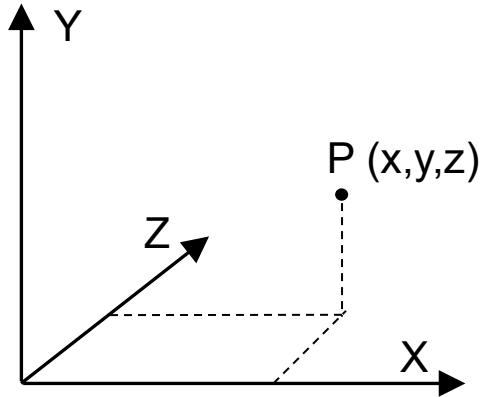


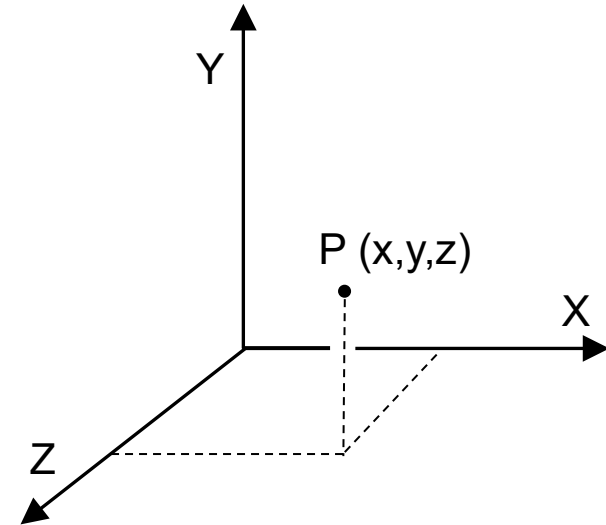


03 - Fundamentals of 3D Systems

3D Coordinate Systems



Left-handed Coordinate System

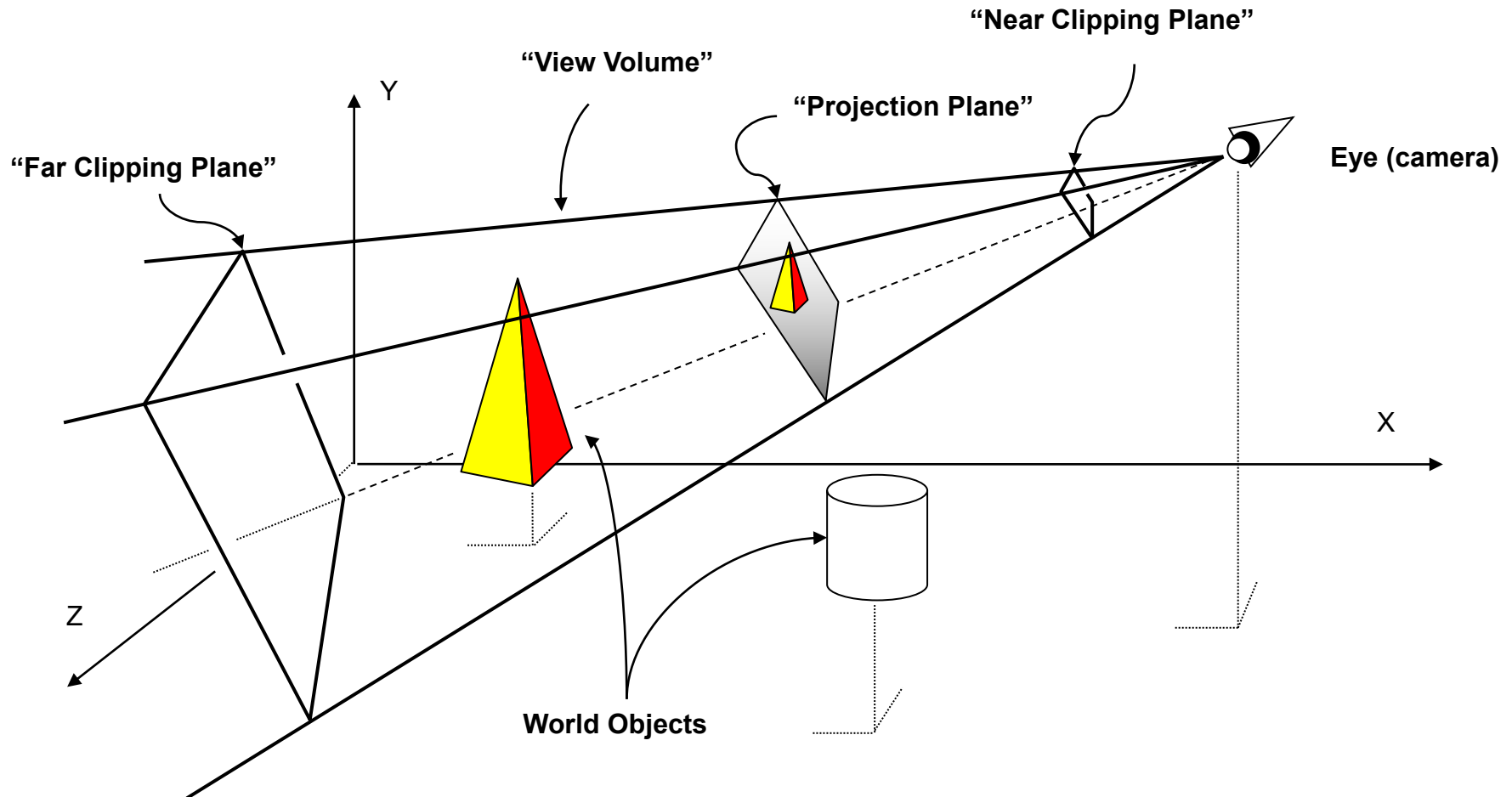


Right-handed Coordinate System

Points can be represented in *homogeneous form*:

$$\mathbf{P} = [\mathbf{x} \ \mathbf{y} \ \mathbf{z} \ 1]$$

“Synthetic Camera” Paradigm

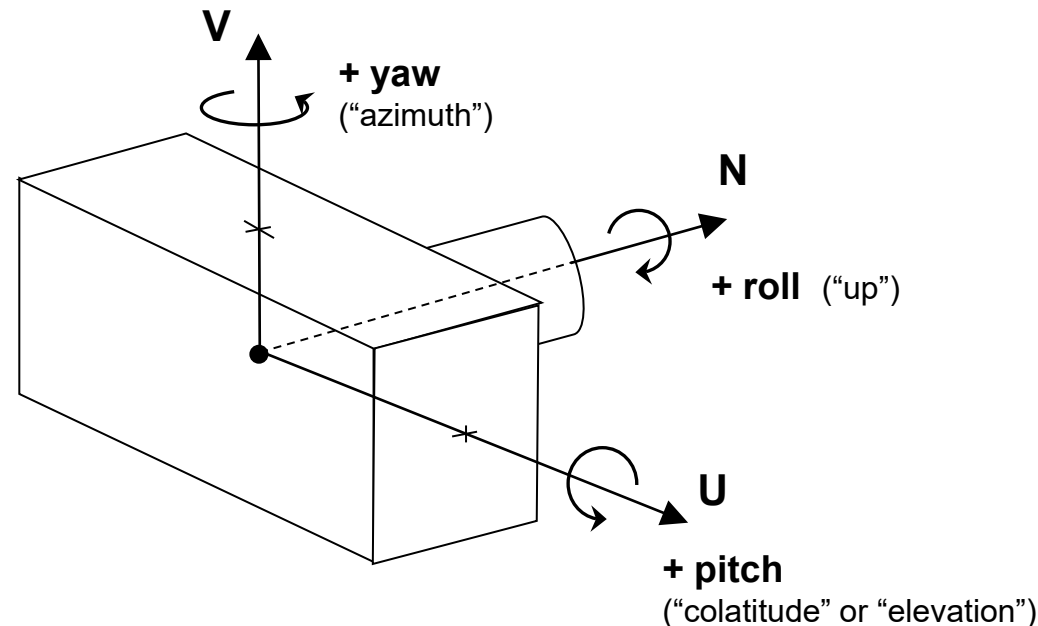


The “UVN” Camera

Two important camera attributes:

- *Location*
- *Orientation of UVN axes*

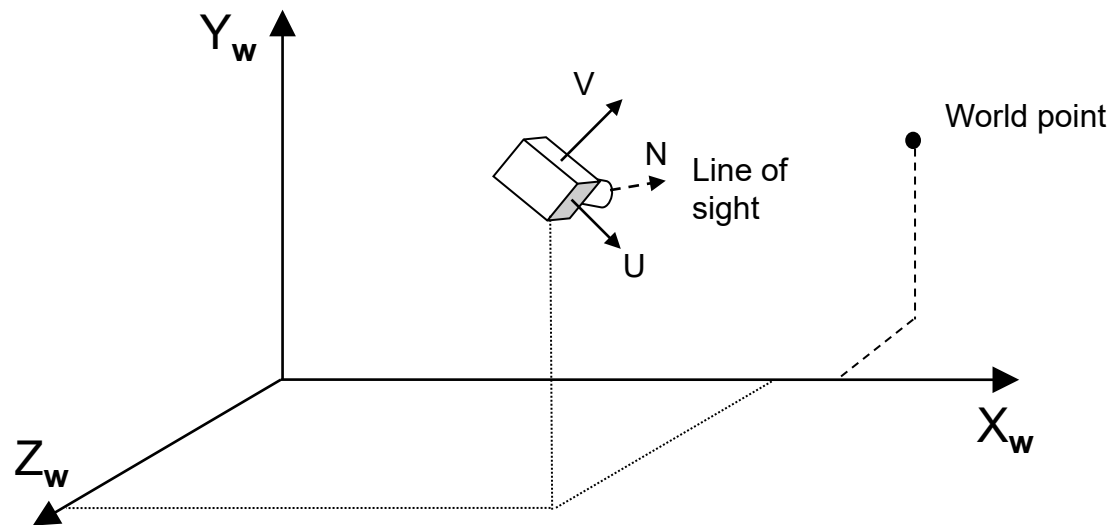
Note the UVN coordinate system is *left-handed*



Generalized Camera Control

Player controls position & orientation

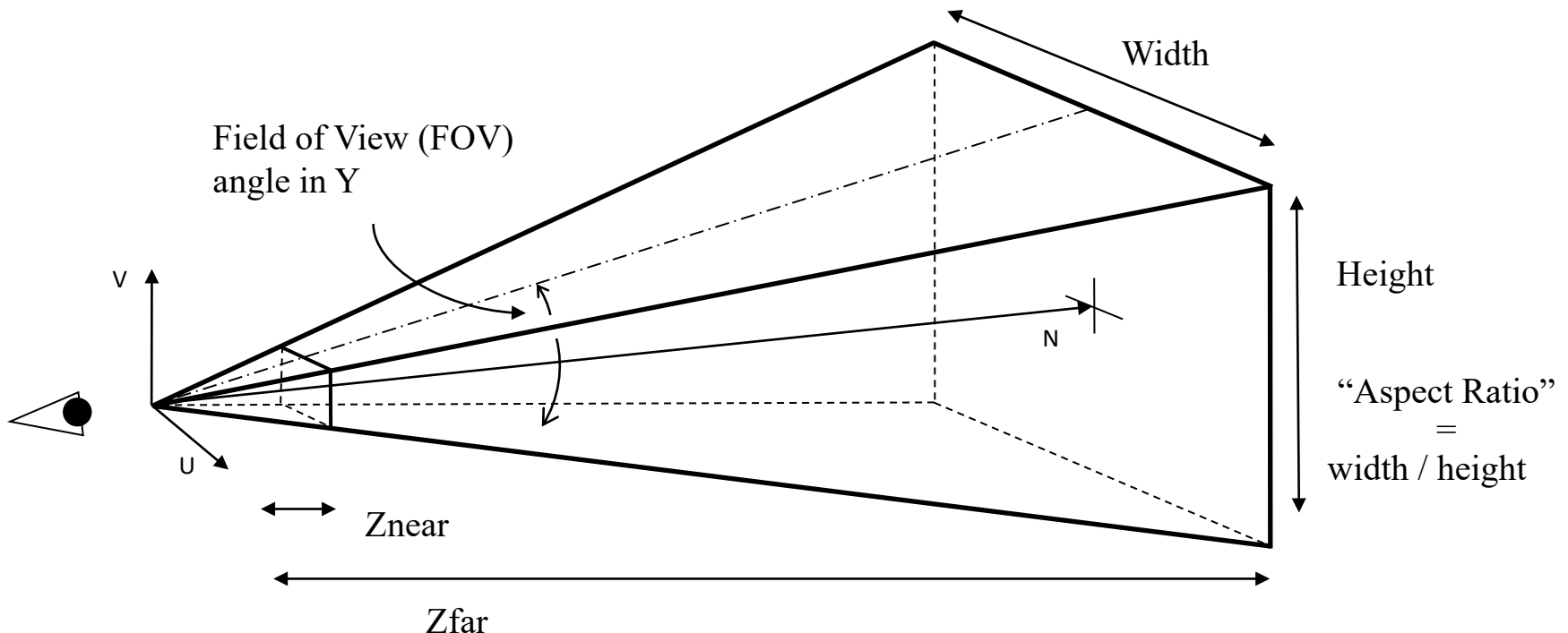
- “World” points must be converted to “camera” points
- Game engine should handle this (it’s game-independent)



Additional Camera Settings

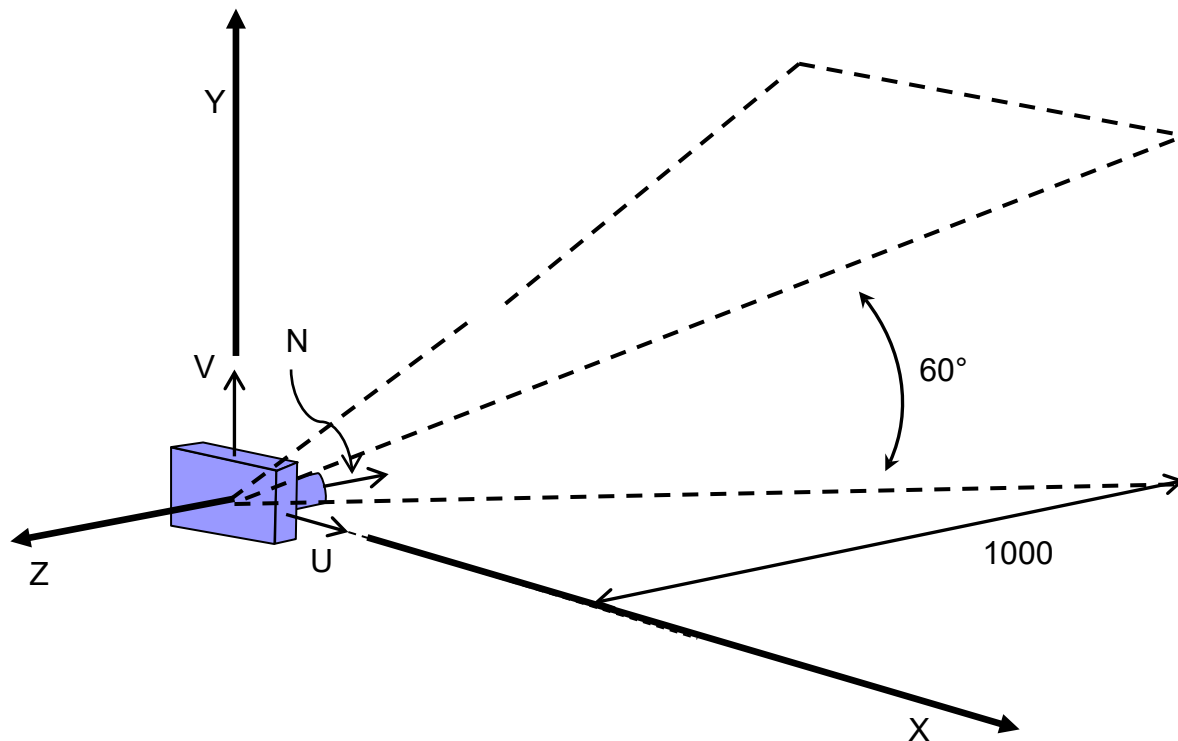
FOVY, Aspect, Near & Far (Clipping)

- Controls “projection” onto 2D plane (& screen)
- Again, game engine should handle details

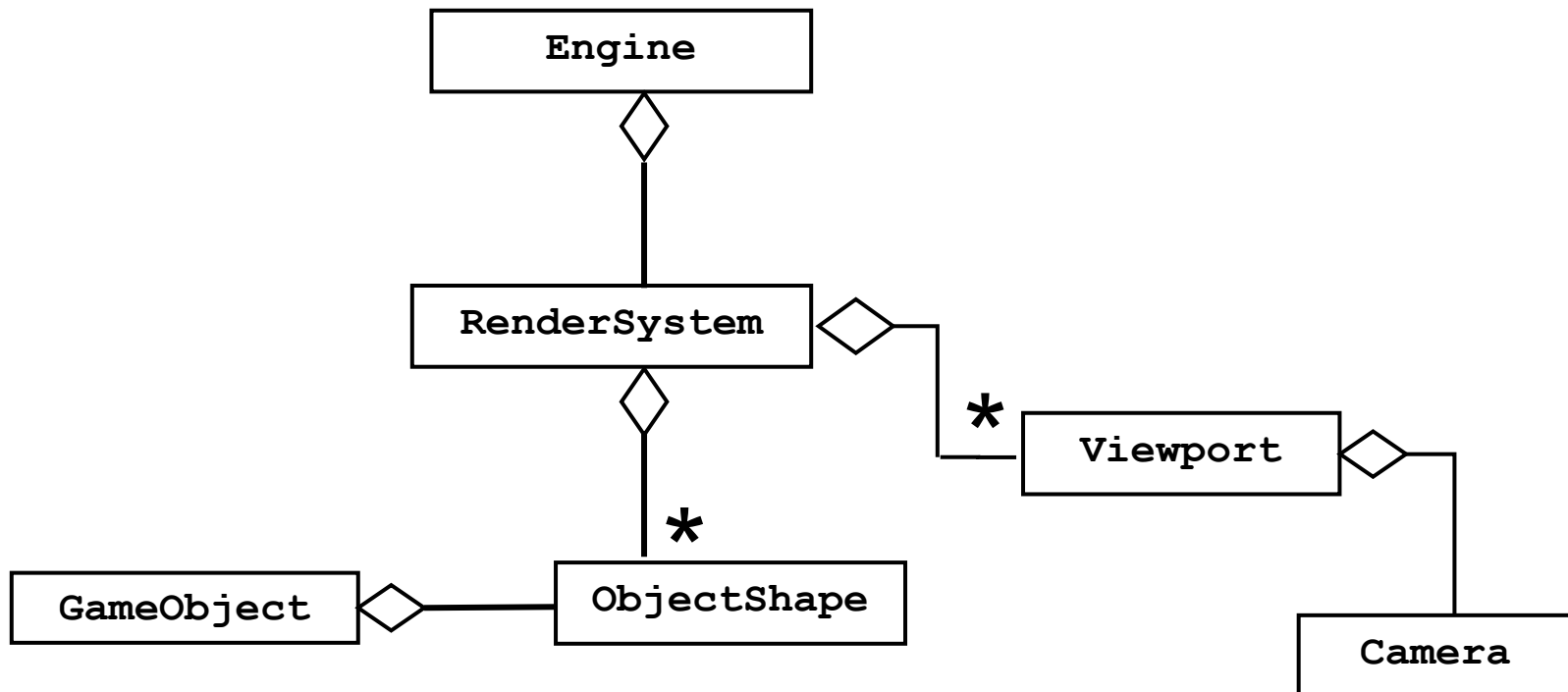


Default Camera Values

- $\text{Loc} = [0 \ 0 \ 0]$, looking down *negative* Z
- $V = Y$, $U = X$, $N = -Z$
- $\text{fovY} = 60^\circ$, $\text{aspect}=1$, $\text{near}=0.01$, $\text{far}=1000$



TAGE Camera/Display Class Structure



TAGE's Camera class

```
public class Camera
{
    private Vector3f u, v, n, location;

    //modify the camera's location/orientation (note that it is the user's
    //responsibility to insure the camera axes remain mutually perpendicular)
    public void setLocation(Vector3f l) {...}
    public void setU(Vector3f newU) {...}
    public void setV(Vector3f newV) {...}
    public void setN(Vector3f newN) {...}

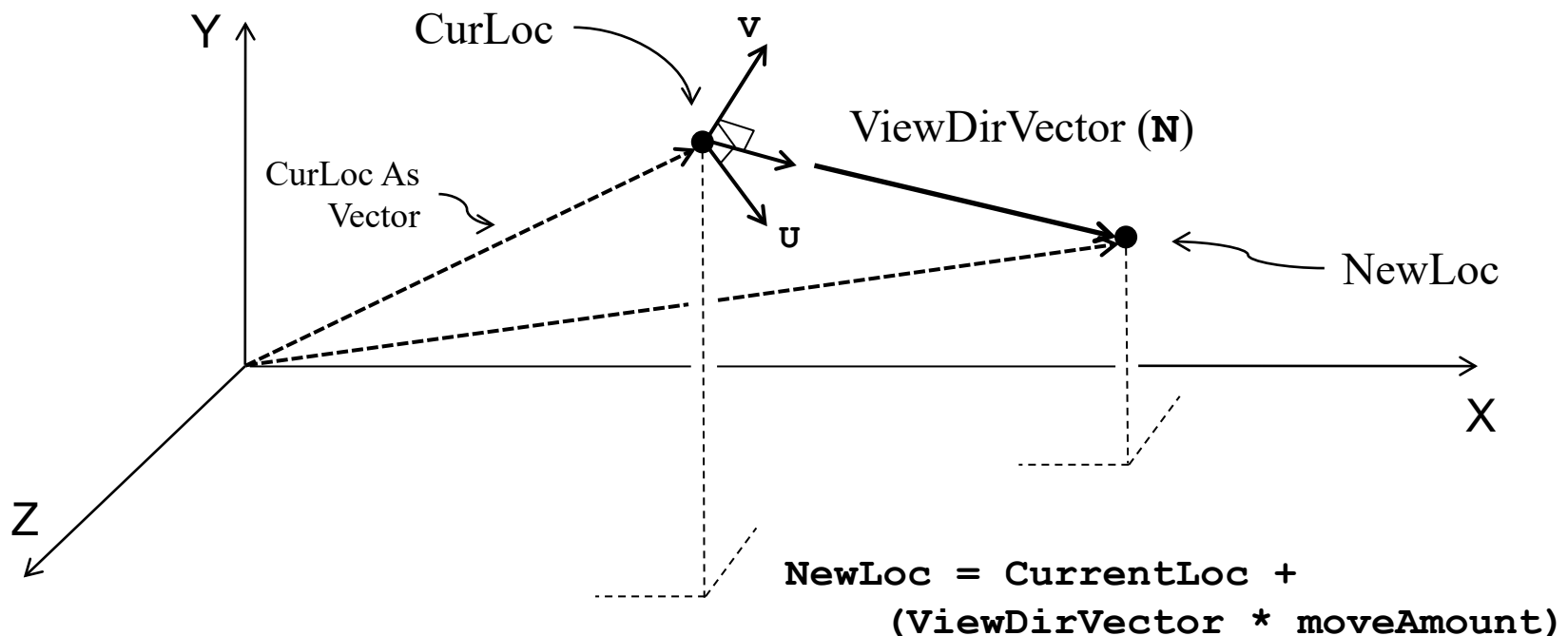
    public Vector3f getLocation() {...}
    public Vector3f getU() {...}
    public Vector3f getV() {...}
    public Vector3f getN() {...}

    public void lookAt(Vector3f target) {...}
    public void lookAt(GameObject go) {...}
    public void lookAt(float x, float y, float z) {...}

    protected Matrix4f getViewMatrix() {...}
}
```

Camera Manipulation

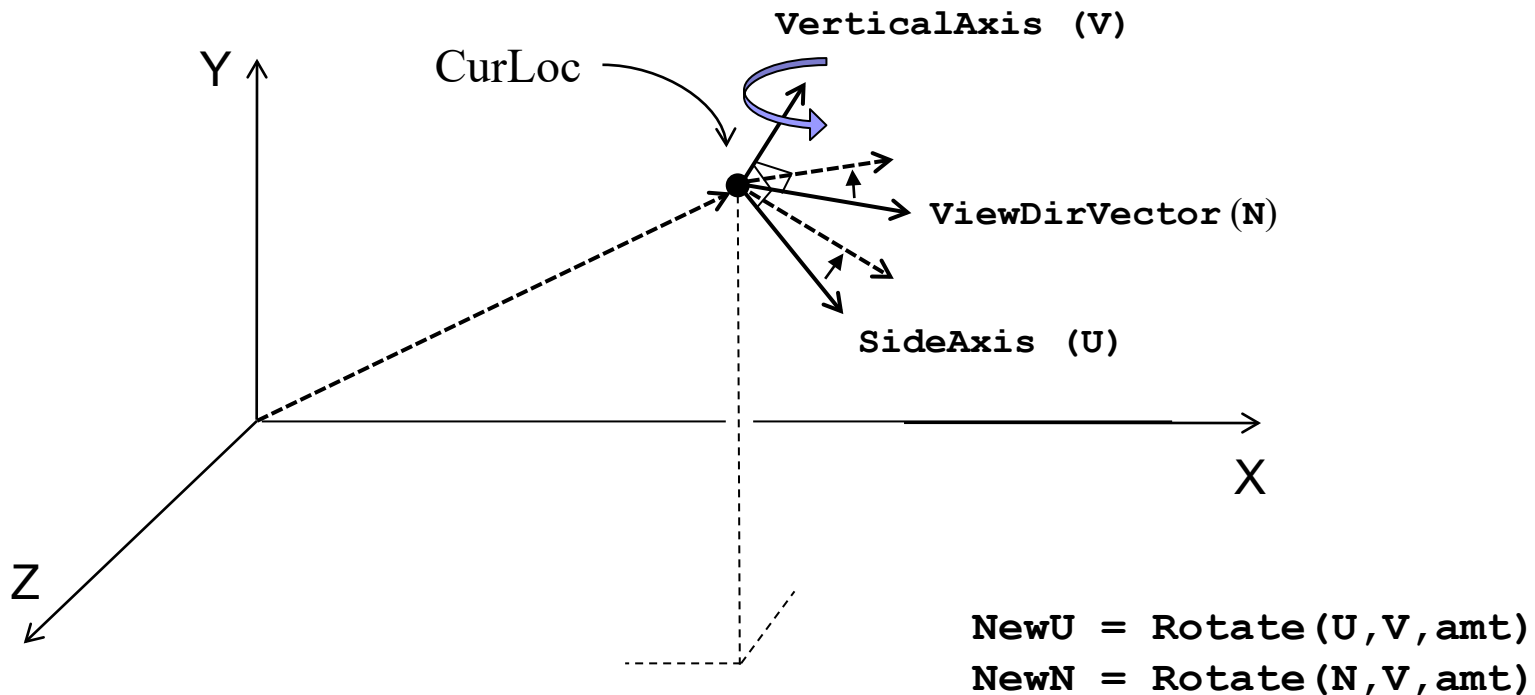
example: “Move Forward” ==
change location along the view direction



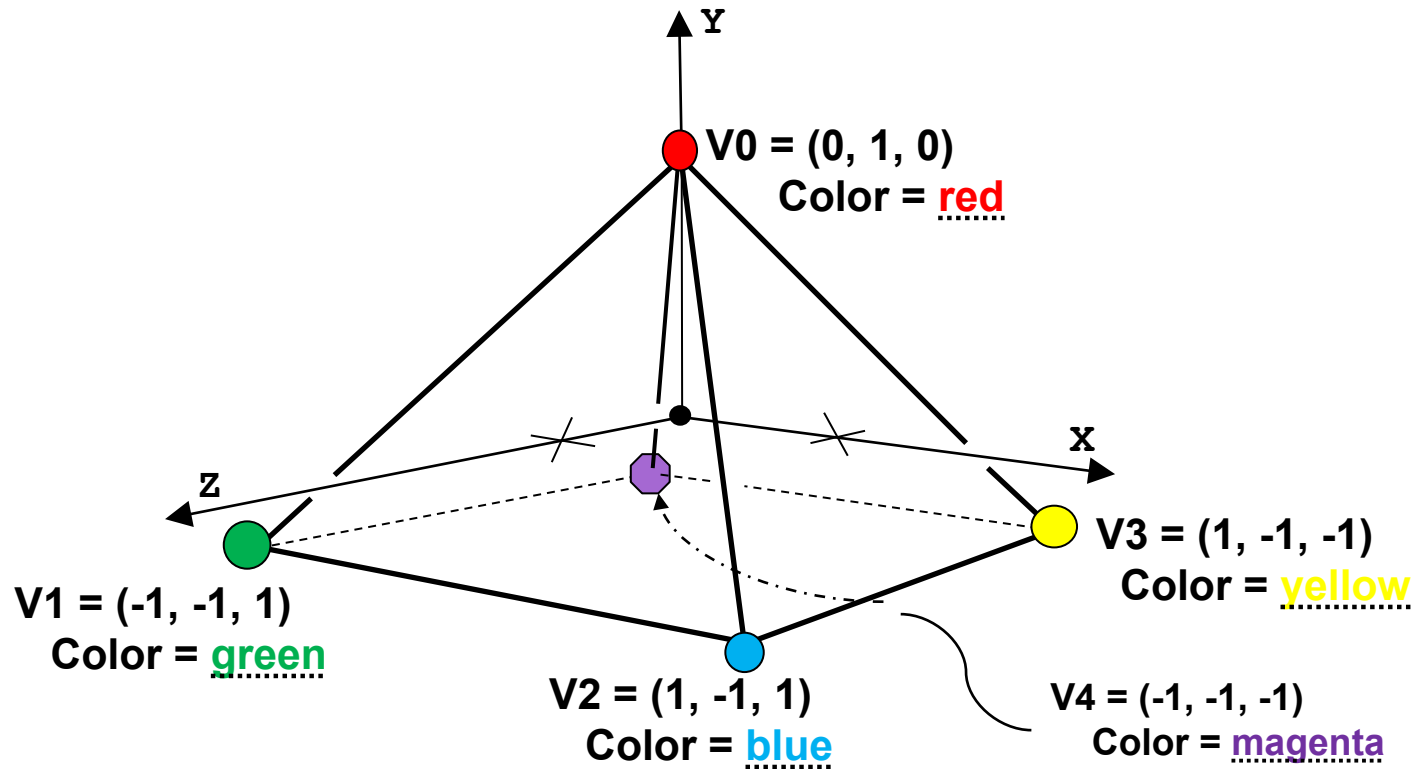
Camera Manipulation

example: "RotateLeft" == (yaw left)

rotate U and N around the V axis

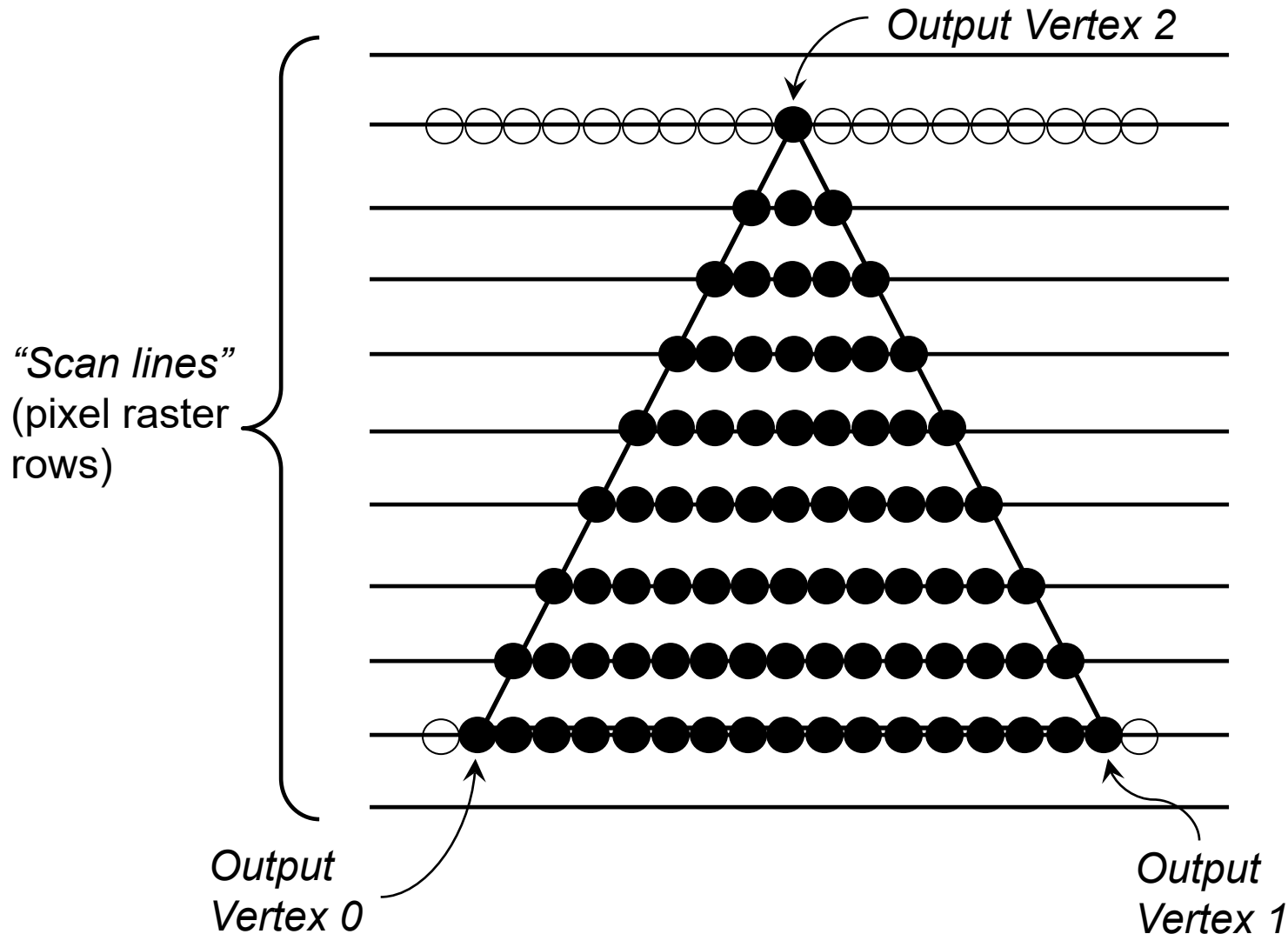


Defining Simple 3D Models

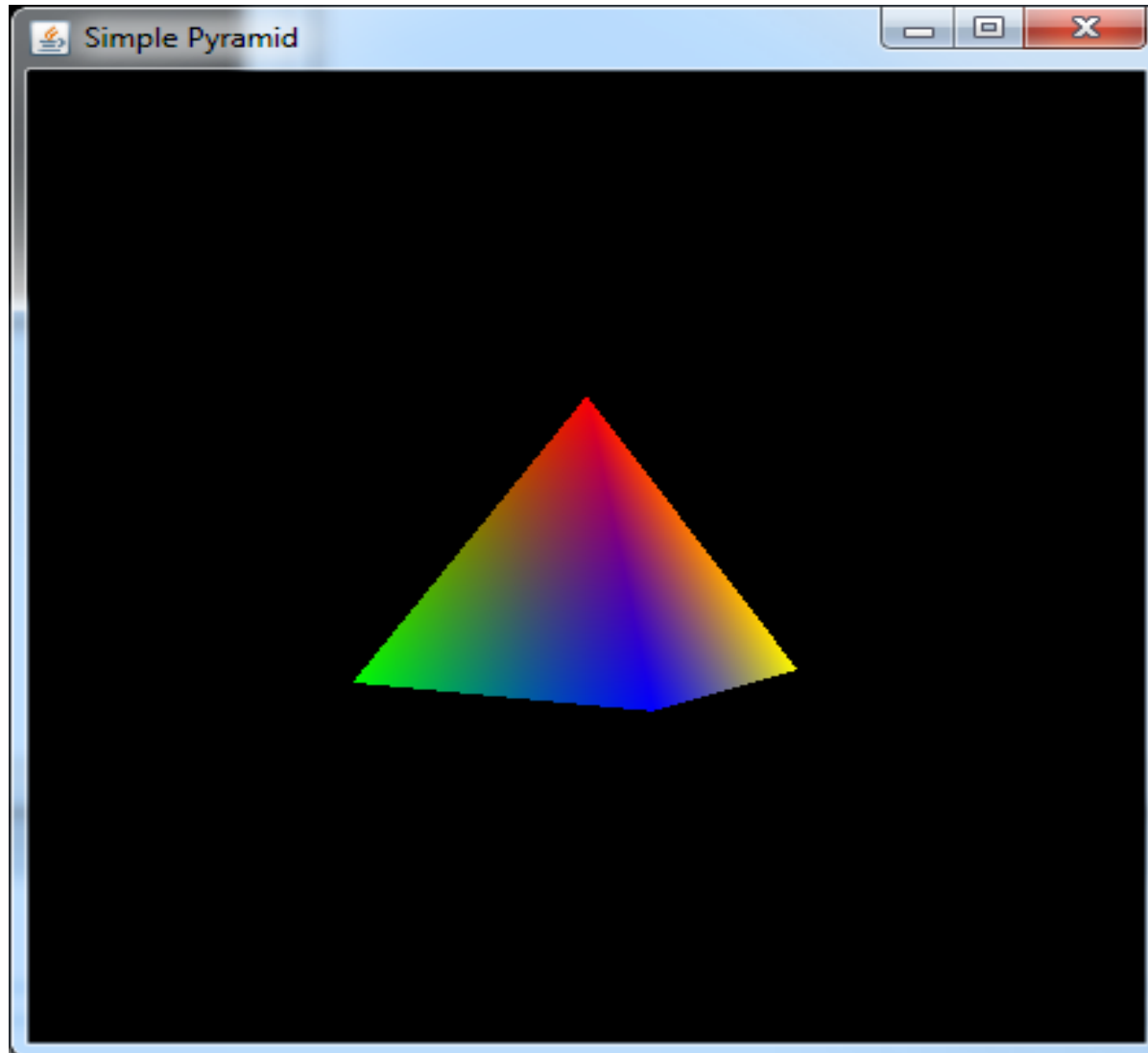


A 2x2x2 “Pyramid” Centered At The Origin

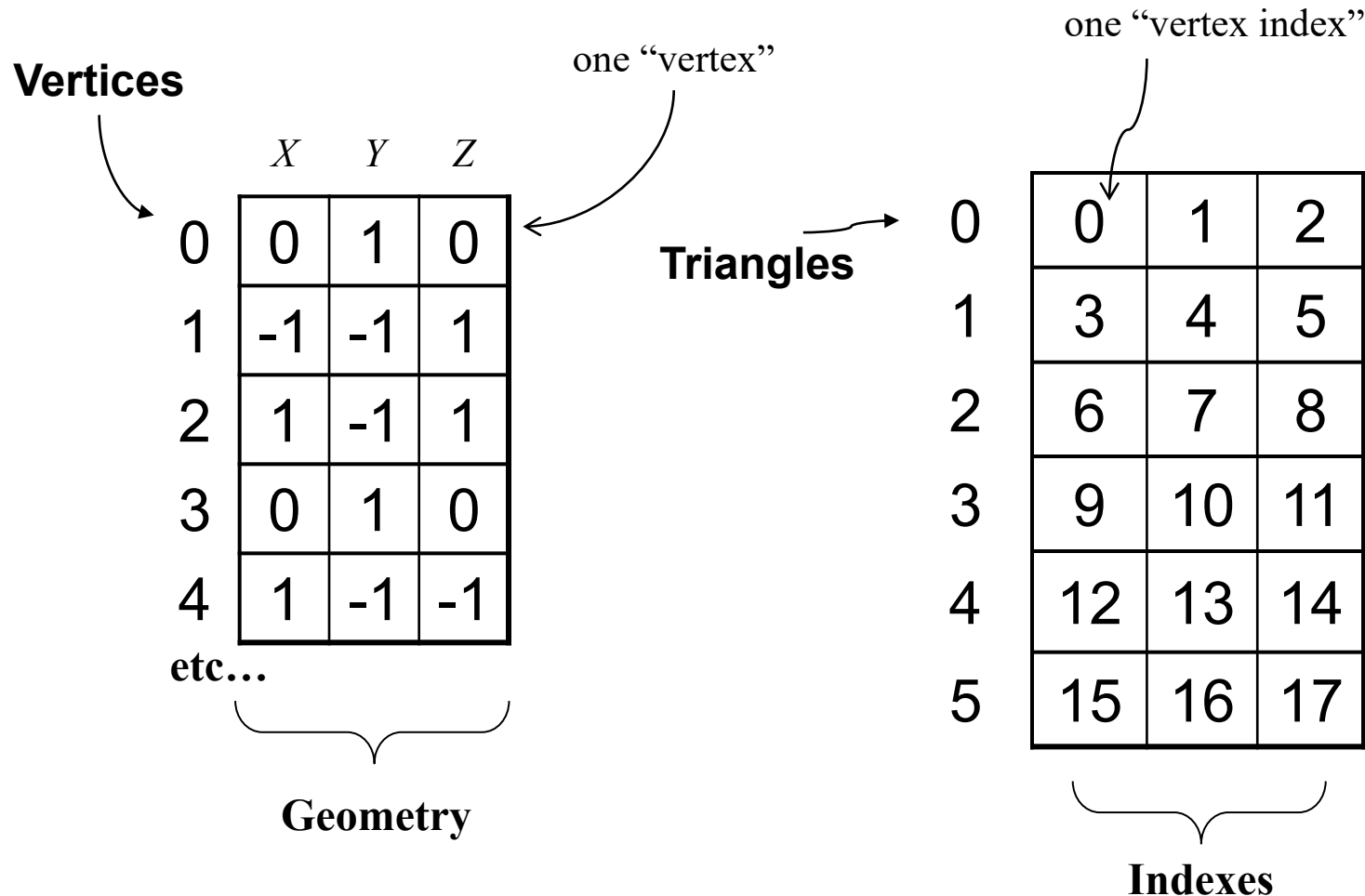
Rasterization



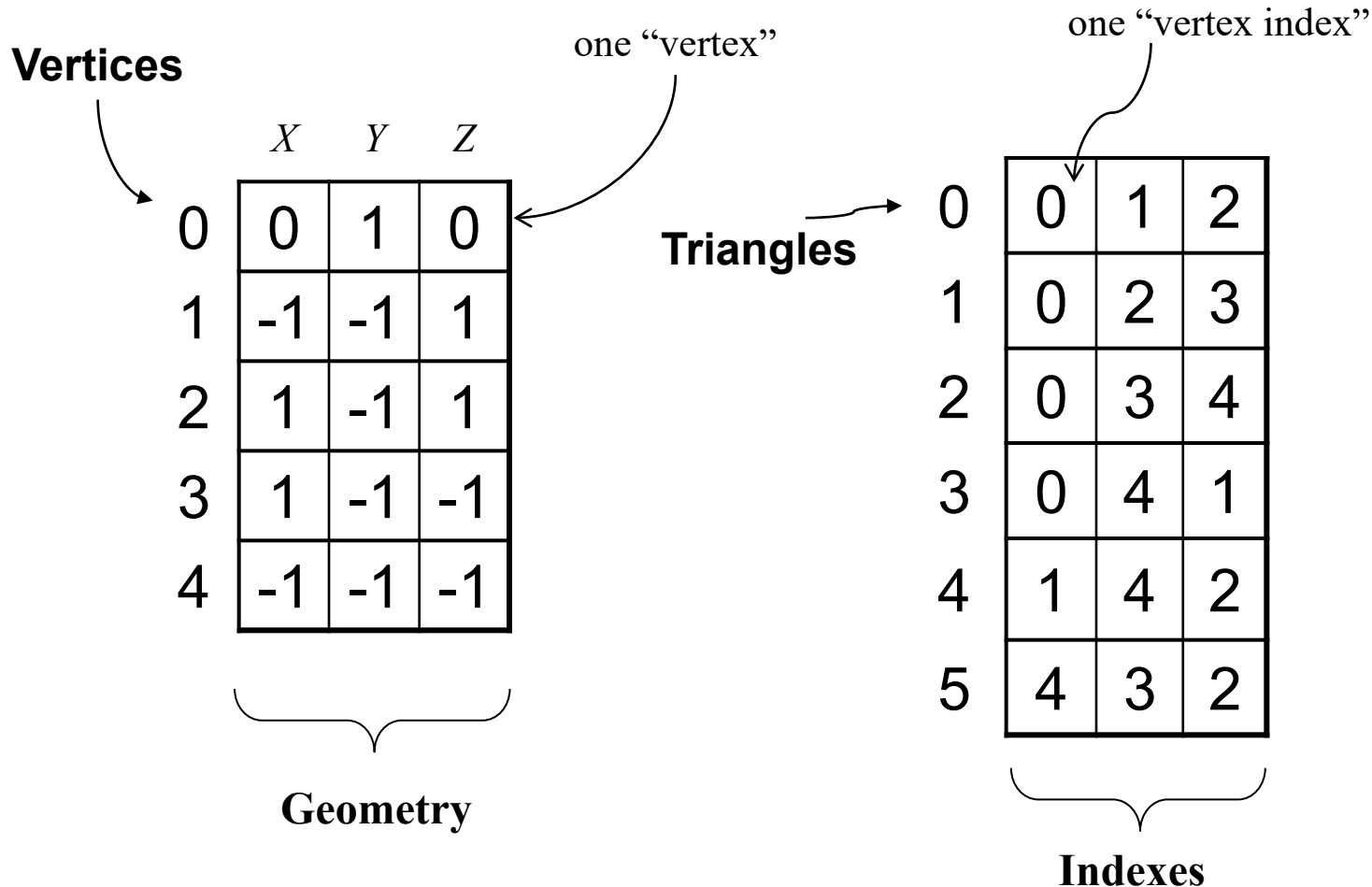
rasterization == interpolation



Pyramid Data Structure (non-indexed)

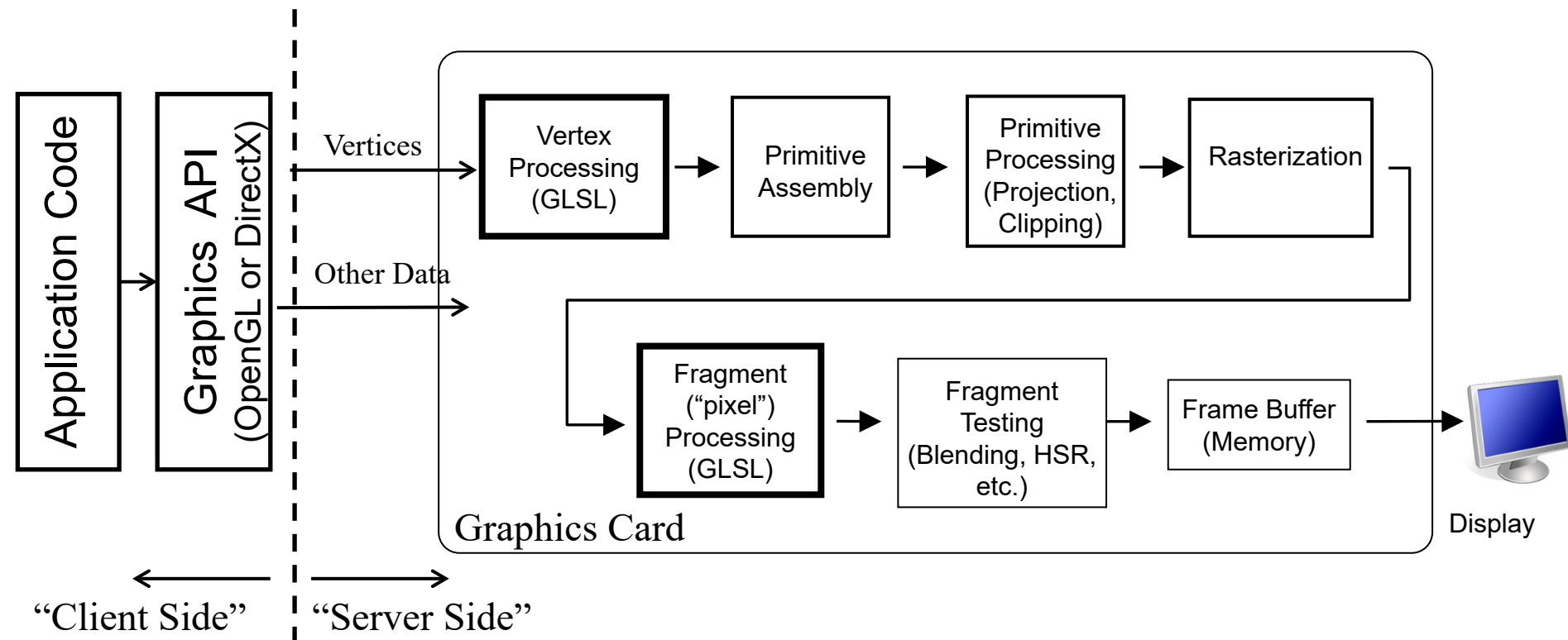


Pyramid Data Structure (indexed)



The Graphics “Pipeline”

- Implemented by the *combination of the graphics driver (software), and graphics (hardware) card*
- Modern pipelines are “shader-based”, meaning that the rendering code resides on the graphics card (for OpenGL, written in GLSL)



3D Transformations

Needed for a wide variety of operations:

- Modeling
- Positioning & orienting objects in the “3D virtual world”
- Camera positioning (“viewing”)
- Creating the 2D screen view of the 3D world view (“projection”)
- Making objects move, grow, spin, fly, etc.

Translation (column-major form):

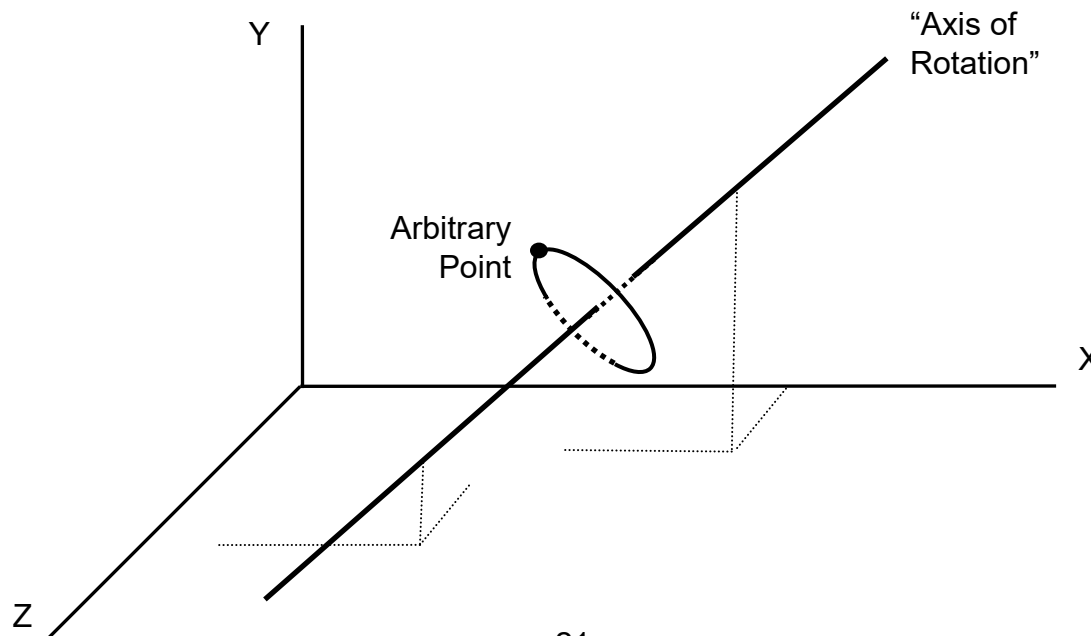
$$\begin{pmatrix} x+T_x \\ y+T_y \\ z+T_z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Scaling (column-major form):

$$\begin{pmatrix} x*S_x \\ y*S_y \\ z*S_z \\ 1 \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

3D Rotation

- Recall **2D** rotations can be “about any point”
 - For simplicity we define only 2D rotation “about the origin”
 - Other rotations require translation to/from the origin
- Similarly, 3D rotations can be “about any line” (any “axis of rotation”) :



Euler's Theorem

“Any rotation (or sequence of rotations) about a point is equivalent to a single rotation about some axis through that point.”

[Leonard Euler, 1707-1783]

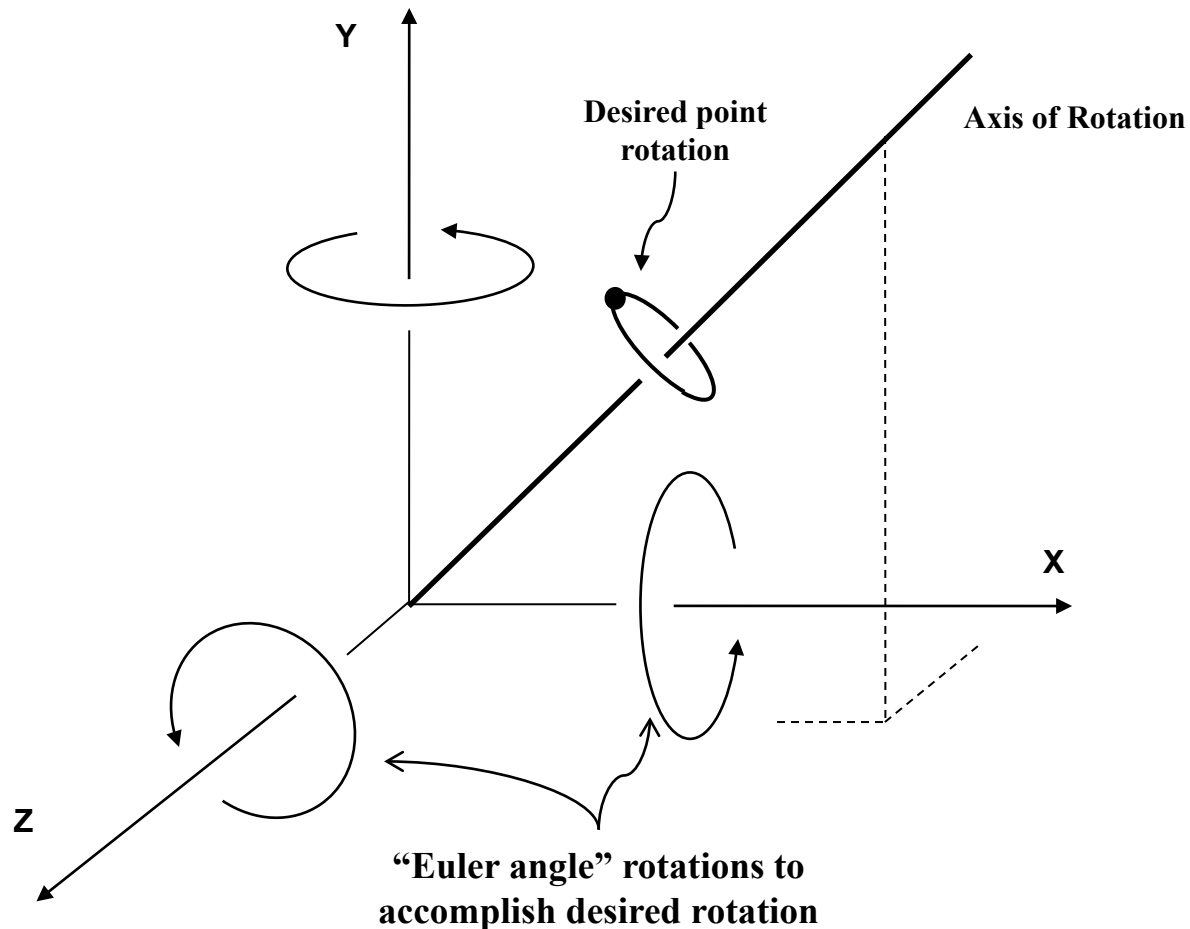
This is equivalent to saying:

Rotation about an arbitrary line through the origin can be accomplished by an equivalent set of rotations about the X, Y, and Z axes.

Thus we can rotate about an arbitrary axis as follows:

1. Translate the axis so it goes through the origin,
2. Rotate by the appropriate “Euler angles” about X, Y, and Z, and
3. “Undo” the translation

Visualizing Euler's Theorem



3D Rotation Transforms

Rotation about X by θ :

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

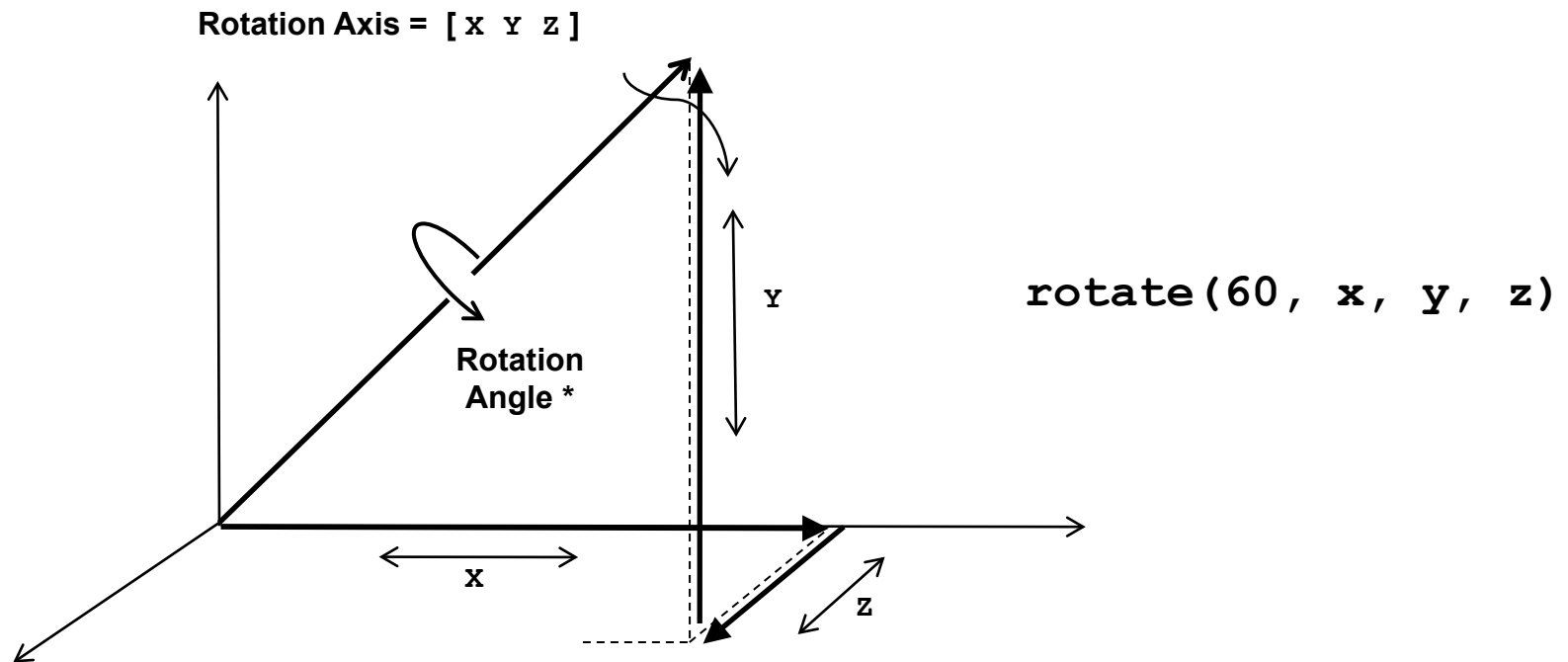
Rotation about Y by θ :

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about Z by θ :

$$\begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Rotation in Angle/Axis Form



* Positive rotation = CCW as seen from vector (axis) head, looking toward tail at origin (right hand rule)

Representing Transforms

JOML (*“Java OpenGL Math Library”*)

- Class **Matrix4f** : a 4x4 (“3D”) matrix

Methods for specifying translation, rotation, & scaling,
obtaining transpose and inverse, etc.

Similar to Java’s **AffineTransform** (but 3D)

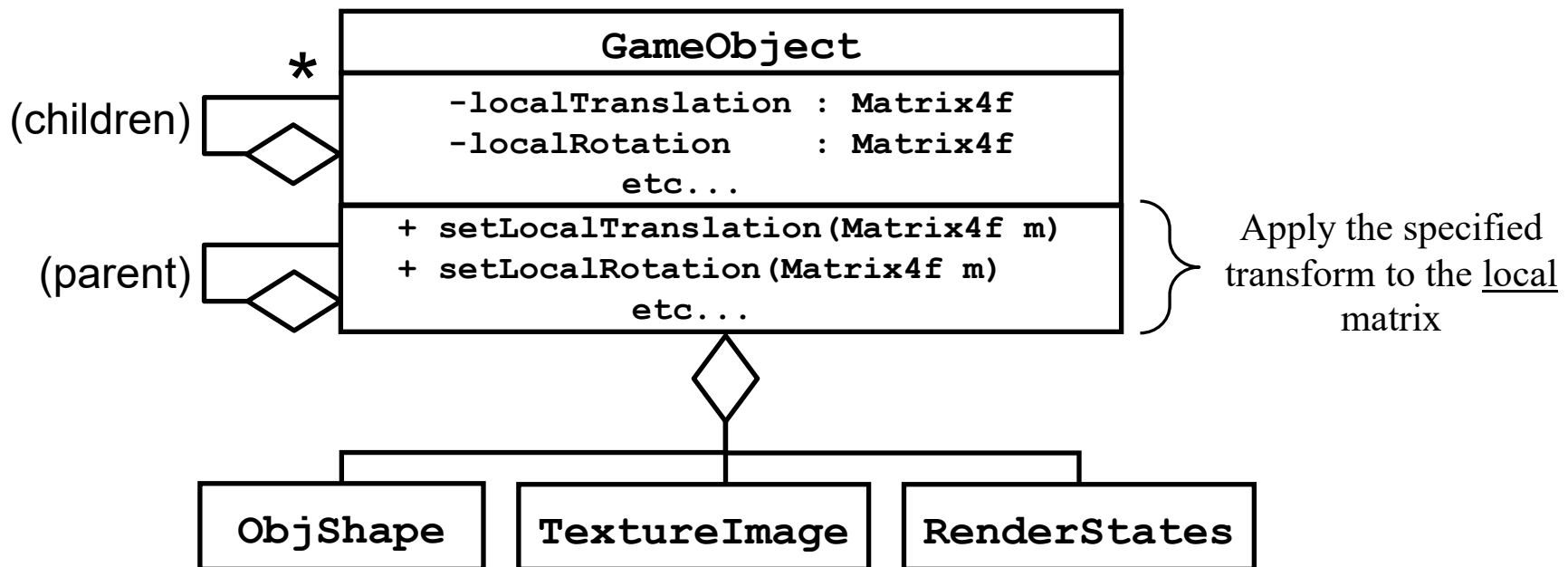
- Class **Vector4f** : a 4-element (“3D”) vector

Methods for most common vector operations: add,
dot- and cross-product, magnitude, normalize...

Useful for representing, for example, a ***rotation axis***

Game Objects (a.k.a. “scene nodes”)

Every object in a scene is an instance of **GameObject**, which provides translate, rotate, and scale matrices.



GameObjects form a tree called a “scene graph”.

This facilitates grouping objects, and building hierarchical objects and systems.

“Perspective” matrix

```
q = 1 / tan(fieldOfView/2);  
A = q / aspectRatio;  
B = (near + far) / (near - far);  
C = (2.0 * near * far) / (near - far);
```

The perspective transformation matrix is then:

$$\begin{pmatrix} A & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & B & C \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Lighting

Real world lights have a *frequency spectrum*

- White light: all (visible) frequencies
- Colored light: restricted frequency distribution

Simplified model:

Light “characteristics”

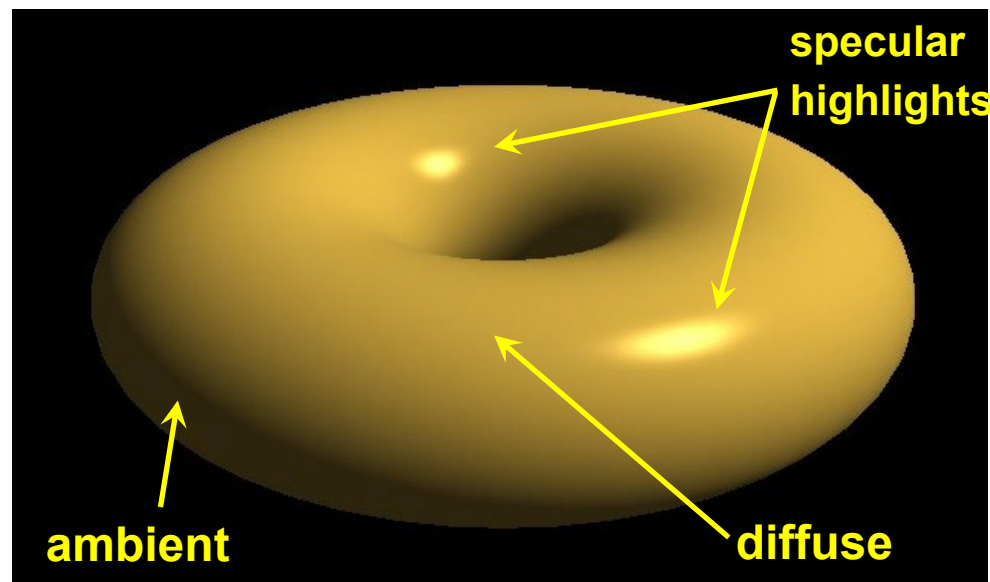
- Ambient, Diffuse, Specular “reflection characteristics”
- Red, Green, Blue “intensities”

Light “type”

- Positional, Directional, ...

The “ADS” lighting model

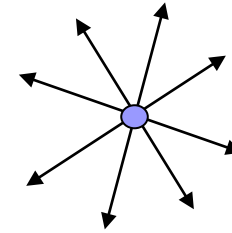
- Ambient reflection simulates a low-level illumination that equally affects everything in the scene.
- Diffuse reflection brightens objects to various degree depending on the light's angle of incidence.
- Specular reflection conveys the shininess of an object by strategically placing a highlight of appropriate size on the object's surface where light is reflected most directly towards our eyes.



Light Types

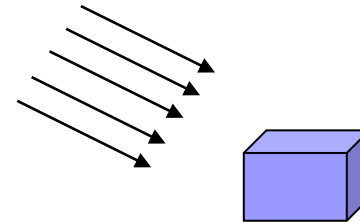
Point source

- Location, intensity



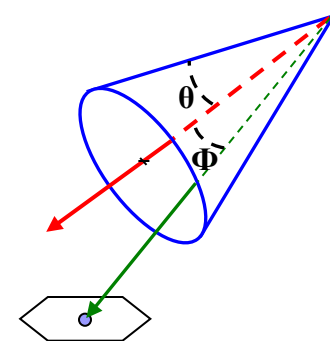
Directional (“distant”)

- Direction, intensity



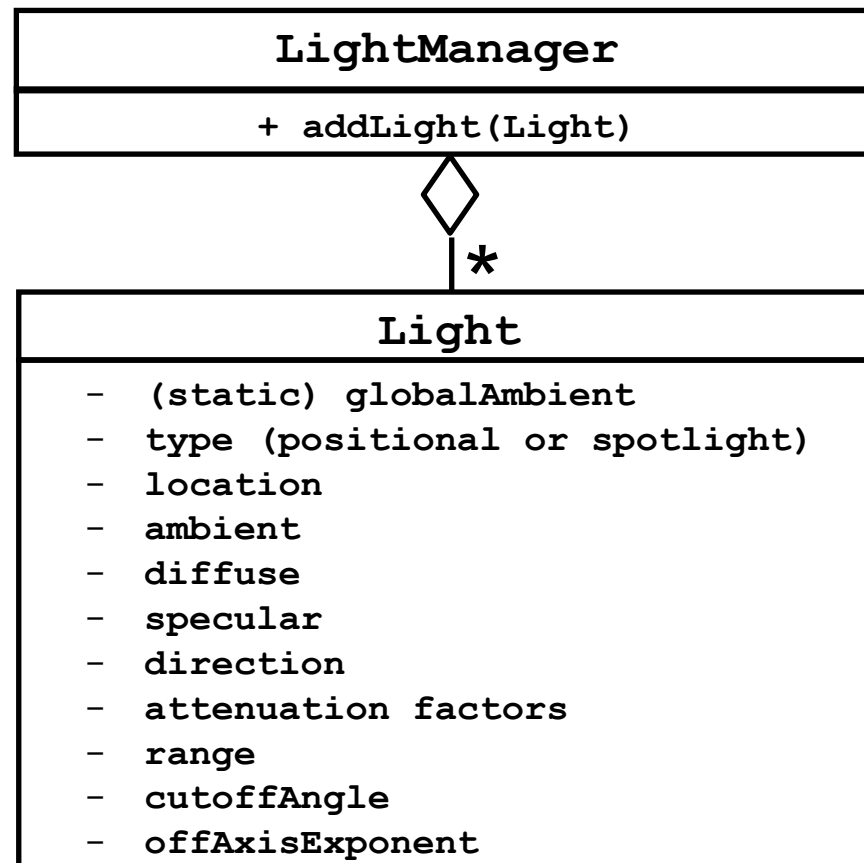
Spot

- Location, direction, intensity, coneAngle, fallOffRate



TAGE Light classes

TAGE allows an unlimited number of lights.



Materials

Models the reflectance characteristics of surfaces.
Usually modeled in ADS with four components:

- Ambient, Diffuse, and Specular
- Shininess (to determine size of specular highlights)

In TAGE, a `GameObject` stores its material characteristics in its `ObjShape`.

some common materials

<i>material</i>	<i>ambient RGBA</i> <i>diffuse RGBA</i> <i>specular RGBA</i>	<i>shininess</i>
Gold	0.24725, 0.1995, 0.0745, 1.0 0.75164, 0.60648, 0.22648, 1.0 0.62828, 0.5558, 0.36607, 1.0	51.2
Jade	0.135, 0.2225, 0.1575, 0.95 0.54, 0.89, 0.63, 0.95 0.3162, 0.3162, 0.3162, 0.95	12.8
Pearl	0.25, 0.20725, 0.20725, 0.922 1.00, 0.829, 0.829, 0.922 0.2966, 0.2966, 0.2966, 0.922	11.264
Silver	0.19225, 0.19225, 0.19225, 1.0 0.50754, 0.50754, 0.50754, 1.0 0.50827, 0.50827, 0.50827, 1.0	51.2

ADS lighting computations

$$I_{observed} = I_{ambient} + I_{diffuse} + I_{specular}$$

Ambient computation is the simplest:

$$I_{ambient} = Light_{ambient} * Material_{ambient}$$

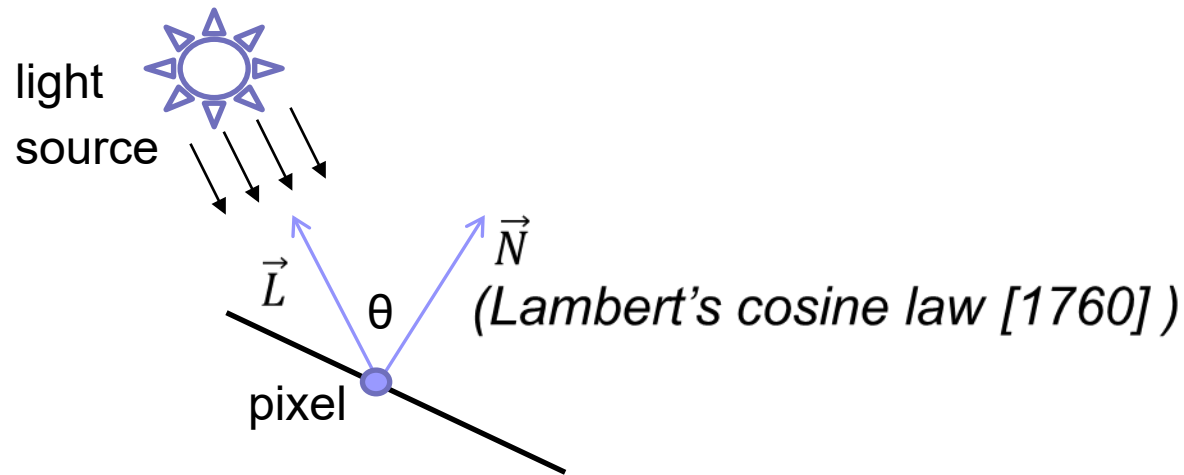
Note that each item has R, G, and B components.
So the computations actually are as follows:

$$I_{ambient}^{red} = Light_{ambient}^{red} * Material_{ambient}^{red}$$

$$I_{ambient}^{green} = Light_{ambient}^{green} * Material_{ambient}^{green}$$

$$I_{ambient}^{blue} = Light_{ambient}^{blue} * Material_{ambient}^{blue}$$

Diffuse computation depends on the angle of incidence between the light and the surface:



$$I_{diffuse} = Light_{diffuse} * Material_{diffuse} * \cos(\theta)$$

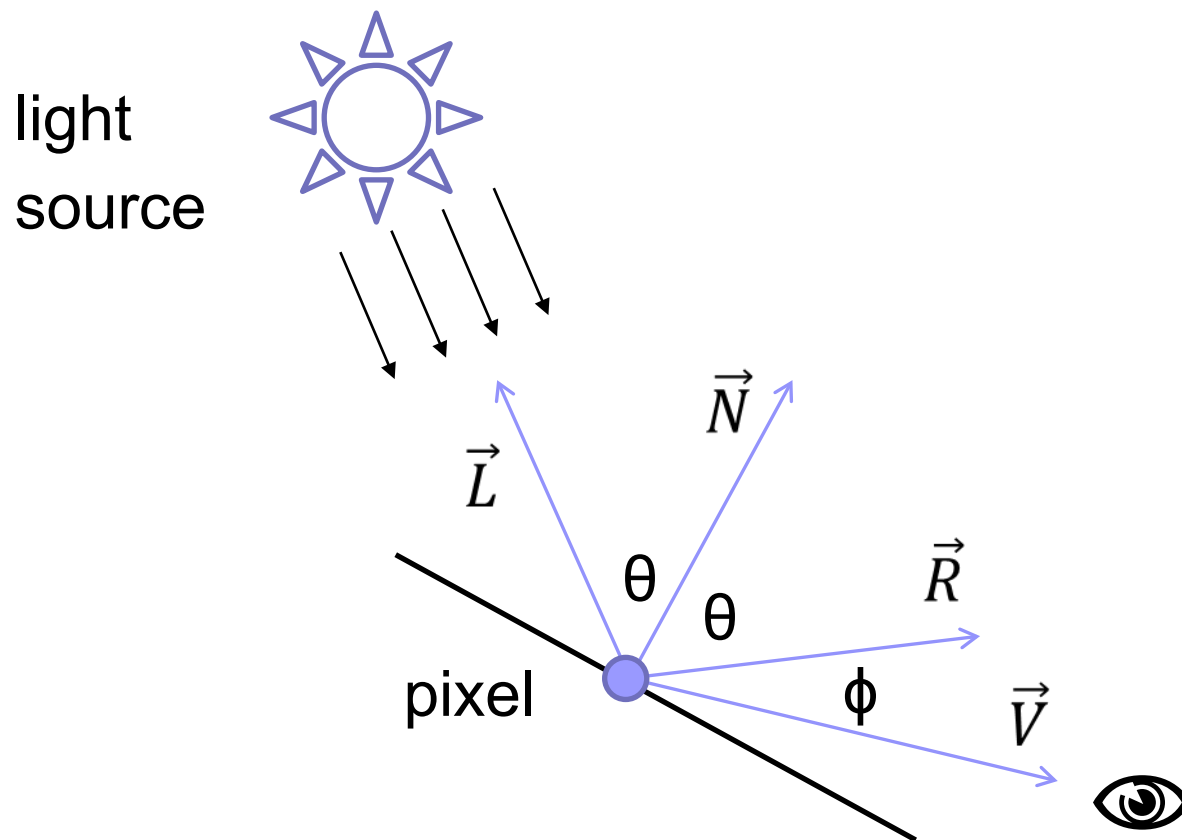
Rightmost term determined simply using dot product:

$$I_{diffuse} = Light_{diffuse} * Material_{diffuse} * (\hat{N} \bullet \hat{L})$$

Only include this term if the surface is exposed to the light:

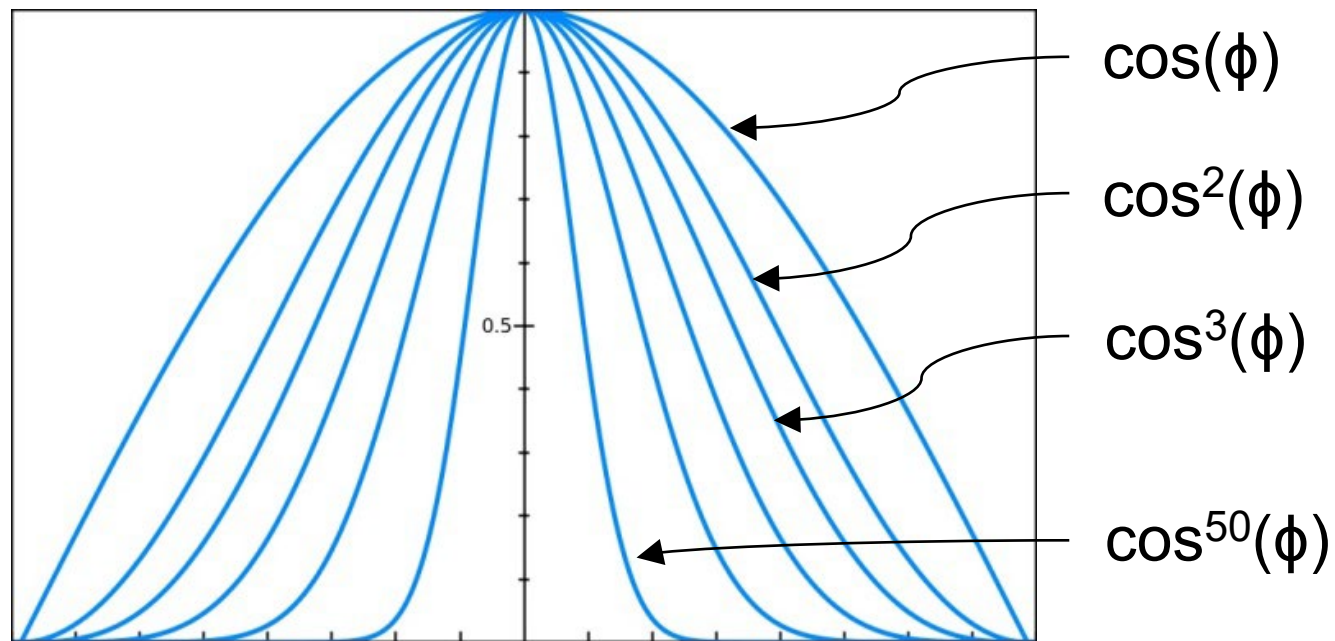
$$I_{diffuse} = Light_{diffuse} * Material_{diffuse} * \max((\hat{N} \bullet \hat{L}), 0)$$

Specular computation depends on the angle of reflection of the light on the surface, and the viewing angle of the eye.

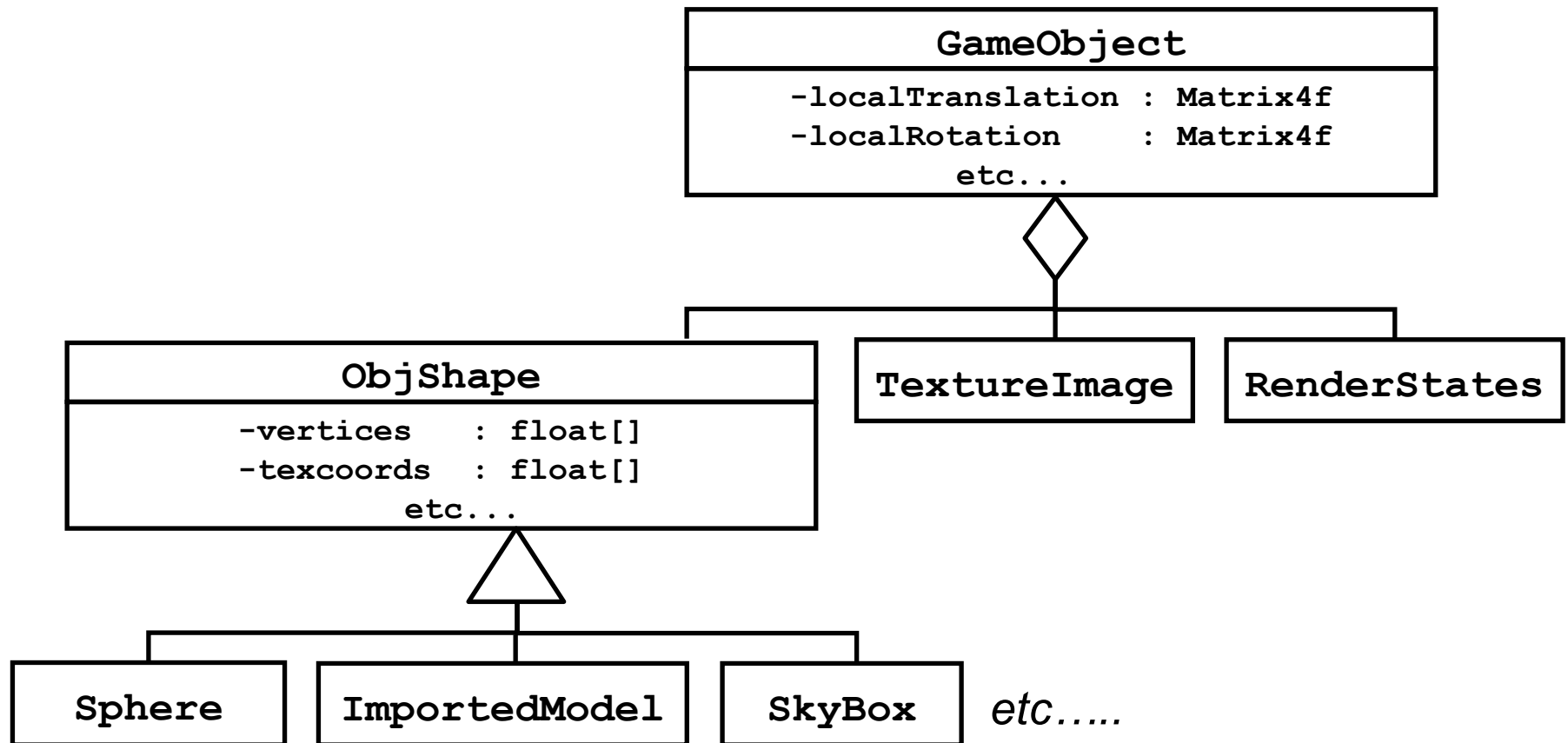


“Shininess” modeled with a falloff function.

Expresses how quickly the specular contribution reduces to zero as the angle ϕ grows.



$$I_{spec} = Light_{spec} * Material_{spec} * \max(0, (\hat{R} \bullet \hat{V})^n)$$



GameObject stores matrix transforms

ObjShape stores the vertex locations, ADS material characteristics, shininess, texture coordinates, normal vectors, and skeleton information (if applicable).