California State University Sacramento - Math 101

Homework Assignment 9

1) Example 2.3.4 on page 74

2) The statement of Theorem 2.8.1

3) Example 2.8.1 on page 99

4) Problem 28 on page 106.

5) Problem 31 on page 106.

6) State the Pigeonhole Principle

7) Example 3.2.4

8) Problem 1 on page 137

9) Problem 3 on page 137

1) **Example 2.3.4.** **(Vandermonde's Identity)** Show that for all $m, n, r \in \mathbf{N}$,

$$\sum_{i=0}^{r} \binom{m}{i}\binom{n}{r-i} = \binom{m}{0}\binom{n}{r} + \binom{m}{1}\binom{n}{r-1} + \cdots + \binom{m}{r}\binom{n}{0}$$
$$= \binom{m+n}{r}. \qquad (2.3.5)$$

2) **Theorem 2.8.1 (The Multinomial Theorem).** *For $n, m \in \mathbf{N}$,*

$$(x_1 + x_2 + \cdots + x_m)^n = \sum \binom{n}{n_1, n_2, \ldots, n_m} x_1^{n_1} x_2^{n_2} \cdots x_m^{n_m}$$

3) **Example 2.8.1.** For $n = 4$ and $m = 3$, we have by Theorem 2.8.1,

$(x_1 + x_2 + x_3)^4 =$

4) **106** ***Exercise 2***

28. $\sum_{r=m}^{n} \binom{n}{r}\binom{r}{m} = 2^{n-m}\binom{n}{m}$ for $m \le n$,

5)

31. $\sum_{r=0}^{n} (-1)^r r \binom{n}{r} = 0,$

6)

3.2. The Pigeonhole Principle

If three pigeons are to be put into two compartments, then you will certainly agree that one of the compartments will accommodate at least two pigeons. A much more general statement of this simple observation, known as the *Pigeonhole Principle*, is given below.

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7) **Example 3.2.4.** Show that for any set of 10 points chosen within a square whose sides are of length 3 units, there are two points in the set whose distance apart is at most $\sqrt{2}$.

What are the objects? What are the boxes? These are the two questions we have to ask beforehand. It is fairly clear that we should treat the 10 given points in the set as our "objects". The conclusion we wish to arrive at is the existence of "2 points" from the set which are "close" to each other (i.e. their distance apart is at most $\sqrt{2}$ units). This indicates that "$k + 1 = 2$" (i.e., $k = 1$), and suggests also that we should partition the 3×3 square into n smaller regions, $n < 10$, so that the distance between any 2 points in a region is at most $\sqrt{2}$.

Exercise 3

8) 1. Show that among any 5 points in an equilateral triangle of unit side length, there are 2 whose distance is at most $\frac{1}{2}$ units apart.

9) 3. Given any set S of 9 points within a unit square, show that there always exist 3 distinct points in S such that the area of the triangle formed by these 3 points is less than or equal to $\frac{1}{8}$. (Beijing Math. Competition, 1963)