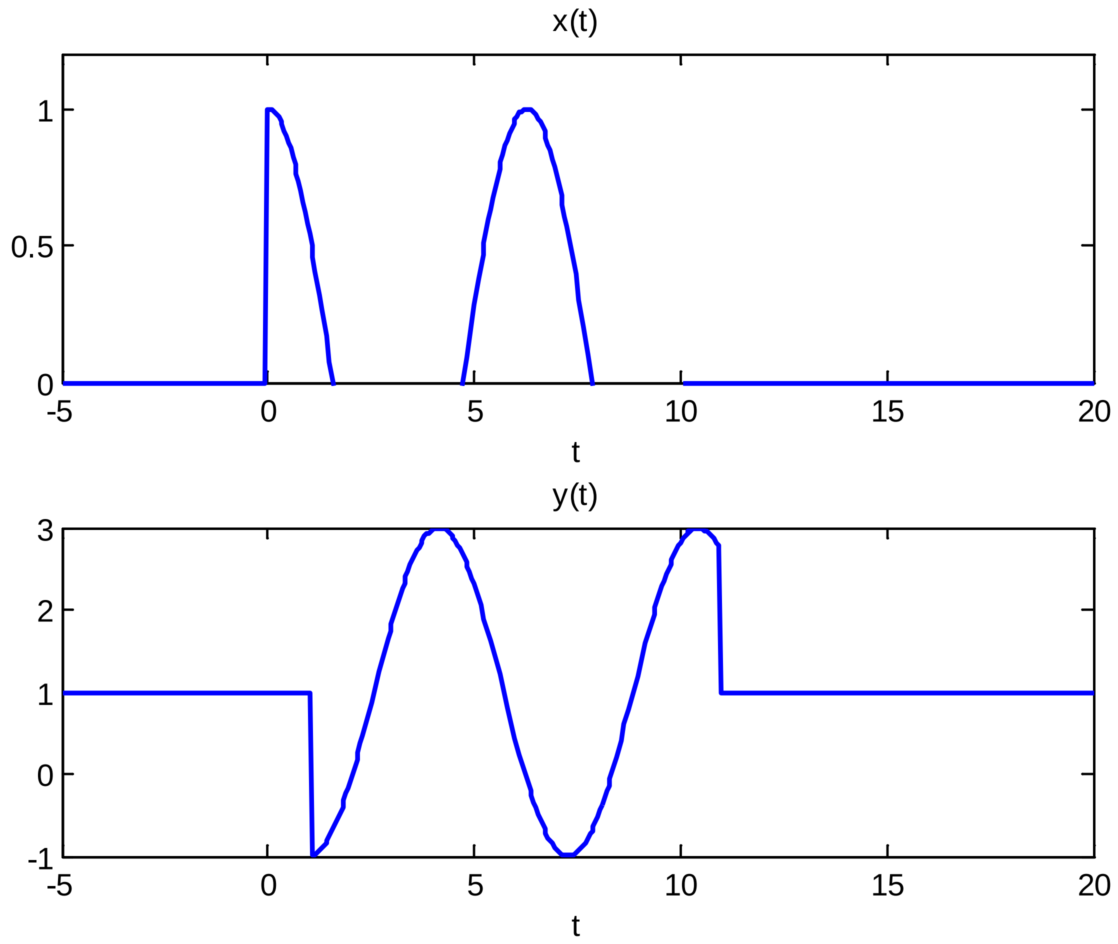
Time-Invariance Test Example 2

Suppose the response of a system to an input signal is $y(t) = 1 - 2x(t-1)$. Determine if the system is time-invariant by using the input signal $x(t) = cos(t)[u(t) - u(t-10)]$



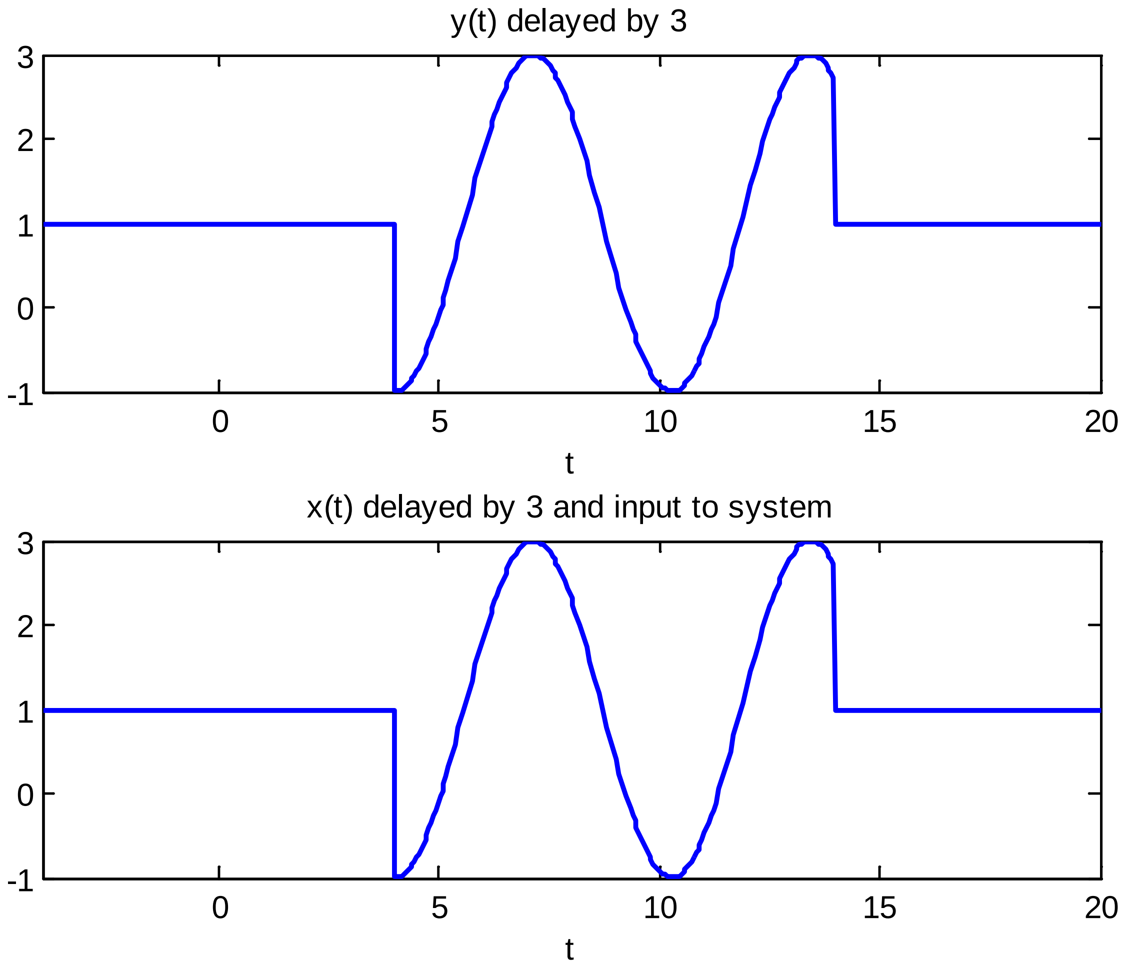
Time-Invariance Test Example 2

```
close all;clear all;clc
u=@(t) t>0;
x=@(t) cos(t)*(u(t)-u(t-10));
y=@(t) 1-2*x(t-1);
figure
subplot(2,1,1)
fplot(x,[-5 20])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('x(t)')
ylim([0 1.2])
subplot(2,1,2)
fplot(y,[-5 20])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('y(t)')
```



Time-Invariance Test Example 2

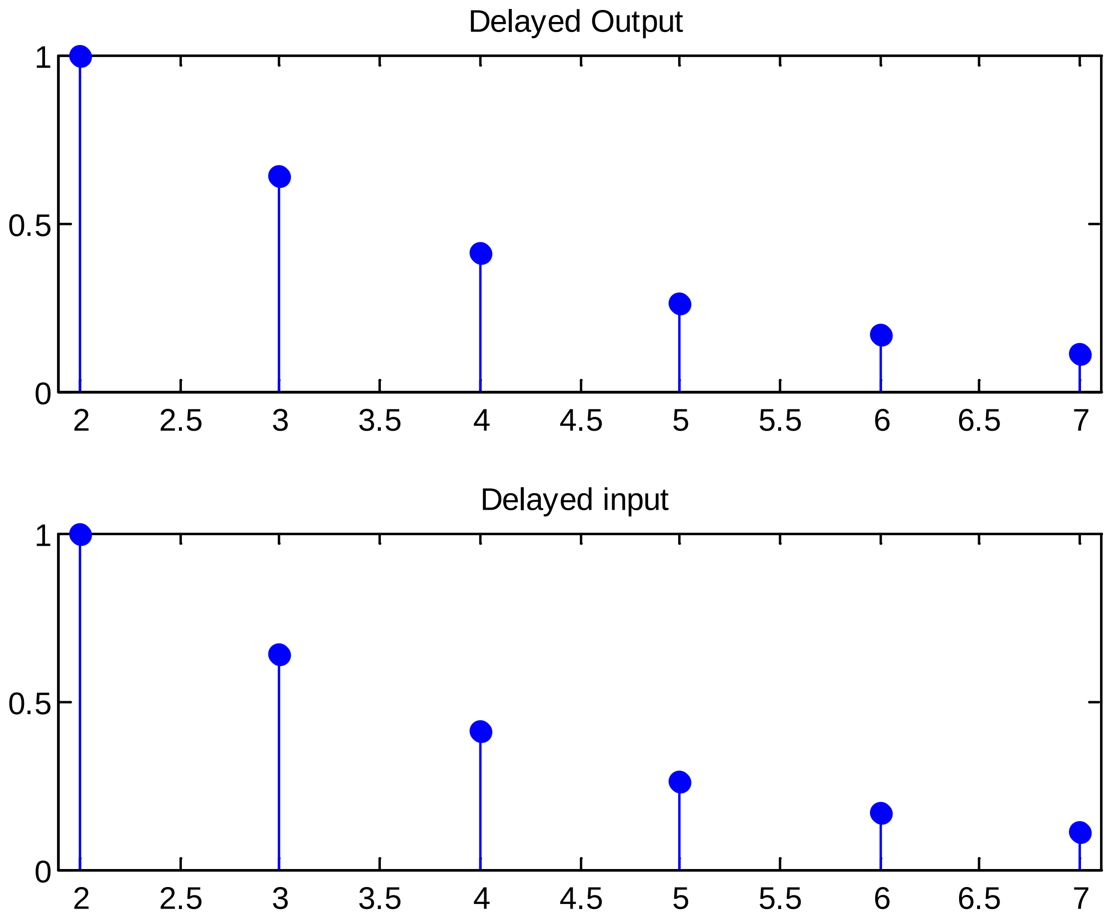
```
%% Delay y by 3
y_delayed=@(t) y(t-3);
%% Delay input by 3 and then put through system
x_delayed=@(t) x(t-3);
y_delayed1=@(t) 1-2*x_delayed(t-1);
%% Plot Results
figure
subplot(2,1,1)
fplot(y_delayed,[-4 20])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('y(t) delayed by 3')
subplot(2,1,2)
fplot(y_delayed1,[-4 20])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('x(t) delayed by 3 and input to system')
```



Because the two results are the same, the system is time-invariant

Time-Invariance Test Example 3

```
close all;clear all;clc
%% Unshifted System Response
n=0:5;
x=0.8.^n;
y=x.^2;
%% Delayed output by 2 units
figure
subplot(2,1,1)
stem(n+2,y,'filled')
xlim([1.9 7.1])
title('Delayed Output')
%% Delayed the input by 2 units first
n=n+2;
y2=(0.8.^(n-2)).^2;
subplot(2,1,2)
stem(n,y2,'filled')
xlim([1.9 7.1])
title('Delayed input')
```



Linearity

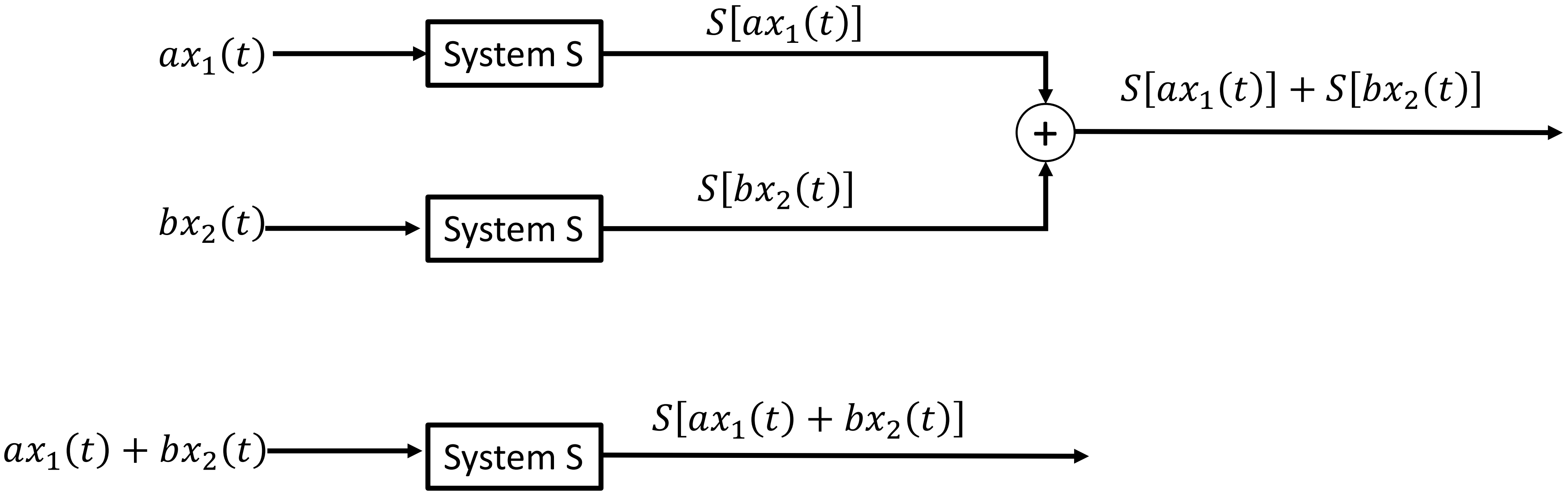
Let $y(t)$ be the response of a system S to an input signal $x(t)$, that is $y(t) = S[x(t)]$.The system S is linear if for any input signals $x_1(t)$and $x_2(t)$ and any scalars a_1 and a_2 the following relationships holds

$$S[x_1(t) + x_2(t)] = S[x_1(t)] + S[x_2(t)]$$

$$S[ax(t)] = aS[x(t)]$$

The response of a linear system to an input that is a linear combination of two signals is the linear combination of two signals is the linear combination of the responses of the systems to each one of these signals

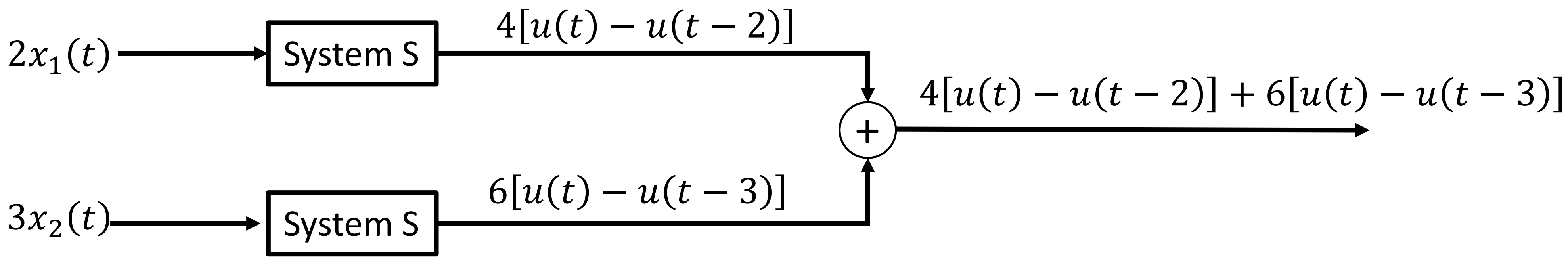
Testing for Linearity

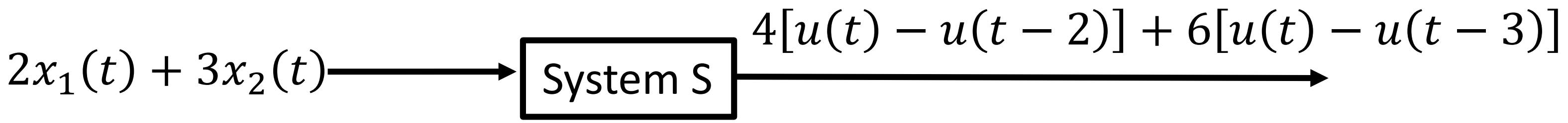


If the resulting outputs are equal, then the system is linear

Test For Linearity Example 1

Let $x_1(t) = u(t) - u(t-2)$ and $x_2(t) = u(t) - u(t-3)$ be the input signals to the system $y(t) = 2x(t)$. Test the system for linearity

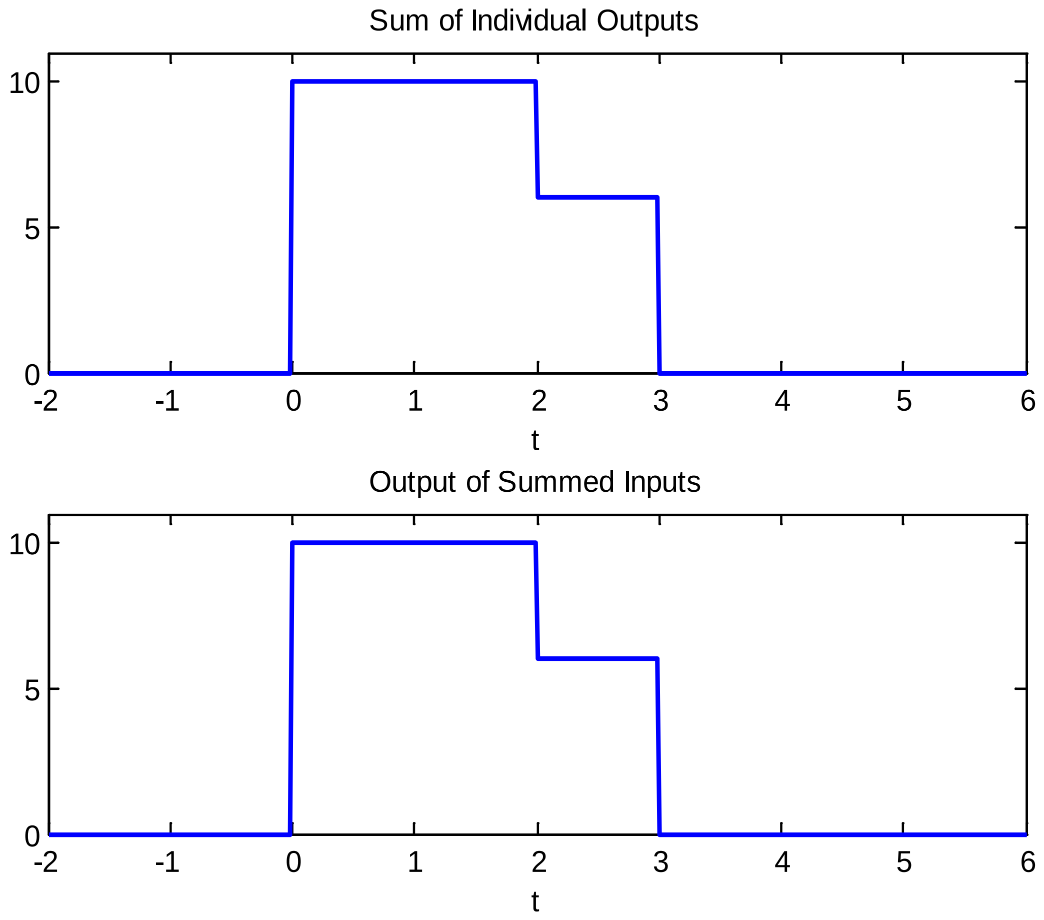




So the system is linear

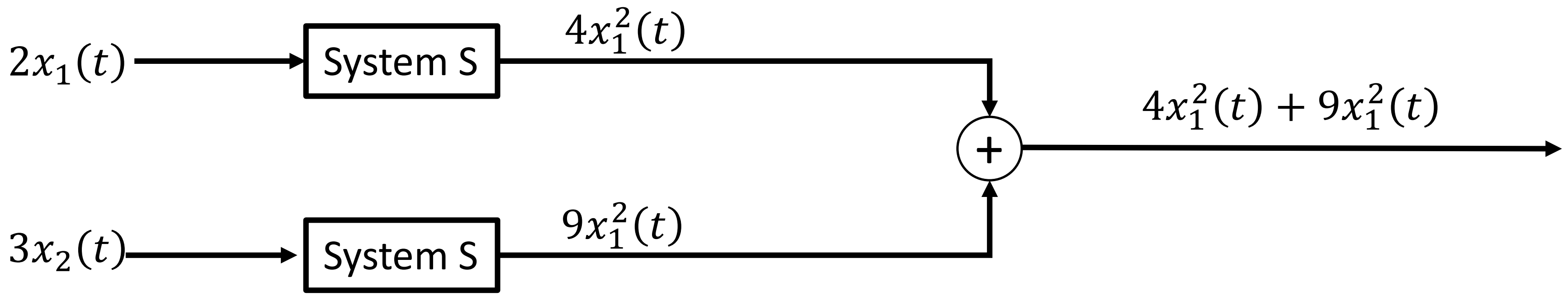
Test For Linearity Example 1

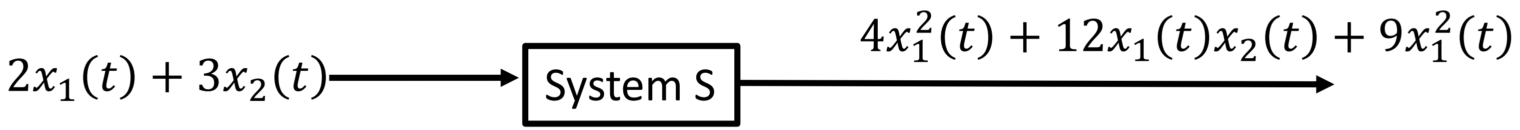
```
close all;clear all;clc
a=2;
b=3;
u=@(t) t>=0;
x1=@(t) u(t)-u(t-2);
x2=@(t) u(t)-u(t-3);
y1=@(t) a*(2*x1(t));
y2=@(t) b*(2*x2(t));
y_combined=@(t) y1(t)+y2(t);
x_combined=@(t) a*x1(t)+b*x2(t);
y=@(t) 2*x_combined(t);
figure
subplot(2,1,1)
fplot(y_combined,[-2 6])
ylim([0 11])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('Sum of Individual Outputs')
subplot(2,1,2)
fplot(y,[-2 6])
ylim([0 11])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('Output of Summed Inputs')
```



Test For Linearity Example 2

Let $x_1(t) = u(t) - u(t-2)$ and $x_2(t) = u(t) - u(t-3)$ be the input signals to the system $y(t) = x^2(t)$. Test the system for linearity

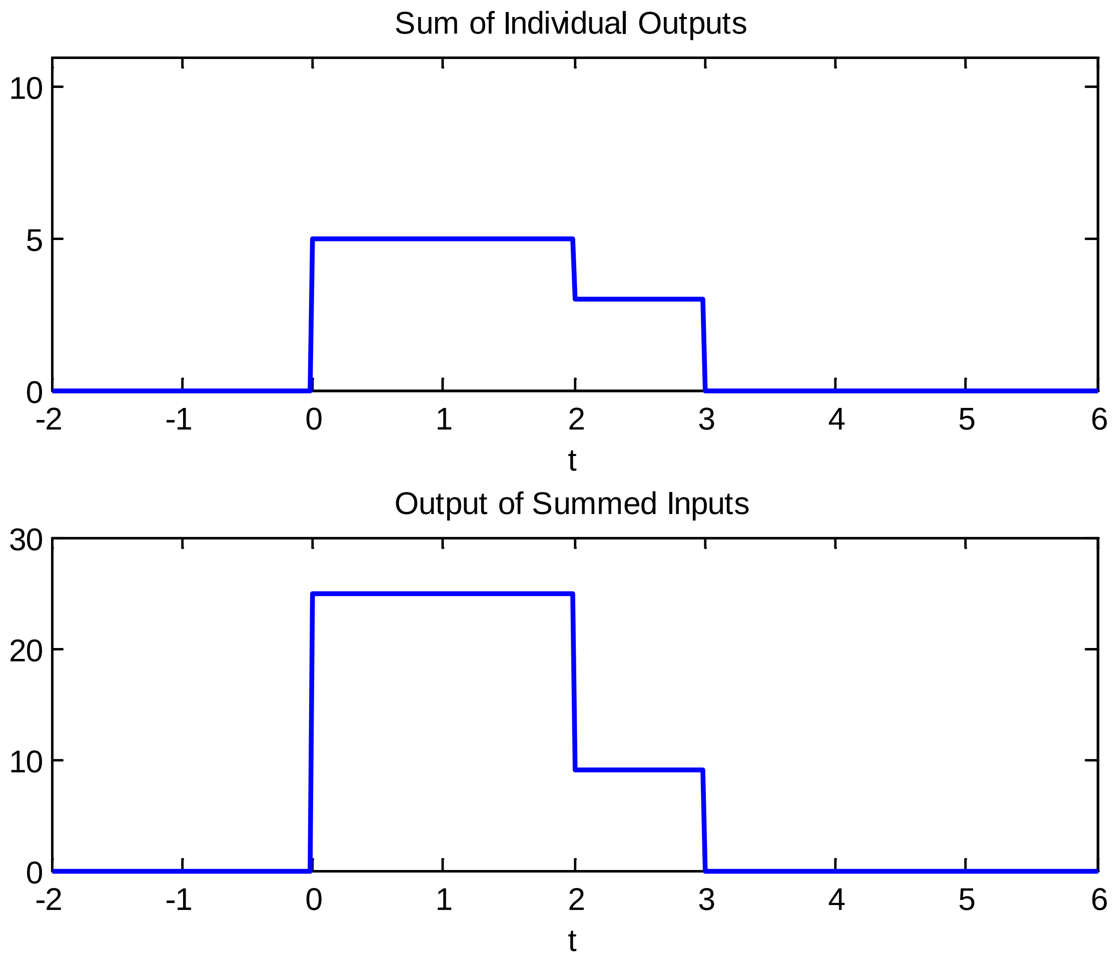




So the system is non-linear

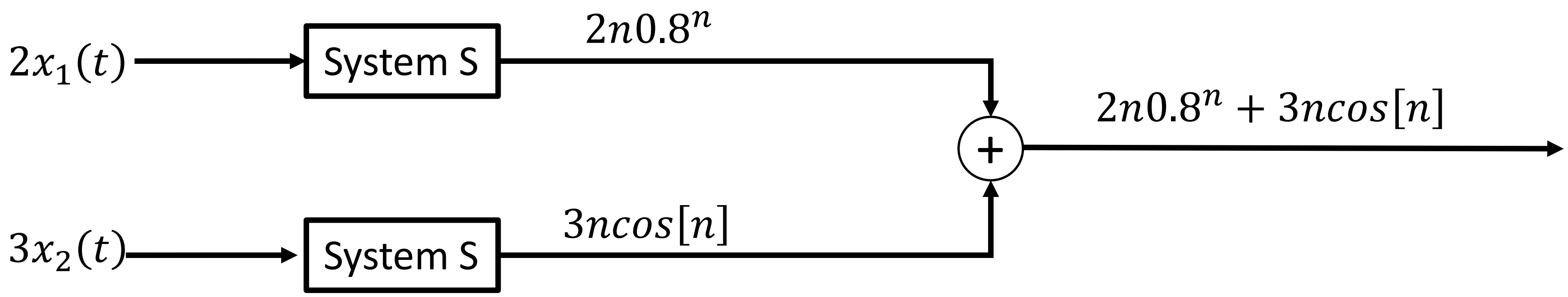
Test For Linearity Example 2

```
close all;clear all;clc
a=2;
b=3;
u=@(t) t>=0;
x1=@(t) u(t)-u(t-2);
x2=@(t) u(t)-u(t-3);
y1=@(t) a*(x1(t)^2);
y2=@(t) b*(x2(t)^2);
y_combined=@(t) y1(t)+y2(t);
x_combined=@(t) a*x1(t)+b*x2(t);
y=@(t) x_combined(t)^2;
figure
subplot(2,1,1)
fplot(y_combined,[-2 6])
ylim([0 11])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('Sum of Individual Outputs')
subplot(2,1,2)
fplot(y,[-2 6])
ylim([0 30])
set(get(gca,'Children'),'Linewidth',2)
xlabel('t')
title('Output of Summed Inputs')
```



Testing for Linearity Example 3

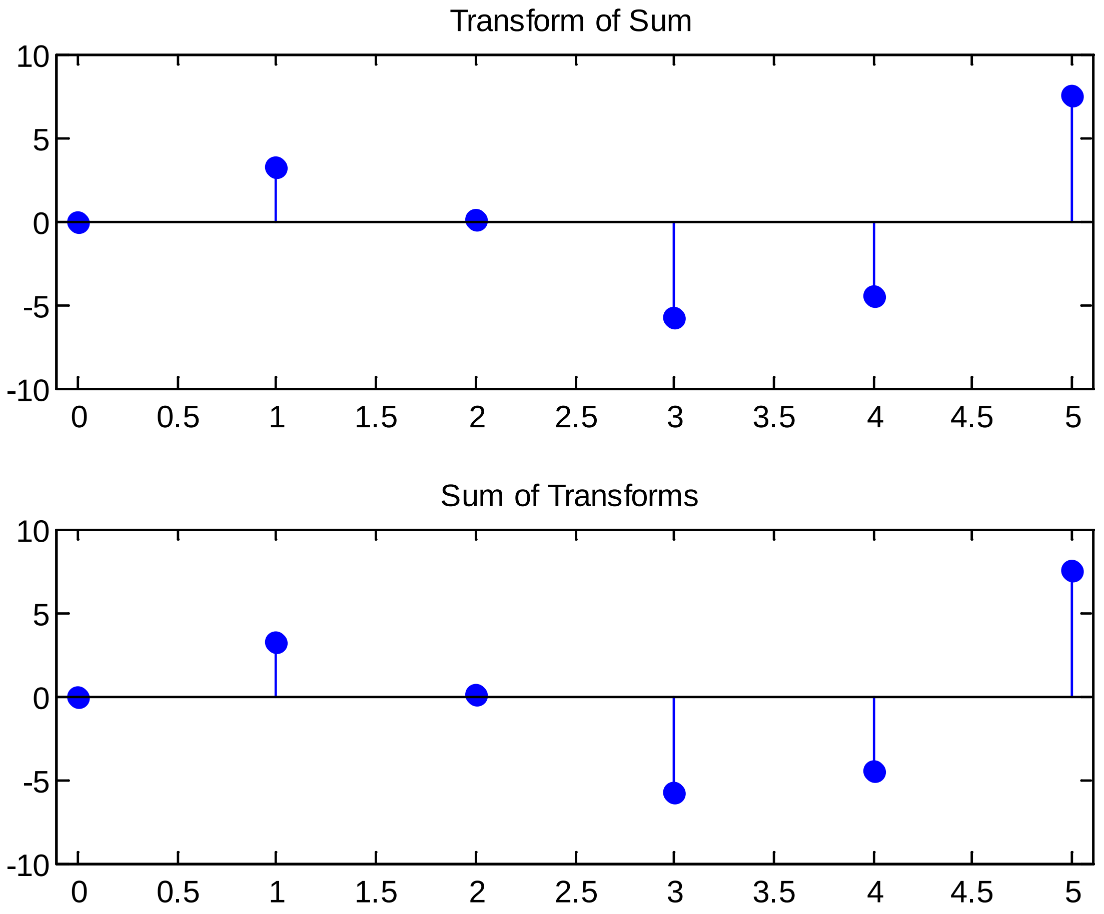
Determine if the linearity property holds for the discrete-time system described by the input/output relationship $y[n] = nx[n]$. Use the input signals $x_1[n] = 0.8^n, 0 \leq n \leq 5$ and $x_2[n] = cos[n], 0 \leq n \leq 5$



So the system is linear

Testing for Linearity Example 3

```
close all;clear all;clc
n=0:5;
x1=0.8.^n;
x2=cos(n);
a1=2;
a2=3;
z=a1*x1+a2*x2;
y1=n.*z;
z1=n.*x1;
z2=n.*x2;
y2=a1*z1+a2*z2;
figure
subplot(2,1,1)
stem(n,y1,'filled')
xlim([-0.1 5.1])
title('Transform of Sum')
subplot(2,1,2)
stem(n,y2,'filled')
xlim([-0.1 5.1])
title('Sum of Transforms')
```



Decomposition of Signals Into Fundamental Components

Why Decompose A Signal?

- Gives fundamental understanding of how signal behaves
- Gives fundamental characteristics that may not be automatically identified
- Allows us to determine how to modify the signal to produce a new signal that is useful for us

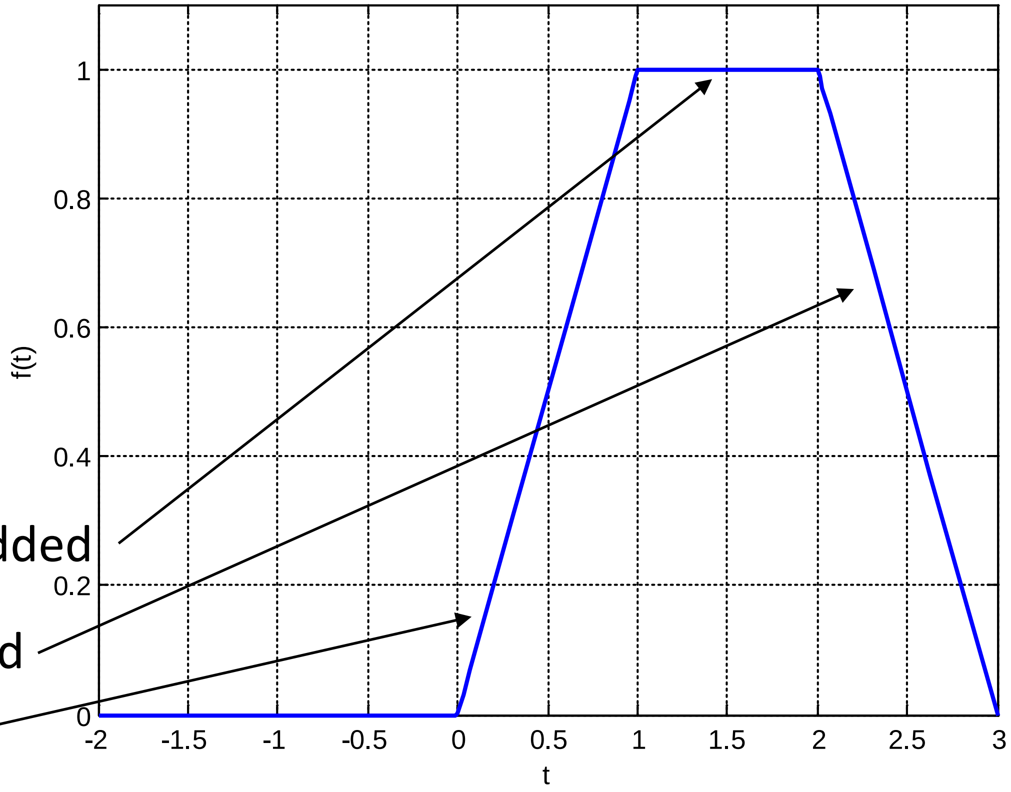
This is the essence of this class: to understand how a signal can be decomposed into a set of other important, meaningful signals/components

Decomposition Example 1

Express the signal shown on the right in terms of a sum of ramp functions

$$r(t) = tu(t)$$

$$f(t) = r(t) - r(t-1) - r(t-2)$$



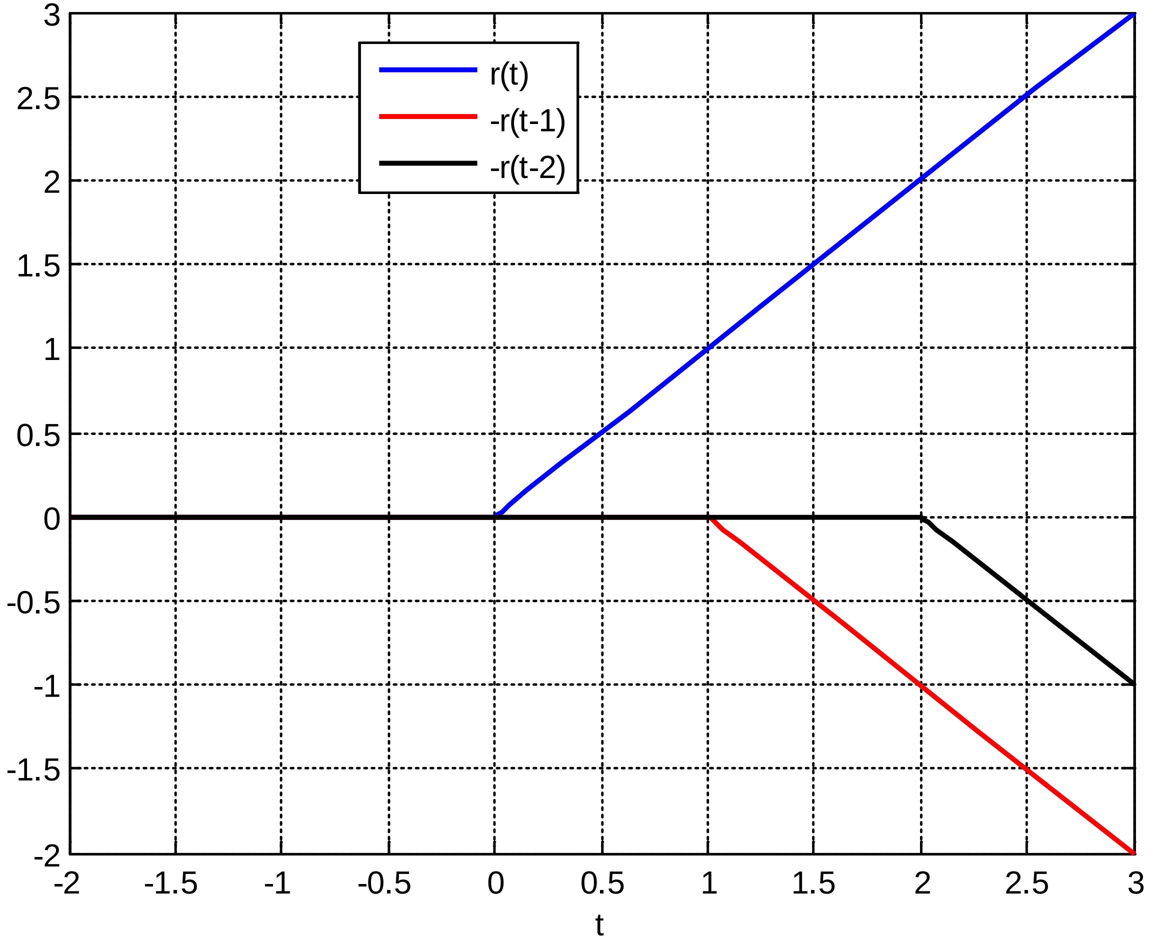
$-r(t-1)$ component added

$-r(t-2)$ component added

$r(t)$ component

Decomposition Example 1

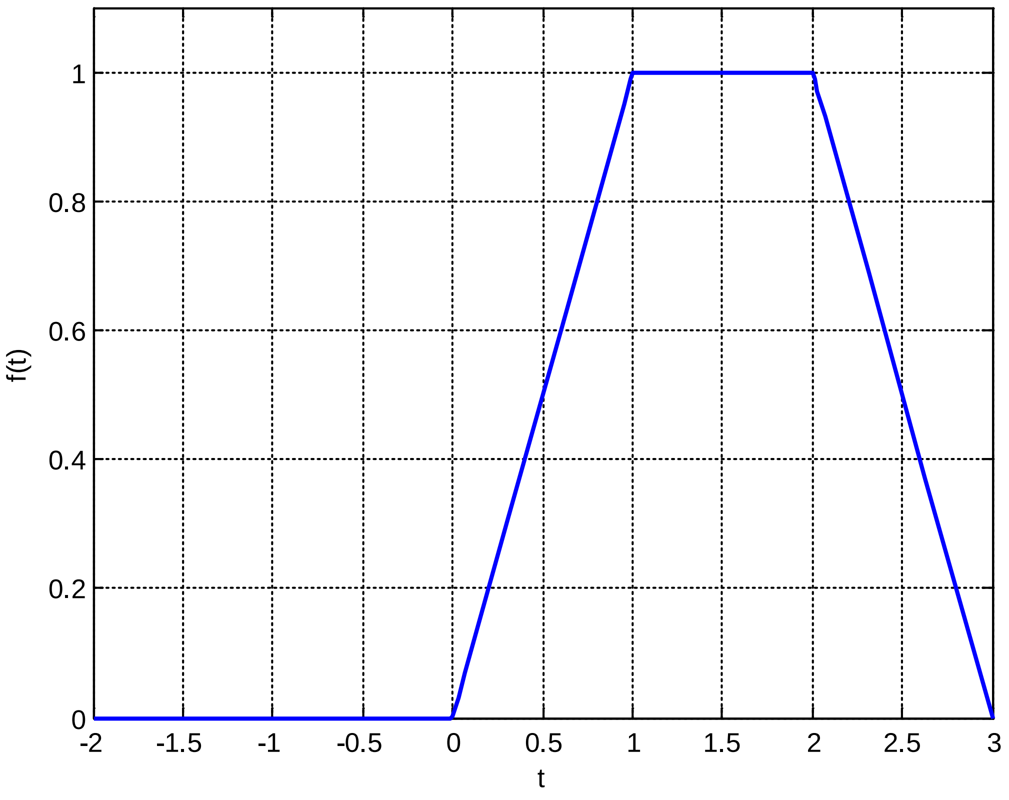
```
close all;clear all;clc
r0=@(t) t*(t>=0);
r1=@(t) -(t-1)*(t>=1);
r2=@(t) -(t-2)*(t>=2);
figure
fplot(r0,[-2 3])
hold on
fplot(r1,[-2 3],'r')
fplot(r2,[-2 3],'k')
grid on
legend('r(t)','-r(t-1)','-r(t-2)',0)
xlabel('t')
a=get(gca,'Children');
set(a,'Linewidth',2)
```



Decomposition Example 1

Express the signal shown on the right in terms of a sum of ramp functions

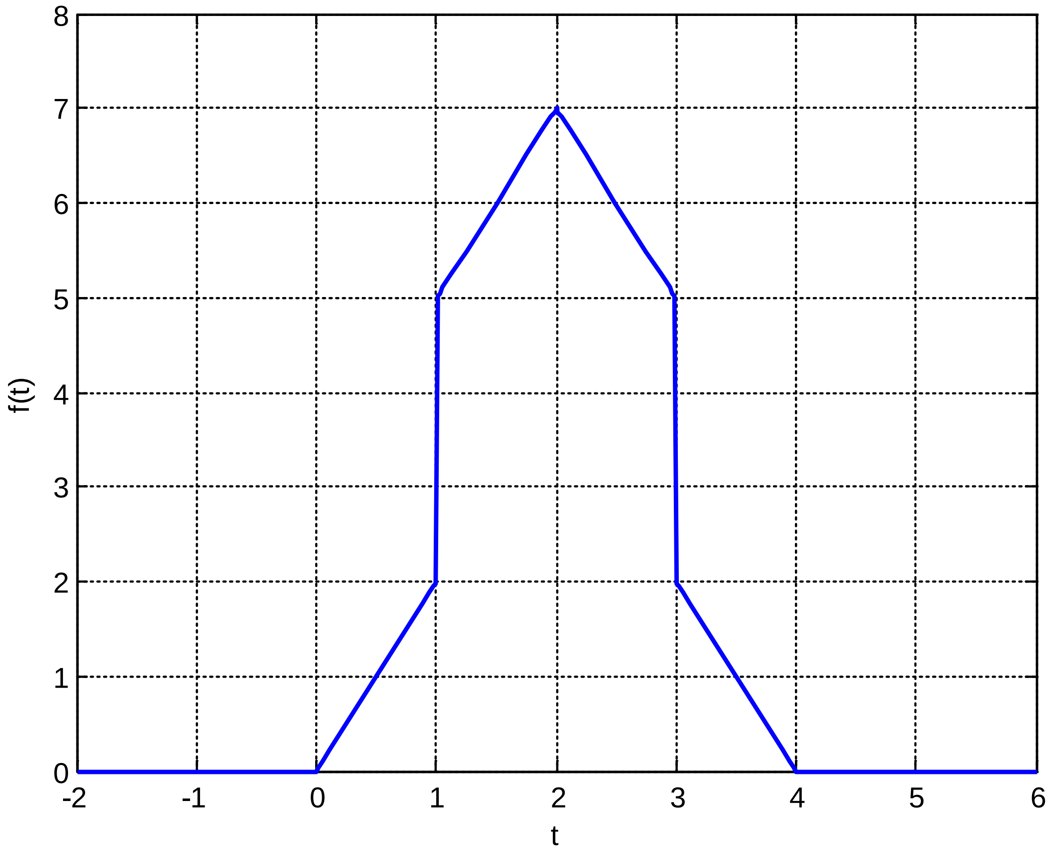
```
close all;clear all;clc
r=@(t) t*(t>=0);
f=@(t) r(t)-r(t-1)-r(t-2);
fplot(f,[-2 3])
grid on
ylim([0 1.1])
xlabel('t')
ylabel('f(t)')
a=get(gca,'Children')
set(a,'Linewidth',2)
```



Decomposition Example 2

Express the function f(t), shown to the right, as a sum of ramps and rectangular pulses

We will approach this from an analytic point of view

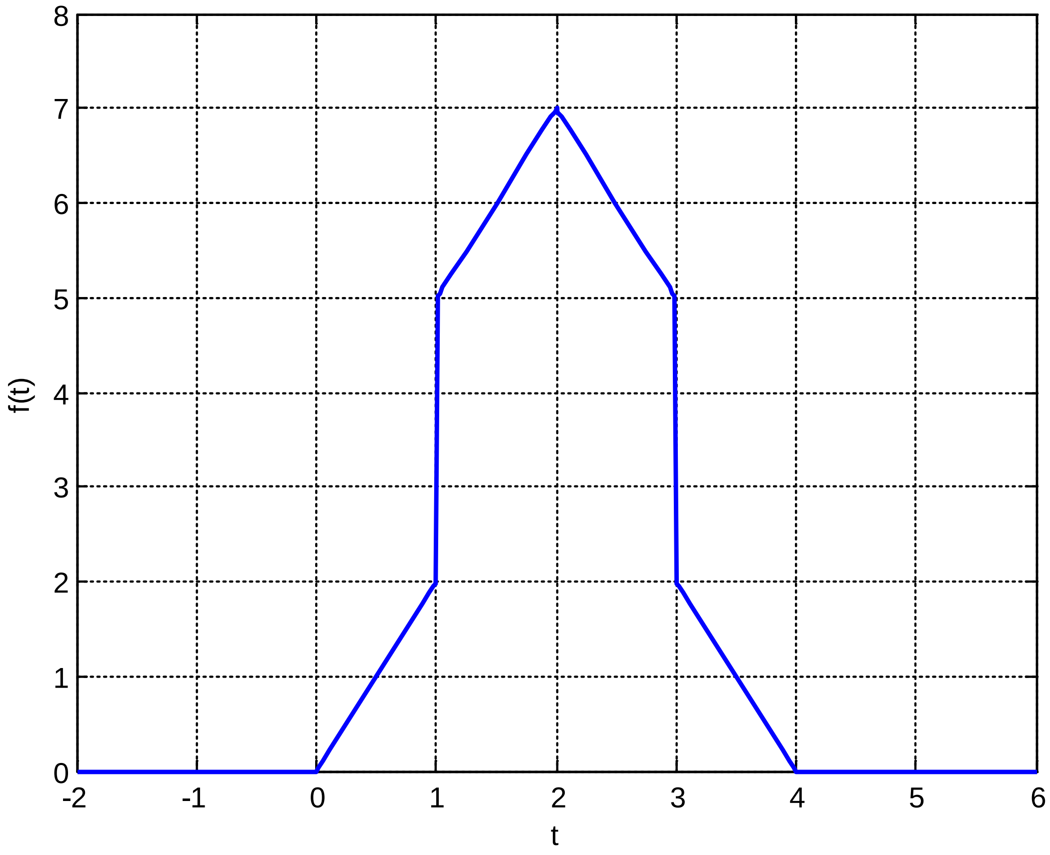


Decomposition Example 2

Observation 1: The first component must be a ramp with a slope of two. Therefore, the first component must be 2r(t)

$$f_1(t) = 2r(t)$$

So let's see how the function is with this function stripped from it

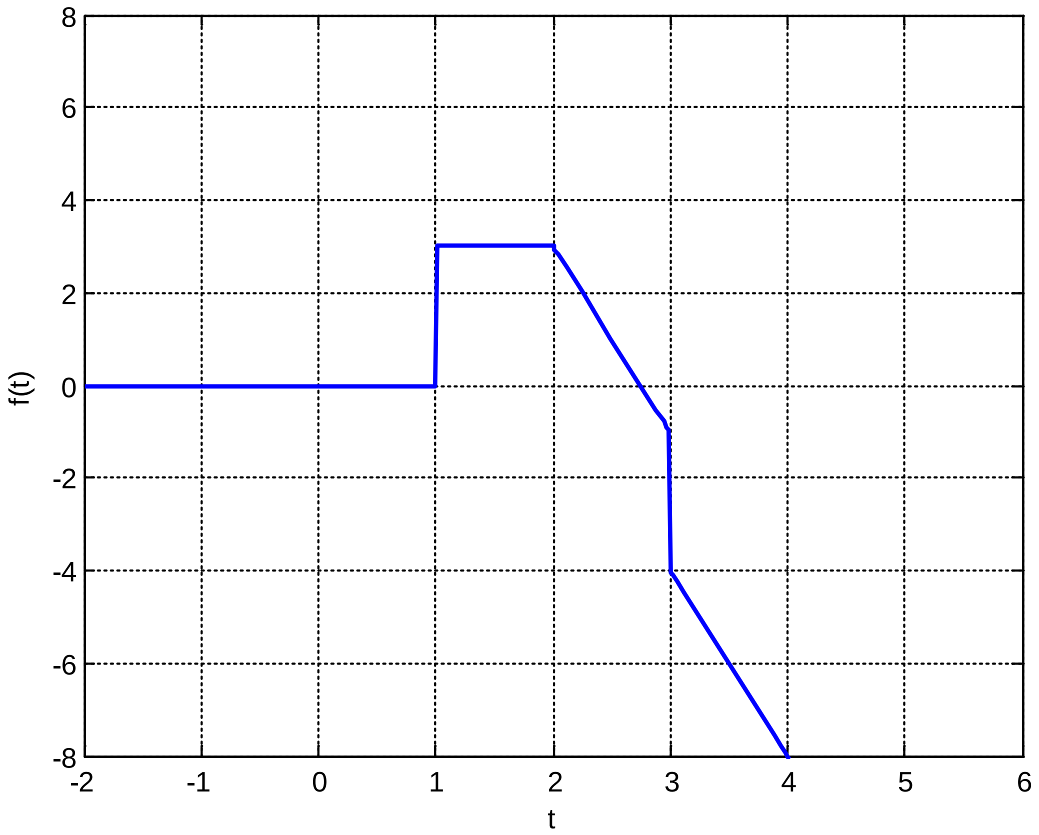


Decomposition Example 2

Observation 2: The second component must be a unit pulse of duration T=2 an amplitude of 3, and a delay of 2. Therefore, the second component must be

$$f_2(t) = 3p2(t-2)$$

So let’s see how the function is with this function stripped from it

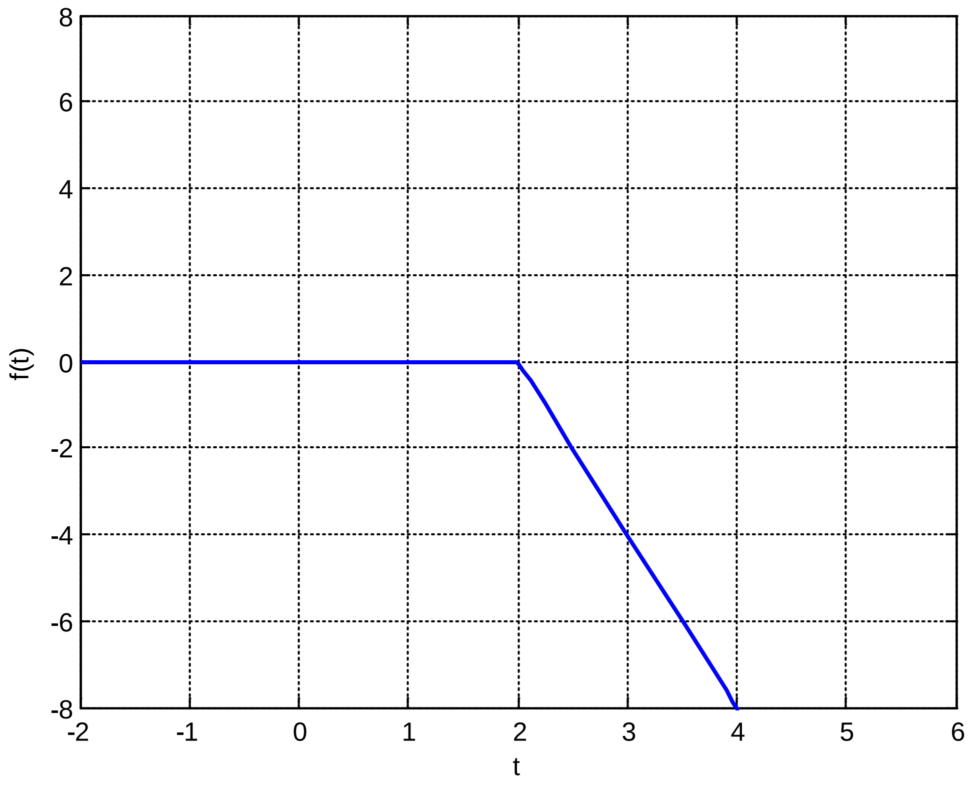


Decomposition Example 2

Observation 3: The third component must be a ramp with a delay of 2 with a slope of -4. Therefore, the third component must be

$$f_3(t) = 4r(t-2)$$

So let’s see how the function is with this function stripped from it

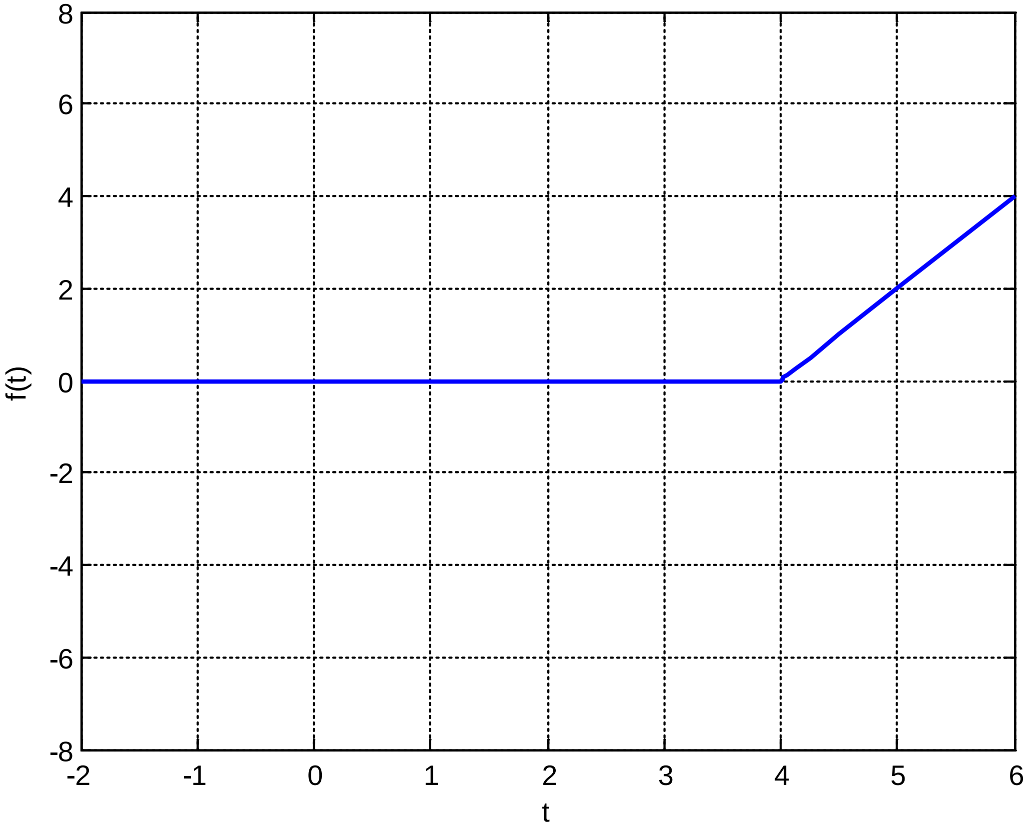


Decomposition Example 2

Observation 4: The fourth component must be a ramp with a delay of 4 with a slope of 2. Therefore, the fourth component must be

$$f_4(t) = 2r(t-4)$$

So let's see how the function is with this function stripped from it



Decomposition Example 2

There are no more components to analyze. So the actual function is

$$f(t) = 2r(t) + 3p2(t-2) - 4r(t-2) + 2r(t-4)$$