

Attempt #1

SUBJECT: Homework Assignment #1

1) Let $A = \{-3, -2, -1, \dots, 5, 6, 7\}$.

(a) Is $1 \in A$? -1 is an element of A . \therefore True

(b) Is $\frac{1}{2} \in A$? $\frac{1}{2}$ is not an element of A . \therefore False

(c) Find $|A|$. \therefore Cardinality of set A is 11

(d) If $B = \{4, 6, 8, 10\}$, find $A \cup B$ and $A \cap B$.

$$A = \{-3, -2, -1, \emptyset, 1, 2, 3, 4, 5, 6, 7\} \quad B = \{4, 6, 8, 10\}$$

(Union)

$$A \cup B = \{-3, -2, -1, \emptyset, 1, 2, 3, 4, 5, 6, 7, 8, 10\}$$

(Intersection)

$$A \cap B = \{4, 6\}$$

2) Suppose $A_1 = \{1, 2, 3\}$, $A_2 = \{3, 4, 5\}$, and $A_3 = \{4, 5, 6\}$.

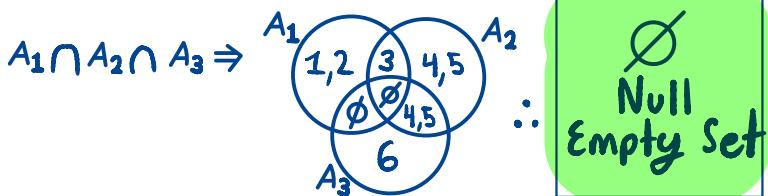
(a) Find $A_1 \cup A_2 \cup A_3$.

$$A_1 = \{1, 2, 3\}, A_2 = \{3, 4, 5\}, A_3 = \{4, 5, 6\}$$

$$A_1 \cup A_2 \cup A_3 \Rightarrow \{1, 2, 3, 4, 5, 6\}$$

(b) Find $A_1 \cap A_2 \cap A_3$.

$$A_1 = \{1, 2, 3\}, A_2 = \{3, 4, 5\}, A_3 = \{4, 5, 6\}$$



(c) True or False: $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$.

$$A_1 = \{1, 2, 3\}, A_2 = \{3, 4, 5\}, A_3 = \{4, 5, 6\}$$

$$\begin{aligned} |A_1 \cup A_2 \cup A_3| &= |A_1| + |A_2| + |A_3| \\ 6 &= 3 + 3 + 3 \\ 6 &\neq 9 \end{aligned}$$

\therefore False.

(d) True or False: $|A_1 \cap A_2 \cap A_3| = |A_1||A_2||A_3|$.

$$A_1 = \{1, 2, 3\}, A_2 = \{3, 4, 5\}, A_3 = \{4, 5, 6\}$$

$$\begin{aligned} |A_1 \cap A_2 \cap A_3| &= |A_1||A_2||A_3| \\ \emptyset &= 3 \cdot 3 \cdot 3 \\ \emptyset &\neq 81 \end{aligned}$$

\therefore False.

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3) Suppose A_1, A_2, \dots, A_5 are pairwise disjoint sets with $|A_i| = i$ for $1 \leq i \leq 5$.

Determine $\left| \bigcup_{i=1}^5 A_i \right|$

$$\left| \bigcup_{i=1}^5 A_i \right| \rightarrow \begin{aligned} A_1 &= \{1, 2, 3, 4, 5\} \\ A_2 &= \{1, 2, 3, 4, 5\} \\ A_3 &= \{1, 2, 3, 4, 5\} \\ A_4 &= \{1, 2, 3, 4, 5\} \\ A_5 &= \{1, 2, 3, 4, 5\} \end{aligned}$$

$$\rightarrow A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$$

$$\therefore \left| \bigcup_{i=1}^5 A_i \right| \rightarrow \{1, 2, 3, 4, 5\} = 5,$$

(Where $|A_i| = i$ for $1 \leq i \leq 5$)

4) Find sets A_1, A_2, \dots, A_5 such that $|A_i| = i$ for $1 \leq i \leq 5$

and $\left| \bigcup_{i=1}^5 A_i \right| = 5$

$$\rightarrow |A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5| = 5$$

(Where $|A_i| = i$ for $1 \leq i \leq 5$)

$$\therefore \left| \bigcup_{i=1}^5 A_i \right| \rightarrow \begin{aligned} A_1 &= \{1\} \\ A_2 &= \{1, 2\} \\ A_3 &= \{1, 2, 3\} \\ A_4 &= \{1, 2, 3, 4\} \\ A_5 &= \{1, 2, 3, 4, 5\} \end{aligned}$$

5) If $A = \{x : 3 \leq x \leq 10\}$ and \mathbb{Z} is the set of all integers, find $|A \cap \mathbb{Z}|$.

$$A = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

$$\therefore |A \cap \mathbb{Z}| \rightarrow \{3, 4, 5, 6, 7, 8, 9, 10\} = 8$$