1. (1 point)
Solve the following differential equation:

$$\left(x - y^5 + y^5 \sin(x)\right) dx = \left(5xy^4 + 5y^4 \cos(x)\right) dy.$$

______________ = constant. help (formulas)

Given

$$(x-y^5+y^5\sin(x))dx - (5xy^4+5y^4\cos(x))dy = 0$$

compose equation with $Mdx + Ndy = 0$, we get

$$(x-y^5+y^5\sin(x))dx - (5xy^4+5y^4\cos(x))dy = 0$$

$M = x-y^5+y^5\sin(x)$ | $N = -5xy^4+5y^4\cos(x)$

$M_y = 0-5y^4+5y^4\sin(x)$ | $N_x = -5(1)y^4+5y^4(-\sin(x))$

$= -5y^4+5y^4\sin(x)$ | $= -5y^4+5y^4\sin(x)$

$M_y = N_x$ is exact

General solution is

$$\int M dx + \int(\text{term free of "}x\text{" in } N)dy = C$$

$$\rightarrow \int(x-y^5+y^5\sin(x))dx + \int(0)dy = C$$

$$\rightarrow \int(x)dx - y^5\int(1)dx + y^5\int(\sin(x))dx + 0 = C$$

$$\rightarrow \frac{x^2}{2} - xy^5 - y^5(\cos(x)) = C$$

* is the required solution

Simular to the homework question

Given $(x-y^5+y^2\sin x)\,dx-(5xy^4+2y\cos x)\,dy=0$ ——— (1)

comparing (1) with $Mdx+Ndy=0$, we get

$M=x-y^5+y^2\sin x$

$M_y=0-5y^4+(\sin x)2y$

$=-5y^4+2y\sin x.$

$N=-5xy^4-2y\cos x$

$N_x=-5y^4(1)-2y(-\sin x)$

$=-5y^4+2y\sin x$

$\boxed{M_y=N_x}$

ie., (1) is Exact

∴ General solution of (1) is

$$\boxed{\int_x M\,dx+\int(\text{term free of 'x' in } N)\,dy=c}$$

$$\Rightarrow \int_x (x-y^5+y^2\sin x)\,dx+\int (0)\,dy=c$$

$$\Rightarrow \int x\,dx-y^5\int 1\,dx+y^2\int \sin x\,dx+0=c$$

$$\Rightarrow \frac{x^2}{2}-y^5(x)+y^2(-\cos x)=c$$

$$\Rightarrow \boxed{\frac{x^2}{2}-xy^5-y^2(\cos x)=c}$$ is the required solution of (1)

Note: $\int_x M\,dx$ means integrating 'M' w.r.t 'x' treating 'y' as constant.

and $M_y=\frac{\partial M}{\partial y}$, $N_x=\frac{\partial N}{\partial x}$.