Review and Preparation for Exam 02

SUBJECT: Review and Preparation for Exam 02 DATE: 2023 / 03 / 10 PAGE#:

Example There are m 0's and n 1's where $m \geq 2$, $n \geq 2$, and $n \geq m$.

Q1) How many ways can the 0's and 1's be put in a line so that the 0's are all consecutive?

scratch work

$m=3$, $n=3$

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$$m+1 \text{ ways}$$

Q2) ... put in a line such that the first position and last position must be a zero

$$\underbrace{0}_{\text{zero}} \underbrace{\binom{m+n-2}{n}}_{m+n-2} \underbrace{0}_{\text{zero}}$$

Q3) ... put in a line so that no two 0's, zeros, are adjacent?

m 0's, n 1's, no adjacent 0's

$$\_\ 1\ \_\ 1 \ldots \_\ 1 \binom{n+1}{m}$$

#3 from HW6

We want to Know how many combination of students with the particular front row girls and back row boys, and the other students

1	G_2	2	3	G_3	4	5	6	G_1	7
8	B_1	9	B_2	10	11	B_3	12	13	B_4

G_1	G_2	G_3	1	2	3	4	5	6	7
B_3	8	B_4	9	B_2	10	B_1	11	12	13

We don't care about ordered positions of the individual students, so we use C^n_r not P^n_r

1st Front row →	G_1	G_2	G_3	1	2	3	4	5	6	7
2nd Back row →	8	9	10	11	12	13	B_1	B_2	B_3	B_4

Approach 01

Because the front row must always be populated by the girls we can pull any 1 of the 13 other students to fill the front 7 spots

G_1	G_2	G_3	1	2	3	4	5	6	7
8	9	10	11	12	13	B_1	B_2	B_3	B_4

$$\binom{13}{7} \times \underbrace{10!}_{\text{1st Front row}} \times \underbrace{10!}_{\text{2nd Back row}}$$

10 seats, so 10! seating arrangements

By default the back row gets chosen automatically

In total $\underbrace{\binom{13}{7}}_{1716} \times \underbrace{10!}_{3628800} \times \underbrace{10!}_{3628800} \cong 22{,}596{,}613{,}080{,}000{,}000$

$$\binom{10}{3}3!\binom{10}{4}4!\,13!$$

$$\rightarrow \frac{10!}{3!7!} \cdot 3! \cdot \frac{10!}{4!6!} \cdot 4! \cdot 13! = \binom{13}{7}(10!)^2$$

Approach 02

Back row choose the 4 boys

Config. of boys amongst themselves

G_1	G_2	G_3	1	2	3	4	5	6	7
8	9	10	11	12	13	B_1	B_2	B_3	B_4

$$\binom{10}{3} \times 3! \times \binom{10}{4} \times 4! \times 13!$$

Front row choose the 3 girls

Config. of girls amongst themselves

Placement of NPCs

In total $\binom{10}{3} \times 3! \times \binom{10}{4} \times 4! \times 13! \cong 22{,}596{,}613{,}080{,}000$