


Exam 1

Question REDO

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Question 3

3. [6pt] Solve the system, and write your answer in parametric vector form.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array} \right]$$

How did it go?



Didn't go so well...

3. [6pt] Solve the system, and write your answer in parametric vector form. Show all work.

REF

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array} \right] \xrightarrow{2r_1 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-1r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-6r_2 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 5 & -5 & 0 \end{array} \right] \xrightarrow{-15^{-1}r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} 2r_3 + r_2 \rightarrow r_2 \\ 7r_3 + r_1 \rightarrow r_1 \end{array}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

careful!

No solution

x₃ is free

3. Parametric vector form

$$\left. \begin{array}{l} x_1 = 0 \\ x_2 = 1 \\ x_3 = 0 \\ x_4 = 0 \end{array} \right\}$$

x₃ is free

correct idea

Solution? *No solution*

$x = 0$

$x_2 = 1$

$x_3 = 0$

x_3 free

$x_4 = 0$

My error?

Basic algebra



Step 1

Original

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array} \right] \xrightarrow{2r_1 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

Redo

Replacement: $r_i + Cr_j \rightarrow r_i$

Replace a row with itself plus any multiple of another row

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array} \right] \xrightarrow{2r_1 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

3. [6pt] Solve the system, and write your answer in parametric vector form. Show all work.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array} \right] \xrightarrow{2r_1 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-1r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-6r_2 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 5 & -5 & 0 \end{array} \right] \xrightarrow{1/5 r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{r_3 + r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{r_2 + r_3 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

REF

3. Parametric vector form

$x_1 = 0$
 $x_2 = 0$
 $x_3 = 0$
 x_3 is free
 $x_4 = 0$

Solution? No Solution

x_3 is free

correct idea

Step 2

Original

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-1r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

Redo

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-1r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

Scaling: $C r_i \rightarrow r_i$
Multiply any row by
a non-zero number

3. [6pt] Solve the system, and write your answer in parametric vector form. Show all work.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{2r_1 + r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-1r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-6r_2 + r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 5 & -5 & 0 \end{array} \right] \xrightarrow{-1/5 r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{r_3 + r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{7r_3 + r_1 \rightarrow r_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

REF

3. Parametric vector form

$x_1 = 0$
 $x_2 = 0$
 $x_3 = 0$
 x_3 is free
 $x_4 = 0$

Solution? No Solution

x_3 is free

No Solution

correct idea

Step 3

Original – Algebra mistake

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-6r_2 + r_3 \rightarrow r_3}$$

Redo – r_{33} : $-6(-1) + (-1) = 5$

Replacement: $r_i + Cr_j \rightarrow r_i$

Replace a row with itself plus any multiple of another row

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

$$\xrightarrow{-6r_2 + r_3 \rightarrow r_3}$$

REF

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 5 & -5 & 0 \end{array} \right]$$

No pivots in column x_4 \uparrow
 x_4 is free

3. [6pt] Solve the system, and write your answer in parametric vector form. Show all work.

REF

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 5 & -5 & 0 \end{array} \right] \xrightarrow{-6r_2 + r_3 \rightarrow r_3}$$

Parametric vector form

$x_1 = 0$
 $x_2 = 1 = 0$
 $x_3 = 0$
 x_4 is free
 $x_4 = 0$

Solution? No Solution

x_3 is free

9. correct idea

Step 4 - REDO

Replacement: $r_i + C_j \rightarrow r_i$
Replace a row with itself plus any multiple of another row

Scaling: $C_i \rightarrow r_i$
Multiply row r_i by a non-zero number

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array} \right] \xrightarrow{2r_1 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-1r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

Replacement: $r_i + C_j \rightarrow r_i$
Replace a row with itself plus any multiple of another row

REF

No pivots in column x_4
 x_4 is free

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-6r_2 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & -7 & -29 & 0 \end{array} \right] \xrightarrow{5^{-1}r_3 + r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & -7 & -29 & 0 \end{array} \right]$$

Replacement: $r_i + C_j \rightarrow r_i$
Replace a row with itself plus any multiple of another row

REF

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{1r_3 + r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 1 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

Parametric Vector Form

$$\left\{ \begin{array}{l} x_1 + (-7)x_4 = 0 \\ x_2 + (-3)x_4 = 0 \\ x_3 + (-1)x_4 = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} x_1 = 7x_4 \\ x_2 = 3x_4 \\ x_3 = 1x_4 \end{array} \right\} \Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7s \\ 3s \\ s \\ s \end{bmatrix} = s \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

REF

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 5 & -5 & 0 \end{array} \right]$$

No pivots in column x_4
 x_4 is free

Replacement: $r_i + C_j \rightarrow r_i$
Replace a row with itself plus any multiple of another row

$$\sim 5^{-1}r_3 + r_2 \rightarrow r_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

Step 4 - REDO

Replacement: $r_i + C_j \rightarrow r_i$
Replace a row with itself plus any multiple of another row

Scaling: $C_i \rightarrow r_i$
Multiply row one by a non-zero number

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array} \right] \xrightarrow{2r_1 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right] \xrightarrow{-1r_2 \rightarrow r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -9 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

Replacement: $r_i + C_j \rightarrow r_i$
Replace a row with itself plus any multiple of another row

REF

No pivots in column x_4
 x_4 is free

Replacement: $r_i + C_j \rightarrow r_i$
Replace a row with itself plus any multiple of another row

RRREF

Parametric Vector Form:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \left\{ \begin{array}{l} x_1 + (-7)x_4 = 0 \\ x_2 + (-3)x_4 = 0 \\ x_3 + (-1)x_4 = 0 \end{array} \right\} \left\{ \begin{array}{l} x_1 = 7x_4 \\ x_2 = 3x_4 \\ x_3 = 1x_4 \end{array} \right\} \left\{ \begin{array}{l} x_4 \text{ is free} \end{array} \right\} \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7s \\ 3s \\ s \\ s \end{bmatrix} = s \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

Replacement: $r_i + C_j \rightarrow r_i$

Replace a row with itself plus any multiple of another row

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

\sim

$$1r_3 + r_2 \rightarrow r_2$$

RRREF

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

Step 5 - REDO

[illegible]

Parametric Vector Form

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \left\{ \begin{array}{l} x_1 + (-7x_4) = 0 \\ x_2 + (-3x_4) = 0 \\ x_3 + (-1x_4) = 0 \\ x_4 \text{ is free} \end{array} \right\} \left\{ \begin{array}{l} x_1 = 7x_4 \\ x_2 = 3x_4 \\ x_3 = 1x_4 \\ x_4 \text{ is free} \end{array} \right\} \bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7s \\ 3s \\ s \\ s \end{bmatrix} = s \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

ALL TOGETHER

Replacement: $r_i + Cr_j \rightarrow r_i$
Replace a row with itself plus
any multiple of another row

Scaling: $Cr_i \rightarrow r_i$
Multiply any row by
a non-zero number

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ -2 & 6 & -1 & -3 & 0 \end{array} \right]$$

$$2r_1 + r_3 \rightarrow r_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

$$-1r_2 \rightarrow r_2$$

Replacement: $r_i + Cr_j \rightarrow r_i$
Replace a row with itself plus
any multiple of another row

Replacement: $r_i + Cr_j \rightarrow r_i$
Replace a row with itself plus
any multiple of another row

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 6 & -1 & -17 & 0 \end{array} \right]$$

$$-6r_2 + r_3 \rightarrow r_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 5 & -5 & 0 \end{array} \right] \quad \text{REF}$$

$$5^{-1}r_3 + r_2 \rightarrow r_2$$

No pivots in column x_4
 x_4 is free

Replacement: $r_i + Cr_j \rightarrow r_i$
Replace a row with itself plus
any multiple of another row

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

$$1r_3 + r_2 \rightarrow r_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \quad \text{RREF}$$

Parametric Vector Form

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

$$\left. \begin{array}{l} x_1 + (-7x_4) = 0 \\ x_2 + (-3x_4) = 0 \\ x_3 + (-1x_4) = 0 \\ x_4 \text{ is free} \end{array} \right\}$$

$$\left. \begin{array}{l} x_1 = 7x_4 \\ x_2 = 3x_4 \\ x_3 = 1x_4 \\ x_4 \text{ is free} \end{array} \right\}$$

$$\left. \begin{array}{l} x_1 = 7x_4 \\ x_2 = 3x_4 \\ x_3 = 1x_4 \\ x_4 \text{ is free} \end{array} \right\} \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7s \\ 3s \\ s \\ s \end{bmatrix} = s \begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$