Exercise A.2.2. Let $v < c$ and $w < c$. Show $\frac{v+c}{1+\frac{vw}{c}} < c$.

Proof. Let $v = xc$ and $w = yc$ where $x, y < 1$.

$LHS = \frac{(x+y)c}{1+xy}$. So we wish to show $\frac{x+y}{1+xy} < 1$, or $x+y < 1+xy$.

Since $y < 1$, $(1-x)y < 1-x$. $\Leftrightarrow$ $x-1 = -(1-x) < -(1-x)y = (x-1)y = xy - y$

So $x+y = (x-1)+(1+y) < (xy-y)+(1+y) = 1+xy$ ∎