# 1.3. Circular Permutations
# 1.4. Combinations

California State University Sacramento - Math 101

**Homework Assignment 4**

**1)** Example 1.3.2

**2)** Example 1.3.3

**3)** Problem 6 on page 51

**4)** Example 1.4.1

**5)** Example 1.4.2

Q1)

**Example 1.3.2.** In how many ways can 5 boys and 3 girls be seated around a table if

(i) there is no restriction?

(ii) boy $B_1$ and girl $G_1$ are not adjacent?

(iii) no girls are adjacent?

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Q2)

**Example 1.3.3.** Find the number of ways to seat $n$ married couples around a table in each of the following cases:

(i) Men and women alternate;

(ii) Every woman is next to her husband.

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Q3)

6. Find the number of *odd* integers between 3000 and 8000 in which no digit is repeated.

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Q4)

**Example 1.4.1.** Prove that

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}, \qquad (1.4.3)$$

where $n, r \in \mathbf{N}$ with $r \leq n$.

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Q5)

**Example 1.4.2.** By Example 1.1.4, there are $2^7$ binary sequences of length 7. How many such sequences are there which contain 3 0's and 4 1's?

## Recall

In general, if $Q^n_r$ is the number of arrangements of 'r' elements placed around a circle chosen from a set of 'n' elements, then

$$Q^n_r = \frac{P^n_r}{r} = \frac{\frac{n!}{(n-r)!}}{r} = \frac{n!}{r\cdot(n-r)!}$$

* Note that if $r=n$, then

$$Q^n_r = \frac{n!}{n(n-n)!} = \frac{n!}{n\cdot 0} = \frac{n!}{n\cdot 1} = \frac{n!}{n} = (n-n)!$$

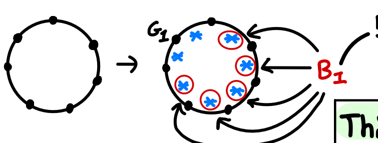
1) Example 1.3.2: In how many ways can 5 boys and 3 girls be seated around a table if

(i) there is no restriction?

$\boxed{\frac{8!}{8} = 7!}$

(ii) boy $B_1$ and girl $G_1$ are not adjacent?

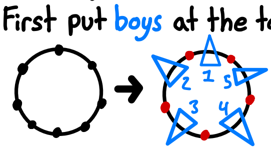
Let us place all except $B_1$ there $\frac{7!}{7} = 6!$ ways to place the seven people around the table (we are not placing $B_1$ yet)




This gives a total count of $6!\times 5$

(iii) no girls are adjacent?

First put boys at the table




$\frac{5!}{5} = 4!$ Possibilites

$5\cdot4\cdot3$ ways to place girls

$\Rightarrow$ Total is $4!\cdot5\cdot4\cdot3 = \boxed{1440}$

2) Example 1.3.3 – Find the number of ways to seat 'n' married couples around a table in each of the following cases:

(i) Men and women alternate

For every partner in a pairing can be arranged $(n-1)!$ ways, for every pairing has 'n' many significant other, so in total we have:

$\boxed{(n-1)!\,n!}$

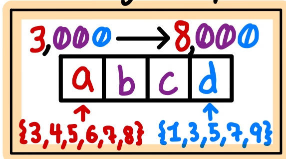
(ii) Every women is next to her husband

Can be thought as binary bits so,

$\boxed{(n-1)!\,2^n}$

Study #3 - Not sure why its 1232 & not 1512

*3) Exercise 1.6 – Find the number of odd integers between 3000 and 8000 in which no digit is repeated




Some case analysis would help...

Case 01: First digit is even — Can't do 8 upper limit

- 2 choices for first digit: 4, 6
- Don't have to worry about 'a' = 'd'
- 'bc' can range from [0,9] but we need to factor that both the leading and tail digit are duplicates

So for case 01: $2\times P^{10-2}_2\times 5 \rightarrow 2\times P^8_2\times 5 = 840$

Case 02: Leading digit is odd

- 3 choices for first digit: 3, 5, 7
- Account for 'a' being one of the five in set {1,3,5,7,9} leaves us with 4 choices
- 'bc' can range from [0,9] but we need to factor that both the leading and tail digit is repeated

So for case 02: $3\times P^{10-2}_2\times 4 \rightarrow 3\times P^8_2\times 4 = 672$

In total $840 + 672 = 1512$

4) Example 1.4.1 – Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$, where 'n', $r\in\mathbb{N}$ with $r\le n$

Recall: $C^n_r = \frac{P^n_r}{r!}$ we can rewrite as $C^n_r = \frac{n!}{r!(n-r)!}$

By algebraic proof...

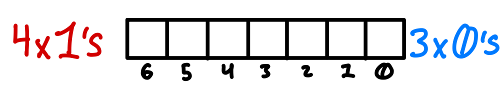
$$\binom{n-1}{r-1} + \binom{n-1}{r} = \frac{(n-1)!}{(r-1)!(n-r)!} + \frac{(n-1)!}{r!(n-1-r)!}$$

$$\rightarrow \frac{(n-r)!\,r + (n-1)!(n-r)}{r!(n-r)!}$$

$$\rightarrow \frac{(n-1)!(r+n-r)}{r!(n-r)!}$$

$$\boxed{\rightarrow \frac{n!}{r!(n-r)!} = \binom{n}{r}}$$

5) Example 1.4.2 – There are $2^7$ binary sequences of length 7. How many sequences are there which contains three 0's and four 1's?




Within a range of 7 bits how many ways can we position three zeros... Once placed we are left with the obvious remaining spaces to put 1's giving us $C^4_4 = 1$, So what we care about are only the zeros $\therefore \boxed{C^7_3 = 35}$