

1. (1 point)

Write the augmented matrix of the system

$$\begin{cases} 20x + 87z = -33 \\ 1x - 7y - 87z = -8 \\ 60x + 90y = 5 \end{cases}$$

$$\left[ \begin{array}{ccc|c} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{array} \right]$$

2. (1 point) Convert the augmented matrix

$$\left[ \begin{array}{cccc} 5 & 1 & 5 & -4 \\ 0 & 2 & -5 & 5 \end{array} \right]$$

to the equivalent linear system. Use **x1**, **x2**, and **x3** to enter the variables  $x_1$ ,  $x_2$ , and  $x_3$ .

$$\_ = \_$$

$$\_ = \_$$

**Solution: SOLUTION:**

$$\begin{array}{rrcrcl} 5x_1 & + & x_2 & + & 5x_3 & = & -4 \\ & & 2x_2 & - & 5x_3 & = & 5 \end{array}$$

3. (1 point) Determine which of the points  $(-6, 3, -2)$ ,  $(-4, -5, -4)$ , and  $(-4, 6, -2)$  satisfy the linear system

$$\begin{array}{rrcrcl} 5x_1 & + & 3x_2 & - & 4x_3 & = & -13 \\ 2x_1 & - & 2x_2 & + & 5x_3 & = & -28 \end{array}$$

Answer: \_\_\_\_\_

**Solution: SOLUTION:**

$5(-6) + 2(3) - 4(-2) = -13$  and  $3(-6) - 2(3) + 5(-2) = -28$ , so  $(-6, 3, -2)$  satisfies the linear system  
 $5(-4) + 2(6) - 4(-2) = 6$  and  $3(-4) - 2(6) + 5(-2) = -30$ , so  $(-4, 6, -2)$  does not satisfy the linear system  
 $5(-4) + 2(-5) - 4(-4) = -19$  and  $3(-4) - 2(-5) + 5(-4) = -18$ , so  $(-4, -5, -4)$  does not satisfy the linear system

4. (1 point) Solve the system using elimination.

$$\begin{cases} -4x + 2y + 5z = -29 \\ 5x - 2y + 2z = -5 \\ -5x - 5y + 6z = -35 \end{cases}$$

$$x = \_$$

$$y = \_$$

$$z = \_$$

5. (1 point) Solve the system using any method

$$\begin{cases} -x + y + z = -7 \\ 4x - 3y - z = 18 \\ x + y + z = -5 \end{cases}$$

Your answer is

$$x = \_$$

$$y = \_$$

$$z = \_$$

6. (1 point)

Solve the system using matrices (row operations)

$$\begin{cases} 2x - 5y + 3z = -13 \\ x + 2y - 4z = 19 \\ -4x - 3y - 4z = -6 \end{cases}$$

How many solutions are there to this system?

- A. None
- B. Exactly 1
- C. Exactly 2
- D. Exactly 3
- E. Infinitely many
- F. None of the above

If there is one solution, give its coordinates in the answer spaces below.

If there are infinitely many solutions, enter  $z$  in the answer blank for  $z$ , enter a formula for  $y$  in terms of  $z$  in the answer blank for  $y$  and enter a formula for  $x$  in terms of  $z$  in the answer blank for  $x$ .

If there are no solutions, leave the answer blanks for  $x$ ,  $y$  and  $z$  empty.

$$x = \_$$

$$y = \_$$

$$z = \_$$

7. (1 point)

The reduced row echelon form of a system of linear equations in  $x$  and  $y$  or in  $x$ ,  $y$  and  $z$  is given. For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

1.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 \end{array} \right]$$

- A. Unique solution:  $x = 1, y = 2, z = 2$
- B. No solutions
- C. Unique solution:  $x = 1, y = 2, z = 0$
- D. Unique solution:  $x = 1, y = 2$
- E. Infinitely many solutions
- F. None of the above

2.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

- A. Infinitely many solutions
- B. Unique solution:  $x = -1, y = -1$
- C. Unique solution:  $x = 0, y = 0, z = 0$
- D. Unique solution:  $x = -1, y = -1$
- E. No solutions
- F. None of the above

3.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

- A. Unique solution:  $x = 2, y = -2$
- B. Unique solution:  $x = 0, y = 2, z = -2$
- C. Unique solution:  $x = 0, y = 2$
- D. Infinitely many solutions
- E. No solutions
- F. None of the above

4.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

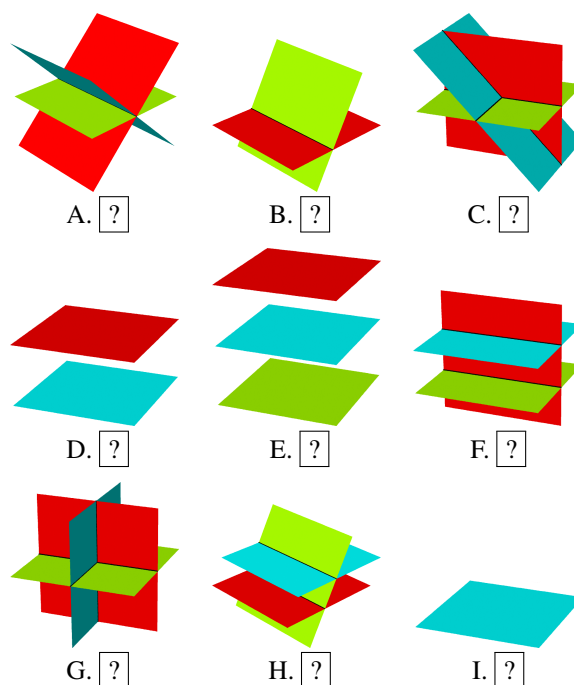
- A. Unique solution:  $x = 1, y = 1, z = 0$
- B. Unique solution:  $x = 0, y = 0, z = 0$
- C. No solutions
- D. Infinitely many solutions
- E. Unique solution:  $x = 0, y = 0$
- F. None of the above

8. (1 point) Determine whether the following system has no solution, an infinite number of solutions or a unique solution.

$$[?]1. \begin{cases} -10x + 10y - 6z = 10 \\ 20x - 20y + 12z = -20 \\ -30x + 30y - 18z = 30 \end{cases}$$

$$\begin{aligned} [?]2. & \begin{cases} -4x - 16y - 61z = 6 \\ 4x + 17y + 63z = 10 \\ x + 4y + 15z = 0 \end{cases} \\ [?]3. & \begin{cases} 3x + 3y - 3z = -5 \\ -3x + 5y + 5z = -3 \\ 9x + 25y - 5z = -28 \end{cases} \\ [?]4. & \begin{cases} 3x + 3y - 3z = -5 \\ -3x + 5y + 5z = -3 \\ 9x + 25y - 5z = -31 \end{cases} \end{aligned}$$

9. (1 point) Each graph below is the graph of a system of three linear equations in three unknowns. Determine which systems are consistent and inconsistent.



(Click on a graph to enlarge it.)

10. (1 point) Consider a linear system whose augmented matrix is

$$\left[ \begin{array}{ccc|c} 1 & 1 & 5 & -3 \\ 1 & 2 & -4 & 1 \\ 7 & 17 & k & 20 \end{array} \right]$$

For what value of  $k$  will the system have no solutions?

$k =$  \_\_\_\_\_

**Solution:**

**SOLUTION:** Performing several elementary row operations, we get that the given augmented matrix is row equivalent to:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 5 & -3 \\ 0 & 1 & -9 & 4 \\ 0 & 0 & k+55 & -20 \end{array} \right]$$

Therefore the system has no solutions if  $k + 55 = 0$ , so  $k = -55$ .