Equitable Active-Reactive Power Envelopes for Managing Distributed Energy Resources in Power Distribution systems

APPENDIX

A. Optimization Model of PQ-E Method

The compact form of allowable power region (10) of DERs is expressed as

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}} + \boldsymbol{B}\boldsymbol{Q}_{E} \le \boldsymbol{W} \tag{A1}$$

The rectangle P-Q envelope of the i_{th} DER is formulated as

$$\Omega_i = \{(P_{\mathrm{E},i}, Q_{\mathrm{E},i}) \mid P_{\mathrm{E},i}^{\mathrm{L}} \le P_{\mathrm{E},i} \le P_{\mathrm{E},i}^{\mathrm{U}},\ Q_{\mathrm{E},i}^{\mathrm{L}} \le Q_{\mathrm{E},i} \le Q_{\mathrm{E},i}^{\mathrm{U}}\} \tag{A2}$$

where $P_{\mathrm{E},i}^{\mathrm{L}}$, $P_{\mathrm{E},i}^{\mathrm{U}}$, $Q_{\mathrm{E},i}^{\mathrm{L}}$, and $Q_{\mathrm{E},i}^{\mathrm{U}}$ are the parameters to be determined.

The PQ-E method establishes an optimization model to obtain the rectangle P-Q envelopes:

$$\max_{(P_i^{\mathrm{U}}, P_i^{\mathrm{L}}, Q_i^{\mathrm{U}}, Q_i^{\mathrm{L}})} \sum\nolimits_{i \in \mathbb{DER}} \left[P_{\mathrm{E},i}^{\mathrm{U}} - P_{\mathrm{E},i}^{\mathrm{L}} \right] \tag{A3a}$$

s.t.

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{U}} + \boldsymbol{B}\boldsymbol{Q}_{E}^{\mathrm{U}} \le \boldsymbol{W} \tag{A3b}$$

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{L}} + \boldsymbol{B}\boldsymbol{Q}_{E}^{\mathrm{L}} \le \boldsymbol{W} \tag{A3c}$$

$$P_{\mathrm{E},j}^{\mathrm{U}} / \gamma_j = P_{\mathrm{E},i}^{\mathrm{U}} / \gamma_i,\ \forall i, j \in \mathbb{DER} \tag{A3d}$$

$$P_{\mathrm{E},j}^{\mathrm{L}} / \gamma_j = P_{\mathrm{E},i}^{\mathrm{L}} / \gamma_i,\ \forall i, j \in \mathbb{DER} \tag{A3e}$$

$$Q_{\mathrm{E},j}^{\mathrm{U}} / \gamma_j = Q_{\mathrm{E},i}^{\mathrm{U}} / \gamma_i,\ \forall i, j \in \mathbb{DER} \tag{A3f}$$

$$Q_{\mathrm{E},j}^{\mathrm{U}} / \gamma_j = Q_{\mathrm{E},i}^{\mathrm{U}} / \gamma_i,\ \forall i, j \in \mathbb{DER} \tag{A3g}$$

$$\left(P_{\mathrm{E},j}^{\mathrm{U}} - P_{\mathrm{E},j}^{\mathrm{L}}\right) \ge \vartheta \left(Q_{\mathrm{E},j}^{\mathrm{U}} - Q_{\mathrm{E},j}^{\mathrm{L}}\right),\ \forall i, j \in \mathbb{DER} \tag{A3h}$$

$$\left(P_{\mathrm{E},j}^{\mathrm{U}} - P_{\mathrm{E},j}^{\mathrm{L}}\right) \le \frac{1}{\vartheta} \left(Q_{\mathrm{E},j}^{\mathrm{U}} - Q_{\mathrm{E},j}^{\mathrm{L}}\right),\ \forall i, j \in \mathbb{DER} \tag{A3i}$$

$$P_{\mathrm{E},i}^{\mathrm{U}} \ge 0,\ Q_{\mathrm{E},i}^{\mathrm{U}} \ge 0, P_{\mathrm{E},i}^{\mathrm{L}} \le 0,\ Q_{\mathrm{E},i}^{\mathrm{L}} \le 0, \forall i \in \mathbb{DER} \tag{A3j}$$

where ϑ is the aspect ratio of rectangle P-Q operating envelopes. The objective function (A3a) is to maximize the total permitted active power range. Inequalities (A3d)-(A3g) are the equitable constraints. Inequalities (A3h)-(A3i) are the aspect ratio constraints of operating envelopes.

B. Optimization Model of PQ-KL Method

The P-Q envelope of the i_{th} DER based on the PQ-KL method is formulated as

$$\Omega_i = \{(P_{\mathrm{E},i}, Q_{\mathrm{E},i}) \mid \boldsymbol{A}_i P_{\mathrm{E},i} + \boldsymbol{B}_i Q_{\mathrm{E},i} \le \boldsymbol{\omega}_i\} \tag{A4}$$

where $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, and $\boldsymbol{\omega}_i$ are the parameters to be determined.

The PQ-KL method establishes an optimization model to obtain the P-Q envelopes:

$$\min_{(\boldsymbol{\omega}, \boldsymbol{p}, \boldsymbol{q})} \mathbb{C}^{\mathrm{net}}(\boldsymbol{p}, \boldsymbol{q}) + \varepsilon g(\boldsymbol{\omega}) \tag{A5a}$$

s.t.

$$\mathbb{C}^{\mathrm{net}}(\boldsymbol{p}, \boldsymbol{q}) = \left\| \upsilon - \upsilon^{nom} \right\|_2 \tag{A5b}$$

$$g(\boldsymbol{\omega}) = \left\| \begin{bmatrix} \boldsymbol{\omega}_1 \oslash \boldsymbol{W} - 1/N_{\mathrm{DER}} \\ \boldsymbol{\omega}_2 \oslash \boldsymbol{W} - 1/N_{\mathrm{DER}} \\ \vdots \\ \boldsymbol{\omega}_{N_{\mathrm{DER}}} \oslash \boldsymbol{W} - 1/N_{\mathrm{DER}} \end{bmatrix} \right\|_2 \tag{A5c}$$

$$\boldsymbol{A}_i p_{\mathrm{E},i} + \boldsymbol{B}_i q_{\mathrm{E},i} \le \boldsymbol{\omega}_i, \forall i \in \mathbb{DER} \tag{A5d}$$

$$\sum_{i \in \mathbb{DER}} \boldsymbol{\omega}_i = \boldsymbol{W} \tag{A5e}$$

$$\begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 & \cdots & \boldsymbol{A}_{N_{\mathrm{DER}}} \end{bmatrix} = \boldsymbol{A} \tag{A5f}$$

$$\begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_{N_{\mathrm{DER}}} \end{bmatrix} = \boldsymbol{B} \tag{A5g}$$

where $\mathbb{C}^{\mathrm{net}}(\boldsymbol{p}, \boldsymbol{q})$ is used to constrain the solution of DNO's optimized objective, which is minimum voltage deviations, inside the P-Q envelopes; $g(\boldsymbol{\omega})$ is the equitableness objective, which is equitably allocating the right-hand side coefficient $\boldsymbol{W}$ to the right-hand side coefficient $\boldsymbol{\omega}_i$ of each DER's P-Q envelope; ε is a weighted coefficient to balance the two objectives. Constraints (A5d)-(A5g) describe the Kornal-Liptak decomposition.

C. Optimization Model of P-E Method

The P envelope of the i_{th} DER is formulated as

$$\Omega_i = \{(P_{\mathrm{E},i}, Q_{\mathrm{E},i}) \mid P_{\mathrm{E},i}^{\mathrm{L}} \le P_{\mathrm{E},i} \le P_{\mathrm{E},i}^{\mathrm{U}},\ Q_{\mathrm{E},i} = 0\} \tag{A6}$$

where $P_{\mathrm{E},i}^{\mathrm{L}}$ and $P_{\mathrm{E},i}^{\mathrm{U}}$ are the parameters to be determined.

The P-E method establishes two optimization models to obtain the P envelopes:

$$\max_{P_i^{\mathrm{U}}} \sum\nolimits_{i \in \mathbb{DER}} P_{\mathrm{E},i}^{\mathrm{U}} \tag{A7a}$$

s.t.

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{U}} \le \boldsymbol{W} \tag{A7b}$$

$$P_{\mathrm{E},j}^{\mathrm{U}} / \gamma_j = P_{\mathrm{E},i}^{\mathrm{U}} / \gamma_i,\ \forall i, j \in \mathbb{DER} \tag{A7c}$$

$$P_{\mathrm{E},i}^{\mathrm{U}} \ge 0,\ \forall i \in \mathbb{DER} \tag{A7d}$$

$$\max_{P_i^{\mathrm{L}}} \sum\nolimits_{i \in \mathbb{DER}} -P_{\mathrm{E},i}^{\mathrm{L}} \tag{A8a}$$

s.t.

$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{L}} \le \boldsymbol{W} \tag{A8b}$$

$$P_{\mathrm{E},j}^{\mathrm{L}} / \gamma_j = P_{\mathrm{E},i}^{\mathrm{L}} / \gamma_i,\ \forall i, j \in \mathbb{DER} \tag{A8c}$$

$$P_{\mathrm{E},i}^{\mathrm{L}} \le 0,\ \forall i \in \mathbb{DER} \tag{A8d}$$

where objective functions (A7a) and (A8a) are to maximize the total exported and imported active power of DERs, respectively. Inequalities (A7c) and (A8c) are the equitable constraints.

D. Optimization Model of P-WE Method

The P-WE method establishes two optimization models to obtain the P envelopes:

$$\max_{P_i^{\mathrm{U}}} \sum\nolimits_{i\in\mathbb{DER}} \gamma_i \log\left(P_{\mathrm{E},i}^{\mathrm{U}}\right) \tag{A9a}$$

s.t.
$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{U}} \le \boldsymbol{W} \tag{A9b}$$

$$P_{\mathrm{E},i}^{\mathrm{U}} \ge 0,\ \forall i \in \mathbb{DER} \tag{A9c}$$

$$\max_{P_i^{\mathrm{L}}} \sum\nolimits_{i\in\mathbb{DER}} \gamma_i \log\left(-P_{\mathrm{E},i}^{\mathrm{L}}\right) \tag{A10a}$$

s.t.
$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{L}} \le \boldsymbol{W} \tag{A10b}$$

$$P_{\mathrm{E},i}^{\mathrm{L}} \le 0,\ \forall i \in \mathbb{DER} \tag{A10c}$$

where logarithmic objective functions (A9a) and (A10a) are to maximize the total exported and imported active power of DERs considering the weighted proportional equitableness, respectively.

E. Optimization Model of P-SE Method

The P-SE method establishes two optimization models to obtain the P envelopes:

$$\max_{P_i^{\mathrm{U}}} \sum\nolimits_{i\in\mathbb{DER}} P_{\mathrm{E},i}^{\mathrm{U}} - \sum\nolimits_{i=1}^{N_{DER}-1} \left(P_{\mathrm{E},i}^{\mathrm{U}}/\gamma_i - P_{\mathrm{E},i+1}^{\mathrm{U}}/\gamma_{i+1}\right)^2 \tag{A11a}$$

s.t.
$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{U}} \le \boldsymbol{W} \tag{A11b}$$

$$P_{\mathrm{E},i}^{\mathrm{U}} \ge 0,\ \forall i \in \mathbb{DER} \tag{A11c}$$

$$\max_{P_i^{\mathrm{L}}} \sum\nolimits_{i\in\mathbb{DER}} -P_{\mathrm{E},i}^{\mathrm{L}} - \sum\nolimits_{i=1}^{N_{DER}-1} \left(P_{\mathrm{E},i}^{\mathrm{L}}/\gamma_i - P_{\mathrm{E},i+1}^{\mathrm{L}}/\gamma_{i+1}\right)^2 \tag{A12a}$$

s.t.
$$\boldsymbol{A}\boldsymbol{P}_{\mathrm{E}}^{\mathrm{L}} \le \boldsymbol{W} \tag{A12b}$$

$$P_{\mathrm{E},i}^{\mathrm{L}} \le 0,\ \forall i \in \mathbb{DER} \tag{A12c}$$

where the first terms of (A11a) and (A12a) are to maximize the total exported and imported active power of DERs, respectively, and the second terms of (A11a) and (A12a) are the equitable objectives.

F. Directly Controlled Device Configuration

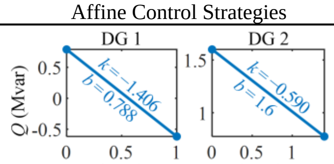
TABLE A.I
DIRECTLY CONTROLLED DEVICE CONFIGURATION

Distribution system	Control Device	Parameter	Placement
33-bus system	OLTC	±1.25%×8	Primary side of the transformer
	CBs 1-4	0.5Mvar×4	Bus 17, 24, 30, 32
	DG 1	1.2MVA	Replace DER 2
	DG 2	1.6MVA	Replace DER 4
135-bus system	OLTC	±1.25%×8	Primary side of the transformer
	CBs 1-10	0.5Mvar×4	Bus 15, 38, 42, 85, 59, 99, 108, 114, 120, 136
	DG 1	1.4MVA	Replace DER 14
	DG 2	1.2MVA	Replace DER 15
	DG 3	1.8MVA	Replace DER 16
	DG 4	2.8MVA	Replace DER 17
	DG 5	1.4MVA	Replace DER 18

G. Decision Results of Control Devices

TABLE A.II
DECISION RESULTS FOR MAXIMIZING ACTIVE POWER EXPORTATION IN THE IEEE 33-BUS SYSTEM

Affine Control Strategies

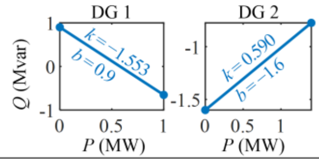


Position Control Strategies

Tap position of OLTC: 3
Position level of CB 1: 1
Position level of CB 2: 4
Position level of CB 3: 3
Position level of CB 4: 1

TABLE A.III
DECISION RESULTS FOR MAXIMIZING ACTIVE POWER IMPORTATION IN THE IEEE 33-BUS SYSTEM

Affine Control Strategies

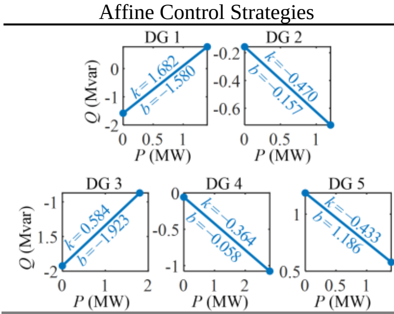


Position Control Strategies

Tap position of OLTC: -3
Position level of CB 1: 0
Position level of CB 2: 0
Position level of CB 3: 4
Position level of CB 4: 2

TABLE A.IV
DECISION RESULTS FOR MAXIMIZING ACTIVE POWER EXPORTATION IN THE REAL-WORLD 135-BUS SYSTEM

Affine Control Strategies

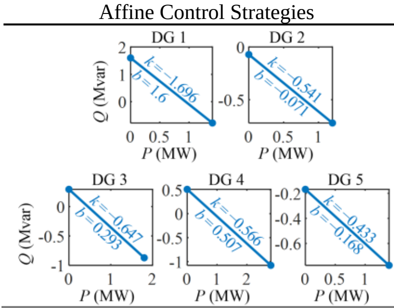


Position Control Strategies

Tap position of OLTC: 2
Position level of CB 1: 4
Position level of CB 2: 4
Position level of CB 3: 0
Position level of CB 4: 4
Position level of CB 5: 2
Position level of CB 6: 4
Position level of CB 7: 0
Position level of CB 8: 4
Position level of CB 9: 4
Position level of CB 10: 4

TABLE A.V
DECISION RESULTS FOR MAXIMIZING ACTIVE POWER IMPORTATION IN THE REAL-WORLD 135-BUS SYSTEM

Affine Control Strategies



Position Control Strategies

Tap position of OLTC: -3
Position level of CB 1: 0
Position level of CB 2: 4
Position level of CB 3: 0
Position level of CB 4: 0
Position level of CB 5: 2
Position level of CB 6: 0
Position level of CB 7: 0
Position level of CB 8: 0
Position level of CB 9: 4
Position level of CB 10: 3