California State University Sacramento - Math 101

Quiz #4

Name: ____________________

1) For integers $1 \le r \le n$, give an algebraic proof that

$$\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}.$$

2) (a) Determine the number of unordered pairs of integers from the set $\{1, 2, 3, \ldots, 6\}$.

(b) Evaluate $2^{\binom{n}{2}}$ for $n = 2, 3, 4, 5, 6$.

3) In how many ways can 7 boys and 3 girls be seated around a table if no girls are adjacent?

4) In a group of 12 students, 7 of them are female. If exactly 3 boys are to be selected, in how many ways can 5 students be chosen from the group to form a committee?

5) In a group of 12 students, 7 of them are female. If at least one boy is to be selected, in how many ways can 4 students be chosen from the group to form a committee?

6) Find the number of ordered pairs of integers (a, b) where $|a - b| = 2$ and $a, b \in \{1, 2, 3, 4, 5, 6, 7, 8\}$.

7) Consider a set of n equally spaced points placed on the unit circle $x^2 + y^2 = 1$ in the x, y-plane. How many triangles are there whose vertices are the points on the circle?

8) (a) How many 0-1 sequences of length 8 have exactly three 0's?
(b) How many 0-1 sequences of length 8 have at most three 0's?
(c) What is the total number of 0-1 sequences of length 8?

9) In a group of ten people, we must form a committee consisting of three people where one of the people is the leader of the committee and the other two people are his/her assistant. How many ways can such a committee be formed?

10) Find the number of nonempty subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ that contain only odd numbers.

11) Let $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z, t\}$.

(a) Find the number of functions from A to B.

(b) Find the number of injective functions from A to B.

(c) Find the probability that a random function from A to B is injective.