2 Linear Equations in 2D

We have

$$2x - y = 0$$
$$-x + 2y = 3$$

We can turn these equations into a 2x2 matrix:

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$Ax = B$$

The matrices are A, X, and B, respectively This can be rearranged to become a **Linear Combination**:

$$x \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

The set of x and y that satisfies that equality is the point of intersection of the system of linear equations.

3 Linear Equations in 3D

Given that

$$2x - y = 0$$
$$-x + 2y - z = -1$$
$$-3y + 4z = 4$$

Let A and B equal:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix}$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

For the equation corresponding to the second row of A, imagine a plane representing all solutions to the equation. Now, imagine each equation corresponding to each row of the matrix has a plane of solutions. The intersection of the three planes is a **point**.

Now,

$$Ax = B$$

becomes

$$x \cdot \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y \cdot \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} + z \cdot \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

For the equality to be satisfied x=0, y=0, z=1

Interdependence

When two or more vectors lie in the same plane, their span is limited to that plane. Those vectors are linearly dependent of each other.

Tips for Visualizing Matrix Multiplication

To multiply this,

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

picture that you are taking 1 of the first column and 2 of the second column and adding the results together:

$$1 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \cdot \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 + 2 * 5 \\ 1 + 2 * 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$