

Exercise 3.2.2. Verify the cyclic identity $R^a_{bcd} + R^a_{cdb} + R^a_{dbc} = 0$

$$\begin{aligned}
 \text{Sum} &= \underline{\partial_c \Gamma^a_{bd}} - \underline{\partial_d \Gamma^a_{bc}} + \underline{\Gamma^e_{bd} \Gamma^a_{ec}} - \underline{\Gamma^e_{bc} \Gamma^a_{ed}} \\
 &+ \underline{\partial_d \Gamma^a_{cb}} - \underline{\partial_b \Gamma^a_{cd}} + \underline{\Gamma^e_{cb} \Gamma^a_{ed}} - \underline{\Gamma^e_{cd} \Gamma^a_{eb}} \\
 &+ \underline{\partial_b \Gamma^a_{dc}} - \underline{\partial_c \Gamma^a_{db}} + \underline{\Gamma^e_{dc} \Gamma^a_{eb}} - \underline{\Gamma^e_{db} \Gamma^a_{ec}} \\
 &= 0 \quad \checkmark
 \end{aligned}$$