California State University Sacramento - Math 101

Quiz #7

Name: _____

1) Let $S = \{1, 2, 3, 4\}$ and $T = \{(x, y, z) : x, y, z \in S, x < z, y < z\}$. Determine $|T|$.

2) (a) Find the number of permutations of the 6 letters a, a, a, b, b, c.

(b) Find the number of permutations of the multi-set $\{r_1 \cdot a_1, r_2 \cdot a_2, \ldots, r_n \cdot a_n\}$.

3) Let $M = \{\infty \cdot a_1, \infty \cdot a_2, \ldots, \infty \cdot a_n\}$. Let H_r^n be the number of r-element multi-subsets of M. Give a formula for H_r^n.

4) Show, using algebra, that for $n \geq r \geq 1$,

$$\binom{n}{r} = \frac{n - r + 1}{r} \binom{n}{r - 1}.$$

5) Let $A = \{a, b, c\}$ and $B = \{u, v, x, y, z\}$.

(a) Find the number of ordered pairs of the form (α, β) where $\alpha \in A$ and $\beta \in B$.

(b) Find the number of ordered pairs of the form (A', B') where A' is a subset of A with two elements, and B' is a subset of B with three elements.

(c) Find the number of 3-combinations of $A \cup B$.

6) Consider the matching shown below.

(a) Find the number of ways to choose three edges from the matching.

(b) Find the number of ways to choose exactly one endpoint (vertex) from every edge. For example, $\{a_1, b_2, b_3, a_4, b_5\}$ contains only one vertex from each edge.

(c) Find the number of ordered pairs of the form (α, β) where α is an edge and β is a set of size 4 containing exactly one vertex from the four edges not equal to α. For example, $(\{a_5, b_5\}, \{a_4, b_3, b_2, a_1\})$.

(d) Find the number of subsets of size 5 from $\{a_1, \ldots, a_5, b_1, \ldots, b_5\}$ that contain exactly one edge.

(e) Find the number of subsets of size 5 from $\{a_1, \ldots, a_5, b_1, \ldots, b_5\}$ that do not contain any of the edges.

(f) Find the number of subsets of size 5 from $\{a_1, \ldots, a_5, b_1, \ldots, b_5\}$ that contain exactly two edges.