



## Midterm 2 Overview

Basic Information

1

## Midterm Information

- 75 minutes
- 150 points
- No notes allowed
- Exams will be taken on Canvas



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## Exam Format

- They will be available for 75 minutes (normal class time)
- If the question has you fill in an answer – use lowercase (they are case sensitive)
- Bring scratch paper!  
**You will need it!**



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## Exam Format

- Have Zoom open during the exam
- I'm not going to watch you
- ... this is so I can talk to the class if necessary and you can ask questions



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## Part 6 – Need to Know

- Boolean logic is set theory
- Boolean operators
- Implication!
- Precedence
- Tautology / contradiction
- Logic equivalences



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## Part 6 – Don't Worry About

- George Boole
- My "Gold Rush" example



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## Part 7 – Need to Know

- Arguments are implications
- Valid arguments
- Logical fallacies



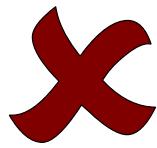
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## Part 7 – Don't Worry About

- *Know it all*



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## Part 8 – Need to Know

- Theorems & Definitions
- Direct proof
- Proof by contrapositive



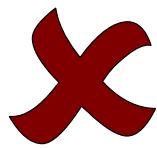
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## Part 8 – Don't Worry About

- Proof by contradiction (still know the approach)



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## Part 9 – Need to Know

- Quantifiers
- Quantifier Equivalencies
- Bound/Free Variables
- Bounded Quantifiers
- Converting to English



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## Part 9 – Don't Worry About

- *Know it all*



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## Part 10 – Need to Know

- Induction!



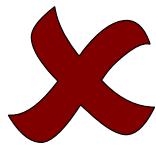
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## Part 10 – Don't Worry About

- Strong Induction



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# Midterm 2 Study Guide

## —Boolean operators—

Operator	Name
$p \text{ and } q$	True <u>only</u> if <u>both</u> p and q are <u>true</u>
$p \text{ or } q$	True if <u>either</u> p or q <u>true</u>
$\text{not } p$	True if p <u>false</u>
$p \text{ xor } q$	True if p and q are <u>different</u>
$p \text{ implies } q$	True <u>unless</u> p is true and q is false

### Precedence Levels Logic Notation of Operators

Logic	C Family	Visual Basic
2) $p \wedge q$	$p \& q$	$p \text{ and } q$
3) $p \vee q$	$p \mid\mid q$	$p \text{ or } q$
1) $\neg p$	$\text{!}p$	$\text{not } p$
3) $p \oplus q$	none	$p \text{ xor } q$

4)  $P \rightarrow q \rightarrow P \text{ implies } q$

Truth Table

		NOT	OR	AND	Implication
P	q	$\neg p$	$p \vee q$	$p \wedge q$	$p \rightarrow q$
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	T	F	T
F	F	T	F	F	T

## —Boolean logic is set theory $\overline{T} = U$

The complement of U is  $\emptyset$

So, naturally, False is represented with  $\emptyset$

$$F = \emptyset$$

### AND - Intersection Comparison

Boolean Logic	Set Theory
$T \wedge T = T$	$U \cap U = U$
$T \wedge F = F$	$U \cap \emptyset = \emptyset$
$F \wedge T = F$	$\emptyset \cap U = \emptyset$
$F \wedge F = F$	$\emptyset \cap \emptyset = \emptyset$

### OR - Union Comparison

Boolean Logic	Set Theory
$T \vee T = T$	$U \cup U = U$
$T \vee F = T$	$U \cup \emptyset = U$
$F \vee T = T$	$\emptyset \cup U = U$
$F \vee F = F$	$\emptyset \cup \emptyset = \emptyset$

## —Tautology/contradiction—

Tautology

If the statement is always true

$$p \vee \neg p$$

$$p \rightarrow p$$

$$p = p$$

Contradiction

If the statement is always false

$$p \wedge \neg p$$

$$p \oplus p$$

# —Logic Equivalencies —

Commutative Law  
 $a \wedge b \equiv b \wedge a$

Identity Law  
 $a \wedge \text{true} \equiv a$

Complement Law  
 $a \wedge \neg a \equiv \text{false}$

$a \vee b \equiv b \vee a$

$a \vee \text{false} \equiv a$

$a \vee \neg a \equiv \text{true}$

Distributive Law

$$a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$$

$$a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$$

Absorption Law

$$a \wedge (a \vee b) \equiv a$$

$$a \vee (a \wedge b) \equiv a$$

Associative Law

$$a \wedge (b \wedge c) \equiv (a \wedge b) \wedge c$$

$$a \vee (b \vee c) \equiv (a \vee b) \vee c$$

Idempotent Law

$$a \wedge a \equiv a$$

$$a \vee a \equiv a$$

Involution Law

$$\neg \neg a \equiv a$$

Domination Law

$$a \vee \text{true} \equiv \text{true}$$

$$a \wedge \text{false} \equiv \text{false}$$

DeMorgan's Law

$$\neg (a \wedge b) \equiv \neg a \vee \neg b$$

$$\neg (a \vee b) \equiv \neg a \wedge \neg b$$

Example

$$(A' \cap B) \cup (C \cap C) \cup (A' \cap B')$$

$$(A' \cap B) \cup \emptyset \cup (A' \cap B')$$

Complement  
Identity

Distributive  
Distributive

Complement  
Complement

Solution

$$(a \wedge b) \vee (\neg a \wedge b) =$$

$$(a \vee \neg a) \wedge b =$$

After using Distributive Law

$$\text{true} \wedge b =$$

After using Complement Law

$$b$$

After using Identity Law

# —Implication — Arguments are implications

For "p implies q" is contradicted (false) when p is true & q is false

antecedent hypothesis assumption

Consequent Conclusion  
Therefore  $\vdash$  or  $\therefore$

A implies B  
 $A \rightarrow B$   
 $B$  if  $A$   
If  $A$  then  $B$   
 $B$  given  $A$

Equivalent

$$p \rightarrow q \equiv \neg(p \wedge \neg q)$$

$$\equiv \neg p \vee q$$

Example

$$\neg \forall x \exists x (\neg B(x) \wedge P(x))$$

$$\neg \forall x \exists x (\neg B(x) \wedge P(x))$$

$$\exists x \neg \exists x (\neg B(x) \wedge P(x))$$

$$\exists x \forall x (\neg B(x) \vee \neg P(x))$$

$$\exists x \forall x (P(x) \rightarrow B(x))$$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

OR Logic

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

AND Logic

p	q	$\neg p$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$p \rightarrow q$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	F	T	T
F	F	T	F	T	T

# - Arguments are implications -

Argument is a collection of statements called premises when all are true implies a consequence

- Arguments are actually implications with each premise connected with  $\wedge$
- So, if you have premises A, premise B, and conclusion C, then it has the following form

**A**  $\wedge$  **B**  $\rightarrow$  **C**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

# - Valid arguments -

Rules of Inference are valid arguments

Modus Ponens (Law of Detachment) \* Most basic rule \*

Based on the logic that if:

- an implication is true
- implication's hypothesis is true
- then the implication's conclusion must be true

If it is a fish, then it lives in water.

It is a fish.

Therefore, it lives in water! **True**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Tollens \* closely related to Modus Ponens \*

Based on the logic that if:

- an implication is true
- implication's hypothesis is false
- then the implication's conclusion must be false

If it is a fish, then it lives in water.

It doesn't live in water.

Therefore, it is not a fish!

$P \rightarrow q$

$\neg q$

$\neg P$  (True)

p	q	$p \rightarrow q$	$\neg p$	$\neg q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Disjunctive Syllogism \* Based on the OR Operator \*

Based on the logic that if:

- an or-statement is true
- one of the operands is false
- then, the other operand must be true

It breathes water or air.

It doesn't breath water.

Therefore, it breathes air.

$P \vee q$

$\neg P$

$q$  (True)

Hypothetical Syllogism \* Based on an implication chain \*

- Gives a logical "chain" of events
- So, if  $a \Rightarrow b$  and  $b \Rightarrow c$  then  $a \Rightarrow c$

If is a trout, then it is a fish

If it is a fish, then it lives in water.

Therefore, a trout lives in water!

$P \rightarrow q$

$q \rightarrow r$

$P \rightarrow r$  (True)

# - Logical fallacies -

## Fallacy of the Converse

Based on assumption that if the conclusion is true then the hypothesis is true

**Also called:**

- affirming the consequent
- converse error

If it is a fish, then it lives in water.

It lives in water.

Therefore, it is a fish!

$$\frac{P \rightarrow q}{q}$$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

## Fallacy of the Inverse

Based on assumption that if the hypothesis is false, then the conclusion is also false

**Also called:**

- denying the antecedent
- inverse error

If it is a cat, then it is furry.

It is not a cat.

Therefore, it is not furry!

$$\frac{P \rightarrow q}{\neg P}$$

p	q	$p \rightarrow q$	$\neg p$	$\neg q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

## Fallacy of Affirming a Disjunct

Based on assumption that if there are two attributes and one is true, the other must be false

**Also called:**

- fallacy of the alternative
- false exclusionary disjunct

Suspect is either a politician or a lawyer.

Suspect is a politician.

Therefore, the suspect isn't a lawyer.

$$\frac{P \vee q}{P}$$

p	q	$p \vee q$	$\neg q$
T	T	T	F
T	F	T	T
F	T	T	F
F	F	F	T

- Start with a
- Try to change it into  $\neg b$
- Use the premises to replace values

$$\begin{aligned} a \vee b &= \text{true} \\ a \wedge b &= \text{false} \end{aligned}$$

$$\begin{aligned} a \vee b &= \text{true} \\ a \wedge b &= \text{false} \end{aligned}$$

$$a = \neg b$$

$a = a$	$= a \wedge \text{true}$	Identity
	$= a \wedge (b \vee \neg b)$	Complement
	$= a \wedge b \vee a \wedge \neg b$	Distributive
	$= \text{false} \vee a \wedge \neg b$	Premise
	$= b \wedge \neg b \vee a \wedge \neg b$	Complement
	$= \neg b \wedge (b \vee a)$	Distributive
	$= \neg b \wedge \text{true}$	Premise
	$= \neg b$	Identity

# Theorems & Definitions —

- A theorem is a statement we intend to prove using existing known facts (called axioms or lemmas)
- Used extensively in all mathematical proofs – which should be obvious

## Example

- Most theorems are of the form: If A, then B
- The theorem below is very easy to interpret

If  $a$  and  $b$  are even integers

then  $a \times b$  is an even integer

# — Direct Proof —

# — Proof by contrapositive —

## Part 9

- Quantifiers
- Quantifier Equivalencies
- Bound/Free Variables
- Converting to English

## Part 10

- Induction