1. (1 point) Determine how many pivots each of the following matrices have.

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

(1) $\begin{bmatrix} 1 & 0 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

(2) $\begin{bmatrix} 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 8 \end{bmatrix}$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

(3) $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix}$

- Choose
- One Pivot
- Two Pivots
- Three Pivots
- Four Pivots

(4) $\begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -7 \end{bmatrix}$

2. (1 point) How many free variables does each augmented matrix have?

(1) [Choose/None/One/Two/Three] $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & -10 \\ 0 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

(2) [Choose/None/One/Two/Three] $\left[\begin{array}{cc|c} 1 & 8 & 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

(3) [Choose/None/One/Two/Three] $\left[\begin{array}{ccc|c} 1 & -5 & 6 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(4) [Choose/None/One/Two/Three] $\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{array}\right]$

3. (1 point) Solve the matrix equation $Ax = b$, where

$$A = \begin{bmatrix} -2 & 2 \\ -3 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \quad b = \begin{bmatrix} -6 \\ -9 \end{bmatrix}.$$

The solution is:
$x_1 =$ __________
$x_2 =$ __________

4. (1 point)

The reduced row echelon form of a system of linear equations in x and y or in x, y and z is given. For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

1.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

- A. Unique solution: $x = 0, y = -1$
- B. Unique solution: $x = 0, y = -1, z = 3$
- C. Infinitely many solutions
- D. No solutions
- E. Unique solution: $x = -1, y = 3$
- F. None of the above

2.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & -4 \end{array}\right]$$

- A. Unique solution: $x = -3, y = 4, z = -4$
- B. No solutions
- C. Unique solution: $x = -3, y = 4, z = 0$
- D. Unique solution: $x = -3, y = 4$
- E. Infinitely many solutions
- F. None of the above

3.

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

- A. Unique solution: $x=0, y=0, z=0$
- B. Unique solution: $x=1, y=1, z=0$
- C. Unique solution: $x=0, y=0$
- D. Infinitely many solutions
- E. No solutions
- F. None of the above

4.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

- A. No solutions
- B. Infinitely many solutions
- C. Unique solution: $x=0, y=0, z=0$
- D. Unique solution:$x=3, y=-2$
- E. Unique solution: $x=-2, y=3$
- F. None of the above

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5. (1 point) For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

(1) $\begin{cases} 3x-4y= & 19 \\ -9x+9y= & -54 \end{cases}$

- A. Unique solution: $x=-1,\ y=5$
- B. Infinitely many solutions
- C. Unique solution: $x=0,\ y=0$
- D. Unique solution: $x=5,\ y=-1$
- E. No solutions
- F. None of the above

(2) $\begin{cases} -8x+4y=0 \\ 9x+4y=0 \end{cases}$

- A. Unique solution: $x=-4,\ y=13$
- B. Unique solution: $x=-4,\ y=-8$
- C. No solutions
- D. Infinitely many solutions
- E. Unique solution: $x=0,\ y=0$
- F. None of the above

(3) $\begin{cases} -2x+2y= & 8 \\ 6x-6y= & -24 \end{cases}$

- A. Unique solution: $x=-4,\ y=0$
- B. Unique solution: $x=8,\ y=-24$
- C. Infinitely many solutions
- D. No solutions
- E. Unique solution: $x=0,\ y=0$
- F. None of the above

(4) $\begin{cases} 5x+4y= & 6 \\ -5x-4y= & -5 \end{cases}$

- A. No solutions
- B. Infinitely many solutions
- C. Unique solution: $x=-5,\ y=6$
- D. Unique solution: $x=6,\ y=-5$
- E. Unique solution: $x=0,\ y=0$
- F. None of the above

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6. (1 point)
Solve the system using matrices (row operations)

$$\begin{cases} 2x-2y-z= -21 \\ x+4y-2z \ = 10 \\ 2x-5y+6z= -15 \end{cases}$$

How many solutions are there to this system?

- A. None
- B. Exactly 1
- C. Exactly 2
- D. Exactly 3
- E. Infinitely many
- F. None of the above

If there is one solution, give its coordinates in the answer spaces below.

If there are infinitely many solutions, enter z in the answer blank for z, enter a formula for y in terms of z in the answer blank for y and enter a formula for x in terms of z in the answer blank for x.

If there are no solutions, leave the answer blanks for x, y and z empty.

$x =$ ____________________

$y =$ ____________________

$z =$ ____________________

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7. (1 point)
The following system has an infinite number of solutions. Write the solution in terms of the free variables y and z.

$$\begin{array}{rcrcrcrcr} 2w & - & x & + & 2y & - & z & = & 1 \\ w & + & x & - & y & + & 4z & = & 3 \\ 3w & & & + & 1y & + & 3z & = & 4 \\ 3w & - & 3x & + & 5y & - & 6z & = & -1 \end{array}$$

$w =$ ____________________

$x =$ ____________________

y is free

z is free

8. (1 point) Solve the system

$$\begin{cases} x_1+x_2 = 3 \\ x_2+x_3 = 2 \\ x_3+x_4=-5 \\ x_1 + x_4=-4 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + s \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

9. (1 point) Solve the system

$$\begin{cases} 4x_1-5x_2+4x_3+3x_4=3 \\ -x_1+x_2+2x_3+3x_4=3 \\ 3x_1-4x_2+6x_3+6x_4=6 \\ -2x_1+2x_2+4x_3+6x_4=6 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + s \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + t \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

10. (1 point) Find the set of solutions for the linear system

$$\begin{array}{rcrcrcr} -3x_1 & - & 6x_2 & + & 3x_3 & = & 13 \\ & & 4x_2 & + & 7x_3 & = & -9 \end{array}$$

Use **s1**, **s2**, etc. for the free variables if necessary.

$(x_1, x_2, x_3) = \left(\_\_\_\_, \_\_\_\_, \_\_\_\_ \right)$