
Which of the following is NOT a differential equation.

- A. $\frac{d^2y}{dx^2} + e^x \frac{dy}{dx} = x$
- B. $\frac{\partial}{\partial x} g - \frac{\partial}{\partial y} g = g$
- C. $y^3 - 2y^2 + y = 0$
- D. $y'y = 7x$

Solution:
SOLUTION:
We have that C is the only equation that does not have a derivative. Thus, the correct answer is C.
Correct Answers:

- C

Which of the following IS a differential equation.

- A. $y^{(3)} - 2y^{(2)} + y = 0$
- B. $y^2 - y + 5 = 0$
- C. $\frac{d^2y}{dx^2} - e^x \frac{dy}{dx}$
- D. $\frac{\partial}{\partial x} g - \frac{\partial}{\partial y} g$

Solution:
SOLUTION:
We have that A is the only equation that has a derivative. The other expressions either do not have derivatives, or are not equations at all. Thus, the correct answer is A.
Correct Answers:

- A

Which of the following IS an ordinary differential equation (ODE)?

- A. $y^2 - y + 5 = 0$
- B. $y^{(3)} - 2y^{(2)} + y$
- C. $\frac{d^2y}{dt^2} - \frac{dx}{dt} = 0$
- D. $\frac{\partial}{\partial x} g - \frac{\partial}{\partial y} g = g$

Solution:
SOLUTION:
We have that C is the only differential equation that has derivatives for a single independent variable. Yes, it does have two functions (x and y), but this is fine for an ODE. The other expressions either are not differential equations, or have (partial) derivatives for more than one independent variable. Thus, the correct answer is C.
Correct Answers:

- C

Which of the following IS a partial differential equation (PDE)?

- A. $\frac{d^2y}{dt^2} - \frac{dx}{dt}$
- B. $\frac{d^2y}{dt^2} - \frac{dx}{dt} = 0$
- C. $y^{(3)} - 2y^{(2)} + y = 0$
- D. $\frac{\partial}{\partial x} g - \frac{\partial}{\partial y} g = g$

Solution:
SOLUTION:
We have that D is the only differential equation that has (partial) derivatives for more than one independent variable. The

other expressions either are not differential equations, or have derivatives for only one independent variable. Thus, the correct answer is D.

Correct Answers:

- D

Which of the following IS a linear differential equation?

- A. $\frac{d^2y}{dt^2} - \frac{dx}{dt} = 0$
- B. $y^{(3)} - 2y^{(2)} + y^2 = 0$
- C. $\left(\frac{\partial}{\partial x}g\right)^2 - \frac{\partial}{\partial y}g = g$
- D. $y''y - y' = 0$

Solution:

SOLUTION:

We have that A is the only differential equation that does not contain a product of the function and/or its derivatives. Thus, the correct answer is A.

Correct Answers:

- A

Which of the following IS a nonlinear differential equation?

- A. $y^{(3)} - 2y^{(2)} + y = 0$
- B. $\frac{\partial}{\partial x}g - \frac{\partial}{\partial y}g = g$
- C. $y''y - y' = 3$
- D. $\frac{d^2y}{dt^2} - \frac{dx}{dt} = 0$

Solution:

SOLUTION:

We have that C is the only differential equation that containa a product of the function and/or its derivatives (in this case $y''y$. Thus, the correct answer is C.

Correct Answers:

- C

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7. (1 point) Select all conditions that the equation $x'' - xx' = 3$ satsifies. There may be more than one correct box that needs to be checked.

- A. It is a differential equation
- B. It is a linear differential equation
- C. It is a nonlinear differential equation
- D. It is an ODE
- E. It is a PDE
- F. None of the above

Solution:

SOLUTION:

The function is x. There are derivatives and it is an equation, and thus the given equation is a differential equation. Additionally, since the derivatives are only with respect to one independent variable (which is not given), we have that it is an ODE. However, due to the product xx'\$ this ODE is nonlinear. Thus, the correct answer is ACD.

Correct Answers:

- ACD

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8. (1 point) Select all conditions that the equation $\frac{\partial g}{\partial y} + \frac{\partial^2 g}{\partial x^2} = 0$ satsifies. There may be more than one correct box that needs to be checked.

- A. It is an ODE
- B. It is a PDE
- C. It is a differential equation
- D. It is a linear differential equation
- E. It is a nonlinear differential equation
- F. None of the above

Solution:

SOLUTION:

The function g is a function of independent variables x and y, which can be seen since we have partial derivatives of each. Therefore it is a differential equation, and in particular a PDE. Additionally, since there are no produces of the function g and/or its derivatives we conclude it is linear. Thus, the correct answer is BCD.

Correct Answers:

- BCD

9. (1 point)

? 1. Which equation below is a linear ODE?

? 2. Which equation below is a linear PDE?

? 3. Which equation below is a nonlinear ODE?

A. $e^{\theta}y'' + \theta\cos(\theta)y' = y$

B. $y'' = \sqrt{1-y'}$

C. $g_x + g_y = 0$

Solution:

SOLUTION

The answer to the first question is A, because the equation consists of partial derivatives of g with respect to both x and y, but there are not products of the functions or derivatives.

The answer to the second question is C, because there are no products of y or its derivatives. Yes, there are products of the θ, and there are nonlinear terms with respect to θ, but this is not the depentent variable.

Correct Answers:

- A
- C
- B

Is the differential equation $\sin(y') = y$ linear?

- A. Yes
- B. No

Solution:

SOLUTION:

To be a linear differential equation we need all degrees of y and its derivatives (including y') to be 1. However, $\sin(y')$ has terms with degree greater than 1. This can be seen by considering the power series expansion of $\sin(y')$ which is

$$\sin(y') = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}(y')^{2k+1} = y' - \frac{(y')^3}{3!} + \frac{(y')^5}{5!} - \cdots.$$

We do not require to consider power series to determine whether a term is nonlinear. However, it is imperative to understand that terms like e^y, $\sin(y'')$, $\ln(y')$, $\sqrt{y'}$, and others are all nonlinear. Thus, the correct answer is B.

Correct Answers:

- B

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11. (1 point)

? 1. Which differential equation below has order two?

? 2. Which differential equation below has order four?

? 3. Which differential equation below has order three?

A. $(1+\sin(\theta))\dfrac{d^2f}{d\theta} = \sqrt{3+f}$

B. $y^{(2)} - y^{(4)} + y' = y$

C. $ty''' - t^3y' + y = t^4$

Solution:

SOLUTION

The answer to the first question is A, because the highest order derivative that occurs is of order two. Even if one squares both sides of the equation, it is then just a product of two order two derivatives on the left. Regardless, the highest ordered derivative which occurs is two.

The answer to the second question is B, because the highest order derivative that occurs is three. The variable t is the independent variable, and we do not care what its powers are..

Correct Answers:

- A
- B
- C