Assume the tanks to be well mixed, of constant volume and of equal size.

(a) Write down the appropriate conservation balances.

(b) Write the state and output equations using perturbation variables where the output is X_2

(c) Write the model in discrete form, where: $F = 2.$
$M = 10.$
$T_s = 0.5.$

(d) Write as an input-output model between X_2 and S.

(e) Write down the corresponding pulse transfer function.

(f) Show that the process is stable.

(g) Show that the closed-loop system is stable with a proportional controller of gain 10. Calculate the gain at which the closed-loop process would be unstable.

1.7 References

Astrom, K. J. and Wittenmark, B. (1984), *Computer Controlled Systems: Theory and Design*, Prentice Hall, Englewood Cliffs, NJ.

2 The evaporator model

2.1 Introduction

The concentration of dilute liquors by evaporating solvent from the feed stream is an important industrial process used in such industries as sugar mills, alumina production and paper manufacture, to name a few. An often used evaporator, known as a "forced circulation evaporator", is shown in Figure 2.1. In this evaporator, feed is mixed with a

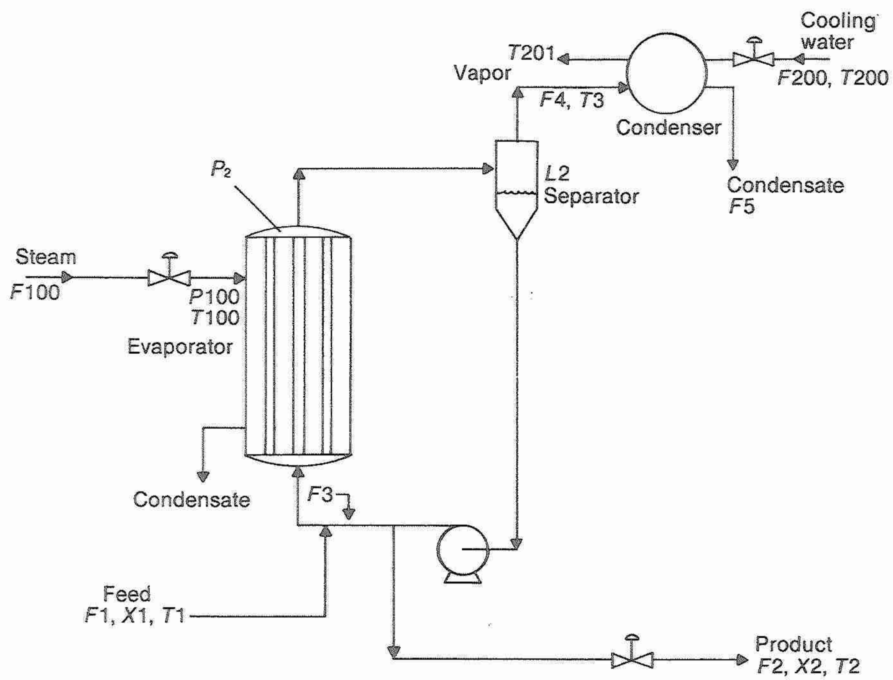

Figure 2.1 Evaporator system

high volumetric flowrate of recirculating liquor and is pumped into a vertical heat exchanger. Often the heat exchanger is heated with steam which condenses on the outside of the tube walls. The liquor which passes up the inside of the tubes, boils and then passes to a separation vessel. In this vessel, liquid and vapor are separated. The liquid is recirculated with some being drawn off as product. The vapor is usually condensed by cooling, with water often being used as the coolant.

This chapter derives a mathematical model of a forced circulation evaporator. There are two major parts to this description of the mathematical model:

- The first part derives a mechanistic nonlinear model with major assumptions stated, variables defined and parameters given.
- The second part derives a linear state space description in normalized form by evaluating the appropriate Jacobian elements.

2.2 The nonlinear model

Evaporator variables

The variable names, descriptions, standard steady state values, and engineering units are shown in Table 2.1 and in Figure 2.1. The solvent is water and the solute is nonvolatile.

Table 2.1 Evaporator variables

Variable	Description	Value	Units
$F1$	feed flowrate	10.0	kg/min
$F2$	product flowrate	2.0	kg/min
$F3$	circulating flowrate	50.0	kg/min
$F4$	vapor flowrate	8.0	kg/min
$F5$	condensate flowrate	8.0	kg/min
$X1$	feed composition	5.0	percent
$X2$	product composition	25.0	percent
$T1$	feed temperature	40.0	deg C
$T2$	product temperature	84.6	deg C
$T3$	vapor temperature	80.6	deg C
$L2$	separator level	1.0	metres
$P2$	operating pressure	50.5	kPa
$F100$	steam flowrate	9.3	kg/min
$T100$	steam temperature	119.9	deg C
$P100$	steam pressure	194.7	kPa
$Q100$	heater duty	339.0	kW
$F200$	cooling water flowrate	208.0	kg/min
$T200$	cooling water inlet temperature	25.0	deg C
$T201$	cooling water outlet temperature	46.1	deg C
$Q200$	condenser duty	307.9	kW

Process liquid mass balance

A mass balance on the total process liquid (solvent and solute) in the system yields:

$$\rho A\, dL2/dt = F1 - F4 - F2 \tag{2.1}$$

where: ρ is the liquid density.
A is the cross-sectional area of the separator.

The product ρA is assumed to be constant at 20 kg/metre.

Process liquid solute mass balance

A mass balance on the solute in the process liquid phase yields:

$$M\, dX2/dt = F1\, X1 - F2\, X2 \tag{2.2}$$

where M is the amount of liquid in the evaporator and is assumed to be constant at 20 kg.

Process vapor mass balance

A mass balance on the process vapor in the evaporator will express the total mass of the water vapor in terms of the pressure that exists in the system:

$$C\, dP2/dt = F4 - F5 \tag{2.3}$$

where C is a constant that converts the mass of vapor into an equivalent pressure and is assumed to have a value of 4 kg/kPa. This constant can be derived from the ideal gas law.

Process liquid energy balance

The process liquid is assumed to always exist at its boiling point and to be perfectly mixed (assisted by the high circulation rate).

The liquid temperature is:

$$T2 = 0.5616\, P2 + 0.3126\, X2 + 48.43 \tag{2.4}$$

which is a linearization of the saturated liquid line for water about the standard steady-state value and includes a term to approximate boiling point elevation due to the presence of the solute.

The vapor temperature is:

$$T3 = 0.507\, P2 + 55.0 \tag{2.5}$$

which is a linearization of the saturated liquid line for water about the standard steady-state value.

The dynamics of the energy balance are assumed to be very fast so that:

$$F4 = (Q100 - F1\,C_P\,(T2 - T1))/\lambda \tag{2.6}$$

where C_P is the heat capacity of the liquor and is assumed constant at a value of 0.07 kW/K (kg/min) and λ is the latent heat of vaporization of the liquor and is assumed to have a constant value of 38.5 kW/(kg/min).

The sensible heat change between $T2$ and $T3$ for $F4$ is considered small compared to the latent heat. It is assumed that there are no heat losses to the environment or heat gains from the energy input of the pump.

Heater steam jacket

Steam pressure $P100$ is a manipulated variable which determines steam temperature under assumed saturated conditions. An equation relating steam temperature to steam pressure can be obtained by approximating the saturated steam temperature–pressure relationship by local linearization about the steady-state value:

$$T100 = 0.1538\,P100 + 90.0 \tag{2.7}$$

The rate of heat transfer to the boiling process liquid is given by:

$$Q100 = UA1\,(T100 - T2) \tag{2.8}$$

where $UA1$ is the overall heat transfer coefficient times the heat transfer area and is a function of the total flowrate through the tubes in the evaporator:

$$UA1 = 0.16\,(F1 + F3)$$

The steam flowrate is calculated from:

$$F100 = Q100/\lambda_s \tag{2.9}$$

where λ_s is the latent heat of steam at the saturated conditions, assumed constant at a value of 36.6 kW/(kg/min).

The dynamics within the steam jacket have been assumed to be very fast.

Condenser

The cooling water flowrate $F200$ is a manipulated variable and the inlet temperature $T200$ is a disturbance variable.

A cooling water energy balance yields:

$$Q200 = F200\,C_P\,(T201 - T200)$$

where C_P is the heat capacity of the cooling water assumed constant at 0.07 kW/(kg/min).

The heat transfer rate equation is approximated by:

$$Q200 = UA2\,(T3 - 0.5\,(T200 + T201))$$

where $UA2$ is the overall heat transfer coefficient times the heat transfer area, which is

assumed constant with a value of 6.84 kW/K.

These two equations can be combined to eliminate $T201$ to give explicitly:

$$Q200 = \frac{UA2\,(T3 - T200)}{1 + UA2/(2\,C_P\,F200)} \tag{2.10}$$

It follows that:

$$T201 = T200 + Q200/(F200\,C_P) \tag{2.11}$$

The condensate flowrate is:

$$F5 = Q200/\lambda \tag{2.12}$$

where λ is the latent heat of vaporization of water assumed constant at 38.5 kW/K(kg/min).

The dynamics within the condenser have been assumed to be very fast.

Degrees of freedom

In any modeling and control problem, it is advisable to check that the problem is well formulated. This can be achieved by performing a degrees of freedom analysis. If the degrees of freedom of the problem is equal to zero, then the problem is well posed and a unique solution of the problem is possible.

For this problem there are 20 variables, which are shown in Table 2.1. There are 12 equations defined above. Therefore:

$$\begin{aligned} \text{DOF} &= \text{Number of variables} - \text{Number of equations} \\ &= 20 - 12 \\ &= 8 \end{aligned}$$

This implies that eight extra pieces of information need to be supplied to define a unique solution. This is achieved by specifying the input variables as follows:

3 manipulated variables ($F2$, $P100$, $F200$)
5 disturbance variables ($F3$, $F1$, $X1$, $T1$, $T200$)

With these variables assigned values, the problem has zero degrees of freedom and a unique solution is possible.

Nonlinear model implementations

Several implementations of the nonlinear model are currently available:

1. a FORTRAN subroutine;
2. an IBM Advanced Control System simulation;
3. a SPEEDUP simulation model;
4. a Bailey Network 90 simulation;
5. a FIX simulation.

Information on the availability of software can be found in Appendix A.