1. (1 point)

Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{7s+8}{s^2+100}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{7s+8}{s^2+100}\right\} =$ ______________ help (formulas)

Solution:

SOLUTION:

Correct Answers:

- `7*cos(10*t)+0.8*sin(10*t)`

2. (1 point)

Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{8}{s^2} + \dfrac{2}{s+9}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{8}{s^2} + \dfrac{2}{s+9}\right\} =$ ______________ help (formulas)

Solution:

SOLUTION:

We have

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{8}{s^2}+\frac{2}{s+9}\right\} &= 8\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}+2\mathcal{L}^{-1}\left\{\frac{1}{s-(-9)}\right\}\\ &= 8t+2e^{-9t}.\end{aligned}$$

Correct Answers:

- `8*t+2*e^(-9*t)`

3. (1 point)

Use translation properties for the Laplace transform to compute $\mathcal{L}\left\{e^{5t}\sin(6t)\right\}$.

- A. $\mathcal{L}\left\{e^{5t}\sin(6t)\right\} = \dfrac{5}{s^2-61}$
- B. $\mathcal{L}\left\{e^{5t}\sin(6t)\right\} = \dfrac{6}{s^2-10s+61}$
- C. $\mathcal{L}\left\{e^{5t}\sin(6t)\right\} = \dfrac{6}{s^2-5}$
- D. $\mathcal{L}\left\{e^{5t}\sin(6t)\right\} = \dfrac{6}{s^2-10s+25}$

Solution:

SOLUTION:

We have

$$\begin{aligned}\mathcal{L}\left\{e^{5t}\sin(6t)\right\} &= F(s-5)\\ &= \frac{6}{(s-5)^2+36}\\ &= \frac{6}{s^2-10s+61}.\end{aligned}$$

Correct Answers:

- `B`

4. (1 point)

Use translation properties for the Laplace transform to compute $\mathcal{L}^{-1}\left\{\dfrac{5}{s^2-12s+61}\right\}$.

- A. $\mathcal{L}^{-1}\left\{\dfrac{5}{s^2-12s+61}\right\} = e^{6t}\cos(5t)$
- B. $\mathcal{L}^{-1}\left\{\dfrac{5}{s^2-12s+61}\right\} = e^{6t}\sin(5t)$
- C. $\mathcal{L}^{-1}\left\{\dfrac{5}{s^2-12s+61}\right\} = e^{5t}\sin(6t)$
- D. $\mathcal{L}^{-1}\left\{\dfrac{5}{s^2-12s+61}\right\} = e^{5t}\cos(6t)$

Solution:

SOLUTION:

Completing the square and using translation properties we find

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{5}{s^2-12s+61}\right\} &= \mathcal{L}^{-1}\left\{\frac{5}{(s-6)^2+25}\right\}\\ &= e^{6t}\sin(5t).\end{aligned}$$

Correct Answers:

- `B`

5. (1 point)

Consider the following initial value problem:

$$y''-5y'-14y=\sin(2t) \qquad y(0)=-2,\ y'(0)=-1$$

Using Y for the Laplace transform of $y(t)$, i.e., $Y=\mathcal{L}\{y(t)\}$, find the equation you get by taking the Laplace transform of the differential equation and solve for

$Y(s) =$ ______________________

To find a solution to the IVP above, what steps must next be performed next?

- A. Plug in the given values to $Y(s)$.
- B. Take the derivative of $Y(s)$.
- C. Apply the inverse Laplace transform to $Y(s)$ (using partial fraction decomposition where necessary).
- D. Apply the Laplace transform to $Y(s)$.

Solution:
SOLUTION:
We begin by applying the Laplace transform to the differential equation. That is,

$$y'' - 5y' - 14y = \sin(2t) \qquad y(0) = -2,\ y'(0) = -1$$

$$\mathcal{L}\left\{y'' - 5y' - 14y\right\} = \mathcal{L}\left\{\sin(2t)\right\}$$

which via the linearity properties becomes

$$\mathcal{L}\left\{y''\right\} - 5\mathcal{L}\left\{y'\right\} - 14\mathcal{L}\left\{y\right\} = \mathcal{L}\left\{\sin(2t)\right\}.$$

Using that

$$\begin{aligned} \mathcal{L}\left\{y\right\} &= Y(s) \\ \mathcal{L}\left\{y'\right\} &= sY(s) - y(0) = sY(s) - -2 \\ \mathcal{L}\left\{y''\right\} &= s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - s - 2 - -1 \end{aligned}$$

and also that

$$\mathcal{L}\left\{\sin(2t)\right\} = \frac{2}{s^2+4},$$

we have the equation becomes

$$s^2Y(s) + 2s + 1 - 5\left(sY(s) + 2\right) - 14Y(s) = \frac{2}{s^2+4}.$$

This can be rewritten as

$$Y(s)\left(s^2 - 5s - 14\right) + 2s + 1 + (5)(-2) = \frac{2}{s^2+4}.$$

Solving for $Y(s)$ gives

$$Y(s) = \frac{-2s+9}{s^2-5s-14} + \frac{2}{((s^2-5s-14)(s^2+4))}.$$

To find a solution to the IVP above, apply the inverse Laplace transform to $Y(s)$ (using partial fraction decomposition where necessary).

Correct Answers:

- `(-2*s+9)/(s^2 - 5*s - 14)+2/((s^2 - 5*s - 14)*(s^2+4))`
- `C`