

You must show and explain your work! The following formulas may, or may not, be useful.

1. (10 points)

a. (2 pts) Circle all of the following expression which are **linear** differential equations.

(i) $y''' - y'' + y = 0$

(iii) $\ln(y') - y = x \cos(x)$

(ii) $(y')^3 = y^2$

(iv) $e^x y^{(7)} - (x^2 - 3)y = \sin(x^2)$

b. (2 pts) Circle all of the following expression which are linear **nonhomogeneous** differential equations.

(i) $y''' - y'' = y - x^2 \sin(x)$

(iii) $y^{(3)} - \sec(x)y' - y = e^x$

(ii) $t^2 \frac{d^3 f}{dt^3} + t \frac{df}{dt} + t^5 = t^5$

(iv) $y^{(3)} - x^5 y' - 2x^2 = 0$

c. (3 pts) Compute $\mathcal{L}\{e^{5t} \sin(3t)\}$.

d. (3 pts) Compute $\mathcal{L}^{-1}\left\{\frac{9}{s+2} + \frac{3}{s^2+16}\right\}$.

2. (10 points) Solve the following differential equations.

a. (5 pts) $e^y dx + (2y + xe^y) dy = 0$

b. (5 pts) $y' - \frac{3}{x+1}y = (x+1)^4$

3. (20 points) Solve the following differential equations. Your answers should be written as real general solutions.

a. (5 pts) $y'' - 4y' + 13y = 0$

b. (5 pts) $x^2 y'' + 5xy' + 4y = 0$

c. (10 pts) $y'' - 8y' + 15y = 8 \cos(x) - 14 \sin(x)$

4. (10 points) Use the Laplace Transform to solve the initial value problem $y'' = 2 + 3t$ with initial conditions $y(0) = 0$, $y'(0) = 0$. (You must use the method of Laplace Transform for credit.)