Matices, vectors, and matrix-vector multiplication

Before we begin the discussion pertaining to systems of differential equations, we need some machinery.

That machinery consist of matrices and vectors, and includes multiplying matrices and vectors. If you are familiar with this, great! If not, all we will need is contained in the following definition.

Definition

(a) We call the arrays $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ a 2×2 **matrix** and 3×3 **matrix**, respectively, where the letter a_{ij} are called the **entries** and in this course are typically real numbers.

(b) We call the arrays $\begin{pmatrix} x \\ y \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ vectors, with entries x, y, z which in this course could be numbers or variables.

(c) we have the following **matrix-vector multiplication**:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{pmatrix}$$

and

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11}x + a_{12}y + a_{13}z \\ a_{21}x + a_{22}y + a_{23}z \\ a_{31}x + a_{32}y + a_{33}z \end{pmatrix}.$$

Note: In exercises and homeworks we may encounter the 3×3 cases above. However, on exams it will only be the 2×2 cases.

We also comment about the addition and scalar multiplication of vectors. In particular, the addition of vectors $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ is given by

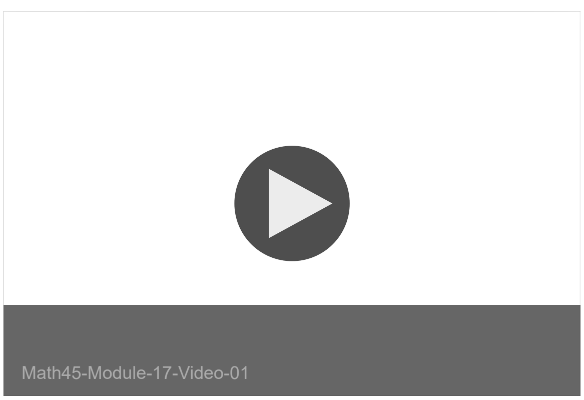
$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix},$$

and have the scalar multiplication

$$\alpha \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \alpha a_1 \\ \alpha a_2 \end{pmatrix},$$

where α can be a number or a function. Such an addition and scalar multiplication also holds for larger vectors.

Discussion, comments, and examples:



WeBWorK module 17 exercises:

- Problems 1, 2, 3, 4

Relevant Wikipedia articles:

- **Linear maps** (https://en.wikipedia.org/wiki/Matrix_multiplication#Fundamental_applications)
- **Vector addition and scalar multiplication** (https://en.wikipedia.org/wiki/Euclidean_vector#Addition_and_subtraction)