Spring 2020 – Assignment #3 – Quantified Logic & Induction   Matthew Mendoza

Quantified Logic

1. Convert the following statement to a quantified expression.

All my cats are asleep.

Then, convert it into an equivalent statement (where exists and for-all are switched). Finally, convert the answer back to English

[Quantifier] [Subject] [Copula] [Predicate]

All my cats are asleep

→ All my cats are asleep *Let asleep be S()

→ $\forall_c S(c)$

1a) $\forall_c S(c)$ *Now convert to equivant statement where Exist and For-All are switched

→ $\forall_c S(c)$

→ $\sim\exists_c \sim S(c)$

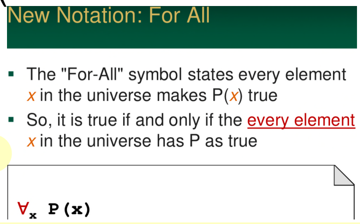
1b) The logical equivant of $\forall_c S(c)$ is $\sim\exists_c \sim S(c)$ ($\forall_c S(c) \equiv \sim\exists_c \sim S(c)$)

$\sim\exists_c \sim S(c)$ *Finally convert the answer back to English

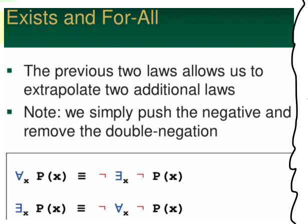
→ $\sim\exists_c \sim S(c)$

→ Not one of my cats are not asleep

1c) Not one of my cats are not asleep



Proposition
All S are P
Meaning in class notation
Every member of the S class is a member of the P class; that is, the S class is included in the P class



Statement form
All S are P
Symbolic Translation
$(x)(S_x \rightarrow P_x)$
Verbal Meaning
For any x, if x is an S, than x is a P

2. Convert the following statement into a quantified expression:

Everyone, who has seen Rick and Morty and has a sense of humor, likes Szechuan Sauce.

Everyone, who has seen Rick and Morty and has a sense of humor, likes Szechuan Sauce.

→ Everyone, who has seen Rick and Morty and has a sense of humor, likes Szechuan Sauce.

→ $\forall_x (R(x) \cap H(x)) \rightarrow S$

∴ $\forall_x (R(x) \cap H(x)) \rightarrow S$ * Everyone who has seen R and H it is implied they like S?

3. Simplify the following Quantified Statement. The result should have no negation symbols.

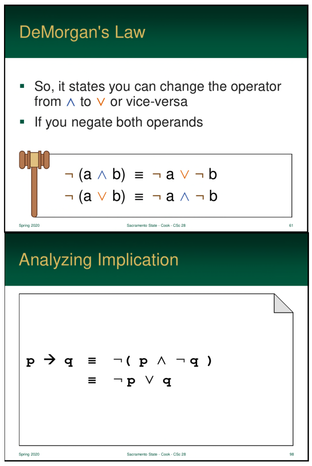
¬∀x ∃x (¬B(x)∧P(x))

→ $\sim\forall_x \exists_x (\sim B(x) \cap P(x))$

→ $\sim(\forall_x \exists_x (\sim B(x) \cap P(x)))$ * Demorgan's Law: $\sim(a \cup b) \equiv \sim a \cap \sim b$

→ $\sim(\forall_x \exists_x (B(x) \cup P(x)))$ * Implication Equivalence: $P \rightarrow q \equiv \sim(P \cap \sim q)$
$\equiv \sim(P \cup q)$

∴ $\forall_x \exists_x (B(x) \rightarrow P(x))$



Induction

4. Prove the following using induction (show your work - both steps):

If x ≥ 2 then 2 + 4 + 6 + ... + 2n = n(n+1)

If $x \geq 2$ then $2 + 4 + 6 + \ldots + 2n = n(n+1)$

→ $P(1) = 2 = 1(1+1)$ True

→ $P(n) = 2+4+6+\cdots+2n = n^2+n$

→ $P(n+1) = 2+4+6+\cdots+2n+2(n+1) = (n+1)(n+2)$

$= \underbrace{2+4+6+\cdots+2n}_{\text{subsitute for } n^2+n} + 2n+2 = n^2+3n+2$ * Assuming P(n) is true and so the equality is true aswell

$= n^2+n+2n+2 = n^2+3n+2 = P(n+1)$

∴ Therefore, when P(n) is true, then P(n+1) is also true.

Symbolic Translation
$P(n) \rightarrow P(n+1)$

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5. Prove the following using induction (show your work - both steps):

If x≥1 then (n)^2 + n is even

If $x \geq 1$ then $n^2 + n$ is even

$\rightarrow P(1) = 1 + 1 = 2$ is even

$\rightarrow P(n) = n^2 + n$

$\rightarrow P(n+1) = (n+1)^2 + (n+1)$

$(n+1)(n+1) + (n+1)$

$(n^2 + n + n + 1) + (n+1)$

$(n^2 + 2n + 1) + (n+1)$

$n^2 + 2n + 1 + n + 1$

$(n^2 + n) + 2(n+1)$

Divisible by two, so it must be even

Assuming that P(n) is true and so it must be even

Therefore $P(n) \rightarrow P(n+1)$