1) $x^2 = x + 6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$

$\Rightarrow x = 3, x = -2$

$\Rightarrow a_n = A \cdot 3^n + B(-2)^n$

$a_0 = 0 \Rightarrow 0 = A + B \Rightarrow B = -A$

$a_1 = 1 \Rightarrow 1 = 3A - 2B$

$1 = 3A + 2A$

$A = 1/5$

$B = -1/5$

$a_n = \frac{1}{5} \cdot 3^n - \frac{1}{5} \cdot (-2)^n$

2) $x^2 = 6x - 9 \Rightarrow x^2 - 6x + 9 = 0$

$\Rightarrow (x-3)^2 = 0 \Rightarrow x = 3$

$\Rightarrow a_n = (A + Bn) \cdot 3^n$

$a_0 = 1 \Rightarrow 1 = A$

$a_n = (1 + Bn) \cdot 3^n$

$a_1 = 2 \Rightarrow 2 = (1 + B) \cdot 3$

$2 = 3 + 3B$

$-\frac{1}{3} = B$

$a_n = \left(1 - \frac{n}{3}\right) \cdot 3^n$

3) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C|$
$+ |A \cap B \cap C|$

4) If A_k is the number of integers in $\{1, 2, \dots, 180\}$ divisible by k, then

$A_{36} = A_4 \cap A_9$

$A_4 = \left\lfloor \frac{180}{4} \right\rfloor = 45, A_9 = \left\lfloor \frac{180}{9} \right\rfloor = 20, A_{36} = \left\lfloor \frac{180}{36} \right\rfloor = 5$

The number of integers divisible by 4 or 9 is

$|A_4 \cup A_9| = 45 + 20 - 5 = 60$

5) Using the same method as in #4, the number of integers in $\{1,2,\dots,180\}$ that are divisible by 2, 3, or 5 is

$$\frac{180}{2}+\frac{160}{3}+\frac{180}{5}-\frac{180}{6}-\frac{180}{10}-\frac{180}{15}+\frac{180}{30}$$

$= 132$

6) Using the obvious notation, we have

$80 = |B\cup S\cup V|$, $20 = |B|$, $30 = |S|$, $45 = |V|$, $5 = |B\cap S\cap V|$.

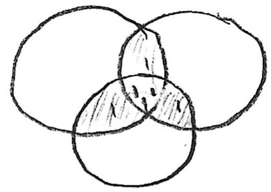
Thus,

$$80 = |B\cup S\cup V| = 20+30+45-|B\cap S|-|B\cap V|-|S\cap V|+5$$

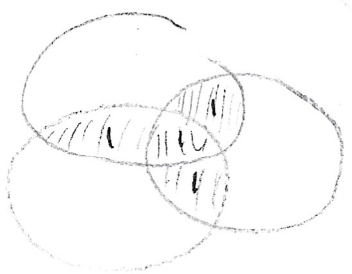
$$= 100 - |B\cap S| - |B\cap V| - |S\cap V|.$$

This implies

$$|B\cap S| + |B\cap V| + |S\cap V| = 20$$



the set $B\cap S\cap V$ is counted two additional times in this sum.



$$20 - 2|B\cap S\cap V|$$

$$= 20 - 2(5) = 10.$$

10 students played at least three sports

7) As in #6,

$$80 = |B \cup S \cup V| = \overbrace{45 + 30 + 45}^{120} - |B \cap S| - |B \cap V| - |S \cap V| + |B \cap S \cap V|$$

therefore,

$$|B \cap S \cap V| = |B \cap S| + |B \cap V| + |S \cap V| - 40$$

$$\leq 15 + 10 + 20 - 40 = 5$$

At most 5 played all three sports

8) (a) $x^3 = x^2 + x - 1 \Rightarrow x^3 - x^2 - x + 1 = 0$

$\Rightarrow x^2(x-1) - 1(x-1) = 0$

$\Rightarrow (x^2-1)(x-1) = 0$

$\Rightarrow (x-1)^2(x+1) = 0$ $\qquad \underbrace{x=1}_{\text{repeat root}} \qquad x = -1$

$$a_n = (A + Bn) \cdot 1^n + C(-1)^n$$

$a_0 = 1 \Rightarrow$ $1 = A + C$

$a_1 = 1$ $\qquad 1 = A + B - C$

$a_2 = 2$ $\qquad 2 = A + 2B + C$

subtract $1 = A + C$ from $2 = A + 2B + C$

$1 = 2B \Rightarrow B = 1/2$

Solving for A and C gives $A = 3/4$, $C = 1/4$ so

$$a_n = \left(\frac{3}{4} + \frac{n}{2}\right) + \frac{1}{4}(-1)^n$$

(b) $$a_{n+3} = a_{n+2} + a_{n+1} + a_n$$

$a_0 = 1$
$a_1 = 1$
$a_2 = 2$
$a_3 = 4$

stair $n+3$

arrive at $n+3$ by taking three steps at once at stair n