Augmented

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & \boxed{-1} & \boxed{0} \end{array}\right] \quad \left\{\begin{array}{l} x_1+(-7x_4) \\ x_2+(-3x_4) \\ x_3+(-1x_4) \\ x_4 \text{ is free} \end{array}\right\} \quad \left\{\begin{array}{l} x_1 = 7x_4 \\ x_2 = 3x_4 \\ x_3 = 1x_4 \\ x_4 \text{ is free} \end{array}\right\}$$

Let x_4 be s

Parametric Vector Form

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7s \\ 3s \\ s \\ s \end{bmatrix} = s\begin{bmatrix} 7 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

Matrix Transpose

No Solution $-1 = 0$ is a contradiction ∴ the system is INCONSISTENT

∞ solution $0 = 0$ (nints) - The matrix has 1 or more solutions ∴ the system is Consistant

When NOT the trivial solution (No free var) (all zeros) - Linearly Dependent (No Free var); else, independent

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} x_1 = -5x_3 - 2x_4 \\ x_2 = 3x_3 - x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{array} \quad \left.\begin{array}{l} x_1 + 5x_3 + 2x_4 = 0 \\ x_2 + (-3x_3) + x_4 = 0 \end{array}\right\}$$

Let x_3 be s, Let x_4 be t

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s\begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Null space = Param $\left\{\begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}\right\}$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 1 & 7 & 5 \\ 0 & 1 & -3 & 1 \\ 2 & 3 & 1 & 7 \end{bmatrix}$$ ← Basis for column space - Matrix A

① Get to RREF ② The pivots reference original matrix are your Basis

Basis for Col(A) $\left\{\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}\right\}$

Is Basis Check

1. Linear Independence Trivial solution
2. Spanning set - span domain, No Combo

Determinant 2x2

$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} \to ad - bc$

Finding A^{-1} of 2x2

if $\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} \neq 0$; $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \frac{1}{\det}$

$\det(A) = 4$, $\det(B) = -9$, Matrix 3x3

- $\det(AB) = (4 \cdot -9)$
- $\det(-2A) = (-2)^3 \det A \to (-2)^3 \cdot 4$
- $\det(A^T) = \det(A)$
- $\det(B^{-1}) = \frac{1}{\det B} \to -\frac{1}{9}$
- $\det(B^2) = -9 \cdot -9 = 81$

Determinant Properties

Linear Transform $R^3 \to R^2$

$A\begin{bmatrix} 8 & -3 & -4 \\ -7 & 5 & 4 \end{bmatrix}, \vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \to \begin{bmatrix} 8a+(-3b)+(-4c) \\ -7a+5b+4c \end{bmatrix}$

Matrix A Matrix B

$3 \times \underbrace{3 \quad\quad 3}_{\text{Match}} \times 2$

- Pivot in all row "onto"
- Pivot in every col 1:1

multiply each element in row A by column B

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \\ -1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 5 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (2\cdot1)+(1\cdot5)+(1\cdot0) & (2\cdot1)+(1\cdot-2)+(1\cdot1) \\ (3\cdot1)+(1\cdot5)+(2\cdot0) & (3\cdot1)+(1\cdot-2)+(2\cdot1) \\ (-1\cdot1)+(0\cdot5)+(5\cdot0) & (-1\cdot1)+(0\cdot-2)+(5\cdot1) \end{bmatrix}$$

Determinant of nxn

$-a_{12}[-a_{22}[\;]]$

$$\begin{matrix} +a_{11} & -a_{12} & +a_{13} \\ -a_{21} & +a_{22} & -a_{23} \\ +a_{31} & -a_{32} & +a_{33} \end{matrix}$$

Approach 1
Just go across the top

Approach 2
Find col w/ most 0s and go down that

Diagonilization A=n x n; If A has n different eiganvalues; then A is diagonaliseable

step1) Find eigenvalues

$A=\begin{bmatrix}-3 & -4\\5 & 6\end{bmatrix}$ $\det(A-\lambda I)=0$

$\rightarrow \begin{bmatrix}-3-\lambda & -4\\5 & 6-\lambda\end{bmatrix}=0 \rightarrow [(-3-\lambda)(6-\lambda)]-(4\cdot 5)=0$

$\rightarrow -18+3\lambda+6\lambda+\lambda^2+20=0$

Factor out

$\rightarrow \lambda^2-3\lambda+2=0 \rightarrow (\lambda-1)(\lambda-2)=0$

$A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

$\begin{vmatrix}1-\lambda & 2\\3 & 4-\lambda\end{vmatrix}=\lambda^2-5\lambda-2$

quad form $\frac{5\pm\sqrt{33}}{2}$

char polyB

eigenvalues are $\lambda=1$ & $\lambda=2$

step 2) Find eigenvectors

Plug-in λ into eq $(A-\lambda I)\vec{x}=\vec{0}$

$\lambda=1$

$\begin{bmatrix}-3 & -4\\5 & 6\end{bmatrix}-\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\rightarrow\begin{bmatrix}-4 & -4\\5 & 5\end{bmatrix}$ $\Big\}$ $-4x_1+-4x_2=0$

$x_1=-x_2$

Now pick easy # to plug in for x_2

*Let $x_2=1$ $x_1=-1$, $x_2=1$; $\vec{x}_1=\begin{bmatrix}-1\\1\end{bmatrix}$

$\lambda=2$; $(A-2I)\vec{x}=\vec{0}$

$\begin{bmatrix}-3 & -4\\5 & 6\end{bmatrix}-\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\rightarrow\begin{bmatrix}-5 & -4\\5 & 4\end{bmatrix}$ $\Big\}$ $-5x_1+(-4x_2)=0$

$x_1=-4/5\,x_2$

*Choosing x_2 be 1

$x_1=-4/5(1)$

$x_2=1$

eigenvectors are

for $\lambda=2$ $\vec{x}_2=\begin{bmatrix}-4/5\\1\end{bmatrix}$

$P=[v_1 \cdots v_n]$ ← made of e-vectors

Diagonal Matrix → $D=\begin{bmatrix}\lambda_1 & 0\\0 & \lambda_n\end{bmatrix}$

made of e-val →

$P=\begin{bmatrix}\vec{x}_1 & \vec{x}_2\end{bmatrix}=\begin{bmatrix}-1 & -4/5\\1 & 1\end{bmatrix}$ $D=\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$

$P^{-1}=\begin{bmatrix}1 & 4/5\\-1 & -1\end{bmatrix}\frac{1}{-1/5}\rightarrow\begin{bmatrix}-5 & -4\\5 & 5\end{bmatrix}$ $\therefore A=PDP^{-1}$

Find Least Sq

1) Show $A\vec{x}=\vec{b}$ has no soln.

Augment Matrix - row reduce

2) Find A^TA & $A^T\vec{b}$

3) Solve sys. $A^TA\vec{x}=A^T\vec{b}$

Augment Matrix - row reduce

Step3 gives soln $\hat{x}$ = [ ] RREF

4) Find Error

$[A\vec{x}][b]\rightarrow\sqrt{(b_1-A_1)^2+(b_2-A_2)^2}$

Dot Product

$u=\begin{bmatrix}3\\-1\\5\end{bmatrix}, V=\begin{bmatrix}0\\7\\-2\end{bmatrix}$

$U\cdot V=[3\cdot 0+-1\cdot 7+5\cdot -2]$

Distance $(v,w)=\|v-w\|$

$V=\begin{bmatrix}1\\3\\2\end{bmatrix}; W=\begin{bmatrix}0\\2\\5\end{bmatrix}$

$\sqrt{(1-0)^2+(3-2)^2+(2-5)^2}$

Ortogonality, $u=\begin{bmatrix}12\\3\\-5\end{bmatrix}$ $v=\begin{bmatrix}1\\-3\\-3\end{bmatrix}$

Step 1) dot product 2 vect

$[(12\cdot 1)+(3\cdot -3)+(-5\cdot -3)]$

$=0$? Yes orthog ; No Not orth

step2) Find θ both vec $\sqrt{\ }$

$\vec{V}_1=\sqrt{(a)^2+(b)^2+(c)^2}$

$u\cdot v=\|u\|\cdot\|v\|\cos\theta$

arcsin

Row operations change e-val but simularity does not

similarity show det A = det B

$B=P^{-1}AP$ for some P

$\det(B)=\det(P^{-1}AP)\rightarrow d(P^{-1})\,dA\,dP$

$\rightarrow \frac{1}{\cancel{\det P}}\cdot\det A\cdot\cancel{\det P}\therefore \det A$

Diagonalization

$\lambda=\#$

Let $T: R^2 \to R^3$ linear transformation $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} -y \\ y-2x \\ 7x \end{bmatrix}$

a) image of vect $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$$T\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -5 \\ 5-2(2) \\ 7(2) \end{bmatrix} \to T = \begin{bmatrix} -5 \\ 1 \\ 14 \end{bmatrix}$$

Std Matrix

$T\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$I\begin{bmatrix} 0 \\ 1 \end{bmatrix}$