

California State University, Sacramento College of Engineering and Computer Science

Computer Science 28: Discrete Mathematics

Spring 2020 - Assignment #3 - Quantified Logic & Induction

Quantified Logic

1. Convert the following statement to a quantified expression.

All my cats are asleep.

Then, convert it into an equivalent statement (where exists and for-all are switched). Finally, convert the answer back to English

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Approach #1
S(x) = x is asleep
x = all my cats
\forall_x S(x)
                         "All my cats are asleep"
\neg \exists_x \neg S(x)
                         "There doesn't exist a cat that's awake"
Approach #2
S(x) = x is asleep
C(x) = x is my cat
x = all cats
\forall_x (C(x) \rightarrow S(x))
                                    "All cats that are mine are asleep"
\neg \exists_x \neg (C(x) \rightarrow S(x))
\neg \exists_x \neg (\neg C(x) \lor S(x))
\neg \exists_x (C(x) \land \neg S(x))
                                     "There doesn't exist a cat that is mine and is awake"
```

2. Convert the following statement into a quantified expression:

Everyone, who has seen Rick and Morty and has a sense of humor, likes Szechuan Sauce.

```
R(x) = x has seen Rick and Morty H(x) = x has a sense of humor S(x) = x likes Szechuan sauce \forall_x (R(x) \land H(x) \rightarrow S(x))
```

3. Simplify the following Quantified Statement. The result should have **no** negation symbols.

$$\neg \forall_{x} \exists_{x} (\neg B(x) \land P(x))$$

$$\neg \forall_{x} \exists_{x} (\neg B(x) \land P(x))$$

$$\exists_{x} \neg \exists_{x} (\neg B(x) \land P(x))$$

$$\exists_{x} \forall_{x} \neg (\neg B(x) \land P(x))$$

$$\exists_{x} \forall_{x} (B(x) \lor \neg P(x))$$

 $\exists_x \forall_x (P(x) \rightarrow B(x))$

Induction

4. Prove the following using <u>induction</u> (show your work - both steps):

```
If n \ge 1 then 2 + 4 + 6 + ... + 2n = n(n+1)
```

```
P(n): 2 + 4 + 6 + ... + 2n = n (n + 1)
P(n + 1): 2 + 4 + 6 + ... + 2n + 2(n + 1) = (n + 1) (n + 2)
\frac{\text{Step 1: Basis}}{P(1): 2 = 1 (1 + 1)} \quad \text{Yes, these equal. Basis is correct!}
\frac{\text{Step 2: Induction}}{2 + 4 + 6 + ... + 2n + 2(n + 1)} = (n + 1) (n + 2)
\frac{n (n + 1) + 2(n + 1)}{n^2 + n + 2n + 2} = \frac{n + 1}{n + 1} \frac{n + 2}{n + 2}
\frac{n^2 + n + 2n + 2}{n^2 + 3n + 2} = \frac{n + 1}{n + 1} \frac{n + 2}{n + 2}
\frac{n^2 + n + 2n + 2}{n^2 + 3n + 2} = \frac{n + 1}{n + 1} \frac{n + 2}{n + 2}
```

5. Prove the following using induction (show your work - both steps):

```
If n \ge 1 then n^2 + n is even
```

```
P(n) : n^2 + n is even

P(n + 1) : (n + 1)^2 + (n + 1) is even

\frac{\text{Step 1: Basis}}{P(1) : 1^2 + 1 = 2}
\frac{\text{Step 2: Induction}}{(n + 1)^2 + (n + 1)}
n^2 + 2n + 1 + n + 1
n^2 + 2n + 2 + n
n^2 + n + 2n + 2
n^2 + n + 2(n + 1)
Simply
Rearrange
```