Matt's Linear Algebra Notes

December 12, 2015

Chapter 1

Material

1.1 Vector Spaces

Definition 1.1.1 (Vector Space)**.** A *vector space* V over a field $\mathbb{F}$ is a set with two binary operations, $+ : V \times V \to V$ and $\cdot : V \times \mathbb{F} \to V$ such that all of the following hold.

1. For all $x, y \in V$, $x + y = y + x$. (Additive Commutativity)
2. For all $x, y, z \in V$, $x + (y + z) = (x + y) + z$. (Additive Associativity)
3. There exists an element, denoted 0, in V such that for all $x \in V$, $x + 0 = x$.
4. For each $x \in V$ there exists a $y \in V$, denoted $-x$, such that $x + y = 0$.
5. For all $x \in V$, $1x = x$.
6. For all $a, b \in \mathbb{F}$ and $x \in V$, $a(bx) = (ab)x$.
7. For all $a \in \mathbb{F}$ and $x, y \in V$, $a(x + y) = ax + ay$.
8. For all $a, b \in \mathbb{F}$ and $x \in V$, $(a + b)x = ax + bx$.

Furthermore, $x + y$ is called the *sum of x and y* while ax is called the *product of x and a*. Moreover, each $x \in V$ is called a *vector* and each $a \in \mathbb{F}$ is called a *scalar*.

Chapter 2

Definitions

2.1 Vector Spaces

Definition 2.1.1 (Vector Space)**.** A *vector space* V over a field $\mathbb{F}$ is a set with two binary operations, $+ : V \times V \to V$ and $\cdot : V \times \mathbb{F} \to V$ such that all of the following hold.

1. For all $x, y \in V$, $x + y = y + x$. (Additive Commutativity)
2. For all $x, y, z \in V$, $x + (y + z) = (x + y) + z$. (Additive Associativity)
3. There exists an element, denoted 0, in V such that for all $x \in V$, $x + 0 = x$.
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5. For all $x \in V$, $1x = x$.
6. For all $a, b \in \mathbb{F}$ and $x \in V$, $a(bx) = (ab)x$.
7. For all $a \in \mathbb{F}$ and $x, y \in V$, $a(x + y) = ax + ay$.
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Chapter 3

Theorems