California State University, Sacramento
College of Engineering and Computer Science

Computer Science 28: Discrete Mathematics

Spring 2020 – Assignment #3 – Quantified Logic & Induction

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Quantified Logic

1. Convert the following statement to a quantified expression.

   All my cats are asleep.

   Then, convert it into an equivalent statement (where exists and for-all are switched). Finally, convert the answer back to English

   Approach #1
   S(x) = x is asleep
   x = all my cats

   $\forall_x S(x)$ "All my cats are asleep"
   $\neg\exists_x \neg S(x)$ "There doesn't exist a cat that's awake"

   Approach #2
   S(x) = x is asleep
   C(x) = x is my cat
   x = all cats

   $\forall_x (C(x) \rightarrow S(x))$ "All cats that are mine are asleep"
   $\neg \exists_x \neg (C(x) \rightarrow S(x))$
   $\neg \exists_x \neg (\neg C(x) \vee S(x))$
   $\neg \exists_x (C(x) \wedge \neg S(x))$ "There doesn't exist a cat that is mine and is awake"

2. Convert the following statement into a quantified expression:

   Everyone, who has seen Rick and Morty and has a sense of humor, likes Szechuan Sauce.

   R(x) = x has seen Rick and Morty
   H(x) = x has a sense of humor
   S(x) = x likes Szechuan sauce

   $\forall_x (R(x) \wedge H(x) \rightarrow S(x))$

3. Simplify the following Quantified Statement. The result should have **no** negation symbols.

   $\neg \forall_x \exists_x (\neg B(x) \wedge P(x))$

   $\neg \forall_x \exists_x (\neg B(x) \wedge P(x))$
   $\exists_x \neg\exists_x (\neg B(x) \wedge P(x))$
   $\exists_x \forall_x \neg (\neg B(x) \wedge P(x))$
   $\exists_x \forall_x (B(x) \vee \neg P(x))$

   $\exists_x \forall_x (P(x) \rightarrow B(x))$

Induction

4. Prove the following using induction (show your work - both steps):

If $n \geq 1$ then $2 + 4 + 6 + \ldots + 2n = n(n+1)$

$P(n) : 2 + 4 + 6 + \ldots + 2n = n(n+1)$
$P(n+1) : 2 + 4 + 6 + \ldots + 2n + 2(n+1) = (n+1)(n+2)$

Step 1: Basis
$P(1) : 2 = 1(1+1)$ Yes, these equal. Basis is correct!

Step 2: Induction
$2 + 4 + 6 + \ldots + 2n + 2(n+1) = (n+1)(n+2)$

$n(n+1) + 2(n+1) = (n+1)(n+2)$
$n^2 + n + 2n + 2 = (n+1)(n+2)$
$n^2 + 3n + 2 = (n+1)(n+2)$

$(n+1)(n+2) = (n+1)(n+2)$

5. Prove the following using induction (show your work - both steps):

If $n \geq 1$ then $n^2 + n$ is even

$P(n)$: $n^2 + n$ is even
$P(n+1)$: $(n+1)^2 + (n+1)$ is even

Step 1: Basis
$P(1)$: $1^2 + 1 = 2$

Step 2: Induction
$(n+1)^2 + (n+1)$
$n^2 + 2n + 1 + n + 1$
$n^2 + 2n + 2 + n$ Simply
$n^2 + n + 2n + 2$ Rearrange
$n^2 + n + 2(n+1)$