

**Homework Assignment 5**

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2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if  
 (iv) between two particular boys  $A$  and  $B$ , there are no boys but exactly 3 girls?

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15. In a group of 15 students, 5 of them are female. If exactly 3 female students are to be selected, in how many ways can 9 students be chosen from the group  
 (i) to form a committee?  
 (ii) to take up 9 different posts in a committee?
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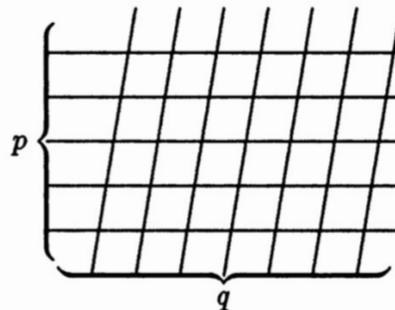
**Example 1.4.3.** In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if

- (i) there is no restriction in the selection?  
 (ii) the committee must include exactly 2 teachers?  
 (iii) the committee must include at least 3 teachers?
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2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if  
 (iii) no 2 girls are adjacent?
- 

20. In a group of 15 students, 3 of them are female. If at least one female student is to be selected, in how many ways can 7 students be chosen from the group  
 (i) to form a committee?  
 (ii) to take up 7 different posts in a committee?
- 

22. Two sets of parallel lines with  $p$  and  $q$  lines each are shown in the following diagram:



Find the number of parallelograms formed by the lines?

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23. There are 10 girls and 15 boys in a junior class, and 4 girls and 10 boys in a senior class. A committee of 7 members is to be formed from these 2 classes. Find the number of ways this can be done if the committee must have exactly 4 senior students and exactly 5 boys.

# Homework Assignment 05

SUBJECT:

DATE: 2023/02/21 PAGE#: 01/01

Q1

2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if  
 (iv) between two particular boys  $A$  and  $B$ , there are no boys but exactly 3 girls?

$$12 \text{ students} - 5 \text{ girls} = 7 \text{ boys in the party.}$$

X	X	X	X	A	G <sub>1</sub>	G <sub>2</sub>	G <sub>3</sub>	B	G <sub>4</sub>	G <sub>5</sub>	X
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"The crew"

- Consider as 1 unit (a group), so 7 single seats & 1 group seating  $\cong$  8 seats the group seating has their own arrangements
  - $A \& B$ -2 configurations ( $AB$ ) & ( $BA$ )
  - Girls in the center  $P_5^5$ , 5 girls total, but 3 can be w/crew
- So...  $(5+2+1)! \times [2! \times P_3^5] = 4,838,400$  ways  
 Not including  $A \& B$   $\downarrow$   
 Girls who aren't w/the crew

15. In a group of 15 students, 5 of them are female. If exactly 3 female students are to be selected, in how many ways can 9 students be chosen from the group

- (i) to form a committee?  
 (ii) to take up 9 different posts in a committee?

Info gathering & initial thoughts

- 15 students, 5 female, 10 male
- Exactly 3 female selected  $C_5^3$
- 9 students can be chosen to form a group
  - 3 female & 6 male students in a group
  - \* Arrangement order  $\sim$  9 member committee  $\sim$  does not matter!

(i) to form a committee (no additional requirements)  
 \*Because arrangement order does not matter we use  $C_9^9$  not  $P_9^9$

$$10 \text{ male, but } 6 \text{ in committee } C_6^{10} \times 5 \text{ female, but only } 3 \text{ in committee } C_3^5 = 2,100$$

(ii) to take up 9 different posts in a committee?

$$(C_6^{10} \times C_3^5) 9! = 76,204,800$$

\*9 different posts\*

- Q3 Example 1.4.3. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if

- (i) there is no restriction in the selection?  
 (ii) the committee must include exactly 2 teachers?  
 (iii) the committee must include at least 3 teachers?

Notes & initial thoughts  $T \quad S \quad T \quad S \quad S$

- Order does not matter, so we use  $C_5^n$  not  $P_5^n$  for the selection of either students & teachers

(iii) At least 3 teachers \*Needs case analysis\*

Case 01 - 3 teachers      Case 02 - 4 teachers

$$\binom{7}{2} \binom{4}{3} = 84 \quad \binom{7}{1} \binom{4}{4} = 7$$

Case 01 + Case 02

$$\text{So in total } \binom{7}{2} \binom{4}{3} + \binom{7}{1} \binom{4}{4} = 91$$

$$84 + 7 = 91$$

(i) No restrictions

$$\binom{11}{5} = 462$$

(ii) Exactly 2 teachers

$$\binom{7}{2} \binom{4}{3} = 210$$

Q4

2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if  
 (iii) no 2 girls are adjacent?

\* No 2 girls adjacent = two girls aren't together  
 $\rightarrow 12 \text{ students} - 5 \text{ girls} = 7 \text{ boys}$

\* Seating position matters, so we want to use  $P_7^n$

0	1	2	3	4	5	6	7	8	9	10	11
B	G	B	G	B	G	B	G	B	G	B	B

• 1 huge group seating & 2 single seating, so 3! of arranging 1 group and 2 single seaters

• Can start w/ B first or G first

B	G	B	G	B	G	B	G	B	B	B
Group Seating										

$$[2! \cdot P_5^7 \cdot P_5^5] \times 3! = 3,628,800$$

20. In a group of 15 students, 3 of them are female. If at least one female student is to be selected, in how many ways can 7 students be chosen from the group

(i) to form a committee?

(ii) to take up 7 different posts in a committee?

Notes - 7 student committee - At least 1 female

- 15 students - 3 female students = 12 male students
- Positional order does not matter, so use  $C_7^n$

(i) No additional restrictions

\* Needs case analysis \*

case 01	case 02	case 03
1 female	2 female	3 female
6 male	5 male	4 male
$\binom{3}{1} \binom{12}{6}$	$\binom{3}{2} \binom{12}{5}$	$\binom{3}{3} \binom{12}{4}$

$$\text{So } \binom{3}{1} \binom{12}{6} + \binom{3}{2} \binom{12}{5} + \binom{3}{3} \binom{12}{4}$$

$$\Rightarrow 2772 + 2376 + 495$$

$$= 5643$$

(ii) Each configuration has 7 different posts

case 01	case 02	case 03
1 female	2 female	3 female
6 male	5 male	4 male
7 Posts	7 Posts	7 Posts

$$\binom{3}{1} \binom{12}{6} ?!$$

$$\binom{3}{2} \binom{12}{5} ?!$$

$$\binom{3}{3} \binom{12}{4} ?!$$

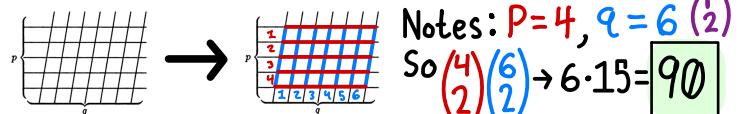
$$\text{So } \binom{3}{1} \binom{12}{6} ?! + \binom{3}{2} \binom{12}{5} ?! + \binom{3}{3} \binom{12}{4} ?!$$

$$\Rightarrow 2772(5040) + 2376(5040) + 495(5040)$$

$$= 28,440,720$$

22. Two sets of parallel lines with  $p$  and  $q$  lines each are shown in the following diagram:

Q6 Find the number of parallelograms formed by the lines?



Notes:  $P=4, Q=6$  (2)

$$\text{So } \binom{4}{2} \binom{6}{2} \rightarrow 6 \cdot 15 = 90$$

Took longer to think through... will this be on the exam?

23. There are 10 girls and 15 boys in a junior class, and 4 girls and 10 boys in a senior class. A committee of 7 members is to be formed from these 2 classes. Find the number of ways this can be done if the committee must have exactly 4 senior students and exactly 5 boys.

10 girls + 15 boys = 25 Juniors, 4 girls + 10 boys = 14 seniors

\* Sequential, arranged order, does not matter:  $C_7^n$

\* Find exactly 4 senior students (boy/girl) & 5 boys (JR/SR)

Approaches...  $\binom{10}{4} \binom{15}{3} \binom{10}{1} + \binom{10}{3} \binom{15}{2} \binom{10}{1} + \binom{10}{2} \binom{15}{3} \binom{10}{2}$

fill SR first?  $\binom{10}{4} \binom{15}{3} \binom{10}{1}$  Completes Committee

Can't have more than 4 seniors  $\uparrow$  5th boy Completes Committee

Completes Senior req.  $\uparrow$  Completes boy req.  $\uparrow$  Completes Committee

Completes Senior req.  $\uparrow$  Completes both boys & Committee req.

In total there are 768600 ways

21. Find the number of  $(m + n)$ -digit binary sequences with  $m$  0's and  $n$  1's such that no two 1's are adjacent, where  $n \leq m + 1$ .

Notes - a set of all strings w/no consecutive 1's where  $\Sigma = \{0, 1\}$   
- Base case:  $\emptyset, \cancel{00}, \cancel{01}, \cancel{10}, \cancel{11}, \cancel{000}, \cancel{001}, \cancel{010}, \cancel{100}, \cancel{110}, \cancel{111} \dots$   
- RegEx:  $\emptyset + [(\emptyset + 10)^* (\lambda + 1)]$