Homework Assignment 04

Matthew Mendoza

Recall

In general, if Q^n_r is the number of arrangements of 'r' elements placed around a circle chosen from a set of 'n' elements, then

$$Q^n_r = \frac{P^n_r}{r} = \frac{\frac{n!}{(n-r)!}}{r} = \frac{n!}{r\cdot(n-r)!}$$

* Note that if $r=n$, then

$$Q^n_r = \frac{n!}{n(n-n)!} = \frac{n!}{n\cdot 0} = \frac{n!}{n\cdot 1} = \frac{n!}{n} = (n-n)!$$

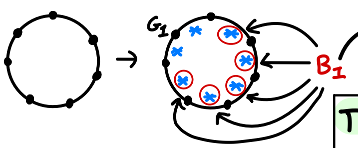
1) Example 1.3.2: In how many ways can 5 boys and 3 girls be seated around a table if

(i) there is no restriction?

$$\boxed{\frac{8!}{8} = 7!}$$

(ii) boy B_1 and girl G_1 are not adjacent?

Let us place all except B_1 there $\frac{7!}{7} = 6!$ ways to place the seven people around the table (we are not placing B_1 yet)

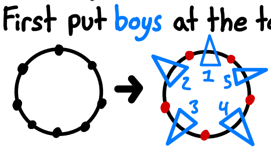


5 choices for B_1

This gives a total count of $6!\times 5$

(iii) no girls are adjacent?

First put boys at the table



$\frac{5!}{5} = 4!$ Possibilites

$5\cdot 4\cdot 3$ ways to place girls

$\Rightarrow$ Total is $4!\cdot 5\cdot 4\cdot 3 = \boxed{1440}$

2) Example 1.3.3 - Find the number of ways to seat 'n' married couples around a table in each of the following cases:

(i) Men and women alternate

For every partner in a pairing can be arranged $(n-1)!$ ways, for every pairing has 'n' many significant other, so in total we have:

$$\boxed{(n-1)!\,n!}$$

(ii) Every women is next to her husband

Can be thought as binary bits so,

$$\boxed{(n-1)!\,2^n}$$

Study #3 - Not sure why its 1232 & not 1512

***3) Exercise 1.6 - Find the number of odd integers between 3000 and 8000 in which no digit is repeated**

Some case analysis would help...

3,000 ⟶ 8,000

a	b	c	d

a: {3,4,5,6,7,8}   d: {1,3,5,7,9}

Can't do 8 upper limit

Case 01: First digit is even

- 2 choices for first digit: 4, 6
- Don't have to worry about 'a' = 'd'
- 'bc' can range from [0,9] but we need to factor that both the leading and tail digit are duplicates

So for case 01: $2\times P^{10-2}_2\times 5 \rightarrow 2\times P^8_2\times 5 = 840$

Case 02: Leading digit is odd

- 3 choices for first digit: 3, 5, 7
- Account for 'a' being one of the five in set {1,3,5,7,9} leaves us with 4 choices
- 'bc' can range from [0,9] but we need to factor that both the leading and tail digit is repeated

So for case 02: $3\times P^{10-2}_2\times 4 \rightarrow 3\times P^8_2\times 4 = 672$

In total $840 + 672 = 1512$

4) Example 1.4.1 - Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$, where 'n', $r\in\mathbb{N}$ with $r\le n$

Recall: $C^n_r = \frac{P^n_r}{r!}$ we can rewrite as $C^n_r = \frac{n!}{r!(n-r)!}$

By algebraic proof...

$$\binom{n-1}{r-1} + \binom{n-1}{r} = \frac{(n-1)!}{(r-1)!(n-r)!} + \frac{(n-1)!}{r!(n-1-r)!}$$

$$\rightarrow \frac{(n-r)!\,r + (n-1)!(n-r)}{r!(n-r)!}$$

$$\rightarrow \frac{(n-1)!(r+n-r)}{r!(n-r)!}$$

$$\boxed{\rightarrow \frac{n!}{r!(n-r)!} = \binom{n}{r}}$$

5) Example 1.4.2 - There are 2^7 binary sequences of length 7. How many sequences are there which contains three 0's and four 1's?

4x1's

6	5	4	3	2	1	0

3x0's

Within a range of 7 bits how many ways can we position three zeros... Once placed we are left with the obvious remaining spaces to put 1's giving us $C^4_4 = 1$, So what we care about are only the zeros $\therefore \boxed{C^7_3 = 35}$