20240507 P1

Last Time:

Theorem

$\hat{x}$ is a best possible approx solution to $A\bar{x} \approx \bar{b}$ $\left\{ \begin{array}{l} \hat{x} \text{ is a least squares solution} \\ \text{to } A\bar{x} = \bar{b} \\ [A \mid \bar{b}] \end{array} \right.$ $\Longleftrightarrow$ $\hat{x}$ is a (actual) solution to $\underbrace{A^T \cdot A}_{\text{new } A}\bar{x} = \underbrace{A^T \bar{b}}_{\text{new } \bar{b}}$

Review

20240509 P1

#11

$\text{proj}_W(\bar{v})$

$$\bar{v} = \begin{bmatrix} -19 \\ -14 \\ -14 \end{bmatrix}$$

$$W = \text{Span}\left(\underbrace{\begin{bmatrix} -2 \\ 6 \\ -2 \end{bmatrix}}_{W_1}, \underbrace{\begin{bmatrix} 6 \\ 6 \\ 12 \end{bmatrix}}_{W_2} \right)$$

$$\text{proj}_W(\bar{v}) = \text{proj}_{w_1}(\bar{v}) + \text{proj}_{w_2}(\bar{v})$$

$$= \frac{\bar{v} \cdot \bar{w}_1}{\bar{w}_1 \cdot \bar{w}_1} \bar{w}_1 + \frac{\bar{v} \cdot \bar{w}_2}{\bar{w}_2 \cdot \bar{w}_2} \bar{w}_2$$

$$= \frac{-18}{44} \begin{bmatrix} -2 \\ 6 \\ -2 \end{bmatrix} + \frac{-366}{216} \begin{bmatrix} 6 \\ 6 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{36}{44} \\ \frac{-108}{44} \\ \frac{36}{44} \end{bmatrix} + \begin{bmatrix} -2196/216 \\ -2196/216 \\ -4392/216 \end{bmatrix} = \Bigg[$$

$$= \begin{bmatrix} 36/44 - 2196/216 \\ -108/44 - 2196/216 \\ 36/44 - 4392/216 \end{bmatrix}$$

Sidework

$\bar{v} \cdot \bar{w}_1 = 19 \cdot 2 - 14 \cdot 6 + 14 \cdot 2 = -18$

$\bar{w}_1 \cdot \bar{w}_1 = (-2)^2 + (6)^2 + (2)^2 = 44$

$\bar{v} \cdot \bar{w}_2 = -19 \cdot 6 - 14 \cdot 6 - 14 \cdot 12 = -366$

$\bar{w}_2 \cdot \bar{w}_2 = 6^2 + 6^2 + (12)^2 = 216$

HW12, Q4

$$A = \begin{bmatrix} 49 & 24 \\ -112 & -55 \end{bmatrix}$$

$$P = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$$

corresponding

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 0 \\ 0 & -1 \end{bmatrix}$$

eigenvalues

$A = PDP^{-1}$

eigenvalues

$$P(\lambda) = \det(A - \lambda I)$$

$$= \det\left(\begin{bmatrix} 49-\lambda & 24 \\ -122 & -55-\lambda \end{bmatrix} \right)$$

$$= (49-\lambda)(-55-\lambda) - 24(-112)$$

$$= \lambda^2 - 6\lambda - 2695 + 2688$$

$$= \lambda^2 - 6\lambda - 7$$

$P(\lambda) = 0$

$\lambda^2 - 6\lambda - 7 = 0$

$(\lambda + 7)(\lambda - 1) = 0$

$\boxed{\lambda = -7, 1}$

eigen vectors

$\mathrm{Nul}(A-\lambda I)$

$\underline{\lambda=7}$

$\mathrm{Nul}(A+7I)$

$$A+7I = \begin{bmatrix} 56 & 24 \\ -112 & -48 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Solve:

$$\left[\begin{array}{cc|c} 42 & 24 & 0 \\ -112 & -62 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 56 & 24 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 7 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

*have to have free variables; otherwise, you know something is wrong

$$\left.\begin{array}{l} 7x_1+3x_2=0 \\ x_2 \text{ free} \end{array}\right\} \left.\begin{array}{l} x_1=-3/7x_2 \\ x_2 \text{ free} \end{array}\right\} \begin{array}{l} x_1=-3/7t \\ x_2=t \end{array}$$

$$\Rightarrow \bar{v}_1 = \begin{bmatrix} -3/7 \\ 1 \end{bmatrix} \Rightarrow \bar{x} = t\begin{bmatrix} -3/7 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} -3/7 & \bar{V}_2 \\ 1 & \end{bmatrix}$$

corresponding eigenvector

$$D = \begin{bmatrix} -7 & 0 \\ 0 & 1 \end{bmatrix}$$

eigenvalues $\lambda = -7, 1$

$\mathrm{Nul}(A-\lambda I)$

$\underline{\lambda=1}$

$\mathrm{Nul}(A-I)$

$$A-I = \begin{bmatrix} 48 & 24 \\ -112 & -56 \end{bmatrix}$$

Solve:

$$\left[\begin{array}{cc|c} 48 & 24 & 0 \\ -112 & -56 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 1 & 0 \\ -112 & -56 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} x_1 = -\tfrac{1}{2}x_2 \\ x_2 \text{ free} \end{array} \Rightarrow \ldots \Rightarrow \bar{x} = t\begin{bmatrix} -1/2 \\ 1 \end{bmatrix} \rightarrow \bar{V}_2 = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

Q1: Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \\ -1 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 5 & -2 \\ 0 & 1 \end{bmatrix}$. Compute AB.

* Multiply the rows of A by the columns of B

$$= \begin{bmatrix} 2\cdot1+1\cdot5+1\cdot0 & 2\cdot1+1\cdot(-2)+1\cdot1 \\ 3\cdot1+1\cdot5+2\cdot0 & 3\cdot1+1\cdot(-2)+2\cdot1 \\ (-1)\cdot1+0\cdot5+5\cdot0 & (-1)\cdot1+0\cdot(-2)+5\cdot1 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 8 & 3 \\ -1 & 4 \end{bmatrix}$$

Q3: Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -y \\ y-2x \\ 7x \end{bmatrix}$

(a) Compute the image of the vector $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \Rightarrow T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 5-2(2) \\ 7(2) \end{bmatrix}$$

* Compute the values

$$T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) = \left(\begin{bmatrix} -5 \\ 5-4 \\ 14 \end{bmatrix}\right)$$

$T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) = \left(\begin{bmatrix} -5 \\ 1 \\ 14 \end{bmatrix}\right)$ So, the image of vector $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ under T is $\begin{bmatrix} -5 \\ 1 \\ 14 \end{bmatrix}$

(b) Find the standard matrix of T

* Find $T(x_1)$ and $T(x_2)$:

$$T(x_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -0 \\ 0-2(1) \\ 7(1) \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 7 \end{bmatrix}$$

$$T(x_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1-2(0) \\ 7(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ Standard Matrix of T $\begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 7 & 0 \end{bmatrix}$