Circuits

Part 11

1

Circuits

Boolean Hardware

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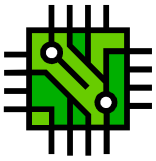
Circuits

- Boolean algebra gives designers tools to design & analyze complex solutions
- Can we create hardware that performs solutions?
- Can electronic circuits allow Boolean logic to exist in the physical world?



3

Two Bit Multiplier? Can we make it?

4

Designing It

- To design a circuit that multiplies two 2-bit numbers, we can use Boolean algebra
- We need to figure the logic – given that bits of 1 and 0 will map directly to truth values
- The result of the algebra will be the desired output

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It Takes the Following Skills

1. Design a truth-table to represent the different inputs and the desired output
2. Convert the truth-table into a Boolean function
3. Simplify the Boolean function
4. Finally, convert it into a circuit

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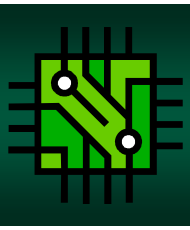

Gates

Boolean Hardware

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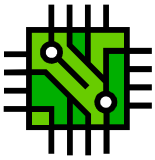
Gates

- Electronic devices are made up of *gates*
- Gates take in two inputs and produce a single output
- This is how hardware is used to implement Boolean logic (or any logic)



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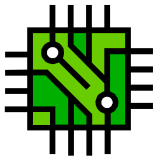
Gates

- Gates can be combined into circuits with *any number of input wires* and a single output wire
- We will chain these together much like subexpressions can be combined into larger expressions



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Graphical Representation

- Gates are typically represented using graphical shapes – much like flowcharts
- There are two different competing symbol standards
- We will use the standard, distinct, symbols rather than the IEC (European) ones

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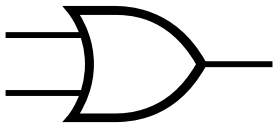
Or Gate



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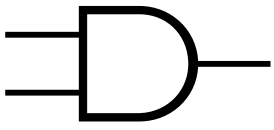
And Gate



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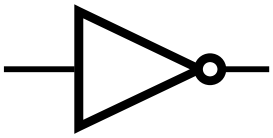
Not Gate



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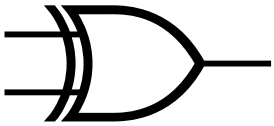
Exclusive Or Gate (aka XOR)



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Some Other Gate Symbols

- There are also gate symbols for negated operators
- I won't use these much in class, but it's good to be aware of them (since they are quite common in computer engineering
- For each, note the circle on the output line – it means "not"

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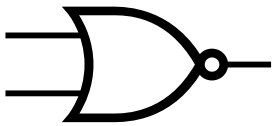
Not Or Gate (aka NOR)



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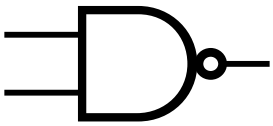
Not And Gate (aka NAND)



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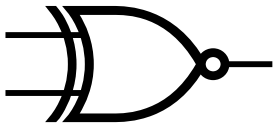
Not Exclusive Or Gate (aka XNOR)



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Computer Engineering Notation

Same thing, different paint job

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Computer Engineering Notation

- Gates, used to computer engineering, form a Boolean Algebra
- i.e. they can define the three operations (and, or, not) and share the same axioms

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Computer Engineering Notation

- The notation used in computer engineering is a *tad different* that what we have covered
- … and certainly different than any programming language you have used

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Computer Engineering Notation

- But is serves the same purpose
- And, not surprisingly, it works better for writing expressions in this discipline

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Computer Engineering Notation

1 ≡ T

0 ≡ F

* ≡ ∧

\+ ≡ ∨

' ≡ ¬

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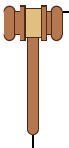
Commutative Law



x + y ≡ y + x

x * y ≡ y * x

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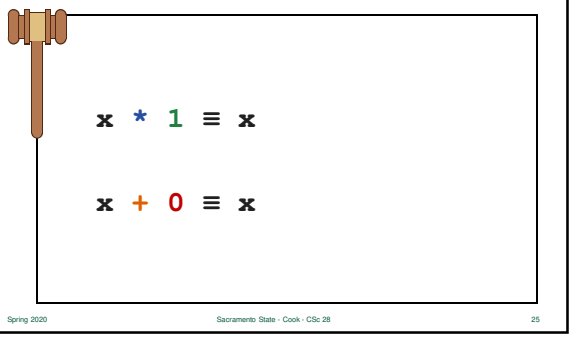
Identity Law



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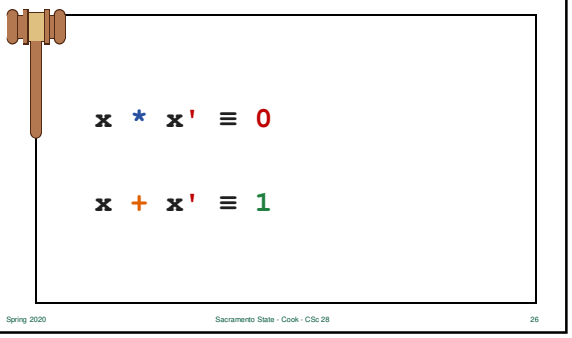
Complement Law



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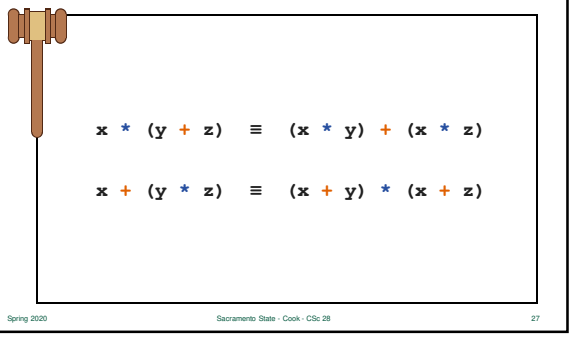
Distributive Law



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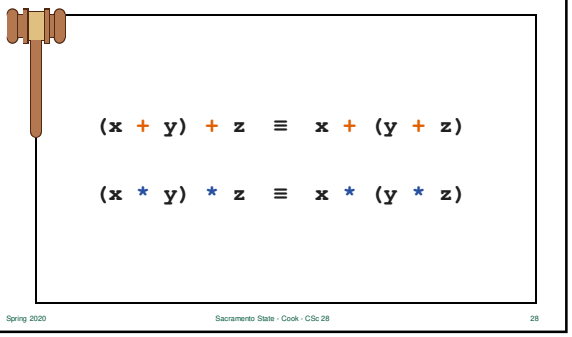
Associative Law



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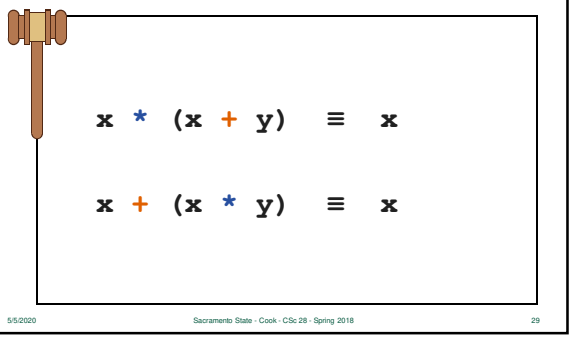
Absorption Law



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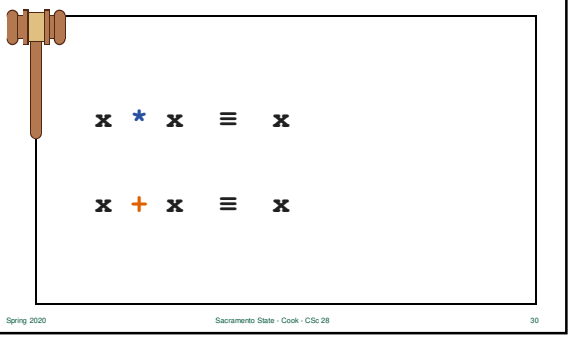
Idempotent Law



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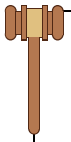
Involution Law



x'' ≡ x

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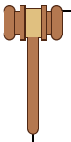
Domination Law



x + 1 ≡ 1

x * 0 ≡ 0

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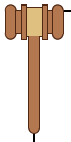
DeMorgan's Law



(x * y)' ≡ x' + y'

(x + y)' ≡ x' * y'

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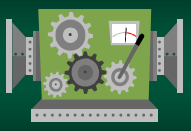

Converting Boolean to Circuits

From Logic to Wires

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Converting Boolean to Circuits

- Converting from Boolean to circuits maintains a *one-to-one* correspondence between gates and operators in the equation
- But, given an *arbitrary* Boolean expression, how do we realize a circuit for it?

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Steps

1. Choose the last operation evaluated
2. Draw a gate and hook up its output
3. Goto 1 until all operations have associated gates
4. Attach the expression inputs

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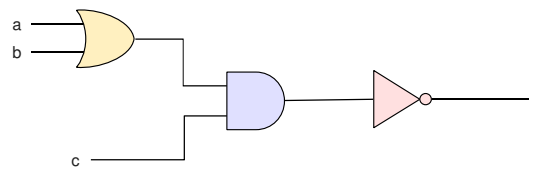
((a + b) * c)'



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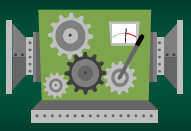

Converting Circuits to Boolean

From Logic to Wires

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Converting Circuits to Boolean

- The other direction is easy too
- Any circuit can be realized as a Boolean expression using the same basic algorithm

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Converting Circuits to Boolean

1. Pick a wire that has known Boolean values
2. Write *on the wire* a Boolean expression for its value
3. Goto 1 until all wires are complete
4. Circuit's expression written on the circuit's output wire

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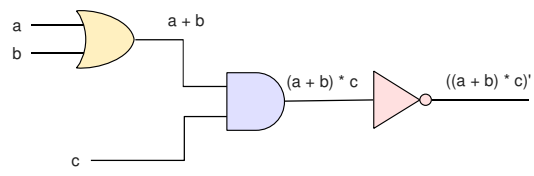
Example Circuit



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