COMP 4200: Assignment 4

Due on February 26, 2024

Professor Kwon 001

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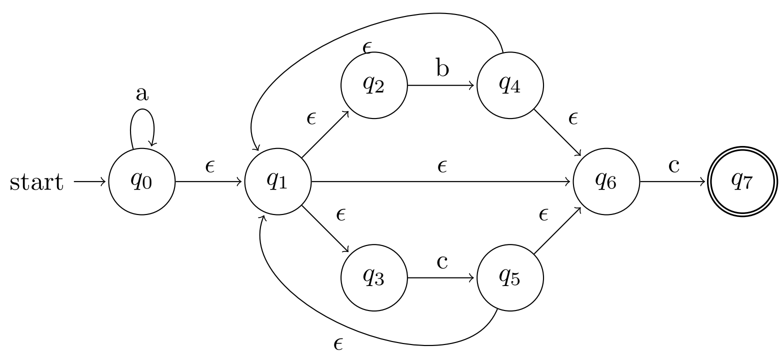
Problem 1

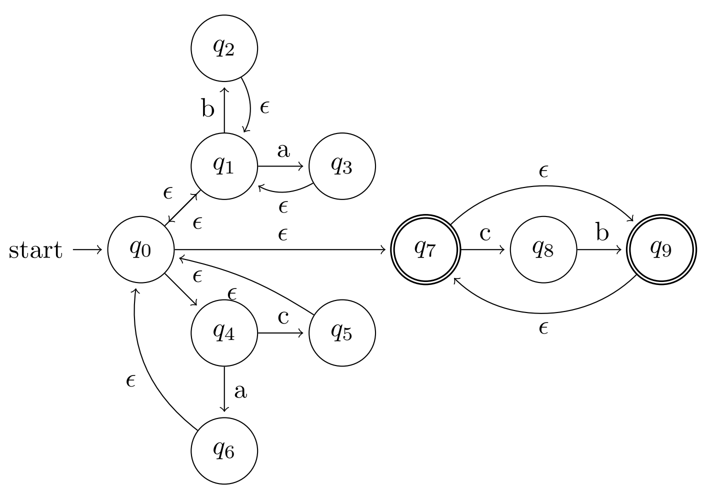
Convert the following regular expressions into equivalent NFAs.

1. $a^*(b \cup c)^*c$
2. $((b \cup a)^* \cup (c \cup a))^*(cb)^*$

Solution
Part One



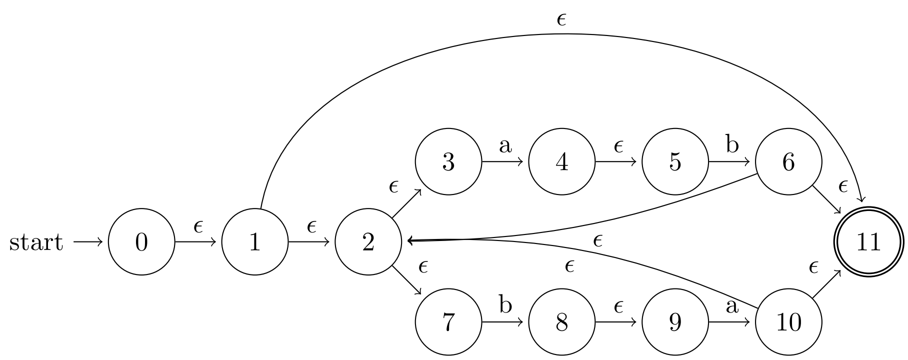
Part Two



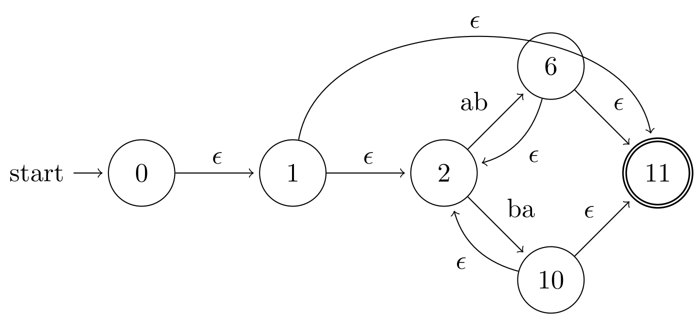
Problem 2

Convert the 2 NFA's to equivalent regular expressions.
Solution

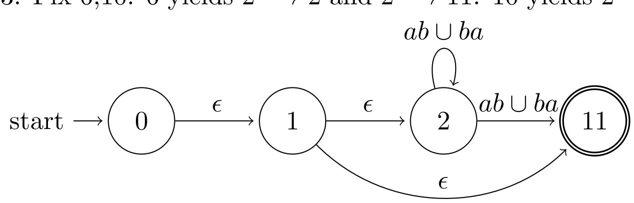
Part One
1. Convert NFA to GNFA



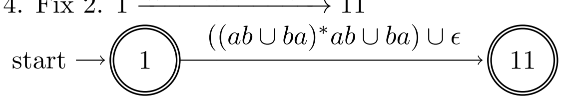
2. Fix 3,4,5 and 7,8,9. These are trivial and can be summarized by $2 \xrightarrow{ab} 6$ and $2 \xrightarrow{ba} 10$



3. Fix 6,10. 6 yields $2 \xrightarrow{ab} 2$ and $2 \xrightarrow{ab} 11$. 10 yields $2 \xrightarrow{ba} 2$ and $2 \xrightarrow{ba} 11$. Finally, $2 \xrightarrow{ab \cup ba} 2$ and $2 \xrightarrow{ab \cup ba} 11$



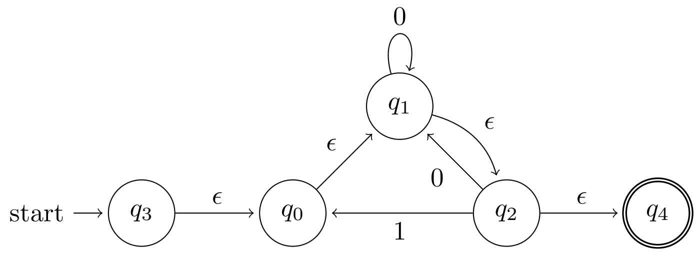
4. Fix 2. $1 \xrightarrow{((ab \cup ba)^* ab \cup ba) \cup \epsilon} 11$



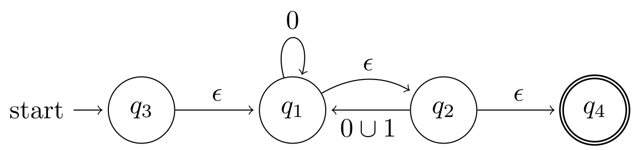
RE: $((ab \cup ba)^* ab \cup ba) \cup \epsilon$

Part Two

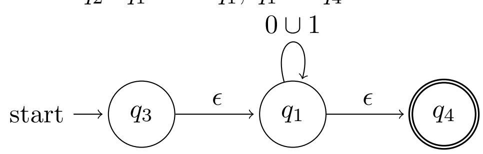
1. Convert NFA to GNFA



2. Fix q_0: $q_3 \xrightarrow{\epsilon} q_1$, $q_2 \xrightarrow{1} q_1$



3. Fix q_2: $q_1 \xrightarrow{0\cup1} q_1$, $q_1 \xrightarrow{\epsilon} q_4$



RE: $(0 \cup 1)^*$

Problem 3

Prove the regularity of the languages. If regular, construct a DFA, NFA, or RE.

1. $X = \{0^m 1^n \mid m > n \geq 0\}$
2. $Y = \{0^n \mid n \text{ is a prime}\}$

Part One

Assume X is regular.
Let $s = 0^{P+1}1^P$. $P + 1 > P$. So for all $P \geq 0$, $s \in X$.
$|s| = 2P + 1$, which is $\geq P$.
Let $xy = 0^P$. So $|xy| = P$. And we can represent s as $xy01^P$.
y will be some amount of 0's. For any $y = 0^n$ where $n > 2$, when you pump down, the number of leading zeros will be less than P. Then $m \not> n$. Then s is not in X, and we arrive at a contradiction. Therefore, X is non-regular.

Part Two

Assume Y is regular.
Let $s = 0^P$, where P is a prime number.
$|s| = P$, which is $\geq P$.
Let $xy = 0^P$. So $|xy| = P$.

Problem 3 continued on next page...

y will be some amount of 0's. After 2, there are no even prime numbers. Therefore for approximately half of all y-pumps, $|xy|$ is not a prime number. Y is not pumpable and we arrive at a contradiction. Therefore, Y is non-regular.