California State University Sacramento - Math 101

Quiz #2

Name: _____

1) How many pairs of distinct integers $\{a, b\}$ with $a, b \in \{1, 2, \ldots, 10\}$ satisfy $|a - b| = 3$?

Let us list all such pairs:

$\{1,4\}, \{2,5\}, \{3,6\}, \{4,7\}, \{5,8\}, \{6,9\}, \{7,10\}$.

There are 7 such pairs.

2) How many pairs of distinct integers $\{a, b\}$ with $a, b \in \{1, 2, \ldots, 10\}$ satisfy $|a - b| \leq 3$?

Let us assume that a is smaller than b. We can do this because $\{a,b\}$ is a set and so we do not count $\{a,b\}$ as being different from $\{b,a\}$.

possible a's →	$a=1$	$a=2$	...	$a=7$	$a=8$	$a=9$
possible b's →	$b=2$ $b=3$ $b=4$	$b=3$ $b=4$ $b=5$	...	$b=8$ $b=9$ $b=10$	$b=9$ $b=10$	$b=10$

There are $7 \cdot 3 + 2 + 1 = 24$ such pairs.

3) Find the number of positive divisor of $1800 = 2^3 \cdot 3^2 \cdot 5^2$ which are multiples of 3.

We want to count numbers of the form

$2^a 3^b 5^c$ where $0 \le a \le 3, 1 \le b \le 2, 0 \le c \le 2$.

There are $4 \cdot 2 \cdot 3 = 24$ such divisors.

4) Find the number of positive divisors of $1800 = 2^3 \cdot 3^2 \cdot 5^2$ that are multiples of 6.

We want to count numbers of the form

$2^a 3^b 5^c$ where $1 \le a \le 3, 1 \le b \le 2, 0 \le c \le 2$.

There are $3 \cdot 2 \cdot 3 = 18$ such divisors.

5) Let $A_1 = \{1, 2, 3\}$, $A_2 = \{2, 3\}$, and $A_3 = \{1, 2, 3, 4\}$. Find the number of 3-tuples (a_1, a_2, a_3) where $a_1 \in A_1$, $a_2 \in A_2$, and $a_3 \in A_3$.

There are $3 \cdot 2 \cdot 4 = 24$ such 3-tuples.

# of choices for a_1

# of choices for a_2

# of choices for a_3