California State University Sacramento - Math 101

**Homework Assignment 2**

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**4)** Problem 12 on page 51

Q1) **1. Find the number of ways to choose a pair $\{a, b\}$ of distinct numbers from the set $\{1, 2, ..., 50\}$ such that**

**(i) $|a - b| = 5$; (ii) $|a - b| \leq 5$.**

Q2) **10. Find the number of common positive divisors of $10^{40}$ and $20^{30}$.**

Q3) **11. In each of the following, find the number of positive divisors of $n$ (inclusive of $n$) which are multiples of 3:**

**(i) $n = 210$; (ii) $n = 630$; (iii) $n = 151200$.**

Q4) **12. Show that for any $n \in \mathbf{N}$, the number of positive divisors of $n^2$ is always odd.**

Q **5)** Find the number of ordered pairs $(x, y)$ of integers such that $x^2 + y^2 \leq 4$.
Remark: This problem is similar to Example 1.1.2.

Q **6)** Find the number of sequences $a_1a_2a_3$ of length 3 where $a_i \in \{0, 1, 2, 3, 4\}$.
Remark: This is a special case of Example 1.1.4.

Q **7)** Let $X = \{1, 2, \ldots, 10\}$ and let

$$S = \{(a, b, c) : a, b, c \in X, a < b \text{ and } a < c\}.$$

Find $|S|$.
Remark: This problem is similar to Example 1.1.6.

SUBJECT: **Homework Assignment 02** **Matthew Mendoza** DATE: 2023/02/05

**Note to Professor**
Noticed it's Unordered $\{a,b\}$ NOT ordered $(a,b)$ So... 1.i = 45 and 1.ii = 235

## (i) $|a-b| = 5$

1. Find the number of ways to choose a pair $\{a, b\}$ of distinct numbers from the set $\{1, 2, ..., 50\}$ such that
(i) $|a - b| = 5$; (ii) $|a - b| \le 5$.

not ordered

a & b can't be the Same number from Set $\{1,2,3,...50\}$

- min: 1, 6
- max: 45, 50
- from min to max we have 45 possibilites of (a,b) pairs

If given a restriction of a>b; then, the solutions we can have follows this tuple pattern: (6,1), (7,2), ...(50,45)

Because we are taking the absolute value of the difference of both 'a' and 'b' the order of either 'a' or 'b' does not matter, that is 45 more possible pairs of (a,b)

So, $\boxed{\text{45 possible ways of (a,b) pairings}}$

## (ii) $|a-b| \le 5$

Let the smaller of both 'a' and 'b' be '$n$' where $1 \le n \le 45$, then there are 5 pairs of distinct numbers from $\{1,2,3,...,50\}$: $\{n,n+1\}$, $\{n,n+2\}$, $\{n,n+3\}$, $\{n,n+4\}$, $\{n,n+5\}$. Gives us: $(45)(5) = 225$

n plus offset 1→5 is to account for distinct numbers for either 'a' or 'b'.
→ 45 (where both 'a' and 'b' are the smallest number in the set)
→ 5 (five pairs of distinct numbers from set $\{1,2,3,...,50\}$

$45-(45+1)=1$
$45-(45+2)=2$
$\vdots$
$45-(45+5)=5$

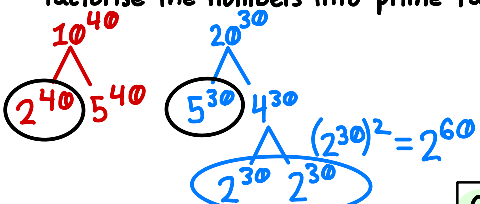
When $46 \ge n < 50$
- $n = 46$ there are 4 pairs: (46,47), (46,48), (46,49), (46,50)
- $n = 47$ there are 3 pairs: (47,48), (47,49), (47,50)
- $n = 48$ there are 2 pairs: (48,49), (48,50)
- $n = 49$ there is 1 pair: (49,50)

$(a<b)$: $[(45)(5)] + 4 + 3 + 2 + 1 = 235$ possibilities.

So in total $\boxed{\text{235 possibilities}}$

10. Find the number of common positive divisors of $10^{40}$ and $20^{30}$.

* Factorise the numbers into prime factors *

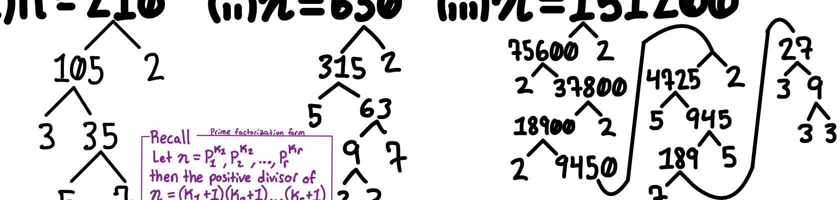



Recall — Prime factorization form
Let $n = P_1^{K_1}, P_2^{K_2}, ..., P_r^{K_r}$
then the positive divisor of $n = (K_1+1)(K_2+1)...(K_r+1)$

So $\boxed{(40+1)(30+1) = 1271}$

11. In each of the following, find the number of positive divisors of $n$ (inclusive of $n$) which are multiples of 3:
(i) $n = 210$; (ii) $n = 630$; (iii) $n = 151200$.

**(i) $n = 210$  (ii) $n = 630$  (iii) $n = 151200$**

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(i)
→ $2^1 \cdot 3^1 \cdot 5^1 \cdot 7^1$
→ $(1+1)(1)(1+1)(1+1)$
→ $\boxed{2 \cdot 1 \cdot 2 \cdot 2 = 8}$

(ii)
→ $2^1 \cdot 3^2 \cdot 5^1 \cdot 7^1$
→ $(1+1)(2)(1+1)(1+1)$
→ $\boxed{2 \cdot 2 \cdot 2 \cdot 2 = 16}$

(iii)
→ $2^5 \cdot 3^3 \cdot 5^2 \cdot 7^1$
→ $(5+1)(3)(2+1)(1+1)$
→ $\boxed{6 \cdot 3 \cdot 3 \cdot 2 = 108}$

## 12) Show that for any $n \in \mathbb{N}$, the number of positive divisors of $n^2$ is always odd

If prime factorization of $n = P_1^{K_1}, P_2^{K_2}, ..., P_r^{K_r}$ then $n^2 = (P_1^{K_1})^2 (P_2^{K_2})^2 ... (P_r^{K_r})^2$ the resulting positive divisor of $n^2 = (2K_1+1)(2K_2+1)...(2K_r+1)$ follows the general form and the definition of odd $(2n+1)$. * Product of odd numbers is odd *

## 5) Find the number of ordered pairs $(x, y)$ of integers such that $x^2+y^2 \le 4$.

| case A | case B | case C | case D | case E |
|---|---|---|---|---|
| $x^2+y^2=0$ | $x^2+y^2=1$ | $x^2+y^2=2$ | $x^2+y^2=3$ | $x^2+y^2=4$ |
| (0,0) | (0,1)(1,0) (0,-1)(-1,0) | (-1,1)(1,1) (-1,-1)(1,-1) | NONE | (0,2)(2,0) (0,-2)(-2,0) |

So in total $\boxed{1+4+4+0+4 = 13}$

## 6) Find the number of sequences $a_1a_2a_3$ of length 3 where $a_i \in \{0,1,2,3,4\}$.

3 groupings $(a_1a_2a_3)$ of 5 elements (0,1,2,3,4):

So $\boxed{5 \cdot 5 \cdot 5 = 125}$

*Think $P_r^n = \frac{n!}{(n-r)!}$ = # of Permutations

## 7) Let $X = \{1,2,3,...,10\}$ and let $S\{(a,b,c): a,b,c \in X, a<b \text{ and } a<c\}$. Find $|S|$.

Key hint is $c \in X$, so we can try

$\sum_{x=1}^{10} (\overset{b}{x} \cdot \overset{c}{x}) \rightarrow [(1 \cdot 1)+(2 \cdot 2)+(3 \cdot 3)+...+(9^2)] = 285$