

Homework Assignment 6

- 1) Example 1.4.6
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Q1

Example 1.4.6. If there must be at least one person in each table, in how many ways can 6 people be seated

- (i) around two tables?
- (ii) around three tables?

(We assume that the tables are indistinguishable.)

Q2

Example 1.4.7. Show that

$$s(r, n) = s(r - 1, n - 1) + (r - 1)s(r - 1, n)$$

where $r, n \in \mathbb{N}$ with $n \leq r$.

Q3

18. A group of 20 students, including 3 particular girls and 4 particular boys, are to be lined up in two rows with 10 students each. In how many ways can this be done if the 3 particular girls must be in the front row while the 4 particular boys be in the back?
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Q4

19. In how many ways can 7 boys and 2 girls be lined up in a row such that the girls must be separated by exactly 3 boys?
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Q5

21. Find the number of $(m + n)$ -digit binary sequences with m 0's and n 1's such that no two 1's are adjacent, where $n \leq m + 1$.
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Q6

26. Find the number of ways of forming a group of $2k$ people from n couples, where $k, n \in \mathbb{N}$ with $2k \leq n$, in each of the following cases:
 - (i) There are k couples in such a group;
 - (ii) No couples are included in such a group;
 - (iii) At least one couple is included in such a group;
 - (iv) Exactly two couples are included in such a group.

Q1) Example 1.4.6. - If there must be at least one person in each table, in how many ways can 6 people be seated (We assume that the tables are indistinguishable)

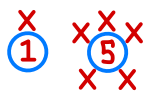
(i) around two tables?

$$S(6, 2) \quad S(r\text{-people}, n\text{-tables})$$

Recall: if $r=n$ then

$$Q_r^n = \frac{n!}{n(n-n)!} = (n-1)!$$

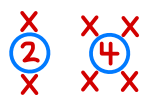
case01



$$(6)Q_1^1 \cdot Q_5^5$$

$$\begin{aligned} & \downarrow \\ & (6) \times 0! \times 4! \\ & \rightarrow 6 \times 1 \times 24 \\ & \rightarrow 144 \end{aligned}$$

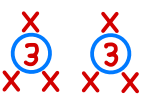
case02



$$(6)Q_2^2 \cdot Q_4^4$$

$$\begin{aligned} & \downarrow \\ & (6) \times 1! \times 3! \\ & \rightarrow 15 \times 1 \times 6 \\ & \rightarrow 90 \end{aligned}$$

case03



$$(6)Q_3^3 \cdot Q_3^3$$

Accounts for the double count the two tables w/ 3 people

$$\begin{aligned} & \downarrow \\ & [(6) \times 2! \times 2!] \cdot \left(\frac{1}{2}\right) \\ & \rightarrow [20 \times 2 \times 2] \cdot \left(\frac{1}{2}\right) \\ & \rightarrow 40 \end{aligned}$$

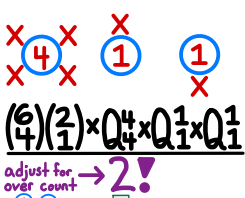
case01 case02 case03

$$\text{In total } 144 + 90 + 40 = 274$$

(ii) around three tables?

$$S(6, 3) \quad S(r\text{-people}, n\text{-tables})$$

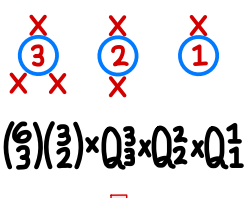
case01



$$(6)Q_4^4 \cdot Q_1^1 \cdot Q_1^1$$

$$\begin{aligned} & \downarrow \\ & [(15)(2) \times 3! \times 0! \times 0!] \cdot \left(\frac{1}{2}\right) \\ & \rightarrow [(30) \times 6 \times 1 \times 1] \cdot \left(\frac{1}{2}\right) \\ & \rightarrow 90 \end{aligned}$$

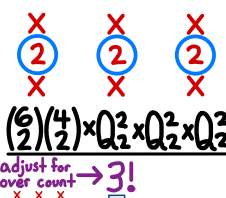
case02



$$(6)Q_3^3 \cdot Q_2^2 \cdot Q_2^2$$

$$\begin{aligned} & \downarrow \\ & [(20)(3) \times 2! \times 1! \times 0!] \\ & \rightarrow [(60) \times 2 \times 1 \times 1] \\ & \rightarrow 120 \end{aligned}$$

case03



$$(6)Q_2^2 \cdot Q_2^2 \cdot Q_2^2$$

$$\begin{aligned} & \downarrow \\ & [(15)(6) \times 0! \times 0! \times 0!] \cdot \left(\frac{1}{6}\right) \\ & \rightarrow [(90) \times 1 \times 1 \times 1] \cdot \left(\frac{1}{6}\right) \\ & \rightarrow 15 \end{aligned}$$

case01 case02 case03

$$\text{In total } 90 + 120 + 15 = 225$$

Q2) Example 1.4.7 Show that

$$s(r, n) = s(r-1, n-1) + (r-1)s(r-1, n)$$

where $r, n \in \mathbb{N}$ with $n \leq r$

$$s(r, n) = s(r-1, n-1) + (r-1)s(r-1, n)$$

For every given table this guarantees a seat at that table for that 1 person (a table for at least 1 person by themselves) OR For person who had their own table else sit at those tables Have everyone sit at those tables



18. A group of 20 students, including 3 particular girls and 4 particular boys, are to be lined up in two rows with 10 students each. In how many ways can this be done if the 3 particular girls must be in the front row while the 4 particular boys be in the back?

Q3)

We want to know how many combination of students with the particular front row girls and back row boys, and the other students

1	G ₁	2	3	G ₂	4	5	6	G ₃	7	G ₁	G ₂	G ₃	1	2	3	4	5	6	7
8	B ₁	9	B ₂	10	11	B ₃	12	13	B ₄	B ₃	8	B ₄	9	B ₂	10	B ₁	11	12	13

We don't care about ordered positions of the individual students, so we use C_r^n not P_r^n

1 st Front row	→	G ₁	G ₂	G ₃	1	2	3	4	5	6	7
2 nd Back row	→	8	9	10	11	12	13	B ₁	B ₂	B ₃	B ₄

Approach 01

Because the front row must always be populated by the girls we can pull any 1 of the 13 other students to fill the front 7 spots

G ₁	G ₂	G ₃	1	2	3	4	5	6	7
8	9	10	11	12	13	B ₁	B ₂	B ₃	B ₄

By default the back row gets chosen automatically

$$\text{In total } \binom{13}{7} \times 10! \times 10! \cong 22,596,613,080,000,000$$

Approach 02

G ₁	G ₂	G ₃	1	2	3	4	5	6	7
8	9	10	11	12	13	B ₁	B ₂	B ₃	B ₄

$$\binom{10}{3} \times 3! \times \binom{10}{4} \times 4! \times 13!$$

$$\text{In total } \binom{10}{3} \times 3! \times \binom{10}{4} \times 4! \times 13! \cong 22,596,613,080,000$$

19. In how many ways can 7 boys and 2 girls be lined up in a row such that the girls must be separated by exactly 3 boys?

Q4)

G	B	B	B	G	B	B	B	B
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Base Case Ways to place the girls 5, but both girls can swap places it's twice that: 10

21. Find the number of $(m+n)$ -digit binary sequences with m 0's and n 1's such that no two 1's are adjacent, where $n \leq m+1$.

Notes: a set of all strings w/no consecutive 1's where $\Sigma = \{0, 1\}$
 → Base cases: 0, 00, 01, 10, 000, 001, 010, 100, 110, 111...
 → RegEx: $0 + [(0+10)^* (\lambda+1)]$

$$\begin{aligned} 0's &= m \\ 1's &= n \end{aligned}$$

restriction $n \leq m+1$

Binary $\binom{m+1}{n}$ of states: 1 & 0 choose a state

$$\begin{aligned} & \text{we apply our restriction so...} \\ & \binom{m+1}{n} \end{aligned}$$

See Back

Q6)

26. Find the number of ways of forming a group of $2k$ people from n couples, where $k, n \in \mathbb{N}$ with $2k \leq n$, in each of the following cases:

- (i) There are k couples in such a group;
- (ii) No couples are included in such a group;
- (iii) At least one couple is included in such a group;
- (iv) Exactly two couples are included in such a group.

(i) No restriction

$$\binom{n}{k}$$

(ii)

$$\binom{n}{2k} \cdot 2^{2k}$$

(iii)

$$\binom{n}{2k} - \binom{n}{k}$$

(iv)

$$\binom{n}{2} \binom{n-2}{2k-4} \cdot 2^{2k-4}$$