MATH 45 – Module 16–17 Review Questions

Dr. Krauel

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1. Rework, study, and understand all of the homework and quiz problems.

2. Study all of the midterm reviews.

3. Use the definition of the Laplace operator to determine $\mathcal{L}(\sin(3t))$.

4. Compute the following.

(a) $\mathcal{L}(2\sin(3t))$

(b) $\mathcal{L}\left(\frac{t^7}{3} + \frac{e^{5t+3}}{2}\right)$

(c) $\mathcal{L}^{-1}\left(\frac{3s}{2s^2+5}\right)$

(d) $\mathcal{L}^{-1}\left(\frac{3}{s^4} - \frac{1}{3s}\right)$

5. Compute the following.

(a) $\mathcal{L}\left(e^{2t}\frac{t^7}{3} + e^{-3t}\frac{e^{5t+3}}{2}\right)$

(b) $\mathcal{L}^{-1}\left(\frac{2}{s^2-4s+13}\right)$

6. Solve the following initial value problems.

(a) $y' - 2y = 2e^{3t}; \quad y(0) = 2.$

(b) $y'' + 5y' + 6y = 0; \quad y(0) = 1,\ y'(0) = 0.$

7.

(a) Write the following system of differential equations in matrix form

$$\frac{dy}{dx} = x + 4y$$
$$\frac{dy}{dy} = 2x + 3y.$$

(b) Write the following system of differential equations as two differential equations

$$\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix}\mathbf{X}.$$

8. Verify the given functions are solutions to the system of differential equations.

(a) The differential equation

$$\frac{dy}{dx} = x + 4y$$
$$\frac{dy}{dy} = 2x + 3y.$$

with $y = \binom{1}{1}e^{5t}$.

(b) The differential equation

$$\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} \mathbf{X}.$$

with $y = \begin{pmatrix} -\frac{2}{3} \\ 1 \end{pmatrix} e^{-t}$.

9. Suppose the following functions are solutions to a differential equation. Determine whether they are linearly independent solutions on the given interval.

(a) $y_1 = \binom{1}{1} e^{5t}$, $y_2 = \binom{2}{2} e^{-3t}$; $\quad I = (0, \infty)$.

(b) $y_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$, $y_2 = \begin{pmatrix} -\frac{2}{3} \\ 1 \end{pmatrix} e^{-t}$; $\quad I = (-\infty, \infty)$.

10. Determine whether the functions $y_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$, $y_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t}$ form a fundamental set of solutions for the differential equation

$$\begin{aligned} \frac{dy}{dx} &= x + 4y \\ \frac{dy}{dy} &= 2x + 3y. \end{aligned}$$

Explain your answer.

11. Verify that $y = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} -\frac{2}{3} \\ 1 \end{pmatrix} e^{-t}$ is the general solution to the differential equation

$$\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} \mathbf{X}.$$

12. Solve the following systems of differential equations.

(a) The system

$$\begin{aligned} \frac{dy}{dx} &= x + 4y \\ \frac{dy}{dy} &= 2x + 3y. \end{aligned}$$

(b) The system

$$\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} \mathbf{X}.$$

13. Explain your answers to the following questions.

(a) Do $y_1 = 1$, $y_2 = 2$, and $y_3 = 3$ form a fundamental set of solutions for the differential equation $y' = 0$?

(b) Are $y_1 = \sin(5x)$ and $y_2 = \sin(-5x)$ linearly independent?

(c) Suppose y_1 and y_2 are solutions to a differential equation. Is it possible for the Wronskian $\mathcal{W}(y_1, y_2)$ to equal 0?

(d) Suppose y_1 and y_2 form a fundamental set of solutions to a differential equation. Is it possible for the Wronskian $\mathcal{W}(y_1, y_2)$ to equal 0?