

Spring 2020 - Assignment #3 - Quantified Logic & Induction Matthew Mendoza

Quantified Logic

1. Convert the following statement to a quantified expression.

All my cats are asleep.

Then, convert it into an equivalent statement (where exists and for-all are switched). Finally, convert the answer back to English

New Notation: For All

- The "For-All" symbol states every element x in the universe makes $P(x)$ true
- So, it is true if and only if the **every element** x in the universe has P as true

$$\forall x P(x)$$

Proposition

All S are P

Meaning in class notation

Every member of the S class is a member of the P class; that is, the S class is included in the P class

[Quantifier] [Subject] [Copula] [Predicate]

All my cats are asleep

→ All my cats are asleep *Let asleep be $S(x)$

→ $\forall x S(x)$

1a) $\forall x S(x)$ * Now convert to equivalent statement where Exist and For-All are switched

→ $\forall x S(x)$

→ $\neg \exists x \neg S(x)$

1b) The logical equivalent of $\forall x S(x)$ is $\neg \exists x \neg S(x)$ ($\forall x S(x) \equiv \neg \exists x \neg S(x)$)

→ $\neg \exists x \neg S(x)$ * Finally convert the answer back to English

→ $\neg \exists x \neg S(x)$

→ Not one of my cats are not asleep

1c) Not one of my cats are not asleep

Exists and For-All

- The previous two laws allows us to extrapolate two additional laws
- Note: we simply push the negative and remove the double-negation

$$\forall x P(x) \equiv \neg \exists x \neg P(x)$$

$$\exists x P(x) \equiv \neg \forall x \neg P(x)$$

Statement form

All S are P

Symbolic Translation

$$(x)(Sx \rightarrow Px)$$

Verbal Meaning

For any x , if x is an S, then x is a P

2. Convert the following statement into a quantified expression:

Everyone, who has seen Rick and Morty and has a sense of humor, likes Szechuan Sauce.

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→ Everyone, who has seen Rick and Morty and has a sense of humor, likes Szechuan Sauce.

→ $\forall x (R(x) \cap H(x)) \rightarrow S$

∴ $\forall x (R(x) \cap H(x)) \rightarrow S$ * Everyone who has seen R and H it is implied they like S?

3. Simplify the following Quantified Statement. The result should have no negation symbols.

$$\neg \forall x \neg \exists x (\neg B(x) \wedge P(x))$$

→ $\neg \forall x \neg \exists x (\neg B(x) \wedge P(x))$

→ $\neg (\forall x \neg \exists x (\neg B(x) \wedge P(x)))$ * Demorgan's Law: $\neg (a \vee b) \equiv \neg a \wedge \neg b$

→ $\neg (\forall x \neg \exists x (B(x) \vee P(x)))$ * Implication Equivalence: $P \rightarrow Q \equiv \neg (P \wedge \neg Q) \equiv \neg (P \vee Q)$

∴ $\forall x \exists x (B(x) \rightarrow P(x))$

DeMorgan's Law

- So, it states you can change the operator from \wedge to \vee or vice-versa
- If you negate both operands

$$\neg (a \wedge b) \equiv \neg a \vee \neg b$$

$$\neg (a \vee b) \equiv \neg a \wedge \neg b$$

Analyzing Implication

$$P \rightarrow Q \equiv \neg (P \wedge \neg Q)$$

$$\equiv \neg P \vee Q$$

Induction

4. Prove the following using induction (show your work - both steps):

If $x \geq 2$ then $2 + 4 + 6 + \dots + 2n = n(n+1)$

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→ $P(2) = 2 = 1(1+1)$ True

→ $P(n) = 2 + 4 + 6 + \dots + 2n = n^2 + n$

→ $P(n+1) = 2 + 4 + 6 + \dots + 2n + 2(n+1) = (n+1)(n+2)$

$= 2 + 4 + 6 + \dots + 2n + 2n + 2 = n^2 + 3n + 2$ * Assuming $P(n)$ is true and so the equality is true as well

$= n^2 + n + 2n + 2 = n^2 + 3n + 2 = P(n+1)$

∴ Therefore, when $P(n)$ is true, then $P(n+1)$ is also true.

Symbolic Translation

$$P(n) \rightarrow P(n+1)$$

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5. Prove the following using induction (show your work - both steps):

If $x \geq 1$ then $n^2 + n$ is even

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$$\rightarrow P(1) = 1 + 1 = 2 \text{ is even}$$

$$\rightarrow P(n) = n^2 + n$$

$$\rightarrow P(n+1) = (n+1)^2 + (n+1)$$

$$(n+1)(n+1) + (n+1)$$

$$(n^2 + n + n + 1) + (n+1)$$

$$(n^2 + 2n + 1) + (n+1)$$

$$n^2 + 2n + 1 + n + 1$$

$$(n^2 + n) + 2(n+1)$$

Divisible by two, so it must be even

Assuming that $P(n)$ is true and so it must be even

Therefore $P(n) \rightarrow P(n+1)$