California State University Sacramento - Math 101

Homework Assignment 11 - Solutions

1) The characteristic equation is $x^2-3x+2=0$ which can be rewritten as $(x-2)(x-1)=0$. The roots of this equation are $x=2$ and $x=1$. Let

$$a_n = A\cdot 2^n + B\cdot 1^n = A\cdot 2^n + B.$$

The condition $a_0=2$ implies $2=A+B$. The condition $a_1=3$ implies $3=2A+B$. The solution to this linear system is $A=B=1$. Therefore,

$$a_n = 2^n + 1.$$

2) The characteristic equation is $x^2-6x+9=0$ which is equivalent to $(x-3)^2=0$. The number 3 is a double root of this quadratic equation. Let

$$a_n = (A+Bn)3^n.$$

The condition $a_0=2$ implies $2=A$ so that $a_n=(2+Bn)3^n$. The condition $a_1=3$ implies $3=(2+B)\cdot 3$ which gives $B=1$. Therefore,

$$a_n = (2-n)3^n.$$

3) The characteristic equation is $2x^2-x-1=0$ which can be rewritten as $(2x+1)(x-1)=0$. The roots of this quadratic are $-1/2$ and 1. Let

$$a_n = A\cdot 1^n + B\cdot(-1/2)^n = A + B\cdot(-1/2)^n.$$

The condition $a_0=0$ implies $0=A+B$. The condition $a_1=1$ implies $1=A-(1/2)B$. The solution to this system is $A=2/3$ and $B=-2/3$. The solution to this recurrence relation is

$$a_n = \frac{2}{3} - \frac{2}{3}\left(-\frac{1}{2}\right)^n.$$

4) The characteristic equation is $x^2-4x+4=0$ which is equivalent to $(x-2)^2=0$ and so 2 is a double root. Let

$$a_n = (A+Bn)2^n.$$

The initial condition $a_0=-1/4$ implies that $-1/4=A$. Hence, $a_n=\left(-\frac{1}{4}+Bn\right)2^n$. The condition $a_1=1$ implies $1=\left(-\frac{1}{4}+B\right)2$. Solving this equation for B gives $B=3/4$. Therefore,

$$a_n = \left(-\frac{1}{4}+\frac{3n}{4}\right)2^n.$$

5) The characteristic equation is $2x^3-x^2-2x+1=0$. Using factoring by grouping,

$$x^2(2x-1)-1(2x-1)=0 \quad\Rightarrow\quad (2x-1)(x^2-1)=0 \quad\Rightarrow\quad (2x-1)(x-1)(x+1)=0.$$

The roots of this equation are $x = 1/2$, $x = 1$, and $x = -1$. Let

$$a_n = A(1)^n + B(-1)^n + C(1/2)^n = A + B(-1)^n + C(1/2)^n.$$

The three initial conditions lead to the system of equations

$$\begin{aligned} A + B + C &= 0 \\ A - B + \frac{1}{2}C &= 0 \\ A + B + \frac{1}{4}C &= 2. \end{aligned}$$

The solution to this system is $A = 5/2$, $B = 1/6$, and $C = -8/3$. The solution to the congruence is

$$a_n = \frac{5}{2} + \frac{1}{6}(-1)^n - \frac{8}{3}(1/2)^n.$$