Expressesions (forms) of differential equations

We conclude the content of this module by discussing various ways in which a DE might be written. Manipulating how a DE is written is useful and important. However, it should quickly be noted that when doing so, care must be taken as you may alter the domain of the functions involved!

For starters, if we want to write an arbitrary DE it is cumbersome to write something like

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^n} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y + b(x) = 0$$

each time. Therefore, we instead sometimes use the notation

$$F(x, y, y' y'', \ldots, y^{(n)})$$

to denote a function in terms of $x, y, y', y'', \ldots, y^{(n)}$. In which case, we could then say something like, "consider the DE $F(x, y, y', y'', \ldots, y^{(n)}) = 0$" instead of the more complicated expression above. We immediately put this new notation to work (where we already modify it a bit).

Definition:

If a DE is written in terms of its highest derivative, i.e., as

$$y^{(n)} = F(x, y, y', \ldots, y^{(n-1)}),$$

we say it is in **normal form**.

On the other hand, we could place the function y and its derivatives on one side of the equation and everything else on the other. If we additionally require that the coefficient of the highest derivative is 1, we obtain another useful form.

Definition:

If a DE is written in the form

$$y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = b(x),$$

we say it is in **standard form**.

The final form we mention here is more peculiar. This is because it rests on the use of *differentials*, which you may not have much experience with. A Wikipedia page is linked to below with more concrete definitions, but for the purpose of this class we simply make the assumption that we are allowed to `separate' the dy and dx terms in the expression $\frac{dy}{dx}$ and manipulate them algebraically. That is, we have that $dy = \frac{dy}{dx}dx$ holds.

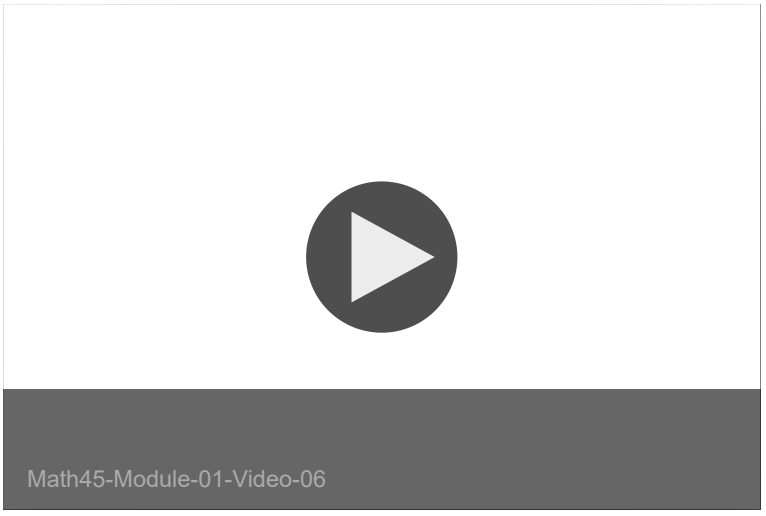
Definition:

If a DE is written as

$$F(x, y)dx + G(x, y)dy = 0$$

for 2-variable functions F and G we say it is in **differential form**.

Discussion, comments, and examples:



WeBWorK module 01 exercises:

- Problem 9

Relevant Wikipedia articles:

- **Differential of a function** (https://en.wikipedia.org/wiki/Differential_of_a_function)
  - See also the **differential as an infinitesimal** (https://en.wikipedia.org/wiki/Differential_(infinitesimal))