Exercise A.1.2. Eq (A.13) for an x-boost in matrix form can be expressed

$$x^{\mu'} = \Lambda^{\mu'}{}_{\nu'} x^{\nu}$$

where

$$\left(\Lambda^{\mu'}{}_{\nu}\right) = \begin{pmatrix} \gamma & -\frac{\gamma v}{c} & 0 & 0 \\ -\frac{\gamma v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Find the inverse matrix $\left(\Lambda^{\nu}{}_{\mu'}\right)$ and the velocity of K' relative to K.

Ans:

$$\left(\Lambda^{\nu}{}_{\mu'}\right) = \begin{pmatrix} \gamma & \frac{\gamma v}{c} & 0 & 0 \\ \frac{\gamma v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Lambda^{1}{}_{1'} = \gamma^2 - \frac{\gamma^2 v^2}{c^2} = \frac{1}{1-\frac{v^2}{c^2}}\left(1 - \frac{v^2}{c^2}\right) = 1 \checkmark$$

$$\Lambda^{1}{}_{2'} = \frac{1}{c}(\gamma^2 v - \gamma^2 v) = 0 \checkmark \qquad \text{similarly for the rest}$$

clearly if K' → K w/ velocity v then K → K' with velocity $-v$.

More formally

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \frac{\gamma v}{c} & 0 & 0 \\ \frac{\gamma v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z \end{pmatrix} = \begin{pmatrix} \gamma ct' + \frac{1}{c}\gamma v x' \\ \gamma v t' + \gamma x' \\ y' \\ z' \end{pmatrix}$$

so $\left\{\begin{array}{l} ct = \gamma c t' + \frac{\gamma v}{c} x' \\ x = \gamma(x' + vt') \end{array}\right\}$ holds for all x and t.

The origin O of K has $x = 0$ at t.

$\Rightarrow \quad 0 = x' + vt' \Rightarrow \quad x' = -vt'$

i.e. K → K' with velocity $-v$ ✓

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