Exam 1 Question REDO

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Exam 01 – February 29th, 2024

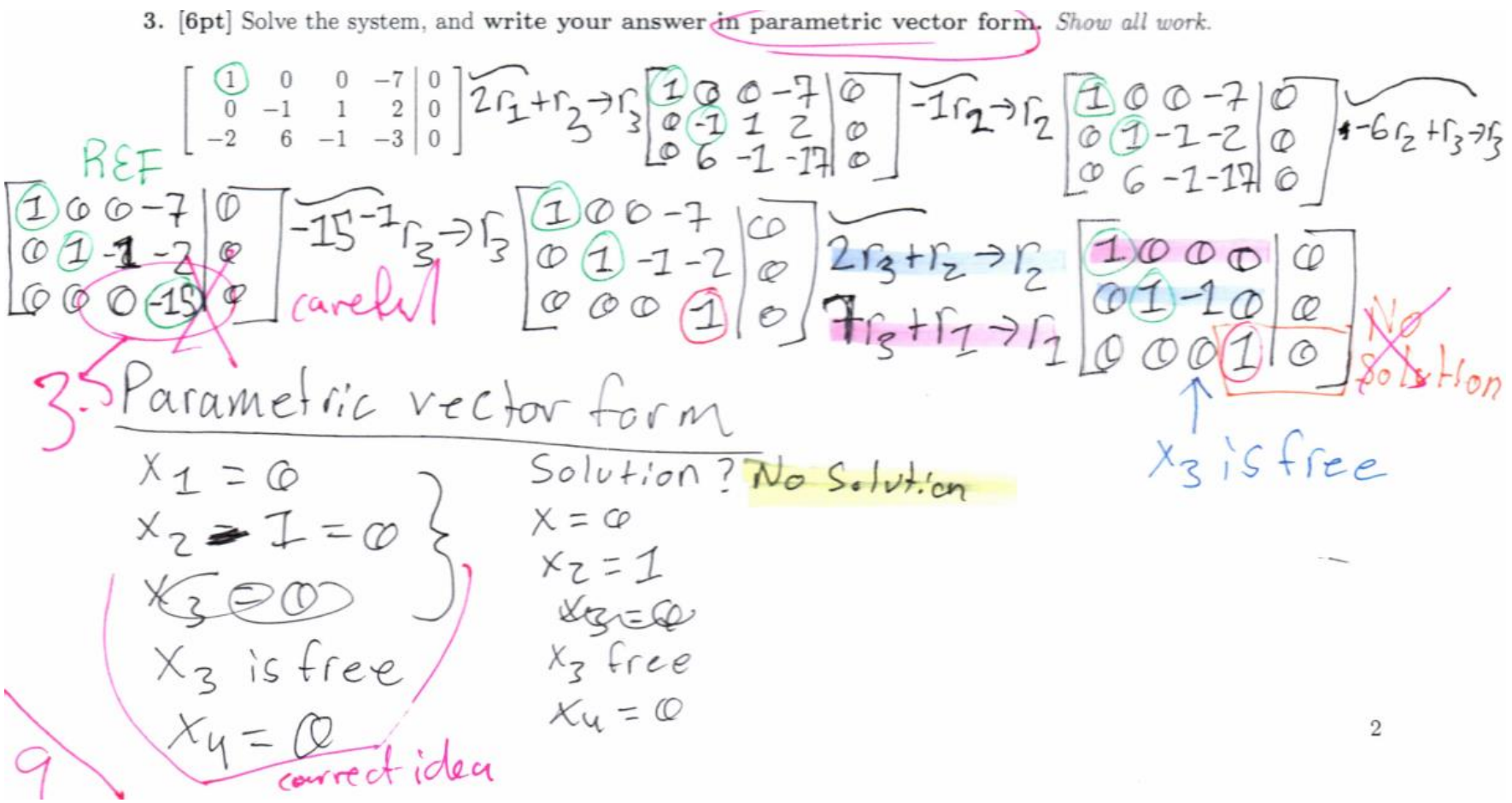
Question 3

3. [6pt] Solve the system, and write your answer in parametric vector form.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array}\right]$$

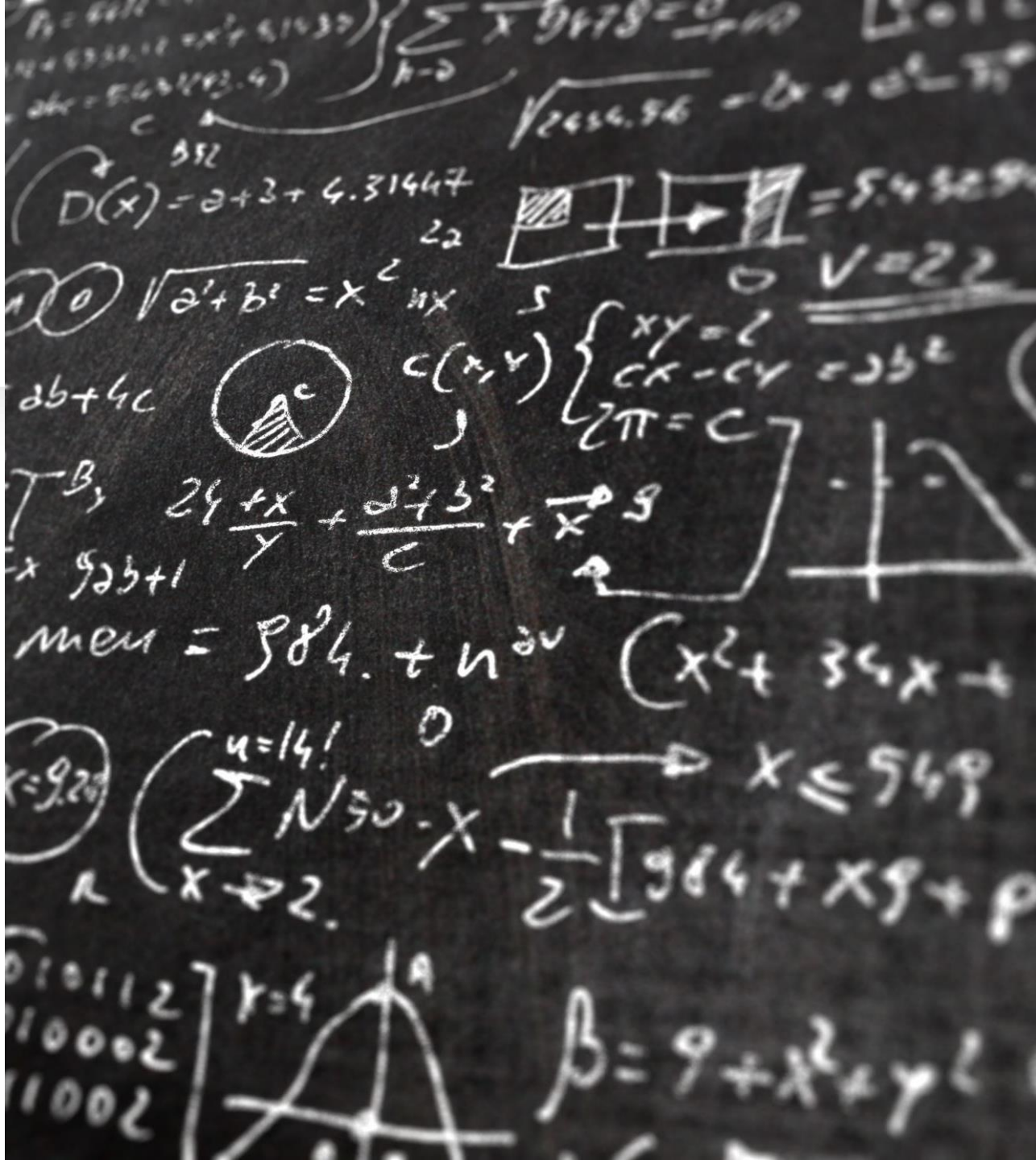
How did it go?

Didn't go so well...

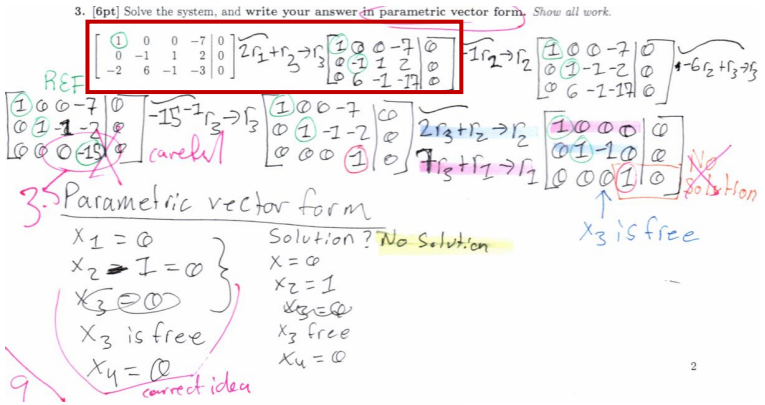


My error?

Basic algebra



Step 1



Original

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array}\right] \overset{2r_1+r_3\to r_3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array}\right]$$

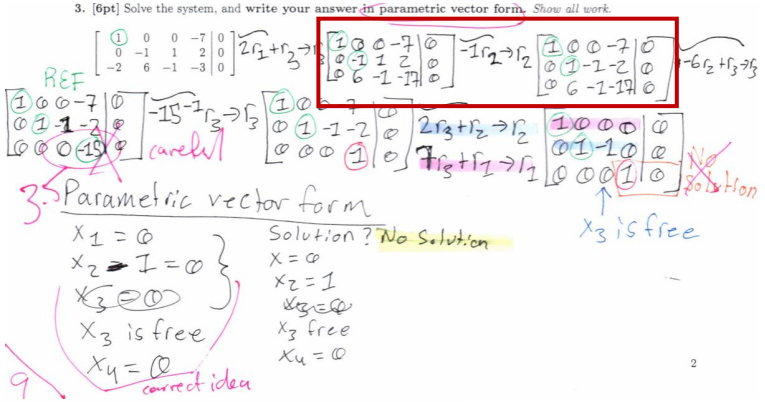
Redo

Replacement: $r_i + Cr_j \to r_i$

Replace a row with itself plus any multiple of another row

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array}\right] \overset{2r_1+r_3\to r_3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array}\right]$$

Step 2



Original

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array}\right] \sim -1r_2 \to r_2 \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array}\right]$$

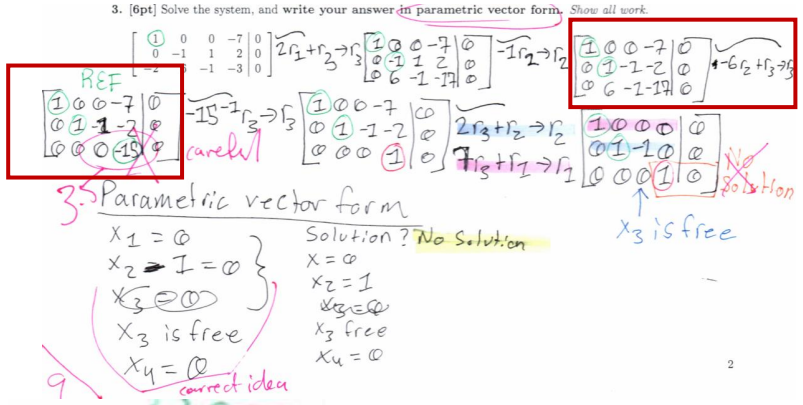
Redo

Scaling: $cr_i \to r_i$

Multiply any row by a non-zero number

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array}\right] \sim -1r_2 \to r_2 \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array}\right]$$

Step 3



Original – Algebra mistake

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 6 & -1 & 0 \end{array}\right] \quad -6r_2 + r_3 \to r_3$$

REF

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -15 & 0 \end{array}\right]$$

Redo – r_{33}: $-6(-1) + (-1) = 5$

Replacement: $r_i + Cr_j \to r_i$

Replace a row with itself plus any multiple of another row

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 6 & -1 & 0 \end{array}\right] \quad -6r_2 + r_3 \to r_3$$

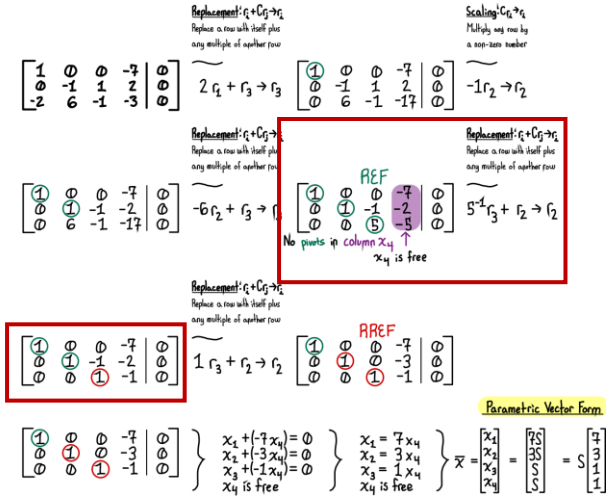
REF

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 5 & 0 \end{array}\right]$$

No pivots in column x_4

x_4 is free

Step 4 - REDO



Replacement: $r_i + Cr_j \to r_i$

Replace a row with itself plus any multiple of another row

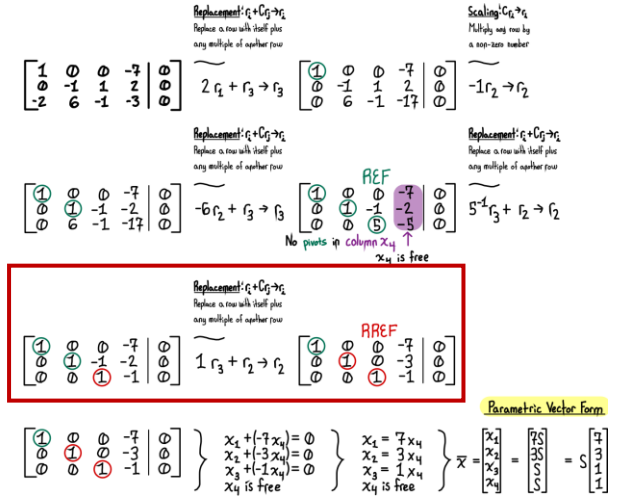
REF

$$\left[\begin{array}{ccc|c} \textcircled{1} & 0 & 0 & -7 & 0 \\ 0 & \textcircled{1} & -1 & -2 & 0 \\ 0 & 0 & \textcircled{5} & -5 & 0 \end{array}\right] \sim 5^{-1}r_3 + r_2 \to r_2 \left[\begin{array}{ccc|c} \textcircled{1} & 0 & 0 & -7 & 0 \\ 0 & \textcircled{1} & -1 & -2 & 0 \\ 0 & 0 & \textcircled{1} & -1 & 0 \end{array}\right]$$

No pivots in column x_4 ↑

x_4 is free

Step 4 - REDO



Replacement: $r_i + Cr_j \to r_i$

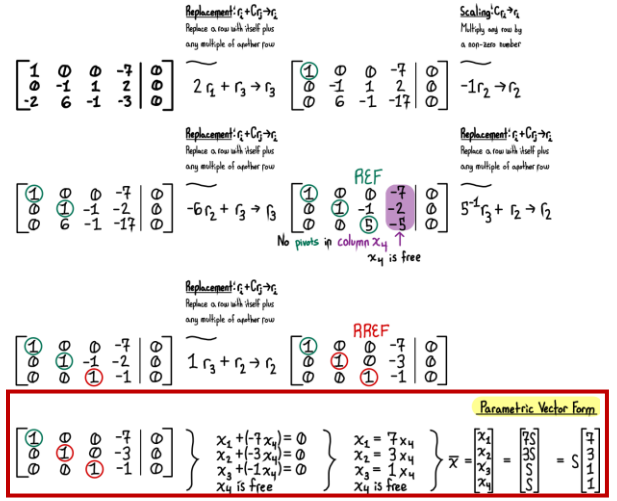
Replace a row with itself plus

any multiple of another row

$$\begin{bmatrix} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \sim 1r_3 + r_2 \to r_2 \begin{bmatrix} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

RREF

Step 5 - REDO



Parametric Vector Form

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 + (-7x_4) = 0 \\ x_2 + (-3x_4) = 0 \\ x_3 + (-1x_4) = 0 \\ x_4 \text{ is free} \end{array} \Rightarrow \begin{array}{l} x_1 = 7x_4 \\ x_2 = 3x_4 \\ x_3 = 1x_4 \\ x_4 \text{ is free} \end{array} \Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7S \\ 3S \\ S \\ S \end{bmatrix} = S\begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

ALL TOGETHER