CSC28 - Discrete Mathematics: Assignment #2 - Basic Logic and Proof

Matthew Mendoza

Boolean Algebra

1 $\cap \wedge \sim \cap \vee (\cap \wedge (q \vee \sim q))$

$\cap \wedge \sim \cap \vee (\cap \wedge (q \vee \sim q))$ ← Tautologies - Complement Law $a \vee \sim a \equiv true$

$\cap \wedge \sim \cap \vee (\cap \wedge (\ T\ ))$

$\cap \wedge \sim \cap \vee (\cap \wedge (\ T\ ))$ ← Identity Law $a \wedge true \equiv a$

$\cap \wedge \sim \cap \vee (\ \cap\ )$

$\cap \wedge \sim \cap \vee (\ \cap\ )$ ← Contradiction - Complement Law $a \wedge \sim a \equiv false$

$F \quad \vee (\ \cap\ )$

$F \quad \vee (\ \cap\ )$ ← Identity Law $false \vee a \equiv a$

$\cap$ Solution.

Note

* Union $A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$

\+ Intersection $A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$

− Difference $A \setminus B = \{x \mid x \in A \text{ AND } x \notin B\}$

2 $(P \vee \sim q) \rightarrow (P \wedge q)$

$(P \vee \sim q) \rightarrow (P \wedge q)$ abstracted

$P \qquad q$

If $P = (P \vee \sim q)$ AND $q = (P \wedge q)$

then $(P \vee \sim q) \rightarrow (P \wedge q) \equiv \sim(P \vee \sim q) \vee (P \wedge q)$

$\sim(P \vee \sim q) \vee (P \wedge q)$

$\sim(P \vee \sim q) \vee (P \wedge q)$ ← DeMorgan's Law $\sim(a \vee b) \equiv \sim a \wedge \sim b$

$(\sim P \wedge q) \vee (P \wedge q)$

$(\sim P \wedge q) \vee (P \wedge q)$ ← Distributive Law $a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$

$q \wedge (\sim P \vee P)$

$q \wedge (\sim P \vee P)$ ← Complement Law $a \vee \sim a \equiv true$

$q \wedge (\ T\ )$

$q \wedge (\ T\ )$ ← Identity Law $a \wedge true \equiv a$

q Solution.

Note

OR $P \rightarrow q \equiv \sim(P \wedge \sim q)$

AND $P \rightarrow q \equiv \sim(P \wedge \sim q)$

$\equiv \sim P \vee q$

Implication

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

P	q	~q	P∨~q	P∧q	(P∨~q)→(P∧q)
F	F	T	T	F	F
F	T	F	F	F	T
T	F	T	T	F	F
T	T	F	T	T	T

3 $\sim a \wedge \sim b \vee c \wedge \sim c \vee \sim a \wedge b$

$\sim a \wedge \sim b \vee c \wedge \sim c \vee \sim a \wedge b$ Ranking Precedence Levels

$(\sim a \wedge \sim b) \vee (c \wedge \sim c) \vee (\sim a \wedge b)$ Rewriten...

$(\sim a \wedge \sim b) \vee (c \wedge \sim c) \vee (\sim a \wedge b)$ ← Complement Law $a \wedge \sim a \equiv false$

$(\sim a \wedge \sim b) \vee (\ F\ ) \vee (\sim a \wedge b)$

$(\sim a \wedge \sim b) \vee (\ F\ ) \vee (\sim a \wedge b)$ ← Distributive Law $a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$

$(\ F\ ) \vee \sim a \wedge (\sim b \vee b)$

$(\ F\ ) \vee \sim a \wedge (\sim b \vee b)$ ← Complement Law $a \vee \sim a \equiv true$

$(\ F\ ) \vee \sim a \wedge (\ T\ )$

$(\ F\ ) \vee \sim a \wedge (\ T\ )$ ← Identity Law $a \wedge true \equiv a$

$(\ F\ ) \vee \sim a$

$(\ F\ ) \vee \sim a$ ← Identity Law $a \vee false \equiv a$

$\sim a$ Solution.

Standard Precedence Levels

0	~ !	Negation
1	∧ +	Intersection
2	∨ ⊕ *	union XOR
3	→	Implication

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Arguments

4 The following argument from the movie Monty Python and the Holy Grail.
"If it's made of wood then it floats. If it's a duck then it floats. THEREFORE, a duck is wood."

Let
Wood be "W"
Floats be "F"
Ducks be "D"

$W \to F$
$D \to F$
$\therefore D \to W$ Consequence

Premise

$W \to F$			$D \to F$			$D \to W$			When the conclusion of both $W \to F$ & $D \to F$ are both TRUE then $D \to W$ is False
T	T	T	T	T	T	T	T	T	
T	T	T	F	T	T	F	T	T	
T	F	F	T	F	F	T	T	T	
T	F	F	F	T	F	F	T	T	
F	T	T	T	T	T	T	F	F	← INVALID
F	T	T	F	T	T	F	T	F	
F	T	F	T	F	F	T	F	F	
F	T	F	F	T	F	F	T	F	

Informal fallacie - Weak Analogy

Premise ⇩ Conclusion — Depends on inadequate analogy

5 "If they are a student, then they'll come to class or use Zoom. Students can't come to class. Therefore, if they are a student, then they will use Zoom."

Let
Student be "S"
Class be "C"
Zoom be "Z"

"If they are a student, then they'll come to class or use Zoom." $S \to (C \vee Z)$
Students can't come to class. $\sim C$
Therefore, if they are a student, then they will use Zoom." $S \to Z$

$S \to (C \vee Z)$					$\sim C$	$S \to Z$		
T	T	T	T	T	F	T	T	T
T	T	T	T	F	F	T	F	F
T	T	F	T	T	T	T	T	T
T	F	F	F	F	T	T	F	F
F	T	T	T	T	F	F	T	T
F	T	T	T	F	F	F	T	F
F	T	F	T	T	T	F	T	T
F	T	F	F	F	T	F	T	F

Looks Like Disjunctive syllogism

$$\frac{(C \vee Z)}{\sim C} \\ \hline Z$$

Valid: CVZ is TRUE and ~C is TRUE; therefore, Z is also TRUE

6 "If its toilet paper, then its being horded. If it's being horded, then it must be guarded. Therefore, toilet paper must be guarded!"

Let
Toilet Paper be "P"
Hoarded be "H"
Garded be "G"

FOR THE HOARD

$P \to H$			$H \to G$			$P \to G$		
T	T	T	T	T	T	T	T	T
T	T	T	T	F	F	T	F	F
T	F	F	F	T	T	T	T	T
T	F	F	F	T	F	T	F	F
F	T	T	T	T	T	F	T	T
F	T	T	T	F	F	F	T	F
F	T	F	F	T	T	F	T	T
F	T	F	F	T	F	F	T	F

Looks like Hypothetical Syllogism

$P \to H$
$H \to G$
$\therefore P \to G$

Valid: When both $P \to H$ & $H \to G$ are TRUE; then, $P \to G$ is TRUE.

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Simple Proofs

7 If a is odd, b is even, and c odd then a × b × c is even

$a = 2i+1 \quad b = 2j \quad C = 2i+1$

$a \times b \times c = (2i+1)(2j)(2i+1)$

$= (4i^2+4i+1)(2j)$

$= 2(4i^2j+4ij+j)$

After a, b, c are rewritten and foiled out we can then factor out a two and any Integer multiplied by two is even.

TRUE

8 If a is divisible by 2 and b is divisible by 6 then a × b is divisible by 4.

$a = 2i \quad b = 6j$

$a \times b = 2i \times 6j$

$= 12ij \therefore 4(3ij)$

When multiplied out + after a multiplied by b are rewritten we can factor out a four, so axb is divisable by four.