1. (1 point) Are the following functions homogeneous? (You have only one attempt! Submit all answers at the same time)

(a)
- Choose
- Homogeneous
- Not Homogeneous

$f(x,y) = x^3y^5$.

(b)
- Choose
- Homogeneous
- Not Homogeneous

$f(x,y) = x\sin(y)$.

(c)
- Choose
- Homogeneous
- Not Homogeneous

$f(x,y) = x + y^2$.

Solution:
SOLUTION:
We have (a) is homogeneous. This is because

$$\begin{aligned} f(tx,ty) &= (tx)^3(ty)^5 = t^3x^3t^5y^5 \\ &= t^8x^3y^5 = t^8 f(x,y). \end{aligned}$$

Meanwhile, (b) and (c) are not homogeneous.
Correct Answers:

- `Homogeneous`
- `Not Homogeneous`
- `Not Homogeneous`

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2. (1 point)
Determine whether the differential equation is homogeneous or not. If it is homogeneous, provide the degree of x^2y^4 and x^6+y^6. If it is not homogeneous, put -1 as the degree. (Note: This is the first definition of homogeneous DE we saw!)

$$\left(x^2y^4\right)\,dx + \left(x^6+y^6\right)\,dy = 0$$

(a) The degree is _____.

(b) The equation is
- Choose
- Homogeneous
- Not Homogeneous

.

Solution:
SOLUTION:
Take $M(x,y) = x^2y^4$ and $N(x,y) = x^6+y^6$. Since

$$\begin{aligned} M(tx,ty) &= (tx)^2(ty)^4 = t^2x^2t^4y^4 \\ &= t^6x^2y^4 = t^6M(x,y) \end{aligned}$$

and

$$\begin{aligned} N(tx,ty) &= (tx)^6+(ty)^6 = t^6x^6+t^6y^6 \\ &= t^6\left(x^6+y^6\right) = t^6N(x,y), \end{aligned}$$

we have that both $M(x,y)$ and $N(x,y)$ are homogeneous of degree 6. Thus, the DE is homogeneous of degree 6.
Homogeneous.
Correct Answers:

- `6`
- `Homogeneous`

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3. (1 point) Use substitution to find the general solution of the differential equation $(7x-y)\,dx + x\,dy = 0$.

(Use C to denote the arbitrary constant and ln| input | if using ln.)
$y =$ __________ help (formulas)

Solution:
SOLUTION:
First we divide the differential equation by dx so that we may write $(7x-y)\,dx + x\,dy = 0$ as

$$x\frac{dy}{dx} + (7x-y) = 0.$$

We take $y = ux$. Then $\frac{dy}{dx} = \left(\frac{du}{dx}\right)x + u\left(\frac{dx}{dx}\right) = x\frac{du}{dx} + u$. Thus, we may replace y with ux and $\frac{dy}{dx}$ with $x\frac{du}{dx}+u$ to get

$$x\left(x\frac{du}{dx}+u\right) + (7x-ux) = 0,$$

or

$$\frac{du}{dx} = -\frac{7}{x}.$$

Separating variables gives

$$du = -\frac{7}{x}\,dx,$$

which we integrate both sides to find

$$u = -7\ln|x| + C.$$

Replacing u with $\frac{y}{x}$ (since we took $y = ux$) we get

$$\frac{y}{x} = -7\ln|x| + C,$$

or

$$y = -7x\ln|x| + Cx.$$

Correct Answers:

- `-7*x*ln(|x|)+C*x`

Solve the differential equation $\left(y^2+xy\right)\,dx - x^2\,dy = 0$.

- A. $y = \dfrac{x}{xC + \ln|x|}$
- B. $y = C - \ln|x|$
- C. $y = \dfrac{x}{C - \ln|x|}$
- D. $y = C + \ln|x|$

Solution:
SOLUTION:
First we divide the differential equation by dx so that we may write $\left(y^2+xy\right)\,dx - x^2\,dy = 0$ as

$$x^2\frac{dy}{dx} - \left(y^2+xy\right) = 0.$$

We take $y = ux$. Then $\frac{dy}{dx} = \left(\frac{du}{dx}\right)x + u\left(\frac{dx}{dx}\right) = x\frac{du}{dx} + u$. Thus, we may replace y with ux and $\frac{dy}{dx}$ with $x\frac{du}{dx}+u$ to get

$$x^2\left(x\frac{du}{dx}+u\right) - \left((ux)^2 + x(ux)\right) = 0,$$

which is equivalent to

$$x^3\frac{du}{dx} + ux^2 - u^2x^2 - ux^2 = 0,$$

or

$$\frac{du}{dx} = \frac{u^2}{x}.$$

Separating variables gives

$$\frac{1}{u^2}\,du = \frac{1}{x}\,dx,$$

which we integrate both sides to find

$$-\frac{1}{u} = \ln|x| + C.$$

Replacing u with $\frac{y}{x}$ (since we took $y = ux$) we get

$$\frac{x}{y} = -\ln|x| + C,$$

or

$$y = \frac{x}{C - \ln|x|}.$$

Thus, the correct answer is C.
Correct Answers:

- `C`

Solve the homogeneous differential equation $-y\,dx + \left(x+\sqrt{xy}\right)\,dy = 0$. (Note: Some algebraic manipulation goes into putting your answer into the form below.)

- A. $y = x\left(\ln|x| - C\right)^2$
- B. $y\left(\ln|y| - C\right)^2 = 4x$
- C. $\sqrt{yx}\ln|y| = C\sqrt{x}$

Solution:
SOLUTION:
First we divide the differential equation by dx so that we may write $-y\,dx + \left(x+\sqrt{xy}\right)\,dy = 0$ as

$$\left(x+\sqrt{xy}\right)\frac{dy}{dx} - y = 0.$$

We take $y = ux$. Then $\frac{dy}{dx} = \left(\frac{du}{dx}\right)x + u\left(\frac{dx}{dx}\right) = x\frac{du}{dx} + u$. Thus, we may replace y with ux and $\frac{dy}{dx}$ with $x\frac{du}{dx}+u$ to get

$$\left(x+\sqrt{ux^2}\right)\left(x\frac{du}{dx}+u\right) - ux = 0,$$

which is equivalent to

$$\left(x^2 + x\sqrt{ux^2}\right)\frac{du}{dx} + u\sqrt{ux^2} = 0,$$

or

$$\begin{aligned}\frac{du}{dx} &= -\frac{u\sqrt{ux^2}}{x^2+x\sqrt{ux^2}}\\ &= -\frac{xu\sqrt{u}}{x^2\left(1+\sqrt{u}\right)}\\ &= -\frac{u\sqrt{u}}{x\left(1+\sqrt{u}\right)}.\end{aligned}$$

Separating variables gives

$$\frac{1+u^{\frac{1}{2}}}{u^{\frac{3}{2}}}\,du = -\frac{1}{x}\,dx,$$

which we integrate both sides to find

$$\int\left(u^{-\frac{3}{2}} + \frac{1}{u}\right)\,du = -\int\frac{1}{x}\,dx,$$

or

$$-2u^{-\frac{1}{2}} + \ln|u| = -\ln|x| + C.$$

Replacing u with $\frac{y}{x}$ (since we took $y = ux$) we get

$$-2\left(\frac{y}{x}\right)^{-\frac{1}{2}} + \ln\left|\frac{y}{x}\right| = -\ln|x| + C,$$

or

$$-\frac{2\sqrt{x}}{\sqrt{y}} + \ln|y| - \ln|x| = -\ln|x| + C,$$

which can be written as

$$-\frac{2\sqrt{x}}{\sqrt{y}} + \ln|y| = C.$$

We could rewrite this in a number of ways. For example, we can turn this into $\sqrt{y}\ln|y| = \sqrt{y}C + 2\sqrt{x}$, or $\sqrt{y}\,(\ln|y\ rvert - C) = 2\sqrt{x}$. Then squaring both sides gives

$$y\,(\ln|y| - C)^2 = 4x.$$

Thus, the correct answer is B.

Correct Answers:

- B

Which of the following is a solution to the IVP consisting of the homogeneous differential equation $-y\,dx + \left(x + \sqrt{xy}\right)\,dy = 0$ with the initial condition $y(4) = 1$. (Note: Some algebraic manipulation goes into putting your answer into the form below.)

- A. $\sqrt{yx}\ln|y| = 4\sqrt{x}$
- B. $y = x\,(\ln|x| + 4)^2$
- C. $y\,(\ln|y| + 4)^2 = 4x$

Solution:

SOLUTION:

From the previous problem we know that

$$y\,(\ln|y| - C)^2 = 4x$$

is a one-parameter family of solutions. If $y = 1$ and $x = 4$ we have this becomes

$$(-C)^2 = 16.$$

Thus, $C = \pm 4$. Of the options, there is one where $C = -4$. Thus, the correct answer is C.

Correct Answers:

- C

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7. (1 point) Note that $y = c_1 e^{4x} + c_2 e^{-x}$ is a general solution for the second-order differential equation $y'' - 3y' - 4y = 0$ on the interval $(-\infty, \infty)$. Find values c_1 and c_2 so that y is a solution to the second-order IVP consisting of the differential equation $y'' - 3y' - 4y = 0$ and the initial condition $y(0) = 3$, $y'(0) = 7$. The values are $c_1 =$ _____ and $c_2 =$ _____.

Solution:

SOLUTION:

We are given that $y = c_1 e^{4x} + c_2 e^{-x}$ is a family of solutions for $y'' - 3y' - 4y = 0$. Since $y(0) = 3$ we have that when $x = 0$ that $y = 3$. Thus,

$$3 = y\,(0) = c_1 e^0 + c_2 e^{-0} = c_1 + c_2.$$

This is not enough information to solve for c_1 or c_2. However, we also note that since $y'(0) = 7$ we have that when $x = 0$ that $y' = 7$. Thus, since $y' = 4c_1 e^{4x} - c_2 e^{-x}$ we have

$$7 = y'\,(0) = 4c_1 e^0 - c_2 e^{-0} = 4c_1 - c_2.$$

Therefore, we have the system of equations

$$3 = c_1 + c_2$$

$$7 = 4c_1 - c_2.$$

Adding these two equations gives $10 = 3 + 7 = 5c_1$. Thus, $c_1 = \frac{10}{5} = 2$. Plugging this value into either of the two equations and solving for c_2 gives $c_2 = 1$. Plugging both of these values into the solution we find

$$y = 2e^{4x} + 1e^{-x}.$$

Correct Answers:

- 2
- 1

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8. (1 point) Note that $y = c_1 x + c_2 x \ln(x)$ is a general solution for the second-order differential equation $x^2 y'' - xy' + y = 0$ on the interval $(0, \infty)$. Find values c_1 and c_2 so that y is a solution to the second-order IVP consisting of the differential equation $x^2 y'' - xy' + y = 0$ and the initial condition $y(1) = 2$, $y'(1) = 7$. The values are $c_1 =$ _____ and $c_2 =$ _____.

Solution:

SOLUTION:

We are given that $y = c_1 x + c_2 x \ln(x)$ is a family of solutions for $x^2 y'' - xy' + y = 0$. Since $y(1) = 2$ we have that when $x = 1$ that $y = 2$. Thus,

$$2 = y\,(1) = c_1 + c_2 \ln(1) = c_1.$$

This is not enough information to solve for c_2. However, we also note that since $y'(1) = 7$ we have that when $x = 1$ that $y' = 7$. Thus, since $y' = c_1 + c_2 \ln(x) + c_2$ we have

$$7 = y'\,(1) = c_1 + c_2 \ln(1) + c_2 = c_1 + c_2.$$

Therefore, using that $c_1 = 2$ we have

$$7 = c_1 + c_2 = 2 + c_2$$

so that $c_2 = 7 - 2 = 5$. Plugging both of these values into the solution we find

$$y = 2x + 5x \ln(x).$$

Correct Answers:

- 2
- 5

We have that $y = c_1 + c_2x^2$ is a two-parameter family of solutions for the differential equation $xy'' - y' = 0$ on the interval $(-\infty, \infty)$. Does there exist values c_1 and c_2 so that y satisfies the initial conditions $y(0) = 0$ and $y'(0) = 1$?

- A. No
- B. Yes

Why does you answer above not violate the theorem in class concerning the existence of a unique solution?

- A. The coefficients are continuous on the interval.
- B. The highest order derivative is two.
- C. The coefficent of the y'' term is 0.
- D. The differential equation is linear.

Solution:
SOLUTION:
We note have that

$$0 = y(0) = c_1 + c_2(0) = c_1$$

and since $y' = 2c_2x$ we have

$$1 = y'(0) = 2c_2(0) = 0,$$

which cannot be. Thus the answer for the first part is no.

The reason the theorem from class does not work is that the coefficient of the x term is 0. Thus, the correct answer for the second part is C.
Correct Answers:

- A
- C

Consider the initial value problem $(x-8)y'' + 3y = x$ with initial conditions $y(0) = 3$ and $y'(0) = 1$. Which of the following is an interval containing 0 for which this IVP has a unique solution on?

- A. $(-\infty, 8)$
- B. $(-8, \infty)$
- C. $(-\infty, -8)$
- D. $(-\infty, 3)$

Solution:
SOLUTION:
Since the leading coefficient $(x-8)$ equals 0 when $x = 8$, we have two intervale where a unique solution exists are $(-\infty, 8)$ and $(8, \infty)$. However, only the first of these contains 0. Thus, the correct answer is A.
Correct Answers:

- A