1. (1 point) Let

$$A = \begin{bmatrix} 8 & -3 & -4 \\ -7 & 5 & 4 \end{bmatrix}.$$

Define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(\vec{x}) = A\vec{x}$.

Find the images of $\vec{u} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ under T.

$T(\vec{u}) = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

$T(\vec{v}) = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

Answer(s) submitted:

- -26
- 8a-3b-4c

(correct)
Correct Answers:

- $\begin{bmatrix} -26 \\ 29 \end{bmatrix}$
- $\begin{bmatrix} 8*a-3*b-4*c \\ 5*b-7*a+4*c \end{bmatrix}$

2. (1 point) Consider a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^3$ for which

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 3 \\ -5 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}.$$

Find the matrix A of T.

$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$.

Answer(s) submitted:

- -5

(correct)
Correct Answers:

- $\begin{bmatrix} -5 & 5 & -3 \\ 3 & -2 & 1 \\ -5 & 2 & 4 \end{bmatrix}$

3. (1 point) Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 5x-9y \\ 3y-7x \\ x-2y \\ 6y-4x \end{bmatrix}.$$

Find its standard matrix A.

$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Answer(s) submitted:

- 5

(correct)
Correct Answers:

- $\begin{bmatrix} 5 & -9 \\ -7 & 3 \\ 1 & -2 \\ -4 & 6 \end{bmatrix}$

4. (1 point)
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points through the x-axis and then then reflects points through the line $y = -x$. Find the standard matrix A for T.

$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.

Answer(s) submitted:

- 0
- -1
- 1
- 0

(correct)
Correct Answers:

- $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

5. (1 point)
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotates points clockwise through $120°$ ($2\pi/3$ radians) and then reflects points through the line $y = x$. Find the standard matrix A for T.

$A = \begin{bmatrix} \_\_\_\_\_ & \_\_ \\ \_\_\_\_ & \_\_ \end{bmatrix}$.

Answer(s) submitted:

- -((sqrt(3))/2)

(correct)
Correct Answers:

- $\begin{bmatrix} -0.866025 & -0.5 \\ -0.5 & 0.866025 \end{bmatrix}$

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6. (1 point) To every linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$, there is an associated 2×2 matrix. Match the following linear transformations with their associated matrix.

___1. Reflection about the line y=x
___2. Clockwise rotation by $\pi/2$ radians
___3. Reflection about the y-axis
___4. The projection onto the x-axis given by T(x,y)=(x,0)
___5. Counter-clockwise rotation by $\pi/2$ radians
___6. Reflection about the x-axis

A. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
B. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
C. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
D. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
E. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
F. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
G. None of the above

Answer(s) submitted:

- f
- e
- c
- d
- b
- a

(correct)
Correct Answers:

- F
- E
- C
- D
- B
- A

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7. (1 point) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -5x - 15y \\ 5x + 15y \end{bmatrix}.$$

Find a vector $\vec{w}$ that is **not** in the range of T.

$\vec{w} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$.

Answer(s) submitted:

- 5

(correct)
Correct Answers:

- $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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8. (1 point) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2x - 2y \\ -4x - 4y \\ 3x - 4y \end{bmatrix}.$$

Find a vector $\vec{w}$ that is **not** in the range of T.

$\vec{w} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Answer(s) submitted:

- 1

(correct)
Correct Answers:

- $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

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9. (1 point) Let T be an linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$. Let A be the matrix associated to T.

Fill in the correct answer for each of the following situations.

___1. The row-echelon form of A has a column corresponding to a free variable.
___2. Every column in the row-echelon form of A is a pivot column.
___3. The row-echelon form of A has no column corresponding to a free variable.
___4. Two columns in the row-echelon form of A are not pivot columns.

A. T is not one-to-one
B. T is one-to-one
C. There is not enough information to tell.

Answer(s) submitted:

- a
- b
- b
- a

(correct)
Correct Answers:

- A
- B
- B
- A

10. (1 point) Let T be an linear transformation from $\mathbb{R}^r$ to $\mathbb{R}^s$. Let A be the matrix associated to T.

Fill in the correct answer for each of the following situations.

___1. Every row in the row-echelon form of A has a pivot.
___2. Two rows in the row-echelon form of A do not have pivots.
___3. The row-echelon form of A has a pivot in every column.
___4. The row-echelon form of A has a row of zeros.

A. T is onto
B. T is not onto
C. There is not enough information to tell.

Answer(s) submitted:

- a
- b
- c
- b

(correct)
Correct Answers:

- A
- B
- C
- B