Homework Assignment #10

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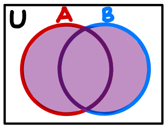
1) Suppose that A and B are finite sets. Give a formula for $|A \cup B|$ that only involves $|A|$, $|B|$, and $|A \cap B|$.



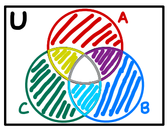
A union of two sets combines all members of each set into a new one

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$$|A \cup B| = \underbrace{|A|}_{\text{set } A} + \underbrace{|B|}_{\text{set } B} - \underbrace{|A \cap B|}_{\text{subtract the "over-counted" intersection}}.$$

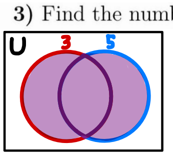
because it's a whole new set and not "both"

2) Suppose that A, B, and C are finite sets. Give a formula for $|A \cup B \cup C|$ that only involves $|A|$, $|B|$, $|C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$, and $|A \cap B \cap C|$.



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

3) Find the number of integers in the set $\{1, 2, \ldots, 90\}$ that are divisible by 3 or 5.



$|A \cup B| = |A| + |B| - |A \cap B|$

$|A_3 \cup B_5| = |A_3| + |B_5| - |A_3 \cap B_5|$

$|A_3| = \lfloor \frac{90}{3} \rfloor = 30$

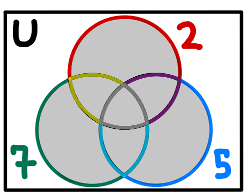
$|B_5| = \lfloor \frac{90}{5} \rfloor = 18$

$|A_3 \cap B_5| = \lfloor \frac{90}{15} \rfloor = 6$

$|A_3 \cup B_5| = 30 + 18 - 6$
$= 48 - 6$
$= 42$

There are 42 integers in set of $\{1, 2, \ldots 90\}$ that are divisible by 3 or 5.

4) Find the number of integers in the set $\{1, 2, 3, \ldots, 140\}$ that are divisible by 2, 5, or 7.



$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$|A_2 \cup B_5 \cup C_7| = |A_2| + |B_5| + |C_7| - |A_2 \cap B_5| - |A_2 \cap C_7| - |B_5 \cap C_7| + |A_2 \cap B_5 \cap C_7|$

70 28 29 14 10 4 2

$|A_2| = \lfloor \frac{140}{2} \rfloor = 70$

$|B_5| = \lfloor \frac{140}{5} \rfloor = 28$

$|C_7| = \lfloor \frac{140}{7} \rfloor = 29$

$|A_2 \cap B_5| = \lfloor \frac{140}{10} \rfloor = 14$

$|A_2 \cap C_7| = \lfloor \frac{140}{14} \rfloor = 10$

$|B_5 \cap C_7| = \lfloor \frac{140}{35} \rfloor = 4$

$|A_2 \cap B_5 \cap C_7| = \lfloor \frac{140}{70} \rfloor = 2$

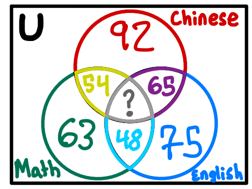
$|A_2 \cup B_5 \cup C_7|$

$\Rightarrow 70 + 28 + 29 - 14 - 10 - 4 + 2$
$= 101$

There are 101 integers in set of $\{1, 2, \ldots, 140\}$ that are divisible by 2, 5, or 7.

Exercise 4

1. A group of 100 students took examinations in Chinese, English and Mathematics. Among them, 92 passed Chinese, 75 English and 63 Mathematics; at most 65 passed Chinese and English, at most 54 Chinese and Mathematics, and at most 48 English and Mathematics. Find the largest possible number of the students that could have passed all the three subjects.



$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

100 92 75 63 65 54 48 ? x

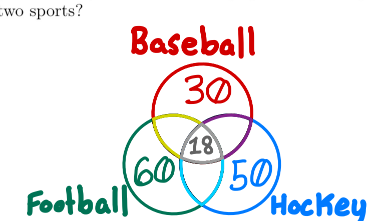
So,

$100 = 92 + 75 + 63 - 65 - 54 - 48 + x$

$\Rightarrow 100 = 230 - 167 + x$

$\therefore x = 37$

6) Suppose 100 students play three sports; baseball, hockey, or football. Each student may play one, two, or all three sports. If 30 students played baseball, 50 students played hockey, 60 students played football, and 18 students played all three sports, how many students played at least two sports?



$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

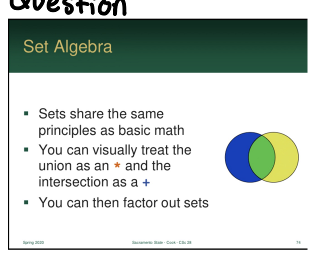
100 30 50 60 ? ? ? 18

So, $100 = 30 + 50 + 60 - 3x + 18$

$\Rightarrow 3x = 258$

$\Rightarrow x =$

Question



Is "$|A| \cup |B| \cup |C|$" the same as "$|A| + |B| + |C|$"

If so, why is

$|B_2 \cap B_3| = \lfloor \frac{500}{6} \rfloor$ not $\lfloor \frac{500}{5} \rfloor$?

$|B_3 \cap B_5| = \lfloor \frac{500}{15} \rfloor$ not $\lfloor \frac{500}{8} \rfloor$?