Exam 2—Math 100 Name Matthew Mendoza

Spring 2024 Instructor: J. Wiscons

23.5 75

Points. The exam is out of **35 points**.

Time. This is an in-class 75-minute exam.

Rules for the exam. *Please read these carefully!* Violation of the rules will be reported to the Sacramento State Office of Student Conduct.

1. You are allowed:
   - to use a calculator for *basic arithmetical* computations;
   - to use one 3×5 notecard of notes.
2. You are not allowed:
   - to use any resources on this exam except those listed above;
   - to look at another person's exam or their work;
   - to let another person see your exam or your work.
3. **Please justify all of your work and show all steps unless indicated otherwise.**
4. Let me know if you have any questions at all!

A $\boxed{\equiv}$ x B $\boxed{||}$

Sum (A (~~column~~ row) x B(~~row~~ col)) r_{ij}

A B

3x3 3x2

match

a_{11} a_{12} a_{13}

a_{21} a_{22} a_{23}

a_{31} a_{32} a_{33}

1. [3pt] Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \\ -1 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 5 & -2 \\ 0 & 1 \end{bmatrix}$. Compute AB.

a_{11}: $2(1) + 3(1) + (-1)0$, $1(1)+1(1)+(-1)(1)$)

a_{12}

a_{13}

a_{21}

a_{22}

a_{23}

a_{31}

a_{32}

a_{33}

0.5

2. [3pt] Let $A = \begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 3 \\ -1 & 0 & -2 \end{bmatrix}$. Find A^{-1} using the row reduction process developed in class.

oops Need identity

$$\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 3 \\ -1 & 0 & -2 \end{bmatrix} \xrightarrow[1r_1+r_3\to r_3]{2r_1+r_2\to r_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 9 \\ 0 & \cancel{1} & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ -2 & 1 & 3 & 0 & 1 & 0 \\ -1 & 0 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[1r_1+r_3\to r_3]{2r_1+r_2\to r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 9 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow[-3r_3+r_1\to r_1]{-9r_3+r_2\to r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \overset{2}{\cancel{-8}} & 0 & -3 \\ 0 & 1 & 0 & -7 & 1 & -9 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} \overset{-2}{\cancel{-8}} & 0 & -3 \\ -7 & 1 & -9 \\ 1 & 0 & 1 \end{bmatrix}$$

2.5

3. [4pt] Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -y \\ y-2x \\ 7x \end{bmatrix}$.

(a) Compute the image of the vector $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

(b) Find the standard matrix of T.

-1

1

3

4. [4pt] Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$. Find A^{-1}. Then use A^{-1} to solve $A\mathbf{x} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$.

$\det(A) \Rightarrow [(1\cdot 8)-(2\cdot 3)] \rightarrow \det(A)=2,\ 2\neq 0$; so, is invertable

$A^{-1} \Rightarrow \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow \frac{1}{2}\cancel{[(8\cdot 1)-(-2\cdot -3)]} \Rightarrow \frac{1}{2}(2) = 1$

1.5

5. [3pt] Determine if $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 3 \end{bmatrix}$ is a basis for $\mathbb{R}^3$ or not. *Please justify all work and explain.*

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 3 & 8 & 0 \\ -1 & -5 & 3 & 0 \end{array}\right] \xrightarrow[1r_1+r_3\rightarrow r_3]{-2r_1+r_2\rightarrow r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & -3 & 6 & 0 \end{array}\right] \xrightarrow{-1r_2\rightarrow r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -3 & 6 & 0 \end{array}\right] \xrightarrow{3r_2+r_3\rightarrow r_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Check
- Linear independence (trivial solution)
- Spans set

x_3 is free

∴ Linearly dependent (Not trivial solution)

Set $\left\{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 3 \end{bmatrix}\right\}$ is NOT a basis for $\mathbb{R}^3$

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6. [5pt] Let $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 1 & 7 & 5 \\ 0 & 1 & -3 & 1 \\ 2 & 3 & 1 & 7 \end{bmatrix}$. You can use that $A \sim \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for Nul A, and determine the dimension of Nul A. *Please justify your answers.*

Nul Space

?

~~Basis Null(A) is~~ $\cancel{\left\{\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}\right\}}$ ~~dimension~~

Null Space

$x_1 = -5x_3 - 2x_4$

$x_2 = 3x_3 - x_4$

$x_1 + 5x_3 + 2x_4 = 0$

$x_2 + (-3x_3) + x_4 = 0$

x_3 is free

x_4 is free

Let x_3 be s.
Let x_4 be t

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s\begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow$

Null space(A)
$\left\{\begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}\right\}$

(b) Find a basis for Col A, and determine the dimension of Col A. *Please justify your answers.*

?

Basis for Col(A)

$\left\{\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}\right\}$

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8.5

7. [5pt] Let $A = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & -3 \\ 2 & 6 & -3 \end{bmatrix}$.

(a) Calculate $\det(A)$. *You may use cofactor expansion or reduction to triangular form.*

$$\begin{bmatrix} +1 & -3 & +2 \\ -1 & +0 & -3 \\ 2 & -6 & -3 \end{bmatrix} \to -\begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & -3 \\ 2 & 6 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & -3 \\ 2 & 6 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & -3 \\ 2 & 6 & -3 \end{bmatrix}$$

$$-3\begin{bmatrix} -1 & -3 \\ 2 & -3 \end{bmatrix} + 0 + \left(-6\begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix}\right)$$

$$-3[(-1\cdot -3)-(-3\cdot 2)] \qquad -6[(1\cdot -3)-(2\cdot -1)]$$

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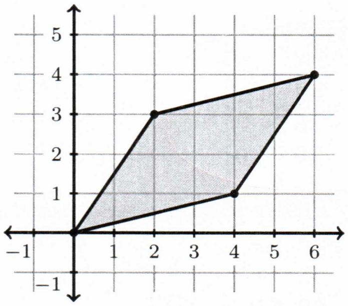
$$-3(9) - 6(-1) \Rightarrow -27+6 = -21.$$

(b) Use your previous answer to determine if A is invertible or not. *Why?*

Matrix A is invertable, for $\det(A) \neq 0$.

8. [3pt] Use a determinant to compute the area of the parallelogram below.



$A=(0,0)$; $B=(2,3)$; $C=(6,4)$; $D=(4,1)$

Area of Parallelogram $|AB \times AD|$

$AB = (0,0)-(2,3) = (-2,-3)$

$AD = (0,0)-(4,1) = (-4,-1)$

3

So, $AB \times AD \begin{vmatrix} i & j & k \\ -2 & -3 & 0 \\ -4 & -1 & 0 \end{vmatrix} \Rightarrow i(0-0) - j(0-0) + k[(-2\cdot -1)-(-3\cdot -4)]$

$\Rightarrow k(2-12)$ OK

$\Rightarrow -10k$

$\Rightarrow$ Area $= 10u$

9. [1pt each] True or False: **Circle one.** You do not need to justify your answer.

— It's not the entries it's the 3x3 determinate if $\neq 0$ that allows it to be invertable

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(a) True or False: If a 3×3 matrix has no entries equal to 0, then the matrix is invertible.

b) True Final answer-matt

(b) True or ~~False~~: If v_1, v_2, v_3, v_4 are linearly independent vectors in $\mathbb{R}^4$, then they form a basis.

↳ AND isn't a linear combination of the other vectors

(c) True or False: If A is a 5×7 matrix and the reduced-row echelon form (RREF) of A has exactly two rows consisting of all zeros, then $\dim(\text{Nul}\, A) = 4$.

(d) True or False: If A and B are 5×5 matrices such that $\det(A)$ and $\det(B)$ are both odd integers, then $\det(AB)$ is also an odd integer. $\det(AB)$; $\det(A)=3$, $\det(B)=7$; $\det(AB)=(3\times 7)\ldots$

(e) True or False: If A is a 5×5 matrix such that $\det(A)$ is positive, then $\det(A^{-1})$ is negative.

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