1. (1 point)
Let $\mathbf{X} = \binom{x}{y}$. Find the matrix form of the linear system

$$\frac{dx}{dt} = 6x + 7y$$
$$\frac{dy}{dt} = 1x + 9y.$$

- A. $\mathbf{X}' = \begin{pmatrix} 6 & 1 \\ 7 & 9 \end{pmatrix} \mathbf{X}$
- B. $\mathbf{X}' \begin{pmatrix} 6 & 1 \\ 7 & 9 \end{pmatrix} = \mathbf{X}$
- C. $\mathbf{X}' = \begin{pmatrix} 6 & 7 \\ 1 & 9 \end{pmatrix} \mathbf{X}$
- D. $\mathbf{X}' = \begin{pmatrix} 1 & 9 \\ 6 & 7 \end{pmatrix} \mathbf{X}$

Solution:
SOLUTION:
The correct answer is $\mathbf{X}' = \begin{pmatrix} 6 & 7 \\ 1 & 9 \end{pmatrix} \mathbf{X}$.
Correct Answers:

- C

2. (1 point)
Let $\mathbf{X} = \binom{x}{y}$. Write the system of differential equations given by $\mathbf{X}' = \begin{pmatrix} 1 & 1 \\ 3 & 8 \end{pmatrix} \mathbf{X}$ without using matrices.

- A. $\frac{dx}{dt} = 1x + 1y$; $\frac{dy}{dt} = 3x + 8y$
- B. $\frac{dx}{dt} = 3x + 8y$; $\frac{dy}{dt} = 1x + 1y$
- C. $\frac{dx}{dt} = 1x - 1y$; $\frac{dy}{dt} = 3x - 8y$
- D. $\frac{dx}{dt} = 1x + 1y$; $\frac{dy}{dt} = 8x + 3y$

Solution:
SOLUTION:
The correct answer is $\frac{dx}{dt} = 1x + 1y$; $\frac{dy}{dt} = 3x + 8y$.
Correct Answers:

- A

3. (1 point)
Which of the following is a solution to the system of differential equations given by $\mathbf{X}' = \begin{pmatrix} 4 & 3 \\ 0 & 6 \end{pmatrix} \mathbf{X}$?

- A. $\mathbf{X} = \binom{1}{0} e^{4t}$
- B. $\mathbf{X} = \binom{1}{-1} e^{6t}$
- C. $\mathbf{X} = \binom{0}{1} e^{4t}$
- D. $\mathbf{X} = \binom{1}{0} e^{6t}$

Solution:
SOLUTION:
The correct answer is $\mathbf{X} = \binom{1}{0} e^{4t}$.
Correct Answers:

- A

4. (1 point)
Which of the following is a solution to the system of differential equations given by $\mathbf{X}' = \begin{pmatrix} 8 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{X}$?

- A. $\mathbf{X} = \binom{1}{0} e^{2t}$
- B. $\mathbf{X} = \binom{1}{0} e^{8t}$
- C. $\mathbf{X} = \binom{0}{1} e^{8t}$
- D. $\mathbf{X} = \binom{0}{2} e^{2t}$

Solution:
SOLUTION:
The correct answer is $\mathbf{X} = \binom{1}{0} e^{8t}$.
Correct Answers:

- B

5. (1 point)

Which of the following vectors forms a fundamental set of solutions to the system of differential equations $\mathbf{X}' = \left(\begin{smallmatrix} 2 & 1 \\ 0 & 6 \end{smallmatrix}\right) \mathbf{X}$?

- A. $\mathbf{y_1} = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) e^{2t}$; $\mathbf{y_2} = \left(\begin{smallmatrix} 2 \\ 0 \end{smallmatrix}\right) e^{6t}$
- B. $\mathbf{y_1} = \left(\begin{smallmatrix} 2 \\ -2 \end{smallmatrix}\right) e^{1t}$; $\mathbf{y_2} = \left(\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right) e^{1t}$; $\mathbf{y_3} = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) e^{2t}$
- C. $\mathbf{y_1} = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) e^{2t}$; $\mathbf{y_2} = \left(\begin{smallmatrix} 1 \\ 4 \end{smallmatrix}\right) e^{6t}$
- D. $\mathbf{y_1} = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) e^{2t}$; $\mathbf{y_2} = \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right) e^{1t}$; $\mathbf{y_3} = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) e^{2t}$; $\mathbf{y_4} = \left(\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right) e^{1t}$

Solution:

SOLUTION:

The correct answer is $\mathbf{y_1} = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right) e^{2t}$; $\mathbf{y_2} = \left(\begin{smallmatrix} 1 \\ 4 \end{smallmatrix}\right) e^{6t}$.

Correct Answers:

- `C`

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6. (1 point)

Is it guarenteed that the initial value problem consisting of the system $\mathbf{X}' = \left(\begin{smallmatrix} 7 & 9 \\ 0 & 6 \end{smallmatrix}\right) \mathbf{X} + \begin{pmatrix} \sin(t) \\ \frac{1}{t} \end{pmatrix}$ and the initial conditions $\mathbf{X}(3) = \begin{pmatrix} \sqrt{49} \\ 49 \end{pmatrix}$ has a unique solution on the interval $I = (0, \infty)$?

- A. Yes
- B. No

Solution:

SOLUTION:

Since the functions $\sin(t)$ and $\frac{1}{t}$ are both continuous on $I = (0, \infty)$ and the point 3 is in I, the correct answer is YES.

Correct Answers:

- `A`

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7. (1 point) Consider the linear system

$$\mathbf{X}' = \begin{bmatrix} 6 & 4 \\ -12 & -8 \end{bmatrix} \mathbf{X}.$$

(1) Find the eigenvalues and eigenvectors for the coefficient matrix.

$\lambda_1 =$ ___, $\vec{v}_1 = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$, and $\lambda_2 =$ ___, $\vec{v}_2 = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

(2) For each eigenpair in the previous part, form a solution of $\mathbf{X}' = A\mathbf{X}$. Use t as the independent variable in your answers.

$\vec{y}_1(t) = \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$ and $\vec{y}_2(t) = \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$

(3) Does the set of solutions you found form a fundamental set (i.e., linearly independent set) of solutions?

- Choose
- Yes, it is a fundamental set
- No, it is not a fundamental set

Correct Answers:

-
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-
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```
- `Yes, it is a fundamental set`

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