MATH 45 – Exam Two Review Questions

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1. Rework, study, and understand all of the homework and quiz problems.

2. For each of the following differential equations, state whether it is linear or nonlinear, and also whether it is homogeneous or nonhomogeneous.

(a) $x^2y''' + xy'' + \sin(x)y' + xy = 0$

(b) $y'' + yy' + 2x^3y = 0$

(c) $y'' + y' - \sin(x) = 0$

(d) $y^{(31)} + x^2e^xy'' = x\cos(x)$

3. Determine whether a unique solution is guaranteed to exist for the following initial value problems on the given interval I. Explain your answer.

(a) $x^2y''' + xy'' + \sin(x)y' + xy = 0; \quad y(0) = 3,\ y'(0) = 0,\ y''(0) = 2,\ I = (-\infty, \infty).$

(b) $x^2y''' + xy'' + \sin(x)y' + xy = 0; \quad y(2) = 3,\ y'(2) = 0,\ y''(2) = 2,\ I = (0, \infty).$

(c) $y'' + \dfrac{1}{x-2}y' + x^2y = x; \quad y(3) = 0,\ y'(3) = 1,\ I = (0, \infty).$

(d) $y' + \tan(x)y = x^2; \quad y(\frac{\pi}{2}) = 2,\ I = \left(0, \frac{\pi}{2}\right).$

4. Find a largest interval for which a solution to the initial value problem $y' + \dfrac{1}{1-e^x}y = \sin(x)$ is guaranteed to exist and be unique?

5. Find the Wronskian of $y_1 = \cos(3t)$ and $y_2 = \sin(3t)$.

6. Suppose the following functions are solutions to a differential equation. Determine whether they are linearly independent solutions on the given interval.

(a) $y_1 = e^{2x}$, $y_2 = e^{-\frac{3}{2}x}$; $I = (-\infty, \infty)$.

(b) $y_1 = e^x$, $y_2 = 3e^{x+2}$; $I = (0, \infty)$.

(c) $y_1 = x$, $y_2 = xe^x$; $I = (-5, -2)$.

7. Verify that $y_1 = 1$, $y_2 = \cos(t)$, and $y_3 = \sin(t)$ are solutions to the differential equation $y''' + y' = 0$. Do these solutions form a fundamental set of solutions for this differential equation? Explain your answer.

8. Solve the following differential equations (finding general solutions).

(a) $y'' + 2y' - 3y = 0$

(b) $y'' + 4y' + 4y = 0$

(c) $y'' - 6y' + 13y = 0$

(d) $y''' - y'' - y' + y = 0$

9. Determine whether $y = c_1x + c_2xe^x$ is a general solution for $x^2y'' - x(x+2)y' + (x+2)y = 0$.

10. Verify that $y = c_1e^{-t} + c_2e^{-2t} + \dfrac{5}{12}e^{2t}$ is the general solution to $y'' + 3y' + 2y = 5e^{2t}$.

11. For the following differential equations we are given that the given y_1 is a solution. Find a second solution y_2.

(a) $t^2y'' - 4ty' + 6y = 0; \quad y_1 = t^2$

(b) $xy'' - y' + 4x^3y = 0; \quad y_1 = \sin(x^2)$

12. Solve the following differential equations.

(a) $3y'' + y' - 2y = 2\cos(x)$

(b) $y'' + 4y' = x^2 + xe^{-4x}$

13. Use variation of parameters to solve the following differential equations.

(a) $y'' + 9y = \tan(3t)$ (It may be useful to know $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$.)

(b) $y'' - y = t + 3$

14. Solve the following differential equations.

(a) $x^2y'' - 30y = 0$

(b) $x^2y'' - 5xy' + 9 = 0$

15. Explain your answers to the following questions.

(a) Do $y_1 = 1$, $y_2 = 2$, and $y_3 = 3$ form a fundamental set of solutions for the differential equation $y' = 0$?

(b) Are $y_1 = \sin(5x)$ and $y_2 = \sin(-5x)$ linearly independent?

(c) Suppose y_1 and y_2 are solutions to a differential equation. Is it possible for the Wronskian $\mathcal{W}(y_1, y_2)$ to equal 0?

(d) Suppose y_1 and y_2 form a fundamental set of solutions to a differential equation. Is it possible for the Wronskian $\mathcal{W}(y_1, y_2)$ to equal 0?