

California State University Sacramento - Math 101

Homework Assignment 1 - Solutions

1) Let $A = \{-3, -2, -1, \dots, 5, 6, 7\}$.

(a) Is $1 \in A$? Yes. The set A contains 1.

(b) Is $\frac{1}{2} \in A$? No. The set A does not contain $\frac{1}{2}$.

(c) Find $|A|$.

$$|A| = | \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\} | = 11$$

(d) If $B = \{4, 6, 8, 10\}$, find $A \cup B$ and $A \cap B$.

$$A \cup B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 10\} \text{ and } A \cap B = \{4, 6\}$$

2) Suppose $A_1 = \{1, 2, 3\}$, $A_2 = \{3, 4, 5\}$, and $A_3 = \{4, 5, 6\}$.

(a) Find $A_1 \cup A_2 \cup A_3$.

$$A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6\}$$

(b) Find $A_1 \cap A_2 \cap A_3$.

$$A_1 \cap A_2 \cap A_3 = \emptyset$$

(c) True or False: $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$.

False since $|A_1 \cup A_2 \cup A_3| = 6$, but $|A_1| + |A_2| + |A_3| = 9$.

(d) True or False: $|A_1 \cap A_2 \cap A_3| = |A_1||A_2||A_3|$.

False since $|A_1 \cap A_2 \cap A_3| = 0$, but $|A_1||A_2||A_3| = 3^3 = 27$.

3) Suppose A_1, A_2, \dots, A_5 are pairwise disjoint sets with $|A_i| = i$ for $1 \leq i \leq 5$. Determine

$$\left| \bigcup_{i=1}^5 A_i \right|.$$

Since the sets A_1, A_2, \dots, A_5 are pairwise disjoint, we have

$$\left| \bigcup_{i=1}^5 A_i \right| = |A_1| + |A_2| + |A_3| + |A_4| + |A_5| = 1 + 2 + 3 + 4 + 5 = 15.$$

4) Find sets A_1, A_2, \dots, A_5 such that $|A_i| = i$ for $1 \leq i \leq 5$ and

$$\left| \bigcup_{i=1}^5 A_i \right| = 5.$$

Let $A_1 = \{1\}$, $A_2 = \{1, 2\}$, $A_3 = \{1, 2, 3\}$, $A_4 = \{1, 2, 3, 4\}$, and $A_5 = \{1, 2, 3, 4, 5\}$. Then

$$\left| \bigcup_{i=1}^5 A_i \right| = |\{1\} \cup \{1, 2\} \cup \{1, 2, 3\} \cup \{1, 2, 3, 4\} \cup \{1, 2, 3, 4, 5\}| = |\{1, 2, 3, 4, 5\}| = 5.$$

5) If $A = \{x : 3 \leq x \leq 10\}$ and \mathbb{Z} is the set of all integers, find

$$|A \cap \mathbb{Z}|$$

$$|A \cap \mathbb{Z}| = |\{3, 4, 5, 6, 7, 8, 9, 10\}| = 8$$