

course: [CSC 135](#)

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related\_notes: [2022-02-24](#)

# Tail Recursion Optimization

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## Agenda

1. Traversing HAMT
2. Tail recursion
3. Code Step-by-step

## Traversing HAMT - Logic

```
visit(self):  
    # Do something here (before visiting children)  
    for i = 0 to deg-1  
        if child[i] not None:  
            visit(child[i])  
    # Do something here (after visiting children)
```

## Tail recursion

- Recursion may break one's stack frame:  $O(n)$  or worse
- **Tail call optimization:** If the last operation is a recursive call, the recursion can be turned into a loop.

```
def foo(x):  
    if x is small:  
        # solve directly  
    else:  
        # ...  
        foo(smaller_x) # very last work done in branch of recursive  
function
```

The sample pseudo-code is effectively the same as the loop-based approach below

```
def foo(x):  
    while True:  
        if x is small:  
            # solve directly  
            return  
        else:  
            # ...  
            x = smaller x
```

Java nor Python will optimize your code; however, GCC, a **C** compiler, will do it for you

## Greatest Common Divisor `gcd(x,y)` - Euclid's Algorithm

Pseudo-code recursion approach

```
def gcd(x,y):  
    if y == 0:  
        return x  
    else:  
        return gcd(y, x % y)
```

Pseudo-code loop-based approach

- Avoid a stack call overhead

```
def gcd(x,y):  
    while True:  
        if y == 0:  
            return x  
        else:  
            (x,y) = (y, x & y)
```

## Factorials

Can't be turned into a loop

```
def fact(x):  
    if x == 0:  
        return 1
```

```
else:
    return x \* fact(x-1)
```

You can get around this via an **accumulator** where it works out the solution as you go down, before the call, and NOT provide a result after the call

## Factorials - Accumulator Version

```
def fact(x, accu):
    if x == 0:
        return accu
    else:
        return fact(x-1, accu \* x)
```

## Factorials - Loop Version of Accumulator Version

```
def fact(x, accu = 1):
    while True:
        if x == 0:
            return accu
        else:
            (x, accu) = (x-1, x \* accu)
```

## Another Example - Rewrite the following function to use-tail recursion

```
# Multiplies a and non-negative b using repeated addition
def mult(a, b):
    if b == 0:
        return 0
    else:
        return a + mult(a, b-1)
```

## Tail Recursion

```
# Multiplies a and non-negative b using repeated addition
def mult(a, b, acc = 0):
    if b == 0:
        return acc
```

```
else:  
    return mult(a, b-1, a + acc)
```

## Cues/Questions

- $O(n)$  stack frames problem for HAMT
  - Test cases professor uses won't blow-up the stack counter
  - Nevertheless, best to implement a loop-based solution