1. Normalize the following vectors ($i = \sqrt{-1}$).

   i. $\mathbf{v} = 3\hat{\mathbf{x}} - 4\hat{\mathbf{z}}$

   ii. $\mathbf{w} = -2\hat{\mathbf{x}} + i\hat{\mathbf{y}} + \sqrt{3}\hat{\mathbf{z}}$

2. Calculate the matrix-matrix and matrix-vector products shown below.

   i. $\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix}$

   ii. $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 2 \\ 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

3. Invert the following matrices. If a matrix has no inverse, explain why.

   i. $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

   ii. $B = \begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}$

4. Solve the following matrix equations. If an equation has no solution, explain why.

   i. $\begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ -11 \end{pmatrix}$

   ii. $\begin{pmatrix} 1 & -2 \\ -4 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

5. Calculate the inner ($\mathbf{u} \cdot \mathbf{v}$) and cross ($\mathbf{u} \times \mathbf{v}$) products of the following pairs of vectors.

i. $\mathbf{a} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

ii. $\mathbf{u} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$, $\mathbf{v} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$

6. Calculate the eigenvalues and eigenvectors for the following matrices.

i. $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

ii. $B = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

7. What can you say about the eigenvalues and eigenvectors of a Hermitian matrix?

8. If a matrix is unitary, what equation does it satisfy?

9. Determine whether or not these sets of vectors form a basis (hint: there are two requirements).

   i. $V = \left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \right\}$ ii. $U = \left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ -4 \end{pmatrix} \right\}$

10. For the two bases shown below

$$B = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \quad , \quad B' = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$$

   determine the transition matrices to go from

   i B' to B ii. B to B'

11. Using the results of the previous problem, find the representations of the following vectors (initially expressed in basis B) in basis B'.

   i $\mathbf{v} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}_B$ ii. $\mathbf{v} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}_B$

12. Find the representation of the matrix $A = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}_B$ in the basis B'.

13. Give the first four Taylor series terms for the following functions. For each function, expand about whatever value of x you see fit.

    i. $f(x) = \sin(x)$ ii. $g(x) = e^x$ iii. $h(x) = (1+x)^n$ for non-integer n

14. Find the Fourier transform of the function $f(x) = \sin(x) + 2\cos(3x)$.