course: CSC 135-01 - Computing Theory and Programming Languages

instructor: Ted Krovetz

related_notes: 2022-03-17

Finite Automata

W11.4 | Thursday, March 17, 2022 | 09:01 AM

Multiple approaches to text search

- Exact search
- Computational Biology: loosely match

Exact Match

Have a pattern `P`, length `M`
Text `T`, with length `N`
Find all occurrences of `P` in `T`

The Naive Approach

`T` [xxxxxxxxxxxxxxxxxxxxxxxx]
`P` [...] ← test to match with the text above
`T` [xxxxxxxxxxxxxxxxxxxxxxxx]
`P` [...][...]
`T` [xxxxxxxxxxxxxxxxxxxxxxxx]
`P` [...][...][...] ← And so on...

The Naive Algorithm

```
for i = 1 to n-m+1 # Time-complication: O([n-m]*m) <- O(n*m) if n > m
        if P == T[i...i+m-i]   # Time-complication: O(m)
                output "match at"
```

Naive Approach Example

```
T: ..abcde](f)g...  # ..abcdef suffix of T upto the mismatch
P: not{abcde}k
```

```
            abcdek <- naive shift
                 abcdek <- clever shift
P: abfdek
     [ab(f)dek] # If a suffix matches the prefix of our pattern...
```

The Clever Approach

Idea: Shift `P` so that we have a match between a prefix of `P` and a suffix of T up to `T`'s mismatch

Clever Approach Example

At each step: we compare prefix of `P` with suffix of `T`

- This is "expensive" time-compilation at O(m*n)

TODO: MAKE SURE TO ASK THE PROFESSOR/DISCORD TO HOW THIS WORKS

```
T: aaabaaabaaab
P: baab

# 1) first `a` of T against `b` of P
T: a]aabaaabaaab
P: [baab


# 2)
T: aaabaaabaaab
P: baab


# 3)
T: aaabaaabaaab
P: baab


# 4)
T: aaabaaabaaab
P: baab
```

Clever Approach via Pre-processing

```
               w     c
T: [     |/////////|X|        ]
P:       [/////////|Y]
```

Pre-computation

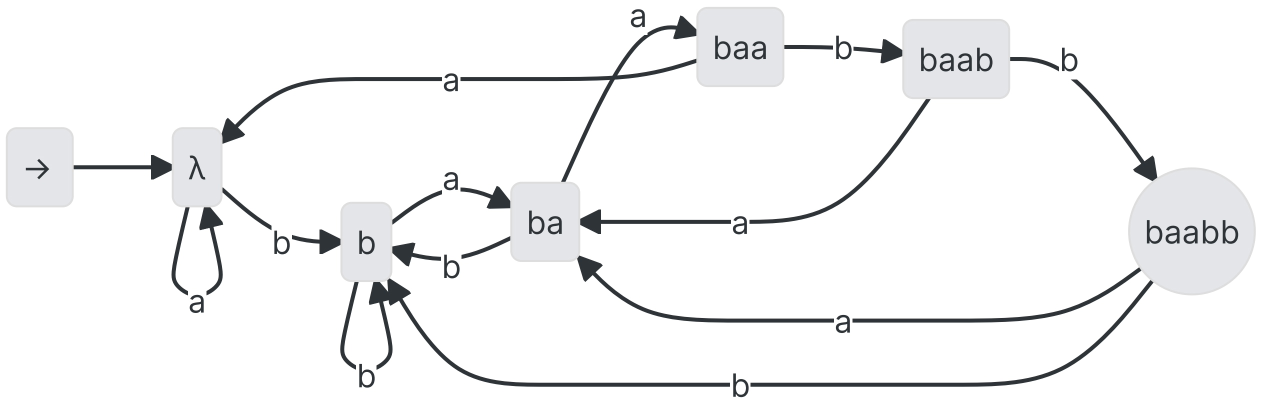
A = {a, b} ← our language
P: baabb

`w` (matches)	`c` (miss-matches)	Biggest Prefix of P = Suffix of wc (DFA Transitions)
λ	a	λ
a	b	b
ba	b	b
baa	b	λ
baab	a	ba
match {baabb}	a	ba
match {baabb}	b	b

This allows us to make into a DFA and can be seen as DFA transitions

- We can name our states with what we matched
- Time-complication for search: O(n)
    - Each time entering accept state output: Found `P`
- Time-complication for preprocessing: multiply the below we get $O\left(m^3 \cdot |A|\right)$, but clever algorithms can do so in $O\left(m \cdot |A|\right)$
    - Lengths of P prefixes: O(m)
    - To find longest prefix = suffix: $O(m^c)$
    - Possible `c` values: $O\left(|A|\right)$
- All together and overall time-complication: $O\left(n + m \cdot |A|\right)$ very good if n >> m

Note: **baabb** is the accepting state


→
λ
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b
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b