Existence and uniqueness of solutions for systems

Here we answer the question: *Does a solution always exist?*

We restrict our answer to initial value problems. We note that we write the initial conditions of a IVP

$$X' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} X + F$$

as $X(t_0) = X_0$. Here, $X(t_0) = X_0$ represents the point t_0 being plugged into each component of X. That is, $X(t_0) = X_0$ could also be written as

$$\begin{pmatrix} x(t_0) \\ y(t_0) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix},$$

where $x(t_0) = x_0$ and $y(t_0) = y_0$ are the initial conditions on the functions x and y, respectively. With the notation behind us, we state the following theorem which provides us with an answer to the question above.

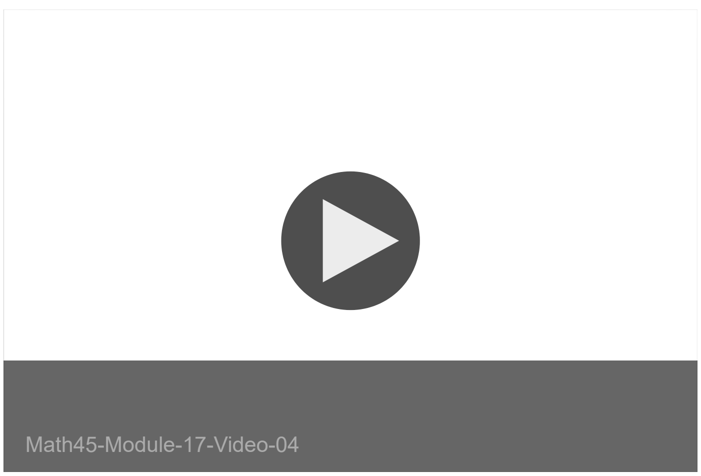
Theorem

Consider an IVP of the form

$$X' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} X + \begin{pmatrix} f \\ g \end{pmatrix}$$

with initial condition $X(t_0) = X_0$. If $f(t)$ and $g(t)$ are continuous on an interval I that contains t_0, then there exists a unique solution to the IVP on I.

Discussion, comments, and examples:



WeBWorK module 17 exercises:

- Problem 7

Relevant Wikipedia articles:

- See Theorem 10.2.1 in the textbook (https://digitalcommons.trinity.edu/mono/8/)