A note on domain and range of functions and solutions

We take a brief moment to review what functions we are interested in this course and to recall their domains.

Note, however, that the domain of a function is not necessarily the domain of the solution of a DE!!

Below is a table with some common functions we will encounter and their domains.

Polynomials (x^2, x^3, 1, $2+3x^5$, etc.)	$(-\infty, \infty)$
$\frac{1}{x-c}$, $\frac{1}{(x-c)^2}$, etc. (c a constant)	$(-\infty, c) \cup (c, \infty)$, i.e., all numbers but c
$\sqrt{x}$, $\sqrt[4]{x}$, $\sqrt[6]{x}$, etc. (even roots)	$[0, \infty)$
$\ln(x)$	$(0, \infty)$
e^x	$(-\infty, \infty)$
$\sin(x), \cos(x)$	$(-\infty, \infty)$
$\tan(x)$	All x except $x = \frac{\pi}{2}k$ for integers k
$\sec(x)$	All x except $x = \frac{\pi}{2}k$ for integers k
$\csc(x)$	All x except $x = \pi k$ for integers k
$\cot(x)$	All x except $x = \pi k$ for integers k

Being able to extend these properties will also be crucial. For example, using the fourth line of the table to deduce the domain of $\ln(x+5)$ is $(-5, \infty)$.

While the domain of a solution to a DE may be different that the domain of the underlying function, it can only be more restrictive. That is, to determine the domain of a solution, we can first consider the domain of the underlying function, and then possibly remove points.

WeBWorK module 02 exercises:

- Problems 4,5

Relevant Wikipedia articles:

- **Domain of a function** (https://en.wikipedia.org/wiki/Domain_of_a_function#Natural_domain)