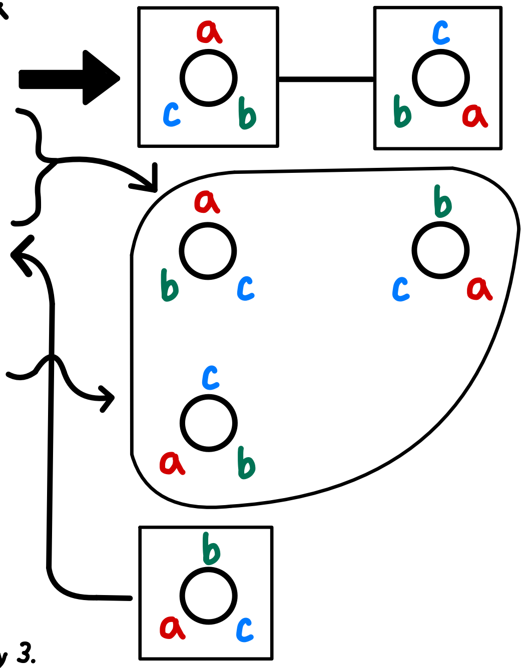
SUBJECT: Section 1.3: Circular Permutations DATE: 2023/02/06 PAGE#:

Problem: How many distinct ways can 'a', 'b', and 'c' be arranged around a circular table where two arrangements are the same if they are rotations of each other?

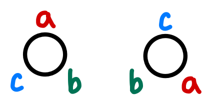
All 3 factorials ways of putting 'a', 'b', 'c' in a line

$$\frac{n!}{n} = (n-1)!$$

Top	Right	Left
a	b	c
a	c	b
b	a	c
b	c	a
c	a	b
c	b	a



Solution: Two distinct ways



One can think of getting this count by first lining up (3! ways) and then dividing by 3.

In general, if Q_r^n is the number of arrangements of 'r' elements placed around a circle chosen from a set of 'n' elements, then

$$Q_r^n = \frac{P_r^n}{r} = \frac{\frac{n!}{(n-r)!}}{r} = \frac{n!}{r\cdot(n-r)!}$$

* Note that if $r=n$, then

$$Q_r^n = \frac{n!}{n(n-n)!} = \frac{n!}{n\cdot 0!} = \frac{n!}{n\cdot 1} = \frac{n!}{n} = (n-1)! \quad \boxed{Q_n^n = \frac{P_n^n}{n} = (n-1)!}$$

Example 01

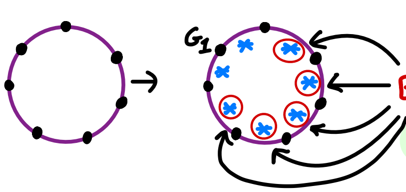
Problem: In how many ways can 5 boys and 3 girls be seated around a table if

(i) There are no restrictions

$\frac{8!}{8} = 7!$

(ii) Boy B_1 and girl G_1 cannot be next to each other

Let us place all except B_1 there $\frac{7!}{7} = 6!$ ways to place the seven people around the table (we are not placing B_1 yet)



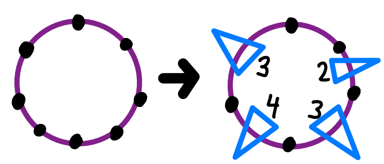
B_1 – 5 choices for B_1

This gives a total count of $6! \times 5$

(iii) No two girls sit next to each other

First put boys at the table

$\frac{5!}{5} = 4!$ Possibilites



$5\cdot4\cdot3$ ways to place girls

$\Rightarrow$ Total is $4!\cdot5\cdot4\cdot3$