

Please show and explain your work where necessary. Good luck!!

1. (6 points) Is a unique solution guaranteed to exist for the following initial value problems on the given intervals. Explain your answers.

Linear
nonhomogeneous
 $\neq 0$

a. (2 pts) $\ln(x)y''' + \frac{e^x}{x-4}y' - y = \cos(x); \quad y(2) = 1, \quad I = (0, 3).$

- (i) coeff of highest order derivative $\neq 0$ on I
- (ii) all coefficient function continuous on I
- (iii) The " x_0 " is in I

$\rightarrow \ln(1) = 0, x=1$

$\textcircled{2} \cdot \ln(x)y''' + \frac{e^x}{x-4}y' - y = \cos(x)$
 $\cdot y(2) = 1$
 $\cdot I = (0, 3)$

No it fails because $x=1$ and the interval contains #'s between 0 & 3

b. (2 pts) $\ln(x)y''' + \frac{e^x}{x-4}y' - y = x^2; \quad y(3) = 1, \quad I = (2, 4).$

- (i) coeff of highest order derivative $\neq 0$ on I
- (ii) all coefficient function continuous on I
- (iii) The " x_0 " is in I

$\rightarrow \ln(1) = 0, x=1$

Yes all pass

$x_0 = 3$ is 3 contained?

(i) Pass

(ii) Pass because 4 isn't included

(iii) Pass

c. (2 pts) $\ln(x)y''' + \frac{e^x}{x-4}y' - y = x^2; \quad y(2) = 1, \quad I = (1, 3).$

- (i) coeff of highest order derivative $\neq 0$ on I

$\rightarrow \ln(1) = 0, x=1$

$x_0 = 2$

Yes all pass

(i) Pass 1 isn't included in I

(ii) Pass 4 isn't included in I

(iii) Pass $x_0 = 2$ is contained in I

2. (2 points) In your own words, describe what it means for functions to be linearly independent.

3. (2 points) Provide an example of the following:

a. (1 pt) A 3rd order linear homogeneous differential equation.

$xy''' - \frac{2}{e^x} + y = 0$

b. (1 pt) A 2nd order linear nonhomogeneous differential equation.

$y'' + \tan(x)y = e^x$