

Homework Assignment 05

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SUBJECT:

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Q1

2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if
(iv) between two particular boys A and B, there are no boys but exactly 3 girls?

12 students - 5 girls = 7 boys in the party.



"The crew"

- Consider as 1 unit (a group),
so 7 single seats & 1 group seating \cong 8 seats
the group seating has their own arrangements
- A & B - 2 configurations (AB) & (BA)
- Girls in the center P_3^3 , 5 girls total, but 3 can be w/crew

So... $(5 + 2 + 1) \times 2! \times P_3^5 = 4,838,400$
Not including A & B ↑ Girls who aren't w/ the crew "The Crew"

Q2

15. In a group of 15 students, 5 of them are female. If exactly 3 female students are to be selected, in how many ways can 9 students be chosen from the group
- (i) to form a committee?
(ii) to take up 9 different posts in a committee?

Info gathering & initial thoughts

- 15 students, 5 female, 10 male
- Exactly 3 female selected C_3^5
- 9 students can be chosen to form a group
- 3 female & 6 male students in a group

* Arrangement order ~ 9 member committee ~ does not matter! M F M F M M M M F

(i) to form a committee (no additional requirements)

* Because arrangement order does not matter we use C_r^n not P_r^n

10 male, but 6 in committee $C_6^{10} \times$ 5 female, but only 3 in committee $C_3^5 = 2,100$

(ii) to take up 9 different posts in a committee?

$(C_6^{10} \times C_3^5) 9! = 76,204,800$

* 9 different posts *

Q3

Example 1.4.3. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if

- (i) there is no restriction in the selection?
(ii) the committee must include exactly 2 teachers?
(iii) the committee must include at least 3 teachers?

Notes & initial thoughts T S T S S

Order does not matter, so we use C_r^n not P_r^n for the selection of either students & teachers

(i) No restrictions

$\binom{11}{5} = 462$

(ii) Exactly 2 teachers

$\binom{7}{3} \binom{4}{2} = 210$

(iii) At least 3 teachers *Needs case analysis*

Case 01 - 3 teachers

$\binom{7}{2} \binom{4}{1} = 84$

Case 02 - 4 teachers

$\binom{7}{1} \binom{4}{2} = 7$

So in total Case 01 + Case 02

$\binom{7}{2} \binom{4}{1} + \binom{7}{1} \binom{4}{2} = 91$

Q4

2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if
(iii) no 2 girls are adjacent?

* No 2 girls adjacent = two girls aren't together
 $\rightarrow 12 \text{ students} - 5 \text{ girls} = 7 \text{ boys}$

* Seating position matters, so we want to use P_r^n



* 1 huge group seating & 2 single seating, so 3! of arranging 1 group and 2 single seaters

* Can start w/ B first or G first



$[2! P_5^7 \cdot P_5^5] \times 3! = 3,628,800$

20. In a group of 15 students, 3 of them are female. If at least one female student is to be selected, in how many ways can 7 students be chosen from the group

Q5

- (i) to form a committee?
(ii) to take up 7 different posts in a committee?

Notes - 7 student committee - At least 1 female

- 15 students - 3 female students = 12 male students
- Positional order does not matter, so use C_r^n

* Needs case analysis *

(i) No additional restrictions

Case 01 1 female 6 male $\binom{3}{1} \binom{12}{6}$
Case 02 2 female 5 male $\binom{3}{2} \binom{12}{5}$
Case 03 3 female 4 male $\binom{3}{3} \binom{12}{4}$

So $\binom{3}{1} \binom{12}{6} + \binom{3}{2} \binom{12}{5} + \binom{3}{3} \binom{12}{4}$
 $\Rightarrow 2772 + 2376 + 495$
 $= 5643$

(ii) Each configuration has 7 different posts

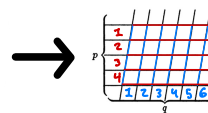
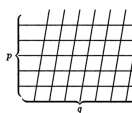
Case 01 1 female 6 male 7 posts $\binom{3}{1} \binom{12}{6} 7!$
Case 02 2 female 5 male 7 posts $\binom{3}{2} \binom{12}{5} 7!$
Case 03 3 female 4 male 7 posts $\binom{3}{3} \binom{12}{4} 7!$

So $\binom{3}{1} \binom{12}{6} 7! + \binom{3}{2} \binom{12}{5} 7! + \binom{3}{3} \binom{12}{4} 7!$
 $\Rightarrow 2772(5040) + 2376(5040) + 495(5040)$
 $= 28,440,720$

22. Two sets of parallel lines with p and q lines each are shown in the following diagram:

Q6

Find the number of parallelograms formed by the lines?



Notes: P = 4, Q = 6

So $\binom{4}{2} \binom{6}{2} \rightarrow 6 \cdot 15 = 90$

Took longer to think through... will this be on the exam?

23. There are 10 girls and 15 boys in a junior class, and 4 girls and 10 boys in a senior class. A committee of 7 members is to be formed from these 2 classes. Find the number of ways this can be done if the committee must have exactly 4 senior students and exactly 5 boys.

Q7

10 girls + 15 boys = 25 Juniors; 4 girls + 10 boys = 14 seniors

* Sequential, arranged order, does not matter: C_r^n

\rightarrow Find exactly 4 senior students (boy/girl) & 5 boys (JR/SR)

Approaches... Fill SR first? $\left[\binom{10}{4} \binom{15}{1} \binom{10}{2} \right] + \left[\binom{10}{3} \binom{4}{1} \binom{15}{2} \binom{10}{1} \right] + \left[\binom{10}{2} \binom{4}{2} \binom{15}{3} \right]$
Can't have more than 4 seniors 5th boy Completes Committee Completes Senior req. Completes boy req. Completes Committee Completes Senior req. both boys & Committee req.

In total there are 768600 ways