2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if
(iv) between two particular boys A and B, there are no boys but exactly 3 girls?

12 students − 5 girls = 7 boys in the party.

×	×	×	×	A	G_1	G_2	G_3	B	G_4	G_5	×

"The crew"

- Consider as 1 unit (a group), so 7 single seats & 1 group seating ≅ 8 seats
the group seating has their own arrangements
- A&B − 2 configurations (AB)&(BA)
- Girls in the center P^5_3, 5 girls total, but 3 can be w/crew

So... $(5+2+1)! \times [2! \times P^5_3] = 4,838,400$ ways

(5: Not including A&B; 2: Girls who aren't w/the crew; $[2! \times P^5_3]$: "The Crew")

15. In a group of 15 students, 5 of them are female. If exactly 3 female students are to be selected, in how many ways can 9 students be chosen from the group
(i) to form a committee?
(ii) to take up 9 different posts in a committee?

Info gathering & inital thoughts
- 15 students, 5 female, 10 male
- Exactly 3 female selected C^5_3
- 9 students can be chosen to form a group
  - 3 female & 6 male students in a group

* Arrangement order does not matter!

~ 9 member committee ~

M	F	M	F	M	M	M	M	F

(i) to form a committee (no additional requirements)

*Because arrangement order does not matter we use C^n_r not P^n_r

10 male, but 6 in committe C^{10}_6 × 5 female, but only 3 in committee C^5_3 = 2,100

(ii) to take up 9 different posts in a committee?

$(C^{10}_6 \times C^5_3)\,9! = 76,204,800$

9 different posts

Example 1.4.3. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if
(i) there is no restriction in the selection?
(ii) the committee must include exactly 2 teachers?
(iii) the committee must include at least 3 teachers?

Notes & initial thoughts

T	S	T	S	S

- Order does not matter, so we use C^n_r not P^n_r for the selection of either students & teachers

(i) No restrictions $\binom{11}{5} = 462$

(ii) Exactly 2 teachers $\binom{7}{3}\binom{4}{2} = 210$

(iii) At least 3 teachers *Needs case analysis*

Case01 − 3 teachers: $\binom{7}{2}\binom{4}{3} = 84$

Case02 − 4 teachers: $\binom{7}{1}\binom{4}{4} = 7$

So in total Case01 + Case02: $\binom{7}{2}\binom{4}{3} + \binom{7}{1}\binom{4}{4} = 84 + 7 = 91$

2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if
(iii) no 2 girls are adjacent?

* No 2 girls adjacent = two girls aren't together
→ 12 students − 5 girls = 7 boys
* Seating position matters, so we want to use P^n_r

0	1	2	3	4	5	6	7	8	9	10	11
B	G	B	G	B	G	B	G	B	G	B	B

- 1 huge group seating & 2 single seating, so 3! of arranging 1 group and 2 single seaters
- Can start w/ B first or G first

B	G	B	G	B	G	B	G	B	G	B	B

Group Seating | Single | Single

$[2! P^7_5 \cdot P^5_5] \times 3! = 3,628,800$

20. In a group of 15 students, 3 of them are female. If at least one female student is to be selected, in how many ways can 7 students be chosen from the group
(i) to form a committee?
(ii) to take up 7 different posts in a committee?

Notes − 7 student committee − At least 1 female
- 15 students − 3 female students = 12 male students
- Positional order does not matter, so use C^n_r

* Needs case analysis *

(i) No additional restrictions

case#1	case#2	case#3
1 female	2 female	3 female
6 male	5 male	4 male
$\binom{3}{1}\binom{12}{6}$	$\binom{3}{2}\binom{12}{5}$	$\binom{3}{3}\binom{12}{4}$

So $\binom{3}{1}\binom{12}{6} + \binom{3}{2}\binom{12}{5} + \binom{3}{3}\binom{12}{4}$

⇒ 2772 + 2376 + 495

= 5643

(ii) Each configuration has 7 different posts

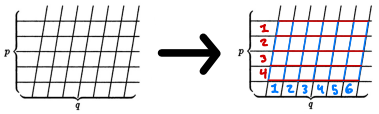
case#1	case#2	case#3
1 female	2 female	3 female
6 male	5 male	4 male
7 Posts	7 Posts	7 Posts
$\binom{3}{1}\binom{12}{6}7!$	$\binom{3}{2}\binom{12}{5}7!$	$\binom{3}{3}\binom{12}{4}7!$

So $\binom{3}{1}\binom{12}{6}7! + \binom{3}{2}\binom{12}{5}7! + \binom{3}{3}\binom{12}{4}7!$

⇒ 2772(5040) + 2376(5040) + 495(5040)

= 28,440,720

22. Two sets of parallel lines with p and q lines each are shown in the following diagram:
Find the number of parallelograms formed by the lines?



Notes: $p = 4$, $q = 6$

So $\binom{4}{2}\binom{6}{2} \to 6 \cdot 15 = 90$

Took longer to think through... will this be on the exam?

23. There are 10 girls and 15 boys in a junior class, and 4 girls and 10 boys in a senior class. A committee of 7 members is to be formed from these 2 classes. Find the number of ways this can be done if the committee must have exactly 4 senior students and exactly 5 boys.

10 girls + 15 boys = 25 Juniors; 4 girls + 10 boys = 14 seniors
- Sequential, arranged order, does not matter: C^n_r

→ Find exactly 4 senior students (boy/girl) & 5 boys (JR/SR)

Approaches...

fill SR first? $\left[\binom{10}{4}\binom{15}{1}\binom{10}{2}\right] + \left[\binom{10}{3}\binom{4}{1}\binom{15}{2}\binom{10}{1}\right] + \left[\binom{10}{2}\binom{4}{2}\binom{15}{3}\right]$

($\binom{10}{4}$: Can't have more than 4 seniors; $\binom{15}{1}$: 5th boy; $\binom{10}{2}$: Completes committee; $\binom{10}{3}$: Completes Senior req.; $\binom{4}{1}\binom{15}{2}$: completes boy req.; $\binom{10}{1}$: completes committee; $\binom{10}{2}$: Completes Senior req.; $\binom{4}{2}\binom{15}{3}$: completes both boys & committee req.)

In total there are 768600 ways