Fundamental sets of solutions for systems

Here we answer the question: *Is there a notion of a fundamental set of solutions, and what constitutes a fundamental set of solutions and a general solution?*

There is a notion of a fundamental set of solutions! Though we restrict our attention to systems of n-many linear differential equations with n-many unknowns.

Theorem

If we have n-many differential equations in a system with n-many unknowns, then n-many linearly independent solutions form a fundamental set of solutions for the system.

However, this begs another question. How do we know if a set of solutions (which are vectors) is linearly independent?

To answer this, we only look at the case of systems of 2 linear differential equations with 2 unknowns. To determine if two solutions of such a system are linearly independent, we use the following theorem.

Theorem

Suppose $y_1 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ and $y_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ are solutions to a 2×2 system of linear differential equations on an interval I. Then y_1 and y_2 are linearly independent if and only if

$$W(y_1, y_2) = \det\left(\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}\right) \neq 0$$

on all of I.

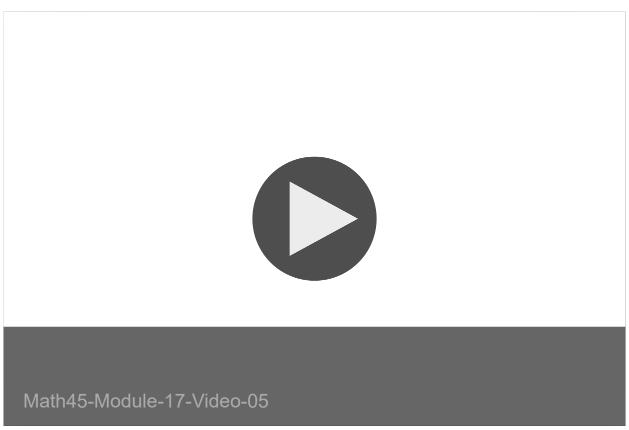
Note: The W above is again called the **Wronskian**, but here there are no derivatives.

Finally, we note that again, if y_1 and y_2 form a fundamental set of solutions for the system of *homogeneous* linear differential equations is given by

$$y = c_1 y_1 + c_2 y_2,$$

just as before. (And a general solution for a nonhomogeneous system also follows the framework from before, though we omit that discussion here.)

Discussion, comments, and examples:



WeBWorK module 17 exercises:

- Problems 8, 9

Relevant Wikipedia articles:

- See Theorems 10.3.1 and 10.3.2 in the [textbook (https://digitalcommons.trinity.edu/mono/8/)](https://digitalcommons.trinity.edu/mono/8/)