

Chapter 1. Permutations and Combinations

California State University Sacramento - Math 101

Homework Assignment 1

1) Let $A = \{-3, -2, -1, \dots, 5, 6, 7\}$.

- (a) Is $1 \in A$?
- (b) Is $\frac{1}{2} \in A$?
- (c) Find $|A|$.
- (d) If $B = \{4, 6, 8, 10\}$, find $A \cup B$ and $A \cap B$.

2) Suppose $A_1 = \{1, 2, 3\}$, $A_2 = \{3, 4, 5\}$, and $A_3 = \{4, 5, 6\}$.

- (a) Find $A_1 \cup A_2 \cup A_3$.
- (b) Find $A_1 \cap A_2 \cap A_3$.
- (c) True or False: $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$.
- (d) True or False: $|A_1 \cap A_2 \cap A_3| = |A_1||A_2||A_3|$.

3) Suppose A_1, A_2, \dots, A_5 are pairwise disjoint sets with $|A_i| = i$ for $1 \leq i \leq 5$. Determine

$$\left| \bigcup_{i=1}^5 A_i \right|.$$

4) Find sets A_1, A_2, \dots, A_5 such that $|A_i| = i$ for $1 \leq i \leq 5$ and

$$\left| \bigcup_{i=1}^5 A_i \right| = 5.$$

5) If $A = \{x : 3 \leq x \leq 10\}$ and \mathbb{Z} is the set of all integers, find

$$|A \cap \mathbb{Z}|$$

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Homework Assignment 2

1) Problem 1 on page 50

2) Problem 10 on page 51

3) Problem 11 on page 51

4) Problem 12 on page 51

Q1) 1. Find the number of ways to choose a pair $\{a, b\}$ of distinct numbers from the set $\{1, 2, \dots, 50\}$ such that
(i) $|a - b| = 5$; (ii) $|a - b| \leq 5$.

Q2) 10. Find the number of common positive divisors of 10^{40} and 20^{30} .

Q3) 11. In each of the following, find the number of positive divisors of n (inclusive of n) which are multiples of 3:
(i) $n = 210$; (ii) $n = 630$; (iii) $n = 151200$.

Q4) 12. Show that for any $n \in \mathbf{N}$, the number of positive divisors of n^2 is always odd.

Q 5) Find the number of ordered pairs (x, y) of integers such that $x^2 + y^2 \leq 4$.

Remark: This problem is similar to Example 1.1.2.

Q 6) Find the number of sequences $a_1a_2a_3$ of length 3 where $a_i \in \{0, 1, 2, 3, 4\}$.

Remark: This is a special case of Example 1.1.4.

Q 7) Let $X = \{1, 2, \dots, 10\}$ and let

$$S = \{(a, b, c) : a, b, c \in X, a < b \text{ and } a < c\}.$$

Find $|S|$.

Remark: This problem is similar to Example 1.1.6.

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Section 1.2. Permutations

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Homework Assignment 3

- 1) Example 1.2.1 on page 6. **Q1)** **Example 1.2.1.** Let $A = \{a, b, c, d\}$. All the 3-permutations of A are
- 2) Example 1.2.2 on page 7.
Q2) **Example 1.2.2.** Let $E = \{a, b, c, \dots, x, y, z\}$ be the set of the 26 English alphabets. Find the number of 5-letter words that can be formed from E such that the first and last letters are distinct vowels and the remaining three are distinct consonants.
- 3) Example 1.2.3 on page 8.
- 4) Example 1.2.4 on page 9.
- 5) Problem 4 on page 50.
- 6) Problem 2(i) and 2(ii) on page 50.
- 7) Problem 14(i), 14(ii), 14(iii), and 14(iv) on page 51.

Q3) **Example 1.2.3.** There are 7 boys and 3 girls in a gathering. In how many ways can they be arranged in a row so that

- (i) the 3 girls form a single block (i.e. there is no boy between any two of the girls)?
- (ii) the two end-positions are occupied by boys and no girls are adjacent?

Q4) **Example 1.2.4.** Between 20000 and 70000, find the number of even integers in which no digit is repeated.

- Q5)** 4. How many 5-letter words can be formed using $A, B, C, D, E, F, G, H, I, J$,
- (i) if the letters in each word must be distinct?
 - (ii) if, in addition, A, B, C, D, E, F can only occur as the first, third or fifth letters while the rest as the second or fourth letters?
- Q6)** 2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if
- (i) there are no restrictions?
 - (ii) the 5 girls must be together (forming a block)?

- Q7)** 14. Let $n, r \in \mathbf{N}$ with $r \leq n$. Prove each of the following identities:
- (i) $P_r^n = n P_{r-1}^{n-1}$,
 - (ii) $P_r^n = (n - r + 1) P_{r-1}^n$,
 - (iii) $P_r^n = \frac{n}{n-r} P_r^{n-1}$, where $r < n$,
 - (iv) $P_r^{n+1} = P_r^n + r P_{r-1}^n$,
 - (v) $P_r^{n+1} = r! + r(P_{r-1}^n + P_{r-1}^{n-1} + \dots + P_{r-1}^r)$.

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1.3. Circular Permutations

1.4. Combinations

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Homework Assignment 4

- 1) Example 1.3.2
- 2) Example 1.3.3
- 3) Problem 6 on page 51
- 4) Example 1.4.1
- 5) Example 1.4.2

Q1) **Example 1.3.2.** In how many ways can 5 boys and 3 girls be seated around a table if

- (i) there is no restriction?
- (ii) boy B_1 and girl G_1 are not adjacent?
- (iii) no girls are adjacent?

Q2) **Example 1.3.3.** Find the number of ways to seat n married couples around a table in each of the following cases:

- (i) Men and women alternate;
- (ii) Every woman is next to her husband.

Q3) 6. Find the number of *odd* integers between 3000 and 8000 in which no digit is repeated.

Q4) **Example 1.4.1.** Prove that

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}, \quad (1.4.3)$$

where $n, r \in \mathbf{N}$ with $r \leq n$.

Q5) **Example 1.4.2.** By Example 1.1.4, there are 2^7 binary sequences of length 7. How many such sequences are there which contain 3 0's and 4 1's?

Math 101 - Quiz 01 (2023-02-03)

1. $A = \{2n : n \text{ is an integer}\}$ and $B = \{1, 2, 3, 4, 5\}$. Find $A \cap B$.

2. For $1 \leq i \leq 10$, let $A_i = \{1, 2, \dots, i\}$. Determine $|\cup_{i=1}^{10} A_i|$.

3. Suppose $A = \{0, 3, 6, 9, 12, 15, 18\}$ and $B = \{0, 2, 4, 6, 8\}$. Find $A \cup B$ and $A \cap B$.

4. Suppose A and B are disjoint sets with $|A| = 5$ and $|B| = 3$. Determine $|A \cup B|$.

5. Suppose that A and B are sets with $|A| = 5$ and $|B| = 3$. Can you say anything about $|A \cup B|$?

Quiz #2 - Page 1 of 2

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Name: _____

1) How many pairs of distinct integers $\{a, b\}$ with $a, b \in \{1, 2, \dots, 10\}$ satisfy $|a - b| = 3$?

2) How many pairs of distinct integers $\{a, b\}$ with $a, b \in \{1, 2, \dots, 10\}$ satisfy $|a - b| \leq 3$?

Quiz #2 Page 2 of 2

3) Find the number of positive divisor of $1800 = 2^3 \cdot 3^2 \cdot 5^2$ which are multiples of 3.

4) Find the number of positive divisors of $1800 = 2^3 \cdot 3^2 \cdot 5^2$ that are multiples of 6.

5) Let $A_1 = \{1, 2, 3\}$, $A_2 = \{2, 3\}$, and $A_3 = \{1, 2, 3, 4\}$. Find the number of 3-tuples (a_1, a_2, a_3) where $a_1 \in A_1$, $a_2 \in A_2$, and $a_3 \in A_3$.

Quiz #3

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Name: _____

- 1)** Determine the number of 3-permutations of a set with 7 elements.
- 2)** List all 2-permutations of the set $A = \{a, b, c\}$.
- 3)** Find the number of sequences (a_1, a_2, a_3, a_4) that consist of distinct elements from the set $\{1, 2, 3, 4, 5\}$ where a_1 is even.
- 4)** Find the number of sequences (a_1, a_2, a_3, a_4) that consist of elements from the set $\{1, 2, 3, 4, 5\}$ where a_1 is even.
- 5)** Find the number of odd integers between 10,000 and 20,000 where no digit is repeated.
- 6)** Find the number of 3-circular permutations of a set with 7 elements.
- 7)** List all 3-circular permutations of the set $A = \{x, y, z, t\}$.
- 8)** Write down formulas for P_r^n , Q_r^n , and write down an equation that relates Q_r^n to P_r^n .
- 9)** Write down a formula for C_r^n in terms of factorials.
- 10)** Find the number of ordered pairs (A, B) where A is a 3-element subset of $\{1, 2, 3, 4, 5\}$, and B is a 2-element subset of $\{6, 7, 8, 9, 10\}$.
- 11)** Find the number of subsets of $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ that consist of exactly three even integers and three odd integers.
- 12)** Find the number of 2×3 rectangles in a 4×5 grid.