California State University Sacramento - Math 101

Homework Assignment 3

5) (i) This is the number of 5-permutations of the set $\{A, B, C, D, E, F, G, H\}$ which is

$$P_5^{10} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6.$$

(ii) There are $6 \cdot 5 \cdot 4$ choices for the first, third, and fifth positions. For positions two and four there are $4 \cdot 3$ choices. This gives a total of

$$(6 \cdot 5 \cdot 4)(4 \cdot 3) = P_3^6 \cdot P_2^4$$

possibilities.

6) (i) This is the number of permutations of a set with 12 elements which is 12!.

(ii) First we choose the position of the first girl in the block of five girls. There are 8 such choices since if we put the first girl in position 9, 10, 11, or 12, then we will not be able to place the remaining four girls so that the girls form a single block. Once we have chosen the position of the first girl, there are 5! ways to arrange the 5 girls and 7! ways to arrange the boys. This gives a total of

$$8(5!)(7!)$$

possibilities.

7) (i) $nP_{r-1}^{n-1} = n \cdot \dfrac{(n-1)!}{(n-1-(r-1))!} = \dfrac{n!}{(n-r)!} = P_r^n.$

(ii) $(n-r+1)P_{r-1}^n = (n-r+1)\dfrac{n!}{(n-(r-1))!} = (n-r+1)\dfrac{n!}{(n-r+1)!} = \dfrac{n!}{(n-r)!} = P_r^n.$

(iii) $\dfrac{n}{n-r}P_r^{n-1} = \dfrac{n}{n-r} \cdot \dfrac{(n-1)!}{(n-1-r)!} = \dfrac{n!}{(n-r)!} = P_r^n.$

(iv) $P_r^n + rP_{r-1}^n = \dfrac{n!}{(n-r)!} + r \cdot \dfrac{n!}{(n-r+1)!} = \dfrac{n!(n-r+1)+r \cdot n!}{(n-r+1)!} = \dfrac{(n+1)!}{(n+1-r)!} = P_r^{n+1}$