1. (1 point)

? 1. Which differential equation below is in normal form?

? 2. Which differential equation below is in differential form?

A. $(y+\sin(\theta))\,dy+y\theta\,d\theta=0$

B. $y'''=ty''-t^3y'+y$

Solution:
SOLUTION
The answer to the first question is B, because of the form $y'''=F(x,y,y',y'')$, where $F(x,y,y',y'')$ is a function in terms of x, y, and derivatives of y with order less than that of the other side.

The answer to the second question is A, because it is of the form $F(y,\theta)\,dy+G(y,\theta)\,d\theta=0$ for functions F and G.

Correct Answers:

- B
- A

2. (1 point)
Determine the order of the given differential equation and state whether the equation is linear or nonlinear.

$$(\sin\theta)y^{(7)}-(\cos\theta)y'=7$$

(a) The order of this differential equation is ____.

(b) The equation is [Choose/Linear/Nonlinear].

Solution:
SOLUTION:
Since the highest order derivative which occurs is 7, we have the order of the differential equation is 7. Meanwhile, since there are no nonlinear terms of the function y and its derivatives, we have that the differential equation is Linear.

Correct Answers:

- 7
- Linear

3. (1 point)
Determine the order of the given differential equation and state whether the equation is linear or nonlinear.

$$\frac{d^4u}{dr^4}+\frac{du}{dr}+6u=\cos(r+u)$$

(a) The order of this differential equation is ____.

(b) The equation is [Choose/Linear/Nonlinear].

Solution:
SOLUTION:
Since the highest order derivative which occurs is 4, we have the order of the differential equation is 4. Meanwhile, since $\cos(r+u)$ is a nonlinear expression of u, we have that the differential equation is Nonlinear.

Correct Answers:

- 4
- Nonlinear

Which of the following functions satisfies the differential equation $(x+1)y'-y+2\ln(1+x)=3$?

- A. $y=\ln(x+x^2)$
- B. $y=e^x$
- C. $y=x+2\ln(1+x)$

Solution:
SOLUTION:
We check each possible solution to see if it satisfiest the differential equation. We find the for $y=x+2\ln(1+x)$ we have $y'=1+\frac{2}{1+x}$. Plugging y and y' into the equation satisfies the equation. Thus, the correct answer is C.

Correct Answers:

- C

Note that $\phi(x)=\ln(1+2x)$ satisfies the differential equation $(2x+1)\ln(1+2x)y'-2y=0$. On what interval is ϕ a solution for this differential equation?

- A. $(-\infty,\infty)$
- B. $(-1,\infty)$
- C. $\left(-\frac{1}{2},\infty\right)$

- D. $[-1,\infty)$

- E. $\left[-\frac{1}{2},\infty\right)$

Solution:
SOLUTION:
We recall that $\ln(w)$ is only defined for $w>0$. Therefore, the function $\phi(x)=\ln(1+2x)$ is defined for all x such that $1+2x>0$, or $x>-\frac{1}{2}$. Thus, the correct answer is C.
Correct Answers:

- `C`

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6. (1 point)

$\boxed{?}$1. Which statement of sets below best describes the domain of the function $f(x)=\dfrac{1}{1-x}$?

$\boxed{?}$2. Which statement of sets below best describes the interval on which the function $f(x)=\dfrac{1}{1-x}$ is a solution to the differential equation $y'=y^2$?

A. $(-\infty,1)$ or $(1,\infty)$

B. $(-\infty,1)$ and $(1,\infty)$

Solution:
SOLUTION
The answer to the first question is B, because the function f is defined for all numbers in $(-\infty,1)$ and in $(1,\infty)$.
The answer to the second question is A, because since a function is a solution only on an interval, and it must be defined on this interval. Therefore, we must pick either of the intervals and say that f is a solution on the chosen interval.
Correct Answers:

- `B`
- `A`

The function $y=c_1e^{3x}+c_2xe^{3x}$ is a two-parameter family of solutions for which of the following differential equations?

- A. $y''-6y'+9y=0$

- B. $y'=y$

- C. $y''+6y'-9y=0$

Solution:
SOLUTION:
We have $y=c_1e^{3x}+c_2xe^{3x}$, $y'=3c_1e^{3x}+c_2e^{3x}+3c_2xe^{3x}$, and $y''=9c_1e^{3x}+6c_2e^{3x}+9c_2xe^{3x}$. Plugging these into the given differential equations we find that y satisfies A for all constants c_1 and c_2. Thus, the correct answer is A. Note that since y is defined on the interval $(-\infty,\infty)$ we have that y is a family of solutions on the interval $(-\infty,\infty)$.
Correct Answers:

- `A`

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8. (1 point)
Find the value k such that $y=e^{kx}$ is a solution to the differential equation $7y'+4y=0$.

The value is $k=$ ______

Solution:
SOLUTION

We note that $y'=ke^{kx}$. Thus, $7y'+4y=0$ becomes $k7e^{kx}=-4e^{kx}$ and we need $k7=-4$, or $k=-\frac{4}{7}$. Thus, the correct answer is $k=-\frac{4}{7}=-0.571429$.

Correct Answers:

- `-4/7`

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9. (1 point) Find the two values of k such that $y=x^k$ is a solution to the differential equation $xy''+9y'=0$. The values are $k=$ _____ and $k=$ _____.
Solution:
SOLUTION:
We note that $y'=kx^{k-1}$ and $y''=k(k-1)x^{k-2}$. Thus, $xy''+9y'=0$ becomes

$$k(k-1)x^{k-1}+9kx^{k-1}=0$$

and we need $k(k-1)+9k=0$, or $k^2-k+9k=0$. That is, we need $k(k+8)=0$. Therefore, $k=0$ and $k=-8$ are the two desired values.
Correct Answers:

- `-(9-1)`
- `0`

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10. (1 point) Find the two values of k such that the constant function $y=k$ is a solution to the differential equation $y'=y^2-10y+21$. The values are $k=$ _____ and $k=$ _____.
Solution:
SOLUTION:
We note that $y'=0$. Thus, $y'=y^2-10y+21$ becomes

$$k^2-10k+21=0.$$

That is,

$$(k-3)(k-7)=0.$$

Therefore, $k=3$ and $k=7$ are the two desired values.
Correct Answers:

- `3`

- 7

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11. (1 point) Find the two values of k such that $y = x^k$ is a solution to the differential equation $xy'' + 9y' = 0$. The values are $k =$ _____ and $k =$ _____.

Solution:

SOLUTION:

We note that $y' = kx^{k-1}$ and $y'' = k(k-1)x^{k-2}$. Thus, $xy'' + 9y' = 0$ becomes

$$k(k-1)x^{k-1} + 9kx^{k-1} = 0$$

and we need $k(k-1) + 9k = 0$, or $k^2 - k + 9k = 0$. That is, we need $k(k+8) = 0$. Therefore, $k = 0$ and $k = -8$ are the two desired values.

Correct Answers:

- -(9-1)
- 0

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12. (1 point)

Let $y' = 2x$.

Find all values of r such that $y = rx^2$ satisfies the differential equation. If there is more than one correct answer, enter your answers as a comma separated list.

$r =$ _________ help (numbers)

Solution:

SOLUTION:

We note that $y' = 2rx$. Therefore, the equation $y' = 2x$ becomes $2rx = 2x$ and we require that $r = \frac{2}{2} = 1$.

Correct Answers:

- 1