Boolean Algebra

1. <u>Use Boolean algebra</u>. Simply the following. Label <u>all</u> the laws you apply.

n ∧ ¬n ∨ (n ∧ (q ∨ ¬q))

```
false ∨ (n ∧ (q ∨ ¬q))    Complement Law
false ∨ (n ∧ true)        Complement Law
false ∨ n                 Identity Law
n                         Identity Law
```

2. <u>Use Boolean algebra</u> to simplify the following. Label <u>all</u> the laws you apply.

(p ∨ ¬q) → (p ∧ q)

```
¬(p ∨ ¬q) ∨ (p ∧ q)       Equivalent
(¬p ∧ q) ∨ (p ∧ q)        DeMorgan's Law
q ∧ (¬p ∨ p)              Distribution Law
q ∧ true                  Complement Law
q                         Identity
```

3. <u>Use Boolean algebra</u>. Simply the following. Label <u>all</u> the laws you apply.

¬ a ∧ ¬ b ∨ c ∧ ¬ c ∨ ¬ a ∧ b

```
(¬ a ∧ ¬ b) ∨ (c ∧ ¬ c) ∨ (¬ a ∧ b)
(¬ a ∧ ¬ b) ∨ false ∨ (¬ a ∧ b)      Complement Law
(¬ a ∧ ¬ b) ∨ (¬ a ∧ b)              Identity Law
¬ a ∧ (¬ b ∨ b)                      Distribution Law
¬ a ∧ true                           Complement Law
¬ a                                  Identity Law
```

Arguments

4. The following argument from the movie *Monty Python and the Holy Grail.*

    "If it's made of wood then it floats. If it's a duck then it floats. <u>Therefore</u>, a duck is wood."

    Convert the English sentence to an argument (using single letters). Show **why** it is valid or invalid. You must use a truth table for credit.

    W = Wood
    B = Floats (buoyant)
    D = Duck

    W → B
    D → B

    D → W

W	B	D	W → B	D → B	D → W
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	F
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

5. Prove (or disprove) the following argument.

"If they are a student, then they'll come to class or use Zoom. Students can't come to class. Therefore, if they are a student, then they will use Zoom."

Convert the English sentence to an argument (using single letters). Show **why** it is valid or invalid. Then use any method you like to prove or disprove it.

```
S = Student
Z = Zoom
C = Class

S → C V Z
¬ C
-----------
S → Z
```

S	C	Z	C∨Z	S→C∨Z	¬C	S→Z
T	T	T	T	T	F	T
T	T	F	T	T	F	F
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	F	T	T	T

6. Prove (or disprove) the following argument by using a truth table.

"If its toilet paper, then it's being hoarded. If it's being hoarded, then it must be guarded. <u>Therefore</u>, toilet paper must be guarded!"

Convert the English sentence to an argument (using single letters). Show **why** it is valid or invalid.

```
P = Toilet paper
H = Hoarded
G = Guarded

P → H
H → G
---------
P → G
```

P	H	G	P → H	H → G	P → G
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Simple Proofs

7. Prove the following (show your work):

    *If **a** is odd, **b** is even, and **c** odd then **a** × **b** × **c** is even*

    ```
    a = 2i + 1
    b = 2j
    c = 2k + 1

    a × b × c

    = (2i + 1) (2j) (2k + 1)
    = (4ij + 2j) (2k + 1)
    = 8ijk + 4ij + 4jk + 2j
    = 2 (4ijk + 2ij + 2jk + j)
    ```

8. Prove the following (show your work):

    *If **a** is divisible by 2 and **b** is divisible by 6 then **a** × **b** is divisible by 4.*

    ```
    a = 2i
    b = 6j

    a × b

    = 2i × 6j
    = 12 ij
    = 4 (3ij)
    ```