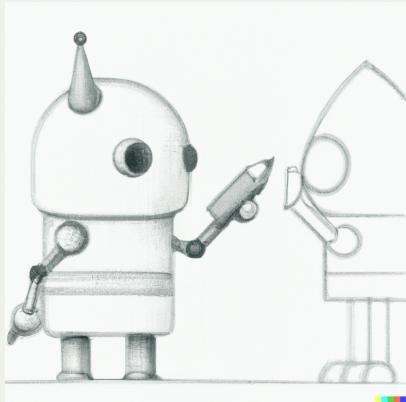


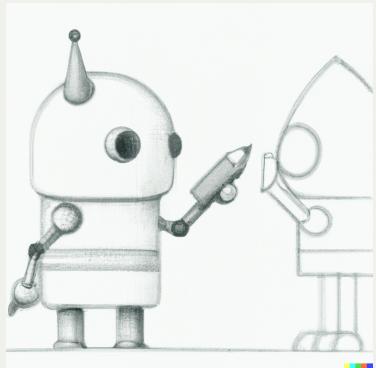
Hierarchical Quantum Circuit Representations



A cute robot building itself with artificial intelligence,
pencil drawing - DALL-E 2



Hierarchical Quantum Circuit Representations



A cute robot building itself with artificial intelligence, pencil drawing - DALL-E 2



A cute robot building itself with artificial intelligence, pencil drawing - DALL-E 3



Outline



HierarQcal

Design and implement hierarchical
compute graphs



Hierarchical Quantum
Circuit Representations

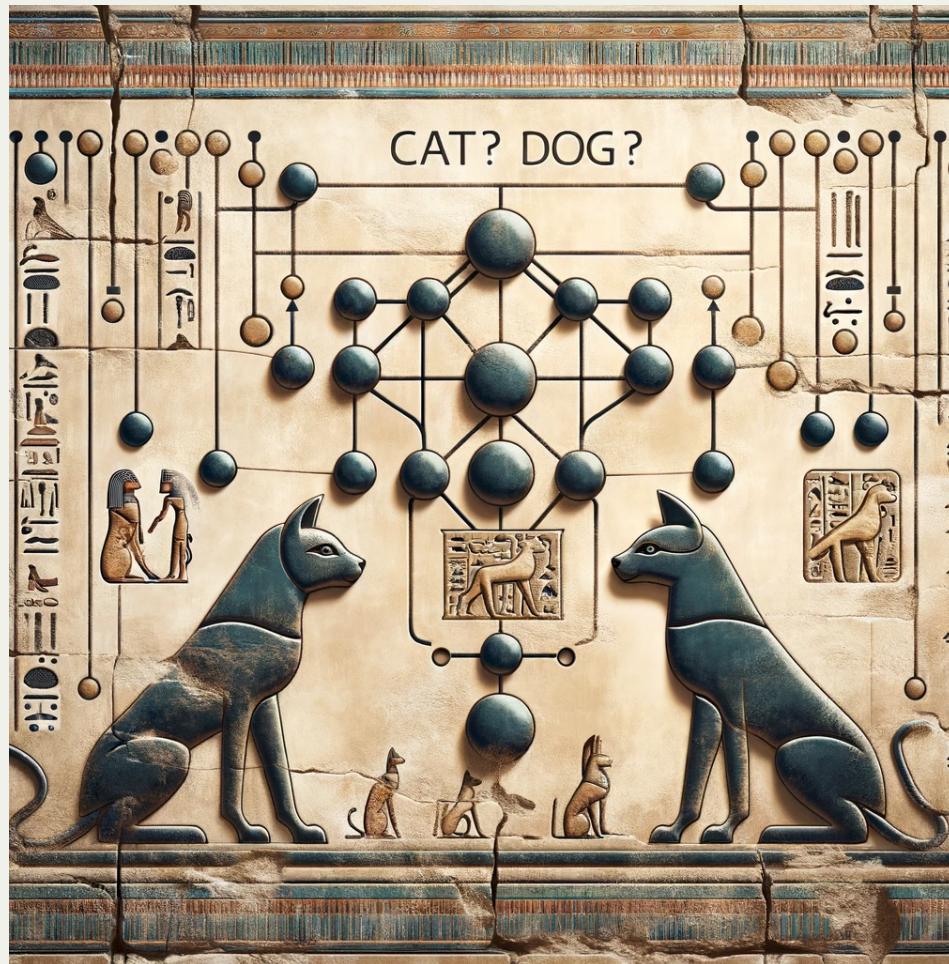
Theoretical framework behind package

This talk

- Background
- Overview of Representation
- Results and usage

Background

Neural Networks



Images Generated with DALL-E 3

AlexNet

ImageNet Classification with Deep Convolutional Neural Networks

Alex Krizhevsky
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kriz@cs.utoronto.ca

Ilya Sutskever
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ilya@cs.utoronto.ca

Geoffrey E. Hinton
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Abstract

We trained a large, deep convolutional neural network to classify the 1.2 million high-resolution images in the ImageNet LSVRC-2010 contest into the 1000 different classes. On the test data, we achieved top-1 and top-5 error rates of 37.5% and 17.0% which is considerably better than the previous state-of-the-art. The neural network, which has 60 million parameters and 650,000 neurons, consists of five convolutional layers, some of which are followed by max-pooling layers, and three fully-connected layers with a final 1000-way softmax. To make training faster, we used non-saturating neurons and a very efficient GPU implementation of the convolution operation. To reduce overfitting in the fully-connected layers we employed a recently-developed regularization method called “dropout” that proved to be very effective. We also entered a variant of this model in the ILSVRC-2012 competition and achieved a winning top-5 test error rate of 15.3%, compared to 26.2% achieved by the second-best entry.

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Model	Top-1	Top-5
<i>Sparse coding [2]</i>	47.1%	28.2%
<i>SIFT + FVs [24]</i>	45.7%	25.7%
CNN	37.5%	17.0%

Table 1: Comparison of results on ILSVRC-2010 test set. In *italics* are best results achieved by others.

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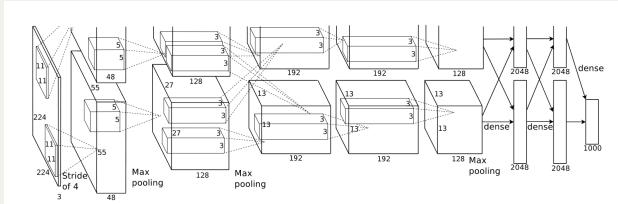
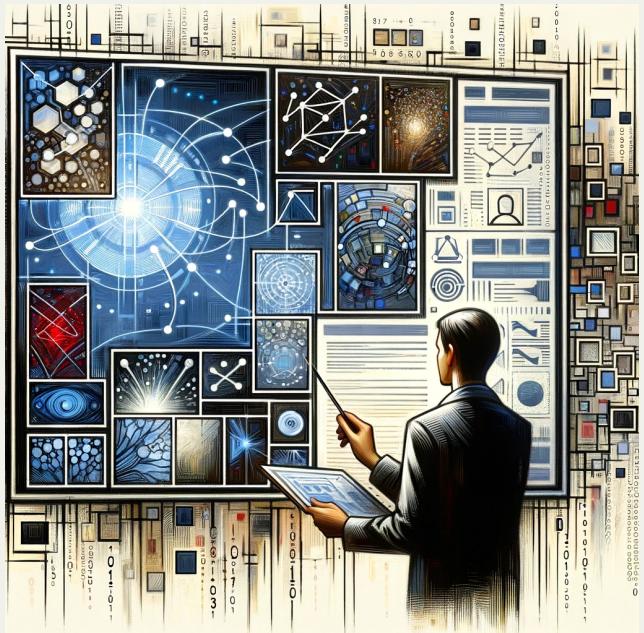


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network’s input is 150,528-dimensional, and the number of neurons in the network’s remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

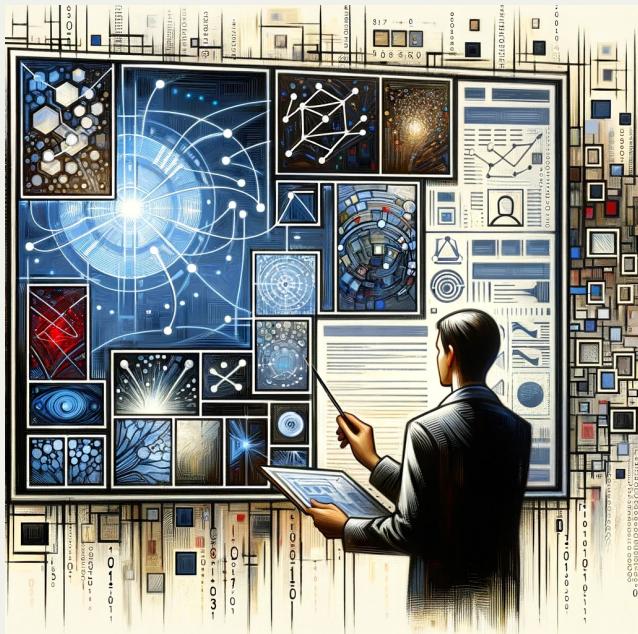
Automation



Images Generated with DALL-E 3

Automation

Manual



Automated

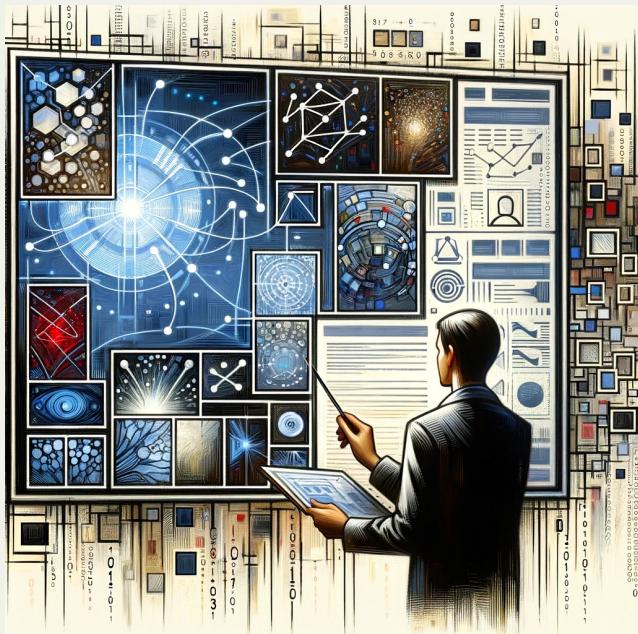


→
Feature
Engineering

Images Generated with DALL-E 3

Automation

Manual



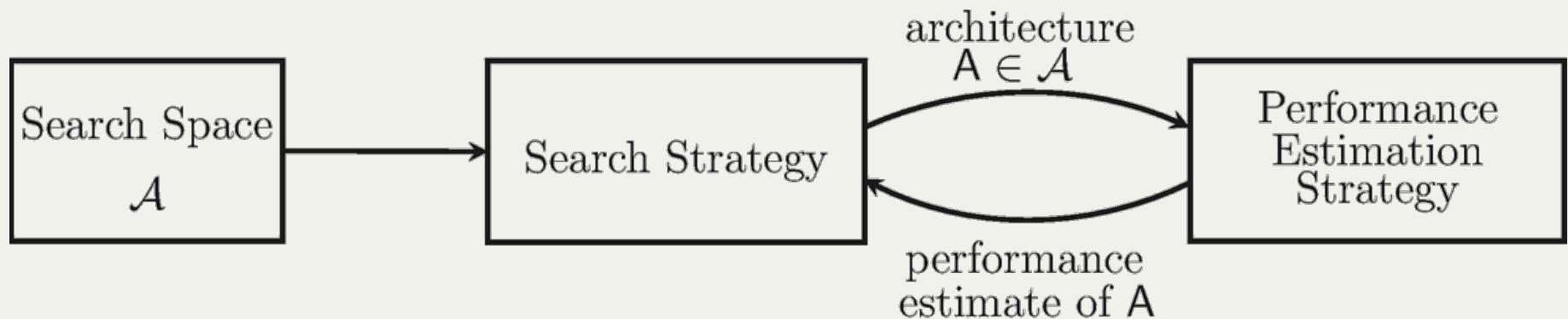
Automated



Architecture
Engineering?

Images Generated with DALL-E 3

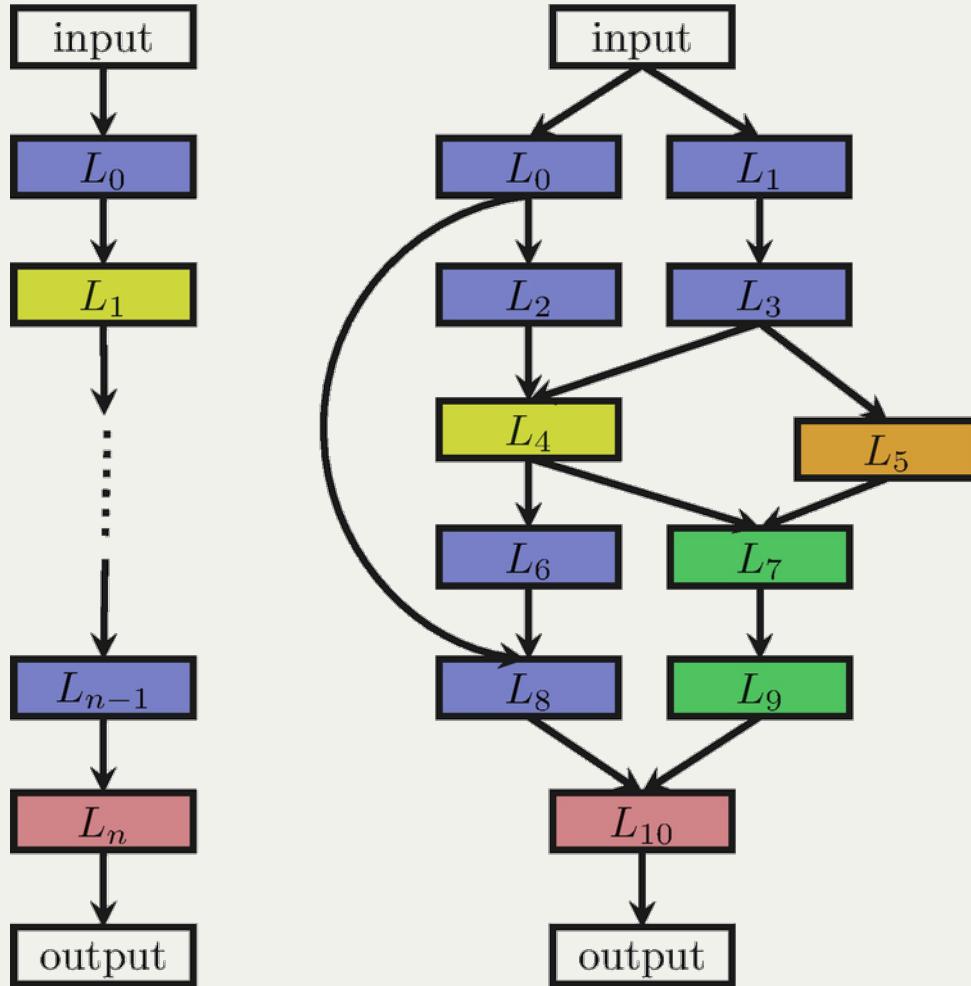
Neural Architecture Search



***Fig:** Abstract illustration of Neural Architecture Search methods. A search strategy selects an architecture A from a predefined search space \mathcal{A} . The architecture is passed to a performance estimation strategy, which returns the estimated performance of A to the search strategy.

*Elsken, T., Metzen, J. H. & Hutter, F. Neural architecture search: a survey. *J. Mach. Learn. Res.* 20, 1-21 (2019).

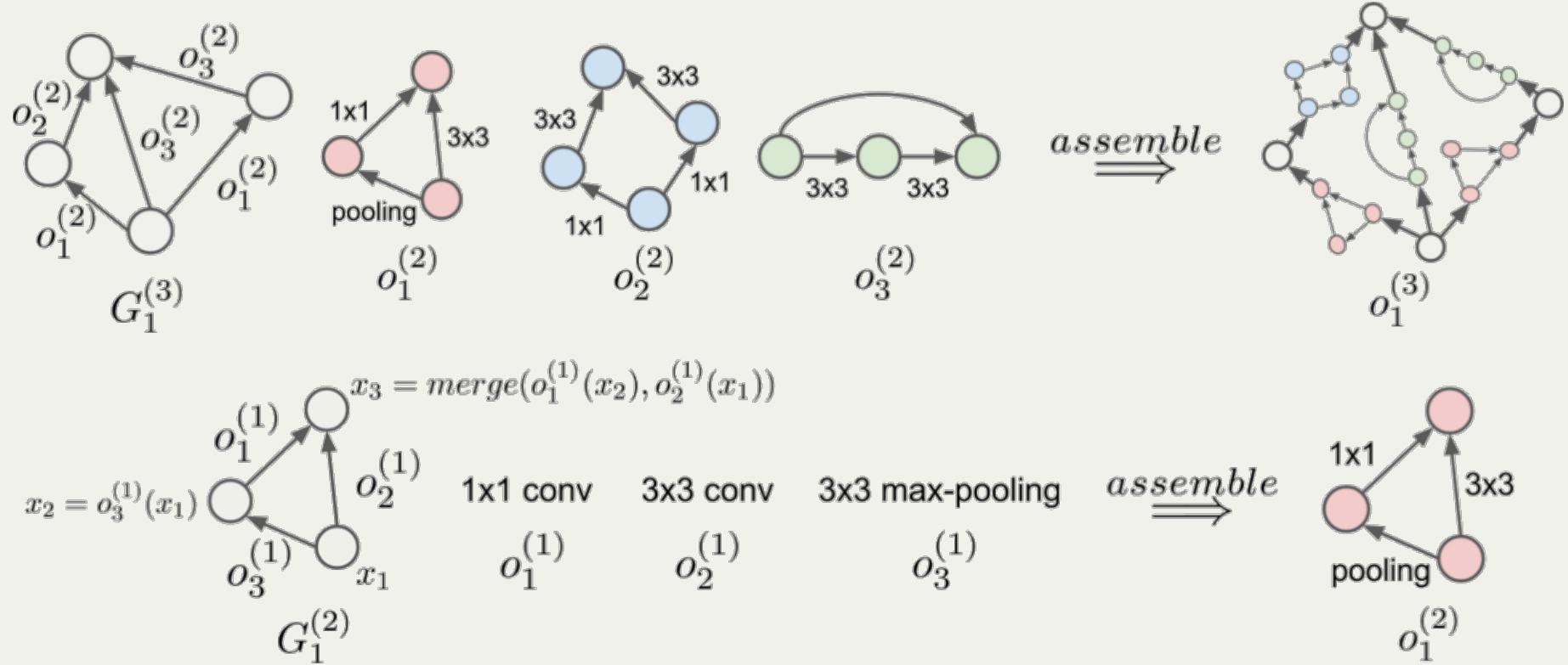
Neural Architecture Search



***Fig:** An illustration of different architecture spaces. Each node in the graphs corresponds to a layer in a neural network, e.g., a convolutional or pooling layer. Different layer types are visualized by different colors. An edge from layer L_i to layer L_j denotes that L_j receives the output of L_i as input. Left: an element of a chain-structured space. Right: an element of a more complex search space with additional layer types and multiple branches and skip connections.

*Elsken, T., Metzen, J. H. & Hutter, F. Neural architecture search: a survey. *J. Mach. Learn. Res.* 20, 1-21 (2019).

Neural Architecture Search

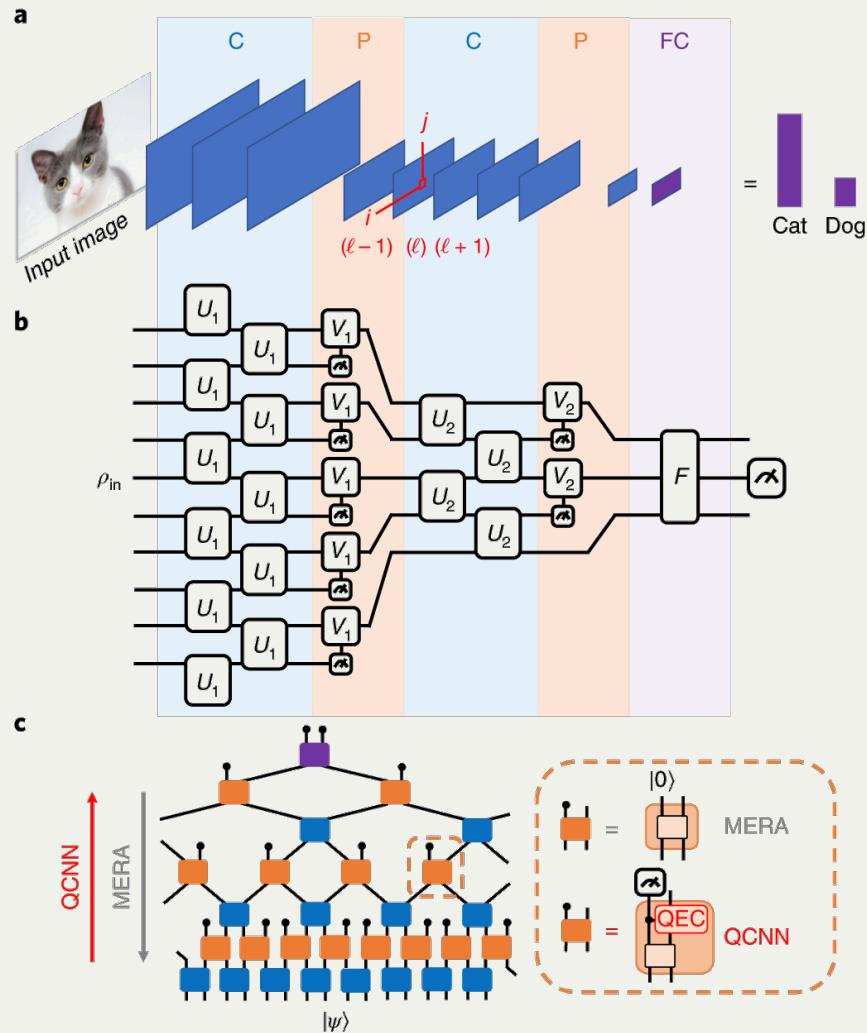


***Fig:** An example of a three-level hierarchical architecture representation. The bottom row shows how level-1 primitive operations $o_1^{(1)}$, $o_2^{(1)}$, $o_3^{(1)}$ are assembled into a level-2 motif $o_1^{(2)}$. The top row shows how level-2 motifs $o_1^{(2)}$, $o_2^{(2)}$, $o_3^{(2)}$ are then assembled into a level-3 motif $o_1^{(3)}$.

*Liu, H., Simonyan, K., Vinyals, O., Fernando, C. & Kavukcuoglu, K. Hierarchical representations for efficient architecture search. Int. Conf. Learn. Represent. (2018).

Ansatz Design

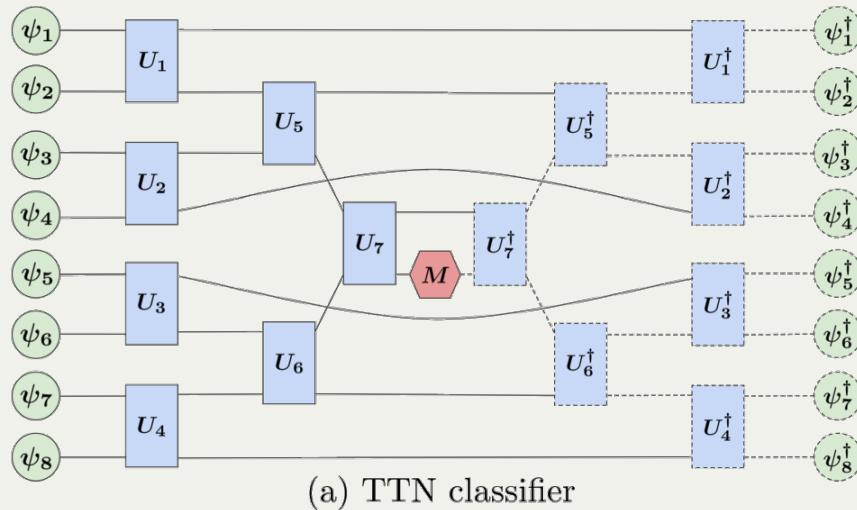
Ansatz Design



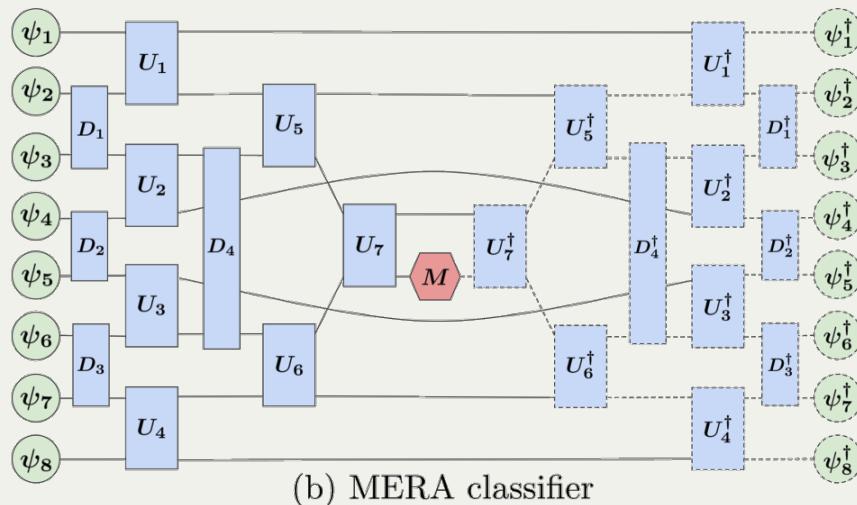
*Fig: The concept of QCNNs. *a*, Simplified illustration of classical CNNs. A sequence of image-processing layers transforms an input image into a series of feature maps (blue rectangles) and finally into an output probability distribution (purple bars). C, convolution; P, pooling; FC, fully connected. *b*, QCNNs inherit a similar layered structure. Boxes represent unitary gates or measurement with feed-forwarding. *c*, The QCNN and the MerA share the same circuit structure, but run in reverse directions.

*Cong, I., Choi, S. & Lukin, M. D. Quantum convolutional neural networks. Nat. Phys. 15, 1273-1278 (2019).
 Image of cat: <https://www.pexels.com/photo/grey-and-white-short-furcat-104827/>.

Ansatz Design



(a) TTN classifier



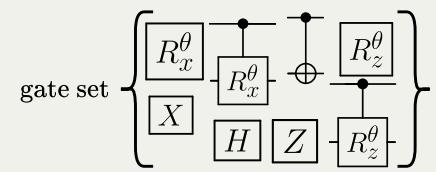
(b) MERA classifier

***Fig:** TTN and MERA classifiers for eight qubits. The quantum circuit is illustrated by the regions outlined in solid lines comprising inputs ψ , unitary blocks $\{U_i\}_{i=1}^7$ and $\{D_i\}_{i=1}^4$, and a measurement operator M . The dashed lines represent its conjugate transpose. The solid and dashed regions together describe a tensor network operating on input ψ_{1-8} and evaluating to the expectation value of observable M

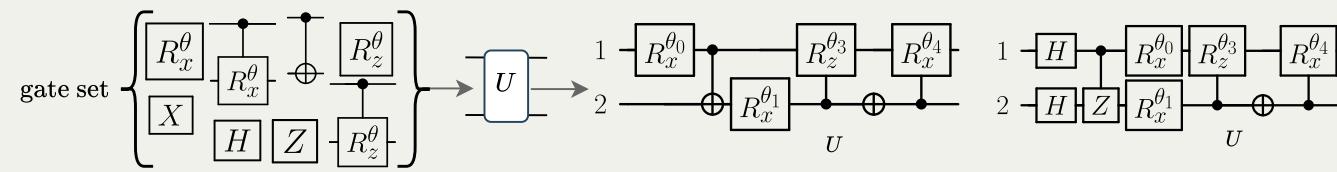
*Grant, E. et al. Hierarchical quantum classifiers. NPJ Quantum Inf. 4, 65 (2018).

Representation

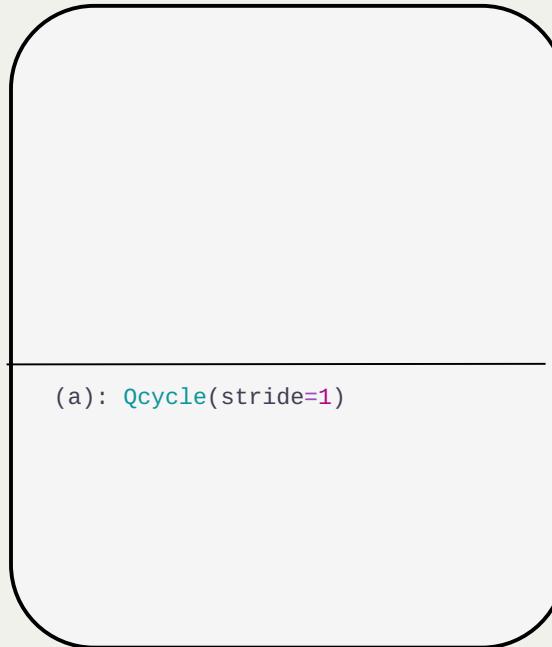
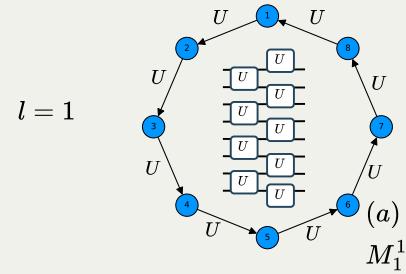
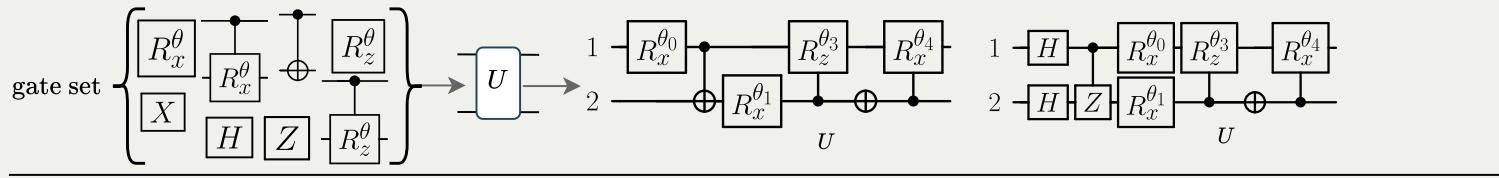
Motifs



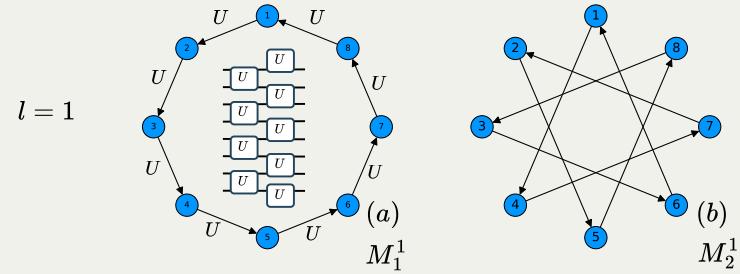
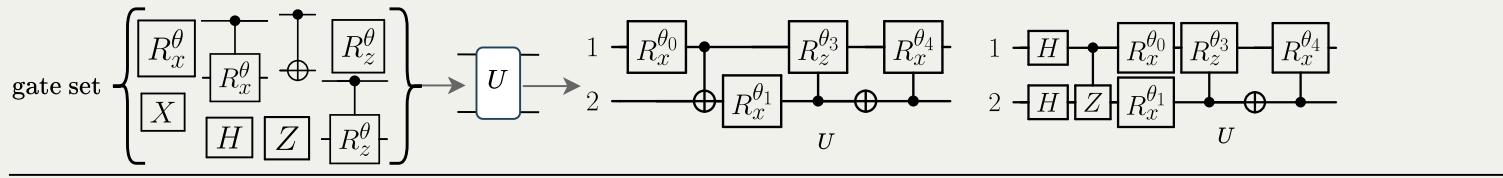
Motifs



Motifs

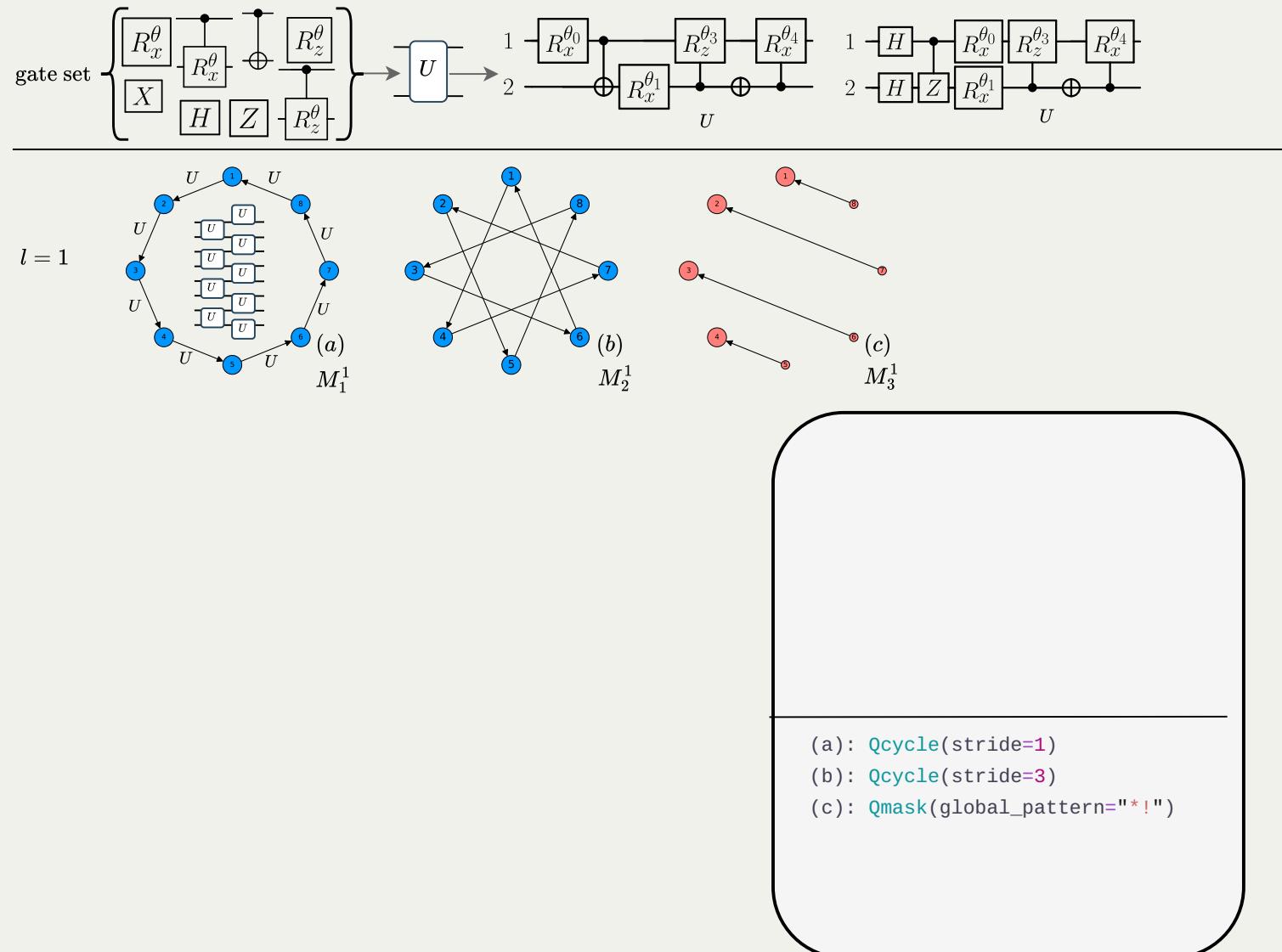


Motifs

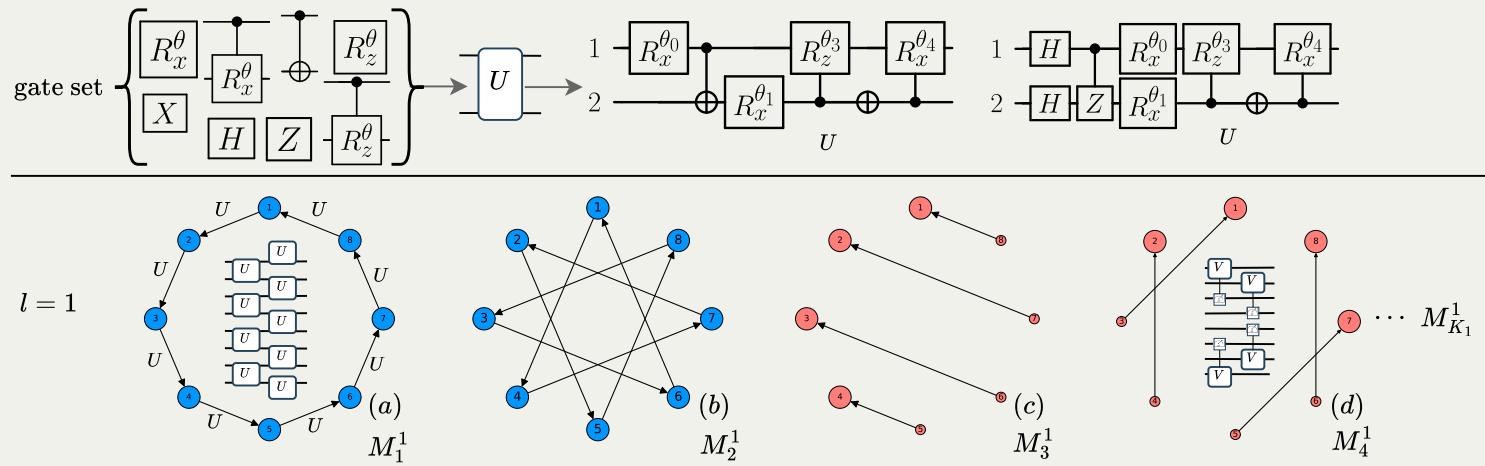


(a): `Qcycle(stride=1)`
 (b): `Qcycle(stride=3)`

Motifs

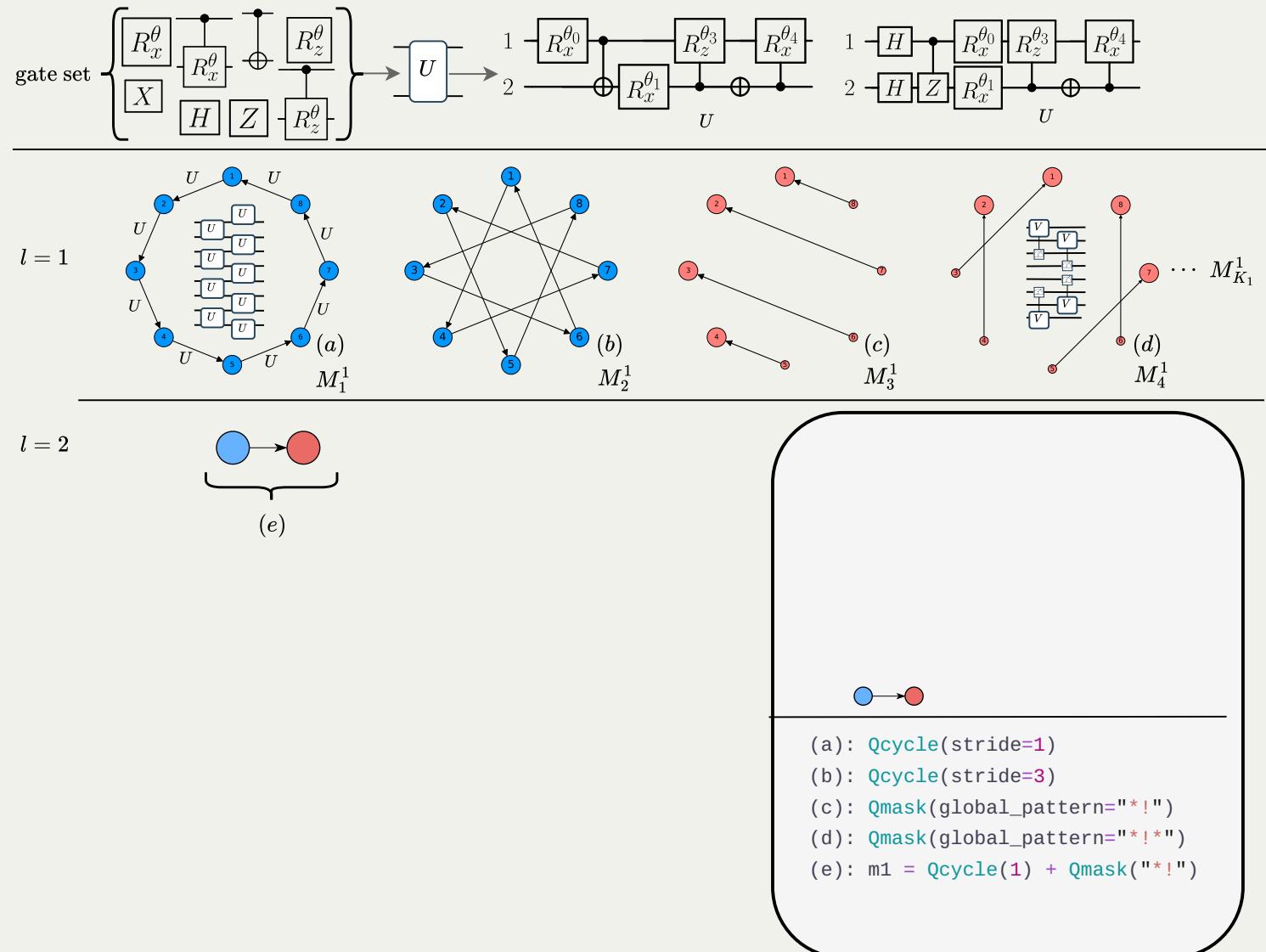


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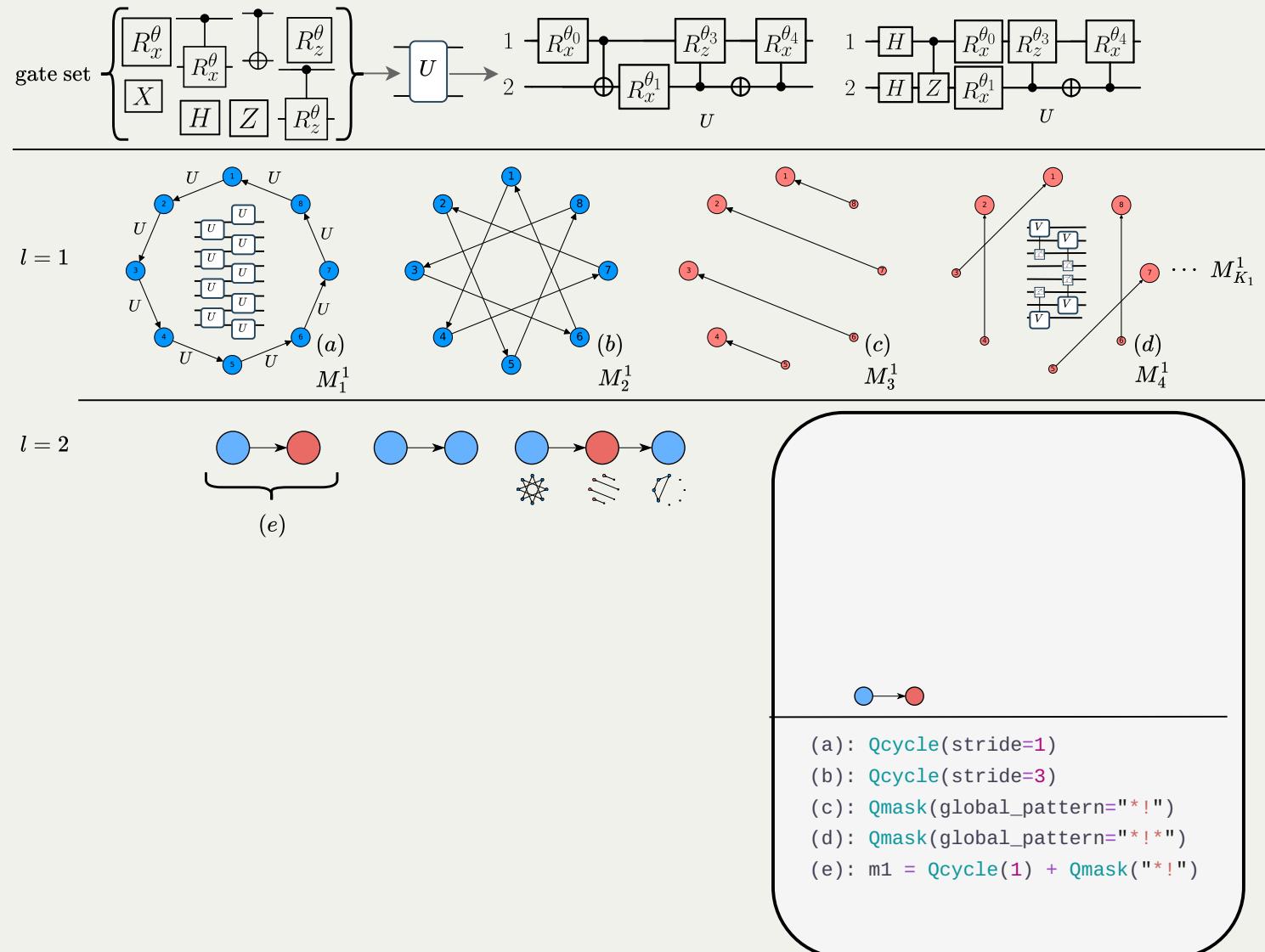


(a): `Qcycle(stride=1)`
 (b): `Qcycle(stride=3)`
 (c): `Qmask(global_pattern="* ! ")`
 (d): `Qmask(global_pattern="* ! *")`

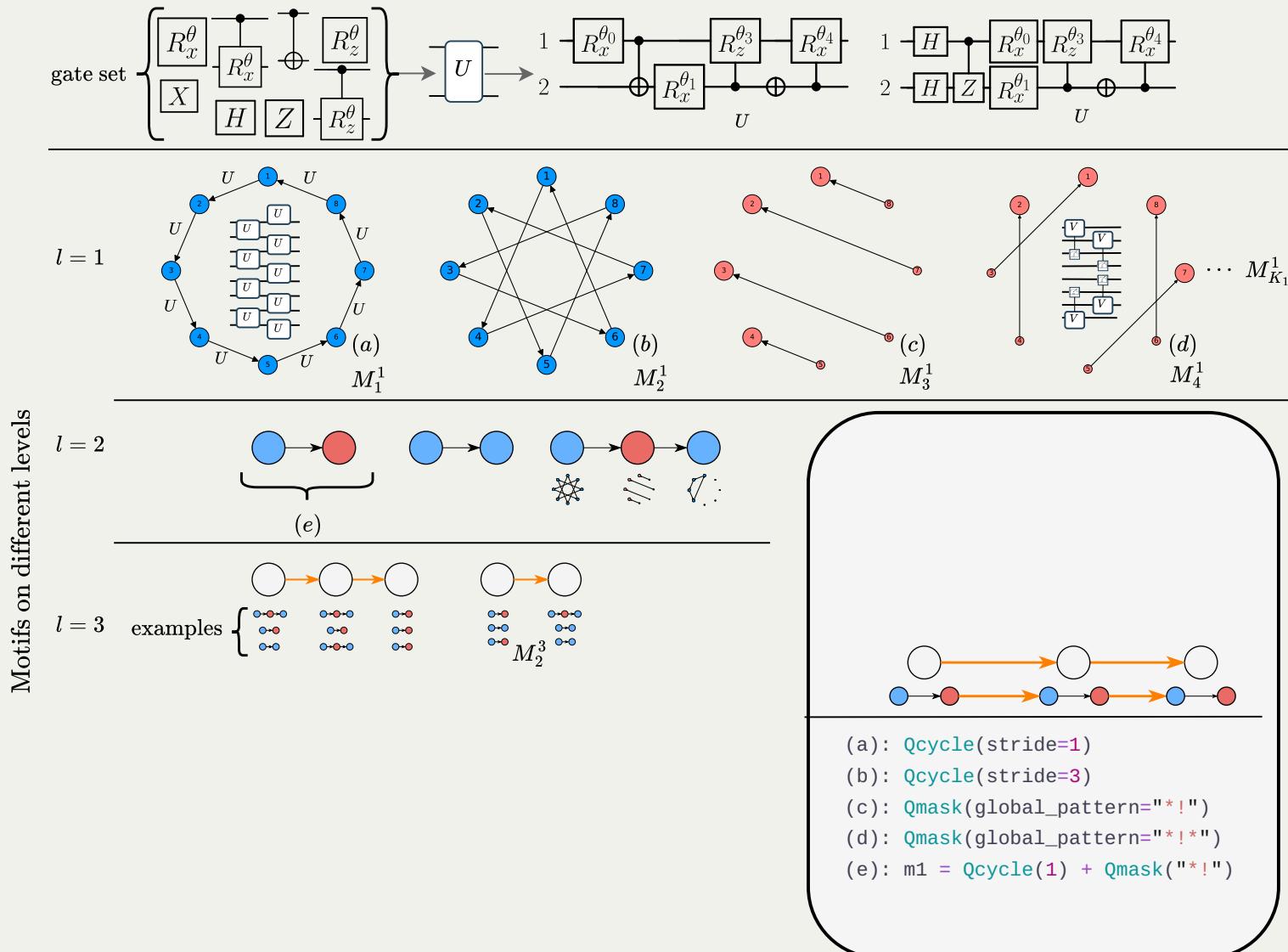
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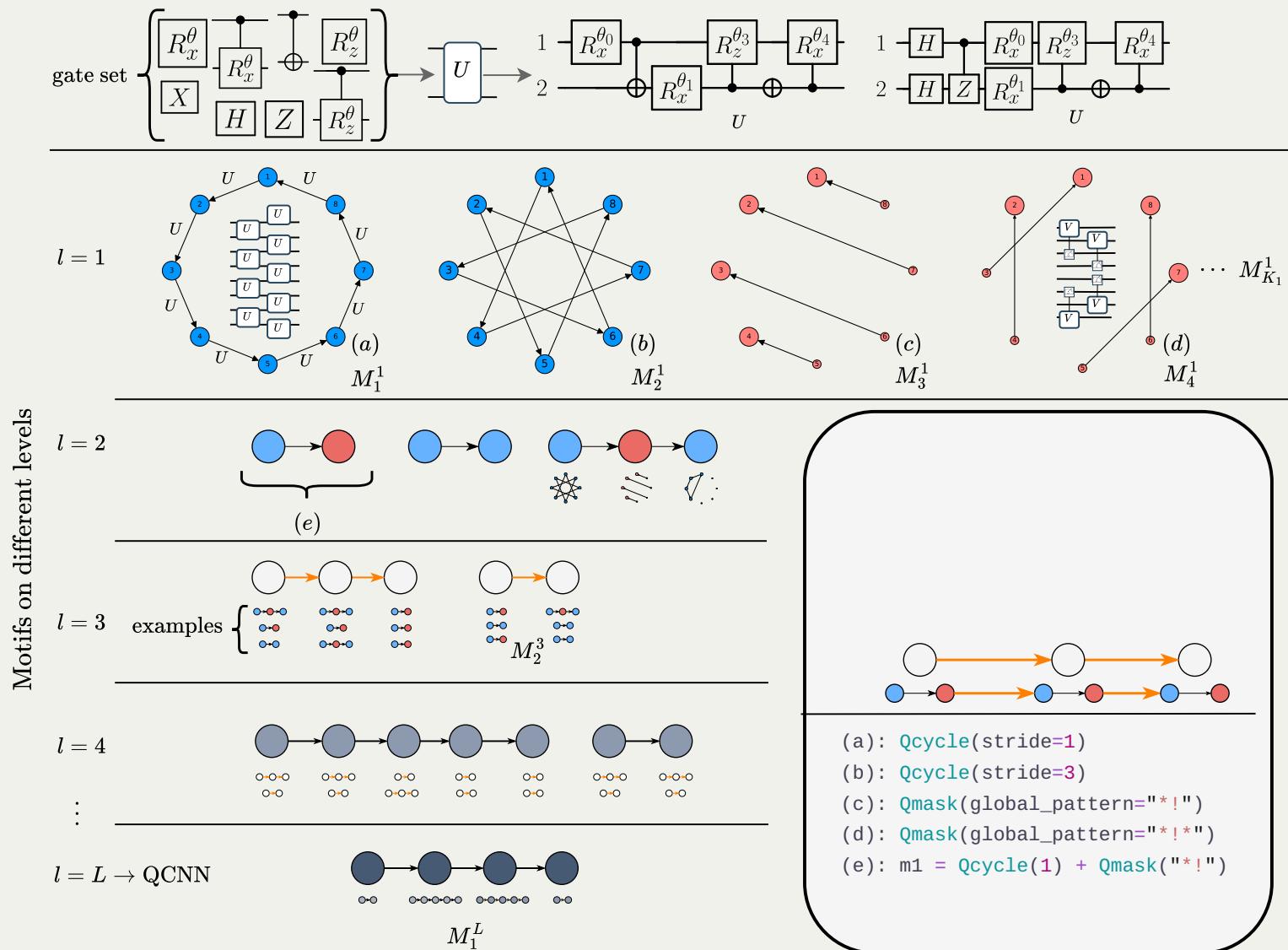
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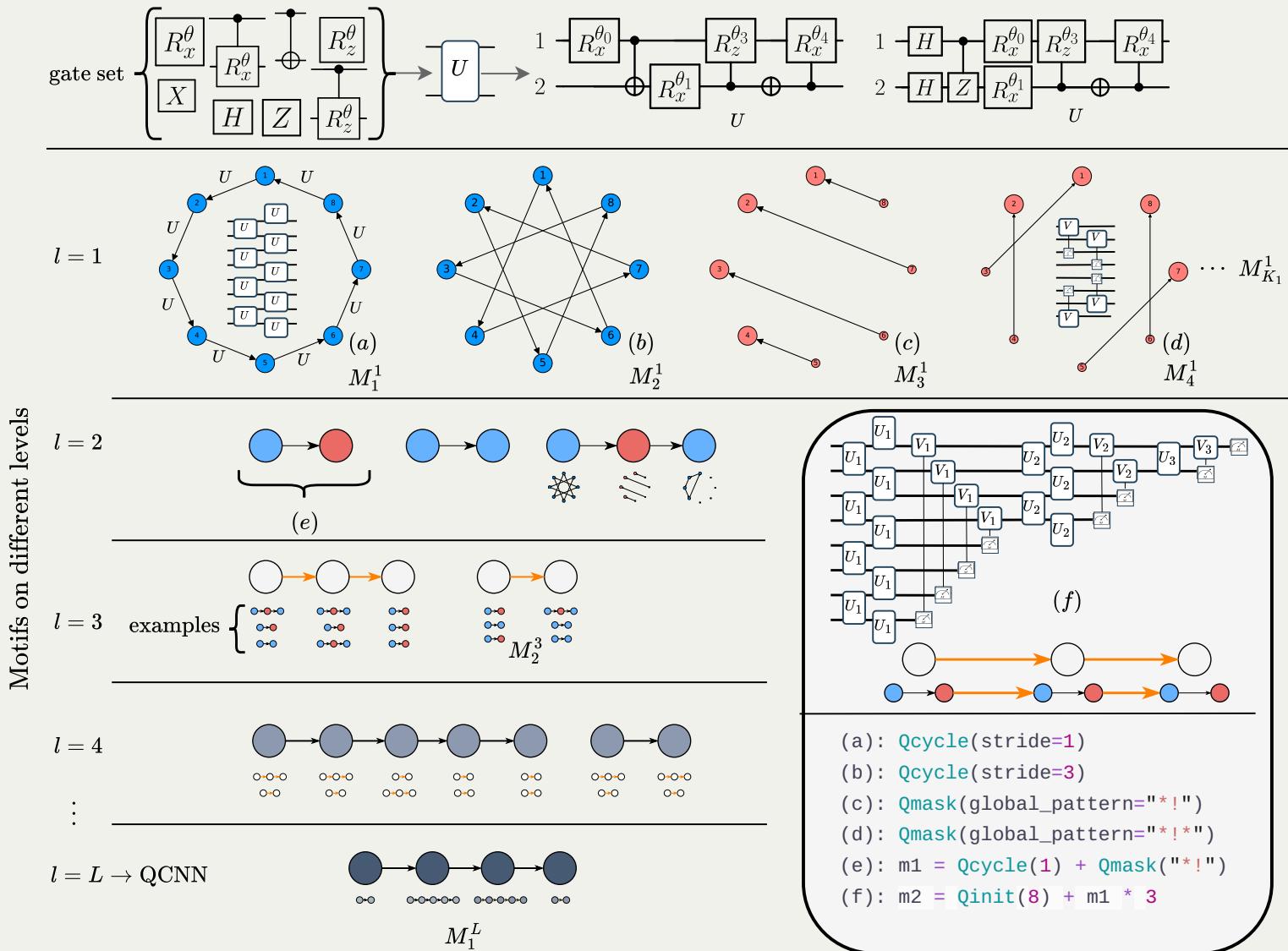
Motifs



Motifs



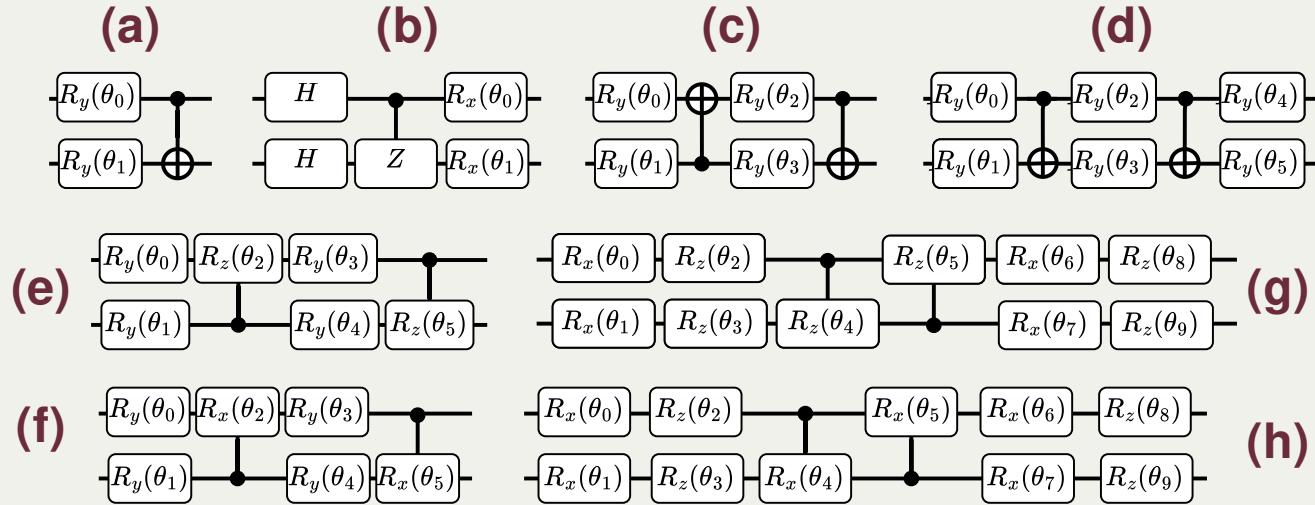
Motifs



Results and Usage

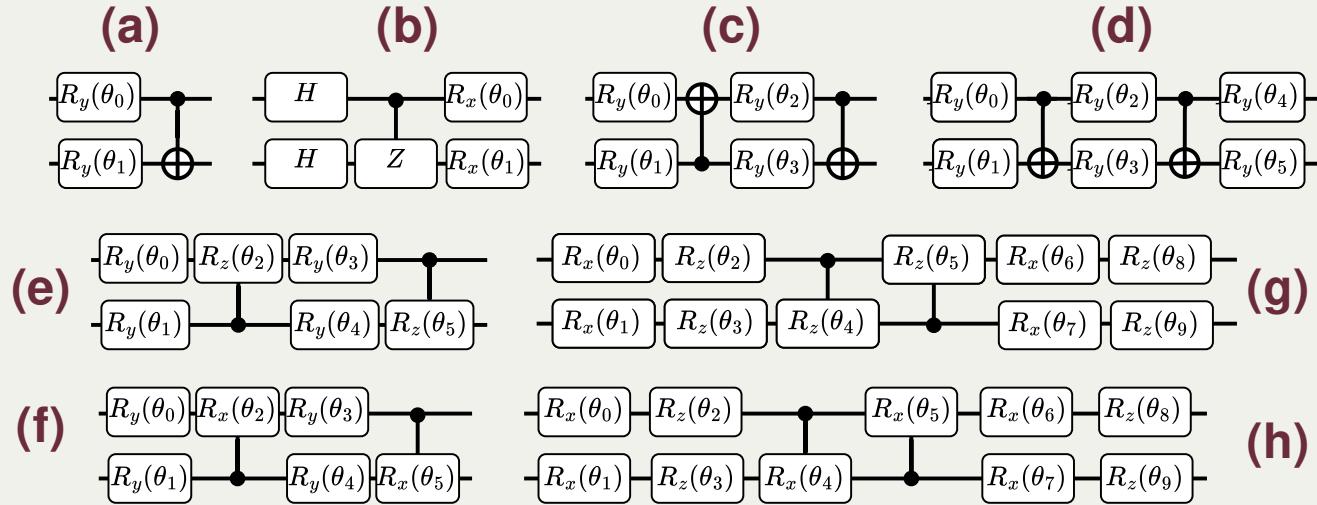
Random Search

```
1 # Example Search Space
2 strides = range(1, 8, 1)
3 steps = range(1, 8, 1)
4 mappings = [a, b, c, d, e, f, g, h]
5 boundaries = ["open", "periodic"]
6 patterns = ["10", "01", "*!", "!*", "!*!", "*!*"]
7
8 qcnn = Qinit(8) + (Qcycle(...) + Qmask(...)) * 3
```



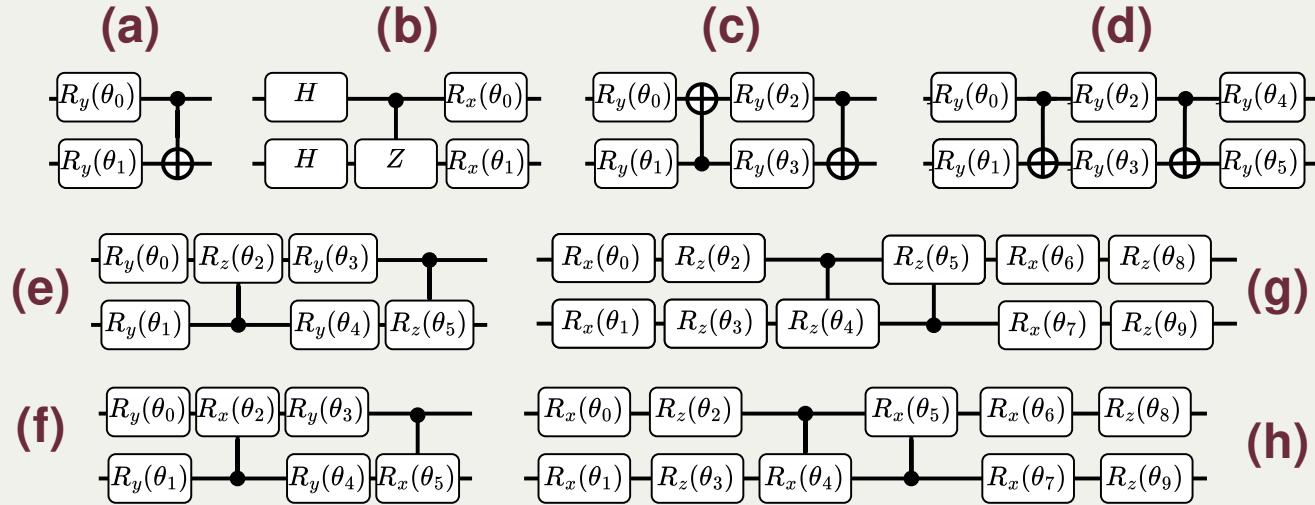
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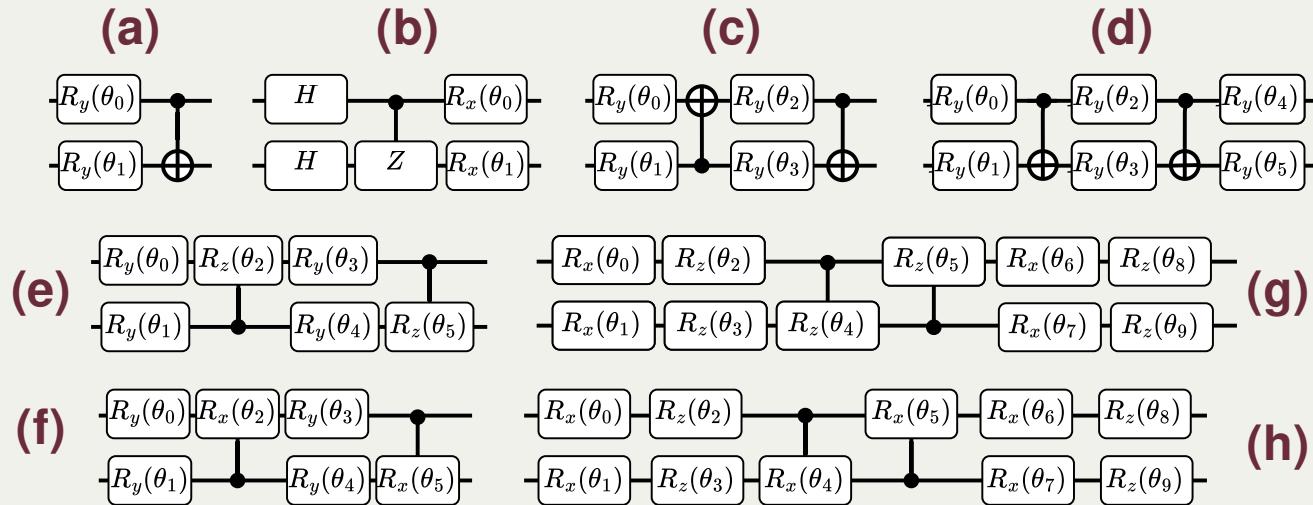
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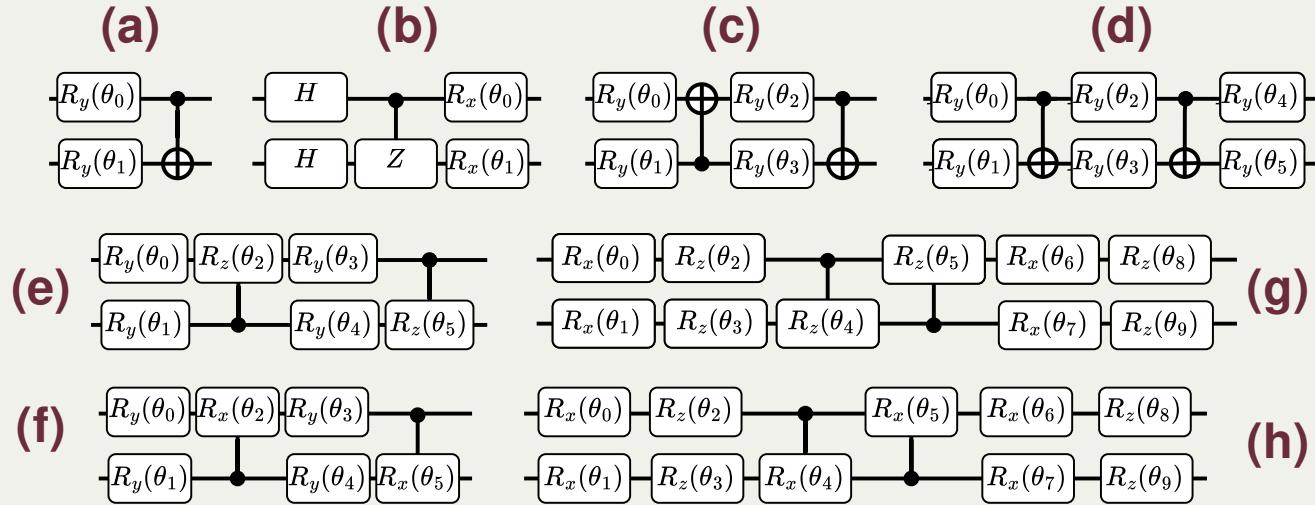
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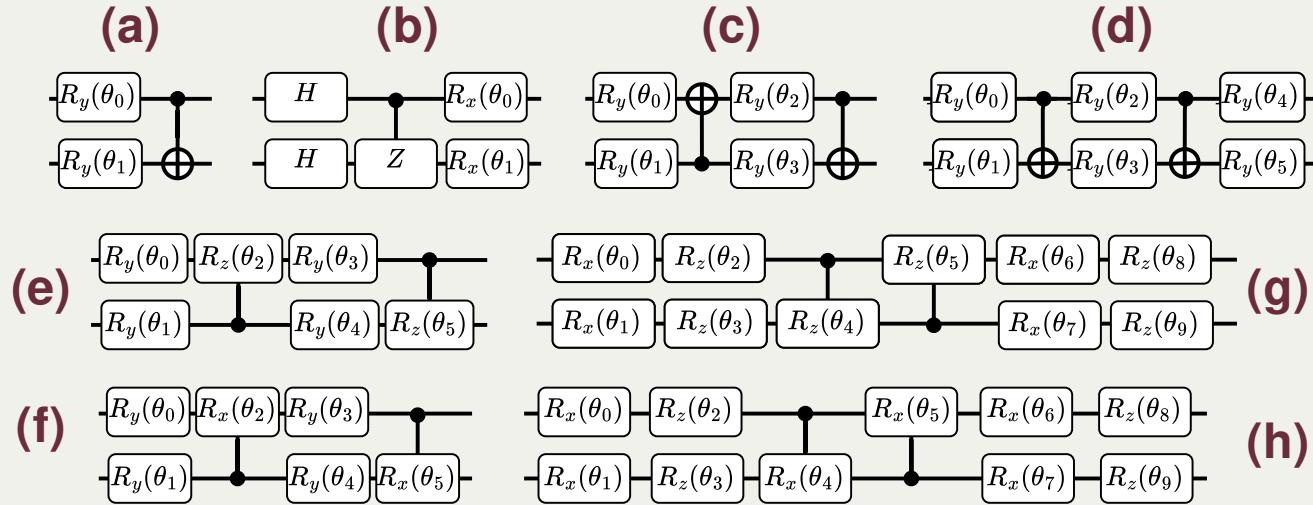
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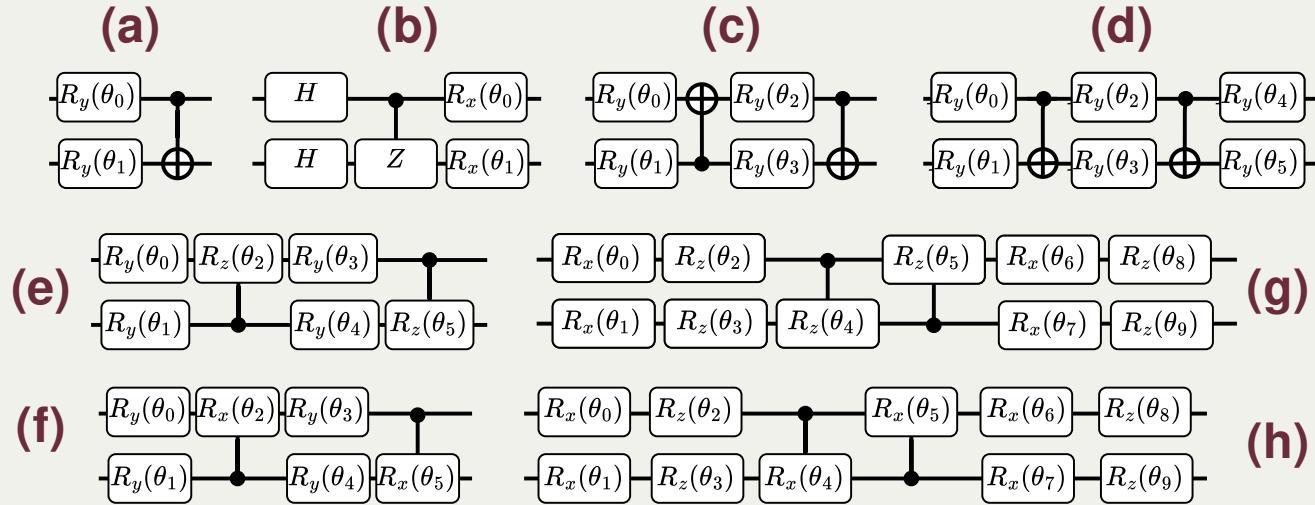
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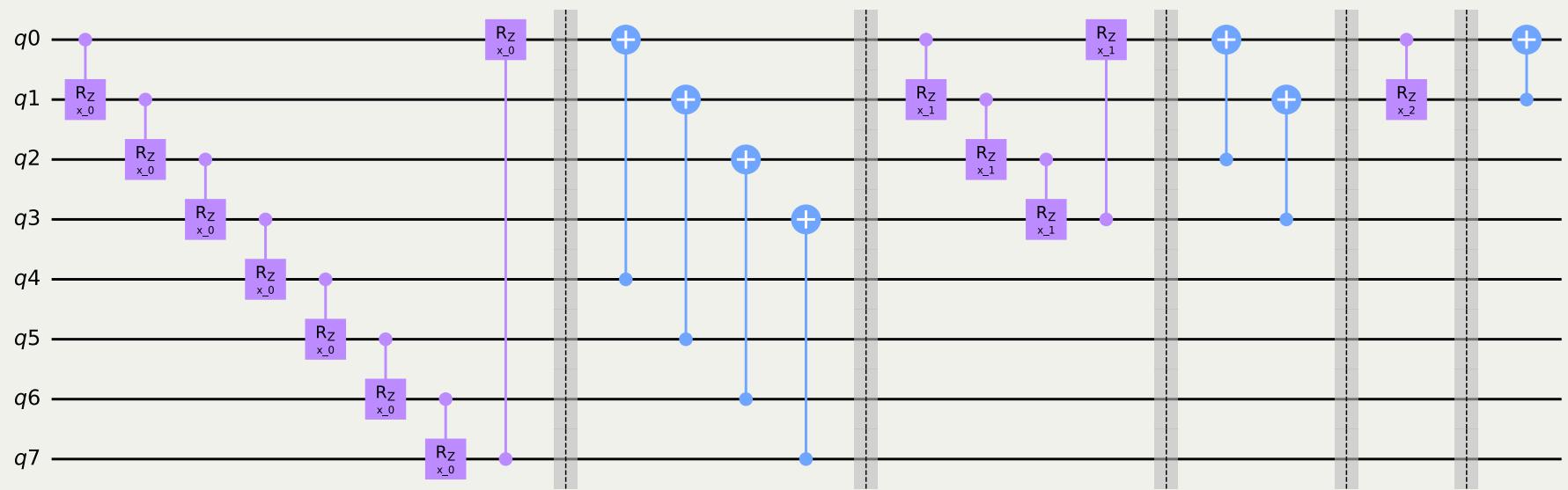
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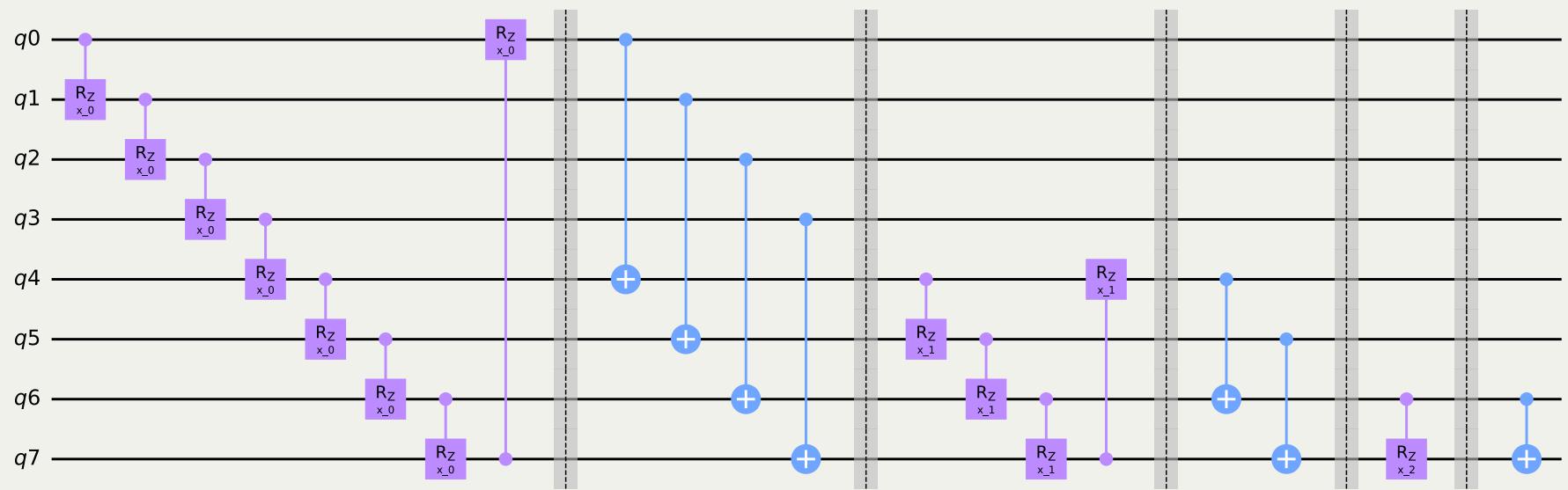
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1 cycle_mask = Qcycle(1, 1, 0, mapping=u2) + Qmask("*!", mapping=v2)
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4 cycle_mask = Qcycle(1, 1, 0, mapping=u2) + Qmask("*!*", mapping=v2)
5 hierq = Qinit(8) + cycle_mask * 3
```



Examples

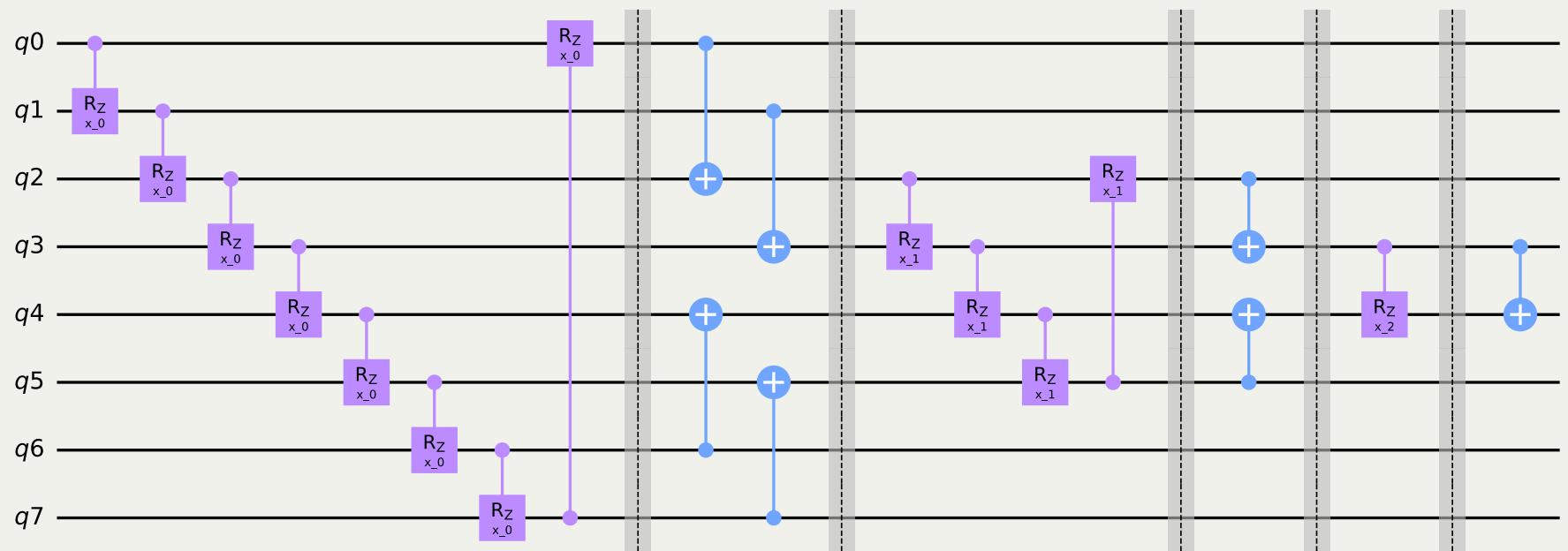
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```



<https://github.com/matt-lourens/hierarqcal>

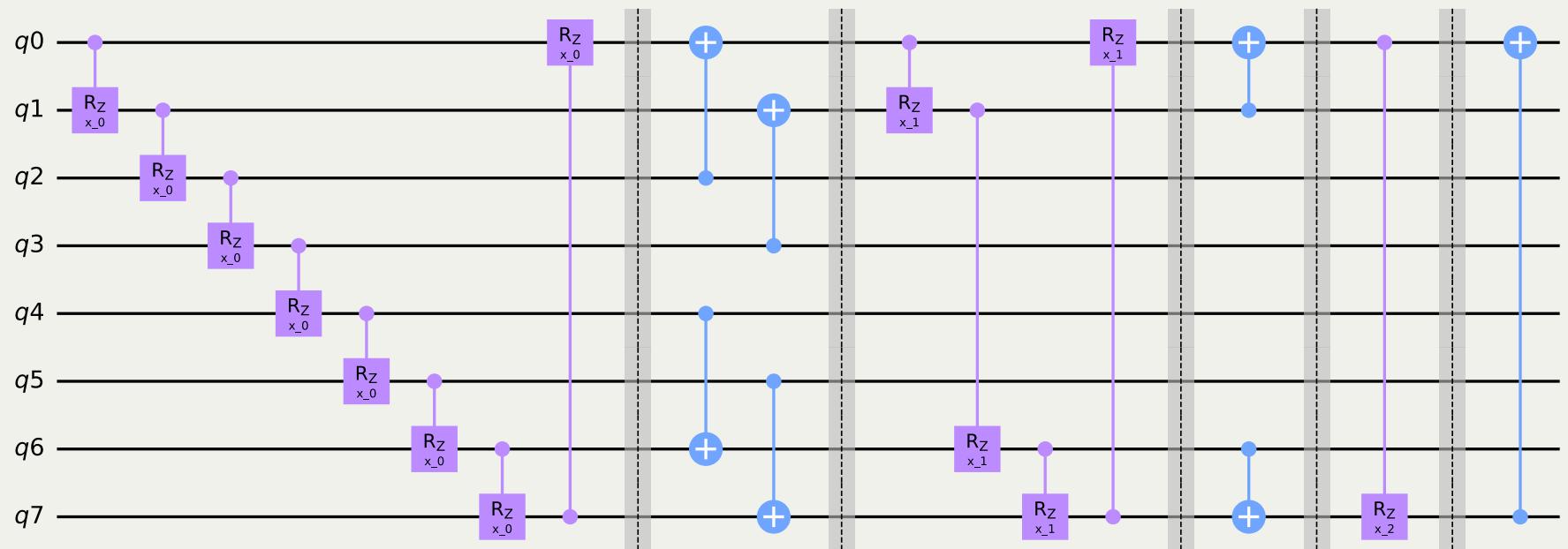
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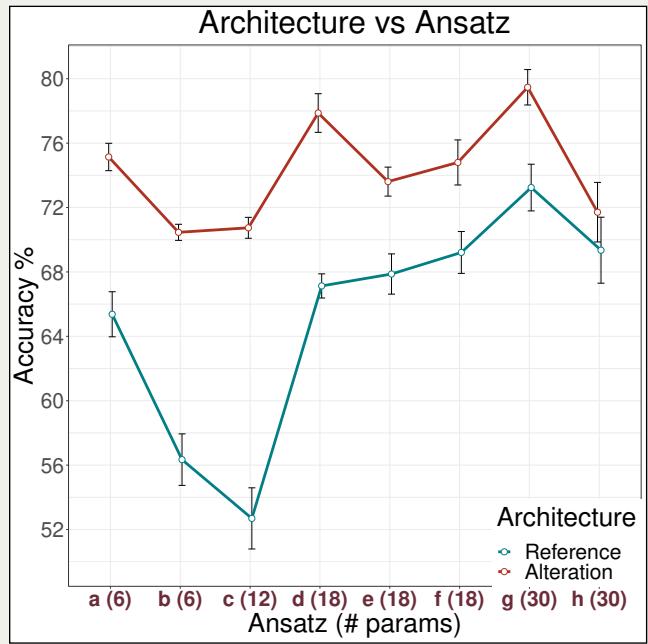


Search space

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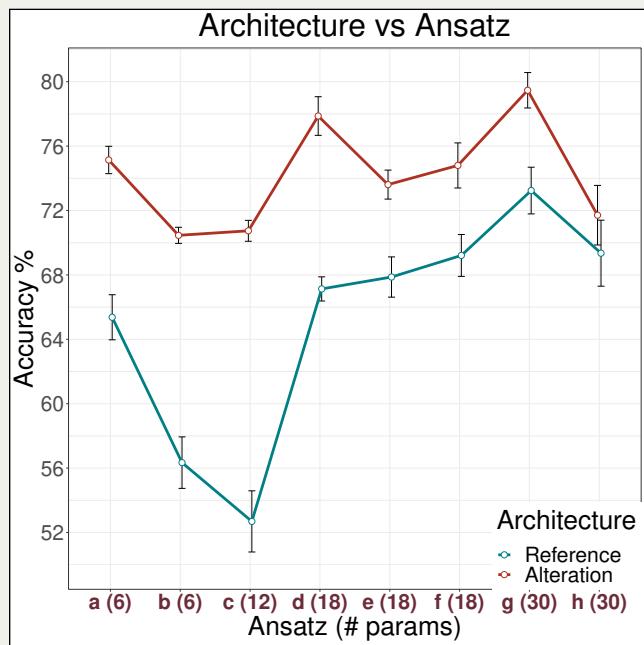
Music Genre Classification



Lourens, M., Sinayskiy, I., Park, D.K. et al. Hierarchical quantum circuit representations for neural architecture search. *npj Quantum Inf* 9, 79 (2023).

*Hur, T., Kim, L. & Park, D. K. Quantum convolutional neural network for classical data classification. *Quantum Mach. Intell.* 4, 3 (2022).

Music Genre Classification

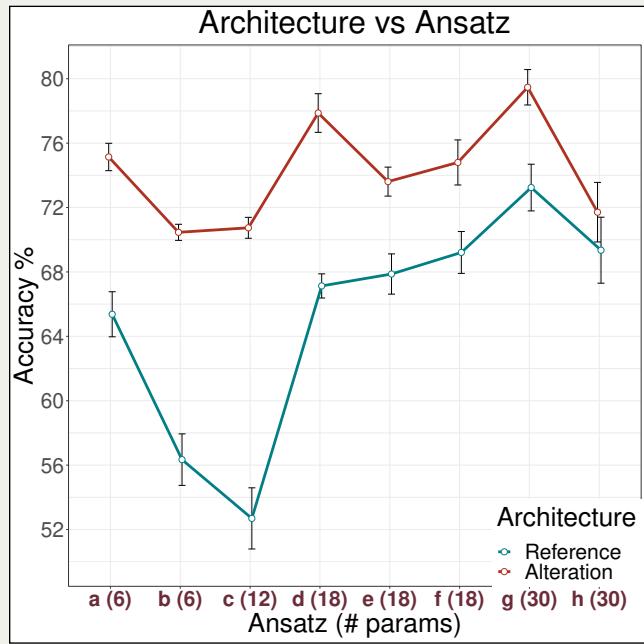


Architecture vs Ansatz					
Ansatz, # Params	Architecture		Δ	Alteration (s_c, F^*, s_p)	
	Reference	New alteration		(s_c, F^*, s_p)	
a, 6	65.37 ± 2.8	75.14 ± 1.7	+9.77	(6, left, 2)	
b, 6	56.34 ± 3.2	70.46 ± 1.0	+14.12	(1, odd, 3)	
c, 12	52.69 ± 3.8	70.74 ± 1.3	+18.05	(1, odd, 0)	
d, 18	67.13 ± 1.5	77.87 ± 2.4	+9.87	(1, outside, 2)	
e, 18	67.87 ± 2.5	73.61 ± 1.8	+5.74	(6, left, 0)	
f, 18	69.21 ± 2.6	74.80 ± 2.8	+5.59	(1, left, 3)	
g, 30	73.24 ± 2.9	79.47 ± 2.2	+6.23	(2, left, 1)	
h, 30	69.35 ± 4.1	71.71 ± 3.7	+2.36	(2, left, 1)	

Lourens, M., Sinayskiy, I., Park, D.K. et al. Hierarchical quantum circuit representations for neural architecture search. *npj Quantum Inf* 9, 79 (2023).

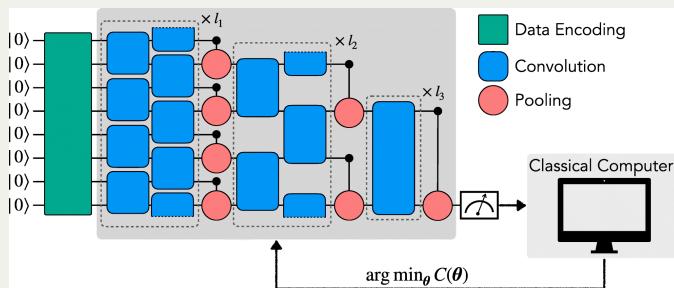
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Architecture vs Ansatz					
Ansatz, # Params	Architecture		Δ	Alteration (s_c, F^*, s_p)	
	Reference	New alteration		(s_c, F^*, s_p)	
a, 6	65.37 ± 2.8	75.14 ± 1.7	+9.77	(6, left, 2)	
b, 6	56.34 ± 3.2	70.46 ± 1.0	+14.12	(1, odd, 3)	
c, 12	52.69 ± 3.8	70.74 ± 1.3	+18.05	(1, odd, 0)	
d, 18	67.13 ± 1.5	77.87 ± 2.4	+9.87	(1, outside, 2)	
e, 18	67.87 ± 2.5	73.61 ± 1.8	+5.74	(6, left, 0)	
f, 18	69.21 ± 2.6	74.80 ± 2.8	+5.59	(1, left, 3)	
g, 30	73.24 ± 2.9	79.47 ± 2.2	+6.23	(2, left, 1)	
h, 30	69.35 ± 4.1	71.71 ± 3.7	+2.36	(2, left, 1)	

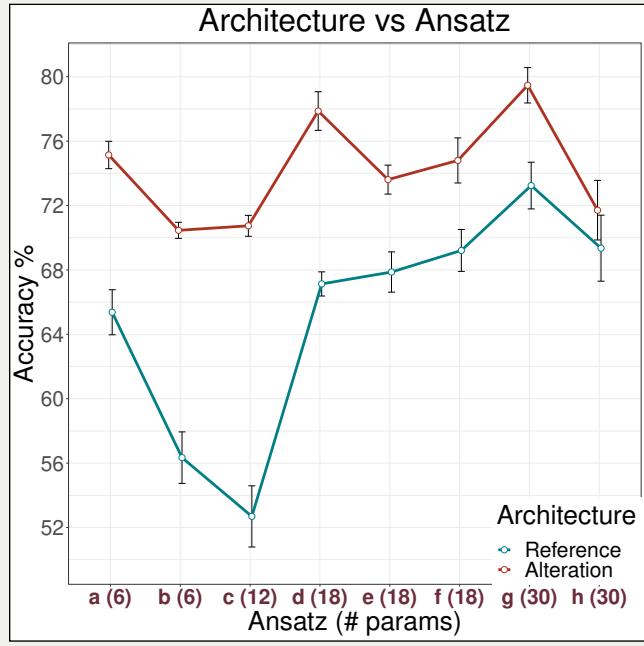
*Reference Architecture:



Lourens, M., Sinayskiy, I., Park, D.K. et al. Hierarchical quantum circuit representations for neural architecture search. npj Quantum Inf 9, 79 (2023).

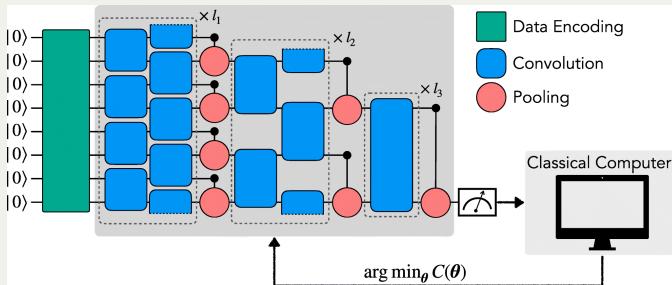
*Hur, T., Kim, L. & Park, D. K. Quantum convolutional neural network for classical data classification. Quantum Mach. Intell. 4, 3 (2022).

Music Genre Classification

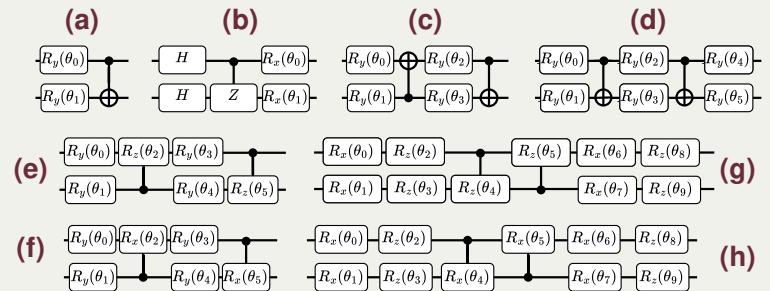


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Ansatz:

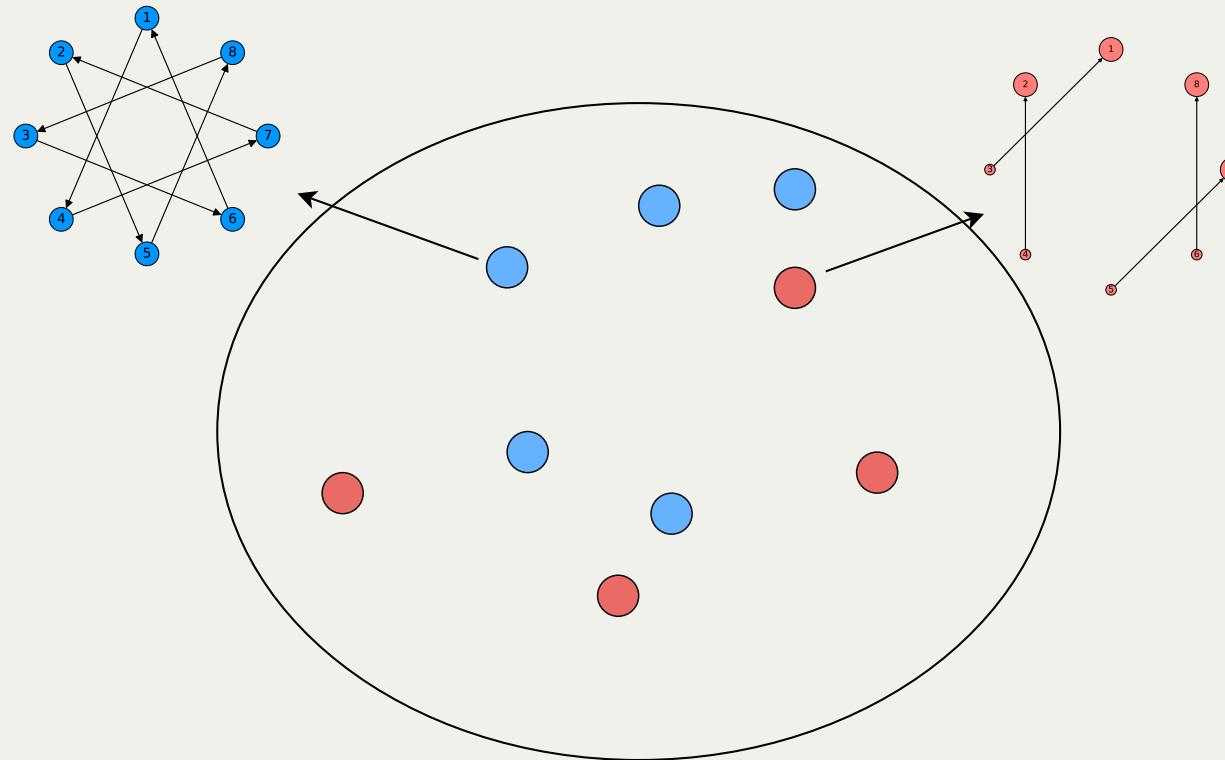


Lourens, M., Sinayskiy, I., Park, D.K. et al. Hierarchical quantum circuit representations for neural architecture search. npj Quantum Inf 9, 79 (2023).

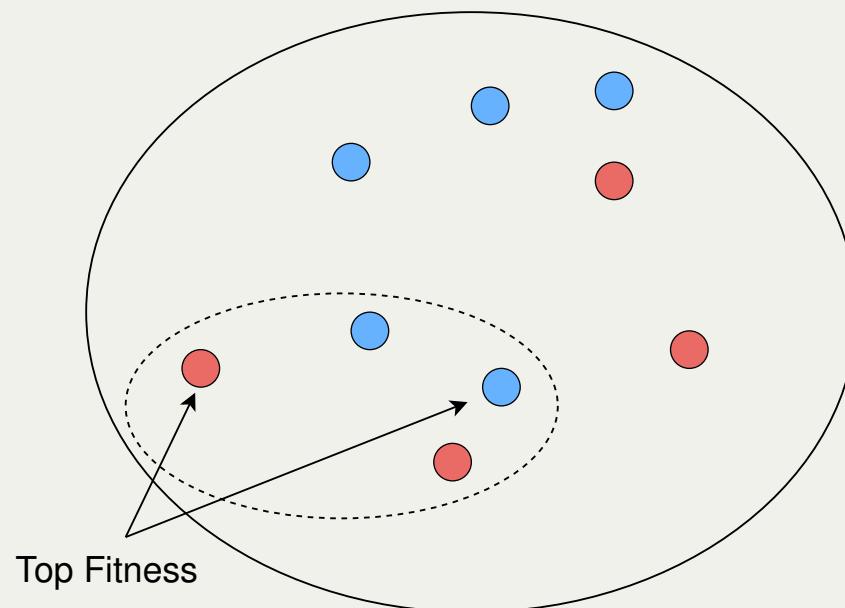
*Hur, T., Kim, L. & Park, D. K. Quantum convolutional neural network for classical data classification. Quantum Mach. Intell. 4, 3 (2022).

Evolutionary Search

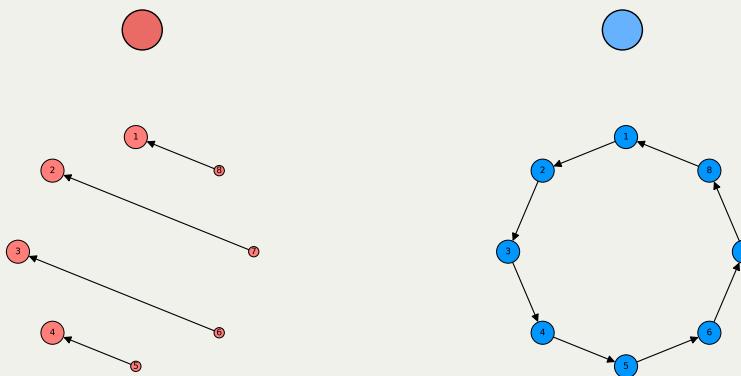
Evolutionary Search



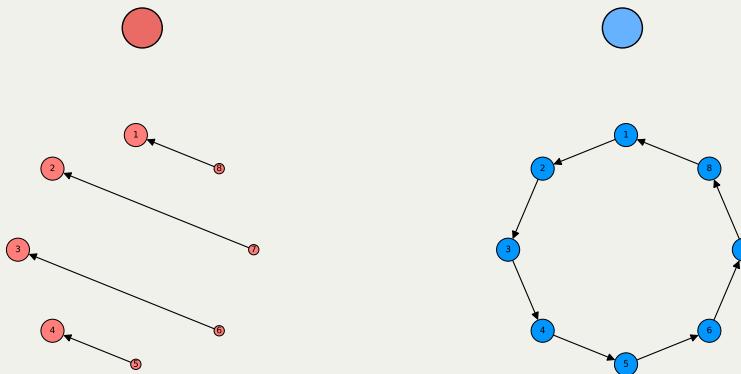
Tournament Selection



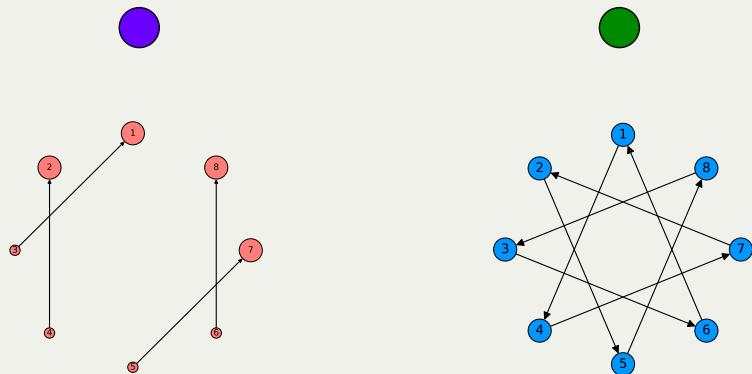
Tournament Selection



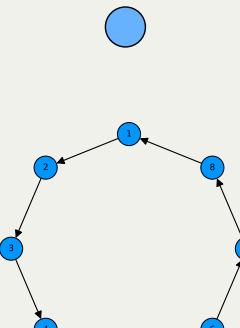
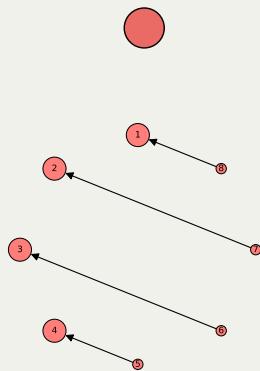
Tournament Selection



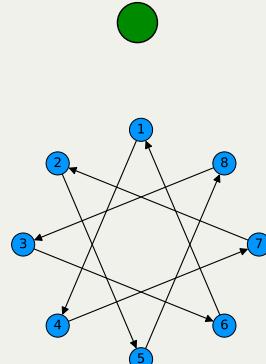
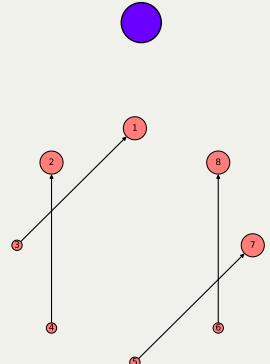
Mutation



Tournament Selection

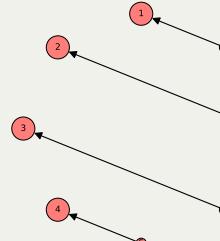


Mutation

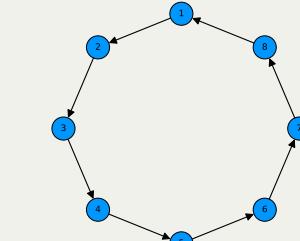


Crossover

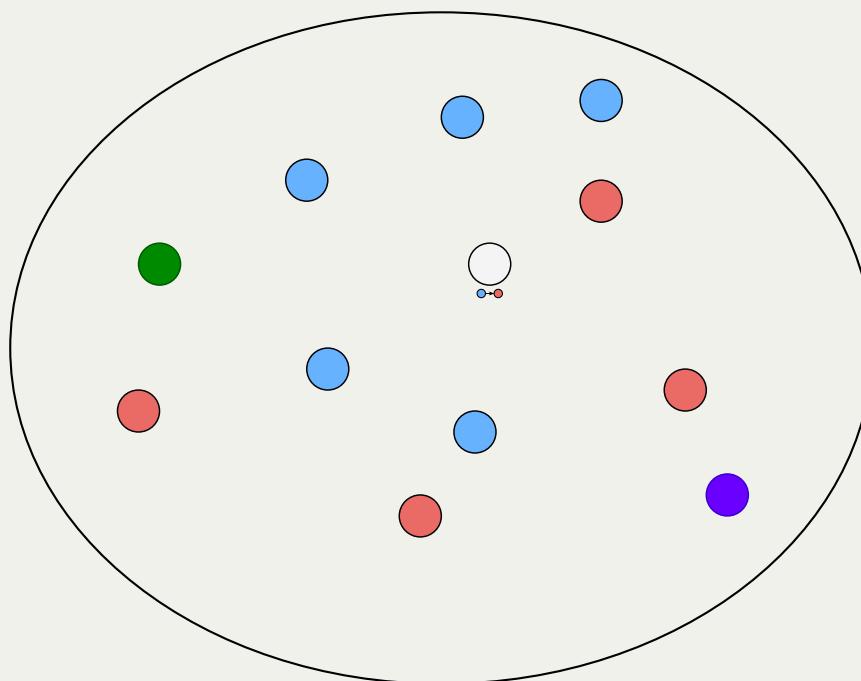
$$\textcircled{1} = \textcircled{1} \rightarrow \textcircled{2}$$



+



Repeat



Quantum Phase Recognition

Cluster-Ising Hamiltonian:

$$H = -J \sum_{i=1}^{N-2} Z_i X_{i+1} Z_{i+2} - h_1 \sum_{i=1}^N X_i - h_2 \sum_{i=1}^{N-1} X_i X_{i+1}.$$

Quantum Phase Recognition

Cluster-Ising Hamiltonian:

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- Given an unknown ground state $|\psi_g\rangle$, what is the phase?

Quantum Phase Recognition

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- Given an unknown ground state $|\psi_g\rangle$, what is the phase?
 - Paramagnetic

Quantum Phase Recognition

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- Given an unknown ground state $|\psi_g\rangle$, what is the phase?
 - Paramagnetic
 - $\mathbb{Z}_2 \times \mathbb{Z}_2$ Symmetry-Protected Topological (SPT)

Quantum Phase Recognition

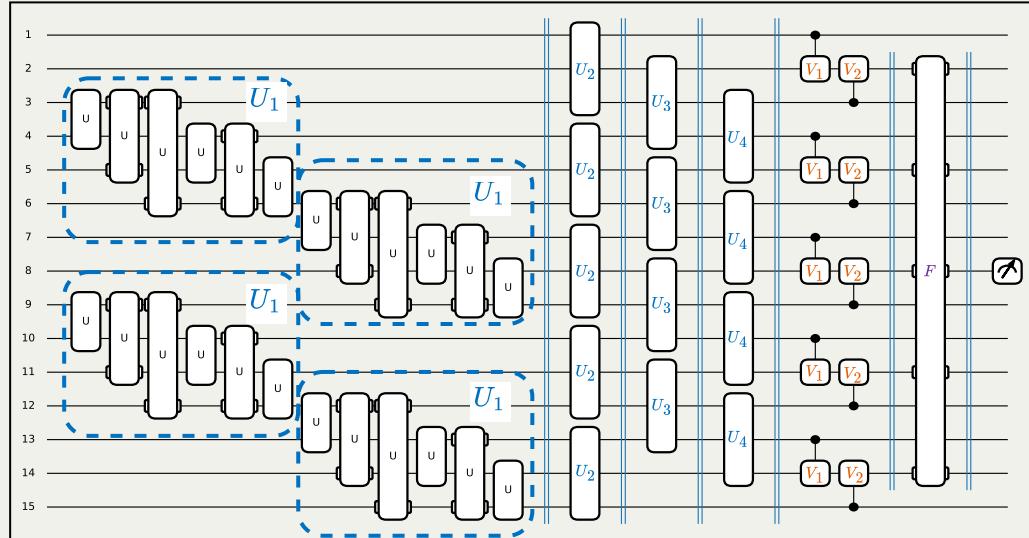
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- Given an unknown ground state $|\psi_g\rangle$, what is the phase?
 - Paramagnetic
 - $\mathbb{Z}_2 \times \mathbb{Z}_2$ Symmetry-Protected Topological (SPT)
 - Antiferromagnetic

Quantum Phase Recognition

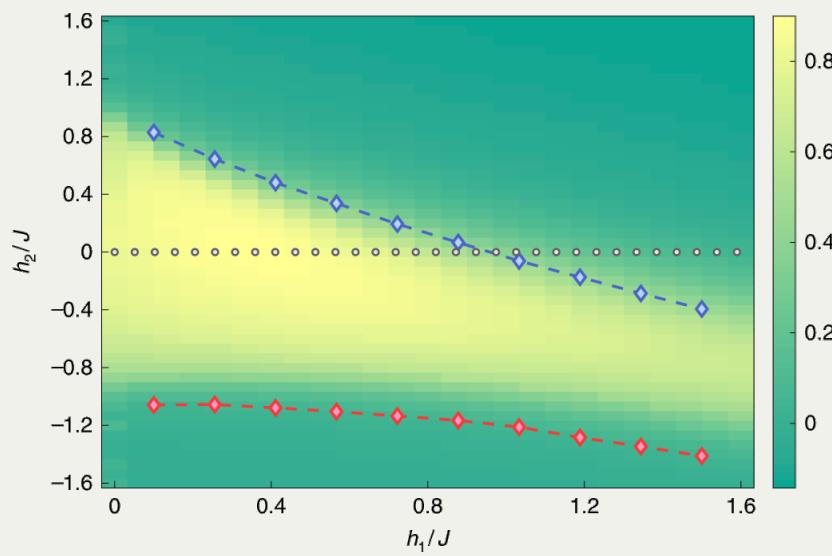
```
1 u0 = Qinit(4) + Qpermute(share_weights=False, mapping=Ugm)
2 u1 = Qcycle(1,3,2, mapping=u0, boundary="open", edge_order=[1,3,2,4])
3 u2 = sum([Qcycle(1,3,offset, mapping=Ugm, boundary="open") for offset in range(3)])
4 u3 = Qmask("101",1,3,0, mapping=Vgm, boundary="open")
5 u4 = Qcycle(1, mapping=Ugm, boundary="open")
6 motif = Qinit(15) + u1 + u2 + u3 + u4
```



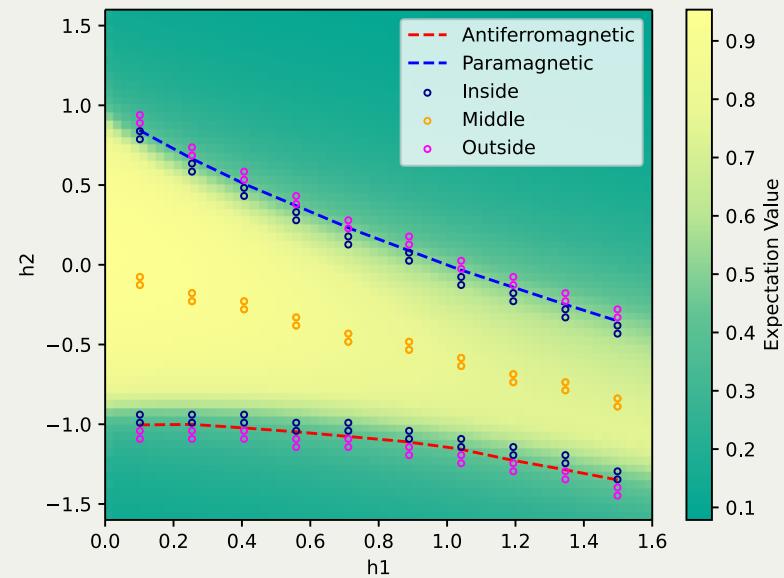
Lourens, M., Sinayskiy, I., Park, D.K. et al. Hierarchical quantum circuit representations for neural architecture search. npj Quantum Inf 9, 79 (2023).

Quantum Phase Recognition

Reference¹



Found²

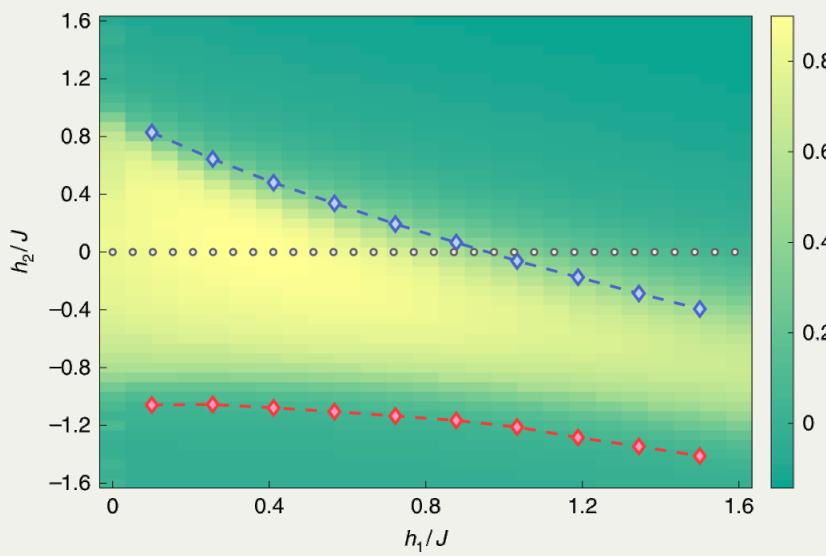


¹Cong, I., Choi, S. & Lukin, M.D. Quantum convolutional neural networks. *Nat. Phys.* 15, 1273-1278 (2019).

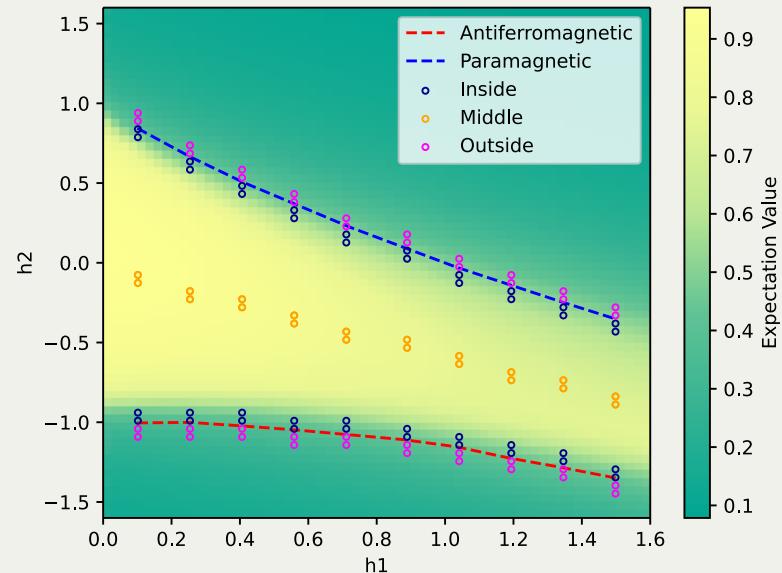
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Quantum Phase Recognition

Reference¹



Found²



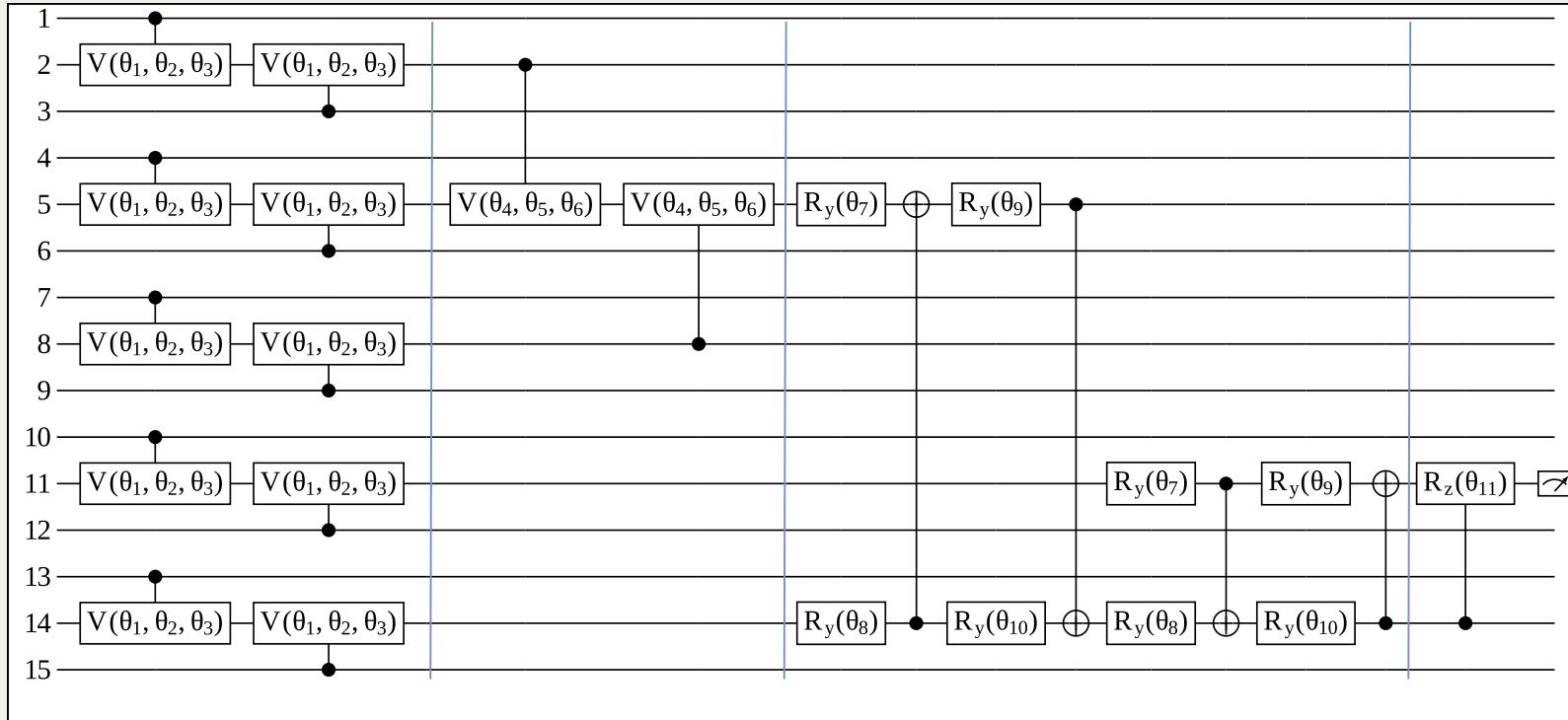
Metric	Reference	Found
Number of parameters	1308	11
Sample Complexity (Inside)	61.523	36.079
Sample Complexity (Middle)	10.992	13.253
MSE (Outside)	0.164	0.167

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Quantum Phase Recognition

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What's next?

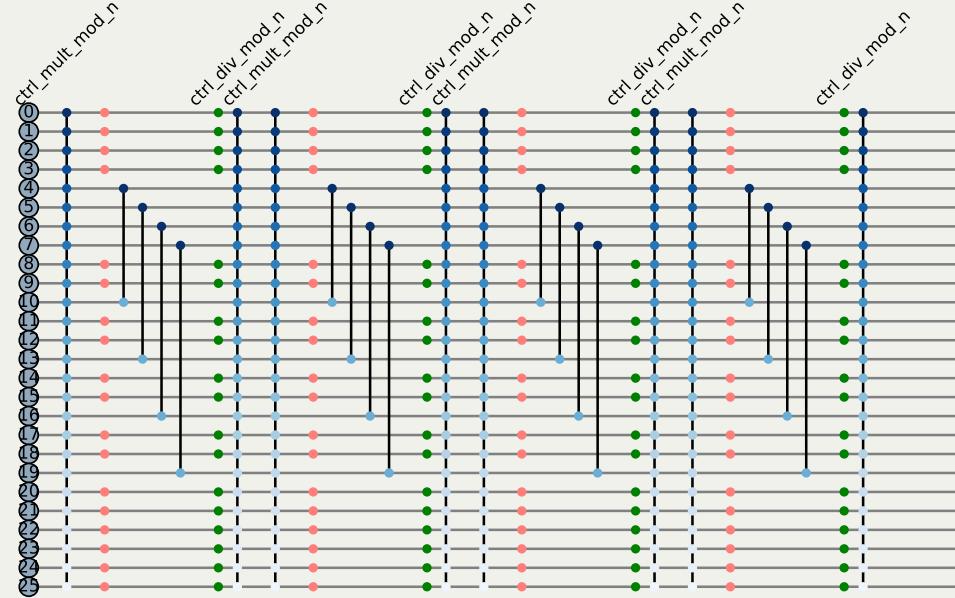
Compute Graph Design

What's next?

Compute Graph Design

- Quantum Circuits

Exponentiation modulo N



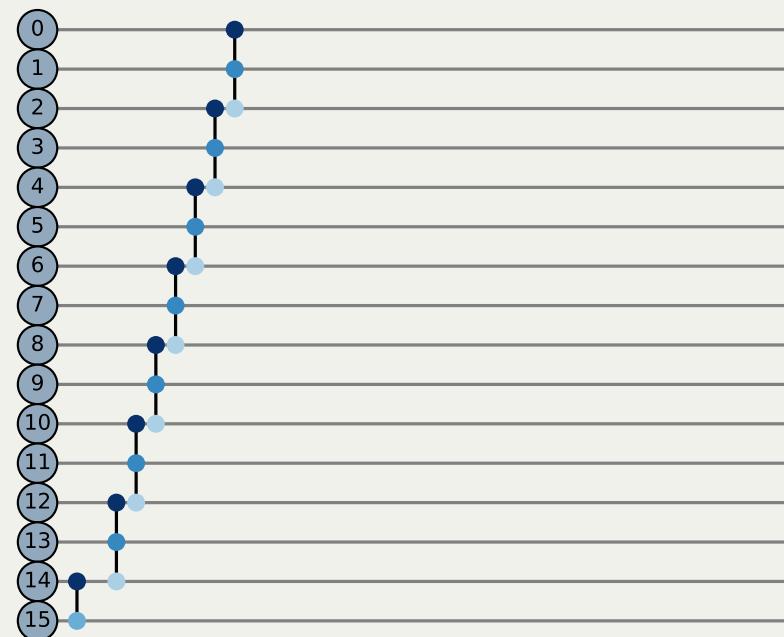
What's next?

Addition

```
1 def or(bits, symbols=None, state=None):
2     b1, b2 = state[bits[0]], state[bits[1]]
3     state[bits[0]] = b1 or b2
4     return state
```

Compute Graph Design

- Quantum Circuits
- Classical Circuits



What's next?

Compute Graph Design

- Quantum Circuits
- Classical Circuits
- Tensor Networks

```
1 tensors = [np.array([1, 0])] * 5
2 hierq = (Qinit(5, tensors=tensors)
3           + mask_anc
4           + U_psi + grover)
5 psi = hierq()
```

What's next?

Compute Graph Design

- Quantum Circuits
- Classical Circuits
- Tensor Networks
- Neural Networks

?

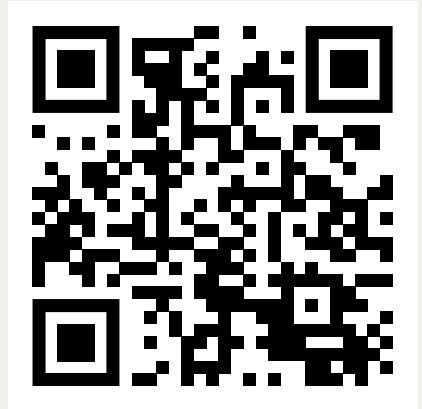
What's next?

Paper

Hierarqcal

Compute Graph Design

- Quantum Circuits
- Classical Circuits
- Tensor Networks
- Neural Networks



Slides



Programming

Shor's Algorithm

- Modular Exponentiation
- Quantum Fourier Transform

Vedral, V., Barenco, A. & Ekert, A. Quantum networks for elementary arithmetic operations. Phys. Rev. A 54, 147-153 (1996)

Shor's Algorithm

- Modular Exponentiation
 - Ctrl-Mult modulo N
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Shor's Algorithm

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 - xor
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Vedral, V., Barenco, A. & Ekert, A. Quantum networks for elementary arithmetic operations. Phys. Rev. A 54, 147-153 (1996)

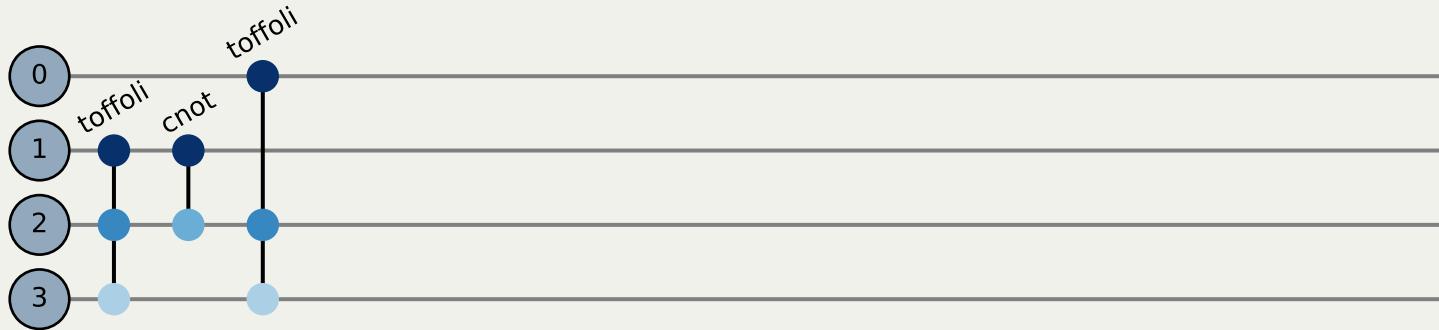
Shor's Algorithm

- Modular Exponentiation
 - Ctrl-Mult modulo N
 - Addition modulo N
 - Addition
 - Carry
 - cnot
 - Toffoli
 - xor
 - cnot
- Quantum Fourier Transform

Vedral, V., Barenco, A. & Ekert, A. Quantum networks for elementary arithmetic operations. Phys. Rev. A 54, 147-153 (1996)

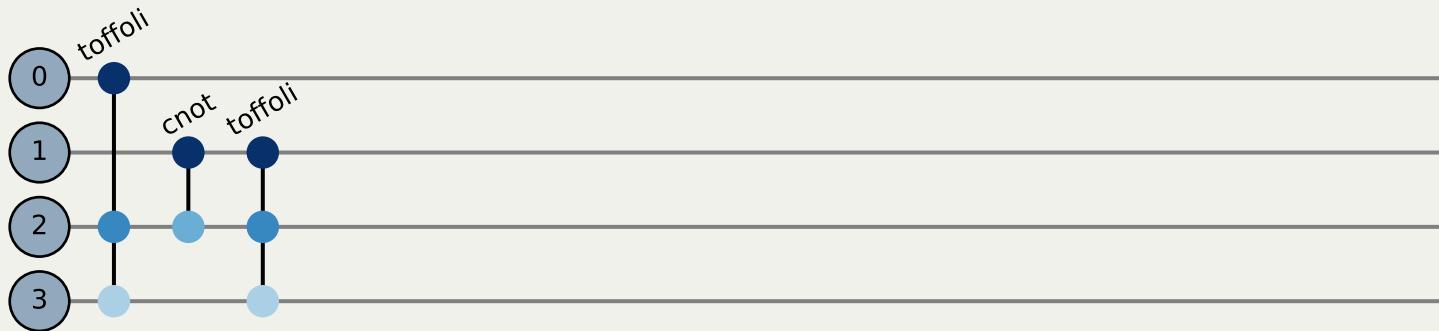
Shor circuit

```
1 # ===== Motifs level 1
2 carry_motif = (
3     Qinit(4)
4     + Qmotif(E=[(1, 2, 3)], mapping=toffoli)
5     + Qmotif(E=[(1, 2)], mapping=cnot)
6     + Qmotif(E=[(0, 2, 3)], mapping=toffoli)
7 )
8
9 # Turn carry into a function to be able to reverse it easily
10 carry = lambda r: carry_motif if r == 1 else carry_motif.reverse()
11 plot_circuit(carry(1))
12 plot_circuit(carry(-1))
```



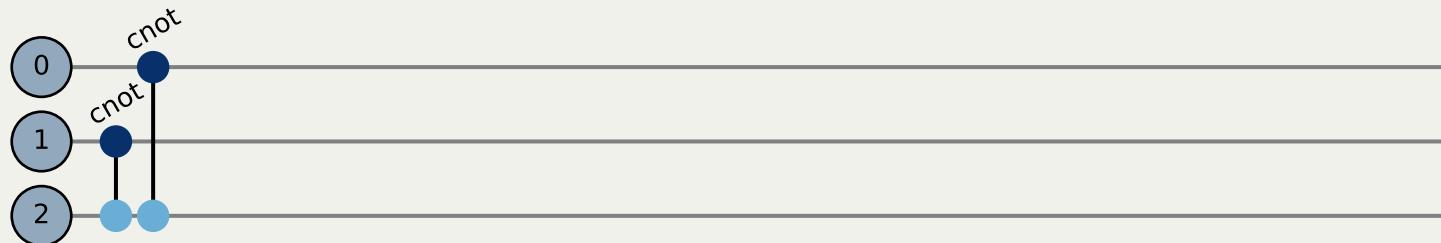
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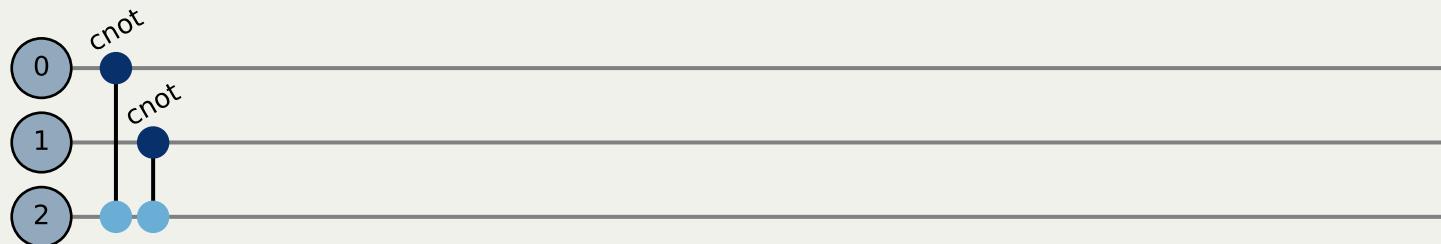
```
1 # ===== Motifs level 1
2 sum = lambda r=1: Qinit(3, name=f"sum") + Qpivot(
3     "*1", merge_within="01", mapping=cnot, edge_order=[-1 * r]
4 )
5
6 plot_circuit(sum(1))
7 plot_circuit(sum(-1))
```



<https://github.com/matt-lourens/hierarqcal>

Shor circuit

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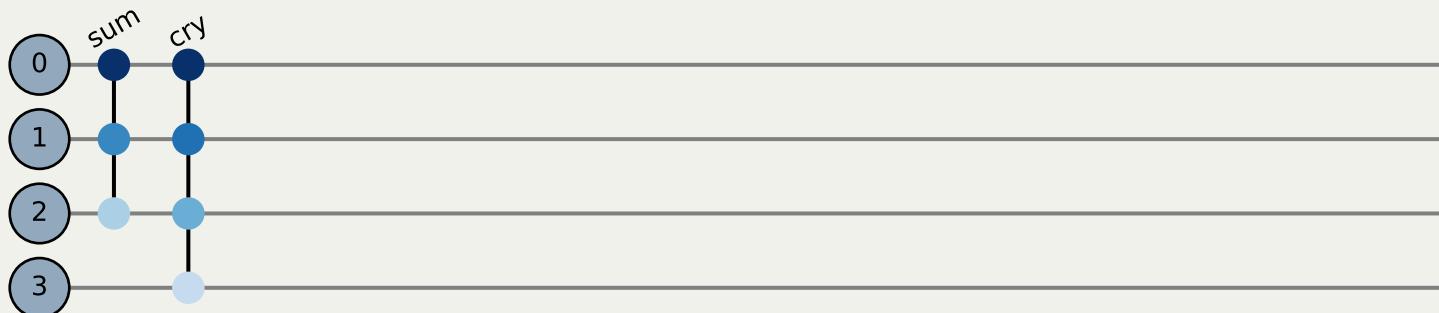
Shor circuit

```
1 # ===== Motifs level 2
2 carry_sum_motif = (
3     lambda r=1: Qinit(4, name=f"crs")
4     + Qpivot("1*", merge_within="1111", mapping=carry(-r))
5     + Qpivot("1*", merge_within="111", mapping=sum(r))
6 )
7
8 carry_sum = lambda r=1: carry_sum_motif(1) if r == 1 else carry_sum_motif(-1).reverse
9
10 plot_circuit(carry_sum(1))
11 plot_circuit(carry_sum(-1))
```



Shor circuit

```
1 # ===== Motifs level 2
2 carry_sum_motif = (
3     lambda r=1: Qinit(4, name=f"crs")
4     + Qpivot("1*", merge_within="1111", mapping=carry(-r))
5     + Qpivot("1*", merge_within="111", mapping=sum(r))
6 )
7
8 carry_sum = lambda r=1: carry_sum_motif(1) if r == 1 else carry_sum_motif(-1).reverse
9
10 plot_circuit(carry_sum(1))
11 plot_circuit(carry_sum(-1))
```



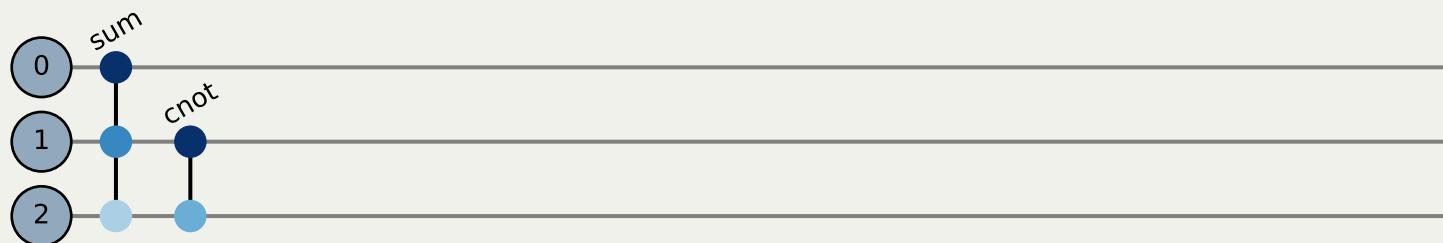
Shor circuit

```
1 cnot_sum_motif = (
2     lambda r=1: Qinit(3, name=f"cns")
3     + Qpivot("*1", merge_within="11", mapping=cnot)
4     + Qpivot("*1", merge_within="111", mapping=sum(r))
5 )
6
7 cnot_sum = lambda r=1: cnot_sum_motif(1) if r == 1 else cnot_sum_motif(-1).reverse()
8
9 plot_circuit(cnot_sum(1))
10 plot_circuit(cnot_sum(-1))
```



Shor circuit

```
1 cnot_sum_motif = (
2     lambda r=1: Qinit(3, name=f"cns")
3     + Qpivot("*1", merge_within="11", mapping=cnot)
4     + Qpivot("*1", merge_within="111", mapping=sum(r))
5 )
6
7 cnot_sum = lambda r=1: cnot_sum_motif(1) if r == 1 else cnot_sum_motif(-1).reverse()
8
9 plot_circuit(cnot_sum(1))
10 plot_circuit(cnot_sum(-1))
```



Shor circuit

```
1 # ===== Motifs level 3
2 carry_layer = lambda r=1: Qcycle(
3     1,
4     3,
5     0,
6     mapping=carry(r),
7     boundary="open",
8     edge_order=[r],
9 )
10 plot_circuit(carry_layer_1)
```



Shor circuit

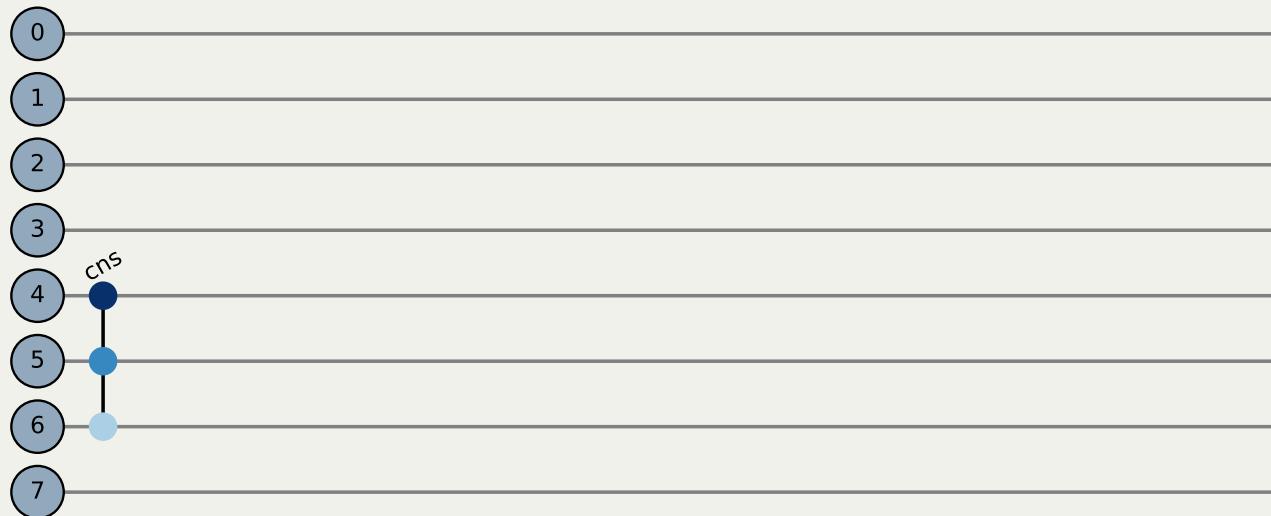
```
1 cnot_sum_pivot = lambda r: Qpivot("*10", merge_within="111", mapping=cnot_sum(r))
2
3 plot_circuit(Qinit(6) + cnot_sum_pivot(1))
4 plot_circuit(Qinit(8) + cnot_sum_pivot(1))
```



<https://github.com/matt-loureens/hierarqcal>

Shor circuit

```
1 cnot_sum_pivot = lambda r: Qpivot("*10", merge_within="111", mapping=cnot_sum(r))
2
3 plot_circuit(Qinit(6) + cnot_sum_pivot(1))
4 plot_circuit(Qinit(8) + cnot_sum_pivot(1))
```



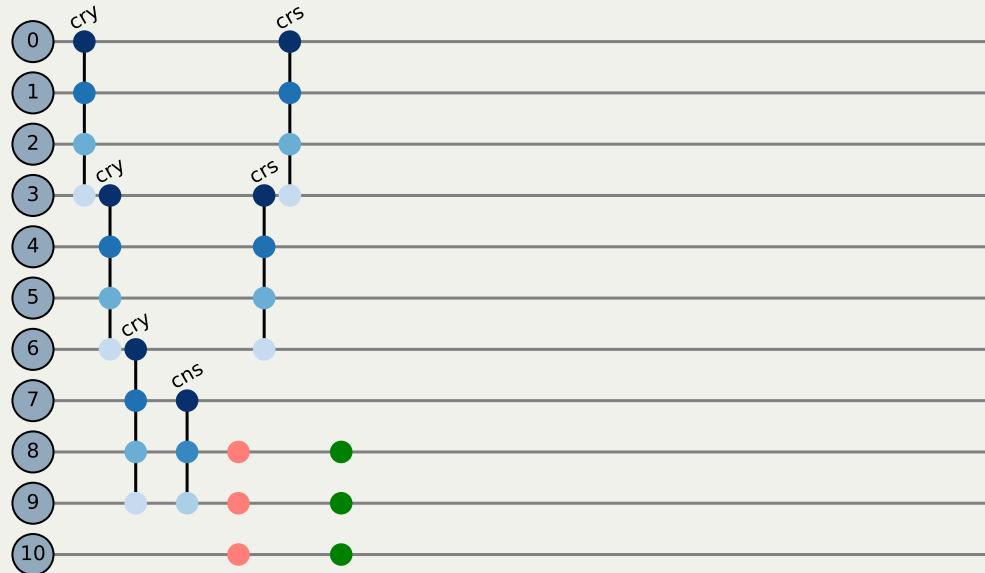
Shor circuit

```
1 carry_sum_layer = lambda r: (
2     Qmask("*111")
3     + Qcycle(1, 3, 0,
4         boundary="open",
5         mapping=carry_sum(r),
6         edge_order=[-r],
7     )
8     + Qunmask("previous")
9 )
10 plot_circuit(Qinit(10) + carry_sum_layer(1))
```



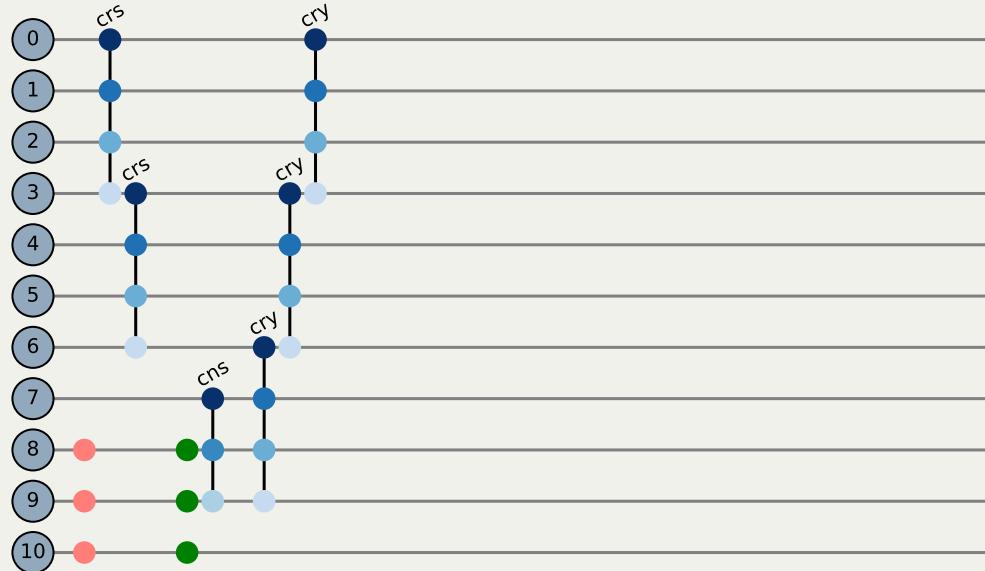
Shor circuit

```
1 addition = carry_layer(1) + cnot_sum_pivot(1) + carry_sum_layer(1)
2 subtraction = carry_sum_layer(-1) + cnot_sum_pivot(-1) + carry_layer(-1)
3
4 plot_circuit(Qinit(11) + addition)
5 plot_circuit(Qinit(11) + subtraction)
6 plot_circuit(Qinit(13) + addition)
7 plot_circuit(Qinit(13) + subtraction)
```



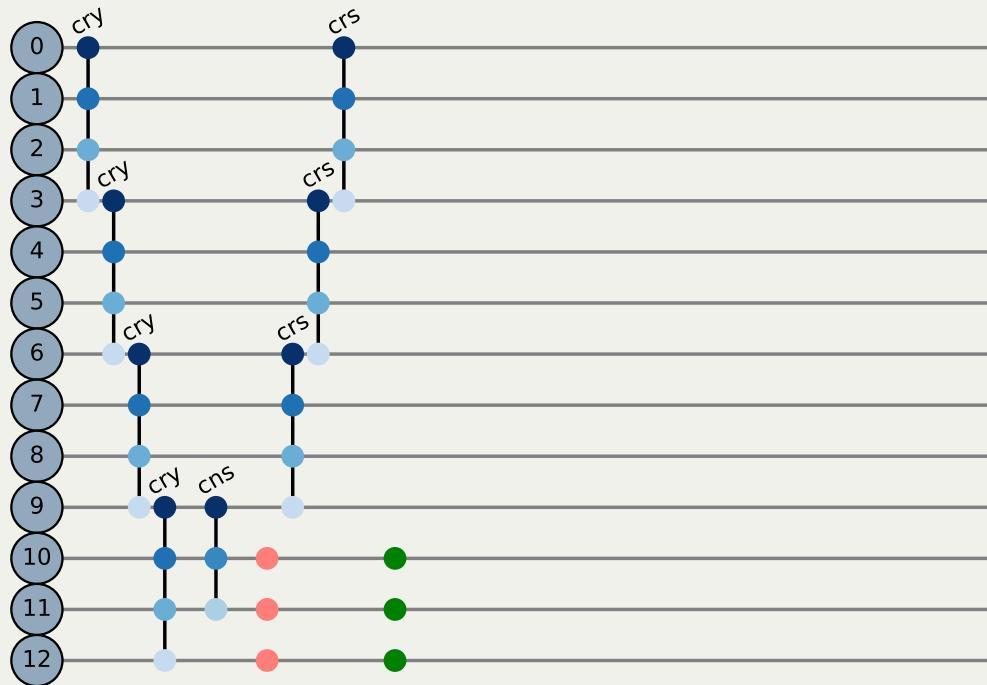
Shor circuit

```
1 addition = carry_layer(1) + cnot_sum_pivot(1) + carry_sum_layer(1)
2 subtraction = carry_sum_layer(-1) + cnot_sum_pivot(-1) + carry_layer(-1)
3
4 plot_circuit(Qinit(11) + addition)
5 plot_circuit(Qinit(11) + subtraction)
6 plot_circuit(Qinit(13) + addition)
7 plot_circuit(Qinit(13) + subtraction)
```



Shor circuit

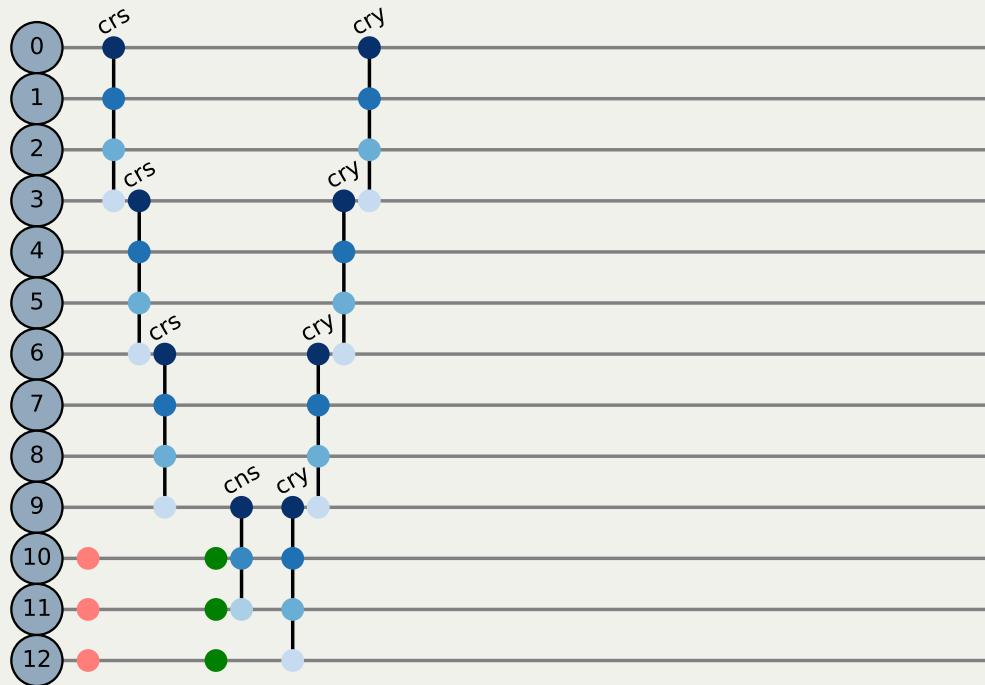
```
1 addition = carry_layer(1) + cnot_sum_pivot(1) + carry_sum_layer(1)
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3
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5 plot_circuit(Qinit(11) + subtraction)
6 plot_circuit(Qinit(13) + addition)
7 plot_circuit(Qinit(13) + subtraction)
```



<https://github.com/matt-lourens/hierarqcal>

Shor circuit

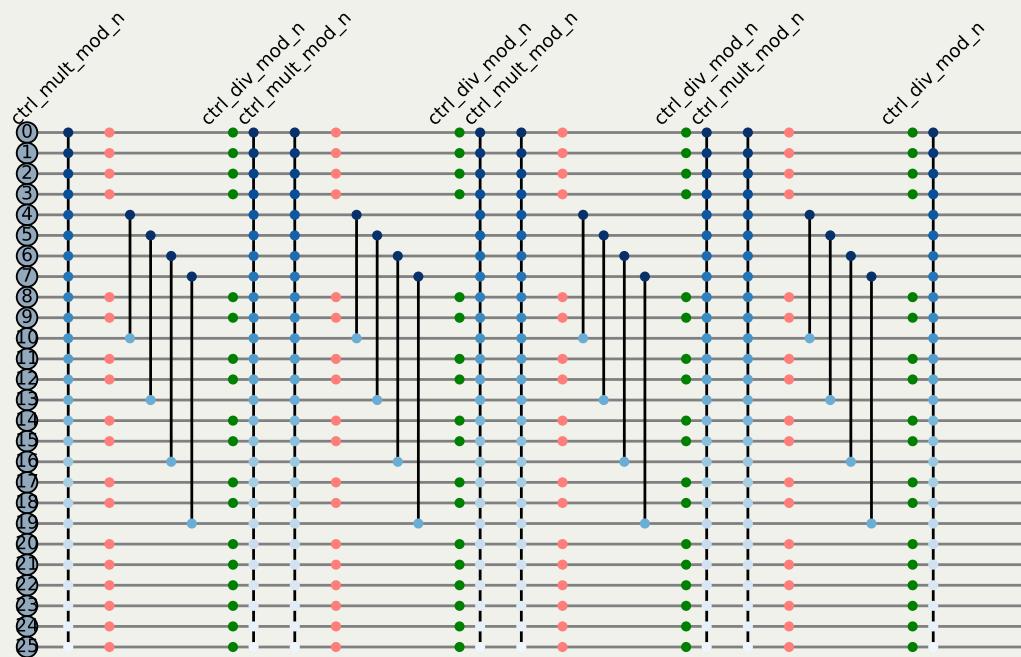
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2 subtraction = carry_sum_layer(-1) + cnot_sum_pivot(-1) + carry_layer(-1)
3
4 plot_circuit(Qinit(11) + addition)
5 plot_circuit(Qinit(11) + subtraction)
6 plot_circuit(Qinit(13) + addition)
7 plot_circuit(Qinit(13) + subtraction)
```



<https://github.com/matt-lourens/hierarqcal>

Shor circuit

```
1 exp_mod_n = tuple()
2 for k in range(n):
3     exp_mod_n += (
4         Qpivot(mapping=ctrl_mult(a ** (2**k), N, n, ctrl=k, divide=False))
5         + swap_bx
6         + Qpivot(mapping=ctrl_mult(a ** (2**k), N, n, ctrl=k, divide=True)))
7     )
8
9 hierq = Qinit(nq, tensors=tensors) + exp_mod_n
```

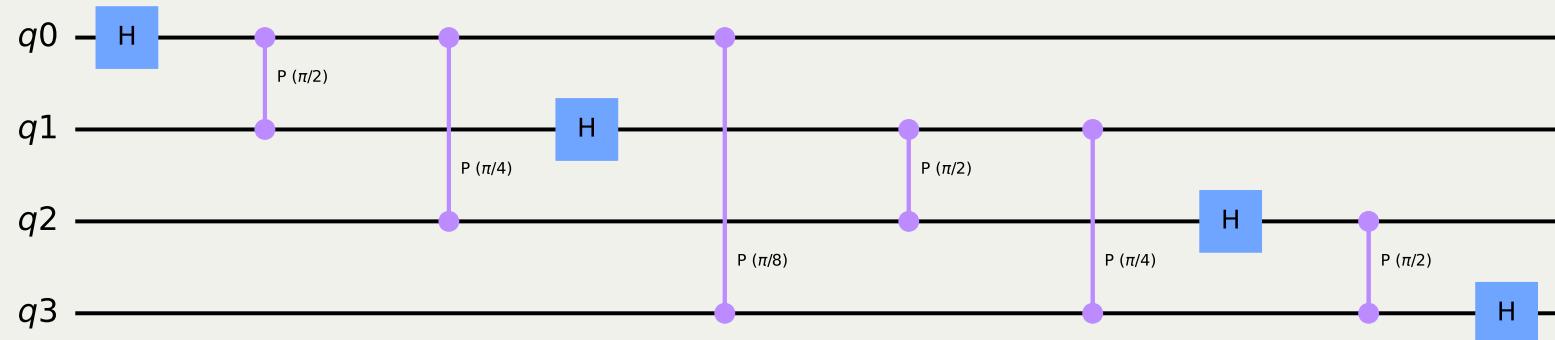


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Quantum Fourier Transform

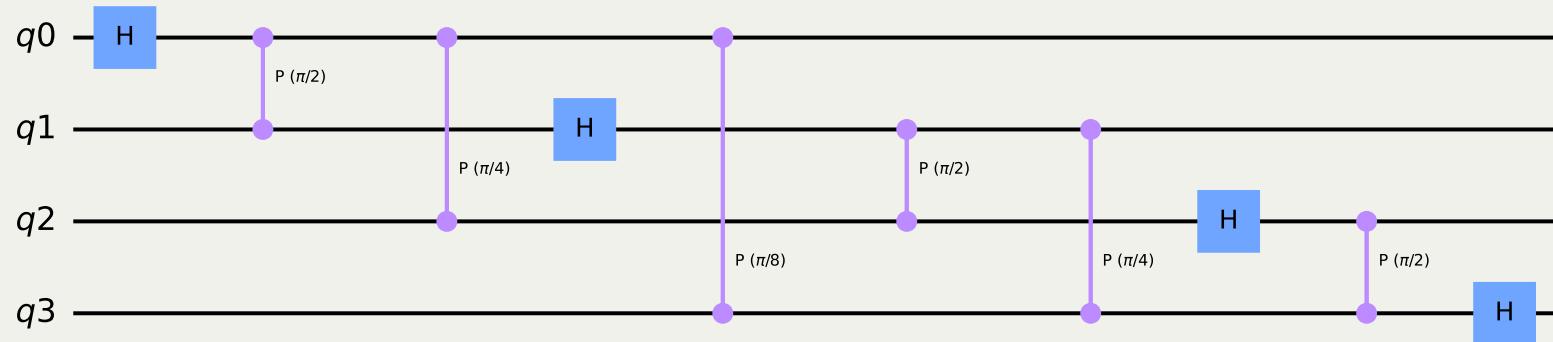
Quantum Fourier Transform

```
1 # QFT circuit
2 n = 4
3 qft = (
4     Qpivot(mapping=Qunitary("h()^0"))
5     + Qpivot(
6         mapping=Qunitary("cp(x)^01"),
7         share_weights=False,
8         symbol_fn=lambda x, ns, ne: np.pi * 2 ** (-ne),
9     )
10    + Qmask("1*")
11 ) * n
12 qft_n = Qinit(n) + qft
13
14 circuit_qft = qft_n(backend="qiskit", barriers=False)
15 circuit_qft.draw("mpl")
```



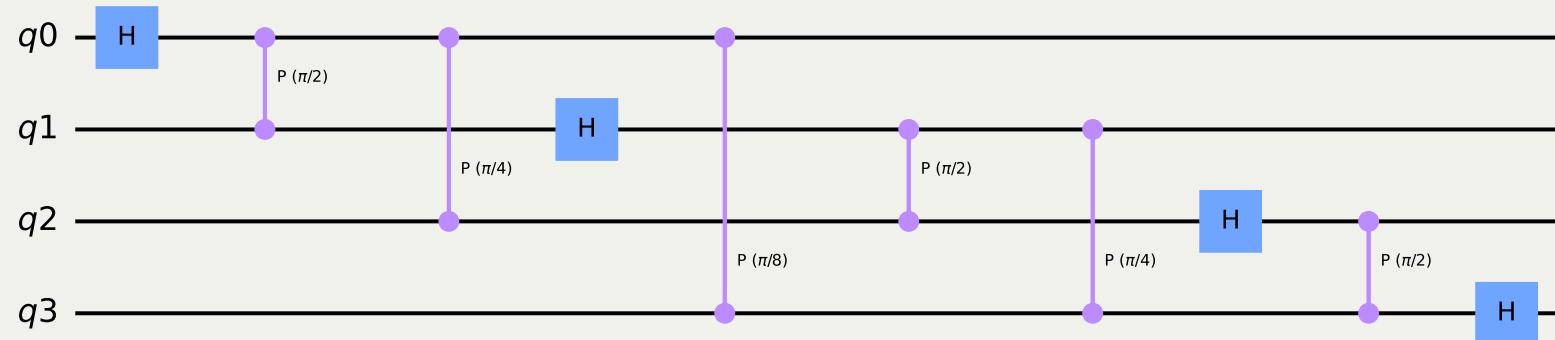
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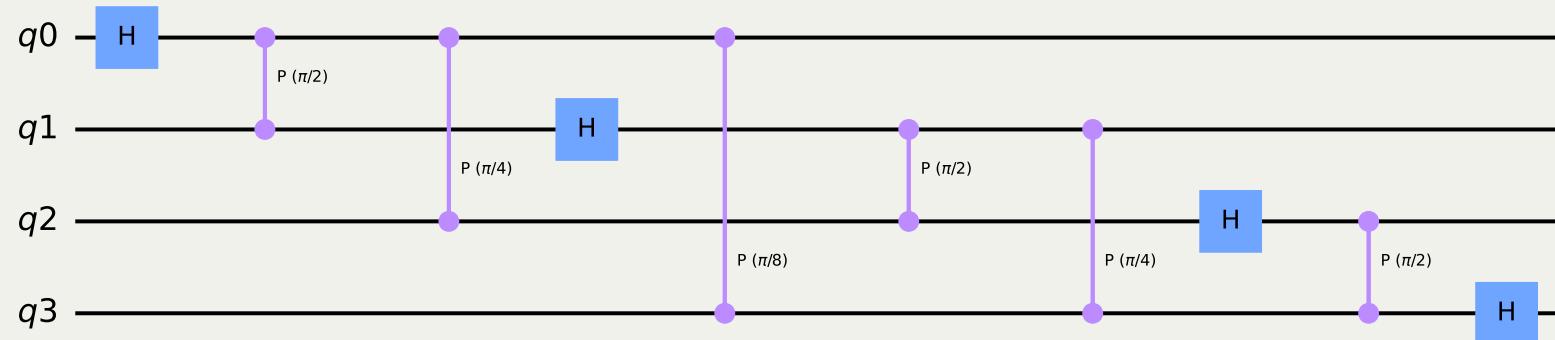
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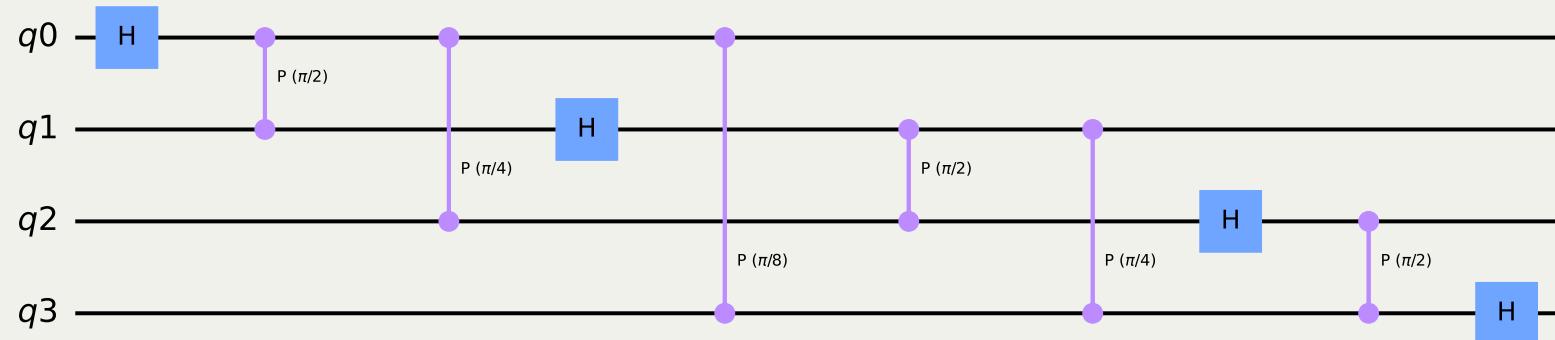
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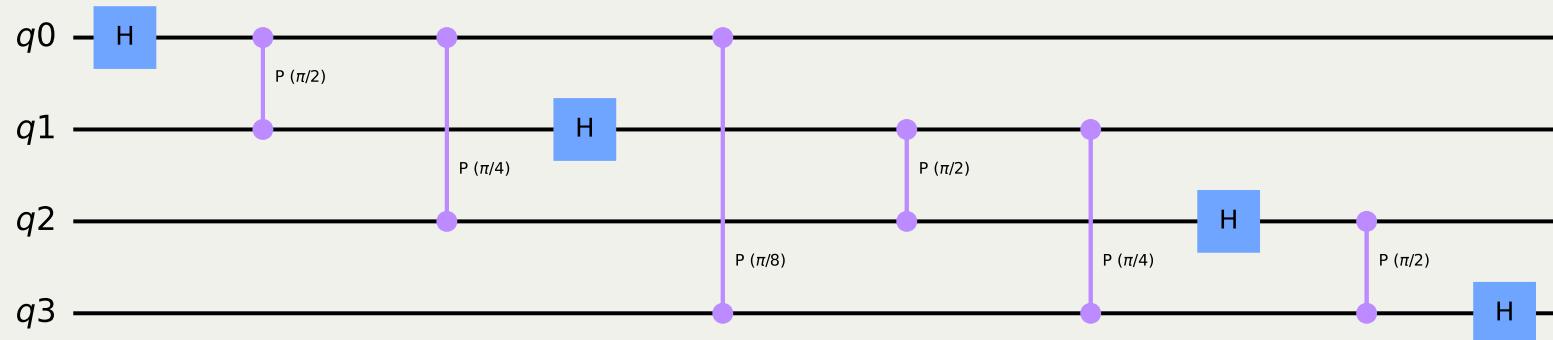
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Classical Adder

Classical Adder

```
1 def half_adder(bits, symbols=None, state=None):
2     b1, b2 = state[bits[0]], state[bits[1]]
3     xor = b1 ^ b2
4     carry = b1 and b2
5     state[bits[0]] = carry
6     state[bits[1]] = xor
7     return state
8
9 def or_top(bits, symbols=None, state=None):
10    b1, b2 = state[bits[0]], state[bits[1]]
11    state[bits[0]] = b1 or b2
12    return state
```

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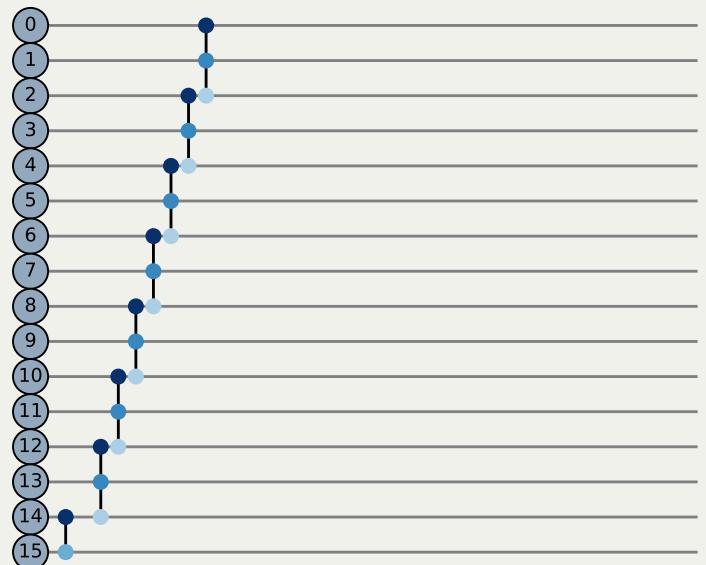
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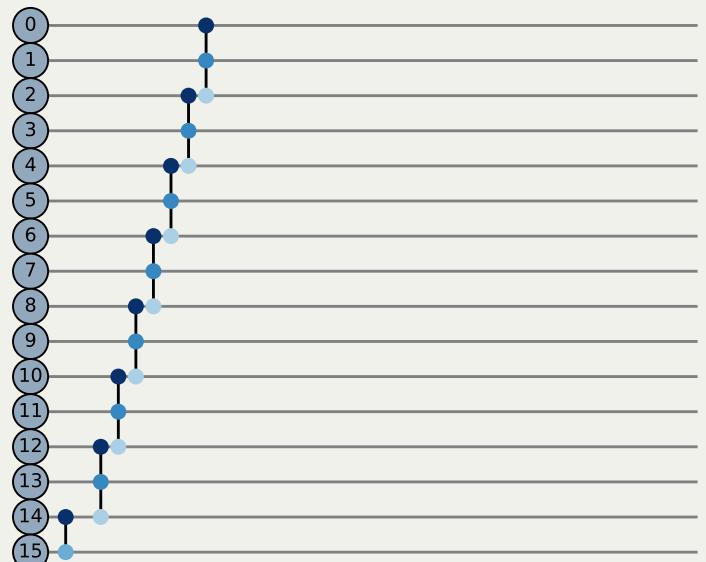
```
1 # program
2 full_adder = (
3     Qinit(3)
4     + Qcycle(mapping=half_adder, boundary="open")
5     + Qpivot(global_pattern="1*", merge_within="11"
6           ,mapping=or_top)
7 )
8 addition = (
9     Qinit(n)
10    + Qpivot("*1", "11", mapping=half_adder)
11    + Qcycle(step=2,
12              edge_order=[-1],
13              mapping=full_adder,
14              boundary="open")
15 )
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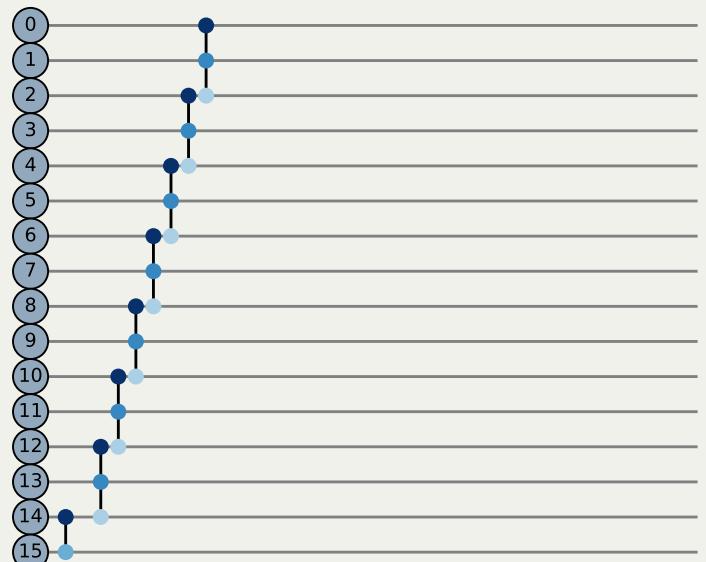
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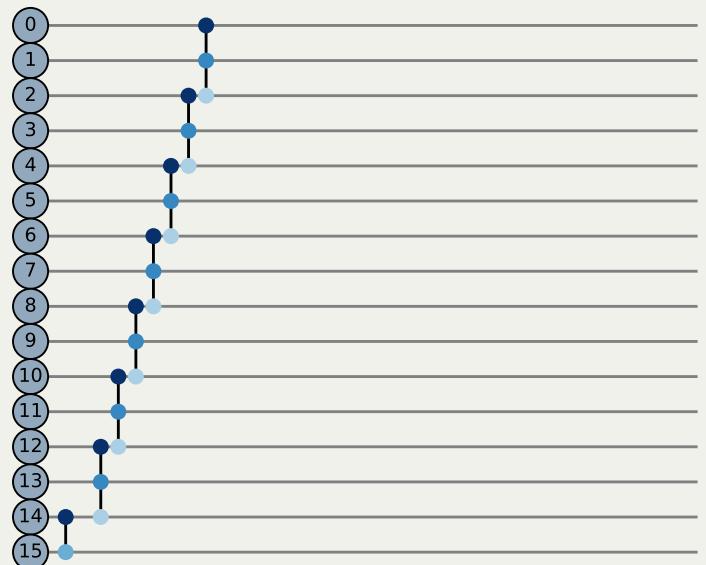
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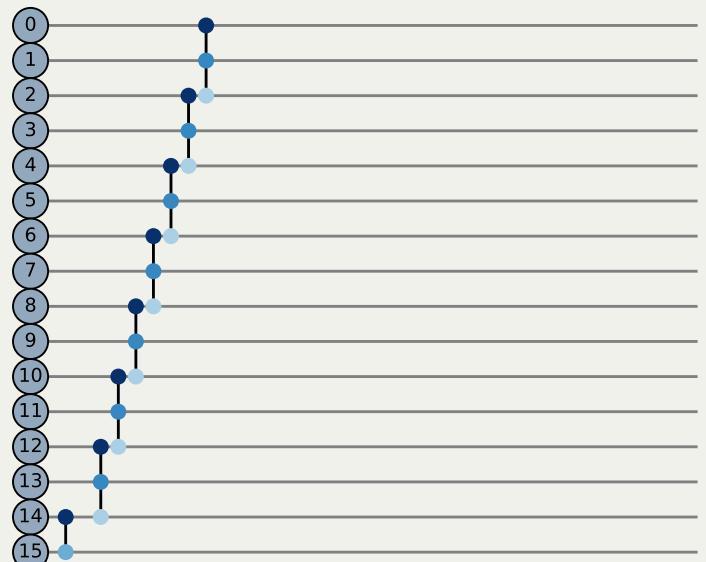
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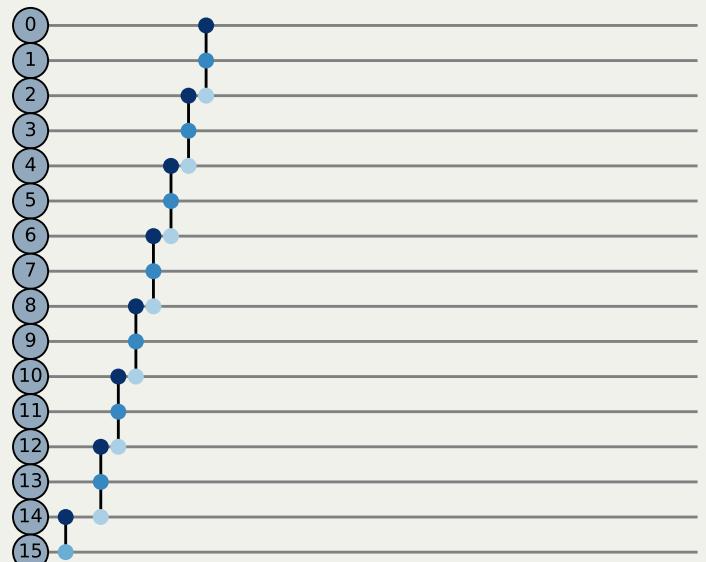
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