California State University Sacramento - Math 101

Quiz #8

Name: _____

1) (a) State the Binomial Theorem.

(b) Prove that $0 = \sum_{k=0}^{n} (-1)^k \binom{n}{k}$ for all integers $n \geq 1$.

2) Let $X = \{1, 2, \ldots, n\}$, $\mathcal{A} = \{A \subseteq X : 1 \notin A\}$ and $\mathcal{B} = \{B \subseteq X : 1 \in B\}$.

(a) In the case that $n = 3$, write down all of the elements of $\mathcal{A}$, all of the elements of $\mathcal{B}$, and a bijection f from $\mathcal{A}$ to $\mathcal{B}$.

(b) Give a formula for a function $f : \mathcal{A} \to \mathcal{B}$ that defines a bijection from $\mathcal{A}$ to $\mathcal{B}$ and is valid for every n.

3) What is the coefficient of x^3 in $(x+1)^{12}$? Simplify your answer as much as possible.

4) Prove that $n2^{n-1} = \sum_{r=1}^{n} r\binom{n}{r}$ for all integers $n \geq 1$.

5) Simplify $\sum_{r=1}^{n} \binom{n}{r}$ as much as possible. Your final answer should involve two terms, one of which depends on n.