Existence and uniqueness of solutions for higher order linear differential equations

RECALL:

(i) Found conditions for when a 1^{st}-order IVP is guaranteed to have a unique soln on an interval.

* Note: DE is not required to be linear.

(ii) Solving some 1st-order DEs:

→ separable
→ linear
→ Exact
→ 1st order homogeneous

<u>Thm</u> Suppose we have the IVP

$$a_n(x)\frac{d^ny}{dx^n}+a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}}+\cdots+a_1(x)\frac{dy}{dx}+a_0(x)y=b(x)$$

with initial conditions

$$y(x_0)=y_0,\ y'(x_0)=y_1,\ \ldots,\ y^{(n-1)}(x_0)=y_{n-1}.$$

Then if:

✓(i) • $a_n(x), a_{n-1}(x), \ldots, a_1(x), a_0(x), b(x)$ are continuous on I (an interval),

✓(ii) • $a_n(x)\neq 0$ for any x in I, and

✓(iii) • x_0 is in I,

we are guarenteed the existence of a unique soln of the IVP on I.

Ex| Consider the IVP $y'' + \tan(x)\, y = e^x$ with $y(0) = 1$, $y'(0) = 0$. Find an interval about $x = 0$ where we are guarenteed a uniqhe soln.

soln|

→ Need $\underline{I}$ such that 0 is in I.

→ $a_2(x) = 1$, $a_1(x) = 0$, $a_0(x) = \tan(x)$, $b(x) = e^x$ are all cont on $(-\infty, \infty)$ except $\tan(x)$ since $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ ← $= 0$ for $x = k + \frac{\pi}{2}$

But $\tan(x)$ is cont on $(-\pi/2, \pi/2) = \underline{I}$.

* And 0 is in $(-\pi/2, \pi/2)$.

→ $a_2(x) = 1 \neq 0$ on $(-\pi/2, \pi/2)$.

Thus $(-\pi/2, \pi/2)$ works!

Homogeneous and nonhomogeneous linear DEs

DEFN (a) The linear DE

$$a_n(x)\,y^{(n)} + a_{n-1}(x)\,y^{(n-1)} + \cdots + a_1(x)\,y' + a_0(x)\,y = 0$$

is called <u>homogeneous</u>, while

(b) the linear DE

$$a_n(x)\,y^{(n)} + a_{n-1}(x)\,y^{(n-1)} + \cdots + a_1(x)\,y' + a_0(x)\,y = b(x),$$

where $b(x) \neq 0$, is called <u>nonhomogeneous</u>.

Ex (a) $y' + y - 3 = 0$ ← nonhomog.

(b) $y' = y$ ← $y' - y = 0$, so homog.