

Assignment Math45-Homework-WEEK-03 due 09/19/2020 at 11:59pm PDT

THEOREM	
$\frac{d}{dx} \sin(x) = \cos(x)$	$\frac{d}{dx} \cos(x) = -\sin(x)$
$\frac{d}{dx} \tan(x) = \sec^2(x)$	$\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$
$\frac{d}{dx} \cot(x) = -\csc^2(x)$	$\frac{d}{dx} \csc(x) = -\csc(x)\cot(x)$

Q1 Which of the following differential equations is $y = c_1 \cos(t) + c_2 \sin(t)$ a two-parameter family of solutions for?

A. $y'' + y = 0$

B. $y'' + y' + y = 0$

C. $y'' - y = 0$

D. $y' + y = 0$

$$y = c_1 \cos(t) + c_2 \sin(t)$$

$$y' = [c_1 \cos(t)]' + [c_2 \sin(t)]' \\ = -c_1 \sin(t) + c_2 \cos(t)$$

$$y'' = [c_1 \cos(t)]'' + [c_2 \sin(t)]'' \\ = [-c_1 \sin(t)]' + [c_2 \cos(t)]' \\ = -c_1 \cos(t) + (-c_2 \sin(t)) \\ = c_1 \cos(t) + c_2 \sin(t)$$

Answer(s) submitted:

(incorrect)

Q2 Note that $x = ce^{-t}$ is a one-parameter solution for the differential equation $x' + x = 0$. Which of the following is a solution to the first-order IVP consisting of the differential equation $x' + x = 0$ and the initial condition $x(0) = 7$. **Initial Value Problem**

A. $x = \frac{1}{7}e^{-t}$

B. $x = 7e^{-t+1}$

C. $x = 0$

D. $x = 7e^{-t}$

Answer(s) submitted:

(incorrect)

Q3 Note that $y = c_1 \cos(-t) + c_2 \sin(-t)$ is a two-parameter solution for the second-order differential equation $y'' + y = 0$. Which of the following is a solution to the second-order IVP consisting of the differential equation $y'' + y = 0$ and the initial condition $y(-\frac{\pi}{2}) = 2$, $y'(-\frac{\pi}{2}) = 3$.

$$2 = y(-\frac{\pi}{2}) = c_1 \cos(-\frac{\pi}{2}) + c_2 \sin(-\frac{\pi}{2})$$

$$2 = c_1 \cos(\frac{\pi}{2}) + c_2 \sin(\frac{\pi}{2})$$

$$2 = c_1(1) + c_2(0)$$

$$c_1 = 2$$

$$y = c_1 \cos(-t) + c_2 \sin(-t) \\ \rightarrow 2 \cos(-t) - 3 \sin(-t)$$

$$y' = [c_1 \cos(-t) + c_2 \sin(-t)]' \\ = c_1 [-\sin(-t) \cdot -1] + c_2 [\cos(-t) \cdot -1] \\ = c_2 \sin(-t) - c_1 \cos(-t)$$

$$3 = y'(-\frac{\pi}{2}) = c_2 \sin(-t) - c_1 \cos(-t)$$

$$3 = c_2 \sin(-\frac{\pi}{2}) - c_1 \cos(-\frac{\pi}{2})$$

$$3 = c_2 \sin(\frac{\pi}{2}) - c_1 \cos(\frac{\pi}{2})$$

$$3 = c_2(1) - c_1(1)$$

$$c_2 = -3$$

Q3 A. $y = 3 \cos(-t) - 2 \sin(-t)$

B. $y = 2 \cos(-t) + 3 \sin(-t)$

C. $y = 3 \cos(-t) + 2 \sin(-t)$

D. $y = \cos(-2) + 2 \sin(-3)$

What is the largest integral I over which the solution from the previous part is defined?

A. $(-\frac{\pi}{2}, \infty)$

B. $(-\infty, \infty)$

C. $(-\infty, -\frac{\pi}{2})$

Answer(s) submitted:

(incorrect)

Q4 Note that $y = \frac{1}{c+x}$ is a one-parameter solution for the differential equation $y' + y^2 = 0$. Which of the following is a solution to the first-order IVP consisting of the differential equation $y' + y^2 = 0$ and the initial condition $y(3) = \frac{1}{5}$.

A. $y = \frac{1}{5+x}$

B. $y = \frac{1}{2+x}$

C. $y = \frac{1}{3+x}$

$$\frac{1}{5} = y(3) = \frac{1}{c+3}$$

$$\frac{1}{5} = \frac{1}{c+3}$$

$$\rightarrow \frac{c+3}{5} = 1$$

$$c+3 = 5$$

$$c = 2$$

- D. $y = \frac{1}{c+x} + 5$

Q4

What is the largest integral I over which the solution from the previous part is defined?

- A. $(-\infty, -2)$

- B. $(-\infty, \infty)$

- C. $(-2, \infty)$

- D. $(3, \infty)$

Answer(s) submitted:

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(incorrect)

Q5

Note that $y = c_1 e^x + c_2 e^{-x}$ is a two-parameter solution for the second-order differential equation $y'' - y = 0$. Which of the following is a solution to the second-order IVP consisting of the differential equation $y'' - y = 0$ and the initial condition $y(1) = 1, y'(1) = 2$.

- A. $y = \frac{3}{2}e^{x-1} - \frac{1}{2}e^{1-x}$

- B. $y = \frac{e}{2}e^x + \frac{1}{2}e^{-x}$

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$$y = c_1 e^x + c_2 e^{-x}$$

$$1 = y(1) = c_1 e^{(1)} + c_2 e^{(-1)}$$

$$\rightarrow 1 = c_1 e^{(1)} + c_2 e^{(-1)}$$

$$\rightarrow 1 = c_1 (2.718) + c_2 (0.367)$$

$$y' = [c_1 e^x + c_2 e^{-x}]'$$

$$y' = c_1 e^x - c_2 e^{-x}$$

$$2 = y'(1) = c_1 e^x - c_2 e^{-x}$$

$$2 = c_1 e^{(1)} - c_2 e^{(-1)}$$

- C. $y = e^x + 2e^{-x}$

- D. $y = \frac{3}{2}e^x - \frac{1}{2}e^x$

Q5

What is the largest integral I over which the solution from the previous part is defined?

- A. $(0, \infty)$

- B. $(-\infty, \infty)$

- C. $(-\infty, 0)$

Answer(s) submitted:

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(incorrect)

6. (1 point) Note that $y = c_1 e^x + c_2 e^{-x}$ is a two-parameter solution for the second-order differential equation $y'' - y = 0$. Find values c_1 and c_2 so that y is a solution to the second-order IVP consisting of the differential equation $y'' - y = 0$ and the initial condition $y(0) = 3, y'(0) = 9$. The values are $c_1 = \underline{\hspace{1cm}}$ and $c_2 = \underline{\hspace{1cm}}$.

Answer(s) submitted: $y'' - y = 0$

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(incorrect)

Solution $y = c_1 e^x + c_2 e^{-x} \leftarrow \textcircled{1}$

Given $y(0) = 3, y'(0) = 9$

when $y(0) = 3$ from $\textcircled{1}$

$$3 = c_1 + c_2 \leftarrow \textcircled{2}$$

$$y'(x) = c_1 e^x - c_2 e^{-x}$$

$$y'(0) = 9$$

$$9 = c_1 - c_2 \leftarrow \textcircled{3}$$

Solve (2) & (3)

$$c_1 + c_2 = 3 \rightarrow 6 + c_2 = 3$$

$$c_1 - c_2 = 9 \quad c_2 = -3$$

$$2c_1 = 12$$

$$c_1 = 6 \quad \& \quad c_2 = -3$$