

1. (1 point) Determine how many pivots each of the following matrices have.

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

$$(1) \begin{bmatrix} 1 & 0 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

$$(2) \begin{bmatrix} 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

$$(3) \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

$$(4) \begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -7 \end{bmatrix}$$

2. (1 point) How many free variables does each augmented matrix have?

$$(1) \text{ [Choose/None/One/Two/Three]} \begin{bmatrix} 1 & 0 & 0 & 5 & -10 \\ 0 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(2) \text{ [Choose/None/One/Two/Three]} \begin{bmatrix} 1 & 8 & 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(3) \text{ [Choose/None/One/Two/Three]} \begin{bmatrix} 1 & -5 & 6 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(4) \text{ [Choose/None/One/Two/Three]} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

3. (1 point) Solve the matrix equation $Ax = b$, where

$$A = \begin{bmatrix} -2 & 2 \\ -3 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -6 \\ -9 \end{bmatrix}.$$

The solution is:

$$x_1 = \underline{\hspace{2cm}}$$

$$x_2 = \underline{\hspace{2cm}}$$

4. (1 point)

The reduced row echelon form of a system of linear equations in x and y or in x , y and z is given. For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

1.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

- A. Unique solution: $x = 0, y = -1$
- B. Unique solution: $x = 0, y = -1, z = 3$
- C. Infinitely many solutions
- D. No solutions
- E. Unique solution: $x = -1, y = 3$
- F. None of the above

2.

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

- A. Unique solution: $x = -3, y = 4, z = -4$
- B. No solutions
- C. Unique solution: $x = -3, y = 4, z = 0$
- D. Unique solution: $x = -3, y = 4$
- E. Infinitely many solutions
- F. None of the above

3.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

- A. Unique solution: $x = 0, y = 0, z = 0$
- B. Unique solution: $x = 1, y = 1, z = 0$
- C. Unique solution: $x = 0, y = 0$
- D. Infinitely many solutions
- E. No solutions
- F. None of the above

4.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

- A. No solutions
- B. Infinitely many solutions
- C. Unique solution: $x = 0, y = 0, z = 0$
- D. Unique solution: $x = 3, y = -2$
- E. Unique solution: $x = -2, y = 3$
- F. None of the above

5. (1 point) For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

$$(1) \begin{cases} 3x - 4y = 19 \\ -9x + 9y = -54 \end{cases}$$

- A. Unique solution: $x = -1, y = 5$
- B. Infinitely many solutions
- C. Unique solution: $x = 0, y = 0$
- D. Unique solution: $x = 5, y = -1$
- E. No solutions
- F. None of the above

$$(2) \begin{cases} -8x + 4y = 0 \\ 9x + 4y = 0 \end{cases}$$

- A. Unique solution: $x = -4, y = 13$
- B. Unique solution: $x = -4, y = -8$
- C. No solutions
- D. Infinitely many solutions
- E. Unique solution: $x = 0, y = 0$
- F. None of the above

$$(3) \begin{cases} -2x + 2y = 8 \\ 6x - 6y = -24 \end{cases}$$

- A. Unique solution: $x = -4, y = 0$
- B. Unique solution: $x = 8, y = -24$
- C. Infinitely many solutions
- D. No solutions
- E. Unique solution: $x = 0, y = 0$
- F. None of the above

$$(4) \begin{cases} 5x + 4y = 6 \\ -5x - 4y = -5 \end{cases}$$

- A. No solutions
- B. Infinitely many solutions
- C. Unique solution: $x = -5, y = 6$
- D. Unique solution: $x = 6, y = -5$
- E. Unique solution: $x = 0, y = 0$
- F. None of the above

6. (1 point)

Solve the system using matrices (row operations)

$$\begin{cases} 2x - 2y - z = -21 \\ x + 4y - 2z = 10 \\ 2x - 5y + 6z = -15 \end{cases}$$

How many solutions are there to this system?

- A. None
- B. Exactly 1
- C. Exactly 2
- D. Exactly 3
- E. Infinitely many
- F. None of the above

If there is one solution, give its coordinates in the answer spaces below.

If there are infinitely many solutions, enter z in the answer blank for z , enter a formula for y in terms of z in the answer blank for y and enter a formula for x in terms of z in the answer blank for x .

If there are no solutions, leave the answer blanks for x, y and z empty.

$x =$ _____

$y =$ _____

$z =$ _____

7. (1 point)

The following system has an infinite number of solutions. Write the solution in terms of the free variables y and z .

$$\begin{array}{rrcrrcrrc} 2w & - & x & + & 2y & - & z & = & 1 \\ w & + & x & - & y & + & 4z & = & 3 \\ 3w & & & + & 1y & + & 3z & = & 4 \\ 3w & - & 3x & + & 5y & - & 6z & = & -1 \end{array}$$

$w =$ _____

$x =$ _____

y is free

z is free

8. (1 point) Solve the system

$$\begin{cases} x_1 + x_2 &= 3 \\ x_2 + x_3 &= 2 \\ x_3 + x_4 &= -5 \\ x_1 &+ x_4 = -4 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + s \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}.$$

9. (1 point) Solve the system

$$\begin{cases} 4x_1 - 5x_2 + 4x_3 + 3x_4 = 3 \\ -x_1 + x_2 + 2x_3 + 3x_4 = 3 \\ 3x_1 - 4x_2 + 6x_3 + 6x_4 = 6 \\ -2x_1 + 2x_2 + 4x_3 + 6x_4 = 6 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + s \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + t \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}.$$

10. (1 point) Find the set of solutions for the linear system

$$\begin{array}{rrcr} -3x_1 & - & 6x_2 & + & 3x_3 & = & 13 \\ & & 4x_2 & + & 7x_3 & = & -9 \end{array}$$

Use **s1**, **s2**, etc. for the free variables if necessary.

$$(x_1, x_2, x_3) = \left(\text{---}, \text{---}, \text{---} \right)$$

Solution: SOLUTION: Note that x_3 is a free variable so let $x_3 = s_1$. The second equation then gives $4x_2 + 7s_1 = -9 \Rightarrow x_2 = -\frac{9}{4} - \frac{7}{4}s_1$. Substitute this into the first equation, $-3x_1 - 6(-\frac{9}{4} - \frac{7}{4}s_1) + 3s_1 = 13 \Rightarrow x_1 = \frac{1}{6} + \frac{9}{2}s_1$.