California State University Sacramento - Math 101

Make Up Quiz

Name: ______________________

1) Solve the recurrence relation $a_{n+2} = 10a_{n+1} \cancel{+} 24a_n$ given that $a_0 = 1$ and $a_1 = 2$.

$x^2 = 10x - 24$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x = 4, \; x = 6$

$a_n = A \cdot 4^n + B \cdot 6^n$

$1 = a_0 = A + B$

$2 = a_1 = 4A + 6B$

$\Longrightarrow$

$-4 = -4A - 4B$

$2 = 4A + 6B$

$-2 = 2B$

$-1 = B$

$1 = A + B$

$2 = A$

$a_n = 2 \cdot 4^n - 6^n$

2) Solve the recurrence $a_{n+2} = 8a_{n+1} - 16a_n$ given that $a_0 = 1$ and $a_1 = 3$.

$x^2 - 8x + 16 = 0$

$(x-4)^2 = 0$

$x = 4$ double root

$a_n = (A + Bn) \cdot 4^n$

$1 = a_0 = A$

$a_n = (1 + Bn) \cdot 4^n$

$3 = (1 + B) \cdot 4$

$\frac{3}{4} = 1 + B$

$-\frac{1}{4} = B$

$a_n = \left(1 - \frac{n}{4}\right) \cdot 4^n$

3) Find the number of integers in the set $\{1, 2, \ldots, 210\}$ that are divisible by 3, 5, or 7.

Let $A_k = \{x \in \{1,2,\ldots,210\} : x \text{ is divisible by } k\}$ Then

$|A_3| = \frac{210}{3} = 70, \ |A_5| = 42, \ |A_7| = 30$

$|A_3 \cap A_5| = \frac{210}{15} = 14 \qquad |A_3 \cap A_7| = 10 \qquad |A_5 \cap A_7| = 6$

$|A_3 \cap A_5 \cap A_7| = \frac{210}{105} = 2$

$\Rightarrow |A_1 \cup A_2 \cup A_3| = 70 + 42 + 30 - 14 - 10 - 6 + 2$

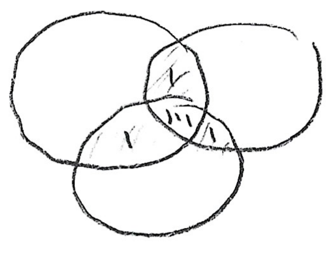
$= 142 - 30 = 114$

4) Suppose that 80 students played three sports; basketball, soccer, or volleyball. Each student may play one, two or all three sports. If 20 students played basketball, 30 played soccer, 45 played volleyball, and ~~5~~ 4 played all three sports, how many students played **exactly two** sports?

$|B| = 20 \quad |S| = 30 \quad |V| = 45 \quad |B \cap S \cap V| = 5$

$\Rightarrow 80 = 20 + 30 + 45 - |B \cap S| - |B \cap V| - |S \cap V|$
$+ 4$

$\Rightarrow |B \cap S| + |B \cap V| + |S \cap V| = 19$



$19 - 3(4) = 7$ played exactly two sports

5) Write out the Inclusion-Exclusion Principle in the case of the three finite sets A_1, A_2, and A_3.

$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3|$
$+ |A_1 \cap A_2 \cap A_3|$