$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$$

<u>Level 1</u>: plugging in #'s for x, y to get identity

$x = 1$
$y = 1$

$x = -1$
$y = 1$

$$2^n = \sum_{r=0}^{n} \binom{n}{r} \qquad 0 = \sum_{r=0}^{n} \binom{n}{r} (-1)^r$$

<u>Level 2</u>: Additional like a derivative or using a previous, or simplifying first

$$\sum_{r=m}^{n} \binom{n}{r}\binom{r}{m} \stackrel{?}{=} 2^{n-m}\binom{n}{m}$$

$$\sum_{r=m}^{n} \frac{n!}{\cancel{r!}\,(n-r)!} \cdot \frac{\cancel{r!}}{m!\,(r-m)!} \stackrel{?}{=} 2^{n-m} \frac{n!}{m!\,(n-m)!}$$

$$\frac{n!}{m!} \sum_{r=m}^{n} \frac{1}{(n-r)!\,(r-m)!} \stackrel{?}{=} \frac{n!}{m!} \cdot 2^{n-m} \cdot \frac{1}{(n-m)!}$$

$$\frac{n!}{m!} \sum_{r=m}^{n} \frac{(n-m)!}{(n-r)!\,(r-m)!} \stackrel{?}{=} \frac{n!}{m!} 2^{n-m}$$

$$\frac{n!}{m!}\sum_{r=m}^{n}\binom{n-m}{n-r} \stackrel{?}{=} 2^{n-m}\frac{\cancel{n!}}{m!}$$

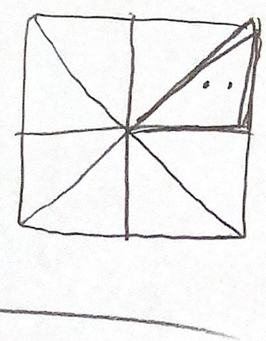
$$\binom{n-m}{n-r} = \frac{(n-m)!}{(n-r)!\,(n-m-(n-r))!}$$

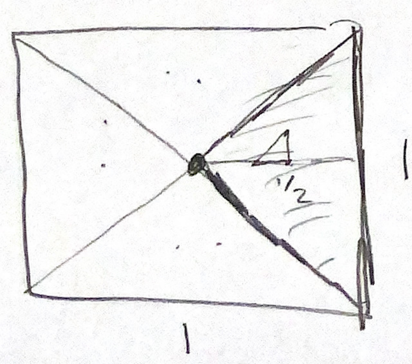
$$= \frac{(n-m)!}{(n-r)!\,(r-m)!}$$

$$\frac{n!}{m!}\Bigg(\underbrace{\binom{n-m}{n-m}+\binom{n-m}{n-m-1}+\dots+\binom{n-m}{0}}_{2^{n-m}}\Bigg)\stackrel{?}{=}2^{n-m}\frac{n!}{m!}$$

$\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

Match!





Area of small triangle formed by 3 points

$\leq$ Area of 

$= \frac{1}{2}(1)\left(\frac{1}{2}\right) = \frac{1}{4}$