Matthew Mendoza **Math-100-J-Wiscons-Sp24**
Assignment HW-12 due 05/06/2024 at 11:59pm PDT

1. (2 points) One of the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

is a real number. Find this eigenvalue and a basis of the eigenspace.

The eigenvalue is _____.

A basis for the eigenspace is $\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

Correct Answers:

- `1`
- `[[0],[1],[1]],` $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

2. (2 points) The matrix

$$A = \begin{bmatrix} 6 & -3 & -9 \\ -3 & 6 & 9 \\ 3 & -3 & -6 \end{bmatrix}$$

has two real eigenvalues, one whose eigenspace has dimension 1 and one whose eigenspace has dimension 2. Find the eigenvalues and a basis for each eigenspace.

The eigenvalue whose eigenspace has dimension 1 is $\lambda_1 =$_____ and a basis for its associated eigenspace is

$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

The eigenvalue whose eigenspace has dimension 2 is $\lambda_2 =$_____ and a basis for its associated eigenspace is

$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

Correct Answers:

- `0`
- $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$
- `3`
- `[[1],[-2],[1]],` $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

3. (2 points) Let

$$A = \begin{bmatrix} 1 & 0.25 \\ -65 & -8 \end{bmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

$$P = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

$$D = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

Correct Answers:

- $\begin{bmatrix} 1 & 1 \\ -26 & -10 \end{bmatrix}$
- $\begin{bmatrix} -5.5 & 0 \\ 0 & -1.5 \end{bmatrix}$

4. (2 points) Let $A = \begin{bmatrix} -2 & -4 \\ 0 & -6 \end{bmatrix}$

Find a matrix P, a diagonal matrix D and P^{-1} such that $A = PDP^{-1}$.

$P = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, D = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, P^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$
- $\begin{bmatrix} -6 & 0 \\ 0 & -2 \end{bmatrix}$
- $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

5. (2 points) Let $A = \begin{bmatrix} 2 & 0 & 0 \\ -12 & 2 & 6 \\ 9 & -3 & -7 \end{bmatrix}$.

Find P and D such that $A = PDP^{-1}$. $P = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$,

$$D = \begin{bmatrix} \_\_ & 0 & 0 \\ 0 & \_\_ & 0 \\ 0 & 0 & \_\_ \end{bmatrix}.$$

Correct Answers:

- `[[0,0,-1],[-1,-2,3],[1,1,-2]]; -4; -1; 2`