California State University Sacramento - Math 101

Quiz #10

Name: ______________________

1) Compute (a) $\binom{11}{3}$ (b) $\binom{8}{0}$

2) Find the coefficient of x^3y^8 in the expansion of $(x+y)^{11}$. Simplify your answer as much as possible.

3) Prove each statement using the Binomial Theorem.

(a) $2^n = \sum_{k=0}^{n} \binom{n}{k}$ (b) $3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k$ (c) $0 = \sum_{k=0}^{n} (-1)^k \binom{n}{k}$

4) Determine the exact value of $\sum_{r=1}^{10} \binom{10}{r}$.

5) Let $X = \{1, 2, 3, 4, 5\}$.

(a) If $A = \{1, 2, 3\}$, find $X \backslash A$.

(b) How many subsets of X have an odd number of elements?

(c) How many subsets of X have an even number of elements?

(d) Let $\mathcal{A}$ be the set of all subsets of X with an odd number of elements, and $\mathcal{B}$ be the set of all subsets of X with an even number of elements. If $f : \mathcal{A} \to \mathcal{B}$ is defined by $f(A) = X \backslash A$, find $f(\{5\})$.

6) Show that for any five points on the unit circle $x^2 + y^2 = 1$, there are at least two points that are within distance $\sqrt{2}$ of each other.

7) Suppose that there are 100 students in a class.

(a) Show that there are at least 9 students who were born in the same month.

(b) Suppose each student selects a 0-1 sequence of length 6. Must there exists two students who selected the same sequence?

(c) The class has a lottery where two numbers are chosen from $\{1, 2, \ldots, 15\}$. The winning ticket gets an A in the course. How many tickets should you purchase to ensure that you have a winning ticket?

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8) (a) Find the coefficient of $x^2y^3z^4$ in the expansion of $(x+y+z)^9$. You may leave your answer as a multinomial coefficent.

(b) Simplify $\binom{6}{1,2,3}$ as much as possible. You may want to use the formula

$$\binom{m}{m_1, m_2, \ldots, m_k} = \frac{(m_1 + m_2 + \cdots + m_k)!}{m_1! m_2! \cdots m_k!}$$

where $m = m_1 + m_2 + \cdots + m_k$.

9) (a) Recall that the power set of a set X is the set of all subsets of X. List all elements of the power set of $X = \{a, b, c\}$.

(b) Draw a bijection between the set of all 0-1 sequences of length 2 and all subsets of $\{x, y\}$.

(c) Draw a bijection between the set of all 0-1 sequences of length 3 and all subsets of $\{x, y, z\}$.

10) Let $X = \{1, 2, 3\}$. Suppose that five distinct subsets of X are chosen. Show that one of those subsets must contain 1.