Homework Assignment 02

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SUBJECT: Homework Assignment 02 DATE: 2023/02/05

Note to Professor
Noticed it's Unordered {a,b} NOT ordered (a,b) So...
1.i = 45
and
1.ii = 235

(i) $|a-b|=5$

1. Find the number of ways to choose a pair $\{a, b\}$ of distinct numbers from the set $\{1, 2, ..., 50\}$ such that
(i) $|a-b| = 5$; (ii) $|a-b| \le 5$.

a & b can't be the same number from set {1,2,3,...50}

- min: 1, 6
- max: 45, 50
- from min to max we have 45 possibilites of (a,b) pairs

If given a restriction of a>b; then, the solutions we can have follows this tuple pattern: (6,1), (7,2), ...(50,45)

Because we are taking the absolute value of the difference of both 'a' and 'b' the order of either 'a' or 'b' does not matter, that is 45 more possible pairs of (a,b)

So, 45 possibilites (a>b) + 45 possibilites (a<b) = $\boxed{\text{90 possible ways of (a,b) pairings}}$

(ii) $|a-b|\le 5$

Let the smaller of both 'a' and 'b' be 'n' where $1 \le n \le 45$, then there are 5 pairs of distinct numbers from $\{1,2,3,...,50\}$: $\{n,n+1\}, \{n,n+2\}, \{n,n+3\}, \{n,n+4\}, \{n,n+5\}$. Gives us: $(45)(5) = 225$

n plus offset 1→5 is to account for distinct numbers for either 'a' or 'b'.
→ 45 (where both 'a' and 'b' are the smallest number in the set)
→ 5 (five pairs of distinct numbers from set {1,2,3,...,50}

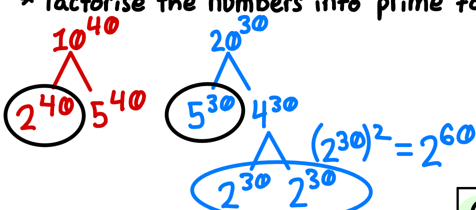
$45-(45+1)=1$
$45-(45+2)=2$
$\vdots$
$45-(45+5)=5$

When $46 \ge n < 50$
- $n=46$ there are 4 pairs: (46,47), (46,48), (46,49), (46,50)
- $n=47$ there are 3 pairs: (47,48), (47,49), (47,50)
- $n=48$ there are 2 pairs: (48,49), (48,50)
- $n=49$ there is 1 pair: (49,50)

(a<b): $[(45)(5)]+4+3+2+1 = 235$ possibilities.
Because we have the absolute value account (a>b):
meaning we have 235 more possibilities. $\boxed{\text{So in total } 235+235=470}$

10. Find the number of common positive divisors of 10^{40} and 20^{30}.

* Factorise the numbers into prime factors *



Recall — Prime factorization form
Let $n = p_1^{k_1}, p_2^{k_2}, ..., p_r^{k_r}$
then the positive divisor of $n = (k_1+1)(k_2+1)...(k_r+1)$

$\boxed{\text{So } (40+1)(30+1) = 1271}$

11. In each of the following, find the number of positive divisors of n (inclusive of n) which are multiples of 3:
(i) $n = 210$; (ii) $n = 630$; (iii) $n = 151200$.

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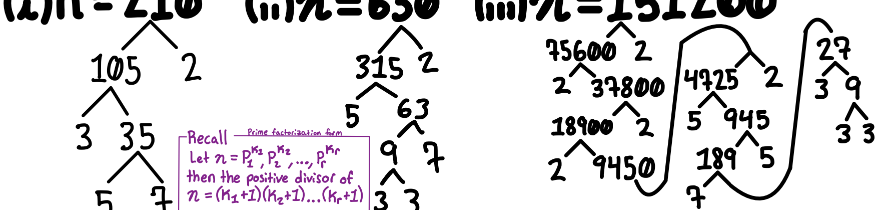

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(i)
→ $2^1 \cdot 3^1 \cdot 5^1 \cdot 7^1$
→ $(1+1)(1)(1+1)(1+1)$
→ $\boxed{2\cdot1\cdot2\cdot2 = 8}$

(ii)
→ $2^1 \cdot 3^2 \cdot 5^1 \cdot 7^1$
→ $(1+1)(2)(1+1)(1+1)$
→ $\boxed{2\cdot2\cdot2\cdot2 = 16}$

(iii)
→ $2^5 \cdot 3^3 \cdot 5^2 \cdot 7^1$
→ $(5+1)(3)(2+1)(1+1)$
→ $\boxed{6\cdot3\cdot3\cdot2 = 108}$

12) Show that for any $n \in \mathbb{N}$, the number of positive divisors of n^2 is always odd

If prime factorization of $n = p_1^{k_1}, p_2^{k_2}, ..., p_r^{k_r}$ then $n^2 = (p_1^{k_1})^2, (p_2^{k_2})^2, ...(p_r^{k_r})^2$ the resulting positive divisor of $n^2 = (2k_1+1)(2k_2+1)...(2k_r+1)$ follows the general form and the definition of odd $(2n+1)$.

5) Find the number of ordered pairs (x,y) of integers such that $x^2+y^2 \le 4$.

case A	case B	case C	case D	case E
$x^2+y^2=0$	$x^2+y^2=1$	$x^2+y^2=2$	$x^2+y^2=3$	$x^2+y^2=4$
(0,0)	(0,1)(1,0) (0,-1)(-1,0)	(-1,1)(1,1) (-1,-1)(1,-1)	NONE	(0,2)(2,0) (0,-2)(-2,0)

So in total $\boxed{1+4+4+0+4 = 13}$

6) Find the number of sequences $a_1a_2a_3$ of length 3 where $a_i \in \{0,1,2,3,4\}$.

3 groupings $(a_1a_2a_3)$ of 5 elements (0,1,2,3,4):

So $\boxed{5\cdot5\cdot5 = 125}$

*Think $P_r^n = \frac{n!}{(n-r)!}$ = # of Permutations

7) Let $X = \{1,2,3,...,10\}$ and let $S\{(a,b,c): a,b,c \in X, a<b \text{ and } a<c\}$. Find $|S|$.

Key hint is $c \in X$, so we can try

$\sum_{x=1}^{10} (x^b \cdot x^c) \to [(1\cdot1)+(2\cdot2)+(3\cdot3)+...+(10\cdot10)] = 385$