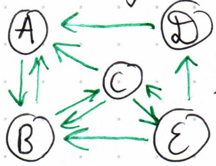
Ranking of web pages



$\bar{x}_0 \leftarrow$ prob. after zero clocks

$T\bar{x}_0 = \bar{x}_1 \leftarrow \cdots$ 1 clocks

$T^2\bar{x}_0 = \bar{x}_2$

$\vdots$

$$T = \begin{array}{c|c} \begin{matrix} A & B & C & D & E \end{matrix} & \\ \begin{bmatrix} 0 & 1/2 & 1/3 & 1 & 0 \\ 1 & 0 & 1/3 & 0 & 1/3 \\ 0 & 1/2 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 1/3 \\ 0 & 0 & 1/3 & 0 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{array}$$

* Prob. of moving from column to row page

- Eiganvalues of T: $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$

$$\begin{bmatrix} \lambda_1 = 1 \\ |\lambda_2| = |\lambda_3| \approx 0.7 \\ |\lambda_4| = |\lambda_5| \approx 0.3 \end{bmatrix}$$

- $T = PDP^{-1}$

$$D = \begin{bmatrix} 1 & & & & \\ & \lambda_2 & & 0 & \\ & & \lambda_3 & & \\ & 0 & & \lambda_4 & \\ & & & & \lambda_5 \end{bmatrix}$$

$$P = [\bar{v}_1 \ \bar{v}_2 \ \bar{v}_3 \ \bar{v}_4 \ \bar{v}_5]$$

$$T^\infty = \lim_{k\to\infty} T^k$$

$$= \lim_{k\to\infty} PD^kP^{-1}$$

$$= \lim_{k\to\infty} P \begin{bmatrix} 1^k & & & & 0 \\ & \cancel{\lambda_2^k} \to 0 & & & \\ & & \cancel{\lambda_3^k} \to 0 & & \\ & & & \cancel{\lambda_4^k} \to 0 & \\ 0 & & & & \cancel{\lambda_5^k} \to 0 \end{bmatrix} P^{-1}$$

$$= P \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & & \vdots \\ \vdots & & 0 & \\ 0 & \cdots & 0 & 0 \end{bmatrix} P^{-1}$$

Thus,

$$\bar{x}_\infty = \lim_{k\to\infty} \bar{x}_k$$

$$= \lim_{k\to\infty} T^k \bar{x}_0$$

$$= T^\infty \cdot \bar{x}_0$$

$$\begin{bmatrix} 0.293 & 0.293 & \cdots \\ 0.390 & 0.390 & \\ 0.220 & 0.220 & \\ 0.024 & 0.024 & \\ 0.073 & 0.073 & \end{bmatrix}$$

$$= \begin{bmatrix} 0.293 \\ 0.390 \\ 0.220 \\ 0.024 \\ 0.073 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix}$$

Ranking
#1 : B
#2 : A
#3 : C
#4 : E
#5 : D

Another view of what happened

The eigenvectors $\bar{v}_1, \ldots, \bar{v}_5$ form a basis for $\mathbb{R}^5$. This means we can write

$$\bar{x}_0 = c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3 + c_4\bar{v}_4 + c_5\bar{v}_5$$

for some choice of scalars $c_1, \ldots, c_5$

Then

$$T^k\bar{x}_0 = T^k(c_1\bar{v}_1 + \cdots + c_5\bar{v}_5)$$

$$= c_1T^k\bar{v}_1 + \cdots + c_5T^k\bar{v}_5$$

$$= c_1 1^k \cdot \bar{v}_1 + c_2\lambda_2^k\bar{v}_2 + \cdots + c_5\lambda_5^k\bar{v}_5$$

So: $\bar{x}_\infty = \lim_{k\to\infty} T^k \bar{x}_0$

$$= \lim_{k\to\infty} (c_1 1^k \bar{v}_1 + \cancel{c_2\lambda_2^k\bar{v}_2} + \cdots + \cancel{c_5\lambda_5^k\bar{v}_5})$$

$$= c_1\bar{v}_1$$

Thus the vector $\bar{x}_\infty$ that we want satisfies

AND
- $\bar{x}_\infty$ is in $E_1(T)$
- $\bar{x}_\infty$ is a probability vector

→ This makes $\bar{x}_\infty$ a steady-state vector for T

Now we'll see that we can find $\bar{x}_\infty$ without doing the full diagonalization process