Chapter 3

Solving Problems by Searching

Topics Covered

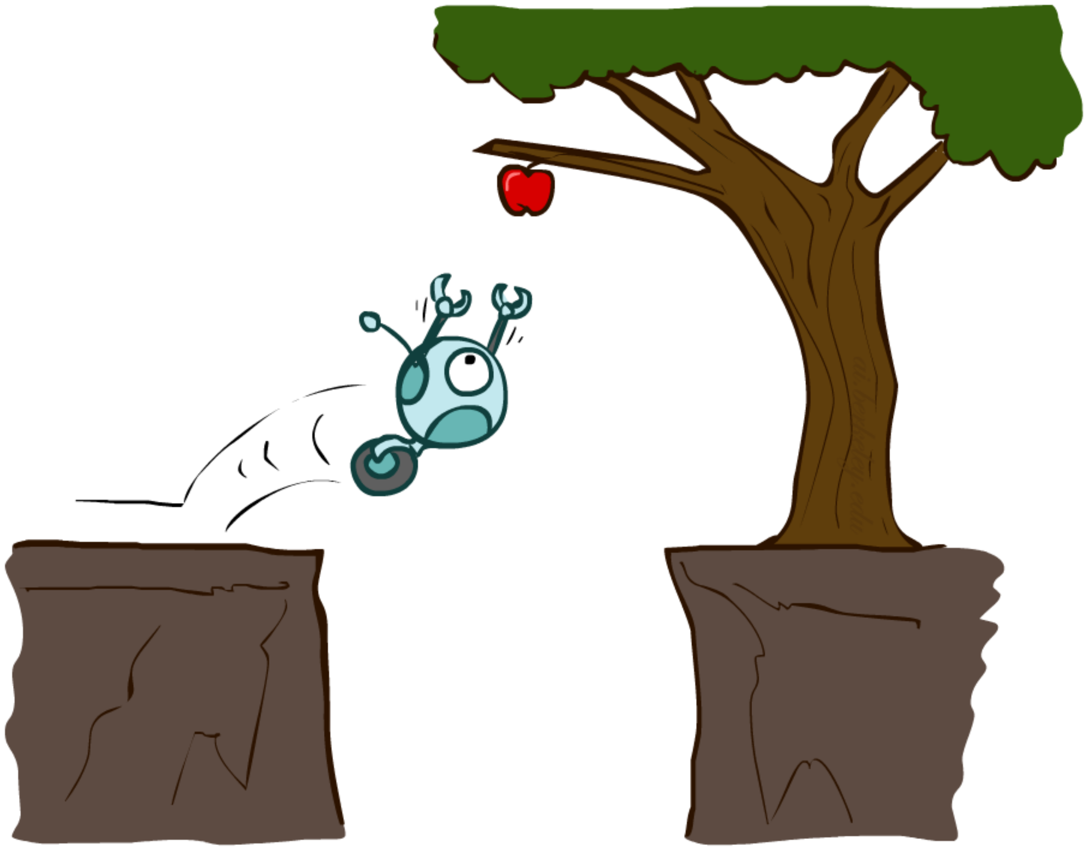
- Solving Problems by Searching
- Problem Solving Agents
- Example Problems
- Search Algorithms
- Uninformed Search Strategies
- Informed (Heuristic) Search Strategies
- Heuristic Functions

Agents that Plan

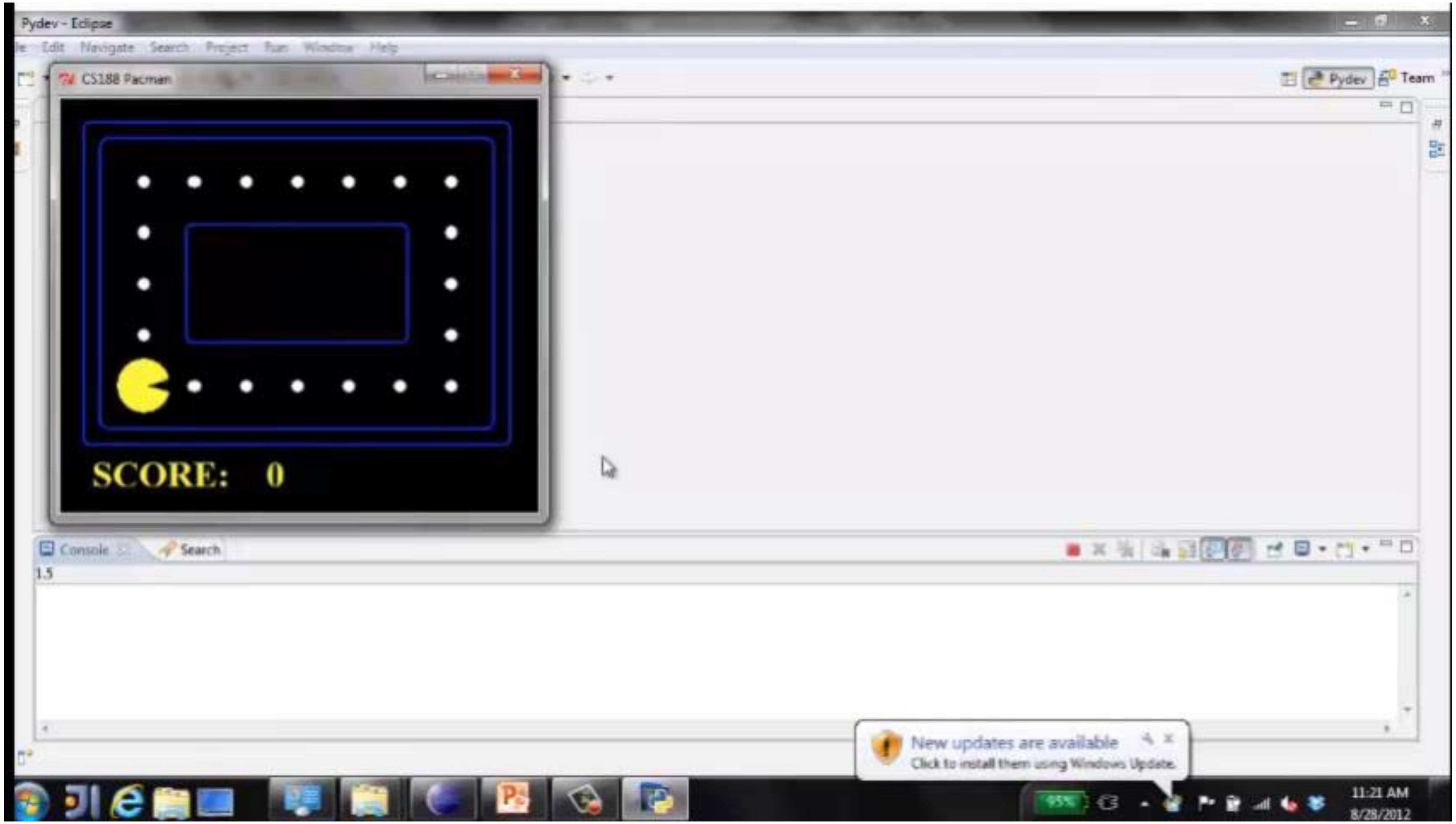
Reflex Agents

- Reflex agents:
  - Choose action based on current percept (and maybe memory)
  - May have memory or a model of the world's current state
  - Do not consider the future consequences of their actions
  - Consider how the world IS

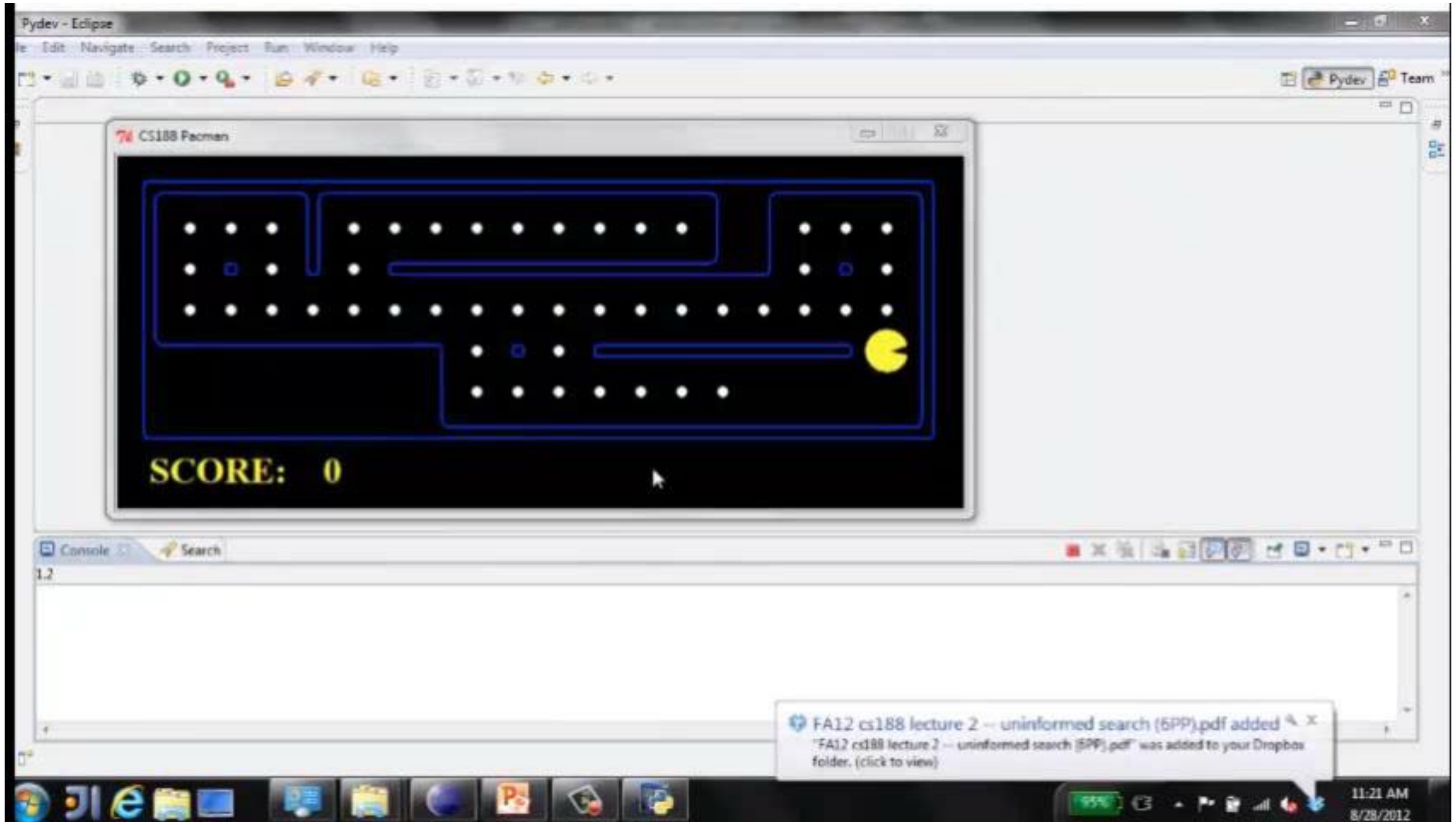
- Can a reflex agent be rational?



Video of Demo Reflex Optimal



Video of Demo Reflex Odd

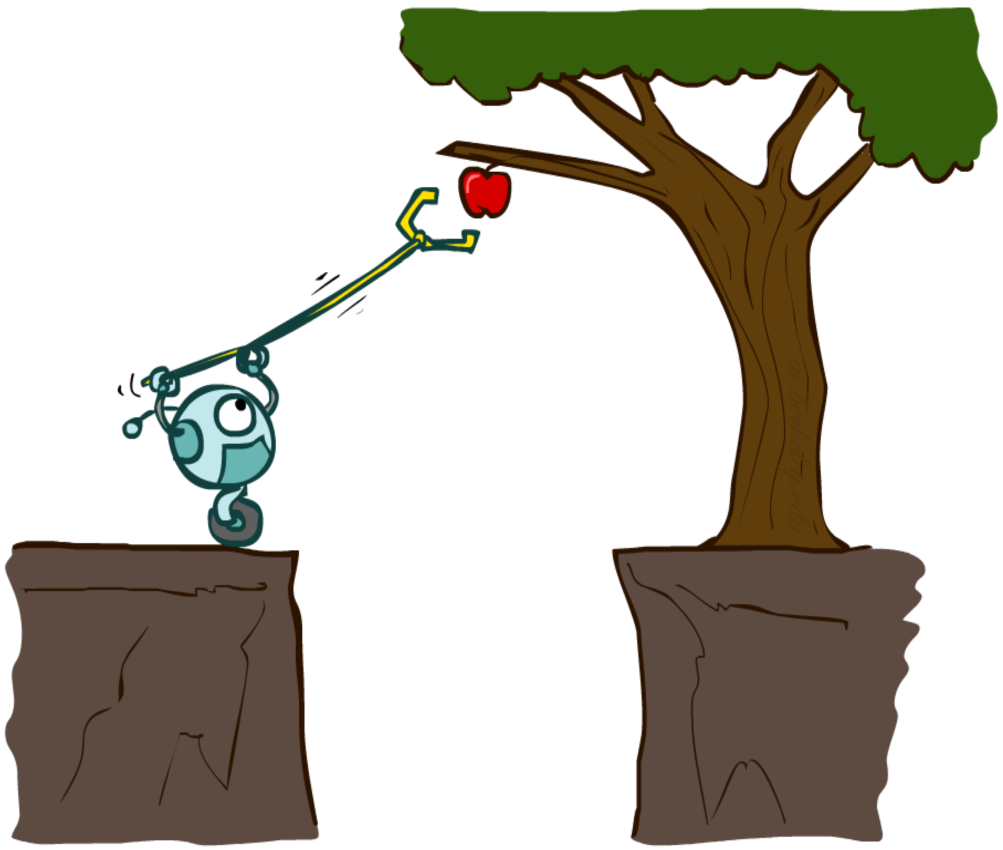


Planning Agents

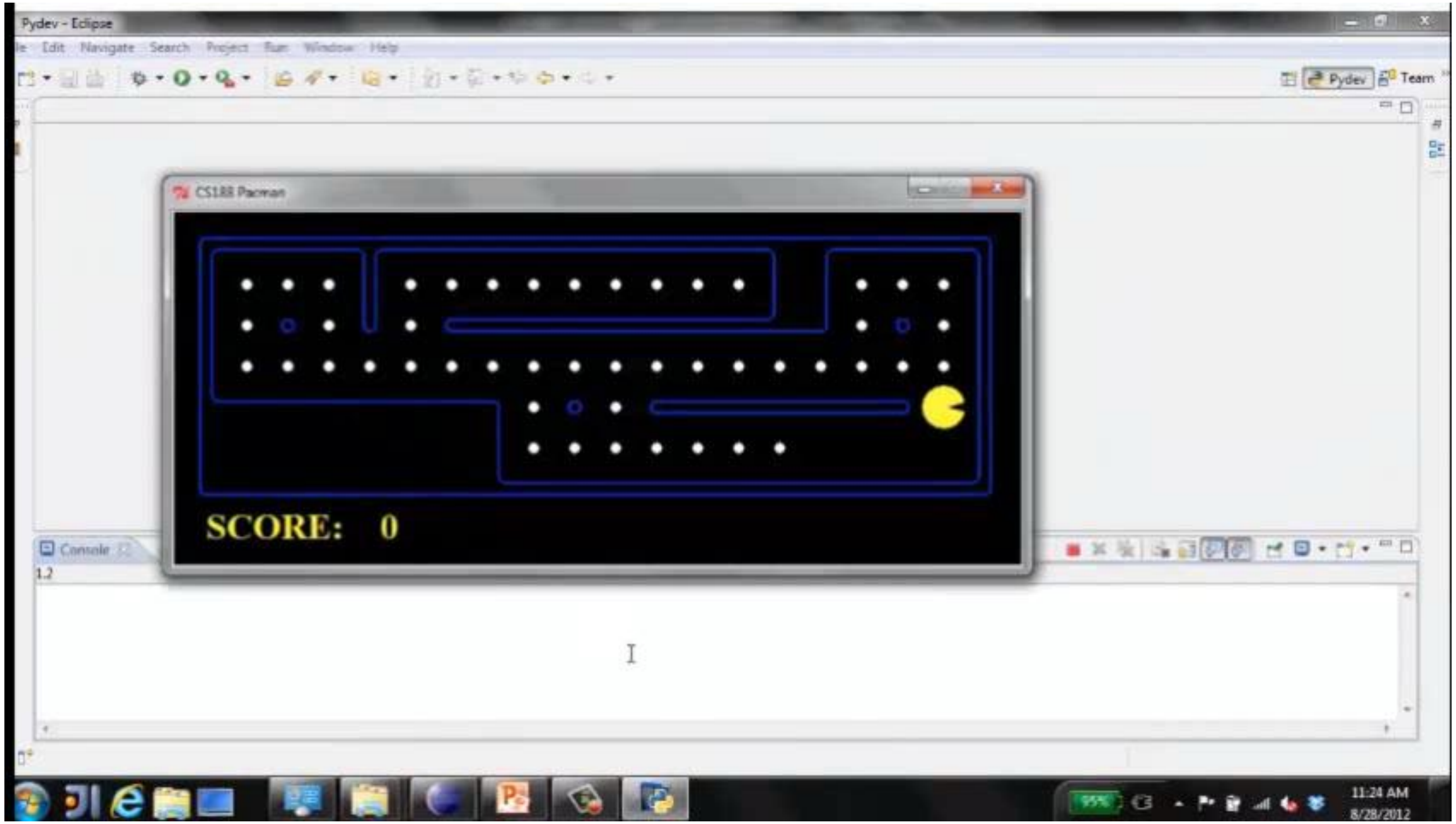
- Planning agents:
  - Ask “what if”
  - Decisions based on (hypothesized) consequences of actions
  - Must have a model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world WOULD BE

- Optimal vs. complete planning

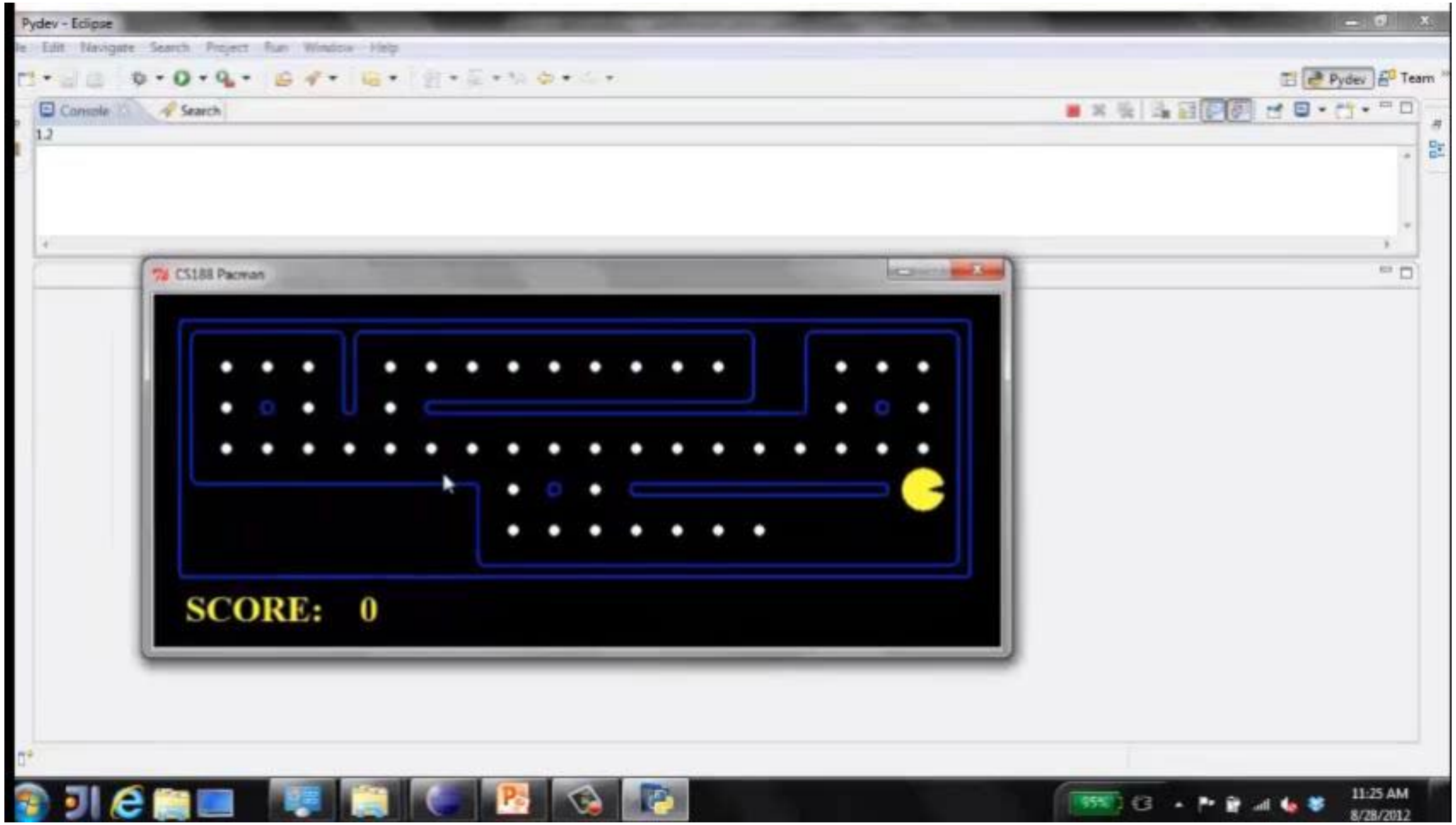
- Planning vs. replanning



Video of Demo Replanning



Video of Demo Mastermind

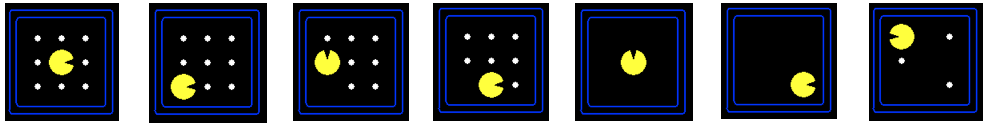


Search Problems

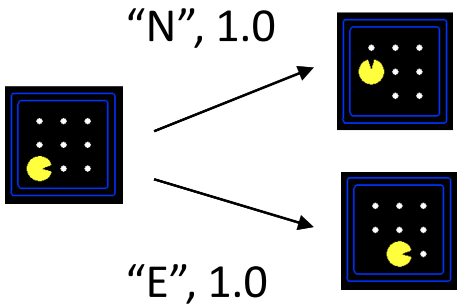
Finding a sequence of actions to get to the goal state

Search Problems

- A search problem consists of:
  - A state space



  - A successor function (with actions, costs)



  - A start state and a goal test

- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

Search Problems Are Models

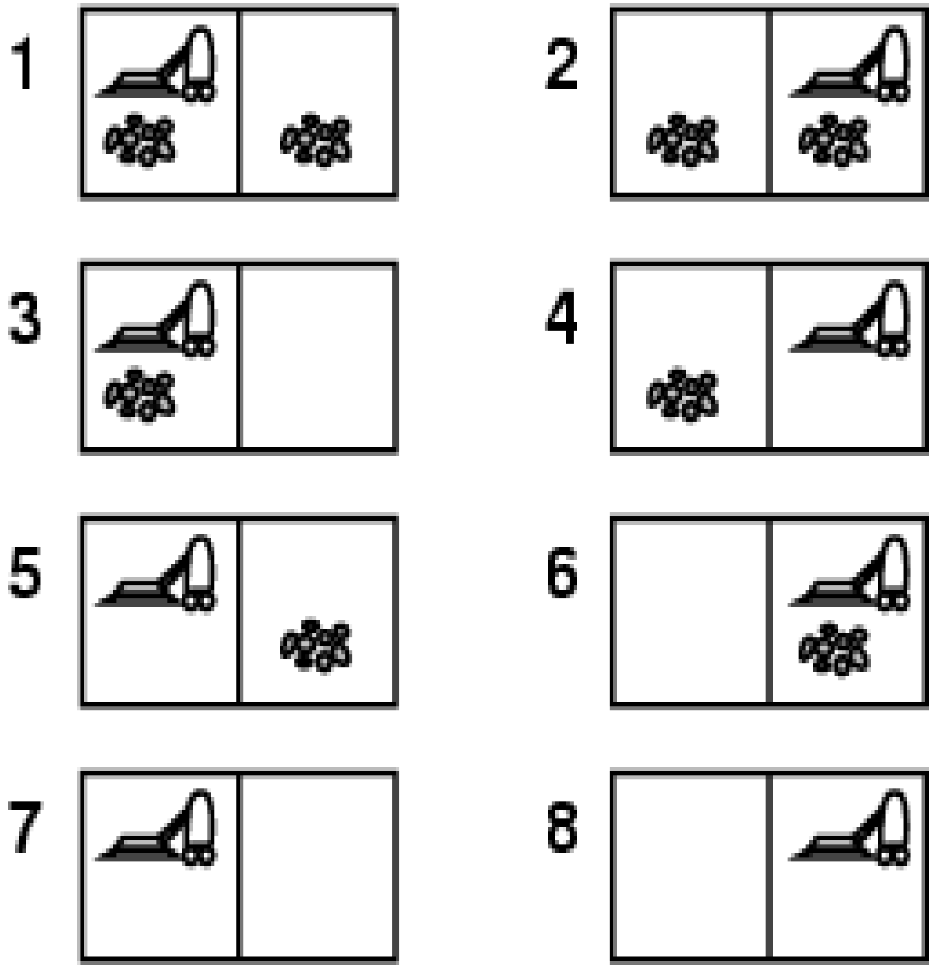
Problem types

- Deterministic, fully observable → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable → sensor less problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable → contingency problem
  - percepts provide new information about current state
  - often interleave} search, execution
- Unknown state space → exploration problem

Example: vacuum world

- Deterministic, fully observable
  → single-state problem
- Agent knows exactly which state it will be in; solution is a sequence
- Single-state
- start in #5

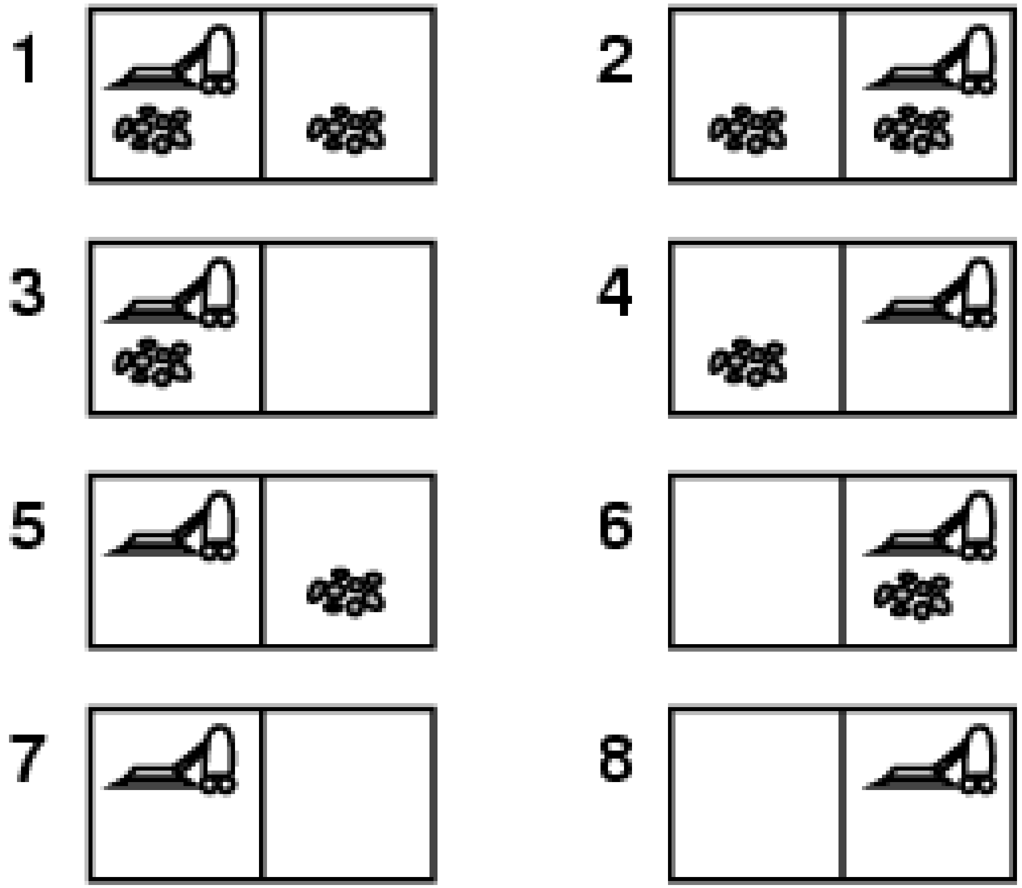
Solution?



Example: vacuum world

- Non-observable → sensor less problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence

- Sensorless
- start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution?

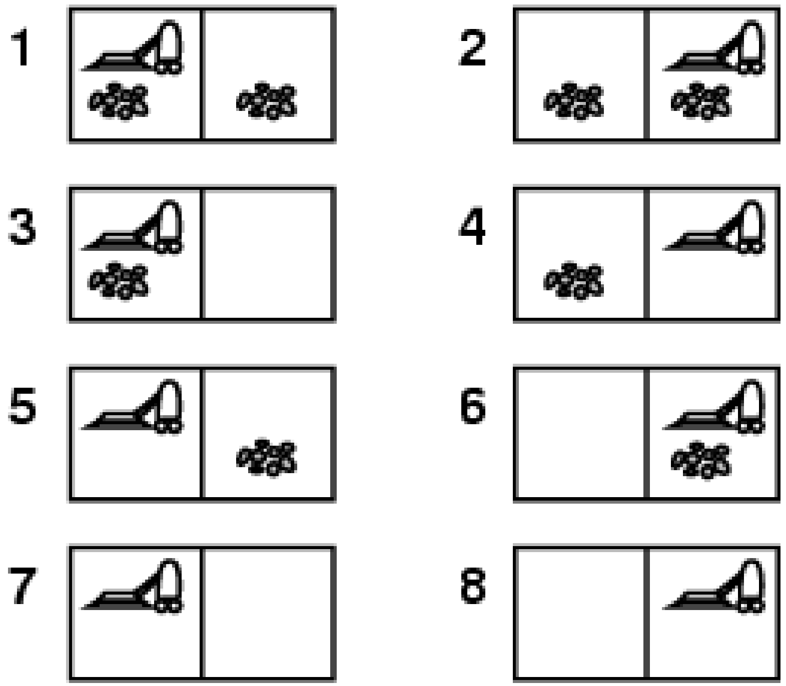


Example: vacuum world

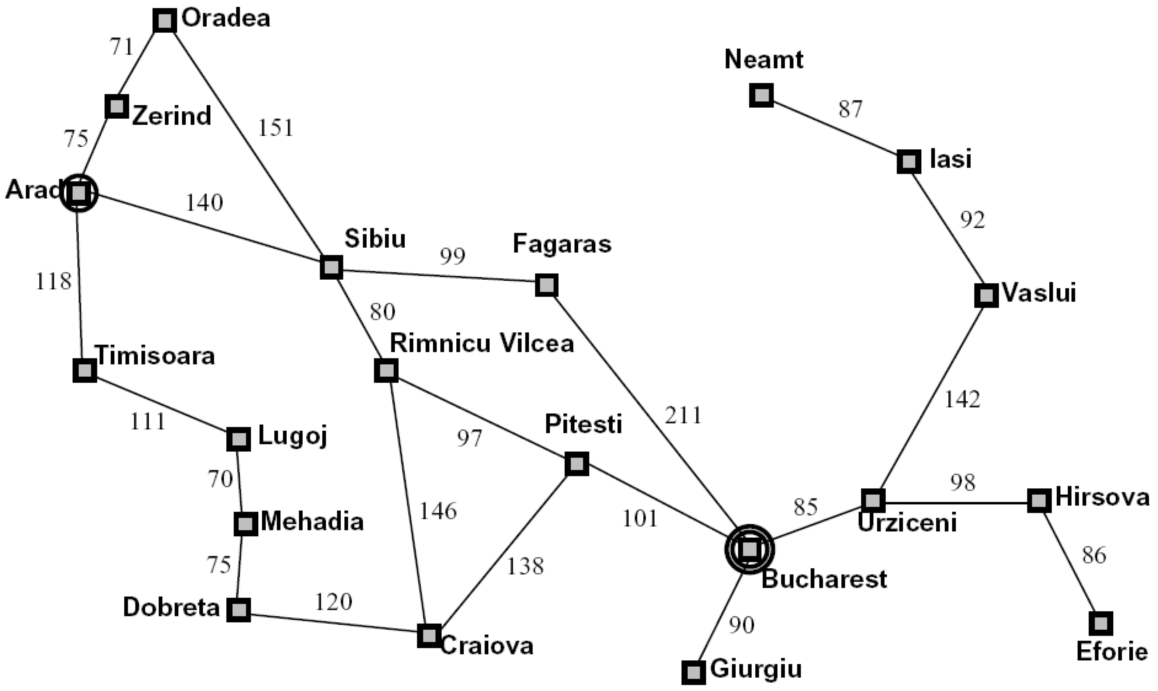
- ❖ Nondeterministic and/or partially observable → contingency problem
  - ➢ percepts provide new information about current state
  - ➢ often interleave search, execution

- ❖ Contingency
  - ➢ Nondeterministic: *Suck* may dirty a clean carpet
  - ➢ Partially observable: location, dirt at current location.
  - ➢ Percept: *[L, Clean],* i.e., start in #5 or #7

Solution?



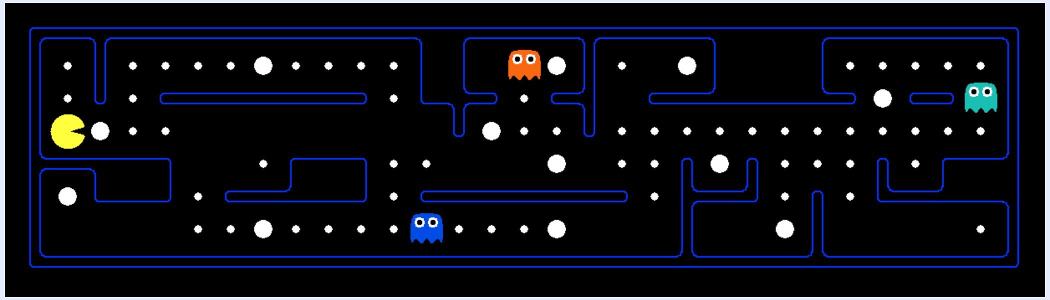
Example: Traveling in Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

What's in a State Space?

The world state includes every last detail of the environment



A search state keeps only the details needed for planning (abstraction)

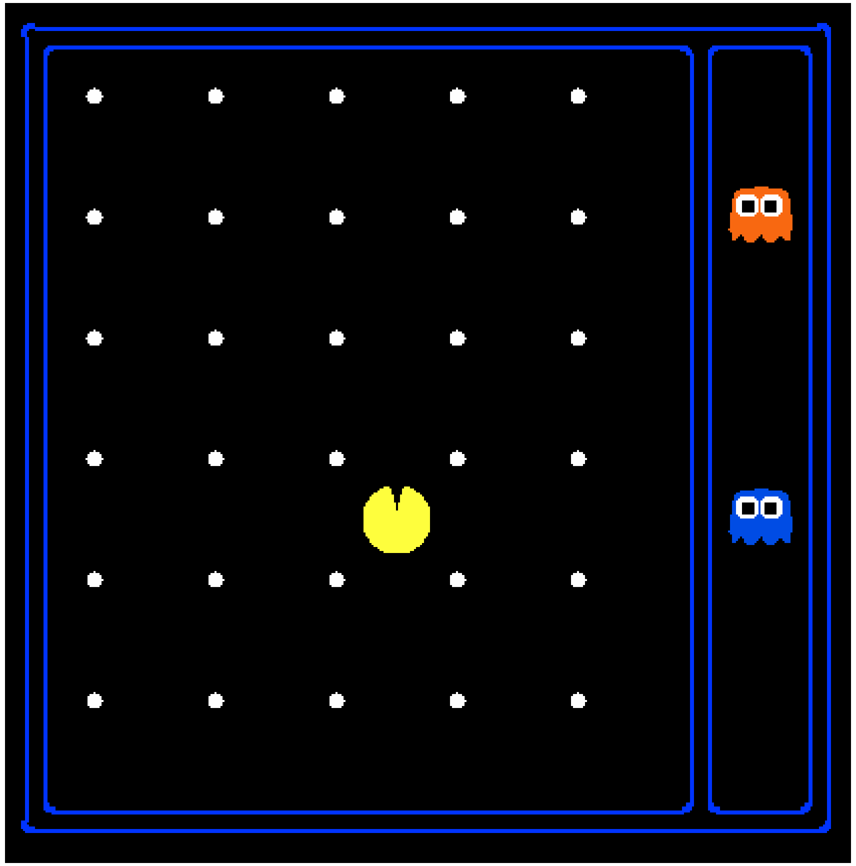
❖ Problem: Pathing
- States: (x,y) location
- Actions: NSEW
- Successor: update location only
- Goal test: is (x,y)=END

❖ Problem: Eat-All-Dots
- States: {(x,y), dot booleans}
- Actions: NSEW
- Successor: update location and possibly a dot boolean
- Goal test: dots all false

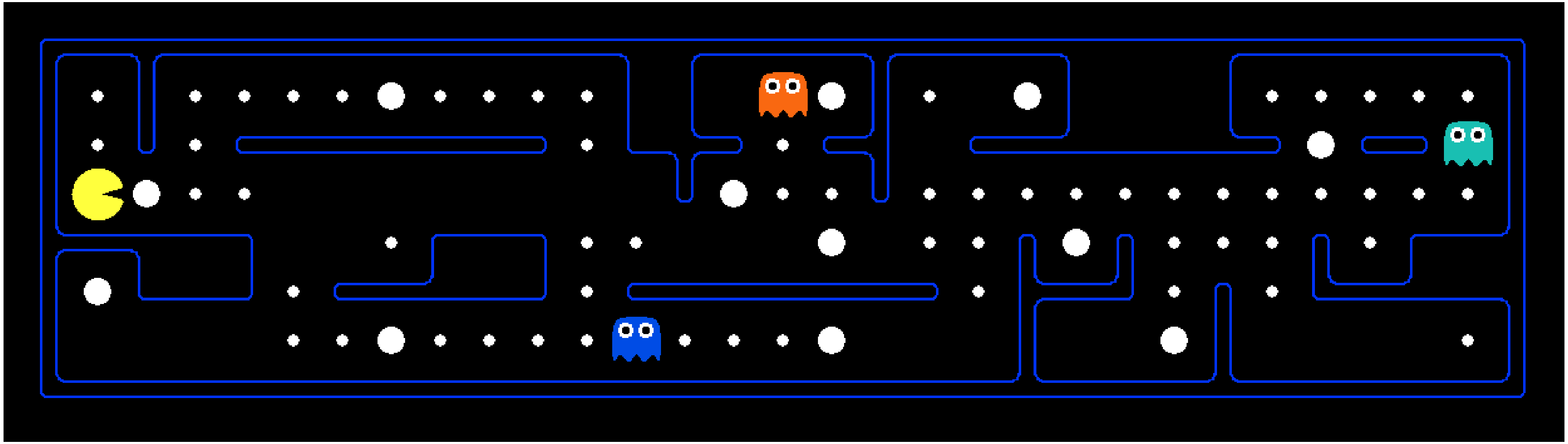
State Space Sizes?

- World state:
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW

- How many
  - World states?
  - States for pathing?
  - States for eat-all-dots?



Quiz: Safe Passage



- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?

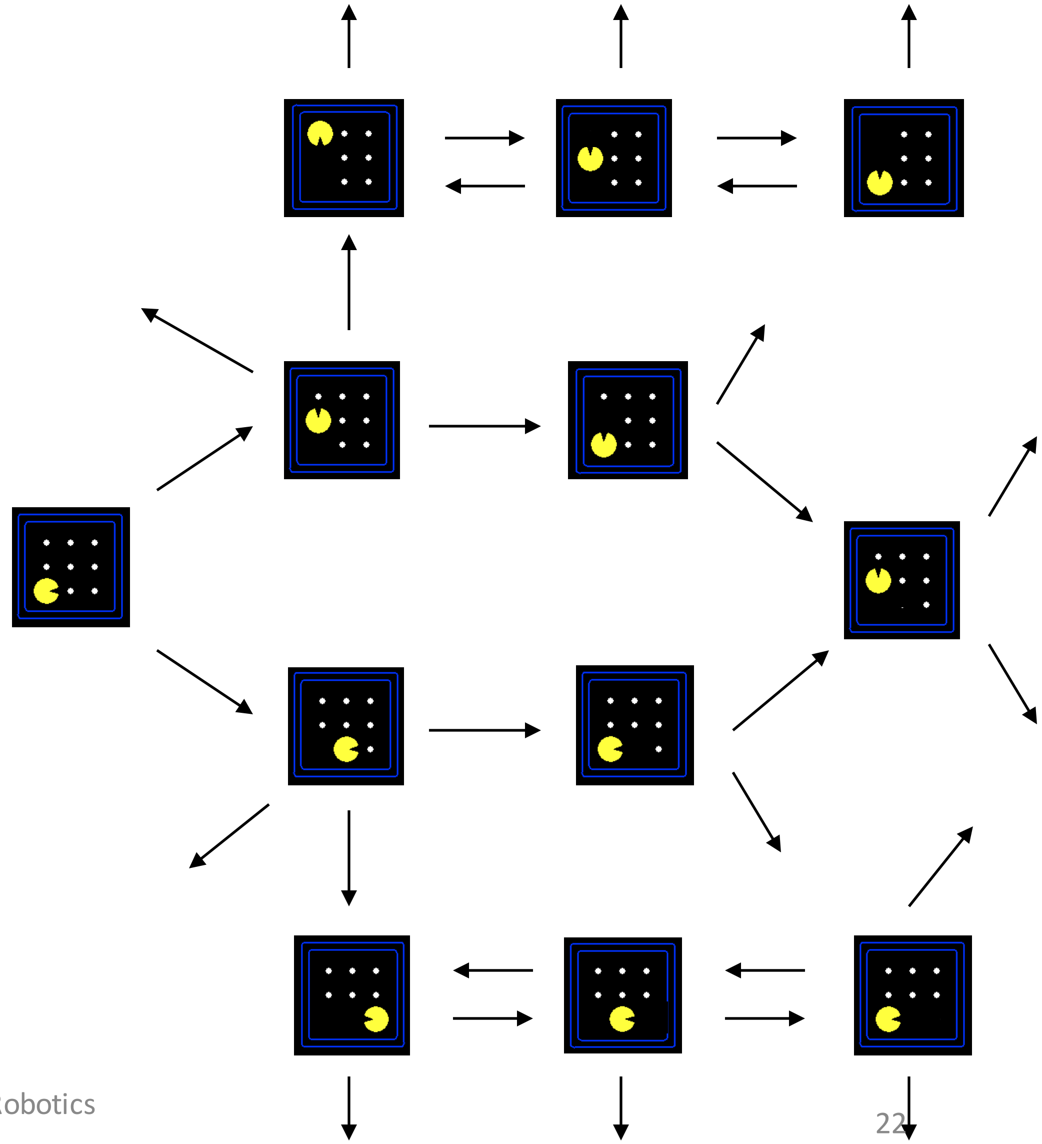
State Space Graphs and Search Trees

State Space Graphs

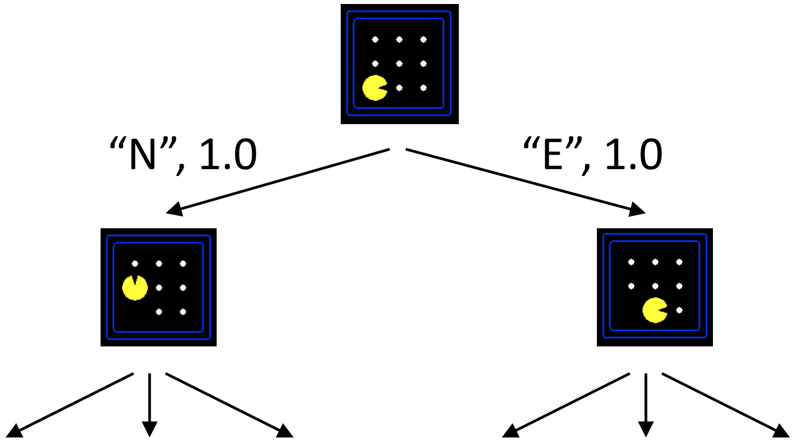
- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)

- In a state space graph, each state occurs only once!

- We can rarely build this full graph in memory (it's too big), but it's a useful idea



Search Trees



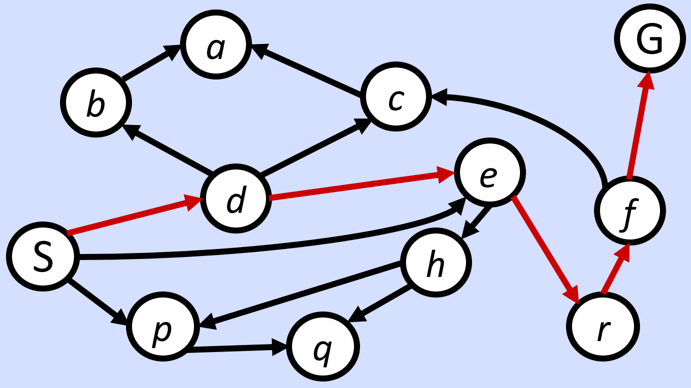
This is now / start

Possible futures

- A search tree:
  - A “what if” tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree

State Space Graphs vs. Search Trees

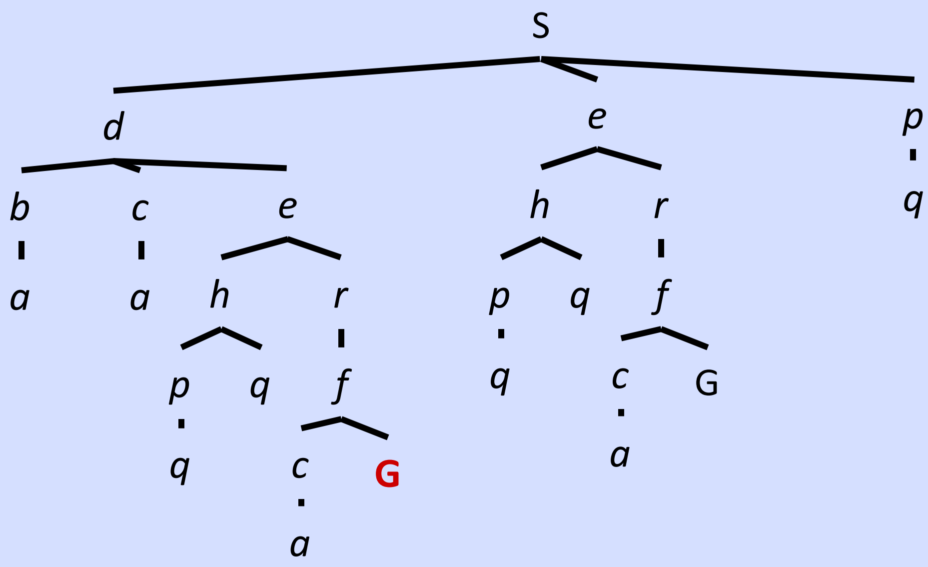
State Space Graph



Each NODE in in the search tree is an entire PATH in the state space graph.

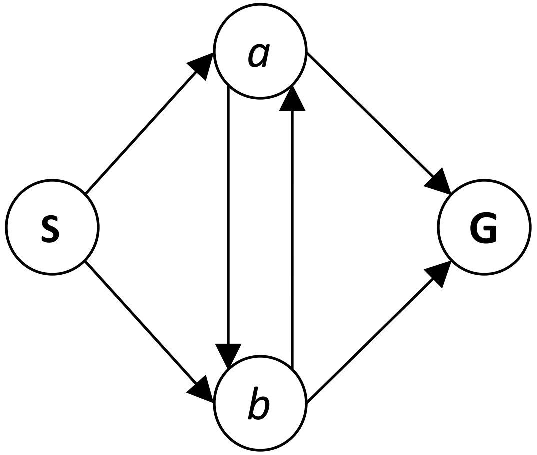
We construct both on demand – and we construct as little as possible.

Search Tree

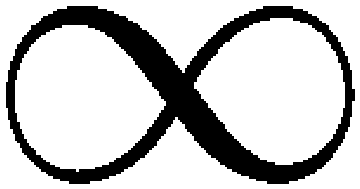


Quiz: State Space Graphs vs. Search Trees

Consider this 4-state graph:



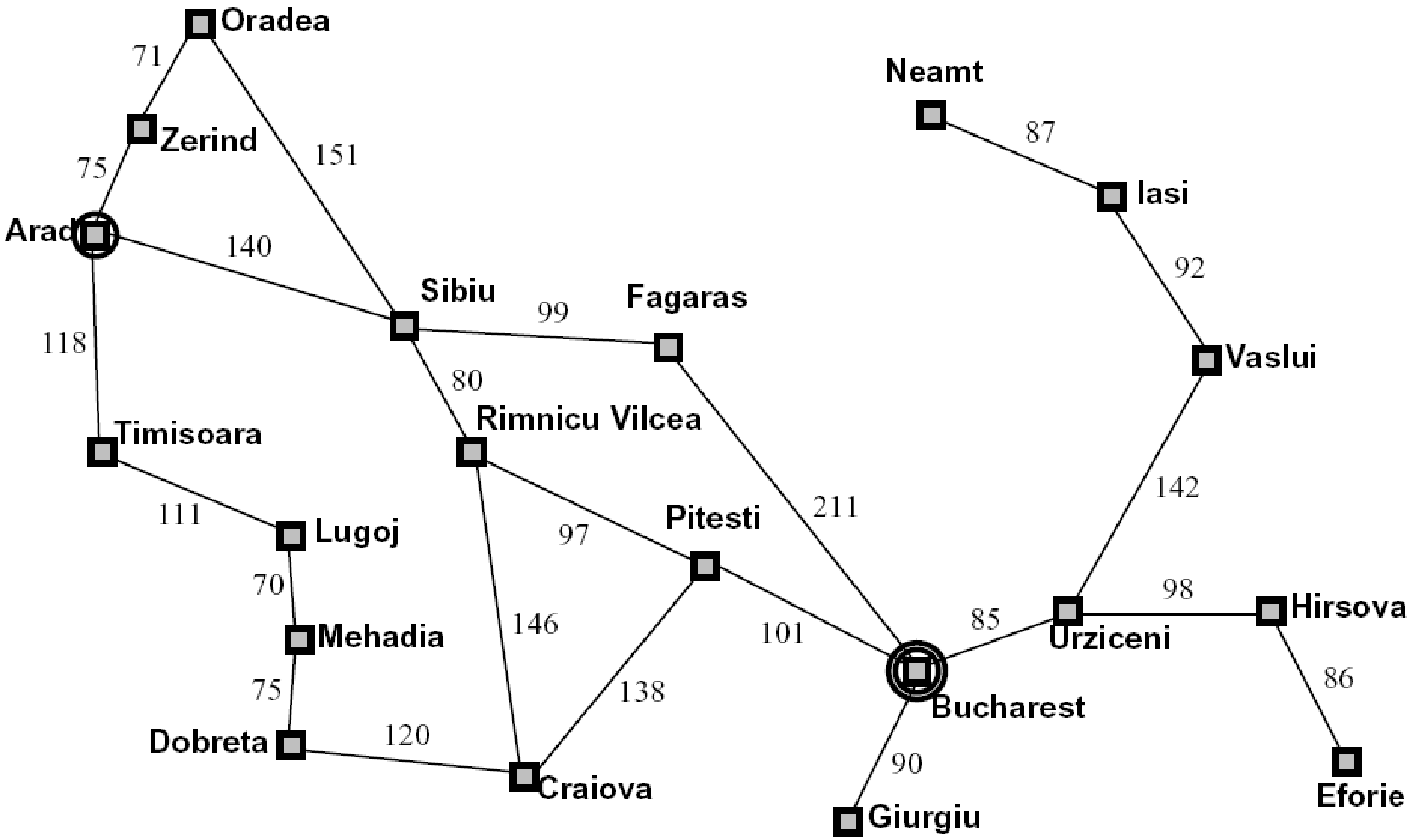
How big is its search tree (from S)?



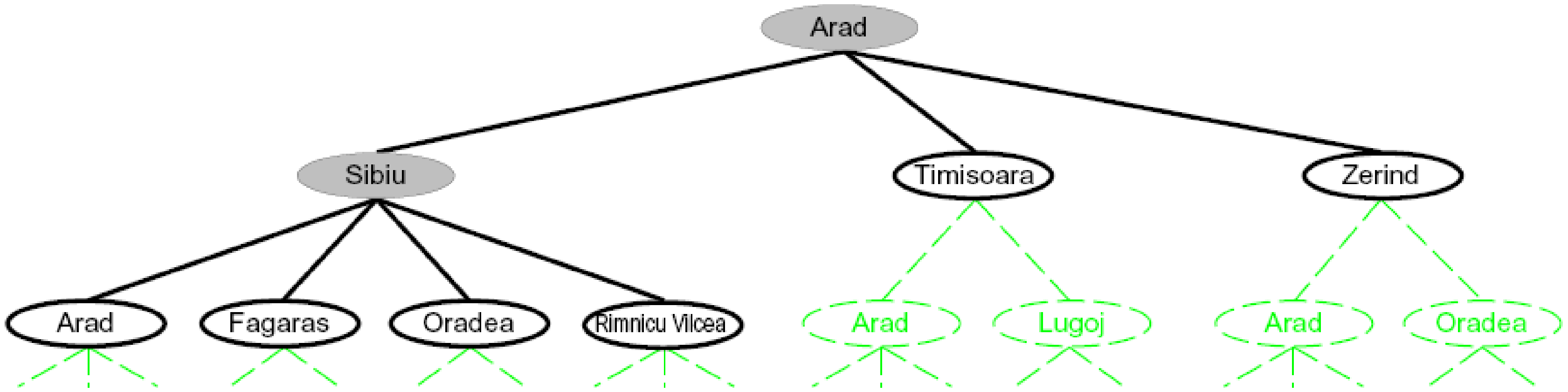
Important: Lots of repeated structure in the search tree!

Tree Search

Search Example: Romania



Searching with a Search Tree



- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a fringe of partial plans under consideration
  - Try to expand as few tree nodes as possible

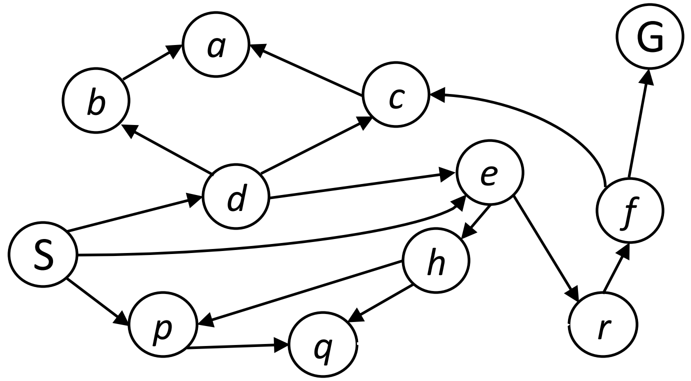
General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Important ideas:
    - Fringe
    - Expansion
    - Exploration strategy

- Main question: which fringe nodes to explore?

Example: Tree Search

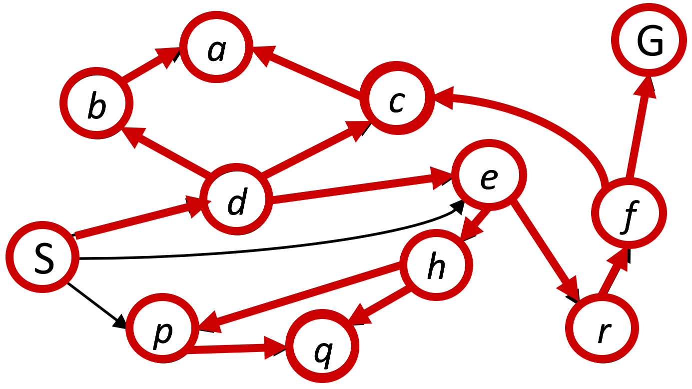


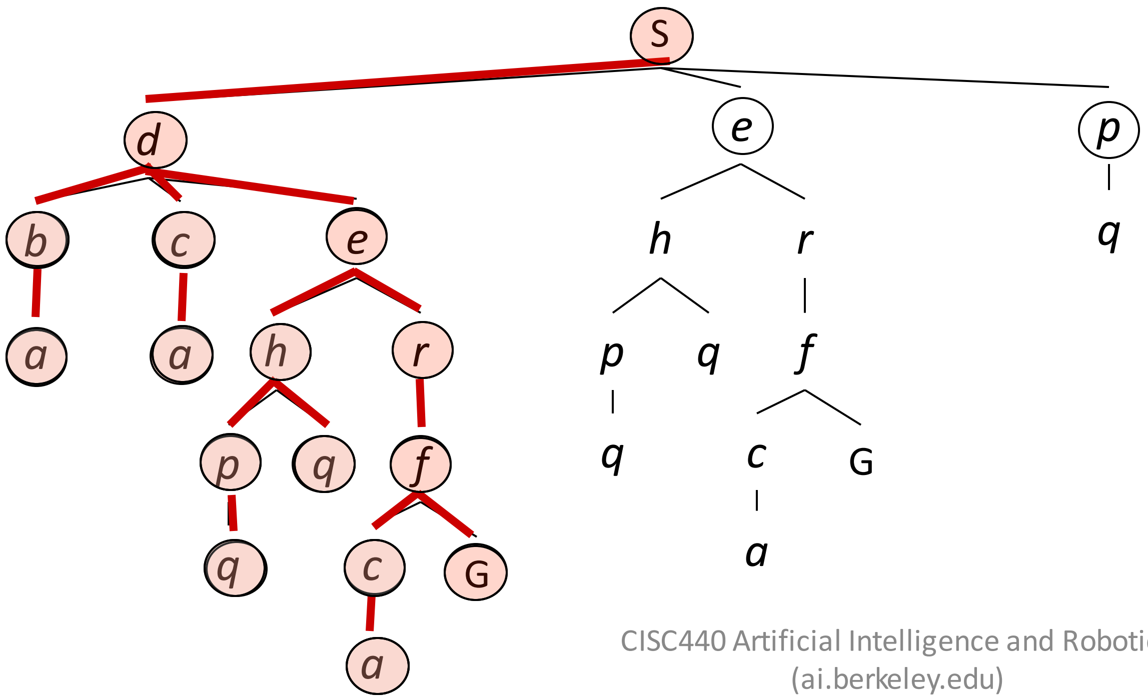
Depth-First Search

Depth-First Search

Strategy: expand a deepest node first

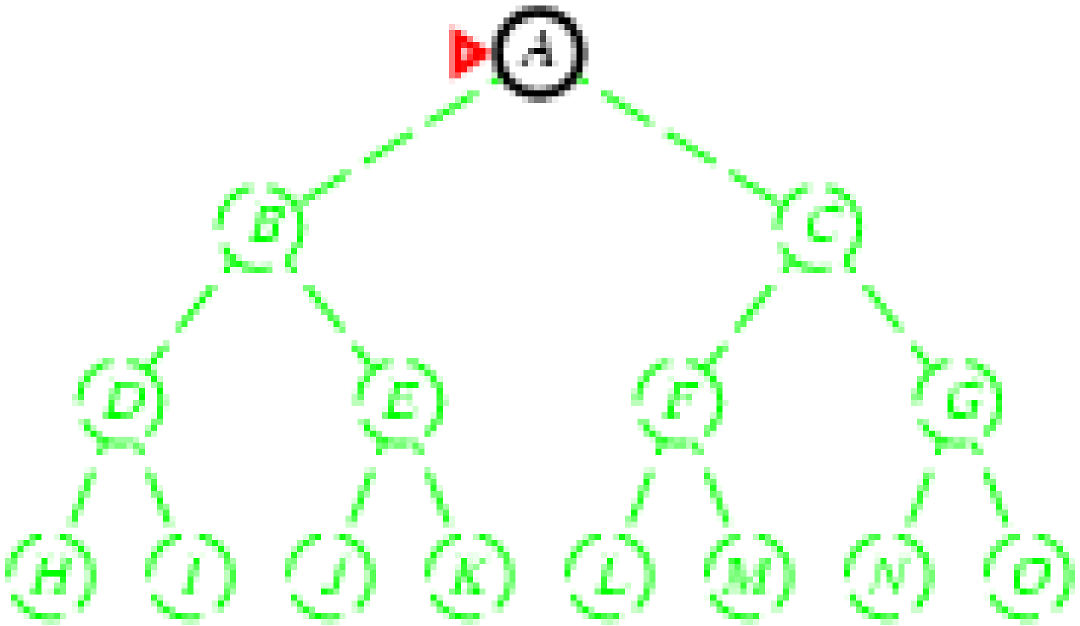
Implementation: Fringe is a LIFO stack





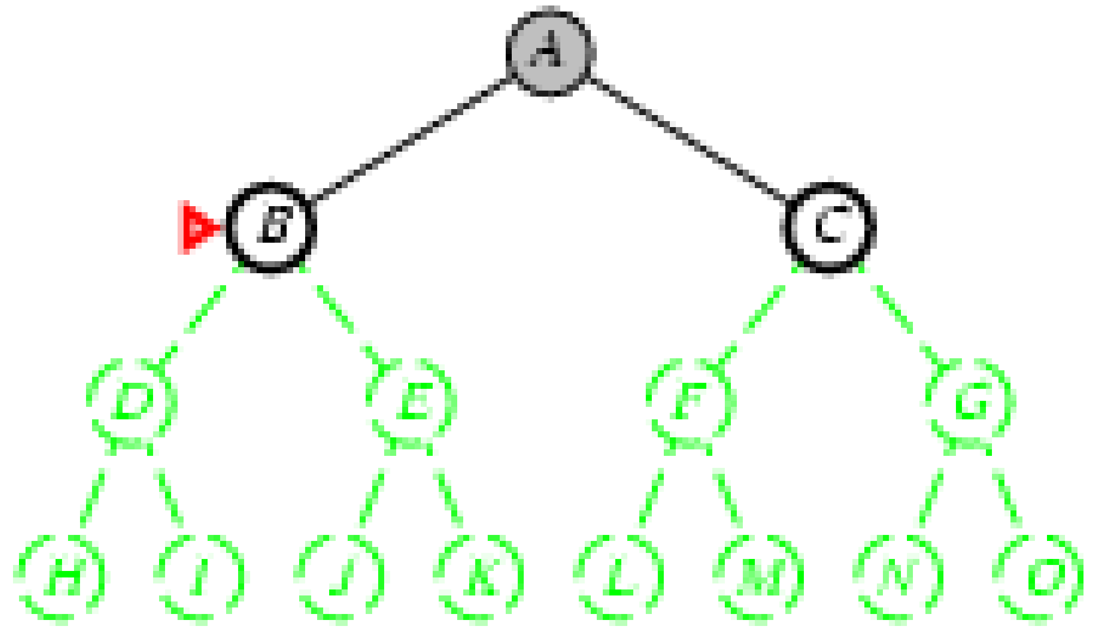
Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

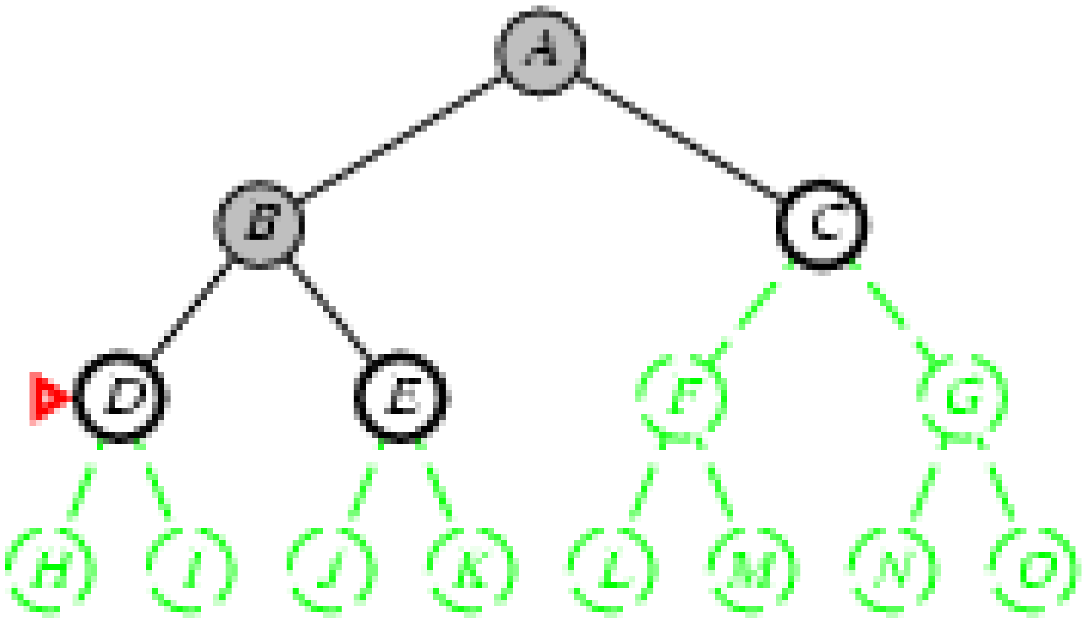


Depth-first search

❖ Expand deepest unexpanded node

❖ Implementation:

➢ *fringe* = LIFO queue, i.e., put successors at front

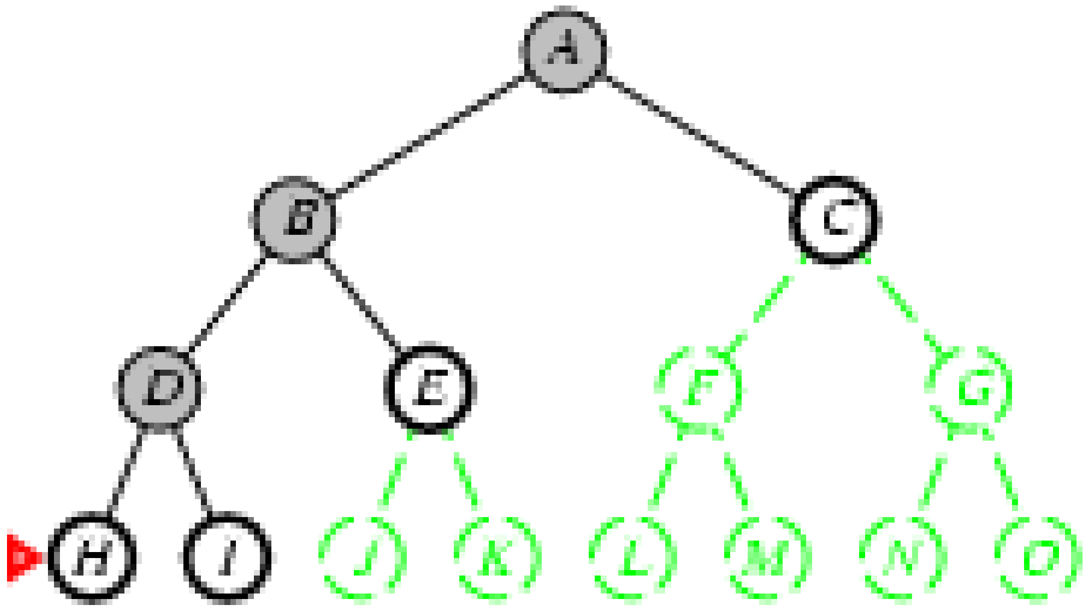


Depth-first search

❖ Expand deepest unexpanded node

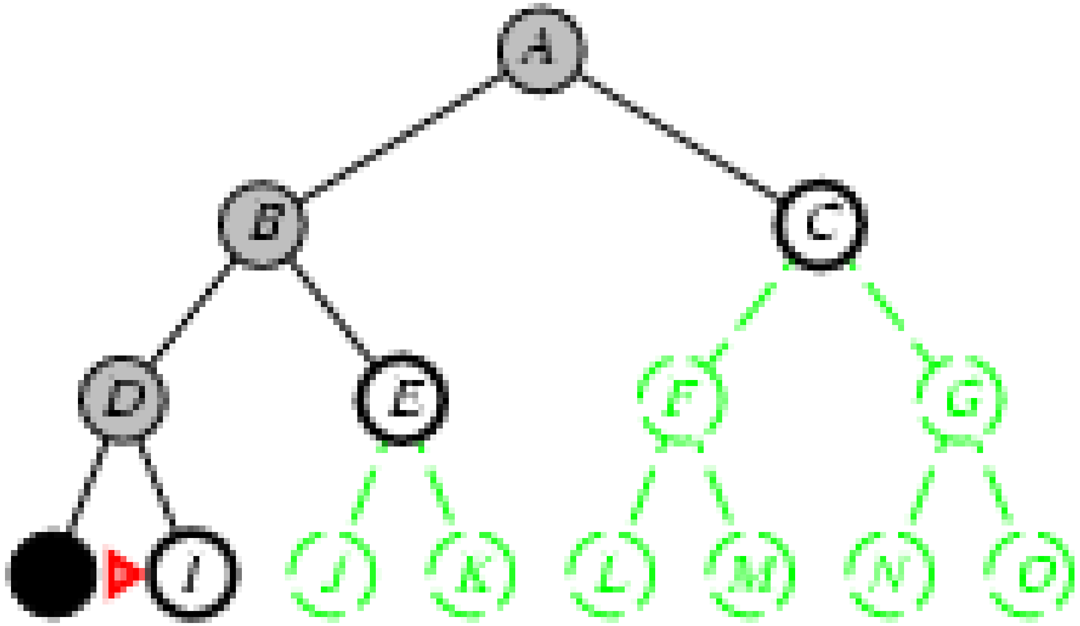
❖ Implementation:

➢ *fringe* = LIFO queue, i.e., put successors at front



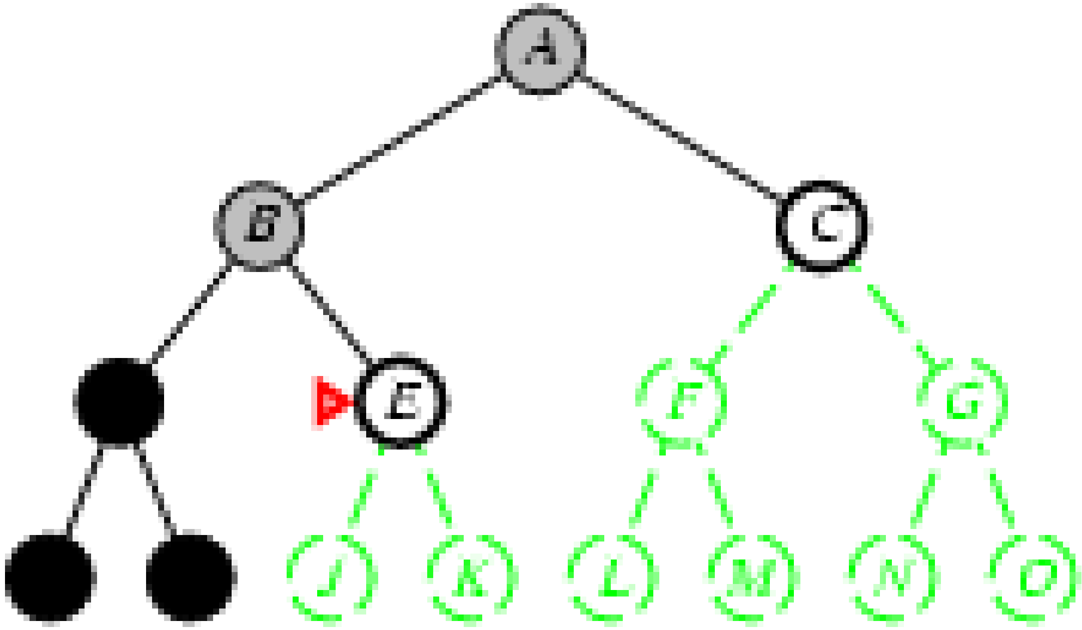
Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



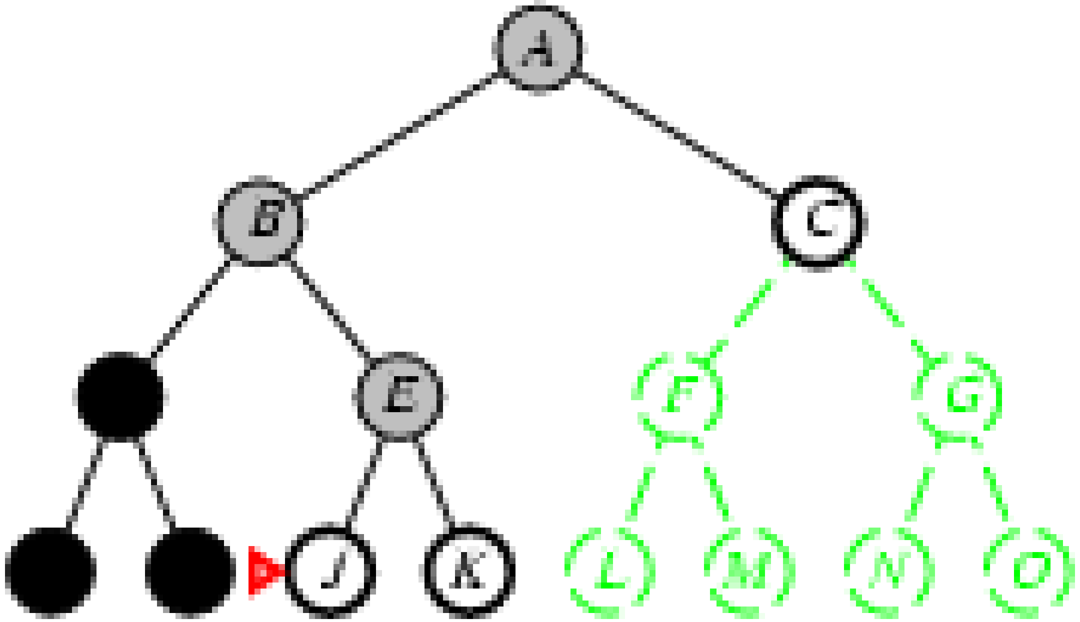
Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



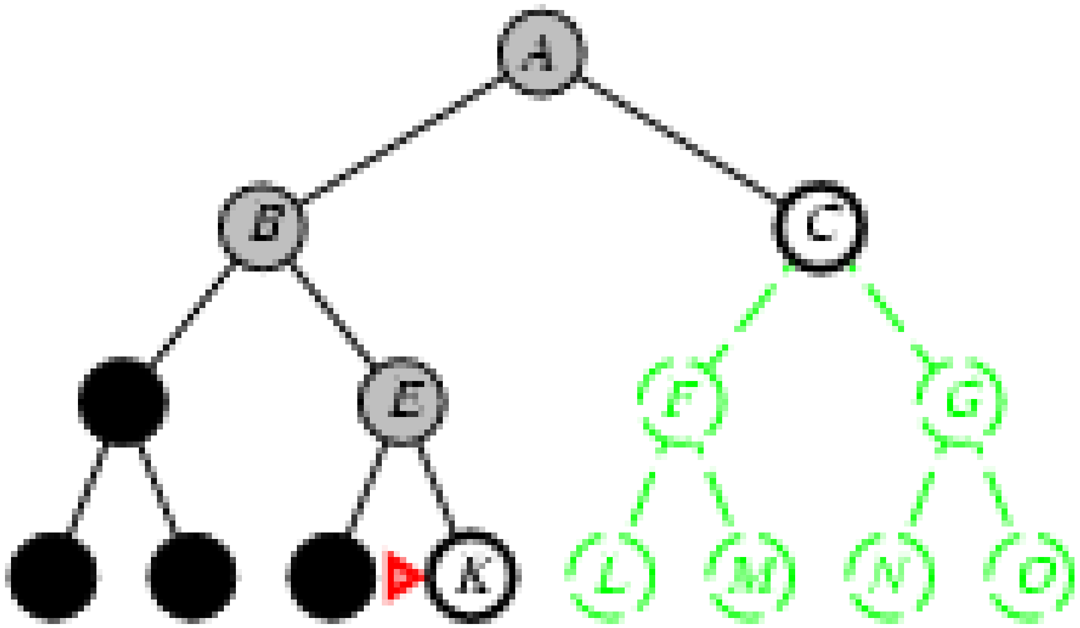
Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



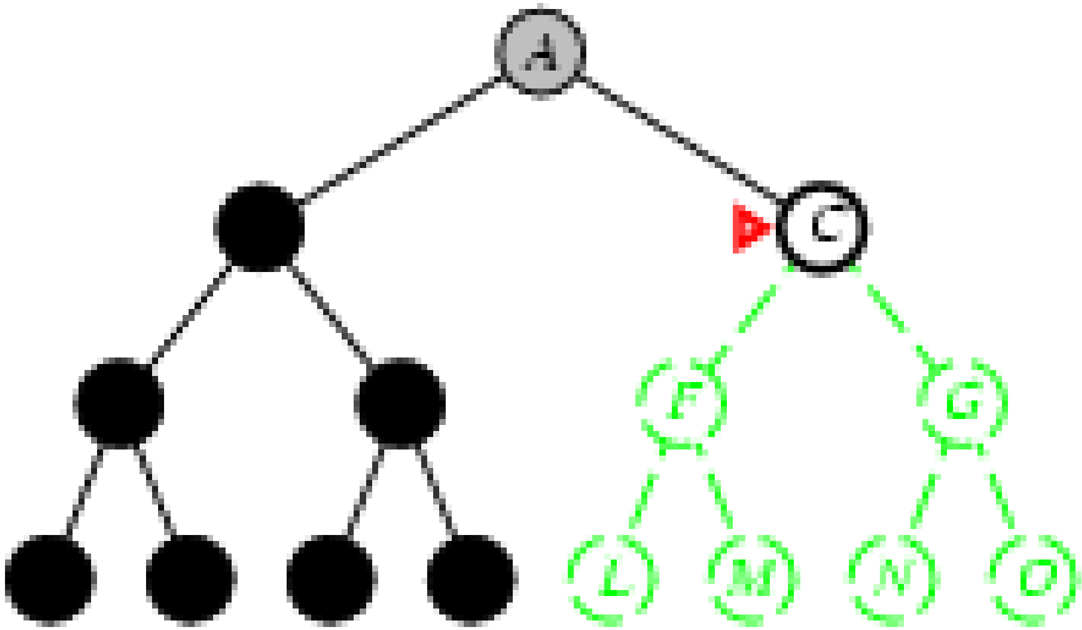
Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

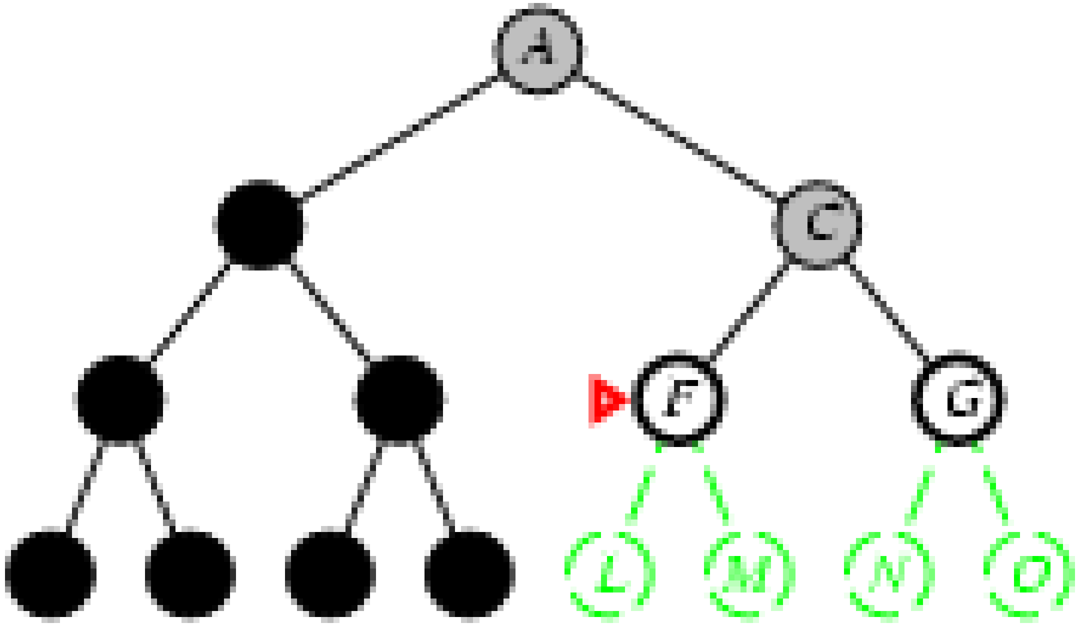


Depth-first search

❖ Expand deepest unexpanded node

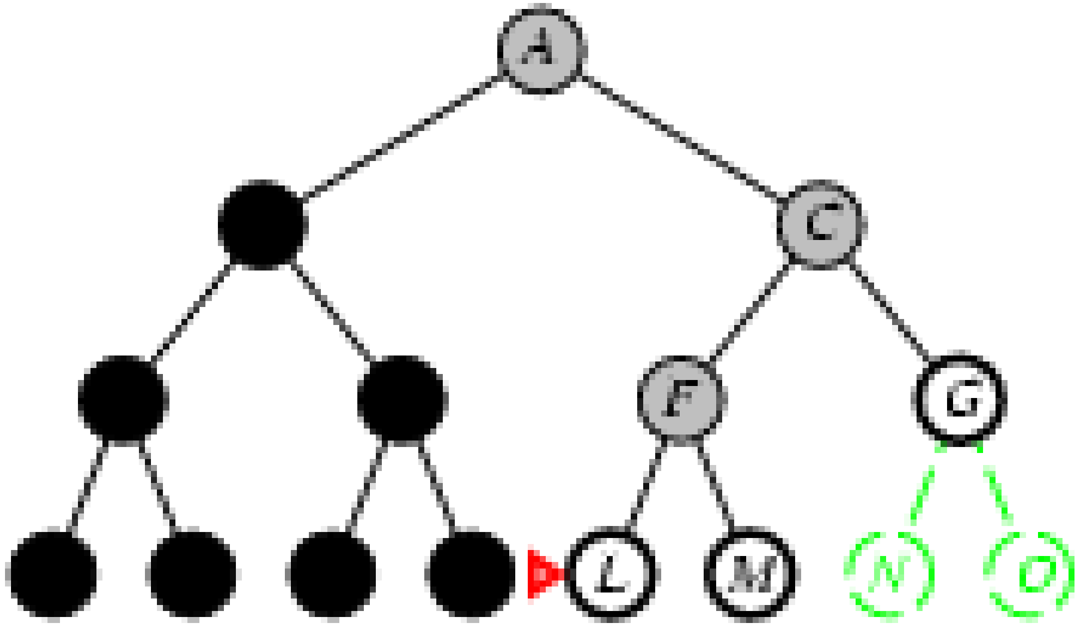
❖ Implementation:

➢ *fringe* = LIFO queue, i.e., put successors at front



Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

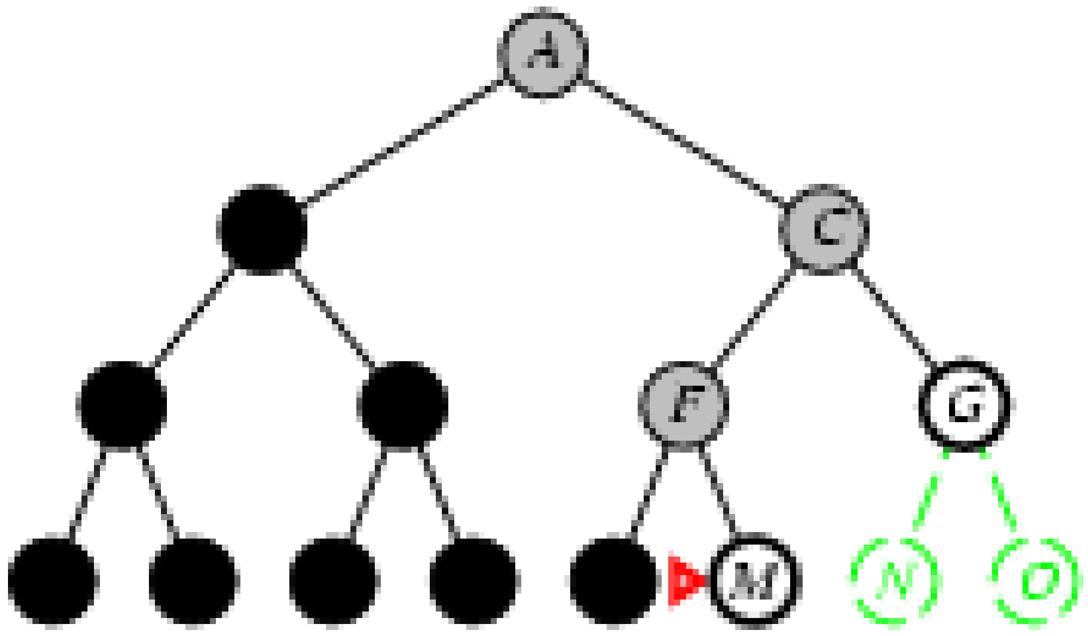


Depth-first search

❖ Expand deepest unexpanded node

❖ Implementation:

➢ *fringe* = LIFO queue, i.e., put successors at front



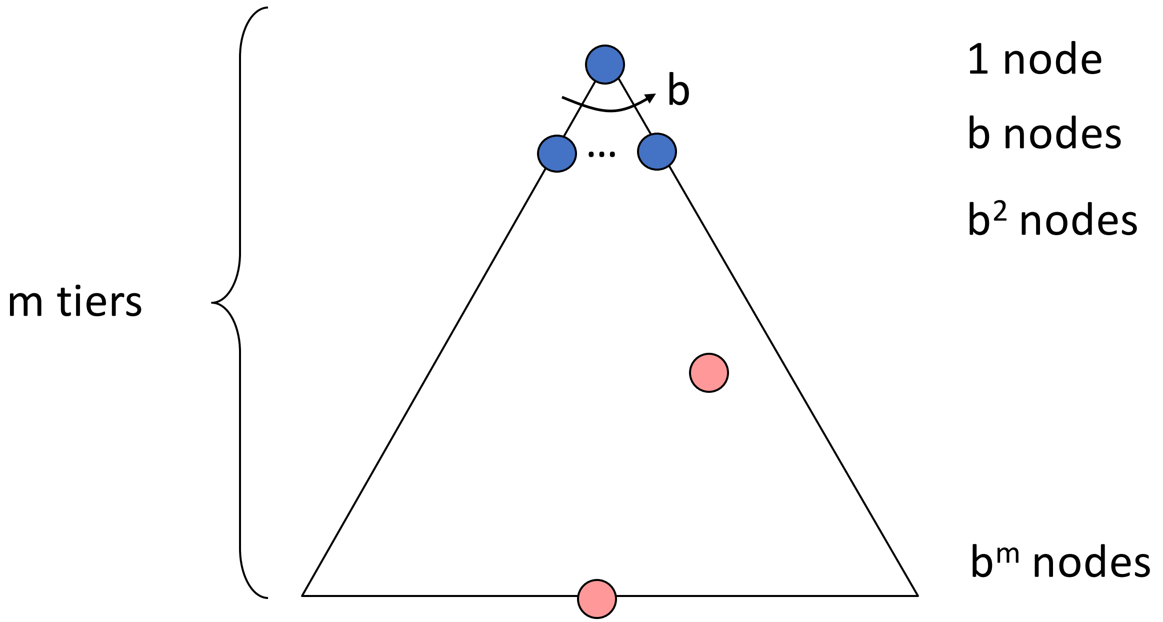
Search Algorithm Properties

Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths

- Number of nodes in entire tree?
  - $1 + b + b^2 + ... b^m = O(b^m)$



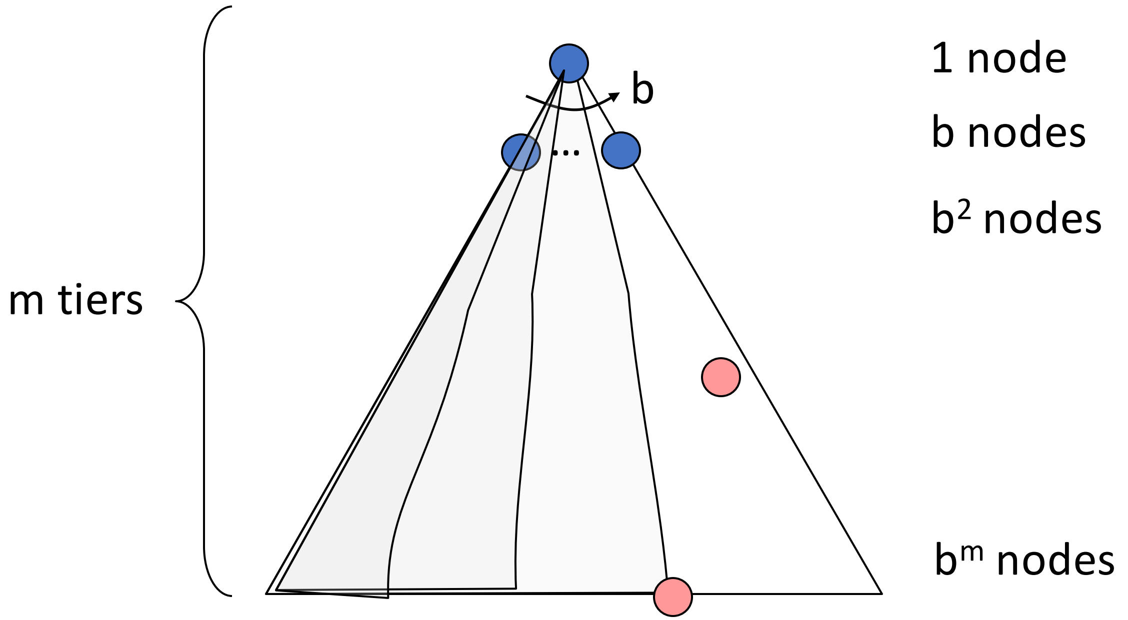
Depth-First Search (DFS) Properties

- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$

- How much space does the fringe take?
  - Only has siblings on path to root, so $O(bm)$

- Is it complete?
  - m could be infinite, so only if we prevent cycles (more later)

- Is it optimal?
  - No, it finds the "leftmost" solution, regardless of depth or cost

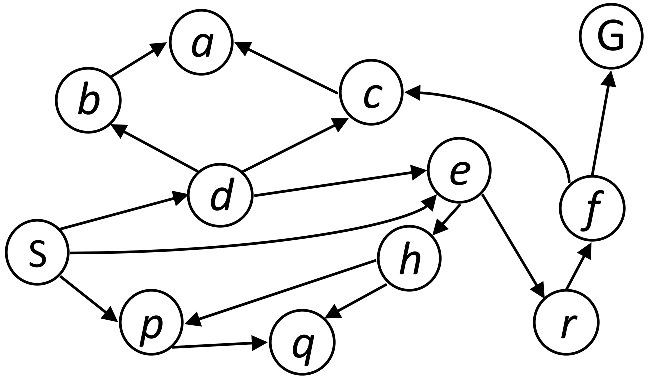


Breadth-First Search

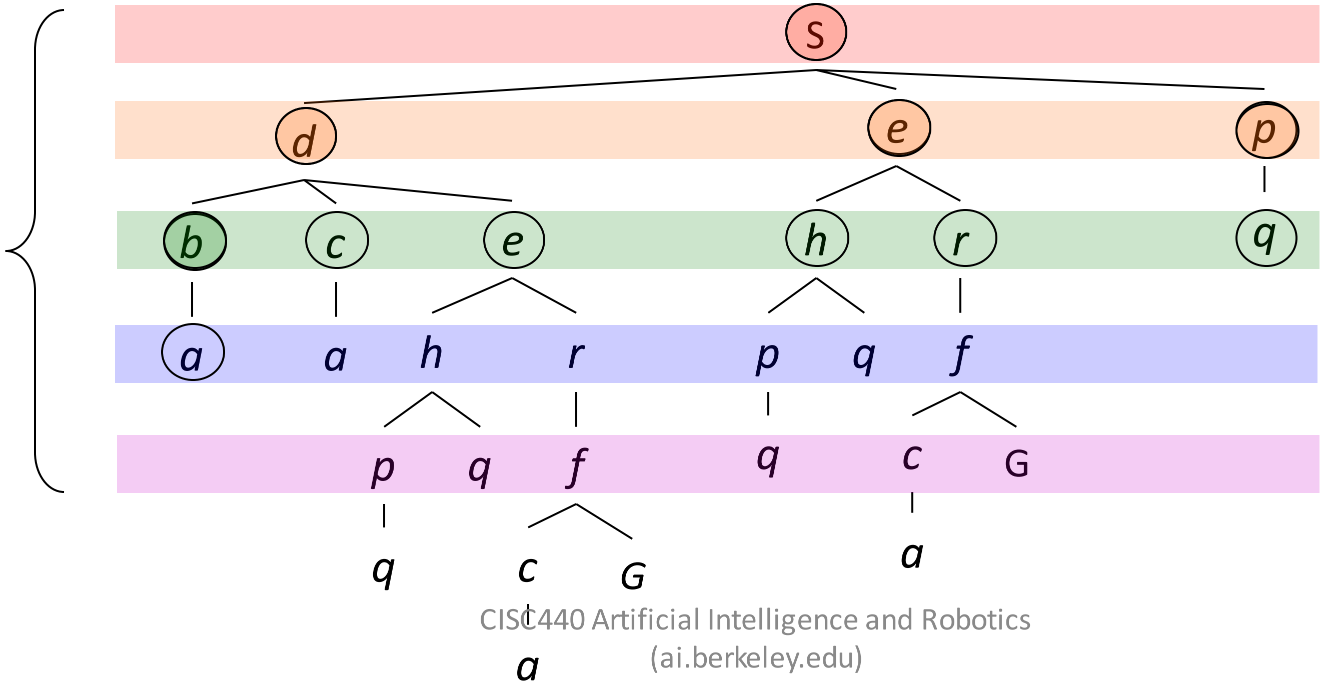
Breadth-First Search

Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue

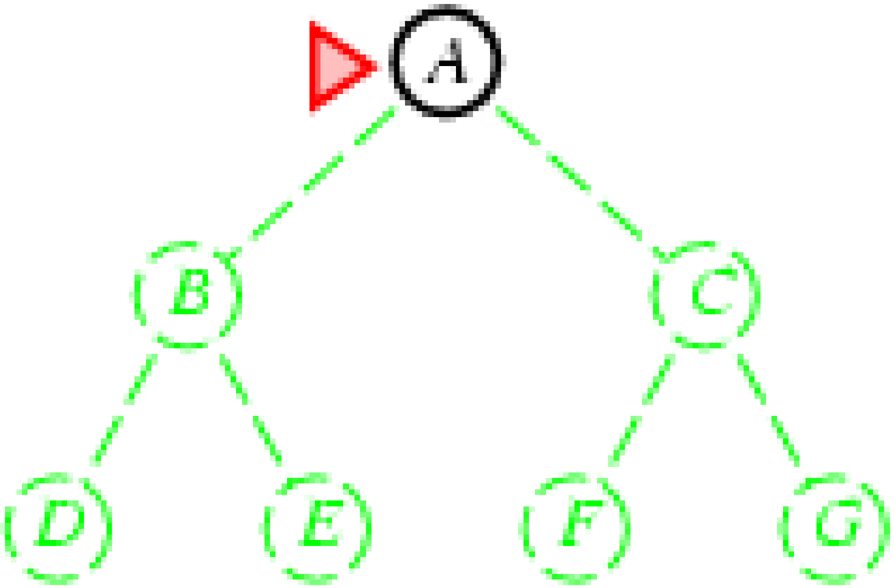


Search Tiers



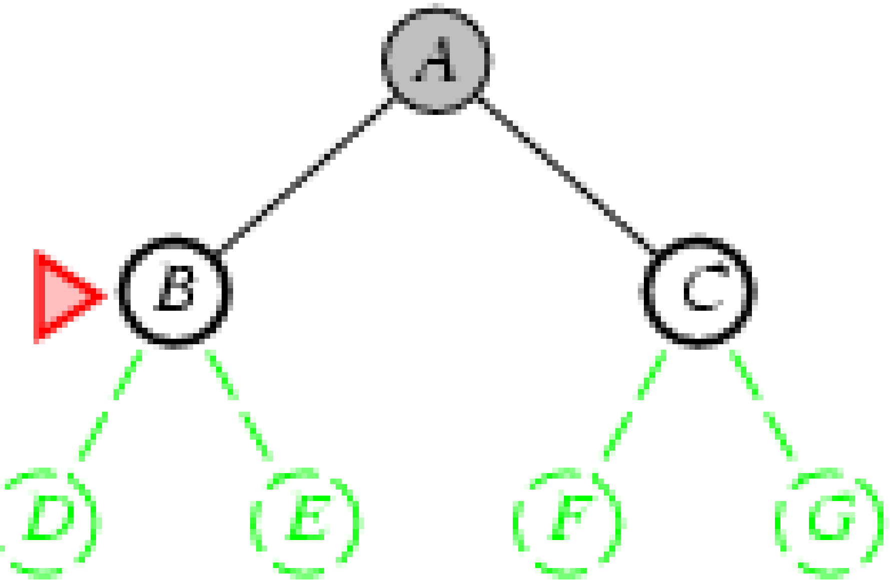
Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



Breadth-first search

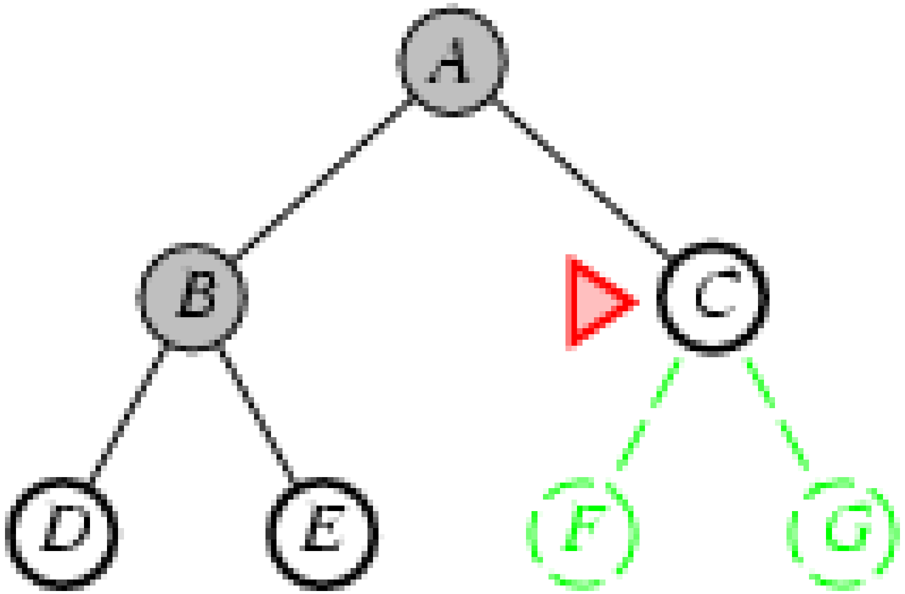
- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



(ai.berkeley.edu)

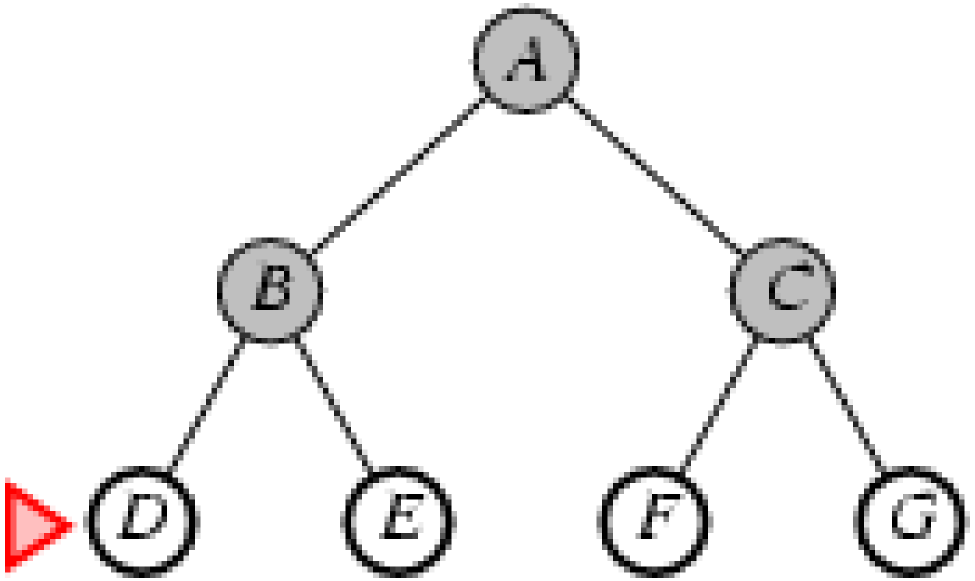
Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



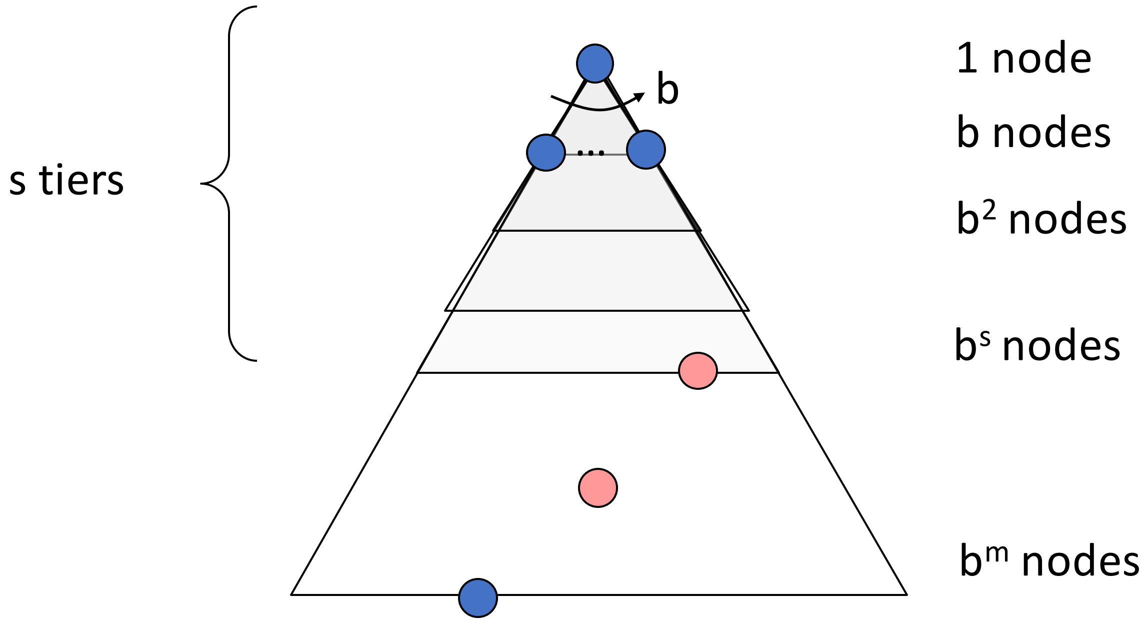
Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

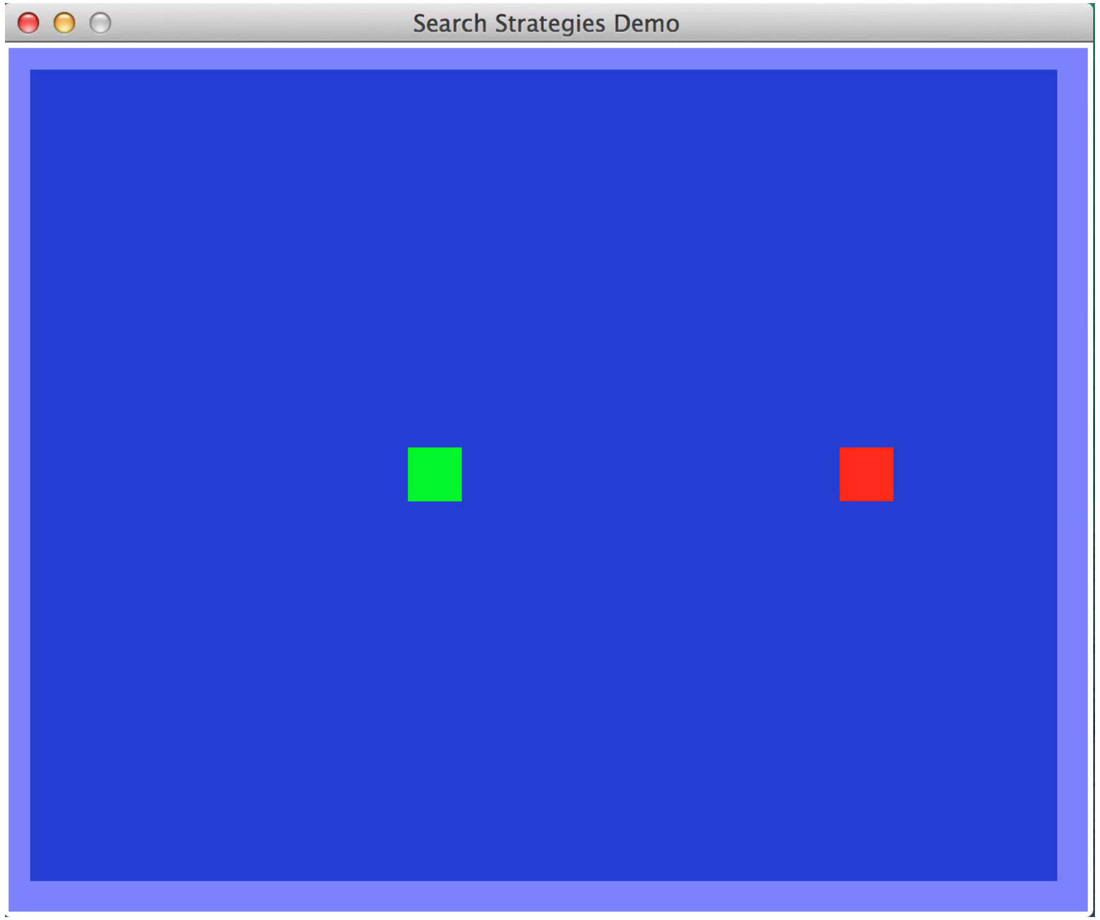


Breadth-First Search (BFS) Properties

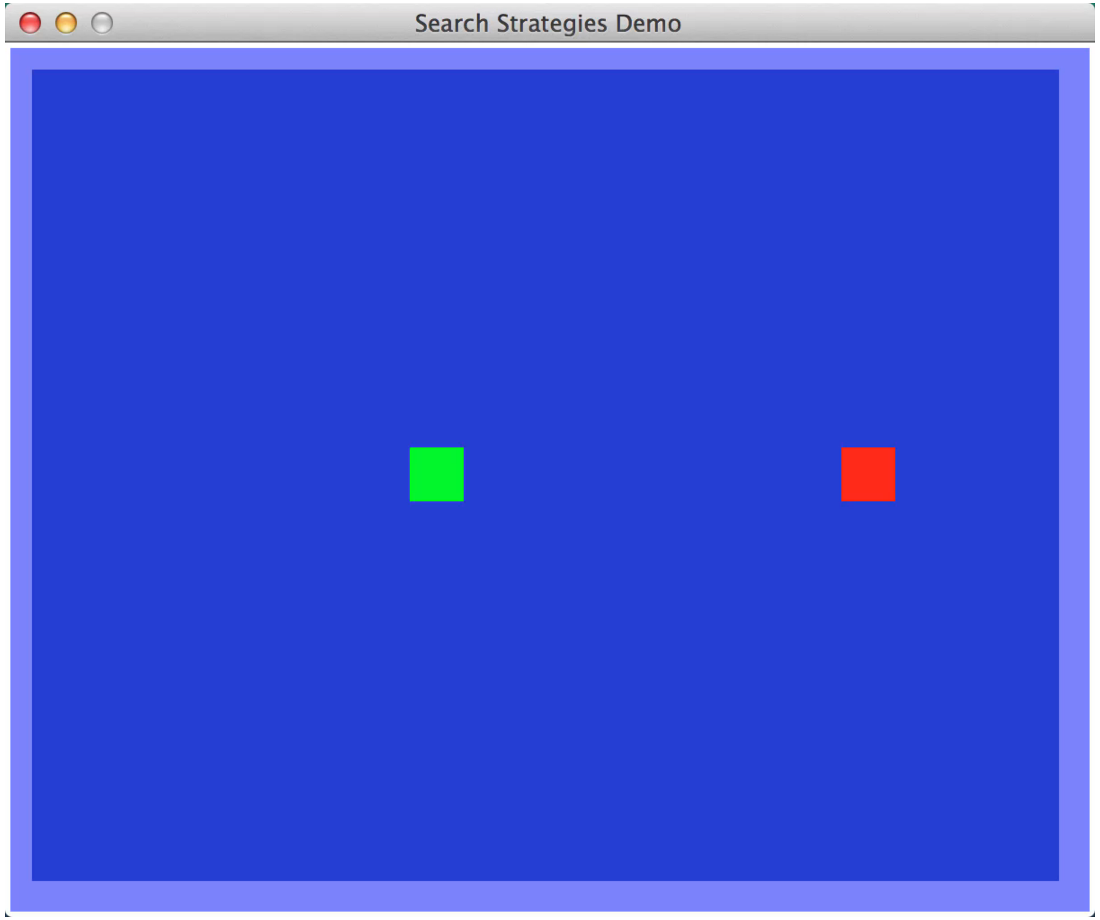
- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s
  - Search takes time $O(b^s)$
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^s)$
- Is it complete?
  - s must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



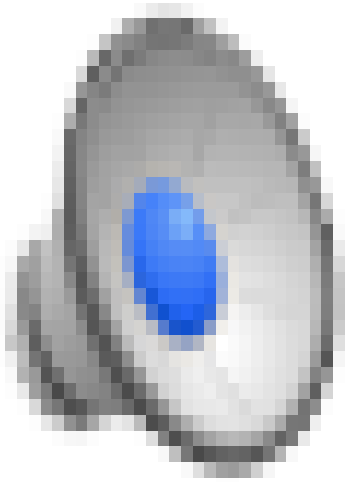
Video of Demo Target DFS/BFS



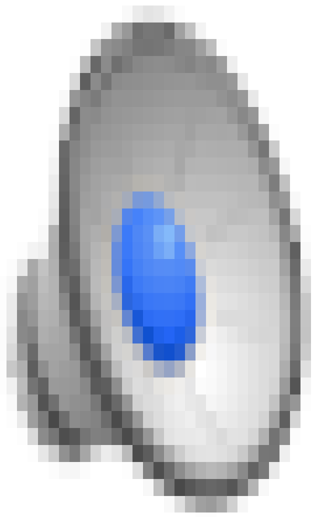
Video of Demo Target DFS/BFS



Video of Demo Maze Water DFS/BFS (part 1)



Video of Demo Maze Water DFS/BFS (part 2)



DFS vs BFS

- When will BFS outperform DFS?
- When will DFS outperform BFS?

- Shallow?
- Deepness w.r.t goal?
- Space?
- Provide shortest path?

Depth-limited search

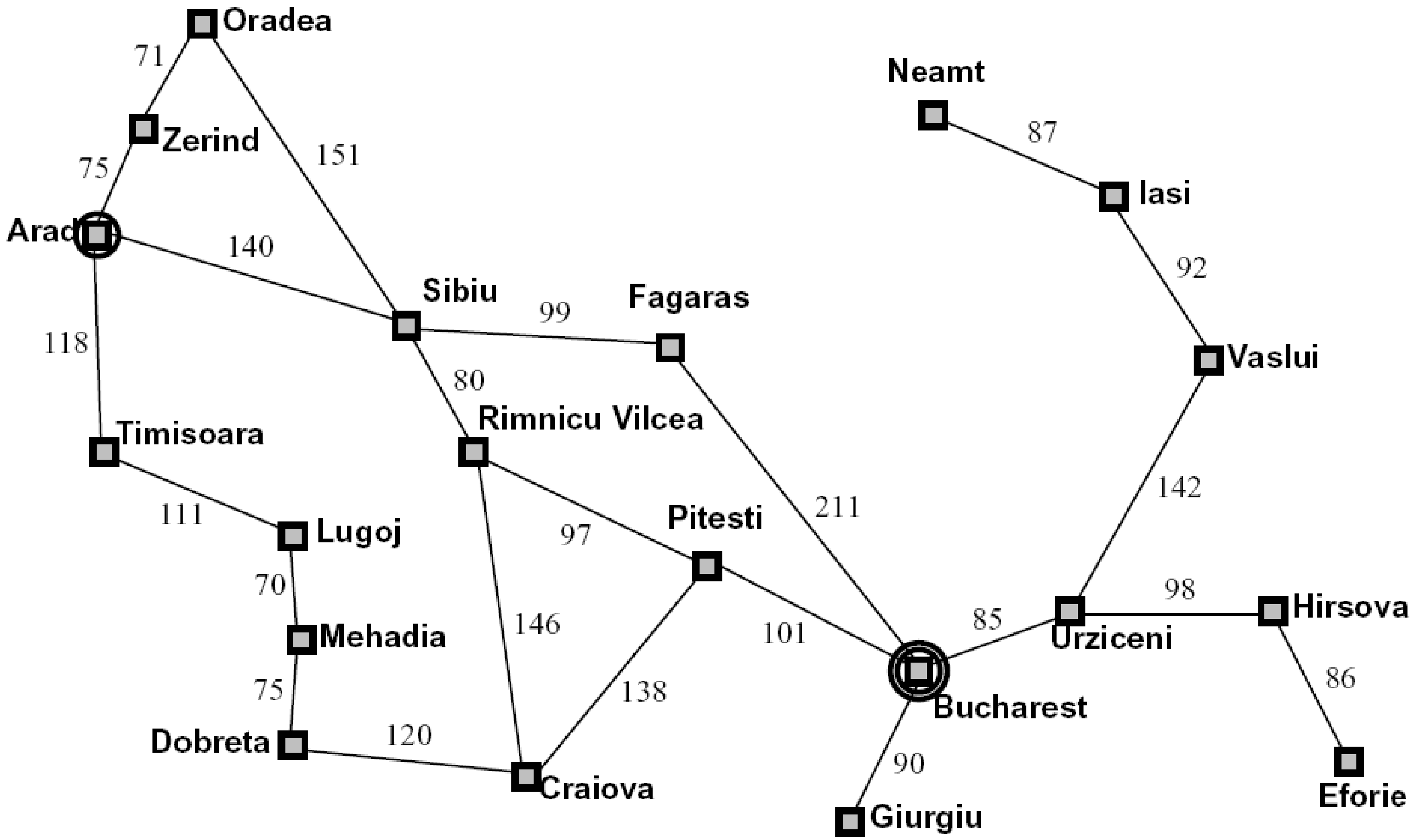
= depth-first search with depth limit *l*,

i.e., nodes at depth *l* have no successors

❖ Recursive implementation:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

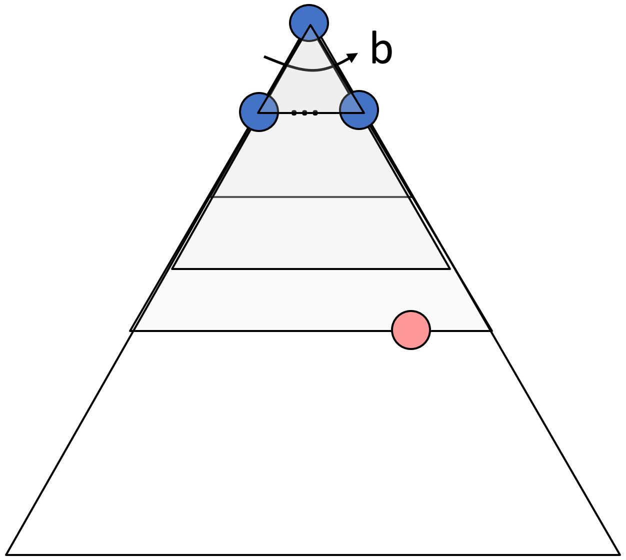
Example: Romania



Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution…
  - Run a DFS with depth limit 2. If no solution…
  - Run a DFS with depth limit 3. …..

- Isn't that wastefully redundant?
  - Generally, most work happens in the lowest level searched, so not so bad!

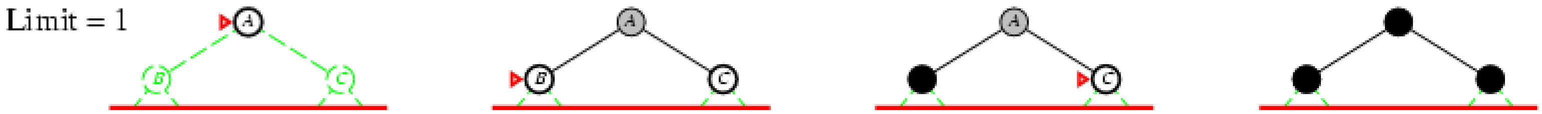


Iterative deepening search

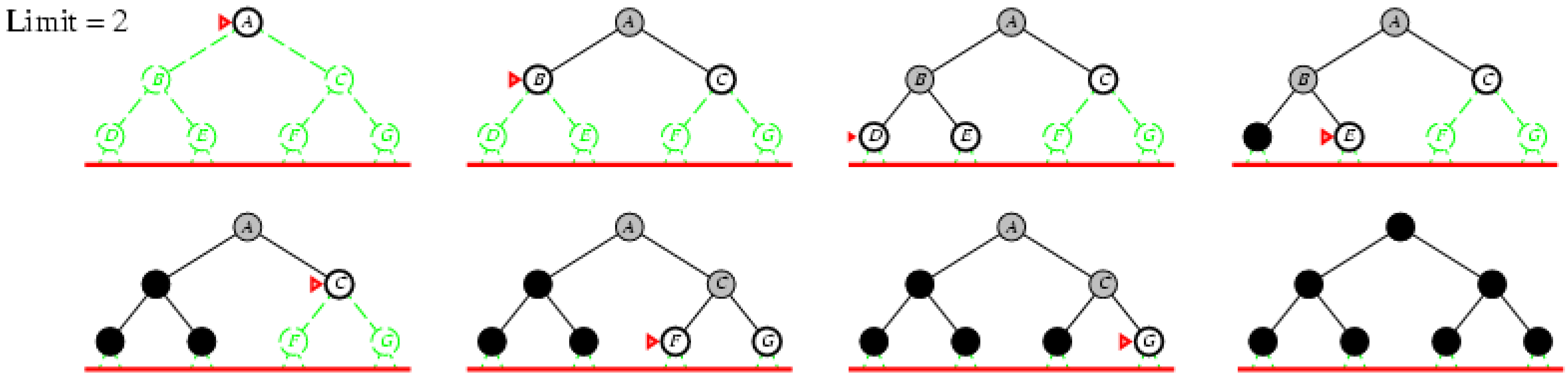
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or fail-
ure
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

Iterative deepening search *l* =0

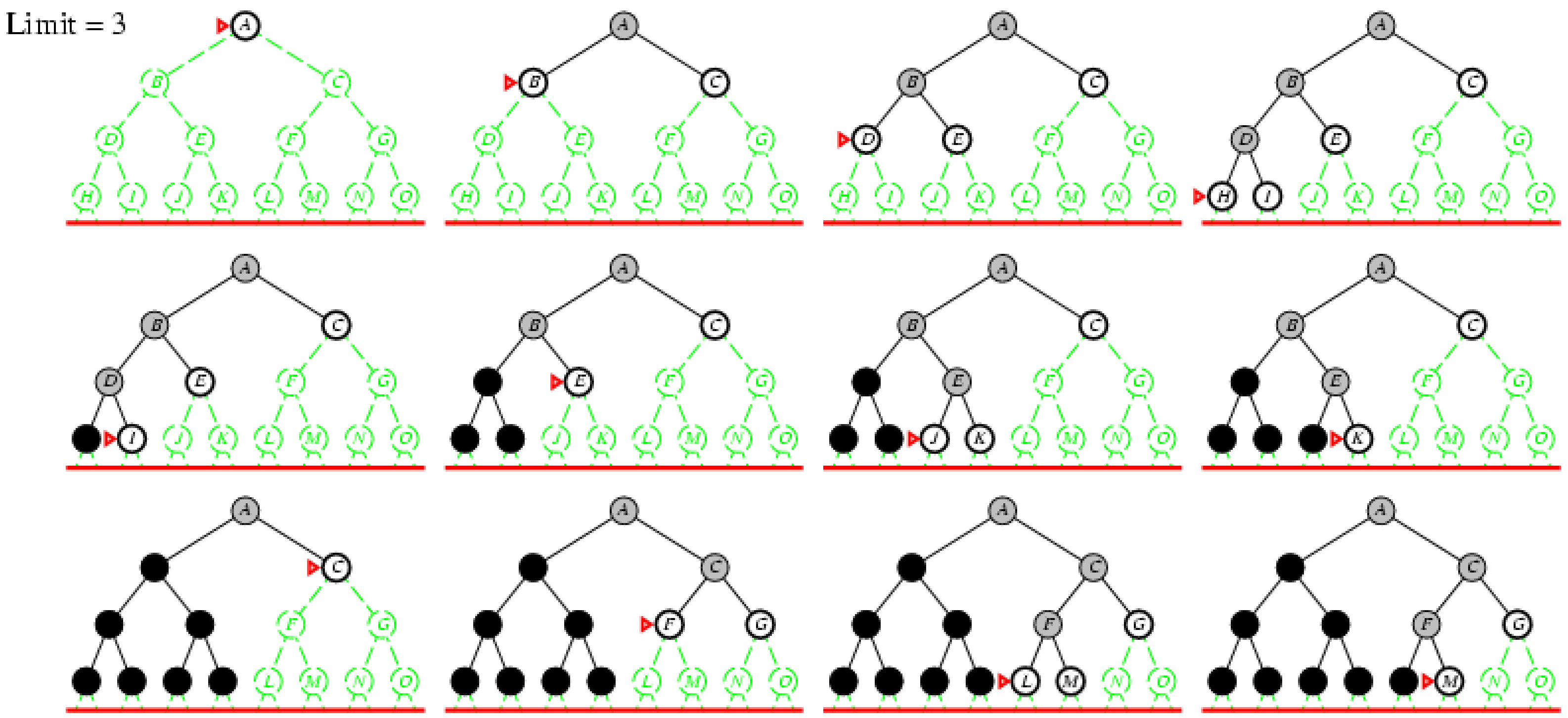
Iterative deepening search *l* =1



Iterative deepening search *l* =2



Iterative deepening search *l* =3



Iterative deepening search

- Number of nodes generated in a depth-limited search (DLS) to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

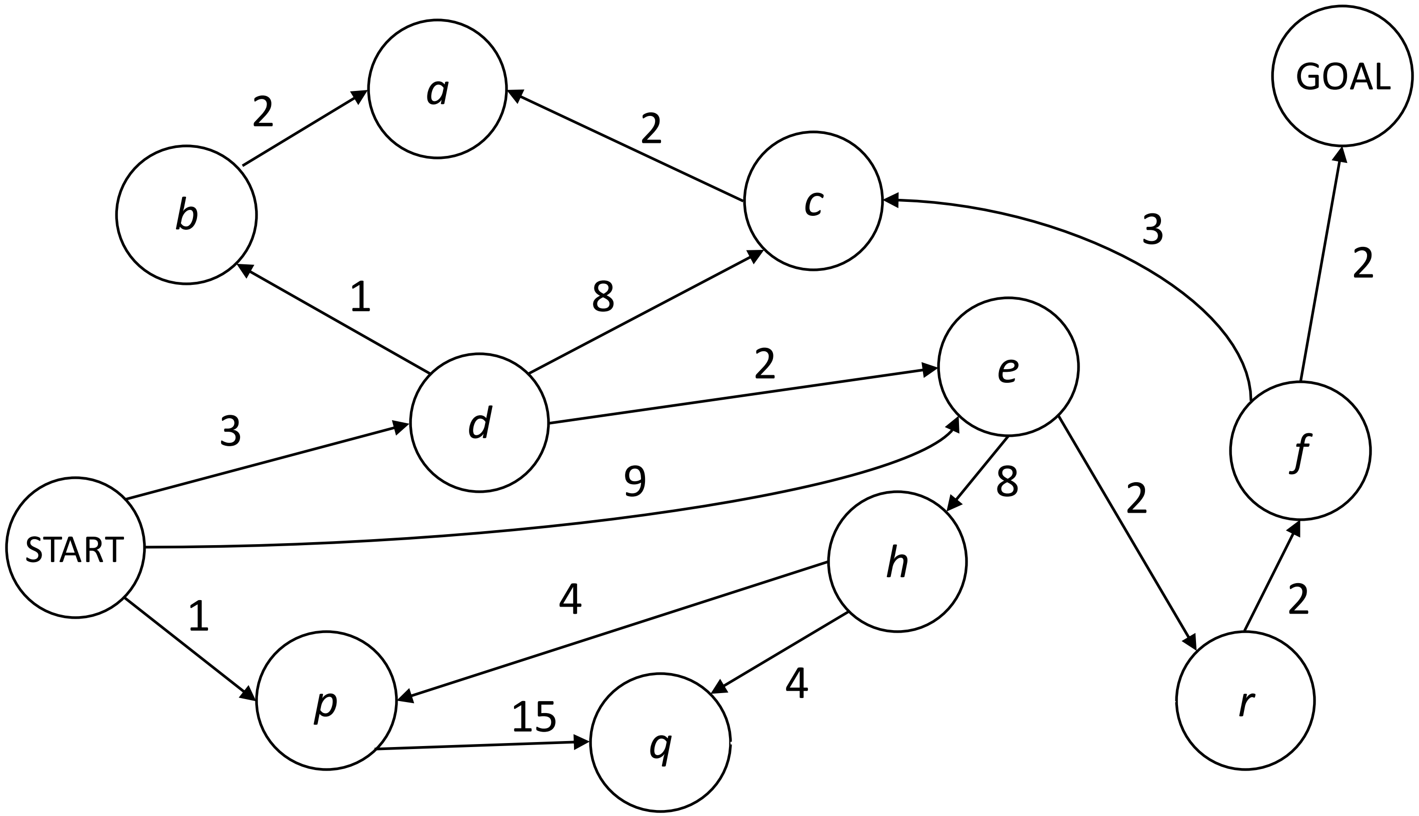
- Number of nodes generated in an iterative deepening search (IDS) to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\ b^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10$, $d = 5$,
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$

- Overhead = (123,456 - 111,111)/111,111 = 11%

Cost-Sensitive Search



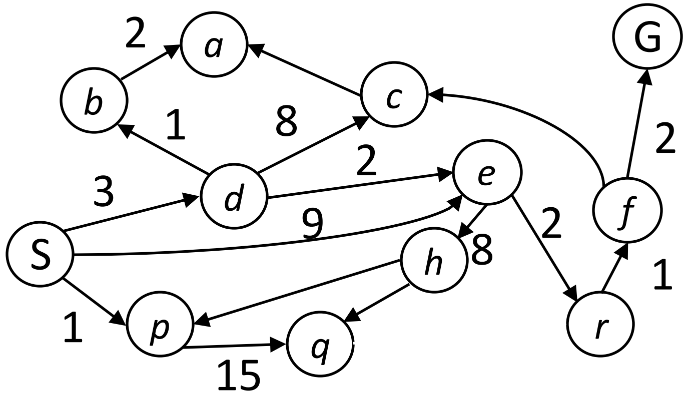
BFS finds the shortest path in terms of number of actions.
It does not find the least-cost path. We will now cover
a similar algorithm which does find the least-cost path.

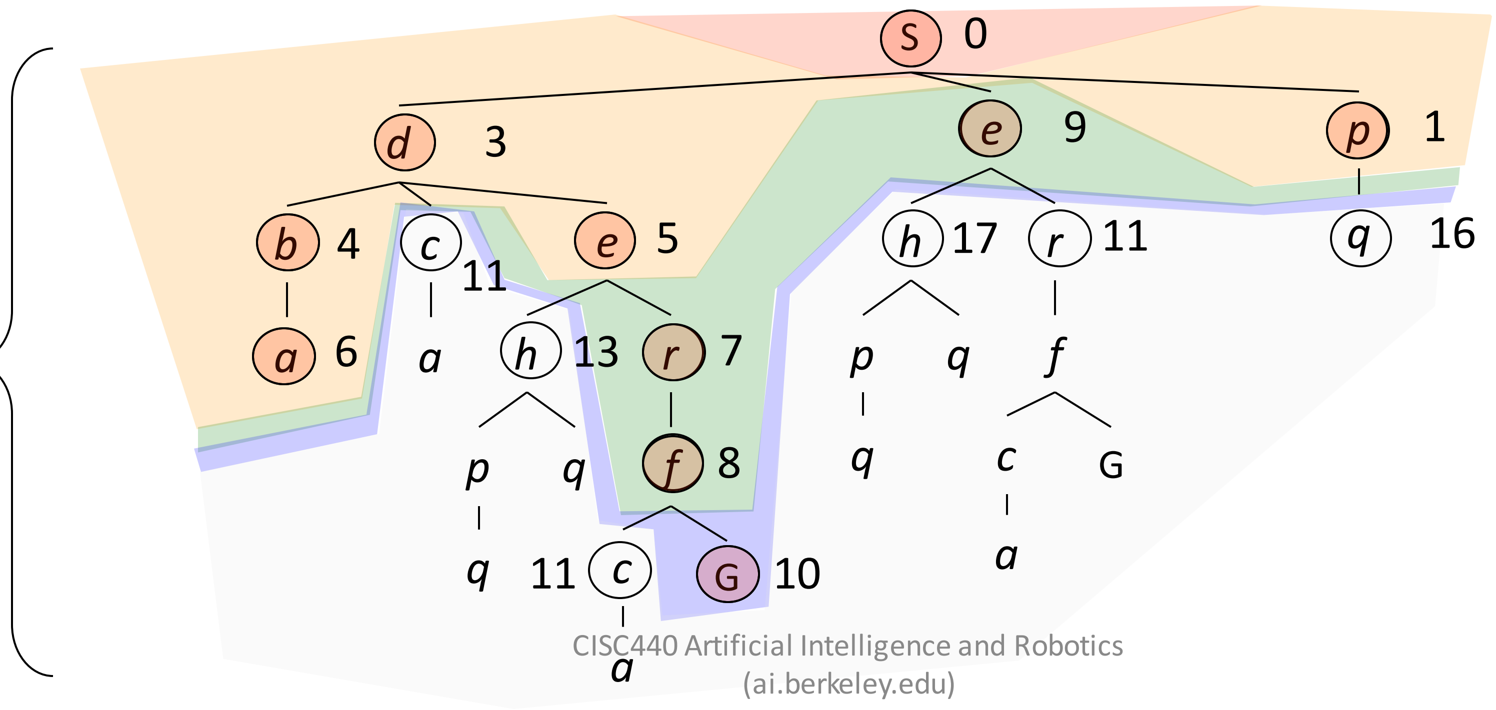
Uniform Cost Search

Uniform Cost Search

Strategy: expand a cheapest node first:

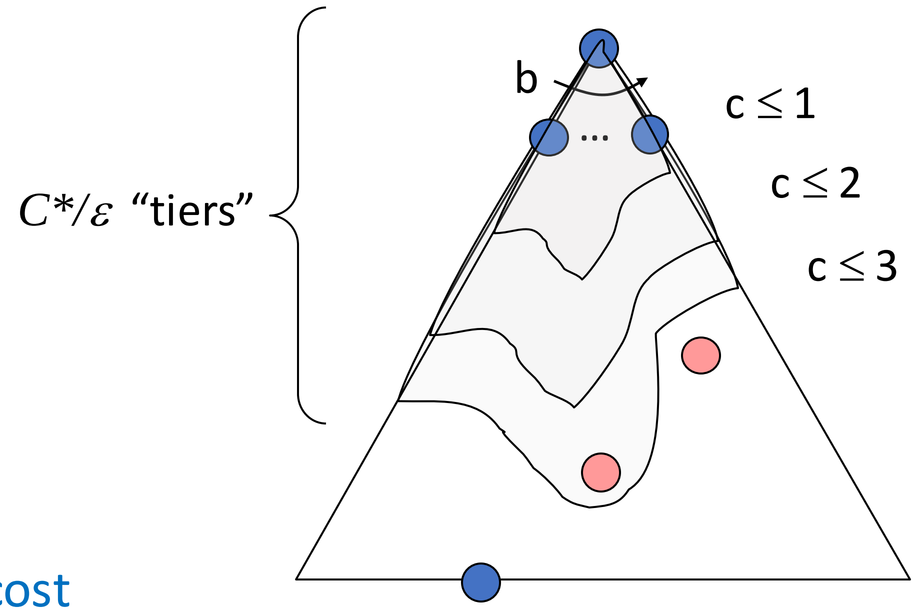
Fringe is a priority queue (priority: cumulative cost)





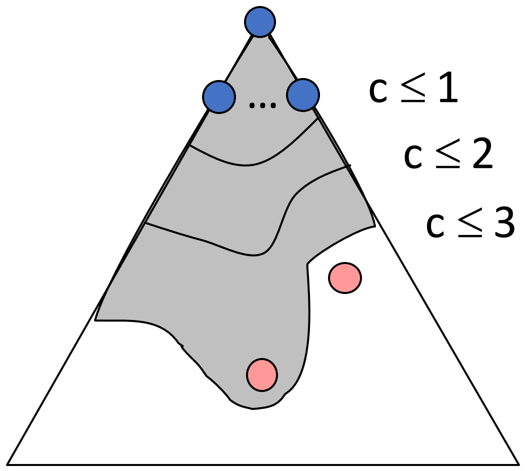
Uniform Cost Search (UCS) Properties

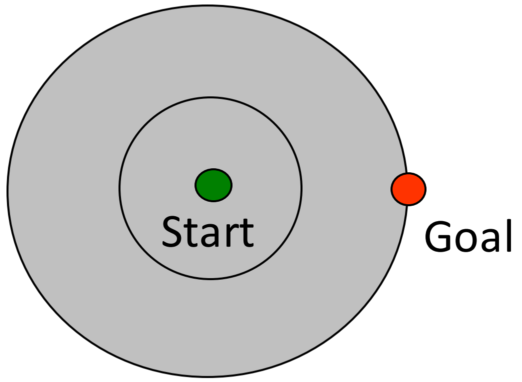
- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε, then the "effective depth" is roughly C^*/ε
  - Takes time O(b$^{C^*/\varepsilon}$) (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so O(b$^{C^*/\varepsilon}$)
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes! (Proof next lecture via A*)



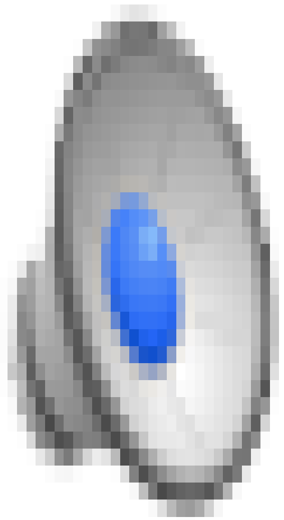
Uniform Cost Issues

- Remember: UCS explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every “direction”
  - No information about goal location

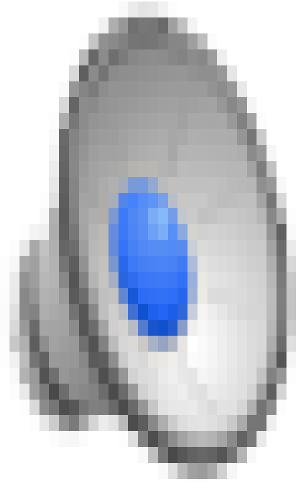




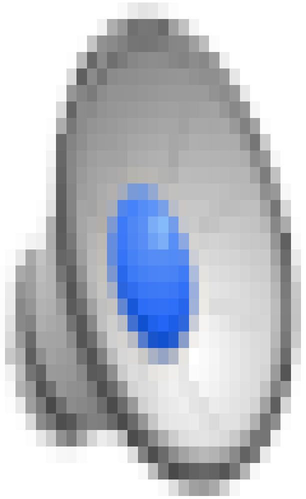
Video of Demo Empty UCS



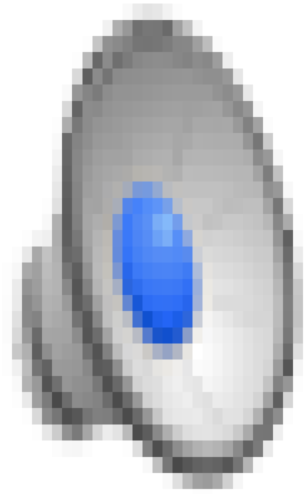
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 1)



Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 2)



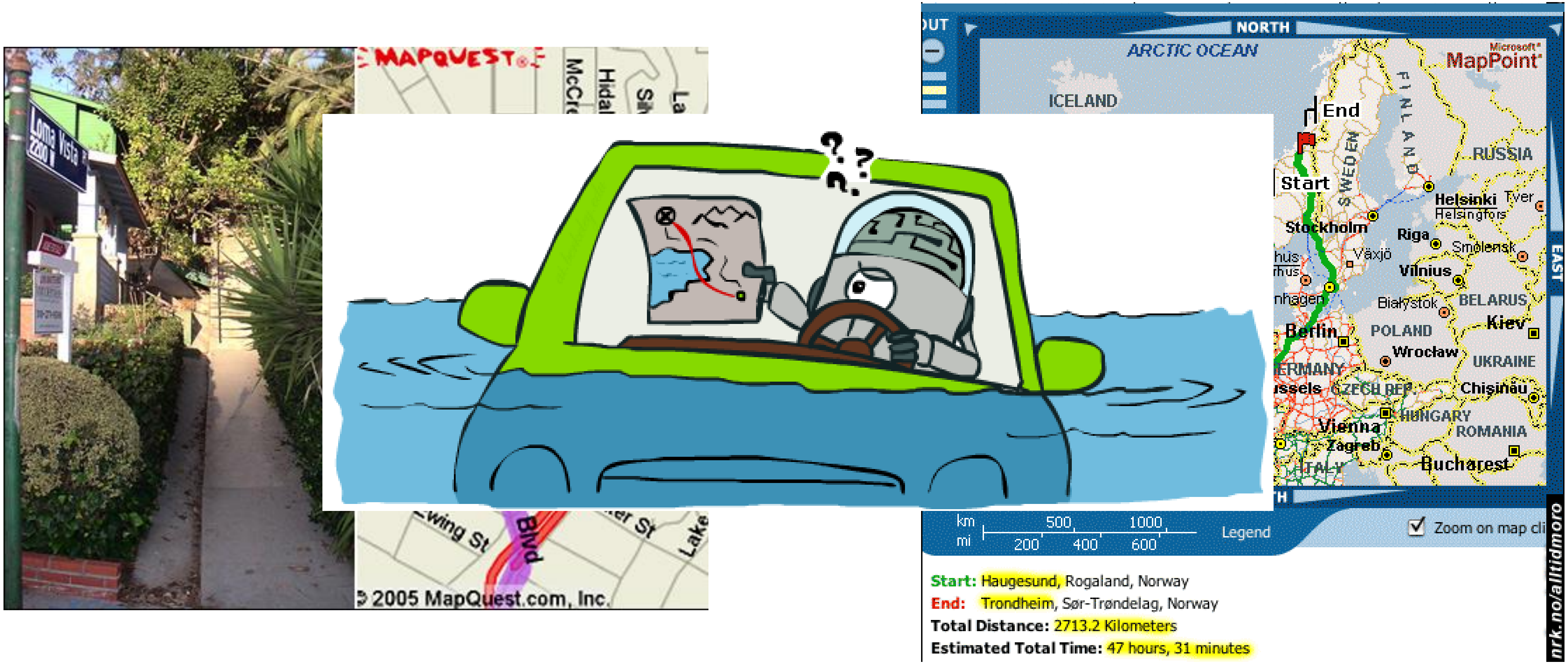
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 3)



The One Queue

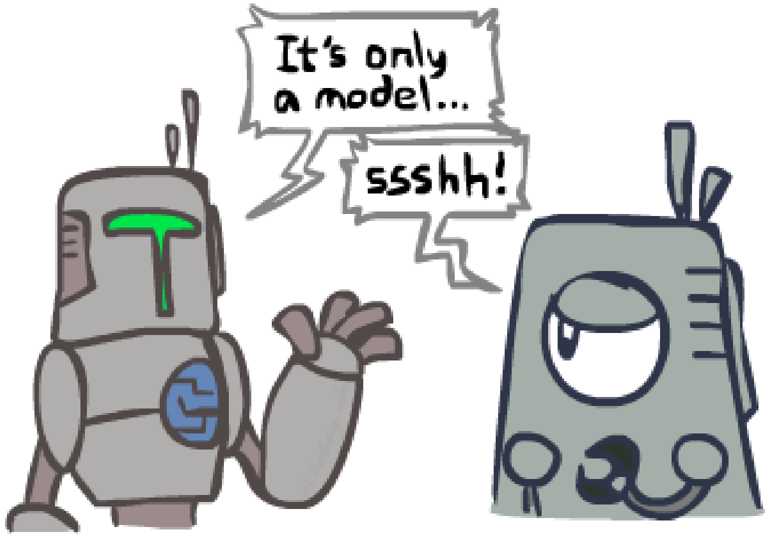
- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object

Search Gone Wrong?



Search and Models

- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all "in simulation"
  - Your search is only as good as your models…



Informed Search

Strategy uses problem-specific knowledge beyond the definition of the problem itself

Best First Search

Best-first search is an instance of the general TREE-SEARCH or GRAPH-SEARCH algorithm in which a node is selected for expansion based on an **evaluation function** $f(n)$

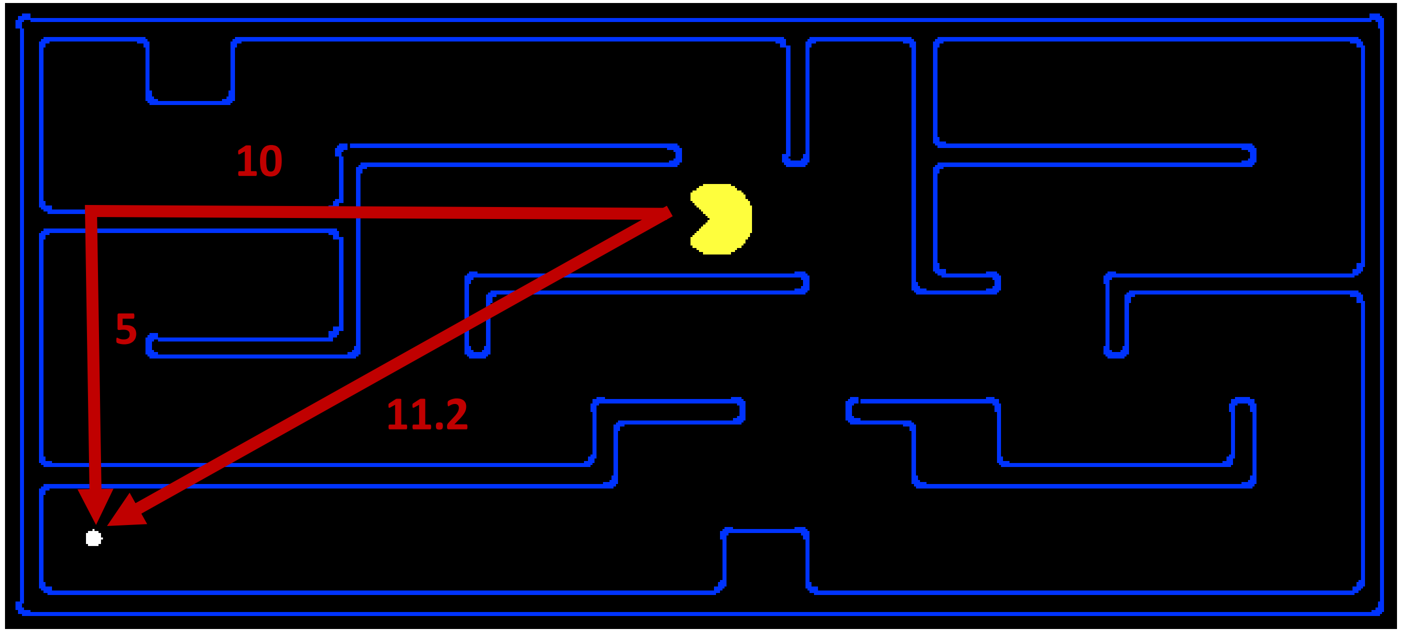
- ➢ Greedy best first Search
- ➢ A* Search

- ❖ $f(n)$ → Evaluation Function
- ❖ $g(n)$ → Cost Function
- ❖ $h(n)$ → Heuristic Function

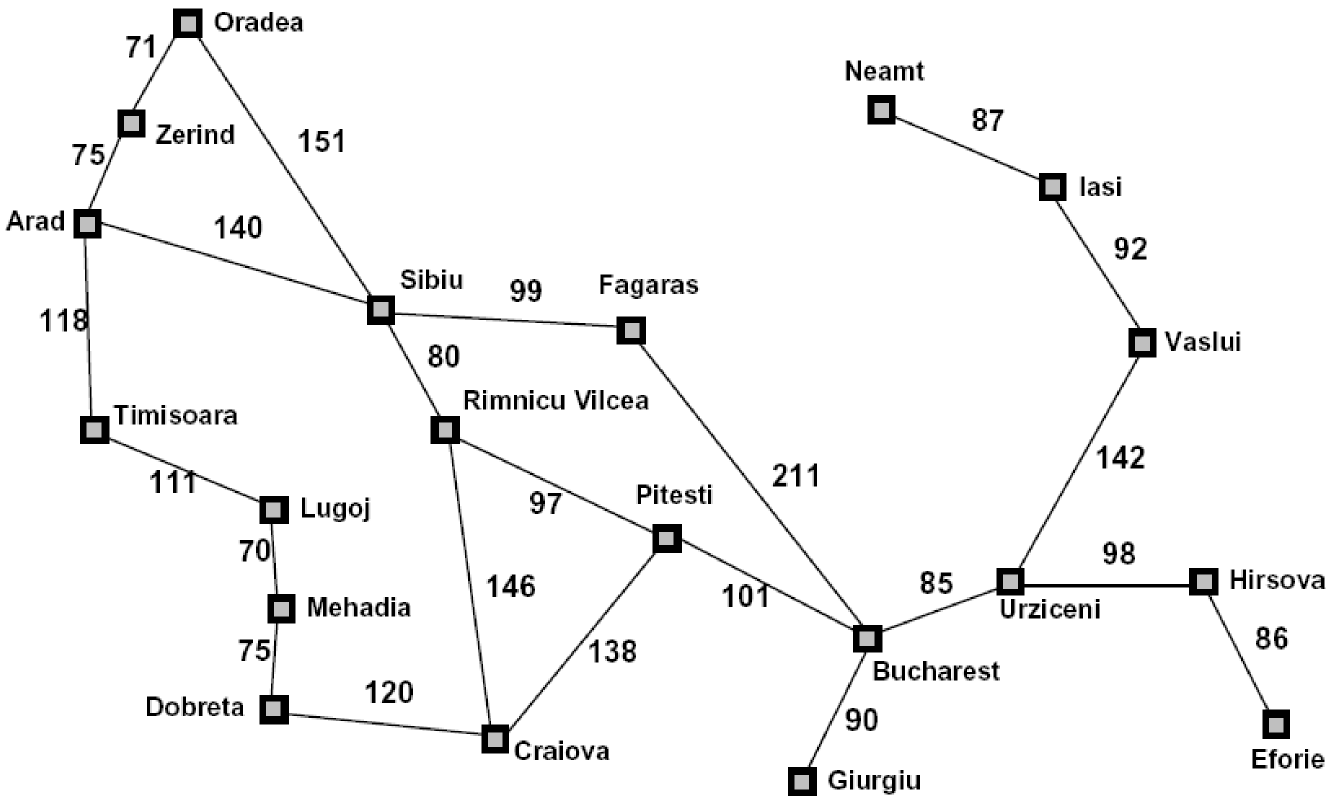
Search Heuristics

A heuristic is:

- A function that *estimates* how close a state is to a goal
- Designed for a particular search problem
- Examples: Manhattan distance, Euclidean distance for pathing



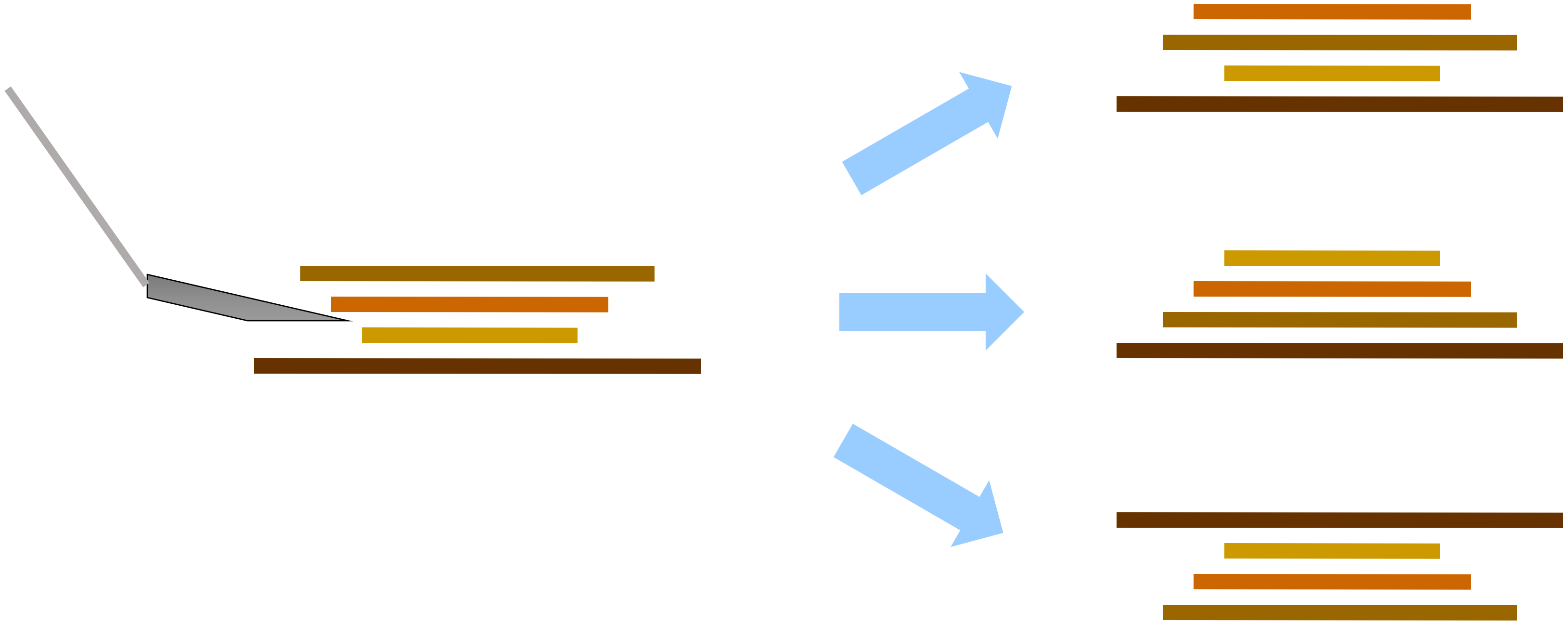
Example: Heuristic Function



Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

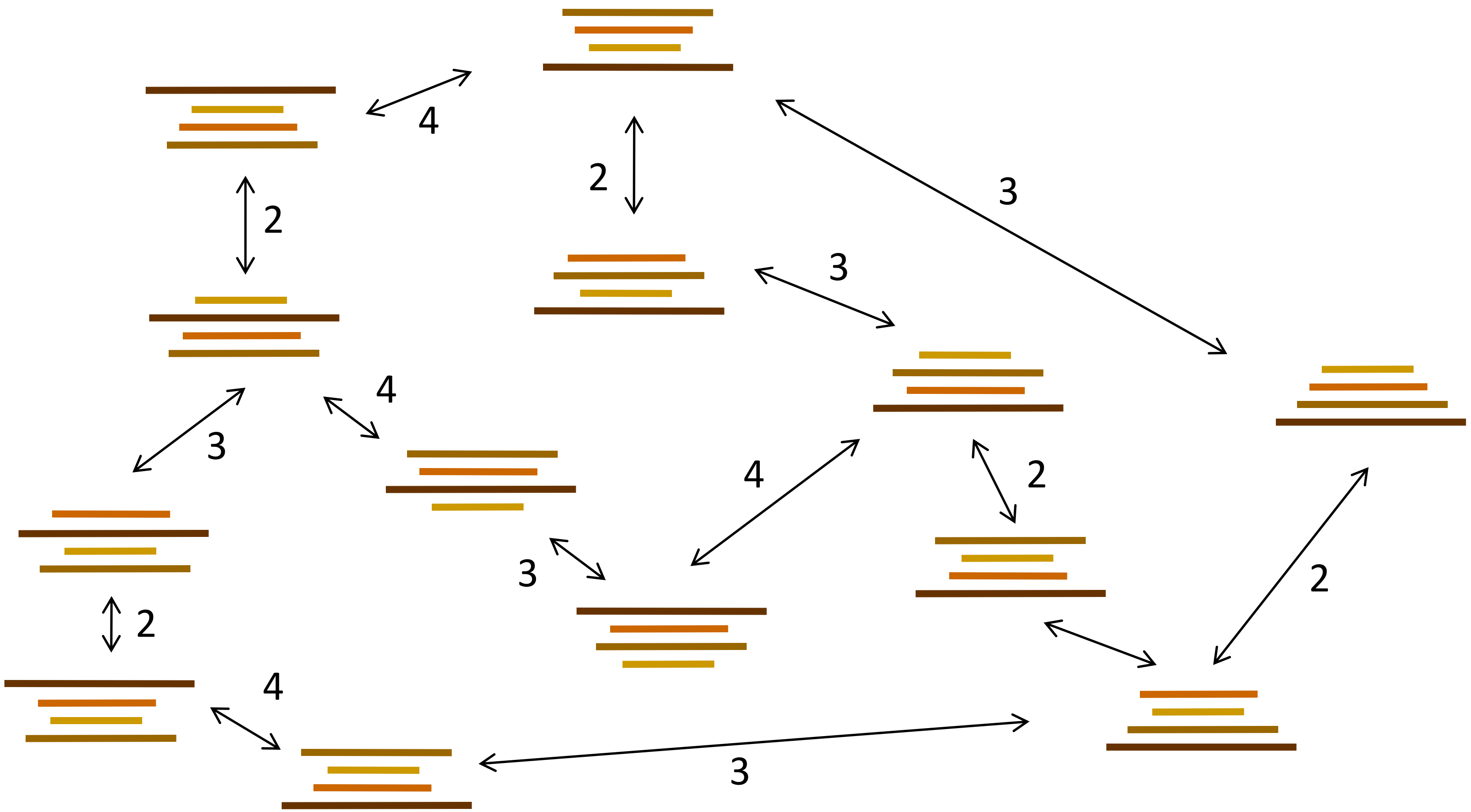
Pancake Problem



Cost: Number of pancakes flipped

Example: Pancake Problem

State space graph with costs as weights

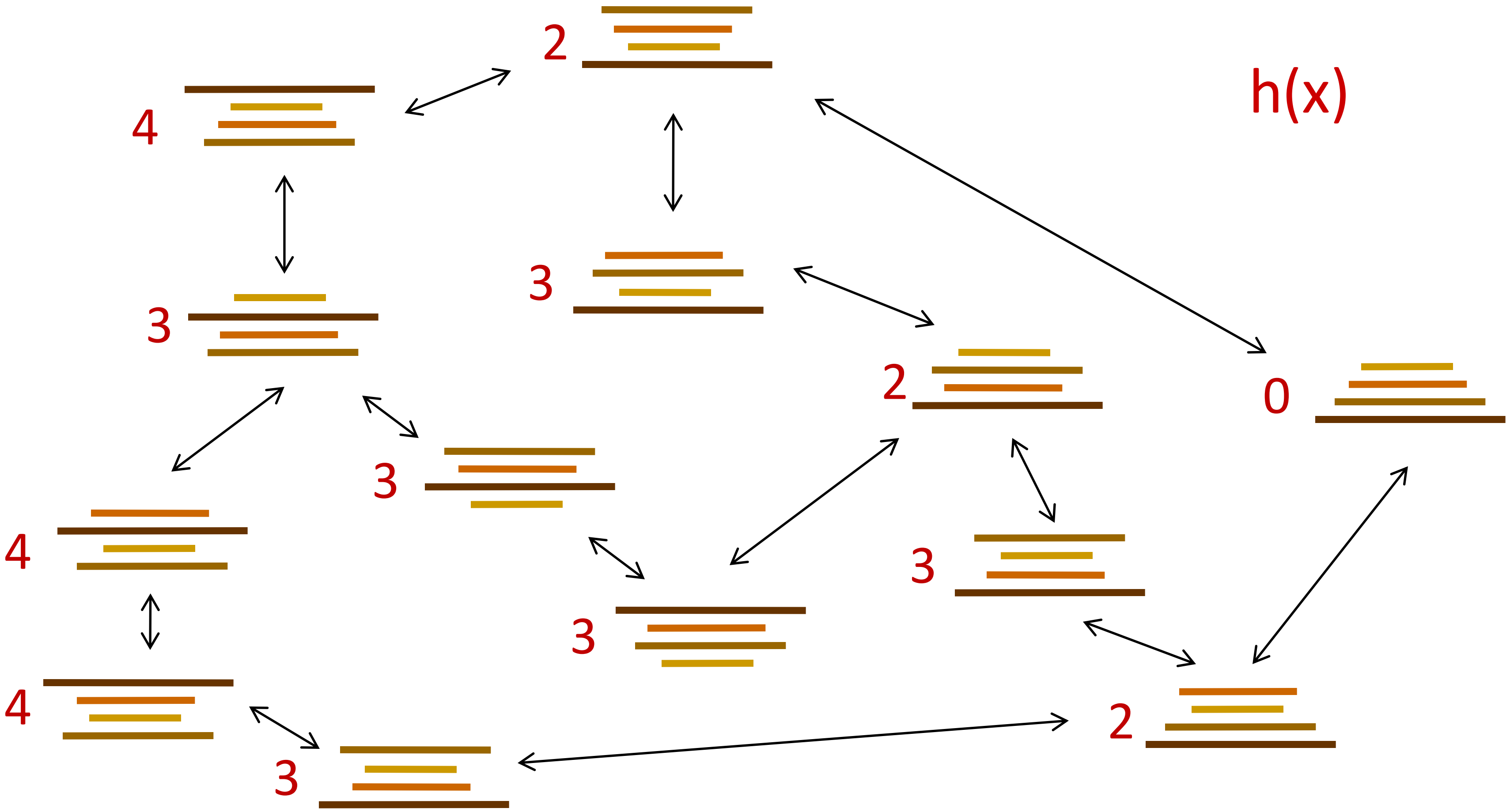


Heuristic Function

Associated with nodes not arcs

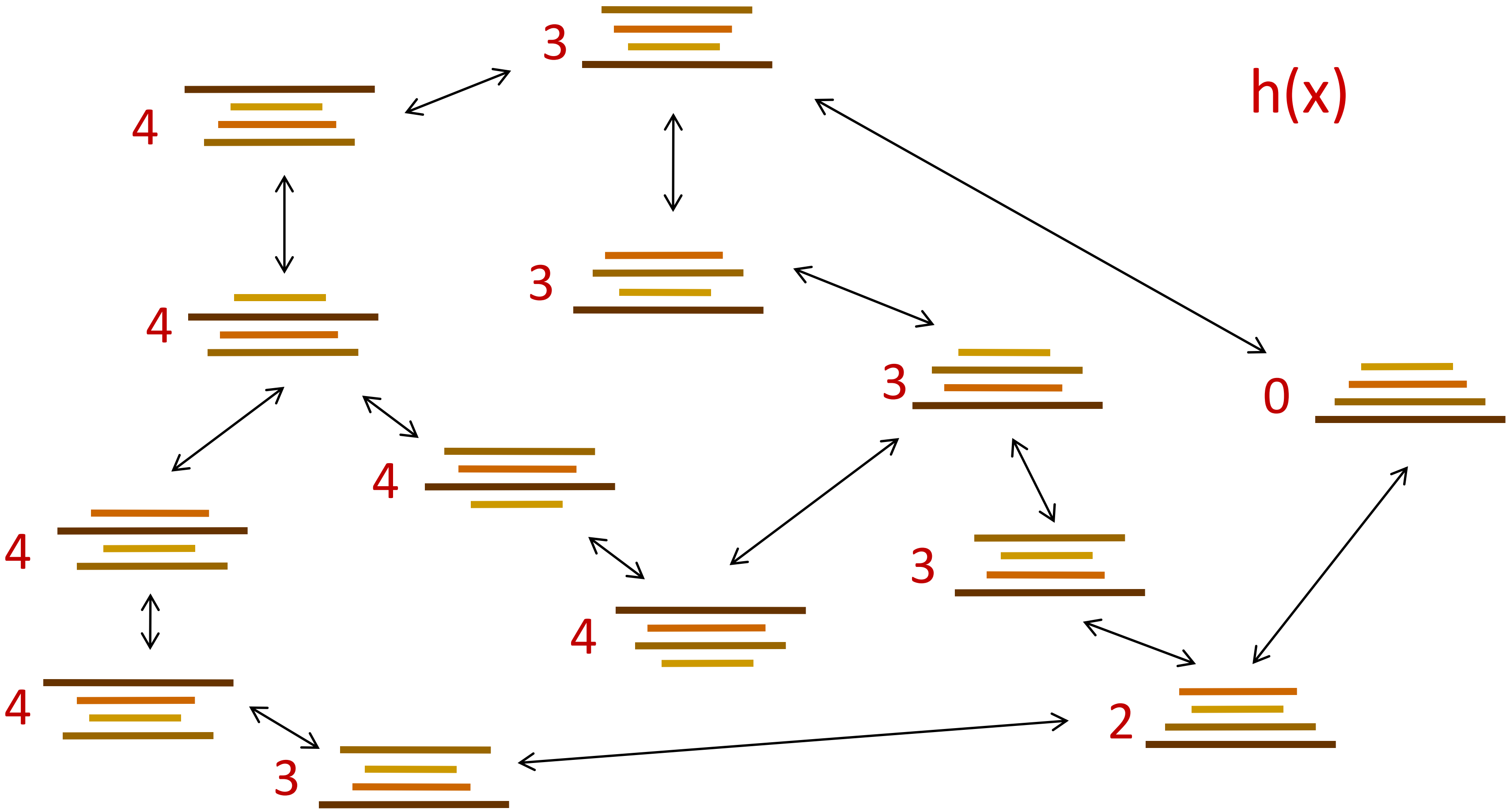
Example: Heuristic Function

Heuristic: the number of the pancake that is still out of place



Example: Heuristic Function

Heuristic: the number of the largest pancake that is still out of place

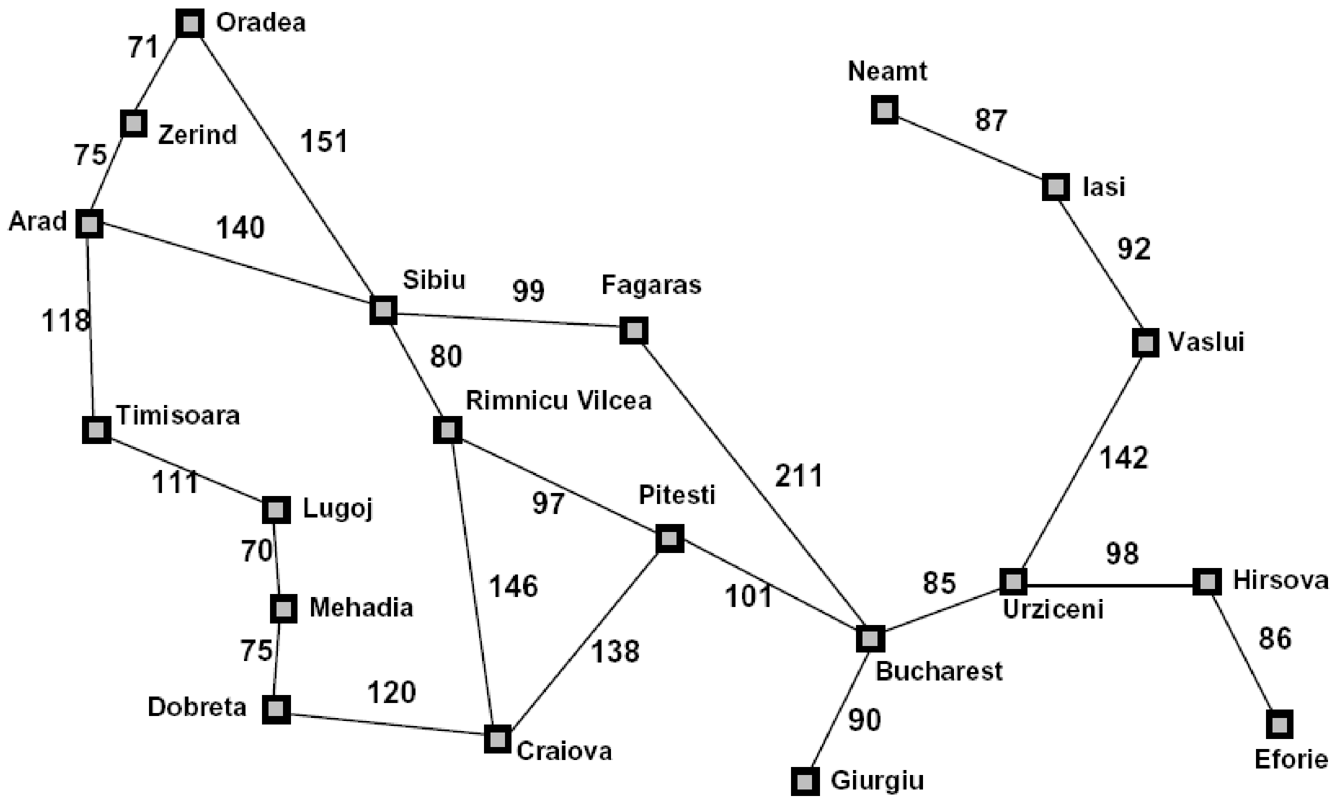


Greedy Search

$$f(n) = h(n)$$

Pick the node with lowest $h(n)$

Heuristic Function



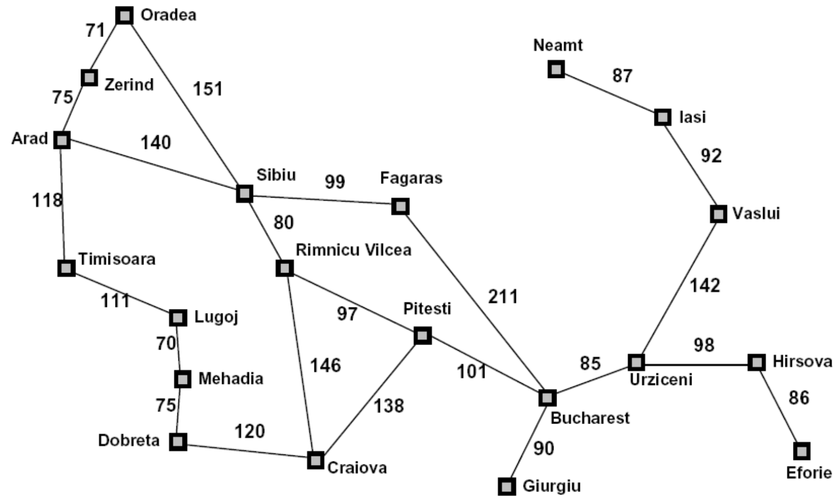
Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

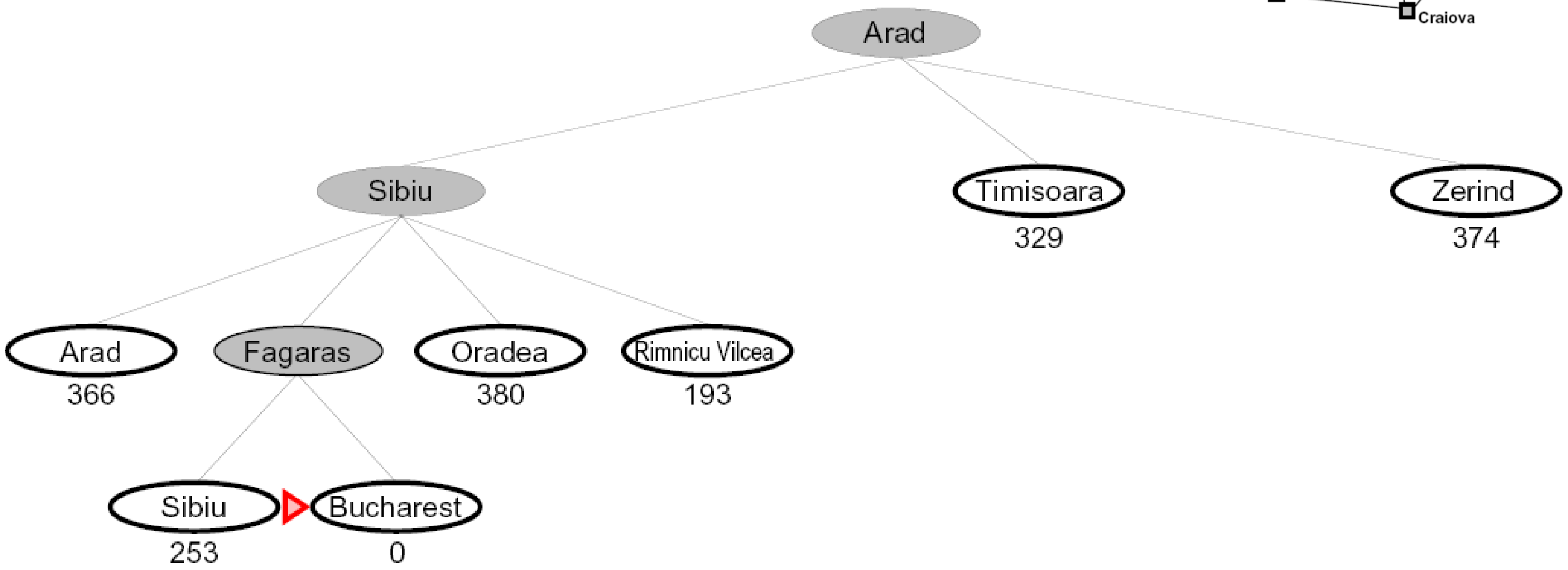
Greedy Search

❖ Expand the node that seems closest…



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374



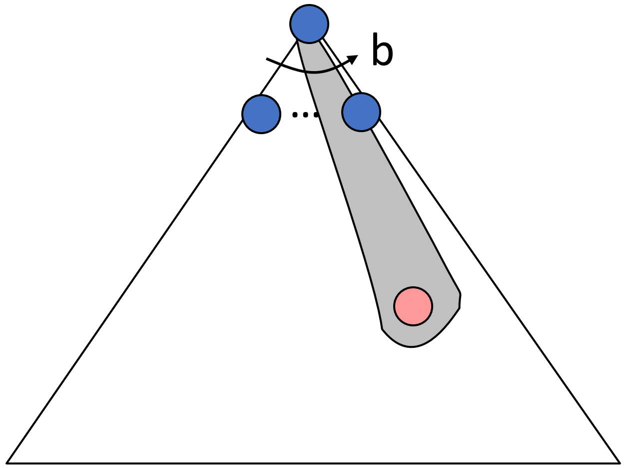
❖ What can go wrong?

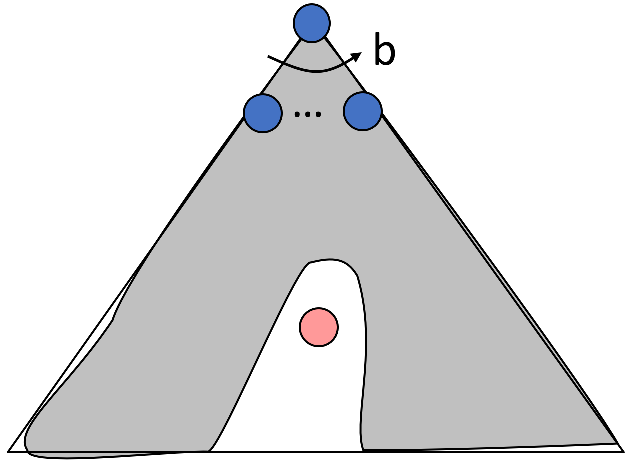
Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state

- A common case:
  - Best-first takes you straight to the (wrong) goal

- Worst-case: like a badly-guided DFS





Video of Demo Contours Greedy (Empty)

Properties of greedy best-first search

- Complete? No – can get stuck in loops
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

A* Search

$$f(n) = g(n) + h(n)$$

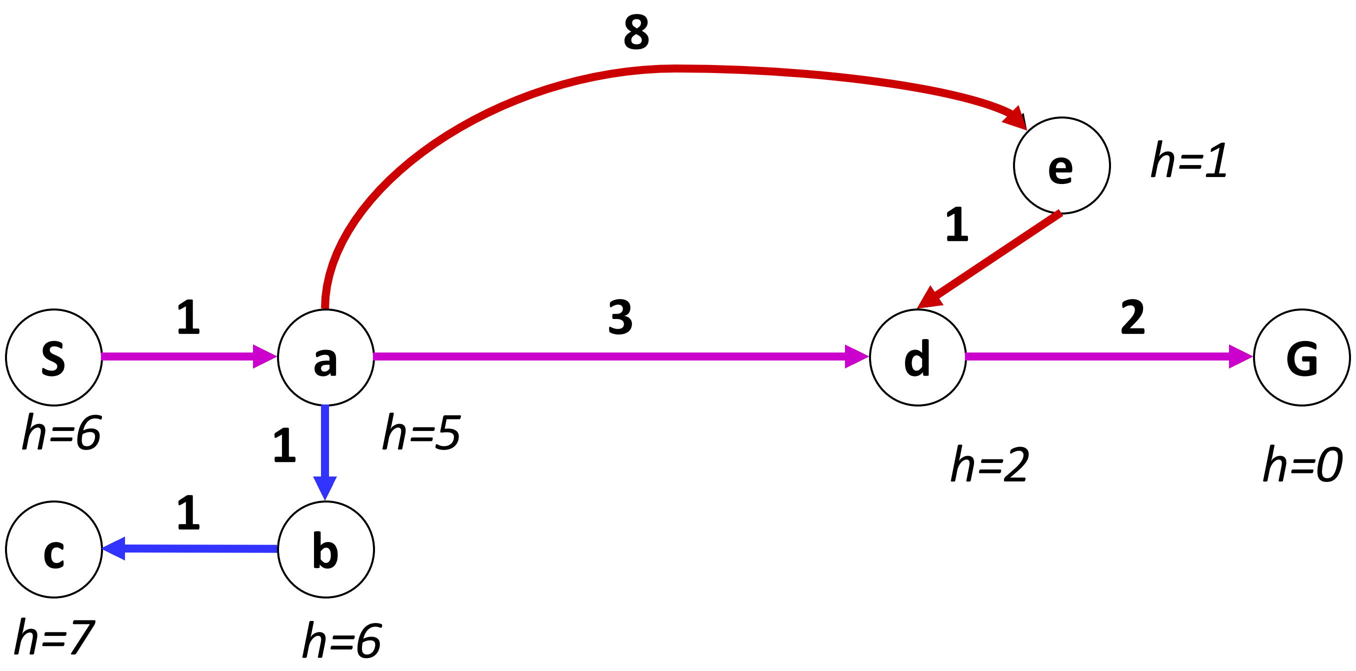
A* Search

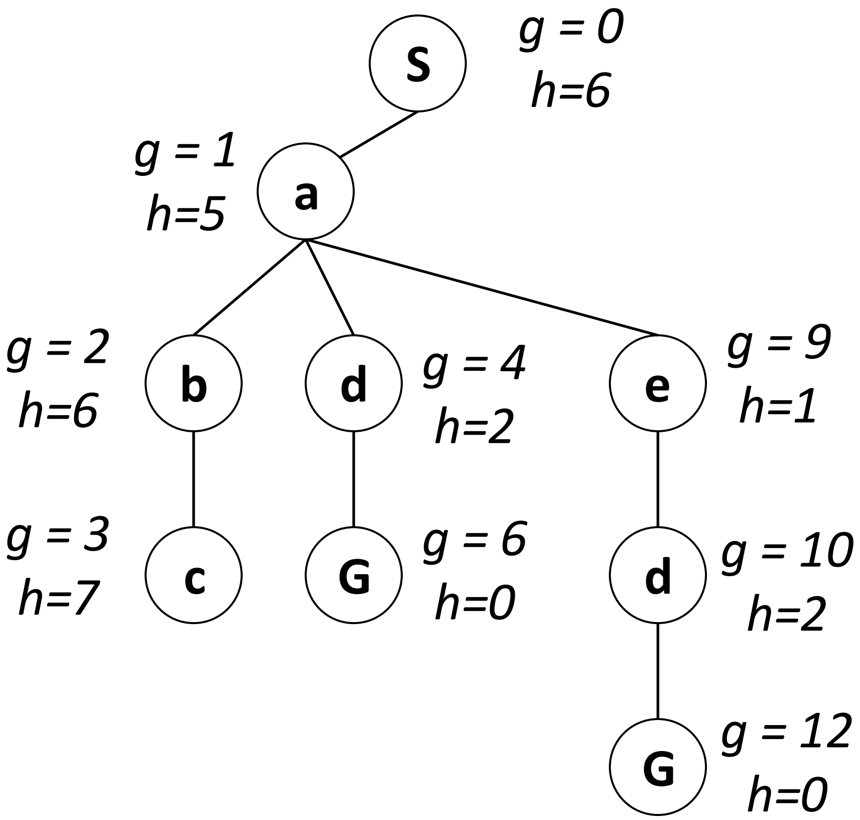
- Avoid expanding paths that are already expensive
- Teamwork approach

UCS + Greedy = A*

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)



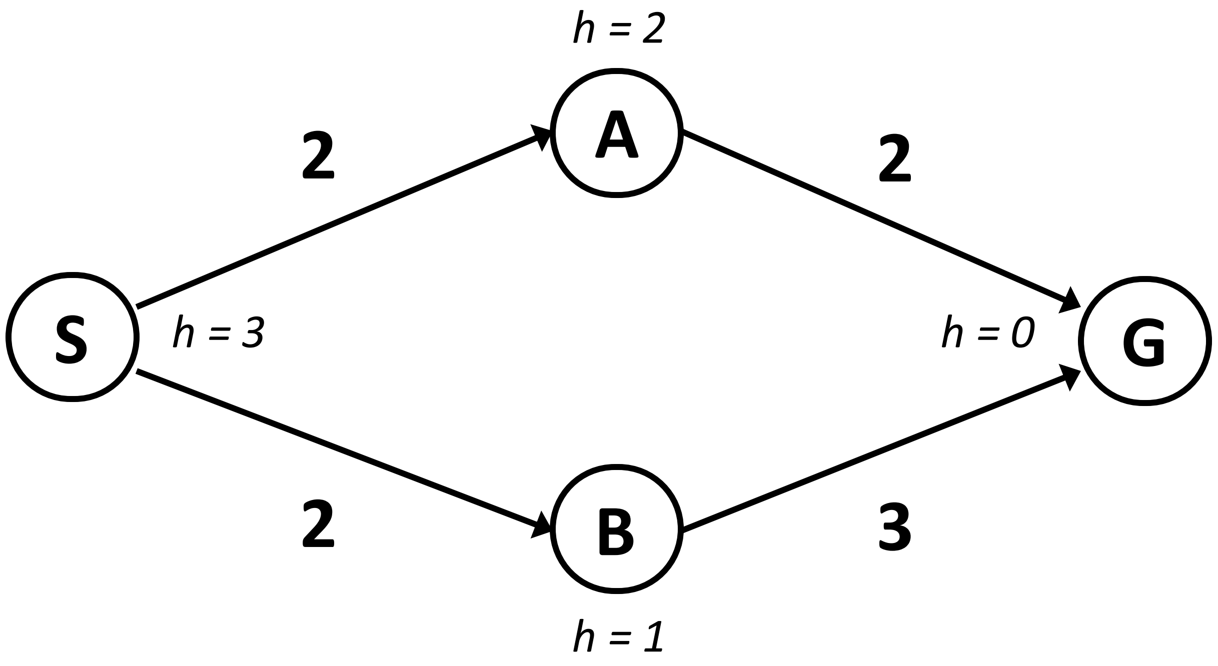


- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

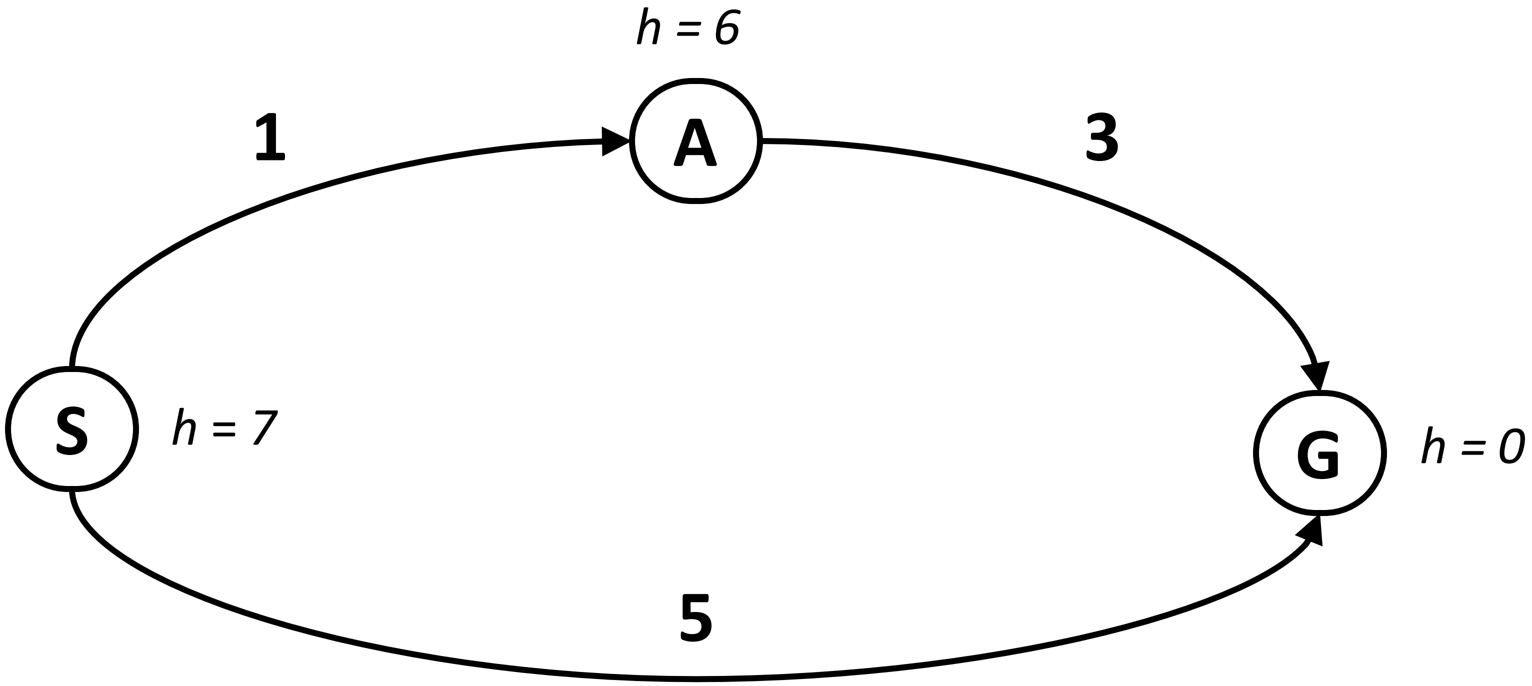
When should A* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

Admissible Heuristics

Admissible heuristic is one that *never overestimates* the cost to reach the goal.

Idea: Admissibility

- Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe
- Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

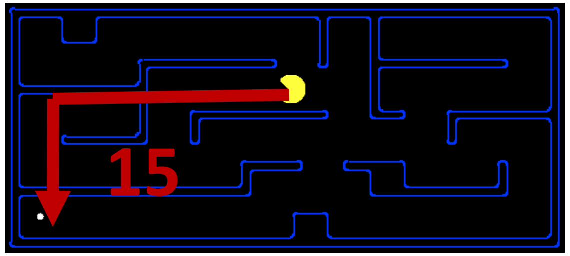
Admissible Heuristics

- A heuristic *h* is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:



- Producing admissible heuristics is most of what's involved in using A* in practice.

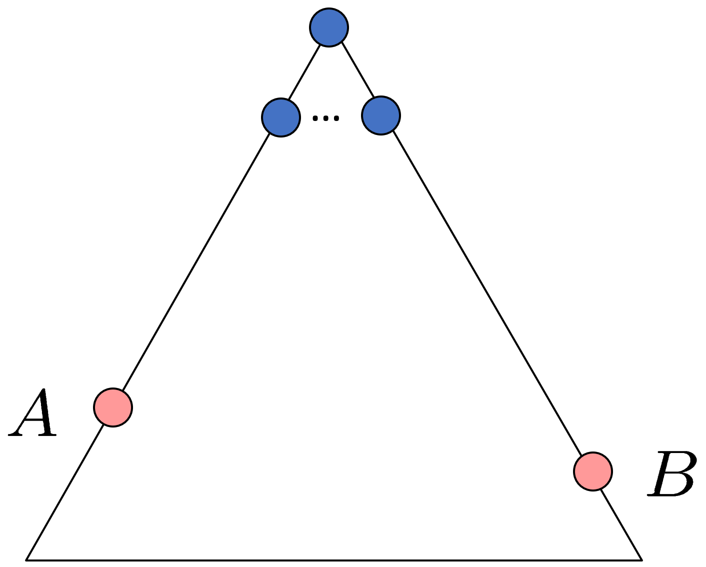
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:

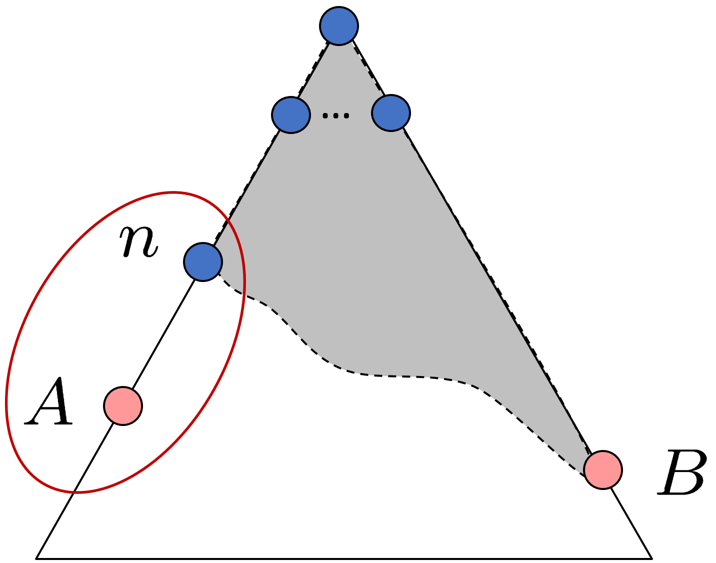
- A will exit the fringe before B



Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)

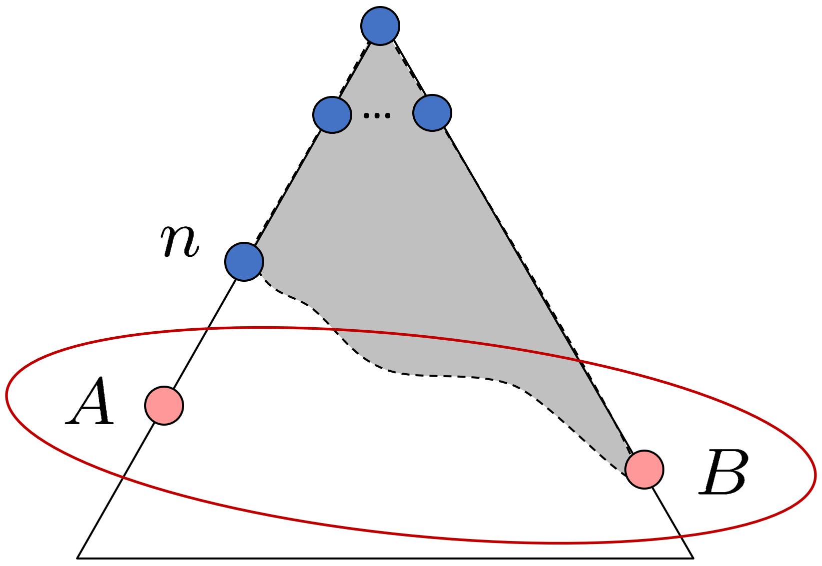


$f(n) = g(n) + h(n)$	Definition of f-cost
$f(n) \leq g(A)$	Admissibility of h
$g(A) = f(A)$	h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)



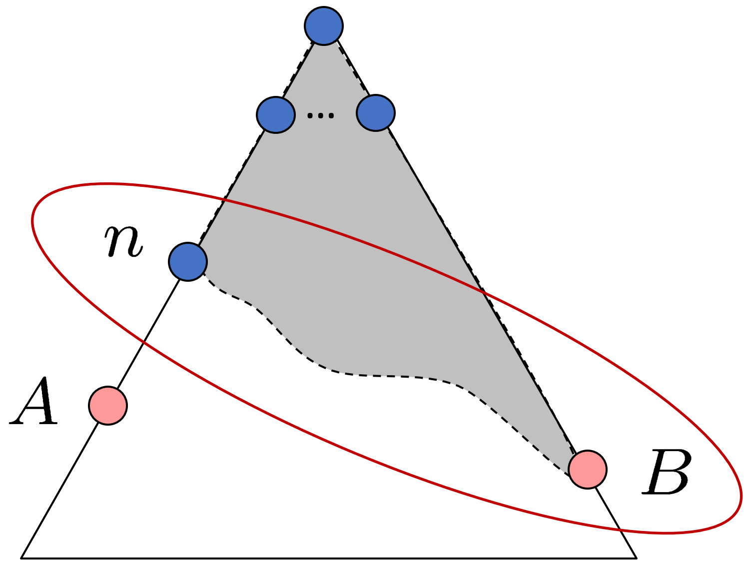
$$g(A) < g(B)$$ B is suboptimal

$$f(A) < f(B)$$ h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal with admissible heuristic

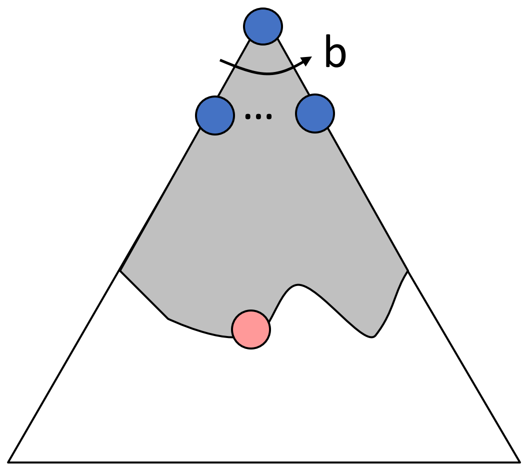


$$f(n) \leq f(A) < f(B)$$

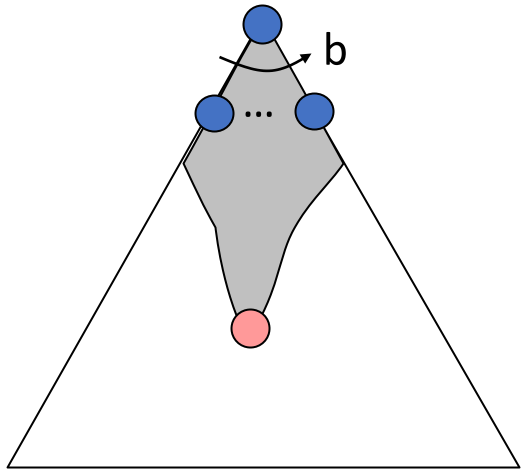
Properties of A*

Properties of A*

Uniform-Cost

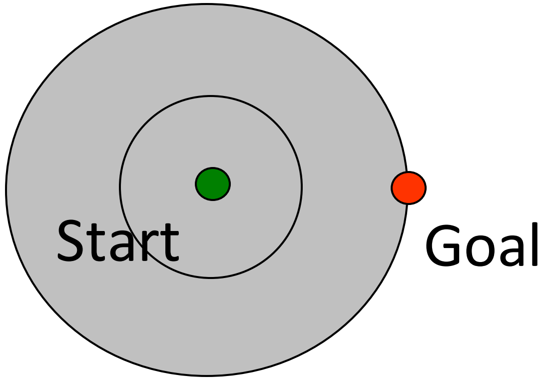


A*

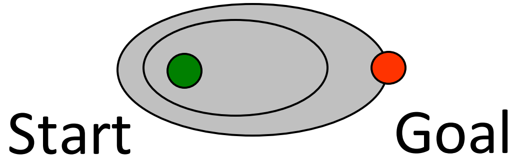


UCS vs A* Contours

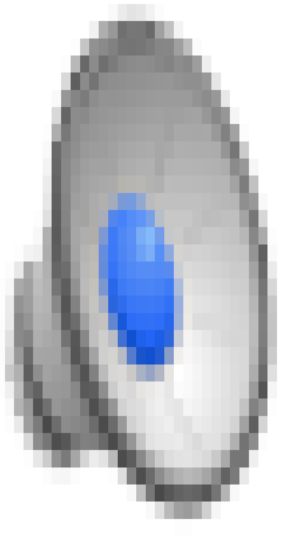
- Uniform-cost expands equally in all "directions"



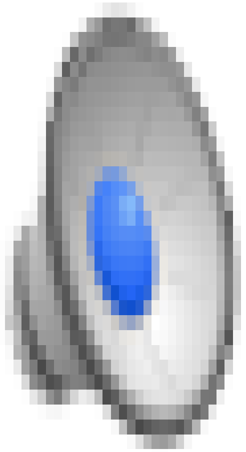
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



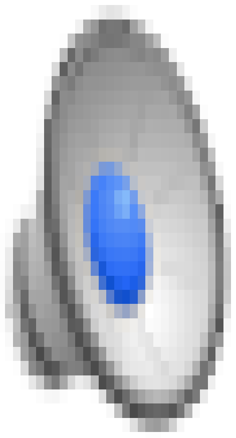
Video of Demo Contours (Empty) -- UCS



Video of Demo Contours (Empty) -- Greedy



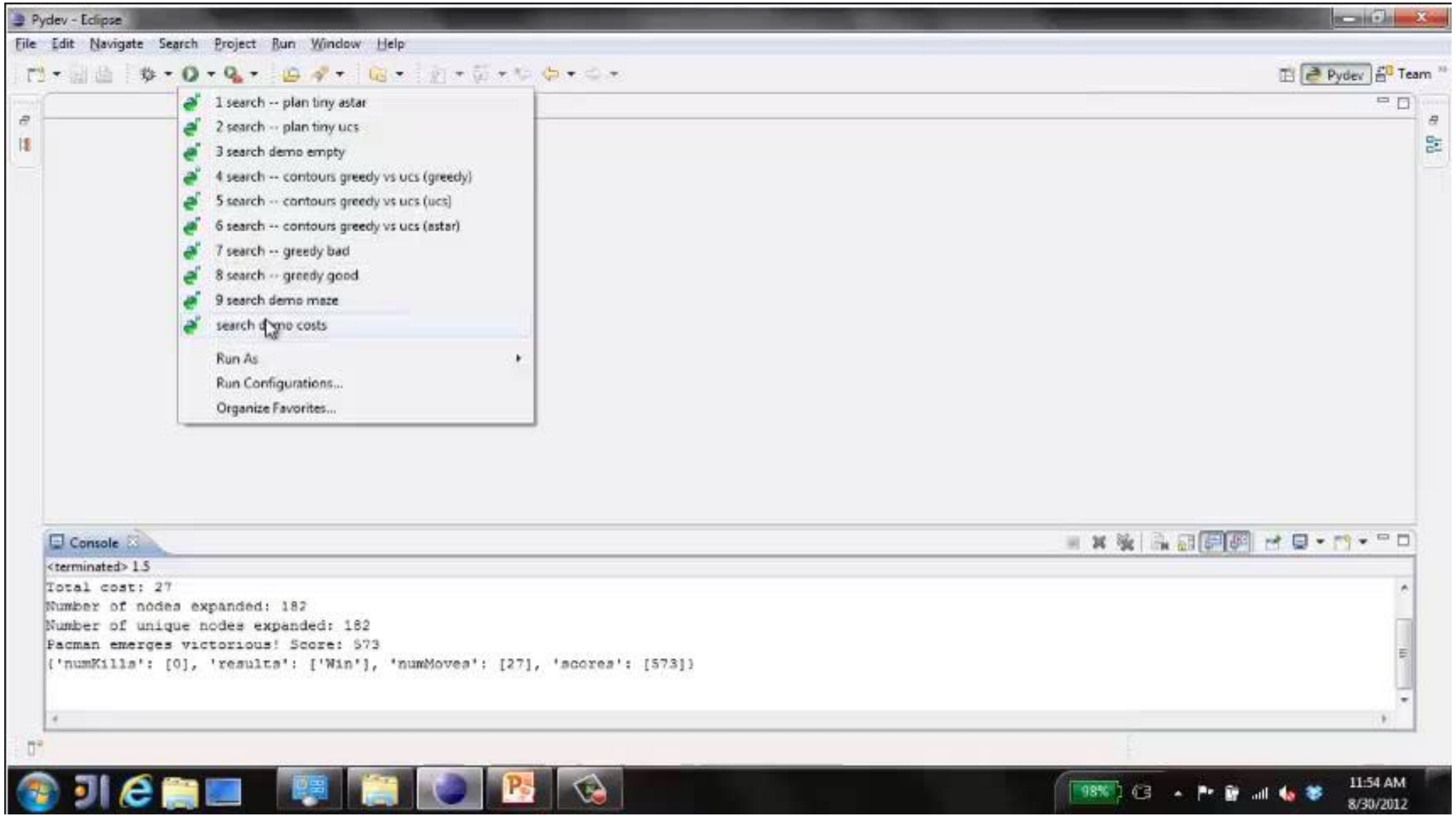
Video of Demo Contours (Empty) – A*



Properties of A*

- Complete? Yes (unless there are infinitely many nodes with $f \leq f(G)$)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

Video of Demo Empty Water Shallow/Deep – Guess Algorithm

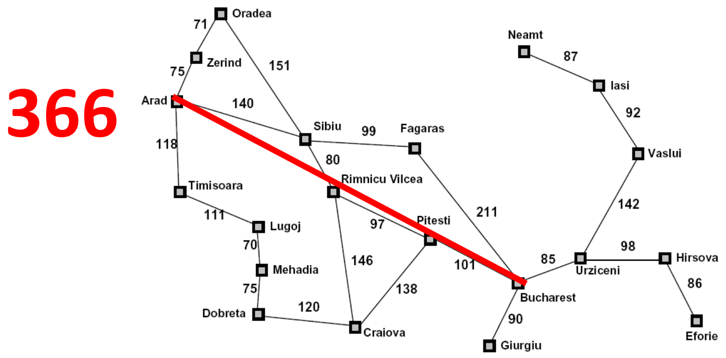


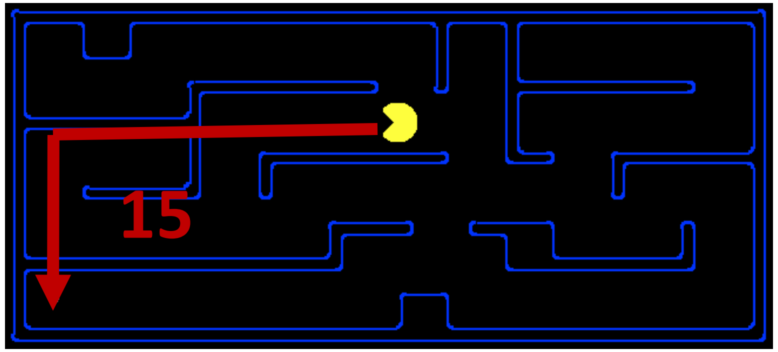
Creating Heuristics

Creating Admissible Heuristics

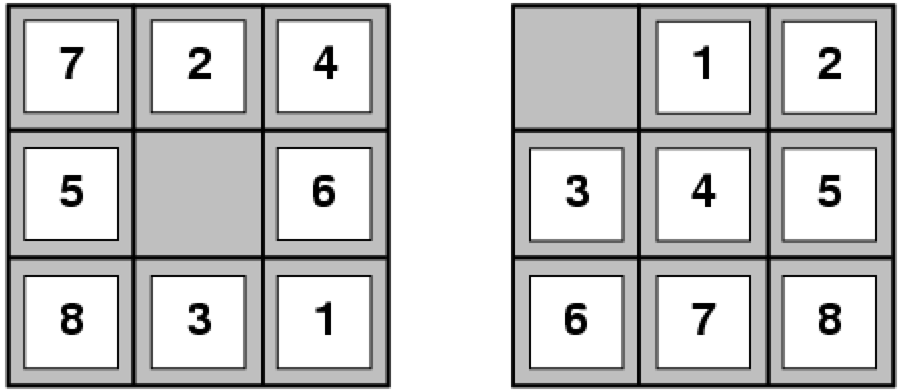
- Most of the work in solving hard search problems optimally is in producing admissible heuristics

- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available





Example: 8 Puzzle

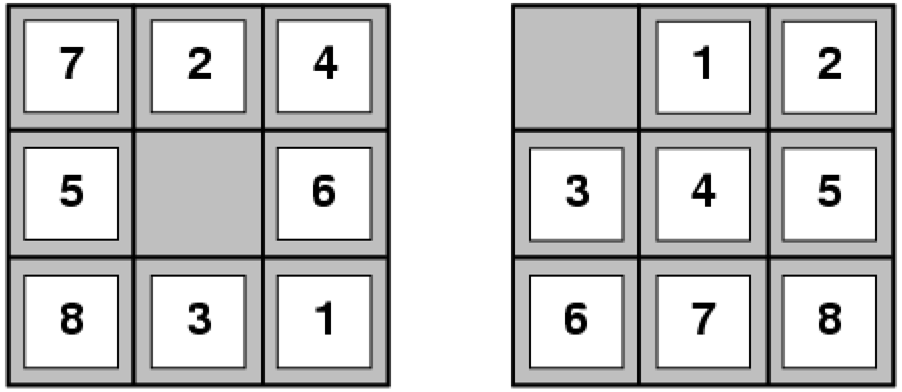


Start State   Goal State

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

- Heuristic: h_1= Number of tiles misplaced
- Why is it admissible?
- h_1(start) =8
- This is a *relaxed-problem* heuristic



Start State   Goal State

	Average nodes expanded when the optimal path has...		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

Statistics from Andrew Moore

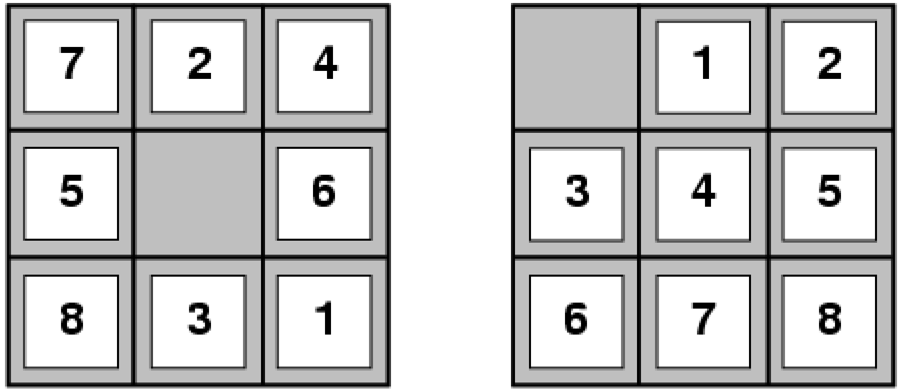
8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

- H_2=Total *Manhattan* distance

- Why is it admissible?

- h_2(start) = 3 + 1 + 2 + … = 18



Start State Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?

- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Semi-Lattice of Heuristics

Trivial Heuristics, Dominance

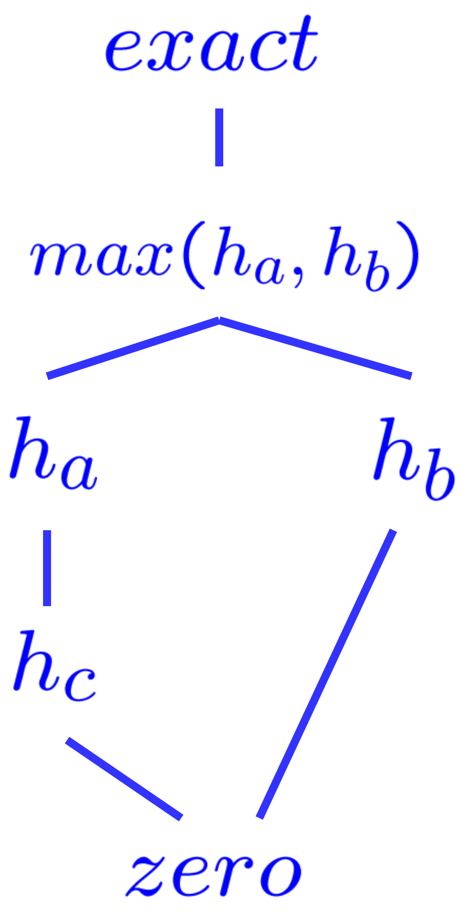
- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

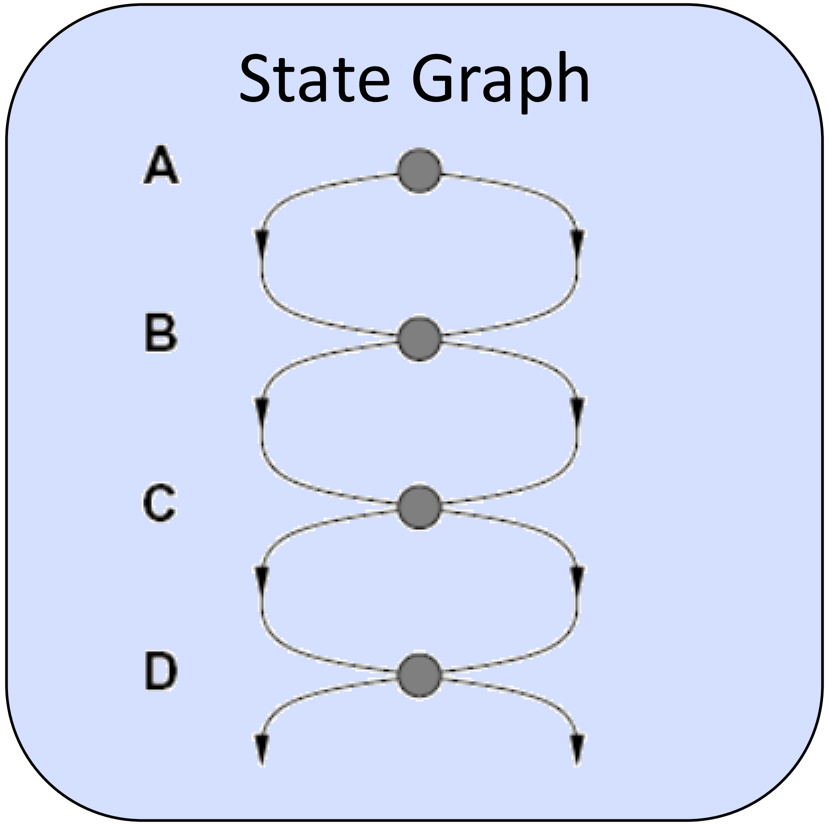
- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic

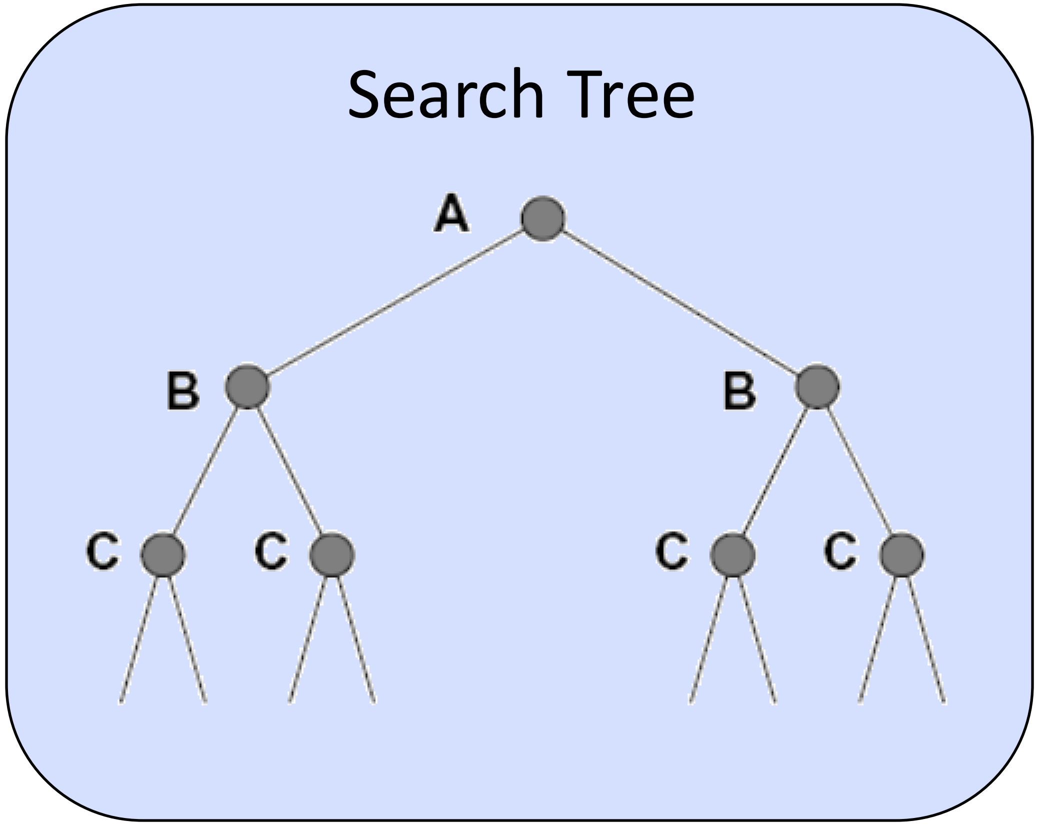


Graph Search

Tree Search: Extra Work!

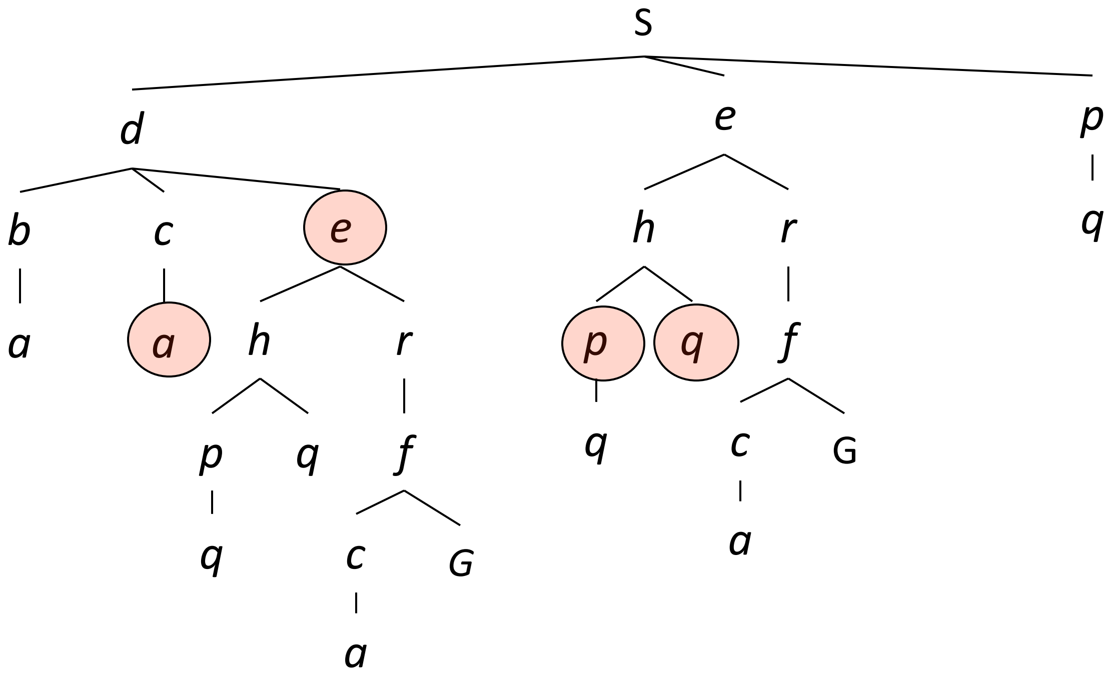
- Failure to detect repeated states can cause exponentially more work.





Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



Graph Search

- Idea: never expand a state twice

- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set

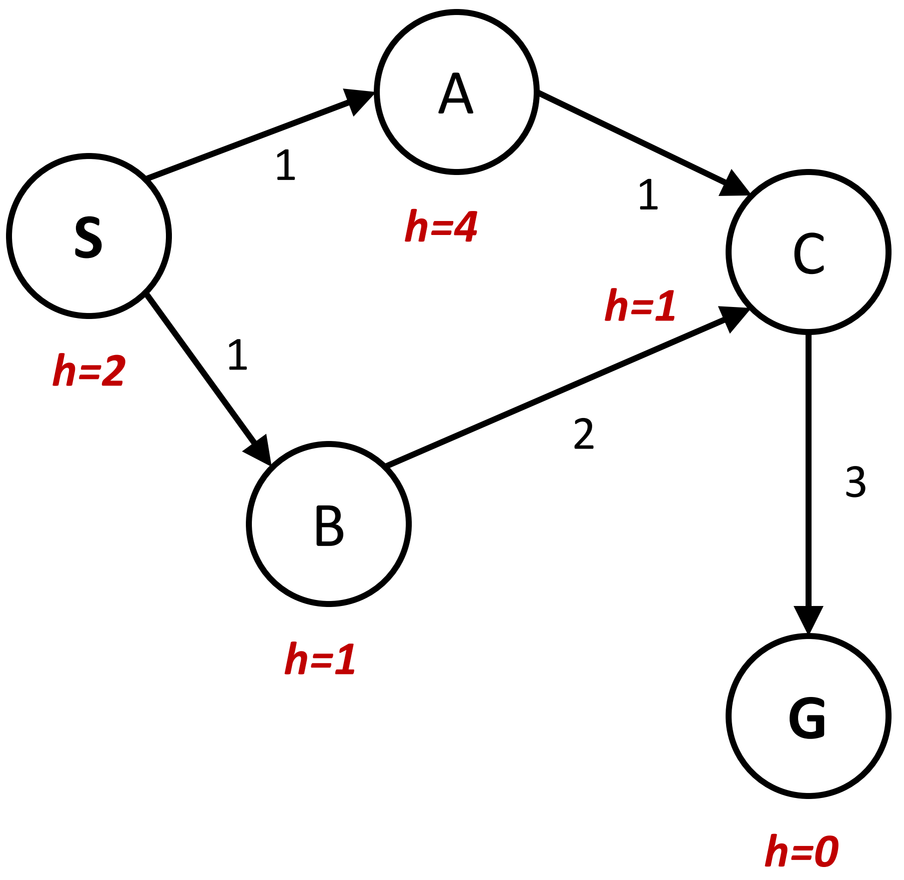
- Important: store the closed set as a set, not a list

- Can graph search wreck completeness? Why/why not?

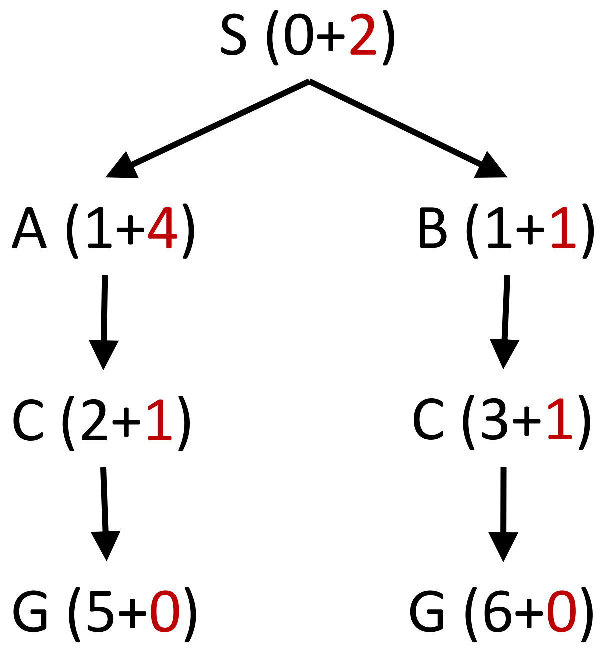
- How about optimality?

A* Graph Search Gone Wrong?

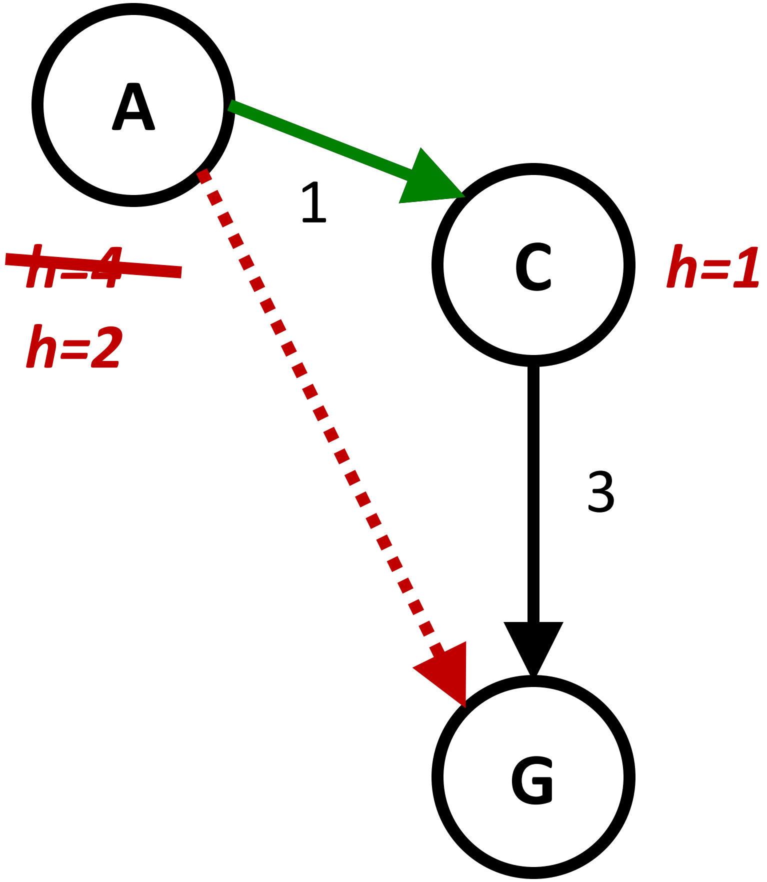
State space graph



Search tree



Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)
  - A* graph search is optimal

A*: Summary

A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems

Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```

Chapter 4

Search in Complex Environments

(Focus on Local Search)

Topic

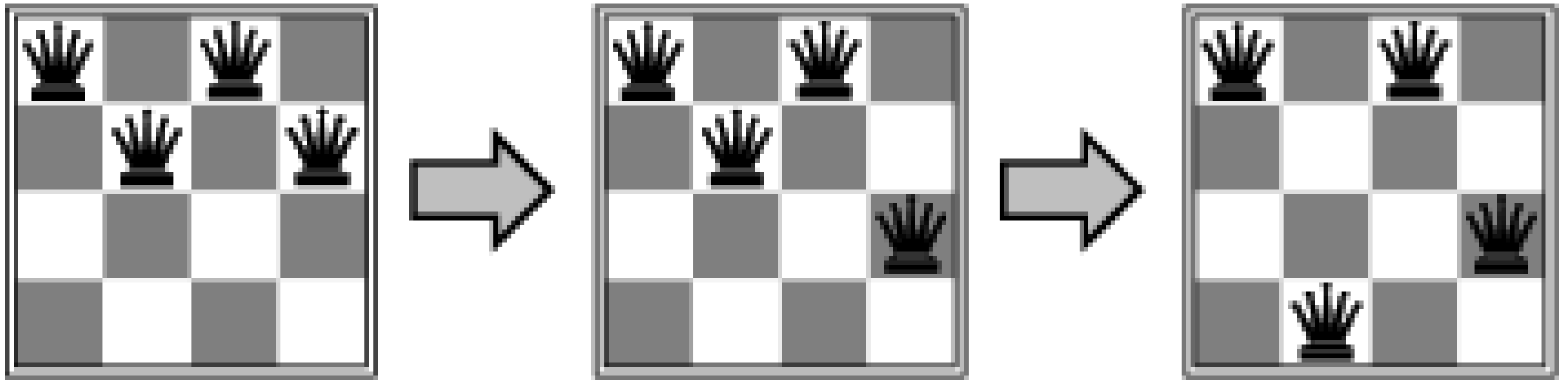
- Local Search and Optimization Problems
- Local search in Continuous spaces
  - Hill Climbing
  - Simulated Annealing
  - Local Beam Search
  - Genetic Algorithm

Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use local search algorithms
- keep a single "current" state, try to improve it

Example: *n*-queens

- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal



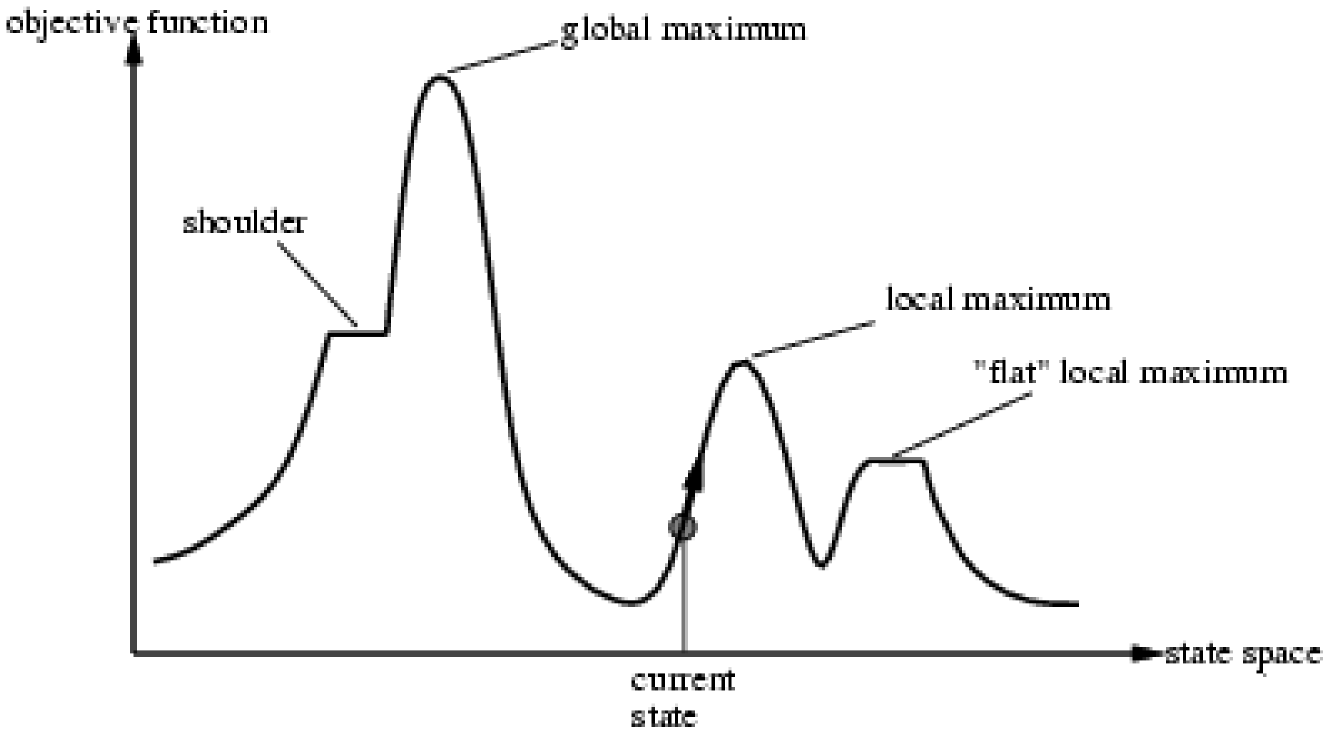
Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"

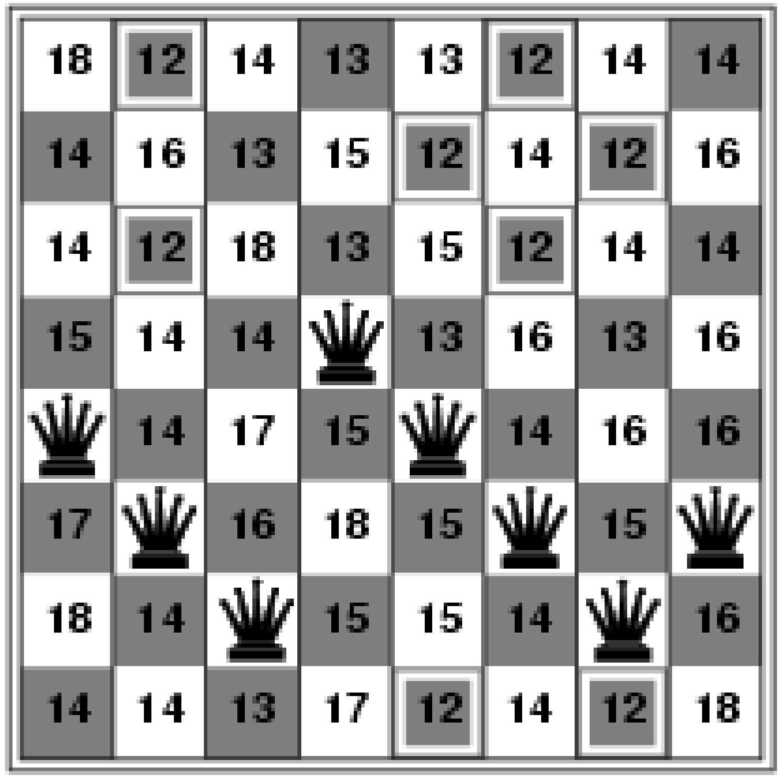
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
      current ← neighbor
```

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima

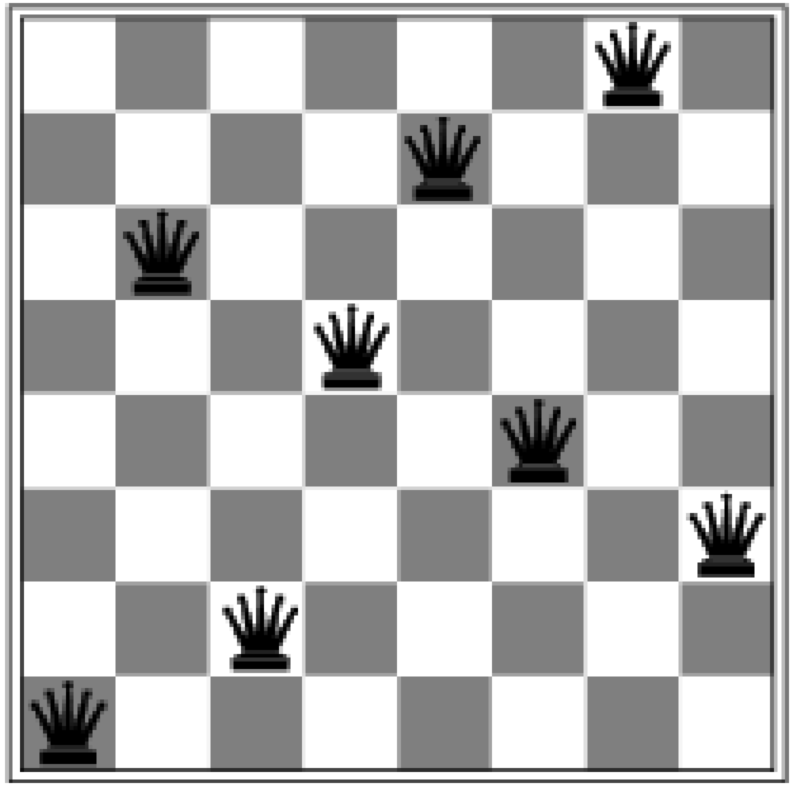


Hill-climbing search: 8-queens problem



- *h* = number of pairs of queens that are attacking each other, either directly or indirectly
- *h = 17* for the above state

Hill-climbing search: 8-queens problem



- A local minimum with *h = 1*

Simulated annealing search

❖ Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency

```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
      T ← schedule[t]
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← VALUE[next] – VALUE[current]
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(Δ E/T)
```

Properties of simulated annealing search

- One can prove: If T decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1
- Widely used in VLSI layout, airline scheduling, etc

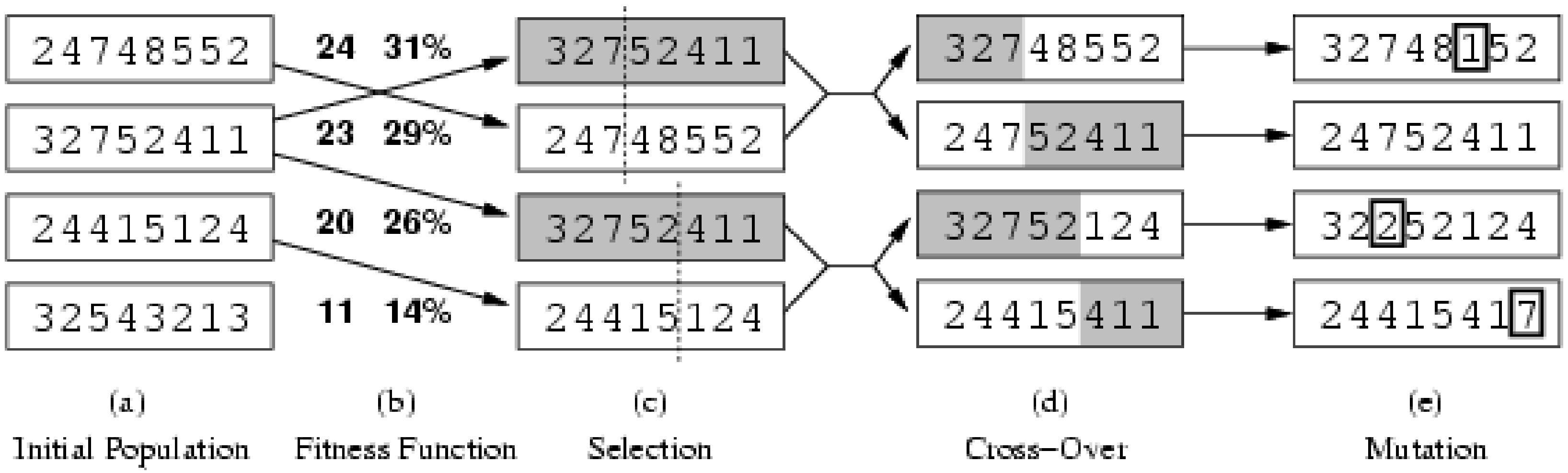
Local beam search

- Keep track of *k* states rather than just one
- Start with *k* randomly generated states
- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat.

Genetic algorithms

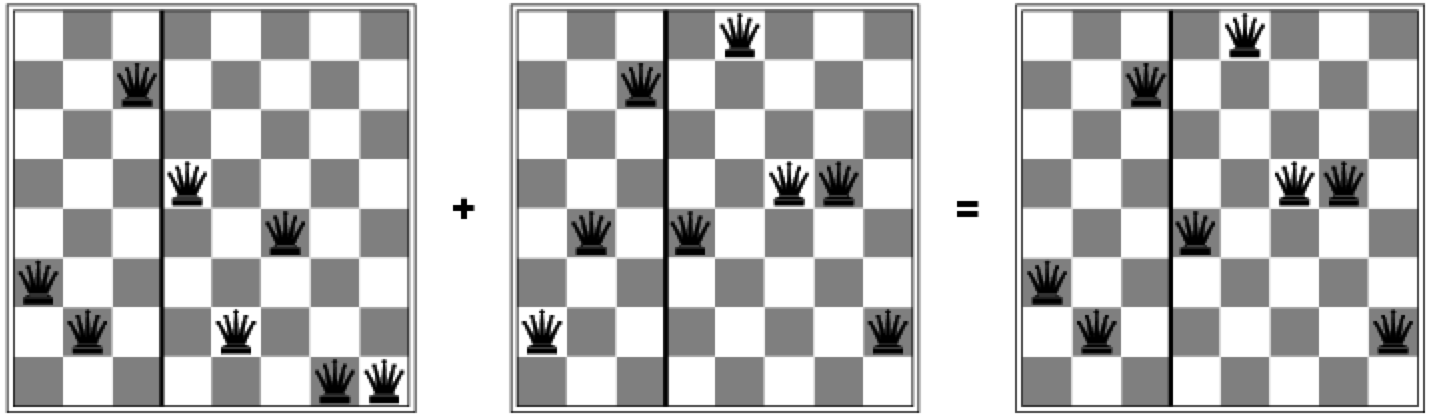
- A successor state is generated by combining two parent states
- Start with *k* randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation

Genetic algorithms



- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

Genetic algorithms



Consent

Data for this course is taken from several books specifically AIMA by Russel Norvig, slides and already similar course taught in other universities specifically MIT, UC-Berkley, and Stanford and is the sole property of the respective owner. The copyright infringement is not intended, and this material is solely used for the academic purpose or as a teaching material.This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted. The work and materials from it should never be made available to students except by instructors using the accompanying text in their classes. All recipients of this work are expected to abide by these restrictions and to honor the intended pedagogical purposes and the needs of other instructors who rely on these materials.