

1. (1 point)

- (1) Set up an integral for finding the Laplace transform of $f(t) = 1$. (Don't forget any dt terms.)

$$F(s) = \mathcal{L}\{f(t)\} = \int_A^B \text{_____ help (formulas)}$$

where $A = \text{_____}$ and $B = \text{_____}$. (Note: use the word INFINITY for ∞ .)

- (2) Find the antiderivative (with constant term 0) corresponding to the previous part.

- (3) Evaluate appropriate limits to compute the Laplace transform of $f(t)$:

$$F(s) = \mathcal{L}\{f(t)\} = \text{_____}$$

- (4) Where does the Laplace transform you found exist? In other words, what is the domain of $F(s)$?

_____ help (inequalities)

2. (1 point)

Use the definition of the Laplace transform to find $F(s) = \mathcal{L}\{f(t)\}$ for the function $f(t) = e^{3t+8}$, for $s > 3$.

$$F(s) = \mathcal{L}\{f(t)\} = \text{_____ help (formulas)}$$

Answers:

Given that:
By the definition of Laplace transforms

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

from question $f(t) = 6$.

$$\begin{aligned} \textcircled{a} F(s) &= \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt \\ &= \int_0^{\infty} e^{-st} 6 dt \\ &= 6 \int_0^{\infty} e^{-st} dt \\ &= 6 \left[\frac{e^{-st}}{-s} \right]_0^{\infty} \\ &= -\frac{6}{s} [e^{-\infty} - e^0] \\ &= -\frac{6}{s} (0 - 1) \\ &= \frac{6}{s} \end{aligned}$$

$$\begin{pmatrix} e^{-\infty} = 0 \\ e^0 = 1 \end{pmatrix}$$

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} 6 dt \quad \text{only want this one for part (a)}$$

$$\text{and } \boxed{A=0} \text{ and } \boxed{B=\infty}$$

(b) finding an antiderivative corresponding to previous part.

$$F(s) = 6 \int_0^{\infty} e^{-st} dt$$

$$\begin{aligned} F(s) &= 6 \left[\frac{e^{-st}}{-s} \right]_0^{\infty} \quad \text{only want this one for part (b).} \\ &= -\frac{6}{s} [e^{-\infty} - e^0] \\ &= -\frac{6}{s} [0 - 1] \\ &= \frac{6}{s} \end{aligned}$$

$$\therefore \boxed{F(s) = \frac{6}{s}}$$

(c) evaluating appropriate limits to be computing h.T of $f(t)$

$$\begin{aligned} F(s) &= \mathcal{L}\{f(t)\} \\ &= -\frac{6}{s} [e^{-\infty} - e^0] \\ &= -\frac{6}{s} (0 - 1) \\ &= -\frac{6}{s} (-1) = \frac{6}{s} \end{aligned}$$

$$\boxed{F(s) = \frac{6}{s}}$$

① where does the laplace transform you found exist and what domain exist

Domain of $f(s)$ is all real numbers except for $s=0$.

Domain of $f(s) = \frac{6}{s} = \frac{6}{s} = (-\infty, 0) \cup (0, +\infty)$

Domain of $f(s) = \frac{6}{s}$ at $(-\infty, 0) \cup (0, +\infty)$

Laplace Transform of e^{3t+8} : $\frac{e^8}{s-3}$

Steps

$$L\{e^{3t+8}\}$$

Use the constant multiplication property of Laplace Transform:

For function $f(t)$ and constant a : $L\{a \cdot f(t)\} = a \cdot L\{f(t)\}$

$$= e^8 L\{e^{3t}\}$$

$$L\{e^{3t}\}: \frac{1}{s-3}$$

Hide Steps

$$L\{e^{3t}\}$$

Use Laplace Transform table: $L\{e^{at}\} = \frac{1}{s-a}$

$$L\{e^{3t}\} = \frac{1}{s-3}$$

$$= \frac{1}{s-3}$$

$$= e^8 \frac{1}{s-3}$$

$$\text{Refine } e^8 \frac{1}{s-3}: \frac{e^8}{s-3}$$

Hide Steps

$$e^8 \frac{1}{s-3}$$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$$= \frac{1 \cdot e^8}{s-3}$$

Multiply: $1 \cdot e^8 = e^8$

$$= \frac{e^8}{s-3}$$

$$= \frac{e^8}{s-3}$$