

# Reconstructing veriT proofs in Isabelle/HOL

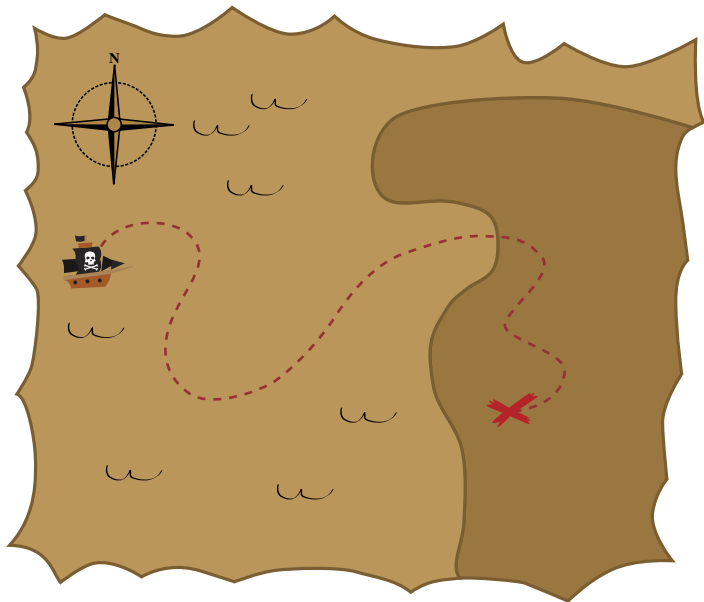
VeriDis Retreat + Matryoshka Workshop 2019

Amsterdam – Pays-Bas

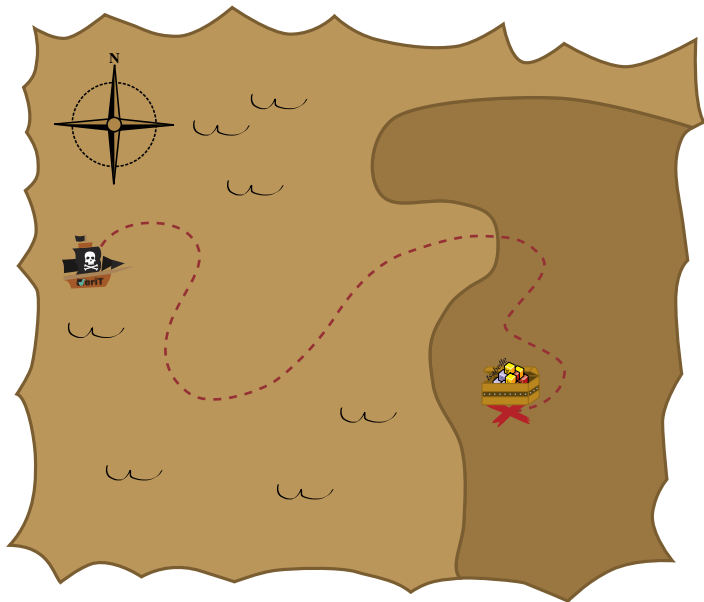
Hans-Jörg Schurr

June 12, 2019

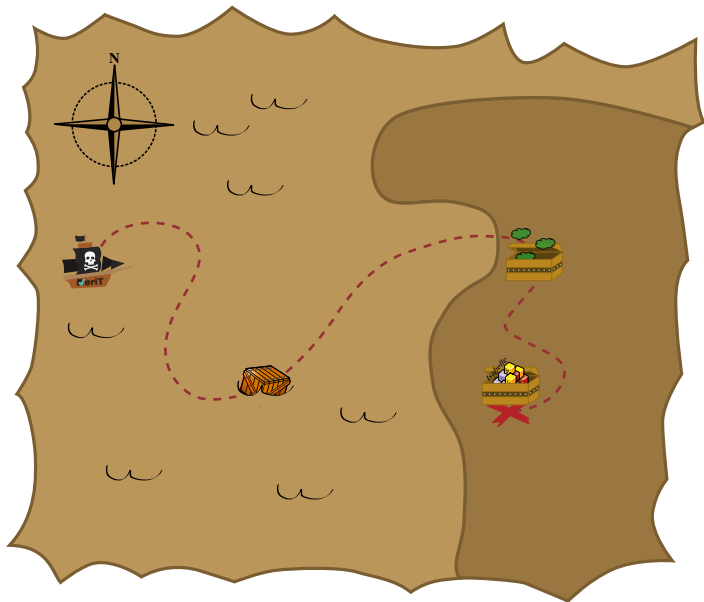
# An Adventure



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# Proof Reconstruction in Isabelle/HOL

- ▶ Proof automation allows faster proof development
- ▶ One approach:
  1. Encode proof obligation into SMT-LIB
  2. Call an ATP
  3. Reconstruct the resulting proof
- ▶ Implemented by the `smt` tactic in Isabelle/HOL using Z3
  - ▶ Reconstruction can fail
  - ▶ Restricted to Z3
  - ▶ We want perfect reconstruction

The screenshot shows the Isabelle/HOL IDE interface. The main window displays a theorem named `Example` with the following code:

```
theory Example
  imports Main
begin

lemma
  assumes
    <math>\langle \wedge x s \ x \ ys. \ xs @ x \ \# \ ys = x1 @ xs2 @ x2 \ \# \ ys' \Rightarrow P \ ys' \rangle</math>
  shows <math>P \ ys'</math>
  using assms [[smt_trace, smt_solver=z3]]
  by (smt append.assoc)

end
```

The `shows` line is highlighted in yellow. The left sidebar shows the 'File Browser' and 'Documentation' panels. The right sidebar shows the 'Output', 'Sidekick', 'State', and 'Theories' panels. The bottom status bar indicates the version '8.16 (127/201)' and the current file 'Example.thy'.

# Assisting Proof Construction

- ▶ Built-in methods
  - ▶ LCF approach
  - ▶ Checked by the prover kernel
  - ▶ In Isabelle: `auto`, `metis`, ...
- ▶ External automation:
  - ▶ `smt` with Z3 in Isabelle, `SMTCoq`
  - ▶ Hammers: `Sledgehammer`, `HOL(y)Hammer`, `CoqHammer`



# The SMT Solver veriT

- ▶ Traditional CDCL(T) solver
- ▶ Supports:
  - ▶ Uninterpreted functions
  - ▶ Linear Arithmetic
  - ▶ Non-Linear Arithmetic
  - ▶ Quantifiers
  - ▶ ...
- ▶ Proof producing
- ▶ SMT-LIB input

```
(set-option :produce-proofs true)
(set-logic AUFLIA)
(declare-sort A$ 0)
(declare-sort A_list$ 0)
(declare-fun p$ (A_list$) Bool)
(declare-fun x1$ () A_list$)
(declare-fun x2$ () A$)
(declare-fun ys$ () A_list$)
(declare-fun xs2$ () A_list$)
(declare-fun cons$ (A$ A_list$) A_list$)
(declare-fun append$ (A_list$ A_list$) A_list$)
(assert (! (forall ((?v0 A_list$) (?v1 A_list$)
  (?v2 A_list$)) (= (append$ (append$ ?v0 ?v1) ?v2)
    (append$ ?v0 (append$ ?v1 ?v2)))) :named a0))
(assert (! (forall ((?v0 A_list$) (?v1 A$)
  (?v2 A_list$)) (=> (= (append$ ?v0 (cons$ ?v1 ?v2))
    (append$ x1$ (append$ xs2$ (cons$ x2$ ys$))))
  (p$ ys$))) :named a1))
(assert (! (not (p$ ys$)) :named a2))
(check-sat)
(get-proof)
```

# Proofs from SMT Solvers

## Use Cases

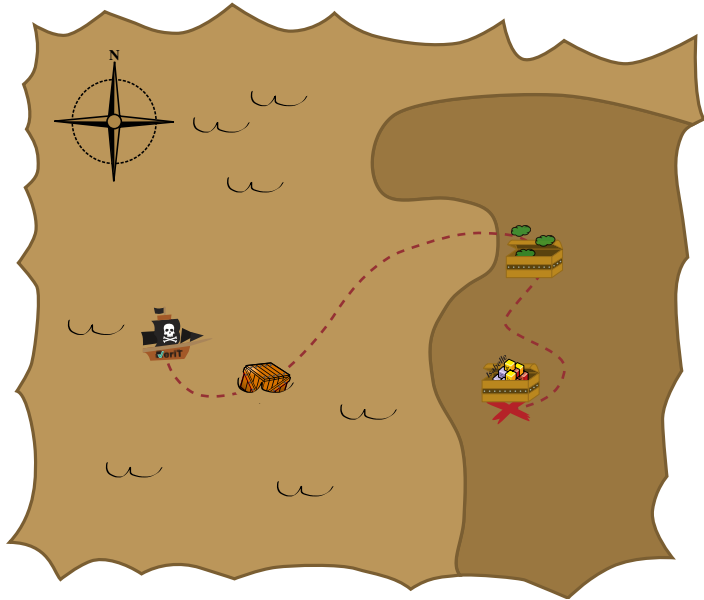
- ▶ Learning from proofs:
  - ▶ guidance: (FE)MaLeCoP, rICoP (reinforcement learning), ...
  - ▶ see also Daniel's talk
- ▶ Unsatisfiable cores
- ▶ Finding interpolants
- ▶ Result certification if the problem is unsatisfiable
- ▶ Debugging

## Proof Generating SMT Solvers

CVC4 (LFSC, no proofs for quantifiers), Z3 (SMT-LIB based proof trees, coarser steps, esp. for skolemization), veriT, ArchSAT, ZenonModulo (Deducti), ...



# Setting Sails



## veriT's Proofs

```
(assume h1 (not (p a)))
(assume h2 (forall ((z1 U)) (forall ((z2 U)) (p z2))))
...
(anchor :step t9 :args ((:= z2 veriT_vr4)))
(step t9.t1 (cl (= z2 veriT_vr4)) :rule refl)
(step t9.t2 (cl (= (p z2) (p veriT_vr4))) :rule cong :premises (t9.t1))
(step t9 (cl (= (forall ((z2 U)) (p z2))
                (forall ((veriT_vr4 U)) (p veriT_vr4)))) :rule bind)
...
(step t14 (cl (forall ((veriT_vr5 U)) (p veriT_vr5)))
          :rule th_resolution :premises (t11 t12 t13))
(step t15 (cl (or (not (forall ((veriT_vr5 U)) (p veriT_vr5))) (p a)))
          :rule forall_inst :args ((:= veriT_vr5 a)))
(step t16 (cl (not (forall ((veriT_vr5 U)) (p veriT_vr5))) (p a))
          :rule or :premises (t15))
(step t17 (cl) :rule resolution :premises (t16 h1 t14))
```

## veriT's Proofs

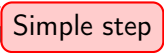
Input assumptions



```
(assume h1 (not (p a)))
(assume h2 (forall ((z1 U)) (forall ((z2 U)) (p z2))))
...
(anchor :step t9 :args ((:= z2 veriT_vr4)))
(step t9.t1 (cl (= z2 veriT_vr4)) :rule refl)
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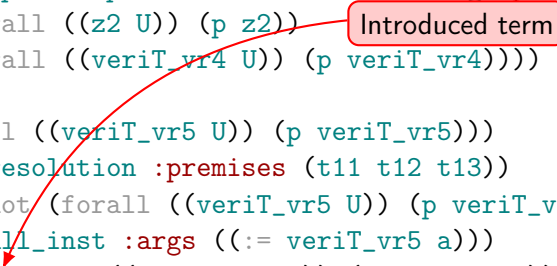
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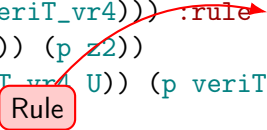
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## veriT's Proofs

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```



A red arrow points from a red-outlined box labeled "Premises" to the argument `(t9.t1)` in the `:premises` field of the `(step t9.t2)` line.



## veriT's Proofs

```
(assume h1 (not (p a)))
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...
(anchor :step t9 :args ((:= z2 veriT_vr4)))
(step t9.t1 (cl (= z2 veriT_vr4)) :rule refl)
(step t9.t2 (cl (= (p z2) (p veriT_vr4))) Context annotation ses (t9.t1))
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          :rule or :premises (t15))
(step t17 (cl) :rule resolution :premises (t16 h1 t14))
```

## veriT's Proofs

```
(assume h1 (not (p a)))
(ass ... Skolemization is done by showing lemmas of the form  $(\exists x.P[x]) = P[(\epsilon x.P)/x]$  ...))
...
(anchor :step to target ((:= veriT_vr4)))
(step t9.t1 (cl (= z2 veriT_vr4)) :rule refl)
(step t9.t2 (cl (= (p z2) (p veriT_vr4))) :rule cong :premises (t9.t1))
(step t9 (cl (= (forall ((z2 U)) (p z2))
                (forall ((veriT_vr4 U)) (p veriT_vr4)))) :rule bind)
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(step t14 (cl (forall ((veriT_vr5 U)) (p veriT_vr5)))
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(step t15 (cl (or (not (forall ((veriT_vr5 U)) (p veriT_vr5))) (p a)))
          :rule forall_inst :args ((:= veriT_vr5 a)))
(step t16 (cl (not (forall ((veriT_vr5 U)) (p veriT_vr5))) (p a))
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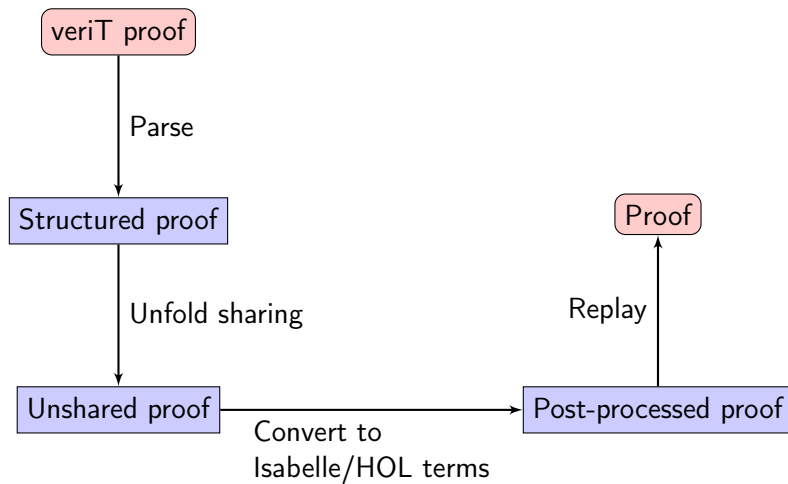
## Collaborate

Given that we are both developers of the SMT solver and the reconstruction, many problems (bugs, unclarities, etc.) can be solved on short notice.

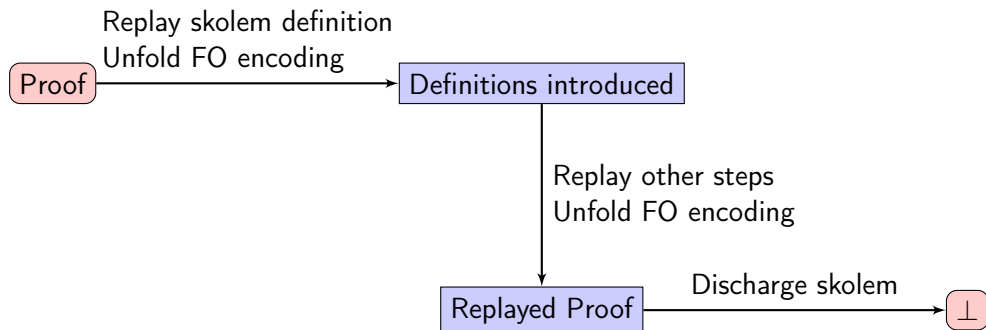
## Documentation

- ▶ Automatically generated: `--proof-format-and-exit`
  - ▶ Necessarily contains all rules
- ▶ Past publications (Besson et al. 2011, Déharbe et al. 2011, Barbosa et al. 2019)

## The Reconstruction Inside Isabelle/HOL



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## Direct Proof Rules

- ▶ Assume  $A \Rightarrow B$  is applied
- ▶ We assume  $A$
- ▶ We derive  $B'$
- ▶ then `simp/fast/blast` to discharge  
 $B' \Rightarrow B$

## Hand-described Rules

- ▶ Call specific tactic for specific rules
- ▶ Some simplification (for speed)
- ▶ Terminal tactics

# Reconstruction

## Direct Proof Rules

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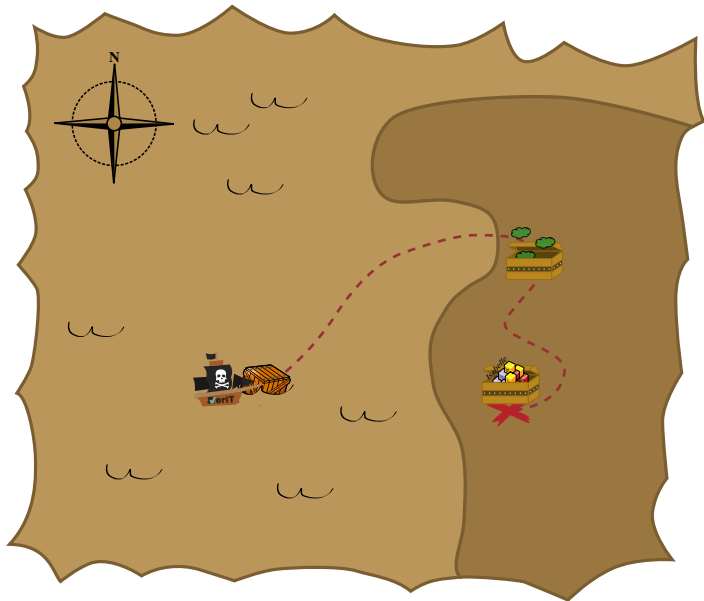
## Hand-described Rules

- ▶ Call specific tactic for specific rules
- ▶ Some simplification (for speed)
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## Challenges

- ▶ `arith` is too weak to reliably reconstruct the current arithmetic step
- ▶ Skolemization
- ▶ The `connective_equiv` rule:
  - ▶ Encodes “trivial” truth about theory connectives
  - ▶ First attempt to solve on the propositional level
  - ▶ Then try automation
- ▶ Implicit steps
  - ▶ Order of  $=$  is freely changed
  - ▶ Step simplification:
$$a \approx b \wedge a \approx b \Rightarrow f(a, a) \approx f(b, b)$$
$$a \approx b \Rightarrow f(a, a) \approx f(b, b)$$
  - ▶ Double negation is eliminated

## Weight: Proof Size







## Weight: Proof Size

- ▶ Proofs are often huge
- ▶ Linear presentation unrolls shared terms
  - ▶ The choice terms introduced by skolemization can be huge
- ▶ veriT proofs support optional sharing
- ▶ Utilizes `(! t :named n)` syntax of SMT-LIB

## Where to introduce names?

- ▶ Perfect solution is hard to find
- ▶ Approximate: Terms which appear with two different parents get a name
  - ▶  $f(h(a), j(x, y)), g(h(a)), g(f(h(a), j(x, y)))$
  - ▶  $[f([h(a)]_{p_2}, j(x, y))]_{p_1}, [g(p_2)]_{p_3}, [g(p_1)]_{p_4}$
- ▶ Can be done in linear time thanks to perfect sharing

## Isabelle/HOL side

- ▶ Isabelle/HOL unfolds everything
- ▶ ... except for skolem terms where the name is used.

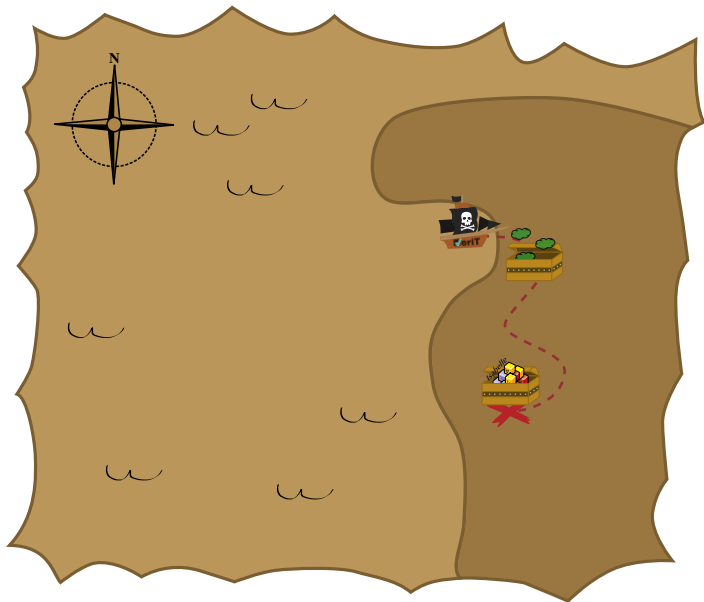
# Proof Without Sharing

```
(assume h1 (and (forall ((?veriT.veriT__4 Client) (?veriT.veriT__3 Client)) (= ?veriT.veriT__4 ?veriT.veriT__3)) (not (=
c1 c2))))
(anchor :step t2 :args ((:= ?veriT.veriT__4 veriT_vr0) (:= ?veriT.veriT__3 veriT_vr1)))
(step t2.t1 (c1 (= ?veriT.veriT__4 veriT_vr0)) :rule refl)
(step t2.t2 (c1 (= ?veriT.veriT__3 veriT_vr1)) :rule refl)
(step t2.t3 (c1 (= (= ?veriT.veriT__4 ?veriT.veriT__3) (= veriT_vr0 veriT_vr1))) :rule cong :premises (t2.t1 t2.t2))
(step t2 (c1 (= (forall ((?veriT.veriT__4 Client) (?veriT.veriT__3 Client)) (= ?veriT.veriT__4 ?veriT.veriT__3)) (forall
((veriT_vr0 Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)))) :rule bind)
(step t3 (c1 (= (and (forall ((?veriT.veriT__4 Client) (?veriT.veriT__3 Client)) (= ?veriT.veriT__4 ?veriT.veriT__3))
(not (= c1 c2))) (and (forall ((veriT_vr0 Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)) (not (= c1 c2)))))) :rule
cong :premises (t2))
(step t4 (c1 (not (= (and (forall ((?veriT.veriT__4 Client) (?veriT.veriT__3 Client)) (= ?veriT.veriT__4 ?veriT.veriT__3))
(not (= c1 c2))) (and (forall ((veriT_vr0 Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)) (not (= c1 c2)))) (not
(and (forall ((?veriT.veriT__4 Client) (?veriT.veriT__3 Client)) (= ?veriT.veriT__4 ?veriT.veriT__3)) (not (= c1 c2))))
(and (forall ((veriT_vr0 Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)) (not (= c1 c2)))) :rule equiv_pos2)
(step t5 (c1 (and (forall ((veriT_vr0 Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)) (not (= c1 c2)))) :rule
th_resolution :premises (h1 t3 t4))
(anchor :step t6 :args ((:= veriT_vr0 veriT_vr2) (:= veriT_vr1 veriT_vr3)))
(step t6.t1 (c1 (= veriT_vr0 veriT_vr2)) :rule refl)
(step t6.t2 (c1 (= veriT_vr1 veriT_vr3)) :rule refl)
(step t6.t3 (c1 (= (= veriT_vr0 veriT_vr1) (= veriT_vr2 veriT_vr3))) :rule cong :premises (t6.t1 t6.t2))
(step t6 (c1 (= (forall ((veriT_vr0 Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)) (forall ((veriT_vr2 Client)
(veriT_vr3 Client)) (= veriT_vr2 veriT_vr3)))) :rule bind)
(step t7 (c1 (= (and (forall ((veriT_vr0 Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)) (not (= c1 c2))) (and (forall
((veriT_vr2 Client) (veriT_vr3 Client)) (= veriT_vr2 veriT_vr3)) (not (= c1 c2)))) :rule cong :premises (t6))
(step t8 (c1 (not (= (and (forall ((veriT_vr0 Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)) (not (= c1 c2))) (and
(forall ((veriT_vr2 Client) (veriT_vr3 Client)) (= veriT_vr2 veriT_vr3)) (not (= c1 c2)))) (not (and (forall ((veriT_vr0
Client) (veriT_vr1 Client)) (= veriT_vr0 veriT_vr1)) (not (= c1 c2))) (and (forall ((veriT_vr2 Client) (veriT_vr3 Client))
(= veriT_vr2 veriT_vr3)) (not (= c1 c2)))) :rule equiv_pos2)
(step t9 (c1 (and (forall ((veriT_vr2 Client) (veriT_vr3 Client)) (= veriT_vr2 veriT_vr3)) (not (= c1 c2)))) :rule
th_resolution :premises (t5 t7 t8))
(step t10 (c1 (forall ((veriT_vr2 Client) (veriT_vr3 Client)) (= veriT_vr2 veriT_vr3))) :rule and :premises (t9))
(step t11 (c1 (not (= c1 c2))) :rule and :premises (t9))
(step t12 (c1 (or (not (forall ((veriT_vr2 Client) (veriT_vr3 Client)) (= veriT_vr2 veriT_vr3)) (= c1 c2))) :rule forall_inst
:args ((:= veriT_vr2 c2) (:= veriT_vr3 c1)))
(step t13 (c1 (not (forall ((veriT_vr2 Client) (veriT_vr3 Client)) (= veriT_vr2 veriT_vr3)) (= c1 c2)) :rule or :premises (t12))
```

# Proof With Sharing

```
(assume h1 (! (and (! (forall ((?veriT.veriT_4 Client) (?veriT.veriT_3 Client)) (! (= ?veriT.veriT_4 ?veriT.veriT_3)
:named @p_3)) :named @p_2) (! (not (! (= c1 c2) :named @p_5)) :named @p_4)) :named @p_1))
(anchor :step t2 :args ((:= ?veriT.veriT_4 veriT_vr0) (:= ?veriT.veriT_3 veriT_vr1)))
(step t2.t1 (c1 (! (= ?veriT.veriT_4 veriT_vr0) :named @p_6)) :rule refl)
(step t2.t2 (c1 (! (= ?veriT.veriT_3 veriT_vr1) :named @p_7)) :rule refl)
(step t2.t3 (c1 (! (= @p_3 (! (= veriT_vr0 veriT_vr1) :named @p_9)) :named @p_8)) :rule cong :premises (t2.t1 t2.t2))
(step t2 (c1 (! (= @p_2 (! (forall ((veriT_vr0 Client) (veriT_vr1 Client)) @p_9) :named @p_11)) :named @p_10)) :rule bind)
(step t3 (c1 (! (= @p_1 (! (and @p_11 @p_4) :named @p_13)) :named @p_12)) :rule cong :premises (t2))
(step t4 (c1 (! (not @p_12) :named @p_14) (! (not @p_1) :named @p_15) @p_13) :rule equiv_pos2)
(step t5 (c1 @p_13) :rule th_resolution :premises (h1 t3 t4))
(anchor :step t6 :args ((:= veriT_vr0 veriT_vr2) (:= veriT_vr1 veriT_vr3)))
(step t6.t1 (c1 (! (= veriT_vr0 veriT_vr2) :named @p_16)) :rule refl)
(step t6.t2 (c1 (! (= veriT_vr1 veriT_vr3) :named @p_17)) :rule refl)
(step t6.t3 (c1 (! (= @p_9 (! (= veriT_vr2 veriT_vr3) :named @p_19)) :named @p_18)) :rule cong :premises (t6.t1 t6.t2))
(step t6 (c1 (! (= @p_11 (! (forall ((veriT_vr2 Client) (veriT_vr3 Client)) @p_19) :named @p_21)) :named @p_20)) :rule bind)
(step t7 (c1 (! (= @p_13 (! (and @p_21 @p_4) :named @p_23)) :named @p_22)) :rule cong :premises (t6))
(step t8 (c1 (! (not @p_22) :named @p_24) (! (not @p_13) :named @p_25) @p_23) :rule equiv_pos2)
(step t9 (c1 @p_23) :rule th_resolution :premises (t5 t7 t8))
(step t10 (c1 @p_21) :rule and :premises (t9))
(step t11 (c1 @p_4) :rule and :premises (t9))
(step t12 (c1 (! (or (! (not @p_21) :named @p_27) @p_5) :named @p_26)) :rule forall_inst :args ((:= veriT_vr2 c2) (:=
veriT_vr3 c1)))
(step t13 (c1 @p_27 @p_5) :rule or :premises (t12))
(step t14 (c1) :rule resolution :premises (t13 t10 t11))
```

# Proof Rot





At the beginning everything was fine and veriT produced the step:

$$\forall x.p[x] \rightarrow p[t]$$



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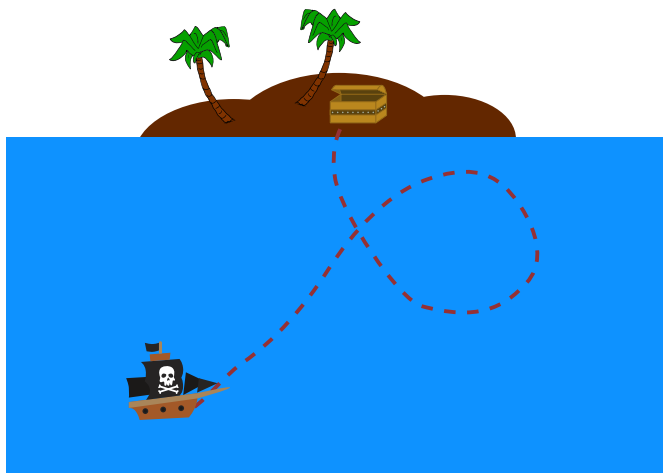
- ▶ Only a few lines of code change
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Since then: Under some circumstances  $p[x]$  is a CNF of another formula.

- ▶ Reconstruction forces you to stay honest



## Where We Are Now



Land in sight!

## Where We Are Now

Test on smt calls in the AFP:

- ▶ Hence, only theorems easy for Z3
- ▶ 498 calls, 447 proofs produced by veriT
- ▶ 443 proofs reconstructed
- ▶ Average solving time 303ms
- ▶ Average reconstruction time 679.4ms

Sledgehammer test:

Theory	Ord. Res. Prover	Formal SSA
Found proofs	5019	5961
Z3-powered	90	109
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Outlook

- ▶ Perfect reconstruction
- ▶ Isabelle/HOL as a certifier
- ▶ Long term: A widely accepted format




Sledgehammer test:

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Found proofs			5019		5961
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# Thank you for your attention!

- ▶ Questions? Suggestions?
- ▶ What would you like to see in the generated proofs?

## References I

-  Besson, Frédéric, Pascal Fontaine, and Laurent Théry (2011). “A Flexible Proof Format for SMT: a Proposal”. In: *PxTP 2011*. Ed. by Pascal Fontaine and Aaron Stump, pp. 15–26.
-  Déharbe, David, Pascal Fontaine, and Bruno Woltzenlogel Paleo (2011). “Quantifier Inference Rules for SMT proofs”. In: *PxTP 2011*. Ed. by Pascal Fontaine and Aaron Stump, pp. 33–39. URL: <https://hal.inria.fr/hal-00642535>.
-  Barbosa, Haniel et al. (2019). “Scalable Fine-Grained Proofs for Formula Processing”. In: *J. Automated Reasoning*.