Translations of the Laplace transform

There are many ways to slightly modify the Laplace transform theory described so far to tackle additional types of problems. We look at one particular case here, that of *frequency shifting*.

The idea centers around the following: if we know the Laplace transform theory surrounding a function $f(t)$, then with very little work, we also know the theory surrounding functions of the form $e^{kt}f(t)$ for any constant k. For instance, if we know the theory for $\sin(2t)$, then we also know the theory for $e^{7t}\sin(2t)$. This detailed in the following theorem.

Theorem

Suppose $\mathcal{L}\{f(t)\} = F(s)$ and k is a real number. Then

$$\mathcal{L}\left\{e^{kt}f(t)\right\} = F(s-k)$$

and

$$\mathcal{L}^{-1}\left\{F(s-k)\right\} = e^{kt}f(t).$$

We apply this to some problems in the video below.

Discussion, comments, and examples:

Math45-Module-16-Video-08

WeBWorK module 16 exercises:

- Problems 14, 15

References:

- See the "Frequency shifting" in the table of Wikipedia here (https://en.wikipedia.org/wiki/Laplace_transform#Properties_and_theorems) .
- See Theorem 8.1.3 in the Trench textbook (https://digitalcommons.trinity.edu/mono/8/) .