Linear and non-linear

Another way in which we can classify differential equations is whether they are linear or non-linear.

<u>Definition:</u>

- A **linear** DE is a DE that is a degree 1 polynomial in terms of its unknown functions and derivatives.
- A **nonlinear** DE is a DE that is not a linear DE.

Let's examine the definition of a linear DE a bit more. In other words, a linear DE is a DE of the form
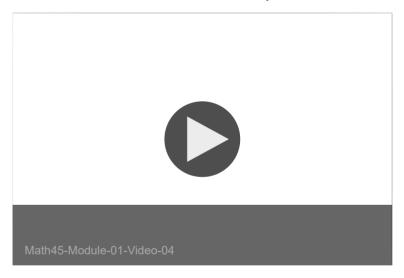
$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x) = b(x),$$

where

- the functions $a_0(x), \ldots, a_n(x)$, and $b(x)$ are differentiable functions *that are non necessarily linear functions*, and
- $y', y^{(2)}, \ldots, y^{(n)}$ are the first, second, ..., and nth derivative of the function y, respectively.

Note: What is important is that we don't have a term of something like $(y')^2$ or $\sqrt{y}$.

Discussion, comments, and examples:

Math45-Module-01-Video-04

WeBWorK module 01 exercises:

- Problem 6

Relevant Wikipedia articles:

- **linear differential equations**   **(https://en.wikipedia.org/wiki/Linear_differential_equation)**
- **nonlinear differential equation**   **(https://en.wikipedia.org/wiki/Nonlinear_partial_differential_equation)**