California State University, Sacramento
College of Engineering and Computer Science

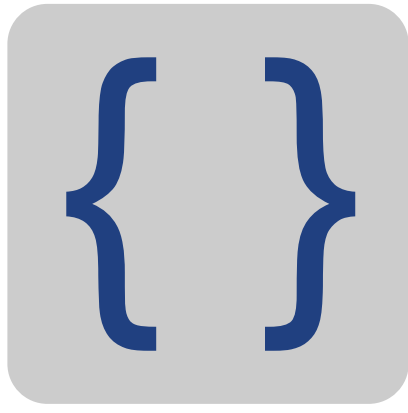
Computer Science 28: Discrete Mathematics

Spring 2020 – Assignment #1 – Sets

About

Homework is due one week after being assigned. It should be ready to turn-in at the beginning of lecture.

If you cannot turn-in your work in class, then you may submit your homework at Riverside Hall 3018 instead, but you must time-stamp and write “Cook CSc 28” across the top of your submission.



Set Notation

For the following set definitions, list all the items in the set.

1. A = { x | x is a letter in "Sacramento State" }
2. B = { x | x is a vowel and x ∈ A} // A = Question #1
3. C = { x | x ∈ Z and 20 < x ≤ 32 }
4. D = { 5x | x ∈ N and x < 7 }

Set Operators

Given the following sets:

U = { rat, ox, tiger, rabbit, dragon, snake, horse, sheep, monkey, rooster, dog, pig }
A = { rat, rabbit, snake, sheep, dog }
B = { tiger, horse, monkey, dog }
C = { rat, tiger, dragon, horse, monkey, pig }

Find the following:

1. A ∩ B
2. A ∩ C
3. B ∪ C
4. A '
5. C \ B
6. (A ∪ B) \ C

Venn Diagrams

Bikini Bottom is having a sale on used boats. Woo hoo! Now, you just have to pass the driving exam!

A survey was taken on **35** boats. Three different options were counted: radios, Krabby Patty holders (much better than cup holders), and jellyfish nets. The following information was gathered:

- 19 have radio (R)
- 17 have Krabby Patty holders (K)
- 15 have jellyfish nets (J)

Also…

- 13 have J and R
- 10 have K and J
- 12 have R and K
- 8 have all three options

Using a Venn Diagram to figure out the following:

1. How many only have R
2. How many only have J
3. How many only have K
4. How many have K and R, but not J
5. How many have J and K, but not R
6. Only one of the options
7. None of these options

Set Algebra

Simply the following. Use set algebra. Label all the laws you apply.

$$(A' \cap B) \cup (C \cap C') \cup (A' \cap B')$$

Floating Point Numbers

1. Encode the number **34.25** (34 and 1/4) to a single-precision (32 bit) floating point. Write down its binary representation. You don't have to convert to hex, but please put spaces either between each byte or each field. Show your work.

2. What is **01000010100110001100000000000000** when interpreted as a floating-point number?

3. What is **01111111100010000000000000000001** when interpreted as a floating-point number?

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Bit Vectors

Convert the following into bit vectors and then solve the questions below. Keep the result in bit-vector form.

U = { 1, 2, 3, … 10 }

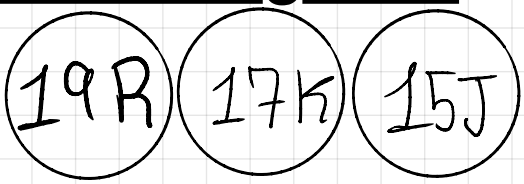
A = { 1, 8, 4, 6 }

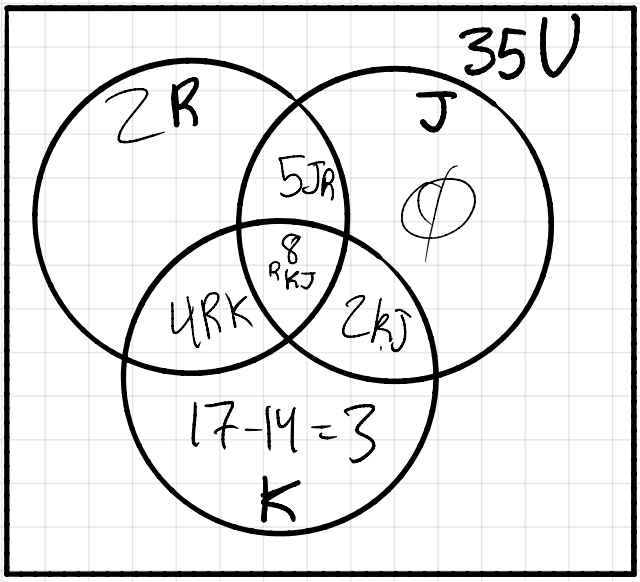
B = { 1, 9, 4, 7 }

Find the following:

1. A ∩ B
2. A ∪ B
3. A'
4. A \ B

Venn Diagrams





35-24
∴11

1) How many only have R
2

2) How many only have J
Zero

3) How many only have K
3

4) How many have K & R, but Not J
4

5) How many have J & K, but not R
2

6) Only one of the options
5 only R & J only have an option of itself 2+3=5

7) None of these options

Of all the cars that have options
2+8+5+2+4+3=24 subtracted from all boats from survey
35-24=11.

Spring 2020 - Assignment #1 - Sets Matthew Mendoza

Set Notation

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2. B = { x | x is a vowel and x ∈ A} // A = Question #1
3. C = { x | x ∈ Z and 20 < x ≤ 32 }
4. D = { 5x | x ∈ N and x < 7 }

1) A={S,a,c,r,m,e,n,t,o}

2) B={a,e,o}

3) Z is all Integers ...-2,-1,0,1,2
x is a member of integers
& $20 < x \leq 32$
∴ x={21,22,23,24,25,26,27,29,30,31,32}

Set Operators

Note
Z is all Int = -2,-1,0,1,2
N is Natural = 1,2,3,4,5
R is Rational = $-\frac{\#}{\#}, \frac{\#}{\#}$
U is Universe = $-\infty, \infty$

1) A∩B Intersection/AND (the two sets overlap)
A=rat, rabbit, snake, sheep, dog
B= tiger, horse, monkey, dog
∴ Dog

2) A∩C Intersection/AND (the two sets overlap)
A=rat, rabbit, snake, sheep, dog
C = rat, tiger, dragon, horse, monkey, pig
∴ rat

3) B∪C Union/OR (is two merged sets)
B= tiger, horse, monkey, dog
C = rat, ~~tiger~~, dragon, ~~horse~~, ~~monkey~~, pig *Duplicates do not matter
∴ tiger, horse, monkey, dog, rat, dragon, pig

4) A' (Elements removed from the universe)
U= ~~rat~~, ox, tiger, ~~rabbit~~, dragon, ~~snake~~, horse, ~~sheep~~, monkey, rooster, ~~dog~~, pig
A'= ~~rat~~, ~~rabbit~~, ~~snake~~, ~~sheep~~, ~~dog~~
∴ ox, tiger, dragon, horse, monkey, rooster, pig

5) C\B Difference (Excludes all items found in one set from another)
C= rat, ~~tiger~~, dragon, ~~horse~~, ~~monkey~~, pig
B= ~~tiger~~, ~~horse~~, ~~monkey~~, dog
∴ rat, dragon, pig

6) [A∪B]\C
A∪B = Dog
C = rat, tiger, dragon, horse, monkey, pig
∴ Dog

Set Algebra

Simply the following. Use set algebra. Label all the laws you apply.

(A' ∩ B) ∪ (C ∩ C') ∪ (A' ∩ B')

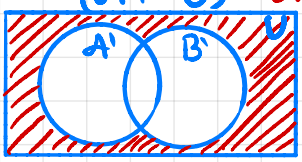
Proof.

$(A' \cap B) \cup (C \cap C') \cup (A' \cap B')$

$(A' \cap B) \cup \varnothing \cup (A' \cap B')$ Complement Law $C \cap C' \equiv \varnothing$

$(A' \cap B) \cup \varnothing \cup (A' \cap B')$ $A' \cap B \equiv B$

$B \cup \varnothing \cup (A' \cap B')$ $(A' \cap B') = \text{Universe } U$



$B \cup \varnothing \cup U$ Identity Law $B \cup \varnothing \equiv B$

$B \cup U$ Domination Law $B \cup U \equiv U$

$\therefore U$

Floating Point Numbers

1. Encode the number **34.25** (34 and 1/4) to a single-precision (32 bit) floating point. Write down its binary representation. You don't have to convert to hex, but please put spaces either between each byte or each field. Show your work.
2. What is **01000010010011000100000000000000** when interpreted as a floating-point number?
3. What is **01111111100010000000000000000001** when interpreted as a floating-point number?

1|34.25 → Binary (Step 1: convert to binary)

$2^7\ 2^6\ 2^5\ 2^4$ $2^3\ 2^2\ 2^1\ 2^0$ $32+2=34$

34 → 0010 0010

$2^0 . 2^{-1} 2^{-2} 2^{-3}$ $\frac{1}{2^2}=0.25$

0.25 → 1/4 → 0.01

0010 0010.01

(Step 2: Scientific Notation)

0010 0010.01 → 1.00010001×2^5

$1.00010 \times 2^5 \rightarrow (1+.0001001) \times 2^5$

→ 0 1000 0100 0010 0010 0000 0000 0000 000

Sign | Exponent | Fraction

Floating Point Numbers

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0 1 2 3 4 5 6 7

2. What is **01000010100110001100000000000000** when interpreted as a floating-point number?

3. What is **01111111100010000000000000000001** when interpreted as a floating-point number?

2) 0100 0010 1001 1001 1100 0000 0000 0000

Positive | Exponent | Fraction

133 − 127 = 6

2^{22}	2^{21}	2^{20}	2^{19}	2^{18}	2^{17}	2^{16}	2^{15}	2^{14}	2^{13}	2^{12}	2^{11}	2^{10}	2^{9}	2^{8}	2^{7}	2^{6}	2^{5}	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}
0	0	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

8,388,608 + 1,048,576 + 524,288 + 65,536 + 32,768 + 16,384

= 1,687,552

$(1 + .1687552) \times 2^{6} = 74.8003328$

3) 0111 1111 1000 1000 0000 0000 0000 0001

Sign | Exponent | Fraction

positive

255 − 127 = 128

2^{22}	2^{21}	2^{20}	2^{19}	2^{18}	2^{17}	2^{16}	2^{15}	2^{14}	2^{13}	2^{12}	2^{11}	2^{10}	2^{9}	2^{8}	2^{7}	2^{6}	2^{5}	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

524,288 + 1

= 524,289

$(1 + .524,289) \times 2^{128} = 5.186886688$

Teacher's Solution

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2) B = {a, e, o}
3) C = {21, 22, 23, ... 32}
4) D = {5, 10, 15, 20, 25, 30}

Set Operators

Given the following sets:

U = { rat, ox, tiger, rabbit, dragon, snake, horse, sheep, monkey, rooster, dog, pig }
A = { rat, rabbit, snake, sheep, dog }
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Find the following:

1. A ∩ B
2. A ∩ C
3. B ∪ C
4. A'
5. C \ B
6. (A ∪ B) \ C

1) {Dog}
2) {rat}
3) {tiger, horse, monkey, dog, rat, dragon, pig}
4) {ox, tiger, dragon, horse, monkey, rooster, pig}
5) {rat, dragon, pig}
6) A ∪ B = {rat, rabbit, snake, sheep, dog, tiger, horse, monkey}
A ∪ B / C = {rabbit, snake, sheep, dog}

Teacher's Solution

Venn Diagrams

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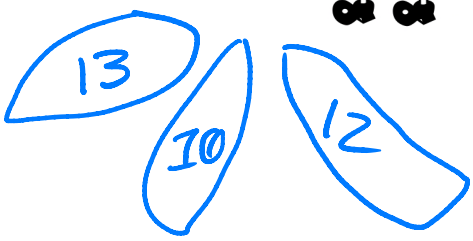

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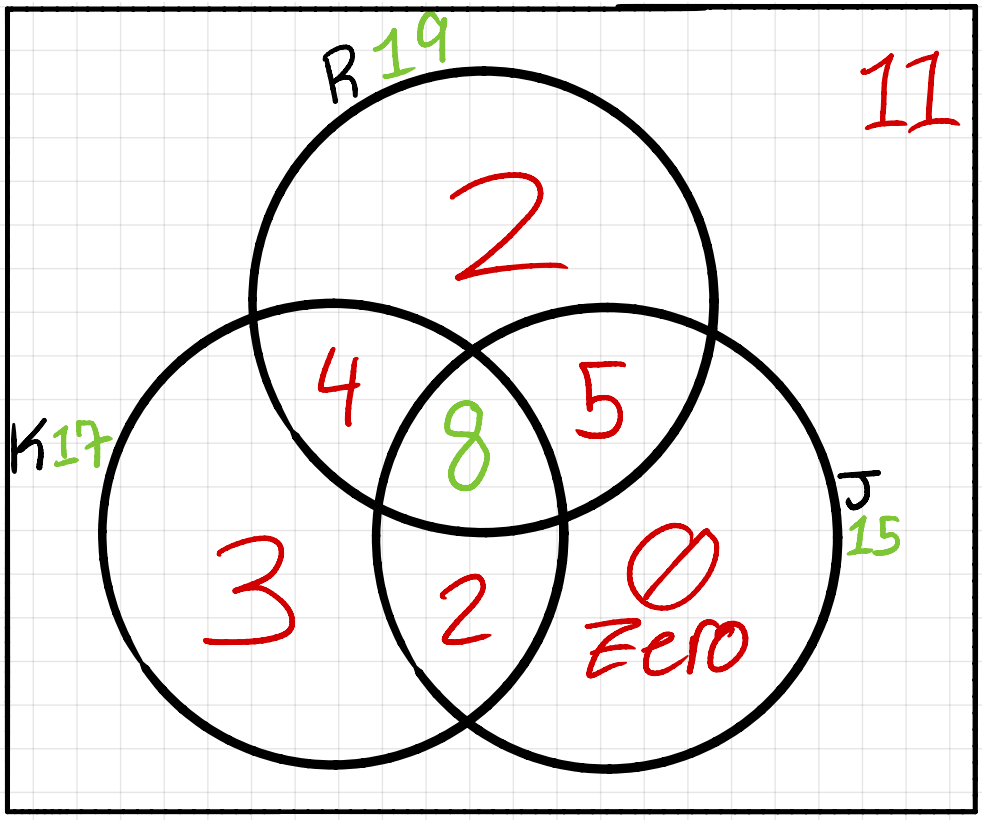

1) 2 Two
2) 0 Zero
3) 3 Three
4) 4 Four
5) 2 Two
6) 5 Five
7) 11 Eleven

Teacher's Solution

Set Algebra

Simply the following. Use set algebra. Label all the laws you apply.

$$(A' \cap B) \cup (C \cap C') \cup (A' \cap B')$$

$(A' \cap B) \cup (C \cap C') \cup (A' \cap B')$

$(A' \cap B) \cup \varnothing \cup (A' \cap B')$ Complement

$(A' \cap B) \cup (A' \cap B')$ Identity

$A' \cup (B \cap B')$ Distributive

$A' \cup U$ Complement

A' Solution

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1) Step 1. 100010.01 ← 34.25

5

Step 2. 1.00010001×2^5

fraction

Step 3.

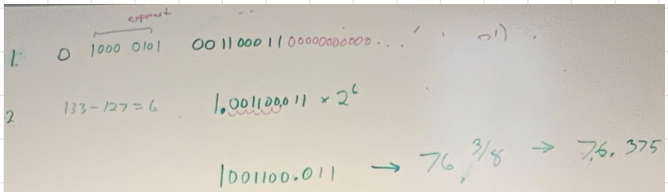
3) NAN

Ask why

2) Step 1. 0 1000 0101 00110001100000000000

Exponent

Step 2.



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Find the following:

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2. A ∪ B
3. A'
4. A \ B

A = 1 0 0 1 0 1 0 1 0 0

B = 1 0 0 1 0 0 1 0 1 0

1) A ∩ B = 1 0 0 1 0 0 0 0 0 0

A ∪ B = 1 0 0 1 0 1 1 1 1 0

A' = 0 1 1 0 1 0 1 0 1 1

A \ B = 0 0 0 0 0 1 0 1 0 0