

1. (1 point) Determine how many pivots each of the following matrices have.

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

$$(1) \begin{bmatrix} 1 & 0 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

$$(2) \begin{bmatrix} 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

$$(3) \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

$$(4) \begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -7 \end{bmatrix}$$

Correct Answers:

- Four Pivots
- Two Pivots
- Two Pivots
- Three Pivots

2. (1 point) How many free variables does each augmented matrix have?

$$(1) \text{ [Choose/None/One/Two/Three]} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & -10 \\ 0 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$(2) \text{ [Choose/None/One/Two/Three]} \left[\begin{array}{cc|c} 1 & 8 & 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$(3) \text{ [Choose/None/One/Two/Three]} \left[\begin{array}{ccc|c} 1 & -5 & 6 & 6 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$(4) \text{ [Choose/None/One/Two/Three]} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{array} \right]$$

Correct Answers:

- Two
- One
- Two
- None

3. (1 point) Solve the matrix equation $Ax = b$, where

$$A = \begin{bmatrix} -2 & 2 \\ -3 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -6 \\ -9 \end{bmatrix}.$$

The solution is:

$$x_1 = \underline{\hspace{2cm}}$$

$$x_2 = \underline{\hspace{2cm}}$$

Correct Answers:

- 3
- 0

4. (1 point)

The reduced row echelon form of a system of linear equations in x and y or in x , y and z is given. For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

1.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

- A. Unique solution: $x = 0, y = -1$
- B. Unique solution: $x = 0, y = -1, z = 3$
- C. Infinitely many solutions
- D. No solutions

- E. Unique solution: $x = -1, y = 3$
- F. None of the above

2.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & -4 \end{array} \right]$$

- A. Unique solution: $x = -3, y = 4, z = -4$
- B. No solutions
- C. Unique solution: $x = -3, y = 4, z = 0$
- D. Unique solution: $x = -3, y = 4$
- E. Infinitely many solutions
- F. None of the above

3.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

- A. Unique solution: $x = 0, y = 0, z = 0$
- B. Unique solution: $x = 1, y = 1, z = 0$
- C. Unique solution: $x = 0, y = 0$
- D. Infinitely many solutions
- E. No solutions
- F. None of the above

4.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

- A. No solutions
- B. Infinitely many solutions
- C. Unique solution: $x = 0, y = 0, z = 0$
- D. Unique solution: $x = 3, y = -2$
- E. Unique solution: $x = -2, y = 3$
- F. None of the above

Correct Answers:

- B
- B
- C
- B

5. (1 point) For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

$$(1) \begin{cases} 3x - 4y = 19 \\ -9x + 9y = -54 \end{cases}$$

- A. Unique solution: $x = -1, y = 5$
- B. Infinitely many solutions
- C. Unique solution: $x = 0, y = 0$
- D. Unique solution: $x = 5, y = -1$
- E. No solutions
- F. None of the above

$$(2) \begin{cases} -8x + 4y = 0 \\ 9x + 4y = 0 \end{cases}$$

- A. Unique solution: $x = -4, y = 13$

- B. Unique solution: $x = -4, y = -8$
- C. No solutions
- D. Infinitely many solutions
- E. Unique solution: $x = 0, y = 0$
- F. None of the above

$$(3) \begin{cases} -2x + 2y = 8 \\ 6x - 6y = -24 \end{cases}$$

- A. Unique solution: $x = -4, y = 0$
- B. Unique solution: $x = 8, y = -24$
- C. Infinitely many solutions
- D. No solutions
- E. Unique solution: $x = 0, y = 0$
- F. None of the above

$$(4) \begin{cases} 5x + 4y = 6 \\ -5x - 4y = -5 \end{cases}$$

- A. No solutions
- B. Infinitely many solutions
- C. Unique solution: $x = -5, y = 6$
- D. Unique solution: $x = 6, y = -5$
- E. Unique solution: $x = 0, y = 0$
- F. None of the above

Correct Answers:

- D
- E
- C
- A

6. (1 point)

Solve the system using matrices (row operations)

$$\begin{cases} 2x - 2y - z = -21 \\ x + 4y - 2z = 10 \\ 2x - 5y + 6z = -15 \end{cases}$$

How many solutions are there to this system?

- A. None
- B. Exactly 1
- C. Exactly 2
- D. Exactly 3
- E. Infinitely many
- F. None of the above

If there is one solution, give its coordinates in the answer spaces below.

If there are infinitely many solutions, enter z in the answer blank for z , enter a formula for y in terms of z in the answer blank for y and enter a formula for x in terms of z in the answer blank for x .

If there are no solutions, leave the answer blanks for x, y and z empty.

$$x = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$

$$z = \underline{\hspace{2cm}}$$

Correct Answers:

- B
- -4
- 5
- 3

7. (1 point)

The following system has an infinite number of solutions. Write the solution in terms of the free variables y and z .

$$\begin{array}{rrrrrrcl} 2w & - & x & + & 2y & - & z & = & 1 \\ w & + & x & - & y & + & 4z & = & 3 \\ 3w & & & + & 1y & + & 3z & = & 4 \\ 3w & - & 3x & + & 5y & - & 6z & = & -1 \end{array}$$

$$w = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

y is free

z is free

Correct Answers:

- $(1+3+(1-2) * y + (1-4) * z) / 3$
- $(-1+2*3+(2+2) * y - (1+2*4) * z) / 3$

8. (1 point) Solve the system

$$\begin{cases} x_1 + x_2 = 3 \\ x_2 + x_3 = 2 \\ x_3 + x_4 = -5 \\ x_1 + x_4 = -4 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix} + s \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}.$$

Correct Answers:

- $\begin{bmatrix} -4 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

9. (1 point) Solve the system

$$\begin{cases} 4x_1 - 5x_2 + 4x_3 + 3x_4 = 3 \\ -x_1 + x_2 + 2x_3 + 3x_4 = 3 \\ 3x_1 - 4x_2 + 6x_3 + 6x_4 = 6 \\ -2x_1 + 2x_2 + 4x_3 + 6x_4 = 6 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix} + s \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix} + t \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}.$$

Correct Answers:

- $\begin{bmatrix} -18 \\ -15 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 14 \\ 12 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 18 \\ 15 \\ 0 \\ 1 \end{bmatrix}$

10. (1 point) Find the set of solutions for the linear system

$$\begin{array}{rrrrcl} -3x_1 & - & 6x_2 & + & 3x_3 & = & 13 \\ & & 4x_2 & + & 7x_3 & = & -9 \end{array}$$

Use s_1, s_2 , etc. for the free variables if necessary.

$$(x_1, x_2, x_3) = \left(\underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right)$$

Correct Answers:

- $1/6 + 9/2 * s_1$
- $-(9/4) - 7/4 * s_1$
- s_1