Alphathon 2024, Question 4: Corner Rank Strategy

Mathew Thiel and Yago Mateos

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Summary

To solve the portfolio selection problem, we choose a subset of alphas based on their relation to the next period's return and use a low turnover optimization framework from Chitsiripanich et al(2024) [1] to calculate portfolio weights, rebalancing weekly. We attempt to preserve portfolio alpha by gradually lowering the weight of old alphas relative to new, and we only remove assets from the portfolio if the direction of their investment changes. With this framework, we captured 5.11% annualized net return and had 84,63% net implied portfolio alpha with minimal transaction cost.

1 Data Insights

Our initial observations found that the accuracy of alpha values to the next period's returns tended to increase as the return calculation window increased. We also found that extreme alpha values were slightly more predictable than low-value alphas, which motivated the selection process for our project. Specifically, extreme alphas alone will not necessarily outperform non-extremes, but we found better performance when focusing our selection methodology around the extreme values relative to their returns.

2 Methodology

2.1 Choosing Significant Alphas

To choose significant alphas, we create a grid of arbitrary size where the x-axis corresponds to alpha values and the y-axis corresponds to return values for the previous period. We organize the grid into n bins and impose a selection area across the diagonal of the grid which emphasizes values at the corners of the grid. From this selection area, we take the top 30% of the top and bottom assets relative to the corners of the grid, where the ranking metric is determined by the distance from the center and to the diagonal of the grid.

2.2 Picking Portfolio Weights

To choose portfolio weights, we adapt the approach from Chitsiripanich et al(2024) [1] to suit this problem. This approach essentially reduces turnover by implementing an L-1 penalty and constraining the optimization function to only remove assets from the universe if the signal direction changes, i.e. long to short. This is desirable not only for the transaction cost reduction but also because a key characteristic of this implementation is that alphas are kept in the portfolio for a longer period, allowing for persistent alphas to have a larger effect on the portfolio. Following [1] and with some inclusion of our own problem-specific modifications, we have:

- $\mathbf{w}$: the vector of portfolio weights for each asset in the universe.
- $\mathbf{\Sigma}_t$: the covariance matrix of the assets, defined as:

$$\mathbf{\Sigma}_t = \beta \mathbf{\Sigma}_f \beta^T + \sigma_r^2 \tag{1}$$

  where β are the factor loadings, $\mathbf{\Sigma}_f$ is the factor covariance matrix, and σ_r^2 is the diagonal matrix of residual variance.

- λ_{L1}: the L1 regularization parameter which controls turnover preference.
- $\mathbf{w}^*$: the target portfolio weights.

2.2.1 Objective Function:

Minimize the following objective function, which balances the portfolio risk and the L1 penalty on the portfolio weights:

$$\min_{\mathbf{w}} \quad \frac{1}{2}\mathbf{w}^T \mathbf{\Sigma}_t \mathbf{w} + \lambda_{L1}\left(\sum_i w_{i,+} + \sum_i w_{i,-}\right) \tag{2}$$

where $\mathbf{w}$ represents the portfolio weights, and $w_{i,+}$ and $w_{i,-}$ represent the positive and negative deviations from the target weights.

2.2.2 Constraints:

1. The sum of the portfolio weights must be net zero:

$$\sum_i w_i = 0 \tag{3}$$

2. The sum of the positive weights must equal 1:

$$\sum_i \max(w_i, 0) = 1 \tag{4}$$

3. The portfolio weights must match the target portfolio $\mathbf{w}^*$:

$$\mathbf{w} + \mathbf{w}_+ - \mathbf{w}_- = \mathbf{w}^* \tag{5}$$

   where $\mathbf{w}_+$ and $\mathbf{w}_-$ are the vectors of positive and negative weight deviations.

4. Weights should be greater or less than a minimum threshold to ensure feasibility:

$$w_i \geq \tau \quad \vee \quad w_i \leq -\tau \quad \vee \quad w_i = 0 \tag{6}$$

for some fixed minimum weight parameter τ.

5. New long assets must have positive weights:

$$0 \leq w_i \leq 1, \quad \forall i \in I_{t+1}^{+} \setminus A_t^s, \tag{7}$$

6. New short assets must have negative weights:

$$-1 \leq w_i \leq 0, \quad \forall i \in I_{t+1}^{-} \setminus A_t^l, \tag{8}$$

7. Assets in the previous portfolio must have weights between 0 and their previous weight:

$$\begin{aligned} w_{t,i}^* \leq w_i \leq 0, &\quad \forall i \in A_t^s \setminus \left(I_{t+1}^{+} \cup I_{t+1}^{-}\right) \\ 0 \leq w_i \leq w_{t,i}^*, &\quad \forall iin A_t^l \setminus \left(I_{t+1}^{+} \cup I_{t+1}^{-}\right) \end{aligned} \tag{9}$$

8. Assets that have a direction change, i.e. long to short, are removed from the portfolio:

$$w_i = 0, \quad \forall i \in \left(I_{t+1}^{+} \cap A_t^s\right) \cup \left(I_{t+1}^{-} \cap A_t^l\right) \tag{10}$$

where:
- I_{t+1}^{+} and I_{t+1}^{-} are the sets of assets to be long and short at the current time.
- A_t^l and A_t^s are the active long and short portfolios at the previous time.

2.3 Jacobians

Due to issues with matrix singularity in optimization sub-problems, we additionally specify simple Jacobians in an effort to guide the optimizer to a solution, defined as follows:

2.3.1 Objective Function Jacobian

$$\nabla f(w, w^+, w^-) = \begin{bmatrix} \Sigma_t w \\ \lambda_{L1} \mathbf{1} \\ \lambda_{L1} \mathbf{1} \end{bmatrix} \tag{11}$$

2.3.2 Constraint Jacobians

Net zero constraint 3:

$$\nabla g_1(w, w^+, w^-) = [\mathbf{1}^T, \mathbf{0}^T, \mathbf{0}^T] \tag{12}$$

Sum of positive weights constraint 4:

$$\nabla g_2(w, w^+, w^-) = [\mathbf{1}_{w>0}^T, \mathbf{0}^T, \mathbf{0}^T] \tag{13}$$

Portfolio weights must match the target portfolio constraint 5:

$$\nabla g_3(w, w^+, w^-) = [I, I, -I] \tag{14}$$

For the previously invested constraint 9:

$$\nabla g_4(w, w^+, w^-) = \begin{cases} [e_i^T, \mathbf{0}^T, \mathbf{0}^T] & \text{if } w_i > 0 \\ [-e_i^T, \mathbf{0}^T, \mathbf{0}^T] & \text{if } w_i < 0 \\ \mathbf{0}^T & \text{if } w_i = 0 \end{cases} \tag{15}$$

Where $\mathbf{1}$ is a vector of ones, I is the identity matrix, $\mathbf{1}_{w>0}$ is a vector with 1 for positive weights and 0 otherwise, and e_i is the i-th standard basis vector.

2.3.3 Bounds:

The weight changes are bounded such that:

$$w_{i,+}, w_{i,-} \geq 0 \tag{16}$$

2.3.4 Post-Processing Weights

To ensure that the portfolio is always invested 100% long and 100% short, we normalize the weights where normalized weights $\tilde{w}_i$ are given by:

$$\begin{aligned}
S_+ &= \sum_i \max(w_i, 0) \quad \text{(sum of positive weights)} \\
S_- &= \sum_i \min(w_i, 0) \quad \text{(sum of negative weights)} \\
\tilde{w}_i &= \begin{cases} \frac{w_i}{S_+} & \text{if } w_i > 0 \\ \frac{w_i}{|S_-|} & \text{if } w_i \leq 0 \end{cases}
\end{aligned} \tag{17}$$

2.3.5 Backtesting

To backtest the strategy, we implemented a simple for-loop backtester that uses only the most recent data relative to the current date to avoid look-ahead bias. We rebalance weekly and consistently check that each asset in the current portfolio is investable at each time step. Trades are made on a next-day basis, meaning if we used alpha data on 10-20-2021, we would trade starting from 10-21-2021.

3 Findings

This method produced a gross annualized return of 5.18%, with annualized transaction and shorting cost percent of 0.07%. The low transaction costs are to be expected due to our implementation methodology, but we can see that this implementation also captures a large percentage of implied portfolio alpha at 84.63%. 13.36% of the returns were due to factors, with the largest exposure to Diversified Financials and Consumer Staples at 0.031 and 0.034 on average, respectively. The strategy maintained 100% long and 100% within 0.03%, where the deviation is likely due to rounding error in the code and/or Excel workbook.

4 Relevance

Our approach is well-suited to the portfolio selection problem as we address two key issues of the problem: turnover and signal preservation. This methodology is also feasible for large problems, which is important in this case, as the optimization framework is formulated as a Quadratic Programming (QP) problem.

5 Conclusion

By allowing assets in previous portfolios to persist between rebalance periods, we maintain a good percentage of the effect given by expected returns with minimal transaction costs. This approach is easily applicable to other formulations of the portfolio selection problem, with constraints on the weighting scheme to impose realistic conditions. In the given dataset, this methodology captured approximately 84.63% of alpha and maintained an annualized return after costs of 5.11%.

References

[1] S. Chitsiripanich, M. S. Paolella, P. Polak, and P. S. Walker. Smoothing Out Momentum and Reversal, Aug. 2024.