California State University Sacramento - Math 101

Homework Assignment 5 - Solutions

1) First we place the two boys. There are 8 possible ways to do this since the first of the two boys can be 1st in the row, 2nd in the row,... all the way up to 8th in the row. There are then two ways to order these two boys (either A comes before B or B comes before A). Next we place the remaining students. There are $\binom{5}{3}$ ways to choose which three girls will go between A and B and 3! ways to order those girls. There are 7! ways to order the remaining students. This gives a total of

$$8 \cdot 2 \cdot \binom{5}{3} 3!7!$$

possibilities.

2) (i) There are $\binom{5}{3}$ ways to choose the female students and $\binom{10}{6}$ ways to choose the male students giving a total of

$$\binom{5}{3}\binom{10}{6}$$

possibilities.

(ii) As shown in part (i), there are $\binom{5}{3}\binom{10}{6}$ ways to choose the students on the committee and then 9! ways to assign these 9 students to the different posts. This gives a total of

$$\binom{5}{3}\binom{10}{6} 9!$$

possibilities.

3) Example 1.4.3 parts (i), (ii), and (iii) - See text book.

4) There are 7! ways to line up the boys. Given an arrangement of the boys, we then count how many ways we can arrange the girls. The are 8 choices for the first girl. She can go before the first boy in line, before the second boy in line, ... , before the seventh boy in line, or behind the seventh boy in line (see page 9 where a similar counting strategy is used). The next girl has 7 choices since she cannot be next to the first girl. Continuing in this fashion, we find that there are $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$ ways to line up the girls once the arrangement of the boys has been given. Therefore, there are

$$7!(8 \cdot 7 \cdot 6 \cdot 5 \cdot 4)$$

possibilities.

5) (i) We divide the problem into three cases. There are $\binom{3}{1}\binom{12}{6}$ ways to form a group with exactly one female. There are $\binom{3}{2}\binom{12}{5}$ ways to form a group with exactly two females and $\binom{3}{3}\binom{12}{4}$ ways to form a group with three females. These cases are disjoint so that we have a count of

$$\binom{3}{1}\binom{12}{6} + \binom{3}{2}\binom{12}{5} + \binom{3}{3}\binom{12}{4}.$$

(ii) Once we have chosen which 7 form a group, there are 7! ways to assign them to different posts so by part (i), we have

$$7!\left(\binom{3}{1}\binom{12}{6}+\binom{3}{2}\binom{12}{5}+\binom{3}{3}\binom{12}{4}\right)$$

possibilities.

6) Choosing two horizontal lines and two vertical lines uniquely determines a rectangle and there are

$$\binom{p}{2}\binom{q}{2}$$

ways to choose a pair of horizontal lines and a pair of vertical lines.

7) We consider three disjoint cases which depend on the number of boys chosen from the senior class.

Case 1: 4 boys are chosen from the senior class

There are $\binom{10}{4}$ ways to choose the 4 boys from the senior class, $\binom{15}{1}$ ways to choose 1 boy from the junior class, and $\binom{10}{2}$ ways to choose 2 girls from the junior class.

Case 2: 3 boys are chosen from the senior class

There are $\binom{10}{3}$ ways to choose 3 boys from the senior class, $\binom{4}{1}$ ways to choose 1 girl from the senior class, $\binom{15}{2}$ ways to choose 2 boys from the junior class, and $\binom{10}{1}$ ways to choose 1 girl from the junior class.

Case 3: 2 boys are chosen from the senior class

There are $\binom{10}{2}$ ways to choose 2 boys from the senior class, $\binom{4}{2}$ ways to choose 2 girls from the senior class, and $\binom{15}{3}$ ways to choose 3 boys from the junior class.

If we combine all three cases, we find that there are

$$\binom{10}{4}\binom{15}{1}\binom{10}{2}+\binom{10}{3}\binom{4}{1}\binom{15}{2}\binom{10}{1}+\binom{10}{2}\binom{4}{2}\binom{15}{3}$$

possibilities.