California State University Sacramento - Math 101

Exam # 2 - Numbers Changed

Name: ________________________

Exam is out of 8 points.

1) Let $X = \{1, 2, 3, 4, 5, 6, 7\}$.

(a) What is the number of 4-permutations of X? (0.25 points)

(b) What is the number of 4-circular permutations of X? (0.25 points)

(c) What is the number of 3-combinations of X? Simplify your answer as much as possible. (0.25 points)

(d) If $\mathcal{P}(X)$ is the power set of X, how many elements are in $\mathcal{P}(X)$? (0.25 points)

(e) Find the number of 2-combinations of X with only odd numbers. (0.25 points)

(f) Find the number of 3-combinations of X that do not contain a pair of consecutive integers. (0.25 points)

2) A box contains 16 distinct marbles. Ten of the marbles are red and six of the marbles are blue. Thus, the elements of the box could be represented as the set

$$\{r_1, r_2, r_3, r_4, r_5, \ldots, r_{10}, b_1, b_2, b_3, b_4, b_5, b_6\}.$$

(a) Find the number of ways to form a combination of three red marbles and five blue marbles. (0.25 points)

(b) Find the number of ways to form a combination of six marbles, at most two of which are blue. (0.5 points)

(c) How many ways can all 16 marbles be arranged around a circle so that no blue marbles are next to each other? (0.25 points)

(d) Find the number of permutations of all 16 marbles such that b_1 immediately precedes b_2. NOTE: This is different from Exam 2 where b_1 could be anywhere before b_2. (0.5 points)

3) Recall that a 0-1 sequence of length n is a sequence of the form $a_1 a_2 \cdots a_n$ such that each a_i is either 0 or 1.

(a) What is the total number of 0-1 sequences of length 8? Simplify your answer as much as possible. (0.25 points)

(b) Find the total number of 0-1 sequences of length 9 that have two, three, or four 0's. Simplify your answer as much as possible. (0.5 points)

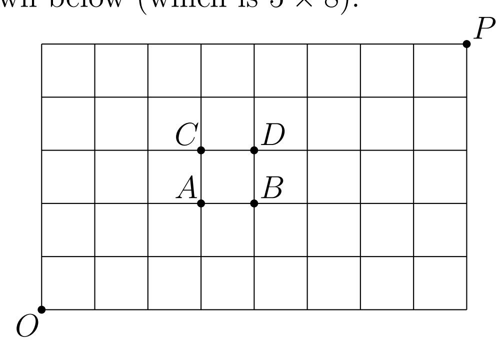
(c) 40 people are going to vote on their preference for a new menu item at Stars and Bars. The options are A, B, C, D, and E. How many possible outcomes are there after the 40 people vote? In other words, how many ways can 40 elements be chosen from A, B, C, D, and E allowing for repetition. (0.5 points)

4) Let $1 \le r \le n$.

(a) Give an algebraic proof that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$. Write your proof carefully and use correct mathematical notation. (0.5 points)

(b) Explain why $2^n = \sum_{j=0}^{n} \binom{n}{k}$ is true by describing a counting process that shows the number of subsets of $\{1, 2, \ldots, n\}$, which you can assume is 2^n, is equal to the right hand side of this equation. (0.5 points)

5) Consider the figure shown below (which is 5×8).



(a) Find the number of shortest routes from O to P. (0.25 points)

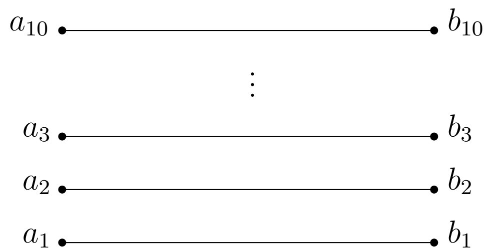
(b) Find the number of shortest routes from O to P that pass through the street AB or the street CD. (0.5 points)

6) Let $s(r, n)$ be the number of ways to arrange r distinct objects around n indistinguishable circles so that every circle has at least one object.

(a) Why is $s(r, r-1) = \binom{r}{2}$? (0.25 points)

(b) Determine, with explanation (which can include pictures) the value of $s(6, 2)$. (0.75 points)

7) Consider the matching shown below.



(a) Find the number of subsets of size 7 from $\{a_1, \ldots, a_{10}, b_1, \ldots, b_{10}\}$ that do not contain any of the edges. (0.25 points)

(b) Find the number of subsets of size 8 from $\{a_1, \ldots, a_{10}, b_1, \ldots, b_{10}\}$ that contain exactly one edge. (0.5 points)

(c) Find the number of subsets of size 12 from $\{a_1, \ldots, a_{10}, b_1, \ldots, b_{10}\}$ that contain exactly three edges. (0.25 points)