Matthew Mendoza
Assignment HW-06 due 03/06/2024 at 11:59pm PST

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1. (1 point) Let

$$A = \begin{bmatrix} 8 & -3 & -4 \\ -7 & 5 & 4 \end{bmatrix}.$$

Define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(\vec{x}) = A\vec{x}$. Find the images of $\vec{u} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ under T.

$T(\vec{u}) = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

$T(\vec{v}) = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

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2. (1 point) Consider a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^3$ for which

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 3 \\ -5 \end{bmatrix},\ T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix},\ T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}.$$

Find the matrix A of T.

$$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}.$$

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3. (1 point) Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 5x-9y \\ 3y-7x \\ x-2y \\ 6y-4x \end{bmatrix}.$$

Find its standard matrix A.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

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4. (1 point)
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points through the x-axis and then then reflects points through the line $y = -x$. Find the standard matrix A for T.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}.$$

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5. (1 point)
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotates points clockwise through 120° ($2\pi/3$ radians) and then reflects points through the line $y = x$. Find the standard matrix A for T.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}.$$

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6. (1 point) To every linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$, there is an associated 2×2 matrix. Match the following linear transformations with their associated matrix.

__1. Reflection about the line y=x
__2. Clockwise rotation by $\pi/2$ radians
__3. Reflection about the y-axis
__4. The projection onto the x-axis given by T(x,y)=(x,0)
__5. Counter-clockwise rotation by $\pi/2$ radians
__6. Reflection about the x-axis

A. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
B. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
C. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
D. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
E. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
F. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
G. None of the above

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7. (1 point) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -5x-15y \\ 5x+15y \end{bmatrix}.$$

Find a vector $\vec{w}$ that is **not** in the range of T.

$$\vec{w} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}.$$

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8. (1 point) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2x-2y \\ -4x-4y \\ 3x-4y \end{bmatrix}.$$

Find a vector $\vec{w}$ that is **not** in the range of T.

$$\vec{w} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

B. T is one-to-one
C. There is not enough information to tell.

9. (1 point) Let T be an linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$. Let A be the matrix associated to T.

Fill in the correct answer for each of the following situations.

___1. The row-echelon form of A has a column corresponding to a free variable.
___2. Every column in the row-echelon form of A is a pivot column.
___3. The row-echelon form of A has no column corresponding to a free variable.
___4. Two columns in the row-echelon form of A are not pivot columns.

A. T is not one-to-one

10. (1 point) Let T be an linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$. Let A be the matrix associated to T.

Fill in the correct answer for each of the following situations.

___1. Every row in the row-echelon form of A has a pivot.
___2. Two rows in the row-echelon form of A do not have pivots.
___3. The row-echelon form of A has a pivot in every column.
___4. The row-echelon form of A has a row of zeros.

A. T is onto
B. T is not onto
C. There is not enough information to tell.