Title: Relation between diameter and flow in major branches of the arch of the aorta

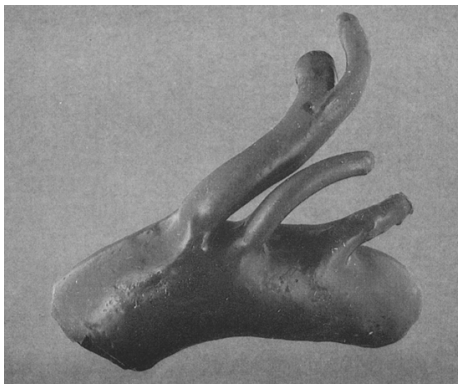
Authors: Mair Zamir, Paula Sinclair, Thomas H. Wonnacott

Journal: Journal of Biomechanics

Year: 1992

Summary

- Relationship between the diameter of a vessel and its flowrate has been studied for a while
  - Many results indicate existence of a power law $q \propto d^x$ where q is the flowrate and x is a number between 1.0 and 5.0.
  - Cube law ($x = 3.0$) (Murray, 1926) one of the most famous
    - Minimizes the sum of two energies: metabolic energy required to maintain volume of blood filling a vessel and pumping energy required to drive blood through vessel
    - Found to match biological data well but may not be applicable for the first few generations of the arterial tree
- Rigid model of arch geometry (arch + major branches) created using corrosion casting
  - Specimens obtained from 91 human subjects
  - Stretched aortas to *in vivo* dimensions by applying 120 mmHg pressure when casting



- Theory of power law is constructed in two basic steps
  - Flow rates at a bifurcation with one parent vessel (q_0) and two branches (q_1 and q_2), by continuity

$$q_0 = q_1 + q_2$$

  - If the flow q_i varies as a function of d_i^x where d_i for $i = 0, 1, 2$ represent the diameters of the three vessels, then

$$d_0^x = d_1^x + d_2^x$$

  - Goal is to find x that best matches biological data
- Measure diameters of brachiocephalic trunk (b), left common carotid (c), and left subclacian arteries (s)

  - Assumption: flow to right and left sides of the head and upper limbs is equal on average
  - Hypothesis: $b^x = c^x + s^x$

- Correlation between b^x and $c^x + s^x$ poor for $x = 1(r^2 = 0.61), 2(r^2 = 0.58), 3(r^2 = 0.53), 4(r^2 = 0.49)$.
  - $x = 1$ has highest correlation but still poor as with all other values of x

- Test of equality (mean squared relative error) of b^x and $c^x + s^x$ determined that $x = 2$ is best

$$MSRE(x) = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{b_i^x - c_i^x - s_i^x}{b_i^x} \right)^2$$

- Relationship between vessel diameter and flow in the aorta and major branching arteries governed by 'square law' as opposed to 'cube law'
  - Though square law may apply at this level of arterial tree, should be noted that this is not true at all levels of the tree