Please show and explain your work. Justification is necessary for credit. Also sign your name here acknowledging you will not use problem solving software (Wolfram Alpha, etc.):\_\_\_\_\_

## **1.** (14 points)

a. (2 pts) Circle all of the following expression which are linear differential equations.

(i) 
$$y''' - 2y''' + 3y = 5$$

(iii) 
$$\sqrt{y'} = x \cos(x)y - 3$$

(ii) 
$$(x^2 - \sin(x)) y^{(5)} - xe^x y + \sin(x^2) = 0$$
 (iv)  $(y')^2 = y^3 + 2$ 

**b.** (2 pts) Circle all of the following expression of the form M(x,y) dx + N(x,y) dy = 0 such that **both** M(x,y) and N(x,y) are homogeneous functions of the **same** degree.

(i) 
$$x^2 dx + yx dy = 0$$

(iii) 
$$\sin(y) dx + (2y + xe^y) dy = 0$$

(ii) 
$$(3xy) dx + e^{\left(\frac{x}{y}\right)} dy = 0$$

(iv) 
$$(x-2y) dx + x dy = 0$$

**c.** (4 pts) Circle whether the following differential equation is an ordinary differential equation (ODE) or a partial differential equation (PDE) and state the **order** of the differential equations (you can do this for any PDEs as well).

(i) 
$$\ln(y')y + x^2y = 1$$

(ii) 
$$\frac{\partial f}{\partial x} + \frac{\partial^2 f}{\partial y^2} + xy = 0$$

(iii) 
$$x^2 \frac{d^3 f}{dx^3} + \frac{df}{dx} = \frac{d^2 f}{dx^2} - \sin(x)$$

(iv) 
$$\left(\frac{d}{dx}\right)^4 f(x) + \left(\frac{d^2}{dx^2}\right) f(x) + f'(x) = 0$$

**d.** (4 pts) Circle all of the following expression which are **exact** differential equations. (Provide work for partial credit!)

(i) 
$$xy dx + xy dy = 0$$

(iii) 
$$xy^2 dx + x^2 y dy = 0$$

(ii) 
$$x^2y \, dx + \frac{1}{3}x^3 \, dy = 0$$

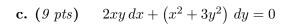
(iv) 
$$\sin(xy) dx + xy \cos(x) dy = 0$$

**e.** (2 pts) Suppose  $y^2 - x = 3$  is a solution to a differential equation. Is this solution implicit or explicit? Explain your answer in one sentence.

2. (27 points) Solve the following differential equations using any technique you like that works. However, you must explain WHY you chose that method to receive credit!

**a.** (9 pts) 
$$y' = \frac{x}{y^2}$$

**b.** (9 pts)  $x \frac{dy}{dx} - 2y = x^3 e^x$ 



**3.** (9 points)

**a.** (8 pts) We note that  $y(x) = c_1 e^{3x} + c_2 e^{-3x}$  is a solution to the differential equation y'' - 9y = 0. Find a solution to the initial conditions y(0) = 0 and y'(0) = 2.

**b.** (1 pt) Does the trivial solution also satisfy the initial value problem given above in 3a? Explain your answer.