

Please show and explain your work where necessary. Good luck!!

1. (10 points)

a. (4 pts) Find a general solution to the DE $y'' - 5y' + 6y = 0$.

Plug-in $y = e^{mx}$; $y' = me^{mx}$; $y'' = m^2e^{mx}$

This gives: $m^2e^{mx} - 5me^{mx} + 6e^{mx} = 0$

Factoring out e^{mx}

$\rightarrow e^{mx}(m^2 - 5m + 6) = 0$

So, we need

$\rightarrow m^2 - 5m + 6 = 0$

$$y_1 = e^{\left(\frac{-(-5) + \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}\right)x}$$

$$y_2 = e^{\left(\frac{-(-5) - \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}\right)x}$$

$$= \frac{5 + \sqrt{25 - 24}}{2} x$$

$$= \frac{5 - \sqrt{25 - 24}}{2} x$$

$$= 6/2 = 3x$$

$$= 4/2 = 2x$$

b. (2 pts) Note that $y_p = e^x$ is a solution to the DE $y'' - 5y' + 6y = 2e^x$ (you do not need to show or verify this). Provide a general solution to this DE.

$$y = C_1 e^{3x} + C_2 e^{2x} + e^x$$

c. (4 pts) We have that e^{2x} is a solution to the DE $y'' - 4y' + 4y = 0$. Use the method of reduction of order to find another (linearly independent) solution to this DE and write the general solution for this DE.

Need 2 soln

$$y = e^{mx}; y' = me^{mx}; y'' = m^2e^{mx}$$

$$y_1 = e^{2x}$$

Too Long

Let $u(x)$ be an arbitrary function, and set $y = u(x)y_1(x) = u(x)e^{2x} = ue^{2x}$.

① Find y' and y''

* Remember ue^{2x} that " u " is a function of " x " so we have to use the product rule.

Recall The Product Rule

$$(uv)' = u'v + uv'$$

$$y' = u'e^{2x} + u2e^{2x}$$

$$u'e^{2x} + 2ue^{2x}$$

$$y'' = u''\sin(x) + u'\cos(x) + u'\cos(x) - u\sin(x)$$

$$= u''\sin(x) + 2u'\cos(x) - u\sin(x)$$

$$y'' - 4y' + 4y = 0$$

substitute $y = e^{kx}$

$$y' = ke^{kx} \text{ and } y'' = k^2e^{kx}$$

substituting, we get

$$k^2 - 4k + 4 = 0$$

$$(k-2)^2 = 0$$

$$k = 2$$

The general solution is $y = (C_1 + C_2x)(e^{2x})$

This is complimentary solution.

there is no particular solution as R.H.S = 0

General solution = complimentary solution + particular solution

The general solution is $y = (C_1 + C_2x)(e^{2x})$