Please show and explain your work. Justification is necessary for credit. Also sign your name here acknowledging you will not use problem solving software (Wolfram Alpha, etc.):________________

1. (*14 points*)

a. (*2 pts*) Circle all of the following expression which are **linear** differential equations.

(i) $y'''' - 2y''' + 3y = 5$

(ii) $\left(x^2 - \sin(x)\right) y^{(5)} - xe^x y + \sin\left(x^2\right) = 0$

(iii) $\sqrt{y'} = x\cos(x)y - 3$

(iv) $(y')^2 = y^3 + 2$

b. (*2 pts*) Circle all of the following expression of the form $M(x,y)\,dx + N(x,y)\,dy = 0$ such that **both** $M(x,y)$ and $N(x,y)$ are homogeneous *functions* of the **same** degree.

(i) $x^2\,dx + yx\,dy = 0$

(ii) $(3xy)\;dx + e^{\left(\frac{x}{y}\right)}\;dy = 0$

(iii) $\sin(y)\,dx + (2y + xe^y)\;dy = 0$

(iv) $(x - 2y)\;dx + x\,dy = 0$

c. (*4 pts*) Circle whether the following differential equation is an ordinary differential equation (ODE) or a partial differential equation (PDE) and state the **order** of the differential equations (you can do this for any PDEs as well).

(i) $\ln(y')y + x^2y = 1$ ODE *PDE* *Order:*________

(ii) $\dfrac{\partial f}{\partial x} + \dfrac{\partial^2 f}{\partial y^2} + xy = 0$ ODE *PDE* *Order:*________

(iii) $x^2\dfrac{d^3 f}{dx^3} + \dfrac{df}{dx} = \dfrac{d^2 f}{dx^2} - \sin(x)$ ODE *PDE* *Order:*________

(iv) $\left(\dfrac{d}{dx}\right)^4 f(x) + \left(\dfrac{d^2}{dx^2}\right) f(x) + f'(x) = 0$ ODE *PDE* *Order:*________

d. (*4 pts*) Circle all of the following expression which are **exact** differential equations. (Provide work for partial credit!)

(i) $xy\,dx + xy\,dy = 0$

(ii) $x^2y\,dx + \frac{1}{3}x^3\,dy = 0$

(iii) $xy^2\,dx + x^2y\,dy = 0$

(iv) $\sin(xy)\,dx + xy\cos(x)\,dy = 0$

e. (*2 pts*) Suppose $y^2 - x = 3$ is a solution to a differential equation. Is this solution implicit or explicit? Explain your answer in one sentence.

2. (*27 points*) Solve the following differential equations using any technique you like that works. However, **you must explain WHY you chose that method** to receive credit!

a. (*9 pts*) $y' = \dfrac{x}{y^2}$

b. (*9 pts*) $x\dfrac{dy}{dx} - 2y = x^3e^x$

c. (*9 pts*) $2xy\,dx + \left(x^2 + 3y^2\right)\,dy = 0$

3. (*9 points*)

a. (*8 pts*) We note that $y(x) = c_1 e^{3x} + c_2 e^{-3x}$ is a solution to the differential equation $y'' - 9y = 0$. Find a solution to the initial conditions $y(0) = 0$ and $y'(0) = 2$.

b. (*1 pt*) Does the trivial solution also satisfy the initial value problem given above in **3a**? Explain your answer.