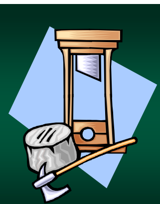


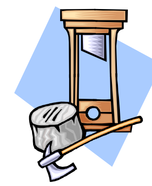
Midterm 2 Overview

Basic Information

1

Midterm Information

- 75 minutes
- 150 points
- No notes allowed
- Exams will be taken on Canvas



2

Exam Format

- They will be available for 75 minutes (normal class time)
- If the question has you fill in an answer – use lowercase (they are case sensitive)
- Bring scratch paper! **You will need it!**

3

Exam Format

- Have Zoom open during the exam
- I'm not going to watch you
- … this is so I can talk to the class if necessary and you can ask questions

4

Part 6 – Need to Know

- Boolean logic is set theory
- Boolean operators
- Implication!
- Precedence
- Tautology / contradiction
- Logic equivalences

5

Part 6 – Don't Worry About

- George Boole
- My "Gold Rush" example

6

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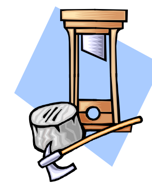

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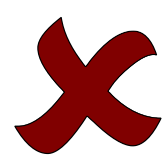

6

Part 7 – Need to Know

- Arguments are implications
- Valid arguments
- Logical fallacies

7

Part 7 – Don't Worry About

- *Know it all*

8

Part 8 – Need to Know

- Theorems & Definitions
- Direct proof
- Proof by contrapositive

9

Part 8 – Don't Worry About

- Proof by contradiction (still know the approach)

10

Part 9 – Need to Know

- Quantifiers
- Quantifier Equivalencies
- Bound/Free Variables
- Bounded Quantifiers
- Converting to English

11

Part 9 – Don't Worry About

- *Know it all*

12

Part 10 – Need to Know

- Induction!

13

Part 10 – Don't Worry About

- Strong Induction

14

Midterm 2 Study Guide

—Boolean operators—

Operator	Name
p and q	True only if both p and q are true
p or q	True if either p or q true
not p	True if p false
p xor q	True if p and q are different
p implies q	True unless p is true and q is false

Precedence Levels
Logic Notation of Operators

Logic	C Family	Visual Basic
2) p ∧ q	p && q	p and q
3) p ∨ q	p || q	p or q
1) ¬p	!p	not p
3) p ⊕ q	*none*	p xor q
4) p → q	→	P implies q

Truth Table

p	q	¬p (NOT)	p ∨ q (OR)	p ∧ q (AND)	p → q (Implication)
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	T	F	T
F	F	T	F	F	T

—Boolean logic is set theory—

T = U

F = ∅

The complement of U is ∅
So, naturally, False is represented with ∅

AND-Intersection Comparison

Boolean Logic	Set Theory
T ∧ T = T	U ∩ U = U
T ∧ F = F	U ∩ ∅ = ∅
F ∧ T = F	∅ ∩ U = ∅
F ∧ F = F	∅ ∩ ∅ = ∅

OR-Intersection Comparison

Boolean Logic	Set Theory
T ∨ T = T	U ∪ U = U
T ∨ F = T	U ∪ ∅ = U
F ∨ T = T	∅ ∪ U = U
F ∨ F = F	∅ ∪ ∅ = ∅

—Tautology/contradiction—

Tautology
If the statement is always true

$p \vee \neg p$

$p \rightarrow p$

$p = p$

Contradiction
If the statement is always false

$p \wedge \neg p$

$p \oplus p$

Logic Equivancies

Communitive Law
$a \land b \equiv b \land a$

$a \lor b \equiv b \lor a$

Identity Law
$a \land \text{true} \equiv a$

$a \lor \text{false} \equiv a$

Complement Law
$a \land \neg a \equiv \text{false}$

$a \lor \neg a \equiv \text{true}$

Distributive Law
$a \land (b \lor c) \equiv (a \land b) \lor (a \land c)$

$a \lor (b \land c) \equiv (a \lor b) \land (a \lor c)$

Absorption Law
$a \land (a \lor b) \equiv a$

$a \lor (a \land b) \equiv a$

Associative Law
$a \land (b \land c) \equiv (a \land b) \land c$

$a \lor (b \lor c) \equiv (a \lor b) \lor c$

Idempotent Law
$a \land a \equiv a$

$a \lor a \equiv a$

Involution Law
$\neg \neg a \equiv a$

Domination Law
$a \lor \text{true} \equiv \text{true}$

$a \land \text{false} \equiv \text{false}$

DeMorgan's Law
$\neg (a \land b) \equiv \neg a \lor \neg b$

$\neg (a \lor b) \equiv \neg a \land \neg b$

Example
$(A' \cap B) \cup (C \cap C') \cup (A' \cap B')$

$(A' \cap B) \cup \varnothing \cup (A' \cap B')$ Complement

$(A' \cap B) \cup (A' \cap B')$ Identity

$A' \cup (B \cap B')$ Distributive

$A' \cup U$ Complement

A' Solution

$(a \land b) \lor (\neg a \land b) =$

$(a \lor \neg a) \land b =$ After using Distributive Law

$\text{true} \land b =$ After using Complement Law

b After using Identify Law

Implication — Arguments are implications

For "p implies q" is contradicted (false) when p is true & q is false

$\underbrace{p}_{\text{antecedent, hypothesis, assumption}}$ — antecedent / hypothesis / assumption

q — Consequent / Conclusion / Therefore ⊢ or ∴

A implies B
$A \to B$
B if A
If A then B
B given A

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Equivalent
$p \to q \equiv \neg (p \land \neg q)$

$\equiv \neg p \lor q$

Example
$\neg \forall_x \exists_x (\neg B(x) \land P(x))$

$\neg \forall_x \exists_x (\neg B(x) \land P(x))$

$\exists_x \neg \exists_x (\neg B(x) \land P(x))$

$\exists_x \forall_x \neg (\neg B(x) \land P(x))$

$\exists_x \forall_x (B(x) \lor \neg P(x))$

$\exists_x \forall_x (P(x) \to B(x))$

OR Logic

p	q	¬p	¬p ∨ q	p → q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

AND Logic

p	q	¬q	p ∧ ¬q	¬(p ∧ ¬q)	p → q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Part 7

—Arguments are implications—

Argument is a collection of stetement called premises when all are true implies a consequence

- Arguments are actually implications with each premise connected with ∧
- So, if you have premises A, premise B, and conclusion C, then it has the following form

$A \wedge B \rightarrow C$

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

—Valid arguments—

Roles of Inference are valid arguments

Modus Pones (Law of Detachment) *Most basic rule*

Based on the logic that if:
- an implication is true
- implication's hypothesis is true
- then the implication's conclusion must be true

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

If it is a fish, then it lives in water.
It is a fish.
∴ Therefore, it lives in water! True

Modus Tollens * closely related to Modus Ponens*

Based on the logic that if:
- an implication is true
- implication's hypothesis is false
- then the implication's conclusion must be false

p	q	p → q	¬ p	¬ q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

If it is a fish, then it lives in water.
It doesn't live in water.
Therefore, it is not a fish!

$P \rightarrow q$
$\sim q$
$\sim P$ (True)

Disjunctive Syllogism *Based on the OR Operator*

Based on the logic that if:
- an or-statement is true
- one of the operands is false
- then, the other operand must be true

It breathes water or air.
It doesn't breath water.
Therefore, it breathes air.

$P \vee q$
$\sim P$
q (True)

Hypothetical Syllogism *Based on an implication chain*

- Gives a logical "chain" of events
- So, if a => b and b => c then a => c

If is a trout, then it is a fish
If it is a fish, then it lives in water.
Therefore, a trout lives in water!

$P \rightarrow q$
$q \rightarrow r$
$P \rightarrow r$ (True)

Part 7

— Logical fallacies —

Fallacy of the Converse

Based on assumption that if the conclusion is true then the hypothesis is true

Also called:
- affirming the consequent
- converse error

If it is a fish, then it lives in water.
It lives in water.
Therefore, it is a fish!

$p \to q$
q
$\overline{p}$

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Fallacy of the Inverse

Based on assumption that if the hypothesis is false, then the conclusion is also false

Also called:
- denying the antecedent
- inverse error

If it is a cat, then it is furry.
It is not a cat.
Therefore, it is not furry!

$p \to q$
$\sim p$
$\overline{\sim q}$

p	q	p → q	¬ p	¬ q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Fallacy of Affirming a Disjunct

Based on assumption that if there are two attributes and one is true, the other must be false

Also called:
- fallacy of the alternative
- false exclusionary disjunct

Suspect is either a politician or a lawyer.
Suspect is a politician.
Therefore, the suspect isn't a lawyer.

$p \vee q$
p
$\overline{\sim q}$

p	q	p ∨ q	¬ q
T	T	T	F
T	F	T	T
F	T	T	F
F	F	F	T

- Start with a
- Try to change it into ¬ b
- Use the premises to replace values

```
a ∨ b = true
a ∧ b = false
_____________
a = ¬b
```

```
a ∨ b = true
a ∧ b = false
```

a =	a	
=	a ∧ true	Identity
=	a ∧ (b ∨ ¬b)	Complement
=	a ∧ b ∨ a ∧ ¬b	Distributive
=	false ∨ a ∧ ¬b	Premise
=	b ∧ ¬b ∨ a ∧ ¬b	Complement
=	¬b ∧ (b ∨ a)	Distributive
=	¬b ∧ true	Premise
=	¬b	Identity

Theorems & Definitions

- A theorem is a statement we intend to prove using existing known facts (called axioms or lemmas)
- Used extensively in all mathematical proofs – which should be obvious

Example

- Most theorems are of the form: If A, then B
- The theorem below is very easy to interpret

If a and b are even integers
then a × b is an even integer

Direct proof

Proof by contrapositive

Part 9
- Quantifiers
- Quantifier Equivalencies
- Bound/Free Variables
- Converting to English

Part 10
- Induction