
Q1 Consider the function $f(x,y) = \frac{y^4}{x}$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

- A. $\frac{\partial f}{\partial x} = \frac{y^4}{x^2}$; $\quad \frac{\partial f}{\partial y} = \frac{y^4}{x}$
- B. $\frac{\partial f}{\partial x} = -\frac{y^4}{x^2}$; $\quad \frac{\partial f}{\partial y} = \frac{y^3}{x}$
- C. $\frac{\partial f}{\partial x} = -\frac{4y^3}{x}$; $\quad \frac{\partial f}{\partial y} = -\frac{4y^3}{x^x}$
- D. $\frac{\partial f}{\partial x} = -\frac{y^4}{x^2}$; $\quad \frac{\partial f}{\partial y} = \frac{4y^3}{x}$

Answer(s) submitted:

- `D`

(correct)

Q2 Consider the first-order differential equation $y' = \frac{y^7}{x}$. Which of the following best describes the regions in the xy-plane for which the differential equation would have a unique solution which passes through a point in the region? $y = \frac{7y^6}{x}$

- A. half-plane defined by either $y < 0$ or $y > 0$
- B. the quadrant with $y < 0$ and $x > 0$
- C. half-plane defined by either $x < 0$ or $x > 0$
- D. the quadrant with $x < 0$ and $y > 0$

$y' = \frac{d}{dy}\left(\frac{y^7}{x}\right) \to \frac{1}{x} \cdot \frac{d}{dy}(y^7)$

$\to \frac{1}{x} \cdot 7y^{7-1}$

$= \frac{7y^6}{x}$

Answer(s) submitted:

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(incorrect)

Q3 Consider the first-order differential equation $y' = y^{\frac{2}{7}}$. Which of the following best describes the regions in the xy-plane for which the differential equation would have a unique solution which passes through a point in the region?

Q3 $y' = y^{2/7} \to y = \frac{2}{7} y^{-5/7}$

- A. half-plane defined by either $x < 0$ or $x > 0$
- B. half-plane defined by either $y < 0$ or $y > 0$
- C. the quadrant with $y < 0$ and $x > 0$
- D. the quadrant with $x < 0$ and $y > 0$

Answer(s) submitted:

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(incorrect)

Q4 Consider the first-order differential equation $(x+y)y' = y^3$. Which of the following best describes the regions in the xy-plane for which the differential equation would have a unique solution which passes through a point in the region?

- A. the quadrant with $y < 0$ and $x > 0$
- B. half-plane defined by either $y < -x$ or $y > -x$
- C. half-plane defined by either $y < x$ or $y > x$
- D. the quadrant with $x < 0$ and $y > 0$

Answer(s) submitted:

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(incorrect)

Q5 Consider the first-order differential equation $y' = \ln(y^2 - 4)$. For which point (x_0, y_0) below is it guaranteed that this differential equation has a unique solution at the point (x_0, y_0)?

$y' = \ln(y^2-4)$

- A. $(x_0, y_0) = (1, 1)$

$\Rightarrow f(x,y) = \ln(y^2-8)$

$f(x,y)$ is continuous if $y^2 - 4 > 0$

$\frac{\partial f}{\partial y} = \frac{1}{y^2-4}(2y) = \frac{2y}{y^2-4}$

$\to y^2 > 4$

$\to y < -2$ or $y > 2$

A

Q5

$y' = \ln(y^2-4)$
$\Rightarrow f(x,y) = \ln(y^2-8)$
$f(x,y)$ is contiuous if $y^2-4>0$

- B. $(x_0, y_0) = (1, 3)$

$\frac{\partial f}{\partial y} = \frac{1}{y^2-4}(2y) = \frac{2y}{y^2-4}$ | $\rightarrow y^2>4$ $\rightarrow y<-2$ or $y>2$

- C. $(x_0, y_0) = (1, 2)$

Here $\frac{\partial f}{\partial y}$ is continuous if $y^2=4\neq\emptyset$

$\rightarrow y^2=4$

- D. $(x_0, y_0) = (2, -2)$

$\rightarrow y\neq\pm2$ (B)

by condition A&B

Answer(s) submitted:

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(incorrect)

Q6

Consider the first-order differential equation $y' = \ln(y^2-4)$. For which point (x_0, y_0) below is it guaranteed that this differential equation has a unique solution at the point (x_0, y_0)?

- A. $(x_0, y_0) = (-2, -5)$

$y' = \ln(y^2-4)$
$\Rightarrow f(x,y) = \ln(y^2-8)$
$f(x,y)$ is contiuous if $y^2-4>0$

- B. $(x_0, y_0) = (5, 1)$

$\frac{\partial f}{\partial y} = \frac{1}{y^2-4}(2y) = \frac{2y}{y^2-4}$ | $\rightarrow y^2>4$ $\rightarrow y<-2$ or $y>2$

- C. $(x_0, y_0) = (0, 1)$

Here $\frac{\partial f}{\partial y}$ is continuous if $y^2=4\neq\emptyset$

- D. $(x_0, y_0) = (1, -2)$

$\rightarrow y^2=4$
$\rightarrow y\neq\pm2$ (B)
by condition A&B

Answer(s) submitted:

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(incorrect)

Q7

You should verify that $y = \frac{1}{x^2+c}$ is a one-parameter family of solutions for the first-order differential equation $y' = -2xy^2$. Setting $f(x,y) = -2xy^2$ note also that $f(x,y)$ and $\frac{\partial f}{\partial y} = -4xy$ are continous thoughout the entire xy-plane. Thus, for any point (x_0, y_0) in the xy-plane there exists an interval I such that there exists a unique solution which passes through (x_0, y_0).

Find a solution from the family $y = \frac{1}{x^2+c}$ and determine the largest interval I of definition for the solution of for the initial value condition $y(0) = -\frac{1}{9}$.

- A. $y = \frac{1}{x^2+\frac{1}{9}}$; $(-\infty, \infty)$

Q7

- B. $y = \frac{1}{x^2-9}$; $(-\infty, -3)$ or $(3, \infty)$
- C. $y = \frac{1}{x^2-9}$; $(-3, 3)$
- D. $y = \frac{1}{x^2-3}$; $(-\infty, -3)$ or $(3, \infty)$

Answer(s) submitted:

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(correct)

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8. (1 point)

You should verify that $y = \frac{1}{x^2+c}$ is a one-parameter family of solutions for the first-order differential equation $y' = -2xy^2$. Setting $f(x,y) = -2xy^2$ note also that $f(x,y)$ and $\frac{\partial f}{\partial y} = -4xy$ are continous thoughout the entire xy-plane. Thus, for any point (x_0, y_0) in the xy-plane there exists an interval I such that there exists a unique solution which passes through (x_0, y_0).

Note, however, that there is no solution from the family $y = \frac{1}{x^2+c}$ which satisfies $y(0) = 0$.

(a) A solution for $y' = -2xy^2$ such that $y(0) = 0$ is $y =$ $\emptyset$

(b) The largest interval of definition for y in part (a) is

- Choose
- All real numbers
- All positive real numbers
- All nonnegative real numbers

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Answer(s) submitted:

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(incorrect)

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9. (1 point)

Solve the differential equation $\frac{dy}{dx} = \cos(5x)$ using separation of variables.

$\int = \cos(5x)$

$y =$ ______ $+C$

[NOTE: Remember to enter all necessary *, (, and) see help (syntax) for more information.]

Answer(s) submitted:

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(incorrect)

$\int \cos(5x)\,du$

$\rightarrow \frac{1}{5}\cdot\int \cos(u)\,du$

$\rightarrow \frac{1}{5}\sin(u)$

$\rightarrow \frac{1}{5}\sin(5x)$

$\therefore \frac{1}{5}\sin(5x)+C$

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10. (1 point)
Solve the differential equation $e^{9x}\,dy + dx = 0$ using separation of variables.

$y = \frac{-e^{-9x}}{-9}$ $+C$

$\rightarrow dx = -e^{9x}dy$

$\rightarrow \frac{dy}{dx} = -\frac{1}{e^{9x}}$

$\rightarrow dy = -e^{-9x}dx$

[NOTE: Remember to enter all necessary *, (, and) see help (syntax) for more information.]

Integrate Both Sides

$\int dy = -\int e^{-9x}dx$

$\rightarrow y = \frac{-e^{-9x}}{-9}$

Answer(s) submitted:

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(incorrect)

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11. (1 point) Find the general solution of the differential equation $y' = e^{4x} - 9x$.

$\int(e^{4x} - 9x)$

(Use C to denote the arbitrary constant.)
$y =$ $\frac{1}{4}e^{4x} - \frac{9x^2}{2} + C$ help (formulas)
Answer(s) submitted:

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(incorrect)

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12. (1 point) Find the general solution of the differential equation $x\dfrac{dy}{dx} = 5y$.

(Use C to denote the arbitrary constant.)
$y =$ __________ help (formulas)
Answer(s) submitted:

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(incorrect)

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13. (1 point) Find the equation of the solution to $\dfrac{dy}{dx} = x^5 y$ through the point $(x, y) = (1, 4)$.

$\frac{dy}{y} = x^5 dx$

(Don't forget to add '$y =$' to your equation!) $\rightarrow \int \frac{dy}{y} = \int x^5 dx$

__________ help (equations)

$\rightarrow \ln(y) = (x^6/6) + C$

$\rightarrow \ln(4) = (1/6) + C$

Answer(s) submitted:

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(incorrect) $\rightarrow C = \ln(4) - (\frac{1}{6}) \therefore y = 5^{((x^7)/7) - (1/7))}$

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14. (1 point) Find the general solution of the differential equation $\dfrac{dy}{dx} = e^{2x-9y}$.

(Use C to denote the arbitrary constant.)
$y =$ __________ help (formulas)
Answer(s) submitted:

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(incorrect)