
Consider the function $f(x,y) = \dfrac{y^4}{x}$. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

- A. $\dfrac{\partial f}{\partial x} = \dfrac{y^4}{x^2}$; $\quad \dfrac{\partial f}{\partial y} = \dfrac{y^4}{x}$
- B. $\dfrac{\partial f}{\partial x} = -\dfrac{y^4}{x^2}$; $\quad \dfrac{\partial f}{\partial y} = \dfrac{y^3}{x}$
- C. $\dfrac{\partial f}{\partial x} = -\dfrac{4y^3}{x}$; $\quad \dfrac{\partial f}{\partial y} = -\dfrac{4y^3}{x^x}$
- D. $\dfrac{\partial f}{\partial x} = -\dfrac{y^4}{x^2}$; $\quad \dfrac{\partial f}{\partial y} = \dfrac{4y^3}{x}$

Solution:
SOLUTION:
To find $\dfrac{\partial f}{\partial x}$ we treat y as a constant and take the derivative with respect to x. Since the derivative of $\dfrac{1}{x}$ is $-\dfrac{1}{x^2}$ and we are treating y as a constant, we find

$$\frac{\partial f}{\partial x} = -\frac{y^4}{x^2}.$$

To find $\dfrac{\partial f}{\partial y}$ we treat x as a constant and take the derivative with respect to y. Since the derivative of y^4 is $4y^3$ and we are treating x as a constant, we find

$$\frac{\partial f}{\partial y} = \frac{4y^3}{x}.$$

Thus, the correct answer is D.
Correct Answers:

- D

Consider the first-order differential equation $y' = \dfrac{y^7}{x}$. Which of the following best describes the regions in the xy-plane for which the differential equation would have a unique solution which passes through a point in the region?

- A. half-plane defined by either $y < 0$ or $y > 0$
- B. the quadrant with $y < 0$ and $x > 0$
- C. half-plane defined by either $x < 0$ or $x > 0$
- D. the quadrant with $x < 0$ and $y > 0$

Solution:
SOLUTION:
Set $f(x,y) = \dfrac{y^7}{x}$ so that the given differential equation can be written as $y' = f(x,y)$. There will be such a unique solution on a region for which $f(x,y) = \dfrac{y^7}{x}$ and $\frac{\partial f}{\partial y} = \dfrac{7y^6}{x}$ are continuous on. We have that $f(x,y)$ and $\frac{\partial f}{\partial y}$ are continuous for all y and x, except for when $x = 0$ since we cannot divide by zero. Thus, either $x < 0$ or $x > 0$. This gives us the two half-planes for which a solution will have a unique solution. One half-plane given by all y and $x < 0$ and the other given by all y and $x > 0$. Thus, the correct answer is C.
Correct Answers:

- C

Consider the first-order differential equation $y' = y^{\frac{2}{7}}$. Which of the following best describes the regions in the xy-plane for which the differential equation would have a unique solution which passes through a point in the region?

- A. half-plane defined by either $x < 0$ or $x > 0$
- B. half-plane defined by either $y < 0$ or $y > 0$
- C. the quadrant with $y < 0$ and $x > 0$
- D. the quadrant with $x < 0$ and $y > 0$

Solution:
SOLUTION:
Set $f(x,y) = y^{\frac{2}{7}}$ so that the given differential equation can be written as $y' = f(x,y)$. There will be such a unique solution on a region for which $f(x,y) = y^{\frac{2}{7}}$ and $\frac{\partial f}{\partial y} = \frac{2}{7}y^{\frac{-5}{7}}$ are continuous on. We have that $f(x,y)$ is continuous for all x and all y. However,

since we cannot have $y=0$ in $\frac{\partial f}{\partial y}=\frac{2}{7}y^{\frac{-5}{7}}$ we have that $\frac{\partial f}{\partial y}$ is continuous for all x and y, except for when $y=0$ since we cannot divide by zero. Thus, either $y<0$ or $y>0$. This gives us the two half-planes for which a region will have a unique solution. One half-plane given by all x and $y<0$ and the other given by all x and $y>0$. Thus, the correct answer is B.

Correct Answers:

- B

Consider the first-order differential equation $(x+y)y'=y^3$. Which of the following best describes the regions in the xy-plane for which the differential equation would have a unique solution which passes through a point in the region?

- A. the quadrant with $y<0$ and $x>0$
- B. half-plane defined by either $y<-x$ or $y>-x$
- C. half-plane defined by either $y<x$ or $y>x$
- D. the quadrant with $x<0$ and $y>0$

Solution:
SOLUTION:
Set $f(x,y)=\dfrac{y^3}{x+y}$ so that the given differential equation can be written as $y'=f(x,y)$. There will be such a unique solution on a region for which $f(x,y)=\dfrac{y^3}{x+y}$ and

$$\frac{\partial f}{\partial y}=\frac{3y^2(x+y)-y^3}{(x+y)^2}=\frac{3y^2\left((x+y)-y\right)}{(x+y)^2}=\frac{3xy^2}{(x+y)^2}$$

are continuous on. We have that $f(x,y)$ and $\frac{\partial f}{\partial y}$ are continuous for all y and x, except for when $y=-x$ since we cannot divide by zero. Thus, either $y<-x$ or $y>-x$. This gives us the two half-planes for which a solution will have a unique solution. One half-plane given by all $y<-x$ and the other given by all $y>-x$. Thus, the correct answer is B.

Correct Answers:

- B

Consider the first-order differential equation $y'=\ln(y^2-4)$. For which point (x_0,y_0) below is it guaranteed that this differential equation has a unique solution at the point (x_0,y_0)?

- A. $(x_0,y_0)=(1,1)$
- B. $(x_0,y_0)=(1,3)$
- C. $(x_0,y_0)=(1,2)$
- D. $(x_0,y_0)=(2,-2)$

Solution:
SOLUTION:
Set $f(x,y)=\ln(y^2-4)$. We need that $f(x,y)$ and $\frac{\partial f}{\partial y}$ are both continuous in a region that contains the point (x_0,y_0). Since $\ln(w)$ is continuous for $w>0$ we have that $f(x,y)$ is continuous for $y^2-4>0$. That is, for $y^2>4$, or when $y>2$ or $y<2$. Meanwhile, since we cannot divide by zero we have that

$$\frac{\partial f}{\partial y}=\frac{2y}{y^2-4}$$

is continuous so long as $y\neq\pm 2$. These points were already excluded with our range of $y>2$ or $y<2$. The only point given so that $y>2$ or $y<2$ is the point $(1,3)$. Thus, the correct answer is B.

Correct Answers:

- B

Consider the first-order differential equation $y'=\ln(y^2-4)$. For which point (x_0,y_0) below is it guaranteed that this differential equation has a unique solution at the point (x_0,y_0)?

- A. $(x_0,y_0)=(-2,-5)$
- B. $(x_0,y_0)=(5,1)$
- C. $(x_0,y_0)=(0,1)$
- D. $(x_0,y_0)=(1,-2)$

Solution:
SOLUTION:
Set $f(x,y)=\ln(y^2-4)$. We need that $f(x,y)$ and $\frac{\partial f}{\partial y}$ are both continuous in a region that contains the point (x_0,y_0). Since $\ln(w)$ is continuous for $w>0$ we have that $f(x,y)$ is continuous for $y^2-4>0$. That is, for $y^2>4$, or when $y>2$ or $y<2$. Meanwhile, since we cannot divide by zero we have that

$$\frac{\partial f}{\partial y}=\frac{2y}{y^2-4}$$

is continuous so long as $y \neq \pm 2$. These points were already excluded with our range of $y > 2$ or $y < 2$. The only point given so that $y > 2$ or $y < 2$ is the point $(-2, -5)$. Thus, the correct answer is A.

Correct Answers:

- `A`

You should verify that $y = \dfrac{1}{x^2 + c}$ is a one-parameter family of solutions for the first-order differential equation $y' = -2xy^2$. Setting $f(x,y) = -2xy^2$ note also that $f(x,y)$ and $\dfrac{\partial f}{\partial y} = -4xy$ are continous thoughout the entire xy-plane. Thus, for any point (x_0, y_0) in the xy-plane there exists an interval I such that there exists a unique solution which passes through (x_0, y_0).

Find a solution from the family $y = \dfrac{1}{x^2 + c}$ and determine the largest interval I of definition for the solution of for the initial value condition $y(0) = -\dfrac{1}{9}$.

- A. $y = \dfrac{1}{x^2 + \frac{1}{9}}; \quad (-\infty, \infty)$
- B. $y = \dfrac{1}{x^2 - 9}; \quad (-\infty, -3)$ or $(3, \infty)$
- C. $y = \dfrac{1}{x^2 - 9}; \quad (-3, 3)$
- D. $y = \dfrac{1}{x^2 - 3}; \quad (-\infty, -3)$ or $(3, \infty)$

Solution:

SOLUTION:

We need $y(0) = -\frac{1}{9}$. Thus, we have $\frac{1}{0^2 + c} = -\frac{1}{9}$, and solving for c gives $c = -9$. Thus, the desired solution is $y = \frac{1}{x^2 - 9}$. This solution is continuous for $x < -3$, $-3 < x < 3$, and $x > 3$. However, the initial condition is in $(-3, 3)$. Thus, the correct answer is C.

Correct Answers:

- `C`

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8. (1 point)

You should verify that $y = \dfrac{1}{x^2 + c}$ is a one-parameter family of solutions for the first-order differential equation $y' = -2xy^2$. Setting $f(x,y) = -2xy^2$ note also that $f(x,y)$ and $\dfrac{\partial f}{\partial y} = -4xy$ are continous thoughout the entire xy-plane. Thus, for any point (x_0, y_0) in the xy-plane there exists an interval I such that there exists a unique solution which passes through (x_0, y_0).

Note, however, that there is no solution from the family $y = \dfrac{1}{x^2 + c}$ which satisfies $y(0) = 0$.

(a) A solution for $y' = -2xy^2$ such that $y(0) = 0$ is $y =$ _____.

(b) The largest interval of definition for y in part (a) is

- Choose
- All real numbers
- All positive real numbers
- All nonnegative real numbers

.

Solution:

SOLUTION:

Note that the trivial solution $y = 0$ is a solution to $y' = -2xy^2$ and satifies $y(0) = 0$. Thus, the answer for part (a) is $y = 0$. Meanwhile, since $y = 0$ is continuous for all real numbers we have the answer to part (b) is All real numbers.

Correct Answers:

- `0`
- `All real numbers`

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9. (1 point)

Solve the differential equation $\dfrac{dy}{dx} = \cos(5x)$ using separation of variables.

$y =$ _______________ $+C$

[NOTE: Remember to enter all necessary *, (, and) see help (syntax) for more information.]

Solution:

SOLUTION:

We 'separate the variables' and rewrite $\dfrac{dy}{dx} = \cos(5x)$ as

$$dy = \cos(5x)\,dx.$$

Integrating both sides (which is done using a u-sub with $u = 5x$ and $du = 5\,dx$ so that $dx = \frac{du}{5}$)

$$\int dy = \int \cos(5x)\,dx$$

gives

$$y + c_1 = \frac{1}{5}\sin(5x) + c_2,$$

for arbitrary constants c_1 and c_2. Combinging these to a single arbitrary constant C we have

$$y = \frac{1}{5}\sin(5x) + C.$$

Thus the solution is $1/5 * sin(5 * x) + C$.

Correct Answers:

- `1/5 * sin(5 *x)`

10. (1 point)
Solve the differential equation $e^{9x}\,dy + dx = 0$ using separation of variables.

$y =$ ______________ $+C$

[NOTE: Remember to enter all necessary *, (, and) see help (syntax) for more information.]

Solution:

SOLUTION:

We 'separate the variables' and rewrite $e^{9x}\,dy + dx = 0$ as $e^{9x}dy = -dx$, or

$$dy = -e^{-9x}dx$$

(where we used that $\frac{1}{e^{9x}} = e^{-9x}$. Integrating both sides (which is done using a u-sub with $u = -9x$ and $du = -9\,dx$ so that $dx = -\frac{du}{9}$)

$$\int dy = -\int e^{-9x}dx$$

gives

$$y + c_1 = \frac{1}{9}e^{-9x} + c_2,$$

for arbitrary constants c_1 and c_2. Combinging these to a single arbitrary constant C we have

$$y = \frac{1}{9}e^{-9x} + C.$$

Thus the solution is $1/9 * exp(-9*x) + C$.

Correct Answers:

- `1/9 * exp(-9 *x)`

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11. (1 point) Find the general solution of the differential equation $y' = e^{4x} - 9x$.

(Use C to denote the arbitrary constant.)

$y =$ __________ help (formulas)

Solution:

SOLUTION:

We 'separate the variables' and rewrite $y' = e^{4x} - 9x$ as

$$dy = \left(e^{4x} - 9x\right)dx.$$

Integrating both sides (which is done using a u-sub with $u = 4x$ and $du = 4\,dx$ so that $dx = \frac{du}{4}$)

$$\int dy = \int e^{4x}\,dx - \int 9x\,dx$$

gives

$$y + c_1 = \frac{1}{4}e^{4x} - \frac{9}{2}x^2 + c_2,$$

for arbitrary constants c_1 and c_2. Combining these to a single arbitrary constant C we have the solution is

$$y = \frac{1}{4}e^{4x} - \frac{9}{2}x^2 + C.$$

Correct Answers:

- `0.25*exp(4*x)-4.5*x^2+C`

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12. (1 point) Find the general solution of the differential equation $x\dfrac{dy}{dx} = 5y$.

(Use C to denote the arbitrary constant.)

$y =$ __________ help (formulas)

Solution:

SOLUTION:

We 'separate the variables' and rewrite $x\dfrac{dy}{dx} = 5y$ as

$$\frac{1}{y}\,dy = \frac{5}{x}\,dx.$$

Integrating both sides

$$\int \frac{1}{y}\,dy = \int \frac{5}{x}\,dx$$

gives

$$\ln(y) + c_1 = 5\ln(x) + c_2,$$

for arbitrary constants c_1 and c_2. Combinging these to a single arbitrary constant k we have

$$\ln(y) = 5\ln(x) + k.$$

Applying e to both sides (that is, $e^{\ln(y)} = e^{5\ln(x)+k}$) we find

$$y = e^{5\ln(x)+k} = e^{5\ln(x)}e^k = e^{\ln\left(x^5\right)}C = Cx^5,$$

where we replace the arbitrary constant e^k with the notation C. Thus the solution is

$$y = Cx^5.$$

Correct Answers:

- `C*x^5`

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13. (1 point) Find the equation of the solution to $\dfrac{dy}{dx} = x^5 y$ through the point $(x,y) = (1,4)$.

(Don't forget to add '$y =$' to your equation!)

______________ help (equations)

Solution:

SOLUTION:

We 'separate the variables' and rewrite $\frac{dy}{dx} = x^5 y$ as

$$\frac{1}{y}\,dy = x^5\,dx.$$

Integrating both sides

$$\int \frac{1}{y}\,dy = \int x^5\,dx$$

gives

$$\ln(y) + c_1 = \frac{1}{5+1}x^{5+1} + c_2,$$

for arbitrary constants c_1 and c_2. Combining these to a single arbitrary constant k we have

$$\ln(y) = \frac{1}{6}x^6 + k.$$

Applying e to both sides (that is, $e^{\ln(y)} = e^{\frac{1}{6}x^6+k}$) we find

$$y = e^{\frac{1}{6}x^6+k} = e^{\frac{1}{6}x^6}e^k = e^{\frac{1}{6}x^6}C = Ce^{\frac{1}{6}x^6},$$

where we replace the arbitrary constant e^k with the notation C.

We want the particular solution that passes through the point $(1,4)$. That is, we want $y(1) = 4$, or using our general solution above

$$4 = y(1) = Ce^{\frac{1}{6}1^6} = Ce^{\frac{1}{6}}.$$

Solving for C we find $C = 4e^{-\frac{1}{6}}$. Thus the solution is

$$y = 4e^{-\frac{1}{6}}e^{\frac{1}{6}x^6} = 4e^{\frac{1}{6}\left(x^6-1\right)}.$$

Correct Answers:

- `y = 4/[e^(1/6)]*e^(x^6/6)`

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14. (1 point) Find the general solution of the differential equation $\frac{dy}{dx} = e^{2x-9y}$.

(Use C to denote the arbitrary constant.)

$y =$ __________ help (formulas)

Solution:

SOLUTION:

We 'separate the variables' and rewrite $\frac{dy}{dx} = e^{2x-9y}$ as

$$\frac{1}{e^{-9y}}\,dy = e^{2x}\,dx.$$

Integrating both sides

$$\int e^{9y}\,dy = \int e^{2x}\,dx$$

(using u-subs) gives

$$\frac{1}{9}e^{9y} + c_1 = \frac{1}{2}e^{2x} + c_2,$$

for arbitrary constants c_1 and c_2. Combining these to a single arbitrary constant C we have

$$e^{9y} = \frac{9}{2}e^{2x} + C.$$

Applying ln to both sides (that is, $\ln(e^{9y}) = \ln(\frac{9}{2}e^{2x} + C)$) we find

$$9y = \ln\left(\frac{9}{2}e^{2x} + C\right),$$

or

$$y = \frac{1}{9}\ln\left(\frac{9}{2}e^{2x} + C\right).$$

Correct Answers:

- `0.111111*ln(4.5*exp(2*x)+C)`