- Come back to
- Make Note of Note-to-self

California State University Sacramento - Math 101

Exam # 2

$P^n_r = \frac{n!}{(n-r)!}$

$Q^n_r = \frac{P^n_r}{r}, Q^n_n = (n-1)$

$C^n_r = \frac{P^n_r}{r!}$

Name: Matthew Mendoza

Exam is out of 8 points.

1) Let $X = \{1, 2, 3, \ldots, 20\}$.

.25 (a) What is the number of 3-permutations of X? (0.25 points)

.25 (b) What is the number of 3-circular permutations of X? (0.25 points)

.2 (c) What is the number of 2-combinations of X? Simplify your answer as much as possible. (0.25 points)

.25 (d) If $\mathcal{P}(X)$ is the power set of X, how many elements are in $\mathcal{P}(X)$? (0.25 points)

.25 (e) Find the number of 5-combinations of X with only odd numbers. (0.25 points)

.2 (f) Find the number of 5-combinations of X that do not contain a pair of consecutive integers. (0.25 points)

a) $n=20, r=3 \rightarrow \boxed{P^{20}_3 = \frac{20!}{20-3!} = \frac{20!}{17!}}$ ✓

b) $n=20, r=3 \rightarrow Q^{20}_3 = \boxed{\frac{20!}{3(17)!}}$ ✓

1 Number?

c) $n=20, r=2; C^{20}_2 \rightarrow \frac{20!}{2!(20-2)!} \rightarrow \frac{20!}{2(18)!} = \frac{20 \cdot 19}{2} = 190$ ✓

d) $P(X)$; 20 elements, so $\boxed{2^{20}}$ ✓

e) 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 sanity check

$\underline{1}$ $\underline{3}$ $\underline{5}$ $\underline{7}$ $\underline{9}$ $\underline{11}$ $\underline{13}$ $\underline{15}$ $\underline{17}$ $\underline{19}$

1 2 3 4 5 6 7 8 9 10

$n=10, r=5$; so $C^{10}_5 \rightarrow \frac{10!}{5!(10-5)!} \Rightarrow \frac{10!}{5!5!}$ ✓

f) $n = \cancel{10}$ 20, $r=5$

so, $\binom{n-r+1}{r} \rightarrow \binom{\cancel{10} - 5 + 1}{5}$ 20 $\rightarrow \boxed{\binom{\cancel{6}}{5}}$

$\binom{16}{5}$

~~5.35~~ 6.35

1.4

12 in All 7 Red 5 blue

2) A box contains 12 distinct marbles. Seven of the marbles are red and five of the marbles are blue. Thus, the elements of the box could be represented as the set

$$\{r_1, r_2, r_3, r_4, r_5, r_6, r_7, b_1, b_2, b_3, b_4, b_5\}.$$

(a) Find the number of ways to form a combination of two red marbles and three blue marbles. (0.25 points)

5 blue

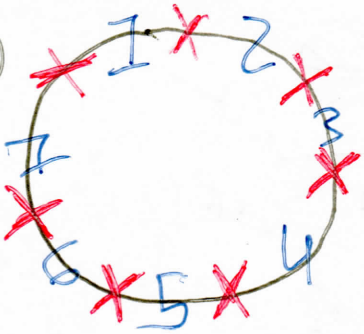
(b) Find the number of ways to form a combination of five marbles, at least three of which are blue. (0.5 points) Case

(c) How many ways can all 12 marbles be arranged around a circle so that no blue marbles are next to each other? (0.25 points)

.25 ~~(d)~~ Find the number of permutations of all 12 marbles such that b_1 comes before b_2. (0.5 points)

a) $\binom{7}{2}\binom{5}{3}$ ✓

b) Case 1: 5: 3b, 2r $\binom{5}{3}\binom{7}{2}$ + Case 2: 5: 4b, 1r $\binom{5}{4}\binom{7}{1}$ + Case 3: all blues $\binom{5}{5}$ ✓

c) 

$Q_7^7 = (7-1)! = 6!$ Red marbles

for every blue marble n-1 open spots

$\underbrace{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}_{\text{5 marbles}}$

$\boxed{\text{So, all together } 6! \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}$ ✓

~~d)~~ 7r, 5b

$$\frac{1}{2} \cdot \frac{(7+5)!}{\cancel{7!}\,\cancel{5!}}$$

half of these!

$\binom{12}{2} 10!$

choose b_1, b_2 location

line up rest

1.1 / .25

n-many

3) Recall that a 0-1 sequence of length n is a sequence of the form $a_1 a_2 \cdots a_n$ such that each a_i is either 0 or 1.

~~(a)~~ What is the total number of 0-1 sequences of length 6? Simplify your answer as much as possible. (0.25 points)

0.45 (b) Find the total number of 0-1 sequences of length 10 that have at most two 0's. Simplify your answer as much as possible. (0.5 points) case

0.4 (c) 37 people are going to vote on their preference for a new menu item at Timmons' Stars and Bars SandwHich Shop. The options are A, B, C, and D. How many possible outcomes are there after the 37 people vote? In other words, how many ways can 37 elements be chosen from A, B, C, and D allowing for repetition. (0.5 points)

one Number ?

a) $n=6$, $r=2$, so $\binom{6}{2} \to \frac{6!}{2!(6-2)!} \to \frac{720}{2(4)!} \to \frac{720}{2(24)} \to \boxed{\frac{720}{48}}$

$6\cdot5\cdot4\cdot3\cdot2\cdot1$

$2^6 = 64$

b) $n=10$, $r \le 2$

Case 1 No Zeros, Case 2 1 zero, Case 3 2 zeros

$\binom{10}{0} + \binom{10}{1} + \binom{10}{2}$

$1 + 10 + 45 = 56$

c) A | B | C | D

$H^n_r = \binom{r+n-1}{r}$ or $\binom{r+n-1}{n-1}$

$n=37$

$r=3$

3 bars

So $\boxed{\binom{3+37-\cancel{1}}{3} \to \binom{39}{3}}$

since 3 bars

$\binom{40}{3}$

0.85

* Note factorial Rule
$n(n-1)! = = n!$

0.5 **4)** Let $1 \leq r \leq n$.

(a) Give an algebraic proof that $\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}$. Write your proof carefully and use correct mathematical notation. (0.5 points)

LHS

definition

$\binom{n}{r} = \frac{n!}{r!(n-r)!}$

RHS

$\frac{n}{r}\binom{n-1}{r-1}$

$\rightarrow \frac{n}{r}\left(\frac{(n-1)!}{(r-1)![(n-1)-(r-1)]!}\right)$ ✓

$\rightarrow \frac{n}{r}\left(\frac{(n-1)!}{(r-1)![n-1-r+1]!}\right)$ ✓

$\rightarrow \frac{n}{r}\left(\frac{(n-1)!}{(r-1)![n-r]!}\right)$ * Factorial Rule

$\rightarrow \frac{n!}{r!(n-r)!}$ LHS = RHS

(b) Explain why $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$ is true by describing a counting process that shows the number of r-combinations of $\{1, 2, \ldots, n\}$ is equal to the right hand side of this equation. (0.5 points)

Explain later

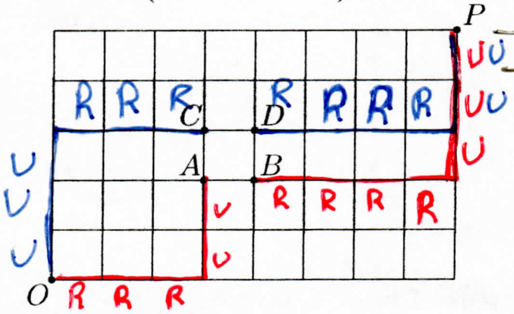
0.5

0.45

$\binom{5}{2}\binom{7}{3}+\binom{6}{3}\binom{6}{2}$

5) Consider the figure shown below (which is 5×8).

add blue count & red count →



Info for Part B

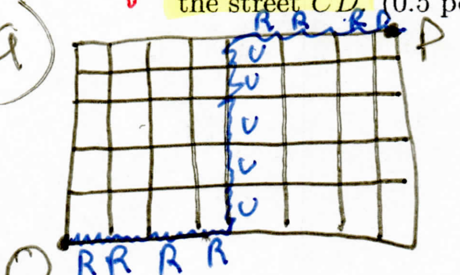
7R, 5U = 12 Mov

7R, 5U = 12 Moves

.25 (a) Find the number of shortest routes from O to P. (0.25 points) — don't use complement rule here

.4 (b) Find the number of shortest routes from O to P that pass through the street AB or the street CD. (0.5 points)

a)



8R's, 5U's = 13 total

$\binom{13}{5}$ ✓ or $\binom{13}{8}$

count up movements

count Right movements

b) $\binom{13}{5} - [AB \text{ or } + CD]$

So

$\binom{13}{5} - \left[\binom{12}{5} + \binom{12}{5}\right]$

Selecting for up Movements

up-Moves

0.85

6) Let $s(r, n)$ be the number of ways to arrange r distinct objects around n indistinguishable circles so that every circle has at least one object.

0.15 (a) Why is $s(r, r) = 1$? Answer in a complete sentence and do not use any math symbols. (0.25 points)

0.70 (b) Determine, with explanation (which can include pictures) the value of $s(6, 2)$. (0.75 points)

a) Reason

A person is seated at a table

need to explain a bit more here

b) $s(6,2)$

* Note to self

$s(r,n)$

6-People, 2 tables

r-people

n-tables

Case 1 | Case 2 | Case 3

$\binom{6}{1}Q_5^5$ ✓

$\binom{6}{2}Q_4^4$ ✓

$\dfrac{\binom{6}{3}Q_3^3 Q_3^3}{2!}$ over count

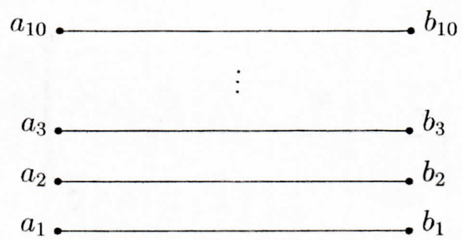
good!

then must add up

1.5

n-Pairs, k-single person in Pairing

7) Consider the matching shown below.



(a) Find the number of subsets of size 5 from $\{a_1, \ldots, a_{10}, b_1, \ldots, b_{10}\}$ that do not contain any of the edges. (0.25 points)

(b) Find the number of subsets of size 5 from $\{a_1, \ldots, a_{10}, b_1, \ldots, b_{10}\}$ that contain exactly one edge. (0.5 points)

(c) Find the number of subsets of size 10 from $\{a_1, \ldots, a_{10}, b_1, \ldots, b_{10}\}$ that contain exactly three edges. (0.25 points)