Please show and explain your work where necessary. Good luck!!

1. (6 points) Is a unique solution guaranteed to exist for the following initial value problems on the given intervals. Explain you answers.

a. (2 pts)
$$\ln(x)y''' + \frac{e^x}{x-4}y' - y = \cos(x); \quad y(2) = 1, \quad I = (0,3).$$

b. (2 pts)
$$\ln(x)y''' + \frac{e^x}{x-4}y' - y = x^2; \quad y(3) = 1, \quad I = (2,4).$$

c. (2 pts)
$$\ln(x)y''' + \frac{e^x}{x-4}y' - y = x^2; \quad y(2) = 1, \quad I = (1,3).$$

2. (2 points) In your own words, describe what it means for functions to be linearly independent.

- 3. (2 points) Provide an example of the following:
 - **a.** (1 pt) A 3rd order linear homogeneous differential equation.
 - **b.** (1 pt) A 2nd order linear nonhomogeneous differential equation.