California State University Sacramento - Math 101

Exam # 1

Name: ______________________

This exam is out of 12 points.

1) (a) Give a formula for P_r^n in terms of factorials. (0.25 points)

(b) Give a formula for Q_r^n in terms of factorials. (0.25 points)

(c) Give a formula for C_r^n in terms of factorials. (0.25 points)

(d) Express Q_r^n in terms of P_r^n. (0.25 points)

2) Find the number of ways that 8 boys and 3 girls can be put in a line such that the 3 girls form a single block. (1 point)

3) Find the number of **ordered** pairs (a, b), with $a, b \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, such that $|a - b| \leq 2$. (1 point)

4) Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$.

(a) List all 2-permutations of A. (0.5 points)

(b) List all 3-circular permutations of B. (0.5 points)

(c) List all subsets of A of size 3. (0.25 points)

5) (a) Find the number of common positive divisors of 10^{30} and 20^{20}. (1 point)

(b) Find the number of positive divisors of $900 = 2^2 \cdot 3^3 \cdot 5^2$ that are multiples of 5. (1 point)

6) Find the number of odd integers between 1000 and 3000 that have no repeated digit. (1 point)

7) (a) Determine the number of 0-1 sequences of length 5. For instance, 10101 and 11001 are two such sequences. (0.25 points)

(b) Determine the number of 0-1 sequences of length 5 that have exactly two 1's. (0.25 points)

(c) Determine the number of 0-1 sequences of length 5 that have at most two 1's (so the sequence can have no 1's, one 1, or two 1's). (0.5 points)

(d) Let n be an arbitrary integer. Determine the number of 0-1 sequences of length n with at most one 1 (so the sequence can have no 1's, or one 1). (0.5 points)

8) Let G be a 5×6 grid.

(a) Determine the number of $1x1$ squares in G. (0.5 points)

(b) Determine the number of 2×3 rectangles in G. (0.5 points)

(c) Determine the total number of squares in G. (0.5 points)

(d) Determine the total number of rectangles in G. (0.5 points)

9) (a) Give a proof, using algebra, that $P^n_r = nP^{n-1}_{r-1}$. (0.75 points)

(b) Give a proof, using algebra, that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$. (1 point)