General solutions for nonhomogeneous equations

In the previous module we learned the ingredients (fundamental set of solutions) to build a general solution for a homogeneous linear differential equation. Here, we add to our previous body of knowledge and learn how to construct general solutions for a *nonhomogeneous* linear differential equation

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = b(x). \qquad (\star)$$

This is done in the following theorem.

Theorem

Suppose that

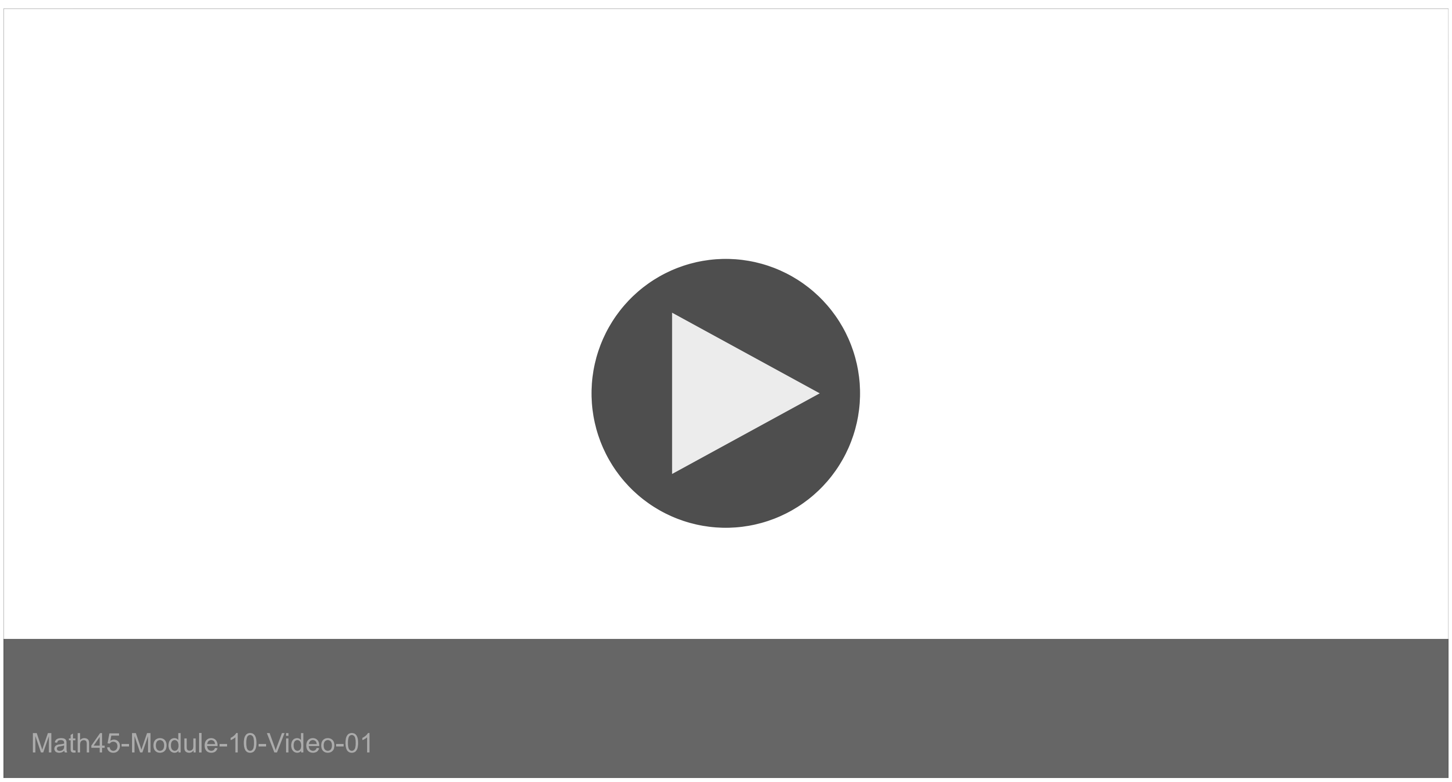
- $y_p(x)$ is a particular solution for the nonhomogeneous linear differential equation given by $(\star)$ above.
- $y_1, \ldots, y_n$ are a fundamental set of solutions for the homogeneous linear differential equation $a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = 0.$

Then the general solution for $(\star)$ is

$$y = c_1 y_1(x) + c_2 y_2(x) + \cdots + c_n y_n(x) + y_p(x),$$

where $c_1, \ldots, c_n$ are arbitrary constants.

Discussion, comments, and examples:



WeBWorK module 10 exercises:

- Problems 1, 2

References:

- The theorem above is **Theorem 9.1.6 in this text** (http://ramanujan.math.trinity.edu/wtrench/texts/TRENCH_DIFF_EQNS_I.PDF) .