Millikan's Oil Drop

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1 Summary

In this experiment, we determined the value of the fundamental electric charge. This was accomplished by suspending oil drops in a chamber, allowing electrons to transfer between them via collisions. An electric field was applied, and the rate at which the droplets fell was used to determine the charge on each one. The fundamental electric charge was calculated as $(1.67 \pm 0.09) \times 10^{-19}\,\mathrm{C}$, which agrees with the known value of $1.60 \times 10^{-19}\,\mathrm{C}$. Though random error due to air currents in the chamber is likely, it does not appear to have significantly affected the results.

2 Introduction

Consider a charged oil drop falling in an air-filled chamber. If the drop is at terminal velocity, then the force of gravity is equal to the drag force. Assuming laminar drag (as the drop has a low Reynolds number due to its size), we can express this relationship as:

$$F_g = mg = F_D = 6\pi\eta' a v_t \tag{1}$$

where v_t is the terminal velocity, η' is the modified viscosity of air, and a is the radius of the drop. We can also express the drop's mass as the product of density (denoted ρ) and volume (equal to $\frac{4}{3}\pi a^3$). The use of η' instead of η is due to the fact that air does not behave continuously at this scale. They are related by

$$\eta' = \frac{\eta}{1 + \frac{b}{pa}} \tag{2}$$

where p is the pressure and b is a correction factor. Because we will be recording the time it takes for drops to fall a given distance, it is also useful to replace v_t with l/t. Making these substitutions to the previous equation and solving for a, we get

$$a = \sqrt{\frac{9\eta' l}{2g\rho t}} \tag{3}$$

The charge on the drop can be determined by applying an electric field to the chamber and balancing gravity against the electrostatic force, so that the drop remains completely stationary. In this case, the following relationship holds (d denotes the plate separation):

$$mg = qE = \frac{qV}{d} \tag{4}$$

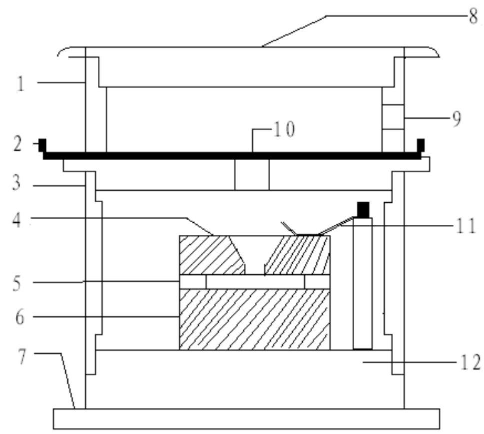
Combining this with equation 1, substituting the expression for a obtained in equation 3, and solving for q, we obtain the following expression for the charge on the droplet:

$$q = \left(\frac{18\pi d}{\sqrt{2\rho g}}\left(\eta' l\right)^{3/2}\right)\frac{1}{t^{3/2}V} \tag{5}$$

3 Procedure

3.1 Experiment

The experimental setup is shown in figure 1, which has been shamelessly copied from the lab manual. An atomizer was used to fill the chamber with oil droplets. A microscope attached to a camera was used to observe the chamber, and a drop was selected. The electric field in the chamber was turned on, and the voltage required for the drop to remain still was recorded. The drop was then allowed to fall a fixed distance, and the time was recorded. This data was used to determine the charge on the droplet, and the procedure was repeated for 16 different droplets.



1. oil mist cup
2. aperture switch
3. windscreen
4. upper electrode
5. drop channel
6. lower electrode
7. stand
8. cover
9. spray aperture
10. mist aperture
11. electrode clamp
12. base

Figure 1

4 Experimental Results

Figure 2: Charge Histogram

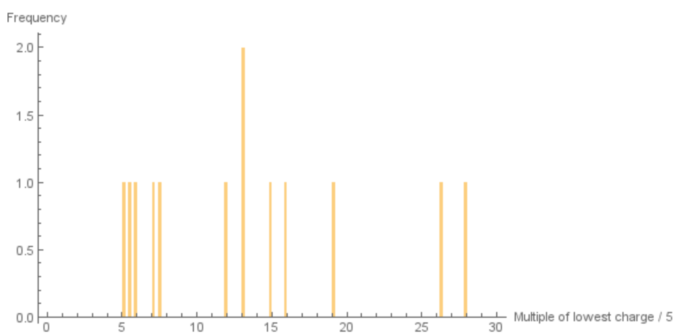
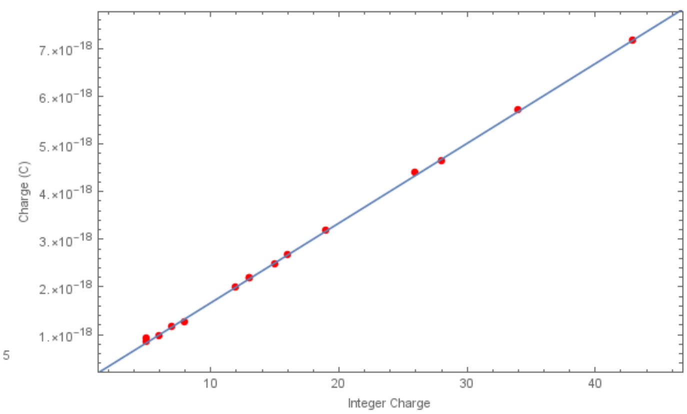


Figure 3: Integer Charge vs Actual Charge



Raw data can be found in Appendix A.

The smallest calculated charge was 8.34×10^{-19} C, but most of the other droplets were nowhere near integer multiples of this charge, suggesting that none of the drops in our sample had the charge of a single electron. It did appear, however, that many charges were close to integer multiples of 1/5 of the smallest charge, indicating that at least 5 electrons were transferred to every drop (this may be due to unintentional bias in the selection process toward drops with higher charges). By rounding all drops to the nearest integer multiple of $q_{min}/5$ and performing a linear regression between the integer charge and the actual charge (see figure 3), we obtain a value of $(1.67 \pm 0.09) \times 10^{-19}$ C. This agrees (0.8σ) with the recognized value of 1.60×10^{-19} C.

5 Discussion of Uncertainty

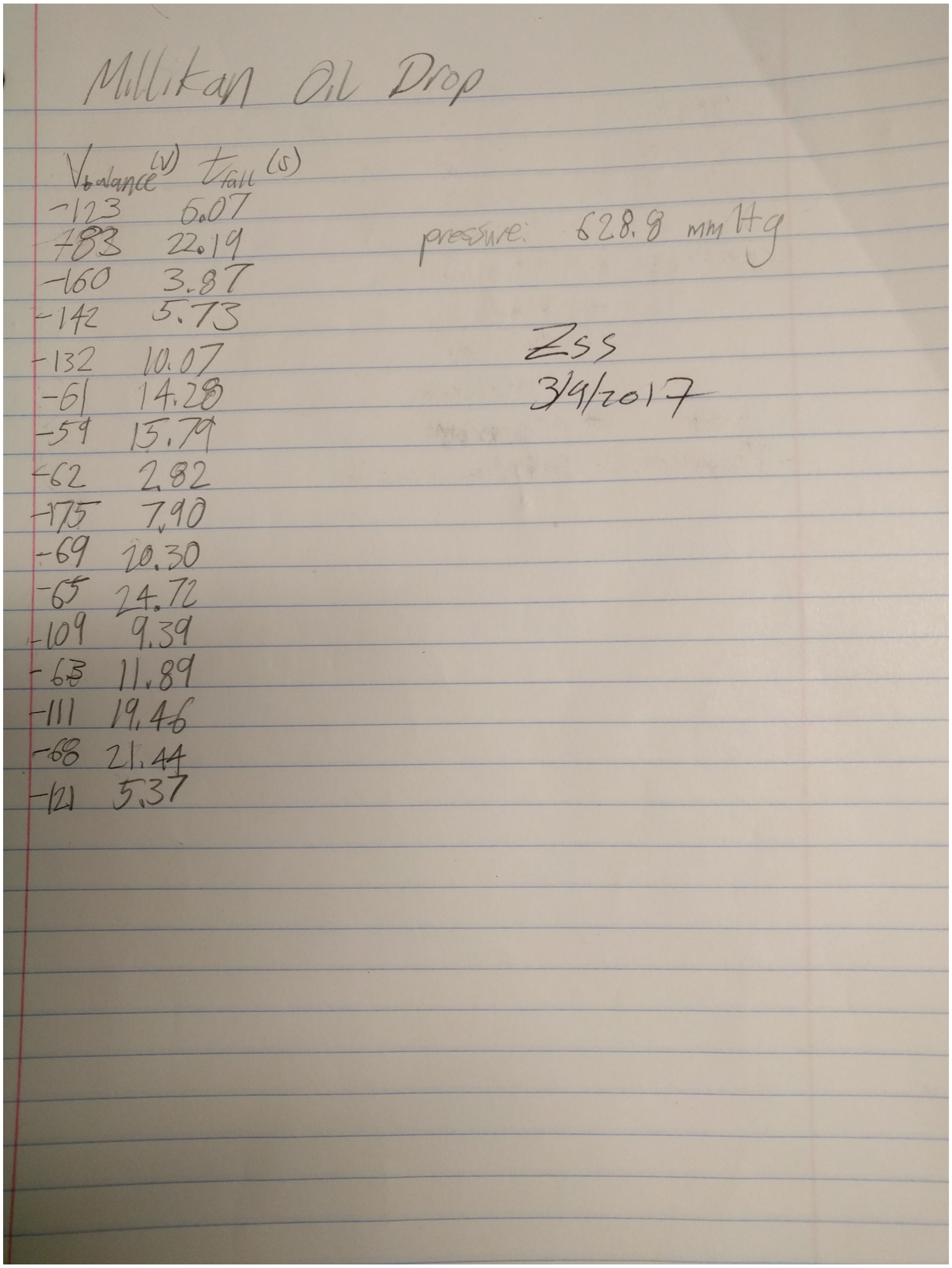
A single outlier was rejected from the data set, as the calculated charge of one droplet was found to be 3.0×10^{-17} C, over 4 times the next highest value. This was most likely due to an error collecting data.

One potential sources of statistical uncertainty was the random fluctuation of air currents in the chamber. This would have altered the fall time of drops, and thus the calculated charge. Possible sources of systematic uncertainty include the effect of human reflex times when starting and stopping the timer, as well as temperature differences between the air in the apparatus and the surrounding room (which would have altered air pressure readings). However, as the result agreed with the known value, these sources of error do not appear to have made a significant contribution.

6 Conclusion

The droplet charges were found to be integer multiples of a single charge, in agreement with theory. The method used was accurate in determining the fundamental charge, producing a value of $(1.67 \pm 0.09) \times 10^{-19}$ C, in agreement with the accepted value. Although it is possible that only highly-charged drops were selected for trials, no sources of error seem to have significantly affected the results.

Appendix A Raw Data



Millikan Oil Drop

$V_{balance}$ (V)	t_{fall} (s)
-123	5.07
-183	22.19
-160	3.87
-142	5.73
-132	10.07
-61	14.28
-59	15.79
-62	2.82
-175	7.90
-69	20.30
-65	24.72
-109	9.39
-63	11.89
-111	19.46
-68	21.44
-121	5.37

pressure: 628.8 mmHg

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