Please show and explain your work. Justification is necessary for credit. Also sign your name here acknowledging you will not use problem solving software (Wolfram Alpha, etc.):__________________

1. (*5 points*)

a. (*2 pts*) Circle all of the following expression which are **homogeneous linear differential equations**.

(i) $y''' - 2y''' + 3y = 5$

(ii) $\left(x^2 - \sin(x)\right) y^{(5)} - xe^x y' + \sin\left(x^2\right) y = 0$

(iii) $\sqrt{y'} = x\cos(x)y - 3$

(iv) $y'' = y + 2y'$

b. (*2 pts*) We have that $y_h = c_1 e^x + c_2 e^{-x}$ is a general solution for the DE $y'' - y = 0$ and that $y_p = x^3$ is a particular solution to $y'' - y = 6x - x^3$ (you do not need to verify these statements). Provide a general solution to the DE $y'' - y = 6x - x^3$.

c. (*1 pt*) Compute the Wronskian of $y_1 = x$ and $y_2 = x^2$.

2. (*10 points*) Given that $y_1 = x\sin(x)$ is a solution to the DE $x^2y'' - 2xy' + (x^2 + 2)y = 0$, find a general solution to this differential equation.

3. (*20 points*) Solve the following differential equations. If a technique is asked for, use it.

a. (*5 pts*) $y'' - 6y' + 9y = 0$
b. (*5 pts*) $y'' + 4y' + 7y = 0$
c. (*10 pts*) $y'' - 4y' + 3y = \cos(x) + \sin(x)$ [using the method of undetermined coefficients]

4. (*5 points*) Suppose $y_1 = x$ and $y_2 = x - 1$ are solutions to a 2nd-order homogeneous linear DE on $I = (-\infty, \infty)$. Do y_1 and y_2 form a fundamental set of solutions for this DE on I? Explain your answer using any math needed.

5. (*10 points*) Use the method of variation of parameters to find a general solution to the differential equation $x^2y'' - 4xy' + 4y = 3x^3$.