

Derivatives of Arctan at 0

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1 Statement of Problem

Define $f(x) := \arctan x$. Find $f^{(n)}(0)$.

2 Background

For this problem we will need two theorems. Firstly, we need Taylor's Theorem.

Theorem 2.1 (Taylor's Theorem on a disk in \mathbb{C}). *Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be analytic on a disk of radius r about z_0 . Then there exists a unique power series such that*

$$f(z) = \sum_{k=0}^{\infty} \frac{f^{(k)}(z_0)}{k!} (z - z_0)^k$$

for all z such that $|z - z_0| < r$.

Furthermore, we need an additional Taylor expansion.

Theorem 2.2. *Define*

$$f(z) := \frac{1}{1 - z^2}.$$

Then,

$$f(z) = \sum_{k=0}^{\infty} z^k$$

for any $z \in \mathbb{C}$ with $|z| < 1$.

3 Solution