California State University Sacramento - Math 101

Quiz #6

Name: ______________________

1) Let $X = \{1, 2, \ldots, 13, 14\}$.

(a) Find the number of 2-combinations of X. Simplify your answer as much as possible. (1 point)

(b) Find the number of 5-combinations of X that do not contain a pair of consecutive integers. Write your answer as a binomial coefficient. (1 point)

(a) $\binom{14}{2} = \frac{14 \cdot 13}{2} = 7 \cdot 13 = 91$

(b) $\binom{14-5+1}{5} = \binom{10}{5}$

2) Find the number of 13-digit binary sequences with nine 0's and four 1's such that no two 1's are adjacent. (2 points)

$\underset{1}{\_}\ \underset{2}{\_}\ \underset{3}{\_}\ \underset{4}{\_}\ \underset{5}{\_}\ \underset{6}{\_}\ \underset{7}{\_}\ \underset{8}{\_}\ \underset{9}{\_}\ \underset{10}{\_}\ \underset{11}{\_}\ \underset{12}{\_}\ \underset{13}{\_}$

Choose four positions for the 1's such that no positions are adjacent.

$\binom{13-4+1}{4} = \binom{10}{4}$

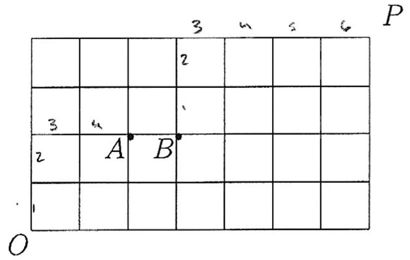
3) (a) Let $X = \{\{1\}, y\}$. Find all elements of $\mathcal{P}(X)$ (the power set of X). (1 point)

(b) If Y is a set with 6 elements, how many elements are in $\mathcal{P}(Y)$? (1 point)

(a) $\{\phi, \{\{1\}\}, \{y\}, \{\{1\}, y\}\}$

(b) $2^6 = 64$

4) Find the number of shortest routes from O to P that pass through the street AB. (2 points)



$$\underbrace{\binom{4}{2}}_{\text{from } O \text{ to } A} \cdot \underbrace{\binom{6}{2}}_{\text{from } B \text{ to } P} \quad \text{or} \quad \binom{4}{2} \cdot \binom{6}{4}$$

5) Suppose that k and n are positive integers with $3 \le k \le n$ and that $a_1, \ldots, a_n, b_1, \ldots, b_n$ are $2n$ distinct elements. Form the n pairs $\{a_1, b_1\}, \{a_2, b_2\}, \ldots, \{a_n, b_n\}$.

(a) Find the number of subsets of $\{a_1, \ldots a_n, b_1, \ldots, b_n\}$ of size k that ~~contain~~ do not contain two elements from the same pair. (1 point)

(b) Find the number of subsets of $\{a_1, \ldots a_n, b_1, \ldots, b_n\}$ of size k that contain exactly one of the pairs. (1 point)

(a) $$\underbrace{\binom{n}{k}}_{\text{choose } k \text{ pairs}} \cdot \underbrace{2^k}_{\text{choose one from each pair}}$$

(b) $$\underbrace{\binom{n}{1}}_{\text{choose one pair so have two people}} \underbrace{\binom{n-1}{k-2}}_{\text{choose } k-2 \text{ pairs}} \underbrace{2^{k-2}}_{\text{choose one from of the } k-2 \text{ pairs}}$$