

Consider the differential equation  $(x - 6)y'' + (x + 5)y' + x^3y = e^x$  with initial conditions  $y(2) = 4$  and  $y'(2) = 3$ . Which of the following is something we do NOT need to check in order to guarantee the existence of a unique solution for the IVP on an interval  $I$ ?

- A. That  $x - 6$ ,  $x + 5$ ,  $x^3$ , and  $e^x$  are continuous on  $I$ .
- B. That 2 is in  $I$ .
- C. That  $x - 6 \neq 0$  for any  $x$  in  $I$ .

- D. That  $x - 6 = x + 5$  for some  $x$  in  $I$ .

Answer(s) submitted:

•  
(incorrect)

Consider the differential equation  $(x - 5)y'' + (x + 7)y' + x^9y = e^x$  with initial conditions  $y(5) = 9$  and  $y'(5) = 5$ . For what reason does our theorem FAIL to guarantee the existence of a unique solution on the interval  $I = (-\infty, \infty)$ ?

- A. Since  $x - 5 = 0$  for some  $x$  in  $I$ .
- B. Since 5 is not in  $I$ .
- C. Since one of  $x - 5$ ,  $x + 7$ ,  $x^9$ , and  $e^x$  are not continuous on  $I$ .

Answer(s) submitted:

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(incorrect)

Consider the differential equation  $(x - 14)y'' + \frac{1}{x + 9}y' + x^7y = e^x$  with initial conditions  $y(-9) = 8$  and  $y'(-9) = 8$ . Why does our theorem fail to guarantee the existence of a unique solution on the interval  $I = (-10, -8)$ .

- A. Since  $x - 14 = 0$  for some  $x$  in  $I$ .
- B. Since  $-9$  is not in  $I$ .

- C. Since  $\frac{1}{x + 9}$  is not continuous on  $I$ .

Answer(s) submitted:

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(incorrect)

4. (1 point) Select the following which are homogeneous linear differential equations.

- A.  $y' = 2y$

- B.  $x'' - 2x' + 3x = 0$

- C.  $y'' + 3 = 0$

- D.  $s^{(2)} + s = 0$

- E.  $y' = x$

- F.  $y^3 - y^2 + y = 0$

- G.  $x^2 \frac{dy^5}{dx^5} + e^x \frac{dy}{dx} = xy$

- H.  $\frac{d^2y}{dx^2} - e^x \frac{dy}{dx} + \sin(x) = 2$

- I. None of the above

Answer(s) submitted:

•  
(incorrect)

equations: dependent variable  $y$  and  
 A linear ordinary differential equation of order  $n$  in the independent variable  $x$  is said to be homogeneous if it is of the form or can be expressed in the form

$$a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1}(x) \frac{dy}{dx} + a_n(x)y = 0 \quad (1)$$

✓ A.  $s^{(2)} + s = 0$  is a second order homogeneous linear equation. Here  $n=2$ ,  $a_0=1$ ,  $a_1=0$  and  $a_2=1$

✓ B.  $y' = 2y \Rightarrow \frac{dy}{dx} - 2y = 0$  is a homogeneous equation with  $n=1$ ,  $a_0(x)=1$  and  $a_1(x)=-2$

✗ C.  $y' = x \Rightarrow \frac{dy}{dx} - x = 0$  not an equation of the form (1) and hence not homogeneous.

✓ D.  $x^2 \frac{d^5 y}{dx^5} + e^x \frac{dy}{dx} = xy$  [ $\frac{dy^5}{dx^5}$  should be corrected as  $\frac{d^5 y}{dx^5}$ ]

This is a homogeneous equation since it can be written as  $x^2 \frac{d^5 y}{dx^5} + e^x \frac{dy}{dx} - xy = 0$

Here  $n=5$ ,  $a_0(x)=x^2$ ,  $a_1(x)=0$ ,  $a_2(x)=0$ ,  $a_3(x)=0$ ,  $a_4(x)=e^x$ ,  $a_5(x)=-x$ .

✓ E.  $x'' - 2x' + 3x = 0$ . Note that here  $x$  is the dependent variable.  
 $n=2$ ,  $a_0=1$ ,  $a_1=-2$ ,  $a_2=3$

□ F.  $y^3 - y^2 + y = 0$  If the exponents of  $y$  indicates derivatives, then it is homogeneous. otherwise not.

✗ G.  $y'' + 3 = 0$  Not homogeneous

✗ H.  $\frac{d^2 y}{dx^2} - e^x \frac{dy}{dx} + \sin(x) = 2$  not homogeneous since the