University of Ottawa

Discrete-Velocity Scheme Project

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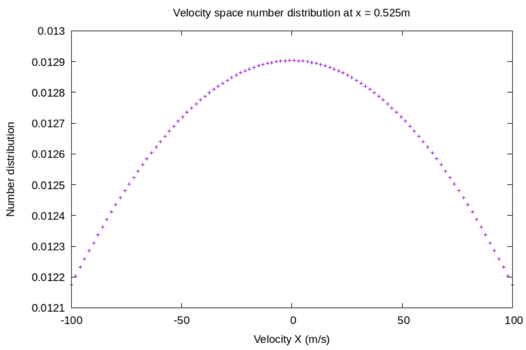
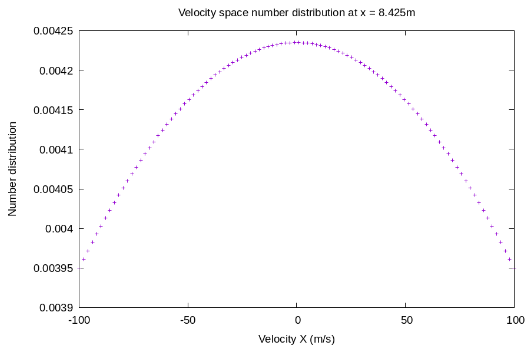
1 Shock-Tube Problem

For the initial conditions of the shock-tube we must select an appropriate velocity space. This velocity space can be determined by observing the distribution function on each side of the shock tube for both initial conditions. The distribution function in velocity space can be determined by applying Maxwell-Boltzmann distribution for various ranges in velocity space.

$$f = \frac{\rho}{m}\left(\frac{\rho}{2\pi p}\right)^{\frac{1}{2}} e^{\frac{\rho}{2p}(u-v)^2}$$



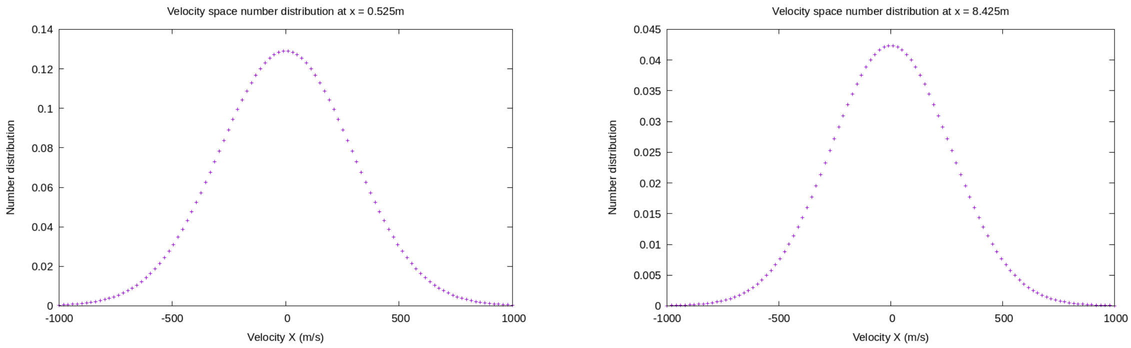
(a) Left initial conditions



(b) Right initial conditions

Figure 1: Distribution function at initial conditions velocity range of $\pm 100\,\mathrm{m\,s^{-1}}$

Figure 1 shows the distribution function for the left and right initial conditions. The velocity space selected was ranging from $\pm 100\,\mathrm{m\,s^{-1}}$. This range of velocity as shown in Figure 1 is insufficient as the tail end of the distribution is cut off for both the left and right ends of the shock tubes.



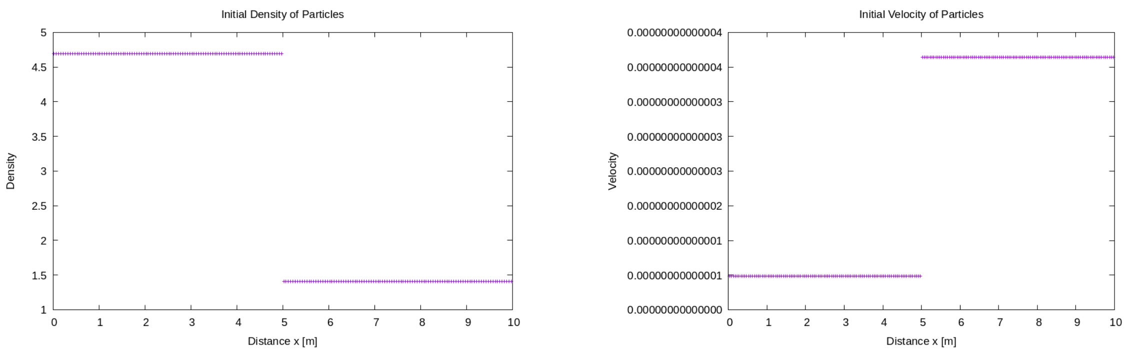
(a) Left initial conditions (b) Right initial conditions

Figure 2: Distribution function at initial conditions velocity range of $\pm 1000\,\mathrm{m\,s^{-1}}$

Selecting a velocity range of $\pm 1000\,\mathrm{m\,s^{-1}}$, as shown in 2 we obtain a velocity space that covers the full distribution of the particle space.

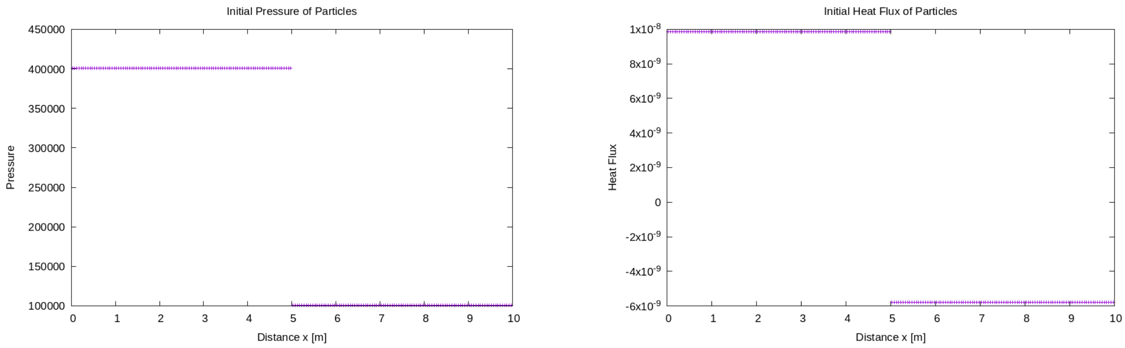
In addition to verifying the velocity space distribution we can also evaluate the properties of the gas at those initial conditions. The properties of the gas can be computed at each point in the x direction by the following equations.

$$
\begin{aligned}
\rho &= \langle mF \rangle & \qquad c &= v - u \\
\rho u &= \langle mvF \rangle \\
p &= \langle mc^2 F \rangle \\
q &= \frac{1}{2} \langle mc^3 F \rangle
\end{aligned}
$$



(a) Left initial conditions (b) Right initial conditions

Figure 3: Initial conditions ρ, u



(a) Left initial conditions

(b) Right initial conditions

Figure 4: Initial conditions p, q

As shown in Figure 3 and 4 the initial density calculated using the set number density of the particles is around the $4.696\,\mathrm{kg/m^3}$ and $1.408\,\mathrm{kg/m^3}$. The initial velocities and heat flux are near zero and the pressure is set to be near $404.4\,\mathrm{kPa}$ and $101.1\,\mathrm{kPa}$.

1.1 Results

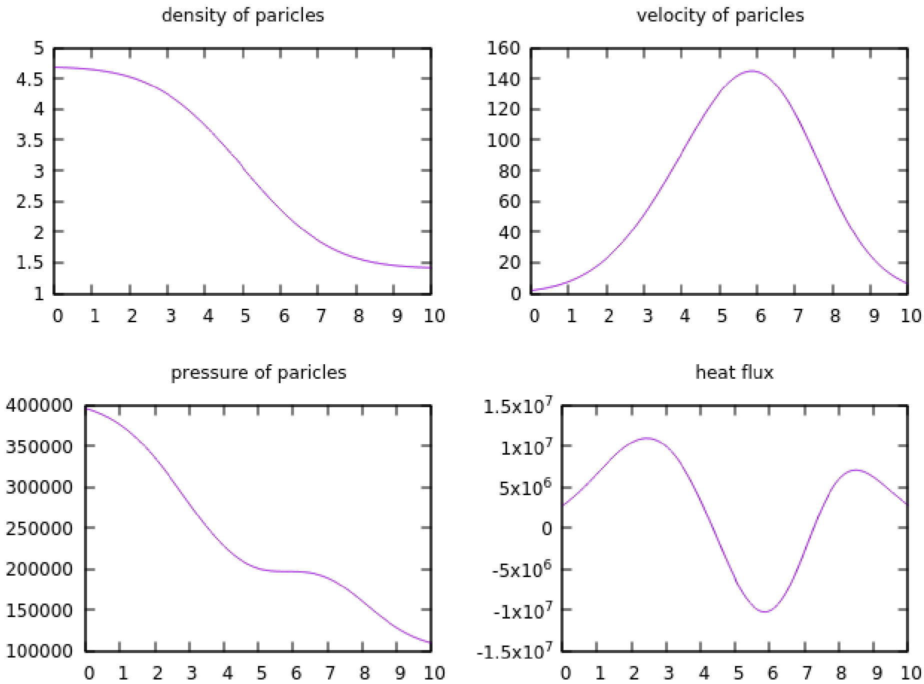


Figure 5: $\tau = 10$

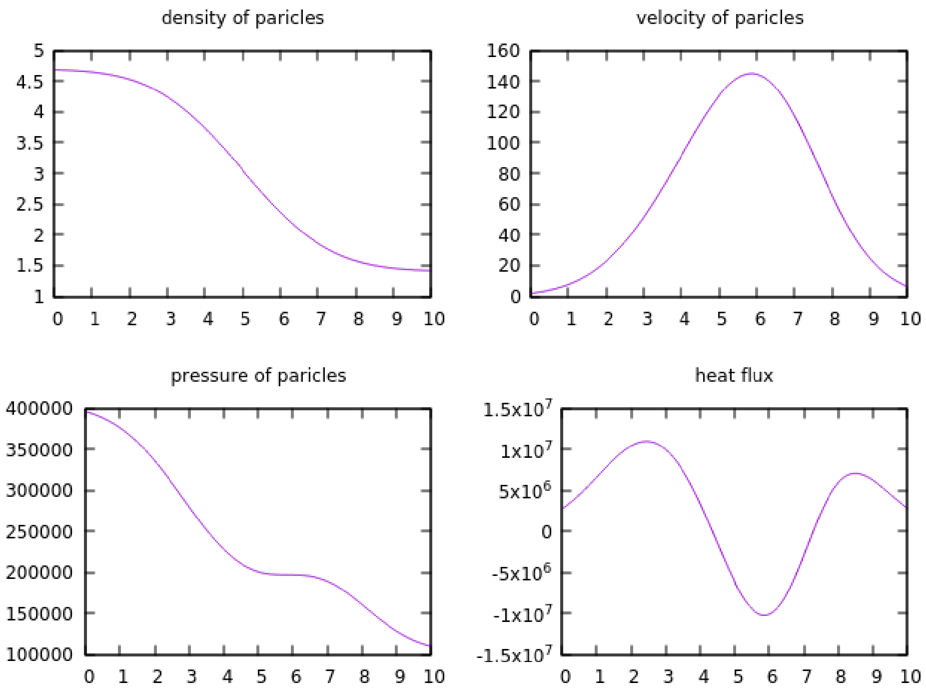


Figure 6: $\tau = 1$

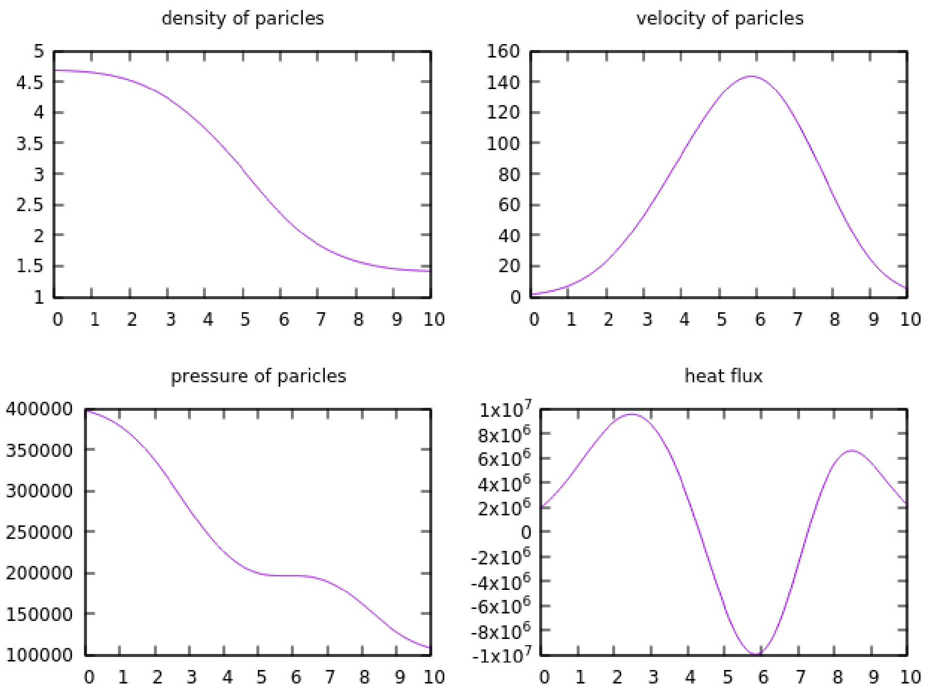


Figure 7: $\tau = 0.01$

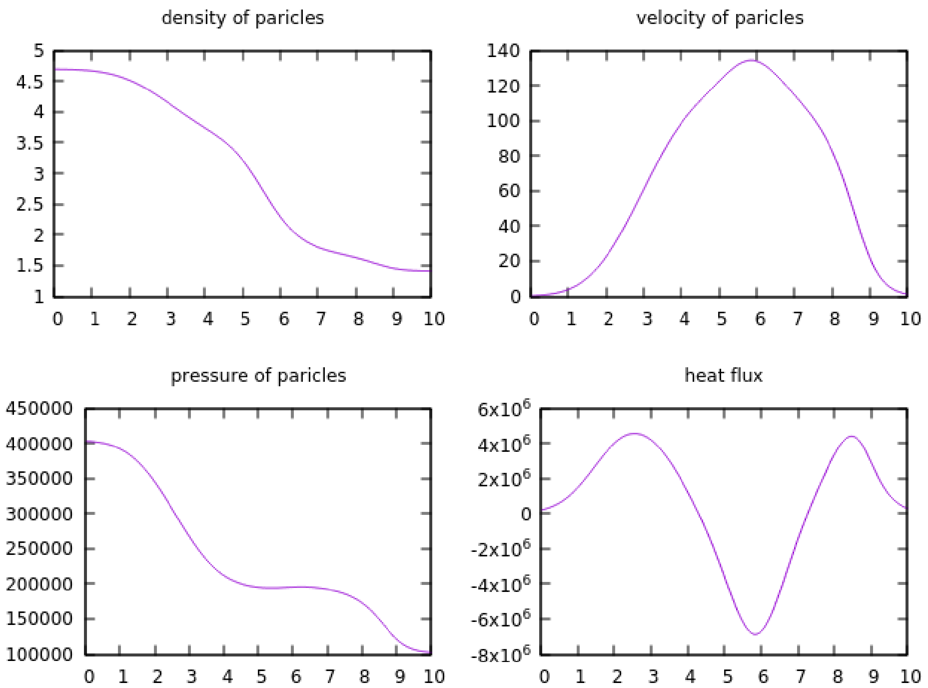


Figure 8: $\tau = 1 \times 10^{-3}$

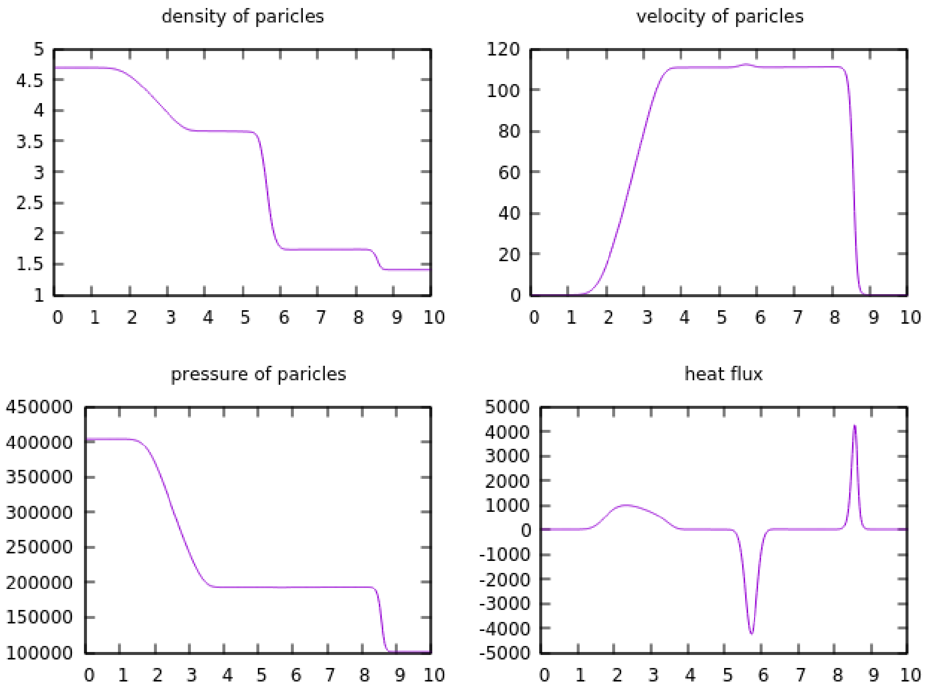


Figure 9: $\tau = 1 \times 10^{-7}$

By looking at Figures 5 - 9 we can attempt to estimate the continuum, transition and free-molecular regimes. In free-molecular regimes particles have little interactions with each other, we seem to be in a free-molecular flow in between $\tau = 10$ to $\tau \approx 0.01$. After which the flow enters a transition zone until it arrives at the continuum region near $\tau \approx 1 \times 10^{-7}$.
In addition to the change of the fluid properties along the x axis we can also evaluate the velocity distribution at certain points along the shock tube. Looking at the probability distribution of the particles at given points lets us observer the most likely velocities particles are to be found at those given points.

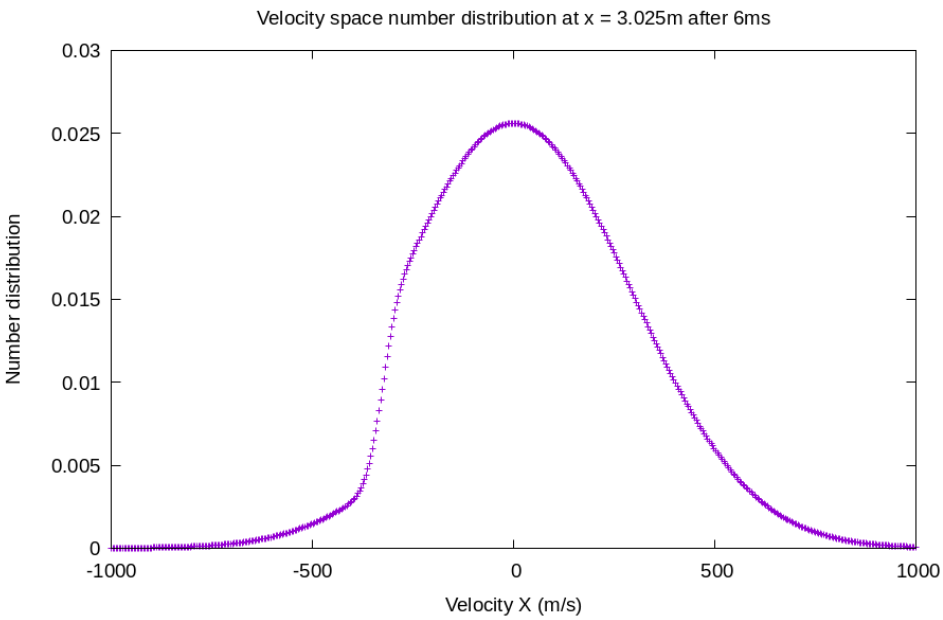


Figure 10: Left velocity distribution at $x = 3.025\,\mathrm{m}, \tau = 1.0$

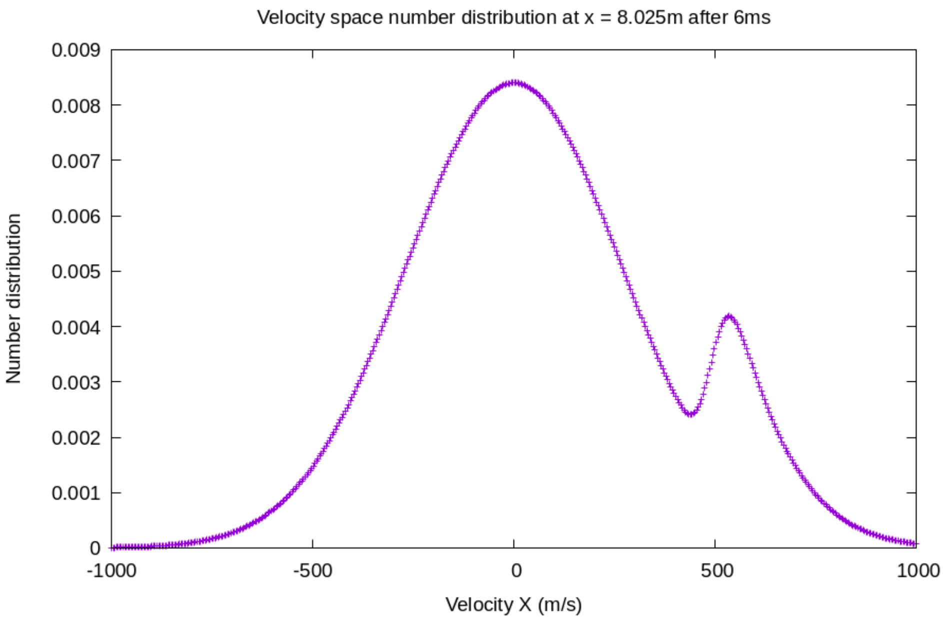


Figure 11: Right velocity distribution at $x = 8.025\,\mathrm{m}, \tau = 1.0$

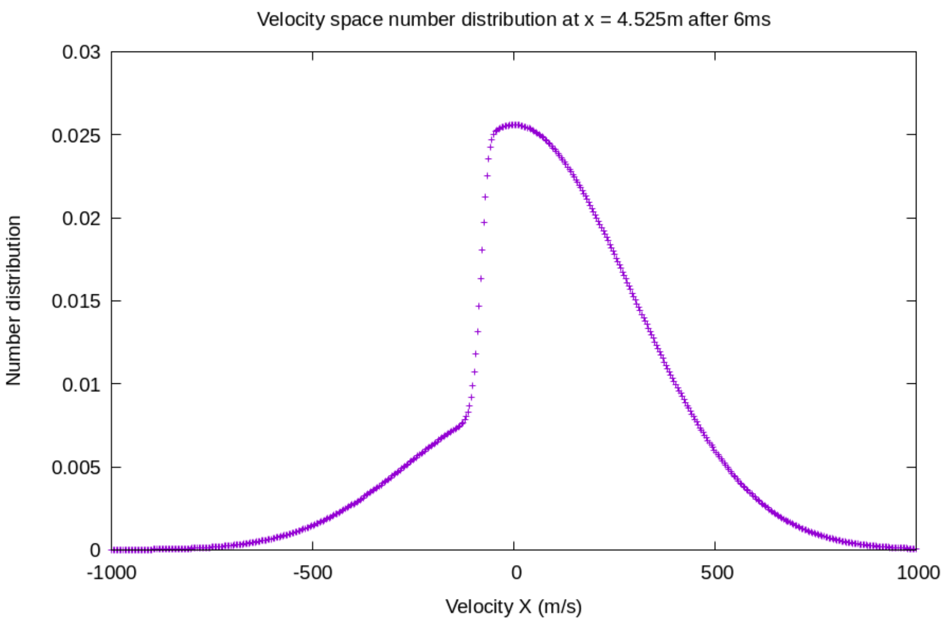


Figure 12: Velocity distribution near center of shock-tube, $\tau = 1.0$

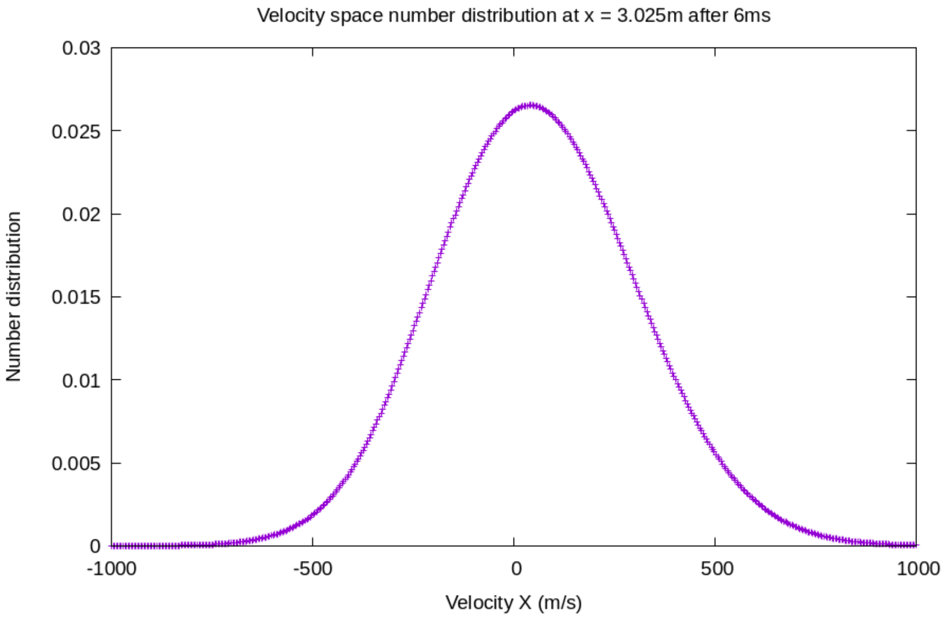


Figure 13: Left velocity distribution at $x = 3.025\,\mathrm{m}, \tau = 1 \times 10^{-3}$

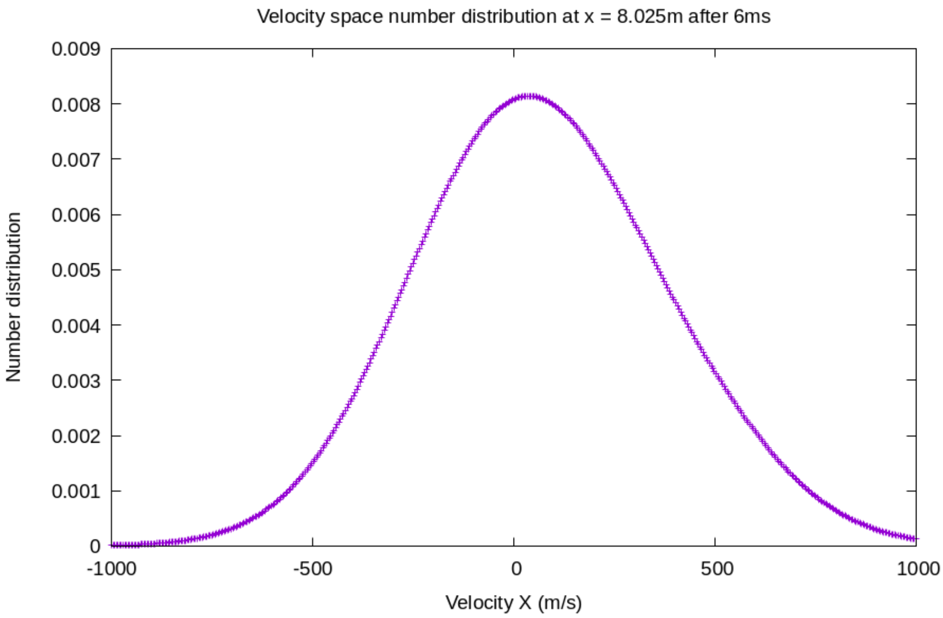


Figure 14: Right velocity distribution at $x = 8.025\,\mathrm{m}, \tau = 1 \times 10^{-3}$

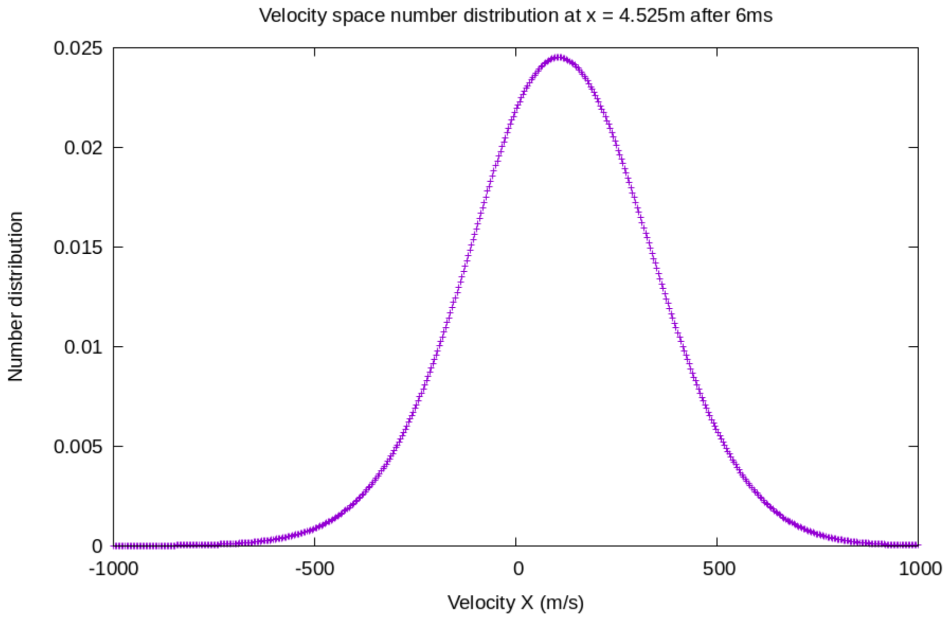


Figure 15: Velocity distribution near center of shock-tube, $\tau = 1 \times 10^{-3}$

2 Shock-wave problem

For this problem the up-steam conditions are set to the following

$$\rho_l = 1.225\,\mathrm{kg/m^3}$$
$$p_l = 101.325\,\mathrm{kPa}$$

Because we know that $\gamma = 3.0$ we can determine the speed of sound for the fluid by the following equation.

$$c = \sqrt{\gamma \frac{p}{\rho}}$$

The velocity of the fluid can then be calculated for a given Mach M, by the following equation.

$$v = M\sqrt{\gamma \frac{p}{\rho}}$$

Given that this is a normal shock we can also calculated to downstream conditions using the following equations.

$$\frac{p_r}{p_l} = \frac{2\gamma M^2 - (\gamma - 1)}{\gamma + 1}$$
$$\frac{\rho_r}{\rho_l} = \frac{(\gamma + 1)M^2}{(\gamma - 1)M^2 + 2}$$
$$M_r^2 = \frac{(\gamma - 1)M^2 + 2}{2\gamma M^2 - (\gamma - 1)}$$

2.1 Results

As this is a normal shock the domain (in the x direction) does not have to be really large as we are only evaluating the conditions before and after the shock. Because of this a domain ranging from 0 meter to 0.5 meter was selected.

For a mach number of 2 the following upstream and downstream conditions were used to initially set the domain.

```
Begin shock time maching
UPSTEAM CONDITIONS
MACH:        2
Rho:         1.225kg/m^3
Pressure:    101325Pa
Velocity:    996.279m/s

DONWSTREAM CONDITIONS
Rho:         1.96kg/m^3
Pressure:    557288Pa
Velocity:    622.674m/s
```

With $\tau = 1 \times 10^{-7}$ and a final time of $tf = 1 \times 10^{-4}$ second the flow properties along the shock are the following.

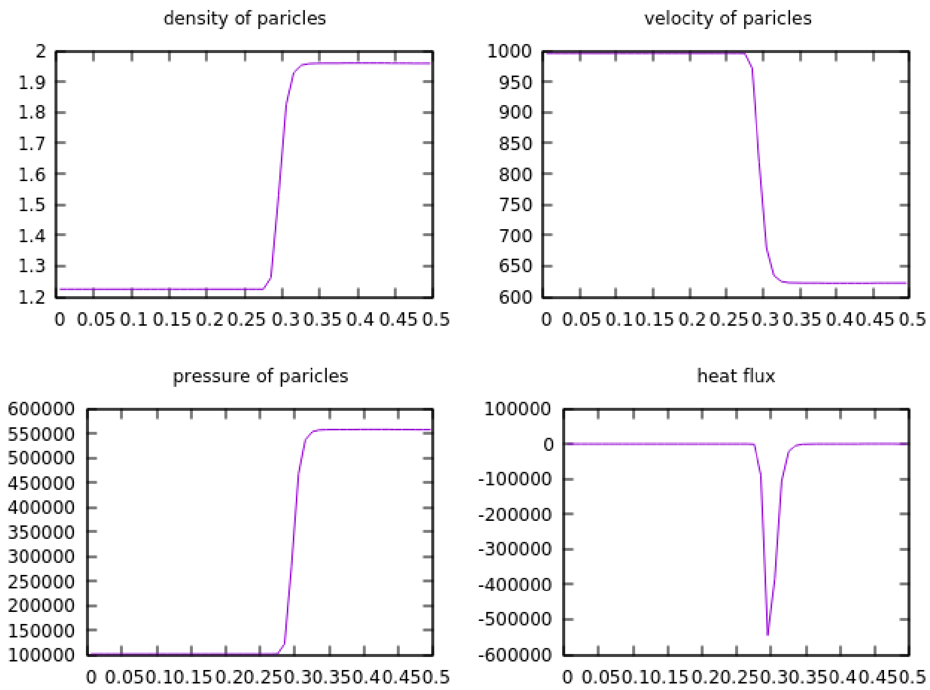


Figure 16: Properties along the shock, $\tau = 1 \times 10^{-7}$

In addition to observing the flow properties along the shock we can also investigate the distribution at certain points along the shock.

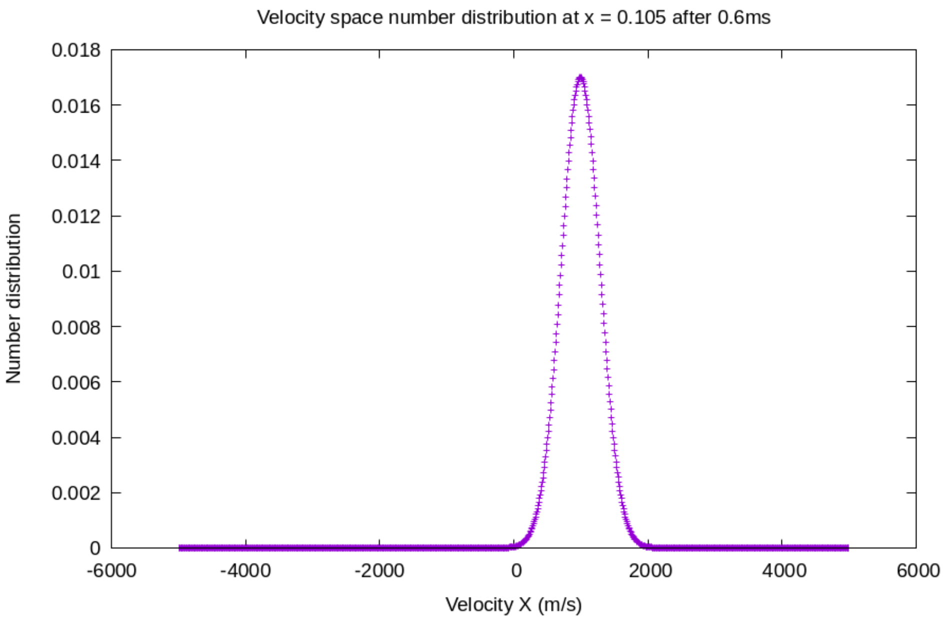


Figure 17: Distribution at $x = 0.105$

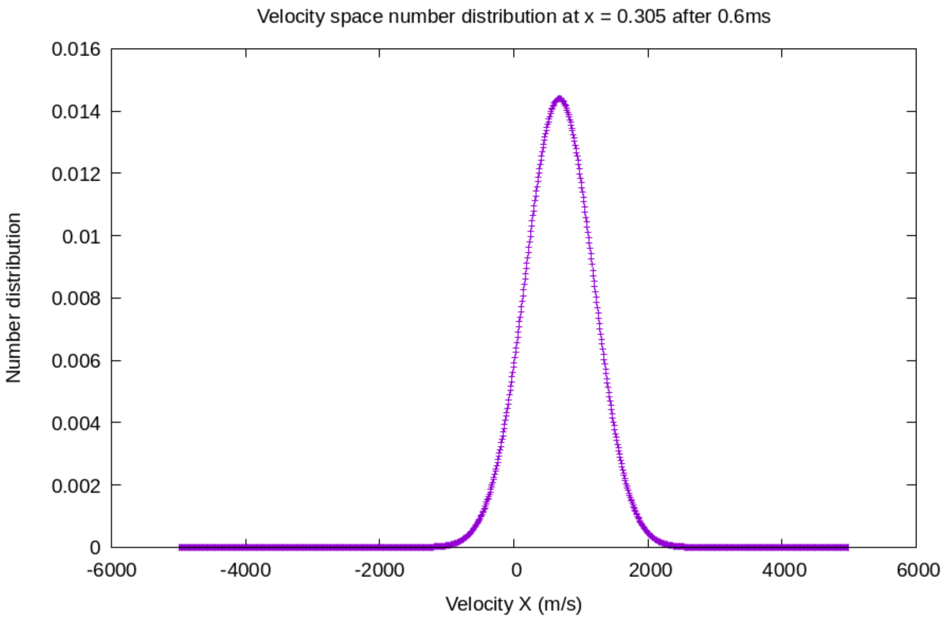


Figure 18: Distribution near shock at $x = 0.305$

For a mach number of 4 the following upstream and downstream conditions were used to initially set the domain.

```
Begin shock time maching
UPSTEAM CONDITIONS
```

```
MACH:        4
Rho:         1.225kg/m^3
Pressure:    101325Pa
Velocity:    1992.56m/s

DONWSTREAM CONDITIONS
Rho:         2.30588kg/m^3
Pressure:    2.38114e+06Pa
Velocity:    1058.55m/s
```

With $\tau = 1 \times 10^{-7}$ and a final time of $tf = 1 \times 10^{-4}$ second the flow properties along the shock are the following.

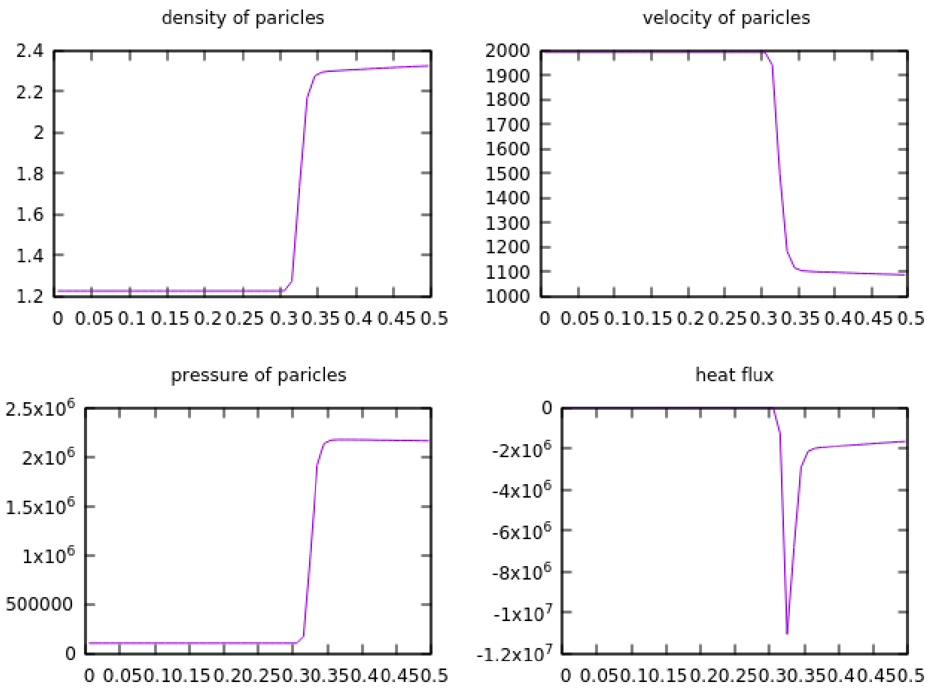


Figure 19: Properties along the shock, $\tau = 1 \times 10^{-7}$

In addition to observing the flow properties along the shock we can also investigate the distribution at certain points along the shock.

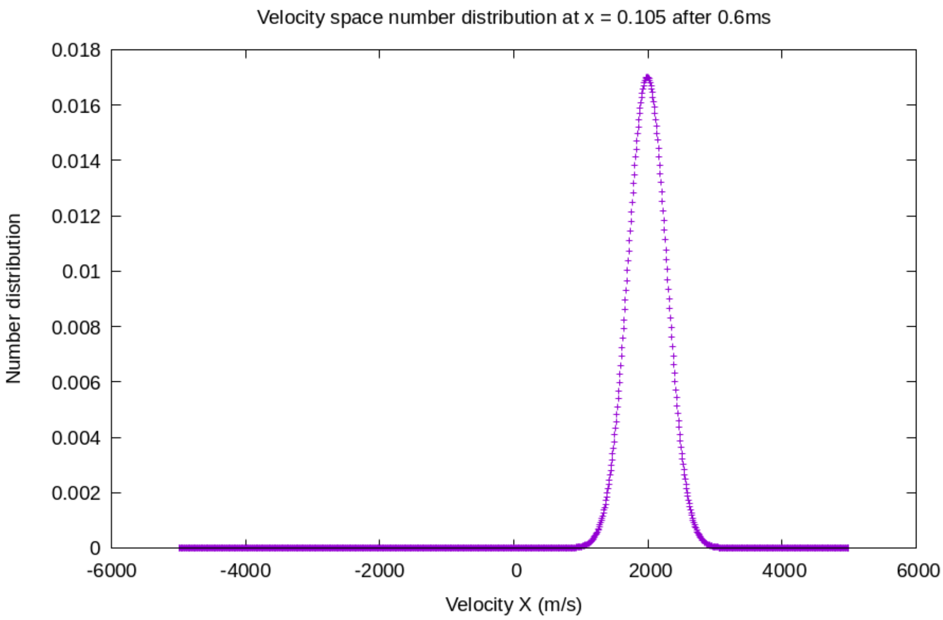


Figure 20: Distribution at $x = 0.105$

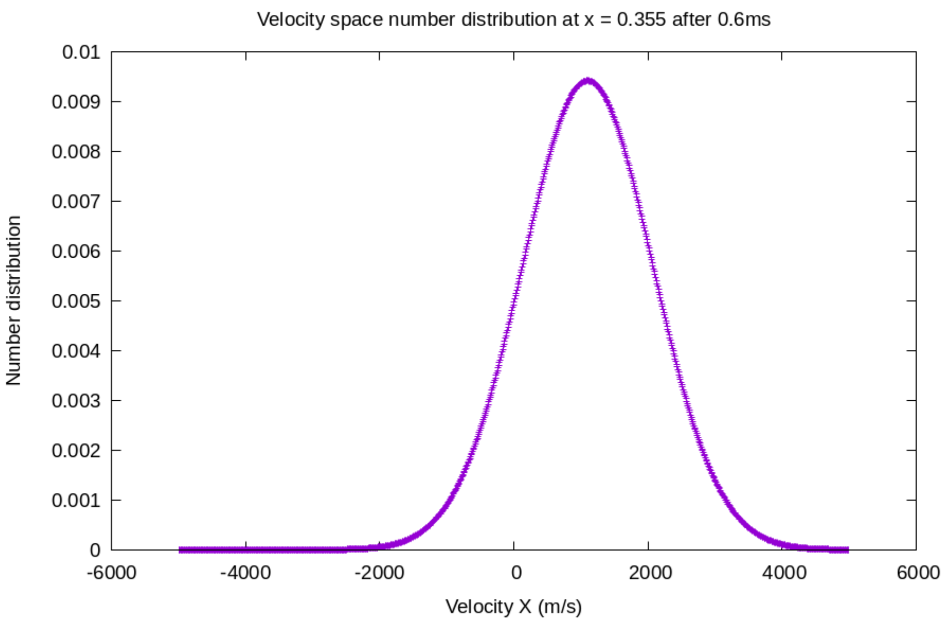


Figure 21: Distribution near shock at $x = 0.305$

It is important to note that as we have significantly decreased τ compared to the problem 1 and the Δx step in physical space we also needed to reduce the time step Δt to still achieve stable time marching.