Turing Machine Example

Write a TM that begins with a number in unary on the tape and doubles it.

Before:

```
111
^
```

After:

```
111111
^
```

Start with pseudocode

Algorithm design always comes before coding.

Think in terms of simple things a TM can do, like:
- Move right to next blank
- Replace 1 with 0
- Erase 1
- etc.

Each takes a few TM instructions.

Double unary number

High level idea:
- delete a 1 at source
- write two 1s at destination
- repeat until source empty

Double unary number

Low-level pseudocode:

```
Erase 1
Move right to next blank (end of source)
Move right to next blank (end of destination)
Write two 1s
Move left to next blank (end of source)
Move left
If blank
   Move right twice and exit (Done! Start of destination)
Else
   Move left to next blank
   Move right (to start of source) and goto beginning
```

Double unary number simulation

```
11
^

Erase 1

B1
^

Move right to next blank (end of source)

B1B
   ^
```

Double unary number simulation

```
B1B
 ^
```

Move right to next blank (end of destination)

```
B1BB
  ^
```

Write two 1s

```
B1B11
   ^
```

Double unary number simulation

```
B1B11
   ^

Move left to next blank (end of source)

B1B11
  ^

Move left

B1B11
  ^
```

Double unary number simulation

```
B1B11
 ^


        If blank
            Move right twice and exit (Done! Start of destination)
        Else
            Move left to next blank
            Move right (to start of source) and goto beginning


B1B11
^

B1B11
 ^
```

Double unary number simulation

```
B1B11
 ^


Erase 1

BBB11
 ^


Move right to next blank (end of source)

BBB11
   ^
```

Double unary number simulation

```
BBB11
  ^
```

Move right to next blank (end of destination)

```
BBB11B
     ^
```

Write two 1s

```
BBB1111
      ^
```

Double unary number simulation

```
BBB1111
      ^
```

Move left to next blank (end of source)

```
BBB1111
   ^
```

Move left

```
BBB1111
  ^
```

Double unary number simulation

```
BBB1111
  ^


      If blank
        Move right twice and exit (Done! Start of destination)
      Else
        Move left to next blank
        Move right (to start of source) and goto beginning


BBB1111
    ^
```
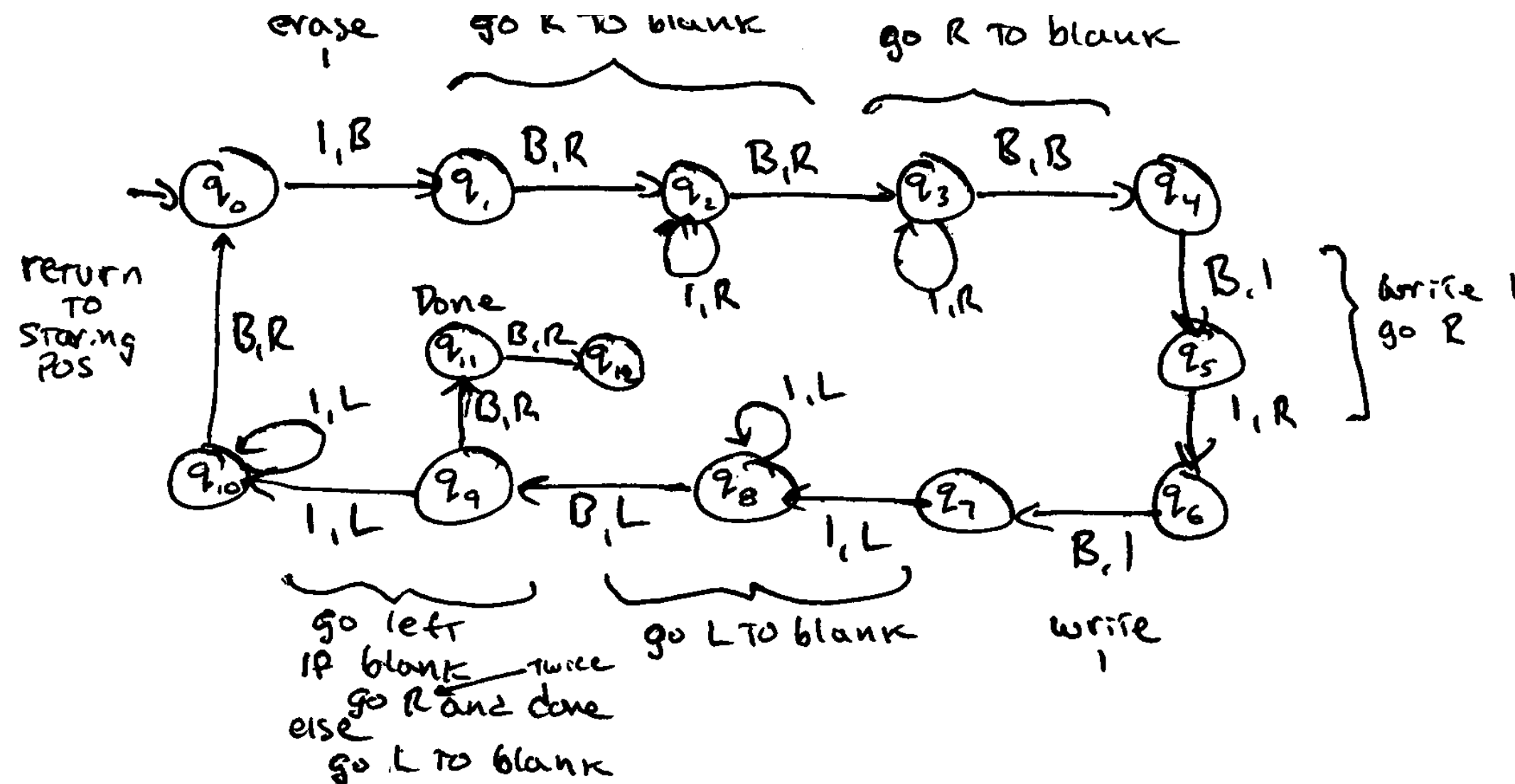
Double unary number TM

Turning each step into TM instructions:

Double unary number TM

Each arrow expressed as a TM instruction:

q0 1 B q1   q1 B R q2   q2 1 R q2
q2 B R q3   q3 1 R q3   q3 B B q4
q4 B 1 q5   q5 1 R q6   q6 B 1 q7
q7 1 L q8   q8 1 L q8   q8 B L q9
q9 1 L q10   q10 1 L q10   q10 B R q0
q9 B R q11   q11 B R q12

If I ask you to give a text representation of a TM, do it this way.

Test

Be sure to give your TM some test inputs to ensure it works correctly.

— Edge cases: zero 1's and one 1.
— General case: two 1's.

Zero 1's

B
^

No instruction for this configuration.

Termination.

0 x 2 = 0

Correct

One 1

```
1
^
```

Erase 1

```
B
^
```

Move right to next blank (end of source)

```
BB
 ^
```

Double unary number simulation

```
BB
 ^


Move right to next blank (end of destination)


BBB
  ^


Write two 1s


BB11
   ^
```

Double unary number simulation

```
BB11
   ^

Move left to next blank (end of source)

BB11
 ^

Move left

BB11
^
```

Double unary number simulation

```
BB11
^


     If blank
         Move right twice and exit (Done! Start of destination)
     Else
         Move left to next blank
         Move right (to start of source) and goto beginning


BB11
  ^
```