Matthew Mendoza **Math-100-J-Wiscons-Sp24**

Assignment HW-05 due 02/26/2024 at 11:59pm PST

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1. (1 point) Let v_1, v_2, v_3 be the vectors in $\mathbb{R}^3$ defined by

$$v_1 = \begin{bmatrix} 25 \\ -8 \\ -12 \end{bmatrix} \quad v_2 = \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} \quad v_3 = \begin{bmatrix} -15 \\ 2 \\ 7 \end{bmatrix}$$

(a) Is $\{v_1, v_2, v_3\}$ linearly independent? Write all zeros if it is or if it is linearly dependent write zero as a non-trivial (not all zero coefficients) linear combination of v_1, v_2, and v_3

$$0 = \_\_\_ \; v_1 + \_\_\_ \; v_2 + \_\_\_ \; v_3$$

(b) Is $\{v_1, v_3\}$ linearly independent? Write all zeros if it is or if it is linearly dependent write zero as a non-trivial (not all zero coefficients) linear combination of v_1 and v_3.

$$0 = \_\_\_ \; v_1 + \_\_\_ \; v_3$$

Solution:

Solution:

(a) Notice that $0 = -1v_1 - 1v_2 - 2v_3$ so that the set is linearly dependent.

(b) Notice that v_3 is not a constant multiple of v_1, so they are linearly independent.

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2. (1 point) Let

$$\vec{v}_1 = \begin{bmatrix} -5 \\ -2 \\ -2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -10 \\ -3 \\ -4 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}.$$

Are the vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ linearly independent?

- choose
- linearly dependent
- linearly independent

If the vectors are independent, enter zero in every answer blank since zeros are only the values that make the equation below true. If they are dependent, find numbers, not all zero, that make the equation below true. You should be able to explain and justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \_\_\_ \begin{bmatrix} -5 \\ -2 \\ -2 \end{bmatrix} + \_\_\_ \begin{bmatrix} -10 \\ -3 \\ -4 \end{bmatrix} + \_\_\_ \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}.$$

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3. (1 point) Let

$$\vec{v}_1 = \begin{bmatrix} 10 \\ -4 \\ -4 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 6 \\ -12 \\ 9 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -1 \\ -6 \\ 8 \end{bmatrix}.$$

Are the vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ linearly independent?

- choose
- linearly dependent
- linearly independent

If the vectors are independent, enter zero in every answer blank since zeros are only the values that make the equation below true. If they are dependent, find numbers, not all zero, that make the equation below true. You should be able to explain and justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \_\_\_ \begin{bmatrix} 10 \\ -4 \\ -4 \end{bmatrix} + \_\_\_ \begin{bmatrix} 6 \\ -12 \\ 9 \end{bmatrix} + \_\_\_ \begin{bmatrix} -1 \\ -6 \\ 8 \end{bmatrix}.$$

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4. (1 point) Let

$$\vec{v}_1 = \begin{bmatrix} -5 \\ -4 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 30 \\ -9 \end{bmatrix}.$$

Are the vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ linearly independent?

- choose
- linearly dependent
- linearly independent

If the vectors are independent, enter zero in every answer blank since those are only the values that make the equation below true. If they are dependent, find numbers, not all zero, that make the equation below true. You should be able to explain and justify your answer.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \_\_\_ \begin{bmatrix} -5 \\ -4 \end{bmatrix} + \_\_\_ \begin{bmatrix} 2 \\ -5 \end{bmatrix} + \_\_\_ \begin{bmatrix} 30 \\ -9 \end{bmatrix}.$$

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5. (1 point) Are the vectors $\begin{bmatrix} -4 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -16 \\ -2 \\ -6 \end{bmatrix}$ linearly independent?

- Choose
- linearly dependent
- linearly independent

If they are linearly dependent, find scalars that are not all zero such that the equation below is true. If they are linearly independent, find the only scalars that will make the equation below true.

$$\underline{\quad}\begin{bmatrix}-4\\0\\-3\end{bmatrix}+\underline{\quad}\begin{bmatrix}0\\-1\\3\end{bmatrix}+\underline{\quad}\begin{bmatrix}-16\\-2\\-6\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

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6. (1 point) The vectors

$$\vec{u}=\begin{bmatrix}-1\\-3\\1\end{bmatrix},\quad \vec{v}=\begin{bmatrix}2\\4\\3+k\end{bmatrix},\quad \vec{w}=\begin{bmatrix}1\\-1\\3\end{bmatrix}$$

are linearly independent if and only if $k \neq$ _____.

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7. (1 point)

Let u, v, w be three linearly independent vectors in $\mathbb{R}^7$. Determine whether the following sets of vectors are linearly independent or dependent.

[?] 1. The set $\{u+v, v+w, w+u\}$

[?] 2. The set $\{u-v, v-w, w-u\}$

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8. (1 point)

Let **S** be a set of m vectors in $\mathbb{R}^n$ with $m > n$.

Select the best statement.

- A. The set **S** is linearly independent, as long as no vector in **S** is a scalar multiple of another vector in the set.
- B. The set **S** is linearly dependent.
- C. The set **S** could be either linearly dependent or linearly independent, depending on the case.
- D. The set **S** is linearly independent.
- E. The set **S** is linearly independent, as long as it does not include the zero vector.
- F. none of the above

Solution:

SOLUTION

By theorem 2.13, a linearly independent set in $\mathbb{R}^n$ can contain no more than n vectors.

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9. (1 point)

Let A be a matrix with more rows than columns.

Select the best statement.

- A. The columns of A could be either linearly dependent or linearly independent depending on the case.
- B. The columns of A are linearly independent, as long as they does not include the zero vector.
- C. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A
- D. The columns of A must be linearly independent.
- E. The columns of A must be linearly dependent.
- F. none of the above

Solution:

SOLUTION

The zero matrix is an example where the columns are linearly dependent. The matrix where the top square portion is the identity matrix and the portion below that is all zeros is an example where the columns are linearly independent.

The columns of A could be either linearly dependent or linearly independent depending on the case.

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10. (1 point)

Let A be a matrix with more columns than rows.

Select the best statement.

- A. The columns of A must be linearly dependent.
- B. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A
- C. The columns of A could be either linearly dependent or linearly independent depending on the case.
- D. The columns of A are linearly independent, as long as they does not include the zero vector.
- E. none of the above

Solution:

SOLUTION

Since there are more columns than rows, when we row reduce the matrix not all columns can have a leading 1.

The columns of A must be linearly dependent.

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11. (1 point)

Let A be a matrix with linearly independent columns.

Select the best statement.

- A. The equation $A\mathbf{x}=\mathbf{0}$ never has nontrivial solutions.
- B. The equation $A\mathbf{x}=\mathbf{0}$ has nontrivial solutions precisely when it has more rows than columns.
- C. The equation $A\mathbf{x}=\mathbf{0}$ always has nontrivial solutions.
- D. The equation $A\mathbf{x}=\mathbf{0}$ has nontrivial solutions precisely when it has more columns than rows.
- E. The equation $A\mathbf{x}=\mathbf{0}$ has nontrivial solutions precisely when it is a square matrix.
- F. There is no easy way to tell if such an equation has nontrivial solutions.
- G. none of the above

Solution:

SOLUTION

The linear independence of the columns does not change with row reduction. Since the columns are linearly independent, after row reduction, each column contains a leading 1. We get nontrivial solutions when we have columns without a leading 1 in the row reduced matrix.

The equation $A\mathbf{x}=\mathbf{0}$ never has nontrivial solutions.