Q1) Example 1.4.6. – If there must be at least one person in each table, in how many ways can 6 people be seated (We assume that the tables are indistinguable)

(i) around two tables?

Recall: if r=n then
$$Q^n_r = \frac{n!}{n(n-n)!} = (n-1)!$$

$S(6,2)$ S(r-people, n-tables)

case01

$\binom{6}{1} Q^1_1 \cdot Q^5_5$

$\binom{6}{1} \times 0! \times 4!$
$\rightarrow 6 \times 1 \times 24$
$\rightarrow 144$

case02

$\binom{6}{2} Q^2_2 \cdot Q^4_4$

$\binom{6}{2} \times 1! \times 3!$
$\rightarrow 15 \times 1 \times 6$
$\rightarrow 90$

case03

$\dfrac{\binom{6}{3} Q^3_3 \cdot Q^3_3}{2}$ ← tables w/ 3 people

Accounts for the double count the two

$[\binom{6}{3} \times 2! \times 2!] \cdot (\frac{1}{2})$
$\rightarrow [20 \times 2 \times 2] \cdot (\frac{1}{2})$
$\rightarrow 40$

case01 case02 case03
In total $144 + 90 + 40 = 274$

(ii) around three tables?

$S(6,3)$ S(r-people, n-tables)

case01

$\binom{6}{4}\binom{2}{1} \times Q^4_4 \times Q^1_1 \times Q^1_1$

adjust for over count → $2!$

$[(15)(2) \times 3! \times 0! \times 0!](\frac{1}{2})$
$\rightarrow [(30) \times 6 \times 1 \times 1](\frac{1}{2})$
$\rightarrow 90$

case02

$\binom{6}{3}\binom{3}{2} \times Q^3_3 \times Q^2_2 \times Q^1_1$

$[(20)(3) \times 2! \times 1! \times 0!]$
$\rightarrow [(60) \times 2 \times 1 \times 1]$
$\rightarrow 120$

case03

$\binom{6}{2}\binom{4}{2} \times Q^2_2 \times Q^2_2 \times Q^2_2$

adjust for over count → $3!$

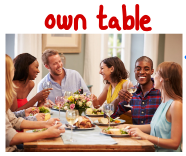
$[(15)(6) \times 0! \times 0! \times 0!](\frac{1}{6})$
$\rightarrow [(90) \times 1 \times 1 \times 1](\frac{1}{6})$
$\rightarrow 15$

case01 case02 case03
In total $90 + 120 + 15 = 225$

Q2) Example 1.4.7 Show that
$s(r,n) = s(r-1,n-1) + (r-1)s(r-1,n)$
where $r,n \in \mathbb{N}$ with $n \le r$

$$s(r,n) = s(r-1,n-1) + (r-1)s(r-1,n)$$

For every given table this guarantees a seat at that table for that 1 person (a table for at least 1 person by themselves) OR + For person who had their own table — Have everyone else sit at those tables



18. A group of 20 students, including 3 particular girls and 4 particular **Q3)** boys, are to be lined up in two rows with 10 students each. In how many ways can this be done if the 3 particular girls must be in the front row while the 4 particular boys be in the back?

We want to know how many combination of students with the particular front row girls and back row boys, and the other students

1	G_2	2	3	G_3	4	5	6	G_1	7
8	B_1	9	B_2	10	11	B_3	12	13	B_4

G_1	G_2	G_3	1	2	3	4	5	6	7
B_3	8	B_4	9	B_2	10	B_1	11	12	13

We don't care about ordered positions of the individual students, so we use C^n_r not P^n_r

1st Front row →	G_1	G_2	G_3	1	2	3	4	5	6	7
2nd Back row →	8	9	10	11	12	13	B_1	B_2	B_3	B_4

Approach 01

Because the front row must always be populated by the girls we can pull any 1 of the 13 other students to fill the front 7 spots

G_1	G_2	G_3	1	2	3	4	5	6	7
8	9	10	11	12	13	B_1	B_2	B_3	B_4

$\binom{13}{7} \times 10! \times 10!$ (1st Front row, 2nd Back row) — 10 seats, so 10! seating arrangements

By default the back row gets chosen automatically

In total $\underbrace{\binom{13}{7}}_{1716} \times \underbrace{10!}_{3628800} \times \underbrace{10!}_{3628800} \approx 22{,}596{,}613{,}080{,}000{,}000$

Approach 02

G_1	G_2	G_3	1	2	3	4	5	6	7
8	9	10	11	12	13	B_1	B_2	B_3	B_4

$\binom{10}{3} \times 3! \times \binom{10}{4} \times 4! \times 13!$

Front row choose the 3 girls — Config. of girls amongst themselves — Back row choose the 4 boys — Config. of boys amongst themselves — Placement of NPCs

In total $\binom{10}{3} \times 3! \times \binom{10}{4} \times 4! \times 13! \approx 22{,}596{,}613{,}080{,}000$

19. In how many ways can 7 boys and 2 girls be lined up in a row such that **Q4)** the girls must be separated by exactly 3 boys?

Base Case

G	B	B	B	G	B	B	B	B

Index 0 1 2 3 4 5 6 7 8

1: 0–4; 2: 4–8; 3: 0–4; 4: 1–5; 5: 2–6 … 3–7

Ways to place the girls 5, but both girls can swap places it's twice that: 10

Ways to config. the boys: $7!$

So...

In total $7!(5 \cdot 2) = 50400$

21. Find the number of $(m+n)$-digit binary sequences with m 0's and n **Q5)** 1's such that no two 1's are adjacent, where $n \le m+1$.

Notes: a set of all strings w/no consecutive 1's where $\Sigma = \{0,1\}$

→ Base cases: 0, ~~00~~, 01, 10, ~~11~~, ~~000~~, 001, 010, 100, ~~110~~, ~~111~~...

→ RegEx: $0 + [(0+10)^*(\lambda+1)]$

0's = m
1's = n

~restriction~
$n \le m+1$

Binary $\binom{2}{1}$ ← number of states: 1 & 0 ← choose a state

We apply our restriction

So...
$$\binom{m+1}{n}$$

See Back

Q6) 26. Find the number of ways of forming a group of $2k$ people from n couples, where $k, n \in \mathbf{N}$ with $2k \leq n$, in each of the following cases:

(i) There are k couples in such a group;

(ii) No couples are included in such a group;

(iii) At least one couple is included in such a group;

(iv) Exactly two couples are included in such a group.

(i) No restriction

$$\boxed{\binom{n}{k}}$$

(ii)

$$\boxed{\binom{n}{2k} \cdot 2^{2k}}$$

(iii)

$$\boxed{\binom{2n}{2k} - \binom{n}{2k} \cdot 2^{2k}}$$

(iv)

$$\boxed{\binom{n}{2}\binom{n-2}{2k-4} \cdot 2^{2k-4}}$$