Analytical supplement to "Public entrepreneurs in Canada: theory and evidence"

Draft —please direct comments and suggestions to matt@mattwilder.net

This supplement reiterates, using simple formal methods, the verbal argument put forth in "Public entrepreneurs in Canada: theory and evidence." For convenience, utility functions presented in the main text are reproduced in the first section *Utility functions*. Elaborations and proofs using spatial modelling and game theoretic techniques comprise the subsections on *Social choice as a function of opportunity costs*, *Mobilization games, Scale conditions and production functions, Scope conditions as production possibilities, Provision subgames, Provision with private entrepreneurs,* and *Provision with public entrepreneurs.*

Utility functions

Modifying slightly the utility functions given by Frohlich, Oppenheimer and Young (1972), let $u_e(l_e)$ represent the utility an entrepreneur receives from leading followers in a collective enterprise, where u represents utility, e stands for entrepreneur, l denotes leadership and f stands for follower:

$$u_e(l_e) = u_e(x)p_e(x) + \sum_{f=1}^{n} t_f(e) + b_e - [c(o_e) + c(x_e) + c(m_e)]$$

Entrepreneurial leadership is considered a function of the utility the entrepreneur receives from the collective good x multiplied by the probability p the entrepreneur assigns to the collective good being provided, plus the sum of transfers t paid to the entrepreneur from followers, the non-material benefits (i.e., pleasure, enjoyment) b the entrepreneur derives from leading, minus the cost c incurred by the entrepreneur for administering the organization o, the cost incurred by the entrepreneur in providing the collective good x, and the cost incurred by the entrepreneur for administering a monitoring apparatus m.

Let $u_f(l_e)$ represent the utility a follower f receives from the entrepreneur's leadership:

$$u_f(l_e) = u_f(x)p_f(x) + g_f(e)[c(o_e) + c(x_e) + (m_e)] + b_f - [t_f(e) + c(x_f) + c(m_f)]$$

Followership is considered a function of the utility the follower receives from the collective good x multiplied by the probability p the follower assigns to the collective good being provided, plus the share of contracts g awarded to the follower by the entrepreneur regarding administering the organization o, the provision of the collective good x and administering the monitoring apparatus m, and non-material benefits b the follower derives from being part of the collective enterprise, minus transfers made by followers to the entrepreneur $t_f(e)$, follower costs of providing collective goods $c(x_f)$ and monitoring costs assumed by the follower $c(m_f)$.

Collective action with entrepreneurial leadership should only be forthcoming if both $u_e(l_e)$ and $u_f(l_e)$ are positive. If $u_e(l_e)$ is negative, the entrepreneur lacks incentive to mobilize. If $u_f(l_e)$ is negative, followers have no incentive to participate in the collective enterprise.

Notice that participation in collective enterprises (i.e., followership, denoted u_f) is related to, but independent from, contributions to the collective enterprise (denoted t_f and $c(x_f)$) . The distinction is warranted, given that one may wish to follow without much or any contribution. Indeed, the lower the value of the contribution terms $\{t_f, c(x_f), c(m_f)\}$ the greater the value of the followership term u_f. The substantive implication is that followers may have incentive to free-ride.

Accounting for variable incentive to free-ride (which, as shown below, is an artefact of the production function of the collective good) is a matter of comparing pre- and post-contribution estimates of $p_f(x)$. If the change in $p_f(x)$ following from contribution changes the sign of $u_f(l_e)$ from negative to positive, we may say the production function is determinative.

Alternatively, if the production function of the collective good is not known and cannot be estimated a priori, as happens when technology is ambiguous, p_i will depend on actor i's tolerance toward uncertainty. Generally, it is assumed that actors have zero tolerance for uncertainty because p_i cannot be estimated (Knight 1921). Rather, positive values for p_i imply some sort of calculation, erroneous as it may be, that converts uncertainty to risk. In such cases, assume i is capable of envisioning a production function for the collective good. When the risk tolerance of actors is not known, it is customary to assume risk-aversion, i.e., a value of p_i is closer to zero than one (Kahneman and Tversky 1979).

Imputing values into the terms in the utility functions above is facilitated by thorough modelling of nested action situations. The following sections elaborate. As demonstrated, the presence of a public entrepreneur may drastically alter the utilities of followers. Although any positive utility implies a rent, when already positive utilities are increased by the presence of a public entrepreneur, we may say that followers extract excessive rents by exploiting moral hazard opportunities endemic to public entrepreneurship.

Social choice as a function of opportunity costs

The panels below represent spatial models of social choice, so called because they invoke Euclidean distance to model group decision-making based on actors' preferences over resource distribution. Borrowing insights from Scharpf (1997), the panel on the left conveys the orthogonal relationship between two dimensions of choice: value creation and distribution. For simplicity's sake, assume two actors, A and B, who may act in concert to create value from which both may benefit. Yet, potential conflict over the distribution of collective gains may render agreement on the initiation of collective action a mixed-motive game.
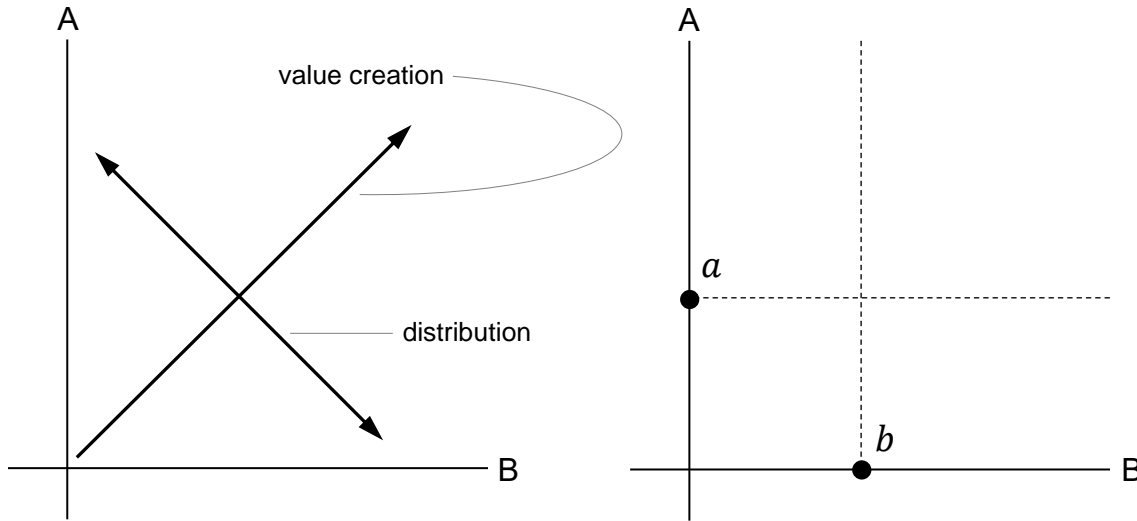
The panel on the right includes two points, *a* and *b*, which represent resource allocations that can be obtained unilaterally by players A and B respectively. These amounts represent the actors' opportunity costs, the intersection of which (represented by dashed lines) constitutes the lower threshold resource allocation that A and B would agree to because anything less would make one or both players worse off than acting unilaterally.

Within the possibility space north-east of the intersection of the dashed lines, the point on which players will agree, if they agree on any point at all, is a function of the properties of the collective good, the properties of assets held by players (i.e., structural considerations), and strategies employed by actors (agential considerations). Conveniently, player strategies may be considered a function of the "market power" accruing to players from the assets held by them (Klein, Crawford and Alchian 1978). As shown below, when players monopolize assets required to produce a collective good, they may extract monopoly rents in exchange for their participation in joint production. Moreover, market power considerations are useful for estimating opportunity costs: i.e., the unilateral yields obtainable by players producing on their own, denoted *a* and *b* above.

Whether or not players consider potential gains from collective action to exceed what can be obtained unilaterally entails foresight regarding what players expect to happen in games to follow. Although mobilization games precede provision games, player strategies in the former depend on what they expect to happen in the latter. This is because expectations about cooperative behaviour affects the probability that the collective good will be provided (term *p* in the utility functions above). It is therefore necessary to consider provision games in tandem with mobilization games.

Mobilization games

Almost any action situation can be modelled as some variant of the "archetypal" games shown in two-player matrix form below. Payoffs for one player (row) are bottom left justified, payoffs for the other player (column) are top right justified. C and D represent strategies "cooperate" and "defect" except for in *battle*, where D represents "defer" (cf. Scharpf 1997).

pure coordination

	C	D
C	3 (row) / 3 (col)	1 (col) / 1 (row)
D	1 (col) / 1 (row)	2 (col) / 2 (row)

stag hunt

	C	D
C	3 / 3	2 / 0
D	0 / 2	1 / 1

battle

	C	D
C	1 / 1	2 / 3
D	3 / 2	0 / 0

chicken

	C	D
C	2 / 2	3 / 1
D	1 / 3	0 / 0

prisoner's dilemma

	C	D
C	2 / 2	3 / 0
D	0 / 3	1 / 1

pure conflict

	C	D
C	1 / 2	2 / 1
D	2 / 1	1 / 2

An equilibrium outcome is defined as a cell in which no player can gain from switching strategies. All of the games except *pure conflict* have two Nash equilibria. In these games, coordinated joint effort is required to move from a suboptimal equilibrium to an optimal equilibrium. For example, in *pure coordination,* if both players defect (D,D), neither has incentive to cooperate because cooperation would entail a lower payoff for that player if the other continues to play defect; yet if both players switched to cooperation (C,C), they could realize the optimal Nash equilibrium solution (3,3).

The figure below models the chicken initiation game, which is a classic bargaining situation with bluffing. Let points *a′* and *b′* represent most-preferred outcomes for players A and B respectively. Both A and B prefer to increase their yields but must accept that their partners will not agree to anything less than what can be obtained unilaterally. The curves represent the preference frontiers for both players, and are interpreted such that each player obtains more utility the closer the decision falls to the player's most-preferred point (i.e., *a′* for player A and *b′* for player B). For ease of representation, let us assume a convex (circular) indifference function.[*] The values in the game matrix correspond with resource allocations in the spatial model. Mutually-cooperative behaviour results in an equal distribution of resources (2,2). Bluffing may result in either an unequal distribution of resources or non-agreement if both parties assume bull-headed strategies. Thus, non-agreement may ensue despite a zone of mutual benefit (i.e., a winset of the status quo). Chicken is a mixed-motive game with two Nash equilibria, (3,1) and (1,3).

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[*] Alternatively, it might be more appropriate to assume that both actors prefer any point in the area north-east of the intersection of the dashed lines, in which case point bundle (3,1) would be at point *a′* and bundle (1,3) would be at point *b′*. The preference envelopes would be kinked (L-shaped) and map directly to the dashed lines.

A
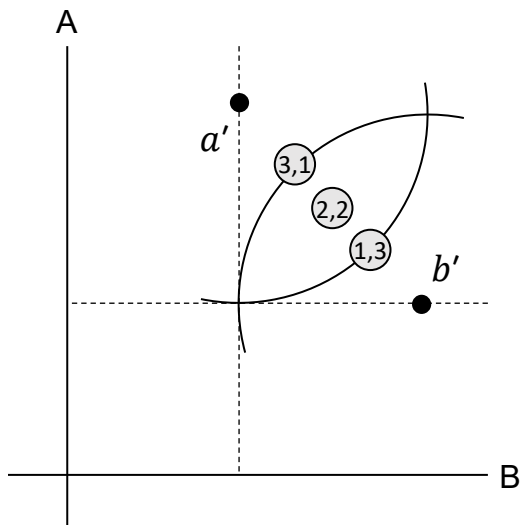
|	A	
	cooperate	bluff
B cooperate	2 2	3 1
B bluff	1 3	0 0

chicken

In pure coordination, distributional concerns do not play a part; actors must benefit proportionately or not at all. Nevertheless, players have agency over whether to coordinate. Mutual coordination is not difficult to achieve, as all parties maximize payoffs by coordinating; mutual coordination (3,3) is a Nash equilibrium solution. If one player coordinates while the other refrains, per bundle (1,1), positive externalities accruing from weak coordination benefit both parties, but payoffs diminish markedly. Failure to coordinate whatsoever results in no value being created, per bundle (0,0).

A
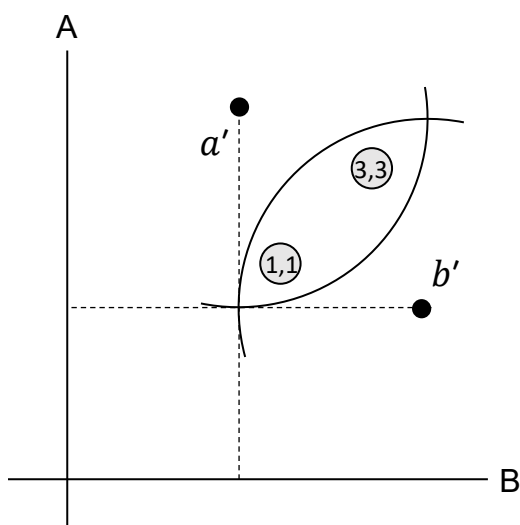
	A	
	coordinate	don't coordinate
B coordinate	3 3	1 1
B don't coordinate	1 1	0 0

pure coordination

Battle represents a bargaining situation wherein one party must benefit disproportionately for value to be maximized. As in the other initiation games, there is a circumstance in which, despite the existence of a

winset, players cannot agree to mobilize for collective action; each player defers to the other with the consequence that neither party assumes the requisite leadership for the project to proceed. Conversely, if both players attempt to lead, inefficiencies result in less value being created, resulting in lower payoffs for both players. Like chicken, battle is a mixed-motive game with two Nash equilibrium solutions (3,2 and 2,3).
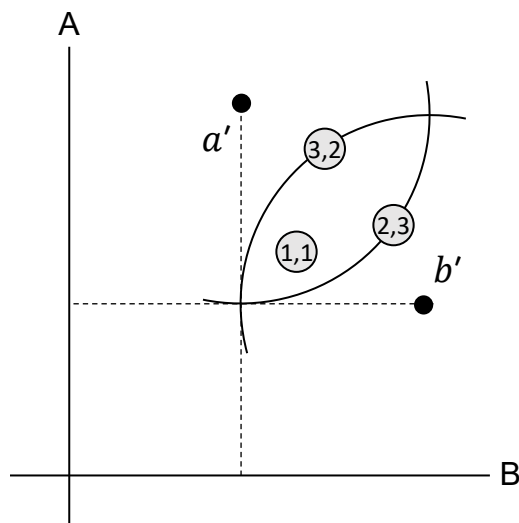

## Scale conditions and production functions

Scale conditions, otherwise known as (dis)economies of scale, determine the rate with which value is created as contributions from partners accumulate. Per the figure to the right, with a linear production function, each contribution has a constant impact on value creation. The slope of the function is determined by the value added to inputs by the collective enterprise. Importantly, the slope of the function is expected to be greater than one, as anything less can be achieved unilaterally.
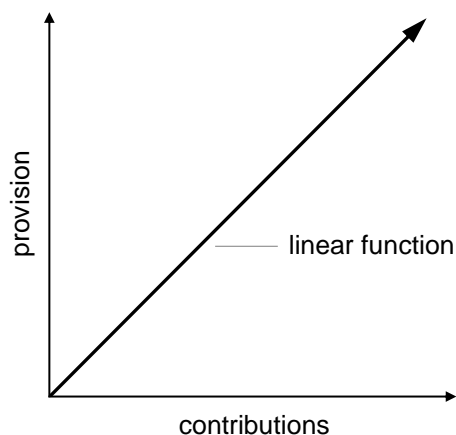

Accelerating, decelerating and logistic production functions are presented below. These functions represent situations in which value creation per unit of contribution is non-constant. Rather, value creation accelerates with each contribution when the production function is accelerating, decelerates when the production function is decelerating, and accelerates then decelerates when the production function is logistic.

| accelerating function | decelerating function | logistic function |
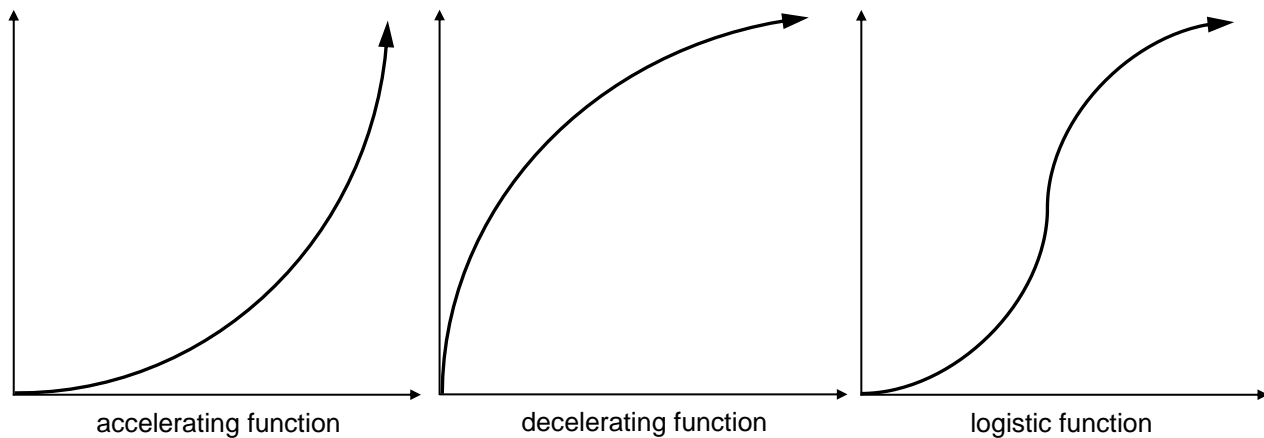
Step functions are unlike the others in that they are discontinuous. No value is created until a threshold is crossed, after which value may be maximized or increase with additional contributions. Flat step functions do not exhibit continuous economies of scale since there are no scale effects after the threshold is crossed and the collective good is provided. However, it is easy to imagine circumstances in which value increases linearly or at a variable rate with additional contributions after the step threshold is crossed.

In situations characterized by step functions, additional precision may be gained from considering the rate at which additional contributions affect the likelihood collective goods will be provided. The task involves consideration of production functions that represent probabilities collective goods will be provided. In the linear case, each contribution increases the probability the collective good will be provided at a constant rate, and so forth for the other four shapes.
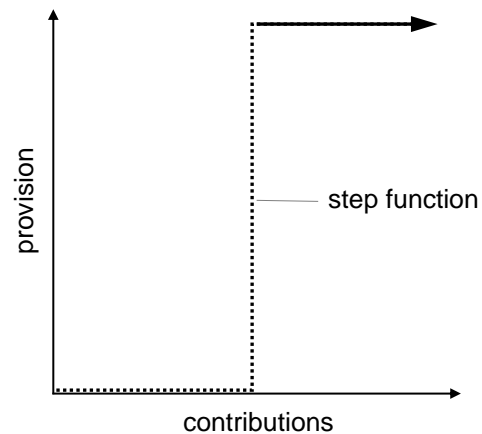
Scope conditions as production possibilities

As portrayed below, scope conditions are modelled by invoking graphical summaries of production possibilities. Axes represent the asset characteristics of contributions (i.e., heterogenous inputs). The production frontier represents what is attainable at different combinations of contribution. Production possibilities are non-discriminating in projects exhibiting neutral scope conditions. In the left panel below, whether production relies primarily on player A's assets, player B's assets or some combination of the two does not affect the scope of production.

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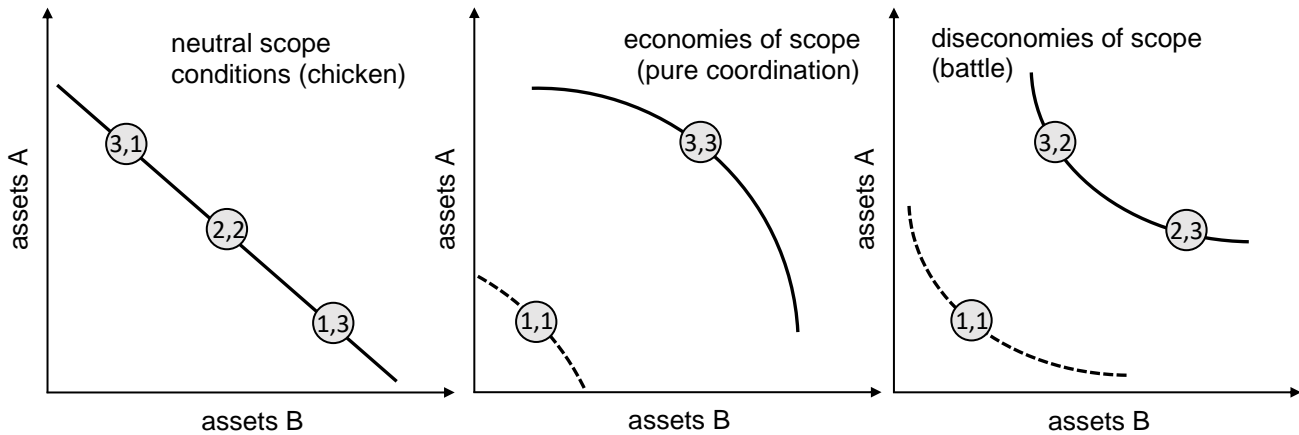
Because relative contributions of heterogenous assets do not affect scope of production, neutral scope conditions are associated with chicken initiation games, the payoffs of which are superimposed at corresponding points in the left-side panel above. Notice that the points mirror those presented above in the corresponding social choice models. This is no coincidence; spatial models of social choice and production possibility frontiers are complementary if we assume that joint production that relies disproportionately on one player's assets results in a greater distributive share accruing to that player.

In projects involving economies of scope, production is maximized when there is equal contribution of heterogenous assets. The corresponding initiation game is thus pure coordination. By contrast, in projects involving diseconomies of scope, production is maximized when the collective good relies disproportionately on a certain type of asset. Consequently, diseconomies of scope correspond with battle initiation games, as the contributor of the more highly-valued asset is expected to require (or demand) a greater share of collective spoils.

Provision sub-games

Provision sub-games can take any of the aforementioned game forms, the most interesting of which for our purposes are stag hunt, prisoner's dilemma, and a hybrid form when three or more players are involved, which we'll call threshold.

As shown in the matrixes below, a prisoner's dilemma is characterized by incentive to refrain from contributing to the provision of collective goods because collective goods will be provided regardless of one's own contributions so long as partners contribute. Contributions are *non-separable* in the sense that unique contributions are not required to instantiate collective goods. Negative payoffs convey losses incurred from redistribution of value actually obtained unilaterally.

Stag hunt represents situations in which collective goods are not provided if one or more parties refrains from contributing to their provision, opting instead to revert to unilateral action. Contributions are

separable; unique contributions are required for collective goods to be provided. Negative payoffs convey losses incurred pursuing collective goods in vain. The payoffs presented are premised on partners choosing mutual cooperation in a chicken initiation game, which can be adjusted to reflect alternative outcomes in initiation games.

prisoner's dilemma

B \ A	A: contribute	A: don't contribute
B: contribute	2 / 2	3 / -1
B: don't contribute	-1 / 3	0 / 0

stag hunt

B \ A	A: joint action	A: unilateral action
B: joint action	2 / 2	1[†] / -1
B: unilateral action	-1 / 1[†]	0 / 0

Cooperation in a prisoner's dilemma is undermined by the fact that mutual non-contribution is a Nash equilibrium solution, which is found by eliminating dominated strategies. In prisoner's dilemma, defection yields a higher payoff than cooperation regardless of what the other partner does. Cooperation is thus a dominated strategy. Whether players intend to gain by exploiting partners or guard against exploitation, the best response is non-contribution. Situations resembling stag hunt are less thorny in the sense that stag hunt features two Nash equilibria: mutual cooperation and mutual reversion to unilateral action. However, absent assurances that partners will commit to joint action, avoiding the worst possible outcome entails opting for unilateral action.

threshold

C (separable)

|	C: contribute		C: don't contribute	
B \ A	A: contribute	A: don't contribute	A: contribute	A: don't contribute
B: contribute	$1\frac{1}{3}$ / $1\frac{1}{3}$ / $1\frac{1}{3}$	1 / 2 / 1	0 / -1 / -1	0 / 0 / -1
B: don't contribute	1 / 1 / 2	-1 / 0 / 0	0 / -1 / 0	0 / 0 / 0

In threshold, collective goods are provided beyond a certain threshold of separable and non-separable contribution, and are not provided otherwise. In the three player matrix above, collective goods are

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[†] In a classic stag hunt, parties who opt for unilateral action while their partners contribute to joint action in vain are assumed to achieve a greater payoff than is the case when both parties revert to unilateral action. Such a payoff scheme is appropriate if partners are otherwise competitors, as the partner contributing to joint action in vain is removed from the competition, which is advantageous for the defecting party. When circumstances do not warrant the assumption, the payoff for unilateral action is zero across board.

forthcoming only if two out of three players contribute. Payoffs for the third player, C, are top-right justified. Let's assume Player C's contribution is separable, and that contributions from Player A and Player B are non-separable. Like in a prisoner's dilemma, payoffs are maximized when players free-ride on the contributions of others. There are multiple pure strategy Nash equilibria, and mixed strategy Nash equilibrium solutions are biased toward non-contribution.

Provision with private entrepreneurs

Rollback analysis from provision subgames reveals that agreement on terms in initiation games will not be automatic even in the most conducive of conditions (i.e., when separability, task programmability and economies of scope obtain). Anticipating exploitation or non-commitment, players will not voluntarily undertake collective action without credible assurances or institutional safeguards.
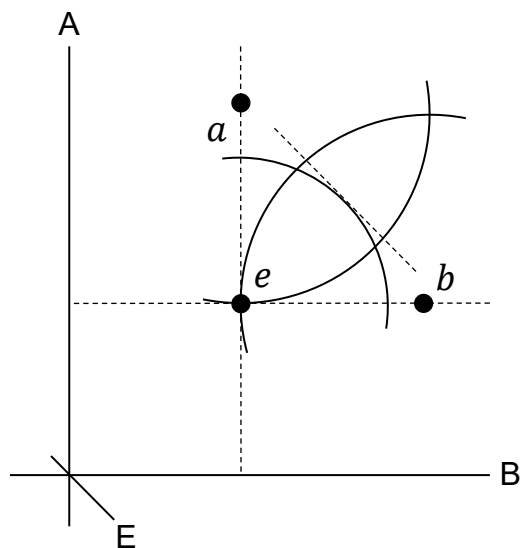
Model Modelling collective action involving entrepreneurs requires accounting for the preferences and payoffs of at least three actors: two or more actors involved in joint production and an entrepreneur, E. Distribution to the entrepreneur in initiation games is modelled on a third axis in the spatial model to the left, which is interpreted as coming off the page toward the reader. Like its followers, the entrepreneur only accepts agreements that make it better off than acting unilaterally (represented as a dashed line). In the spatial model, the entrepreneur prefers agreements closer to point *e*, as it is the point closest to the E axis that actors A and B will consider given what they can obtain unilaterally.

Recalling the utility functions presented above, entrepreneurial benefits are not restricted to utility obtained from the provision of collective goods, but also include payments received from followers (and non-material benefits derived from leading). The fact that entrepreneurial services are independent from the provision of collective goods is consequential, as it raises the possibility of exploitation on the part of entrepreneurs.

		E cooperate — A cooperate	E cooperate — A bluff	E bluff — A cooperate	E bluff — A bluff
B	cooperate	$1\frac{1}{3}$ $1\frac{1}{3}$ $1\frac{1}{3}$	1 2 1	2 1 1	2 2 0
B	bluff	1 1 2	0 2 2	2 0 2	0 0 0

chicken

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The game matrixes to the right model three player chicken and battle games with an entrepreneur. Pure coordination is not shown here, as it is not a mixed-motive game and is therefore uninteresting; entrepreneurship under conditions of pure coordination is not problematic. Payoffs for the entrepreneur are top-right justified. Cells in both matrixes correspond to different points in the winset represented by the intersection of the three preference curves in the spatial model above. Which game applies depends on the situation. Chicken is a classic bargaining game. Battle

		E cooperate — A		E bluff — A	
		lead	defer	lead	defer
B	lead	$\frac{2}{3}$; $\frac{2}{3}$; $\frac{2}{3}$	$1\frac{1}{2}$; $1\frac{1}{2}$; 2	1 ; $\frac{1}{2}$; $\frac{1}{2}$	2 ; $1\frac{1}{3}$; $1\frac{2}{3}$
B	defer	$1\frac{1}{2}$; 2 ; $1\frac{1}{2}$	0 ; 0 ; 0	2 ; $1\frac{2}{3}$; $1\frac{1}{3}$	0 ; 0 ; 0

battle

represents situations in which one party must benefit disproportionately for the group to obtain maximum gain. Nevertheless, the darkly-shaded cells in both matrixes represent situations in which the entrepreneur plays a bluff strategy during initiation games: the entrepreneur insists on an allocation of resources closer to its preferred point than the entrepreneur would be willing to accept.

As explained above, most provision subgames of interest to researchers can be represented as a prisoner's dilemma, stag hunt or threshold. Which game depends on whether collective goods will be provided regardless of (non)contribution. If so, the game form is prisoner's dilemma. If collective goods will not be provided without contributions from others, the game form is stag hunt. If collective goods are provided only if a certain number of actors with a mix of separable and non-separable assets contribute, the game form is threshold. The payoffs presented are premised on entrepreneurs contributing to the provision of collective goods, either directly or by awarding contracts to partners. Payoffs are also premised on partners choosing mutual cooperation in a chicken initiation game, which can be adjusted to reflect alternative outcomes in initiation games.

The addition of entrepreneurial contribution to the provision of collective goods has no effect on threshold games, but it has significant impacts on prisoner's dilemma and stag hunt games. Interestingly, some problems normally associated with prisoner's dilemma are resolved with the addition of an entrepreneur, while entrepreneurs may introduce problems to stag hunt that are normally associated with prisoner's dilemma if entrepreneurs subsidize production by partners (i.e., award contracts). Only instances in which the entrepreneur is capable of providing the collective good unassisted qualify as prisoner's dilemmas when entrepreneurs are involved, however. Other instances of collective action involving entrepreneurs take the form of stag hunt, but stag hunt with characteristics normally associated with prisoner's dilemma if the entrepreneur subsidizes followers.

prisoner's dilemma

	E cooperate		E defect	
	A cooperate	A defect	A cooperate	A defect
B cooperate	1⅓ 1⅓ 1⅓	0 3 1⅓	3 0 0	3 3 -1
B defect	0 1⅓ 3	-1 3 3	3 -1 3	0 0 0

In the top left cell of the three person prisoner's dilemma matrix on the left, all parties cooperate to produce the collective good and split the dividends evenly. In the remaining cells, defecting parties obtain a rent from free-riding on the provision of the collective good, unless all three parties defect. Of course, if the purpose of the entrepreneur is to enforce contracts or otherwise monitor free-riding, then the involvement of the entrepreneur will encourage mutual cooperation. If, on the other hand, monitoring is difficult or impossible, there is strong incentive to free-ride. Unlike a typical prisoner's dilemma, the threat of free-riding by one's partner is allayed by the presence of an entrepreneur. However, players A and B have such a strong incentive to exploit the entrepreneur that no reasonable person would accept the role if one's own time and resources were at stake. The Nash equilibrium solution is mutual defection, which can only be avoided with effective monitoring.

Provision with public entrepreneurs

Public entrepreneurs do not expend their own resources on the provision of collective goods. Consequently, situations resembling prisoner's dilemma in which monitoring is difficult or impossible lend themselves to public entrepreneurship. The rationale is better known in the context of public goods and common pool resource problems; because the collective good will be provided regardless of contribution, market actors have no incentive to contribute to its supply.

stag hunt

	E cooperate		E defect	
	A joint	A unilateral	A joint	A unilateral
B joint	1⅓ 1⅓ 1⅓	-3 2 1	2 1 1	0 0 -1
B unilateral	-3 1 2	-3 2 2	0 -1 0	0 0 0

Situations resembling stag hunt are similarly prone to exploitation when an entrepreneur is involved. This is especially true if entrepreneurs subsidize followers only to have them revert to unilateral action, dooming the collective enterprise. As shown in the matrix on the left, when entrepreneurship entails subsidies (i.e., ex ante production contracts), it introduces moral hazard to the stag hunt game, which incentivizes defection thereby making the game resemble a conventional prisoner's dilemma. This is because partners receive a payoff as a matter of contract, regardless of whether the collective good is provided (unless

delivery of the good is a component of the contract, of course). Like a conventional prisoner's dilemma, mutual defection is the Nash equilibrium solution in stag hunt featuring an entrepreneur if the entrepreneurial role involves subsidizing followers.

Again, things are not so dire if the entrepreneurial function is limited to facilitating contracts and monitoring defection. But, as before, if monitoring is difficult or impossible due to lack of task programmability, the threat of defection would be too high for any reasonable person to privately assume the entrepreneurial role. Public entrepreneurs, on the other hand, can afford the risk since they do not put up their own resources, but costs associated with risk are merely shifted on to society.