Quiz #7

1) If $z=1$, then there are no choices for x or y.
If $z>1$, then x and y can be any of

$$\underbrace{1, 2, 3, \ldots, z-1}_{(z-1)\cdot(z-1)}$$

$(z-1)$ ↑ choices for x ; $(z-1)$ ↑ choices for y

$$|T| = \sum_{z=2}^{4} (z-1)^2 = 1^2 + 2^2 + 3^2 = 14$$

2) (a) $\dfrac{6!}{3!\,2!}$ (b) $\dfrac{(r_1 + r_2 + \ldots + r_n)}{r_1!\, r_2! \cdots r_n!}$

3) $H_r^n = \binom{n+r-1}{r}$

4) $$\frac{n-r+1}{r}\binom{n}{r-1} = \frac{n-r+1}{r} \cdot \frac{n!}{(r-1)!\,(n-(r-1))!}$$

$$= \frac{n-r+1}{r} \cdot \frac{n!}{(r-1)!\,(n-r+1)!}$$

$$= \frac{n!}{r!} \cdot \frac{n-r+1}{(n-n)!\,(n-r+1)}$$

$$= \frac{n!}{r!\,(n-r)!} = \binom{n}{r}$$

5) (a) $|A||B| = 3 \cdot 5 = 15$

(b) there are $\binom{3}{2}$ choices for A'

and $\binom{5}{2}$ choices for B'.

$$\Rightarrow \binom{3}{2}\binom{5}{2} = 3 \cdot 10 = 30$$

(c) $|A \cup B| = 8$ so $\binom{8}{3}$

6) (a) $\binom{5}{3} = 10$ (b) 2^5

(c) There are $\binom{5}{1} = 5$ choices for α

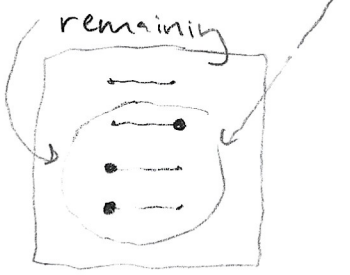
and then $2^{5-1} = 2^4$ choices for β.

(d) $5 \cdot \binom{4}{3} \cdot 2^3$

choose one edge say $a_i \text{---} b_i$ (two elements)

choose three more edges from the four remaining

then choose one endpoint from each edge



(e) 2^5

(e) $\binom{5}{2} \cdot 6$

choose edges which gives four elements $a_i \text{---} b_i$, $a_j \text{---} b_j$

pick one more element