Study the HW math

California State University Sacramento - Math 101

Exam # 1

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This exam is out of 12 points.

1) (a) Give a formula for P^n_r in terms of factorials. (0.25 points)

(b) Give a formula for Q^n_r in terms of factorials. (0.25 points)

(c) Give a formula for C^n_r in terms of factorials. (0.25 points)

(d) Express Q^n_r in terms of P^n_r. (0.25 points)

(a) $P^n_r = \frac{n!}{(n-r)!}$

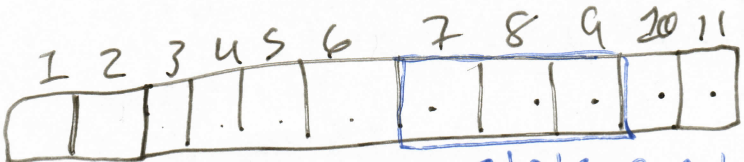
(b) $Q^n_r = \frac{P^n_r}{r} \Rightarrow \frac{\frac{n!}{(n-r)!}}{r} \Rightarrow \frac{n!}{r(n-r)!}$; $n=r$ $Q^n_n = (n-n)!$

(c) $C^n_r = \frac{P^n_r}{r!} \rightarrow \frac{\frac{n!}{(n-r)!}}{r!} \rightarrow \frac{n!}{r!(n-r)!}$

(d) See Part (b)

2) Find the number of ways that 8 boys and 3 girls can be put in a line such that the 3 girls form a single block. (1 point)

seating arrangements



8 single boys, 1 girls only

girls only

3! ways

Boys to seats
Permutations

$P^{9 \text{ seats}}_{8 \text{ boys to seat}}$

So $P^9_8 \times 3!$

4.5

4.9

2

3) Find the number of **ordered** pairs (a, b), with $a, b \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, such that $|a - b| \leq 2$. (1 point)

Absolute valu

Range

combos

$$\boxed{C^8_2 + 2}$$

9 & 10

4) Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$.

(a) List all 2-permutations of A. (0.5 points)

(b) List all 3-circular permutations of B. (0.5 points)

.25 (c) List all subsets of A of size 3. (0.25 points)

12 13 14

21 23 24

31 32 34

41 42 43

(a) (1,2,3,4) original

(4,3,2,1), (2,3,1,4)

(b) [a|b,c,d]

*abcd original

bcda

cdab

dabc

should be 8

c) C^4_3, Subsets of A size 3 $\{1, 2, 3, 4\}$

$\frac{4!}{3!(4-3)!} \to \frac{24}{6}$

$= 4$ combos

1. $\{1, 2, 3\}$
2. $\{2, 3, 4\}$
3. $\{3, 4, 1\}$
4. $\{4, 1, 2\}$

.25

5) (a) Find the number of common positive divisors of 10^{30} and 20^{20}. (1 point)

(b) Find the number of positive divisors of $900 = 2^2 \cdot 3^3 \cdot 5^2$ that are multiples of 5. (1 point)

a) 10^{30} 20^{20}

5^{30} 2^{30} 2^{20} 10^{20}

2 5

b) $900 = 2^2 \cdot 3^3 \cdot 5^2$

trying to recall

r+1, r-1 ...

6) Find the number of odd integers between 1000 and 3000 that have no repeated digit. (1 point)

.5

max

duplicates

odd

1 | $\underbrace{X\ X}_{P^{10-2}_{2}}$ | {1, 3, 5, 7, 9}

5-1

accounted for 1

So

$\boxed{1 \times P^{8}_{2} \times 4}$ □

this is the case when $a_1 = 1$ but also need to consider $a_1 = 2$

$\boxed{.5}$

7) (a) Determine the number of 0-1 sequences of length 5. For instance, 10101 and 11001 are two such sequences. (0.25 points)

(b) Determine the number of 0-1 sequences of length 5 that have exactly two 1's. (0.25 points)

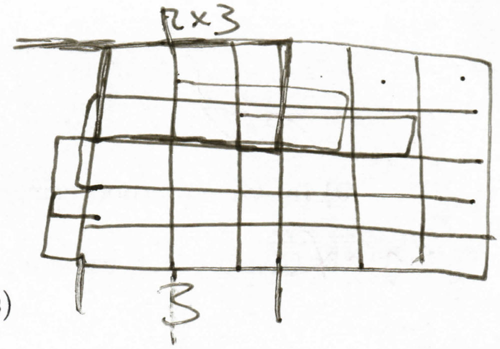
(c) Determine the number of 0-1 sequences of length 5 that have at most two 1's (so the sequence can have no 1's, one 1, or two 1's). (0.5 points)

(d) Let n be an arbitrary integer. Determine the number of 0-1 sequences of length n with at most one 1 (so the sequence can have no 1's, or one 1). (0.5 points)

(a) 2^5 seq? what is being asked

(b)

0



8) Let G be a 5×6 grid.

(a) Determine the number of $1x1$ squares in G. (0.5 points)

(b) Determine the number of 2×3 rectangles in G. (0.5 points)

(c) Determine the total number of squares in G. (0.5 points)

(d) Determine the total number of rectangles in G. (0.5 points) 2x3

(a) $5 \times 6 = 30$

(b) $3 \times 4 = 12$ .25

(c) 4×4

(d)

.75

.75

9) (a) Give a proof, using algebra, that $P^n_r = nP^{n-1}_{r-1}$. (0.75 points)

.25 (b) Give a proof, using algebra, that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$. (1 point)

(a) $P^n_r = nP^{n-1}_{r-1}$

LHS $\qquad\qquad$ $\boxed{\text{RHS}}$

P^n_r

$\rightarrow \dfrac{n(n-1)!}{[(n-1)-(r-1)]!}$

Def of $n!$

$n! = n(n-1)!$

$\rightarrow \dfrac{n(n-1)!}{[n+(\cancel{-1})+(-r)+\cancel{1}]!} \rightarrow \dfrac{n(n-1)!}{[n-r]!}$

$$\boxed{\frac{n!}{(n-r)!} = \frac{n!}{(n-r)!}}$$

b) $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$

LHS $\qquad\qquad$ RHS

$\dfrac{n!}{r!(n-r)!}$ $\qquad$ $\dfrac{(n-1)!}{(r-1)![(n-1)-(r-1)]!} + \dfrac{(n-1)!}{r!(n-1-r)!}$

$\rightarrow \dfrac{(n-1)!(n-1)!}{[[(r-1)!(n-r)!][r!(n-1-r)!]]}$