Arguments

Part 7

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Arguments

Proving a Point

2

Arguments

- A combination of true statements can be used to claim another as true
- An *argument* is a collection of statements (called *premises*), which, when all are true, imply a consequence

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Example Argument

```
raining → wet outside
not wet outside
_____________________
?
```

Our conclusion goes here

4

Example Argument 2

```
raining → wet outside
not raining
_____________________
?
```

What conclusion goes here?
Does it work?

5

Example Argument 3

```
x is duck or x is swan
x isn't a swan
_____________________
?
```

Obvious! But why?

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When an Argument is Valid

- When all the premises are true then the consequence must be true
- If all the premises are true, but the conclusion can be false, the argument is disproven

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When an Argument is Valid

- However, if any premise is false, then the argument is not disproven – *it is still valid*
- We can often prove arguments by building truth tables

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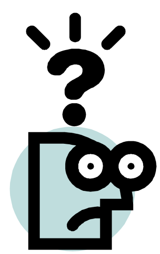
Argument Notation

- Arguments can be written out several ways
- The most common approach is to write each premise on a different line
- The consequence is written below the premises separated with horizontal line



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Common Notation

$p \to q$
$q \to r$ — Premises

$p \to r$ — Consequence

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Another Argument Notation

- Arguments can be written on a single line
- Premises are separated with commas
- Consequence can use the symbol ⊦ or ∴

"Therefore"

$p \to q,\ q \to r \ \vdash\ p \to r$

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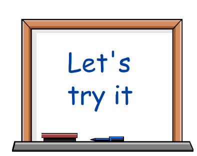
Let's Try This

$p \to q$
p

---

q — Valid!



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Arguments and Implication

- Arguments are actually implications with each premise connected with $\wedge$
- So, if you have premises A, premise B, and conclusion C, then it has the following form

$$A \wedge B \rightarrow C$$

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Valid Arguments

Proving a Point

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Valid Arguments

- *Rules of Inference* are valid arguments that are commonly used in proofs
- Most of these are obvious to you… it is natural logical thought

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Rules of Inference

- *Modus Pones*
  (aka Law of Detachment)
- *Modus Tollens*
- *Disjunctive Syllogism*
- *Hypothetical Syllogism*

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Modus Ponens

- *Modus Ponens* is the most basic Rule of Inference
- Based on the logic that if:
  - an implication is true
  - implication's hypothesis is true
  - then the implication's conclusion must be true

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Modus Ponens Example

If it is a fish, then it lives in water.

It is a fish.

Therefore, it lives in water!

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Modus Ponens

Rules of Inference

p → q
p

---

q

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Modus Ponens

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

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Modus Tollens

- *Modus Tollens* is closely relate to modus ponens
- Based on the logic that if:
  - an implication is true
  - implication's conclusion is false
  - then the implication's hypothesis must be false

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Modus Tollens Example

If it is a fish, then it lives in water.

It doesn't live in water.

Therefore, it is not a fish!

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Modus Tollens

Rules of Inference

p → q
¬ q

¬ p

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Modus Tollens

p	q	p → q	¬ p	¬ q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

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Disjunctive Syllogism

- *Disjunctive Syllogism* is based on the $\vee$ operator that
- Based on the logic that if:
  - an or-statement is true
  - one of the operands is false
  - then, the other operand must be true

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Disjunctive Syllogism Example

It breathes water or air.

It doesn't breath water.

Therefore, it breathes air.

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Disjunctive Syllogism

Rules of Inference

$p \vee q$
$\neg p$

---

q

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Hypothetical Syllogism

- *Hypothetical Syllogism* is based on an implication chain
- Gives a logical "chain" of events
- So, if a → b and b → c then a → c

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Hypothetical Syllogism Example

If is a trout, then it is a fish

If it is a fish, then it lives in water.

Therefore, a trout lives in water!

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Hypothetical Syllogism

Rules of Inference

$p \rightarrow q$
$q \rightarrow r$

$p \rightarrow r$

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Let's Apply The Logic

- Let's these rules on the argument below
- We can use either a truth table or logical deduction

```
If I study then I will an A
If I don't watch Netflix then I will study
I didn't get an A
________________________
I watched Netflix
```


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Simply Logic: letters

- First, let's simply the structure of the argument so we can see the logic
- We will assign each part a letter

```
s = studied
a = got an A
n = watched Netflix
```


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Simply Logic: New Form

1. $s \rightarrow a$
2. $\neg n \rightarrow s$
3. $\neg a$

---

n

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Modus Tollens – study, no A

1. $s \rightarrow a$
2. $\neg n \rightarrow s$
3. $\neg a$

---

n

1 and 3: Modus Tollens

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Modus Tollens – study, no A

1. $\neg s$
2. $\neg n \rightarrow s$
3. $\neg a$

---

n

1 and 3: Modus Tollens

"Did not study"

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Modus Tollens – study, Netflix

1. $\neg s$
2. $\neg n \rightarrow s$
3. $\neg a$

---

n

1 and 2: Modus Tollens

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Modus Tollens – study, Netflix

1. ¬ s
2. ¬ ¬ n
3. ¬ a

---

n

1 and 2: Modus Tollens

"Did not *not* watch Netflix"

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Double Negation

1. ¬ s
2. ¬ ¬ n
3. ¬ a

---

n

Double negation

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Double Negation

1. ¬ s
2. n
3. ¬ a

---

n

2: Double Negation

"Watched Netflix"

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Logical Fallacies

"This is most illogical" – Mr. Spock

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Logical Fallacies

- There are a number of fallacious arguments that, while they might look logical, are wrong
- The following slides contain some of them
- For fun, apply them to current political discourse or *History Channel 2*

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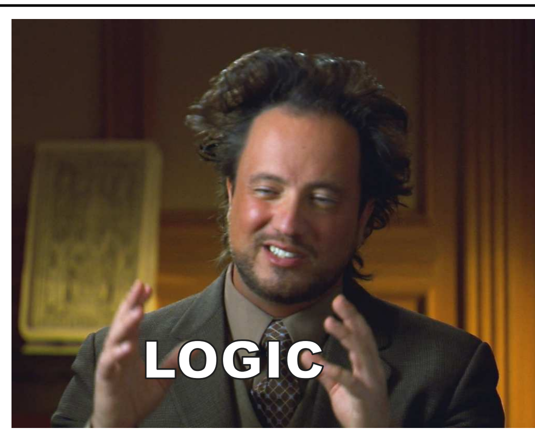

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Fallacy of the Converse

- *Fallacy of the Converse* is based on assumption that if the conclusion is true then the hypothesis is true
- Also called:
  - *affirming the consequent*
  - *converse error*

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Fallacy of the Converse Example

If it is a fish, then it lives in water.

It lives in water.

Therefore, it is a fish!

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Fallacy of the Converse

$p \rightarrow q$
q
___
p

p can still be false

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Fallacy of the Converse

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

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Fallacy of the Inverse

- *Fallacy of the Inverse* is based on assumption that if the hypothesis is false, then the conclusion is also false
- Also called:
  - denying the antecedent
  - inverse error

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Fallacy of the Inverse Example

If it is a cat, then it is furry.

It is not a cat.

Therefore, it is not furry!

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Fallacy of the Inverse

$p \rightarrow q$
$\neg p$

$\neg q$

q can be either true or false

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Fallacy of the Converse

p	q	p → q	¬ p	¬ q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

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Fallacy of Affirming a Disjunct

- *Fallacy of Affirming a Disjunct* is based on assumption that if there are two attributes and one is true, the other must be false
- Other names:
  - *fallacy of the alternative*
  - *false exclusionary disjunct*

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Affirming a Disjunct Example

Suspect is either a politician or a lawyer.

Suspect is a politician.

Therefore, the suspect isn't a lawyer.

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Fallacy of Affirming a Disjunct

$p \vee q$
p

$\neg q$

Just because p is true, doesn't mean q has to be false.

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Fallacy of the Converse

p	q	p ∨ q	¬ q
T	T	T	F
T	F	T	T
F	T	T	F
F	F	F	T

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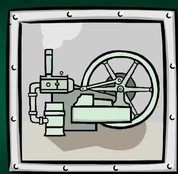
Boolean Algebra & Proofs



Proofs and Logic are one

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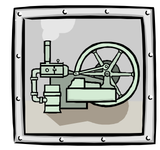
Boolean Algebra

- Remember Boolean algebra laws: Associative, Commutative, etc…
- These can be used to expand an expression… and then simplify it in a different form



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Example

if "a or b" is true and
"a and b" is false

then
a = not b

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Example (with a rewrite)

We can rewrite it as an argument:

$a \vee b = \mathtt{true}$
$a \wedge b = \mathtt{false}$
―――――――
$a = \neg b$

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The Strategy

- We could use a Truth Table to prove this
- Let's use Boolean Algebra to prove if this is correct

$a \vee b = \mathtt{true}$
$a \wedge b = \mathtt{false}$
―――――――
$a = \neg b$

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The Approach

- Start with a
- Try to change it into ¬ b
- Use the premises to replace values

$a \vee b = \mathtt{true}$
$a \wedge b = \mathtt{false}$
―――――――
$a = \neg b$

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$a \lor b = \text{true}$
$a \land b = \text{false}$

$a = a$	
$= a \land \text{true}$	Identity
$= a \land (b \lor \neg b)$	Complement
$= a \land b \lor a \land \neg b$	Distributive
$= \text{false} \lor a \land \neg b$	Premise
$= b \land \neg b \lor a \land \neg b$	Complement
$= \neg b \land (b \lor a)$	Distributive
$= \neg b \land \text{true}$	Premise
$= \neg b$	Identity

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