

1. (1 point) Find the dot product of

$$\vec{x} = \begin{bmatrix} 1 \\ 7 \end{bmatrix} \text{ and } \vec{y} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}.$$

$$\vec{x} \cdot \vec{y} = \underline{\hspace{2cm}}.$$

2. (1 point) Find the dot product of

$$\vec{x} = \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix} \text{ and } \vec{y} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}.$$

$$\vec{x} \cdot \vec{y} = \underline{\hspace{2cm}}.$$

3. (1 point) Find the length of the vector $\vec{x} = \begin{bmatrix} -4 \\ -6 \\ -3 \end{bmatrix}$.

$$\|\vec{x}\| = \underline{\hspace{2cm}}.$$

4. (1 point) Find the length of \vec{x} and the unit vector \vec{u} in the direction of \vec{x} if

$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}.$$

$$\|\vec{x}\| = \underline{\hspace{2cm}},$$

$$\vec{u} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}$$

5. (1 point)

Suppose $\vec{u} = \langle -2, -6, -3 \rangle$. Mark each vector below with a “**T**” if it is orthogonal to \vec{u} , and an “**F**” if it is not orthogonal to \vec{u} :

- ___ 1. $\langle -2, 5, 3 \rangle$
- ___ 2. $\langle -3, -16, 34 \rangle$
- ___ 3. $\langle 1, 5, -5 \rangle$
- ___ 4. $\langle -3, 1, 0 \rangle$

6. (1 point)

Find the value of k for which the vectors

$$\begin{bmatrix} -4 \\ 0 \\ 4 \\ 4 \end{bmatrix} \text{ and } \begin{bmatrix} -1 \\ 4 \\ 4 \\ k \end{bmatrix}$$

are orthogonal.

$$k = \underline{\hspace{2cm}}.$$

7. (1 point) Find a non-zero vector \vec{v} orthogonal to the vector

$$\vec{u} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}.$$

$$\vec{v} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}$$

8. (1 point) Find the angle α between the vectors

$$\begin{bmatrix} -4 \\ 1 \\ 4 \end{bmatrix} \text{ and } \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}.$$

Please give your answer in radians.

$$\alpha = \underline{\hspace{2cm}}.$$

9. (1 point) Let L be the line in \mathbb{R}^3 that consists of all scalar multiples of the vector $\begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$. Find the orthogonal projection

of the vector $\vec{x} = \begin{bmatrix} 9 \\ 3 \\ 1 \end{bmatrix}$ onto L .

$$\text{proj}_L \vec{x} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}$$

10. (1 point) Compute the orthogonal projection of $\vec{v} =$

$\begin{bmatrix} -5 \\ 7 \\ 6 \end{bmatrix}$ onto the line L through $\begin{bmatrix} -7 \\ 6 \\ -4 \end{bmatrix}$ and the origin.

$$\text{proj}_L(\vec{v}) = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}.$$

11. (1 point) Find the orthogonal projection of

$$\vec{v} = \begin{bmatrix} -20 \\ -10 \\ 4 \end{bmatrix}$$

onto the subspace W of \mathbb{R}^3 spanned by

$$\begin{bmatrix} -7 \\ 4 \\ -2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ -4 \\ -15 \end{bmatrix}.$$

$$\text{proj}_W(\vec{v}) = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}$$

12. (1 point) Find the orthogonal projection of

$$\vec{v} = \begin{bmatrix} -20 \\ -16 \\ 8 \\ 3 \end{bmatrix}$$

onto the subspace W of \mathbb{R}^4 spanned by

$$\begin{bmatrix} -2 \\ 4 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 4 \\ 5 \\ -6 \\ 0 \end{bmatrix}.$$

$$\text{proj}_W(\vec{v}) = \begin{bmatrix} \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt} \end{bmatrix}$$

13. (1 point) Find the least-squares solution $\hat{\mathbf{x}}$ of the system

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ 3 & 3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 14 \\ -2 \\ 6 \end{bmatrix}.$$

$$\hat{\mathbf{x}} = \begin{bmatrix} \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt} \end{bmatrix}$$

14. (1 point) Find the least-squares solution $\hat{\mathbf{x}}$ of the system

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -10 \\ -4 \\ 6 \\ 0 \end{bmatrix}.$$

$$\hat{\mathbf{x}} = \begin{bmatrix} \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt} \end{bmatrix}$$

15. (1 point) Fit a quadratic function of the form $f(t) = c_0 + c_1t + c_2t^2$ to the data points $(0, -8)$, $(1, -11)$, $(2, -8)$, $(3, -19)$, using least squares.

$$f(t) = \rule{2cm}{0.4pt}$$