Implicit and explicit solutions

We tend to think of functions as explicit functions. That is, it has the expression $y = f(x)$. In other words, y is defined explicitly in terms of x.

Definition:

A solution in which the dependent variable can be expressed entirely in terms of the independent variable is called an **explicit solution**.

However, some solutions are not explicit functions. That is, we encounter solutions that are relations in which the dependent variable cannot be isolated and expressed entirely in terms of the independent variables.

An example is the circle of radius r equation given by $y^2 + x^2 = r^2$, which we examine closer in the video.

To give a more precise definition of an implicit solution for a DE we recall that the notation $F(x, y) = 0$ from before would represent an arbitrary equation relating y and x. For example, taking $F(x, y) = y^2 - x^2$ means that $F(x, y) = 0$ represent the equation $y^2 - x^2 = 0$.

Definition:

A relation $F(x, y) = 0$ (for example, $y^2 - x^2 = 0$) is an **implicit solution** to the DE if there is at least one function y, which is not an explicit solution for the DE, and satisfies the DE and $F(x, y) = 0$.

To determine whether y satisfies the DE in an implicit solution, we need implicit differentiation.

Discussion, comments, and examples:

WeBWorK module 02 exercises:

- Problems 6,7

Relevant Wikipedia articles:

- **Implicit functions** (https://en.wikipedia.org/wiki/Implicit_function)