Problem 1.

0.1f: In the set constructor, the conditional $n = n + 1$ is a contradiction, which always evaluates to false. The set is never populated and thus is $\emptyset$.

0.1d: The set will contain palindromes, as each string $s = \text{reverse}(s)$, with each string being defined over the binary alphabet.

0.6d: The domain of g is the cross product $X \times Y$ where Y is the range of f. The range of g is also Y.

0.6e: $g(4, f(4)) = 8$

Problem 2.

1. Base: $n = 1$

$$\begin{aligned} 5^1 + 5 &< 5^{1+1} \\ 10 &< 5^2 \\ 10 &< 25 \end{aligned}$$

   Thus, the base case holds
   Induction Hypothesis: $n = k \implies 5^k + 5 < 5^{k+1}$ when $k \geq 1$

$$\begin{aligned} 5^{k+1} + 5 &< 5^{k+1+1} \\ 5^k + 5 &< 5^{k+1} \cdot 5 \\ 5(5^k + 1) &< 5(5^{k+1}) \\ 5^k + 1 &< 5^{k+1} \\ \text{IS:} \quad 5^k + 5 &< 5^{k+1} \end{aligned}$$

$$\square$$

2. Base: $n = 1$

$$\begin{aligned} \sum_{i=1}^{n} (-1)^i i^2 &= (-1)^n \frac{n(n+1)}{2} \\ \sum_{i=1}^{1} (-1)^i i^2 &= (-1)^1 \frac{1(1+1)}{2} \\ -1 &= -1 \end{aligned}$$

   Thus, the base case holds

Induction Hypothesis: $n = k \implies \sum_{i=1}^{k}(-1)^i i^2 = (-1)^k \frac{k(k+1)}{2}$ when $k \geq 1$

$$\sum_{i=1}^{k+1}(-1)^i i^2 = (-1)^{k+1}\frac{(k+1)(k+1+1)}{2}$$

$$\sum_{i=1}^{k}(-1)^i i^2 + (-1)^{k+1}(k+1)^2 = (-1)^{k+1}\frac{(k+1)(k+1+1)}{2}$$

$$\text{IS:} \quad (-1)^k\frac{k(k+1)}{2} + (-1)^{k+1}(k+1)^2 = (-1)^{k+1}\frac{(k+1)(k+1+1)}{2}$$

$$(-1)^k\frac{k^2+k}{2} + (-1)^{k+1}(k^2+2k+1) = (-1)^{k+1}\frac{k^2+3k+2}{2}$$

$$(-1)^k(\frac{k^2}{2} + \frac{k}{2}) + (-1)^{k+1}(k^2+2k+1) = (-1)^{k+1}(\frac{k^2}{2} + \frac{3k}{2} + 1)$$

$$(-1)^k(\frac{k^2}{2} + \frac{k}{2}) + (-1)^k(-k^2-2k) = (-1)^k(\frac{-k^2}{2} + \frac{-3k}{2})$$

$$(-1)^k(\frac{k^2}{2} + \frac{k}{2} + -k^2 - 2k) = (-1)^k(\frac{-k^2}{2} + \frac{-3k}{2})$$

$$(-1)^k(\frac{-k^2}{2} + \frac{-3k}{2}) = (-1)^k(\frac{-k^2}{2} + \frac{-3k}{2})$$

$\square$