DEs in Dr. Krauel's interests

My research focuses on an intersection of algebra, number theory, and physics.

(Note: I have some research projects and topics I like to look at if folks are interested!)

Vertex operator algebras

In particular, I work often in the theory of **vertex operator algebras (VOAs)** **(https://en.wikipedia.org/wiki/Vertex_operator_algebra)**. Here, there are both algebraic (pure math) and physical (physics) motivations to study certain functions attached to VOAs. It had been conjectured in the 1980s that these functions

1. converge (like the convergence of series in Calc II!), and
2. are **modular forms** **(https://en.wikipedia.org/wiki/Modular_form)** (a number theoretic function -- important in the proof of Fermat's Last Theorem and other things).

An ingenious solution to these problems was to use the theory of VOAs to construct a homogeneous linear differential equations, where the functions being discussed above happen to be solutions. This was done by Zhu in this **paper (pdf)** **(http://www.ams.org/journals/jams/1996-9-01/S0894-0347-96-00182-8/S0894-0347-96-00182-8.pdf)**. Then some differential equations theory (which we did not cover in this class) states that

- the solutions of a differential equation converge if the coefficients of the differential equations converge.

This gave the answer to issue 1 above. As for 2, Zhu essentially found a fundamental set of solutions which were modular forms, and this then implies that all of the solutions (and thus the functions of interest) are modular forms.

In conclusion, the theory of linear ODEs was used in theoretical math and physics to solve some good stuff!!

Number Theory

Speaking of modular forms. Zhu's work somewhat sparked a resurgence of of the study of something called vector-valued modular forms.

Recent work in the last decade studies vector-valued modular forms via studying homogeneous linear differential equations (that have some modular properties). In fact, the Wronskian even plays a fundamental role! For instance, it can be found in this paper by **Franc and Mason** **(https://arxiv.org/abs/1503.05519)**.