California State University Sacramento - Math 101

Exam # 3

Name: ________________________

1) (a) State the Binomial Theorem. (4 points)

(b) Use the Binomial Theorem to determine the coefficient of x^7y^{12} in the expansion of $(x+y)^{19}$. (3 points)

2) Write the sum $\binom{10}{3} + \binom{10}{4}$ as a single binomial coefficient. (3 points)

3) Find the coefficient of $x_1^3x_2^2x_3$ in the expansion of $(x_1+x_2+x_3)^6$. (3 points)

4) Show that $\sum_{r=0}^{n} 3^r \binom{n}{r} = 4^n$ for all integers $n \geq 1$. (4 points)

5) Write the sum $\sum_{r=0}^{6} \binom{12}{r} \binom{10}{6-r}$ as a single binomial coefficient. (4 points)

6) Write the sum $\binom{4}{4} + \binom{5}{4} + \binom{6}{4} + \binom{7}{4}$ as a single binomial coefficient. (3 points)

7) Find the coefficient of x^7 in the expansion of $(1 + x + x^3)^{12}$. (5 points)

8) State the Pigeonhole Principle as stated in class. (4 points)

9) Suppose that 37 people are in a classroom. At least how many people have their birthdays in the same month? (3 points)

10) Show that for any set of five points in a 2 by 2 square, there are at least two points in the set whose distance apart is at most $\sqrt{2}$. (4 points)

11) Show that for any set of 5 points on the unit circle, there are at least two points in the set whose distance apart is at most $\sqrt{2}$. (3 points)

12) Determine the exact value of $\displaystyle\sum_{r=0}^{10} \binom{10}{r}$. (3 points)

13) Show that $\sum_{r=0}^{n} \frac{n+1}{r+1} \binom{n}{r} = 2^{n+1} - 1$ for all integers $n \geq 1$. (4 points)