Section Z.8 (continued)

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· Subspaces of IR" · What is a subspace · How to check if something is in subspace · Example of subspaces: Span {VI, ..., VK}, NUL(A), R, col(A)

· Basis for a subspace. · About efficiently describing subspaces · Example: determining if something a basis for Rn

· Finding a basis for Nul(A)
· Finding a basis for col(A)
· Finding a basis for Span {r_1,...v_k}

Section 2.9 Dimension and Rank

Section 3. 1 Determinants

- Recall: If A = [ab] then we define def A = ad-bc and saw that A = exists () def A + 0 (Zero)

We want to generalize this to larger matries.

Notation: We'll write a b to mean delt ([ab] Definition: Let A b nxn

Majoris (n-1)x(n-1) the matrix obtained from A matrix by deleting the it row and it column

Number (i, j) - cofactor of A, denoted Cij, is

(i) = (-I) i+j det (Aij)

3 x 3 Determinants Def If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{bmatrix}$

Then we define

det A = a 11 C12 + a12 C12 + a13 C13

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Example

 $A = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 1 & 7 \\ -0 & 4 & 0 \end{bmatrix}$ A32 = [10]

C37 = (-1) 3+2 ede+ A37

= (-1) · [1.7-0-3]

$$I = a_{11}C_{11} + a_{12}C_{12} + a_{1}$$

NXN Determonates

Def If
$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & \alpha_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ A_{n1} & a_{n1} & \cdots & \alpha \end{bmatrix}$$

along 3nd row

$$def A = 3C_{13} + 0C_{23} + 6C_{33}$$

$$= 3(-1)^{4} | \frac{1}{6} \frac{1}{7} + 0 + 8(-1)^{6} | \frac{1}{4} \frac{2}{5} |$$

$$= 3(28-30) + 8(5-8)$$

$$= -6 - 24 = -60 - 30$$

Example Let
$$\begin{bmatrix} 0 & 4 & 6 & 5 \\ 2 & 2 & 3 & 0 \\ 6 & 6 & -3 & 0 \\ 2 & -1 & -2 & 0 \end{bmatrix}$$
 Compute det A along 4 column Pg 3

along 4 column

$$de + A = 5C_{14} + 0C_{24} + 0_{34} + 0C_{44}$$

$$= 5(-1)^{9} \cdot \frac{2}{0} \cdot \frac{3}{0}$$

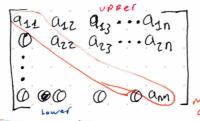
$$= -5 \left[0C_{21} + 0C_{22} - 3C_{23} \right]$$

$$= -5 \left[-3(-1)^{5} \cdot \frac{2}{2} \right]$$

$$= (-5)(-3)(-1)(-2-4)$$

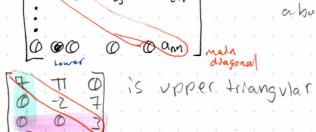
$$= (-15)(-6) = 90$$

Def A square matrix A is called "upper triangular" if all enteries below the main diagonal one (C (Zero)).



We similarly define lower friangular matrices to have all zeros above the main diagonal

For Example,



Theorem . If 'A' is upper as lower trangular, then det A = product of numb on main along

Last Time

- · Intro to determinate via cofactor expansion

$$\det \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ o & a_{2n} & \dots & a_{2n} \\ o & & \vdots & & \vdots \\ o & \dots & & \ddots & \vdots \\ o & \dots & & \ddots & \vdots \\ o & \dots & & \ddots & \vdots \\ o & \dots & & \ddots & \vdots \\ o & \dots & & \ddots & \vdots \\ o & \dots & & \ddots & \vdots \\ o & \dots & & \ddots & \vdots \\ o & \dots & & \dots & \vdots \\ o & \dots & & \dots & \vdots \\ o & \dots & & \dots & \dots \\ o & \dots &$$

Section 3.2 Properties of Determinants

We'll first study how rows operations affect determinants. Let's investigate 2x2 case

Interchange Rows

Consider Beab

def B = cb-da

= bc-ad

= -ad+bc

= - (ad-bc)

= -defA

Thus A-B then -detA=detB

Scaling a Row

Consider B = [ka kb]

detB=ka·d-kb·c

= K(ad-bc)

= R. det A

Thus

A Kry 1 B => Kodet A = det B

Replacement

Consider B=[a b | Katc kbtd]

detB= a(kb+d)-b(ka+c)

= akb + ad - bka - bc

= ad-bc

= detA

Thus,

A Krz+rz + rz B => det A = det B

Theorem & Let A be nxn

- DReplacement: If B is the result of performing a replacement row operation on A, then A being B = detA = detB
- 2) Interchange: If B is the result of interchanging 2 rows of A then,

 A THOSE = -det A = -det B
- 3 Scaling: If B is the vesult of Scaling one row at A by K, then

A K. JETA = detB (detA = = detB) Mirke K**