**1.** (1 point)
Solve the following differential equation:

$$\left(x - y^5 + y^5\sin(x)\right)\,dx = \left(5xy^4 + 5y^4\cos(x)\right)\,dy.$$

______________ = constant. help (formulas)

**Solution:**
SOLUTION:
First we verify that

$$\left(x - y^5 + y^4\sin(x)\right)\,dx - \left(5xy^4 + 5y^4\cos(x)\right)\,dy = 0$$

is an exact differential equation. Since

$$\frac{\partial\left(x - y^5 + y^4\sin(x)\right)}{\partial y} = -5y^4 + 4y^3\sin(x) = \frac{\partial\left(-5xy^4 - 5y^4\cos(x)\right)}{\partial x}$$

this is indeed an exact DE. Thus, there exists a function $f(x,y)$ such that

$$\frac{\partial f}{\partial x} = x - y^5 + y^4\sin(x).$$

Thus, we integrate $x - y^5 + y^4\sin(x)$ with respect to $x$ by treating $y$ as a constant. We find

$$f(x,y) = \int\left(x - y^5 + y^4\sin(x)\right)\,dx \tag{1}$$

$$= \frac{x^2}{2} - xy^4 - y^4\cos(x) + g(y), \tag{2}$$

where $g(y)$ is some function of $y$ (or a constant). To continue to find the true $f(x,y)$ we need to determine what $g(y)$ is. To do so, we differentiate the previous display with respect to $y$ to find

$$\frac{\partial f}{\partial y} = -4xy^3 - 4y^3\cos(x) + g'(y).$$

Since $\frac{\partial f}{\partial y} = -5xy^4 - 5y^4\cos(x)$, we have

$$-4xy^3 - 4y^3\cos(x) + g'(y) = -5xy^4 - 5y^4\cos(x),$$

so that $g'(y) = 0$. Integrating $g'(y)$ with respect $y$ we find

$$\begin{aligned} g(y) &= \int g'(y)\,dy = \int (0)\,dy \\ &= C. \end{aligned}$$

Plugging this into the equation above for $f(x,y)$ we find

$$f(x,y) = \frac{x^2}{2} - xy^4 - y^4\cos(x) + C.$$

Thus, the equation

$$\frac{x^2}{2} - xy^4 - y^4\cos(x) = \text{constant} \qquad \text{or}$$
$$x^2 - 2xy^4 - 2y^4\cos(x) = \text{constant}$$

is the desired answer. The second solution here comes from the fact that we could multiply the entire expression by 2 since it is still an arbitrary constant on the right side.

*Correct Answers:*

- `x^2-2*x*y^5-2*y^5*cos(x)`