Chapter 6

20240425 P2
Final Exam
40 algan stuff
60% cumulative

Motivation: Suppose you are modeling a solution and need to solve a linear system $A\bar{x}=\bar{b}$ ← $[A|\bar{b}]$
However, you find out that the system is inconsistant (has no solution).
What do you do?!

Option 1: Try to find the "best possible" approximate solution. That is, try to find a vector $\bar{x}$ such that $A\bar{x}$ is a "as close as possible" to $\bar{b}$ (even though it might not be equal $\bar{b}$)

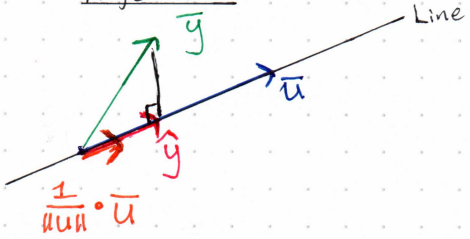
This is where we are going, but to do it, we first need to discuss distance

Chapter 6, Section 1: Inner product, linear, distance

Worksheet 13

Chapter 6.2 & 6.3 orthogonal sets and orthoganal projections

Projections



- Want $\hat{y}$: the projection of $\bar{y}$ onto the line spanned by $\bar{u}$
- Note: This will be the vector on Line that is as close as possible to $\bar{y}$

How to find $\hat{y}$ (given $\bar{y}$ and $\bar{u}$)

Idea: $\hat{y} = \frac{\|\hat{y}\|\cdot\frac{1}{\|u\|}\bar{u}}{\text{scale } \bar{u}}$

So we can find $\hat{y}$ once we know $\|\hat{y}\|$
So, need to find $\|\hat{y}\|$ in terms of $\bar{y}$ and $\bar{u}$

$\cos\theta = \frac{\|\hat{y}\|}{\|\bar{y}\|}$

Also, θ is angle b/w $\bar{y}$ and $\bar{u}$

So,

$$\cos\theta = \frac{\bar{y}\cdot\bar{u}}{\|\bar{y}\|\,\|\bar{u}\|}$$

Combining, we get

$$\frac{\|\hat{y}\|}{\|\bar{y}\|} = \frac{\bar{y}\cdot\bar{u}}{\|\bar{u}\|\,\|\bar{u}\|} \Rightarrow \|\hat{y}\| = \frac{\bar{y}\cdot\bar{u}}{\|\bar{u}\|}$$

Thus,

$$\hat{y} = \frac{\bar{y}\cdot\bar{u}}{\|\bar{u}\|\,\|\bar{u}\|}\bar{u} = \frac{\bar{y}\cdot\bar{u}}{\bar{u}\cdot\bar{u}}\bar{u}$$