Last Time

- + Time ants Intro to determinate via cofactor expansion

$$\det \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ o & a_{2n} & \dots & a_{2n} \\ o & & \vdots & \vdots \\ o & \dots & o & a_{nn} \end{pmatrix} = \begin{pmatrix} \rho & \text{roduct of } \\ en & \text{tres} \\ = a \end{pmatrix}$$

Section 3.2 Properties of Determinants

We'll first study how rows operations affect determinants. Let's investigate 2x2 case

Interchange Rows

Thus And then -det A = det B

Scaling a Row

Replacement

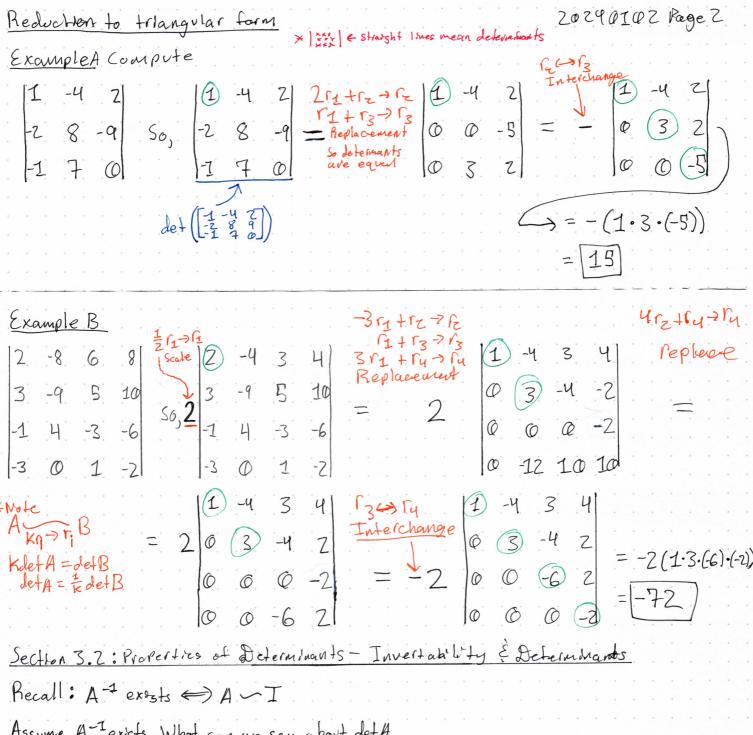
Thus,

Amrz+rz-12B=) det A=det B

Theorem & Let A be nxn

- Deplacement: If B is the result of performing a replacement row operation on A, then A being B = detA = detB
- 2) Interchange: If B is the result of interchanging 2 rows of A then,

 A THIS B = det A = det B
- (3) Scaling: If B is the vesult of Scaling one row at A by K, then



Assume A Texists. What can we say about det A.

 A^{-1} exists $\Rightarrow A \sim A_1 \sim A_2 \sim \cdots \sim A_K = I$

=) detA = C1detA1 = C1C2 detA2 = ··· = C1C2··· CxdetI

for some C1, Cz, ..., CK

Note: det I = 1 and none of c1, C2, ... Ck are equal to zero

Recap: A-1 exists => detA = C162...Cx where C1, ..., Cx are not zero

Similar idea shows detA + 0 = A = exists

*Theorem A is invertable = detA + 0.

Other Properties of the determinant

Theorem

Transpose:

· det(A) = det A

det ([a b]) = det ([a c])

X · det(A·B) = (det A)(det B) · for k = 1,2,3,..., det(A=) = (det A) K Note: Typically det(A+B) = det A + det B f det A 7 0 then

det (A = 1) = I

det A = (det A)

Example Suppose you know detA=7, det B= 1/2

(a) det(BA) = detB·detA (b) det(B3) = Compute = 1/2.7 (a) det(BA)

(b) det(B3)

(c) det (A-1) (C) Since detA + 0, A-1 exists

A · A -1 = I

detA·A·I) = detI

(det A). (det A-1)=1

 $det A^{-1} = \frac{1}{det A} = \frac{1}{4}$