Applied Linear Algebra — Outline for Exam 1

Sections 1.1–1.7

Main ideas

A. Solving linear systems, vector equations, and matrix equations

B. Row reduction process and REF/RREF forms

C. Representing solution sets parametrically and interpreting geometrically

D. Linear combinations, span, and linear independence

E. Application: network flow

Skills you should have

1. Be able to use the row reduction process to transform a matrix to RREF (and REF)

- Make sure you can recognize when a matrix is in REF and RREF

2. Be able to solve linear systems.

- *Usual process:* (1) write as augmented matrix, (2) row reduce to REF or RREF, (3) write the solution set (using free variables if necessary).
- Be able to determine if the system is consistent/inconsistent and if there are infinitely many solutions.
- Be able to write the solution set in parametric vector form.
- Be able to interpret solution sets geometrically: point, line, or plane.

3. Be able to multiply a matrix by a vector

4. Be able to solve vector and matrix equations of the form $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_n\mathbf{v}_n = \mathbf{b}$ and $A\mathbf{x} = \mathbf{b}$

- *Usual process:* convert to an augmented matrix and then solve as a linear system

5. Be able to determine if a vector $\mathbf{b}$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$; this is the same as determining if $\mathbf{b}$ is in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$

- *Usual process:* determine if $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_k\mathbf{v}_k = \mathbf{b}$ is consistent by converting to an augmented matrix and solving as a linear system.
- Make sure you understand the definitions of "linear combination" and "span"

6. Be able to determine if *every* $\mathbf{b} \in \mathbb{R}^m$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$; this is the same as determining if *every* $\mathbf{b} \in \mathbb{R}^m$ is in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$

- Usual process: (1) make the "coefficient" matrix $\begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_k\end{bmatrix}$, (2) row reduce to REF, (3) if there is a pivot in every *row*, then YES, every $\mathbf{b} \in \mathbb{R}^m$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$; if there is NOT a pivot in every *row*, then NO.
- Note: for this type of problem, you really only need to work with the coefficient matrix $\begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_k\end{bmatrix}$ instead of the full augmented matrix $\left[\begin{array}{cccc|c}\mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_k & \mathbf{b}\end{array}\right]$

7. Be able to determine if $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent or linearly dependent

- The definition: $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly *independent* if and only if $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_k\mathbf{v}_k = \mathbf{0}$ has only one solution (i.e. if there are no free variables).
- *Usual process:* solve the system $\begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_k \mid \mathbf{0}\end{bmatrix}$ and determine how many solutions there are
- There are also some theorems that can sometimes help:
  - Assume $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are in $\mathbb{R}^m$. If $k > m$, then the vectors must be linearly dependent.
  - If one of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ is the zero vector, then the vectors must be linearly dependent.
  - Two vectors $\mathbf{v}_1, \mathbf{v}_2$ are linearly dependent if and only if one is a scalar multiple of the other.

8. Be able to solve network flow problems

- The main principle is that "flow in = flow out" at every intersection. Also, the flow into the entire network must equal the flow out of the entire network.

How to study

I. Review core topics—make sure to have a working understanding of definitions and theorems

II. Work lots of problems all of the way through—focus on WeBWorK problems and Handout problems

- WeBWork #1–5, Handout #1–6
- You can also look in other books for problems to try

III. Practice doing several problems in a short amount of time

IV. Come talk with me if you have any questions