Math 45, Fall 2020
August 31, Quiz 01

Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

1. (*5 points*) For each of the following,
Circle all of the following expression which are differential equations.

(i) $g'(x) + g(x) = 0$

(ii) $\left(\frac{d}{dt}\right)^5 f(t) + \frac{d}{dt} f(t) + f(t) = 0$

(iii) $\sin(x)\frac{d^2 f}{dx^2} + \frac{df}{dx} + e^x = \frac{d^3 f}{dx^3}$

(iv) $\cancel{y''' + y' + x}$ Not an equation

(v) $f'(x) = f(x)$

(vi) $y^2 x = x^2$

(vii) $\csc(y'') + \sin(x) - y = 0$

(viii) $x^2 \frac{\partial^2 y}{\partial t^2} + y^2 \frac{\partial x}{\partial s} = s + t$

(ix) $e^{y''} + e^x = 3y$

(x) $x\frac{\partial^2 y}{\partial t^2} = y\frac{\partial x}{\partial s}$

2. (*3 points*) For the following equations, provide the *dependent variable.*

a. (*1 pt*) $f'(x) - f(x) = 0$ $f(x)$

b. (*1 pt*) $\frac{d^2 g}{dt^2} - e^t g(t) = 3$ g

c. (*1 pt*) $\sin(x)y' + y = 0$ y ?

$\frac{dx}{dt}$ & $\frac{dy}{dt}$ $\frac{\text{Dependent}}{\text{Independent}}$

3. (*2 points*) Consider the function $y = x^3$.

a. (*1 pt*) Compute y' and y''.

$[x^3]' = 3x^2$, $[x^3]'' \to [3x^2]' = 6x$

b. (*1 pt*) Does y satisfy the differential equation $x^2 y'' - 5y = 0$? Justify your answer.

?

then check

8.5

Math 45, Fall 2020 September 9, Quiz 02 Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

1. (*3 points*) Circle all of the following expression which are ordinary differential equations. 0.5

(i) $\ln(y')y + x^2y = 1$

(ii) $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + x^2y = 0$

(iii) $x^2\frac{d^3f}{dx^3} + \frac{df}{dx} + \cos(x) = \frac{d^2f}{dx^2}$

(iv) $e^{y''} - y' + y - 1 = 0$

(v) $\sin\left(\frac{\partial g}{\partial x}\right)\frac{\partial f}{\partial y} = e^{x^2+y^2}$

(vi) $y^{(2)} + y' - y = 0$

does not have $\frac{dy}{dx}$

2. (*3 points*) Circle all of the following expression which are linear differential equations.

(i) $\cos(y)y + y = e^x$ Power series

(ii) $\sin(x)\frac{d^4f}{dx^4} + \frac{df}{dx} + x^3 + \frac{d^2f}{dx^2} = 0$

(iii) $y^{(3)}y - y = 3$ of form $y''\cdot y'$

(iv) $e^{y'} - y'' + y - 1 = 0$

(v) $(y')^2 + y = 0$

(vi) $g(t)\left(\frac{dg}{dt}\right) - g(t) = 0$

Powers more than one

3. (*3 points*) State the order of the following differential equations.

(i) $e^{y'}y + y = x$  first Order

(ii) $\frac{d^5g}{dt^5} - \frac{dg}{dt} + g(t) = 0$ *Order:* fifth

(iii) $\left(\frac{df}{dx}\right)^3 + \frac{df}{dx} + \cos(x) = \frac{d^2f}{dx^2}$ *Order:* Second

(iv) $e^{y'''} - y' + y - 1 = 0$ *Order:* third

(v) $(y'')y + yx^2 = 0$ *Order:* second

(vi) $y^{(4)} + y' - y^2 = 0$ *Order:* Fourth

4. (*1 point*) Provide an example of a nonlinear partial differential equation.

∂

$$\left(\frac{d^2y}{dx^2}\right)^2 + f = 0$$

Need another partial deriv.

Math 45, Fall 2020
September 16, Quiz 03

7

Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

1. (*3 points*)

(i) Is the function e^x a solution to the differential equation $y' - y = 0$? (Circle your answer.)

Yes No

(ii) Circle the following that is most likely to be a trivial solution to a DE.

$y = e^x$ $y = c$ $y = 0$ $y = Ce^x$

(iii) Circle the following that is most likely to be a particular solution to a DE. (C, c_1, c_2, k are arbitrary constants.)

$y = e^x$ $y = c$ $y = c_1x + c_2xe^x$ $y = Ce^x$

(iv) Circle the following that is most likely to be a 2-parameter family of solutions to a DE. (C, c_1, c_2, k are arbitrary constants.)

$y = e^x$ $y = c$ $y = c_1x + c_2xe^x$ $y = Ce^x$

(v) Circle the following that is most likely to be a general solution to a DE. (C, c_1, c_2, k are arbitrary constants.)

$y = e^x$ $y = 0$ $y = c_1x + c_2xe^x$ $y = \cos(x)$

(vi) Suppose $y = \ln(x-3)$ is a solution to a DE. Circle the following which would best represent its interval of validity (or domain of the solution).

y=ln(x-3

$(-\infty, \infty)$ $(-\infty, 3]$ $(-\infty, 3)$ $(3, \infty)$

2. (*2 points*) Suppose $y = \dfrac{1}{x-3}$ is a solution to a differential equation. Is $(-\infty, 3) \cup (3, \infty)$ the interval of validity for the solution (or the domain of the solution)? If so, explain why. If not, provide a possible domain.

Yes, ~~for~~, the interval is 3 non-inclusive.

when X=3 the denom. is zero.

Can't ~~divide~~ $\frac{1}{\emptyset}$

due on other

(There is another problem on the next page!)

3. (*5 points*) Consider the function $f = c_1\cos(3t) + c_2\sin(3t)$, where c_1 and c_2 are arbitrary constants. We are given that $f = c_1\cos(3t) + c_2\sin(3t)$ is a 2-parameter family of solutions to the differential equation $f'' + f = 0$. Find a solution to the IVP consisting of this differential equation and the following initial conditions:

$$f\left(\frac{\pi}{3}\right) = \sqrt{2}, \qquad f'\left(\frac{\pi}{3}\right) = \sqrt{3}.$$

Initial conditions

$$\sqrt{2} = f(\pi/3) = C_1\cos(\cancel{3}(\pi/\cancel{3})) + C_2\sin(\cancel{3}(\pi/\cancel{3}))$$

$$\sqrt{2} = C_1(-1) + C_2(0)$$

$$\therefore \sqrt{2} = -C_1, \quad C_1 = -\sqrt{2}$$

$$f' = C_1 - \sin(3t)[3t]' + \cos(3t)[3t]'$$

$$\rightarrow f' = -3\sin(3t)C_1 + 3\cos(3t)$$

$$\rightarrow \sqrt{3} = f'(\tfrac{\pi}{3}) = -3\sin(3(\tfrac{\pi}{3}))C_1 + 3\cos(3(\pi/3))C_2$$

$$\rightarrow \sqrt{3} = -3\sin(\pi)C_1 + 3\cos(\pi)C_2$$

$$\therefore \sqrt{3} = 3C_2, \quad C_2 = \frac{\sqrt{3}}{3}$$

$= -1$

OK

Math 45, Fall 2020
September 23, Quiz 04

8

Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

1. (*7 points*) Consider the differential equation $y' = y\sqrt{y-x}$. ← already in $f(x,y)$

(i) For what (x, y) is it guaranteed that the differential equation above has a unique solution?

• continuous (x_0, y_0)? not con't at $-x$

• $\frac{d}{dy}F(x,y)$?

$y\sqrt{y-x} \to \cancel{y}-x \le 0$

$y \ge x$

$[y\sqrt{y-x}]'$

$\to y[(y-x)^{1/2}]$

$\to -\frac{y}{2\sqrt{y-x}}$

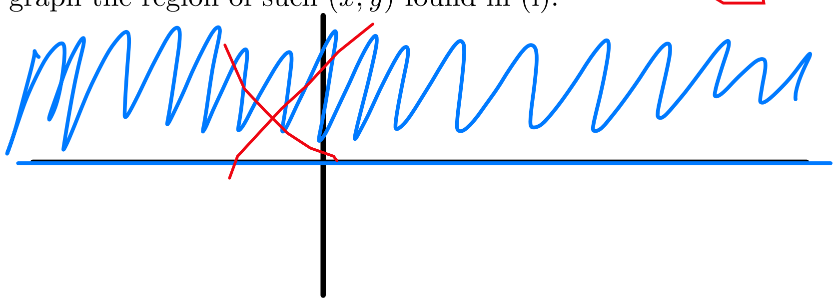
all x, y s.t. $y \ge x$

?

$y > x$

(ii) In the xy-plane, graph the region of such (x, y) found in (i).

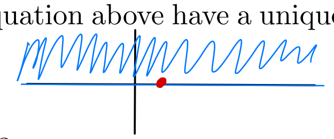
−1



(iii) Is it guaranteed that the differential equation above have a unique solution at the point $(1, 0)$?

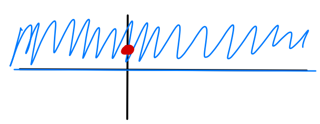
−1

~~Yes~~ ?



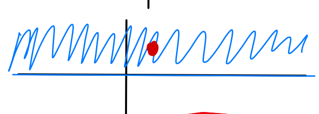
(iv) Same problem as in (iii) but for $(0, 1)$?

Yes



(v) Same problem as in (iii) but for $(1, 1)$?

~~Yes~~



2. (*3 points*) Circle all of the following differential equations which are separable equations.

(i) $\frac{dy}{dx} = x^2y^3$

(ii) $\frac{dy}{dx} = \ln(xy)$

(iii) $w\frac{dw}{dt} = 10 + t$

(iv) $\sqrt{y'} + xy = 0$

(v) $y' + xy + x = 0$

(vi) $xy\frac{dy}{dx} + 1 = 0$

can make x, y on opposite sides of equal sign

Math 45, Fall 2020
September 30, Quiz 05

6.5

Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

1. (*8 points*) For each of the following differential equations, determine whether it is exact or not. (Use math to justify your answer.)

a. (*2 pts*) $x\,dx - y\,dy = 0 \rightarrow x\,dx + (-y)\,dy = 0$

$M(x,y) = x$
$N(x,y) = -y$

$\frac{d}{dx} M = \frac{d}{dx}(x) = 1x^0 = 1$

$\frac{d}{dy} N = \frac{d}{dy}(-y) = -1y^0 = -1$

~~Not~~ same
Not exact

-1

b. (*3 pts*) $y\,dx + x\,dy = 0$

$M(x,y) = \cancel{y} \rightarrow \frac{d}{dx}(y) \cancel{=} 1$

$\cancel{N(x,y) = x} \rightarrow \frac{d}{dy}(x) = 1$

Exact

c. (*3 pts*) $(y - x)\,dx + (x - y)\,dy = 0$

$M(x,y) = (y-x) \rightarrow \frac{d}{dx}(y-x) \rightarrow \frac{d}{dx}(y) - \frac{d}{dx}(x) \rightarrow 0 - 1 = -1$

$N(x,y) = (x-y) \rightarrow \frac{d}{dy}(x-y) \rightarrow \frac{d}{dy}(x) - \frac{d}{dy}(y) \rightarrow 0 - 1 =$

Exact

$-d, 5$

2. (*2 points*) Compute the integrating factor for the differential equation $xy' = 5 - 2y$.

yeah

Need to review this Section More...

Let me know if you have problems

-2

7

Math 45, Fall 2020
October 14, Quiz 06

Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

1. (*6 points*) Is a unique solution guaranteed to exist for the following initial value problems on the given intervals. Explain you answers.

(2) • $\ln(x)y''' + \frac{e^x}{x-4}y' - y = \cos(x)$
• $y(2) = 1$
• $I = (0, 3)$

Linear nonhomogeneous $\neq 0$

a. (*2 pts*) $\ln(x)y''' + \frac{e^x}{x-4}y' - y = \cos(x); \quad y(2) = 1, \quad I = (0, 3).$

(i) coeff of highest order derivative $\neq 0$ on I
(ii) all coefficient function continuous on I
(iii) The "x_0" is in I

$\to \ln(1) = 0,\ x = 1$

No it fails because $x = 1$ and the Interval contains #'s between 0 & 3

b. (*2 pts*) $\ln(x)y''' + \frac{e^x}{x-4}y' - y = x^2; \quad y(3) = 1, \quad I = (2, 4).$

(i) coeff of highest order derivative $\neq 0$ on I
(ii) all coefficient function continuous on I
(iii) The "x_0" is in I

$\to \ln(1) = 0,\ x = 1$

$x_0 = 3$ is 3 contained?

Yes all Pass

(i) Pass
(ii) Pass because 4 isn't included
(iii) Pass

c. (*2 pts*) $\ln(x)y''' + \frac{e^x}{x-4}y' - y = x^2; \quad y(2) = 1, \quad I = (1, 3).$

(i) coeff of highest order derivative $\neq 0$ on I

$\to \ln(1) = 0,\ x = 1$

$x_0 = 2$

Yes all pass

(i) Pass 1 isn't included in I
(ii) Pass 4 isn't included in I
(iii) Pass $x_0 = 2$ is contained in I

2. (*2 points*) In your own words, describe what it means for functions to be *linearly independent*.

-2

3. (*2 points*) Provide an example of the following:

a. (*1 pt*) A 3rd order linear homogeneous differential equation.

$xy''' - \frac{2}{e^x} + y = 0$

b. (*1 pt*) A 2nd order linear nonhomogeneous differential equation.

$y'' + \tan(x)y = e^x$

I'M going to spend more time studying this section | yes!

2

Math 45, Fall 2020
October 21, Quiz 07

Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

1. (*8 points*) Let $f_1(x) = e^x$ and $f_2(x) = e^{x+2}$. Note that both functions are solutions to $y' = y$.

a. (*4 pts*) Compute the *Wronskian* $\mathcal{W}(f_1, f_2)$.

$$W(e^x, e^{x+2}) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \to \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2}[x+2]' \end{vmatrix} \to \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2}[\cancel{x}+\cancel{2}]' \end{vmatrix} \to \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2} \end{vmatrix}$$

1 0

-2

$$\to e^x \cdot e^{x+2} \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2} \end{vmatrix} \to e^{2x+2} \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2} \end{vmatrix}$$

$$\to e^{2x+2}$$

7

b. (*2 pts*) Are $f_1(x) = e^x$ and $f_2(x) = e^{x+2}$ linearly independent? Explain.

-2

c. (*2 pts*) Do $f_1(x) = e^x$ and $f_2(x) = e^{x+2}$ form a fundamental set of solutions for $y' = y$? Explain.

-2

2. (*2 points*) It is true that $y_1 = \sin(x)$ and $y_2 = \sin(x)$ form a fundamental set of solutions to the differential equation $y'' + y = 0$. Meanwhile we have $y_p = e^x$ is a solution to the differential equation $y'' + y = 2e^x$. (You do not need to show either of these previous two statements.) Provide the *general* solution to the differential equation $y'' + y = 2e^x$.

-2

8

Math 45, Fall 2020
October 28, Quiz 08

Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

1. (*10 points*)

a. (*4 pts*) Find a general solution to the DE $y'' - 5y' + 6y = 0$.

Plug-in $y = e^{mx}$; $y' = me^{mx}$; $y'' = m^2e^{mx}$

This gives: $m^2e^{mx} - 5me^{mx} + 6e^{mx} = 0$

Factoring out e^{mx}

$\rightarrow e^{mx}(m^2 - 5m + 6) = 0$

So we need

$\rightarrow m^2 - 5m + 6 = 0$

$y_1 = e^{\left(\frac{-(-5)+\sqrt{(-5)^2-4(1)(6)}}{2(1)}\right)x}$ $= \frac{5+\sqrt{25-24}}{2}x$ $= 6/2 = 3x$

$y_2 = e^{\left(\frac{-(-5)-\sqrt{(-5)^2-4(1)(6)}}{2(1)}\right)x}$ $= \frac{5-\sqrt{25-24}}{2}x$ $= 4/2 = 2x$

$$y = C_1e^{3x} + C_2e^{2x}$$

b. (*2 pts*) Note that $y_p = e^x$ is a solution to the DE $y'' - 5y' + 6y = 2e^x$ (you do not need to show or verify this). Provide a general solution to this DE.

$$y = C_1e^{3x} + C_2e^{2x} + e^x$$

c. (*4 pts*) We have that e^{2x} is a solution to the DE $y'' - 4y' + 4y = 0$. Use the method of reduction of order to find another (linearly independent) solution to this DE and write the general solution for this DE. Need 2 soln

ok!

$Y_1 = e^{2x}$

Too Long

Let $u(x)$ be an arbitrary function, and set

$Y = u(x)Y_1(x) = u(x)e^{2x} = ue^{2x}$.

(a) Find y' and y''

* Remember ue^{2x} that "u" is a function of "x" so we have to use the product rule.

Recall The Product Rule

$\boxed{(uv)' = u'v + uv'}$

$y' = u'e^{2x} + u2e^{2x}$

$u'e^{2x} + u2e^{2x}$

$y'' = u''\sin(x) + u'\cos(x) + u'\cos(x) - u\sin(x)$

$= u''\sin(x) + 2u'\cos(x) - u\sin(x)$

can use the formula too.

-2

Math 45, Fall 2020
November 4, Quiz 09

10

Name: Matthew Mardoye

Please show and explain your work where necessary. Good luck!! You may use the formulas

$$u_1' = -\frac{y_2 f(x)}{W} \qquad \text{and} \qquad u_2' = \frac{y_1 f(x)}{W}$$

sin csc $\to \frac{1}{\sin}$
cos sec $\to \frac{1}{\cos}$
tan cot $-\frac{\cos}{\sin}$

if you so desire.

1. (*10 points*) Use the method of variation of parameters to solve the differential equation $y'' + y = \sec(x)$.

$y'' + y = \sec(x)$

$e^{mx}(y^2+1)=0$ 2nd-order linear nonhomog

$\to$ First, homog: $y''+y=0 \leftrightarrow m^2+1=0$

$m = -\sqrt{-1}, \sqrt{1}$; $\sqrt{-1} \neq \sqrt{1}$

$e^{0}(C_1\cos(x) + C_2\sin(x))$

$y = C_1\cos(x) + C_2\sin(x)$

---

variation of parameters

$$w(f_1,f_2) = \det\left(\begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix}\right)$$

$= (\cos\cdot\cos) + (-(-\sin)\cdot\sin)$

$= 1 \leftarrow$ Pythag

$$u' = \frac{-\sin(x)\sec(x)}{1}$$

~~x~~ still need antideriv

$$u_1' = \int \frac{-\sin(x)\sec(x)}{1}\,dx$$

$$= -\int \sin(x)\sec(x)\,dx$$

$$= -\int \frac{\sin}{\cos}\,dx$$ u-sub: cos(x)

good

$$= \int -\frac{1}{u}\,du = -\ln|u| = -\ln|\cos(x)|$$

$$u_2 = \int \frac{\cos(x)\sec(x)}{1}\,dx$$

$$= \int \left(\frac{\cos(x)}{1} \cdot \frac{1}{\cos(x)}\right) dx$$

$$= x$$

Particular solution

$$y_p = u_1 y_1 + u_2 y_2$$

$$= \ln|\cos(x)|\cos(x) + x\sin(x)$$

Particular solution

$$y_p = u_1 y_1 + u_2 y_2$$

$$= \ln|\cos(x)|\cos(x) + x\sin(x)$$

$\therefore$ General solution

$$y = y_h + y_p$$

$$y_h = C_1\cos(x) + C_2\sin(x)$$

$$y_p = \ln|\cos(x)|\cos(x) + x\sin(x)$$

$$y = C_1\cos(x) + C_2\sin(x) + \ln|\cos(x)|\cos(x) + x\sin(x)$$

40.5/50

Please show and explain your work. Justification is necessary for credit. Also sign your name here acknowledging you will not use problem solving software (Wolfram Alpha, etc.): Matthew Mendez ICe

1. (*14 points*)

a. (*2 pts*) Circle all of the following expression which are **linear** differential equations.

(i) ~~$y'''' - 2y''' + 3y = 5$~~

(iii) $\sqrt{y'} = x\cos(x)y - 3$

* Can't have 'n' be any higher power greater than one *

(ii) ~~$(x^2 - \sin(x))\, y^{(5)} - xe^x y + \sin(x^2) = 0$~~

(iv) ~~$(y')^2 = y^3 + 2$~~

-1.5

b. (*2 pts*) Circle all of the following expression of the form $M(x,y)\,dx + N(x,y)\,dy = 0$ such that **both** $M(x,y)$ and $N(x,y)$ are homogeneous *functions* of the **same** degree.

$\frac{\partial}{\partial y}x^2 = x^2$

(i) ~~$x^2\,dx + yx\,dy = 0$~~

(iii) $\sin(y)\,dx + (2y + xe^y)\,dy = 0$

? -1

(ii) $(3xy)\,dx + e^{\left(\frac{x}{y}\right)}\,dy = 0$

(iv) $(x - 2y)\,dx + x\,dy = 0$

c. (*4 pts*) Circle whether the following differential equation is an ordinary differential equation (ODE) or a partial differential equation (PDE) and state the **order** of the differential equations (you can do this for any PDEs as well).

(i) $\ln(y')y + x^2y = 1$ — ODE *PDE* *Order:* 2 — -0

(ii) $\frac{\partial f}{\boxed{\partial x}} + \frac{\partial^2 f}{\boxed{\partial y^2}} + xy = 0$ — ODE *PDE* *Order:* 1 — least one multivariable function.

(iii) $x^2\frac{d^3 f}{dx^3} + \frac{df}{dx} = \frac{d^2 f}{dx^2} - \sin(x)$ — ODE *PDE* *Order:* 2 — least one function of one independent variable.

(iv) $\left(\frac{d}{dx}\right)^4 f(x) + \left(\frac{d^2}{dx^2}\right) f(x) + f'(x) = 0$ — ODE *PDE* *Order:* 4 — least one function of one independent variable.

d. (*4 pts*) Circle all of the following expression which are **exact** differential equations. (Provide work for partial credit!)

(i) ~~$\overset{M}{xy\,dx} + \overset{N}{xy\,dy} = 0$~~ X

$\frac{\partial}{\partial y}xy = x,\ \frac{\partial}{\partial x}xy = y$

(iii) $\overset{M}{xy^2\,dx} + \overset{N}{x^2y\,dy} = 0$

$\frac{\partial}{\partial y}xy^2 = 2xy,\ \frac{\partial}{\partial x}x^2y = 2xy$ ✓

(ii) $x^2y\,dx + \frac{1}{3}x^3\,dy = 0$

$\frac{\partial}{\partial y}x^2y = x^2\quad \frac{\partial}{\partial x}x^2$ ✓

(iv) ~~$\sin(xy)\,dx + xy\cos(x)\,dy = 0$~~

cos

No grad!

e. (*2 pts*) Suppose $y^2 - x = 3$ is a solution to a differential equation. Is this solution implicit or explicit? Explain your answer in one sentence.

Explicit; for, you can solve for y: $y = \cancel{\pm}\sqrt{3+x}$

No I

-0.5

2. (*27 points*) Solve the following differential equations using any technique you like that works. However, **you must explain WHY you chose that method** to receive credit!

a. (*9 pts*) $y' = \dfrac{x}{y^2}$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$$\rightarrow y^2\,dy = x\,dx$$

$$\rightarrow \int y^2\,dy = \int x\,dx$$

$$\rightarrow \frac{1}{3}y^3 = \frac{1}{2}x^2 + C$$

$$\rightarrow y^3 = \frac{3}{2}x^2 + 3C$$

$$\therefore y = \sqrt[3]{\frac{3}{2}x^2 + 3C}$$

dx'

Why: seprable $\rightarrow \dfrac{dy}{dx} = y'$

b. (*9 pts*) $x\dfrac{dy}{dx} - 2y = x^3e^x$

~~why: It looks separable~~

$$\frac{x\frac{dy}{dx}}{x} - \frac{2y}{x} = \frac{x^3e^x}{x}$$

$$\rightarrow \frac{dy}{dx} - \frac{2y}{x} = x^2e$$

$$\rightarrow \frac{dy}{dx} = \frac{2y}{x} + x^2e$$

$$\rightarrow \frac{dy}{dx} = \frac{1}{x}2y + x^2e$$

$$x\frac{dy}{dx} - 2y = x^3e^x$$
$$+2y \qquad +2y$$
$$\rightarrow x\frac{dy}{dx} = x^3e^x + 2y$$
$$-x^3e^x \qquad -x^3e^x$$
$$\rightarrow x - x^3e^x\frac{dy}{dx} = 2y$$

wrong approach

1st-order linear equation

why: it follows form "$\frac{dy}{dx} + Py = Q$"

$$\frac{x\frac{dy}{dx}}{x} - \frac{2y}{x} = \frac{x^3e^x}{x}$$

standard form

$$\rightarrow \frac{dy}{dx} - \frac{2y}{x} = \frac{x^3e^x}{x}$$

* Can't have $x = \emptyset$ anymore *

$$\rightarrow \frac{dy}{dx} - \underbrace{\frac{2}{x}}_{P(x)}y = \underbrace{\frac{x^3e^x}{x}}_{Q(x)}$$

$$\rightarrow \frac{dy}{dx} - \underbrace{\frac{2}{x}}_{P(x)}y = \underbrace{\frac{x^3e^x}{x}}_{Q(x)}$$

I think this is where I'm confusing myself

$$\rightarrow \frac{dy}{dx} + \left(\underbrace{-\frac{2}{x}}_{P(x)}y\right) = \underbrace{\frac{x^3e^x}{x}}_{Q(x)}$$

Calc Integrating factor

$$e^{\int P\,dx} = e^{\int -\frac{2}{x}dx} = e^{-2\ln|x|}$$

$$\rightarrow e^{-2\ln|} = e^{\ln|x^{-2}|} = x^{-2}$$

good!!

Multiply left & Right by Integrating factor

$$\underbrace{x^{-2}\left(\frac{dy}{dx} - \frac{2y}{x}\right)}_{\text{Product Rule}} = x^{-2}\left(\frac{x^3e^x}{x}\right)$$

Rest?

-2

$$\rightarrow \frac{d}{dx}\left(x^{-2}y\right)$$

c. (*9 pts*) $\underbrace{2xy\,dx}_{M} + \underbrace{(x^2+3y^2)}_{N}\,dy = 0$

Exact equation?
why: follows form $M(x,y)dx + N(x,y)dy = 0$

Step 0
$\frac{\partial}{\partial y}(2xy) = 2x$
$\frac{\partial}{\partial x}(x^2+3y^2) = 2x$ } Yes ✓

Step 1 Anti-deriv w/ respect to "x"
$F(x,y) = \int (xy)\,dx$
$\to \frac{1}{2}x^2y$
$\to \frac{1}{2}x^2y + g(y)$

Step 2 take partial deriv of F w/ y
$\frac{\partial}{\partial y}\left(\frac{1}{2}x^2y + g(y)\right) = \frac{1}{2}(x^2y^2) + g'(y)$

Step 3 set partial-deriv of F w/ "y" equal to N(x,y) & solve for g'(y)
$\frac{\frac{1}{2}(x^2y^2) + g'(y)}{x^2y^2} = \frac{\cancel{x^2}}{x^2y^2} + \frac{3y^2}{x^2y^2}$
$\to 2 \cdot 1/2 + g'(y) = (y^{-2} + 3)\,2$
$\to g'(y) = 2(y^{-2} + 3)$

Step 4 Integrate g'(y)
$g'(y) = 2\left(\frac{1}{y^2}\right) + 3$
$\to g(y) = 2\ln|y^2| + C$

Step 5 Plug step 4 in step 1 anti-derive w/r to "x"
$F(x,y) = \frac{1}{2}x^2y + g(y)$
$\to \frac{1}{2}x^2y + 2\ln|y^2| + C$

Step 6 Thus we have In form $\boxed{f(x,y) = C}$
$C = \frac{1}{2}x^2y + 2\ln|y^2|$ * Implicit solution

—3

3. (*9 points*)

a. (*8 pts*) We note that $y(x) = c_1e^{3x} + c_2e^{-3x}$ is a solution to the differential equation $y'' - 9y = 0$. Find a solution to the initial conditions $y(0) = 0$ and $y'(0) = 2$.

From Notes

Ex (i) Verify that $y = K\cos(2x)$ satisfies the DE $y'' + 4y = 0$.

(ii) Find a solution for the IVP consisting of $y'' + 4y = 0$ and the conditions $y(0) = 3$, $y'(0) = 0$

Soln

(i) $y = K\cos(2x)$, $y' = -2K\sin(2x)$, $y'' = -4K\cos(2x)$.
Thus, $y'' + 4y = -4K\cos(2x) + 4K\cos(2x) = 0$ ✓

(ii) $y(0) = 3$ | $y'(0) = 0$
$\to K\cos(0) = 3$ | $\to -2K\sin(0) = 0$
$\to K = 3$ | $\to 0 = 0$

Thus, $y = 3\cos(2x)$ works

Blanking out for some reason But it looks like it checks-out ✓

$y = c_1e^{3x} + c_2e^{-3x}$
$y' = (c_1e^{3x} \cdot 3) + (c_2e^{-3x} \cdot -3)$
$y'' =$

Chain Rule
$e^x \cdot \boxed{x}$

$y(0) = 0$
$\to c_1e^{3(0)} + c_2e^{-3(0)} = 0$
$\to c_1(1) + c_2(1) = 0$
$\to c = 0$

1 ✓ ⟷ grad. and?

$y'(0)$

b. (*1 pt*) Does the trivial solution also satisfy the initial value problem given above in **3a**? Explain your answer.

Yes

The Trival Solution
Definition: If the function $y = 0$ is a solution on an interval, we can call this the Trival Solution.

	Solution to a DE	Particular (one solution)	Parameter	Could it be general	trival
$y = Ce^{2x}$	✗	✓ - one para	✓ - (?)	✗
$y = 3\cos(4x)$	✓	✗	✗ - (?)	✗
$y = 0$	✓	✗	✗ - (?)	✓

21.5/50

Please show and explain your work. Justification is necessary for credit. Also sign your name here acknowledging you will not use problem solving software (Wolfram Alpha, etc.): Matthew Mendoza

1. (*5 points*)

a. (*2 pts*) Circle all of the following expression which are **homogeneous linear differential equations**.

(i) ~~$y''' - 2y''' + 3y = 5$~~ $b(x) \neq 0$

(iii) ~~$\sqrt{y'} = x\cos(x)y - 3$~~ $b(x) \neq 0$

(ii) $\left(x^2 - \sin(x)\right) y^{(5)} - xe^x y' + \sin\left(x^2\right) y = 0$

(iv) $y'' = y + 2y'$ Rewritten as $y'' - 2y' - y = 0$

4.5 Nice

b. (*2 pts*) We have that $y_h = c_1e^x + c_2e^{-x}$ is a general solution for the DE $y'' - y = 0$ and that $y_p = x^3$ is a particular solution to $y'' - y = 6x - x^3$ (you do not need to verify these statements). Provide a general solution to the DE $y'' - y = 6x - x^3$.

$$y = y_h + y_p$$

$$y = C_1e^x + C_2e^{-x} + x^3$$

Recall-Module10

General Solutions for Nonhomogeneous Linear Differential Equations

Given (a, b, c are constants)

(*) $ay'' + by' + cy = f(x)$

with $f(x) \neq 0$, the general solution is of the form

$y = y_h + y_p$

Where

→ y_h is the general solution of $ay'' + by' + cy = 0$, and

We are familiar with finding the general solution of the homogeneous piece

This is true for higer order as well

??? → y_p is a particular solution of (*)

c. (*1 pt*) Compute the Wronskian of $y_1 = x$ and $y_2 = x^2$.

$$w(f_1, f_2) = \det\begin{pmatrix} f_1 & f_2 \\ f_1' & f_2' \end{pmatrix} \rightarrow (1\cdot 2x) - (1\cdot x^2)$$

$$w(x, x^2) = \det\begin{pmatrix} x & x^2 \\ 1 & 2x \end{pmatrix} = -x^2 + 2x$$

2

2. (*10 points*) Given that $y_1 = x\sin(x)$ is a solution to the DE $x^2y'' - 2xy' + (x^2 + 2)y = 0$, find a general solution to this differential equation.

$$y_2 = y_1(x)\int \frac{e^{-\int P(x)dx}}{(y_1(x))^2}dx$$

$$= x\sin(x)\int \frac{e^{-\int(-2x)dx}}{(x\sin(x))^2}dx$$

$$= x\sin(x)\int \frac{e^{+2}}{x^2\sin^2(x)}dx$$

$$=$$

$$y = x\sin(x) +$$

1.5

0

Reduction of Order - Summary

When to use: Given a 2nd-order homogeneous linear differential equation and ONLY ONE solution.

Why to use: Find the other linearly independent solution to form a fundamental set of solutions.

How to use: TWO OPTIONS

ⓐ Long route above (don't memorize)

ⓑ The formula

$$y_1(x)\int \frac{e^{-\int P(x)dx}}{(y_1(x))^2}dx$$

Summary

The most important thing with these problems is (1) Plug-in $y=e^{mx}$ and (2) Finding out if $m_1=\frac{-b+\sqrt{b^2-4ac}}{2a}$ & $m_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$ are of

Case 1) Two Distinct Roots

Case 2) Repeated Root

Case 3) Complex Conjugate Roots

3. (*20 points*) Solve the following differential equations. If a technique is asked for, use it.

a. (*5 pts*) $y'' - 6y' + 9y = 0$

1) Plug-in $y=e^{mx}$, $y'=me^{mx}$, $y''=m^2e^{mx}$

Skipping a step, instead of plugging-in and refactoring out, we know are able to factor out "e^{mx}"

This gives us

$$e^{mx}(m^2-6m+9)=0$$

So we need

$$m^2-6m+9=0$$

2) Find m_1 & m_2

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{+6\pm\sqrt{6^2-4\cdot 9}}{2}$$

$$=\frac{6\pm\sqrt{36-36}}{2}$$

$$=\frac{6\pm 0}{2}=3$$

Case 2 Repeated Root

$$y=C_1e^{3x}+xC_2e^{3x}$$

In summary: To get y_2 all you need to do is multiply y_1 by 'x'.

b. (*5 pts*) $y'' + 4y' + 7y = 0$

$$e^{mx}(m^2+4m+7)=0$$

$$\rightarrow m^2+4m+7=0$$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-4\pm\sqrt{4^2-4\cdot 7}}{2}$$

$$=\frac{-4\pm\sqrt{16-28}}{2}$$

$$=\frac{-4\pm\sqrt{-12}}{2}$$

$$=\frac{-4\pm i\sqrt{12}}{2}=-2\pm\frac{i\sqrt{12}}{2}$$

*** Recall ***

$e^{i\theta}=\cos(\theta)+i\sin(\theta)$

$$e^{(\underbrace{3/2}_{\alpha}+\underbrace{\frac{i\sqrt{7}}{2}}_{i\beta})x}=e^{3/2x}e^{\frac{i\sqrt{7}}{2}x}$$

$$\rightarrow e^{3/2x}\Big(\underbrace{\cos\left(\tfrac{\sqrt{7}}{2}x\right)}_{\text{Real}}+\underbrace{i\sin\left(\tfrac{\sqrt{7}}{2}x\right)}_{\text{Complex}}\Big)$$

Euler's Formula — Focus on the real — Ignore the complex

3) Using the formula (Euler's Formula)

$$y_1=e^{\alpha x}\cos(\beta x),\quad y_2=e^{\alpha x}\sin(\beta x)$$

We find

$$y_1=e^{-2x}\cos\left(\tfrac{\sqrt{12}}{2}x\right)$$

$$y_2=e^{-2x}\sin\left(\tfrac{\sqrt{12}}{2}x\right)$$

$$\therefore\ y=C_1e^{-2x}\cos\left(\tfrac{\sqrt{12}}{2}x\right)+C_2e^{-2x}\sin\left(\tfrac{\sqrt{12}}{2}x\right)$$

ok

c. (*10 pts*) $y'' - 4y' + 3y = \cos(x) + \sin(x)$ [using the method of undetermined coefficients]

Lets first consider the homogeneous part

General solution for

$$y''-4y'+3y=0$$

1) Plug-in $y=e^{mx}$, $y'=me^{mx}$, $y''=m^2e^{mx}$

Skipping a step, instead of plugging-in and refactoring out, we know are able to factor out "e^{mx}"

This gives us

$$e^{mx}(m^2-4m+3m)=0$$

So we need

$$m^2-4m+3=0$$

2) Find m_1 & m_2

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{4\pm\sqrt{16-12}}{2}$$

$$=\frac{4\pm\sqrt{4}}{2}=2+1,\ 2-1$$

$$m_1=3,\ m_2=1$$

$$y_h=C_1e^{3x}+C_2e^{x}$$

Recall-Module10

General Solutions for Nonhomogeneous Linear Differential Equations

Given (a,b,c are constants)

(*) $ay''+by'+cy=f(x)$

with $f(x)\neq 0$, the general solution is of the form

$$y=y_h+y_p$$

Where

🙂 → y_h is the general solution of $ay''+by'+cy=0$, and

We are familiar with finding the general solution of the homogeneous piece

This is true for higer order as well

??? → y_p is a particular solution of (*)

Case 2) f(x) consists of sin(x) and/or cos(x) terms.

Any sines or cosines (not products of them i.e. $\sin^2$ or $\cos^2$)

$$y_{p_2}=A\cos(x)+B\sin(x)$$

$$y_{p_2}'=-A\sin(x)+B\cos(x)$$

$$y_{p_2}''=-A\cos(x)-B\sin(x),$$

So $y''-4y'+3y$

$$\rightarrow(-A\cos(x)-B\sin(x))-4(-A\sin(x)+B\cos(x))+3(A\cos(x)+B\sin(x))$$

$$\rightarrow(-A\cos(x)-B\sin(x))+(4A\sin(x)-4B\cos(x))+(3A\cos(x)+3B\sin(x))=\cos(x)+\sin(x)$$

$$2A\cos(x)-4B\cos(x)+4A\sin(x)+2B\sin(x)$$

Cosine (LHS = RHS) — Add up all cosine terms

$$2A-4B=1$$

$$A=\frac{1+4B}{2}$$

$$A=\frac{1+4(-1/10)}{2}$$

Sine (LHS = RHS) — Add up all sine terms

$$4A+2B=1$$

$$2(4B+1)+2B=1$$

$$8B+2+2B=1$$

$$-2\qquad -2$$

$$8B+2B=-1$$

$$10B=-1$$

$$B=-1/10$$

$$y_p=\frac{1+4(-1/10)}{2}\cos+-1/10\sin$$

10

10

Nice

augh running out of time...

4. (*5 points*) Suppose $y_1 = x$ and $y_2 = x - 1$ are solutions to a 2nd-order homogeneous linear DE on $I = (-\infty, \infty)$. Do y_1 and y_2 form a fundamental set of solutions for this DE on I? Explain your answer using any math needed.

0/5

5. (*10 points*) Use the method of variation of parameters to find a general solution to the differential equation $x^2y'' - 4xy' + 4y = 3x^3$.

Consider: $a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x)$

In particular before we'd have just constants. Where $a_2(x)y''$, $a_1(x)y'$, $a_0(x)y$, $g(x)$ are functions

The Method:

Step 1 First turn

$a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x)$

↓ into ↓

$y'' + P(x)y' + Q(x) = f(x)$

The idea is to first put it in to this form

$f(x) = \frac{g(x)}{a_2(x)}$, $P(x) = \frac{a_1(x)}{a_2(x)}$ So you divide everything by whatever function is in y''

Step 2 Find our homogeneous peice ($Y_h = c_1y_1 + c_2y_2$), which is the general solution for $y'' + P(x)y' + Q(x)y = 0$

Step 3 Then $Y_p = u_1y_1 + u_2y_2$, depends on the homogeneous peice (Y_1 & Y_2) where u_1 and u_2 are found by integrating

$$u_1' = \frac{-y_2 f(x)}{W} \text{ and } u_2' = \frac{y_1 f(x)}{W},$$

$$\underbrace{\text{and } W(y_1, y_2) \text{ is the wronskian of } y_1, y_2}$$

$$Y_p = u_1y_1 + u_2y_2$$

so? But right?

3/10