Neural Network learning → finding the right weights & biases

Neural Network function
input = 784 numbers (pixels)
output = 10 numbers
Parameters = 13002 weights / biases

Cost function
input = 13002 weights / biases
output = 1 number (cost)
Parameters = many training examples

Cost → Sum of Squared difference from output to correct

↓

small Cost = high accuracy output

↓

minimize Cost function

↓

! to find the optimal weights & biases

Cost of 3

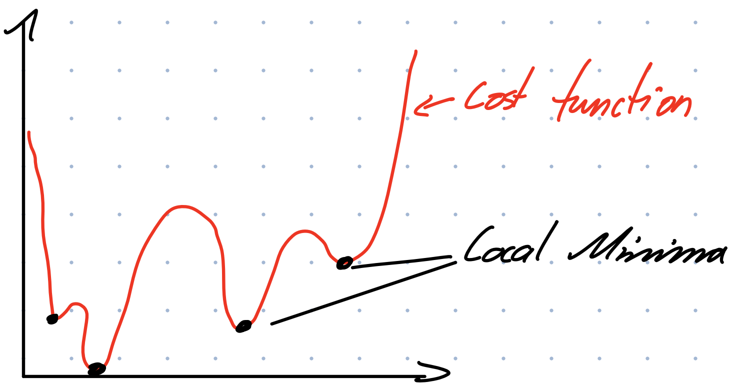
$$0.03 \left\{ \begin{array}{l} 0.0006 \leftarrow (0.02 - 0.00)^2 + \\ 0.0007 \leftarrow (0.03 - 0.00)^2 + \\ 0.0039 \leftarrow (0.06 - 0.00)^2 + \\ 0.0009 \leftarrow (0.97 - 1.00)^2 + \\ 0.0055 \leftarrow (0.07 - 0.00)^2 + \\ 0.0004 \leftarrow (0.02 - 0.00)^2 + \\ 0.0022 \leftarrow (0.05 - 0.00)^2 + \\ 0.0033 \leftarrow (0.06 - 0.00)^2 + \\ 0.0072 \leftarrow (0.08 - 0.00)^2 + \\ 0.0018 \leftarrow (0.04 - 0.00)^2 \end{array} \right.$$

Low cost
correct output

Cost of 3

$$3.37 \left\{ \begin{array}{l} 0.1863 \leftarrow (0.43 - 0.00)^2 + \\ 0.0809 \leftarrow (0.28 - 0.00)^2 + \\ 0.0357 \leftarrow (0.19 - 0.00)^2 + \\ 0.0138 \leftarrow (0.88 - 1.00)^2 + \\ 0.5242 \leftarrow (0.72 - 0.00)^2 + \\ 0.0001 \leftarrow (0.01 - 0.00)^2 + \\ 0.4079 \leftarrow (0.64 - 0.00)^2 + \\ 0.7388 \leftarrow (0.86 - 0.00)^2 + \\ 0.9817 \leftarrow (0.99 - 0.00)^2 + \\ 0.3998 \leftarrow (0.63 - 0.00)^2 \end{array} \right.$$

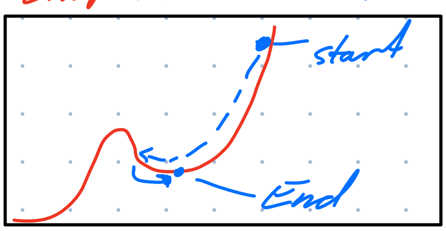
high cost
incorrect output



Minimization Options

Hill climb → step where function decreases most
→ repeat

Endpoint = startpoint +/- step size




```
# Hill climbing algorithm
for i in range(num_iterations):
    current_val = f(x)
    candidate_x1 = x + alpha
    candidate_x2 = x - alpha
    candidate_val1 = f(candidate_x1)
    candidate_val2 = f(candidate_x2)

    if candidate_val1 > current_val:
        x = candidate_x1
    elif candidate_val2 > current_val:
        x = candidate_x2
```

things to consider:
↳ step size

↳ starting point / parameters
↳ number of steps / iterations

Gradient decent

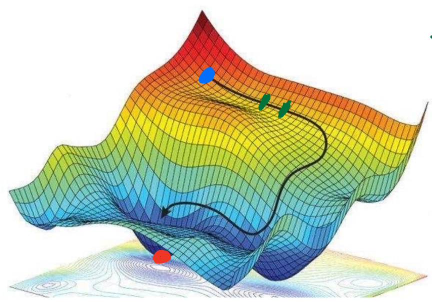
↳ (think multi-dimensional Hill climb)

$$\boxed{\text{Minimum / Endpoint} = \text{Startpoint} - \text{Learning rate} \cdot \text{Gradient}}$$

Start point = random guess / informed point

Learningrate = think step size → it just dotproducts the Gradient

Gradient = direction of steepest increase
↳ calculated by partial derivatives for each variable



→ repeat the formula until
1. max number of iterations
2. tolerence is achieved (Difference of outputs is small

things to consider:
1. learningrate size
2. tolerance
3. number of iterations

Gradient Vector of Costfunction encodes:

$$\vec{\mathbf{W}} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{13{,}000} \\ w_{13{,}001} \\ w_{13{,}002} \end{bmatrix}$$

$$-\nabla C(\vec{\mathbf{W}}) = \begin{bmatrix} 0.31 \\ 0.03 \\ -1.25 \\ \vdots \\ 0.78 \\ -0.37 \\ 0.16 \end{bmatrix} \begin{array}{l} w_0 \text{ should increase somewhat} \\ w_1 \text{ should increase a little} \\ w_2 \text{ should decrease a lot} \\ \\ w_{13{,}000} \text{ should increase a lot} \\ w_{13{,}001} \text{ should decrease somewhat} \\ w_{13{,}002} \text{ should increase a little} \end{array}$$

1. if weight / biases should increase / decrease
2. the importance of each weight / bias