



Graph Shortest Path

Section 4.1 – 4.3

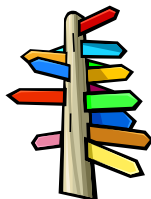


Dijkstra's Algorithm

Finding the best route!

Getting the Shortest Path

- The Kruskal and Prim Algorithms can efficiently create minimum spanning trees
- And these are the "best" trees that *span all nodes*



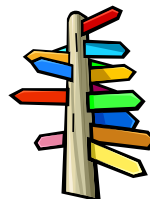
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Getting the Shortest Path

- However, it **doesn't** necessarily produce the minimum path for *each* vertex to reach other vertices
- In other words, the shortest path for each might **not** be reflected in the MST

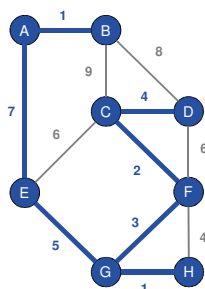


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Prim's Algorithm Example



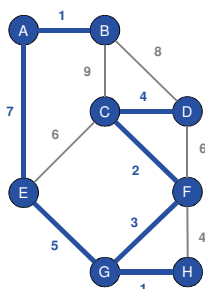
- This is the spanning tree created by both the Prim and Kruskal Algorithms
- Let's say we want to get the minimal path from B to D

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Prim's Algorithm Example



- The incident edge from vertex B to D has a weight of **8**
- However, following the tree, the total weight would be **22**

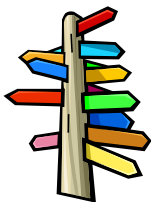
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Getting the Shortest Path

- We need to compute the best path for each node
- This is known as the *distance*
- Given 2 vertices u and v ...
 - find a path with the "best" total weight between u and v
 - e.g. the minimum total is the sum of all the edges in the path



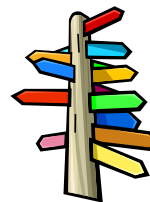
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Some Real World Examples

- Internet packet routing
 - fastest route – faster downloads
 - load distribution
- Driving directions
 - shortest route in miles
 - fastest route – given traffic and speed limit data



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Shortest Path Attributes

- A sub-path of a shortest path
 - ... is itself a shortest path
 - so, *any subset of a optimal solution is optimal*
- There is a tree of shortest paths
 - there are multiple solutions from the start vertex to all the other vertices
 - might be multiple equally-optimal solutions

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Edsger Dijkstra

- The algorithm used today to find the optimal path was written by Edsger Dijkstra
- In fact, his algorithm runs the Internet – a self healing, load distributioning network
- **No Dijkstra → no Internet**



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We're riding on the Internet!



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Dijkstra's Algorithm

- *Dijkstra's Shortest Path First Algorithm* computes the distances of all the vertices from a given start vertex s
- Works on directed and undirected graphs
- However
 - all edges must have nonnegative weight
 - the graph is connected

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Dijkstra's Algorithm

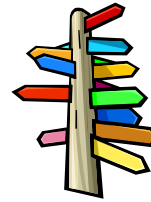
- Dijkstra's Algorithm is *greedy*
 - *always* takes the *best* immediate or local solution while finding an answer
 - they find optimal solutions for some optimization problems very efficiently
 - but may find less-than-optimal solutions
- Greedy algorithms require extra analysis to determine if they work – Dijkstra's does

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Dijkstra's Algorithm Logic



- *Each* vertex has its *own* "distance" table
- The table contains the best weight to each other vertex - as well as the best path

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Dijkstra's Algorithm Logic

1. Initialize a distance table
 - table represents the best distance (weight) to get to the specific vertices
 - set all to *infinity* (worst possible)
 - it will be updated as we see more vertices
2. Recurse until are *settled* (visited)
 - depth-first search the graph
 - advance on smallest weighted edge

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Dijkstra's Algorithm Logic

- Given table D , vertex v , edge weight w with target t : $D(t) = D(v) + w$
- For example:
 - if it takes 12 to get to v
 - ... and an edge of 6 goes to t
 - ... then it will take $12 + 6$ total to get to t

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Dijkstra's Algorithm Logic

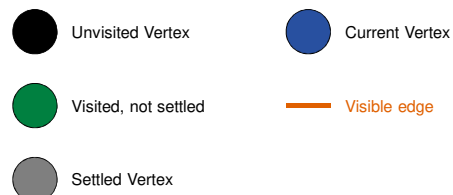
- For each vertex v
 - compute the total distance to get each vertex t that v is adjacent to (v edges point to t)
 - if the value is better than the current table value *then* update
 - don't update t if it will cause a cycle – i.e. t is already in the path

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Notation



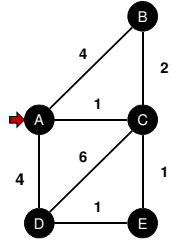
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Example 1: Vertex A Table

Vertex	Best Path	Distance	Look
A		0	
B		∞	
C		∞	
D		∞	
E		∞	



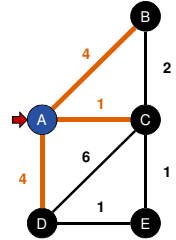
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Example 1: Look Adjacent

Vertex	Best Path	Distance	Look
A	A	0	
B		∞	$0 + 4$
C		∞	$0 + 1$
D		∞	$0 + 4$
E		∞	



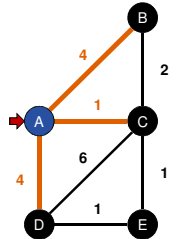
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Example 1: Set Distance/Path

Vertex	Best Path	Distance	Look
A	A	0	
B	A B	4	$0 + 4$
C	A C	1	$0 + 1$
D	A D	4	$0 + 4$
E		∞	



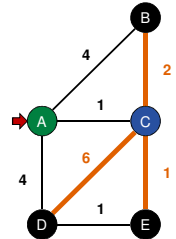
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Example 1: Take Best Edge

Vertex	Best Path	Distance	Look
A	A	0	
B	A B	4	$1 + 2$
C	A C	1	
D	A D	4	$1 + 6$
E		∞	$1 + 1$



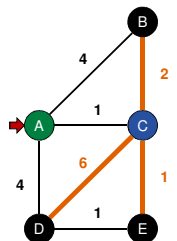
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Example 1: Set Distance/Path

Vertex	Best Path	Distance	Look
A		0	
B	A C B	3	$1 + 2$
C	A C	1	
D	A D	4	$1 + 6$
E	A C E	2	$1 + 1$



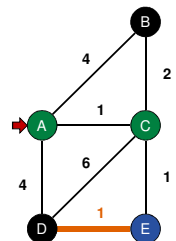
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Example 1: Take Best Edge

Vertex	Best Path	Distance	Look
A		0	
B	A C B	3	
C	A C	1	
D	A D	4	$2 + 1$
E	A C E	2	



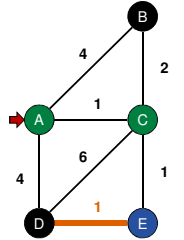
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Example 1: Set Distance/Path

Vertex	Best Path	Distance	Look
A		0	
B	A C B	3	
C	A C	1	
D	A C E D	3	← 2 + 1
E	A C E	2	



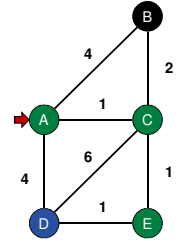
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Example 1: Take Best Edge

Vertex	Best Path	Distance	Look
A		0	
B	A C B	3	
C	A C	1	
D	A C E D	3	
E	A C E	2	



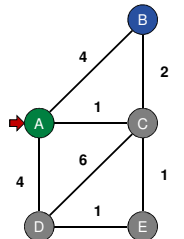
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Example 1: Recurse up, try B

Vertex	Best Path	Distance	Look
A		0	
B	A C B	3	
C	A C	1	
D	A C E D	3	
E	A C E	2	



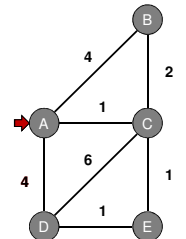
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Example 1: Done

Vertex	Best Path	Distance	Look
A		0	
B	A C B	3	
C	A C	1	
D	A C E D	3	
E	A C E	2	



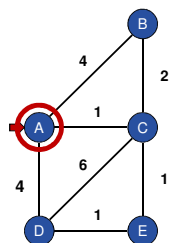
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Example 1: Solution A → D

Vertex	Best Path	Distance
A		0
B	A C B	3
C	A C	1
D	A C E D	3
E	A C E	2



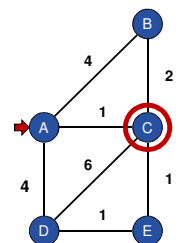
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Example 1: Solution A → D

Vertex	Best Path	Distance
A		0
B	A C B	3
C	A C	1
D	A C E D	3
E	A C E	2



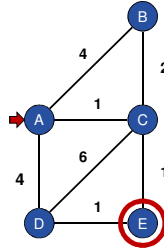
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Example 1: Solution A → D

Vertex	Best Path	Distance
A		0
B	A C B	3
C	A C	1
D	A C E D	3
E	A C E	2



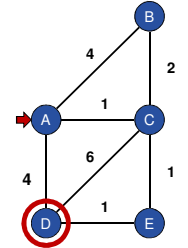
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Example 1: Solution A → D

Vertex	Best Path	Distance
A		0
B	A C B	3
C	A C	1
D	A C E D	3
E	A C E	2



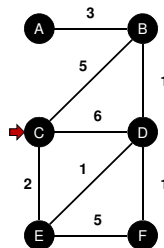
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Example 2: Vertex C Table

Vertex	Best Path	Distance	Look
A		∞	
B		∞	
C		0	
D		∞	
E		∞	
F		∞	



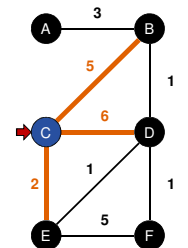
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Example 2: Look Adjacent

Vertex	Best Path	Distance	Look
A		∞	
B		∞	$0 + 5$
C	C	0	
D		∞	$0 + 6$
E		∞	$0 + 2$
F		∞	



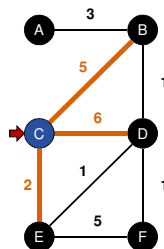
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Example 2: Set Distance/Path

Vertex	Best Path	Distance	Look
A		∞	
B	C B	5	$0 + 5$
C	C	0	
D	C D	6	$0 + 6$
E	C E	2	$0 + 2$
F		∞	



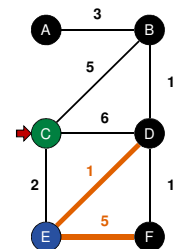
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Example 2: Take Best Edge

Vertex	Best Path	Distance	Look
A		∞	
B	C B	5	
C	C	0	
D	C D	6	$2 + 1$
E	C E	2	
F		∞	$2 + 5$



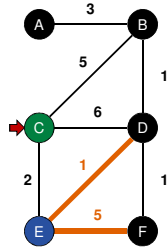
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Example 2: Set Distance/Path

Vertex	Best Path	Distance	Look
A		∞	
B	C B	5	
C	C	0	
D	C E D	3	$2 + 1$
E	C E	2	
F	C E F	7	$2 + 5$



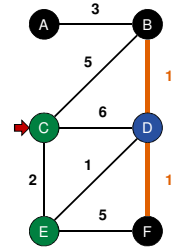
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Example 2: Take Best Edge

Vertex	Best Path	Distance	Look
A		∞	
B	C B	5	$3 + 1$
C	C	0	
D	C E D	3	
E	C E	2	
F	C E F	7	$3 + 1$



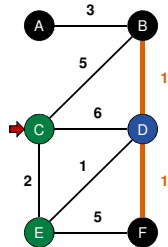
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Example 2: Set Distance/Path

Vertex	Best Path	Distance	Look
A		∞	
B	C E D B	4	$3 + 1$
C	C	0	
D	C E D	3	
E	C E	2	
F	C E D F	4	$3 + 1$



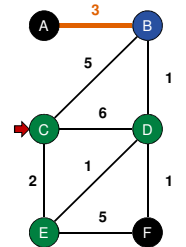
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Example 2: Take Best Edge

Vertex	Best Path	Distance	Look
A		∞	$4 + 3$
B	C E D B	4	
C	C	0	
D	C E D	3	
E	C E	2	
F	C E D F	4	



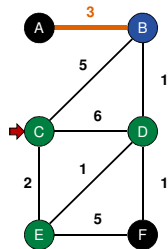
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Example 2: Set Distance/Path

Vertex	Best Path	Distance	Look
A	C E D B A	7	$4 + 3$
B	C E D B	4	
C	C	0	
D	C E D	3	
E	C E	2	
F	C E D F	4	



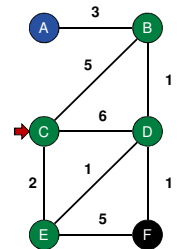
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Example 2: Take Best Path

Vertex	Best Path	Distance	Look
A	C E D B A	7	
B	C E D B	4	
C	C	0	
D	C E D	3	
E	C E	2	
F	C E D F	4	



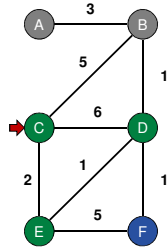
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Example 2: Recursed Up

Vertex	Best Path	Distance	Look
A	CEDBA	7	
B	CEDB	4	
C	C	0	
D	CED	3	
E	CE	2	
F	CEDF	4	



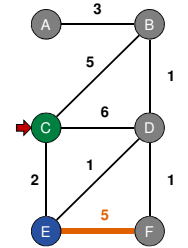
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Example 2: Look as Recurse Up

Vertex	Best Path	Distance	Look
A	CEDBA	7	
B	CEDB	4	
C	C	0	
D	CED	3	
E	CE	2	
F	CEDF	4	2 + 5



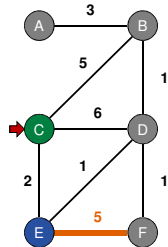
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Example 2: Look as Recurse Up

Vertex	Best Path	Distance	Look
A	CEDBA	7	
B	CEDB	4	
C	C	0	
D	CED	3	
E	CE	2	
F	CEDF	4	2 + 5



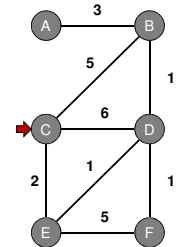
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Example 2: Complete

Vertex	Best Path	Distance	Look
A	CEDBA	7	
B	CEDB	4	
C	C	0	
D	CED	3	
E	CE	2	
F	CEDF	4	



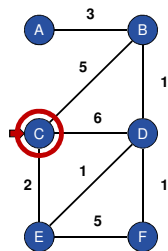
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Example 2 Solution: C → A

Vertex	Best Path	Distance
A	CEDBA	7
B	CEDB	4
C	C	0
D	CED	3
E	CE	2
F	CEDF	4



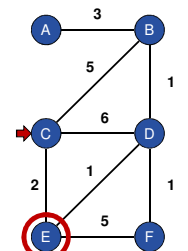
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Example 2 Solution: C → A

Vertex	Best Path	Distance
A	CEDBA	7
B	CEDB	4
C	C	0
D	CED	3
E	CE	2
F	CEDF	4



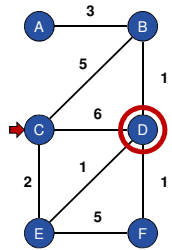
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Example 2 Solution: C → A

Vertex	Best Path	Distance
A	CEDBA	7
B	CEDB	4
C	C	0
D	CED	3
E	CE	2
F	CEDF	4



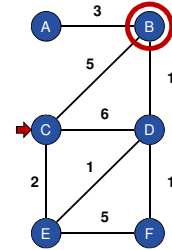
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Example 2 Solution: C → A

Vertex	Best Path	Distance
A	CEDBA	7
B	CEDB	4
C	C	0
D	CED	3
E	CE	2
F	CEDF	4



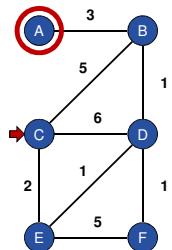
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Example 2 Solution: C → A

Vertex	Best Path	Distance
A	CEDBA	7
B	CEDB	4
C	C	0
D	CED	3
E	CE	2
F	CEDF	4

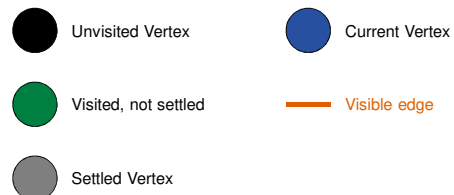


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Notation



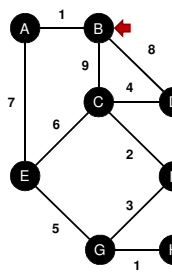
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Example 3: Vertex B Table

Vertex	Best Path	Distance	Look
A		∞	
B		0	
C		∞	
D		∞	
E		∞	
F		∞	
G		∞	
H		∞	



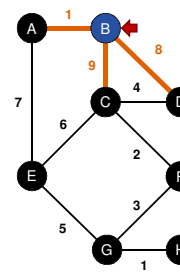
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Example 3: Vertex B Table

Vertex	Best Path	Distance	Look
A		∞	0 + 1
B	B	0	
C		∞	0 + 9
D		∞	0 + 8
E		∞	
F		∞	
G		∞	
H		∞	



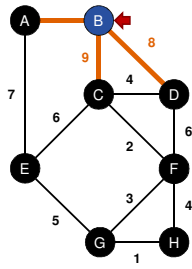
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Example 3: Set Distance/Path

Vertex	Best Path	Distance	Look
A	B A	1	$0 + 1$
B	B	0	
C	B C	9	$0 + 9$
D	B D	8	$0 + 8$
E		∞	
F		∞	
G		∞	
H		∞	



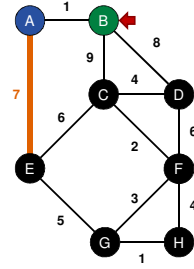
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Example 3: Take Best Edge

Vertex	Best Path	Distance	Look
A	B A	1	
B	B	0	
C	B C	9	
D	B D	8	
E		∞	$1 + 7$
F		∞	
G		∞	
H		∞	



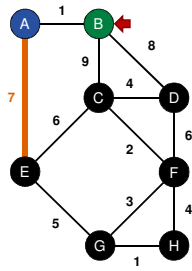
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Example 3: Set Distance/Path

Vertex	Best Path	Distance	Look
A	B A	1	
B	B	0	
C	B C	9	
D	B D	8	
E	B A E	8	$1 + 7$
F		∞	
G		∞	
H		∞	



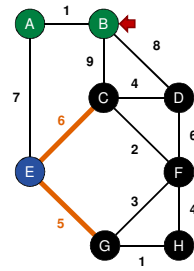
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Example 3: Take Best Edge

Vertex	Best Path	Distance	Look
A	B A	1	
B	B	0	
C	B C	9	$8 + 6$
D	B D	8	
E	B A E	8	
F		∞	
G		∞	$8 + 5$
H		∞	



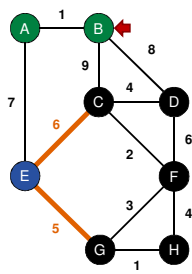
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Example 3: Set Distance/Path

Vertex	Best Path	Distance	Look
A	B A	1	
B	B	0	
C	B C	9	$8 + 6$
D	B D	8	
E	B A E	8	
F		∞	
G	B A E G	13	$8 + 5$
H		∞	



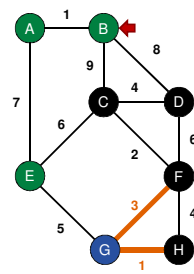
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Example 3: Take Best Edge

Vertex	Best Path	Distance	Look
A	B A	1	
B	B	0	
C	B C	9	
D	B D	8	
E	B A E	8	
F		∞	$13 + 3$
G	B A E G	13	
H		∞	$13 + 1$



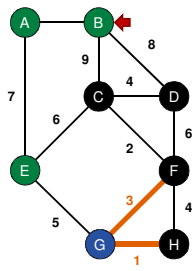
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Example 3: Set Distance/Path

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BAEGF	16	13 + 3
G	BAEG	13	
H	BAEGH	14	13 + 1



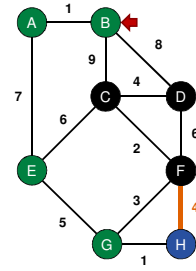
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Example 3: Take Best Edge

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BAEGF	16	14 + 4
G	BAEG	13	
H	BAEGH	14	



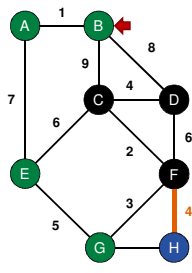
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Example 3: Set Distance/Path

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BAEGF	16	14 + 4
G	BAEG	13	
H	BAEGH	14	



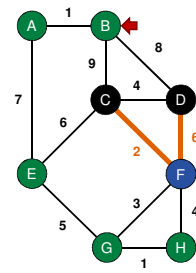
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Example 3: Take Best Edge

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	16 + 2
D	BD	8	16 + 6
E	BAE	8	
F	BAEGF	16	
G	BAEG	13	
H	BAEGH	14	



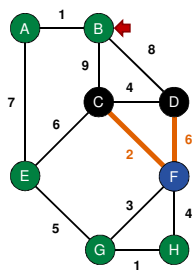
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Example 3: Set Distance/Path

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	16 + 2
D	BD	8	16 + 6
E	BAE	8	
F	BAEGF	16	
G	BAEG	13	
H	BAEGH	14	



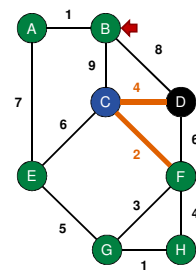
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Example 3: Take Best Edge

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	9 + 4
E	BAE	8	
F	BAEGF	16	9 + 2
G	BAEG	13	
H	BAEGH	14	



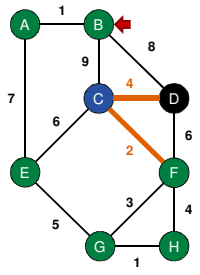
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Example 3: Set Distance/Path

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	\times $9 + 4$
E	BAE	8	
F	BCF	11	$\leftarrow 9 + 2$
G	BAEG	13	
H	BAEGH	14	



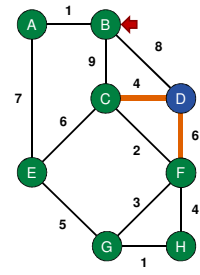
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Example 3: Take Best Edge

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	$8 + 4$
D	BD	8	
E	BAE	8	
F	BCF	11	$8 + 6$
G	BAEG	13	
H	BAEGH	14	



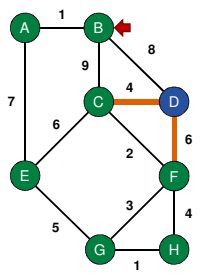
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Example 3: Take Best Edge

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	\times $8 + 4$
D	BD	8	
E	BAE	8	
F	BCF	11	\times $8 + 6$
G	BAEG	13	
H	BAEGH	14	



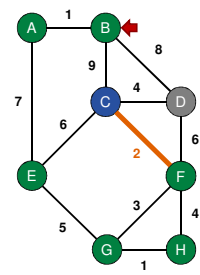
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Example 3: Recurse Back & Check

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BCF	11	\times $9 + 2$
G	BAEG	13	
H	BAEGH	14	



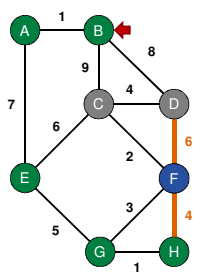
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Example 3: Recurse Back & Check

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	\times $11 + 6$
E	BAE	8	
F	BCF	11	
G	BAEG	13	
H	BAEGH	14	\times $11 + 4$



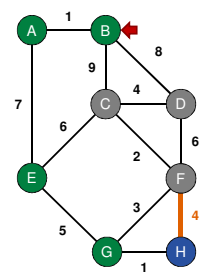
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Example 3: Recurse Back & Check

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BCF	11	\times $14 + 4$
G	BAEG	13	
H	BAEGH	14	



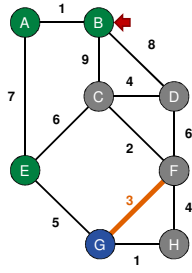
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Example 3: Recurse Back & Check

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BCF	11	13 + 3
G	BAEG	13	
H	BAEGH	14	



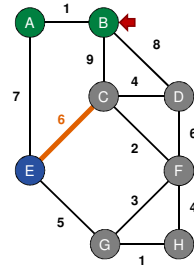
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Example 3: Recurse Back & Check

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	8 + 6
D	BD	8	
E	BAE	8	
F	BCF	11	
G	BAEG	13	
H	BAEGH	14	



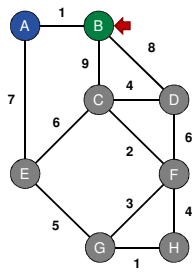
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Example 3: Recurse Back & Check

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BCF	11	
G	BAEG	13	
H	BAEGH	14	



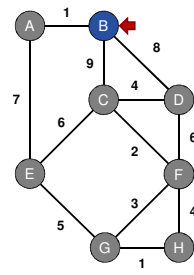
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Example 3: Recurse Back & Check

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BCF	11	
G	BAEG	13	
H	BAEGH	14	



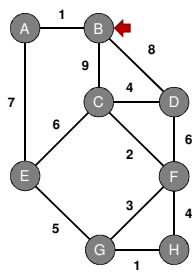
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Example 3: Done

Vertex	Best Path	Distance	Look
A	BA	1	
B	B	0	
C	BC	9	
D	BD	8	
E	BAE	8	
F	BCF	11	
G	BAEG	13	
H	BAEGH	14	



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Let's Look At
California

Driving Around the Golden State

Let's Look At California

- The cities of California are connected in many different ways
- Whether it is communications, water, power, transportation, etc... each is its own graph



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Let's Look At California

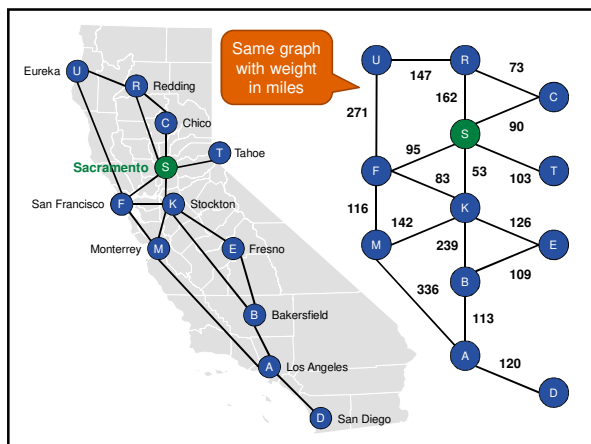
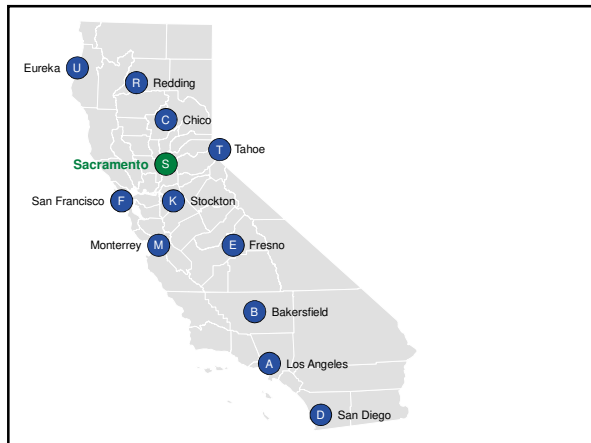
- Let's look at the California freeway system and run the Prim and Dijkstra algorithms on it
- So, let's get a MST and a nice shortest-path for Sacramento



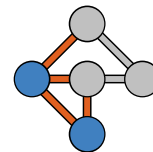
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Minimal Spanning Tree



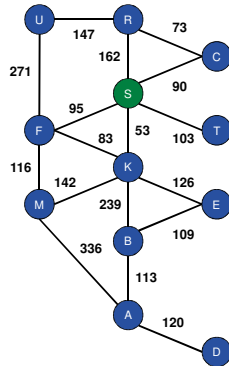
- Now that we have a graph representation of California, let's use Prim's Algorithm to compute the minimal spanning tree
- We can start any node, but let's use Sacramento (S)

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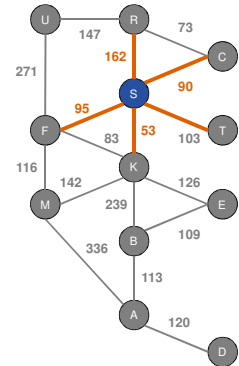
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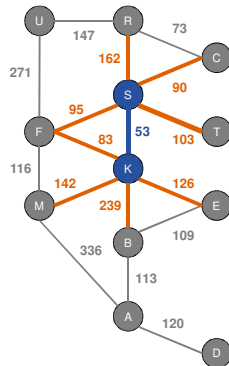
Prim's Minimal
Spanning Tree
Algorithm



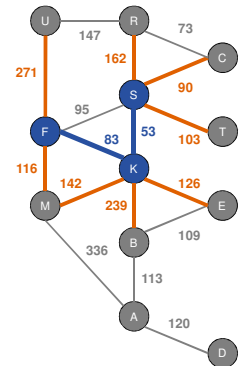
Prim's Minimal
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Algorithm



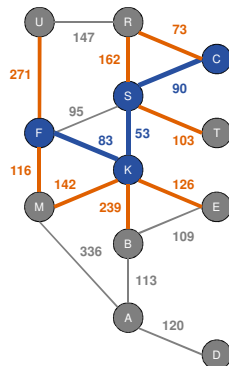
Prim's Minimal
Spanning Tree
Algorithm



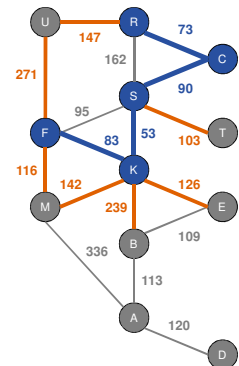
Prim's Minimal
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Algorithm



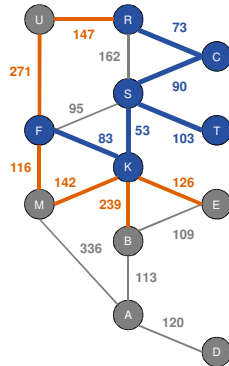
Prim's Minimal
Spanning Tree
Algorithm



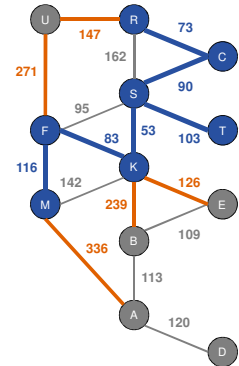
Prim's Minimal
Spanning Tree
Algorithm



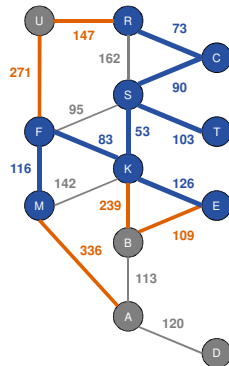
Prim's Minimal
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Algorithm



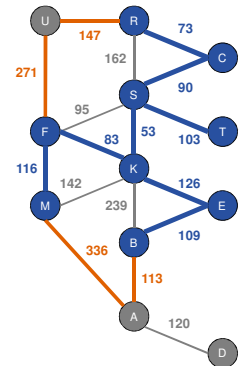
Prim's Minimal
Spanning Tree
Algorithm



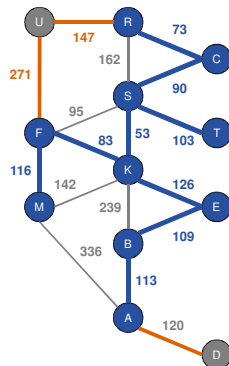
Prim's Minimal
Spanning Tree
Algorithm



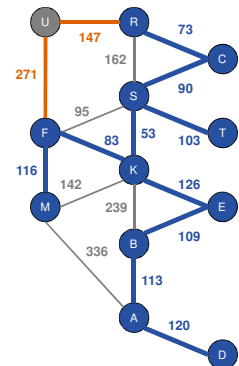
Prim's Minimal
Spanning Tree
Algorithm



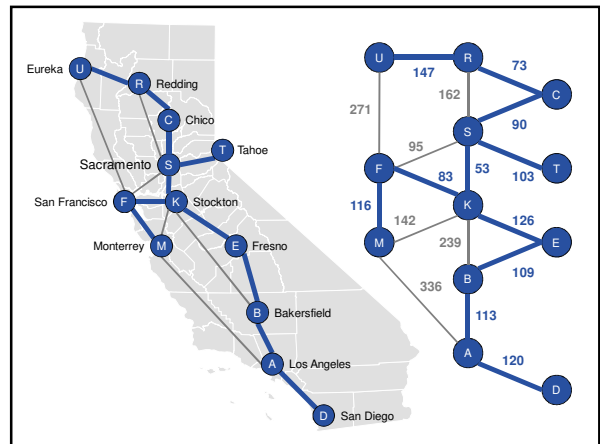
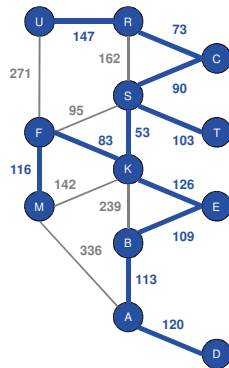
Prim's Minimal
Spanning Tree
Algorithm



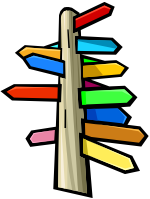
Prim's Minimal
Spanning Tree
Algorithm



Prim's Minimal Spanning Tree Algorithm



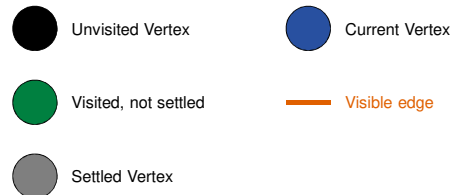
Let's Compute the Best Path



- Given we are in Sacramento, let's compute a shortest path table for it using Dijkstra's Algorithm
- This will take awhile... but it will work

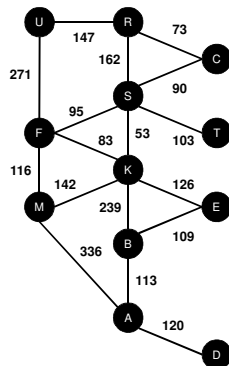
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Notation

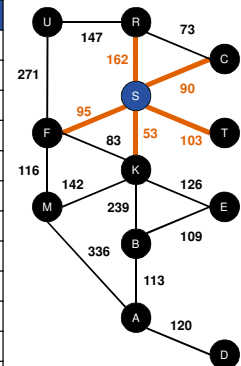


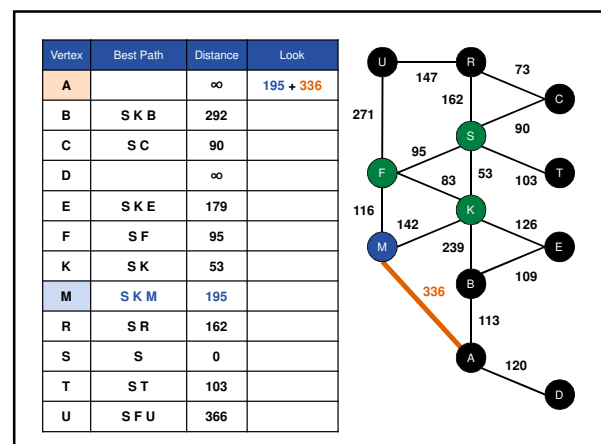
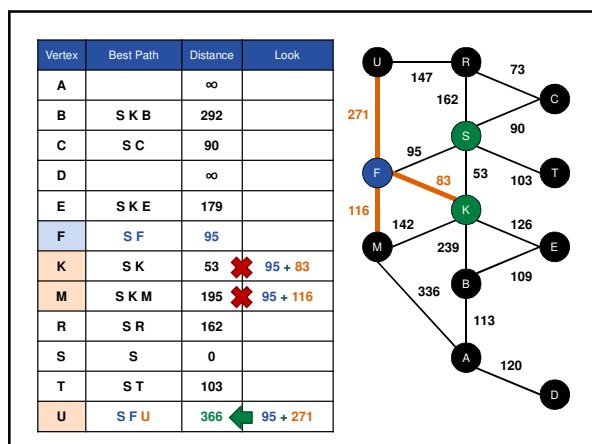
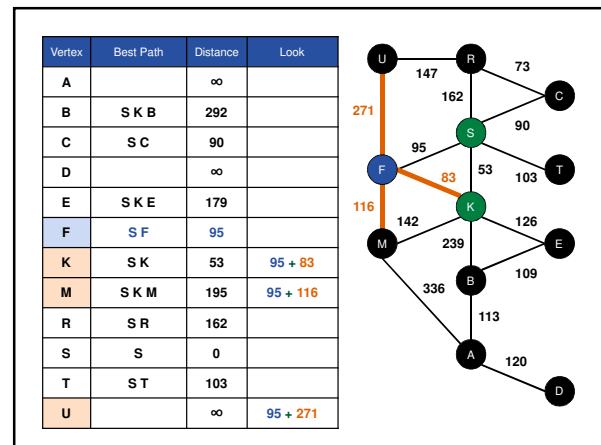
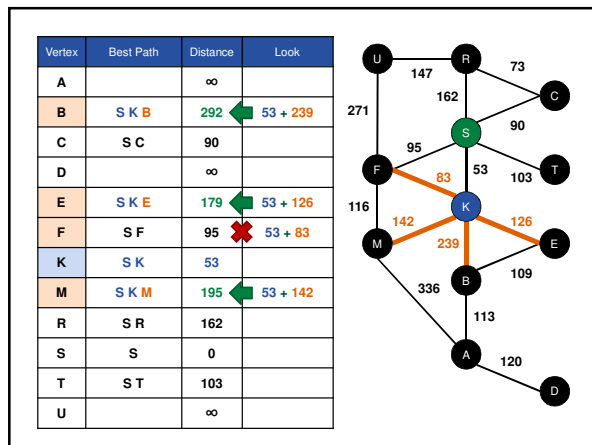
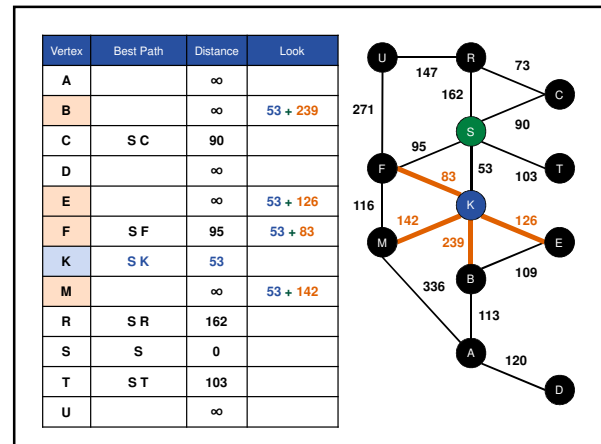
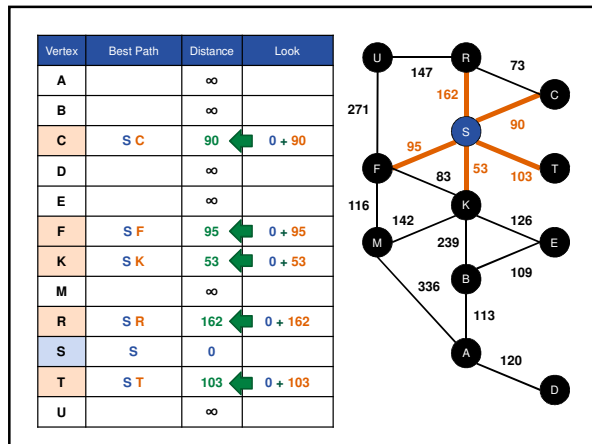
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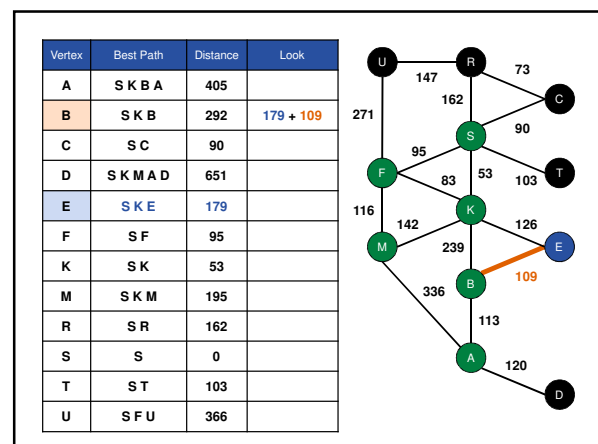
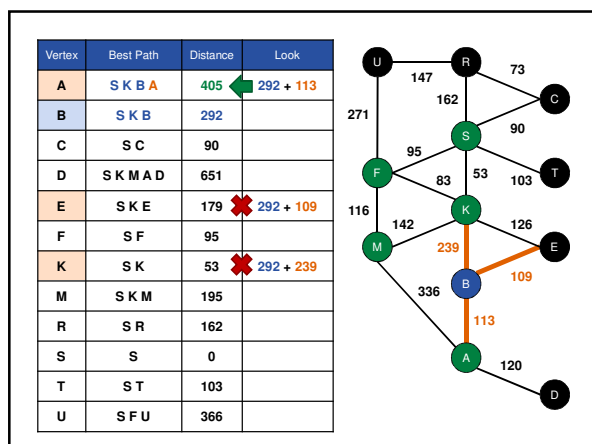
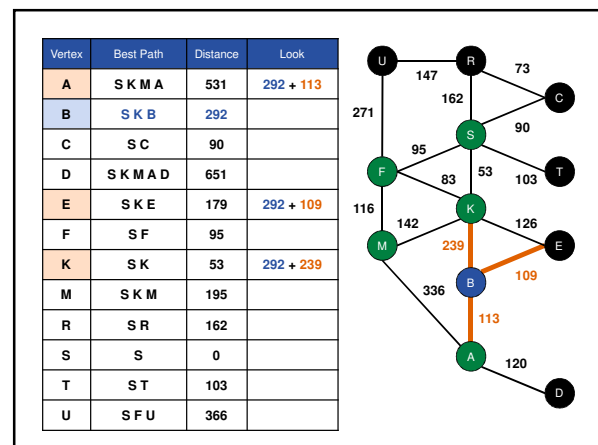
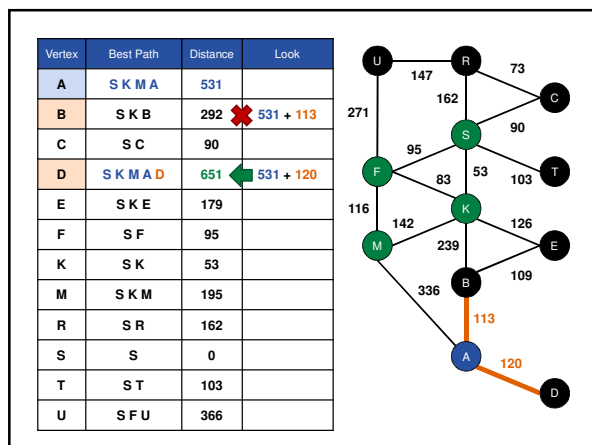
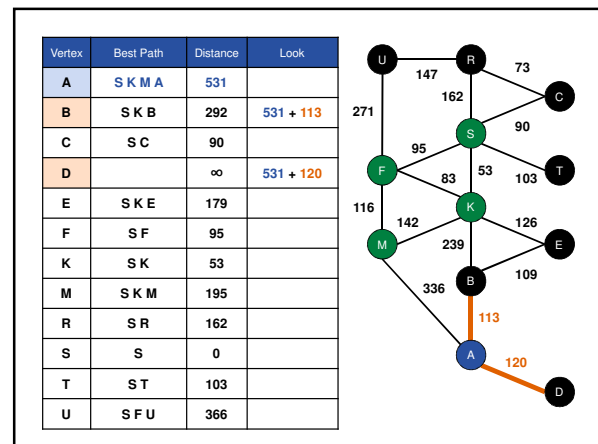
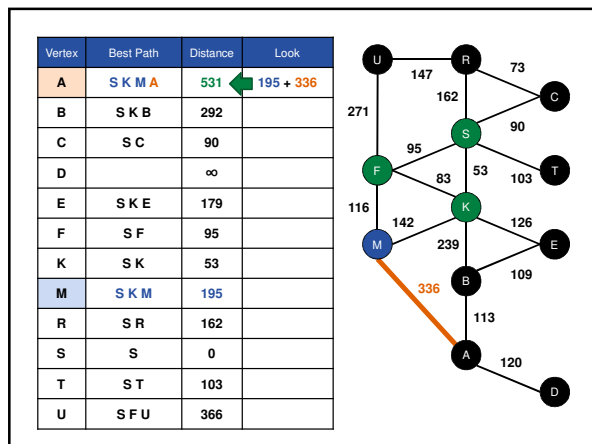
Vertex	Best Path	Distance	Look
A		∞	
B		∞	
C		∞	
D		∞	
E		∞	
F		∞	
K		∞	
M		∞	
R		∞	
S		0	
T		∞	
U		∞	

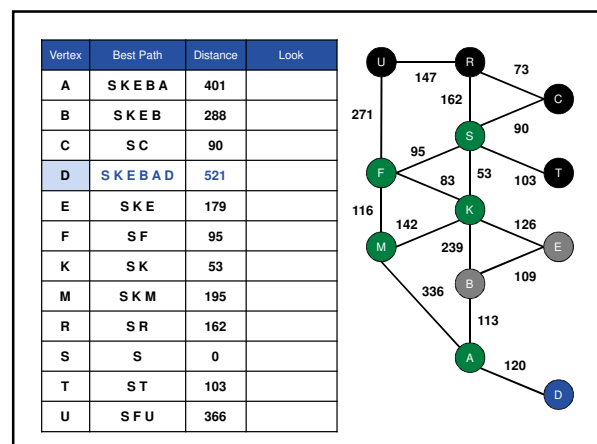
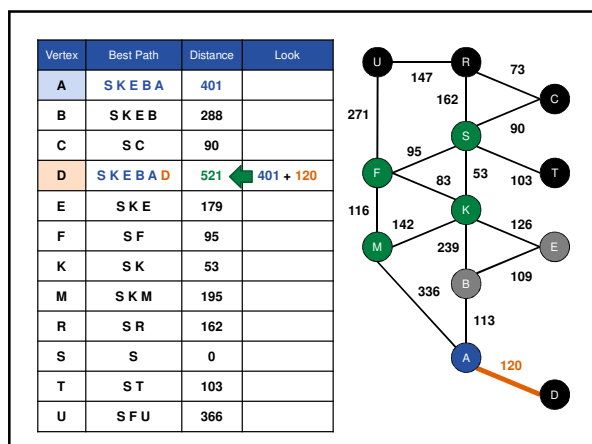
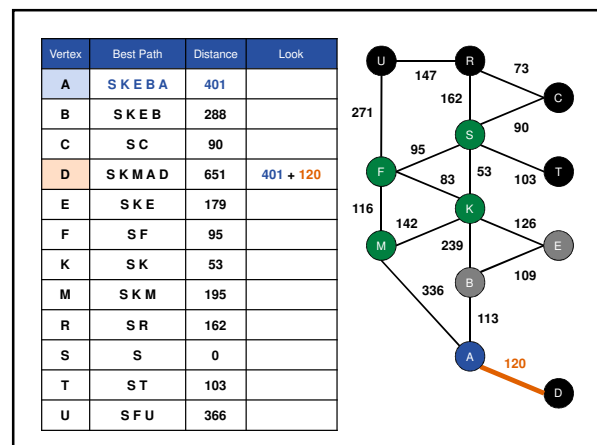
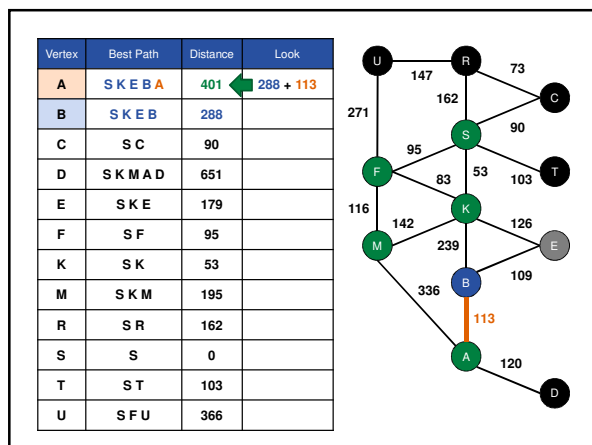
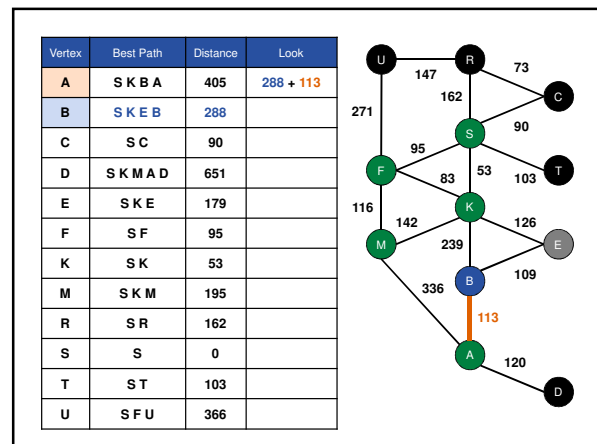
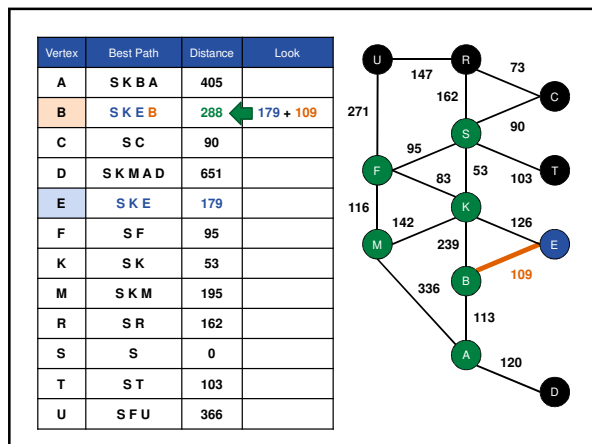


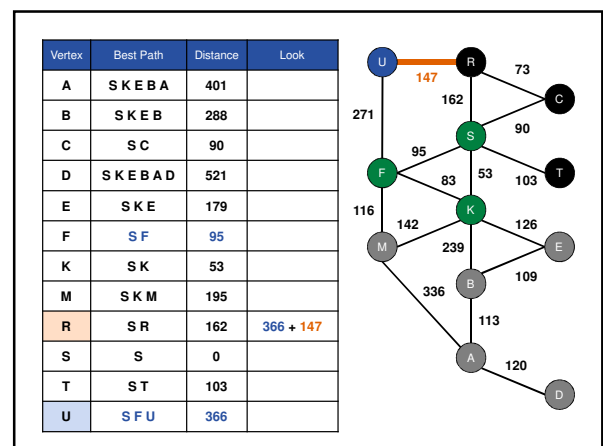
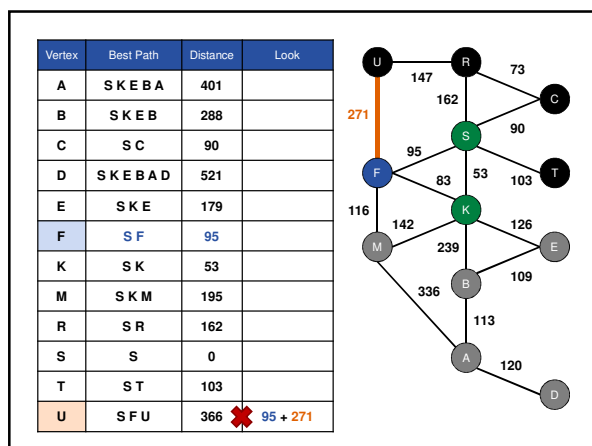
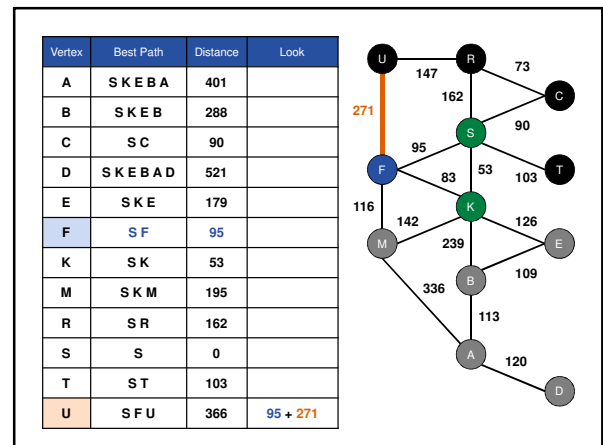
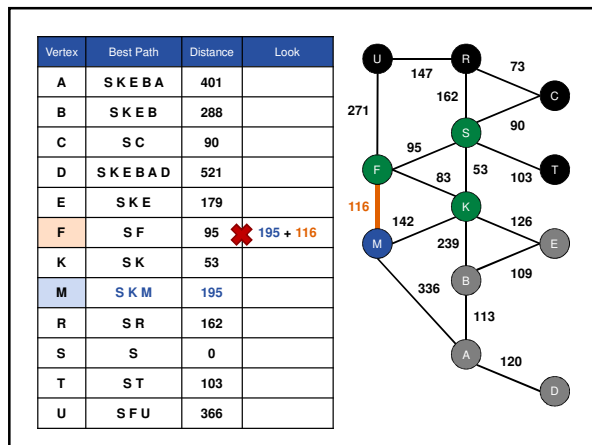
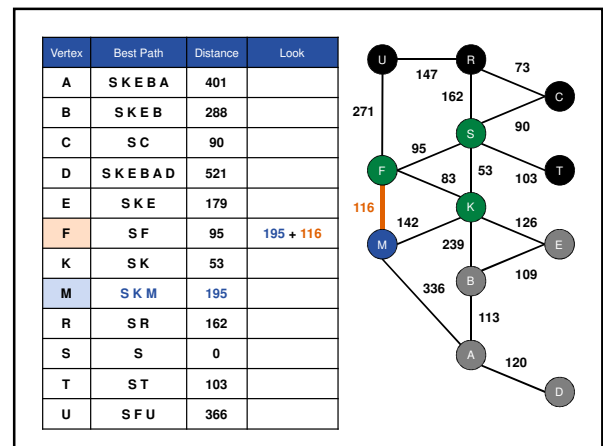
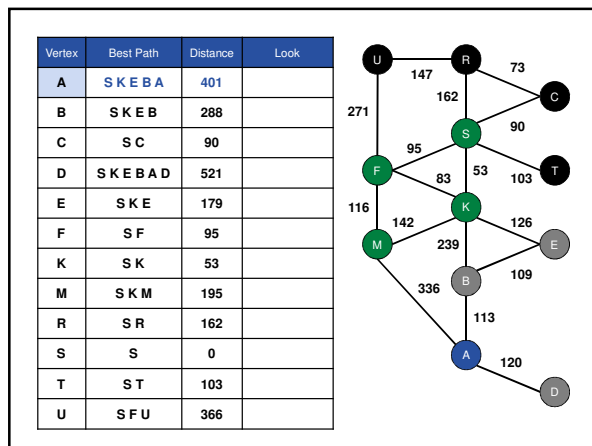
Vertex	Best Path	Distance	Look
A		∞	
B		∞	
C		∞	0 + 90
D		∞	
E		∞	
F		∞	0 + 95
K		∞	0 + 53
M		∞	
R		∞	0 + 162
S	S	0	
T		∞	0 + 103
U		∞	

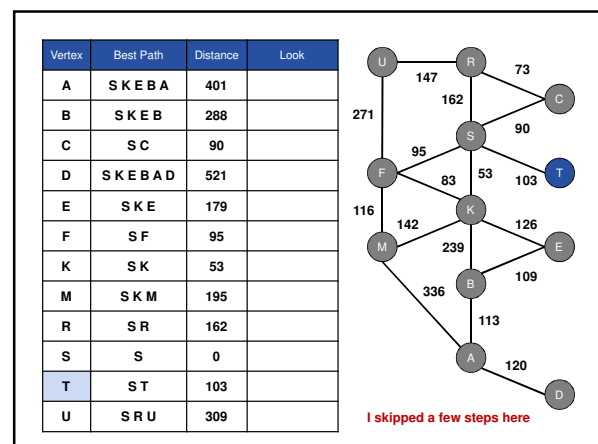
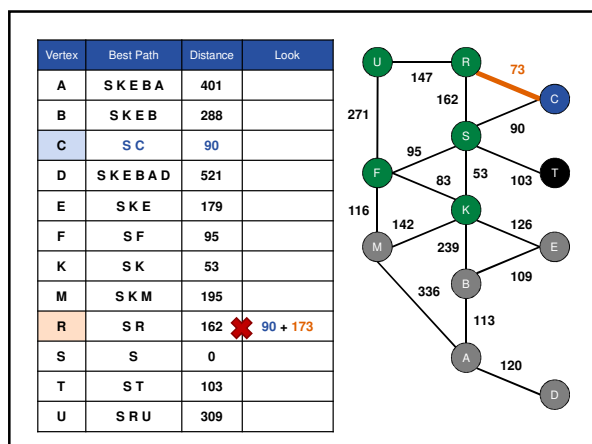
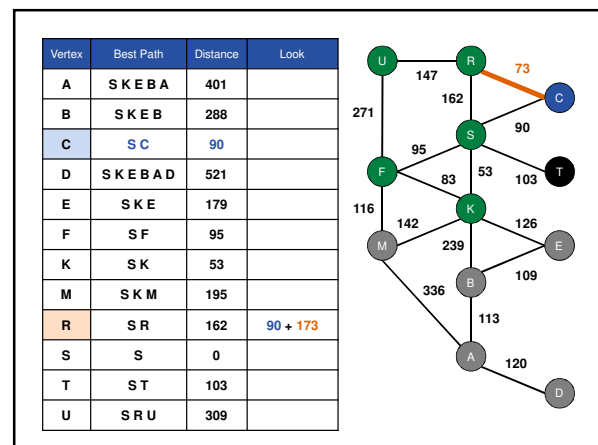
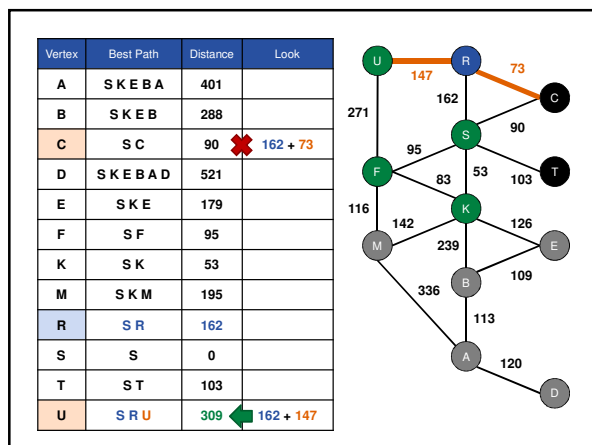
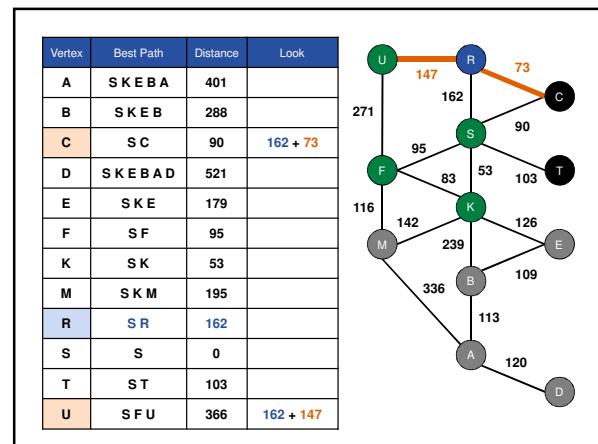
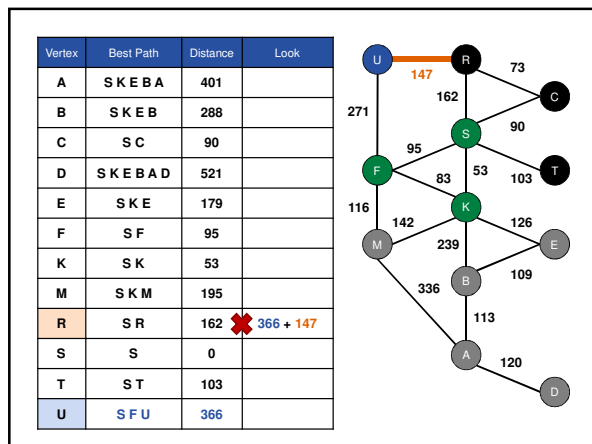


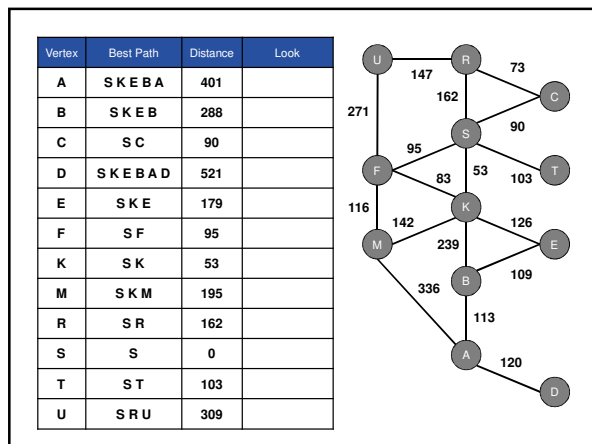








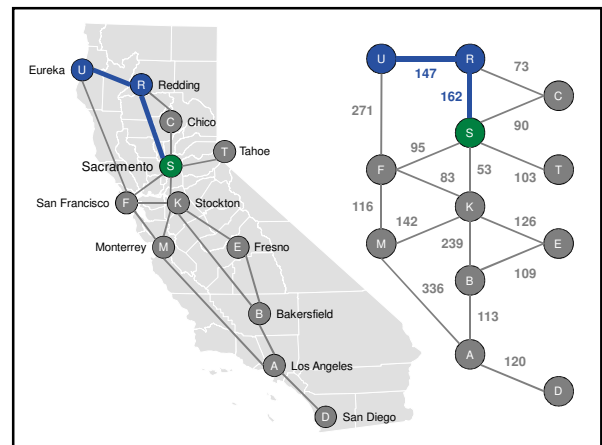
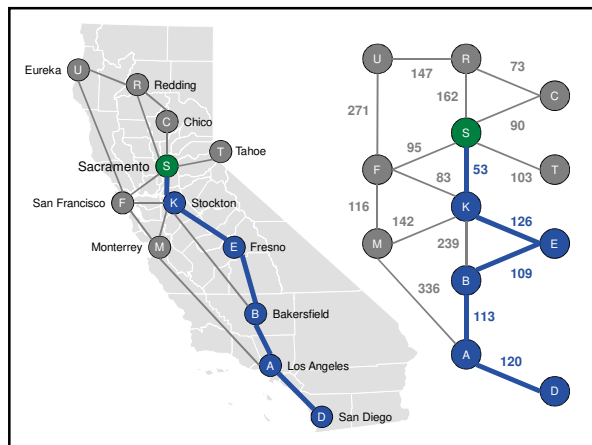




Best Path Complete

- It appears like the best route from Sacramento to San Diego is on the minimal spanning tree
- However, our best path to Eureka isn't (it skips Chico)

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California Result

- But, of course, if you going to go to San Diego, then you might want to take Hwy 5
- It goes around Fresno – which while a great cities – does not some traffic (*which will slow you down*)

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