STAT 360: Computational Statistics and Data Analysis

Project - EFA

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Load Relevant R Libraries

```
library(rmarkdown); library(knitr); library(moments);
library(scatterplot3d); library(corrplot); library(pso)
library(psych); library(GPArotation); library(lavaan); library(readxl); library(dplyr)
```

Matrix Construction

```
data_matrix_csv <- read.csv("C:/Users/User/OneDrive - University of St. Thomas/Classes/STAT360/STAT360 I
life_expectancy <- as.matrix(data_matrix_csv)

col_names_unedited <- c(
  "Life expectancy at birth, total (years)",
  "CO2 emissions (metric tons per capita)",
  "Access to electricity (% of population)",
  "Current health expenditure (% of GDP)",
  "Out-of-pocket expenditure (% of current health expenditure)",
  "Domestic private health expenditure per capita, PPP (current international $)",
  "Domestic general government health expenditure per capita, PPP (current international $)",
  "Renewable internal freshwater resources per capita (cubic meters)",
  "Prevalence of HIV, total (% of population ages 15-49)",
  "Unemployment, total (% of total labor force) (national estimate)",
  "Government Effectiveness: Estimate",
  "Income share held by highest 10%",
  "Prevalence of current tobacco use (% of adults)",
  "Total alcohol consumption per capita (liters of pure alcohol, projected estimates, 15+ years of age)
  "Political Stability and Absence of Violence/Terrorism: Estimate",
  "Population density (people per sq. km of land area)"
)

# Better for displaying
col_names <- c(
  "Life_expectancy",
  "CO2_emissions",
  "Electricity",
  "Health_expenditure",
  "Out_of_pocket",
  "Private_health_expenditure",
  "Govt_health_expenditure",
  "Freshwater_resources",
  "HIV_prevalence",
  "Unemployment",
  "Govt_effectiveness",
  "Income_share",
  "Tobacco_use",
```

```
  "Alcohol_consumption",
  "Political_stability",
  "Population_density"
)

colnames(life_expectancy) <- col_names
```

Variance-Covariance Matrix

```
SIGMA <- cov(life_expectancy, use = "pairwise.complete.obs")
SIGMA
```

```
##                            Life_expectancy CO2_emissions Electricity
## Life_expectancy                  56.613599     19.656000   143.90904
## CO2_emissions                    19.656000     21.957050    53.47077
## Electricity                     143.909039     53.470768   600.25600
## Health_expenditure                7.036888      1.901462    20.16410
## Out_of_pocket                   -45.276102    -27.520381   -83.98652
## Private_health_expenditure     2998.698602   1402.368831  6592.53067
## Govt_health_expenditure        7099.652598   3461.022535 14659.67137
## Freshwater_resources          27420.830532  -3207.162537 62163.79862
## HIV_prevalence                  -12.667762     -2.318291   -32.35660
## Unemployment                     -7.344768     -5.174830   -15.24628
## Govt_effectiveness                5.801790      2.442816    14.96731
## Income_share                    -10.354057     -6.177143   -25.58321
## Tobacco_use                      20.834585      2.787175    95.61148
## Alcohol_consumption              11.493254      3.239246    27.62556
## Political_stability               3.952635      1.505941    11.46460
## Population_density             2567.323142    329.437798  4734.90380
##                            Health_expenditure Out_of_pocket
## Life_expectancy                     7.0368878 -4.527610e+01
## CO2_emissions                       1.9014623 -2.752038e+01
## Electricity                        20.1640954 -8.398652e+01
## Health_expenditure                  9.0758757 -1.700235e+01
## Out_of_pocket                     -17.0023465  3.667401e+02
## Private_health_expenditure        937.5047198 -2.023200e+03
## Govt_health_expenditure          2040.0529666 -1.220182e+04
## Freshwater_resources             6400.0362573 -1.133978e+05
## HIV_prevalence                      0.4001537 -2.014671e+01
## Unemployment                        1.0674188 -4.466286e-01
## Govt_effectiveness                  0.9289957 -8.159019e+00
## Income_share                       -1.7383318  1.388092e+01
## Tobacco_use                         7.2202365 -2.657624e+01
## Alcohol_consumption                 2.9649858 -1.592692e+01
## Political_stability                 0.8223059 -9.718820e+00
## Population_density               -555.9827547 -2.308275e+03
##                            Private_health_expenditure Govt_health_expenditure
## Life_expectancy                              2998.6986              7099.6526
## CO2_emissions                                1402.3688              3461.0225
## Electricity                                  6592.5307             14659.6714
## Health_expenditure                            937.5047              2040.0530
```

```
## Out_of_pocket                        -2023.2004            -12201.8233
## Private_health_expenditure          467350.7680             606905.1703
## Govt_health_expenditure             606905.1703            1913107.4928
## Freshwater_resources                 80433.4890            8820802.0892
## HIV_prevalence                        -464.1102               -905.2394
## Unemployment                          -500.0547              -1593.9605
## Govt_effectiveness                     416.1200               1020.3935
## Income_share                          -510.8805              -3343.4847
## Tobacco_use                            998.9118               1982.9352
## Alcohol_consumption                   1088.5776               2719.3186
## Political_stability                    223.1083                687.2039
## Population_density                   81818.8445             252013.8840
##                           Freshwater_resources HIV_prevalence  Unemployment
## Life_expectancy                   2.742083e+04  -1.266776e+01 -7.344768e+00
## CO2_emissions                    -3.207163e+03  -2.318291e+00 -5.174830e+00
## Electricity                       6.216380e+04  -3.235660e+01 -1.524628e+01
## Health_expenditure                6.400036e+03   4.001537e-01  1.067419e+00
## Out_of_pocket                    -1.133978e+05  -2.014671e+01 -4.466286e-01
## Private_health_expenditure        8.043349e+04  -4.641102e+02 -5.000547e+02
## Govt_health_expenditure           8.820802e+06  -9.052394e+02 -1.593960e+03
## Freshwater_resources              2.036196e+09  -1.292663e+04 -1.048168e+04
## HIV_prevalence                   -1.292663e+04   1.741906e+01  8.907005e+00
## Unemployment                     -1.048168e+04   8.907005e+00  2.682782e+01
## Govt_effectiveness                4.203043e+03  -4.820353e-01 -1.439840e+00
## Income_share                      1.660246e+04   1.490213e+00  3.039868e+00
## Tobacco_use                      -3.566012e+04  -6.987135e+00  3.968488e+00
## Alcohol_consumption               8.432471e+03   1.294147e+00 -1.816340e+00
## Political_stability               7.230459e+03   3.476909e-02 -1.214624e+00
## Population_density               -2.534178e+06  -1.748864e+02 -1.438084e+03
##                           Govt_effectiveness Income_share   Tobacco_use
## Life_expectancy                    5.8017902   -10.354057     20.834585
## CO2_emissions                      2.4428155    -6.177143      2.787175
## Electricity                       14.9673140   -25.583207     95.611484
## Health_expenditure                 0.9289957    -1.738332      7.220237
## Out_of_pocket                     -8.1590193    13.880924    -26.576241
## Private_health_expenditure       416.1200285  -510.880549    998.911818
## Govt_health_expenditure         1020.3934947 -3343.484653   1982.935156
## Freshwater_resources            4203.0430596 16602.459395 -35660.123627
## HIV_prevalence                    -0.4820353     1.490213     -6.987135
## Unemployment                      -1.4398405     3.039868      3.968488
## Govt_effectiveness                 0.9865725    -1.921819      1.960994
## Income_share                      -1.9218194    26.068093    -24.833573
## Tobacco_use                        1.9609940   -24.833573     97.527764
## Alcohol_consumption                2.0353220    -7.687166      7.125095
## Political_stability                0.7214839    -1.900931      1.833900
## Population_density               301.9096685  -206.206758     30.179168
##                           Alcohol_consumption Political_stability
## Life_expectancy                     11.493254          3.95263462
## CO2_emissions                        3.239246          1.50594139
## Electricity                         27.625558         11.46460215
## Health_expenditure                   2.964986          0.82230593
## Out_of_pocket                      -15.926924         -9.71881976
## Private_health_expenditure        1088.577629        223.10829419
## Govt_health_expenditure           2719.318552        687.20389160
```

```
## Freshwater_resources               8432.470830       7230.45946128
## HIV_prevalence                        1.294147          0.03476909
## Unemployment                         -1.816340         -1.21462410
## Govt_effectiveness                    2.035322          0.72148389
## Income_share                         -7.687166         -1.90093069
## Tobacco_use                           7.125095          1.83390009
## Alcohol_consumption                  15.794414          1.73172224
## Political_stability                   1.731722          1.00018271
## Population_density                 -218.842196        345.62908802
##                            Population_density
## Life_expectancy                  2.567323e+03
## CO2_emissions                    3.294378e+02
## Electricity                      4.734904e+03
## Health_expenditure              -5.559828e+02
## Out_of_pocket                   -2.308275e+03
## Private_health_expenditure       8.181884e+04
## Govt_health_expenditure          2.520139e+05
## Freshwater_resources            -2.534178e+06
## HIV_prevalence                  -1.748864e+02
## Unemployment                    -1.438084e+03
## Govt_effectiveness               3.019097e+02
## Income_share                    -2.062068e+02
## Tobacco_use                      3.017917e+01
## Alcohol_consumption             -2.188422e+02
## Political_stability              3.456291e+02
## Population_density               3.938961e+06
```

Covariance Matrix (Response/Predictors)

```
SIGMA_p <- SIGMA[2:16,2:16]
SIGMA_p
```

```
##                            CO2_emissions Electricity Health_expenditure
## CO2_emissions                  21.957050    53.47077          1.9014623
## Electricity                    53.470768   600.25600         20.1640954
## Health_expenditure              1.901462    20.16410          9.0758757
## Out_of_pocket                 -27.520381   -83.98652        -17.0023465
## Private_health_expenditure   1402.368831  6592.53067        937.5047198
## Govt_health_expenditure      3461.022535 14659.67137       2040.0529666
## Freshwater_resources        -3207.162537 62163.79862       6400.0362573
## HIV_prevalence                 -2.318291   -32.35660          0.4001537
## Unemployment                   -5.174830   -15.24628          1.0674188
## Govt_effectiveness              2.442816    14.96731          0.9289957
## Income_share                   -6.177143   -25.58321         -1.7383318
## Tobacco_use                     2.787175    95.61148          7.2202365
## Alcohol_consumption             3.239246    27.62556          2.9649858
## Political_stability             1.505941    11.46460          0.8223059
## Population_density            329.437798  4734.90380       -555.9827547
##                            Out_of_pocket Private_health_expenditure
## CO2_emissions              -2.752038e+01                  1402.3688
## Electricity                -8.398652e+01                  6592.5307
## Health_expenditure         -1.700235e+01                   937.5047
```

```
## Out_of_pocket               3.667401e+02                -2023.2004
## Private_health_expenditure -2.023200e+03               467350.7680
## Govt_health_expenditure    -1.220182e+04               606905.1703
## Freshwater_resources       -1.133978e+05                80433.4890
## HIV_prevalence             -2.014671e+01                 -464.1102
## Unemployment               -4.466286e-01                 -500.0547
## Govt_effectiveness         -8.159019e+00                  416.1200
## Income_share                1.388092e+01                 -510.8805
## Tobacco_use                -2.657624e+01                  998.9118
## Alcohol_consumption        -1.592692e+01                 1088.5776
## Political_stability        -9.718820e+00                  223.1083
## Population_density         -2.308275e+03                81818.8445
##                           Govt_health_expenditure Freshwater_resources
## CO2_emissions                          3461.0225        -3.207163e+03
## Electricity                           14659.6714         6.216380e+04
## Health_expenditure                     2040.0530         6.400036e+03
## Out_of_pocket                        -12201.8233        -1.133978e+05
## Private_health_expenditure            606905.1703         8.043349e+04
## Govt_health_expenditure              1913107.4928         8.820802e+06
## Freshwater_resources                 8820802.0892         2.036196e+09
## HIV_prevalence                         -905.2394        -1.292663e+04
## Unemployment                          -1593.9605        -1.048168e+04
## Govt_effectiveness                     1020.3935         4.203043e+03
## Income_share                          -3343.4847         1.660246e+04
## Tobacco_use                            1982.9352        -3.566012e+04
## Alcohol_consumption                    2719.3186         8.432471e+03
## Political_stability                     687.2039         7.230459e+03
## Population_density                   252013.8840        -2.534178e+06
##                           HIV_prevalence  Unemployment Govt_effectiveness
## CO2_emissions              -2.318291e+00 -5.174830e+00          2.4428155
## Electricity                -3.235660e+01 -1.524628e+01         14.9673140
## Health_expenditure          4.001537e-01  1.067419e+00          0.9289957
## Out_of_pocket              -2.014671e+01 -4.466286e-01         -8.1590193
## Private_health_expenditure -4.641102e+02 -5.000547e+02        416.1200285
## Govt_health_expenditure    -9.052394e+02 -1.593960e+03       1020.3934947
## Freshwater_resources       -1.292663e+04 -1.048168e+04       4203.0430596
## HIV_prevalence              1.741906e+01  8.907005e+00         -0.4820353
## Unemployment                8.907005e+00  2.682782e+01         -1.4398405
## Govt_effectiveness         -4.820353e-01 -1.439840e+00          0.9865725
## Income_share                1.490213e+00  3.039868e+00         -1.9218194
## Tobacco_use                -6.987135e+00  3.968488e+00          1.9609940
## Alcohol_consumption         1.294147e+00 -1.816340e+00          2.0353220
## Political_stability         3.476909e-02 -1.214624e+00          0.7214839
## Population_density         -1.748864e+02 -1.438084e+03        301.9096685
##                           Income_share   Tobacco_use Alcohol_consumption
## CO2_emissions                -6.177143      2.787175            3.239246
## Electricity                 -25.583207     95.611484           27.625558
## Health_expenditure           -1.738332      7.220237            2.964986
## Out_of_pocket                13.880924    -26.576241          -15.926924
## Private_health_expenditure -510.880549    998.911818         1088.577629
## Govt_health_expenditure    -3343.484653   1982.935156         2719.318552
## Freshwater_resources       16602.459395 -35660.123627         8432.470830
## HIV_prevalence                 1.490213     -6.987135            1.294147
## Unemployment                   3.039868      3.968488           -1.816340
```

```
## Govt_effectiveness            -1.921819      1.960994           2.035322
## Income_share                  26.068093    -24.833573          -7.687166
## Tobacco_use                  -24.833573     97.527764           7.125095
## Alcohol_consumption           -7.687166      7.125095          15.794414
## Political_stability           -1.900931      1.833900           1.731722
## Population_density          -206.206758     30.179168        -218.842196
##                           Political_stability Population_density
## CO2_emissions                      1.50594139       3.294378e+02
## Electricity                       11.46460215       4.734904e+03
## Health_expenditure                 0.82230593      -5.559828e+02
## Out_of_pocket                     -9.71881976      -2.308275e+03
## Private_health_expenditure       223.10829419       8.181884e+04
## Govt_health_expenditure          687.20389160       2.520139e+05
## Freshwater_resources            7230.45946128      -2.534178e+06
## HIV_prevalence                     0.03476909      -1.748864e+02
## Unemployment                      -1.21462410      -1.438084e+03
## Govt_effectiveness                 0.72148389       3.019097e+02
## Income_share                      -1.90093069      -2.062068e+02
## Tobacco_use                        1.83390009       3.017917e+01
## Alcohol_consumption                1.73172224      -2.188422e+02
## Political_stability                1.00018271       3.456291e+02
## Population_density               345.62908802       3.938961e+06
```

```
SIGMA_r  <- SIGMA[1,1]
SIGMA_r  <- as.matrix(SIGMA_r)
SIGMA_r
```

```
##          [,1]
## [1,] 56.6136
```

Correlation Matrix (Predictor/Response)

```
R_p <- (solve(sqrt(diag(diag(SIGMA_p))))) %*% SIGMA_p %*%
t(solve(sqrt(diag(diag(SIGMA_p)))))
dimnames(R_p) <- list(c("CO2_emissions", "Electricity", "Health_expenditure", "Out_of_pocket", "Private_
  "Govt_health_expenditure","Freshwater_resources","HIV_prevalence","Unemployment", "Govt_effectiveness
  "Income_share","Tobacco_use","Alcohol_consumption", "Political_stability", "Population_density"),
c("CO2_emissions", "Electricity", "Health_expenditure", "Out_of_pocket", "Private_health_expenditure",
  "Freshwater_resources", "HIV_prevalence", "Unemployment", "Govt_effectiveness", "Income_share", "Toba
  "Alcohol_consumption","Political_stability", "Population_density"))
R_p
```

```
##                           CO2_emissions Electricity Health_expenditure
## CO2_emissions                1.00000000  0.46575891         0.13469655
## Electricity                  0.46575891  1.00000000         0.27319089
## Health_expenditure           0.13469655  0.27319089         1.00000000
## Out_of_pocket               -0.30668199 -0.17900377        -0.29470359
## Private_health_expenditure   0.43777784  0.39360665         0.45520572
## Govt_health_expenditure      0.53400798  0.43259978         0.48958430
## Freshwater_resources        -0.01516786  0.05622889         0.04707913
## HIV_prevalence              -0.11854103 -0.31643340         0.03182513
```

```
## Unemployment                 -0.21321433 -0.12014420         0.06840657
## Govt_effectiveness            0.52485479  0.61505096         0.31045946
## Income_share                 -0.25819392 -0.20451827        -0.11301440
## Tobacco_use                   0.06023004  0.39516427         0.24268505
## Alcohol_consumption           0.17394230  0.28372072         0.24764330
## Political_stability           0.32135229  0.46789786         0.27292887
## Population_density            0.03542384  0.09737603        -0.09298785
##                           Out_of_pocket Private_health_expenditure
## CO2_emissions              -0.306681993                0.437777843
## Electricity                -0.179003767                0.393606646
## Health_expenditure         -0.294703586                0.455205721
## Out_of_pocket               1.000000000               -0.154539057
## Private_health_expenditure -0.154539057                1.000000000
## Govt_health_expenditure    -0.460655088                0.641844273
## Freshwater_resources       -0.131224750                0.002607388
## HIV_prevalence             -0.252065082               -0.162662457
## Unemployment               -0.004502717               -0.141222299
## Govt_effectiveness         -0.428937758                0.612819570
## Income_share                0.141966077               -0.146367028
## Tobacco_use                -0.140523917                0.147959051
## Alcohol_consumption        -0.209267115                0.400669390
## Political_stability        -0.507451668                0.326328169
## Population_density         -0.060731989                0.060303293
##                           Govt_health_expenditure Freshwater_resources
## CO2_emissions                           0.5340080         -0.015167857
## Electricity                             0.4325998          0.056228895
## Health_expenditure                      0.4895843          0.047079129
## Out_of_pocket                          -0.4606551         -0.131224750
## Private_health_expenditure              0.6418443          0.002607388
## Govt_health_expenditure                 1.0000000          0.141328142
## Freshwater_resources                    0.1413281          1.000000000
## HIV_prevalence                         -0.1568128         -0.068637790
## Unemployment                           -0.2224924         -0.044846505
## Govt_effectiveness                      0.7427343          0.093775542
## Income_share                           -0.4734510          0.072062289
## Tobacco_use                             0.1451693         -0.080021920
## Alcohol_consumption                     0.4946963          0.047021161
## Political_stability                     0.4967939          0.160219909
## Population_density                      0.0918045         -0.028296754
##                           HIV_prevalence Unemployment Govt_effectiveness
## CO2_emissions               -0.118541027 -0.213214335         0.52485479
## Electricity                 -0.316433404 -0.120144200         0.61505096
## Health_expenditure           0.031825130  0.068406573         0.31045946
## Out_of_pocket               -0.252065082 -0.004502717        -0.42893776
## Private_health_expenditure  -0.162662457 -0.141222299         0.61281957
## Govt_health_expenditure     -0.156812767 -0.222492374         0.74273430
## Freshwater_resources        -0.068637790 -0.044846505         0.09377554
## HIV_prevalence               1.000000000  0.412027888        -0.11627919
## Unemployment                 0.412027888  1.000000000        -0.27987051
## Govt_effectiveness          -0.116279194 -0.279870508         1.00000000
## Income_share                 0.069932862  0.114949559        -0.37896005
## Tobacco_use                 -0.169520673  0.077583302         0.19991603
## Alcohol_consumption          0.078022415 -0.088237377         0.51560467
## Political_stability          0.008329928 -0.234481992         0.72631072
```

```
## Population_density           -0.021113139 -0.139894553         0.15315165
##                            Income_share  Tobacco_use Alcohol_consumption
## CO2_emissions               -0.25819392  0.060230042          0.17394230
## Electricity                 -0.20451827  0.395164270          0.28372072
## Health_expenditure          -0.11301440  0.242685047          0.24764330
## Out_of_pocket                0.14196608 -0.140523917         -0.20926711
## Private_health_expenditure  -0.14636703  0.147959051          0.40066939
## Govt_health_expenditure     -0.47345098  0.145169270          0.49469627
## Freshwater_resources         0.07206229 -0.080021920          0.04702116
## HIV_prevalence               0.06993286 -0.169520673          0.07802241
## Unemployment                 0.11494956  0.077583302         -0.08823738
## Govt_effectiveness          -0.37896005  0.199916032          0.51560467
## Income_share                 1.00000000 -0.492516119         -0.37884352
## Tobacco_use                 -0.49251612  1.000000000          0.18154102
## Alcohol_consumption         -0.37884352  0.181541018          1.00000000
## Political_stability         -0.37228195  0.185682881          0.43569924
## Population_density          -0.02034967  0.001539757         -0.02774524
##                            Political_stability Population_density
## CO2_emissions                        0.321352294        0.035423837
## Electricity                          0.467897863        0.097376032
## Health_expenditure                   0.272928871       -0.092987852
## Out_of_pocket                       -0.507451668       -0.060731989
## Private_health_expenditure           0.326328169        0.060303293
## Govt_health_expenditure              0.496793898        0.091804496
## Freshwater_resources                 0.160219909       -0.028296754
## HIV_prevalence                       0.008329928       -0.021113139
## Unemployment                        -0.234481992       -0.139894553
## Govt_effectiveness                   0.726310723        0.153151648
## Income_share                        -0.372281948       -0.020349673
## Tobacco_use                          0.185682881        0.001539757
## Alcohol_consumption                  0.435699241       -0.027745239
## Political_stability                  1.000000000        0.174132479
## Population_density                   0.174132479        1.000000000
```

```
R_r <- cor(SIGMA_r, use = "pairwise.complete.obs")
R_r
```

```
##      [,1]
## [1,]   NA
```

Eigenvalues for response/predictor

```
predictor <- eigen(R_p)$values
predictor
```

```
##  [1] 4.82235433 1.63890455 1.39841899 1.18562862 1.04308940 0.96650885
##  [7] 0.86493263 0.73485866 0.66266370 0.42142900 0.39900019 0.37160620
## [13] 0.24974275 0.14573850 0.09512362
```

Intrinsic Dimensionality: Predictors

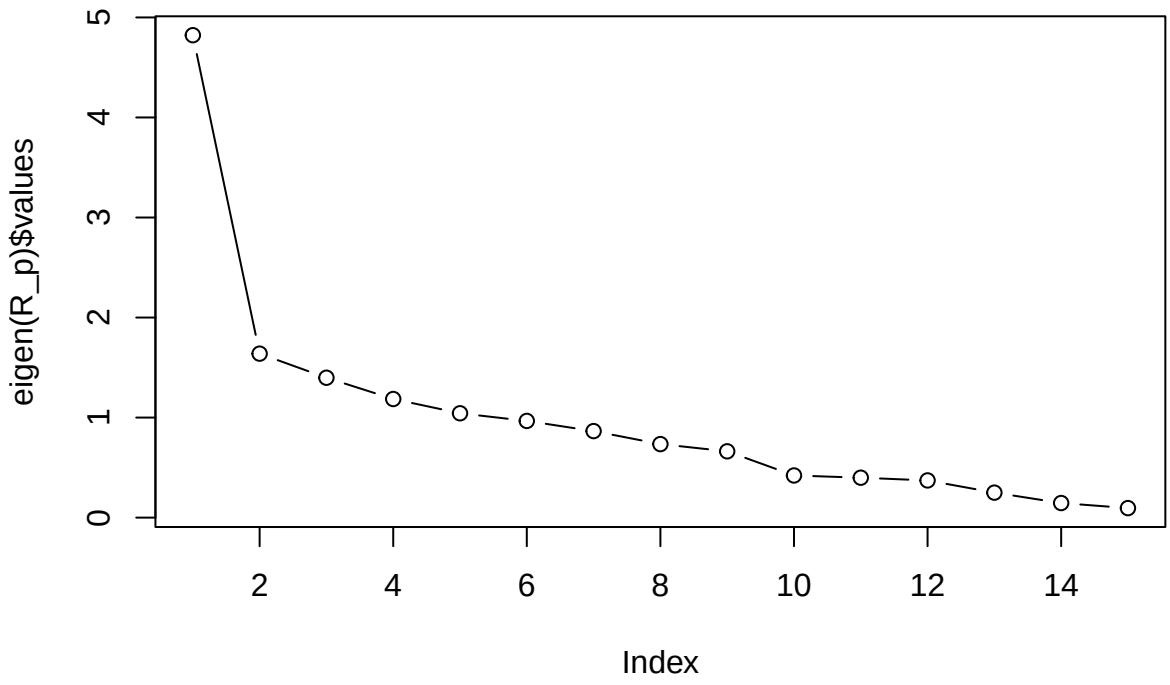
```
sum(eigen(R_p)$values>1.0)
```

```
## [1] 5
```

```
sum(eigen(R_p)$values>0.7)
```

```
## [1] 8
```

```
plot(eigen(R_p)$values, type = "b")
```



We applied 3 types of criterion to the response variable.

For Kaiser's criterion, 5 values are retained since only 5 values are above 1.

For Jolliffe's criterion, 8 values are retained because they are above the threshold of 0.7.

We think that Kaiser's criterion and Jolliffe's criterion represents our dimensionality the best. The average of these is 6.5, which we will round to an intrinsic dimensionality of 7.

Intrinsic Dimensionality: Response

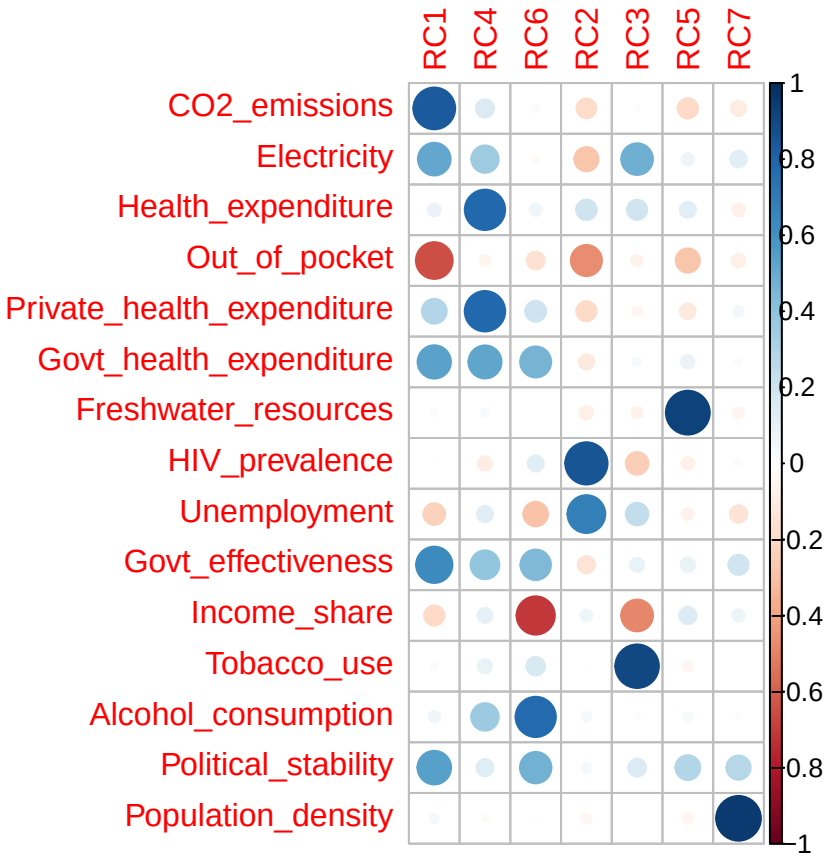
Since we only have a since response dimension are intrinsic dimensionality will be 1 no matter what criterion is applied.

Rotated Loading Matrix: Orthogonal

```
A <- pca(r = R_p, nfactors = 7, rotate = "varimax")$loadings[]
A
```

```
##                                    RC1         RC4           RC6          RC2
## CO2_emissions                0.838863469  0.15655189  0.0250232501 -0.186141974
## Electricity                  0.513183429  0.35953960 -0.0224677469 -0.279170326
## Health_expenditure           0.082302220  0.78090107  0.0666188458  0.201506319
## Out_of_pocket               -0.647539853 -0.05834038 -0.1551741701 -0.464377555
## Private_health_expenditure   0.294858420  0.78579417  0.2089348629 -0.194018364
## Govt_health_expenditure      0.531525704  0.52508965  0.4606357946 -0.114325812
## Freshwater_resources         0.020561709  0.03065211  0.0006122306 -0.085187608
## HIV_prevalence              -0.005978193 -0.09723961  0.1249349452  0.856226022
## Unemployment                -0.224721202  0.12602433 -0.2887392784  0.686701478
## Govt_effectiveness           0.631623417  0.39841327  0.4455757991 -0.142494502
## Income_share                -0.195819426  0.10642848 -0.7015406305  0.066027268
## Tobacco_use                  0.022878027  0.09670803  0.1679373941 -0.005447113
## Alcohol_consumption          0.063461299  0.36201641  0.7776371656  0.045564988
## Political_stability          0.543312666  0.13817200  0.4744371596  0.048291089
## Population_density           0.041983359 -0.02036066 -0.0070765113 -0.048971079
##                                    RC3         RC5           RC7
## CO2_emissions               -0.012699993 -0.19780021 -0.1090446216
## Electricity                  0.481598350  0.07059113  0.1296169655
## Health_expenditure           0.195517869  0.12444446 -0.0785017889
## Out_of_pocket               -0.061688755 -0.27959349 -0.0871601818
## Private_health_expenditure -0.048708961 -0.11954524  0.0525814408
## Govt_health_expenditure      0.033591766  0.08732441  0.0222214798
## Freshwater_resources        -0.060820052  0.92261949 -0.0565467485
## HIV_prevalence              -0.245676004 -0.07897422  0.0236646026
## Unemployment                 0.242474018 -0.06653961 -0.1479993671
## Govt_effectiveness           0.094975286  0.09725086  0.1945884382
## Income_share                -0.485358948  0.14303710  0.0731714592
## Tobacco_use                  0.909030697 -0.04828197  0.0005949483
## Alcohol_consumption          0.014501672  0.03910762 -0.0127114543
## Political_stability          0.152541291  0.29487405  0.2823518120
## Population_density           0.001777788 -0.05475001  0.9592584002
```

```
corrplot(A)
```



Rotated Loading Matrix: Oblique

```
A_1 <- pca(r = R_p, nfactors = 7, rotate = "oblimin")$loadings[]
A_1
```

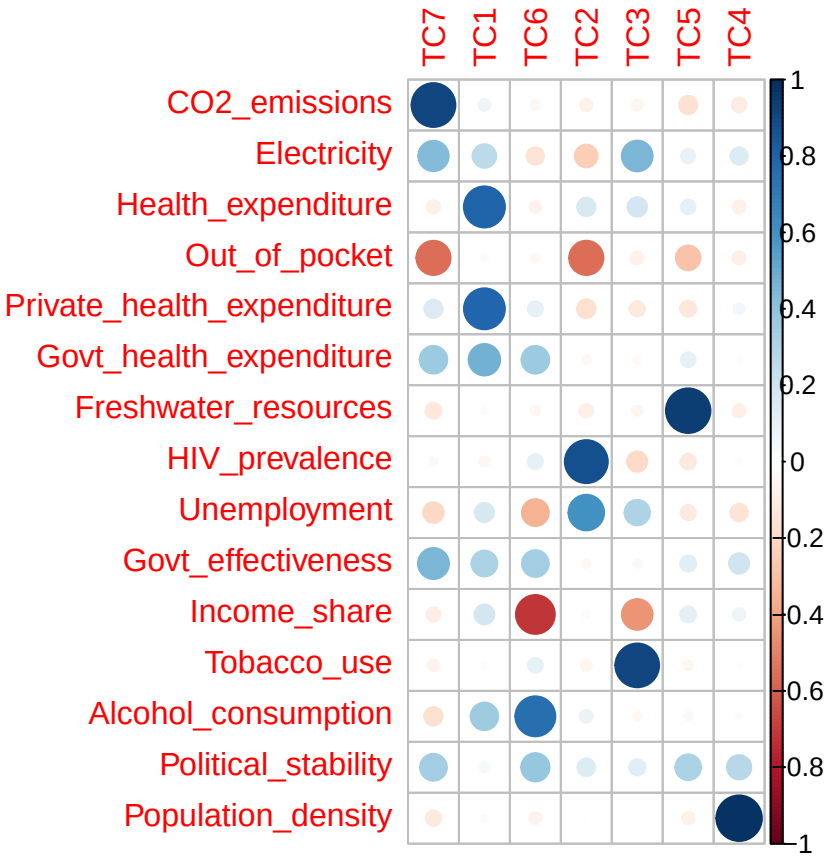
```
##                                 TC7         TC1         TC6          TC2
## CO2_emissions                0.91292105  0.06824601 -0.04368914 -0.070692456
## Electricity                  0.43810561  0.26430140 -0.14527759 -0.246144418
## Health_expenditure          -0.08143382  0.80039874 -0.06493078  0.162009475
## Out_of_pocket               -0.55495965  0.02503986 -0.03992621 -0.557601765
## Private_health_expenditure   0.15798584  0.79759735  0.10827996 -0.165090703
## Govt_health_expenditure      0.36178028  0.47121841  0.36456718 -0.035018485
## Freshwater_resources        -0.12053484 -0.02095155 -0.04080737 -0.089934349
## HIV_prevalence              -0.03105566 -0.04884959  0.10928640  0.876910548
## Unemployment                -0.20104884  0.17425831 -0.34337380  0.601571582
## Govt_effectiveness           0.45680440  0.31836595  0.34101473 -0.039168821
## Income_share                -0.09429021  0.18547755 -0.71740121  0.027917919
## Tobacco_use                 -0.06363029  0.02228116  0.10566238 -0.058676563
## Alcohol_consumption         -0.16097971  0.35973429  0.75958810  0.080508193
## Political_stability          0.34141558  0.04334593  0.38121050  0.148706222
## Population_density          -0.11208442 -0.02373132 -0.06822284  0.007060591
##                                 TC3         TC5         TC4
## CO2_emissions               -0.055741399 -0.15950296 -0.10425962
## Electricity                  0.450325302  0.09461502  0.14154164
```

```
## Health_expenditure          0.180220891  0.10033562 -0.07988782
## Out_of_pocket              -0.077097205 -0.28834776 -0.08084324
## Private_health_expenditure -0.111624503 -0.12434116  0.05774399
## Govt_health_expenditure    -0.027237273  0.10459421  0.01893359
## Freshwater_resources       -0.050869867  0.94342928 -0.08095606
## HIV_prevalence             -0.195333118 -0.11773996  0.01702314
## Unemployment                0.306808747 -0.11039307 -0.14656897
## Govt_effectiveness          0.038470457  0.12007778  0.19633490
## Income_share               -0.447761837  0.11821412  0.07285263
## Tobacco_use                 0.910329164 -0.04302551  0.01263035
## Alcohol_consumption        -0.038004530  0.03788881 -0.02199420
## Political_stability         0.125065796  0.31531745  0.27996805
## Population_density          0.006382124 -0.07516463  0.98504437
```

```
F_1 <- pca(r = R_p, nfactors = 7, rotate = "oblimin")$Phi
F_1
```

```
##             TC7          TC1         TC6          TC2          TC3         TC5
## TC7  1.00000000  0.311026653  0.30493317 -0.092059528  0.123704638 0.15510938
## TC1  0.31102665  1.000000000  0.22001416 -0.001240454  0.155344528 0.13779566
## TC6  0.30493317  0.220014162  1.00000000 -0.017428959  0.155290960 0.09355230
## TC2 -0.09205953 -0.001240454 -0.01742896  1.000000000 -0.001664951 0.03276992
## TC3  0.12370464  0.155344528  0.15529096 -0.001664951  1.000000000 0.01162580
## TC5  0.15510938  0.137795661  0.09355230  0.032769923  0.011625804 1.00000000
## TC4  0.19865602  0.067611664  0.13001281 -0.058940602  0.010842147 0.07899436
##             TC4
## TC7  0.19865602
## TC1  0.06761166
## TC6  0.13001281
## TC2 -0.05894060
## TC3  0.01084215
## TC5  0.07899436
## TC4  1.00000000
```

```
corrplot(A_1)
```



There are correlations among factors above 0.3 , so oblique rotation is indeed necessary.

Communality

```
pca(r = R_p, nfactors = 7, rotate = "oblimin")$communality
```

```
##              CO2_emissions                Electricity
##                  0.8146524                  0.7247875
##         Health_expenditure               Out_of_pocket
##                  0.7214992                  0.7520119
## Private_health_expenditure    Govt_health_expenditure
##                  0.8051393                  0.7927422
##       Freshwater_resources             HIV_prevalence
##                  0.8667430                  0.8253767
##               Unemployment         Govt_effectiveness
##                  0.7064360                  0.8328664
##               Income_share                Tobacco_use
##                  0.8075781                  0.8667768
##        Alcohol_consumption         Political_stability
##                  0.7437802                  0.7316449
##         Population_density
##                  0.9278028
```

For a seven factor solution, all the dimensions retain above 70% of the variance of the original dimensions. This is a good because we avoid excessive data loss.

Variance

```
pca(r = R_p, nfactors = 7, rotate = "oblimin")
```

```
## Principal Components Analysis
## Call: principal(r = r, nfactors = nfactors, residuals = residuals,
##     rotate = rotate, n.obs = n.obs, covar = covar, scores = scores,
##     missing = missing, impute = impute, oblique.scores = oblique.scores,
##     method = method, use = use, cor = cor, correct = 0.5, weight = NULL)
## Standardized loadings (pattern matrix) based upon correlation matrix
##                              TC7   TC1   TC6   TC2   TC3   TC5   TC4   h2    u2
## CO2_emissions               0.91  0.07 -0.04 -0.07 -0.06 -0.16 -0.10 0.81 0.185
## Electricity                 0.44  0.26 -0.15 -0.25  0.45  0.09  0.14 0.72 0.275
## Health_expenditure         -0.08  0.80 -0.06  0.16  0.18  0.10 -0.08 0.72 0.279
## Out_of_pocket              -0.55  0.03 -0.04 -0.56 -0.08 -0.29 -0.08 0.75 0.248
## Private_health_expenditure  0.16  0.80  0.11 -0.17 -0.11 -0.12  0.06 0.81 0.195
## Govt_health_expenditure     0.36  0.47  0.36 -0.04 -0.03  0.10  0.02 0.79 0.207
## Freshwater_resources       -0.12 -0.02 -0.04 -0.09 -0.05  0.94 -0.08 0.87 0.133
## HIV_prevalence             -0.03 -0.05  0.11  0.88 -0.20 -0.12  0.02 0.83 0.175
## Unemployment               -0.20  0.17 -0.34  0.60  0.31 -0.11 -0.15 0.71 0.294
## Govt_effectiveness          0.46  0.32  0.34 -0.04  0.04  0.12  0.20 0.83 0.167
## Income_share               -0.09  0.19 -0.72  0.03 -0.45  0.12  0.07 0.81 0.192
## Tobacco_use                -0.06  0.02  0.11 -0.06  0.91 -0.04  0.01 0.87 0.133
## Alcohol_consumption        -0.16  0.36  0.76  0.08 -0.04  0.04 -0.02 0.74 0.256
## Political_stability         0.34  0.04  0.38  0.15  0.13  0.32  0.28 0.73 0.268
## Population_density         -0.11 -0.02 -0.07  0.01  0.01 -0.08  0.99 0.93 0.072
##                             com
## CO2_emissions               1.1
## Electricity                 3.8
## Health_expenditure          1.3
## Out_of_pocket               2.6
## Private_health_expenditure  1.3
## Govt_health_expenditure     3.0
## Freshwater_resources        1.1
## HIV_prevalence              1.2
## Unemployment                2.9
## Govt_effectiveness          3.4
## Income_share                2.0
## Tobacco_use                 1.1
## Alcohol_consumption         1.6
## Political_stability         4.5
## Population_density          1.0
##
##                        TC7  TC1  TC6  TC2  TC3  TC5  TC4
## SS loadings           2.25 2.14 1.93 1.61 1.52 1.25 1.22
## Proportion Var        0.15 0.14 0.13 0.11 0.10 0.08 0.08
## Cumulative Var        0.15 0.29 0.42 0.53 0.63 0.71 0.79
## Proportion Explained  0.19 0.18 0.16 0.13 0.13 0.11 0.10
## Cumulative Proportion 0.19 0.37 0.53 0.67 0.79 0.90 1.00
##
##  With component correlations of
##       TC7  TC1   TC6   TC2  TC3  TC5   TC4
## TC7  1.00 0.31  0.30 -0.09 0.12 0.16  0.20
```

```
## TC1  0.31 1.00  0.22  0.00 0.16 0.14  0.07
## TC6  0.30 0.22  1.00 -0.02 0.16 0.09  0.13
## TC2 -0.09 0.00 -0.02  1.00 0.00 0.03 -0.06
## TC3  0.12 0.16  0.16  0.00 1.00 0.01  0.01
## TC5  0.16 0.14  0.09  0.03 0.01 1.00  0.08
## TC4  0.20 0.07  0.13 -0.06 0.01 0.08  1.00
##
## Mean item complexity =  2.1
## Test of the hypothesis that 7 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.06
##
## Fit based upon off diagonal values = 0.95
```

For a seven factor solution,

- TC7 accounts for 19% of the entire variance
- TC1 accounts for 18% of the entire variance
- TC6 accounts for 16% of the entire variance
- TC2 accounts for 13% of the entire variance
- TC3 accounts for 13% of the entire variance
- TC5 accounts for 11% of the entire variance
- TC4 accounts for 10% of the entire variance