Hall Effect

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1 Summary

In this experiment, we measure the strength of the Hall effect in bismuth. This was accomplished by running a fixed current through a bismuth strip in a magnetic field, varying the strength of the field, and measuring the transverse voltage at each point. The Hall coefficient was found to be $(-6.4 \pm 0.2) \times 10^{-7}\,\mathrm{m^3\,C^{-1}}$ (disagreeing with previous findings), and the majority charge carriers were found to be electrons. Discrepancies were most likely due to impurities in the sample or misalignment of the field lines with the crystal lattice.

2 Introduction

Figure 1

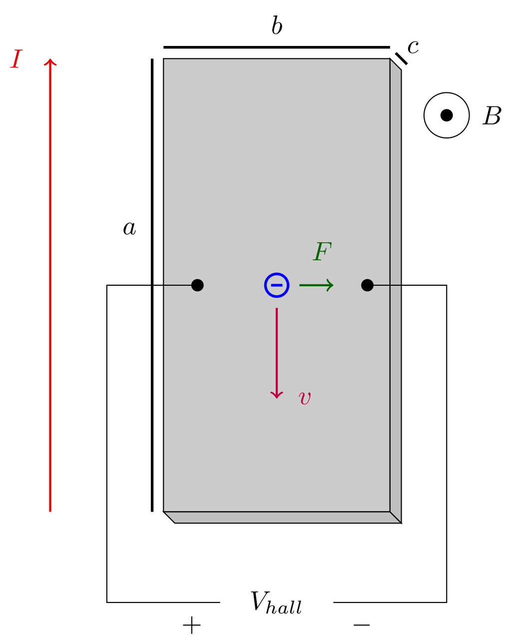
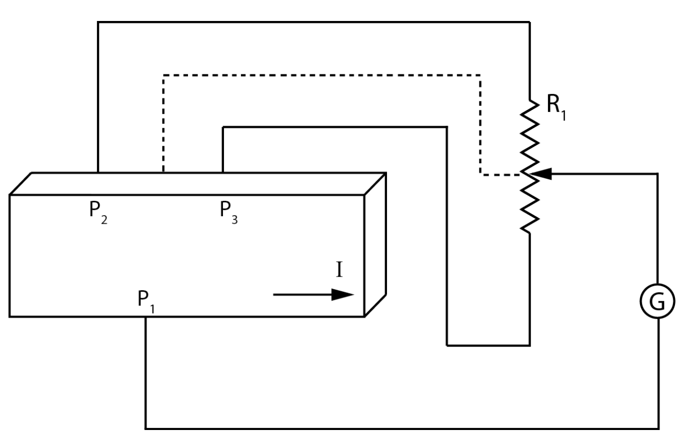


Figure 2



Consider a conducting sheet with current traveling through it, as shown in figure 1. If a magnetic field is oriented normal to the sheet, the individual charge carriers—electrons in this case—will experience a force with magnitude given by $F = qvB$ (where v is the velocity of an individual carrier). This causes an electric field to be generated in the direction of the force, with magnitude

$$E_H = \frac{F}{q} = vB \tag{1}$$

The charge carrier velocity is related to the current density j by $j = nqv$, where n is the density of charge carriers. The current density can be found by dividing I by the cross-sectional area of the strip. In this scenario, it becomes helpful to define a quantity R_H for a given material (termed the *Hall coefficient*) such that $v = jR_H$, giving us

$$R_H = \frac{1}{nq} \tag{2}$$

By substituting $v = jR_H$ and $j = \frac{I}{bc}$ into equation 1, we get

$$E_H = R_H \frac{IB}{bc} \tag{3}$$

The transverse voltage (known as the Hall voltage) can be found by multiplying the electric field by the distance between the terminals of the voltmeter, giving us

$$V_H = R_H \frac{IB}{bc} b' \tag{4}$$

In the absence of a magnetic field, the total electric field would be parallel to the magnetic field (and equal to the product of current density and resistivity). However, because E_{hall} is perpendicular to the current, the total electric field will point in a different direction. We can define the Hall angle, θ, as the angle between $\vec{j}$ and $\vec{E}$. We can determine θ from

$$\tan\theta = \frac{E_H}{E_y} \tag{5}$$

where E_y is the field generating the current in the strip. For example, a 500 mA current in the strip corresponds to an E_y of $(0.81 \pm 0.02)\,\mathrm{V\,m^{-1}}$. If the Hall voltage is 500 mA, then based on the dimensions of the strip (see Appendix B), the Hall angle is $(12.3 \pm 0.3)^\circ$.

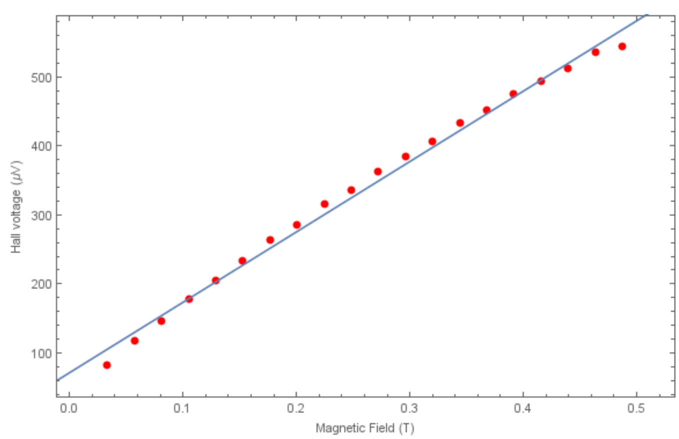
3 Procedure

A bismuth strip (see Appendix B for dimensions) with 3 terminals connected to it (figure 2) was placed between the coils of an electromagnet. A current of 500 mA was passed through the strip, and a potentiometer attached to the two right terminals was adjusted until the measured transverse voltage was 0 V. This was done to prevent a offset between the terminals from affecting the measured Hall voltage. The current in the electromagnet was then increased, and the Hall voltage was measured at various increments. The orientation of the strip relative to the magnetic field was also noted, and was used to determine the sign of the charge carriers.

4 Experimental Results

Raw data can be found in Appendix A.

Figure 3: Magnetic Field vs Hall Voltage



The magnetic field was plotted against the Hall voltage (figure 3) in order to determine the Hall coefficient via equation 4. Electrons were determined to be the majority charge carriers from the polarity of the Hall voltage, and the Hall coefficient was calculated as $(-6.4 \pm 0.2) \times 10^{-7}\,\mathrm{m^3\,C^{-1}}$. This corresponds to a charge carrier density of $(9.7 \pm 0.3) \times 10^{24}\,\mathrm{m^{-3}}$ (for comparison, the density of atoms in bismuth is $2.82 \times 10^{28}\,\mathrm{m^{-3}}$).

From the existing literature [1], the actual Hall coefficient of room-temperature bismuth is $-1.1 \times 10^{-6}\,\mathrm{m^3\,C^{-1}}$. The magnitude is significantly greater (23σ) than that of our measured value, and the two results cannot be said to agree by any reasonable standard (though both have the same sign).

It is also worth noting that the B vs V_H plot is slightly concave down, rather than linear. This trend is to be expected and suggests that the Hall coefficient decreases at higher magnetic fields.

5 Discussion of Uncertainty

We assumed that the magnetic field produced is linearly related to the current in the coil. While this is a fairly reasonable approximation (see calibration table in Appendix C), it is possible that the slight nonlinearity in the magnetic field contributed to the measured nonlinearity in the Hall voltage. Another possibility is that the temperature of the strip changed over time, which would result in a change in electron mobility (and therefore the Hall coefficient).

Several factors could be responsible for the discrepancy between the measured and actual value of R_H. The Hall effect in bismuth is known to be fairly anisotropic, and the alignment of the magnetic field with the crystal lattice could significantly alter the Hall coefficient. It is also possible that impurities in our sample altered the density of charge carriers.

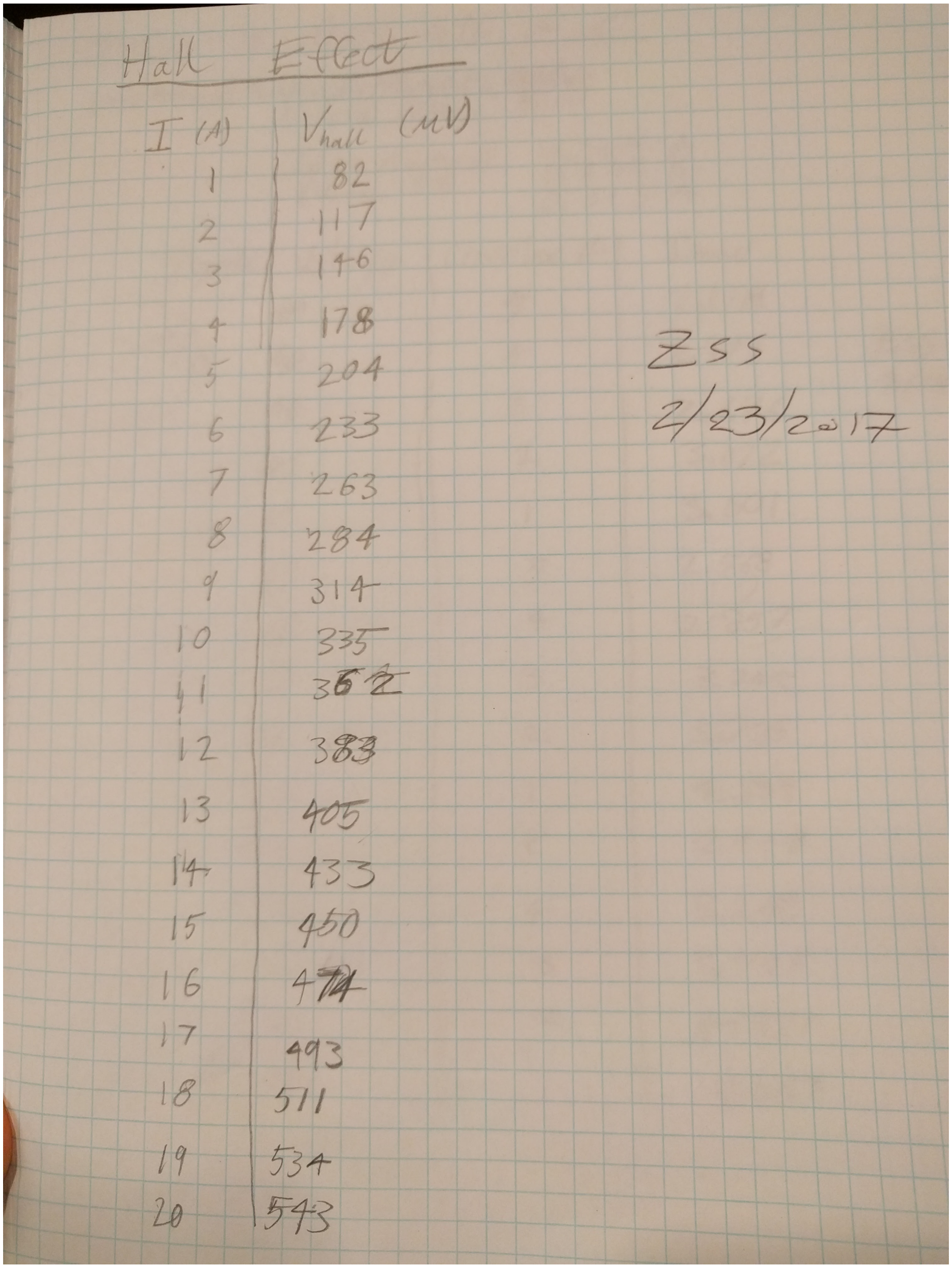
6 Conclusion

The Hall voltage was found to increase roughly (though not quite) linearly with the applied magnetic field, in agreement with theory. We obtained a value of $(-6.4 \pm 0.2) \times 10^{-7}\,\mathrm{m}^3\,\mathrm{C}^{-1}$ for the Hall coefficient, with electrons as the majority carriers. This value does not agree with the one found in literature, but is within an order of magnitude. However, the discrepancy can be fairly easily explained by the presence of impurities and the anisotropy of the Hall effect in bismuth.

References

[1] Y. Hasegawa, Y. Ishikawa, T. Saso, H. Shirai, H. Morita, T. Komine, and H. Nakamura. A method for analysis of carrier density and mobility in polycrystalline bismuth. *Physica B: Condensed Matter*, 382(12):140 – 146, 2006.

Appendix A   Raw Data



Hall Effect

I (A)	V_{hall} (mV)
1	82
2	117
3	146
4	178
5	204
6	233
7	263
8	284
9	314
10	335
11	362
12	383
13	405
14	433
15	450
16	474
17	493
18	511
19	534
20	543

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Appendix B Strip Dimensions

$$a = (6.8 \pm 0.2) \times 10^{-2}\,\mathrm{m}$$
$$b = (6.41 \pm 0.03) \times 10^{-3}\,\mathrm{m}$$
$$b' = (2.84 \pm 0.03) \times 10^{-3}\,\mathrm{m}$$
$$c = (1.40 \pm 0.01) \times 10^{-4}\,\mathrm{m}$$

Appendix C Electromagnet Calibration Data