1) (a) $P_r^n = \frac{n!}{(n-r)!}$ (b) $Q_r^n = \frac{n!}{r\cdot(n-r)!}$ (c) $C_r^n = \frac{n!}{r!\,(n-r)!}$

(d) $Q_r^n = \frac{1}{r} P_r^n$

2) $\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{}$ $\_\ \_$
1 2 3 4 5 6 7 8 9 10 11

9 choices for position of first girl

$$9 \cdot P_8^8 \cdot P_3^3 = 9 \cdot 8! \cdot 3!$$
$$= 2,177,280$$

3)

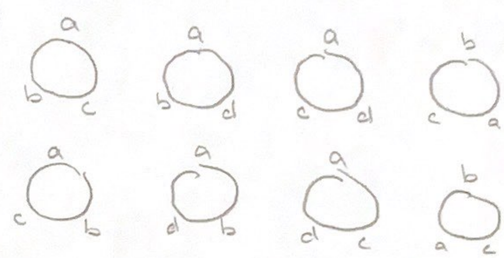
| a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| possible b | 1,2,3 | 1,2,3,4 | 1,2,3,4,5 | 2,3,4,5,6 | 3,4,5,6,7 | 4,5,6,7,8 | 5,6,7,8,9 | 6,7,8,9,10 | 7,8,9,10 | 8,9,10 |

The number of pairs is

$$3 + 4 + 6\cdot 5 + 4 + 3 = 44$$

4) (a) 12, 13, 14, 21, 23, 24, 31, 32, 34, 41, 42, 43

(b)



(c) $\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}$

5) (a) $10^{30} = 2^{30}5^{30}$ and $20^{20} = 2^{40}5^{20}$

must divide $2^{30}$ must divide $5^{20}$

31 choices 21 choices

$2^{\square}\,5^{\square}$

$21 \cdot 31 = 651$ common positive divisors

(b) $2^{\overset{0,1,2}{\square}}\,3^{\overset{0,1,2,3}{\square}}\,5^{\overset{1,2}{\square}}$ There are $3 \times 4 \times 2 = 24$ such divisors

6) $\underset{1 \text{ or } 2}{\underline{\ a_1\ }}\ \underline{\ a_2\ }\ \underline{\ a_3\ }\ \underset{1,3,5,7 \text{ or } 9}{\underline{\ a_4\ }}$

If $a_1 = 2$, then there are 5 choices for $a_4$, then $8 \cdot 7$ choices for $a_2$ and $a_3$.

If $a_1 = 1$, then there are 4 choices for $a_4$, and $8 \cdot 7$ choices for $a_2$ and $a_3$.

Total: $5 \times 8 \times 7 + 4 \times 8 \times 7$

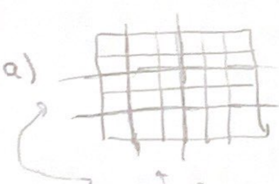
7) (a) $2^5 = 32$ (b) $\binom{5}{2} = 10$ (c) $\binom{5}{0} + \binom{5}{1} + \binom{5}{2} = 16$

choose positions for 1's

_ _ _ _ _

(d) $n+1$ since only one sequence of all 0's, and $\binom{n}{1} = n$ sequences with exactly one 1.

8) (a)



$5 \times 6 = 30$ is the number of $1 \times 1$ squares

(b) $4 \times 4 = 16$

(c)

$$\underbrace{5 \times 6}_{\substack{1\times1 \\ \text{squares}}} + \underbrace{4 \times 5}_{\substack{2\times2 \\ \text{squares}}} + \underbrace{3 \times 4}_{\substack{3\times3 \\ \text{squares}}} + \underbrace{2 \times 3}_{\substack{4\times4 \\ \text{squares}}} + \underbrace{1 \times 2}_{\substack{5\times5 \\ \text{squares}}}$$

$$= 30 + 20 + 12 + 6 + 2$$

$$= 70$$

(d) $\binom{6}{2}\binom{7}{2}$ is the number of rectangles

9) (a) $n P_{r-1}^{n-1} = n \cdot \frac{(n-1)!}{(n-1-(r-1))!} = \frac{n!}{(n-1-r+1)!} = \frac{n!}{(n-r)!} = P_r^n$

(b) $\binom{n-1}{r-1} + \binom{n-1}{r}$

$$= \frac{(n-1)!}{(n-1-(r-1))!\,(r-1)!} + \frac{(n-1)!}{(n-1-r)!\,r!}$$

multiplied first fraction by $\frac{r}{r}$

multiplied second fraction by $\frac{n-r}{n-r}$

$$= \frac{r \cdot (n-1)!}{(n-r)!\,r!} + \frac{(n-1)! \cdot (n-r)}{(n-r)!\,r!}$$

$$= \frac{r \cdot (n-1)! + (n-1)! \cdot (n-r)}{(n-r)!\,r!}$$

$$= \frac{(n-1)!\,(r+n-r)}{(n-r)!\,r!} = \frac{n!}{(n-r)!\,r!} = \binom{n}{r}$$