Ryerson University

Department of Electrical, Computer and Biomedical Engineering

BME639: CONTROL SYSTEMS BIOROBOTICS

Lab Project 3

Introduction to PI, PD and PID Controllers, State-Space Modeling of Physical Systems and Control (Rotary Inverted Pendulum)

Introduction and Objectives

Each lab requires four weeks and the tasks for Part one (two weeks) and Part two (two weeks) are provided explicitly. The Lab project is required to provide complete answer to the questions. Note that formatting has 10% of the Lab grades and, 15% of each of your Lab grades is on your summary of the Lab which has to be provided individually. Your summary has to explain clearly what you have learned in the Lab.

Before submitting the report the TA asks questions about your report. If there is no consistency between your oral answer and the report, you will lose %50 of your total mark.

In the first part, you become familiar with principles and application of PI, PD and PID controller. Then you use the controllers to enhance the response of a third order system.

In the second part, you become familiar with modeling of a rotary inverted pendulum system to obtain the state space representation using linearized differential equations. Then you design and implement a stabilizting state feedback controller for the rotary inverted pendulum using pole placement methodologies.

Part One (two weeks)

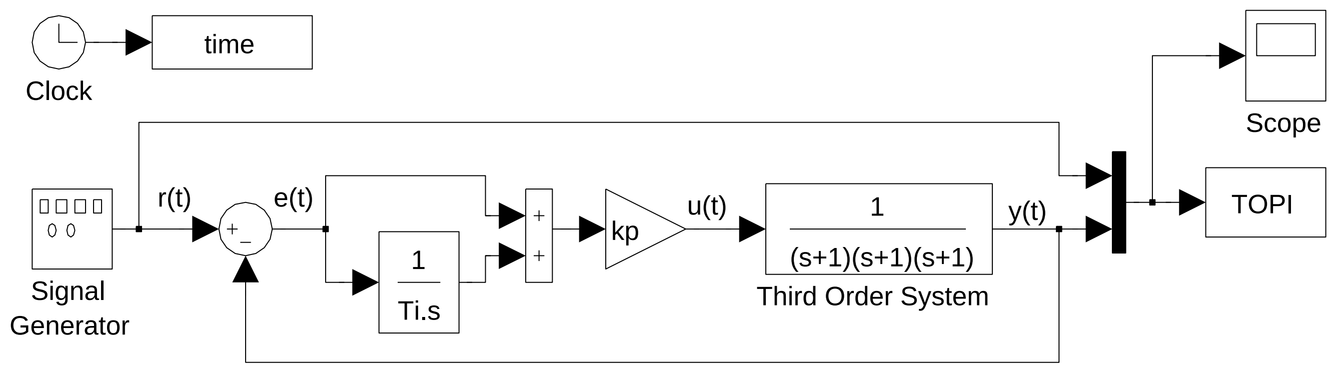
A. Proportional-Integral Controller (PI)

The output of a PI controller in the time-domain is as follows:

$$u(t) = k_p[e(t) + \frac{1}{T_i}\int e(t)dt]$$

A.1:[**3 Marks**] Find the transfer function of a PI controller, $\frac{U(s)}{E(s)}$, and provide its poles and zeros. Find gain of the PI controller for $s \to 0$ (DC gain) and $s \to \infty$ (High frequency gain). Explain how variation of k_p and T_i can affect the poles and zeros location.

A.2:[**2 Marks**] Consider the following third order system with PI controller,



Generate a square wave signal with amplitude 1 and frequency 0.01Hz. Simulate the system with PI controller for $T_i = 4$ and $k_p = 2$. Plot reference input and output of the closed-loop system (Run the simulation for 100 seconds). Provide a print of your Simulink model.

A.3:[**4 Marks**] Find closed-loop transfer function of the system, $\frac{Y(s)}{R(s)}$ by using `tf` and `feedback` functions in MATLAB. Find the time domain specifications of the output, rise time (t_r), percentage of maximum overshoot ($\%O.S.$), settling time (t_s), steady state error (e_{ss}) by using `stepplot` function in MATLAB. Compare the time domain specifications with only proportional controller system, $k_p = 2$. Explain in which term PI controller could enhance the step response, transient response or steady-state response?

	Rise time (t_r)	% Max. overshoot ($\%O.S.$)	Settling time (t_s)	Steady-state error (e_{ss})
PI controller $T_i = 4$, $K_p = 2$				
P controller $K_p = 2$				

A.4:[3 Marks] Plot reference input and output of the closed-loop system for $T_i = 2$ and $T_i = 8$. Find the time domain specifications, rise time (t_r), percentage of maximum overshoot ($\%O.S.$), settling time (t_s), steady state error (e_{ss}) by using the `stepplot` function in MATLAB.

	Rise time (t_r)	% Max. overshoot ($\%O.S.$)	Settling time (t_s)	Steady-state error (e_{ss})
PI controller $T_i = 2$, $K_p = 2$				
PI controller $T_i = 8$, $K_p = 2$				

Compare the results with the PI controller in Part A.3 and explain the effect of increasing and decreasing T_i on transient response and steady-state response.

A.5:[3 Marks] Plot Bode diagram of the PI controllers in Parts A.3 and A.4 ($T_i = 4$, $k_p = 2$), ($T_i = 2$, $k_p = 2$) and ($T_i = 8$, $k_p = 2$) using `bode` function in a same figure in MATLAB. Compare them and explain the effect of changing T_i in frequency characteristics of PI controller. **NOTE:** You need to plot Bode diagram of the controllers ($\frac{U(s)}{E(s)}$) not the closed-loop system.

B. Proportional-Derivative Controller (PD)

The output of a PD controller in the time-domain is as follows:

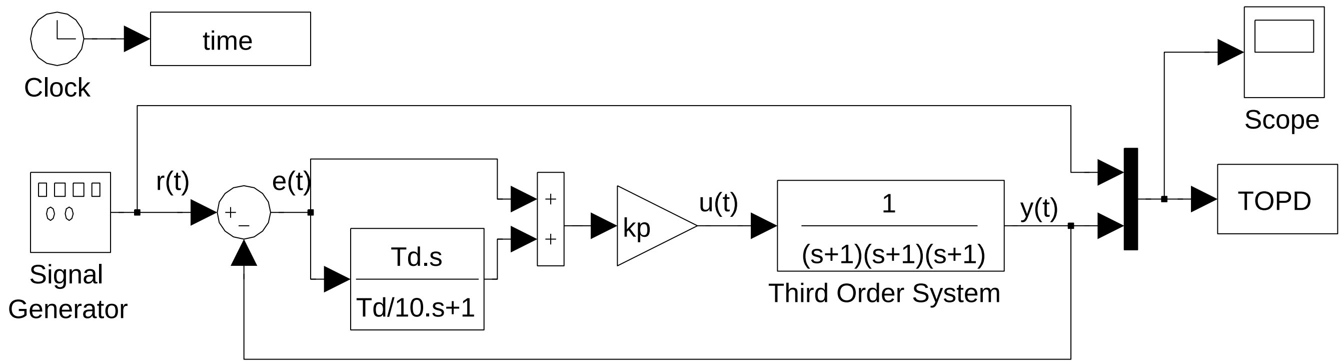
$$u(t) = k_p[e(t) + T_d\frac{de(t)}{dt}]$$

Since the pure derivative term has only one zero and no poles, it cannot be easily simulated, so can approximate it with lead filter. Therefore, the transfer function of a PD controller can be written as,

$$\frac{U(s)}{E(s)} = kp(1 + \frac{T_d s}{\frac{T_d}{\beta}s + 1})$$

B.1:[2 Marks] Find the poles and zeros of the PD controller. Find the gain of the PD controller for $s \to 0$ (DC gain) and $s \to \infty$ (High frequency gain). Explain if β is a constant value, how the variation of T_d can effect the poles and zeros location.

B.2:[2 Marks] Consider the following third order system with PD controller. Generate a square wave signal with amplitude 1 and frequency 0.02Hz. Simulate the system with PD controller for $T_d = 1$ and $k_p = 4$. Plot reference input and output of the closed-loop system (Run the simulation for 75 seconds). Provide a print of your Simulink model.



B.3:[4 Marks] Find closed-loop transfer function of the system, $\frac{Y(s)}{R(s)}$ by using `tf` and `feedback` functions in MATLAB. Find the time domain specifications of the output, rise time (t_r), percentage of maximum overshoot ($\%O.S.$), settling time (t_s), steady state error (e_{ss}) by using `stepplot` function in MATLAB. Compare transient response and steady state response with only proportional controller system, $k_p = 4$. Explain in which term PD controller could enhance the response, transient response or steady-state response?

	Rise time (t_r)	% Max. overshoot ($\%O.S.$)	Settling time (t_s)	Steady-state error (e_{ss})
PD controller $T_d = 1$, $K_p = 4$				
P controller $K_p = 4$				

B.4:[3 Marks] Plot reference input and output of the closed-loop system for $T_d = 0.1$ and $T_d = 2$. Find the time domain specifications, rise time (t_r), percentage of maximum overshoot ($\%O.S.$), settling time (t_s), steady state error (e_{ss}) by using `stepplot` in MATLAB.

	Rise time (t_r)	% Max. overshoot ($\%O.S.$)	Settling time (t_s)	Steady-state error (e_{ss})
PD controller $T_d = 2$, $K_p = 4$				
PD controller $T_d = 0.1$, $K_p = 4$				

Compare the results with the PD controller in Part B.3 and explain the effect of increasing and decreasing T_d on transient response and steady-state response.

B.5:[**3 Marks**] Plot Bode diagram of the PD controllers in Parts B.3 and B.4 ($T_d = 1$, $k_p = 4$), ($T_d = 2$, $k_p = 4$) and ($T_d = 0.1$, $k_p = 4$) using `bode` function in a same figure in MATLAB. Compare them and explain the effect of changing T_d in frequency characteristics of PD controller. **Note:** Plot Bode diagram of the controllers ($\frac{U(s)}{E(s)}$) not the closed-loop system.

C. Proportional-Integral-Derivative Controller (PID)

The output of a PID controller in the time-domain is as follows:

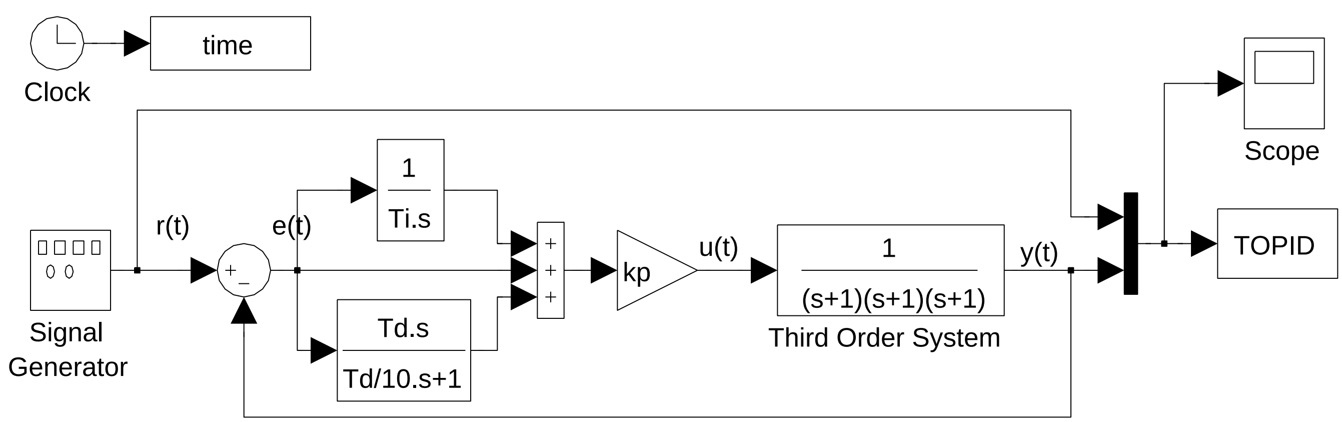
$$u(t) = k_p[e(t) + \frac{1}{T_i}\int e(t)dt + T_d\frac{de(t)}{dt}]$$

and the transfer function of a PID controller can be written as,

$$\frac{U(s)}{E(s)} = k_p(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{\beta}s + 1})$$

C.1:[**1 Mark**] Find the gain of the PID controller for $s \to 0$ (DC gain) and $s \to \infty$ (High frequency gain), then compare them with gains for PI and PD controllers in Part A.1 and B.1.

C.2:[**2 Marks**] Consider the following third order system with PID controller,



Generate a square wave signal with amplitude 1 and frequency 0.02Hz. Simulate the system with PID controller for $T_i = 4$, $T_d = 1$ and $k_p = 4$. Plot the output of the closed-loop system (Run the simulation for 75 seconds). Provide a print of your Simulink model.

C.3:[**4 Marks**] Find closed-loop transfer function of the system, $\frac{Y(s)}{R(s)}$ using the `tf` and `feedback` functions in MATLAB. Find the time domain specifications of the output, rise time (t_r), percentage of maximum overshoot ($\%O.S.$), settling time (t_s), steady state error (e_{ss}) by using `stepplot` function in MATLAB.

	Rise time (t_r)	% Max. overshoot ($\%O.S.$)	Settling time (t_s)	Steady-state error (e_{ss})
PID controller $T_d = 1$, $T_i = 4$, $K_p = 4$				

Compare transient response and steady-state response with PI and PD controllers in Parts A.3 and B.3. Explain in which term PID controller could enhance the response, transient or steady-state?

C.4:[4 Marks] Plot Bode diagram of the PID ($T_i = 4$, $T_d = 1$, $k_p = 4$), PI ($T_i = 4$, $k_p = 4$) and PD ($T_d = 1$, $k_p = 4$) controllers using `bode` function in a same figure in MATLAB and compare them. **Note:** You need to plot Bode diagram of the controllers ($\frac{U(s)}{E(s)}$) not the closed-loop system.

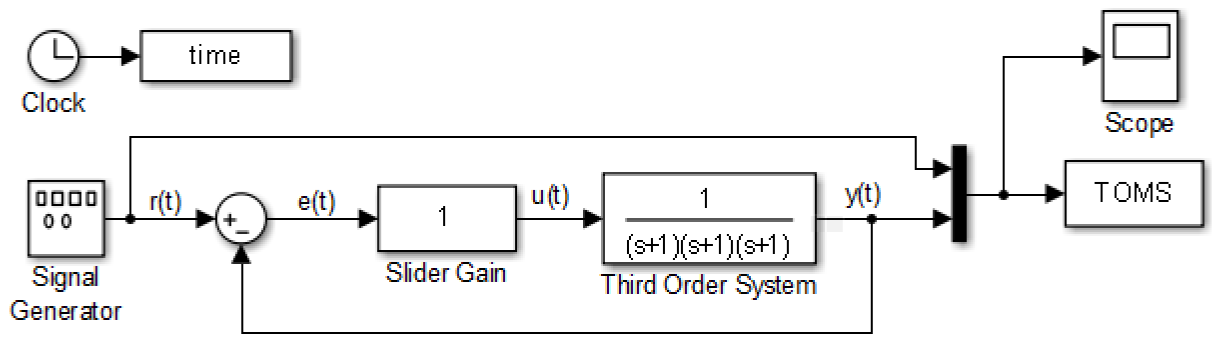
D. PID Controller Tuning via Ziegler-Nichols Approach

This method was developed by Ziegler and Nichols in 1940s to control physical systems with PID controllers. In Ziegler-Nichols approach the PID controller parameters are calculated as follows:

$$k_p = 0.6K_u; \qquad T_i = \frac{T_u}{2}; \qquad T_d = \frac{T_u}{8}$$

where K_u is the marginal stability gain of the system and T_u is the period of marginal stability oscillations.

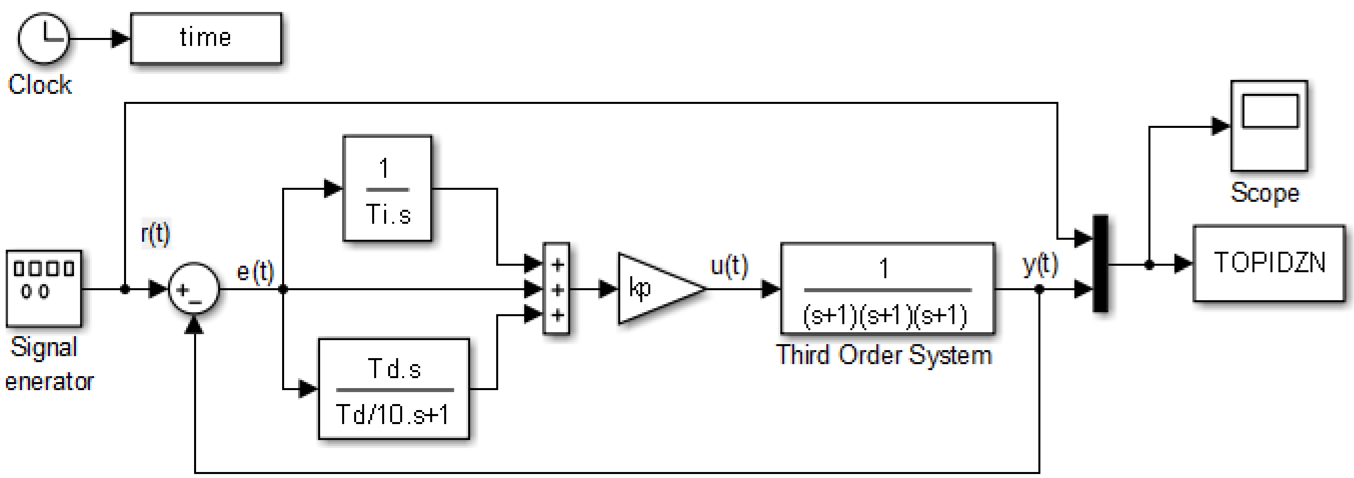
D.1:[3 Marks] Consider the following third-order system with variable proportional gain.



In Simulink generate a square wave signal with amplitude 1 and frequency 0.01Hz. Simulate the system with a `Slider Gain`. First, set the `Slider Gain` to zero, then increase the gain value gradually. You will notice that the step-response becomes more oscillatory by increasing the gain. Keep continue to increase the gain until the closed-loop systems becomes marginally stable. Determine the marginal stability gain K_u and the period of oscillations T_u. Plot the output of the closed-loop system at the marginal stability.(Run the simulation for 100 seconds). Provide a print of your Simulink model.

D.2:[**1 Mark**] Calculate the PID controller parameters k_p, T_i and T_d by using Ziegler-Nichols tuning method, by using the obtained K_u and T_u from Part D.1.

D.3:[**4 Marks**] In Simulink generate a square wave signal with amplitude 1 and frequency 0.01Hz. Simulate the system with PID controller which is obtained by Ziegler-Nichols tuning method. Plot the output of the closed-loop system (Run the simulation for 100 seconds).



Find the time domain specifications of the output, rise time, (t_r), percentage of maximum overshoot, ($\%O.S.$), settling time, (t_s), steady state error, (e_{ss}) using the `stepplot` function in MATLAB.

	Rise time (t_r)	% Max overshoot ($\%O.S.$)	Settling time (t_s)	Steady-state error (e_{ss})
PID controller Ziegler-Nichols method				

Compare the transient response and steady-state response with PID controller in Part C.3. Explain which one of the PID controllers has better response? Provide a print of your Simulink model.

Part Two (two weeks)

A. Linear State-Space Model of Rotary Inverted Pendulum

Consider the Rotary Inverted Pendulum (RIP) model illustrated in the following figure,

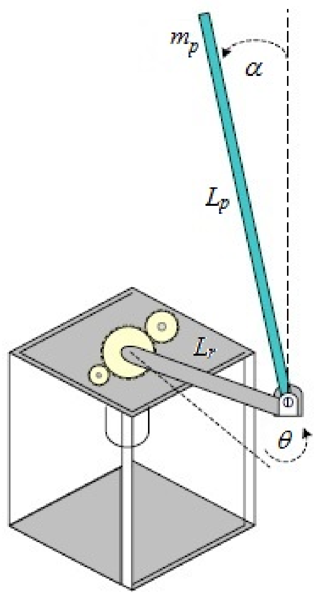


Figure 1: SRV02 Rotary Inverted Pendulum

The rotary arm pivot is attached to the SRV02 system and is actuated. The arm has a length of L_r, a moment of inertia of J_r, and its angle, θ, increases positively when it rotates counter-clockwise (CCW). The servo (and thus the arm) should turn in the CCW direction when the control voltage is positive, i.e., $V_m > 0$.

The pendulum link is connected to the end of the rotary arm. It has a total length of L_p and it center of mass is $\frac{L_p}{2}$. The moment of inertia about its center of mass is J_p. The inverted pendulum angle, α, is zero when it is perfectly upright in the vertical position and increases positively when rotated CCW. The linearized differential equations about an operating point, $\alpha \approx 0$, for zero initial conditions can be written as below,

$$(m_p L_r^2 + J_r)\ddot{\theta} - \frac{1}{2} m_p L_p L_r \ddot{\alpha} = \tau - B_r \dot{\theta}$$

$$-\frac{1}{2} m_p L_p L_r \ddot{\theta} + (J_p + \frac{1}{4} m_p L_p^2)\ddot{\alpha} - \frac{1}{2} m_p L_p g \alpha = -B_p \dot{\alpha}$$

The torque applied at the base of the rotary arm (i.e., at the load gear) is generated by the servo motor as described by the equation:

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta})}{R_m}$$

The parameters of the RIP are defined as follows:

θ Servo gear angular displacement
α Pendulum angular deflection
L_p Total length of pendulum (0.2 m)
m_p Pendulum mass (0.0970 kg)
J_p Pendulum Moment Inertia about center of mass($3.2341 \times 10^{-4} kgm^2$)
B_p Pendulum viscous damping coefficient (0.0024 Nms/rad)
L_r Rotary arm length from pivot to tip (0.216 m)
l_r Rotary arm length from pivot to center of mass (0.0619 m)
B_r Rotary arm viscous damping coefficient(0.1135 Nms/rad)
J_r Rotary arm moment of inertia about its center of mass ($9.98 \times 10^{-4} kgm^2$)
g Gravitational acceleration (9.81 m/s^2)

Considering the $x_1 = \theta$, $x_2 = \alpha$, $x_3 = \dot{\theta}$, and $x_4 = \dot{\alpha}$ as the state variables with zero initial conditions, the θ and α as the output variables and V_m as the input, the linear state-space model of RIP is obtained as below.

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

where

$$\mathbf{x}(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix}, \qquad \mathbf{y}(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \alpha \end{bmatrix}, \qquad \mathbf{u}(t) = V_m$$

$$A = \frac{1}{J_T} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4}m_p^2 L_p^2 L_r g & -(B_r + \frac{K_g^2 k_t k_m}{R_m})(J_p + \frac{1}{4}m_p L_p^2) & -\frac{1}{2}m_p L_p L_r B_p \\ 0 & \frac{1}{2}m_p L_p g(J_r + m_p L_r^2) & -\frac{1}{2}m_p L_p L_r(B_r + \frac{K_g^2 k_t k_m}{R_m}) & -B_p(J_r + m_p L_r^2) \end{bmatrix}$$

$$B = \frac{K_g k_t}{J_T R_m} \begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4}m_p L_p^2 \\ \frac{1}{2}m_p L_p L_r \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where $J_T = J_p m_p L_r^2 + J_r J_p + \frac{1}{4} J_r m_p L_p^2$

A.1:[6 Marks] Write a script file in MATLAB to calculate numerical values of A, B, C and D matrices using the given system parameters. Provide a print of your MATLAB script file and its execute.

A.2:[6 Marks] Find characteristic equation and eigenvalues of the open-loop RIP system from the system matrix (A) by using MATLAB functions `poly` and `eig`. Explain the relation between the eigenvalues in state-space model and poles in transfer function model. What is order of the modeled RIP system? Is the open-loop RIP system stable? Explain your answer.

B. Rotary Inverted Pendulum Balance Control

B.1:[10 Marks] Consider the obtained linear state-space model of RIP system in Part A.

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$

$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

Design a state-feedback controller $\mathbf{u} = -K\mathbf{x}$ to stabilize the RIP system to achieve the following time response requirements,

Damping ration, $\zeta = 0.7$,
Natural frequency, $\omega_n = 4rad/sec$,
Maximum pendulum angle deflection, $|\alpha| < 15deg$,
Maximum control effort voltage, $|V_m| < 10volt$.

Follow the steps to design a balance control:

Step 1: Check controllability of the RIP system
Calculate the controllability matrix (Q_c) for RIP model by using `ctrb` in MATLAB. Here n is the system order.

$$Q_c = [B \;\; AB \;\; A^2B \;\; \dots \;\; A^{n-1}B]$$

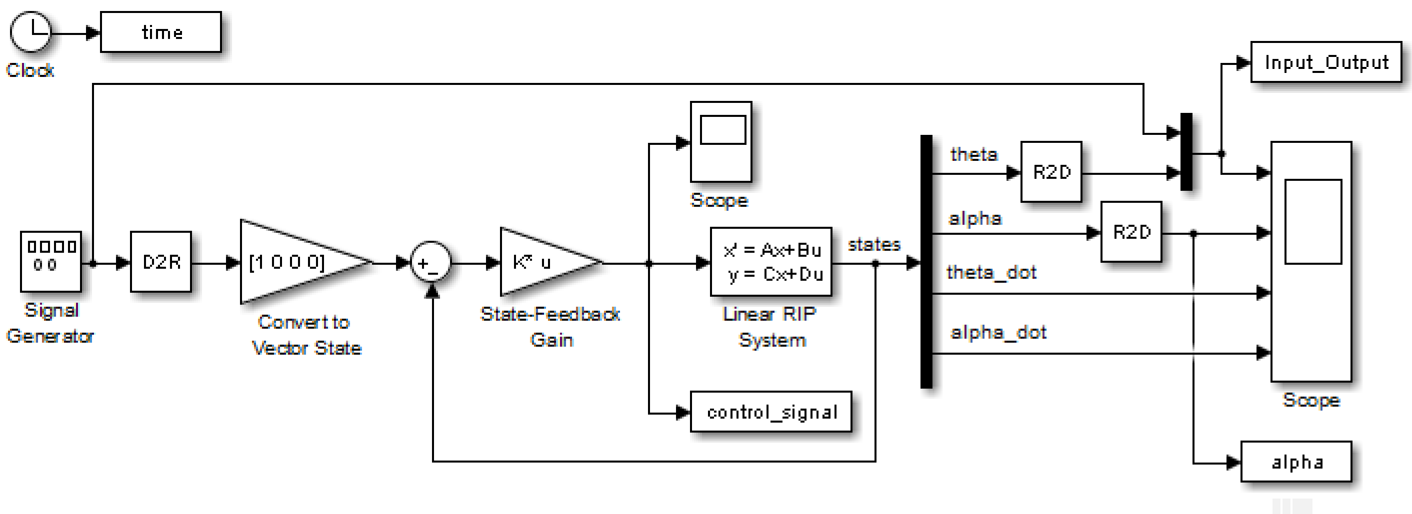
Determine controllability of the RIP system by checking rank of the controllability matrix (Q_c). Use function `rank` in MATLAB. Is the open-loop RIP system controllable?

Step 2: Determine desired closed-loop eigenvalues (poles) location
Calculate location of the two dominant poles, s_1 and s_2, based on the given specifications. Place the other poles at $s_3 = -30$ and $s_4 = -40$ (**Hint:** $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$). Find the desired closed-loop characteristic equation.

Step 3: Calculate state-feedback gain
Determine and provide the state-feedback gain (K) by using `place` function in MATLAB.

Step 4: Verify the closed-loop eigenvalues
Calculate and provide the closed-loop system matrix by $A_{cl} = A - BK$ in MATLAB. Provide eigenvalues of the closed-loop system from A_{cl} using `eig` in MATLAB. Is the closed-loop RIP system stable? Are the closed-loop poles placed at the desired locations? Explain your answer.

B.2:[**5 Marks**] Consider the linear state-space model of RIP system from Part A. In Simulink generate a square wave signal with amplitude 20 and frequency 0.1Hz. Simulate the linear state-space model of RIP system with the designed state-feedback K in Part B.1. Provide your Simulink model.



Here reference input ($\pm 20 degrees$) is applied to command the rotary arm angle (θ). Plot the reference input and the rotary arm angle (θ) at the same graph. Does rotary arm follow the applied reference input?

Plot the pendulum angular deflection (α) and the control signal (V_m) each separately. Measure the maximum pendulum deflection ($|\alpha|_{max}$) and the maximum voltage used ($|V_m|_{max}$). Are the required specifications given in Part B.1 satisfied? Explain your answer.