1.7 Linear Independence (cont'd)

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Final Remark

Often Linear Ind/Dep. comes up when talking about the columns of a matrix. Notice that

columns of A are Lin. Ind $\Leftrightarrow$ $x_1 col_1(A) + x_2 col(A) + \dots + x_n col_n(A) = \bar{0}$

has only one solution

$\Leftrightarrow$ $A\bar{x} = \bar{0}$ has only one solution

$[A \mid \bar{0}]$

$\Leftrightarrow$ A has a pivot position in every column

1.8 Introduction to Linear Transformations

- You're familiar with functions from $\mathbb{R}$ to $\mathbb{R}$, e.g. $f(x) = e^x$, $g(x) = \sin x$
- It's not hard to create functions say from $\mathbb{R}^2$ to $\mathbb{R}^2$ or $\mathbb{R}^2$ to $\mathbb{R}^3$...

$\mathbb{R}^2$ to $\mathbb{R}^2$

$$\underset{\mathbb{R}^2}{g}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \underset{\mathbb{R}^2}{\begin{bmatrix} e^x + \sin(y) \\ 3 x^2 \end{bmatrix}}$$

* Prerend

$g : \underset{\text{Domain}}{\mathbb{R}^2} \to \underset{\text{codomain}}{\mathbb{R}^2}$

$T : \underset{\text{Domain}}{\mathbb{R}^2} \to \underset{\text{codomain}}{\mathbb{R}^3}$

$$T\left(\underset{\mathbb{R}^2}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}\right) = \underset{\mathbb{R}^3}{\begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix}}$$

Recall: the domain of a function is the set of allowable inputs

The codomain of a function is a set that contains all outputs

Def: A function $T : \mathbb{R}^n \to \mathbb{R}^m$ (domain $\mathbb{R}^n$, codomain $\mathbb{R}^m$) will be called a transformation.

- If $\bar{x}$ is in $\mathbb{R}^n$, then the output $T(x)$ is called the image of $\bar{x}$ under T
- The collection of all outputs (i.e. all images) is called the range of T

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A picture