Which of the following is the definition of the Laplace transform on the function $\sin(t)$?

- A. $\int_0^\infty e^{-st} e^{7t}\, dt$
- B. $\int_0^\infty e^{-st} \sin(s))\, dt$
- C. $\int_0^\infty e^{-st} \sin(t)\, dt$
- D. $\int_0^\infty e^{-st} \cos(t)\, dt$

Solution:
SOLUTION:
The correct answer is C.
Correct Answers:

- C

The expression $\lim_{a\to\infty} \int_0^a e^{-st} f(t)\, dt$ is a function of which variable?

- A. k
- B. t
- C. a
- D. s

Solution:
SOLUTION:
The correct answer is D.
Correct Answers:

- D

We have $\mathcal{L}(\cos(8t))$ equals which of the following?

- A. $\frac{1}{s-8}$
- B. $\frac{8}{64+s}$
- C. $\frac{s}{8+s}$
- D. $\frac{s}{64+s^2}$

Solution:
SOLUTION:
The correct answer is D.
Correct Answers:

- D

The expression $\mathcal{L}^{-1}\{F(s)\}$ is a function of which variable?

- A. a
- B. t
- C. k
- D. s

Solution:
SOLUTION:
The correct answer is B.
Correct Answers:

- B

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5. (1 point)
Use the theorem for the inverse Laplace transform to compute the inverse Laplace operator $\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$.

(Note: your answer should be a function of t.)

$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} =$__________

Solution:

SOLUTION:

The answer is $\frac{t^1}{1}$.

Correct Answers:

- `t^1/1`

Recall that $\mathcal{L}\{e^{8t}\} = \dfrac{1}{s-8}$ and $\mathcal{L}\{t\} = \dfrac{1}{s^2}$. Using this and linearity, we find $\mathcal{L}\{7e^{8t} + 4t\}$ is which of the following?

- A. $\dfrac{8}{s-7} + \dfrac{4}{s^2}$
- B. $\dfrac{7}{s-8} + \dfrac{4}{s^2}$
- C. $\dfrac{7}{s-4} + \dfrac{8}{s^2}$
- D. $\dfrac{4}{s-8} + \dfrac{7}{s^2}$

Solution:

SOLUTION:

The correct answer is B.

Correct Answers:

- `B`

Recall that $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2}\right\} = t$ and $\mathcal{L}^{-1}\left\{\dfrac{1}{s-2}\right\} = e^{2t}$. Using this and linearity, we find $\mathcal{L}^{-1}\left\{\dfrac{5}{s^2} + \dfrac{7}{s-2}\right\}$ is which of the following?

- A. $2t + 7e^{5t}$
- B. $7t + 2e^{5t}$
- C. $7t + 5e^{2t}$
- D. $5t + 7e^{2t}$

Solution:

SOLUTION:

The correct answer is D.

Correct Answers:

- `D`

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8. (1 point) Select all of the Laplace transforms of derivatives that are CORRECT.

- A. $\mathcal{L}\{f^{(4)}(t)\} = s^4F(s) - s^3F(s) - s^2F(s) - sF(s) - f(0)$
- B. $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$
- C. $\mathcal{L}\{f^{(3)}(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$
- D. $\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$
- E. $\mathcal{L}\{f^{(6)}(t)\} = s^6f(0) - s^5f'(0) - s^4f''(0) - s^3f'''(0) - s^2f^{(4)}(0) - sf^{(5)}(0) - f^{(6)}(0)$
- F. $\mathcal{L}\{f'(t)\} = f'(0) - sF(s)$
- G. $\mathcal{L}\{f''(t)\} = f''(0) - sf'(0) - s^2F(s)$
- H. $\mathcal{L}\{f^{(5)}(t)\} = s^5F(s) - s^4f(0) - s^3f'(0) - s^2f''(0) - sf'''(0) - f^{(4)}(0)$
- I. None of the above

Solution:

SOLUTION:

The correct answer is BCDH.

Correct Answers:

- `BCDH`

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9. (1 point)

Which of the following is the correct form of the partial fraction decomposition of $\dfrac{2x}{(x+3)(3x+1)}$?

- A. $\dfrac{A}{x+3} + \dfrac{B}{3x+1}$
- B. $\dfrac{Ax+B}{x+3} + \dfrac{C}{3x+1}$
- C. $\dfrac{Ax+B}{x+3} + \dfrac{Cx+D}{3x+1}$
- D. $\dfrac{A}{x+3} + \dfrac{Bx+C}{3x+1}$

Correct Answers:

- `A`

10. (1 point)
Consider the rational function

$$F(s) = \frac{s^3 - 3}{(s^2+6)^2(s+10)^2}.$$

Select ALL terms below that occur in the general form of the complete partial fraction decomposition of $F(s)$. The capital letters A, B, C, . . . , L denote constants.

- A. $\frac{Hs+I}{s+10}$
- B. $\frac{Ks+L}{(s+10)^2}$
- C. $\frac{A}{s^2+6}$
- D. $\frac{G}{s+10}$
- E. $\frac{J}{(s+10)^2}$
- F. $\frac{Bs+C}{s^2+6}$
- G. $\frac{D}{(s^2+6)^2}$
- H. $\frac{Es+F}{(s^2+6)^2}$

Correct Answers:

- `DEFH`

11. (1 point) Find the constants needed in the partial fraction decomposition of the following rational function.

$$\frac{-\left(5x^2+2x+51\right)}{(x+3)(x^2+9)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+9}$$

$A =$ ____ $B =$ ____ $C =$ ____

Correct Answers:

- `-5`
- `0`
- `-2`

12. (1 point) Transform the differential equation

$$4y'' + 2y' - y = t^5$$

$$y(0) = -1$$

$$y'(0) = 5$$

into an algebraic equation by taking the Laplace transform of each side. (Be sure to plug in the initial conditions. Also, denote $Y(s)$ by just Y.)

__________________ = _________

Therefore,
$Y(s) =$ _________

Correct Answers:

- `-4*s*(-1)-20-(-2)+[4*s^2+2*s+(-1)]*Y`
- `120/(s^6)`
- `[120+s^6*(20-4*s-2)]/[(4*s^2+2*s-1)*s^6]`

13. (1 point)
Consider the initial value problem

$$y' + 2y = 16t, \qquad y(0) = 7.$$

(1) Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

__________________ = __________________ help (formulas)

(2) Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________________________

(3) Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$ ___________________________

Correct Answers:

- `s*Y(s)-7+2*Y(s)`
- `16/(s^2)`
- `16/[s^2*(s+2)]+7/(s+2)`
- `8*t-4+11*e^(-2*t)`

Which of the following is $\mathcal{L}\left\{e^{7t}\sin(3t)\right\}$?

- A. $\frac{3}{s^2-58}$
- B. $\frac{s}{s^2-14s+58}$
- C. $\frac{3}{s^2-14s+58}$
- D. $\frac{7}{s^2-14s+58}$

Solution:
SOLUTION:
The correct answer is C.
Correct Answers:

- C

Which of the following is $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2-18s+97}\right\}$?

- A. $e^{9t}\sin(4t)$
- B. e^{9t-4t}
- C. $e^{4t}\cos(9t)$
- D. $e^{9t}\cos(4t)$

Solution:
SOLUTION:
The correct answer is D.
Correct Answers:

- D