California State University Sacramento - Math 101

Homework Assignment 8 - Solutions

6) If we take $x = 1$ and $y = 3$ in the Binomial Theorem, then we have

$$\sum_{r=0}^{n} \binom{n}{r} 1^{n-r} 3^{r} = (1+3)^{n}$$

which can be rewritten as

$$\sum_{r=0}^{n} \binom{n}{r} 3^{n} = 4^{n}.$$

7) From Examples 2.3.1 and 2.3.3 we know that

$$\sum_{r=0}^{n} r\binom{n}{r} = n2^{n-1} \quad \text{and} \quad \sum_{r=0}^{n} \binom{n}{r} = 2^{n}.$$

Adding these two equations together gives

$$\sum_{r=0}^{n} (r+1)\binom{n}{r} = n2^{n-1} + 2^{n}$$

but the right hand side is equal to $(n+2)2^{n-1}$.

8) We have

$$\sum_{r=0}^{n} \frac{n+1}{r+1}\binom{n}{r} = \sum_{r=0}^{n} \binom{n+1}{r+1} = \sum_{r=1}^{n+1} \binom{n+1}{r}. \tag{1}$$

We know from Example 2.3.1 that

$$2^{n+1} = \sum_{r=0}^{n+1} \binom{n+1}{r} = \binom{n+1}{0} + \sum_{r=1}^{n+1} \binom{n+1}{r}.$$

Since $\binom{n+1}{0} = 1$, we can rewrite this last equation as

$$2^{n+1} - 1 = \sum_{r=1}^{n+1} \binom{n+1}{r}. \tag{2}$$

Combining (1) and (2) we get

$$\sum_{r=0}^{n} \frac{n+1}{r+1}\binom{n}{r} = 2^{n+1} - 1$$

which can be rewritten as $\displaystyle\sum_{r=0}^{n} \frac{1}{r+1}\binom{n}{r} = \frac{1}{n+1}(2^{n+1} - 1)$.