California State University Sacramento - Math 101

Homework Assignment 1

1) Let $A = \{-3, -2, -1, \ldots, 5, 6, 7\}$.

(a) Is $1 \in A$?

(b) Is $\frac{1}{2} \in A$?

(c) Find $|A|$.

(d) If $B = \{4, 6, 8, 10\}$, find $A \cup B$ and $A \cap B$.

2) Suppose $A_1 = \{1, 2, 3\}$, $A_2 = \{3, 4, 5\}$, and $A_3 = \{4, 5, 6\}$.

(a) Find $A_1 \cup A_2 \cup A_3$.

(b) Find $A_1 \cap A_2 \cap A_3$.

(c) True or False: $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$.

(d) True or False: $|A_1 \cap A_2 \cap A_3| = |A_1||A_2||A_3|$.

3) Suppose $A_1, A_2, \ldots, A_5$ are pairwise disjoint sets with $|A_i| = i$ for $1 \leq i \leq 5$. Determine

$$\left| \bigcup_{i=1}^{5} A_i \right|.$$

4) Find sets $A_1, A_2, \ldots, A_5$ such that $|A_i| = i$ for $1 \leq i \leq 5$ and

$$\left| \bigcup_{i=1}^{5} A_i \right| = 5.$$

5) If $A = \{x : 3 \leq x \leq 10\}$ and $\mathbb{Z}$ is the set of all integers, find

$$|A \cap \mathbb{Z}|$$

Attempt 02

SUBJECT: Homework Assignment 01

Matthew Mendoza

DATE: 2023/01/27 PAGE#:

1) Let $A = \{-3, -2, -1, \ldots, 5, 6, 7\}$.

(a) Is $1 \in A$? -1 is an element of A ∴ True

(b) Is $\frac{1}{2} \in A$? $-1/2$ is not an element of A ∴ False

(c) Find $|A|$. Cardinality of set A is 11

(d) If $B = \{4, 6, 8, 10\}$, find $A \cup B$ and $A \cap B$.

$A = \{-3,-2,-1,\emptyset,1,2,3,4,5,6,7\}$ $B = \{4, 6, 8, 10\}$

(Union)
$A \cup B = \{-3,-2,-1,\emptyset,1,2,3,4,5,6,7,8,10\}$

(Intersection)
$A \cap B = \{4,6\}$

2) Suppose $A_1 = \{1, 2, 3\}$, $A_2 = \{3, 4, 5\}$, and $A_3 = \{4, 5, 6\}$.

(a) Find $A_1 \cup A_2 \cup A_3$.

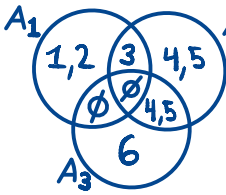
$A_1 = \{1,2,3\}$, $A_2 = \{3,4,5\}$, $A_3 = \{4,5,6\}$

$A_1 \cup A_2 \cup A_3 \Rightarrow \{1,2,3,4,5,6\}$

(b) Find $A_1 \cap A_2 \cap A_3$.

$A_1 = \{1,2,3\}$, $A_2 = \{3,4,5\}$, $A_3 = \{4,5,6\}$

$A_1 \cap A_2 \cap A_3 \Rightarrow$



∴ ∅ Null Empty Set

(c) True or False: $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$.

$A_1 = \{1,2,3\}$, $A_2 = \{3,4,5\}$, $A_3 = \{4,5,6\}$

cardinality
$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$

$6 = 3 + 3 + 3$

$6 \neq 9$

∴ False.

(d) True or False: $|A_1 \cap A_2 \cap A_3| = |A_1||A_2||A_3|$.

$A_1 = \{1,2,3\}$, $A_2 = \{3,4,5\}$, $A_3 = \{4,5,6\}$

cardinality
$|A_1 \cap A_2 \cap A_3| = |A_1||A_2||A_3|$

$\emptyset = 3 \cdot 3 \cdot 3$

$\emptyset \neq 81$

∴ False.

*Study this question

3) Suppose $A_1, A_2, \ldots, A_5$ are pairwise disjoint sets with $|A_i| = i$ for $1 \leq i \leq 5$.

Determine $\left|\bigcup_{i=1}^{5} A_i\right|$ Pairwise Disjoint

$\left|\bigcup_{i=1}^{5} A_i\right| \rightarrow$
$A_1 = \{1\} \rightarrow |A_i| = 1$
$A_2 = \{2,3\} \rightarrow |A_i| = 2$
$A_3 = \{4,5,6\} \rightarrow |A_i| = 3$
$A_4 = \{7,8,9,10\} \rightarrow |A_i| = 4$
$A_5 = \{11,12,13,14,15\} \rightarrow |A_i| = 5$

$\rightarrow A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$

∴ $\left|\bigcup_{i=1}^{5} A_i\right| \rightarrow |\{1, 2, 3, \ldots, 13, 14, 15\}| = 15$.

(Where $|A_i| = i$ for $1 \leq i \leq 5$)*

4) Find sets $A_1, A_2, \ldots A_5$ such that $|A_i| = i$ for $1 \leq i \leq 5$

and $\left|\bigcup_{i=1}^{5} A_i\right| = 5$

$\rightarrow |A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5| = 5$

(Where $|A_i| = i$ for $1 \leq i \leq 5$)*

∴ $\left|\bigcup_{i=1}^{5} A_i\right| \rightarrow$
$A_1 = \{1\}$
$A_2 = \{1,2\}$
$A_3 = \{1,2,3\}$
$A_4 = \{1,2,3,4\}$
$A_5 = \{1,2,3,4,5\}$

5) If $A = \{x : 3 \leq x \leq 10\}$ and $\mathbb{Z}$ is the set of all integers, find $|A \cap \mathbb{Z}|$.

$A = \{3,4,5,6,7,8,9,10\}$

$\mathbb{Z} = \{\ldots, -3, -2, -1, \emptyset, 1, 2, 3, \ldots\}$

∴ $|A \cap \mathbb{Z}| \rightarrow \{3,4,5,6,7,8,9,10\} = 8$

Attempt 01

Matthew Mendoza

SUBJECT: Homework Assignment 01 DATE: 2023/01/27 PAGE#:

1) Let $A = \{-3, -2, -1, \ldots, 5, 6, 7\}$.

(a) Is $1 \in A$? -1 is an element of A ∴ True

(b) Is $\frac{1}{2} \in A$? $-1/2$ is not an element of A ∴ False

(c) Find $|A|$. – Cardinality of set A is 11

(d) If $B = \{4, 6, 8, 10\}$, find $A \cup B$ and $A \cap B$.

$A = \{-3,-2,-1,\emptyset,1,2,3,4,5,6,7\}$ $B = \{4, 6, 8, 10\}$

(Union)
$A \cup B = \{-3,-2,-1,\emptyset,1,2,3,4,5,6,7,8,10\}$

(Intersection)
$A \cap B = \{4,6\}$

2) Suppose $A_1 = \{1, 2, 3\}$, $A_2 = \{3, 4, 5\}$, and $A_3 = \{4, 5, 6\}$.

(a) Find $A_1 \cup A_2 \cup A_3$.

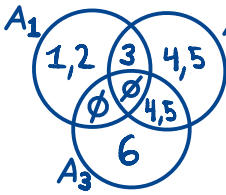
$A_1 = \{1,2,3\}$, $A_2 = \{3,4,5\}$, $A_3 = \{4,5,6\}$

$A_1 \cup A_2 \cup A_3 \Rightarrow \{1,2,3,4,5,6\}$

(b) Find $A_1 \cap A_2 \cap A_3$.

$A_1 = \{1,2,3\}$, $A_2 = \{3,4,5\}$, $A_3 = \{4,5,6\}$

$A_1 \cap A_2 \cap A_3 \Rightarrow$



∴ $\emptyset$ Null Empty Set

(c) True or False: $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$.

$A_1 = \{1,2,3\}$, $A_2 = \{3,4,5\}$, $A_3 = \{4,5,6\}$

cardinality
$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$

$6 = 3 + 3 + 3$

$6 \neq 9$

∴ False.

(d) True or False: $|A_1 \cap A_2 \cap A_3| = |A_1||A_2||A_3|$.

$A_1 = \{1,2,3\}$, $A_2 = \{3,4,5\}$, $A_3 = \{4,5,6\}$

cardinality
$|A_1 \cap A_2 \cap A_3| = |A_1||A_2||A_3|$

$\emptyset = 3 \cdot 3 \cdot 3$

$\emptyset \neq 81$

∴ False.

3) Suppose $A_1, A_2, \ldots, A_5$ are pairwise disjoint sets with $|A_i| = i$ for $1 \leq i \leq 5$.

Determine $\left|\bigcup_{i=1}^{5} A_i\right|$

$\left|\bigcup_{i=1}^{5} A_i\right| \rightarrow$ $A_1 = \{1,2,3,4,5\}$, $A_2 = \{1,2,3,4,5\}$, $A_3 = \{1,2,3,4,5\}$, $A_4 = \{1,2,3,4,5\}$, $A_5 = \{1,2,3,4,5\}$

$\rightarrow A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$

∴ $\left|\bigcup_{i=1}^{5} A_i\right| \rightarrow |\{1,2,3,4,5\}| = 5$.

(Where $|A_i| = i$ for $1 \leq i \leq 5$)*

4) Find sets $A_1, A_2, \ldots A_5$ such that $|A_i| = i$ for $1 \leq i \leq 5$

and $\left|\bigcup_{i=1}^{5} A_i\right| = 5$

$\rightarrow |A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5| = 5$

(Where $|A_i| = i$ for $1 \leq i \leq 5$)*

∴ $\left|\bigcup_{i=1}^{5} A_i\right| \rightarrow$ $A_1 = \{1\}$, $A_2 = \{1,2\}$, $A_3 = \{1,2,3\}$, $A_4 = \{1,2,3,4\}$, $A_5 = \{1,2,3,4,5\}$

5) If $A = \{x : 3 \leq x \leq 10\}$ and $\mathbb{Z}$ is the set of all integers, find $|A \cap \mathbb{Z}|$.

$A = \{3,4,5,6,7,8,9,10\}$

$\mathbb{Z} = \{\ldots, -3, -2, -1, \emptyset, 1, 2, 3, \ldots\}$

∴ $|A \cap \mathbb{Z}| \rightarrow \{3,4,5,6,7,8,9,10\} = 8$