Exercise 2.1.3 Use result of Exercise 2.1.2 to show that if γ is non-null then $u = As + B$ for constants A and B, $A \neq 0$

From Exercise 2.1.2, the length L of $\dot{x}^a = \frac{dx^a}{du}$ is constant. So $\exists$ constant $A \ni$

$0 \neq A^2 \equiv L^2 = \pm g_{ab}\dot{x}^a\dot{x}^b = g_{ab}\frac{dx^a}{ds}\frac{ds}{du}\frac{dx^b}{ds}\frac{ds}{du} = g_{ab}\frac{dx^a}{ds}\frac{dx^b}{ds}\left(\frac{ds}{du}\right)^2.$

$\lambda^a = \frac{dx^a}{ds}$ and $\lambda^b = \frac{dx^b}{ds}$. Then $g_{ab}\frac{dx^a}{ds}\frac{dx^b}{ds} = g_{ab}\lambda^a\lambda^b \overset{(1.79)}{=} \lambda^a\lambda_a = \frac{dx^a}{ds}\frac{dx_a}{ds} = \frac{ds^2}{ds^2} = 1$

So $0 \neq A^2 = \left(\frac{ds}{du}\right)^2$, or $ds = A\,du \Rightarrow s = Au + B$ and $A \neq 0$ $\blacksquare$