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2.1 Cont'd, 2.2 Inverse of a matrix

Def: I_n is the $n \times n$ matrix with 1's on the main diagonal and 0's everywhere else

$$I = I_n \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* For example,

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* I_n is called the <u>identity matrix</u>

! Notice: If A is $m \times n$

$\underline{A\,col_1(I_n)}$ $A\,col_2(I_n)$ ...

$\uparrow$ $I\,col_1(A$ [Missing]

Powers of a matrix

<u>Ex</u> Let $A = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

Compute A^2 and A^3

$$A^2 = AA = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 = A\underbrace{AA}_{AA^2} = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\uparrow$ Zero matrix often simply denoted as 0

So, $A^3 = 0$ ← Zero matrix

2.2 Inverse of a matrix

Question: How would you solve

$$5x = 7$$

$$5x = 7 \Rightarrow \tfrac{1}{5} 5x = \tfrac{1}{5} 7$$

$$\Rightarrow 1x = \tfrac{7}{5}$$

$$\Rightarrow x = \tfrac{7}{5}$$

Q: Can we apply a similar method to solve $A\bar{x} = \bar{b}$? For example

$$\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \bar{x} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Can we multiply both sides by A^{-1}?

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A^{-1} exists $\Longleftrightarrow A \underset{P_1}{\sim} A_1 \underset{P_2}{\sim} A_2 \sim \cdots \underset{P_n}{\sim} I$

$I \underset{P_1}{\sim} B_1 \underset{P_2}{\sim} B_2 \sim \cdots \underset{P_n}{\sim} A^{-1}$

$[A \mid I] \sim \cdots \sim [I \mid A^{-1}]$