Solutions, domain of solution, trivial solution

Definition:

A function ϕ that satisfies a DE for all input of ϕ is called a **solution** to the DE.

Note: When talking about a solution of a differential equation then, we are talking *not* just about the function, but concurrently about what input of the function work!

Read that last sentence again.

The question arises, based on our last definition, what if a function does not satisfy a DE for all of its input? Well, it can still be a solution, but we need to also highlight what values of input it is a solution for.

Definition:

A function ϕ defined on an interval I for which ϕ satisfies a DE for all input from I is called a **solution of the DE on the interval I.**

Really, this last definition is the more appropriate definition and the first one is simply a special case if it. In practice:

- Be sure to describe the interval I for which a function is a solution for a DE.
- In the event the interval is all input, then it is common to omit this discussion.

Therefore, it makes sense to provide some terminology for discussing the interval above.

Definition:

The interval I occurring above is called the **interval of existence**, or also the **domain of the solution**.

We finish this initial study of solutions for DEs with the introduction of a distinguished solution we often encounter.

Definition:

If the function $y = 0$ is a solution on an interval I, we call this the **trivial solution**.

Discussion, comments, and examples:

Math45-Module-02-Video-01

WeBWorK module 02 exercises:

- Problems 1,2,3

Relevant Wikipedia articles:

- **Solutions to DEs** **(https://en.wikipedia.org/wiki/Ordinary_differential_equation#Solutions)**