Math 45, Fall 2020
September 16, Quiz 03

Name: Matthew Mendoza

Please show and explain your work where necessary. Good luck!!

**1.** (*3 points*)

(i) Is the function $e^x$ a solution to the differential equation $y' - y = 0$? (Circle your answer.)

Yes No

(ii) Circle the following that is most likely to be a trivial solution to a DE.

$y = e^x$ $y = c$ $y = 0$ $y = Ce^x$

(iii) Circle the following that is most likely to be a particular solution to a DE. ($C, c_1, c_2, k$ are arbitrary constants.)

$y = e^x$ $y = c$ $y = c_1 x + c_2 x e^x$ $y = Ce^x$

(iv) Circle the following that is most likely to be a 2-parameter family of solutions to a DE. ($C, c_1, c_2, k$ are arbitrary constants.)

$y = e^x$ $y = c$ $y = c_1 x + c_2 x e^x$ $y = Ce^x$

(v) Circle the following that is most likely to be a general solution to a DE. ($C, c_1, c_2, k$ are arbitrary constants.)

$y = e^x$ $y = 0$ $y = c_1 x + c_2 x e^x$ $y = \cos(x)$

(vi) Suppose $y = \ln(x-3)$ is a solution to a DE. Circle the following which would best represent its interval of validity (or domain of the solution).

y=ln(x-3

$\cancel{(-\infty, \infty)}$ $\cancel{(-\infty, 3]}$ $\cancel{(-\infty, 3)}$ $(3, \infty)$

3

**2.** (*2 points*) Suppose $y = \dfrac{1}{x-3}$ is a solution to a differential equation. Is $(-\infty, 3) \cup (3, \infty)$ the interval of validity for the solution (or the domain of the solution)? If so, explain why. If not, provide a possible domain.

Yes; for, the interval is 3 non-inclusive. when x=3 the denom. is zero.

(There is another problem on the next page!)

Can't divide $\frac{1}{\emptyset}$

**3.** (*5 points*) Consider the function $f = c_1 \cos(3t) + c_2 \sin(3t)$, where $c_1$ and $c_2$ are arbitrary constants. We are given that $f = c_1 \cos(3t) + c_2 \sin(3t)$ is a 2-parameter family of solutions to the differential equation $f'' + f = 0$. Find a solution to the IVP consisting of this differential equation and the following initial conditions:

$$f\left(\frac{\pi}{3}\right) = \sqrt{2}, \qquad f'\left(\frac{\pi}{3}\right) = \sqrt{3}.$$

Initial conditions

$$\sqrt{2} = f(\pi/3) = C_1 \cos(\cancel{3}(\pi/\cancel{3})) + C_2 \sin(\cancel{3}(\pi/\cancel{3}))$$

$$\sqrt{2} = C_1(-1) + C_2(0)$$

$$\therefore \sqrt{2} = -C_1, \quad C_1 = -\sqrt{2}$$

$$f' = C_1 - \sin(3t)[3t]' + \cos(3t)[3t]'$$

$$\to f' = -3\sin(3t)C_1 + 3\cos(3t)$$

$$\to \sqrt{3} = f'\left(\frac{\pi}{3}\right) = -3\sin\left(3\left(\frac{\pi}{3}\right)\right)C_1 + 3\cos(3(\pi/3))C_2$$

$$\to \sqrt{3} = -3\sin(\pi)c_1 + 3\cos(\pi)c_2$$

$$\therefore \sqrt{3} = 3C_2, \quad C_2 = \frac{\sqrt{3}}{3}$$