1. (1 point) Evaluate the following matrix-vector product.

$$\begin{bmatrix} 5 & -2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 9 \\ -3 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

2. (1 point) Perform the following matrix operation:

$$\begin{bmatrix} -6 \\ 5 \\ -9 \end{bmatrix} - \begin{bmatrix} -9 \\ -8 \\ 7 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

3. (1 point) Perform the following matrix operation:

$$\begin{bmatrix} -5 & -9 \\ 1 & -2 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -5 \\ -6 & -9 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

4. (1 point)
If

$$A = \begin{bmatrix} -8 \\ -5 \\ -4 \end{bmatrix}$$

then

$$5A = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

Consider the system of differential equations

$$\frac{dx}{dt} = 7x + 2y + e^{8t}$$
$$\frac{dy}{dt} = 5x + 7y + \sin(7t).$$

Which of the following expressions is the matrix form of this system of DEs?

- A. $\mathbf{X}' = \begin{pmatrix} 7 & 2 \\ 5 & 7 \end{pmatrix}\mathbf{X} + \begin{pmatrix} \sin(7t) \\ e^{8t} \end{pmatrix}$
- B. $\mathbf{X}' = \begin{pmatrix} 7 & 2 \\ 5 & 7 \end{pmatrix}\mathbf{X} + \begin{pmatrix} e^{8t} \\ \sin(7t) \end{pmatrix}$
- C. $\mathbf{X}' = \begin{pmatrix} 7 & 2 \\ 4 & 7 \end{pmatrix}\mathbf{X} + \begin{pmatrix} e^{8t} \\ \sin(7t) \end{pmatrix}$
- D. $\mathbf{X}' = \begin{pmatrix} 7 & 2 \\ 5 & 7 \end{pmatrix}\mathbf{X}$

6. (1 point) *This is the first part of a four-part problem.*

Let

$$P = \begin{bmatrix} 9 & -4 \\ 15 & -7 \end{bmatrix},$$

$$\vec{y}_1(t) = \begin{bmatrix} 2e^{3t} + 8e^{-t} \\ 3e^{3t} + 20e^{-t} \end{bmatrix}, \quad \vec{y}_2(t) = \begin{bmatrix} -4e^{3t} + 2e^{-t} \\ -6e^{3t} + 5e^{-t} \end{bmatrix}.$$

(1) Show that $\vec{y}_1(t)$ is a solution to the system $\vec{y}\,' = P\vec{y}$ by evaluating derivatives and the matrix product

$$\vec{y}_1\,'(t) = \begin{bmatrix} 9 & -4 \\ 15 & -7 \end{bmatrix}\vec{y}_1(t)$$

Enter your answers in terms of the variable t.

$$\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(2) Show that $\vec{y}_2(t)$ is a solution to the system $\vec{y}\,' = P\vec{y}$ by evaluating derivatives and the matrix product

$$\vec{y}_2\,'(t) = \begin{bmatrix} 9 & -4 \\ 15 & -7 \end{bmatrix}\vec{y}_2(t)$$

Enter your answers in terms of the variable t.

$$\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

7. (1 point) Select all of system of linear differential equation IVPs which are guaranteed the existence of a unique solution on the interval $I = (0, \infty)$.

- A. $\mathbf{X}' = \begin{pmatrix} 4 & 6 \\ 4 & 6 \end{pmatrix}\mathbf{X}$
- B. $\mathbf{X}' = \begin{pmatrix} 4 & 6 \\ 4 & 6 \end{pmatrix}\mathbf{X} + \begin{pmatrix} \frac{1}{(t-4)^6} \\ t^3 \end{pmatrix}$
- C. $\mathbf{X}' = \begin{pmatrix} 4 & 6 \\ 4 & 6 \end{pmatrix}\mathbf{X} + \begin{pmatrix} e^{1t} \\ \sin(3t) \end{pmatrix}$
- D. $\mathbf{X}' = \begin{pmatrix} 6 & 4 \\ 6 & 6 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 0 \\ 3t \end{pmatrix}$

- E. $\mathbf{X}' = \begin{pmatrix} 4 & 6 \\ 4 & 6 \end{pmatrix} \mathbf{X} + \begin{pmatrix} \sqrt{t} \\ \frac{1}{t} \end{pmatrix}$

- F. $\mathbf{X}' = \begin{pmatrix} 4 & 6 \\ 4 & 6 \end{pmatrix} \mathbf{X} + \begin{pmatrix} \sqrt{t-1} \\ t^3 \end{pmatrix}$

- G. $\mathbf{X}' = \begin{pmatrix} 4 & 6 \\ 4 & 6 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{1t} \\ \frac{1}{t-3} \end{pmatrix}$

- H. None of the above

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8. (1 point)

Suppose $y_1 = \begin{pmatrix} 8e^{6t} \\ 9e^{6t} \end{pmatrix}$ and $y_2 = \begin{pmatrix} 2e^{5t} \\ 7e^{5t} \end{pmatrix}$. Compute the Wronskian $W(y_1, y_2)$.

(Note: your answer should be a function of t.)
$W(y_1, y_2) =$__________

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9. (1 point) Consider the vector functions $y_1(t) = \begin{pmatrix} f_1(t) \\ g_1(t) \end{pmatrix}$ and $y_2(t) = \begin{pmatrix} f_2(t) \\ g_2(t) \end{pmatrix}$. Mark all of the following which must be true if $y = c_1 y_1 + c_2 y_2$ is a general solution to the system of linear equations $\mathbf{X}' = A\mathbf{X}$, where A is a matrix.

- A. y_1, y_2 are solutions to the system of differential equation.

- B. The system is nonhomogeneous.

- C. The system consists of two equations with two unknown functions.

- D. y_1, y_2 are linearly dependent.

- E. y_1, y_2 are linearly independent.

- F. y_1, y_2 are linear combinations.

- G. None of the above

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10. (1 point) Suppose that the matrix A has the following eigenvalues and eigenvectors:
$\lambda_1 = 3$ with $\vec{v}_1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$.
and
$\lambda_2 = 4$ with $\vec{v}_2 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

Write the solution to the linear system $\mathbf{X}' = A\mathbf{X}$ in the following forms.

A. In eigenvalue/eigenvector form:
$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix} e^{\_\_t} + c_2 \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix} e^{\_\_t}$

B. As two equations: *(write "c1" and "c2" for c_1 and c_2)*
$x(t) =$__________
$y(t) =$__________

Hint: For part B, you can multiply out part A and solve for $x(t)$ and $y(t)$.

Note: Both forms of writing the solution above are correct and acceptable.