Solving systems of linear DEs

Finally, we answer the question: *How do we solve such systems of differential equations?*

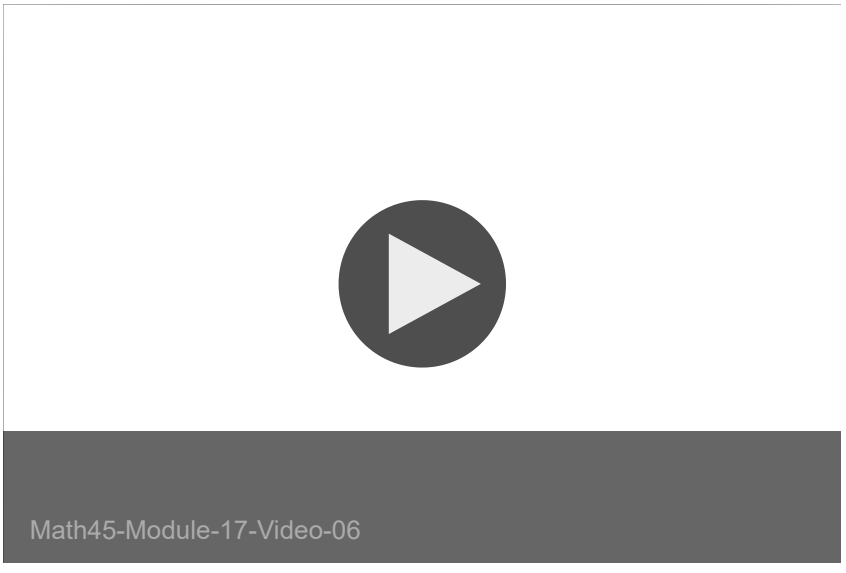
We lay out the steps for answering this question for a 2×2 system of the form $\boldsymbol{X}' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \boldsymbol{X}$. They are

- Find eigenvalues for the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, which we denote λ_1, λ_2;
  - *Note:* We will only look at the case involving two distinct real eigenvalues.
- Find eigenvectors for $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, which we denote $\overrightarrow{v_1}, \overrightarrow{v_2}$;
- Then a fundamental set of solutions is given by $y_1 = \overrightarrow{v_1} e^{\lambda_1 t}$ and $y_2 = \overrightarrow{v_2} e^{\lambda_2 t}$, and the general solution is given by

$$y = c_1 \overrightarrow{v_1} e^{\lambda_1 t} + c_2 \overrightarrow{v_2} e^{\lambda_2 t}.$$

Unfortunately, this means we need to learn how to find eigenvalues and eigenvectors!! This is no easy task. However, in the video below we look at a detailed example of solving a system of linear differential equations. Any problems you will encounter will be similar to this example.

Discussion, comments, and examples:



WeBWorK module 17 exercises:

- Problem 10

Relevant Wikipedia articles:

- See Theorem 10.4.1 and Exercise 10.4.1 in the **textbook (https://digitalcommons.trinity.edu/mono/8/)**