

1st-order linear equations

We are interested in 1st-order linear DEs. Recall the following definition.

Definition:

A differential equation of the form

$$c(x)\frac{dy}{dx} + b(x)y = a(x) \quad (*)$$

is called a **(1st-order) linear differential equation**.

Here, $a(x)$, $b(x)$, $c(x)$ are arbitrary functions of the variable x . Note, that a 1st-order linear differential equation could also be written in a form such as $c(x)y' + b(x)y + d(x) = 0$ as well, where $d(x)$ is a function.

As we will see shortly, to solve such a DE, it will be useful to put a DE of the form (*) into standard form. Thus, we rewrite (*) as

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where we divided both sides of the equation by the function $c(x)$ so that we now have $P(x) = \frac{b(x)}{c(x)}$ and $Q(x) = \frac{a(x)}{c(x)}$. More succinctly, we can set

$P = P(x)$ and $Q = Q(x)$ and write

$$\frac{dy}{dx} + Py = Q.$$

Placing 1st-order linear differential equations into this form will be the first step in solving them.

Discussion, comments, and examples:



Math45-Module-05-Video-01

WeBWork module 05 exercises:

- Problems 1, 2

Relevant Wikipedia articles:

- [Linear differential equations](https://en.wikipedia.org/wiki/Linear_differential_equation) [. \(https://en.wikipedia.org/wiki/Linear_differential_equation\)](https://en.wikipedia.org/wiki/Linear_differential_equation)
- [Inseparable differential equation](https://en.wikipedia.org/wiki/Inseparable_differential_equation) [. \(https://en.wikipedia.org/wiki/Inseparable_differential_equation\)](https://en.wikipedia.org/wiki/Inseparable_differential_equation)