Math 45, Fall 2020
November 4, Quiz 09

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Please show and explain your work where necessary. Good luck!! You may use the formulas

$$u_1' = -\frac{y_2 f(x)}{W} \qquad \text{and} \qquad u_2' = \frac{y_1 f(x)}{W}$$

if you so desire.

sin csc → $\frac{1}{\sin}$
cos sec → $\frac{1}{\cos}$
tan cot → $\frac{\cos}{\sin}$

1. (*10 points*) Use the method of variation of parameters to solve the differential equation $y'' + y = \sec(x)$.

$y'' + y = \sec(x)$ 2nd-order linear nonhomog

$e^{MX}(y^2+1)=0$

→ First, homog: $y'' + y = 0 \leftrightarrow M^2 + 1 = 0$

$M = \sqrt{-1}, \sqrt{1}$; $\sqrt{-1} \neq \sqrt{1}$

$e^0(C_1\cos(x) + C_2\sin(x))$

$y = C_1\cos(x) + C_2\sin(x)$

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variation of parameters

$$w(f_1, f_2) = \det\begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix}$$

$= (\cos\cdot\cos) + (-(-\sin)\cdot\sin)$

$= 1$ ← Pythag

$$u' = \frac{-\sin(x)\sec(x)}{1}$$

~~sin~~ need antiderv

$$u_1' = \int \frac{-\sin(x)\sec(x)}{1}\,dx$$

$$= -\int \sin(x)\sec(x)\,dx$$

$$= -\int \frac{\sin}{\cos}\,dx \quad u\text{-sub: } \cos(x)$$

$$= \int -\frac{1}{u}\,du = -\ln|u| = -\ln|\cos(x)|$$

$$u_2 = \int \frac{\cos(x)\sec(x)}{1}\,dx$$

$$= \int \left(\frac{\cos(x)}{1}\cdot\frac{1}{\cos(x)}\right)dx$$

$$= x$$

Particular solution

$$y_p = u_1 y_1 + u_2 y_2$$

$$= \ln|\cos(x)|\cos(x) + x\sin(x)$$

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$$y_p = u_1 y_1 + u_2 y_2$$

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∴ General solution

$$y = y_h + y_p$$

$$y_h = C_1\cos(x) + C_2\sin(x)$$

$$y_p = \ln|\cos(x)|\cos(x) + x\sin(x)$$

$$y = C_1\cos(x) + C_2\sin(x) + \ln|\cos(x)|\cos(x) + x\sin(x)$$