Ryerson University

Department of Electrical, Computer and Biomedical Engineering

BME639: CONTROL SYSTEMS BIOROBOTICS

Lab Project 2

Transfer Function Modeling of Physical Systems and Control Introduction to Lead/Lag Compensators

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Introduction and Objectives

Each lab requires four weeks and the tasks for Part one (two weeks) and Part two (two weeks) are provided explicitly. The Lab project is required to provide complete answer to the questions. Note that formatting has 10% of the Lab grades and, 15% of each of your Lab grades is on your summary of the Lab which has to be provided individually. Your summary has to explain clearly what you have learned in the Lab.

Before submitting the report the TA asks questions about your report. If there is no consistency between your oral answer and the report, you will lose %50 of your total mark.

In the first part of this project, you become familiar with the basics of modelling of LTI systems in Simulink, to find the mathematical model and transfer function of a DC servo motor. Then you design and implement a PI controller for the DC servo motor.

In the second part of this project, you study the lead and lag compensator, and their time domain and frequency domain characteristics. Then you use these compensators to enhance the response of a second order system.

Part One (two weeks)

A. DC Servo Motor Modeling Using First-Principles

Consider the DC servo motor model illustrated in Figure(1),

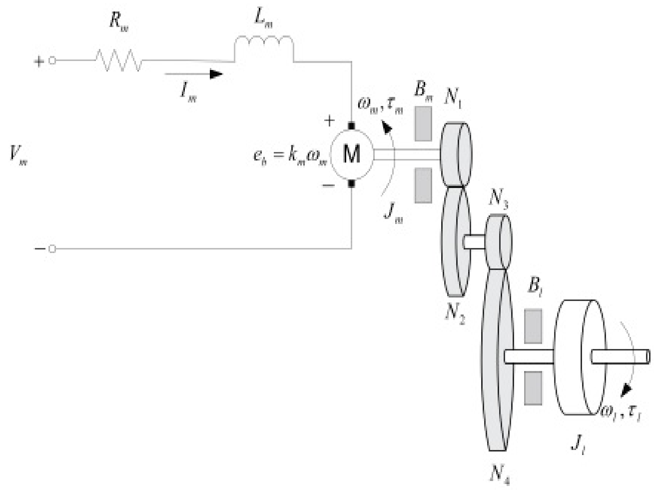


Figure 1: SRV02 DC servo motor armature circuit and gear train

The differential equation that describes the dynamic behaviour of the load shaft speed $\omega_l(t)$ as a function of the motor input voltage $v_m(t)$ can be written as:

$$J_{eq}\dot{\omega}_l(t) + B_{eq}\omega_l(t) = A_m v_m(t)$$

$$J_{eq} = \eta_g K_g^2 J_m + J_l \ , \ B_{eq} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_{eq1} R_m}{R_m} \ , \ A_m = \frac{\eta_g K_g \eta_m k_t}{R_m}$$

The parameters of the servo motor are defined as follows:

v_m Motor input voltage
R_m Motor armature resistance (2.6Ω)
L_m Motor armature inductance (0.18 mH)
J_m Rotor moment of inertia about the motor shaft ($4.61 \times 10^{-7} kgm^2$)
B_m Viscous friction acting on the motor shaft
k_t Motor current-torque constant (7.68×10^{-3} Nm/A)
k_m Motor back-emf constant (7.68×10^{-3} V/(rad/s))
K_g High-gear total gear ratio (14×5)
η_g Gearbox efficiency (0.90)
η_m Motor efficiency (0.69)
J_l Total load moment of inertia ($1.03 \times 10^{-4} kgm^2$)
B_l Viscous friction acting on the load shaft
$B_{eq1} = \eta_g K_g^2 B_m + B_l$ Equivalent viscous damping coefficient ($0.015 \ Nm/(rad/s)$)

A.1:[3 Marks] Find the voltage-to-speed transfer function of the SRV02 servo motor $\frac{\Omega_l(s)}{V_m(s)}$ in terms of J_{eq}, B_{eq}, and A_m parameters by using the differential equation of the servo motor system. What is order of the system? What is type of the system? Find the DC gain (k) and the time constant (τ) of the process model in terms of J_{eq}, B_{eq}, and A_m parameters.

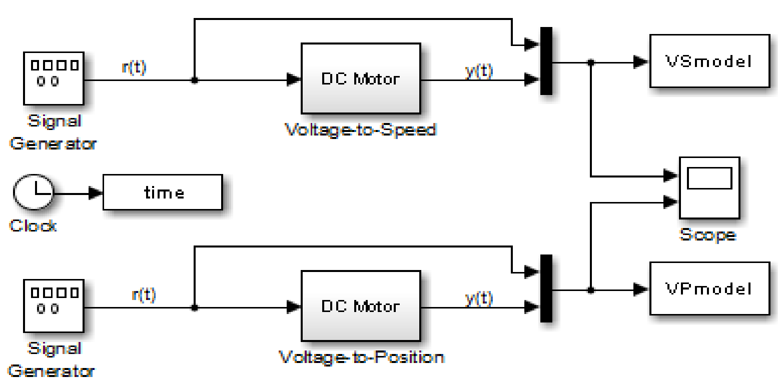
	Transfer function $\frac{\Omega_l(s)}{V_m(s)}$	**System order**	**System type**	**DC-gain** (K)	**Time constant** (τ)
Voltage-to Speed					

A.2:[3 Marks] Find the voltage-to-position transfer function of the SRV02 servo motor, $\frac{\Theta_l(s)}{V_m(s)}$ in terms of J_{eq}, B_{eq}, and A_m parameters. What is order of the system? What is type of the system? Find poles and zeros of the system.

	$\frac{\Theta_l(s)}{V_m(s)}$	System order	System type	poles	zeros
voltage-to-position					

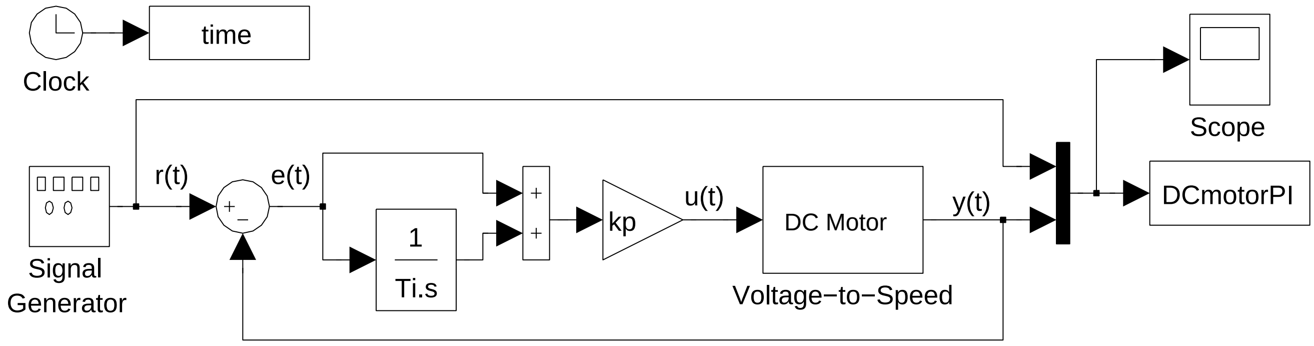
A.3:[2 Marks] Calculate J_{eq}, B_{eq}, and A_m parameters, and DC gain (k) and the time constant (τ) using the system specification. These are the nominal model parameters and will be used to compare with parameters that are later found experimentally. Is the DC motor you modeled stable? Explain your answer.

A.4:[4 Marks] Generate the voltage-to-speed and voltage-to-position transfer functions of the servo motor in the same Simulink model. Plot input and output of square wave response of the open-loop systems with amplitude one and frequency 0.5Hz (Run the simulation for 5 seconds). Provide a print of your Simulink model.



B. DC Servo Motor Speed Control

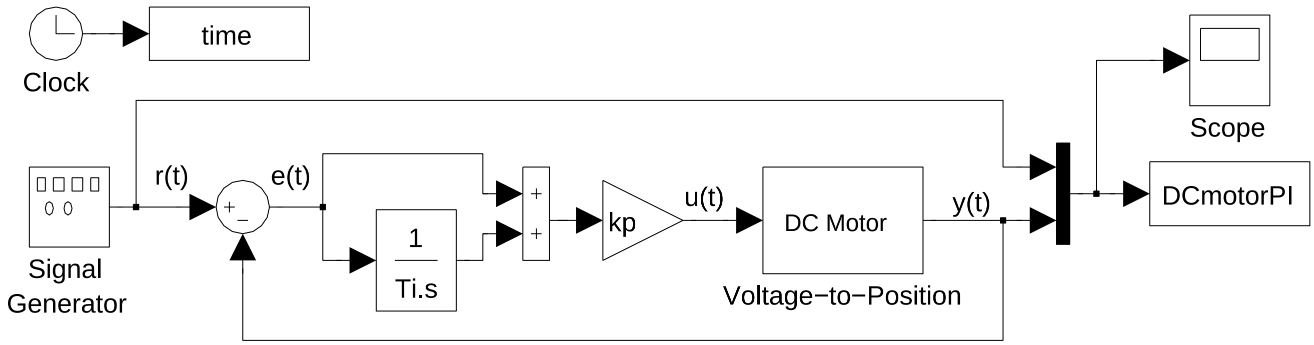
B.1:[2 Marks] Consider voltage-to-speed transfer function $\frac{\Omega_l(s)}{V_m(s)}$ of the DC motor from Part A with a PI controller.



Generate a square wave signal with amplitude one and frequency 2Hz. Simulate the voltage-to-speed transfer function of the servo motor with PI controller for $T_i = 0.02$ and $k_p = 2$. Plot reference input and output of the closed-loop system (Run the simulation for 2 seconds). Provide a print of your Simulink file.

C. DC Servo Motor Position Control

C.1:[2 Marks] Consider the DC motor voltage-to-position transfer function $\frac{\Theta_l(s)}{V_m(s)}$ from Part A. In Simulink generate a sawtooth wave signal with amplitude one and frequency 0.1Hz. Simulate the voltage-to-position transfer function of the servo motor with PI controller for $T_i = 2$ and $k_p = 5$.



Plot reference input and output of the closed-loop system (Run the simulation for 30 seconds). Provide a print of your Simulink model.

Part Two (two weeks)

A. Time Response of a Lead Compensator

A.1:[4 Marks] In Simulink, generate the following lead compensator,

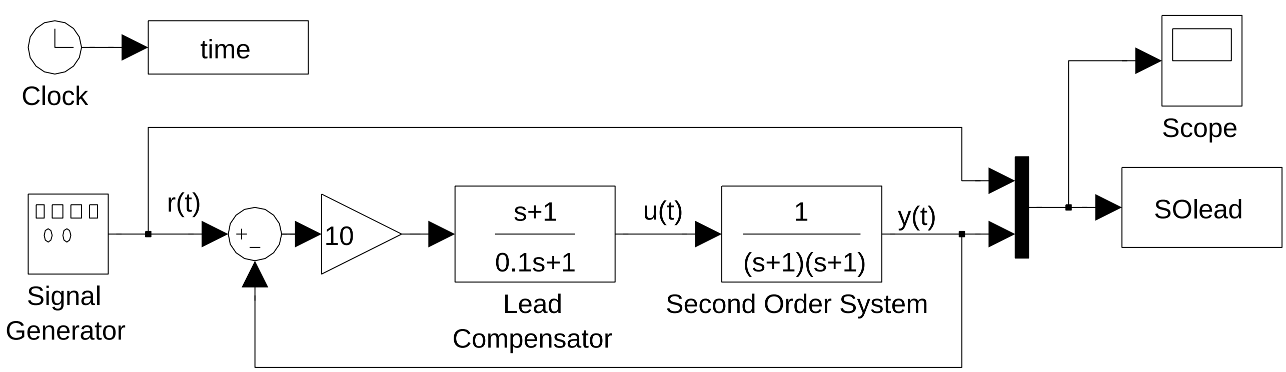
$$\frac{s+1}{0.1s+1}$$

Generate a square wave with amplitude one and frequency 0.2Hz. Plot input and output of the lead compensator to the square wave (Run the simulation for 5 seconds). Find time constant (τ), and gain of the system for $s \to 0$ (DC gain) and $s \to \infty$ (High frequency gain), then compare them with output of the system.

A.2:[5 Marks] Find pole and zero of the lead compensator. Plot Bode diagram of the lead compensator (Magnitude and Phase) using `bode` function in MATLAB. Determine maximum and minimum phase which could obtain by using this lead compensator.

B. Lead Compensator and Second-Order Systems

B.1:[2 Marks] Generate the following closed-loop system in Simulink,



Plot input and output of the system to the square wave signal with amplitude one and frequency 0.15Hz (Run the simulation for 10 seconds). Provide a print of your Simulink model.

B.2:[3 Marks] Find closed-loop transfer function for the system, $\frac{Y(s)}{R(s)}$. Find time domain specifications of the output, rise time (t_r), percentage of maximum overshoot ($\%O.S.$), settling time (t_s), steady state error (e_{ss}) by using `tf` and `stepplot` functions in MATLAB.

B.3:[3 Marks] Compare the time domain specification with results of the close-loop system with only proportional control for $k = 10$. Explain the effect of lead compensator on transient response and steady-state response of the system.

	Rise time (t_r)	% Max. overshoot ($\%O.S.$)	Settling time (t_s)	Steady-state error (e_{ss})
K = 10 with lead				
K = 10				

B.4:[3 Marks] Plot Bode diagram of the system with lead compensator and with only proportional gain $k = 10$ by using `bode` function in MATLAB. Show Gain Margin (GM) and Phase Margin (PM) on the bode plot and compare them. Explain how the lead compensator could increase stability of the closed-loop system.

C. Time Response of a Lag Compensator

C.1:[4 Marks] In Simulink, generate the following lag compensator,

$$\frac{s+3}{s+0.3}$$

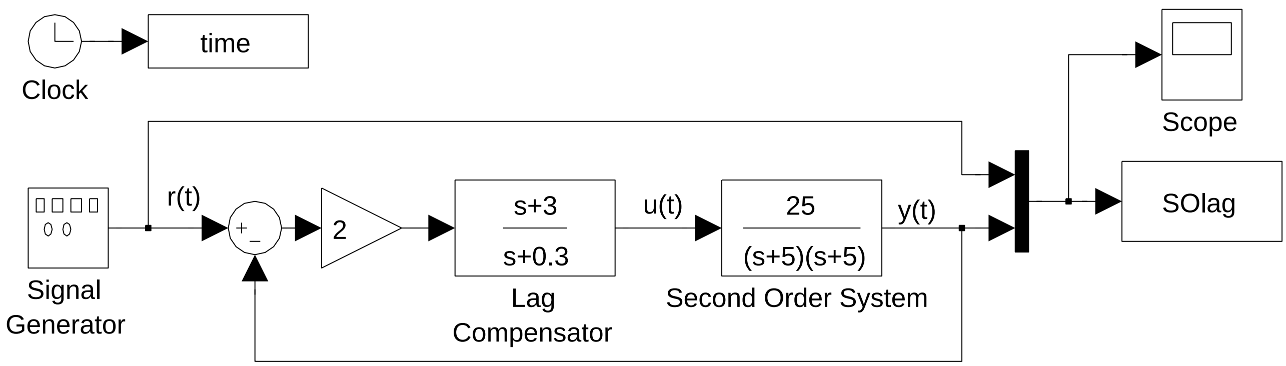
Generate a square wave with amplitude one and frequency 0.02Hz. Plot input and output of the lag compensator to the square wave (Run the simulation for 100 seconds). Find time constant (τ), and the gain of the system for $s \to 0$ (DC gain) and $s \to \infty$ (High frequency gain), then compare them with output of the system.

C.2:[5 Marks] Find pole and zero of the lag compensator. Plot Bode diagram of the lag compensator (Magnitude and Phase) using `bode` function in MATLAB. Determine the maximum and minimum phase which could obtain by using this lag compensator.

D. Lag Compensator and Second-Order Systems

D.1:[2 Marks] Generate the following closed-loop system in Simulink,
Generate a square wave with amplitude one and frequency 0.15Hz. Plot the input signal and output of the system to the square wave signal (Run the simulation for 10 seconds). Provide a print of your Simulink model.

D.2:[3 Marks] Find closed-loop transfer function for the system, $\frac{Y(s)}{R(s)}$. Find the time domain specifications, rise time (t_r), percentage of maximum overshoot ($\%O.S.$), settling time (t_s), steady state error (e_{ss}) by using `tf` and `stepplot` functions in MATLAB.



D.3:[**4 Marks**] Compare the time domain specification with results of the close-loop system with only proportional control, for $k = 2$ and $k = 20$. Explain the effect of lag compensator on transient response and steady-state response of the system.

	Rise time (t_r)	% Max. overshoot ($\%O.S.$)	Settling time (t_s)	Steady-state error (e_{ss})
K = 2 with lag				
K = 2				
K = 20				

D.4:[**3 Marks**] Plot Bode diagram of the system with lag compensator and with only proportional gain $k = 2$ by using `bode` function in MATLAB. Show Gain Margin (GM) and Phase Margin (PM) on the Bode diagram and compare them. Explain how the lag compensator could effect stability of the closed-loop system.