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1. (1 point) Which of the following DEs can be solved using the method of separable equations?

- A. $y' - 5y = x + 9$
- B. $\frac{dS}{dt} = rS$, where r is a constant
- C. $\frac{dy}{dx} + y = e^{2x}$
- D. $\frac{dy}{dx} = e^{2x+6y}$

Solution:
SOLUTION:
We have that the DEs $\frac{dy}{dx} = e^{2x+6y}$ and $\frac{dS}{dt} = rS$, where r is a constant, are both solvable by the use of separation of variables. On the other hand, $y' - 5y = x + 9$ and $\frac{dy}{dx} + y = e^{2x}$ are not.

Correct Answers:

- `BD`

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2. (1 point) Which of the following DEs can be solved using the method developed for linear first order DEs?

- A. $\frac{dy}{dx} + y = e^{3x}$
- B. $\frac{dS}{dt} = rS$, where r is a constant
- C. $y' - 3y = x + 8$
- D. $\frac{dy}{dx} = e^{2x+8y}$

Solution:
SOLUTION:
We have that the DEs $y' - 3y = x + 8$, $\frac{dy}{dx} + y = e^{3x}$, and $\frac{dS}{dt} = rS$, where r is a constant, are all solvable by the use of the technique for linear DEs. On the other hand, $\frac{dy}{dx} = e^{2x+8y}$ is not since it is not linear.

Correct Answers:

- `ABC`

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3. (1 point) Find the general solution of the differential equation $\frac{dS}{dt} = rS$, where r is a constant.

(Use C to denote the arbitrary constant.)
$S =$ __________ help (formulas)

Solution:
SOLUTION:
We 'separate the variables' and rewrite $\frac{dS}{dt} = rS$ as

$$\frac{1}{S}\, dS = r\, dt.$$

Integrating both sides

$$\int \frac{1}{S}\, dS = \int r\, dt$$

gives

$$\ln(S) + c_1 = rt + c_2,$$

for arbitrary constants c_1 and c_2. Combinging these to a single arbitrary constant r we have

$$\ln(S) = rt + k.$$

Applying e to both sides (that is, $e^{\ln(S)} = e^{rt+k}$) we find

$$S = e^{rt+k} = e^{rt}e^{k} = e^{rt}C = Ce^{rt},$$

where we replace the arbitrary constant e^k with the notation C. Thus the solution is

$$Ce^{rt}.$$

Note: One could also solve this DE using the technique developed for linear first order DEs.

Correct Answers:

- `C*exp(r*t)`

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4. (1 point) Find the general solution of the differential equation $\frac{dy}{dx} = e^{2x-5y}$.

(Use C to denote the arbitrary constant.)
$y =$ __________ help (formulas)

Solution:
SOLUTION:
We 'separate the variables' and rewrite $\frac{dy}{dx} = e^{2x-5y}$ as

$$\frac{1}{e^{-5y}}\, dy = e^{2x}\, dx.$$

Integrating both sides

$$\int e^{5y}\,dy = \int e^{2x}\,dx$$

(using u-subs) gives

$$\frac{1}{5}e^{5y} + c_1 = \frac{1}{2}e^{2x} + c_2,$$

for arbitrary constants c_1 and c_2. Combinging these to a single arbitrary constant C we have

$$e^{5y} = \frac{5}{2}e^{2x} + C.$$

Applying ln to both sides (that is, $\ln(e^{5y}) = \ln(\frac{5}{2}e^{2x} + C)$) we find

$$5y = \ln\left(\frac{5}{2}e^{2x} + C\right),$$

or

$$y = \frac{1}{5}\ln\left(\frac{5}{2}e^{2x} + C\right).$$

Correct Answers:

- `0.2*ln(2.5*exp(2*x)+C)`

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5. (1 point) Find the general solution of the differential equation $\dfrac{dy}{dx} + y = e^{5x}$.

(Use C to denote the arbitrary constant.)
$y =$ ________ help (formulas)

Solution:

SOLUTION:

We note this DE is a first order linear DE, and also not separable. Thus, we use the technique developed for linear first order DEs. We have the DE $\dfrac{dy}{dx} + y = e^{5x}$ is already in the form

$$\frac{dy}{dx} + y = e^{5x}.$$

We find the integrating factor to be

$$e^{\int(1)dx} = e^{x}.$$

Multiplying this by both sides of the equation we have

$$e^{x}\left(\frac{dy}{dx} + y\right) = e^{x}e^{5x}.$$

The key idea here is that the integrating factor is specifically formed so that the left side is the product rule of $\frac{d}{dx}\left(e^{x}y\right)$ using implicit differentiation (this is always the case with this technique!). Thus, we can replace the left side with this expression and we get

$$\frac{d}{dx}\left(e^{x}y\right) = e^{6x}.$$

Integrating both sides gives

$$\begin{aligned} e^{x}y + c_1 &= \int e^{6x}\,dx \\ &= \frac{1}{6}e^{6x} + c_2, \end{aligned}$$

for arbitrary constants c_1 and c_2. Combining these to a single arbitrary constant C we have

$$e^{x}y = \frac{1}{6}e^{6x} + C.$$

Solving for y we find

$$\begin{aligned} y &= \frac{1}{6}e^{-x}e^{6x} + Ce^{-x} \\ &= \frac{1}{6}e^{(6-1)x} + Ce^{-x} \\ &= \frac{1}{6}e^{5x} + Ce^{-x} \end{aligned}$$

Correct Answers:

- `0.166667*exp(5*x)+C*exp(-x)`

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6. (1 point) Find the general solution of the differential equation $y' - 4y = x + 7$.

(Use C to denote the arbitrary constant.)
$y =$ ________ help (formulas)

Solution:

SOLUTION:

We note this DE is a first order linear DE, and also not separable. Thus, we use the technique developed for linear first order DEs. We have the DE $y' - 4y = x + 7$ is already in the form

$$\frac{dy}{dx} - 4y = x + 7.$$

We find the integrating factor to be

$$e^{\int(-4)dx} = e^{-4x}.$$

Multiplying this by both sides of the equation we have

$$e^{-4x}\left(\frac{dy}{dx} - 4y\right) = e^{-4x}\left(x + 7\right).$$

The key idea here is that the integrating factor is specifically formed so that the left side is the product rule of $\frac{d}{dx}\left(e^{-}y\right)$ using implicit differentiation (this is always the case with this technique!). Thus, we can replace the left side with this expression and we get

$$\frac{d}{dx}\left(e^{-4x}y\right) = e^{-4x}\left(x + 7\right).$$

Integrating both sides gives

$$\begin{aligned} e^{-4x}y + c_1 &= \int e^{-4x}\left(x + 7\right)dx \\ &= \int xe^{-4x}\,dx + 7\int e^{-4x}\,dx. \end{aligned}$$

Using integration by parts (using $u = x$ and $dv = e^{-4x}$ so that $du = dx$ and $v = -\frac{1}{4}e^{-}$, we find

$$\begin{aligned} \int xe^{-4x}\,dx &= -\frac{1}{4}xe^{-4x} + \frac{1}{4}\int e^{4x}\,dx \\ &= -\frac{1}{4}xe^{-4x} - \frac{1}{(4)^2}e^{-4x} \\ &= -\frac{1}{4}e^{-4x}\left(x + \frac{1}{4}\right). \end{aligned}$$

Plugging this into the equation above, we find

$$\begin{aligned}
e^{-4x}y+c_1 &= \int xe^{-4x}\,dx+7\int e^{-4x}\,dx \\
&= -\frac{1}{4}e^{-4x}\left(x+\frac{1}{4}\right)-\frac{7}{4}e^{-4x}+c_2 \\
&= -\frac{1}{4}e^{-4x}\left(x+\frac{1}{4}+7\right)+c_2 \\
&= -\frac{1}{4}e^{-4x}\left(x+\frac{1+(4)(7)}{4}\right)+c_2
\end{aligned}$$

for arbitrary constants c_1 and c_2. Combining these to a single arbitrary constant C we have

$$e^{-4x}y=-\frac{1}{4}e^{-4x}\left(x+\frac{29}{4}\right)+C.$$

Solving for y we find

$$\begin{aligned}
y &= -\frac{1}{4}\left(x+\frac{29}{4}\right)+Ce^{4x} \\
&= -\frac{x}{4}-\frac{29}{16}+Ce^{4x} \\
&= -\frac{x}{4}-1.8125+Ce^{4x}.
\end{aligned}$$

Correct Answers:

- `(-0.25)*x-1.8125+C*e^(4*x)`

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7. (1 point) Are the following differential equations exact? (You have only one attempt! Submit all answers at the same time)

(a) [Choose/Exact/Not Exact] $\left(1-\frac{5}{y}+x\right)\frac{dy}{dx}+y=\frac{2}{x}-1.$

(b) [Choose/Exact/Not Exact] $(2y-6x)y'-5y=0.$

(c) [Choose/Exact/Not Exact] $\left(5y\sin(x)\cos(x)-y+3y^2e^{xy^2}\right)dx=(x-\sin^2(x)-5xye^{xy^2})dy.$

Solution:

SOLUTION:

(a) is exact, while (b) and (c) are not exact.

Correct Answers:

- `Exact`
- `Not Exact`
- `Not Exact`

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8. (1 point) Are the following differential equations exact? (You have only one attempt! Submit all answers at the same time)

(a) [Choose/Exact/Not Exact] $\left(x^8-y^8\right)dx+\left(x^8-8xy\right)dy=0.$

(b) [Choose/Exact/Not Exact] $(2y-4x)y'-4y-8x=0.$

(c) [Choose/Exact/Not Exact] $\left(5y\sin^4(x)\cos(x)-y+4y^2e^{xy^2}\right)dx=(x-\sin^5(x)-8xye^{xy^2})dy.$

Solution:

SOLUTION:

(a) is not exact, while (b) and (c) are exact.

Correct Answers:

- `Not Exact`
- `Exact`
- `Exact`

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9. (1 point)

Solve the following differential equation:

$(8x+7y)dx+(7x-9y^3)dy=0.$

____________ = constant. help (formulas)

Solution:

SOLUTION:

First we verify that this is an exact differential equation. Since

$$\frac{\partial(8x+7y)}{\partial y}=4=\frac{\partial\left(7x-9y^3\right)}{\partial x}$$

this is indeed an exact DE. Thus, there exists a function $f(x,y)$ such that

$$\frac{\partial f}{\partial x}=8x+7y.$$

Thus, we integrate $8x+7y$ with respect to x by treating y as a constant. We find

$$f(x,y)=\int(8x+7y)\,dx \tag{1}$$

$$=\frac{8}{2}x^2+7yx+g(y), \tag{2}$$

where $g(y)$ is some function of y (or a constant). To continue to find the true $f(x,y)$ we need to determine what $g(y)$ is. To do so, we differentiate the previous display with respect to y to find

$$\frac{\partial f}{\partial y}=7x+g'(y).$$

Since $\frac{\partial f}{\partial y}=7x-9y^3$, we have

$$7x+g'(y)=7x-9y^3,$$

so that $g'(y)=-9y^3$. Integrating $g'(y)$ with respect y we find

$$\begin{aligned}
g(y) &= \int g'(y)\,dy=\int\left(-9y^3\right)dy \\
&= -\frac{9}{4}y^4+C.
\end{aligned}$$

Plugging this into the equation above for $f(x,y)$ we find

$$f(x,y)=\frac{8}{2}x^2+7xy-\frac{9}{4}y^4+C.$$

Thus, the solution we desire is

$$\frac{8}{2}x^2+7xy-\frac{9}{4}y^4=\text{constant} \qquad \text{or}$$

$$4x^2+7xy-2.25y^4=\text{constant}$$

is the desired answer.

Note, we could have also integrated $7x-9y^3$ with respect to y to find

$$\int \left(7x-9y^3\right)\,dy = 7xy - 2.25y^4 + C$$

and combined this with our integral of $8x+7y$ with respect to x.

Correct Answers:

- `8/2*x*x+7*x*y-9/4*y^4`

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10. (1 point)

Solve the following differential equation:

$(y-x^2)dx+(x+y^2)dy=0.$

______________ = constant. help (formulas)

Solution:

SOLUTION:

First we verify that this is an exact differential equation. Since

$$\frac{\partial\left(y-x^2\right)}{\partial y} = 1 = \frac{\partial\left(x+y^2\right)}{\partial x}$$

this is indeed an exact DE. Thus, there exists a function $f(x,y)$ such that

$$\frac{\partial f}{\partial x} = y - x^2.$$

Thus, we integrate $y-x^{3-1}$ with respect to x by treating y as a constant. We find

$$f(x,y) = \int \left(y-x^2\right)\,dx \tag{3}$$

$$= xy - \frac{x^3}{3} + g(y), \tag{4}$$

where $g(y)$ is some function of y (or a constant). To continue to find the true $f(x,y)$ we need to determine what $g(y)$\$ is. To do so, we differentiate the previous display with respect to y to find

$$\frac{\partial f}{\partial y} = x + g'(y).$$

Since $\frac{\partial f}{\partial y} = x+y^2$, we have

$$x+g'(y) = x+y^2,$$

so that $g'(y) = y^2$. Integrating $g'(y)$ with respect y we find

$$g(y) = \int g'(y)\,dy = \int \left(y^2\right)\,dy$$
$$= \frac{y^3}{3} + C.$$

Plugging this into the equation above for $f(x,y)$ we find

$$f(x,y) = xy - \frac{x^3}{3} + \frac{y^3}{3} + C.$$

Thus, the equation

$$xy - \frac{x^3}{3} + \frac{y^3}{3} = \text{constant} \qquad \text{or}$$
$$3xy - x^3 + y^3 = \text{constant}$$

is the desired answer. The second solution here comes from the fact that we could multiply the entire expression by 3 since it is still an arbitrary constant on the right side.

Correct Answers:

- `3*x*y-x^3+y^3`

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11. (1 point)

Solve the following differential equation:

$$\left(1-\frac{3}{y}+x\right)\frac{dy}{dx} + y = \frac{3}{x} - 1.$$

(If you need ln, use absolute value signs. For example, ln| input |.)

______________ = constant. help (formulas)

Solution:

SOLUTION:

We begin by rewriting the equation as

$$\left(y-\frac{3}{x}+1\right)dx + \left(1-\frac{3}{y}+x\right)dy = 0.$$

We verify that this is an exact differential equation. Since

$$\frac{\partial}{\partial y}\left(y-\frac{3}{x}+1\right) = 1 = \frac{\partial}{\partial x}\left(1-\frac{3}{y}+x\right)$$

this is indeed an exact DE. Thus, there exists a function $f(x,y)$ such that

$$\frac{\partial f}{\partial x} = y - \frac{3}{x} + 1.$$

Thus, we integrate $y-\frac{3}{x}+1$ with respect to x by treating y as a constant. We find

$$f(x,y) = \int\left(y-\frac{3}{x}+1\right)dx \tag{5}$$

$$= xy - 3\ln|x| + x + g(y) \tag{6}$$

$$= x(y+1) - 3\ln|x| + g(y), \tag{7}$$

where $g(y)$ is some function of y (or a constant). To continue to find the true $f(x,y)$ we need to determine what $g(y)$ is. To do so, we differentiate the previous display with respect to y to find

$$\frac{\partial f}{\partial y} = x + g'(y).$$

Since $\frac{\partial f}{\partial y} = 1-\frac{3}{y}+x$, we have

$$x+g'(y) = 1-\frac{3}{y}+x,$$

so that $g'(y) = 1-\frac{3}{y}$. Integrating $g'(y)$ with respect y we find

$$g(y) = \int g'(y)\,dy = \int\left(1-\frac{3}{y}\right)dy$$
$$= y - 3\ln|y| + C.$$

Plugging this into the equation above for $f(x,y)$ we find

$$\begin{aligned} f(x,y) &= x(y+1) - 3\ln|x| + y - 3\ln|y| + C \\ &= xy + x + y - 3\left(\ln|x| + \ln|y|\right) + C \\ &= xy + x + y - 3\ln|xy| + C. \end{aligned}$$

Thus, the equation

$$xy + x + y - 3\ln|xy| = \text{constant} \qquad \text{or}$$
$$xy + x + y - 3\ln|xy| = \text{constant}$$

is the desired answer.

Correct Answers:

- `x+y+x*y-3*ln(|x*y|)`