

Homework Assignment 4

6) Write $a_1a_2a_3a_4$ for such a four digit integer. First, a_1 can be any of 3, 4, 5, 6, or 7, while a_4 can be 1, 3, 5, 7, or 9. These sets overlap so we will consider two cases.

If a_4 is 1 or 9 (2 choices), then a_1 can be 3, 4, 5, 6, or 7 (5 choices). We then have 8 choices for a_2 and 7 choices for a_3 because they must be distinct and chosen from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \setminus \{a_1, a_4\}$. In all, there are $2 \times 5 \times 8 \times 7 = 560$ possibilities.

Now suppose that a_4 is one of 3, 5, or 7 (3 choices). In this case there are only 4 choices for a_1 . There are still 8 choices for a_2 and 7 for a_3 giving a total of $3 \times 4 \times 8 \times 7 = 672$ possibilities.

Combining the two cases gives a total count of $560 + 672 = 1232$.