California State University, Sacramento
College of Engineering and Computer Science

Computer Science 28: Discrete Mathematics

Spring 2020 – Midterm 1

93 +7 / 150

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1. Given the following functions, determine the following. You don't have to simply the result. *(15 points, 5 each)*

$f(n) = n^2 - 3$
$g(n) = n + 5$

Composition

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a) $g \circ f(x)$ → $g(f(x))$ $(x^2-3)+5$

b) $f \circ g(i)$ $f(g(i))$ $(i+5)^2-3$

c) $f \circ f(e)$ $(e^2-3)-3$

2. What is the difference between a tuple and set? *(5 points)* tuple = order matters & set • Duplicates don't count • Can be any order

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The difference between a tuple and a set beyond notation (Parentheses/angle bracket for tuple and curley brace for sets is that for tuples order matters where sets order does not matter.

3. For the set {1, 2, 3, 4, 5}, which are the following are partitions. If not, why? *(10 points, 2 each)* → unique to each set

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a) { {1, 2}, {3}, {4} } Yes does not union to 5

b) { {1, 5}, {4}, {3, 2} } Yes

c) { {5}, {4}, {2}, {3}, {1} } Yes

yes d) { {4}, {3, 2}, {5}, {2, 3}, {1} } No, 2&3 violates uniqueness to set

e) { {2, 4, 3}, {5}, {3, 1} } No, 3 appears twice and thus not unique in the set

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4. Simply the following. Identify each law you apply. If you don't remember, please still try to simply it. *(10 points)*

$(G' \cap G) \cup (F \cup F')$

$(G' \cap G) \cup (F \cup F')$

$\emptyset \cup (F \cup F')$ Complement

$\emptyset \cup U$ Complement

$\therefore U$ Identity or domination

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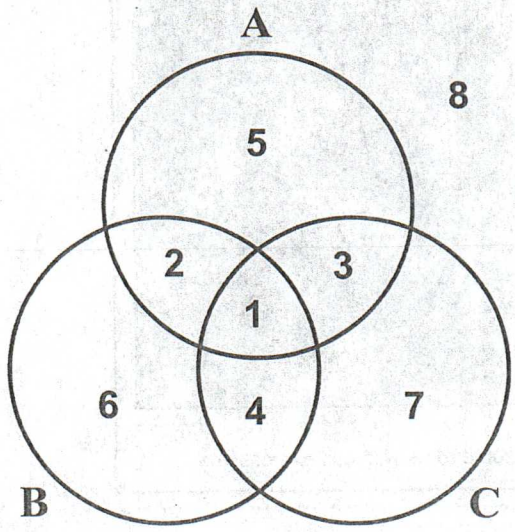
5. What is the set equation for each fundamental product listed below? *(16 points, 2 each)*



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1) $A \cap B \cap C$
2) $(A \cap B) \cup C'$ $A \cap B \cap C'$ or $A \cap B \setminus C$
3) $(A \cap C) \cup B'$ $A \cap B' \cap C$ or $A \cap C \setminus B$
4) $(B \cap C) \cup A'$ $A' \cap B \cap C$ or $B \cap C \setminus A$
5) $(B \cup C)' \cup A$ $A \cap B' \cap C'$ or $A \setminus B \setminus C$
6) $(A \cup C)' \cup B$ $A' \cap B \cap C'$ or $B \setminus A \setminus C$
7) $(A \cup B)' \cup C$ $A' \cap B' \cap C$ or $C \setminus A \setminus B$
8) $(A \cup B \cup C)'$ or $A' \cap B' \cap C'$

6. When you write the following code, what are you staying in terms of set theory? *(5 points)*

```
int x;
```

Long Int

5 X is a subset of integer.

7. Are the following expressions true or false? Fill in the circle if true *(5 points, 1 each)*.

a) { 1, 2, 3 } = { 3, 2, 1 } set order does not matter — True?: ● True

3 b) (a, b, c) ≠ (a, c, b) Tuple order matters — ○ Not true

Does NOT equal so true

c) { a, b } ⊂ (a, b, b) order remains the same, does not count — ● True

10110 or 11010 = 11111

d) 10110 ∪ 11010 = 00010 False ∧ — ○ Not True

e) 12 ∈ { x^2 | x ∈ Z } — ○ False

x is a element/member of int (..., -3, -2, -1, 0, 1, 2, 3, ...)

$X \cdot X = 12$ Plug in

$\sqrt{12}$ → 3,4 / 2,6 / 1,12 Prime

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Best Rock!

And now – for no apparent reason – here are some rocks with odd expressions.

8. The following is a relation, over the domain {1, 2, 3, 4, 5}, below: *(15 points)*

{ (1, 2), (3, 1), (1, 1), (5, 4), (2, 5), (3, 3) }

What are the following closure sets:

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Reflexive: already have (1,1) & (3,3) so (2,2), (4,4), (5,5)

already have (1,1), (3,3)

Symmetric: (2,1), (1,3), (4,5), (5,2)

Transitive: (1,5), (2,4), ~~(3,2)~~ {(1,5) (2,4) (3,2) (1,4) (3,4) (3,5)}

Come Back and check

(1,1), (1,2), (2,5), (3,1), (3,3), (5,4)

9. Are the following countable, countably infinite, or uncountable? *(4 points, 1 each)*

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a) { f | f ∈ N and f < 100000000 } Natural — Countable

b) { g | g ∈ Z and g < 3 } — Countably Infinite

c) { t | t ∈ R and t > 2 } — Uncountable

d) { sonic, knuckles, tails, eggman } — Countable?

cardinality 4?

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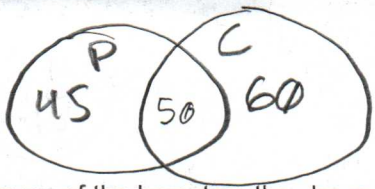

10. A local pet store took at survey of the hamsters they have for sale. Of them, 45 hamsters enjoy eating peanuts and 60 hamsters like constant cardio exercise, and 50 hamsters like both.

How hamsters are there? Solve this using the inclusion-exclusion equation. *(15 points)*

$45+60-(50)$

$=55$ hamsters? I'm probably missing something

$|A|+|B|-(A\cap B)'$

$|P\cup C| = |P|+|C|-|P\cap C|$

$= 45+60-50$

$= 55$

45 +60 105 −50 55 15

11. Write out items in the set created by the following set builder expressions. If the set is infinitely large, show it with ellipsis *(10 points, 2 each)*:

a) { x | x ∈ N and x < 7 } (0,1,2,...) ← Non inclusive

b) { x | sqrt(x) ∈ Z and 1 ≤ x ≤ 20 } ← Plug in

c) { 2x | x ∈ N and x < 6 } 1·2, 2·2, Natural, Non-inclusive

d) { x | 1 ≤ x ≤ 5 and x ∈ Z and 1/x ∈ Q } 1/1, 1/2,

e) { x ÷ 2 | x ∈ Z and x < 6 } −1/2, Int

√4=2 √9=3 6

1,2,3,4,5,6

2,3 1,4,9,16

2,4,6,8,10

1/1, 1/2, 1/3, 1/4, 1/5 1,2,3,4,5

...,$-\frac{3}{2}$,-1,$-\frac{1}{2}$,$\frac{0}{2}$,$\frac{1}{2}$,1,...,$\frac{5}{2}$ ← Stops here

12. What is A × B? *(10 points)*

A = { sonic, robotnik, spongbob } ← domain (x) (S, R, SP)

B = { dance, burgers, beer, life } ← Range (y) (d, b, be, L)

that works! :)

$A\times B = \{(S,d),(S,b),(S,be),(S,L),(R,d),(R,b),(R,be),(R,L),(Sp,d),(Sp,b),(Sp,be),(Sp,L)\}$

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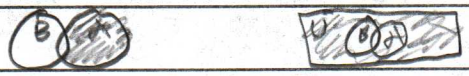
13. Simply the following. Identify each law you apply. If you don't remember, please still try to simply it. *(20 points)*

$(B' \cap A) \cup (B \cup A)'$



$(B' \cap A) \cup (B \cup A)'$

$(A) \cup (B \cup A)'$?

$(A) \cup (\top)$

$\therefore \top$ Domination

$(B' \cap A) \cup (B \cup A)'$

$(B' \cap A) \cup (B' \cap A')$ DeMorgan's Law

$B' \cap (A \cup A')$ Distributive

$B' \cap \top$ Complement

$\therefore B'$ Identity

1. Fill in the Blank: Come to think about it.... what caused the rocks to have that expression?

14. What is the cardinality of the following sets? *(10 points, 2 each)*

a) { ~~spam, spam~~, sausage, spam } Duplicate — 2

b) { {1, 5}, {4}, {3, 2} } — 3

c) { rick, morty, jerry, beth, summer } — 5

d) { {4}, {3, 2}, {5}, {2, 3}, {1} } — ~~5~~ 4

order does not matter

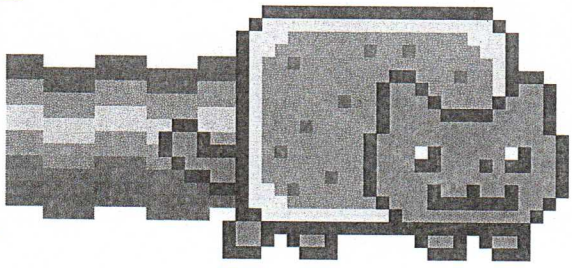
e) {2, 4, 3} × {1, 2 ,3 .. 9} — ~~12~~ 27

why do this

3 x 9 = 27

Why Math

3 x 9 ≠ 12



Have a great weekend!
(Do something fun!)

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