Equitable Active-Reactive Power Envelopes for Managing Distributed Energy Resources in Power Distribution systems

Appendix

A. Optimization Model of PQ-E Method

The compact form of allowable power region (10) is expressed as

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}} + \boldsymbol{B}\boldsymbol{Q}_{E} \le \boldsymbol{W} \tag{A1}$$

The rectangle P-Q envelope of the i_{th} DER is formulated as

$$\Omega_i = \{(P_{\mathrm{E},i}, Q_{\mathrm{E},i}) \mid P_{\mathrm{E},i}^{\mathrm{L}} \le P_{\mathrm{E},i} \le P_{\mathrm{E},i}^{\mathrm{U}},\ Q_{\mathrm{E},i}^{\mathrm{L}} \le Q_{\mathrm{E},i} \le Q_{\mathrm{E},i}^{\mathrm{U}}\} \tag{A2}$$

where $P_{\mathrm{E},i}^{\mathrm{L}}$, $P_{\mathrm{E},i}^{\mathrm{U}}$, $Q_{\mathrm{E},i}^{\mathrm{L}}$, and $Q_{\mathrm{E},i}^{\mathrm{U}}$ are the parameters to be determined.

The following optimization problem is established to obtain the rectangle P-Q envelopes:

$$\max_{(P_i^{\mathrm{U}}, P_i^{\mathrm{L}}, Q_i^{\mathrm{U}}, Q_i^{\mathrm{L}})} \sum\nolimits_{i\in\mathbb{DER}} \left(P_{\mathrm{E},i}^{\mathrm{U}} - P_{\mathrm{E},i}^{\mathrm{L}}\right) \tag{A3a}$$

s.t.

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{U}} + \boldsymbol{B}\boldsymbol{Q}_{E}^{\mathrm{U}} \le \boldsymbol{W} \tag{A3b}$$

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{L}} + \boldsymbol{B}\boldsymbol{Q}_{E}^{\mathrm{L}} \le \boldsymbol{W} \tag{A3c}$$

$$P_{\mathrm{E},j}^{\mathrm{U}} \big/ \gamma_j = P_{\mathrm{E},i}^{\mathrm{U}} \big/ \gamma_i,\ \forall i,j \in \mathbb{DER} \tag{A3d}$$

$$P_{\mathrm{E},j}^{\mathrm{L}} \big/ \gamma_j = P_{\mathrm{E},i}^{\mathrm{L}} \big/ \gamma_i,\ \forall i,j \in \mathbb{DER} \tag{A3e}$$

$$Q_{\mathrm{E},j}^{\mathrm{U}} \big/ \gamma_j = Q_{\mathrm{E},i}^{\mathrm{U}} \big/ \gamma_i,\ \forall i,j \in \mathbb{DER} \tag{A3f}$$

$$Q_{\mathrm{E},j}^{\mathrm{L}} \big/ \gamma_j = Q_{\mathrm{E},i}^{\mathrm{L}} \big/ \gamma_i,\ \forall i,j \in \mathbb{DER} \tag{A3g}$$

$$\left(P_{\mathrm{E},j}^{\mathrm{U}} - P_{\mathrm{E},j}^{\mathrm{L}}\right) \ge \vartheta\left(Q_{\mathrm{E},j}^{\mathrm{U}} - Q_{\mathrm{E},j}^{\mathrm{L}}\right),\ \forall i,j \in \mathbb{DER} \tag{A3h}$$

$$\left(P_{\mathrm{E},j}^{\mathrm{U}} - P_{\mathrm{E},j}^{\mathrm{L}}\right) \le \frac{1}{\vartheta}\left(Q_{\mathrm{E},j}^{\mathrm{U}} - Q_{\mathrm{E},j}^{\mathrm{L}}\right),\ \forall i,j \in \mathbb{DER} \tag{A3i}$$

$$P_{\mathrm{E},i}^{\mathrm{U}} \ge 0,\ Q_{\mathrm{E},i}^{\mathrm{U}} \ge 0, P_{\mathrm{E},i}^{\mathrm{L}} \le 0,\ Q_{\mathrm{E},i}^{\mathrm{L}} \le 0, \forall i \in \mathbb{DER} \tag{A3j}$$

where ϑ is the aspect ratio of rectangular P-Q envelopes. The objective function (A3a) is to maximize the total range of active power. Constraints (A3d)-(A3g) are the equitable conditions. Constraints (A3h)-(A3i) are the aspect ratio constraints of rectangles.

B. Optimization Model of PQ-KL Method

The P-Q envelope of the i_{th} DER is formulated as

$$\Omega_i = \{(P_{\mathrm{E},i}, Q_{\mathrm{E},i}) \mid \boldsymbol{A}_i P_{\mathrm{E},i} + \boldsymbol{B}_i Q_{\mathrm{E},i} \le \boldsymbol{\omega}_i\} \tag{A4}$$

where $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, and $\boldsymbol{\omega}_i$ are the parameters to be determined.

The following optimization model is established to obtain the P-Q envelopes:

$$\min_{(\boldsymbol{\omega},\boldsymbol{p},\boldsymbol{q})} \mathbb{C}^{\mathrm{net}}(\boldsymbol{p},\boldsymbol{q}) + \varepsilon g(\boldsymbol{\omega}) \tag{A5a}$$

s.t.

$$\mathbb{C}^{\mathrm{net}}(\boldsymbol{p},\boldsymbol{q}) = \left\|\boldsymbol{\upsilon} - \upsilon^{\mathrm{nom}}\right\|_2 \tag{A5b}$$

$$g(\boldsymbol{\omega}) = \left\| \begin{bmatrix} \boldsymbol{\omega}_1 \oslash \boldsymbol{W} - 1/N_{\mathrm{DER}} \\ \boldsymbol{\omega}_2 \oslash \boldsymbol{W} - 1/N_{\mathrm{DER}} \\ \vdots \\ \boldsymbol{\omega}_{N_{\mathrm{DER}}} \oslash \boldsymbol{W} - 1/N_{\mathrm{DER}} \end{bmatrix} \right\|_2 \tag{A5c}$$

$$\boldsymbol{A}_i p_{\mathrm{E},i} + \boldsymbol{B}_i q_{\mathrm{E},i} \le \boldsymbol{\omega}_i, \forall i \in \mathbb{DER} \tag{A5d}$$

$$\sum_{i\in\mathbb{DER}} \boldsymbol{\omega}_i = \boldsymbol{W} \tag{A5e}$$

$$\begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 & \cdots & \boldsymbol{A}_{N_{\mathrm{DER}}} \end{bmatrix} = \boldsymbol{A} \tag{A5f}$$

$$\begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_{N_{\mathrm{DER}}} \end{bmatrix} = \boldsymbol{B} \tag{A5g}$$

where $\mathbb{C}^{\mathrm{net}}(\boldsymbol{p},\boldsymbol{q})$ represents the objective of DNO, which is designed as minimizing voltage deviations, $g(\boldsymbol{\omega})$ represents the equitableness objective, which is achieved by allocating the coefficient $\boldsymbol{W}$ into the coefficient $\boldsymbol{\omega}_i$ of the P-Q envelope of each DER, ε is a weighting coefficient to balance the two objectives. Constraints (A5d)-(A5g) indicate the Kornal-Liptak decomposition.

C. Optimization Model of P-E Method

The P envelope of the i_{th} DER is formulated as

$$\Omega_i = \{(P_{\mathrm{E},i}, Q_{\mathrm{E},i}) \mid P_{\mathrm{E},i}^{\mathrm{L}} \le P_{\mathrm{E},i} \le P_{\mathrm{E},i}^{\mathrm{U}},\ Q_{\mathrm{E},i} = 0\} \tag{A6}$$

where $P_{\mathrm{E},i}^{\mathrm{L}}$ and $P_{\mathrm{E},i}^{\mathrm{U}}$ are the parameters to be determined.

The following two optimization models are established to obtain the P envelopes:

$$\max_{P_i^{\mathrm{U}}} \sum\nolimits_{i\in\mathbb{DER}} P_{\mathrm{E},i}^{\mathrm{U}} \tag{A7a}$$

s.t.

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{U}} \le \boldsymbol{W} \tag{A7b}$$

$$P_{\mathrm{E},j}^{\mathrm{U}} \big/ \gamma_j = P_{\mathrm{E},i}^{\mathrm{U}} \big/ \gamma_i,\ \forall i,j \in \mathbb{DER} \tag{A7c}$$

$$P_{\mathrm{E},i}^{\mathrm{U}} \ge 0,\ \forall i \in \mathbb{DER} \tag{A7d}$$

$$\max_{P_i^{\mathrm{L}}} \sum\nolimits_{i\in\mathbb{DER}} -P_{\mathrm{E},i}^{\mathrm{L}} \tag{A8a}$$

s.t.

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{L}} \le \boldsymbol{W} \tag{A8b}$$

$$P_{\mathrm{E},j}^{\mathrm{L}} \big/ \gamma_j = P_{\mathrm{E},i}^{\mathrm{L}} \big/ \gamma_i,\ \forall i,j \in \mathbb{DER} \tag{A8c}$$

$$P_{\mathrm{E},i}^{\mathrm{L}} \le 0,\ \forall i \in \mathbb{DER} \tag{A8d}$$

where objective functions (A7a) and (A8a) are to maximize the total exported and imported active power of DERs, respectively. Constraints (A7c) and (A8c) are the equitable conditions.

D. Optimization Model of P-WE Method

The following two optimization models are established to obtain the P envelopes:

$$\max_{P_i^{\mathrm{U}}} \sum\nolimits_{i\in\mathbb{DER}} \gamma_i \log\left(P_{\mathrm{E},i}^{\mathrm{U}}\right) \tag{A9a}$$

s.t.

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{U}} \le \boldsymbol{W} \tag{A9b}$$

$$P_{\mathrm{E},i}^{\mathrm{U}} \ge 0,\ \forall i \in \mathbb{DER} \tag{A9c}$$

$$\max_{P_i^{\mathrm{L}}} \sum\nolimits_{i\in\mathbb{DER}} \gamma_i \log\left(-P_{\mathrm{E},i}^{\mathrm{L}}\right) \tag{A10a}$$

s.t. $$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{L}} \le \boldsymbol{W} \tag{A10b}$$

$$P_{\mathrm{E},i}^{\mathrm{L}} \le 0,\ \forall i \in \mathbb{DER} \tag{A10c}$$

where logarithmic objective functions (A9a) and (A10a) are to maximize the total exported and imported active power of DERs considering the weighted proportional equitableness, respectively.

E. Optimization Model of P-SE Method

The following two optimization models are established to obtain the P envelopes:

$$\max_{P_i^{\mathrm{U}}} \sum\nolimits_{i\in\mathbb{DER}} P_{\mathrm{E},i}^{\mathrm{U}} - \sum\nolimits_{i=1}^{N_{DER}-1}\left(P_{\mathrm{E},i}^{\mathrm{U}}/\gamma_i - P_{\mathrm{E},i+1}^{\mathrm{U}}/\gamma_{i+1}\right)^2 \tag{A11a}$$

s.t. $$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{U}} \le \boldsymbol{W} \tag{A11b}$$

$$P_{\mathrm{E},i}^{\mathrm{U}} \ge 0,\ \forall i \in \mathbb{DER} \tag{A11c}$$

$$\max_{P_i^{\mathrm{L}}} \sum\nolimits_{i\in\mathbb{DER}} -P_{\mathrm{E},i}^{\mathrm{L}} - \sum\nolimits_{i=1}^{N_{DER}-1}\left(P_{\mathrm{E},i}^{\mathrm{L}}/\gamma_i - P_{\mathrm{E},i+1}^{\mathrm{L}}/\gamma_{i+1}\right)^2 \tag{A12a}$$

s.t. $$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{L}} \le \boldsymbol{W} \tag{A12b}$$

$$P_{\mathrm{E},i}^{\mathrm{L}} \le 0,\ \forall i \in \mathbb{DER} \tag{A12c}$$

where the first terms of (A11a) and (A12a) are to maximize the total exported and imported active power of DERs, respectively, and the second terms of (A11a) and (A12a) are the equitable objectives.

F. Configuration of Devices Controlled by DNO

TABLE A.I
CONFIGURATION INFORMATION OF DEVICES

Distribution system	Device	Parameter	Placement
33-bus system	OLTC	±1.25%×8	Primary side of the transformer
	CBs 1-4	0.5Mvar×4	Bus 17, 24, 30, 32
	DG 1	1.2MVA	Replacing DER 2
	DG 2	1.6MVA	Replacing DER 4
135-bus system	OLTC	±1.25%×8	Primary side of the transformer
	CBs 1-10	0.5Mvar×4	Buses 15, 38, 42, 85, 59, 99, 108, 114, 120, 136
	DG 1	1.4MVA	Replacing DER 14
	DG 2	1.2MVA	Replacing DER 15
	DG 3	1.8MVA	Replacing DER 16
	DG 4	2.8MVA	Replacing DER 17
	DG 5	1.4MVA	Replacing DER 18

G. Control Strategies of Devices

TABLE A.II
CONTROL STRATEGIES FOR MAXIMIZING ACTIVE POWER IMPORTATION IN THE IEEE 33-BUS SYSTEM

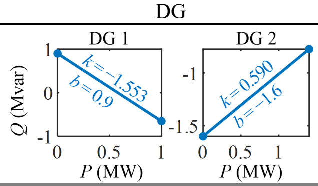
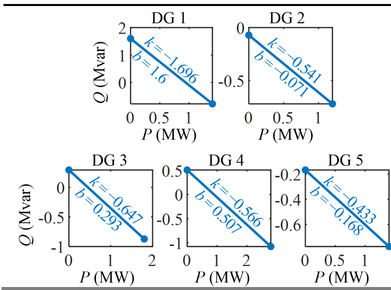
DG	OLTC and CB
	Tap position of OLTC: -3 Position level of CB 1: 0 Position level of CB 2: 0 Position level of CB 3: 4 Position level of CB 4: 2

TABLE A.III
CONTROL STRATEGIES FOR MAXIMIZING ACTIVE POWER IMPORTATION IN THE REAL-WORLD 135-BUS SYSTEM

DG	OLTC and CB
	Tap position of OLTC: -3 Position level of CB 1: 0 Position level of CB 2: 4 Position level of CB 3: 0 Position level of CB 4: 0 Position level of CB 5: 2 Position level of CB 6: 0 Position level of CB 7: 0 Position level of CB 8: 0 Position level of CB 9: 4 Position level of CB 10: 3