

1. a , b and ϵ are real numbers and $\epsilon \ll 1$. Evaluate $e^{i\epsilon a} b e^{-i\epsilon a}$ to first order in ϵ . (hint: start by Taylor expanding $f(a) = e^{i\epsilon a}$ to first order)
2. A and B are $n \times n$ matrices and $\epsilon \ll 1$ is a real number. Evaluate $e^{i\epsilon A} B e^{-i\epsilon A}$ to first order in ϵ . (hint: your answer should be written in terms of the commutator $[A, B]$)

3. Consider the matrix

$$A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (1)$$

(a) Is A Hermitian? Explain your answer.

(b) Find the eigenvalues and *normalized* eigenvectors of A .

(c) Construct a unitary matrix U whose columns are the normalized eigenvectors of A , and show by explicit matrix multiplication that U is unitary.

(d) Show by explicit matrix multiplication that some product involving A and U produces a diagonal matrix. (hint: the diagonal elements should be the eigenvalues from part (3b))