Definition and computation of the Laplace transform

We first introduce the Laplace operator.

Definition

Let $f(t)$ be a function defined for $t \geq 0$. The integral

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

is called the **Laplace transform** of f, so long as the integral exists (converges).

Recall: An *improper integral*, such as the one above, is defined as

$$\int_0^\infty e^{-st} f(t)\, dt = \lim_{a \to 0} \int_0^a e^{-st} f(t)\, dt,$$

and converges if this limit is a finite number.

Note: While we plug a function of t into the Laplace transform, what we get out is a function of s!

In the video below we compute some examples of the the Laplace transform. Additionally, we note that knowing the Laplace transform for a few key functions can be useful. We therefore collect some import examples in the following theorem for future reference.

Theorem

We have

(a) $\mathcal{L}\{C\} = \dfrac{C}{s}$

(b) $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$

(c) $\mathcal{L}\{e^{kt}\} = \dfrac{1}{s-k}$

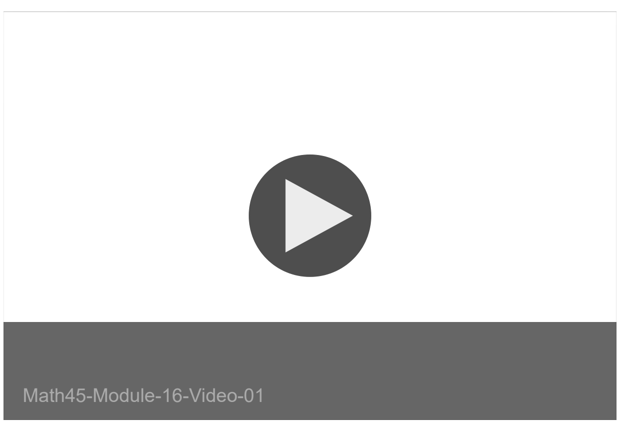
(d) $\mathcal{L}\{\cos(kt)\} = \dfrac{s}{s^2+k^2}$

(e) $\mathcal{L}\{\sin(kt)\} = \dfrac{k}{s^2+k^2}$

where C is an arbitrary constant, k is a given number, and n is any nonzero integer.

We will use all of these shortly.

Discussion, comments, and examples:



WeBWorK module 16 exercises:

- Problems 1, 2, 3

Relevant Wikipedia articles:

- [The Laplace transform](https://en.wikipedia.org/wiki/Laplace_transform) (https://en.wikipedia.org/wiki/Laplace_transform)
- [List of Laplace transforms](https://en.wikipedia.org/wiki/List_of_Laplace_transforms#Table) (https://en.wikipedia.org/wiki/List_of_Laplace_transforms#Table)