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1. (2 points) One of the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

is a real number. Find this eigenvalue and a basis of the eigenspace.

The eigenvalue is ____.

A basis for the eigenspace is $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

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2. (2 points) The matrix

$$A = \begin{bmatrix} 6 & -3 & -9 \\ -3 & 6 & 9 \\ 3 & -3 & -6 \end{bmatrix}$$

has two real eigenvalues, one whose eigenspace has dimension 1 and one whose eigenspace has dimension 2. Find the eigenvalues and a basis for each eigenspace.

The eigenvalue whose eigenspace has dimension 1 is $\lambda_1 =$____ and a basis for its associated eigenspace is

$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

The eigenvalue whose eigenspace has dimension 2 is $\lambda_2 =$____ and a basis for its associated eigenspace is

$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

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3. (2 points) Let

$$A = \begin{bmatrix} 1 & 0.25 \\ -65 & -8 \end{bmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

$$P = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

$$D = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

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4. (2 points) Let $A = \begin{bmatrix} -2 & -4 \\ 0 & -6 \end{bmatrix}$

Find a matrix P, a diagonal matrix D and P^{-1} such that $A = PDP^{-1}$.

$P = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$, $D = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$, $P^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

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5. (2 points) Let $A = \begin{bmatrix} 2 & 0 & 0 \\ -12 & 2 & 6 \\ 9 & -3 & -7 \end{bmatrix}$.

Find P and D such that $A = PDP^{-1}$. $P = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$,

$D = \begin{bmatrix} \_\_ & 0 & 0 \\ 0 & \_\_ & 0 \\ 0 & 0 & \_\_ \end{bmatrix}$.