University of Ottawa

MCG 5141, Assignment I

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Question 1

a)

$$
\begin{aligned}
0 &= \vec{\nabla} \cdot (\vec{\nabla} \times \vec{v}) \\
0 &= \vec{\nabla} \cdot (\epsilon_{ijk} \frac{\partial v_k}{\partial x_j}) \\
0 &= \frac{\partial}{\partial x_i} \cdot (\epsilon_{ijk} \frac{\partial v_k}{\partial x_j}) \\
0 &= \epsilon_{ijk} \frac{\partial^2 v_k}{\partial x_i \partial x_j} \\
0 &= \epsilon_{123} \frac{\partial^2 v_3}{\partial x_1 \partial x_2} + \epsilon_{213} \frac{\partial^2 v_3}{\partial x_2 \partial x_1} + \epsilon_{312} \frac{\partial^2 v_2}{\partial x_3 \partial x_1} \\
&\quad + \epsilon_{132} \frac{\partial^2 v_2}{\partial x_1 \partial x_3} + \epsilon_{231} \frac{\partial^2 v_1}{\partial x_2 \partial x_3} + \epsilon_{321} \frac{\partial^2 v_1}{\partial x_3 \partial x_2} \\
0 &= \frac{\partial^2 v_3}{\partial x_1 \partial x_2} - \frac{\partial^2 v_3}{\partial x_2 \partial x_1} + \frac{\partial^2 v_2}{\partial x_3 \partial x_1} \\
&\quad - \frac{\partial^2 v_2}{\partial x_1 \partial x_3} + \frac{\partial^2 v_1}{\partial x_2 \partial x_3} - \frac{\partial^2 v_1}{\partial x_3 \partial x_2} \\
0 &= 0
\end{aligned}
$$

b)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v})$$
$$\epsilon_{ijk} \frac{\partial}{\partial x_j} \epsilon_{klm} \frac{\partial v_m}{\partial x_l} = \vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v}$$
$$\epsilon_{ijk} \epsilon_{klm} \frac{\partial}{\partial x_j} \frac{\partial v_m}{\partial x_l} = \vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v}$$

iterate over i, j, k, l, m
the solution for $i = 1$

$$\epsilon_{123}\epsilon_{321} \frac{\partial}{\partial x_2} \frac{\partial v_1}{\partial x_2} + \epsilon_{123}\epsilon_{312} \frac{\partial}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \epsilon_{132}\epsilon_{231} \frac{\partial}{\partial x_3} \frac{\partial v_1}{\partial x_3} + \epsilon_{132}\epsilon_{213} \frac{\partial}{\partial x_3} \frac{\partial v_3}{\partial x_1} = \left[\vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v})\right]_i$$
$$(+1)(-1) \frac{\partial}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (+1)(+1) \frac{\partial}{\partial x_2} \frac{\partial v_2}{\partial x_1} + (-1)(+1) \frac{\partial}{\partial x_3} \frac{\partial v_1}{\partial x_3} + (-1)(-1) \frac{\partial}{\partial x_3} \frac{\partial v_3}{\partial x_1} = \left[\vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v})\right]_i$$
$$- \frac{\partial}{\partial x_2} \frac{\partial v_1}{\partial x_2} + \frac{\partial}{\partial x_2} \frac{\partial v_2}{\partial x_1} - \frac{\partial}{\partial x_3} \frac{\partial v_1}{\partial x_3} + \frac{\partial}{\partial x_3} \frac{\partial v_3}{\partial x_1} = \left[\vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v})\right]_i$$
$$\frac{\partial}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \frac{\partial}{\partial x_3} \frac{\partial v_3}{\partial x_1} - \frac{\partial^2 v_1}{\partial x_2^2} - \frac{\partial^2 v_1}{\partial x_3^2} = \left[\vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v})\right]_i$$

The same procedure can be done for j and k

$$\frac{\partial}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \frac{\partial}{\partial x_3} \frac{\partial v_3}{\partial x_2} - \frac{\partial^2 v_2}{\partial x_1^2} - \frac{\partial^2 v_2}{\partial x_3^2} = \left[\vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v})\right]_j$$
$$\frac{\partial}{\partial x_1} \frac{\partial v_1}{\partial x_3} + \frac{\partial}{\partial x_1} \frac{\partial v_3}{\partial x_2} - \frac{\partial^2 v_3}{\partial x_2^2} - \frac{\partial^2 v_3}{\partial x_1^2} = \left[\vec{\nabla}(\vec{\nabla} \cdot \vec{v} - \nabla^2 \vec{v})\right]_k$$