course: CSC 135-01 - Computing Theory and Programming Languages

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related_notes: 2022-03-29

Push-down Automata (PDA - "recognizer"): A Finite Automata, But With a stack

W13.2 | Tuesday, March 29, 2022 | 09:02 AM

Problem:a finite automata isn't able to count
Solution:

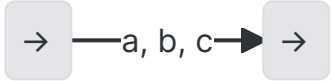
- If the 1's run out or if the stack runs out (typically both runs out)

Push-down Automata (PDA) is a Non-Finite Automata (NFA)

- String is accepted if and only if a path to an accept state exist that consumes entire input.

BUT WITH A STACK:

- Labels have triples



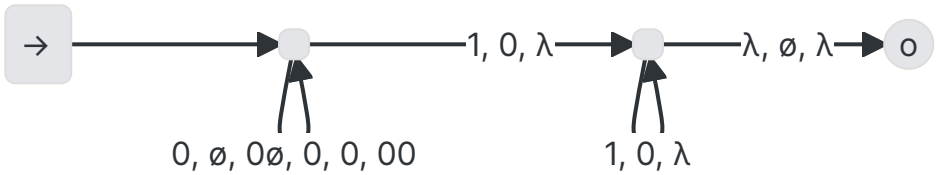
"**a**": Consumed from input or λ ("λ" means don't consume anything)
"**b**": Top of stack, always gets popped
"**c**": String of characters (chars) that gets pushed RIGHT-TO-LEFT

- Cross arrow if and only if "a" is consumed and "b" matches TOP OF STACK

    - "a" is consumed
    - "b" is popped
    - "c" is pushed right-to-left
- Stack begins with special char ∅

Example 01 $\{0^n1^n|n \geq 1\}$

> **Info**
>
> The accept state can get there on 0^n1^m where $0 < n \leq m$ but... input accept ONLY if entirely consumed



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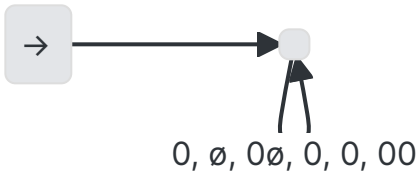
ø
=====
STACK

Example 01 - Simulated: 0011 (TODO-FLUSH OUT NOTE SECTION)

STEP 01.1 : 0011



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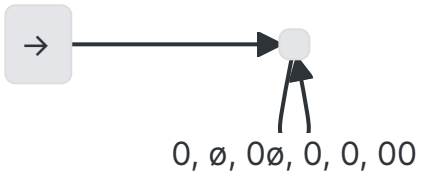
STACK

STEP 01.2: ~~0~~ 011

Example 02 - Simulated: 1100 (TODO-FLUSH OUT NOTE SECTION)

> ✕ **Failure**
>
> NOT EXCEPTED BECAUSE IT STARTS WITH A 1

STEP 02.1



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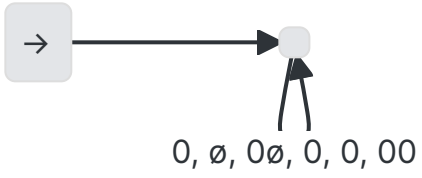
STACK

Example 03 - Simulated: 011 (TODO-FLUSH OUT NOTE SECTION)

ALTHOUGH ENDS AT THE ACCEPT STATE. THE STRING IS NOT ACCEPTED, FOR IT DID NOT CONSUME THE WHOLE INPUT.

STEP 03.1

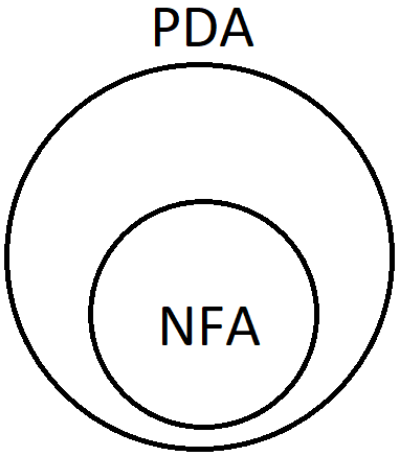


To write a Push-down Automata (PDA)

1. Think stack algorithm
2. Write the PDA to implement the algorithm
3. Test good and bad strings

Theorem: PDA is more powerful than an NFA

1. A PDA can do anything an NFA can



Let M be an arbitrary NFA convert all arrows in NFA

TO

in PDA

"**x,** $\varnothing$**,** $\varnothing$": Don't use the stack.

2. Show the PDA can solve a problem the NFA can't

    1. $\{0^n 1^n | n \geq 1\}$

Context Free Grammar ("generator")

- Regular Expression ← "generator"
- DFA/NFA ← "recognizers"

Context Free Grammar Example 01: $S \rightarrow aSa \ |bSb| \ x$

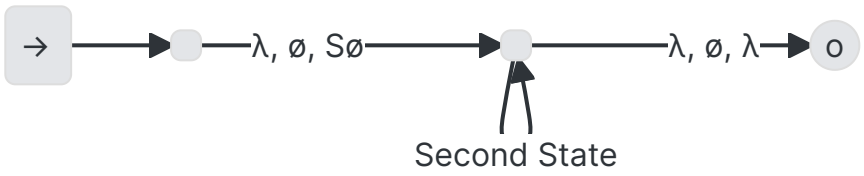
$S \rightarrow aSa \ |bSb| \ x$

> **Idea**
>
> PDA simulate a left-most derivation for a string

$abxba$

1. $S \rightarrow aSa$
2. $S \rightarrow aSa \rightarrow abSba$
3. $S \rightarrow aSa \rightarrow abSba \rightarrow abxba$

One triple per production in grammar



Second state

- a, a, λ
- b, b, λ
- x, x, λ
- λ, S, aSa
- λ, S, bSb
- λ, S, λ

Context Free Grammar Example 02

Grammar:

- $S \rightarrow AE$
- $A \rightarrow aA|a$
- $E \rightarrow aEb|\lambda$

Produces Language: $L = \{a^n b^m | n > m\}$