1. (1 point) The function $y_1(x) = x^4$ is a solution to the differential equation $x^2y'' - 7xy' + 16y = 0$. Use reduction of order to find another solution y_2 to this differential equation.

(If you need ln, use absolute value signs. For example, ln| input |.)

$y_2 =$ ________ help (formulas)

2. (1 point) The function $y_1(x) = \ln|7x|$ is a solution to the differential equation $xy'' + y' = 0$. Use reduction of order to find another solution y_2 to this differential equation.

(If you need ln, use absolute value signs. For example, ln| input |.)

$y_2 =$ ________ help (formulas)

3. (1 point) Find the two values of m for which

$$y(x) = e^{mx}$$

is a solution of the differential equation

$$y'' - 4y' + 3y = 0.$$

smaller value = ____
larger value = ____

4. (1 point) Find the general solution to $3y'' + y' = 0$.
Enter your answer as $y = \ldots$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

________________ help (equations)

5. (1 point) Find the general solution to $6y'' + 18y' - 24y = 0$.
Enter your answer as $y = \ldots$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

________________ help (equations)

6. (1 point) Find the general solution to $y'' + 12y' + 36y = 0$.
Enter your answer as $y = \ldots$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

________________ help (equations)

7. (1 point) Find the general solution to $2y'' + 8y = 0$. Give your answer as $y = \ldots$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

________________ help (equations)

8. (1 point) Find the general solution to $y'' + 4y' + 29y = 0$.
Give your answer as $y = \ldots$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

________________ help (equations)

9. (1 point) Find the general solution to $y''' + 3y'' + 25y' - 29y = 0$. Give your answer as $y = \ldots$. In your answer, use c_1, c_2, and c_3 to denote arbitrary constants and x the independent variable. Enter c_1 as c1, c_2 as c2, and c_3 as c3.

(Hint: Note $m^3 + 3m^2 + 25m - 29 = (m-1)(m^2 + 4m + 29)$.)

________________ help (equations)

10. (1 point) Find the general solution to $y^{(4)} - 7y''' + 10y'' = 0$. In your answer, use c_1, c_2, c_3 and c_4 to denote arbitrary constants and x the independent variable. Enter c_1 as c1, c_2 as c2, etc.

____________________ help (equations)

11. (1 point) Find the particular solution to $y'' + 4y' + 4y = 0$ which satisfies the initial conditions $y(0) = 2$ and $y'(0) = 2$.
Enter your answer as $y = \ldots$. In your answer, use x to denote the independent variable.

________________ help (equations)

Q1

Sol. $x^2 y'' - 7xy' + 16y = 0 \quad —(i)$

As given $\boxed{y_1 = x^4}$

From (i), $y'' - \frac{7}{x} y' + \frac{16}{x^2} y = 0$

Here $P(x) = \frac{-7}{x}$, $Q(x) = \frac{16}{x^2}$

Now By Reduction of order,

$$y_2(x) = c\, y_1(x) \int \frac{e^{-\int p(x)\,dx}}{y_1^2}\, dx \quad — (*)$$

$$\Rightarrow e^{-\int p(x)\,dx} = e^{-\int \frac{-7}{x}dx} = e^{\int \frac{7}{x}dx} = e^{7\ln(x)} = e^{\ln x^7} = \boxed{x^7}$$

Also $\int \frac{e^{-\int p(x)\,dx}}{y_1^2}\,dx = \int \frac{x^7}{(x^4)^2}\,dx = \int \frac{1}{x}\,dx = \ln x$

Plug above these values in (*), we get

$y_2(x) = c \cdot x^4 \ln(x)$ and let $\boxed{c=1}$

$\Rightarrow y_2(x) = x^4 \ln(x)$

b. General solution of (i) is

$$y(x) = c_1 y_1(x) + c_2 y_2(x)$$

or $y(x) = c_1 x^4 + c_2 \ln(x) \cdot x^4$ where c_1 and c_2 are arbitary constant.

c. $y(1) = 10$, $y'(1) = 3$

As $y(1) = 10 \Rightarrow 10 = c_1 (1)^4 + c_2 \ln(1) \cdot (1)^4$

or $\boxed{c_1 = 10}$ $[\because \ln(1) = 0]$

and $y'(x) = 4c_1 x^3 + c_2 \ln(x) \cdot 4x^3 + c_2 x^4 \cdot \frac{1}{x}$

and $y'(1) = 3 \Rightarrow 3 = 4 \times 10 \times (1)^3 + c_2 \ln(1) \cdot 4(1)^3 + c_2 (1)^3$

or $3 - 40 = c_2$ or $\boxed{c_2 = -37}$

$$\Rightarrow \boxed{y(x) = 10x^4 - 37x^4 \ln(x)}$$

Q3

SOLUTION:

For $y(x) = e^{mx}$ we have $y'(x) = me^{mx}$ and $y''(x) = m^2 e^{mx}$. Plugging these into the differential equation $y'' - 4y' + 3y = 0$ gives

$$m^2 e^{mx} - 4me^{mx} + 3e^{mx} = 0$$

or

$$\left(m^2 - 4m + 3\right) e^{mx} = 0.$$

Since $e^{mx} \neq 0$ for any x we have

$$\left(m^2 - 4m + 3\right) = 0.$$

Noting that this is

$$(m-1)(m-3) = 0$$

or solving the quadratic formula, we find $m = 1, 3$.

Q4

$3y'' + y' = 0: \quad y = c_1 + c_2 e^{-\frac{t}{3}}$

Steps

$3y'' + y' = 0$

Second order linear homogeneous differential equation with constant coefficients

A second order linear, homogeneous ODE has the form of $ay'' + by' + cy = 0$

For an equation $ay'' + by' + cy = 0$, assume a solution of the form $e^{\gamma t}$

Rewrite the equation with $y = e^{\gamma t}$

$3\left(\left(e^{\gamma t}\right)\right)'' + \left(\left(e^{\gamma t}\right)\right)' = 0$

Simplify $3\left(\left(e^{\gamma t}\right)\right)'' + \left(\left(e^{\gamma t}\right)\right)' = 0: \quad e^{\gamma t}\left(3\gamma^2 + \gamma\right) = 0$

$e^{\gamma t}\left(3\gamma^2 + \gamma\right) = 0$

Solve $e^{\gamma t}\left(3\gamma^2 + \gamma\right) = 0: \quad \gamma = 0, \gamma = -\frac{1}{3}$

$\gamma = 0, \gamma = -\frac{1}{3}$

For two real roots $\gamma_1 \neq \gamma_2$, the general solution takes the form: $y = c_1 e^{\gamma_1 t} + c_2 e^{\gamma_2 t}$

$c_1 e^0 + c_2 e^{-\frac{1}{3}t}$

Refine

$y = c_1 + c_2 e^{-\frac{t}{3}}$

Q5

$6y'' + 18y' - 24y = 0: \quad y = c_1 e^t + c_2 e^{-4t}$

Steps

$6y'' + 18y' - 24y = 0$

Second order linear homogeneous differential equation with constant coefficients

A second order linear, homogeneous ODE has the form of $ay'' + by' + cy = 0$

For an equation $ay'' + by' + cy = 0$, assume a solution of the form $e^{\gamma t}$

Rewrite the equation with $y = e^{\gamma t}$

$6\left(\left(e^{\gamma t}\right)\right)'' + 18\left(\left(e^{\gamma t}\right)\right)' - 24e^{\gamma t} = 0$

Simplify $6\left(\left(e^{\gamma t}\right)\right)'' + 18\left(\left(e^{\gamma t}\right)\right)' - 24e^{\gamma t} = 0: \quad e^{\gamma t}\left(6\gamma^2 + 18\gamma - 24\right) = 0$

$e^{\gamma t}\left(6\gamma^2 + 18\gamma - 24\right) = 0$

Solve $e^{\gamma t}\left(6\gamma^2 + 18\gamma - 24\right) = 0: \quad \gamma = 1, \gamma = -4$

$\gamma = 1, \gamma = -4$

For two real roots $\gamma_1 \neq \gamma_2$, the general solution takes the form: $y = c_1 e^{\gamma_1 t} + c_2 e^{\gamma_2 t}$

$y = c_1 e^t + c_2 e^{-4t}$

Q6

$y'' + 12y' + 36y = 0: \quad y = c_1 e^{-6t} + c_2 t e^{-6t}$

Steps

$y'' + 12y' + 36y = 0$

Second order linear homogeneous differential equation with constant coefficients

A second order linear, homogeneous ODE has the form of $ay'' + by' + cy = 0$

For an equation $ay'' + by' + cy = 0$, assume a solution of the form $e^{\gamma t}$

Rewrite the equation with $y = e^{\gamma t}$

$\left(\left(e^{\gamma t}\right)\right)'' + 12\left(\left(e^{\gamma t}\right)\right)' + 36e^{\gamma t} = 0$

Simplify $\left(\left(e^{\gamma t}\right)\right)'' + 12\left(\left(e^{\gamma t}\right)\right)' + 36e^{\gamma t} = 0: \quad e^{\gamma t}\left(\gamma^2 + 12\gamma + 36\right) = 0$

$e^{\gamma t}\left(\gamma^2 + 12\gamma + 36\right) = 0$

Solve $e^{\gamma t}\left(\gamma^2 + 12\gamma + 36\right) = 0: \quad \gamma = -6$ with multiplicity of 2

$\gamma = -6$ with multiplicity of 2

For one real root γ, the general solution takes the form: $y = c_1 e^{\gamma t} + c_2 t e^{\gamma t}$

$y = c_1 e^{-6t} + c_2 t e^{-6t}$

Q7

$2y'' + 8y = 0: \quad y = c_1 \cos(2t) + c_2 \sin(2t)$

Steps

$2y'' + 8y = 0$

Second order linear homogeneous differential equation with constant coefficients

A second order linear, homogeneous ODE has the form of $ay'' + by' + cy = 0$

For an equation $ay'' + by' + cy = 0$, assume a solution of the form $e^{\gamma t}$

Rewrite the equation with $y = e^{\gamma t}$

$2\left(\left(e^{\gamma t}\right)\right)'' + 8e^{\gamma t} = 0$

Simplify $2\left(\left(e^{\gamma t}\right)\right)'' + 8e^{\gamma t} = 0: \quad e^{\gamma t}\left(2\gamma^2 + 8\right) = 0$

$e^{\gamma t}\left(2\gamma^2 + 8\right) = 0$

Solve $e^{\gamma t}\left(2\gamma^2 + 8\right) = 0: \quad \gamma = 2i, \gamma = -2i$

$\gamma = 2i, \gamma = -2i$

For two complex roots $\gamma_1 \neq \gamma_2$, where $\gamma_1 = \alpha + i\beta$, $\gamma_2 = \alpha - i\beta$

the general solution takes the form: $y = e^{\alpha t}\left(c_1 \cos(\beta t) + c_2 \sin(\beta t)\right)$

$e^0\left(c_1 \cos(2t) + c_2 \sin(2t)\right)$

Refine

$y = c_1 \cos(2t) + c_2 \sin(2t)$

Q8

$y'' + 4y' + 29y = 0: \quad y = e^{-2t}\left(c_1 \cos(5t) + c_2 \sin(5t)\right)$

Steps

$y'' + 4y' + 29y = 0$

Second order linear homogeneous differential equation with constant coefficients

A second order linear, homogeneous ODE has the form of $ay'' + by' + cy = 0$

For an equation $ay'' + by' + cy = 0$, assume a solution of the form $e^{\gamma t}$

Rewrite the equation with $y = e^{\gamma t}$

$\left(\left(e^{\gamma t}\right)\right)'' + 4\left(\left(e^{\gamma t}\right)\right)' + 29e^{\gamma t} = 0$

Simplify $\left(\left(e^{\gamma t}\right)\right)'' + 4\left(\left(e^{\gamma t}\right)\right)' + 29e^{\gamma t} = 0: \quad e^{\gamma t}\left(\gamma^2 + 4\gamma + 29\right) = 0$

$e^{\gamma t}\left(\gamma^2 + 4\gamma + 29\right) = 0$

Solve $e^{\gamma t}\left(\gamma^2 + 4\gamma + 29\right) = 0: \quad \gamma = -2 + 5i, \gamma = -2 - 5i$

$\gamma = -2 + 5i, \gamma = -2 - 5i$

For two complex roots $\gamma_1 \neq \gamma_2$, where $\gamma_1 = \alpha + i\beta$, $\gamma_2 = \alpha - i\beta$

the general solution takes the form: $y = e^{\alpha t}\left(c_1 \cos(\beta t) + c_2 \sin(\beta t)\right)$

$y = e^{-2t}\left(c_1 \cos(5t) + c_2 \sin(5t)\right)$

Q9

$y''' + 3y'' + 25y' - 29y = 0: \quad y = c_1 e^t + e^{-2t}\left(c_2 \cos(5t) + c_3 \sin(5t)\right)$

Steps

$y''' + 3y'' + 25y' - 29y = 0$

Linear homogeneous differential equation with constant coefficients — Hide Definition

A linear homogeneous ODE with constant coefficients has the form of $a_n y^{(n)} + \ldots + a_1 y' + a_0 y = 0$

For an equation $a_n y^{(n)} + \ldots + a_1 y' + a_0 y = 0$, assume a solution of the form $e^{\gamma t}$

Rewrite the equation with $y = e^{\gamma t}$

$\left(\left(e^{\gamma t}\right)\right)''' + 3\left(\left(e^{\gamma t}\right)\right)'' + 25\left(\left(e^{\gamma t}\right)\right)' - 29e^{\gamma t} = 0$

Show Steps

Simplify $\left(\left(e^{\gamma t}\right)\right)''' + 3\left(\left(e^{\gamma t}\right)\right)'' + 25\left(\left(e^{\gamma t}\right)\right)' - 29e^{\gamma t} = 0: \quad e^{\gamma t}\left(\gamma^3 + 3\gamma^2 + 25\gamma - 29\right) = 0$

$e^{\gamma t}\left(\gamma^3 + 3\gamma^2 + 25\gamma - 29\right) = 0$

Show Steps

Solve $e^{\gamma t}\left(\gamma^3 + 3\gamma^2 + 25\gamma - 29\right) = 0: \quad \gamma = 1, \gamma = -2 + 5i, \gamma = -2 - 5i$

$\gamma = 1, \gamma = -2 + 5i, \gamma = -2 - 5i$

Show Steps

Find solution for $\gamma = 1$: $c_1 e^t$

Show Steps

Find solution for $\gamma = -2 + 5i, \gamma = -2 - 5i$: $e^{-2t}\left(c_2 \cos(5t) + c_3 \sin(5t)\right)$

The general solution has the form of $y = y_1 + y_2 + \ldots + y_{n-1} + y_n$ where $y_1, y_2, \ldots, y_{n-1}, y_n$ are linearly independent solutions of the equation

$y = c_1 e^t + e^{-2t}\left(c_2 \cos(5t) + c_3 \sin(5t)\right)$

Q10

$y''''(t) - 7y''' + 10y'' = 0: \quad y = c_1 e^{2t} + c_2 e^{5t} + c_3 + c_4 t$

Steps

$y''''(t) - 7y''' + 10y'' = 0$

Linear homogeneous differential equation with constant coefficients — Hide Definition

A linear homogeneous ODE with constant coefficients has the form of $a_n y^{(n)} + \ldots + a_1 y' + a_0 y = 0$

For an equation $a_n y^{(n)} + \ldots + a_1 y' + a_0 y = 0$, assume a solution of the form $e^{\gamma t}$

Rewrite the equation with $y = e^{\gamma t}$

$\left(\left(e^{\gamma t}\right)\right)'''' - 7\left(\left(e^{\gamma t}\right)\right)''' + 10\left(\left(e^{\gamma t}\right)\right)'' = 0$

Show Steps

Simplify $\left(\left(e^{\gamma t}\right)\right)'''' - 7\left(\left(e^{\gamma t}\right)\right)''' + 10\left(\left(e^{\gamma t}\right)\right)'' = 0: \quad e^{\gamma t}\left(\gamma^4 - 7\gamma^3 + 10\gamma^2\right) = 0$

$e^{\gamma t}\left(\gamma^4 - 7\gamma^3 + 10\gamma^2\right) = 0$

Show Steps

Solve $e^{\gamma t}\left(\gamma^4 - 7\gamma^3 + 10\gamma^2\right) = 0: \quad \gamma = 0$ with multiplicity of 2, $\gamma = 2, \gamma = 5$

$\gamma = 0$ with multiplicity of 2, $\gamma = 2, \gamma = 5$

Show Steps

Find solution for $\gamma = 2, \gamma = 5$: $c_1 e^{2t} + c_2 e^{5t}$

Show Steps

Find solution for $\gamma = 0$ with multiplicity of 2: $c_3 + c_4 t$

The general solution has the form of $y = y_1 + y_2 + \ldots + y_{n-1} + y_n$ where $y_1, y_2, \ldots, y_{n-1}, y_n$ are linearly independent solutions of the equation

$y = c_1 e^{2t} + c_2 e^{5t} + c_3 + c_4 t$

SOLUTION:

First, we find the general solution.

Consider $y = e^{mx}$ so that $y' = me^{mx}$ and $y'' = m^2 e^{mx}$. Plugging these into the differential equation $y'' + 4y' + 4y = 0$ gives

$$m^2 e^{mx} + 4me^{mx} + 4e^{mx} = 0$$

or

$$\left(m^2 + 4m + 4\right) e^{mx} = 0.$$

Since $e^{mx} \neq 0$ for any x we have that

$$m^2 + 4m + 4 = 0,$$

or

$$(m+2)^2 = 0$$

so that

$$m = -2.$$

We note that this is a real repeated root. Therefore, $y_1 = e^{-2x}$ is a solution and reduction of order finds that $y_2 = xe^{-2x}$ is another solution such that y_1 and y_2 are linearly independent. It follows that

$$y = c_1 e^{-2x} + c_2 x e^{-2x}$$

is the general solution.

Now we solve the initial value problem. Note that

$$\begin{aligned} y'(x) &= -2c_1 e^{-2x} + c_2 e^{-2x} - 2c_2 x e^{-2x} \\ &= e^{-2x}\left(c_2 - 2c_2 x - 2c_1\right). \end{aligned}$$

Therefore, the condition $y(0) = 2$ gives

$$2 = c_1 + c_2(0) = c_1$$

while the condition $y'(0) = 2$ gives

$$2 = c_2 - 2c_1 = c_2 - (2)(2)$$

so that

$$c_2 = 2 + (2)(2) = 6.$$

Thus, the desired particular solution is

$$y = 2e^{-2x} + 6xe^{-2x}$$

Question 4

Symbolab

Solution

$3y'' + y' = 0: \quad y = c_1 + c_2 e^{-\frac{t}{3}}$

Steps

$3y'' + y' = 0$

Second order linear homogeneous differential equation with constant coefficients

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For an equation $ay'' + by' + cy = 0$, assume a solution of the form $e^{\gamma t}$

Rewrite the equation with $y = e^{\gamma t}$

$3\left(\left(e^{\gamma t}\right)\right)'' + \left(\left(e^{\gamma t}\right)\right)' = 0$

Simplify $3\left(\left(e^{\gamma t}\right)\right)'' + \left(\left(e^{\gamma t}\right)\right)' = 0: \quad e^{\gamma t}\left(3\gamma^2 + \gamma\right) = 0$ *Hide Steps*

$3\left(\left(e^{\gamma t}\right)\right)'' + \left(\left(e^{\gamma t}\right)\right)' = 0$

$\left(e^{\gamma t}\right)'' = \gamma^2 e^{\gamma t}$ *Show Steps*

$3\gamma^2 e^{\gamma t} + \left(e^{\gamma t}\right)' = 0$

$\left(e^{\gamma t}\right)' = e^{\gamma t}\gamma$ *Hide Steps*

$\left(e^{\gamma t}\right)'$

Apply the chain rule: $\quad e^{\gamma t}(\gamma t)'$ *Show Steps*

$= e^{\gamma t}(\gamma t)'$

$(\gamma t)' = \gamma$ *Show Steps*

$= e^{\gamma t}\gamma$

$3\gamma^2 e^{\gamma t} + e^{\gamma t}\gamma = 0$

Factor $e^{\gamma t}$

$e^{\gamma t}\left(3\gamma^2 + \gamma\right) = 0$

$e^{\gamma t}\left(3\gamma^2 + \gamma\right) = 0$

Solve $e^{\gamma t}\left(3\gamma^2 + \gamma\right) = 0: \quad \gamma = 0, \gamma = -\frac{1}{3}$ *Show Steps*

$\gamma = 0, \gamma = -\frac{1}{3}$

For two real roots $\gamma_1 \neq \gamma_2$, the general solution takes the form: $\quad y = c_1 e^{\gamma_1 t} + c_2 e^{\gamma_2 t}$

$c_1 e^0 + c_2 e^{-\frac{1}{3}t}$

Refine

$y = c_1 + c_2 e^{-\frac{t}{3}}$