COMP 4200: Assignment 3

Due on February 12, 2024

Professor Kwon 001

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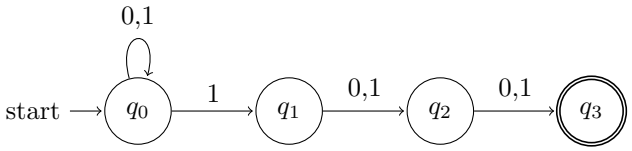
Problem 1

Construct NFAs that recognize the following languages:

1. All binary numbers that contain a 1 in the 3rd location from the right.
2. All binary numbers that contain at most two 1's or contain at most two 0's.
3. All binary numbers that can be divided by 4.

Solution

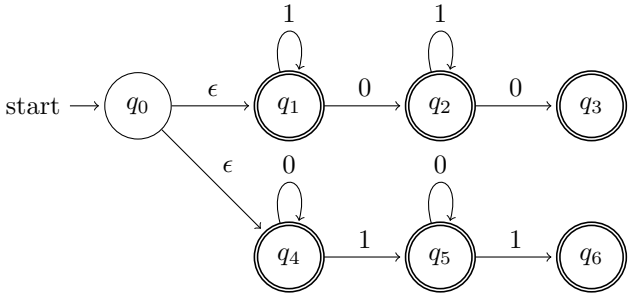
Part One



Justification

Since q_0 loops back on 0,1, each bit in the string will begin to be processed by the rest of the NFA. If this bit is 1, this thread will proceed to q_1. If there are more than 2 bits after the current one, this thread will crash. However, if 1 is only succeeded by any two bits, then the string will be accepted.

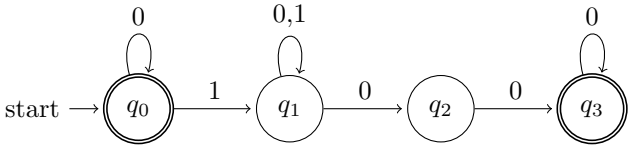
Part Two



Justification

q_0 initiates two branches: one that restricts the number of 0's in the string, and one that restricts the number of 1's. If the string contains ≤ 2 instances of 0 or 1, then at least one branch will end on an accept state. If both branches detect > 2 0's and 1's, the branches will crash on q_3 and q_6.

Part Three



Justification

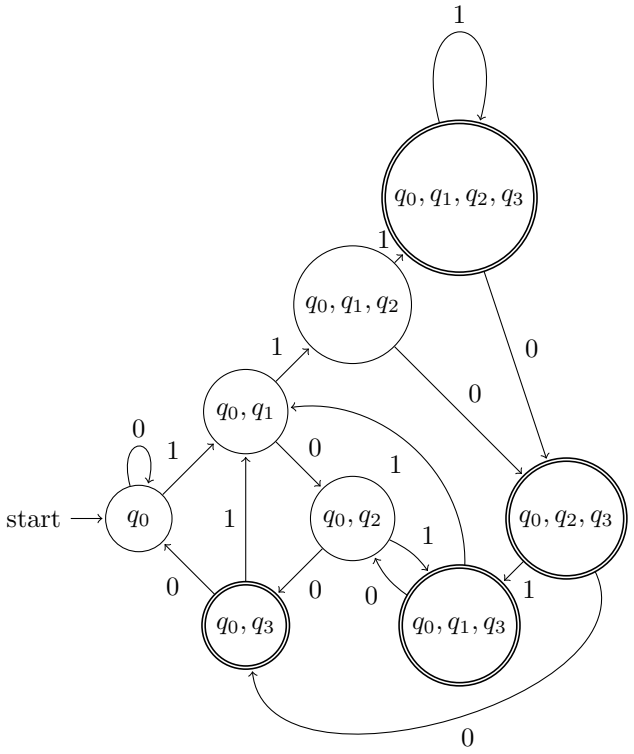
Binary multiples of 4 are either a string of all 0's, or a string containing at least one 1 where the last two bits are 0's. q_0 covers the first case, and also accomodates for any 0's at the beginning of our nonzero multiple of 4. If our thread is at q_1, we only want to accept zero or more 0's or 1's, strictly followed by two 0's. The logic for this segment of the NFA is similar to that in part one.

Problem 2

Via subset construction, provide the corresponding DFAs for the problem 1 NFAs.
Solution

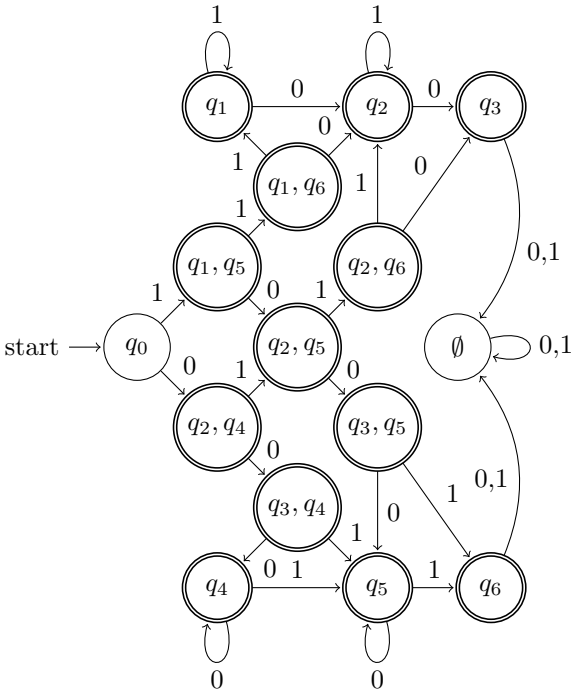
Part One

State	0	1
q_0	q_0	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
$\{q_0, q_3\}$	q_0	$\{q_0, q_1\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$



Part Two

State	0	1
q_0	$\{q_2, q_4\}$	$\{q_1, q_5\}$
$\{q_2, q_4\}$	$\{q_3, q_4\}$	$\{q_2, q_5\}$
$\{q_1, q_5\}$	$\{q_2, q_5\}$	$\{q_1, q_6\}$
$\{q_3, q_4\}$	q_4	q_5
$\{q_2, q_5\}$	$\{q_3, q_5\}$	$\{q_2, q_6\}$
$\{q_1, q_6\}$	q_2	q_1
q_4	q_4	q_5
q_5	q_5	q_6
$\{q_3, q_5\}$	q_5	q_6
$\{q_2, q_6\}$	q_3	q_2
q_2	q_3	q_2
q_1	q_2	q_1
q_6	$\emptyset$	$\emptyset$
q_3	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$



Part Three

State	0	1
q_0	q_0	q_1
q_1	$\{q_1, q_2\}$	q_1
$\{q_1, q_2\}$	$\{q_1, q_2, q_3\}$	q_1
$\{q_1, q_2, q_3\}$	$\{q_1, q_2, q_3\}$	q_1