

For $0 \leq r \leq n$,

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

is the number of r -combinations of an n -element set.

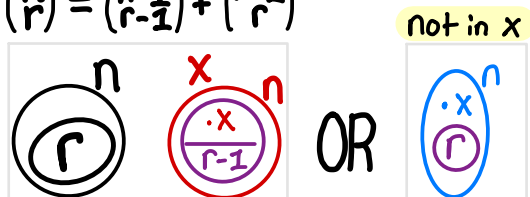
For $r > 0$ or $r \leq 0$.

define $\binom{n}{r} = 0$

The numbers $\binom{n}{r}$ are called Binomial Coefficients

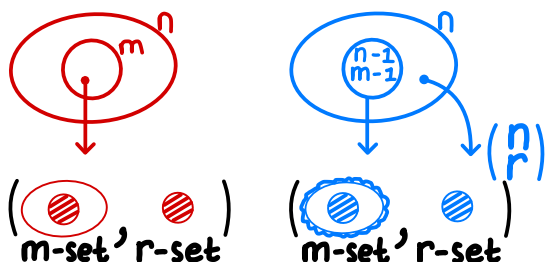
Relations satisfied by $\binom{n}{r}$:

- $\binom{n}{r} = \binom{n}{n-r}$
- $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$



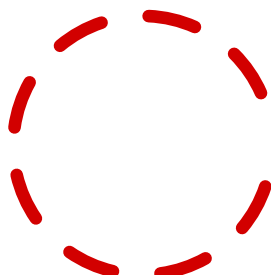
- $\binom{n}{r} = \frac{n}{r} \binom{n-1}{r-1}$
- $\binom{n}{r} = \frac{n-r+1}{r} \binom{n-1}{r}$
- $\binom{n}{m} \binom{m}{r} = \binom{n}{r} \binom{n-1}{m-1}$

$$\binom{n}{m} \binom{m}{r} = \binom{n}{r} \binom{n-1}{m-1}$$



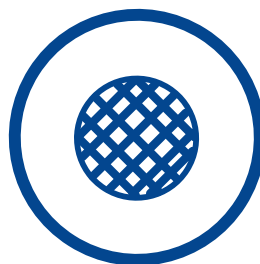
Line-up items

$$P_r^n = n(n-1)\dots(n-r+1)$$



Circle

$$Q_r^n = \frac{1}{r} P_r^n$$



Line-up & divide up with symmetry

$$C_r^n = \frac{P_r^n}{r!}$$