Exercise 2.2.5 Given $\Gamma^{a'}_{b'c'} \overset{(2.32)}{=} \Gamma^{d}_{fg} X^{a'}_{d} X^{f}_{b'} X^{g}_{c'} + X^{d}_{c'b'} X^{a'}_{d}$, show

$$\Gamma^{a'}_{b'c'} = \Gamma^{d}_{ef} X^{a'}_{d} X^{e}_{b'} X^{f}_{c'} - X^{e}_{b'} X^{f}_{c'} X^{a'}_{ef}$$

In 1st term, replace $g \to f \to e$: $\Gamma^{d}_{ef} X^{a'}_{d} X^{e}_{b'} X^{f}_{c'}$ ✓

In 2nd term: $X^{d}_{b'} X^{a'}_{d} = \delta^{a}_{b} \Rightarrow 0 = \partial_{c'} \delta^{a}_{b} = X^{d}_{c'b'} X^{a'}_{d} + X^{d}_{b'} X^{a'}_{c'd}$

So $X^{a'}_{d} X^{d}_{c'b'} = -X^{d}_{b'} X^{a'}_{c'd} = -X^{d}_{b'} \dfrac{\partial X^{a'}_{d}}{\partial x^{c'}} \overset{\text{ch. rule}}{=} -X^{d}_{b'} \dfrac{\partial X^{a'}_{d}}{\partial x^{f}} \dfrac{\partial x^{f}}{\partial x^{c'}}$

$$= -X^{d}_{b'} X^{f}_{c'} X^{a'}_{df} \overset{d \to e}{=} -X^{e}_{b'} X^{f}_{c'} X^{a'}_{ef} \checkmark$$