

Problem. For $x, r \in \mathbb{R}$ find

$$\sum_{k=0}^n r^k \cos kx \text{ and } \sum_{k=0}^n r^k \sin kx.$$

Then, assuming $|r| < 1$ take their limits as $n \rightarrow \infty$.