Matthew Mendoza **Math-100-J-Wiscons-Sp24**
Assignment HW-09 due 04/03/2024 at 11:59pm PDT

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1. (1 point) Let

$$B = \begin{bmatrix} 9 & 11 & -14 \\ 7 & -1 & 5 \\ -5 & -9 & 10 \\ -3 & 4 & -5 \end{bmatrix}.$$

(a) Find the reduced row echelon form of the matrix B. To help with the row reduction, you can use a tool like the **Linear Algebra Toolkit**.

$$\text{rref}(B) = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

(b) How many pivot columns does B have?

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(c) Do the vectors in the set $\left\{ \begin{bmatrix} 9 \\ 7 \\ -5 \\ -3 \end{bmatrix}, \begin{bmatrix} 11 \\ -1 \\ -9 \\ 4 \end{bmatrix}, \begin{bmatrix} -14 \\ 5 \\ 10 \\ -5 \end{bmatrix} \right\}$ span $\mathbb{R}^4$? Be sure you can explain and justify your answer.

- choose
- `the vectors span R^4`
- `the vectors do not span R^4`

(d) Are the vectors in the set $\left\{ \begin{bmatrix} 9 \\ 7 \\ -5 \\ -3 \end{bmatrix}, \begin{bmatrix} 11 \\ -1 \\ -9 \\ 4 \end{bmatrix}, \begin{bmatrix} -14 \\ 5 \\ 10 \\ -5 \end{bmatrix} \right\}$ linearly independent? Be sure you can explain and justify your answer.

- choose
- linearly dependent
- linearly independent

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2. (1 point) Let

$$B = \begin{bmatrix} 11 & 7 & 3 & -1 \\ -26 & -17 & -8 & 1 \end{bmatrix}.$$

(a) Find the reduced row echelon form of the matrix B. To help with the row reduction, you can use a tool like the **Linear Algebra Toolkit**.

$$\text{rref}(B) = \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

(b) How many pivot columns does B have?

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(c) Do the vectors in the set $\left\{ \begin{bmatrix} 11 \\ -26 \end{bmatrix}, \begin{bmatrix} 7 \\ -17 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ span $\mathbb{R}^2$? Be sure you can explain and justify your answer.

- choose
- `the vectors span R^2`
- `the vectors do not span R^2`

(d) Are the vectors in the set $\left\{ \begin{bmatrix} 11 \\ -26 \end{bmatrix}, \begin{bmatrix} 7 \\ -17 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ linearly independent? Be sure you can explain and justify your answer.

- choose
- linearly dependent
- linearly independent

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3. (1 point) Let

$$B = \begin{bmatrix} 6 & -3 & -4 \\ 3 & -1 & -2 \\ -3 & 0 & 2 \end{bmatrix}.$$

(a) Find the reduced row echelon form of the matrix B. To help with the row reduction, you can use a tool like the **Linear Algebra Toolkit**.

$$\text{rref}(B) = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

(b) How many pivot columns does B have?

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(c) Do the vectors in the set $\left\{ \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix} \right\}$ span $\mathbb{R}^3$? Be sure you can explain and justify your answer.

- choose
- `the vectors span R^3`
- `the vectors do not span R^3`

(d) Are the vectors in the set $\left\{ \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix} \right\}$ linearly independent? Be sure you can explain and justify your answer.

- choose
- linearly dependent
- linearly independent

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4. (1 point)

Let W_1 be the set: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is not a basis because it does not span $\mathbb{R}^3$.
- B. W_1 is not a basis because it is linearly dependent.
- C. W_1 is a basis.

Let W_2 be the set: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is not a basis because it is linearly dependent.
- B. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- C. W_2 is a basis.

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5. (1 point)

Let W_1 be the set: $\left\{ \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix} \right\}$

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is not a basis because it is linearly dependent.
- B. W_1 is a basis.
- C. W_1 is not a basis because it does not span $\mathbb{R}^3$.

Let W_2 be the set: $\left\{ \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} \right\}$

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- B. W_2 is a basis.
- C. W_2 is not a basis because it is linearly dependent.

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6. (1 point) The vectors

$$\vec{v}_1 = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 3 \\ -2 \\ k \end{bmatrix}$$

form a basis for $\mathbb{R}^3$ if and only if $k \neq$ ____.

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7. (1 point) Let

$$A = \begin{bmatrix} 2 & -6 & -4 & -6 \\ 3 & -9 & -6 & -9 \end{bmatrix}.$$

Find a basis for the null space of A.

$$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$$

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8. (1 point) Find a basis for the column space of

$$A = \begin{bmatrix} 4 & 0 & -1 & -3 \\ 2 & 1 & -1 & 4 \\ 6 & -1 & -1 & -10 \end{bmatrix}.$$

Basis = $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

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9. (1 point) Find bases for the column space and the null space of matrix A.

$$A = \begin{bmatrix} 1 & 4 & -1 & 1 \\ 3 & 15 & -1 & 6 \\ 4 & 22 & 0 & 10 \end{bmatrix}$$

Basis for the column space of A = $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$

Basis for the null space of A = $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$

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10. (1 point) Let

$$A = \begin{bmatrix} -4 & 2 & -4 & -2 & 4 \\ -1 & 1 & -1 & -4 & 4 \\ -4 & -1 & -5 & -5 & 2 \end{bmatrix}.$$

Give a non-zero vector $\vec{x}$ in the null space of A.

$$\vec{x} = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}.$$

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11. (1 point) Let

$$A = \begin{bmatrix} -12 & -8 & 16 & -12 \\ 6 & -4 & -8 & -6 \\ 0 & 12 & 0 & 18 \end{bmatrix}.$$

Find a non-zero vector in the column space of A.

$$\begin{bmatrix} \_\_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

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12. (1 point) Determine the dimensions of the subspaces below by first finding a basis for each.

A. The dimension of span $\left\{ \begin{bmatrix} 2 \\ -6 \end{bmatrix}, \begin{bmatrix} -5 \\ 15 \end{bmatrix} \right\}$ is ___.

B. The dimension of span $\left\{ \begin{bmatrix} 5 \\ -9 \end{bmatrix}, \begin{bmatrix} -16 \\ 27 \end{bmatrix} \right\}$ is ___.

C. The dimension of span $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$ is ___.

D. The dimension of span $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ 5 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ 8 \end{bmatrix} \right\}$ is ___.

E. The dimension of span $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix} \right\}$ is ___.