Toronto Metropolitan University
Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Toronto Metropolitan University

Department of Electrical & Computer Engineering

BME 639 - Control Systems & Bio-Robotics
Lab Report

Semester/Year	W2024
Lab Number:	2

Instructor:	Dr. Owais Khan
Section:	02
Submission Date:	Mar 12, 2024
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Student Name (ID):	Mathew Szymanowski (501094808)
Signature:	
Student Name (ID):	Tanvir Hassan (501104056)
Signature:	

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PART 1

A: DC Servo Motor Modeling Using First-Principles

A.1

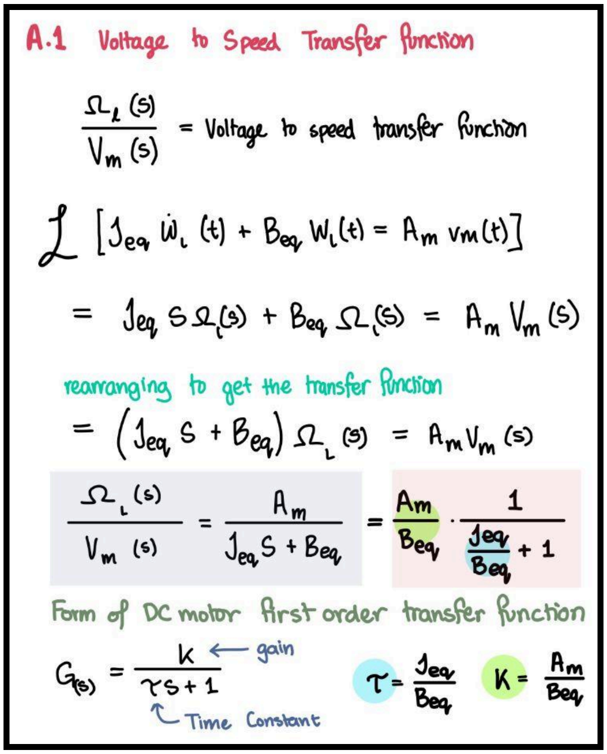

Figure 1: Voltage to Speed Transfer Function Calculation of Servo Motor

Table 1: Voltage to Speed Transfer Function Parameters

	Transfer function $\frac{\Omega_l(s)}{V_m(s)}$	**System order**	**System type**	**DC-gain (K)**	**Time constant (τ)**
Voltage-to Speed	$\frac{A_m}{B_{eq}} \cdot \frac{1}{\frac{J_{eq}}{B_{eq}}s+1}$	1	0	$\frac{A_m}{B_{eq}}$	$\frac{J_{eq}}{B_{eq}}$

A.2

A.2 $\theta(t) = W_L(t)$

$$J_{eq}\,\ddot{\theta}(t) + B_{eq}\,\dot{\theta}(t) = A_m V_m(t)$$

$$\mathcal{L}\left[J_{eq}\,\ddot{\theta}(t) + B_{eq}\,\dot{\theta}(t) = A_m V_m(t)\right]$$

$$\left(J_{eq}\,s^2 + B_{eq}\,s\right)\theta(s) = A_m V_m(s)$$

$$\frac{\theta(s)}{V_m(s)} = \frac{A_m}{J_{eq}s^2 + B_{eq}s}$$

Calculating Poles & Zeros

$$J_{eq} = \eta_g K_g^2 J_m + J_L = (0.9)(19.5^2)(4.61\times10^{-7}) + 1.03\times10^{-4} = 0.00214\ kgm^2$$

$$B_{eq} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_{eq,l} R_m}{R_m} = \frac{(0.9)(14\times5)^2(0.69)(7.68\times10^{-3})^2 + (0.015)(0.26)}{2.6}$$

$B_{eq} = 0.0840$

$\therefore\ 0.00214\,s^2 + 0.0840\,s = 0 \Rightarrow s(0.00214\,s + 0.084)$

Poles:

$s = 0$ $\quad s = -39.3$

Zeros:

No Zeros

Figure 2: Voltage to Position Transfer Function Calculation of Servo Motor

Table 2: Voltage to Position Transfer Function Parameters

	$\frac{\Theta_l(s)}{V_m(s)}$	**System order**	**System type**	**poles**	**zeros**
Voltage-to-position	$\frac{A_m}{J_{eq}s^2+B_{eq}s}$	2	1	0, -39.3	None

A.3

Table 3: Transfer Function Nominal Parameter Values

	Transfer function $\frac{\Omega_l(s)}{V_m(s)}$	DC-gain (K)	Time constant (τ)
Voltage-to Speed	$\frac{1.5238}{0.0255s+1}$	$\frac{A_m}{B_{eq}} = 1.5238$	$\frac{J_{eq}}{B_{eq}} = 0.0255$
Voltage-to-Position	$\frac{1.5238}{s(0.0255s+1)}$	-	-

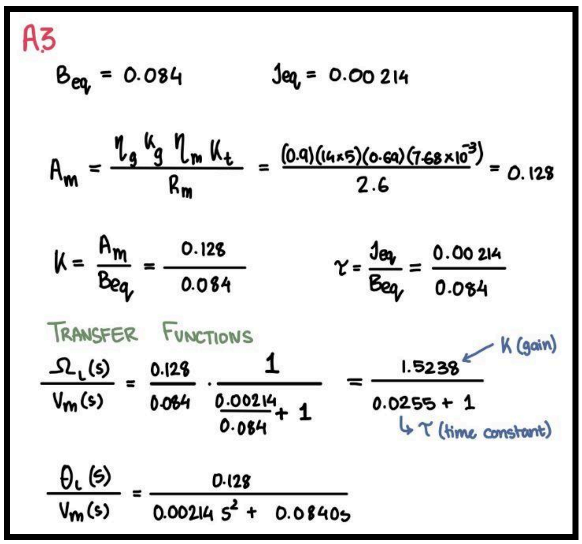

Figure 3: Calculation of the Gain (k)*, Time Constant* (τ)*,* $B_{eq} = 0.084$ *&* $J_{eq} = 0.00214$ *(From Figure 2),* $A_m = 0.128$

Stability of DC motor

A system's stability can be determined by the position of its poles, the first pole is at -39.3 which is left of the imaginary axis therefore it cannot make the system unstable, the second pole is at 0 which makes the system marginally stable.

A.4

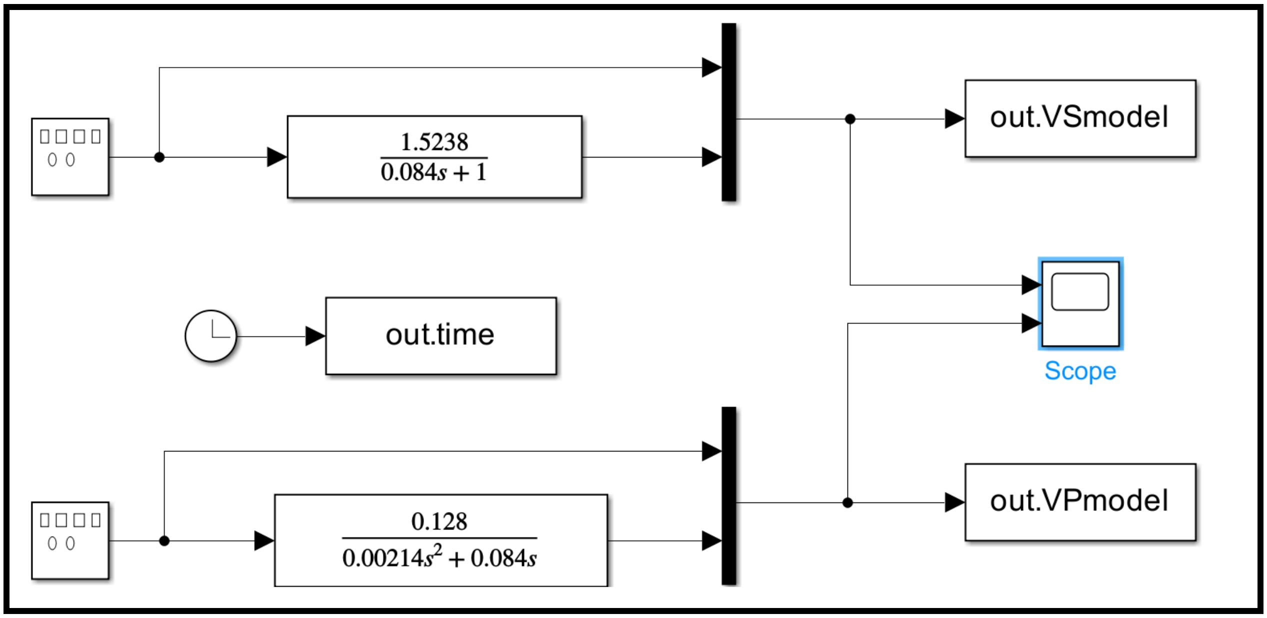

Figure 4: Voltage to Speed and Position Simulink Model

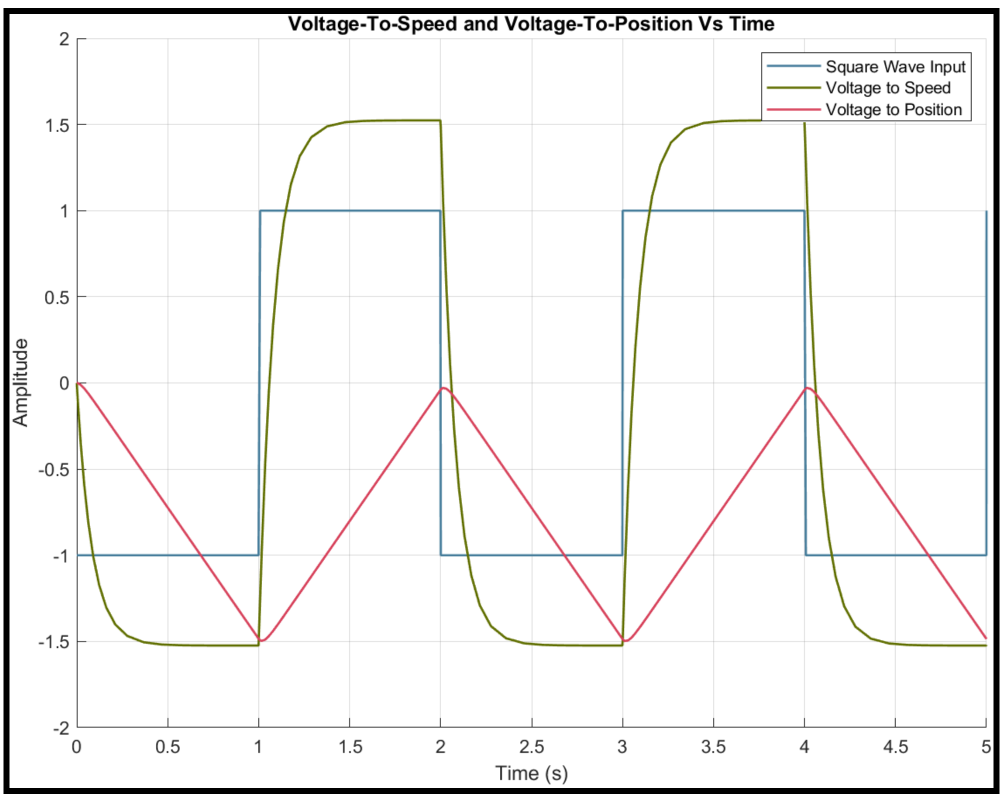

Figure 5: Graphed Simulink Input and Outputs of the Voltage Speed and Position Transfer Functions

B. DC Servo Motor Speed Control

B.1

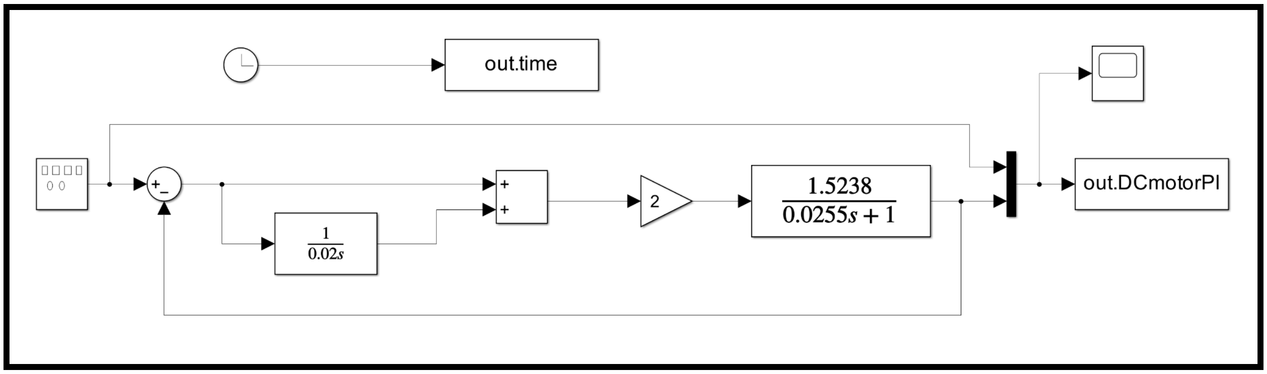

Figure 6: Simulink Model of the DC Servo Motor Speed with PI Controller

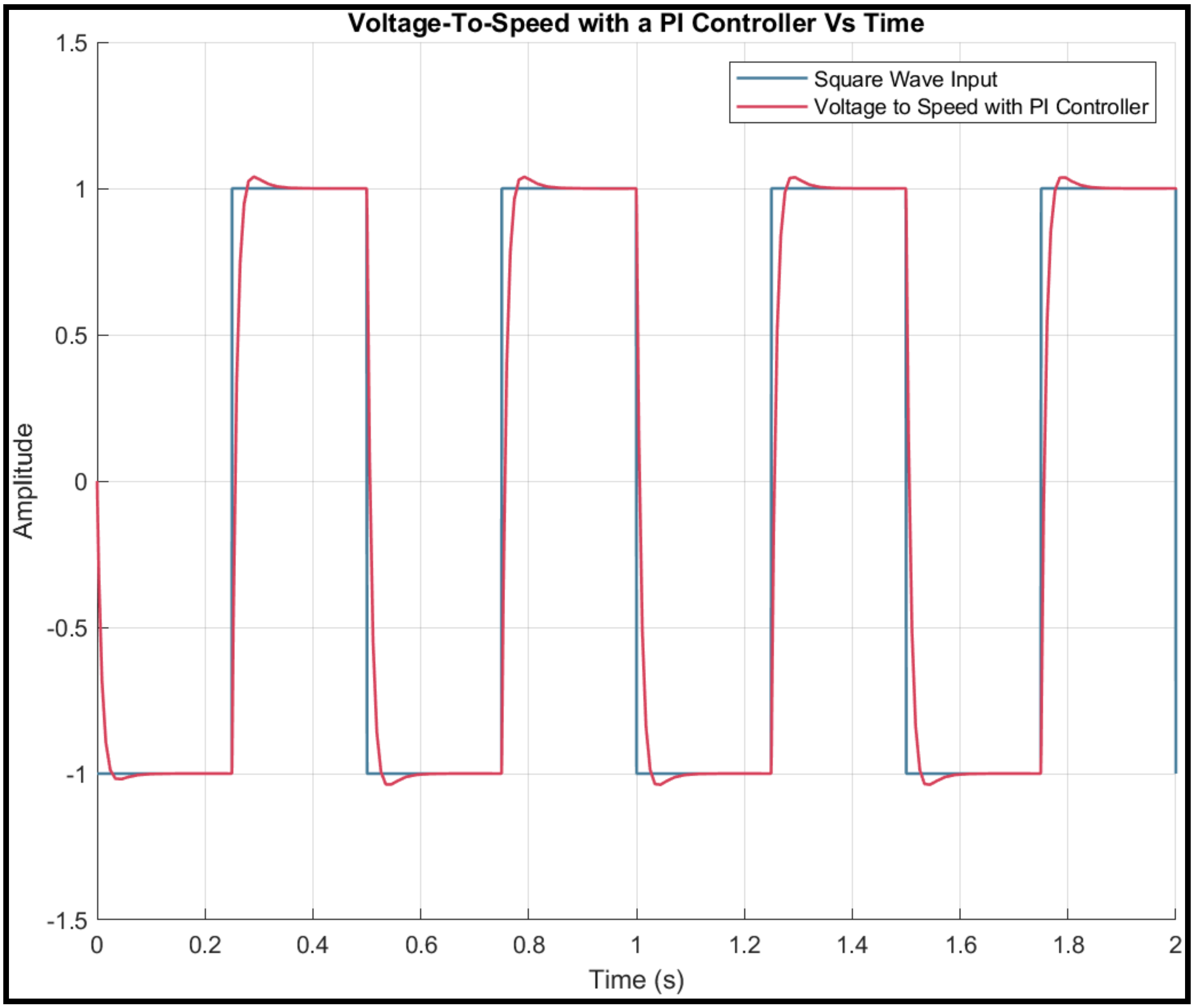

Figure 7: Output of Figure 6 System with 2Hz Input

C. DC Servo Motor Position Control

C.1

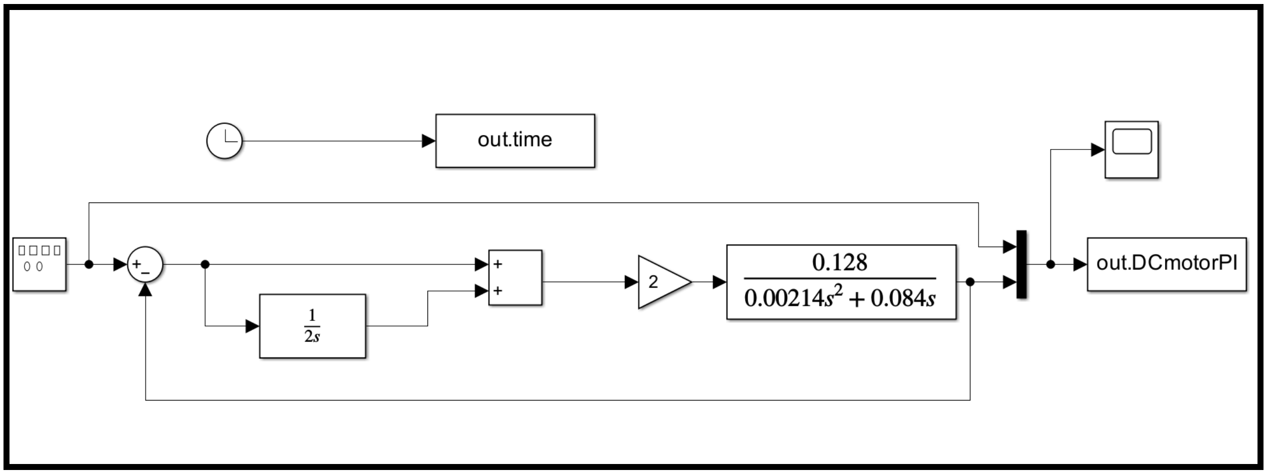

Figure 8: Simulink Model of the Servo Motor Position with PI Controller

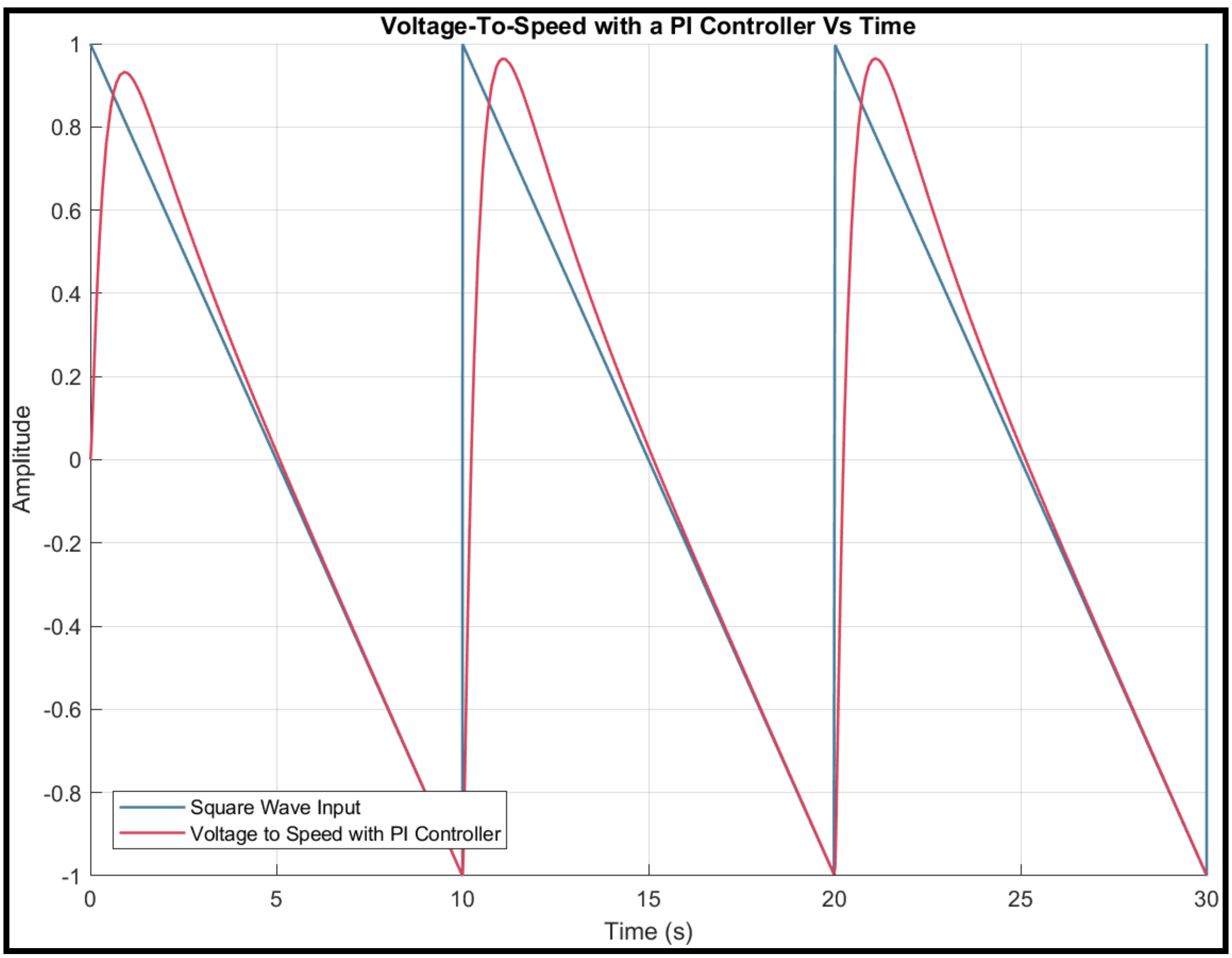

Figure 9: Output of Figure 8 System from a Sawtooth 0.1 Hz Input

PART 2

A: Time Response of a Lead Compensator

A.1

A1.

$G(s) = \frac{s+1}{0.1s+1}$ Where lead compensator formula is $G_c(s) = K\frac{(\tau s+1)}{(\alpha\tau s+1)}$

$\therefore G(s) = (1)\frac{(s+1)}{(0.1\cdot 1\cdot s+1)}$

$\therefore \tau = 1s$

DC Gain: $\lim_{s\to 0} G(s) = \lim_{s\to 0}\frac{s+1}{0.1s+1} = \frac{1}{1} = 1$

High Frequency Gain: $\lim_{s\to\infty} G(s) = \lim_{s\to\infty}\frac{s+1}{0.1s+1} = \frac{1+\cancel{\frac{1}{s}}}{0.1+\cancel{\frac{1}{s}}}$

$= 10$

Figure 10: Time Constant, DC Gain and High-Frequency Gain Calculations for Lead Compensator

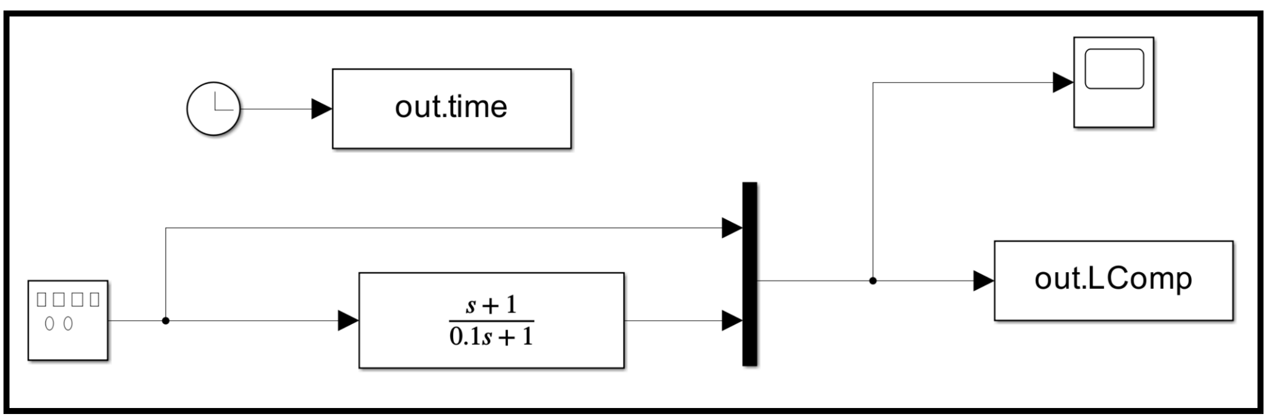

Figure 11: Simulink model of the Lead compensator

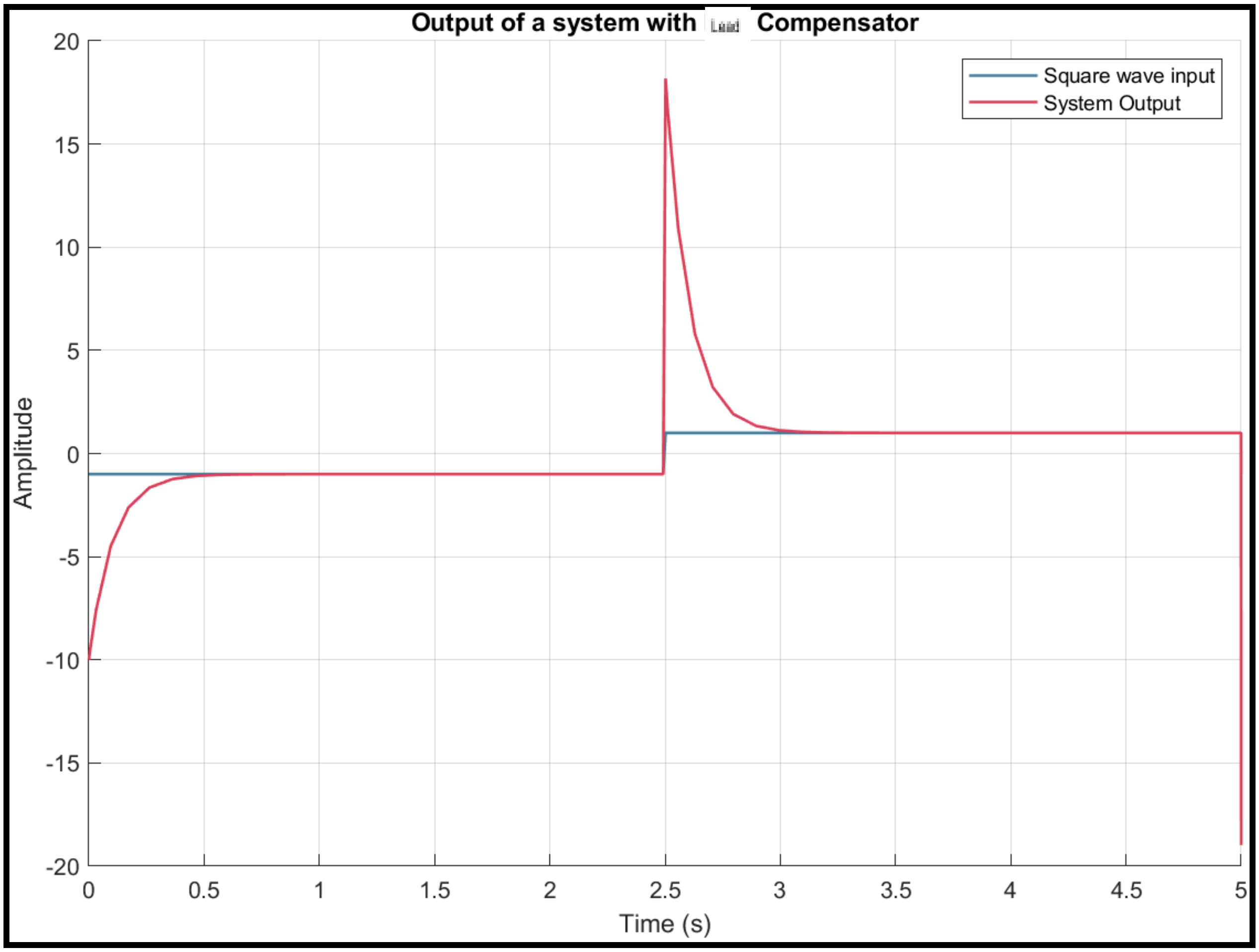

Figure 12: System Output of Lead Compensator from 0.5 HZ Square Wave Input

Table 4: Lead Compensator Parameters from Figure 10

	Lead Compensator	**Time Constant (τ) (s)**	**DC Gain ($s \to 0$)**	**High Frequency Gain ($s \to \infty$)**
Voltage-to-position	$\frac{s+1}{0.1\,s+1}$	1	1	10

A.2

```
>> numerator = [1 1];
denominator = [0.1 1];
Gc = tf(numerator,denominator);

pole(Gc)
zero(Gc)

ans =

   -10


ans =

    -1
```

Figure 13: MATLAB Pole and Zero Lead Compensator Values

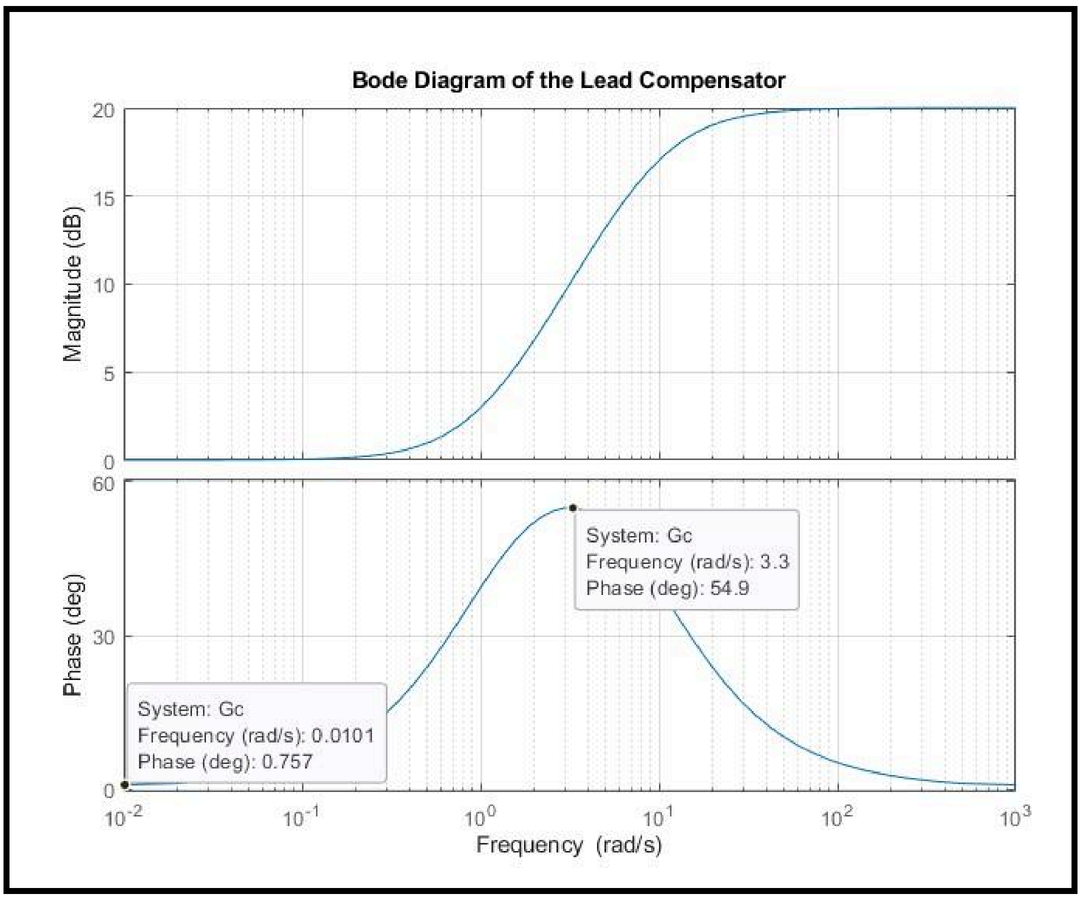

Figure 14: Bode Diagram of the Lead Compensator with Labeled Phase Limits

The maximum and minimum phase shift that can be obtained using this lead compensator is 54.9° and 0.757° respectively.

B: Lead Compensator & Second-Order Systems

B.1

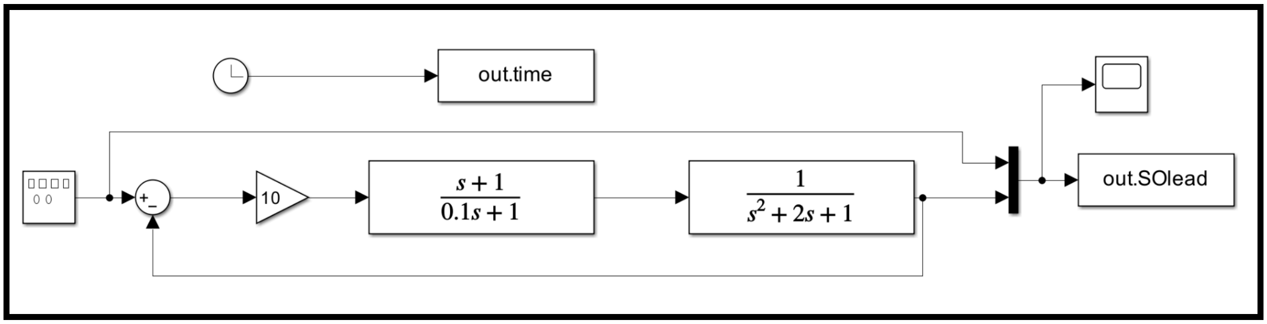

Figure 15: Simulink Model of Second-Order Transfer Function with a Lead Compensator

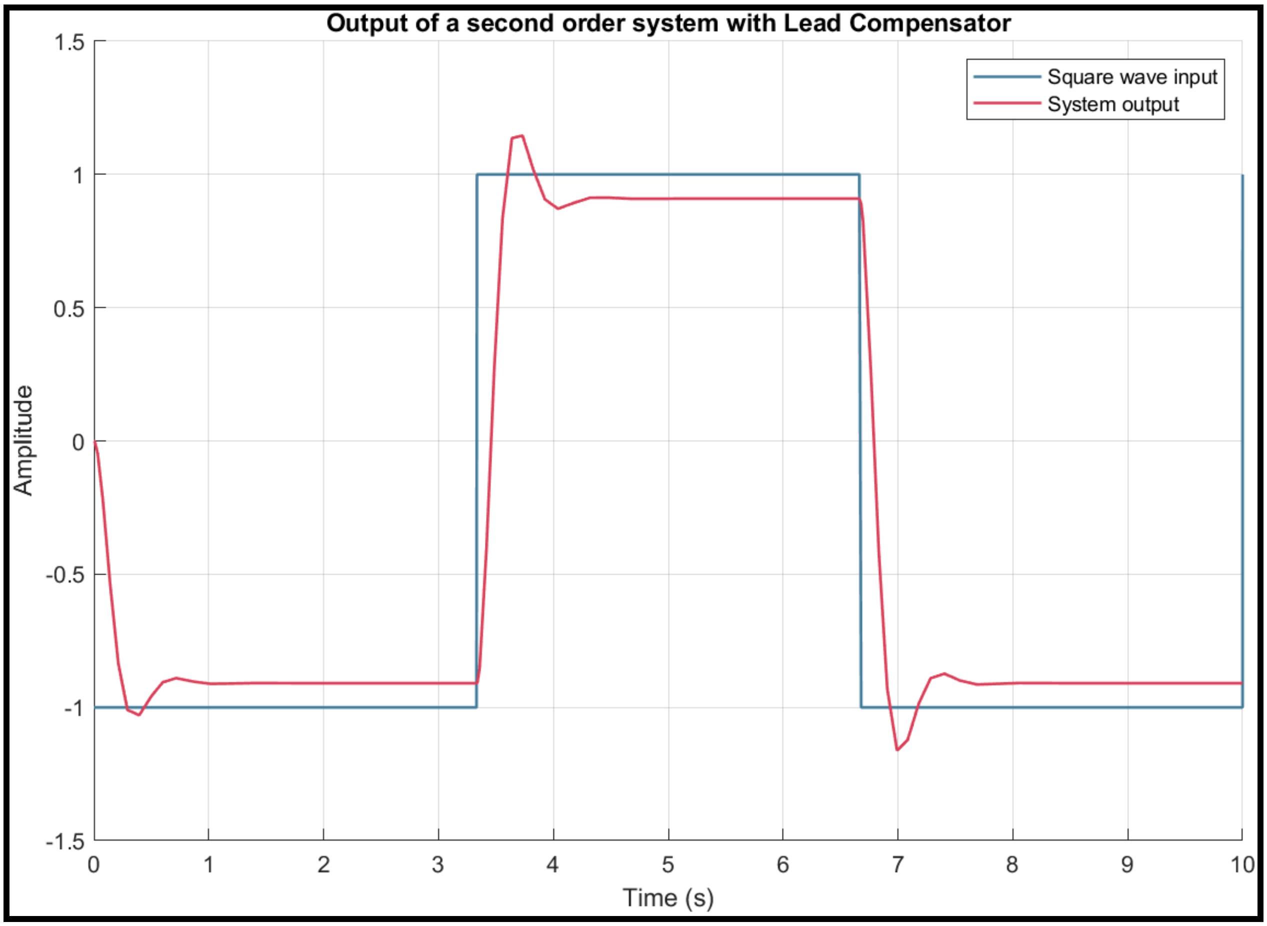

Figure 16: Output of Figure 15 System from Square Wave Input

B.2

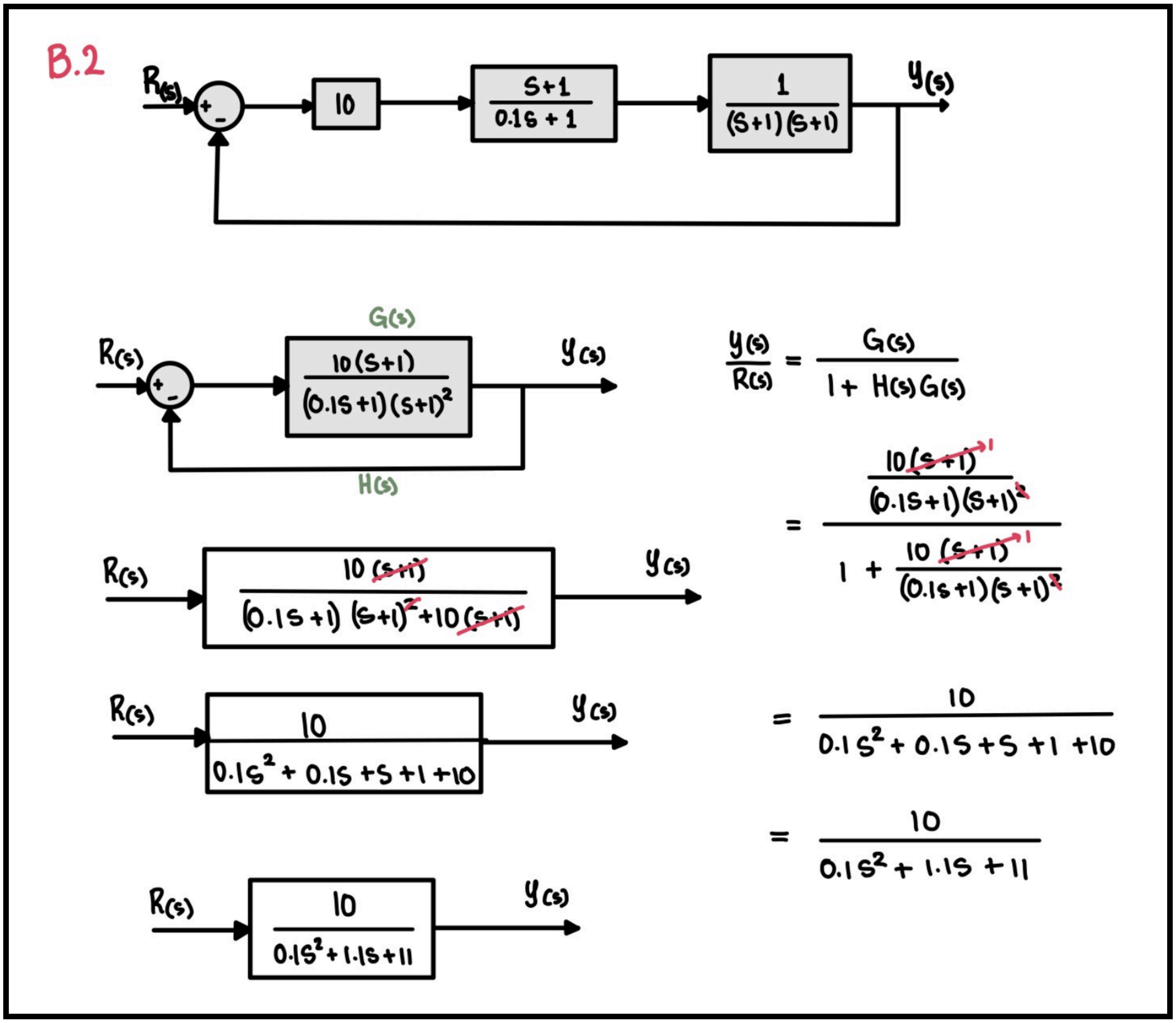

Figure 17: Transfer Function Calculation of Figure 15 Control System

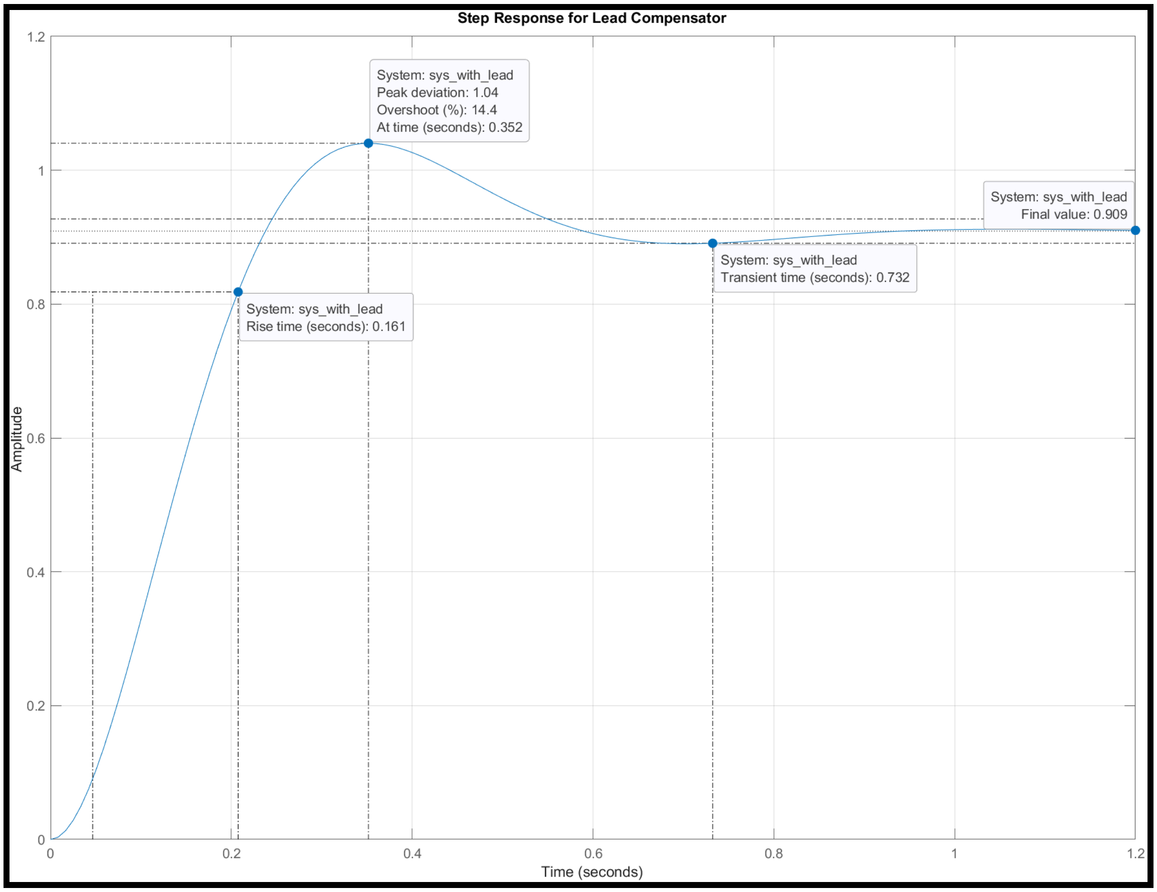

Figure 18: Rise Time $t_r = 0.161s$*, Maximum Overshoot* $\%O.S. = 14.4\%$*, Settling Time* $t_s = 0.732s$ *and Steady-State Error* $e_{ss} = 0.091s$ *Time Domain Specifications of Figure 15 Control System*

B.3

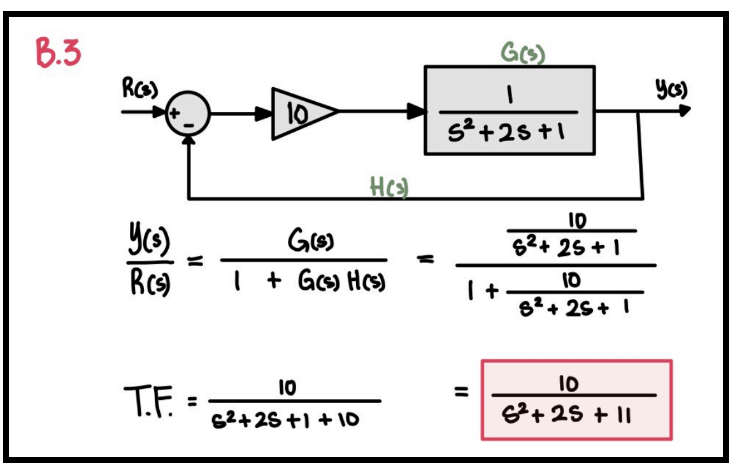

Figure 19: Transfer Function Calculation of Figure 15 System without Lead Compensator

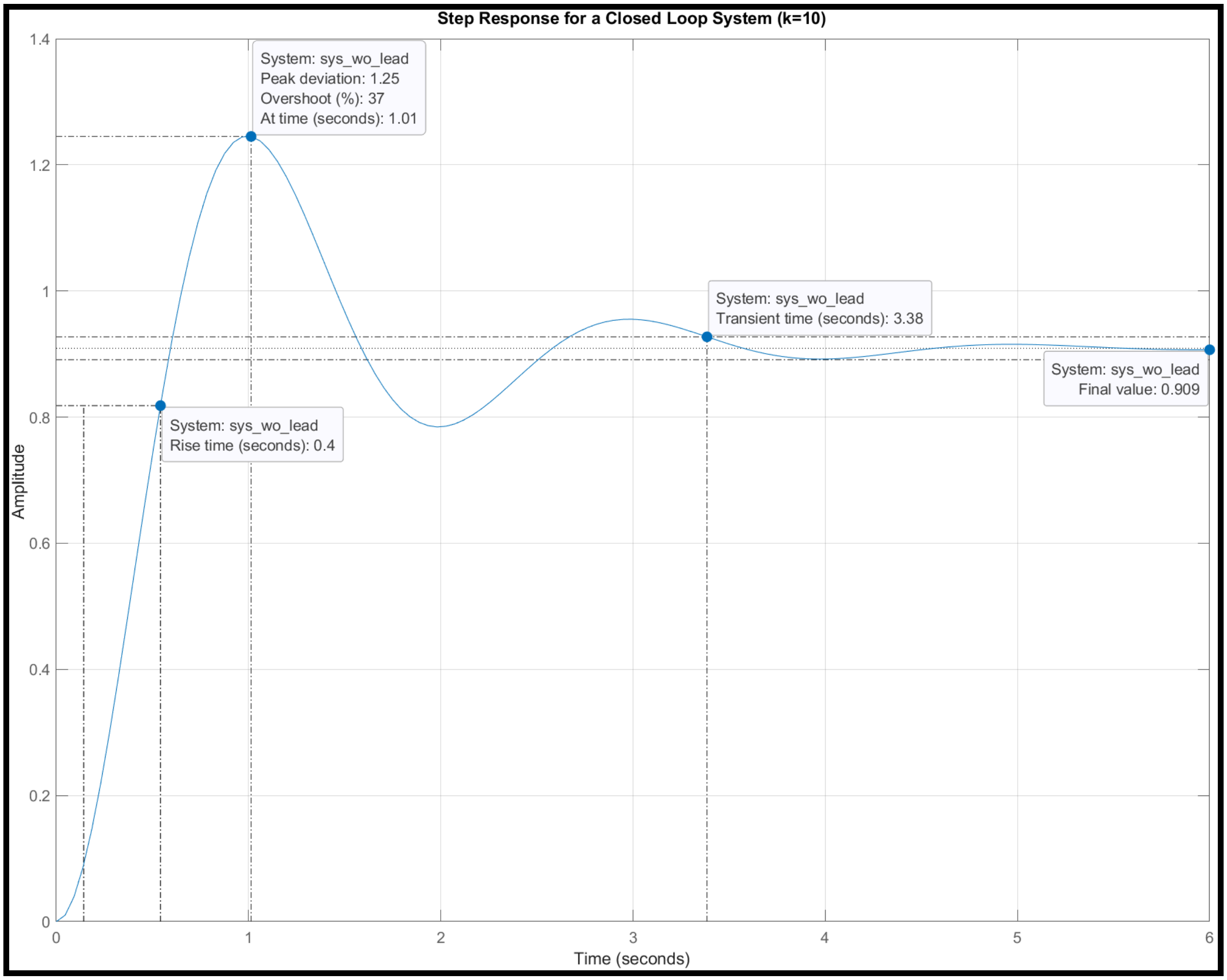

Figure 20: Rise Time $t_r = 0.4s$, *Maximum Overshoot* $\%O.S. = 37\%$, *Settling Time* $t_s = 3.38s$ *and Steady-State Error* $e_{ss} = 0.091s$ *Time Domain Specifications of Figure 19 Control System*

Table 5: Figure 15 and 19 System Time Domain Specifications Comparison

	Transfer function $\frac{Y(s)}{R(s)}$	**Rise Time** (t_r)	**% Max Overshoot** $(\%O.S.)$	**Settling Time** (t_s)	**Steady-State Error** (e_{ss})
K=10 With Lead	$\frac{10}{0.1s^2+1.1s+11}$	0.161s	14.4%	0.732s	9.1%
K=10 Without Lead	$\frac{10}{s^2+2s+11}$	0.400s	37.0%	3.380s	9.1%

Effect of lead compensator on transient response and steady-state response

The lead compensator increases the system's responsiveness, leading to a quicker reaction. This results in a shorter rise time, a reduction in maximum overshoot, and a shorter settling time. These changes highlight the lead compensator's impact on the system's transient behavior. The lead compensator does not affect the steady-state because both systems have a steady-state error of 9.1%.

B.4

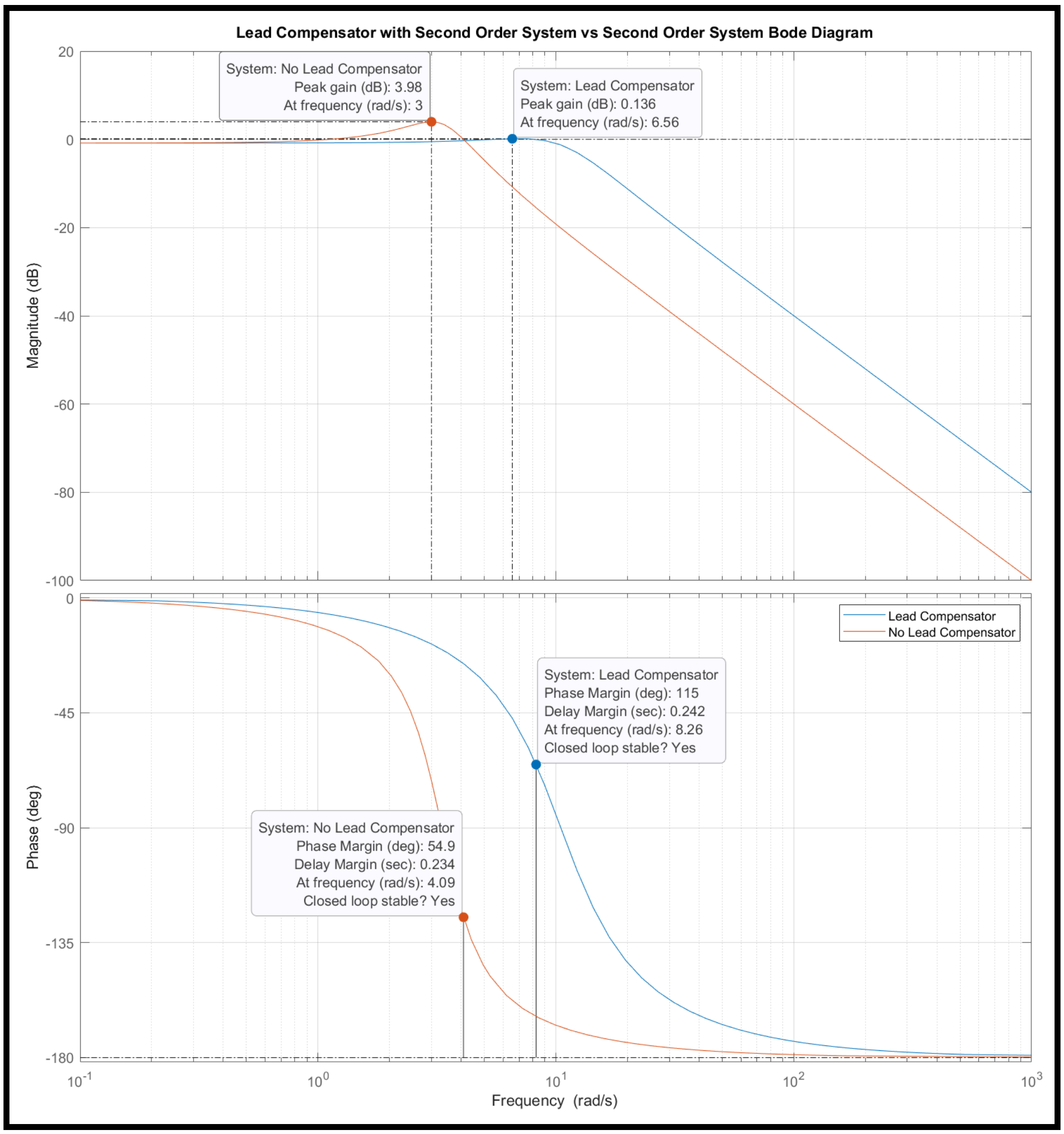

Figure 21: Bode Plots of Lead Compensator and Only Proportional Gain Systems

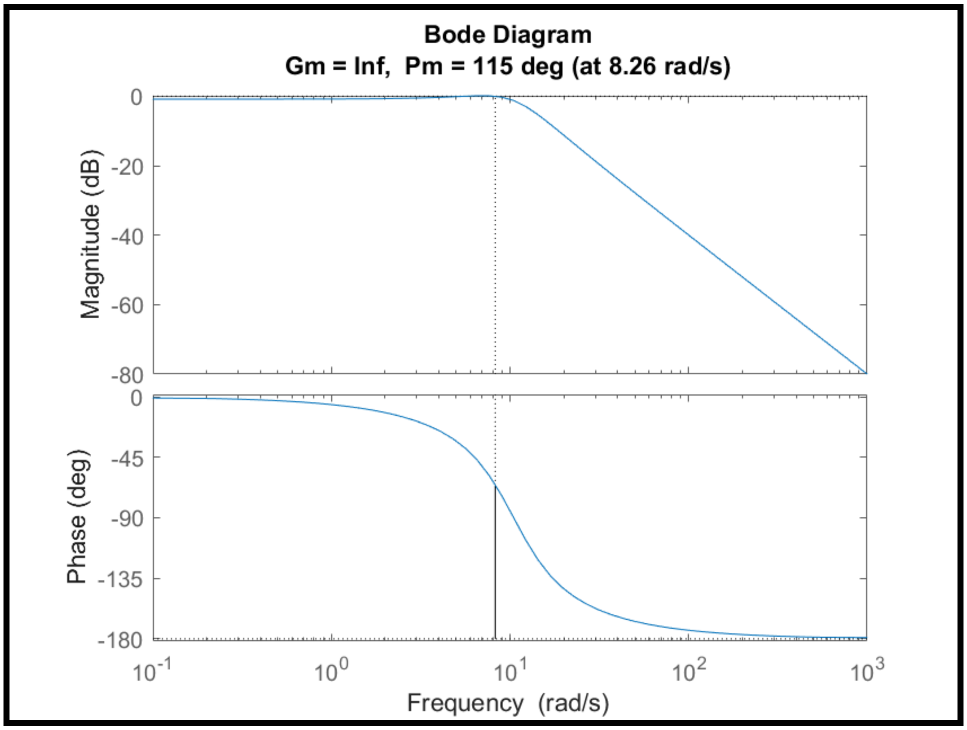

Figure 22: Bode Plot of Lead Compensator System

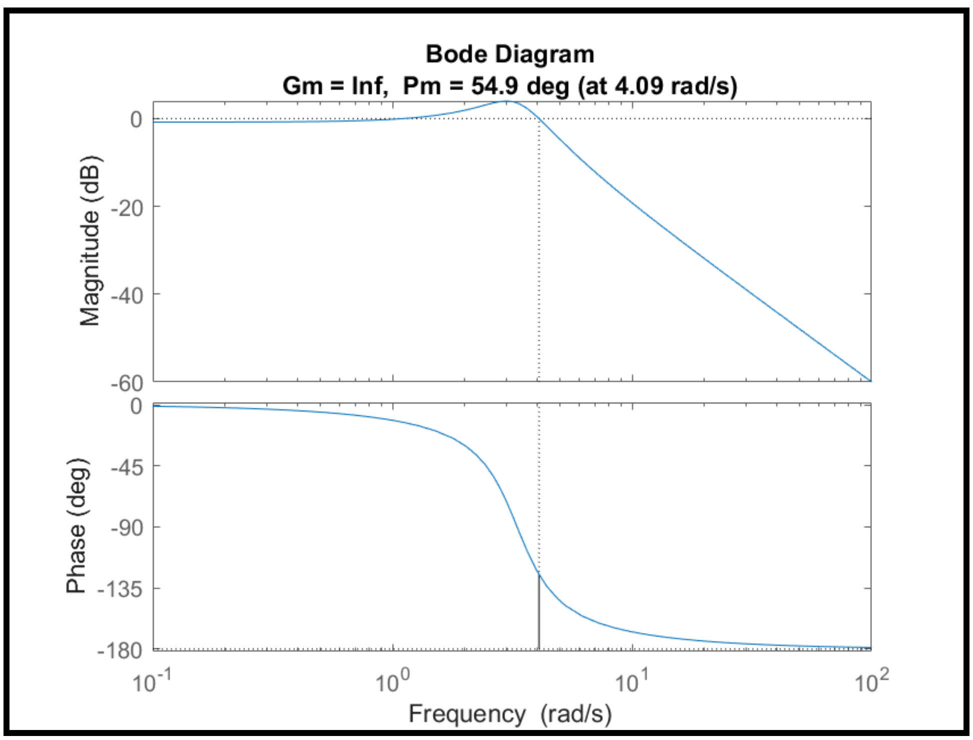

Figure 23: Bode Plot of Portional Gain System

Comparing the phase & gain margins.

Both systems exhibit a gain margin of infinity, indicating their stability, as no value of gain increase could destabilize the system. The system with a lead compensator has a greater phase margin than the system without. A higher phase margin means a more stable system against a phase-shifted input, indicating that, overall, a lead compensator improves the stability and adaptability of the system.

Increasing stability of the closed-loop system using lead compensator.

The lead compensator enhances the closed-loop system's stability by increasing the phase margin. Introducing a lead compensator increases the phase margin, elevating the system's overall stability.

C: Time response of a Lag Compensator

C.1

C1.

$$G(s) = \frac{s+3}{s+0.3}$$ Where lag compensator formula is $G_c(s) = K\frac{(\tau s+1)}{(\beta\tau s+1)}$

$$\therefore\ G(s) = \frac{3\left(\frac{s}{3}+1\right)}{0.3\left(\frac{10}{3}s+1\right)} = 10\,\frac{\left(\frac{1}{3}s+1\right)}{\left(10\cdot\frac{1}{3}s+1\right)}$$

$$\therefore\ \tau = \frac{1}{3}s$$

DC Gain: $\lim_{s\to 0} G(s) = \lim_{s\to 0}\frac{s+3}{s+0.3} = \frac{3}{0.3} = 10$

High Frequency Gain: $\lim_{s\to\infty} G(s) = \lim_{s\to\infty}\frac{s+3}{s+0.3} = \frac{1+\frac{3}{s}}{1+\frac{0.3}{s}}$

$= 1$

Figure 24: Time Constant, DC Gain and High-Frequency Gain Calculations for Lag Compensator

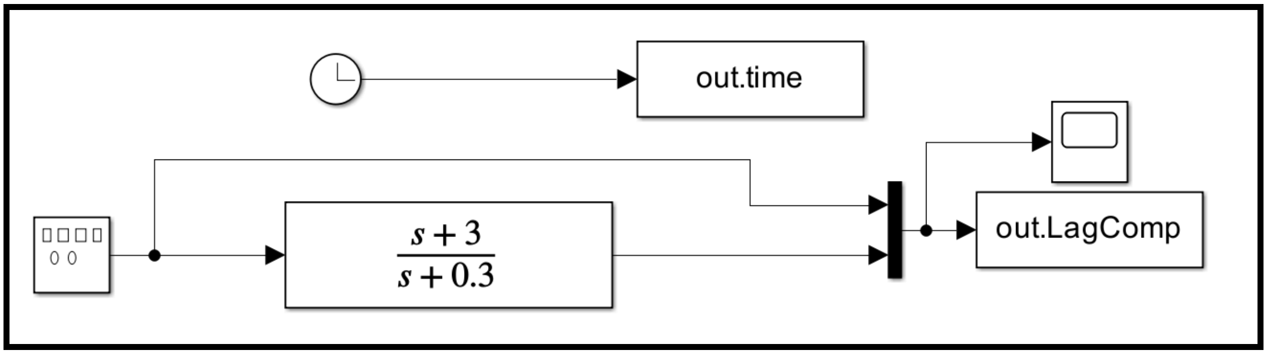

Figure 25: Simulink model of the Lag compensator

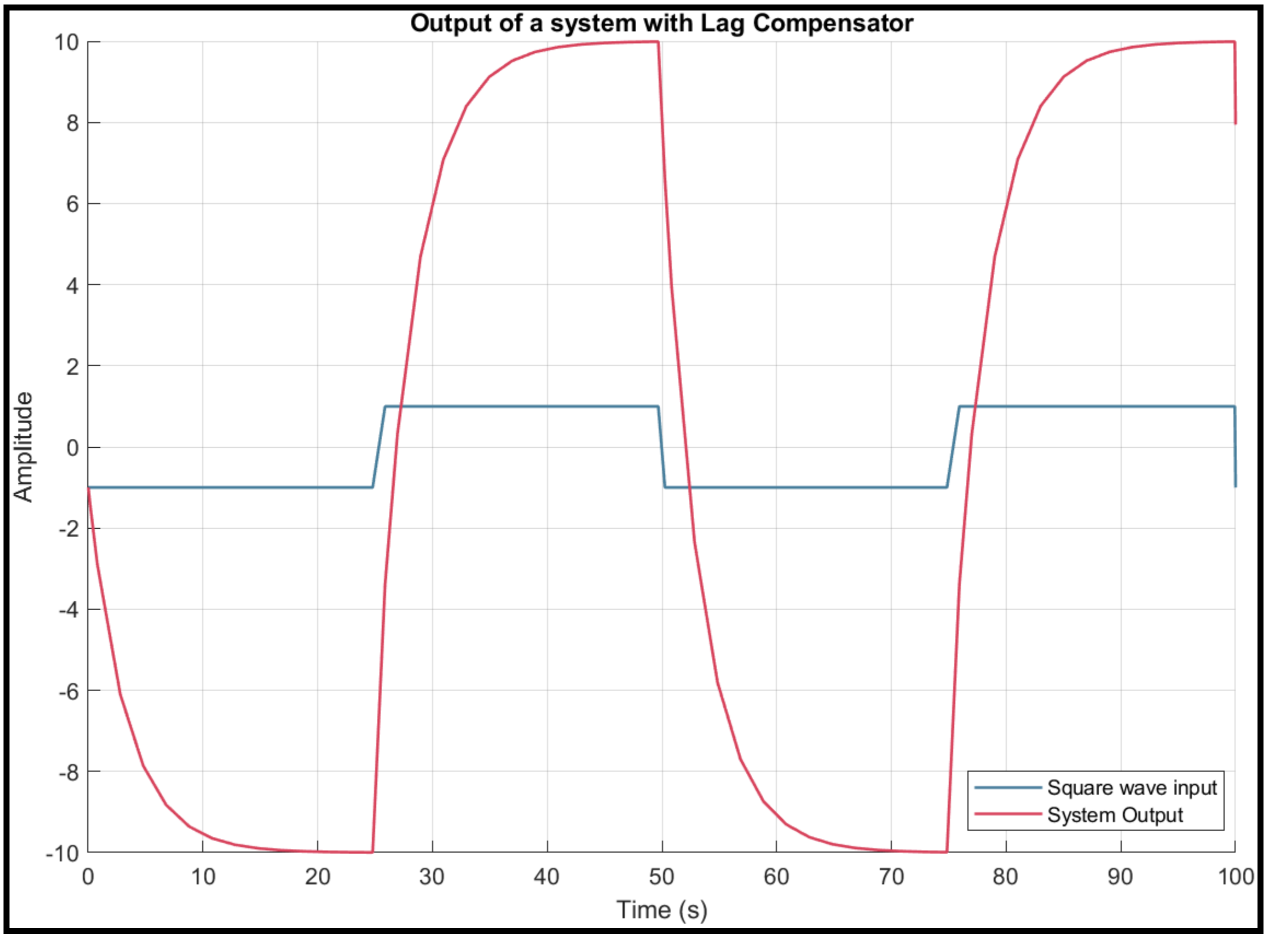

Figure 26: System Output of Lag Compensator from 0.02 HZ Square Wave Input

C.2

```
>> numerator = [1 3];
denominator = [1 0.3];
Gc = tf(numerator,denominator);
pole(Gc)
zero(Gc)

ans =

   -0.3000


ans =

    -3
```

Figure 27: MATLAB Pole and Zero Lag Compensator Values

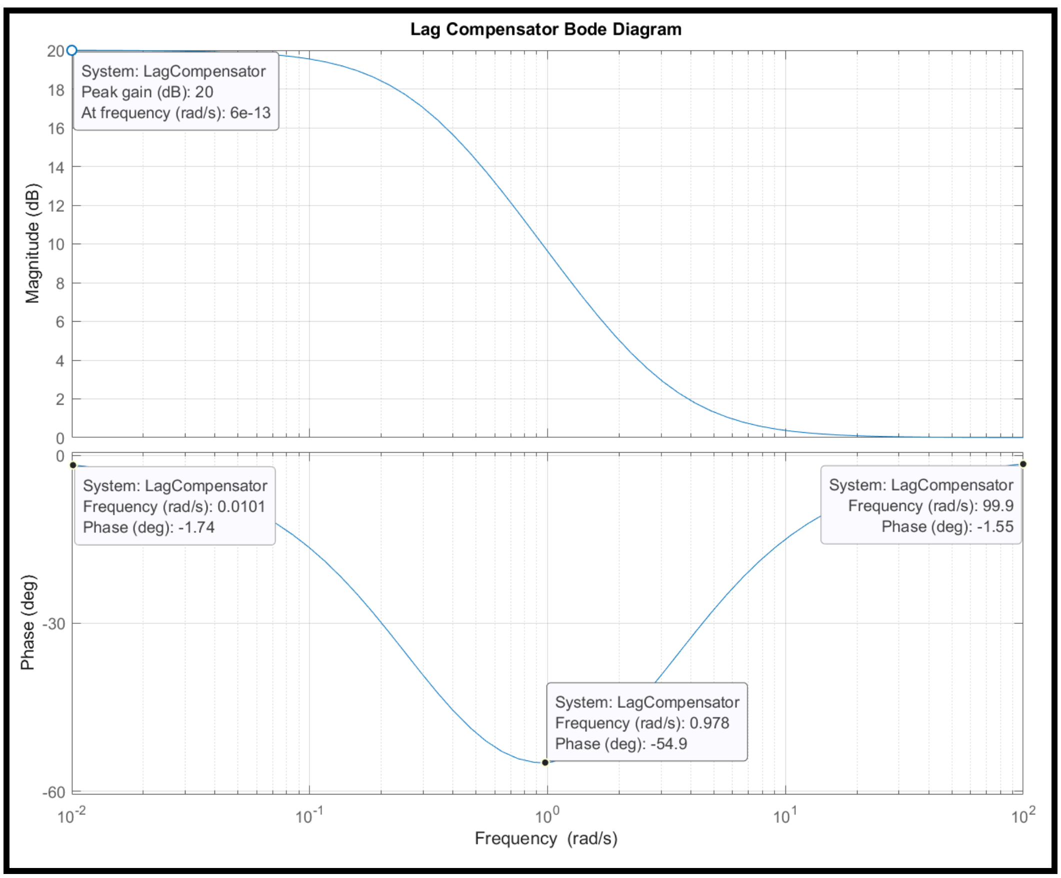

Figure 28: Bode Diagram of the Lag Compensator with Labelled Phase Limits

The maximum and minimum phase shift that can be obtained using this Lag compensator is -1.55° and 54.9° respectively.

D: Lag Compensator & Second-Order Systems

D.1

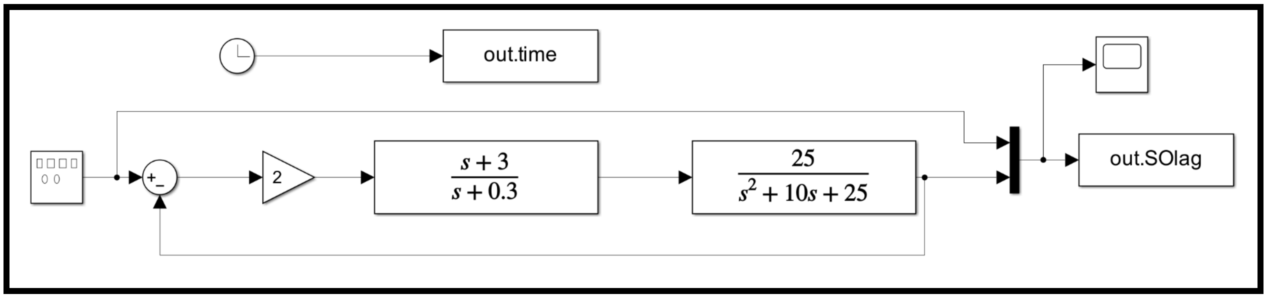

Figure 29: Simulink Model of Second-Order Transfer Function with a Lag Compensator

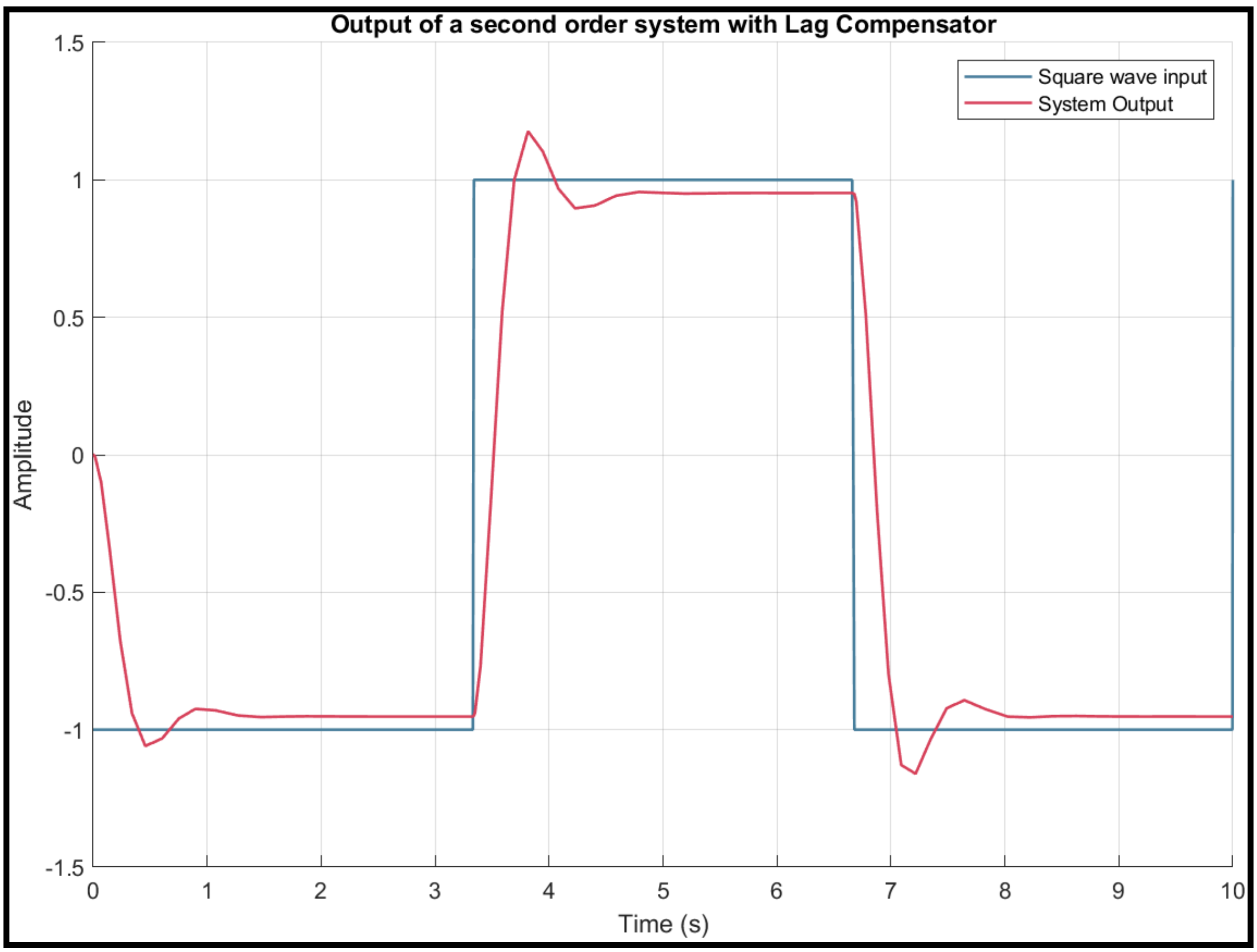

Figure 30: Output of Figure 29 System from Square Wave Input

D.2

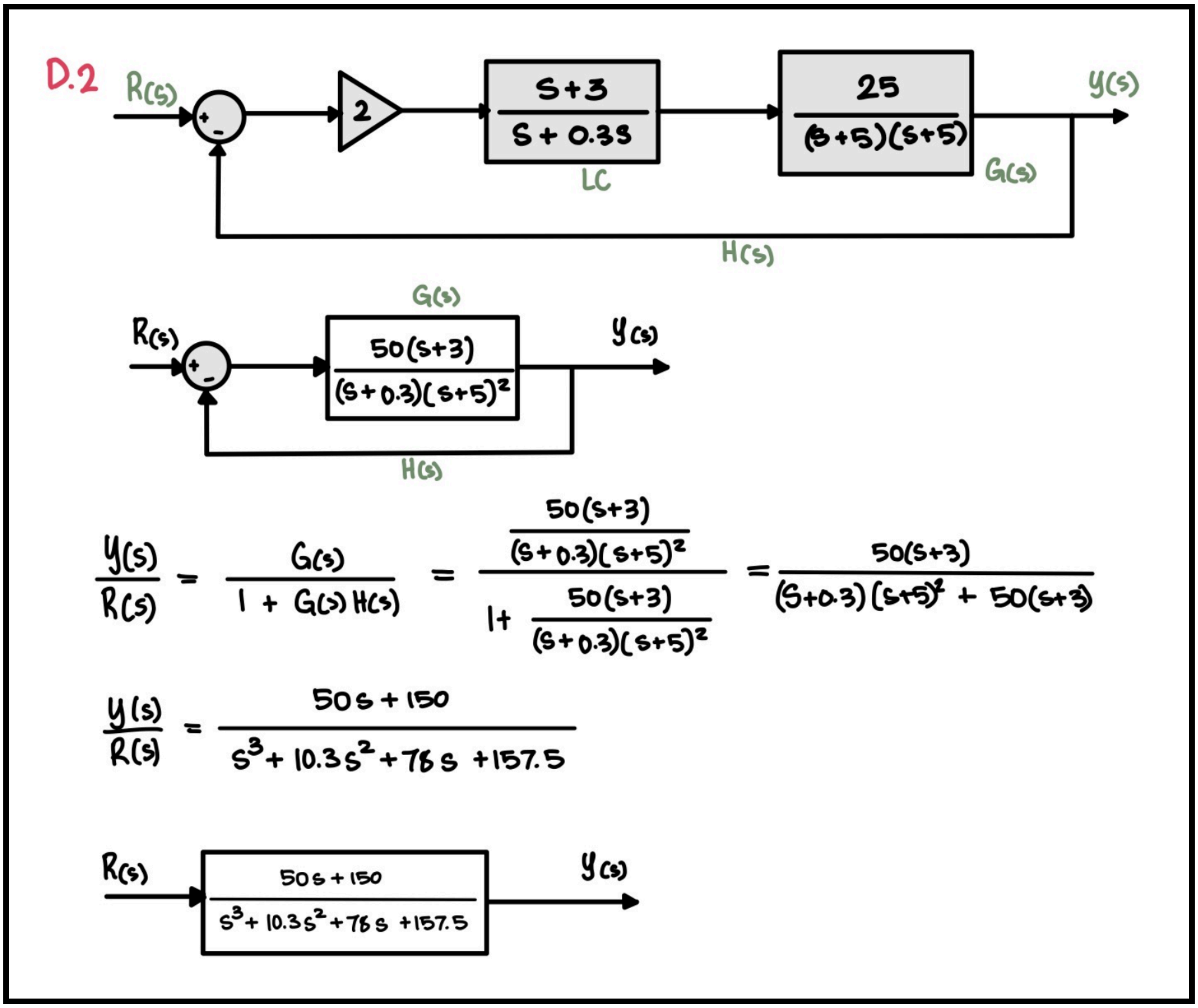

Figure 31: Transfer Function Calculation of Figure 29 Control System

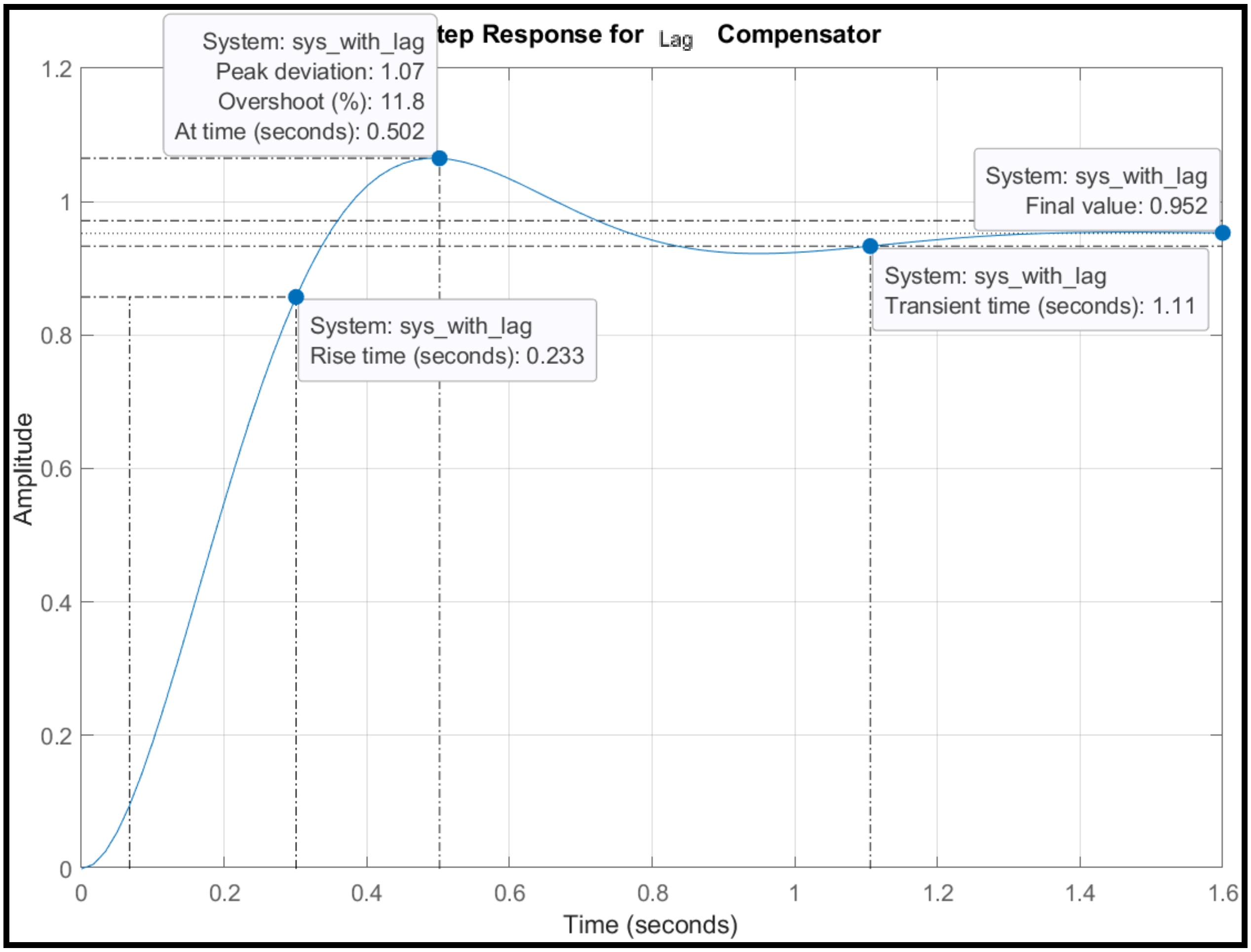

Figure 32: Rise Time $t_r = 0.233s$, *Maximum Overshoot* $\%O.S. = 11.8\%$, *Settling Time* $t_s = 1.11s$ *and Steady-State Error* $e_{ss} = 0.048s$ *Time Domain Specifications of Figure 29 Control System*

D.3

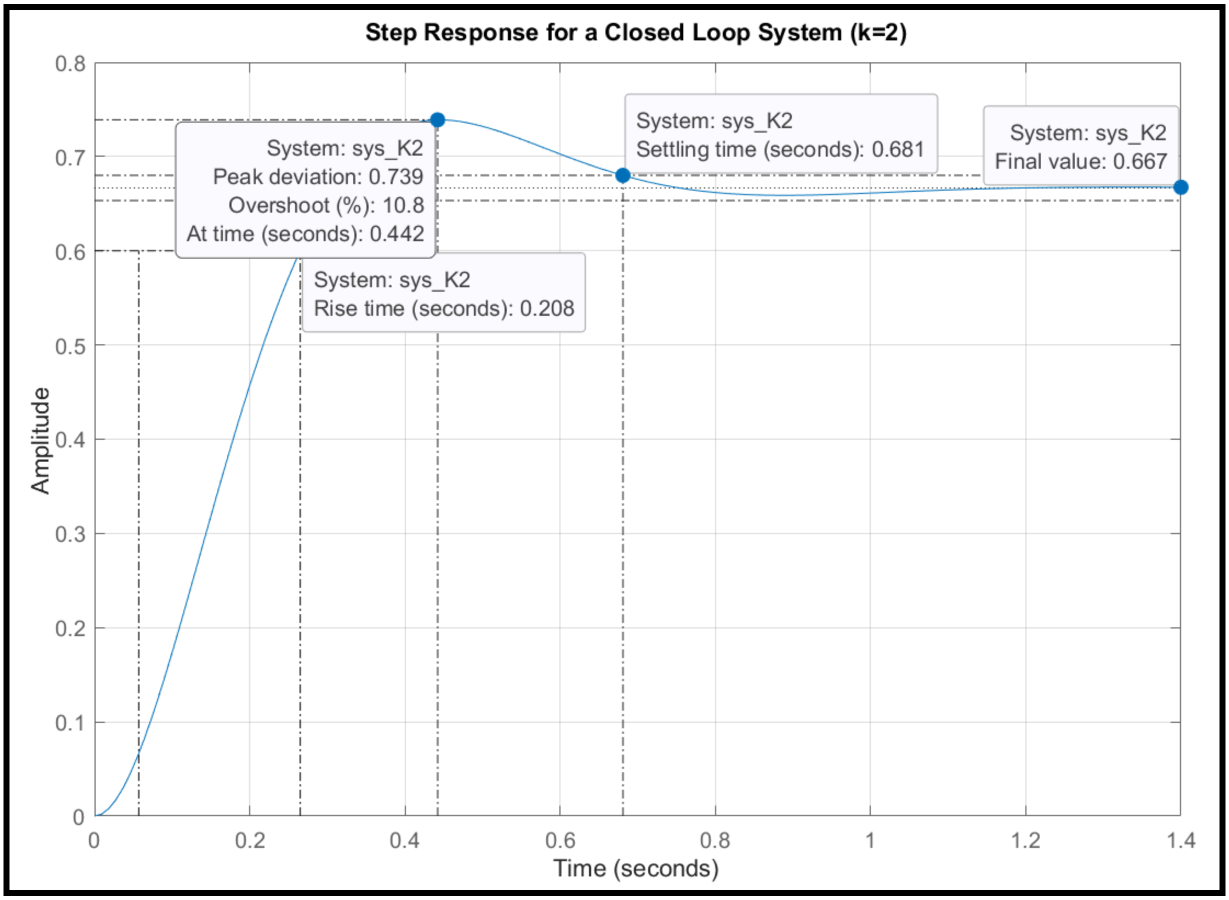

Figure 33: Rise Time $t_r = 0.208s$*, Maximum Overshoot* $\%O.S. = 10.8\%$*, Settling Time* $t_s = 0.681s$ *and Steady-State Error* $e_{ss} = 0.334s$ *Time Domain Specifications of Figure 29 System without Lag Compensator and K=2*

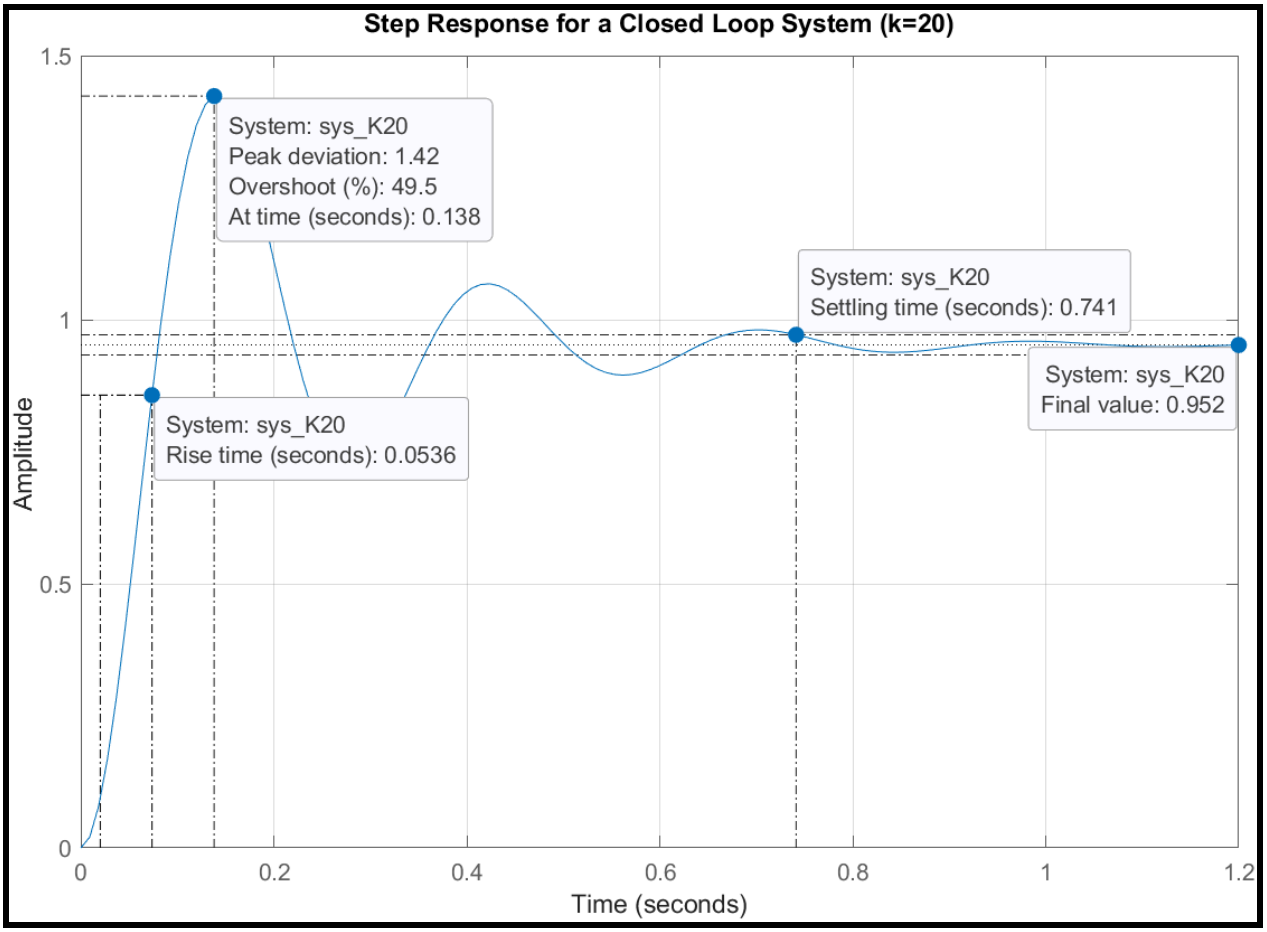

Figure 34: Rise Time $t_r = 0.0536s$*, Maximum Overshoot* $\%O.S. = 49.5\%$*, Settling Time* $t_s = 0.741s$ *and Steady-State Error* $e_{ss} = 0.048s$ *Time Domain Specifications of Figure 29 System without Lag Compensator and K=20*

Table 6: Figure 32, 33 and 34 System Time Domain Specifications Comparison

	Transfer function $\frac{Y(s)}{R(s)}$	**Rise Time** (t_r)	**% Max Overshoot** ($\%O.S.$)	**Settling Time** (t_s)	**Steady-State Error** (e_{ss})
K=2 With Lag	$\frac{50s+150}{0.1s^2+1.1s+11}$	0.2330s	11.8%	1.110s	4.80%
K=2 Without Lag	$\frac{50}{s^2+10s+75}$	0.2080s	10.8%	0.681s	33.3%
K=20 Without Lag	$\frac{10}{s^2+10s+525}$	0.0536s	49.5%	0.741s	4.80%

Effect of lag compensator on transient response and steady-state response.

The lag compensator primarily influences the system's steady-state response, markedly reducing the steady-state error. Although it slightly alters the transient response across various parameters, these changes are not substantial enough to attribute directly to the lag compensator's influence.

In contrast to systems with higher gain, the lag compensator offers slower transient responses but significantly less maximum overshoot for the same steady-state error. Similar to the effects seen with the lead compensator, the lag compensator's impact on the steady-state error of the system is negligible.

D.4

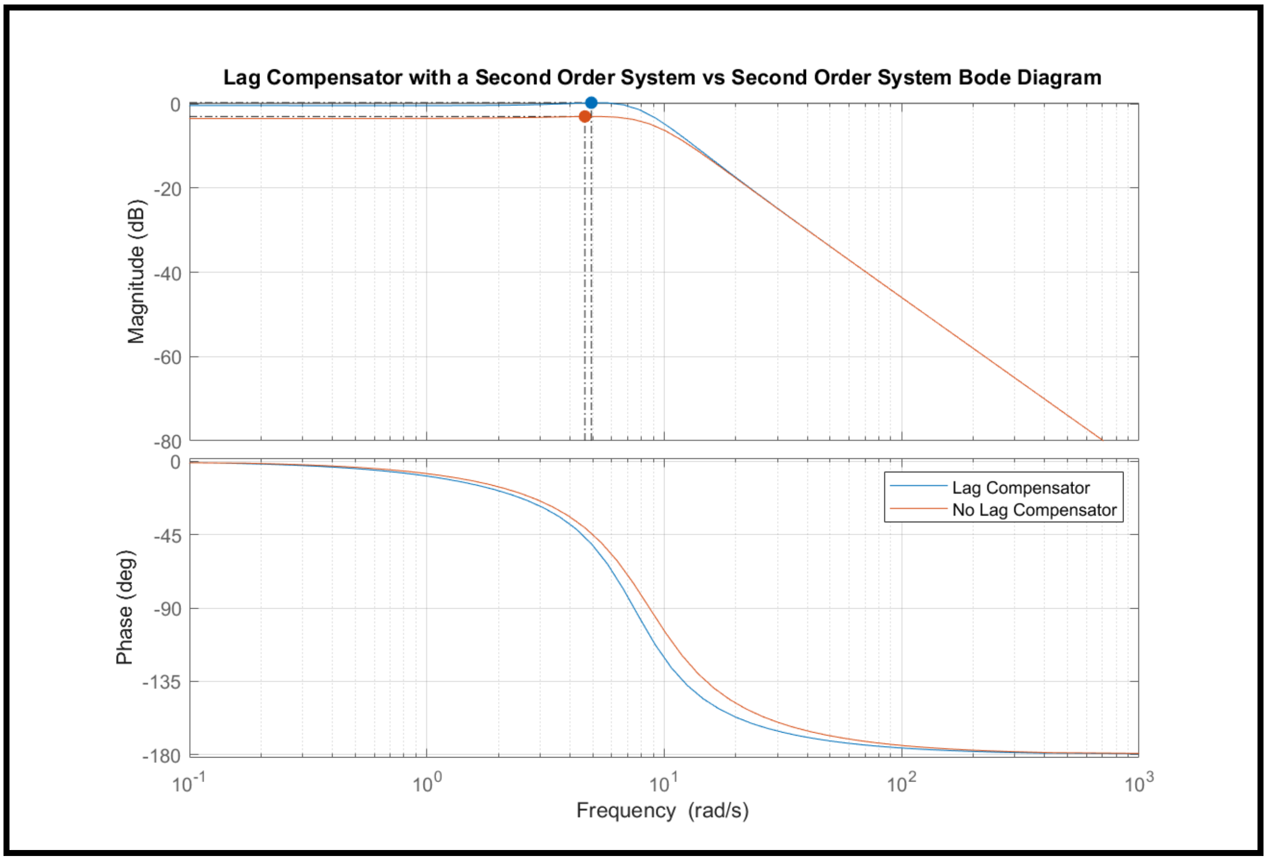

Figure 35: Bode Plots of Lag Compensator and k=2 Proportional Gain Systems

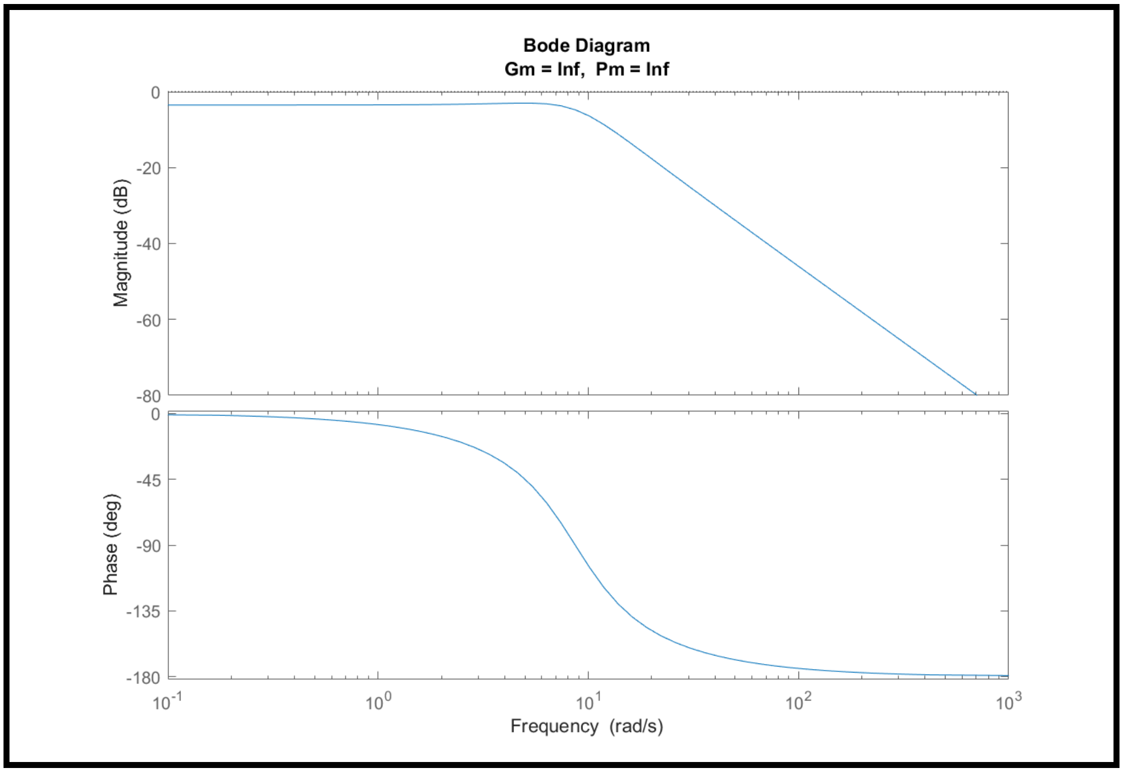

Figure 36: Bode Plot of k=2 Portional Gain System

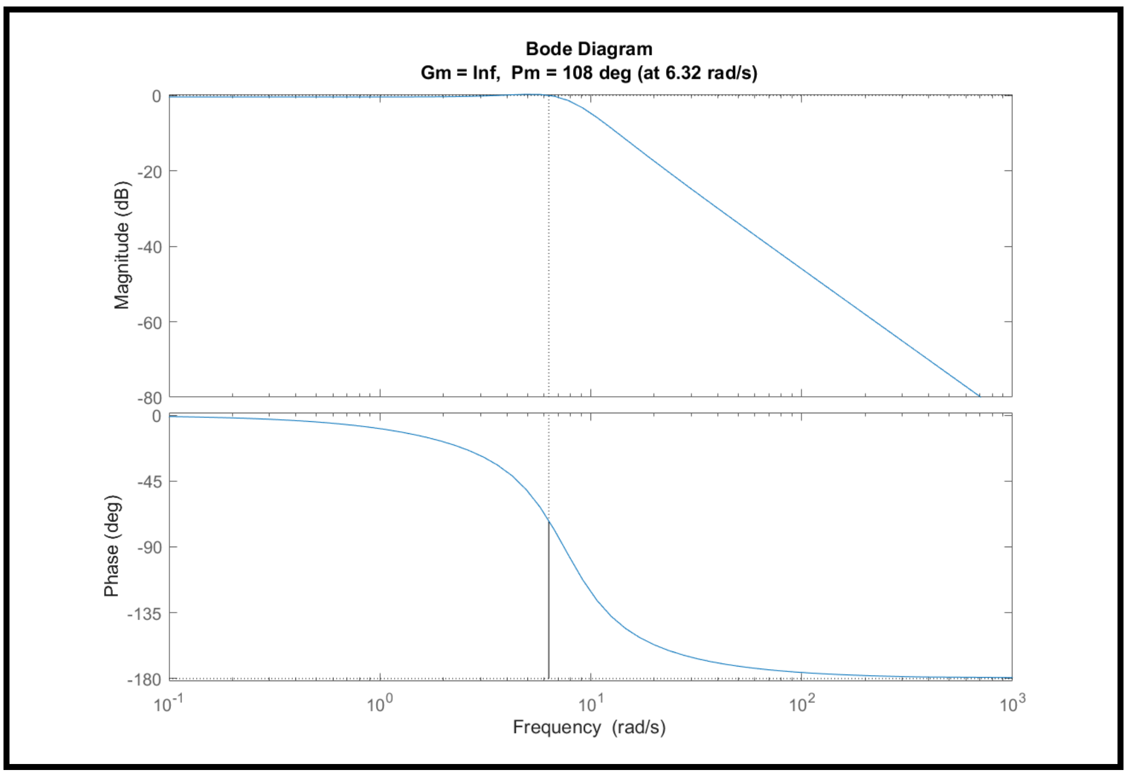

Figure 37: Bode Plot of Lag Compensator System

Compare the phase and gain margins

Both systems possess an infinite gain margin, indicating their absolute stability; no level of gain can destabilize them. The lag compensator features a phase of 108 degrees, signifying stability due to its high phase value. Conversely, the system lacking a compensator exhibits an infinite phase, indicating stability regardless of the input delay. The instability arises from the lag compensator adjusting the system for the desired output.

Stability of the closed-loop system affected by lag compensator.

The lag compensator may impact the stability of the closed-loop system by affecting the phase margin, which, as mentioned previously, contributes to system stability. Moreover, as seen in Table 6, it can increase the overshoot from 10.8% to 11.8%, further indicating increasing instability.

Summaries

Tanvir Hassan *501104056*

In our second lab, we focused on modeling physical systems through transfer functions and applying lead and lag compensators. We better understood transfer function concepts and their utility in representing and predicting physical systems' behaviors. The design and deployment of lead and lag compensators to enhance these systems' efficacy was another key learning area. This lab introduced theoretical foundations with practical applications, modeling essential principles and their real-world applications. We analyzed physical systems via transfer functions and compensators through calculations and the construction of various Simulink diagrams. The initial segment featured an application involving a DC Servo Motor and the role of PI controllers. The following segment delved into the practicalities of lead and lag compensators.

In part 1 of the lab, we employed specific servo metrics to deduce the voltage-to-speed and voltage-to-position transfer functions for the SRV02 servo motor. Laplace transforms were used to correlate voltage to position transfer functions and delineate them within the frequency domain. Referencing the servo metrics, we cataloged the system's order, type, DC gain, and time constants. These parameters were then utilized to ascertain stability. The servo motor's parameters were used to implement the transfer functions in a Simulink model using a square wave as the input. Introducing a PI controller to the Simulink models, through the integration of proportional and integral gains, drastically reduced steady-state errors compared to the original transfer function systems, as illustrated in sections B and C. This initial part established a foundational comprehension of physical systems' transfer function modeling and the integration of controllers.
The second section delves into the effects of lead and lag compensators on second-order systems, examining rise time, percent maximum overshoot, settling time, and steady-state error. It was observed that while compensators enhance certain metrics, they may detract from others. Occasionally, a closed-loop system might exhibit a significantly shifted time domain specification. Based on the specific outcome, a compensator can be integrated to influence a particular time domain specification.

The first part of the lab explored modelling a DC servomotor using control systems. Equations that included variables and constants describing the motor speed and distance were arranged into a transfer function and modelled in Simulink. A PI controller was added to the servomotor control system. The addition was analyzed by plotting the input and output of the system. Adding the controller and a closed-loop system resulted in more predictable and desired outputs that closely modelled the inputs.

Part two explores the impact of lead and lag compensators on second-order systems. The systems' rise time, percent max overshoot, settling time and steady-state error were analyzed. Overall, it was found that compensators improved some of the desired metrics while decreasing the quality of others. A closed-loop system sometimes has at least one highly skewed time domain specification. Depending on the desired output, a compensator may be added to impact a desired time domain specification.

Appendix

```
%% Part 1
%% A.4
figure
hold on
plot(out.time,out.VSmodel(:,1) , 'Color', '#487F9C', 'lineWidth', 1.25)
plot(out.time,out.VSmodel(:,2), 'Color','#637000' , 'lineWidth', 1.25)
plot(out.time,out.VPmodel(:,2), 'Color', '#DB435E', 'lineWidth', 1.25)
xlabel('Time (s)')
ylabel('Amplitude')
title ('Voltage-To-Speed and Voltage-To-Position Vs Time')
legend('Square Wave Input','Voltage to Speed','Voltage to Position')
grid;
hold off
%% B.1
figure
hold on
plot(out.time,out.DCmotorPI(:,1) , 'Color', '#487F9C', 'lineWidth', 1.25)
plot(out.time,out.DCmotorPI(:,2), 'Color','#DB435E' , 'lineWidth', 1.25)
xlabel('Time (s)')
ylabel('Amplitude')
title ('Voltage-To-Speed with a PI Controller Vs Time')
legend('Square Wave Input','Voltage to Speed with PI Controller')
grid;
hold off
%% C.1
figure
hold on
plot(out.time,out.DCmotorPI(:,1) , 'Color', '#487F9C', 'lineWidth', 1.25)
plot(out.time,out.DCmotorPI(:,2), 'Color','#DB435E' , 'lineWidth', 1.25)
xlabel('Time (s)')
ylabel('Amplitude')
title ('Voltage-To-Speed with a PI Controller Vs Time')
legend('Square Wave Input','Voltage to Speed with PI Controller')
grid;
hold off
%% Part 2
%% A.1
figure
hold on
plot(out.time,out.LComp(:,1) , 'Color', '#487F9C', 'lineWidth', 1.25)
plot(out.time,out.LComp(:,2), 'Color','#DB435E' , 'lineWidth', 1.25)
xlabel('Time (s)')
ylabel('Amplitude')
title ('Output of a system with Lag Compensator')
legend('Square wave input','System Output')
grid;
hold off
%% A.2
numerator = [1 1];
denominator = [0.1 1];
Gc = tf(numerator,denominator);
% Find poles and zeros
poles = pole(Gc);
zeros = zero(Gc);
% Display poles and zeros
disp('Poles:');
disp(poles);
disp('Zeros:');
disp(zeros);
% Plot Bode diagram
figure;
bode(Gc);
grid on;
```

```matlab
title('Bode Diagram of the Lead Compensator');
%% B.1
figure
hold on
plot(out.time,out.SOlead(:,1) , 'Color', '#487F9C', 'lineWidth', 1.25)
plot(out.time,out.SOlead(:,2), 'Color','#DB435E' , 'lineWidth', 1.25)
xlabel('Time (s)')
ylabel('Amplitude')
title ('Output of a second order system with Lead Compensator')
legend('Square wave input','System output')
grid;
hold off
%% B.2
num = 10;
den = [0.1 1.1 11];
sys_with_lead = tf (num, den);
figure
stepplot(sys_with_lead)
title ("Step Response for Lead Compensator")
grid on;
%% B.3
num1 = 10;
den1 = [1 2 11];
sys_wo_lead = tf (num1, den1);
figure
stepplot(sys_wo_lead)
title ("Step Response for a Closed Loop System (k=10)")
grid on;
%% B.4
%Bode plot
figure
bode (sys_with_lead)
hold on
bode (sys_wo_lead)
title('Lead Compensator with Second Order System vs Second Order System Bode Diagram')
grid on
legend ('Lead Compensator', 'No Lead Compensator')
hold off;
% Phase and Gain Margin
figure
title ('Lead Compensator in a Closed Loop System')
margin (sys_with_lead);
figure
title ('Closed Loop System without Lead Compensator')
margin (sys_wo_lead);
%% C.1
figure
hold on
plot(out.time,out.LagComp(:,1) , 'Color', '#487F9C', 'lineWidth', 1.25)
plot(out.time,out.LagComp(:,2), 'Color','#DB435E' , 'lineWidth', 1.25)
xlabel('Time (s)')
ylabel('Amplitude')
title ('Output of a system with Lag Compensator')
legend('Square wave input','System Output')
grid;
hold off
%% C.2
%Poles and Zeros
num = [1 3];
den = [1 0.3];
roots(num), roots(den)
%Bode diagram
LagCompensator = tf (num, den);
figure
bode (LagCompensator)
```

```
title('Lag Compensator Bode Diagram')
grid on;
```

```
%% D.1
```

```
figure
hold on
plot(out.time,out.SOlag(:,1) , 'Color', '#487F9C', 'lineWidth', 1.25)
plot(out.time,out.SOlag(:,2), 'Color','#DB435E' , 'lineWidth', 1.25)
xlabel('Time (s)')
ylabel('Amplitude')
title ('Output of a second order system with Lag Compensator')
legend('Square wave input','System Output')
grid;
hold off
```

```
%% D.2
```

```
num = [50 150];
den = [1 10.3 78 157.5];
sys_with_lag = tf (num, den);
figure
stepplot(sys_with_lag)
title ("Step Response for Lead Compensator")
grid on;
```

```
%% D.3
```

```
%Step Response for k=2 and Lag Compensator
sys_K2lag = tf (num, den);
figure
stepplot(sys_K2lag)
title('Step Response with Lag Compensator (k=2)')
grid on;
%Step Response for k=2 Closed Loop System
num1 = 50;
den1 = [1 10 75];
sys_K2 = tf (num1, den1);
figure
stepplot(sys_K2)
title('Step Response for a Closed Loop System (k=2)')
grid on;
%Step Response for k=20 Closed Loop System
num2 = 500;
den2 = [1 10 525];
sys_K20= tf (num2, den2);
figure
stepplot(sys_K20)
title('Step Response for a Closed Loop System (k=20)')
grid on;
```

```
%% D.4
```

```
sys_K2lag = tf (num, den);
figure
bode (sys_K2lag)
hold on
bode (sys_K2)
title('Lag Compensator with a Second Order System vs Second Order System Bode Diagram')
grid on
legend('Lag Compensator', 'No Lag Compensator')
hold off;
%Phase and Gain Margin
figure
title ('Lag Compensator in a Closed Loop System')
margin (sys_K2lag);
figure
title ('Closed Loop System with a Gain of 2')
margin (sys_K2);
```