Boolean Algebra

1. <u>Use Boolean algebra</u>. Simply the following. Label <u>all</u> the laws you apply.

    n ∧ ¬n ∨ (n ∧ (q ∨ ¬q))

false ∨ (n ∧ (q ∨ ¬q))	**Complement Law**
false ∨ (n ∧ true)	**Complement Law**
false ∨ n	**Identity Law**
n	**Identity Law**

2. <u>Use Boolean algebra</u> to simplify the following. Label <u>all</u> the laws you apply.

    (p ∨ ¬q) → (p ∧ q)

¬(p ∨ ¬q) ∨ (p ∧ q)	**Equivalent**
(¬p ∧ q) ∨ (p ∧ q)	**DeMorgan's Law**
q ∧ (¬p ∨ p)	**Distribution Law**
q ∧ true	**Complement Law**
q	**Identity**

3. <u>Use Boolean algebra</u>. Simply the following. Label <u>all</u> the laws you apply.

    ¬ a ∧ ¬ b ∨ c ∧ ¬ c ∨ ¬ a ∧ b

(¬ a ∧ ¬ b) ∨ (c ∧ ¬ c) ∨ (¬ a ∧ b)	
(¬ a ∧ ¬ b) ∨ false ∨ (¬ a ∧ b)	**Complement Law**
(¬ a ∧ ¬ b) ∨ (¬ a ∧ b)	**Identity Law**
¬ a ∧ (¬ b ∨ b)	**Distribution Law**
¬ a ∧ true	**Complement Law**
¬ a	**Identity Law**

Arguments

4. The following argument from the movie *Monty Python and the Holy Grail.*

   "If it's made of wood then it floats. If it's a duck then it floats. Therefore, a duck is wood."

   Convert the English sentence to an argument (using single letters). Show **why** it is valid or invalid. You must use a truth table for credit.

   W = Wood
   B = Floats (buoyant)
   D = Duck

   W → B
   D → B

   D → W

W	B	D	W → B	D → B	D → W
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	T	T	F
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

5. Prove (or disprove) the following argument.

"If they are a student, then they'll come to class or use Zoom. Students can't come to class. <u>Therefore</u>, if they are a student, then they will use Zoom."

Convert the English sentence to an argument (using single letters). Show **why** it is valid or invalid. Then use any method you like to prove or disprove it.

```
S = Student
Z = Zoom
C = Class

S → C V Z
¬ C
-----------
S → Z
```

S	C	Z	C∨Z	S→C∨Z	¬ C	S→Z
T	T	T	T	T	F	T
T	T	F	T	T	F	F
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	F	T	T	T

6. Prove (or disprove) the following argument by using a truth table.

   "If its toilet paper, then it's being hoarded. If it's being hoarded, then it must be guarded. <u>Therefore</u>, toilet paper must be guarded!"

   Convert the English sentence to an argument (using single letters). Show **why** it is valid or invalid.

   ```
   P = Toilet paper
   H = Hoarded
   G = Guarded

   P → H
   H → G
   ---------
   P → G
   ```

P	H	G	P → H	H → G	P → G
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Simple Proofs

7. Prove the following (show your work):

    *If **a** is odd, **b** is even, and **c** odd <u>then</u> **a** × **b** × **c** is even*

    ```
    a = 2i + 1
    b = 2j
    c = 2k + 1

    a × b × c

    = (2i + 1) (2j) (2k + 1)
    = (4ij + 2j) (2k + 1)
    = 8ijk + 4ij + 4jk + 2j
    = 2 (4ijk + 2ij + 2jk + j)
    ```

8. Prove the following (show your work):

    *If **a** is divisible by 2 and **b** is divisible by 6 <u>then</u> **a** × **b** is divisible by 4.*

    ```
    a = 2i
    b = 6j

    a × b

    = 2i × 6j
    = 12 ij
    = 4 (3ij)
    ```