California State University Sacramento - Math 101

Exam #3

Name: ______________________

This exam is out of 7 points.

1) (a) Determine the exact value of the coefficient of x^3 in the expansion of $(x+1)^{13}$. (0.5 points)

(b) Determine the exact value of the coefficient of $x^3y^4z^2$ in the expansion of $(x+y+z)^9$. (0.5 points)

2) In the Binomial Theorem $(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$, state a value for x and value for y that produces the given formula. (0.25 points each)

(a) $0 = \sum_{r=0}^{n} \binom{n}{r} (-1)^r$

(b) $2^n = \sum_{r=0}^{n} \binom{n}{r}$

3) Show that for all integers $n \geq m \geq 1$, $\sum_{r=m}^{n} \binom{n-m}{n-r} = 2^{n-m}$. (0.5 points)

4) Prove $\sum_{r=0}^{n} r\binom{n}{r} = n2^{n-1}$ for all $n \geq 1$. (0.75 points)

5) Prove that $\binom{n}{r}\binom{n-r}{m-r} = \binom{n}{m}\binom{m}{r}$ for all integers $n \geq m \geq r \geq 1$. (0.75 points)

6) Let $X = \{1, 2, 3, 4\}$. Let $\mathcal{A}$ be the collection of all subsets of X with an even number of elements, and let $\mathcal{B}$ be the collection of all subsets of X with an odd number of elements. Remark: The empty set $\emptyset$ is one of the sets in $\mathcal{A}$ since it has 0 elements and 0 is even.

(a) List all of the elements of $\mathcal{A}$. (0.5 points)

(b) List all of the elements of $\mathcal{B}$. (0.5 points)

(c) Is the function $f(C) = X \backslash C$ a bijection from $\mathcal{A}$ to $\mathcal{B}$? Recall that $X \backslash C$ is the complement of C in X. (0.25 points)

(d) Is the function $f(C) = C \cup \{1\}$ a bijection from $\mathcal{A}$ to $\mathcal{B}$? (0.25 points)

(e) Draw a bijection between $\mathcal{A}$ and $\mathcal{B}$. Represent your bijection using an arrow diagram. (0.25 points)

7) (a) There are 50 jobs that must be assigned to 7 processors. Explain why there must be a processor that is assigned at least 8 jobs. (0.5 points)

(b) A list $\mathcal{L}$ contains 134 elements. Each element of $\mathcal{L}$ is a subset of $\{1, 2, 3, 4, 5, 6, 7\}$. Explain why the list $\mathcal{L}$ must contain two elements that are the same. (0.5 points)

(c) Suppose it takes a program 1 second to find the determinant of a 2×2 matrix. If S is the set of all 2×2 matrices of the form $\begin{pmatrix} x & y \\ z & t \end{pmatrix}$ where $x, y, z, t \in \{0, 1\}$, what is the minimum amount of time it would take the program to find the determinant of every matrix in S? (0.25 points)

8) Prove that $\sum_{r=0}^{n-1} \binom{2n-1}{r} = 2^{2n-2}$. (0.5 points)