Recall $\frac{dy}{dx} = f(x,y)$

1. (1 point) Are the following functions homogeneous? (You have only one attempt! Submit all answers at the same time)

$f(x,y) = x^3y^5$

(a)
- Choose
- Homogeneous
- Not Homogeneous

$\to f(tx,ty) = tx^3ty^5 \to t(x^3y^5) \to tf(x,y)$

homogeneous of deg 1

$f(x,y) = x^3y^5$.

(b)
- Choose
- Homogeneous
- Not Homogeneous

$f(x,y) = x\sin(y)$.

(c)
- Choose
- Homogeneous
- Not Homogeneous

$f(x,y) = x + y^2$.

Answer(s) submitted:
- Homogeneous
- Not Homogeneous
- Not Homogeneous

(incorrect)

Sol^n:

Def^n: Mathematically a function $f(x,y)$ is homogeneous if it can ~~do~~ be written as
$f(x,y) = x^n f(y/x)$
this is homogeneous eq^n of degree n.

(a) $f(x,y) = x^3y^6$
$= x^3 \times \frac{x^6}{x^6} y^6$
$= x^9 \left(\frac{y}{x}\right)^6$
$= x^9 f(y/x)$
Hence this is homogeneous function.

(b) $f(x,y) = x\sin y$
this can not be written in the form
$f(x,y) = x^n f(y/x)$
Hence it is not homogeneous function.

(c) $f(x) = x + y^2$
This can not be written in the form
$f(x,y) = x^n f(y/x)$
Hence it is not homogeneous function.

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2. (1 point)
Determine whether the differential equation is homogeneous or not. If it is homogeneous, provide the degree of x^2y^4 and x^6+y^6. If it is not homogeneous, put -1 as the degree.

$$\left(x^2y^4\right)\,dx + \left(x^6+y^6\right)\,dy = 0$$

(a) The degree is 6.

(b) The equation is
- Choose
- Homogeneous
- $\boxed{\text{Not Homogeneous}}$

Answer(s) submitted:
-
-

(incorrect)

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3. (1 point) Use substitution to find the general solution of the differential equation $(7x-y)\,dx + x\,dy = 0$.

(Use C to denote the arbitrary constant and ln| input | if using ln.)
$y =$ ______ help (formulas)
Answer(s) submitted:
-

(incorrect)

Solve the differential equation $\left(y^2+xy\right)\,dx - x^2\,dy = 0$.

- A. $y = \dfrac{x}{xC + \ln|x|}$
- B. $y = C - \ln|x|$
- C. $y = \dfrac{x}{C - \ln|x|}$
- D. $y = C + \ln|x|$

Answer(s) submitted:
-

(incorrect)

Solve the homogeneous differential equation $-y\,dx + \left(x+\sqrt{xy}\right)\,dy = 0$. (Note: Some algebraic manipulation goes into putting your answer into the form below.)

- A. $y = x\left(\ln|x| - C\right)^2$
- B. $y\left(\ln|y| - C\right)^2 = 4x$
- C. $\sqrt{yx}\ln|y| = C\sqrt{x}$

SOLUTION:

First we divide the differential equation by dx so that we may write $-y\,dx + \left(x+\sqrt{xy}\right)\,dy = 0$ as

$$\left(x+\sqrt{xy}\right)\frac{dy}{dx} - y = 0.$$

We take $y = ux$. Then $\frac{dy}{dx} = \left(\frac{du}{dx}\right)x + u\left(\frac{dx}{dx}\right) = x\frac{du}{dx} + u$. Thus, we may replace y with ux and $\frac{dy}{dx}$ with $x\frac{du}{dx} + u$ to get

$$\left(x+\sqrt{ux^2}\right)\left(x\frac{du}{dx}+u\right) - ux = 0,$$

which is equivalent to

$$\left(x^2 + x\sqrt{ux^2}\right)\frac{du}{dx} + u\sqrt{ux^2} = 0,$$

or

$$\frac{du}{dx} = -\frac{u\sqrt{ux^2}}{x^2+x\sqrt{ux^2}} = -\frac{xu\sqrt{u}}{x^2\left(1+\sqrt{u}\right)} = -\frac{u\sqrt{u}}{x\left(1+\sqrt{u}\right)}.$$

Separating variables gives

$$\frac{1+u^{\frac{1}{2}}}{u^{\frac{3}{2}}}\,du = -\frac{1}{x}\,dx,$$

which we integrate both sides to find

$$\int\left(u^{-\frac{3}{2}} + \frac{1}{u}\right)du = -\int\frac{1}{x}\,dx,$$

or

$$-2u^{-\frac{1}{2}} + \ln|u| = -\ln|x| + C.$$

Replacing u with $\frac{y}{x}$ (since we took $y = ux$) we get

$$-2\left(\frac{y}{x}\right)^{-\frac{1}{2}} + \ln\left|\frac{y}{x}\right| = -\ln|x| + C,$$

or

$$-\frac{2\sqrt{x}}{\sqrt{y}} + \ln|y| - \ln|x| = -\ln|x| + C,$$

which can be written as

$$-\frac{2\sqrt{x}}{\sqrt{y}} + \ln|y| = C.$$

We could rewrite this in a number of ways. For example, we can turn this into $\sqrt{y}\ln|y| = \sqrt{y}C + 2\sqrt{x}$, or $\sqrt{y}\left(\ln|y\ rvert - C\right) = 2\sqrt{x}$. Then squaring both sides gives

$$y(\ln|y| - C)^2 = 4x.$$

Thus, the correct answer is B.

Answer(s) submitted:
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(incorrect)

Which of the following is a solution to the IVP consisting of the homogeneous differential equation $-y\,dx + \left(x + \sqrt{xy}\right)\,dy = 0$ with the initial condition $y(4) = 1$. (Note: Some algebraic manipulation goes into putting your answer into the form below.)

- A. $\sqrt{yx}\ln|y| = 4\sqrt{x}$
- B. $y = x\left(\ln|x| + 4\right)^2$
- C. $y\left(\ln|y| + 4\right)^2 = 4x$

Answer(s) submitted:

-

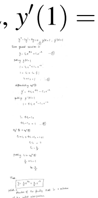
(incorrect)

7. (1 point) Note that $y = c_1e^{4x} + c_2e^{-x}$ is a general solution for the second-order differential equation $y'' - 3y' - 4y = 0$ on the interval $(-\infty, \infty)$. Find values c_1 and c_2 so that y is a solution to the second-order IVP consisting of the differential equation $y'' - 3y' - 4y = 0$ and the initial condition $y(0) = 3$, $y'(0) = 7$. The values are $c_1 =$ _____ and $c_2 =$ _____.
Answer(s) submitted:

-
-

(incorrect)

8. (1 point) Note that $y = c_1x + c_2x\ln(x)$ is a general solution for the second-order differential equation $x^2y'' - xy' + y = 0$ on the interval $(0, \infty)$. Find values c_1 and c_2 so that y is a solution to the second-order IVP consisting of the differential equation $x^2y'' - xy' + y = 0$ and the initial condition $y(1) = 2$, $y'(1) = 7$. The values are $c_1 =$ _____ and $c_2 =$ _____.
Answer(s) submitted:



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(incorrect)

We have that $y = c_1 + c_2x^2$ is a two-parameter family of solutions for the differential equation $xy'' - y' = 0$ on the interval $(-\infty, \infty)$. Does there exist values c_1 and c_2 so that y satisfies the initial conditions $y(0) = 0$ and $y'(0) = 1$?

SOLUTION:

We note have that

$$0 = y(0) = c_1 + c_2(0) = c_1$$

and since $y' = 2c_2x$ we have

$$1 = y'(0) = 2c_2(0) = 0,$$

which cannot be. Thus the answer for the first part is no.

The reason the theorem from class does not work is that the coefficient of the x term is 0. Thus, the correct answer for the second part is C.

- A. No

SOLUTION:

We note have that

$$0 = y(0) = c_1 + c_2(0) = c_1$$

and since $y' = 2c_2x$ we have

$$1 = y'(0) = 2c_2(0) = 0,$$

which cannot be. Thus the answer for the first part is no.

The reason the theorem from class does not work is that the coefficient of the x term is 0. Thus, the correct answer for the second part is C.

- B. Yes

Why does you answer above not violate the theorem in class concerning the existence of a unique solution?

- A. The coefficients are continuous on the interval.
- B. The highest order derivative is two.
- C. The coefficent of the y'' term is 0.
- D. The differential equation is linear.

Answer(s) submitted:

-
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(incorrect)

Consider the initial value problem $(x-8)y'' + 3y = x$ with initial conditions $y(0) = 3$ and $y'(0) = 1$. Which of the following is an interval containing 0 for which this IVP has a unique solution on?

- A. $(-\infty, 8)$
- B. $(-8, \infty)$
- C. $(-\infty, -8)$
- D. $(-\infty, 3)$

Answer(s) submitted:

-

(incorrect)

1b $f(x,y) = x\sin(y)$

$f(x,y) = x\sin(y)$

$\rightarrow f(tx,ty) = tx\sin(ty)$

No?

1c| $f(x,y) = x + y^2$

$\rightarrow f(tx,ty) = (tx + ty^2)$

$\rightarrow t(x+y^2)$

$\therefore t^1 f(x,y)$

$f(x,y)$ is homogeneous of degree 1

2| $(x^2y^4)dx + (x^6+y^6)dy = 0$

$\rightarrow \frac{dy}{dx} = \frac{x^2y^4}{x^6+y^6}$

6^{th}-order homogeneous?

Step 1| Subsitute $y=ux$

$$\frac{dy}{dx} = \frac{x^2(ux)^4}{x^6+(ux)^6}$$

Step 2| Replace "$\frac{dy}{dx}$" with "$x\frac{du}{dx}+u$"

$$x\frac{du}{dx} + u = \frac{x^2(ux)^4}{x^6+(ux)^6}$$