# Goal and idea - Module 17

## GOAL:

It is not uncommon to want to find multiple functions which *simultaneously* solve multiple differential equations. In other words, we may want to find solutions for a *system of differential equations*. As most of our recent knowledge centers around linear differential equations, we look at what a theory of systems of linear differential equations would look like. In particular, we

- define what systems of linear differential equations are, as well as IVPs in this setting;
- find how to verify if a given *vector* is a solution to a system if differential equations;
- learn when a unique solution to an IVP exists;
- learn what constitutes a fundamental set of solutions and a general solution for systems of linear differential equations; and
- solve systems of homogeneous linear differential equations!

## IDEA:

We essentially translate the theory we have seen before to the word of matrices and vectors!

To study systems of differential equations...

**DEFN**

(a) We call $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ a 2x2 and 3x3 matrix respectively. Here the letters are entries which for us are real numbers.

(b) We call $\begin{pmatrix} x \\ y \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ vectors with entries $x, y, z$

(c) We have

$\rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$

$\rightarrow \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{pmatrix}$

Also: Need

(i) Addition of vectors $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

(ii) Scalar multiplication of vector $\begin{pmatrix} a \\ b \end{pmatrix}$ and number or function $\alpha$:

$$\alpha \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \alpha a \\ \alpha b \end{pmatrix}$$

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**Ex** $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

matrix times vector = vector

$\rightarrow \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} (1)(-1) + (2)(2) \\ (3)(-1) + (4)(2) \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

**Ex** $\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 10 \end{pmatrix}$

$\rightarrow \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 10 \end{pmatrix} = \begin{pmatrix} -1 + 8 \\ 2 + 10 \end{pmatrix} = \begin{pmatrix} 7 \\ 12 \end{pmatrix}$

**Ex** $\left(e^{2t}\right)\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\rightarrow \left(e^{2t}\right)\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} e^{2t} \\ 2e^{2t} \end{pmatrix}$

**Ex** $(5)\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\rightarrow (5)\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \end{pmatrix}$

# Expectation checklist - Module 17

**At the completion of this module, you should:**

- know how to express a $2 \times 2$ system of linear differential equations in matrix form (both homogeneous and nonhomogeneous);
- be able to verify whether a given vector is a solution to a system of linear differential equations;
- be able to determine whether we are guaranteed the existence of a unique solution for an system IVP on a given interval;
- Determine whether a given 2-parameter family of solutions is a general solution by
  - finding if the functions within it form a fundamental set of solutions (this requires the Wronskian seen in this module);
- Solve a system of linear differential equations similar to the one seen in this module.

**Coming up next, we:**

- Nothing. Such emptiness.