California State University Sacramento - Math 101

Quiz #5

Name: ______________________

1) If there must be at least one person at each table, in how many ways can five people be seated around two tables where the tables are indistinguishable?

2) (a) Let $s(r, n)$ be the number of ways to arrange r distinct objects around n indistinguishable circles so that every circle has at least one object. The numbers $s(r, n)$ are called the Stirling numbers of the first kind. Problem 1 is asking for $s(5, 2)$. For part (a) of this problem, compute $s(4, n)$ for $n = 1, 2, 3, 4$ (we define $s(4, 0) = 0$).

(b) Expand the polynomial $x(x + 1)(x + 2)(x + 3)$ as much as possible.

3) Given that $s(6, 1) = 120$ and $s(6, 2) = 274$, use the formula

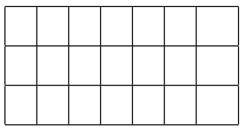
$$s(r, n) = s(r - 1, n - 1) + (r - 1)s(r - 1, n)$$

to determine $s(7, 2)$

4) Find the number of 4-combinations of $\{1, 2, \ldots, 12\}$ that contain no consecutive integers.

5) Suppose that k and n are positive integers with $k \leq n$ and that $a_1, \ldots, a_n, b_1, \ldots, b_n$ are $2n$ distinct elements. Consider the n pairs $\{a_1, b_1\}, \ldots, \{a_n, b_n\}$. Find the number of subsets of $\{a_1, \ldots a_n, b_1, \ldots, b_n\}$ of size k that do not contain two elements from the same pair.

6) Find the number of shortest paths from the bottom left corner to the top right corner in the grid below.



7) (a) Let $X = \{1, b, R\}$. Find all elements of $\mathcal{P}(X)$. Recall that this is the power set of X and it is the set of all possible subets of X.

(b) Suppose X is a set with n elements. What is the cardinality of $\mathcal{P}(X)$?

8) Find the number of 12-digit binary sequences with eight 0's and four 1's such that no two 1's are adjacent.

9) (a) Let t_n be the number of ways to pave a $1 \times n$ rectangle using 1×1 and 1×2 blocks. Determine t_1, t_2, t_3, t_4, and t_5.

(b) Can you determine t_6 using what you know about t_5 and t_4?