Solving order 2 homogeneous linear DEs with constant coefficients

Let's actually begin by solving order 1 homogeneous linear differential equations with constant coefficients. Such differential equations have the form

$$ay' + by = 0$$

for a constant a. We could solve this by noticing it is a separable equation and using the methods of Module 4, in which case we would obtain $y = Ce^{-\frac{b}{a}x}$. However, we point out another way. In particular, plugging in the function $y = e^{mx}$ for an unknown number m, the equation becomes (recalling $y' = me^{mx}$)

$$e^{mx}(am + b) = 0.$$

Since e^{mx} never equals 0, we must have $am + b = 0$, or $m = -\frac{b}{a}$. Thus we find that $y = e^{-\frac{b}{a}x}$. From here we can check it is a fundamental set of solutions, and thus, $y = Ce^{-\frac{b}{a}x}$ is the general solution.

We spent some time going over the 1st-order case, as this method generalizes to higher order.

We now turn to order 2. Such differential equations

$$ay'' + by' + cy = 0,$$

where a, b, and c are constants. Plugging in $y = e^{mx}$ gives

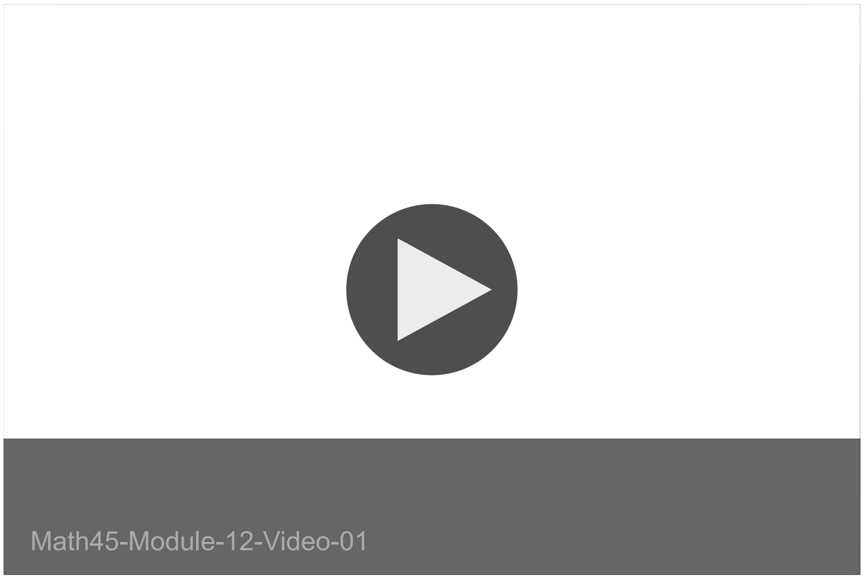
$$e^{mx}(am^2 + bm + c) = 0,$$

and again this means $am^2 + bm + c = 0$. From here, there are three possibilies:

- two distinct real roots,
- one real root that is repeated, or
- two complex roots (which are conjugates).

We examine how all three of these cases plays out in the video below.

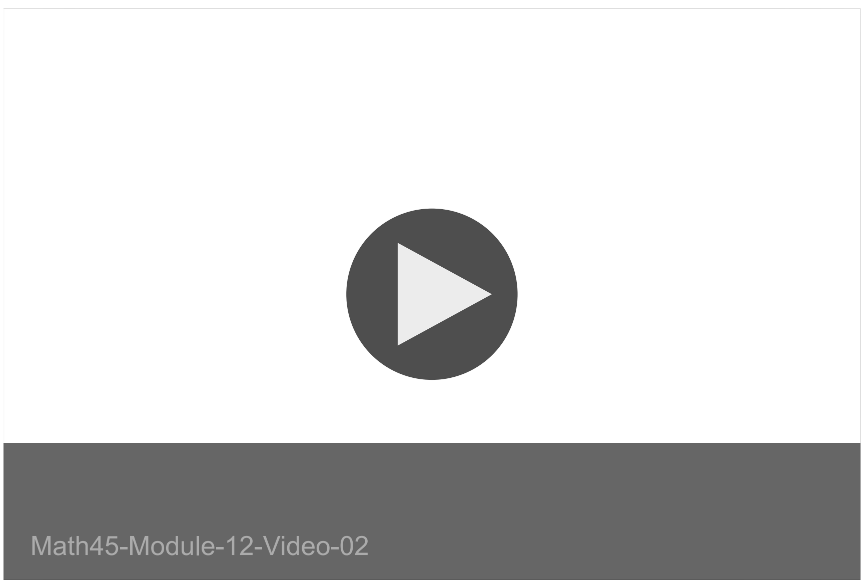
Discussion, comments, and examples:



Click on each of the tabs below to view a different example.

Example One | Example Two | Example Three

Example 1:



WeBWorK module 12 exercises:

- Problems 1, 2, 3, 4, 5, 6

Relevant Wikipedia articles:

- **Homogeneous linear DEs with constant coefficients (https://en.wikipedia.org/wiki/Linear_differential_equation#Homogeneous_equation_with_constant_coefficients)**