Section 2.8 (continued) 2024/03/26

- Subspaces of $\mathbb{R}^n$
  - What is a subspace
  - How to check if something is in subspace
  - Example of subspaces: Span$\{\bar{v}_1, \dots, \bar{v}_k\}$, NUL(A), $\mathbb{R}^n$, col(A)

- Basis for a subspace
  - About efficiently describing subspaces
  - Example: determining if something a basis for $\mathbb{R}^n$
  - Finding a basis for Nul(A)
  - Finding a basis for col(A)
  - Finding a basis for Span$\{\bar{v}_1, \dots \bar{v}_k\}$

Section 2.9 Dimension and Rank 2024/03/26

Section 3.1 Determinants 2024/03/28

Recall: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then we define $\det A = ad - bc$

and saw that $A^{-1}$ exists $\Leftrightarrow$ $\det A \neq 0$ (zero)

We want to generalize this to larger matries.

Notation: we'll write $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ to mean $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$

Definition: Let A b $n \times n$

① $A_{ij}$ is $(n-1)\times(n-1)$ the matrix obtained from A by deleting the $i^{th}$ row and $j^{th}$ column

Matrix

② The $(i,j)$-cofactor of A, denoted $C_{ij}$, is

Number

$$C_{ij} = (-1)^{i+j} \cdot \det(A_{ij})$$

Example

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 1 & 7 \\ 0 & 4 & 0 \end{bmatrix}$$

$$A_{32} = \begin{bmatrix} 1 & 0 \\ 3 & 7 \end{bmatrix}$$

$$C_{32} = (-1)^{3+2} \cdot \det A_{32}$$
$$= (-1) \cdot [1 \cdot 7 - 0 \cdot 3]$$
$$= \boxed{-7}$$

3 x 3 Determinants

Def If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{bmatrix}$

Then we define

$$\det A = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$\det\left(\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}\right) \quad = a_{11}\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{31} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{bmatrix} a_{21} & a_{22} \\ a_{31} + a_{32} \end{bmatrix}$$