Mathematics Workshop

- Slopes
- Equations of Lines
- Graphing Lines

Spring, 2012

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Agenda

Today we will review how to

1. Calculate slopes
2. Develop equations of lines
3. Graph linear equations
   - Fancy way of saying "graph lines"

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Introduction

- There are only 2 things to memorize:
  1. Formula for slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{Rise}}{\text{Run}}$
  2. Slope-Intercept Form of equation of a line: $y = mx + b$
- You will also need to understand
  1. The meaning of slope m
  2. The meaning of y-intercept b
- Helpful Hint:
  - For line equation problems, always start with $y = mx + b$

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Section 1: Slopes of Lines

Example 1: Find the slope of the line passing through (2,5) and (3,8)

We let $P_1 = (x_1, y_1) = (2,5)$ and $P_2 = (x_2, y_2) = (3,8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\boxed{8} - \boxed{5}}{\boxed{3} - \boxed{2}}$$

$$m = \frac{3}{1}$$

$$m = 3$$

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Slopes of Lines (Con't)

Example 2: Find the slope of the line passing through (-1,3) and (2,-5)

$P_1 = (x_1, y_1) = (\boxed{-1}, \boxed{3})$ and $P_2 = (x_2, y_2) = (\boxed{2}, \boxed{-5})$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\boxed{-5} - \boxed{3}}{\boxed{2} - \boxed{-1}}$$

$$m = \frac{\boxed{-8}}{\boxed{3}}$$

$$m = \boxed{}$$

There is nothing further to reduce

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Slopes of Lines (Con't)

Example 3: Find the slope of the line passing through (5,0) and (5,11)

$P_1 = (x_1, y_1) = (\boxed{5}, \boxed{0})$ and $P_2 = (x_2, y_2) = (\boxed{5}, \boxed{11})$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\boxed{11} - \boxed{0}}{\boxed{5} - \boxed{5}}$$

$$m = \frac{\boxed{11}}{\boxed{0}}$$

$m =$ **Undefined**

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Slopes of Lines (Con't)

Example 4: Find the slope of the line passing through (-1,8) and (2,8)

$P_1 = (x_1, y_1) = (\boxed{2}, \boxed{8})$ and $P_2 = (x_2, y_2) = (\boxed{-1}, \boxed{8})$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\boxed{8} - \boxed{8}}{\boxed{-1} - \boxed{2}}$$

$$m = \frac{\boxed{0}}{\boxed{-3}}$$

$$m = 0$$

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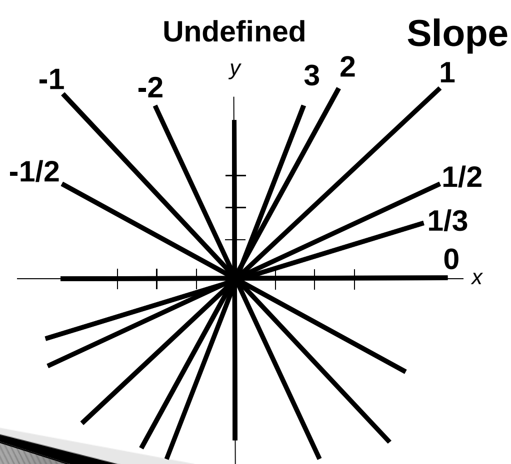
Understanding Slopes

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}}$$

Slope is a measure of the steepness of a line



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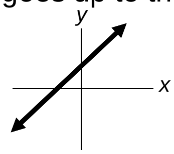
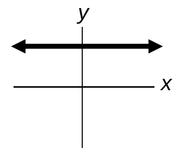
Summarizing Slopes

$$m = \frac{\text{Rise}}{\text{Run}}$$

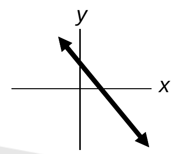
1. Positive Slope: m > 0
Line goes up to the right



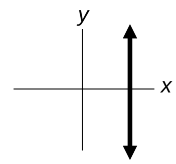
3. m = 0
Line is horizontal



2. Negative Slope: m < 0
Line goes down to the right



4. m is undefined
Line is vertical



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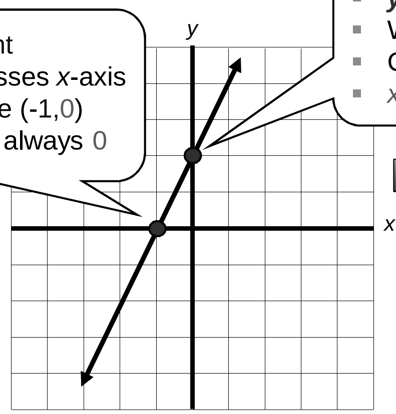
Understanding Intercepts

There are 2 kinds of intercepts

- ***x*-intercept point**
- Where line crosses *x*-axis
- Coordinates are (-1,0)
- *y*-coordinate is always 0



- ***y*-intercept point**
- Where line crosses *y*-axis
- Coordinates are (0,2)
- *x*-coordinate is always 0

Slope

y-intercept

$y = mx + b$

$b = 2$

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Section 2: Equations of Lines

- You need to be able to work 2 types of problems
  1. Find slope *m* and *y*-intercept *b* given equation of a line:
     a. Given ***y = mx+b*****,** the **Slope-intercept form**
     b. Or, given ***Ax+By = C***, the **Standard form**

  2. Find equation of a line given
     a. Slope *m* and y-intercept *b*
     b. Slope *m* and one point on the line
     c. 2 points on the line

Don't worry about which is which. Just start with *y = mx + b*

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Equations of Lines (Types 1a & 1b)

In Examples below, find the slope and *y*-intercept:

Example 5: $y = 2x - 3$ Answer: Start with $y = mx + b$

$m = 2,\ b = -3,\ y$-intercept coordinates $= (0, -3)$

Example 6: $y = \frac{1}{2}x$ Answer: Start with $y = mx + b$

$m = \frac{1}{2},\ b = 0,\ y$-intercept coordinates $= (0, 0)$

Example 7: $2x + 3y = 9$ Answer: Start with $y = mx + b$

$-2x \quad -2x$ We *must* solve for *y* in terms of *x*

$\frac{3y}{3} = \frac{-2x}{3} + \frac{9}{3}$

So, $y = -\frac{2}{3}x + 3$ Thus $m = -\frac{2}{3}$ and $b = 3$

The *y*-intercept coordinates are (0, 3)

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Equations of Lines (Types 1b & 2a)

Example 8: Find the slope and *y*-intercept of $4x - y = 10$

We start with: $y = mx + b$

We *must* solve for *y* in terms of *x*

$$4x - y = 10$$
$$-4x \qquad -4x$$
$$(-1)\ [-y = -4x + 10\ ]$$
$$y = 4x - 10$$

So, $m = 4$ and $b = -10$.

This is the end of the 1st type of line problems. Now we do the Type 2 problems.

Example 9: Find the equation of the line whose slope is $\frac{1}{2}$ and *y*-intercept is $\frac{3}{5}$

Answer: We start with $y = mx + b$

$$y = \frac{1}{2}x + \frac{3}{5}$$

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Equations of Lines (Type 2b)

Example 10: Find the equation of the line whose slope is 2 and that passes through (-2,3)

Start with equation $y = mx + b$ to get $y = \boxed{2}\, x + b$

Then, since the line passes through (-2,3),

plug $x = -2$ and $y = 3$ into the equation:

$$\boxed{3} = 2\,(\boxed{-2}) + b$$
$$3 = -4 + b$$
$$7 = b$$

So the equation becomes $y = 2x + 7$

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Equations of Lines (Type 2b)

Example 11: Find the equation of the line whose slope is ⅓ and that passes through (-3,6)

Start with equation $y = mx + b$ with $m = ⅓$ to get $y = \boxed{⅓}x + b$
Then, since the line passes through (-3,6),
plug $x = -3$ and $y = 6$ into the equation:

$$\boxed{6} = ⅓\,(\boxed{-3}) + b$$
$$6 = -1 + b$$
$$7 = b$$

So the equation becomes $y = ⅓\,x + 7$

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Equations of Lines (Type 2c)

Example 12: Find the equation of the line passing through (-3,4) and (-1,2)

We start with $y = mx + b$

First, we must find *m:*

Let P_1 = (-1,2) and P_2 = (-3,4)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{2}{-2}$$

$$m = \frac{4-2}{-3-(-1)} \qquad m = -1$$

So, $y = -x + b$

Next, we must find b:
Plug either ordered pair into the equation $y = -x + b$.
Let's use (-3,4):

$$4 = -(-3) + b$$
$$4 = 3 + b$$
$$1 = b$$

So, our equation becomes
$y = -x + 1$

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Equations of Lines (Type 2c)

Example 13: Find the equation of the line passing through (0,5) and (4,3)

We start with $y = mx + b$

First, we must find *m:*

Let $P_1 = (0,5)$ and $P_2 = (4,3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{-2}{4}$$

$$m = \frac{3-5}{4-0} \qquad m = -\frac{1}{2}$$

So, $y = -\frac{1}{2}x + b$

Next, we must find b:
Plug either ordered pair into the equation $y = -\frac{1}{2}x + b$

Let's use (4,3):

$$3 = -\frac{1}{2}(4) + b$$

$$3 = -2 + b$$

$$5 = b$$

So, our equation becomes

$$y = -\frac{1}{2}x + 5$$

A quicker way to find *b* is to notice that (0,5) is the *y*-intercept point

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Section 3: Graphs of Linear Equations

The graph of a linear equation in 2 variables is a line

You must know the following 3 ways of graphing lines:

1. Plotting 3 different ordered pairs
2. Using Intercepts
3. Using slope and *y*-intercept

We will also learn how to graph vertical and horizontal lines

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Graphs of Linear Equations

Method 1: Plot 3 Different Ordered Pairs

Example 14: Graph $y = 2x + 1$ by plotting 3 different ordered pairs

x	*y*
0	1
1	3
2	5

T-Chart

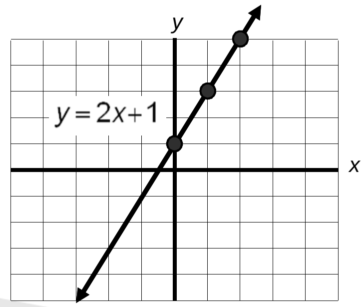
Choose any 3 values for *x* and find corresponding *y*'s

$y = 2\ (0) + 1 = 1$

$y = 2\ (1) + 1 = 3$

$y = 2\ (2) + 1 = 5$

Visual Check:
Does this look like a line with slope of 2?



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Graphs of Linear Equations

Method 1: Plot 3 Different Ordered Pairs

Example 15: Graph $y = -3x + 2$ by plotting 3 different ordered pairs

x	*y*
0	2
1	−1
2	−4

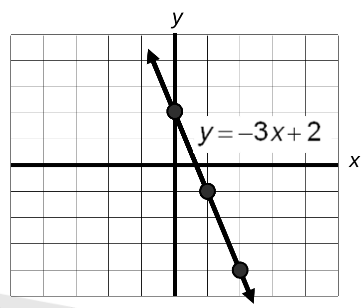
Choose any 3 values for *x* and find corresponding *y*'s

$y = -3\ (0) + 2 = 2$

$y = -3\ (1) + 2 = -3 + 2 = -1$

$y = -3\ (2) + 2 = -6 + 2 = -4$

Visual Check:
Does this look like a line with slope of -3?



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Graphs of Linear Equations

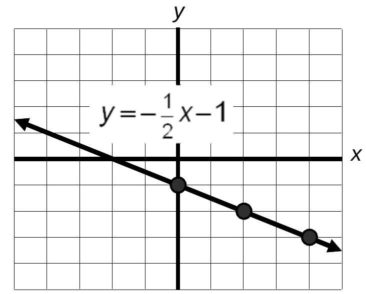
Method 1: Plot 3 Different Ordered Pairs (Con't)

Example 16: Graph $y = -\frac{1}{2}x - 1$ by plotting 3 different ordered pairs

- Because of "½ times x" we choose multiples of 2.

x	*y*
0	−1
2	−2
4	−3

y = -½ (0) – 1 = -1
y = -½ (2) – 1 = -2
y = -½ (4) – 1 = -3



Visual Check:
Does this look like a line with slope of -½?

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Graphs of Linear Equations

Method 1: Plot 3 Different Ordered Pairs

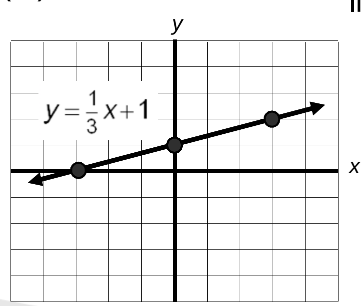
Example 17: Graph $y = \frac{1}{3}x + 1$ by plotting 3 different ordered pairs

Because of "⅓ times x" we choose multiples of 3.

x	*y*
0	1
3	2
−3	0

$y = \frac{1}{3}(0) + 1 = 1$
$y = \frac{1}{3}(3) + 1 = 2$
$y = \frac{1}{3}(-3) + 1 = 0$

Visual Check:
Does this look like a line with slope of $\frac{1}{3}$?



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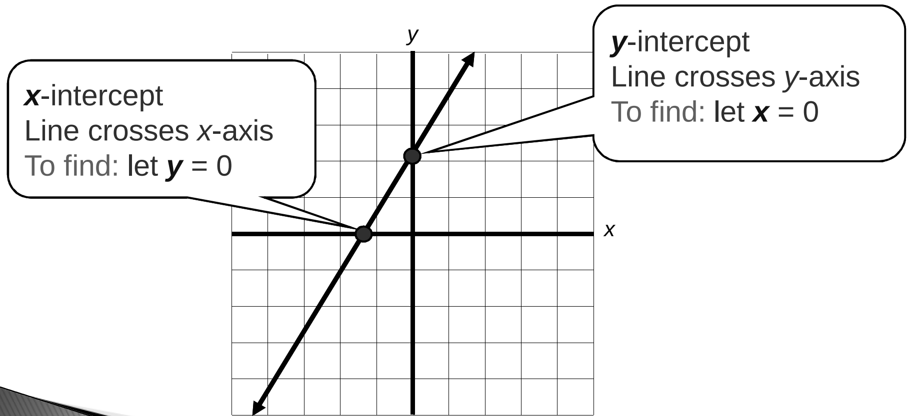
Graphs of Linear Equations
Method 2: Graph Using Intercepts

Recall there are 2 kinds of intercepts



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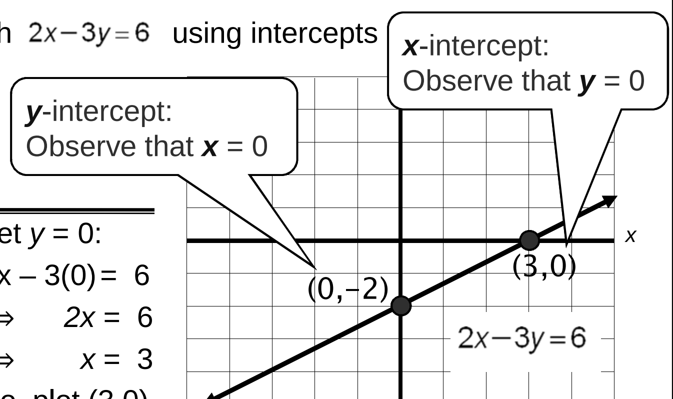
Graphs of Linear Equations
Method 2: Graph Using Intercepts (Con't)

Example 18: Graph $2x - 3y = 6$ using intercepts

x	***y***
0	−2
3	0

Let $x = 0$:	Let $y = 0$:
$2(0) - 3y = 6$	$2x - 3(0) = 6$
$\Rightarrow \quad -3y = 6$	$\Rightarrow \quad 2x = 6$
$\Rightarrow \quad y = -2$	$\Rightarrow \quad x = 3$
So, plot (0,-2)	So, plot (3,0)



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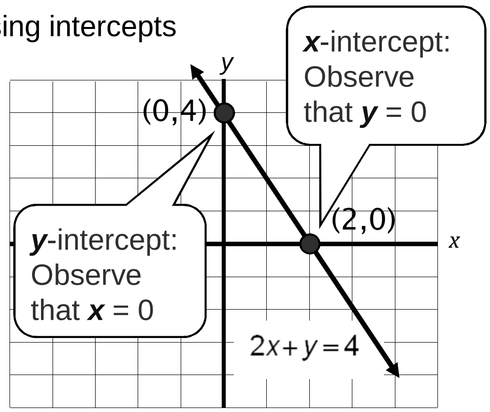
Graphs of Linear Equations
Method 2: Graph Using Intercepts (Con't)

Example 19: Graph $2x + y = 4$ using intercepts

x	*y*
0	4
2	0

Let $x = 0$:	Let $y = 0$:
$2(0) + y = 4$	$2x + 0 = 4$
$\Rightarrow \quad y = 4$	$\Rightarrow \quad 2x = 4$
So, plot (0,4)	$\Rightarrow \quad x = 2$
	So, plot (2,0)

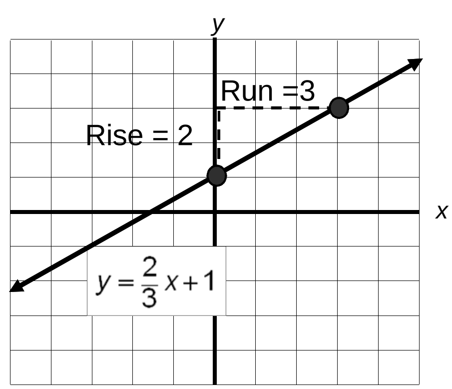


Graphs of Linear Equations
Method 3: Graph Using Slope and *y*-Intercept

Example 20: Graph $y = \frac{2}{3}x + 1$ using slope and *y*-intercept

- $y = mx + b$
- $m = \frac{2}{3}$ and *y*-intercept coordinates = (0,1)
- First, plot the *y*-intercept point
- Then break out slope into rise over run to find another point on the line.

$$m = \frac{2}{3} = \frac{\text{Rise}}{\text{Run}}$$

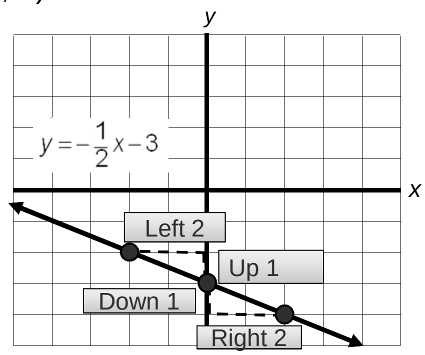


Graphs of Linear Equations

Method 3: Graph Using Slope and *y*-Intercept

Example 21: Graph $y = -\frac{1}{2}x - 3$ using slope and *y*-intercept

- Begin with *y* = *mx* + *b*. Thus, $m = -\frac{1}{2}$ and $b = -3$
- We first plot the *y*-intercept point (0,-3)
- What do we do when *m* < 0?
- When *m* < 0, we can go
  - Up and left: $m = \frac{1}{-2}$
  - Down and right: $m = \frac{-1}{2}$
- Decide for yourself which to do



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Graphs of Linear Equations

Method 3: Graph Using Slope and *y*-Intercept

Example 22: Graph $2x + 3y = 6$ using slope and *y*-intercept

We first rewrite equation as $y = mx + b$

$$2x + 3y = 6$$

$$3y = -2x + 6$$

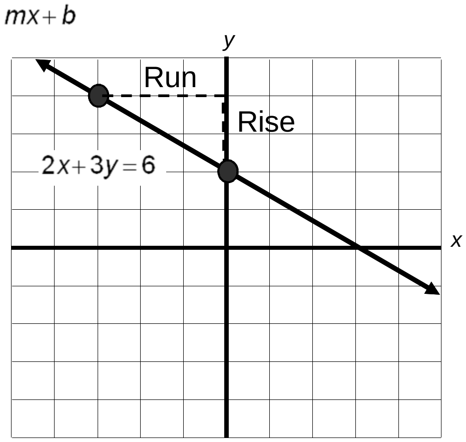
$$y = -\frac{2}{3}x + 2$$

$m = -\frac{2}{3}$ $b = 2$

y-intercept point is (0,2)

Plot the *y*-intercept point

$m = -\frac{2}{3} = \frac{\text{Rise}}{\text{Run}}$



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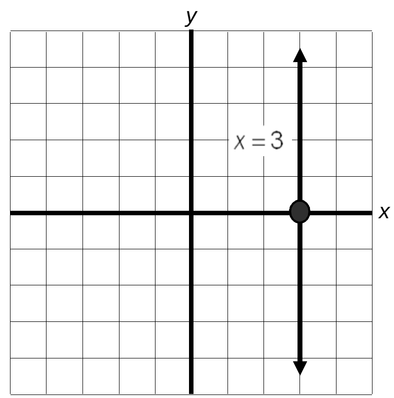
Graphing Vertical & Horizontal Lines

- ***Example 23:*** Graph $x = 3$
- Where is 3 on the x-axis?
- If you move to the left or right, is x still = 3?
- If you move up or down, is $x = 3$?
- Therefore $x = 3$ is a vertical line



Problem can also be worked using a t-chart

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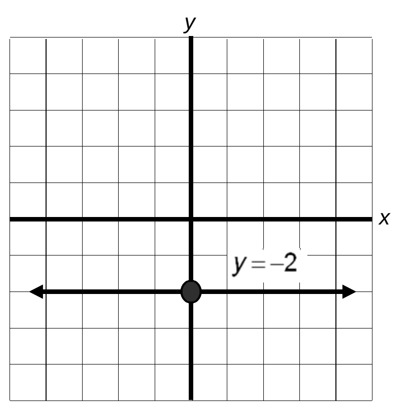
Graphing Vertical & Horizontal Lines

- ***Example 24:*** Graph $y = -2$
- Where is -2 on the y-axis?
- If you move up or down, is $y = -2$?
- If you move to the left or right, is y still = -2?
- Is this line horizontal or vertical?



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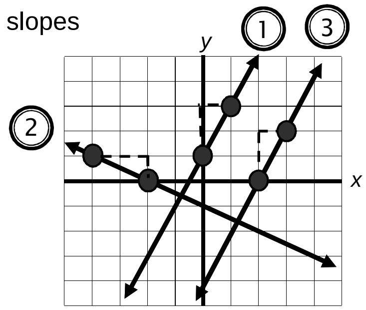
Parallel and Perpendicular Lines

Parallel lines have equal slopes ($m_1 = m_2$)
Perpendicular lines have negative reciprocal slopes

i.e. $m_2 = \frac{-1}{m_1}$, $m_1 = \frac{-1}{m_2}$,

or $m_1 \bullet m_2 = -1$

Example:
Line 1: $m_1 = 2$
Line 2: $m_2 = -\frac{1}{2}$
Line 3: $m_3 = 2$



Now, without using the graph…
Lines 1 and 3 are parallel. Why? $m_1 = m_3$
Lines 1 and 2 are perpendicular because … $2 \bullet \left[-\frac{1}{2}\right] = -1$
Lines 2 and 3 are perpendicular

Lines can be parallel, perpendicular, or neither

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Summary – We learned …

- In order to do equations of lines, we only need to memorize 2 formulas
  - $y = mx + b$ Always start with this for equations of lines
  - $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}}$ Use whenever 2 points are given
- The meaning of slope
  - We learned about positive, negative, zero, and undefined slopes
  - We learned the meaning of rise-over-run
  - We learned about steep and shallow slopes
  - We learned about the slopes of parallel and perpendicular lines
- The *y*-intercept is where the line intercepts the *y*-axis
- The *x*-intercept is where the line intercepts the *x*-axis
- How to graph lines by plotting 3 points, plotting intercepts, and using slope and the *y*-intercept

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Concluding Note

- There are other formulas for lines besides the Slope-Intercept formula $y = mx + b$
  - *In particular, the Point-Slope formula* $y - y_1 = m(x - x_1)$ *is very useful for problems where you are given the slope and one point*
- However, all line problems can be solved using just the Slope-Intercept formula
- We recommend waiting until after you have mastered lines using the Slope-Intercept formula before you learn to use the Point-Slope formula
- For some students, this may not occur until Precalculus or another math course

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