MATH 45 – Exam One Review Questions

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1. Rework, study, and understand all of the homework and quiz problems.

2. Determine whether or not the following expression is a differential equation.

(a) $\dfrac{d\phi}{dy} + \phi$

(b) $\ln(y') = 1$

(c) $y + x = 0$

(d) $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} = 0$

3. Classify the following differential equations as ODE or PDE, linear or nonlinear, and separable or not. If the differential equation is an ODE, also provide its order.

(a) $\dfrac{dy}{dx} + 10\sqrt{x} = x^2$

(b) $x^3 - \dfrac{db}{dx} = 0$

(c) $\dfrac{\partial^2 \psi}{\partial t^2} - \dfrac{\partial^2 \psi}{\partial x^2} - 2 = 0$

(d) $y'' + xy' - y = \sin(x)$

(e) $x^{(3)} + 4ytx' = 0$

(f) $3\cos(3x) + (10w - 4)w' = 0$

(g) $\dfrac{dy}{dx} - \dfrac{y}{x} + e^{\frac{y}{x}} = 0$

(h) $\dfrac{dx}{dt} - 5x = te^{-t}$

(i) $\dfrac{1}{y'} + \dfrac{1}{x} = 5$

(j) $\dfrac{\partial \Phi}{\partial t} = \dfrac{\partial^2 \Phi}{\partial x^2} + x^2\Phi$

4. Determine whether the function $f(x) = x^2 + 1$ is a solution to each of the following differential equations.

(a) $f'' = 0$

(b) $f'' = 2$

(c) $f'' + f' = 1$

(d) $xf'' = f'$

5. Consider the differential equation $(y')^3 + 4y - 4 = 0$. Is $y(x) = 1$ a solution? Is $y(x) = 1 + x^2$ a solution? How about $y(x) = 1 - x^2$?

6. In each case, verify that the the given function satisfies the differential equation for any parameter C. Then find a solution to the given initial conditions.

(a) $y' + y = 0$, $y = Ce^{-x}$, $y(0) = 3$,

(b) $y' + 2xy = 0$, $y = Ce^{-x^2}$, $y(0) = -1$,

(c) $y' = x - y$, $y = Ce^{-x} + x - 1$, $y(0) = 1$,

7. Consider the function $f(t) = c_1e^{-4t} + c_2e^{3t} - \dfrac{1}{10}e^t - \dfrac{1}{6}e^{2t} + \dfrac{1}{12}$, which is a family of solutions for the differential equation $y'' + y' - 12y = e^t + e^{2t} - 1$. Find a solution to this differential equation which satisfies the initial conditions $y(0) = 1$ and $y'(0) = 3$.

8. Solve the following differential equations.

(a) $\dfrac{dy}{dx} + y^2 \sin(x) = 0$ (b) $\dfrac{dy}{dx} = \dfrac{x^2}{y\,(1+x^3)}$

9. Find the solution to the given initial value problem.

(a) $y' = (1-2x)y^2$, $y(0) = -\dfrac{1}{6}$ (b) $\dfrac{dy}{dt} = e^{t+y}$, $y(0) = 0$

10. Solve the differential equation $y' = \dfrac{x^2}{y}$, giving the solution in both implicit and explicit form.

11. Find a solution to the initial value problem $y' = \dfrac{3x^2 - e^x}{2y-5}$ with conditions $y(0) = 1$. Provide the solution in implicit form, and then in explicit form.

12. For each of the following differential equations determine whether or not the existence and uniqueness of a solution at the given points is guaranteed by the theorem discussed in class.

(a) $y' = x\ln(y)$ at $(1,1)$ and at $(1,0)$ (b) $y' = \dfrac{x-1}{y}$ at $(0,1)$ and at $(1,0)$

13. Find a family of solutions for $(1+e^x)\cos(y)\dfrac{dy}{dx} = \dfrac{e^x}{y}$. Then find a solution to the initial value problem $y(0) = \dfrac{\pi}{2}$.

14. Solve the differential equation $y' = x - xy$ in two ways. First, by using the theory of separable equations. Secondly, by using the theory of linear equations.

15. Solve the differential equation $ty' + y = 2t$.

16. Determine whether or not the following differential equations are exact.

(a) $(-4xy^2 + y)\,dx + (-4x^2y + x)\,dy = 0$

(b) $(4e^x \sin(y) - 3y) + (-3x + 4e^x\cos(y))\dfrac{dy}{dx} = 0$

(c) $y^2\,dx + x^2\,dy = 0$

17. Solve the differential equations in Problem 16 which are exact equations.

18. Consider the differential equations in the form $M(x,y)\,dx + N(x,y)\,dy = 0$ below. For each one, determine whether $M(x,y)$ and $N(x,y)$ are homogeneous functions. If they are homogeneous, state their degrees.

(a) $(x+y)\,dx + x\,dy = 0$ (c) $y\,dx + x(\ln(x) - \ln(y) - 1)\,dy = 0$; $y(1) = e$

(b) $x^2\,dx + x^2y\,dy = 0$ (d) $e^x\,dx - e^y\,dy = 0$

(Note that (c) is an initial value problem.)

19. Solve the differential equations in Problem 18 that consist of homogeneous functions of the same degree. (Is it clear which method can be used?)

20. Explain your answer to the following questions.

(a) What is the difference between a particular solution, a family of solutions, and a general solution.

(b) Is $y = 1$ the trivial solution of the differential equation $y' + y - 1 = 0$.

(c) Suppose $f(x) = \sqrt{x^2 - 1}$ satisfies a differential equation. Can we say $f(x) = \sqrt{x^2 - 1}$ is a solution to the differential equation?