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Project Euler

Problem 1

05 October 2001

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

2009-2-15

Haskell:

sum [n | n<-[1..1000-1],mod n 5 == 0 || mod n 3 == 0]

Mathematica:★★★★★

```
Plus @@ Select[Range[999],Mod[#,3]==0||Mod[#,5]==0&]
```

233168

Problem 2

19 October 2001

Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

Find the sum of all the even-valued terms in the sequence which do not exceed four million.

Mathematica:

```
Clear[i,j,k];
For[i=1,i<1000,i++,
  If[Fibonacci[i]>4000000,Break[]]
  ];
s=0;
For[j=3,j<i,j+=3,
  s+=Fibonacci[j];
  ];
Print[s];
```

4613732

2009 年 3 月 11 日星期三，再次精简代码

```
i=3;s=0;
While[Fibonacci[i]<4*10^6,s+=Fibonacci[i];i+=3]//Timing
s
```

{0.016,Null}

4613732

Problem 3

02 November 2001

The prime factors of 13195 are 5, 7, 13 and 29.

What is the largest prime factor of the number 600851475143 ?

Mathematica :

```
FactorInteger[600851475143][[-1,1]]
```

6857

Problem 4

16 November 2001

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is 9009 = 91 ×99.

Find the largest palindrome made from the product of two 3-digit numbers.

Mathematica :

```
[In 401]:=Clear[i,j,k,s,a,b,pallst,L1];
pallst={};
For[i=9,i>0,i--,
  For[j=9,j□0,j--,
   For[k=9,k□0,k--,
    s=i*10^5+j*10^4+k*10^3+k*10^2+j*10+i;
    If[PrimeQ[s]□False,pallst = Append[pallst,s];]
    ]
   ]
  ];
For[i=999,i>900,i--,
```

```
 For[j=999,j>900,j--,
  If[MemberQ[pallst,i*j],Print["i=",i," j=",j,"
i*j=",i*j];Return[]]
  ]
 ]
```

[Out 404]:i= 993 j= 913 i*j= 906609

Return[]

2009年3月11日星期三

不容易看懂的code：★★★★★

```
pQ=Boole[#Reverse@#]&@IntegerDigits@#&;
Array[pQ[1 ##] ##&,{100,100},900,Max]
```

906609

Problem 5

30 November 2001

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest number that is evenly divisible by all of the numbers from 1 to 20?

2009-2-15

Mathematica：

```
LCM @@Range[10]
```

2520

```
Apply[LCM,Range[20]]
```

232792560

Problem 6

14 December 2001

The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + ... + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + ... + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is 3025 − 385 = 2640.

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

Mathematica:

(* 自然数和公式 1+2+..+n □ (1+n)*n/2 *)

(* 平方和公式 1^2+2^2+3^2+...+n^2 == n (n+1)(2n+1)/6 *)

```
n=100;
((1+n)*n/2)^2-(n (n+1)(2n+1)/6)
25164150
```

Problem 7

28 December 2001

By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13.

What is the 10001st prime number?

Mathematica :

```
Print[Prime[6]];
Print[Prime[10001]];
13
104743
```

Problem 8

11 January 2002

Find the greatest product of five consecutive digits in the 1000-digit number.

```
73167176531330624919225119674426574742355349194934
96983520312774506326239578318016984801869478851843
85861560789112949495459501737958331952853208805511
12540698747158523863050715693290963295227443043557
```

```
66896648950445244523161731856403098711121722383113
62229893423380308135336276614282806444486645238749
30358907296290491560440772390713810515859307960866
70172427121883998797908792274921901699720888093776
65727333001053367881220235421809751254540594752243
52584907711670556013604839586446706324415722155397
53697817977846174064955149290862569321978468622482
83972241375657056057490261407972968652414535100474
82166370484403199890008895243450658541227588666881
16427171479924442928230863465674813919123162824586
17866458359124566529476545682848912883142607690042
24219022671055626321111109370544217506941658960408
07198403850962455444362981230987879927244284909188
84580156166097919133875499200524063689912560717606
05886116467109405077541002256983155200055935729725
71636269561882670428252483600823257530420752963450
```

Mathematica:

t=7316717653133062491922511968Problem 8744265747423553491949349698352031277450632623957831801698480186947885184385861560789112949495459501737958331952853208805511125406987471585238630507156932909632952274430435576689664895044524452316173185640309871112172238311362229893423380308135336276614282806444486645238749303589072962904915604407723907138105158593079608667017242712188399879790879227492190169972088809377665727333001053367881220235421809751254540594752243525849077116705560136048395864467063244157221553975369781797784617406495514929086256932197846862248283972241375657056057490261407972968652414535100474821663704844031998900088952434506585412275886668811642717147992444292823086346567481391912316282458617866458359124566529476545682848912883142607690042242190226710556263211111093705442175069416589604080719840385096245544436298123098787992724428490918884580156166097919133875499200524063689912560717606058861164671094050775410022569831552000559357297257163626956188267042825248360082325753042075296345

lst=IntegerDigits[t];
s=0;
MAX=0;
k=0;
For[j=1,j<=1000-5,j++,
 s=Product[i,{i, Take[lst,{j,j+4}]}];
 If[s>MAX,MAX=s;k=j;];

```
  ];
Print["No.",k];
Print[Take[lst,{k,k+4}]];
Print["MaxValue=",MAX];
```

No. 365
{9,9,8,7,9}
MaxValue= 40824

2009 年 3 月 11 日星期三

另外的简介的：（看懂了）★★★★★

```
t=7316717653133062491922511967442657474235534919493496983520312774506
326239578318016984801869478851843858615607891129494954595017379583319
528532088055111254069874715852386305071569329096329522744304355766 89
66489504452445231617318564030987111217223831136222989342338030813533
62766142828064444866452387493035890729629049156044077239071381051585
930796086670172427121883998797908792274921901699720888093776657273330
010533678812202354218097512545405947522435258490771167055601360483958
644670632441572215539753697817977846174064955149290862569321978468622
482839722413756570560574902614079729686524145351004748216637048440319
989000889524345065854122758866688116427171479924442928230863465674 81
391912316282458617866458359124566529476545682848912883142607690042242
190226710556263211111093705442175069416589604080719840385096245544436
298123098787992724428490918884580156166097919133875499200524063689 91
256071760605886116467109405077541002256983155200055935729725716362695
61882670428252483600823257530420752963450;
digits=IntegerDigits[t];
Max[Map[Apply[Times,Take[#,5]]&,NestList[RotateLeft[#,1]&,digits,L
ength[digits]-1]]]
```

40824

说明：

1. Apply[Times,Take[#,5]]&，说明这个是参数用#表示的纯函数，Take[#,5]获取

   前面的 5 个元素，Times 表示连乘，即得到前 5 个元素的乘积
2. NestList[RotateLeft[#,1]&,digits,Length[digits]-1]]
3. RotateLeft[#,1]&，即循环移位，最左边的移出到最右边

4. NestList[f,expr,n] 表示计算函数 f 取 exper 值 n 次，并且是下一次使用上一次的计算；如 `NestList[f,x,4]`，输出 `{x,f[x],f[f[x]],f[f[f[x]]],f[f[f[f[x]]]]}`

Problem 9

25 January 2002

A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which,

$$a^2 + b^2 = c^2$$

For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

There exists exactly one Pythagorean triplet for which $a + b + c = 1000$.
Find the product abc.

Mathematica:

```
Reduce[a^2+b^2==c^2&&1000>a>b>1&&1000>c>1&&a+b+c□1000,
{a,b,c},Integers]
a□375&&b□200&&c□425
```

375*200*425

31875000

2009 年 3 月 11 日星期三

别人写的 code：★★★

```
Reduce[{a^2+b^2□c^2,a+b+c□1000,0<a<b<c},{a,b,c},Integers]
Times@@%[[All,2]]
```

a□ 200&&b□375&&c□425

31875000

主要是这句Times@@%[[All,2]]比较精彩，即取出所有结果中的第二个元素连乘

Problem 10

08 February 2002

The sum of the primes below 10 is 2 + 3 + 5 + 7 = 17.

Find the sum of all the primes below two million.

Mathematica:

```
For[i=1,i<2000000,i++,
  If[Prime[i]>2000000,Break[]]
  ];
Plus @@ Table[Prime[n],{n,i-1}]
```

142913828922

2009 年 3 月 11 日星期三另外一种：

```
i=1;s=0;While[Prime[i]<2*10^6,s+=Prime[i];i++];s
```

142913828922

Problem 11

22 February 2002

In the 20×20 grid below, four numbers along a diagonal line have been marked in red.

```
08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 26 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 63 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 78 78 96 83 14 88 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 14 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48
```

The product of these numbers is 26 × 63 × 78 × 14 = 1788696.

What is the greatest product of four adjacent numbers in any direction (up, down, left, right, or diagonally) in the 20×20 grid?

```
lst={
  {08,02,22,97,38,15,00,40,00,75,04,05,07,78,52,12,50,77,91,08},
  {49,49,99,40,17,81,18,57,60,87,17,40,98,43,69,48,04,56,62,00},
  {81,49,31,73,55,79,14,29,93,71,40,67,53,88,30,03,49,13,36,65},
  {52,70,95,23,04,60,11,42,69,24,68,56,01,32,56,71,37,02,36,91},
  {22,31,16,71,51,67,63,89,41,92,36,54,22,40,40,28,66,33,13,80},
  {24,47,32,60,99,03,45,02,44,75,33,53,78,36,84,20,35,17,12,50},
  {32,98,81,28,64,23,67,10,26,38,40,67,59,54,70,66,18,38,64,70},
  {67,26,20,68,02,62,12,20,95,63,94,39,63,08,40,91,66,49,94,21},
  {24,55,58,05,66,73,99,26,97,17,78,78,96,83,14,88,34,89,63,72},
  {21,36,23,09,75,00,76,44,20,45,35,14,00,61,33,97,34,31,33,95},
  {78,17,53,28,22,75,31,67,15,94,03,80,04,62,16,14,09,53,56,92},
  {16,39,05,42,96,35,31,47,55,58,88,24,00,17,54,24,36,29,85,57},
  {86,56,00,48,35,71,89,07,05,44,44,37,44,60,21,58,51,54,17,58},
  {19,80,81,68,05,94,47,69,28,73,92,13,86,52,17,77,04,89,55,40},
  {04,52,08,83,97,35,99,16,07,97,57,32,16,26,26,79,33,27,98,66},
  {88,36,68,87,57,62,20,72,03,46,33,67,46,55,12,32,63,93,53,69},
  {04,42,16,73,38,25,39,11,24,94,72,18,08,46,29,32,40,62,76,36},
  {20,69,36,41,72,30,23,88,34,62,99,69,82,67,59,85,74,04,36,16},
  {20,73,35,29,78,31,90,01,74,31,49,71,48,86,81,16,23,57,05,54},
  {01,70,54,71,83,51,54,69,16,92,33,48,61,43,52,01,89,19,67,48}};
len=Length[lst]
max=0;
a=0;
b=0;

(*calc 行*)

For[i=1,i≤len,i++,
 For[j=1,j≤len-4,j++,
  s=Times@@Take[lst[[i]],{j,j+3}];
  If[s>max,max=s;a=i;b=j;];
  ];
 ];
max
Take[lst[[a]],{b,b+3}]

(*calc 列*)

For[i=1,i≤len-4,i++,
```

```
  For[j=1,j≤len,j++,
    s=Times@@{lst[[i,j]],lst[[i+1,j]],lst[[i+2,j]],lst[[i+3,j]]};
    If[s>max,max=s;a=i;b=j;];
    ];
  ];
max
{lst[[a,b]],lst[[a+1,b]],lst[[a+2,b]],lst[[a+3,b]]}
```

(*calc 斜线*)

```
For[i=1,i≤len-4,i++,
  For[j=1,j≤len,j++,
    s=Times@@{lst[[i,j]],lst[[i+1,j]],lst[[i+2,j]],lst[[i+3,j]]};
    If[s>max,max=s;a=i;b=j;];
    ];
  ];
20
48477312
{78,78,96,83}
51267216
{66,91,88,97}
```

斜线的计算比较烦

Problem 12

08 March 2002

The sequence of triangle numbers is generated by adding the natural numbers. So the 7^{th} triangle number would be 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28. The first ten terms would be:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Let us list the factors of the first seven triangle numbers:

1: 1
3: 1,3
6: 1,2,3,6
10: 1,2,5,10
15: 1,3,5,15
21: 1,3,7,21
28: 1,2,4,7,14,28

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

Mathematica:

(* 1,2,...,n的和 (1+n)*n/2 *)

```
For[i=1,i<10^10,i++,
 If[Length[Divisors[(1+i)*i/2]]>500,Print[i];Break[]]
 ]
```

12375

Problem 13

22 March 2002

Work out the first ten digits of the sum of the following one-hundred 50-digit numbers.

```
37107287533902102798797998220837590246510135740250
46376937677490009712648124896970078050417018260538
74324986199524741059474233309513058123726617309629
91942213363574161572522430563301811072406154908250
23067588207539346171171980310421047513778063246676
89261670696623633820136378418383684178734361726757
28112879812849979408065481931592621691275889832738
44274228917432520321923589422876796487670272189318
47451445736001306439091167216856844588711603153276
70386486105843025439939619828917593665686757934951
62176457141856560629502157223196586755079324193331
64906352462741904929101432445813822663347944758178
92575867718337217661963751590579239728245598838407
58203565325359399008402633568948830189458628227828
80181199384826282014278194139940567587151170094390
35398664372827112653829987240784473053190104293586
86515506006295864861532075273371959191420517255829
71693888707715466499115593487603532921714970056938
54370070576826684624621495650076471787294438377604
53282654108756828443191190634694037855217779295145
36123272525000296071075082563815656710885258350721
45876576172410976447339110607218265236877223636045
```

```
17423706905851860660448207621209813287860733969412
81142660418086830619328460811191061556940512689692
51934325451728388641918047049293215058642563049483
62467221648435076201727918039944693004732956340691
15732444386908125794514089057706229429197107928209
55037687525678773091862540744969844508330393682126
18336384825330154686196124348767681297534375946515
80386287592878490201521685554828717201219257766954
78182833757993103614740356856449095527097864797581
16726320100436897842553539920931837441497806860984
48403098129077791799088218795327364475675590848030
87086987551392711854517078544161852424320693150332
59959406895756536782107074926966537676326235447210
69793950679652694742597709739166693763042633987085
41052684708299085211399427365734116182760315001271
65378607361501080857009149939512557028198746004375
35829035317434717326932123578154982629742552737307
94953759765105305946966067683156574377167401875275
88902802571733229619176668713819931811048770190271
25267680276078003013678680992525463401061632866526
36270218540497705585629946580636237993140746255962
24074486908231174977792365466257246923322810917141
91430288197103288597806669760892938638285025333403
34413065578016127815921815005561868836468420090470
23053081172816430487623791969842487255036638784583
11487696932154902810424020138335124462181441773470
63783299490636259666498587618221225225512486764533
67720186971698544312419572409913959008952310058822
95548255300263520781532296796249481641953868218774
76085327132285723110424803456124867697064507995236
37774242535411291684276865538926205024910326572967
23701913275725675285653248258265463092207058596522
29798860272258331913126375147341994889534765745501
18495701454879288984856827726077713721403798879715
38298203783031473527721580348144513491373226651381
34829543829199918180278916522431027392251122869539
40957953066405232632538044100059654939159879593635
29746152185502371307642255121183693803580388584903
41698116222072977186158236678424689157993532961922
62467957194401269043877107275048102390895523597457
23189706772547915061505504953922979530901129967519
```

```
86188088225875314529584099251203829009407770775672
11306739708304724483816533873502340845647058077308
82959174767140363198008187129011875491310547126581
97623331044818386269515456334926366572897563400500
42846280183517070527831839425882145521227251250327
55121603546981200581762165212827652751691296897789
32238195734329339946437501907836945765883352399886
75506164965184775180738168837861091527357929701337
62177842752192623401942399639168044983993173312731
32924185707147349566916674687634660915035914677504
99518671430235219628894890102423325116913619626622
73267460800591547471830798392868535206946944540724
76841822524674417161514036427982273348055556214818
97142617910342598647204516893989422179826088076852
87783646182799346313767754307809363333018982642090
10848802521674670883215120185883543223812876952786
71329612474782464538636993009049310363619763878039
62184073572399794223406235393808339651327408011116
66627891981488087797941876876144230030984490851411
60661826293682836764744779239180335110989069790714
85786944089552990653640447425576083659976645795096
66024396409905389607120198219976047599490197230297
64913982680032973156037120041377903785566085089252
16730939319872750275468906903707539413042652315011
94809377245048795150954100921645863754710598436791
78639167021187492431995700641917969777599028300699
15368713711936614952811305876380278410754449733078
40789923115535562561142322423255033685442488917353
44889911501440648020369068063960672322193204149535
41503128880339536053299340368006977710650566631954
81234880673210146739058568557934581403627822703280
82616570773948327592232845941706525094512325230608
22918802058777319719839450180888072429661980811197
77158542502016545090413245809786882778948721859617
72107838435069186155435662884062257473692284509516
20849603980134001723930671666823555245252804609722
53503534226472524250874054075591789781264330331690
```

Mathematica:

a={37107287533902102798797998220837590246510135740250,
463769376774900097126481248969700780504170182605 38,
74324986199524741059474233309513058123726617309629,

91942213363574161572522430563301811072406154908250,
23067588207539346171171980310421047513778063246676,
89261670696623633820136378418383684178734361726757,
28112879812849979408065481931592621691275889832738,
44274228917432520321923589422876796487670272189318,
47451445736001306439091167216856844588711603153276,
70386486105843025439939619828917593665686757934951,
62176457141856560629502157223196586755079324193331,
64906352462741904929101432445813822663347944758178,
92575867718337217661963751590579239728245598838407,
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80181199384826282014278194139940567587151170094390,
35398664372827112653829987240784473053190104293586,
86515506006295864861532075273371959191420517255829,
71693888707715466499115593487603532921714970056938,
54370070576826684624621495650076471787294438377604,
53282654108756828443191190634694037855217779295145,
36123272525000296071075082563815656710885258350721,
45876576172410976447339110607218265236877223636045,
17423706905851860660448207621209813287860733969412,
81142660418086830619328460811191061556940512689692,
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78182833757993103614740356856449095527097864797581,
16726320100436897842553539920931837441497806860984,
48403098129077791799088218795327364475675590848030,
87086987551392711854517078544161852424320693150332,
59959406895756536782107074926966537676326235447210,
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41052684708299085211399427365734116182760315001271,
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35829035317434717326932123578154982629742552737307,
94953759765105305946966067683156574377167401875275,
88902802571733229619176668713819931811048770190271,
25267680276078003013678680992525463401061632866526,
36270218540497705585629946580636237993140746255962,
24074486908231174977792365466257246923322810917141,

91430288197103288597806669760892938638285025333403,
34413065578016127815921815005561868836468420090470,
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11487696932154902810424020138335124462181441773470,
63783299490636259666498587618221225225512486764533,
67720186971698544312419572409913959008952310058822,
95548255300263520781532296796249481641953868218774,
76085327132285723110424803456124867697064507995236,
37774242535411291684276865538926205024910326572967,
23701913275725675285653248258265463092207058596522,
29798860272258331913126375147341994889534765745501,
18495701454879288984856827726077713721403798879715,
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34829543829199918180278916522431027392251122869539,
40957953066405232632538044100059654939159879593635,
29746152185502371307642255121183693803580388584903,
41698116222072977186158236678424689157993532961922,
62467957194401269043877107275048102390895523597457,
23189706772547915061505504953922979530901129967519,
86188088225875314529584099251203829009407770775672,
11306739708304724483816533873502340845647058077308,
82959174767140363198008187129011875491310547126581,
97623331044818386269515456334926366572897563400500,
42846280183517070527831839425882145521227251250327,
55121603546981200581762165212827652751691296897789,
32238195734329339946437501907836945765883352399886,
75506164965184775180738168837861091527357929701337,
62177842752192623401942399639168044983993173312731,
32924185707147349566916674687634660915035914677504,
99518671430235219628894890102423325116913619626622,
73267460800591547471830798392868535206946944540724,
76841822524674417161514036427982273348055556214818,
97142617910342598647204516893989422179826088076852,
87783646182799346313767754307809363333018982642090,
10848802521674670883215120185883543223812876952786,
71329612474782464538636993009049310363619763878039,
62184073572399794223406235393808339651327408011116,
66627891981488087797941876876144230030984490851411,
60661826293682836764744779239180335110989069790714,
85786944089552990653640447425576083659976645795096,
66024396409905389607120198219976047599490197230297,

64913982680032973156037120041377903785566085089252,
16730939319872750275468906903707539413042652315011,
94809377245048795150954100921645863754710598436791,
78639167021187492431995700641917969777599028300699,
15368713711936614952811305876380278410754449733078,
40789923115535562561142322423255033685442488917353,
44889911501440648020369068063960672322193204149535,
41503128880339536053299340368006977710650566631954,
81234880673210146739058568557934581403627822703280,
82616570773948327592232845941706525094512325230608,
22918802058777319719839450180888072429661980811197,
77158542502016545090413245809786882778948721859617,
72107838435069186155435662884062257473692284509516,
20849603980134001723930671666823555245252804609722,
53503534226472524250874054075591789781264330331690
};

Plus @@ a

5537376230390876637302048746832985971773659831892672

Problem 14

05 April 2002

The following iterative sequence is defined for the set of positive integers:

$n \to n/2$ (n is even)
$n \to 3n + 1$ (n is odd)

Using the rule above and starting with 13, we generate the following sequence:

$$13 \to 40 \to 20 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$$

It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.

Which starting number, under one million, produces the longest chain?

NOTE: Once the chain starts the terms are allowed to go above one million.

Mathematica:

方法一：**用时太长，未解决**

```
(*
n = n/2 (n is even)
n = 3n+1 (n is odd)
*)
max=10^6;
num=0;
longest=0;
m=0;
lst=Table[i,{i,max}];
For[i=max/2,i<max,i++,
  n=Part[lst,i];
  num=0;
  While[n>1,num+
+;If[EvenQ[n],n=n/2,n=n+n+n+1];If[i<n<max,lst=ReplacePart
[lst,n□1]]];
  If[num>longest,longest=num;m=i];
  ]//Timing
Print[m," ",longest];
```

方法二：用时太长，未解决

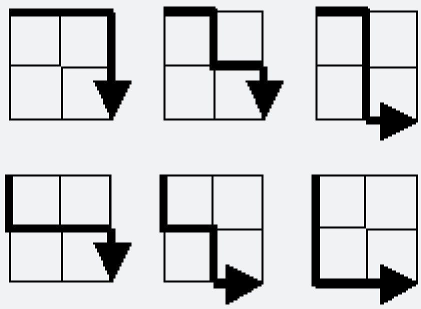
```
(*n=n/2 (n is even) n=3n+1 (n is odd)*)
max=10^6;
num=0;
longest=0;
m=0;
lstnum={1,2};
lstnum=Join[lstnum,Table[0,{max-2}]];
lst=Table[i,{i,1,max}];
For[i=3,i<max,i++,
  n=i;
  num=1;
  While[n>1,num++;

If[EvenQ[n],n=n/2,n=n+n+n+1];If[n<i,lstnum=ReplacePart[lstnum,i□(num+=Part[lstn
um,n]-1)];Break[];];
  If[n□ 1,lstnum=ReplacePart[lstnum,i□num]];
  If[num>longest,longest=num;m=i];]//Timing
Print[m," ",longest];
```

Problem 15

19 April 2002

Starting in the top left corner of a 2×2 grid, there are 6 routes (without backtracking) to the bottom right corner.



How many routes are there through a 20×20 grid?

思路：昨晚在家看了一下算法技巧一书，也说到这个问题，思路就一下打开了

1 1 1 1
1 2 3 4
1 3 6 10
1 4 10 20

从这些数据看出规律来，就简单多了

```
n=21;
lst=Table[1,{n}];
lst1=lst;
For[i=1,i<n,i++,
 For[j=1,j<=n,j++,
  t=Plus@@Take[lst,{1,j}];
  lst1=ReplacePart[lst1,j→t];
  ];
 lst=lst1;
 Print[lst];
 ]
lst[[-1]]
{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21}
{1,3,6,10,15,21,28,36,45,55,66,78,91,105,120,136,153,171,190,210,231}
{1,4,10,20,35,56,84,120,165,220,286,364,455,560,680,816,969,1140,1330,1540,1771
```

}
{1,5,15,35,70,126,210,330,495,715,1001,1365,1820,2380,3060,3876,4845,5985,7315,8855,10626}
{1,6,21,56,126,252,462,792,1287,2002,3003,4368,6188,8568,11628,15504,20349,26334,33649,42504,53130}
{1,7,28,84,210,462,924,1716,3003,5005,8008,12376,18564,27132,38760,54264,74613,100947,134596,177100,230230}
{1,8,36,120,330,792,1716,3432,6435,11440,19448,31824,50388,77520,116280,170544,245157,346104,480700,657800,888030}
{1,9,45,165,495,1287,3003,6435,12870,24310,43758,75582,125970,203490,319770,490314,735471,1081575,1562275,2220075,3108105}
{1,10,55,220,715,2002,5005,11440,24310,48620,92378,167960,293930,497420,817190,1307504,2042975,3124550,4686825,6906900,10015005}
{1,11,66,286,1001,3003,8008,19448,43758,92378,184756,352716,646646,1144066,1961256,3268760,5311735,8436285,13123110,20030010,30045015}
{1,12,78,364,1365,4368,12376,31824,75582,167960,352716,705432,1352078,2496144,4457400,7726160,13037895,21474180,34597290,54627300,84672315}
{1,13,91,455,1820,6188,18564,50388,125970,293930,646646,1352078,2704156,5200300,9657700,17383860,30421755,51895935,86493225,141120525,225792840}
{1,14,105,560,2380,8568,27132,77520,203490,497420,1144066,2496144,5200300,10400600,20058300,37442160,67863915,119759850,206253075,347373600,573166440}
{1,15,120,680,3060,11628,38760,116280,319770,817190,1961256,4457400,9657700,20058300,40116600,77558760,145422675,265182525,471435600,818809200,1391975640}
{1,16,136,816,3876,15504,54264,170544,490314,1307504,3268760,7726160,17383860,37442160,77558760,155117520,300540195,565722720,1037158320,1855967520,3247943160}
{1,17,153,969,4845,20349,74613,245157,735471,2042975,5311735,13037895,30421755,67863915,145422675,300540195,601080390,1166803110,2203961430,4059928950,7307872110}
{1,18,171,1140,5985,26334,100947,346104,1081575,3124550,8436285,21474180,51895935,119759850,265182525,565722720,1166803110,2333606220,4537567650,8597496600,15905368710}
{1,19,190,1330,7315,33649,134596,480700,1562275,4686825,13123110,34597290,86493225,206253075,471435600,1037158320,2203961430,4537567650,9075135300,17672631900,33578000610}
{1,20,210,1540,8855,42504,177100,657800,2220075,6906900,20030010,54627300,141120525,347373600,818809200,1855967520,4059928950,8597496600,17672631900,35345263800,68923264410}
{1,21,231,1771,10626,53130,230230,888030,3108105,10015005,30045015,8467231

5,225792840,573166440,1391975640,3247943160,7307872110,15905368710,33578000610,68923264410,137846528820}

137846528820

2009年3月11日星期三

其实代码只有这些：

```
n=21;
lst=Table[1,{n}];
lst1=lst;
For[i=1,i<n,i++,For[j=1,j□n,j++,t=Plus@@Take[lst,{1,j}];
  lst1=ReplacePart[lst1,j□t];];
 lst=lst1;
 ]
lst[[-1]]
```

137846528820

Problem 16

03 May 2002

2^{15} = 32768 and the sum of its digits is 3 + 2 + 7 + 6 + 8 = 26.

What is the sum of the digits of the number 2^{1000}?

Mathematica:

```
Plus @@ IntegerDigits[2^1000]
```

1366

Problem 17

17 May 2002

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are 3 + 3 + 5 + 4 + 4 = 19 letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

NOTE: Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of "and" when writing out numbers is in compliance with British usage.

```
lst={"one","two","three","four","five","six","seven","eight","nine","ten","eleven","twelve","thirteen","fourteen","fifteen","sixteen","seventeen","eighteen","nineteen","twenty","thirty","forty","fifty","sixty","seventy","eighty","ninety","onehundred","twohundred","threehundred","fourhundred","fivehundred","sixhundred","sevenhundred","eighthundred","ninehundred","thousand"};
lst1={};
lst2={};
lstnum= {};
For[i=1,i□ Length[lst],i++,
  AppendTo[lstnum,StringLength[lst[[i]]]];
  ];
lst1=Take[lstnum,{1,9}];
lst10=Take[lstnum,{10,19}];
lst2=Take[lstnum,{20,27}];
lst3=Take[lstnum,{28,36}];
lstnum
s=0;
s=Plus@@lst10;
For[i=0,i<10,i++,
 For[j=0,j<10,j++,
  For[k=0,k<10,k++,
   (*3为and*)
   If[i□ 0,s+=lst3[[i]]+3];
   If[i□0&&j□0&&k□0,s-=3];
   If[j□1,s+=lst10[[k+1]];Continue[]];
   If[j□0,s+=lst2[[j-1]]];
   If[k□0,s+=lst1[[k]]];
   ]
  ]
 ]
s+=lstnum[[-1]]
{3,3,5,4,4,3,5,5,4,3,6,6,8,8,7,7,9,8,8,6,6,5,5,5,7,6,6,10,10,12,11,11,10,12,12,11,8}
21191
```

答案不对

Problem 18

31 May 2002

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

```
   3
  7 4
 2 4 6
8 5 9 3
```

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom of the triangle below:

```
75
95 64
17 47 82
18 35 87 10
20 04 82 47 65
19 01 23 75 03 34
88 02 77 73 07 63 67
99 65 04 28 06 16 70 92
41 41 26 56 83 40 80 70 33
41 48 72 33 47 32 37 16 94 29
53 71 44 65 25 43 91 52 97 51 14
70 11 33 28 77 73 17 78 39 68 17 57
91 71 52 38 17 14 91 43 58 50 27 29 48
63 66 04 68 89 53 67 30 73 16 69 87 40 31
04 62 98 27 23 09 70 98 73 93 38 53 60 04 23
```

NOTE: As there are only 16384 routes, it is possible to solve this problem by trying every route. However, Problem 67, is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

先把数据保存到 Mathematica 的默认路径(路径可以通过 Directory[]查到)，文件名为 triangle18.txt

Directory[]
C:\Documents and Settings\Owner\My Documents

下面的这段代码也是通过网络参考人家做 PR67 的 code

```
First[First[Import["triangle18.txt","Table"]//.
{x___,a_,b_}{x,a+Max/@Partition[b,2,1]}]]
1074
```

Problem 19

14 June 2002

You are given the following information, but you may prefer to do some research for yourself.

- 1 Jan 1900 was a Monday.
- Thirty days has September,
  April, June and November.
  All the rest have thirty-one,
  Saving February alone,
  Which has twenty-eight, rain or shine.
  And on leap years, twenty-nine.
- A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

Problem 20

21 June 2002

$n!$ means $n \times (n - 1) \times \ldots \times 3 \times 2 \times 1$

Find the sum of the digits in the number 100!

Mathematica:

```
Plus @@ IntegerDigits[100!]
648
```

Problem 21

05 July 2002

Let d(*n*) be defined as the sum of proper divisors of *n* (numbers less than *n* which divide evenly into *n*).
If d(*a*) = *b* and d(*b*) = *a*, where *a* ≠ *b*, then *a* and *b* are an amicable pair and each of *a* and *b* are called amicable numbers.

For example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; therefore d(220) = 284. The proper divisors of 284 are 1, 2, 4, 71 and 142; so d(284) = 220.

Evaluate the sum of all the amicable numbers under 10000.

Mathematica：

```
Clear[i,j,k,s];
s=0;
For[i=1,i<10000,i++,
  j=Plus @@ Divisors[i] - i;
  If[j<=i,Continue[]];
  k=Plus @@ Divisors[j] - j;
  If[i□k,s+=i;Print[i]];
  ]
s
 220
 1184
 2620
 5020
 6232
```

31626

2009年3月11日星期三使用Total代替Plus的code：★★★

```
totalsum=0;
For[i=0,i<10000,i++,j=Total[Divisors[i]]-i;
 If[j□i,Continue[]];
 test=Total[Divisors[j]]-j;
 If[i□test,totalsum+=i]]
totalsum
```

15276

Problem 22

19 July 2002

Using names.txt (right click and 'Save Link/Target As...'), a 46K text file containing over five-thousand first names, begin by sorting it into alphabetical order. Then working out the alphabetical value for each name, multiply this value by its alphabetical position in the list to obtain a name score.

For example, when the list is sorted into alphabetical order, COLIN, which is worth 3 + 15 + 12 + 9 + 14 = 53, is the 938th name in the list. So, COLIN would obtain a score of 938 $\times$53 = 49714.

What is the total of all the name scores in the file?

Problem 23

02 August 2002

A perfect number is a number for which the sum of its proper divisors is exactly equal to the number. For example, the sum of the proper divisors of 28 would be 1 + 2 + 4 + 7 + 14 = 28, which means that 28 is a perfect number.

A number whose proper divisors are less than the number is called deficient and a number whose proper divisors exceed the number is called abundant.

As 12 is the smallest abundant number, 1 + 2 + 3 + 4 + 6 = 16, the smallest number that can be written as the sum of two abundant numbers is 24. By mathematical analysis, it can be shown that all integers greater than 28123 can be written as the sum of two abundant numbers. However, this upper limit cannot be reduced any further by analysis even though it is known that the greatest number that cannot be expressed as the sum of two abundant numbers is less than this limit.

Find the sum of all the positive integers which cannot be written as the sum of two abundant numbers.

```
Clear[i,j,c,s,upper];
s=0;
c=0;
upper=28123;
For[i=1,i≤100,i++,
  j=Plus @@ Divisors[i]-i;
  If[j>i,s+=i;Print[i];c++];
  ]
Print["Find: ",c];
```

Problem 24

16 August 2002

A permutation is an ordered arrangement of objects. For example, 3124 is one possible permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically or alphabetically, we call it lexicographic order. The lexicographic permutations of 0, 1 and 2 are:

012   021   102   120   201   210

What is the millionth lexicographic permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9?

Mathematica :

```
Timing[lst=Permutations[{0,1,2,3,4,5,6,7,8,9}];]
Part[lst,10^6]
 {2.032 Second,Null}
 {2,7,8,3,9,1,5,4,6,0}
```

So the answer is : 2783915460

Problem 25

30 August 2002

The Fibonacci sequence is defined by the recurrence relation:

$F_n = F_{n-1} + F_{n-2}$, where $F_1 = 1$ and $F_2 = 1$.

Hence the first 12 terms will be:

$F_1 = 1$
$F_2 = 1$
$F_3 = 2$
$F_4 = 3$
$F_5 = 5$
$F_6 = 8$
$F_7 = 13$
$F_8 = 21$
$F_9 = 34$
$F_{10} = 55$
$F_{11} = 89$
$F_{12} = 144$

The 12th term, F_{12}, is the first term to contain three digits.

What is the first term in the Fibonacci sequence to contain 1000 digits?

Mathematica :

```
i=1;
n=1;
While[n<1000,i++;n=Length[IntegerDigits[Fibonacci[i]]]];
i
```

4782

Problem 26

13 September 2002

A unit fraction contains 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

$^1/_2$	=	0.5
$^1/_3$	=	0.(3)
$^1/_4$	=	0.25
$^1/_5$	=	0.2
$^1/_6$	=	0.1(6)
$^1/_7$	=	0.(142857)
$^1/_8$	=	0.125
$^1/_9$	=	0.(1)

$^1/_{10}$ = 0.1

Where 0.1(6) means 0.166666..., and has a 1-digit recurring cycle. It can be seen that $^1/_7$ has a 6-digit recurring cycle.

Find the value of *d* <1000 for which $^1/_d$ contains the longest recurring cycle in its decimal fraction part.

某些循环小数由此性质：

$$10^N \times 0.\overline{M} = M.\overline{M}$$
$$(10^N - 1) \times 0.\overline{M} = M.\overline{M} - 0.\overline{M} = M$$
故，$0.\overline{M} = \frac{M}{10^N - 1}$

也就是1/d=M/(10^N-1)，接着可以得到，M=(10^N-1)/d，而且N即是M的长度

思路一：(思路基本思路二，但不够好)

```
lst=Table[10^i-1,{i,1500}];
lstrep={};
m=0;
max=0;
For[i=3,i<1000,i+=2,n=1;
  While[n<Length[lst]&&Mod[lst[[n]],i]□0,n++;];
  If[n□Length[lst],Continue[]];
  For[j=1,j<Length[lst],j++,If[10^j-1□lst[[n]],Break[]];];
  lstrep=Append[lstrep,j];
  If[j>max,max=j;m=i];
  ]//Timing
(*lstrep*)
Sort[lstrep][[-1]]
m
{2.734,Null}
982
983
```

思路二：（偶数可以除 2 变为更小的奇数或者 2，所以不用计算偶数，当然如果得到的奇数假设是 491 是最长的，那么他的 2 倍，即 982 是最长的。如果在

1000 内，最长数大于 500 的奇数(假设是 a)的话，那么就最长就是 a 了；另外在奇数中，能被 5 整除的都不用计算，假设 a 能被 5 正常，a/5=b,那么 1/a==1/(5*b)==1/5*1/b==0.2*1/b，1/a 的循环数长度也就比 b 长一位。所以如果计算出来的最长数假设是 t，那么如果 5*t<1000 的话，最长的是 5t）

```
lst={};
m=0;
max=0;
For[j=3,j<1000,j+=2,
  If[Mod[j,5]□0,Continue[]];
  For[i=1,i<1500,i++,
   If[Mod[10^i-1,j]□0,Break[]];
   ];
  If[i>max,max=i;m=j;];
  lst=Append[lst,i];
  ]//Timing
Sort[lst][[-1]]
m
{0.953,Null}
982
983
```

2009 年 3 月 11 日星期三美妙的 code：★★★★★

```
f[x_]:=Length[Level[RealDigits[x^-1],{3}]]
f/@Range[1,999];
Ordering[%,-1]
{983}
```

Problem 27

27 September 2002

Euler published the remarkable quadratic formula:

$$n^2 + n + 41$$

It turns out that the formula will produce 40 primes for the consecutive values n = 0 to 39. However, when n = 40, $40^2 + 40 + 41 = 40(40 + 1) +$

41 is divisible by 41, and certainly when $n = 41$, $41^2 + 41 + 41$ is clearly divisible by 41.

Using computers, the incredible formula $n^2 - 79n + 1601$ was discovered, which produces 80 primes for the consecutive values $n = 0$ to 79. The product of the coefficients, −79 and 1601, is −126479.

Considering quadratics of the form:

$n^2 + an + b$, where $|a| < 1000$ and $|b| < 1000$

where $|n|$ is the modulus/absolute value of n
e.g. $|11| = 11$ and $|-4| = 4$

Find the product of the coefficients, a and b, for the quadratic expression that produces the maximum number of primes for consecutive values of n, starting with $n = 0$.

```
lst={};
i=1;
While[Prime[i]<1000,lst=Append[lst,Prime[i]];i++];
len=Length[lst];
max=0;
a1=0;
b1=0;
For[i=1,i≤len,i++,b=lst[[i]];
  For[a=-1000,a≤1000,a++,For[n=0,n<10^3,n+
+,If[PrimeQ[n^2+a*n+b],If[n>max,max=n;a1=a;b1=b];Continue[],Break[]];
     ];
   ];
  ];//Timing
a1
b1
a1*b1
max

{10.969,Null}
-61
971
-59231
70
```

Problem 28

11 October 2002

Starting with the number 1 and moving to the right in a clockwise direction a 5 by 5 spiral is formed as follows:

21	22	23	24	25
20	7	8	9	10
19	6	1	2	11
18	5	4	3	12
17	16	15	14	13

It can be verified that the sum of both diagonals is 101.

What is the sum of both diagonals in a 1001 by 1001 spiral formed in the same way?

Mathematica:

(*主要是找出数字的间隔规律*)

```
Clear[i,j,s];
s=1;
For[i=3,i≤1001,i+=2,
  j=4;
  While[j>0,s+=i^2-(i-1)*(j-1);j--]
  ]
s
 669171001
```

Problem 29

25 October 2002

Consider all integer combinations of a^b for 2 ≤ a ≤ 5 and 2 ≤ b ≤ 5:

2^2=4, 2^3=8, 2^4=16, 2^5=32
3^2=9, 3^3=27, 3^4=81, 3^5=243
4^2=16, 4^3=64, 4^4=256, 4^5=1024
5^2=25, 5^3=125, 5^4=625, 5^5=3125

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms:

4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125

How many distinct terms are in the sequence generated by a^b for $2 \le a \le 100$ and $2 \le b \le 100$?

```
lst={};
max=100;
For[a=2,a≤max,a++,
  For[b=2,b≤max,b++,
    t=a^b;
    If[MemberQ[lst,t],Continue[],lst=Append[lst,t]];
    ];
  ];//Timing
Length[lst]
{6.266,Null}
9183
```

穷举了，数量比较少，不过应该可以分析一下的。

Problem 30

08 November 2002

Surprisingly there are only three numbers that can be written as the sum of fourth powers of their digits:

$1634 = 1^4 + 6^4 + 3^4 + 4^4$
$8208 = 8^4 + 2^4 + 0^4 + 8^4$
$9474 = 9^4 + 4^4 + 7^4 + 4^4$

As $1 = 1^4$ is not a sum it is not included.

The sum of these numbers is 1634 + 8208 + 9474 = 19316.

Find the sum of all the numbers that can be written as the sum of fifth powers of their digits.

开始没能解答是思路错了，以为计算出来的n=7，是表示最大值是到10^7-1，其

实是到10^6-1而已。

技巧{1,2,3,4}^2等于{1,4,9,16}

Mathematica:
```
(*method of exhaustion*)
Clear[i,j,k,t,s,n,lst];
(*get max length*)
For[n=1,n<10,n++,
  If[Length[IntegerDigits[n*9^5]]<n,Break[]];
  ];
s=0;
t=0;
For[i=2,i<10^(n-1),i++,
  If[Plus @@(IntegerDigits[i]^5)□ i,s+=i;Print[i]];
  ]//Timing
s
4150
4151
54748
92727
93084
194979
{20.703,Null}
443839
```

Problem 31

22 November 2002

In England the currency is made up of pound, £, and pence, p, and there are eight coins in general circulation:

1p, 2p, 5p, 10p, 20p, 50p, £1 (100p) and £2 (200p).

It is possible to make £2 in the following way:

1×£1 + 1×50p + 2×20p + 1×5p + 1×2p + 3×1p

How many different ways can £2 be made using any number of coins?

```
lst=Reduce[a+2b+5c+10d+20e+50f+100g□200&&a□ 0&&b□0&&c□0&&d□0&&e□0&
```

```
&f□0&&g□0,{a,b,c,d,e,f,g},Integers];//Timing
Length[lst]
{13.343,Null}
73681
```

差点被题目蒙了，以为答案就是 73681，其实还有一种就是已有一枚硬币，就是£2，所以总共 73681+1 即 73682 种

Problem 32

06 December 2002

We shall say that an n-digit number is pandigital if it makes use of all the digits 1 to n exactly once; for example, the 5-digit number, 15234, is 1 through 5 pandigital.

The product 7254 is unusual, as the identity, 39 × 186 = 7254, containing multiplicand, multiplier, and product is 1 through 9 pandigital.

Find the sum of all products whose multiplicand/multiplier/product identity can be written as a 1 through 9 pandigital.

HINT: Some products can be obtained in more than one way so be sure to only include it once in your sum.

思路:

a*b=c,只存在两种情况,a 一位数，b 四位数，c 四位数，或者 a 两位数，b 三位数，c 四位数。

所以做法是先过滤出所有可能的 a，b，然后再计算出 c，接着判断 abc 是否出现 1 到 9 有且只有 1 次

Mathematica:

```
c2=0;
c3=0;
c4=0;
lst1=Table[i,{i,2,9}];
```

```
lst2={};
lst3={};
lst4={};
For[i=1,i□9,i++,
  For[j=1,j□9,j++,If[i□j,
     c2++;lst2=Append[lst2,i*10+j]
     ];
    For[k=1,k□9,k++,
     If[i□j&&i□k&&j□k,c3++;lst3=Append[lst3,i*100+j*10+k]
      ];
     For[m=1,m□9,m++,
      If[i□j&&i□k&&j□k&&i□m&&j□m&&k□m,
        c4++;lst4=Append[lst4,i*1000+j*100+k*10+m]
        ];
      ];
     ];
    ];
  ];
t=0;
lstresult={};
(*calc 1digits multiply 4digits,result 4digits*)
s=0;
For[i=1,i□Length[lst1],i++,
  For[j=1,j□Length[lst4],j++,
    If[MemberQ[IntegerDigits[Part[lst4,j]],Part[lst1,i]],Continue[]];
    t=Part[lst1,i]*Part[lst4,j];
    lst=Join[IntegerDigits[Part[lst4,j]],IntegerDigits[Part[lst1,i]],IntegerDigits[t]];
    If[Count[lst,0]□ 0,Continue[]];
    For[k=1,k<10,k++,
     If[Count[lst,k]□1,Break[];];
     ];
    If[k□ 10,Print[Part[lst1,i],"*",Part[lst4,j],"==",t],Continue[]];
    If[MemberQ[lstresult,t]□False,lstresult=Append[lstresult,t]];
    ];
  ];
(*calc 2digits multiply 3digits,result 4digits*)
For[i=1,i□ Length[lst2],i++,
  For[j=1,j□Length[lst3],j++,
    lstj1=Join[IntegerDigits[Part[lst2,i]],IntegerDigits[Part[lst3,j]]];
    flag=0;
    For[k=1,k<10,k++,
```

```
     If[Count[lstj1,k]>1,flag=1;Break[];];
     ];
    If[flag==1,Continue[]];

    t=Part[lst2,i]*Part[lst3,j];
    lst=Join[IntegerDigits[Part[lst2,i]],IntegerDigits[Part[lst3,j]],IntegerDigits[t]];
    If[Count[lst,0]□0,Continue[]];
    For[k=1,k<10,k++,
     If[Count[lst,k]□1,Break[];];
     ];
    If[k□10,Print[Part[lst2,i],"*",Part[lst3,j],"==",t],Continue[]];
    If[MemberQ[lstresult,t]□False,lstresult=Append[lstresult,t]];
    ];
  ];
Plus@@lstresult

4 * 1738 == 6952
4 * 1963 == 7852
12 * 483 == 5796
18 * 297 == 5346
27 * 198 == 5346
28 * 157 == 4396
39 * 186 == 7254
42 * 138 == 5796
48 * 159 == 7632
45228
```

Problem 33

20 December 2002

The fraction $^{49}/_{98}$ is a curious fraction, as an inexperienced mathematician in attempting to simplify it may incorrectly believe that $^{49}/_{98} = {}^{4}/_{8}$, which is correct, is obtained by cancelling the 9s.

We shall consider fractions like, $^{30}/_{50} = {}^{3}/_{5}$, to be trivial examples.

There are exactly four non-trivial examples of this type of fraction, less than one in value, and containing two digits in the numerator and denominator.

If the product of these four fractions is given in its lowest common terms, find the value of the denominator.

看不懂题目

Problem 34

03 January 2003

145 is a curious number, as 1! + 4! + 5! = 1 + 24 + 120 = 145.

Find the sum of all numbers which are equal to the sum of the factorial of their digits.

Note: as 1! = 1 and 2! = 2 are not sums they are not included.

Mathematica函数技巧使用Plus@@(IntegerDigits[j]!)，很好！

```
For[i=1,i<10,i++,
  If[10^i>8!,Break[]];
  ];
n=i;
n
s=0;
For[j=3,j<10^n,j++,
  If[j==Plus@@(IntegerDigits[j]!),s+=j;Print[j]];
  ];
s

5
145
40585
40730
```

Problem 35

17 January 2003

The number, 197, is called a circular prime because all rotations of the digits: 197, 971, and 719, are themselves prime.

There are thirteen such primes below 100: 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, and 97.

How many circular primes are there below one million?

```
Timing[
 n=PrimePi[10^6];
 lstp={2};
 lstt={{2}};
 For[i=1,i<=n,i++,
  t=Prime[i];
  If[Count[IntegerDigits[t],0]>0,Continue[]];
  If[Count[IntegerDigits[t],2]>0,Continue[]];
  If[Count[IntegerDigits[t],4]>0,Continue[]];
  If[Count[IntegerDigits[t],6]>0,Continue[]];
  If[Count[IntegerDigits[t],8]>0,Continue[]];
  lstp=Append[lstp,t];
  lstt=Append[lstt,Sort[IntegerDigits[t]]];
  ];
 n=Length[lstp];
 For[i=1,i≤n,i++,
  t=lstp[[i]];
  len=Length[IntegerDigits[t]];
  If[Count[lstt,Sort[IntegerDigits[t]]]!=len,lstp=Delete[lstp,i];n--;i--];
  ];

 s=0;
 lstj={};
 For[i=1,i<10,i++,
  s=0;
  For[j=0,j<6,j++,
   s+=i*10^j;
   If[PrimeQ[s],lstj=Append[lstj,s]];
   ];
  ];
 ]
lstp
lstj
Length[lstp]

{21.687,Null}
```

{2,3,5,7,13,17,31,37,71,73,79,97,113,131,137,157,173,199,311,317,337,359,373,571,593,733,751,919,953,991,1153,1531,1553,1559,1571,1753,1951,3511,3517,3571,5113,5119,5153,5171,5351,5519,5531,5591,5711,5737,7151,7351,7537,7573,7753,7993,9151,9397,9511,9551,9739,9973,11173,11177,11317,11717,11731,13171,13553,13711,17117,17977,19777,19919,19991,33359,35153,35339,35393,35531,35933,55313,55331,59333,71171,71711,77719,91199,97177,97771,99119,99191,379999,399979,793999,799993,939997,993997}
{11,2,3,5,7}
98

做就是这样做了吧，不知为何不对，应该是 98+1(漏的 11)即是 99 个。

Problem 36

31 January 2003

The decimal number, 585 = 1001001001_2 (binary), is palindromic in both bases.

Find the sum of all numbers, less than one million, which are palindromic in base 10 and base 2.

(Please note that the palindromic number, in either base, may not include leading zeros.)

Problem 37

14 February 2003

The number 3797 has an interesting property. Being prime itself, it is possible to continuously remove digits from left to right, and remain prime at each stage: 3797, 797, 97, and 7. Similarly we can work from right to left: 3797, 379, 37, and 3.

Find the sum of the only eleven primes that are both truncatable from left to right and right to left.

NOTE: 2, 3, 5, and 7 are not considered to be truncatable primes.

```
lstp={2};
```

```
(*获取可能的素数*)
For[i=1,i<10^6,i++,
  lst=IntegerDigits[Prime[i]];
  If[lst[[-1]]□9,Continue[]];
  If[lst[[1]]□9,Continue[]];
  If[Count[lst,0]□0,Continue[]];
  If[Count[lst,1]□0,Continue[]];
  If[Count[lst,2]□0,Continue[]];
  If[Count[lst,4]□0,Continue[]];
  If[Count[lst,6]□0,Continue[]];
  If[Count[lst,8]□0,Continue[]];
  lstp=Append[lstp,Prime[i]];
  ];//Timing
s=0;
For[a=5,a□Length[lstp],a++,
  n=lstp[[a]];
  len=Length[IntegerDigits[n]];
     (*from left to right*)
  m=n;
  For[i=1,i<len,i++,
   m=Mod[m,10^(len-i)];
   If[PrimeQ[m],Continue[],Break[]];
   ];
  If[i!=len,Continue[]];
     (*from rigth to left*)
  m=n;
  For[i=1,i<len,i++,
   m=Floor[m/10];
   If[PrimeQ[m],Continue[],Break[]];
   ];
  If[i!=len,Continue[]];
  Print[n];
  s+=n;
  ];//Timing
s
{28.609,Null}
37
53
73
373
```

797
3797
739397
{0.047,Null}
744527

答案不对，不知为什么？加上 2，3，5，7 就总共 11 个素数了，和就变成 744544，但是提交这个数字还是不对。不知是否题目理解错了？

Problem 38

28 February 2003

Take the number 192 and multiply it by each of 1, 2, and 3:

192 ×1 = 192
192 ×2 = 384
192 ×3 = 576

By concatenating each product we get the 1 to 9 pandigital, 192384576. We will call 192384576 the concatenated product of 192 and (1,2,3)

The same can be achieved by starting with 9 and multiplying by 1, 2, 3, 4, and 5, giving the pandigital, 918273645, which is the concatenated product of 9 and (1,2,3,4,5).

What is the largest 1 to 9 pandigital 9-digit number that can be formed as the concatenated product of an integer with (1,2, ... , n) where n >1?

```
For[i=1,i<10^4,i++,
   lst=IntegerDigits[i];
   If[Count[lst,0]≠0,Continue[]];
   lst={};
   For[j=1,j≤9,j++,
    t=i*j;
    lst=Join[lst,IntegerDigits[t]];
    If[Count[lst,0]≠0,Break[]];
    flag=0;
    c=0;
    For[k=1,k<10,k++,
     c=Count[lst,k];
```

```
     If[c>1,flag=1;Break[]];
     ];
    If[flag≠1,Break[]];
    c=0;
    For[k=1,k<10,k++,
     c=Count[lst,k];
     If[c==0,Break[]];
     ];
    If[k≠10,Continue[]];
    Print[i," ",j," ",lst];
    Break[];
    ];
   ];//Timing
1   9   {1,2,3,4,5,6,7,8,9}
9   5   {9,1,8,2,7,3,6,4,5}
192   3   {1,9,2,3,8,4,5,7,6}
219   3   {2,1,9,4,3,8,6,5,7}
273   3   {2,7,3,5,4,6,8,1,9}
327   3   {3,2,7,6,5,4,9,8,1}
6729   2   {6,7,2,9,1,3,4,5,8}
6792   2   {6,7,9,2,1,3,5,8,4}
6927   2   {6,9,2,7,1,3,8,5,4}
7269   2   {7,2,6,9,1,4,5,3,8}
7293   2   {7,2,9,3,1,4,5,8,6}
7329   2   {7,3,2,9,1,4,6,5,8}
7692   2   {7,6,9,2,1,5,3,8,4}
7923   2   {7,9,2,3,1,5,8,4,6}
7932   2   {7,9,3,2,1,5,8,6,4}
9267   2   {9,2,6,7,1,8,5,3,4}
9273   2   {9,2,7,3,1,8,5,4,6}
9327   2   {9,3,2,7,1,8,6,5,4}
{1.14,Null}
```

开始问题没有看懂，以为是要求 9327，其实是求最大的 932718654

Problem 39

14 March 2003

If p is the perimeter of a right angle triangle with integral length sides, $\{a,b,c\}$, there are exactly three solutions for $p = 120$.

{20,48,52}, {24,45,51}, {30,40,50}

For which value of *p* ≤1000, is the number of solutions maximised?

Mathematica的解方程和列表操作

```
Timing[
 (*求解出所有P≤1000内所有可能的解
*)lst=Reduce[a^2+b^2==c^2&&a+b+c≤1000&&a>0&&b>0&&c>0&&a<b,
{a,b,c},Integers];
 (*使用列表来存储每个P对应的解的个数*)
 lstp=Table[0,{1000}];
 For[i=1,i≤Length[lst],i++,s=Plus@@(lst[[i,All,2]]);
   t=lstp[[s]]+1;
   lstp=ReplacePart[lstp,s→ t];]
  (*遍历列表,得到最多解对应的P*)
  max=0;
 j=0;
 For[i=1,i≤Length[lstp],i++,If[lstp[[i]]>max,max=lstp[[i]];j=i;];];
 j
 ]
```

{19.046,840}

开始理解错题目了，以为是求最大的 P(P 的最大值可以取到 1000)；提交答案时显示不对，后面再仔细研读题目，说的应该是求 P，使得符合 P 的解决方案最多！

Problem 40

28 March 2003

An irrational decimal fraction is created by concatenating the positive integers:

0.123456789101112131415161718192021...

It can be seen that the 12th digit of the fractional part is 1.

If d_n represents the n^{th} digit of the fractional part, find the value of the following expression.

$$d_1 \times d_{10} \times d_{100} \times d_{1000} \times d_{10000} \times d_{100000} \times d_{1000000}$$

思路一：(数据量太多，无法解决)

```
lst=Table[i,{i,9}];
For[i=10,i<10^6,i++,
 lst=Join[lst,IntegerDigits[i]];
 ]
s=1;
For[i=0,i<7,i++,
 s*=lst[[10^i]];
 ]
```

方法二：计算出把 10 内，100 内，1000 内....的数编程连续小数总共的位数

```
s=0;
For[n=0,n<100,n++,
 s+=9*10^n*(n+1);
 Print[10^(n+1)," ",s];
 If[s>10^6,Break[]];
 ]
10   9
100   189
1000   2889
10000   38889
100000   488889
1000000   5888889
```

就是 10 内的数共 9 个，第 9 个就是 10-1 即是 9

100 内的数共 189 个，第 189 个数是 100-1 的最后一位数，也是 9，

1000 内的数共 2889 个，第 2889 个数是 1000-1 的最后一位数

后面的照推即可

现在要求

明显：d1=1，d10=1

求 d100,

99 是第 188 和 189 个数字，98 是第 186 和 187 个数字，99 和 98 差值是 1，而 188 和 186 差值是 2；

先求出 100 附近的数，188-2x=100，x=44，所以第 100 开始的数字是 99-44=55，所以 d100=5

求 d1000，

999 是第 2887，2888 和 2889 个数字，998 是第 2884，2885 和 2886 个数字，差值是 1 对 3；

先求出 1000 附近的数，2887-3x=1000，x=1887/3=629，所以第 1000 开始的数字是 999-629=370，所以 d1000=3

求 d10000，...，得到 d10000=7

求 d100000，...，得到 d100000=2

求 d1000000，...，得到 d1000000=1

所以 p=1*1*5*3*7*2*1=210

当然可以把思路变成程序计算的方式

Problem 41

11 April 2003

We shall say that an *n*-digit number is pandigital if it makes use of all the digits 1 to *n* exactly once. For example, 2143 is a 4-digit pandigital and is also prime.

What is the largest *n*-digit pandigital prime that exists?

可能有用的

```
Reduce[Prime[x]>9*10^9&&x>1,x,Integers]
x□ Integers&&x□ 411523196
```

运算时间太久，没有计算出！

```
lst=Permutations[{9,8,7,6,5,4,3,2,1}];
len=Length[lst];
For[i=1,i□len,i++,
 If[Mod[lst [[i,-1]],2]□0,Continue[]];
 j=1;
 t=0;
 While[j□9,t+=lst [[i,j]]*10^(9-j);j++];
 If[PrimeQ[t],Print[t];Break[]];
 ]
```

思路三：（耗时已经超过 1 分钟，应该优化，而且这里是穷举法 method of exhaustion）

```
lst={};
For[a1=9,a1>0,a1--,
  For[a2=9,a2>0,a2--,
   If[a2□a1,Continue[]];
   For[a3=9,a3>0,a3--,
    If[a1□a3||a2□a3,Continue[]];
    For[a4=9,a4>0,a4--,
     If[a1□a4||a2□a4||a3□a4,Continue[]];
     For[a5=9,a5>0,a5--,
      If[a1□a5||a2□a5||a3□a5||a4□a5,Continue[]];
      For[a6=9,a6>0,a6--,
       If[a1□a6||a2□a6||a3□a6||a4□a6||a5□a6,Continue[]];
       For[a7=9,a7>0,a7--,
        If[a1□a7||a2□a7||a3□a7||a4□a7||a5□a7||a6□a7,Continue[]];
```

```
        For[a8=9,a8>0,a8--,
         If[a1□a8||a2□a8||a3□a8||a4□a8||a5□a8||a6□a8||a7□a8,Continue[]];
         For[a9=9,a9>0,a9-=2,
          If[a1□a9||a2□a9||a3□a9||a4□a9||a5□a9||a6□a9||a7□a9||a8□a9,Continue[]];

t=a1*10^8+a2*10^7+a3*10^6+a4*10^5+a5*10^4+a6*10^3+a7*10^2+a8*10^1+a9;
          If[PrimeQ[t],Print[t];Return[]];
          ]
         ]
        ]
       ]
      ]
     ]
    ]
   ]
  ]//Timing
{58.875,Null}
max=8;
For[a1=max,a1>0,a1--,
  For[a2=max,a2>0,a2--,
   If[a2□a1,Continue[]];
   For[a3=max,a3>0,a3--,
    If[a1□a3||a2□a3,Continue[]];
    For[a4=max,a4>0,a4--,
     If[a1□a4||a2□a4||a3□a4,Continue[]];
     For[a5=max,a5>0,a5--,
      If[a1□a5||a2□a5||a3□a5||a4□a5,Continue[]];
      For[a6=max,a6>0,a6--,
       If[a1□a6||a2□a6||a3□a6||a4□a6||a5□a6,Continue[]];
       For[a7=max,a7>0,a7--,
        If[a1□a7||a2□a7||a3□a7||a4□a7||a5□a7||a6□a7,Continue[]];
        For[a9=max,a9>0,a9-=2,
         If[a1□a9||a2□a9||a3□a9||a4□a9||a5□a9||a6□a9||a7□a9,Continue[]];
         t=a1*10^7+a2*10^6+a3*10^5+a4*10^4+a5*10^3+a6*10^2+a7*10^1+a9;
         If[PrimeQ[t],Print[t];Return[]];
         ]
        ]
       ]
      ]
     ]
    ]
```

```
   ]
  ]//Timing
{5.312,Null}
max=7;
For[a1=max,a1>0,a1--,
  For[a2=max,a2>0,a2--,
   If[a2□a1,Continue[]];
   For[a3=max,a3>0,a3--,
    If[a1□a3||a2□a3,Continue[]];
    For[a4=max,a4>0,a4--,
     If[a1□a4||a2□a4||a3□a4,Continue[]];
     For[a5=max,a5>0,a5--,
      If[a1□a5||a2□a5||a3□a5||a4□a5,Continue[]];
      For[a6=max,a6>0,a6--,
       If[a1□a6||a2□a6||a3□a6||a4□a6||a5□a6,Continue[]];
       For[a9=max,a9>0,a9-=2,
        If[a1□a9||a2□a9||a3□a9||a4□a9||a5□a9||a6□a9,Continue[]];
        t=a1*10^6+a2*10^5+a3*10^4+a4*10^3+a5*10^2+a6*10^1+a9;
        If[PrimeQ[t],Print[t];Return[]];
        ]
       ]
      ]
     ]
    ]
   ]
  ]//Timing
7652413
{0.015,Return[]}
```

Problem 43

09 May 2003

The number, 1406357289, is a 0 to 9 pandigital number because it is made up of each of the digits 0 to 9 in some order, but it also has a rather interesting sub-string divisibility property.

Let d_1 be the 1st digit, d_2 be the 2nd digit, and so on. In this way, we note the following:

- $d_2d_3d_4$=406 is divisible by 2
- $d_3d_4d_5$=063 is divisible by 3

- $d_4d_5d_6$=635 is divisible by 5
- $d_5d_6d_7$=357 is divisible by 7
- $d_6d_7d_8$=572 is divisible by 11
- $d_7d_8d_9$=728 is divisible by 13
- $d_8d_9d_{10}$=289 is divisible by 17

Find the sum of all 0 to 9 pandigital numbers with this property.

```
Clear[i,j,k,t,lst2,lst3,lst5,lst7,lst11,lst13,lst17];
lst2={};
lst3={};
lst5={};
lst7={};
lst11={};
lst13={};
lst17={};
For[i=0,i<10,i++,
 For[j=0,j<10,j++,
  If[ij,Continue[]];
  For[k=0,k<10,k++,
   If[ik||jk,Continue[]];
   t=i*100+j*10+k;
   If[Mod[t,2]0,lst2=Append[lst2,t]];
   If[Mod[t,3]0,lst3=Append[lst3,t]];
   If[Mod[t,5]0,lst5=Append[lst5,t]];
   If[Mod[t,7]0,lst7=Append[lst7,t]];
   If[Mod[t,11]0,lst11=Append[lst11,t]];
   If[Mod[t,13]0,lst13=Append[lst13,t]];
   If[Mod[t,17]0,lst17=Append[lst17,t]];
   ]
  ]
 ]
```

思路二：(穷举的累，代码写的烂，也叫做“死路二”,不过可以很快得到结果，几乎不要时间啊，哈！思路应该不错，应该可以在思路上优化代码)

```
s=0;
lst={};
lst2={};
lst3={};
```

```
lst5={};
lst7={};
lst11={};
lst13={};
lst17={};
lst01={};lst02={};lst03={};lst04={};lst05={};lst06={};
For[i=Ceiling[100/17],i<Ceiling[1000/17],i++,
  lst={};
  t=17*i;
  flag=0;
  For[j=0,j<10,j++,
   If[Count[IntegerDigits[t],j]>1,flag=1;Break[];]
   ];
  If[flag==1,Continue[]];
  lst=IntegerDigits[t];

  (*calc lst13*)
  t1=Floor[t/10];
  For[j1=0,j1<10,j1++,
   If[MemberQ[lst,j1],Continue[]];
   t2=100*j1+t1;
   If[Mod[t2,13]==0,lst13=Append[lst13,t2],Continue[]];
   lst01={};
   lst01=Join[IntegerDigits[j1],lst];

   (*calc lst11*)
   t3=Floor[t2/10];
   For[j2=0,j2<10,j2++,
    If[MemberQ[lst01,j2],Continue[]];
    t4=100*j2+t3;
    If[Mod[t4,11]==0,lst11=Append[lst11,t4],Continue[]];
    lst02={};
    lst02=Join[IntegerDigits[j2],lst01];

    (*calc lst7*)
    t5=Floor[t4/10];
    For[j3=0,j3<10,j3++,
     If[MemberQ[lst02,j3],Continue[]];
     t6=100*j3+t5;
     If[Mod[t6,7]==0,lst7=Append[lst7,t6],Continue[]];
     lst03={};
```

```
     lst03=Join[IntegerDigits[j3],lst02];

     (*calc lst5*)
     t7=Floor[t6/10];
     For[j4=0,j4<10,j4++,
      If[MemberQ[lst03,j4],Continue[]];
      t8=100*j4+t7;
      If[Mod[t8,5]==0,lst5=Append[lst5,t8],Continue[]];
      lst04={};
      lst04=Join[IntegerDigits[j4],lst03];

      (*calc lst3*)
      t9=Floor[t8/10];
      For[j5=0,j5<10,j5++,
       If[MemberQ[lst04,j5],Continue[]];
       t10=100*j5+t9;
       If[Mod[t10,3]==0,lst3=Append[lst3,t10],Continue[]];
       lst05={};
       lst05=Join[IntegerDigits[j5],lst04];

       (*calc lst2*)
       t11=Floor[t10/10];
       For[j6=0,j6<10,j6++,
        If[MemberQ[lst05,j6],Continue[]];
        t12=100*j6+t11;
        If[Mod[t12,2]==0,lst2=Append[lst2,t12],Continue[]];
        lst06={};
        lst06=Join[IntegerDigits[j6],lst05];

        (*get result*)
        For[j7=0,j7<10,j7++,
         If[MemberQ[lst06,j7],Continue[]];
         Print[Join[{j7},lst06]];
         Print["sum=
",s+=j7*10^9+j6*10^8+j5*10^7+j4*10^6+j3*10^5+j2*10^4+j1*10^3+t];
         Break[];
         ];

        ];
       ];
      ];
```

```
     ];
    ];
   ];
  lst17=Append[lst17,t];
  ]//Timing
{4,1,6,0,3,5,7,2,8,9}
sum=  4160357289
{1,4,6,0,3,5,7,2,8,9}
sum=  5620714578
{4,1,0,6,3,5,7,2,8,9}
sum=  9727071867
{1,4,0,6,3,5,7,2,8,9}
sum=  11133429156
{4,1,3,0,9,5,2,8,6,7}
sum=  15264382023
{1,4,3,0,9,5,2,8,6,7}
sum=  16695334890
{0.047,Null}
```

方法三：(exhaustion)

```
lst=Permutations[{0,1,2,3,4,5,6,7,8,9}];
len=Length[lst];
s=0;
For[i=9!+1,i<len,i++,
  lst1=Part[lst,i];
  (*fliter*)
  If[lst1[[6]]□5,Continue[]];
  If[Mod[lst1[[4]],2]□0,Continue[]];

  For[j=2,j□8,j++,
   t=lst1[[j]]*100+lst1[[j+1]]*10+lst1[[j+2]];
   If[Mod[t,Prime[j-1]]□0,Break[];];
   ];
  If[j□9,Print[lst1],Continue[]];
  For[j=1,j<=10,j++,
   s+=(lst1[[11-j]])*10^(j-1);
   ];
  ]//Timing
Print["sum= ",s];
{1,4,0,6,3,5,7,2,8,9}
```

```
{1,4,3,0,9,5,2,8,6,7}
{1,4,6,0,3,5,7,2,8,9}
{4,1,0,6,3,5,7,2,8,9}
{4,1,3,0,9,5,2,8,6,7}
{4,1,6,0,3,5,7,2,8,9}
{52.125,Null}
sum= 16695334890
```

Problem 44

23 May 2003

Pentagonal numbers are generated by the formula, $P_n=n(3n-1)/2$. The first ten pentagonal numbers are:

1, 5, 12, 22, 35, 51, 70, 92, 117, 145, ...

It can be seen that $P_4 + P_7 = 22 + 70 = 92 = P_8$. However, their difference, $70 - 22 = 48$, is not pentagonal.

Find the pair of pentagonal numbers, P_j and P_k, for which their sum and difference is pentagonal and $D = |P_k - P_j|$ is minimised; what is the value of D?

```
len=10^3;
min=0;
lstpn=Table[n (3 n-1)/2,{n,len}];
interval=Floor[len/2];
For[i=2,i<interval,i++,
 For[j=1,j≤interval-i,j++,
  s=Part[lstpn,j+i]+Part[lstpn,j];
  d=Part[lstpn,j+i]-Part[lstpn,j];

If[MemberQ[lstpn,s]&&MemberQ[lstpn,d],If[min<d,min=d];Print["Find:",Part[lstpn,
j]," ",Part[lstpn,j+i]]];
  If[MemberQ[lstpn,s],Print["inter:",i," ",Part[lstpn,j]," ",Part[lstpn,j+i]]];
  ];
 ]//Timing
min
```

思路，应该可以用数学的方法得到一些比较好的结论

```
Clear[i,j,k,m];
```

Reduce[3*(i^2)-i+3*(j^2)-j3*(k^2)-k&&3*(i^2)-i-3*(j^2)+j3*(m^2)-m&&i>j&&1<i&&1<j&&1<k<50&&1<m,{i,j,k,m},Integers]

(i|j|k|m)Integers&&((2i35&&1<j<1/6+1/6 23 12 i 36 i 2

&&k1/6+1/6 1 12 i 36 i 2 12 j 36 j 2 &&m1/6+1/6

1 12 i 36 i 2 12 j 36 j 2)||(36i49&&1<j<1/6+1/6

8 9 4 0 1 12 i 36 i 2 &&k1/6+1/6

1 12 i 36 i 2 12 j 36 j 2 &&m1/6+1/6

1 12 i 36 i 2 12 j 36 j 2))

Problem 45

06 June 2003

Triangle, pentagonal, and hexagonal numbers are generated by the following formulae:

Triangle	$T_n=n(n+1)/2$	1, 3, 6, 10, 15, ...
Pentagonal	$P_n=n(3n-1)/2$	1, 5, 12, 22, 35, ...
Hexagonal	$H_n=n(2n-1)$	1, 6, 15, 28, 45, ...

It can be verified that $T_{285} = P_{165} = H_{143} = 40755$.

Find the next triangle number that is also pentagonal and hexagonal.

max=10^10;Reduce[a (a+1)/2==b (3*b-1)/2&&a (a+1)/2c (2*c-1)&&1<a<max&&1<b<max&&1<c<max,{a,b,c},Integers]
(a285&&b165&&c143)||(a55385&&b31977&&c27693)||
(a10744501&&b6203341&&c5372251)||
(a2084377905&&b1203416145&&c104219
a=55385;
a (a+1)/2
1533776805

Problem 46

20 June 2003

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square.

$9 = 7 + 2\times1^2$
$15 = 7 + 2\times2^2$
$21 = 3 + 2\times3^2$
$25 = 7 + 2\times3^2$
$27 = 19 + 2\times2^2$
$33 = 31 + 2\times1^2$

It turns out that the conjecture was false.

What is the smallest odd composite that cannot be written as the sum of a prime and twice a square?

思路：存储平方数的两倍到列表lst中，从3开始穷举非素数的奇数a，中间把搜索a加上一个素数得到的值是否在lst中，如果不在，那么就得到结果，在的话就跳出并得到下一个奇数并进入下一个循环

```
lst=Table[2*i^2,{i,1000}];
lstprime=Table[Prime[i],{i,1000}];
len=Length[lstprime];
For[i=3,i<10^10,i+=2,
  If[PrimeQ[i],Continue[]];
  For[j=1,j<len,j++,
   If[MemberQ[lst,i-Part[lstprime,j]],Break[]]
   ];
  If[j□len,Print[i];Break[]];
  ]//Timing
5777
{25.641,Null}
```

Problem 47

04 July 2003

The first two consecutive numbers to have two distinct prime factors are:

$14 = 2 \times 7$
$15 = 3 \times 5$

The first three consecutive numbers to have three distinct prime factors are:

$644 = 2^2 \times 7 \times 23$
$645 = 3 \times 5 \times 43$
$646 = 2 \times 17 \times 19$.

Find the first four consecutive integers to have four distinct primes factors. What is the first of these numbers?

```
c=0;
For[i=1,i<10^6,i++,
  If[Length[FactorInteger[i]]⩵4,c++,c=0];
  If[c⩵4,Break[]]
  ]
i-3
i-2
i-1
i
 134043
 134044
 134045
 134046
```

Problem 48

18 July 2003

The series, $1^1 + 2^2 + 3^3 + ... + 10^{10} = 10405071317$.

Find the last ten digits of the series, $1^1 + 2^2 + 3^3 + ... + 1000^{1000}$.

```
s=0;
For[i=1,i<1000,i++,
    s+=i^i;
    ];
lst=IntegerDigits[s];
```

```
Take[lst,-10]
 {9,1,1,0,8,4,6,7,0,0}
9110846700
```

Problem 49

01 August 2003

The arithmetic sequence, 1487, 4817, 8147, in which each of the terms increases by 3330, is unusual in two ways: (i) each of the three terms are prime, and, (ii) each of the 4-digit numbers are permutations of one another.

There are no arithmetic sequences made up of three 1-, 2-, or 3-digit primes, exhibiting this property, but there is one other 4-digit increasing sequence.

What 12-digit number do you form by concatenating the three terms in this sequence?

还没有想到解决方法

```
(*search 4digits prime*)
lstp={};
For[i=10^3+1,i<10^4,i+=2,
  If[PrimeQ[i],lstp=Append[lstp,i]];
  ];
len=Length[lstp];
len
lstt={};
For[i=1,i□len,i++,
 lstt=Append[lstt,Sort[IntegerDigits[lstp[[i]]]]];
 ]
Length[lstt]
lstresult={};
t=1487;
For[i=1,i□len,i++,
 (*c=Count[lstt,Sort[IntegerDigits[lstp[[i]]]]];*)

If[Sort[IntegerDigits[lstp[[i]]]]==Sort[IntegerDigits[t]],Print[lstp[[i]]];If[PrimeQ[lstp[[i]]],Print["Prime:",lstp[[i]]]]];
```

```
 ]
1061
1061
1487
Prime: 1487
1847
Prime: 1847
4817
Prime: 4817
4871
Prime: 4871
7481
Prime: 7481
7841
Prime: 7841
8147
Prime: 8147
8741
Prime: 8741
```

Problem 50

15 August 2003

The prime 41, can be written as the sum of six consecutive primes:

$$41 = 2 + 3 + 5 + 7 + 11 + 13$$

This is the longest sum of consecutive primes that adds to a prime below one-hundred.

The longest sum of consecutive primes below one-thousand that adds to a prime, contains 21 terms, and is equal to 953.

Which prime, below one-million, can be written as the sum of the most consecutive primes?

思路：(有点取巧)求出的结果差不多接近 10^6 就有 9 成把握对

```
For[n=1,n<10^10,n++,
  If[Prime[n]>10^6,Break[]];
```

```
  ];
n=n-1;
lstp=Table[Prime[i],{i,n}];
i=Floor[n/100];
i=800;
i
flag=0;
For[j=i-1,j>0,j--,
  For[k=1,k<=i-j,k++,
   s=Plus@@Take[lstp,{k,k+j}];
   If[PrimeQ[s]&&s<10^6,Print["terms:",j+1," sum=",s,"
lst=",lst[[k+j]]];flag=1;Break[]];
   ];
  If[flag⩵1,Break[]];
  ]//Timing
800
terms: 543  sum= 997651  lst= 3931
{5.469,Null}
```

Problem 51

29 August 2003

By replacing the 1st digit of *57, it turns out that six of the possible values: 157, 257, 457, 557, 757, and 857, are all prime.

By replacing the 3rd and 4th digits of 56**3 with the same digit, this 5-digit number is the first example having seven primes, yielding the family: 56003, 56113, 56333, 56443, 56663, 56773, and 56993. Consequently 56003, being the first member of this family, is the smallest prime with this property.

Find the smallest prime which, by replacing part of the number (not necessarily adjacent digits) with the same digit, is part of an eight prime value family.

网络代码：

```
base[k_]:=Array[a,k]
space[k_,i_]:=Subsets[Range[k-1],{i}]
remain[k_,i_]:=Select[IntegerDigits/@Range[10^(k-2-i),10^(k-1-i)-
1],Mod[Total[#],3]≠0&]
```

```
remplace[k_,i_]:=Complement[Range[k-1],#]&/@space[k,i]
last={1,3,7,9};
tab[k_,i_]:=Table[base[k]/.(Thread[Rule[base[k][[#]]&/@o,m]]),{o,remplace[k,i]},
{m,remain[k,i]}]//Flatten[#,1]&
lastnum[n_,k_]:=Table[tab[n,k]/.a[n]→ z,{z,last}]//Flatten[#,1]&
d=Dispatch[Table[a[_]→i,{i,0,9}]];
f=DeleteCases[#,a_/;First[a]==0]&;
end[n_,k_]:=Map[FromDigits,#,{2}]&@(f/@Outer[ReplaceAll,lastnum[n,k],d,1])

Sort[Select[end[6,3],Total@Boole@PrimeQ@#==8&]]//Select[#[[1]],PrimeQ]&//Timin
g

{0.491,{121313,222323,323333,424343,525353,626363,828383,929393}}
```

Problem 52

12 September 2003

It can be seen that the number, 125874, and its double, 251748, contain exactly the same digits, but in a different order.

Find the smallest positive integer, *x*, such that 2*x*, 3*x*, 4*x*, 5*x*, and 6*x*, contain the same digits.

```
lst={};
For[i=1,i<10^6,i++,
  lst=IntegerDigits[i];
  (*如果最高位大于4，跳到下一长度整数*)
  If[lst[[1]]>4,i=10^Length[lst]];
  lst=Sort[IntegerDigits[i]];
  For[j=2,j≤6,j++,
   If[lst≠Sort[IntegerDigits[j*i]],Break[]];
   ];
  If[j==7,Print[i];Break[]];
  ];//Timing

142857
{3.797,Null}
Timing[f1=SameQ@@Sort/@IntegerDigits[# Range@6]&;

 Do[If[f1@#,Return@#]&[10^i+j],{i,5},{j,10^i-1}]]
```

{2.937,142857}

Problem 53

26 September 2003

There are exactly ten ways of selecting three from five, 12345:

123, 124, 125, 134, 135, 145, 234, 235, 245, and 345

In combinatorics, we use the notation, ${}^5C_3 = 10$.

In general,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$,where $r \leq n$, $n! = n\times(n-1)\times\ldots\times3\times2\times1$, and $0! = 1$.

It is not until $n = 23$, that a value exceeds one-million: ${}^{23}C_{10} = 1144066$.

How many, not necessarily distinct, values of ${}^nC_r$, for $1 \leq n \leq 100$, are greater than one-million?

```
c=0;
max=100;
For[n=1,n≤max,n++,
  For[r=1,r≤n,r++,
    If[n!/(r!*(n-r)!)>10^6,c++;(*Print["C[",n,",",r,"]"]*)];
    ];
  ];//Timing
c
{0.109,Null}
4075
Length[Select[Flatten[Table[Binomial[n,m],{n,0,100},
{m,0,n}]],#>1000000&]]//Timing
{0.016,4075}
```

Problem 56

07 November 2003

A googol (10^{100}) is a massive number: one followed by one-hundred zeros; 100^{100} is almost unimaginably large: one followed by two-hundred zeros. Despite their size, the sum of the digits in each number is only 1.

Considering natural numbers of the form, a^b, where $a, b < 100$, what is the maximum digital sum?

```
max=0;
For[i=1,i<100,i++,
 For[j=1,j<100,j++,
  s=Plus@@IntegerDigits[i^j];
  If[s>max,max=s];
  ]
 ]
max
972
```

Problem 60

02 January 2004

The primes 3, 7, 109, and 673, are quite remarkable. By taking any two primes and concatenating them in any order the result will always be prime. For example, taking 7 and 109, both 7109 and 1097 are prime. The sum of these four primes, 792, represents the lowest sum for a set of four primes with this property.

Find the lowest sum for a set of five primes for which any two primes concatenate to produce another prime.

Problem 63

13 February 2004

The 5-digit number, $16807=7^5$, is also a fifth power. Similarly, the 9-digit number, $134217728=8^9$, is a ninth power.

How many n-digit positive integers exist which are also an nth power?

幂最高次设为 100

```
c=0;
For[i=1,i<10,i++,
 For[j=1,j<100,j++,
  t=i^j;
  lst=IntegerDigits[t];
  If[Length[lst]==j,c++;Print[t,"=",i,"^",j]];
  ]
 ]
c
1 = 1 ^ 1
2 = 2 ^ 1
3 = 3 ^ 1
4 = 4 ^ 1
16 = 4 ^ 2
5 = 5 ^ 1
25 = 5 ^ 2
125 = 5 ^ 3
6 = 6 ^ 1
36 = 6 ^ 2
216 = 6 ^ 3
1296 = 6 ^ 4
7 = 7 ^ 1
49 = 7 ^ 2
343 = 7 ^ 3
2401 = 7 ^ 4
16807 = 7 ^ 5
117649 = 7 ^ 6
8 = 8 ^ 1
64 = 8 ^ 2
512 = 8 ^ 3
4096 = 8 ^ 4
32768 = 8 ^ 5
262144 = 8 ^ 6
2097152 = 8 ^ 7
16777216 = 8 ^ 8
134217728 = 8 ^ 9
1073741824 = 8 ^ 10
9 = 9 ^ 1
81 = 9 ^ 2
729 = 9 ^ 3
6561 = 9 ^ 4
```

59049 = 9 ^ 5
531441 = 9 ^ 6
4782969 = 9 ^ 7
43046721 = 9 ^ 8
387420489 = 9 ^ 9
3486784401 = 9 ^ 10
31381059609 = 9 ^ 11
282429536481 = 9 ^ 12
2541865828329 = 9 ^ 13
22876792454961 = 9 ^ 14
205891132094649 = 9 ^ 15
1853020188851841 = 9 ^ 16
16677181699666569 = 9 ^ 17
150094635296999121 = 9 ^ 18
1350851717672992089 = 9 ^ 19
12157665459056928801 = 9 ^ 20
109418989131512359209 = 9 ^ 21
49

Problem 67

09 April 2004

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

3
7 5
2 4 6
8 5 9 3

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom in triangle.txt (right click and 'Save Link/Target As...'), a 15K text file containing a triangle with one-hundred rows.

NOTE: This is a much more difficult version of Problem 18. It is not possible to try every route to solve this problem, as there are 2^{99} altogether! If you could check one trillion (10^{12}) routes every second it would take over twenty billion years to check them all. There is an efficient algorithm to solve it. ;o)

网上的答案，不懂这段 code

First[First[Import["triangle.txt","Table"]//.
{x___,a_,b_}{x,a+Max/@Partition[b,2,1]}]]
7273

Problem 92

01 April 2005

A number chain is created by continuously adding the square of the digits in a number to form a new number until it has been seen before.

For example,

44 →32 →13 →10 →**1** →**1**
85 →**89** →145 →42 →20 →4 →16 →37 →58 →**89**

Therefore any chain that arrives at 1 or 89 will become stuck in an endless loop. What is most amazing is that EVERY starting number will eventually arrive at 1 or 89.

How many starting numbers below ten million will arrive at 89?

Problem 97

10 June 2005

The first known prime found to exceed one million digits was discovered in 1999, and is a Mersenne prime of the form $2^{6972593}-1$; it contains exactly 2,098,960 digits. Subsequently other Mersenne primes, of the form 2^p-1, have been found which contain more digits.

However, in 2004 there was found a massive non-Mersenne prime which contains 2,357,207 digits: $28433\times2^{7830457}+1$.

Find the last ten digits of this prime number.

Mod[28433*(2^7830457)+1,10^10]
8739992577

Problem 108

04 November 2005

In the following equation x, y, and n are positive integers.

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{n}$$

For $n = 4$ there are exactly three distinct solutions:

$$\frac{1}{5}+\frac{1}{20}=\frac{1}{4}$$
$$\frac{1}{6}+\frac{1}{12}=\frac{1}{4}$$
$$\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$$

What is the least value of n for which the number of distinct solutions exceeds one-thousand?

NOTE: This problem is an easier version of problem 110; it is strongly advised that you solve this one first.

```
n=2*3*5*7*11*13*6;
k=1;c=0;M=2*n;
For[i=1,i≤n,i++,
  t=Mod[n*(n+i),i];
  If[t⩵0,c++;]
  ]//Timing
Print[n,",",c];
{1.723,Null}
180180 , 1013
```

上面的想法是蒙的，不算；

1/x+1/y=1/n

x 最小值为 n+1，最大值为 2*n，主要看 1/n-1/x 能否表示为 1/y 的形式；

推出 1/y=1/n-1/x=(x-n)/x*n

即 y = x*n/(x-n)

由于 x 可以用 n+i(i 的值从 1 到 n)表示，

所以 y = n*(n+i)/i = (n*n/i)+n

所以 y 的个数解，其实就是 n 的平方能除尽 i 的个数

```
n=2;
len=0;
k=2;
While[len<10^3,n*=Prime[k];k++;
 len=Length[Select[Divisors[n^2],#≤n&]];
 Print[len,",",n];
 Print[FactorInteger[n]];
 ]
5 , 6
{{2,1},{3,1}}
14 , 30
{{2,1},{3,1},{5,1}}
41 , 210
{{2,1},{3,1},{5,1},{7,1}}
122 , 2310
{{2,1},{3,1},{5,1},{7,1},{11,1}}
365 , 30030
{{2,1},{3,1},{5,1},{7,1},{11,1},{13,1}}
1094 , 510510
{{2,1},{3,1},{5,1},{7,1},{11,1},{13,1},{17,1}}

n=2;
len=0;
k=2;
While[len<10^3,n*=Prime[k];k++;
 n=2*3*5*7*11*13*2*3;
```

```
 len=Length[Select[Divisors[n^2],#□n&]];
 Print[len,",",n];
 Print[FactorInteger[n]];
 Break[];
 ]
1013 , 180180
{{2,2},{3,2},{5,1},{7,1},{11,1},{13,1}}
```

要找出素因素（prime factors）分解与因素分解之间的关系，才能做的比较快，也比较好着手做 Problem 110

网友提交的解决代码：

方法一：

```
sol[n_]:=For[m=1,DivisorSigma[0,m^2]<2*n-1,m++]//m&
sol[1000]//Timing
{13.94,180180}
```

方法二：

```
n=1;
While[Fold[2 ##+##&,0,FactorInteger[++n][[All,2]]]<999]//Timing
n
{4.776,Null}
180180
```