Magnetic Torque

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1 Summary

In this experiment, we determine the magnetic moment of a magnet by examining its response to a magnetic field. We accomplish this in three different ways. First, we allow the magnet to oscillate within the field, and determine the moment by measuring the period. We also determining the torque on it via precession, and measure the force in a non-uniform field. We find that all of these methods agree, with no statistically significant difference between the various values of u (even with a small angle approximation in the first case).

2 Introduction

When a magnet is placed in a magnetic field, it experiences a torque given by

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{1}$$

Therefore, if we have a magnetic field of known strength, knowing the torque on the ball enables us to calculate μ. Consider the case of a magnet that is free to rotate in space. If we assume a uniform magnetic field in the $+z$ direction, equation 1 becomes $\tau = I\ddot{\theta} = -\mu B sin(\theta)$ where I is the moment of inertia ($\frac{2}{5}MR^2$ for a uniform sphere) and θ is the angle between the magnetic moment vector and the field lines (figure 1). The right hand side is negative because the torque acts to align the dipole with the field. If we assume that θ is small, we can approximate this as:

$$\ddot{\theta} \approx -\frac{\mu B}{I}\theta \tag{2}$$

By treating the system as a simple harmonic oscillator, we can approximate the magnetic moment by allowing the object to oscillate and measuring the period, which is given by

$$T = 2\pi\sqrt{\frac{I}{\mu B}} \tag{3}$$

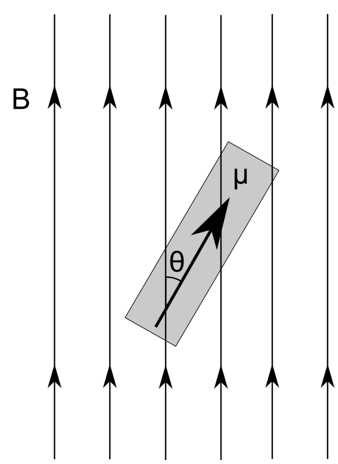


Figure 1

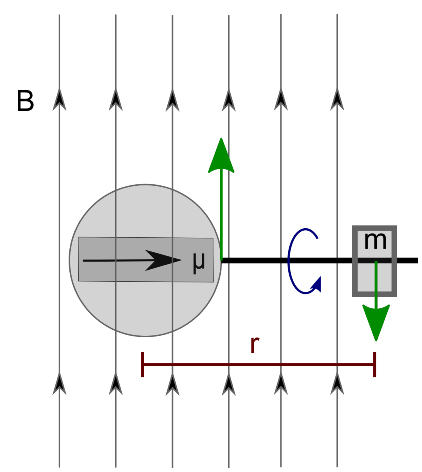


Figure 2

We can also find the torque by spinning the object, applying a counter-torque, and varying the field strength until the torques cancel and the precession becomes 0. If the magnet is perpendicular to the field and the counter-torque is provided by a weight on a rod (figure 2), μ can be calculated from:

$$\mu B = g(mr + m_{rod}\bar{r}_{rod}) \tag{4}$$

The moment of an unknown magnet can also be calculated by placing it in a nonuniform finding the force on it, which is given by

$$\vec{F} = \vec{\mu} \cdot \nabla \vec{B} \tag{5}$$

If the magnet is parallel to the field, equation 5 becomes $F = \mu \frac{\partial B}{\partial z}$. By suspending the magnet on a spring and measuring the displacement at various field gradients, we can determine the force, and therefore the moment.

3 Procedure

3.1 Part 1

A ball containing a magnet with an unknown moment was suspended on an air bearing between Helmholtz coils oriented so that the magnetic field pointed up, with the magnetic field given by $B = ((1.36 \pm 0.03) \times 10^{-3}\,\mathrm{T\,A^{-1}}) \cdot I$. Current was passed through the coils and the ball aligned with the magnetic field. The ball was then given a slight push, allowing it to oscillate at a small angle, and the time for 20 complete oscillations was recorded. This process was repeated for currents ranging from 0.5 to 3.5 A. The moment of inertia was calculated from the diameter and mass of the sphere, and the magnetic moment was determined via equation 3.

3.2 Part 2

The ball was placed so that the magnet was horizontal, and a rod with a small weight on it was attached. The ball was spun about an axis parallel to the rod and allowed to precess. The current was increased from 0 A until torque from the field balanced the torque from the weight, and the precession halted (figure 2). This was repeated 5 times, each with the weight at a different position on the rod.

3.3 Part 3

A magnet identical to the one in the ball was suspended on a spring, and weights were incrementally attached to it to determine the spring constant. The Helmholtz coils were configured to produce a nonuniform field, with a roughly constant gradient give by $\frac{\partial B}{\partial z} = (1.69 \times 10^{-2}\,\mathrm{T\,m^{-1}\,A}) \cdot I$. The magnet was then placed in the nonuniform field, and the displacement was measured at several different values of I. This was used to determine the force on the magnet, which was used to determine the magnetic moment via equation 5.

4 Experimental Results

Raw data can be found in Appendix A.

4.1 Part 1

B was plotted against T^{-2} (figure 3). By equation 3, the slope must equal $\mu/(4\pi I)$. After taking a linear regression, the magnetic moment was calculated as $(0.406 \pm 0.006)\,\mathrm{N\,m\,T^{-1}}$.

4.2 Part 2

mgr (where m is the mass of the weight) was plotted against B (figure 4). By equation 4, the slope should be equal to μ. After taking a linear regression, the magnetic moment was calculated as $(0.393 \pm 0.009)\,\mathrm{N\,m\,T^{-1}}$.

4.3 Part 3

The spring constant was determined to be $2.821(9)\,\mathrm{N\,m^{-1}}$. To determine μ, the force on the spring (given by $k\Delta y$) was plotted against ∇B (figure 5). The slope should be equal to μ (equation 5). The magnetic moment was calculated as $(0.411 \pm 0.014)\,\mathrm{N\,m\,T^{-1}}$.

Figure 3: T^{-2} vs Magnetic Field

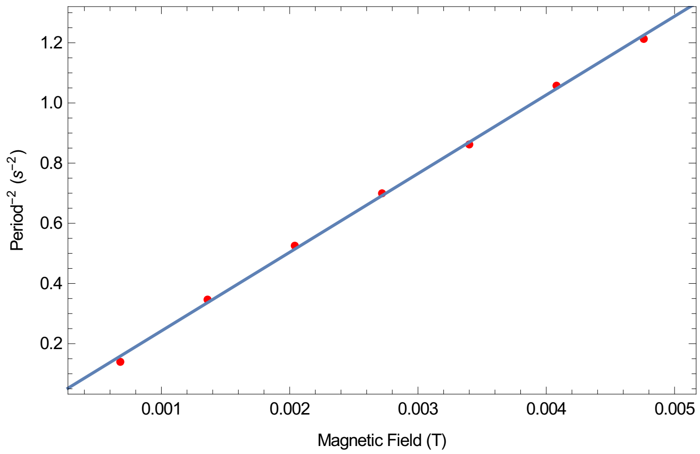


Figure 4: Torque vs Magnetic Field

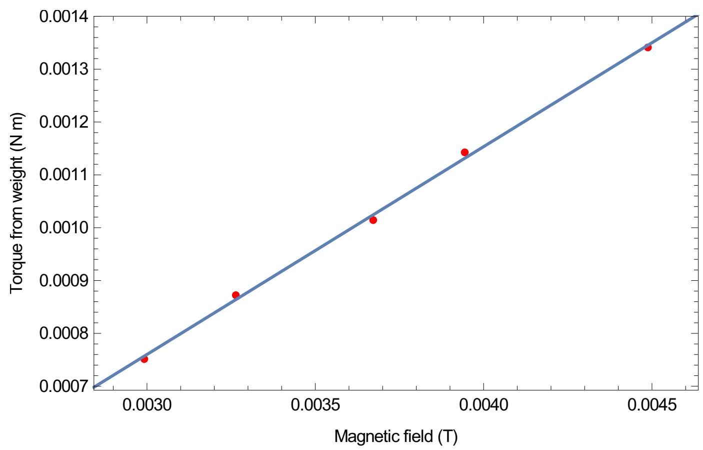
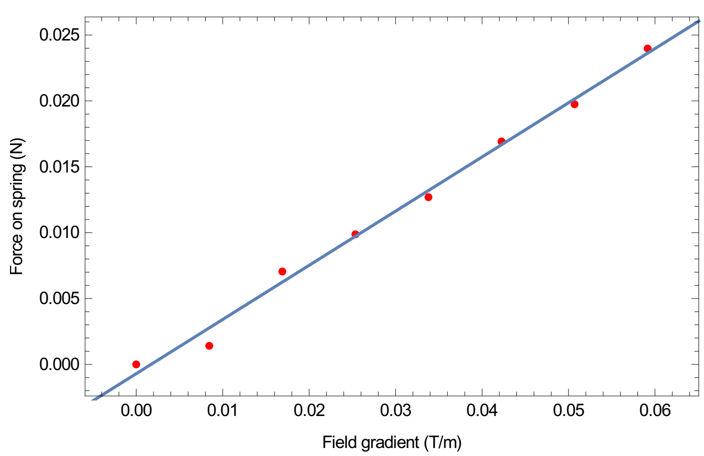


Figure 5: Force vs Field Gradient



5 Discussion of Uncertainty

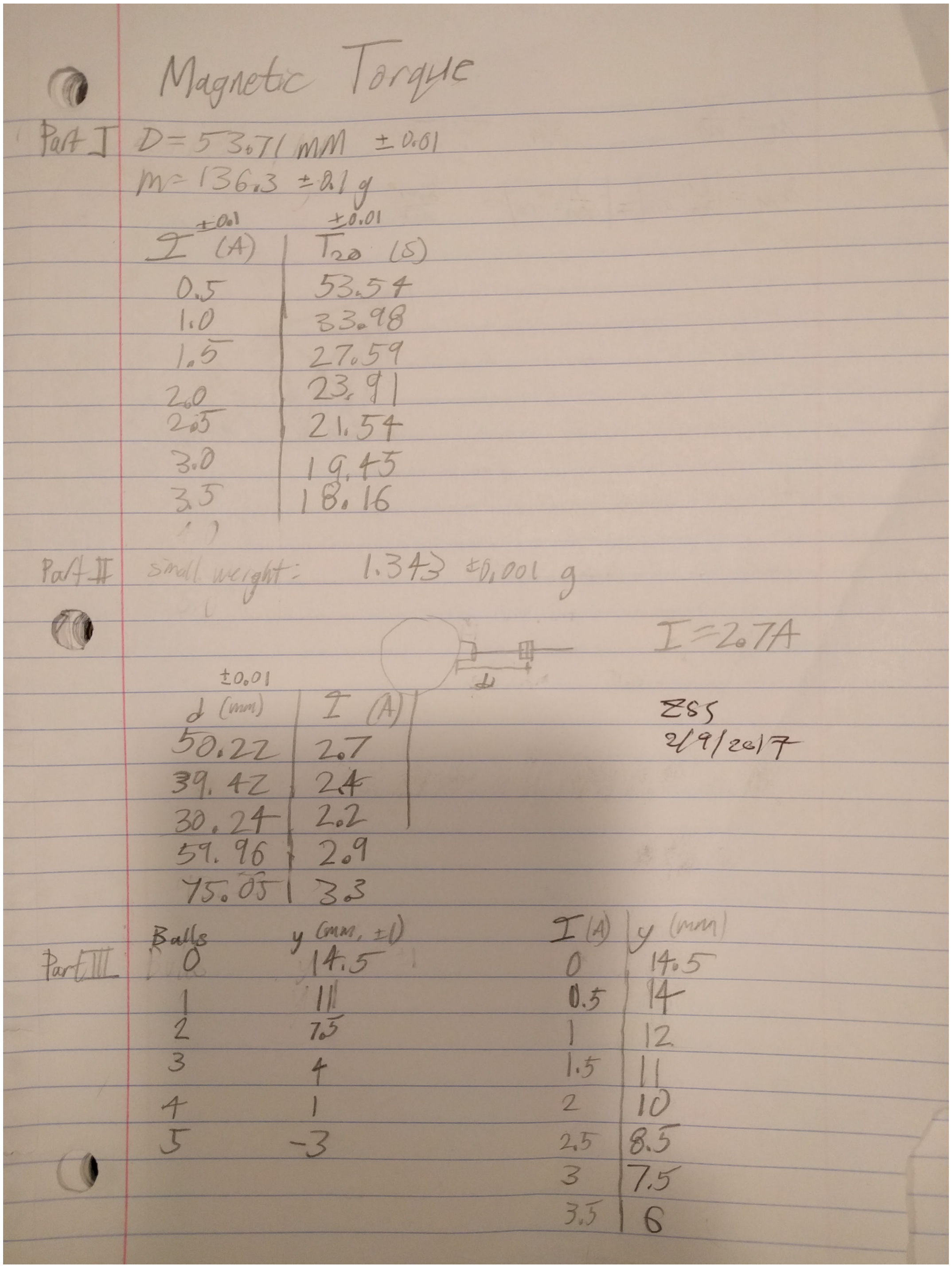
One potential source of statistical uncertainty was the possibility that the weight used in Part 2 moved on the rod during trials. It is also possible, though unlikely to affect our results, that the magnetic field was not entirely uniform (or lacked a uniform gradient in Part 3).

One possible source of systematic uncertainty was the small angle approximation used in Part 1 (a more accurate model would result in a slightly higher μ). Though the bearing was assumed to be frictionless, the presence of friction could also alter the results. Another possibility is that the airflow was not entirely symmetric, which would have altered the torque in part 2.

6 Conclusion

All three methods were found to be reliable procedures for determining magnetic moment. The values were found to be $(0.406 \pm 0.006)\,\mathrm{N\,m\,T^{-1}}$ for Part 1, $(0.393 \pm 0.009)\,\mathrm{N\,m\,T^{-1}}$ for Part 2, and $(0.411 \pm 0.014)\,\mathrm{N\,m\,T^{-1}}$ for Part 3. All calculated values agreed within a single standard deviation. The torque on a dipole was found to be proportional to B, and the force was found to be proportional to ∇B (in agreement with theory). In addition, the small-angle approximation was found to be consistently accurate in determining the period of an oscillating dipole. No major sources of uncertainty were found (at least not to the extent that the results were affected).

Appendix A Raw Data



Magnetic Torque

Part I $D = 53.71$ mm ± 0.01

$m = 136.3 \pm 0.1$ g

I (A) ± 0.01	T_{20} (s) ± 0.01
0.5	53.54
1.0	33.98
1.5	27.59
2.0	23.91
2.5	21.54
3.0	19.45
3.5	18.16

Part II small weight: 1.343 ± 0.001 g

$I = 2.7$A

ZSS 2/9/2017

d (mm) ± 0.01	I (A)
50.22	2.7
39.42	2.4
30.24	2.2
59.96	2.9
75.05	3.3

Part III

Balls	y (mm, ± 1)
0	14.5
1	11
2	7.5
3	4
4	1
5	-3

I (A)	y (mm)
0	14.5
0.5	14
1	12
1.5	11
2	10
2.5	8.5
3	7.5
3.5	6