Algebraic Properties of the Gaussian Integers

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Theorem. *The Gaussian Integers, denoted $\mathbb{Z}(i)$, form a Euclidean domain.*

1 Background

Before we can talk about Euclidean domains, we first need to introduce the definition of a ring.

Definition 1 (Ring)**.** Let R be a nonempty set, and let $+ : R^2 \to R$ and $\cdot : R^2 \to R$ be binary operations on R. Then we say R is a *ring* if all of the following hold.

1. The structure $(R, +)$ is an abelian group.
2. For any $a, b, c \in R$, $a(bc) = (ab)c$ (Multiplicative Associativity).
3. For any $a, b, c \in R$, $a(b + c) = ab + ac$ (Left Distributivity).
4. For any $a, b, c \in R$, $(a + b)c = ac + bc$ (Right Distributivity).