Quiz 4

1) (a) $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

and

$$\frac{n}{r}\binom{n-1}{r-1} = \frac{n}{r} \cdot \frac{(n-1)!}{(r-1)!(n-1-(r-1))!}$$

$$= \frac{n!}{r!(n-1-r+1)!} = \frac{n!}{r!(n-r)!}$$

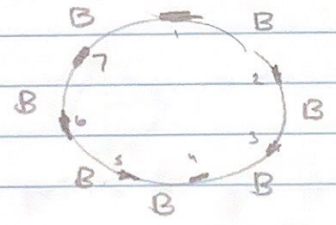
equal

2) (a) $\binom{6}{2} = 15$

(b)

n	2	3	4	5	6
$2^{\binom{n}{2}}$	$2^1 = 2$	$2^{\binom{3}{2}} = 2^3 = 8$	$2^{\binom{4}{2}} = 2^6 = 64$	$2^{\binom{5}{2}} = 2^{10} = 1024$	$2^{\binom{6}{2}} = 2^{15} = 32,768$

3) $Q_7^7 = 6!$ ways for boys to be placed



7·6·5 ways to place the girls at the open seats

$\Rightarrow 6! \cdot 7 \cdot 6 \cdot 5$

4) $\binom{5}{3}\binom{7}{2}$

↑ choose boys ↑ choose girls

5) $\binom{5}{1}\binom{7}{3} + \binom{5}{2}\binom{7}{2} + \binom{5}{3}\binom{7}{1} + \binom{5}{4}$

↑ ↑ ↑ ↑

# of ways to form committee with

1 B, 3 G 2B, 2G 3B, 1G 4B

6)

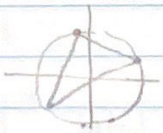
a	b
1	3
2	4
3	1 or 5
4	2 or 6
5	3 or 7
6	4 or 8
7	5
8	6

Total count is

$1+1+2+2+2+2+1+1$

$= 12$

7) $\binom{n}{3}$



choose 3 points to get a triangle

8) (a) $\binom{8}{3}$ ← choose position of the 0's

(b) $\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3}$

(c) 2^8 since either 0 or 1 in the 8 positions

9) $\binom{10}{3} \cdot 3$

↑ choose 3 from 10 ↑ choose the leader

10) We are choosing from $\{1,3,5,7\}$ so

$$\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 2^4 - 1 = 15$$

11) (a) There are 4 choices for where to send 1 to in B
4 " 2
" 4 " 3 "
4 4

$\Rightarrow 4^4$ total functions

(b) $4!$ since 4 choices for 1, then 3 for 2, then 2 for 3.

(c) $\frac{4!}{4^4} = \frac{24}{256} = \frac{3}{32} \approx 0.09375$