California State University Sacramento - Math 101

Practice for Permutations and Circular Permutations

1) Determine the number of 3-permutations of a set with 7 elements.

2) List all 2 permutations of the set $A = \{a, b, c\}$.

3) Find the number of sequences of length 4 that consist of distinct elements of the set $\{1, 2, 3, 4, 5\}$ where the first number in the sequence is even.

4) Find the number of sequences of length 4 whose entries come from the set $\{1, 2, 3, 4, 5\}$ where the first number in the sequence is even.

5) Find the number of odd integers between 10,000 and 20,000 where no digit is repeated.

6) Find the number of 3-circular permutations of a set with 7 elements.

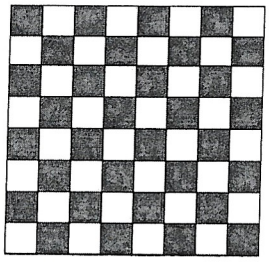
7) List all 3-circular permutations of the set $A = \{x, y, z, t\}$.

8) Write down formulas for P_r^n, Q_r^n, and write down an equation that relates Q_r^n to P_r^n.

For Fun Start with an 8×8 chessboard (see below). Define three operations

1. choose a column and interchange the colors in that column,
2. choose a row and interchange the colors in that row, or
3. choose a 2×2 square and interchange the colors in that square.

Is it possible to perform some sequence of these operations so that there is only one black square?



1) $7 \cdot 6 \cdot 5 = 210$

2) ab ba ca
ac bc cb

3) $2 \cdot 4 \cdot 3 \cdot 2 = 48$

	↑	↑	↑	↑
# of choices	2	4	3	2

↑ must be even so is 2 or 4

4) $2 \cdot 5 \cdot 5 \cdot 5 = 250$

	↑	↑	↑	↑
# of choices	2	5	5	5

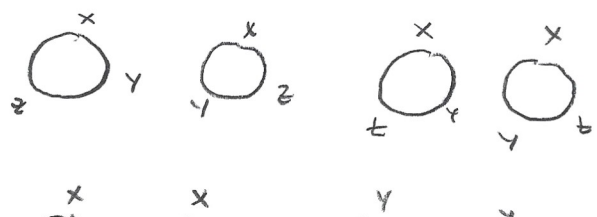
5) _ _ _ _ _ _

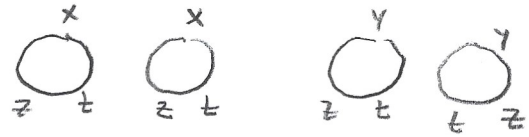
↑ must be 1 ↑ must be 3, 5, 7, or 9

The first entry must be 1.
the last entry is 3, 5, 7, or 9
Since no digit is repeated,
we have
$8 \times 7 \times 6$ choices for
entries 2, 3, and 4.

Total count : $4 \times 8 \times 7 \times 6 = 1344$

6) $\frac{7 \cdot 6 \cdot 5}{3} = 70$

7) 



8) $P_r^n = \frac{n!}{(n-r)!}$ $Q_r^n = \frac{1}{r} \cdot \frac{n!}{(n-r)!}$

$Q_r^n = \frac{1}{r} P_r^n$