I'm going to spend more time studying this section

Math 45, Fall 2020
October 21, Quiz 07

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Please show and explain your work where necessary. Good luck!!

1. (*8 points*) Let $f_1(x) = e^x$ and $f_2(x) = e^{x+2}$. Note that both functions are solutions to $y' = y$.

a. (*4 pts*) Compute the *Wronskian* $\mathcal{W}(f_1, f_2)$.

$$W(e^x, e^{x+2}) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \to \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2}[x+2]' \end{vmatrix} \to \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2}[x+2]' \end{vmatrix} \to \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2} \end{vmatrix}$$

$1 \neq 0$

$$\to e^x \cdot e^{x+2} \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2} \end{vmatrix} \to e^{2x+2} \begin{vmatrix} e^x & e^{x+2} \\ e^x & e^{x+2} \end{vmatrix}$$

$$\to e^{2x+2}$$

b. (*2 pts*) Are $f_1(x) = e^x$ and $f_2(x) = e^{x+2}$ linearly independent? Explain.

c. (*2 pts*) Do $f_1(x) = e^x$ and $f_2(x) = e^{x+2}$ form a fundamental set of solutions for $y' = y$? Explain.

2. (*2 points*) It is true that $y_1 = \sin(x)$ and $y_2 = \sin(x)$ form a fundamental set of solutions to the differential equation $y'' + y = 0$. Meanwhile we have $y_p = e^x$ is a solution to the differential equation $y'' + y = 2e^x$. (You do not need to show either of these previous two statements.) Provide the *general* solution to the differential equation $y'' + y = 2e^x$.