1) Let X = {1, 2, . . ., 13, 14 }

a. Find the number of 2-combinations of X. Simplify your answer as much as possible.

b. Find the number of 5-combinations of X that do not contain a pair of consecutive integers. White your answer as a binomial coefficient. (1 point)

a| $X=\{1,2,3,\ldots,14\}$, subset of 2 combinations

- $n=14$
- $r=2$

So, $\binom{14}{2}=\frac{n!}{r!(n-r)!}=\frac{14!}{2!(14-2)!}=\frac{14!}{2(12)!}=91$

b| $n=14$

$r=5$

$\binom{n-r+1}{r}\rightarrow\binom{14-5+1}{5}$

$\rightarrow\binom{10}{5}=252$

Recall: Section 1.5

General Case — The biggest it can be is $n-r+1$

Let $x=\{1,2,3,4,\ldots,n\}$ and Let $1\le r\le n$

Given a subset $\{S_0,S_1,\ldots,S_r\}$ of x with no consecutive elements define,

$f(\{S_0,S_1,\ldots,S_r\})=\{S_0,S_1-1,S_2-2,\ldots,S_r-(r-1)\}$

This output is a subset of size 'r' since $\{S_0,S_1,\ldots,S_r\}$ has no consecutive elements

$$\binom{n-r+1}{r}$$

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2) Find the number of 13-digit binary sequences with nine 0's and four 1's such that no two 1's are adjacent. (2 points)

- 13-digits – binary 9: 0's, 4: 1's, remove 2: 1's

So $\binom{n-r+1}{r}\rightarrow\binom{13-4+1}{4}=\binom{10}{4}=210$

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3) Let X = { {1}, y }.

a. Find all elements of P(X) (the power set of X). (1 point)

b. If Y is a set with 6 elements, how many elements are in P(Y)? (1 point)

a| All elements of P(X)

$\{\emptyset,\{\{1\}\},\{y\},\{\{1\},y\}\}$

b| Y = 6 elements

So, $2^6=64$

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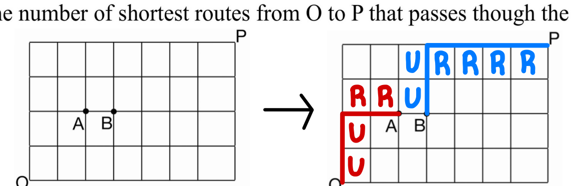
4) Find the number of shortest routes from O to P that passes though the street AB.



From O→A – 4 steps total: 2 R's, 2 U's

From B→P – 6 steps total: 4 R's, 2U's

$\underbrace{\binom{4}{2}}_{\text{Selecting for}}\cdot\underbrace{\binom{6}{2}}_{\text{Selecting for}}$ up-movements OR $\underbrace{\binom{4}{2}}_{\text{Selecting for}}\cdot\underbrace{\binom{6}{4}}_{\text{Selecting for}}$ Right-movements OR $\underbrace{\binom{4}{2}}_{\text{Selecting for up}}\cdot\underbrace{\binom{6}{4}}_{\text{Selecting for Right}}$ movements OR $\underbrace{\binom{4}{2}}_{\text{Selecting for Right}}\cdot\underbrace{\binom{6}{2}}_{\text{Selecting for up}}$ movements

90 ways

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5) Suppose that k and n are positive integers with $3\le k\le n$ and that $a_1,\ldots,a_n,b_1,\ldots,b_n$ are $2n$ distinct elements. Form the n pairs $\{a_1,b_1\},\{a_2,b_2\},\ldots,\{a_n,b_n\}$.

(a) Find the number of subsets of $\{a_1,\ldots a_n,b_1,\ldots,b_n\}$ of size k that do not contain two elements from the same pair. (1 point)

(b) Find the number of subsets of $\{a_1,\ldots a_n,b_1,\ldots,b_n\}$ of size k that contain exactly one of the pairs. (1 point)

a| $\underbrace{\binom{n}{k}}_{\text{choose K-pairs}}\cdot\underbrace{2^k}_{\text{choose one from each pair}}$

b| $\underbrace{\binom{n}{1}}_{\text{Choose one Pair so have two People}}\underbrace{\binom{n-1}{k-2}}_{\text{choose K-2 pairs}}\underbrace{2^{k-2}}_{\text{Choose one from of the K-2 pairs}}$