California State University Sacramento - Math 101

Homework Assignment 11

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Exercise 6

1. Solve
$$a_n = 3a_{n-1} - 2a_{n-2},$$
given that $a_0 = 2$ and $a_1 = 3$.

2. Solve
$$a_n - 6a_{n-1} + 9a_{n-2} = 0,$$
given that $a_0 = 2$ and $a_1 = 3$.

3. Solve
$$a_n = \frac{1}{2}(a_{n-1} + a_{n-2}),$$
given that $a_0 = 0$ and $a_1 = 1$.

4. Solve
$$a_n - 4a_{n-1} + 4a_{n-2} = 0,$$
given that $a_0 = -\frac{1}{4}$ and $a_1 = 1$.

5. Solve
$$2a_n = a_{n-1} + 2a_{n-2} - a_{n-3},$$

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1. Solve

$$a_n = 3a_{n-1} - 2a_{n-2},$$

given that $a_0 = 2$ and $a_1 = 3$.

Recurrence relation written as $a_n - 3a_{n-1} + 2a_{n-2} = 0$
- Charactistic equation: $x^2 - 3x + 2 = 0$
  - Characteristic roots: $a_1 = 1$, $a_2 = 2$
- $a_n = A + B \cdot 2^n$; $A + B = 2$, $A + 2B = 3$
  - We get $A = B = 1$ by solving the system of A&B

So, $a_n = 1 + 2^n$.

2. Solve

$$a_n - 6a_{n-1} + 9a_{n-2} = 0,$$

given that $a_0 = 2$ and $a_1 = 3$.

Recurrence relation written as $x^2 - 6x + 9 = 0$
- Characteristic roots: $a = 3$, multiplicity 2
- General solution: $a_n = (A + Bn) \cdot 3^n$
  - $a_0 = 2$, $a_1 = 3 \Rightarrow A = 2$ & $3(A+B) = 3$
  - $A = 2$, $B = -1$

So, $a_n = (2-n)3^n$

3. Solve

$$a_n = \frac{1}{2}(a_{n-1} + a_{n-2}),$$

given that $a_0 = 0$ and $a_1 = 1$.

Recurrence relation written as $a_n - \frac{1}{2}a_{n-1} - \frac{1}{2}a_{n-2} = 0$
- Characteristic equation: $x^2 - \frac{1}{2}x - \frac{1}{2} = 0$
- Characteristic roots: $a_1 = 1$, $a_2 = -1/2$
- General solution: $a_n = A + B(-\frac{1}{2})^n$
  - $a_0 = 0$, $a_1 = 1$; $A + B = 0$ & $A - \frac{1}{2}B = 1$
  - $\rightarrow A = 2/3$ & $B = -2/3$

So, $a_n = \frac{2}{3}\left[1 - (-\frac{1}{2})^n\right]$

Recall

Repeated Roots

$$\frac{Ax + B}{x^2 + 1}$$

Recurrence relation of the form

$$C_0 a_n + C_1 a_{n-1} + \dots + C_r a_{n-r} = 0$$

4. Solve

$$a_n - 4a_{n-1} + 4a_{n-2} = 0,$$

given that $a_0 = -\frac{1}{4}$ and $a_1 = 1$.

Recurrence relation written as $x^2 - 4x + 4 = 0$
- Characteristic roots: $a = 2$, multiplicity 2
- General solution: $a_n = (A + Bn)2^n$
  - Initial conditions: $a_0 = -\frac{1}{4}$ & $a_1 = 1$
  - $\Rightarrow A = -1/4$ & $2(A+B) = 1 \Rightarrow A = -1/4$ & $B = 3/4$

So, $a_n = (-1/4 + 3n/4)2^n = (3n-1)2^{n-2}$

5. Solve

$$2a_n = a_{n-1} + 2a_{n-2} - a_{n-3},$$

given that $a_0 = 0$, $a_1 = 1$ and $a_2 = 2$.

Recurrence relation written as: $2a_n - a_{n-1} - 2a_{n-2} - a_{n-3} = 0$
- Charactistic equation: $2x^3 - x^2 - 2x + 1 = 0$
  - Characteristic roots: $a_1 = 1$, $a_2 = -1$, and $a_3 = \frac{1}{2}$
- $a_n = A + B(-1)^n + C(\frac{1}{2})^n$; $a_0 = 0$, $a_1 = 1$, and $a_2 = 2$

$\Rightarrow A + B + C = 0$, $A - B + 1/2\, C = 1$, and $A + B + \frac{1}{4}C = 2$

$\rightarrow A = 5/2$, $B = 1/6$, $C = -8/3$

So, $a_n = 5/2 + 1/6(-1)^n - 8/3(\frac{1}{2})^n$