The Laplace transform of derivatives

Essentially--though we have not utilized it in this way yet--we have learned how to apply the Laplace transform to functions, and then also use the inverse Laplace transform to map back to the original variables. Part of this process, however, will also require us knowing the Laplace transform for an arbitrary function and its derivatives.

We begin by supposing $y = f(t)$ is the unknown function.

- As a matter of notation, we let $F(s)$ denote the Laplace transform of $f(t)$. That is, $\mathcal{L}\{f(t)\} = F(s)$.

With this notation in hand, we are able to find the Laplace transform of derivatives in terms of the underlying functions f and F.

Theorem

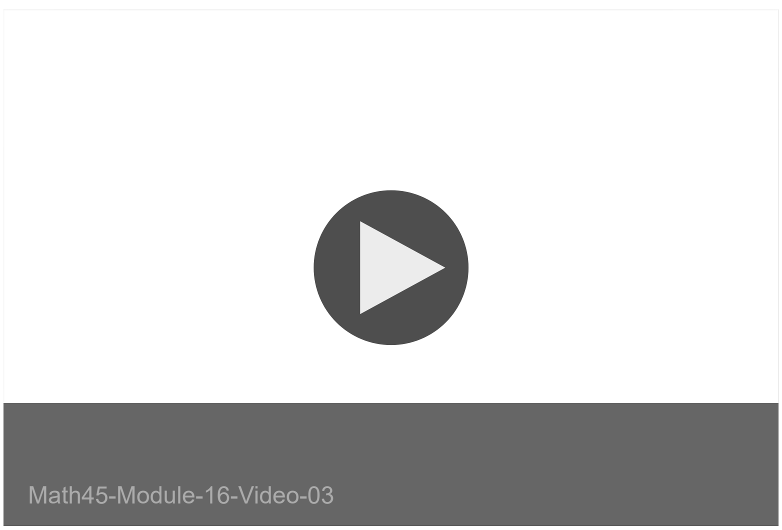
For any $n \geq 0$ we have

$$\mathcal{L}\left\{f^{(n)}(t)\right\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - s f^{(n-2)}(0) - f^{(n-1)}(0).$$

Note: Technically, this theorem only holds for functions that are continuous on $[0, \infty)$ and grow slower than e^{st}, though we don't dwell on this here.

In the video below we make note of the first and second derivatives as special cases. These are the two most common cases of the theorem that we need.

Discussion, comments, and examples:



WeBWorK module 16 exercises:

- Problem 8

Relevant Wikipedia articles:

- [The Laplace transform of derivatives (https://en.wikipedia.org/wiki/Laplace_transform#Computation_of_the_Laplace_transform_of_a_function's_derivative)](https://en.wikipedia.org/wiki/Laplace_transform#Computation_of_the_Laplace_transform_of_a_function's_derivative)