California State University Sacramento - Math 101

Homework Assignment 6 - Solutions

3) There are $\binom{10}{3}$ ways to pick which seats the 3 particular girls will sit in in the first row. There are 3! ways to order these 3 particular girls in the seats that we have chosen for them. Similarly, there are $\binom{10}{4}4!$ ways to choose which seats the 4 particular boys will sit in. After these 7 students have been assigned seats, there are 13! ways to assign seats to the remaining students. Therefore, the total number of possibilities is $\binom{10}{3}3!\binom{10}{4}4!13!$.

4) There are 5 choices for which positions to put the two girls in. Once we have chosen which position that they will go in, there are 2! ways to order these two girls and 7! ways to order the 7 boys. This gives a count of $5 \cdot 2!7! = 10 \cdot 7!$.

5) We pick the positions for the n ones. Choosing the positions for the n ones is the same as choosing an n-combination from $\{1, 2, \ldots, m+n\}$ such that no two of the chosen numbers are consecutive. By Example 1.5.3, there are $\binom{n+m-n+1}{n} = \binom{m+1}{n}$ such n-combinations.

6) (i) In this case, we just need to choose k of the couples to form a group of $2k$ people consisting of k couples. Since there are n couples and we need to choose k of them, we have $\binom{n}{k}$ possibilities.

(ii) Here we will form such a group by first choosing $2k$ couples, and then choosing exactly one person from each couple. In this way, our group of $2k$ people will not include a couple. There are $\binom{n}{2k}$ ways to choose $2k$ couples from n couples, and then 2^{2k} ways to choose one person from each of these $2k$ couples giving a count of $\binom{n}{2k}2^{2k}$.

(iii) We use the Complementation Principle. The total number of ways to choose $2k$ people is $\binom{2n}{2k}$. Here we have $2n$ people to choose from since there are n couples and each couple consists of two people. By part (ii), we know that there are $\binom{n}{2k}2^{2k}$ ways to form a group where there are no couples. The opposite of a group with no couples is a group with at least one couple. Thus, by the Complementation Principle, there are

$$\binom{2n}{2k} - \binom{n}{2k}2^{2k}$$

possibilities.

(iv) First we choose which two couples will be in the group. There are $\binom{n}{2}$ ways to do this. So far we have chosen two couples and so we have four people in our group. We need to choose the remaining $2k-4$ people so that this set of people contains no couple. There are

$$\binom{n-2}{2k-4}2^{2k-4}$$

ways to do this (see part (ii)). This gives a count of

$$\binom{n}{2}\binom{n-2}{2k-4}2^{2k-4}.$$