**1.** (1 point)
Let
$\mathbf{u} = \begin{bmatrix} 9 \\ 3 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ 9 \\ 5 \end{bmatrix}$

Compute $8\mathbf{u} - 3\mathbf{v} - 4\mathbf{w} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$

**Solution:**
SOLUTION

$$8\mathbf{u} - 3\mathbf{v} - 4\mathbf{w} = \begin{bmatrix} 8*9-3*5-4*0 \\ 8*3-3*4-4*9 \\ 8*1-3*9-4*5 \end{bmatrix} = \begin{bmatrix} 57 \\ -24 \\ -12 \end{bmatrix}$$

**2.** (1 point)
Express the following system of linear equations as a vector equation.

$$3x_1 + 5x_2 + 8x_3 = -9$$

$$5x_1 + 6x_2 + 2x_3 = -8$$

$$1x_1 - 8x_2 + 5x_3 = 9$$

$$\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} x_1 + \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} x_2 + \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} x_3 = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

**Solution:**
SOLUTION

$$\begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 5 \\ 6 \\ -8 \end{bmatrix} x_2 + \begin{bmatrix} 8 \\ 2 \\ 5 \end{bmatrix} x_3 = \begin{bmatrix} -9 \\ -8 \\ 9 \end{bmatrix}$$

**3.** (1 point)
Express the following vector equation as a system of linear equations.

$$x_1 \begin{bmatrix} 4 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$$

(Keep the equations in order.)
___ $x_1+$ ___ $x_2 =$ ___.
___ $x_1+$ ___ $x_2 =$ ___.

**Solution:**
SOLUTION

$$4x_1 + 4x_2 = 9$$
$$-3x_1 + 2x_2 = -4$$

**4.** (1 point)
Let $\mathbf{a}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} -4 \\ 16 \end{bmatrix}$.
Is $\mathbf{b}$ a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$?

- A. $\mathbf{b}$ is not a linear combination.
- B. Yes $\mathbf{b}$ is a linear combination.
- C. We cannot tell if $\mathbf{b}$ is a linear combination.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.
$\mathbf{b} =$___ $\mathbf{a}_1+$ ___$\mathbf{a}_2$

**Solution:**
SOLUTION
$\mathbf{b}$ is a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$.
Using row reduction, we see

$$\begin{bmatrix} 2 & 2 & -4 \\ 2 & -3 & 16 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -2 \\ 0 & -5 & 20 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \end{bmatrix}$$

so

$$\mathbf{b} = 2\mathbf{a}_1 - 4\mathbf{a}_2$$

**5.** (1 point)
Let $\mathbf{a}_1 = \begin{bmatrix} -7 \\ 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 14 \\ -12 \end{bmatrix}$.
Is $\mathbf{b}$ in the span of of $\mathbf{a}_1$?

- A. Yes, $\mathbf{b}$ is in the span.
- B. No, $\mathbf{b}$ is not in the span.
- C. We cannot tell if $\mathbf{b}$ is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.
$\mathbf{b} =$___ $\mathbf{a}_1$

**Solution:**
SOLUTION
From the first component we see $14 = -2*-7$.
From the second component we see $-12 = -2*6$.

Thus $\mathbf{b} = -2\mathbf{a}_1$ is in the span of $\mathbf{a}_1$.

**6.** (1 point)

Let $\mathbf{a}_1 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -9 \\ -14 \\ -8 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}$.

Is $\mathbf{b}$ a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$?

- A. Yes, $\mathbf{b}$ is a linear combination.
- B. No, $\mathbf{b}$ is not a linear combination.
- C. We cannot tell if $\mathbf{b}$ is a linear combination.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$\mathbf{b} =$ ___ $\mathbf{a}_1 +$ ___ $\mathbf{a}_2$

**Solution:**

SOLUTION

Using row reduction,

$$\begin{bmatrix} 3 & -9 & 3 \\ 5 & -14 & 6 \\ 1 & -8 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & 1 \\ 0 & -5 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

We get a last row of (0 0 1).

$\mathbf{b}$ is not a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$.

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**7.** (1 point)

Let $\mathbf{a}_1 = \begin{bmatrix} -9 \\ 6 \\ 8 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} -57 \\ 26 \\ 30 \end{bmatrix}$.

Is $\mathbf{b}$ in the span of of $\mathbf{a}_1$ and $\mathbf{a}_2$?

- A. Yes, $\mathbf{b}$ is in the span.
- B. No, $\mathbf{b}$ is not in the span.
- C. We cannot tell if $\mathbf{b}$ is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$\mathbf{b} =$ ___ $\mathbf{a}_1 +$ ___ $\mathbf{a}_2$

**Solution:**

SOLUTION

Using row reduction, we see

$$\begin{bmatrix} -9 & 6 & -57 \\ 6 & 2 & 26 \\ 8 & 5 & 30 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $\mathbf{b} = 5\mathbf{a}_1 - 2\mathbf{a}_2$.

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**8.** (1 point)

Let $\mathbf{a}_1 = \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -7 \\ 3 \\ -2 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 13 \\ -24 \\ 30 \end{bmatrix}$.

Is $\mathbf{b}$ in the span of of $\mathbf{a}_1$ and $\mathbf{a}_2$?

- A. Yes, $\mathbf{b}$ is in the span.
- B. No, $\mathbf{b}$ is not in the span.
- C. We cannot tell if $\mathbf{b}$ is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$\mathbf{b} =$ ___ $\mathbf{a}_1 +$ ___ $\mathbf{a}_2$

**Solution:**

SOLUTION

Using row reduction, we see

$$\begin{bmatrix} -2 & -7 & 13 \\ 7 & 3 & -24 \\ -9 & -2 & 30 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus $\mathbf{b}$ is not in the span of $\mathbf{a}_1$ and $\mathbf{a}_2$.

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**9.** (1 point)

Let $\mathbf{u}_1 = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$, and $\mathbf{u}_2 = \begin{bmatrix} -20 \\ 25 \end{bmatrix}$.

Select all of the vectors that are in the span of $\{\mathbf{u}_1, \mathbf{u}_2\}$. (Check every statement that is correct.)

- A. The vector $7\begin{bmatrix} -20 \\ 25 \end{bmatrix} - 3\begin{bmatrix} 4 \\ -4 \end{bmatrix}$ is in the span.
- B. The vector $-3\begin{bmatrix} 4 \\ -4 \end{bmatrix}$ is in the span.
- C. All vectors in $\mathbb{R}^2$ are in the span.
- D. The vector $\begin{bmatrix} -20 \\ 25 \end{bmatrix}$ is in the span.
- E. The vector $\begin{bmatrix} 4 \\ -4 \end{bmatrix}$ is in the span.
- F. The vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is in the span.
- G. We cannot tell which vectors are in the span.

**Solution:**

SOLUTION

Any linear combination of the vectors is in the span. This always includes the zero vector and the original vectors. Since the two vectors are not on the same line through the origin, all vectors in $\mathbb{R}^2$ are in the span.

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**10.** (1 point) Evaluate the following matrix product.

$$\begin{bmatrix} -3 & -4 & 0 \\ -4 & -1 & 3 \\ -3 & 1 & -4 \end{bmatrix} \begin{bmatrix} -5 \\ 4 \\ -5 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

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**11.** (1 point)

Find $A$, and $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ corresponds to the given linear system.

$$5x_1 - 3x_2 - 1x_3 = -5$$

$$9x_1 + 7x_2 - 2x_3 = 8$$

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

**Solution:**

SOLUTION

$$\begin{bmatrix} 5 & -3 & -1 \\ 9 & 7 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 8 \end{bmatrix}$$

**12.** (1 point)

Find $A$ and $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ corresponds to the given linear system.

$$2x_1 - 1x_2 - 8x_3 = -8$$

$$1x_1 + 6x_2 + 8x_3 = 9$$

$$-3x_1 + 9x_2 + 4x_3 = -3$$

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

**Solution:**

SOLUTION

$$\begin{bmatrix} 2 & -1 & -8 \\ 1 & 6 & 8 \\ -3 & 9 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -8 \\ 9 \\ -3 \end{bmatrix}$$