Solving 1st-order homogeneous DEs

To solve 1st-order homogeneous differential equations as seen in this class (there are other deviations of such functions that can be tackled with deviations of this method, which will not be covered here), we will utilize a substitution of variables.

In particular, given a 1st-order homogeneous differential equation of the form

$$\frac{dy}{dx} = f(x, y)$$

(if it is in a different form we can put it into this form), we will make the substitution $y = ux$, where we treat u as a function of x. The fact that u is a function of x is important, as it means that we can take the derivative of u with respect to x.

Indeed, by the product rule of derivatives, we have

$$\frac{dy}{dx} = \frac{d}{dx}(ux) = \left(\frac{du}{dx}\right)x + u\left(\frac{dx}{dx}\right) = x\frac{du}{dx} + u.$$

We can now outline the general steps to solving a 1st-order homogeneous differential equation:

1. Make the substitution $y = ux$.
2. Replace $\frac{dy}{dx}$ with $x\frac{du}{dx} + u$.
3. Solve the resulting separable equation.
4. Now make the substitution $u = \frac{y}{x}$ (which comes from $y = ux$).

Discussion, comments, and examples:

Math45-Module-07-Video-02

WeBWorK module 07 exercises:

- Problems 4

Relevant Wikipedia articles:

- [1st-order homogeneous differential equations (https://en.wikipedia.org/wiki/Homogeneous_differential_equation#Homogeneous_first-order_differential_equations)](https://en.wikipedia.org/wiki/Homogeneous_differential_equation#Homogeneous_first-order_differential_equations)