California State University Sacramento - Math 101

Homework Assignment 7

1) Example 1.5.1

2) Example 1.5.2

3) Example 1.5.3 (know the result, not the proof)

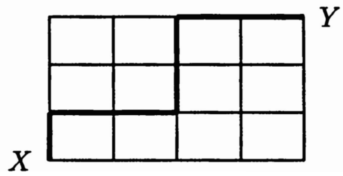
4) Problem 25 on page 53

5) Problem 27 on page 53

6) Problem 40 on page 55

7) Problem 41 on page 55

Q1 **Example 1.5.1.** A student wishes to walk from the corner X to the corner Y through streets as given in the street map shown in Figure 1.5.1. How many shortest routes are there from X to Y available to the student?



Q2 **Example 1.5.2.** Show that if $|X| = n$, then $|\mathcal{P}(X)| = 2^n$ for all $n \in \mathbf{N}$.

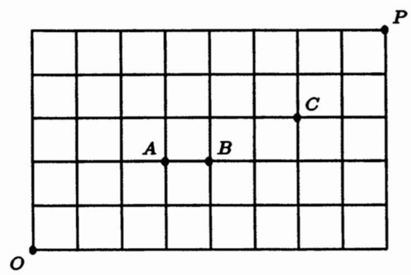
Q3 **Example 1.5.3.** Let $X = \{1, 2, ..., n\}$, where $n \in \mathbf{N}$. Show that the number of r-combinations of X which contain no consecutive integers is given by

$$\binom{n-r+1}{r},$$

where $0 \le r \le n - r + 1$.

Q4

25. In each of the following cases, find the number of shortest routes from O to P in the street network shown below:



(i) The routes must pass through the junction A;
(ii) The routes must pass through the street AB;
(iii) The routes must pass through junctions A and C;
(iv) The street AB is closed.

Q5

27. Let $S = \{1, 2, \ldots, n+1\}$ where $n \ge 2$, and let

$$T = \{(x, y, z) \in S^3 \mid x < z \quad \text{and} \quad y < z\}.$$

Show by counting $|T|$ in two different ways that

$$\sum_{k=1}^{n} k^2 = |T| = \binom{n+1}{2} + 2\binom{n+1}{3}.$$

Q6

40. Prove the identity $\binom{n}{r} = \binom{n}{n-r}$ by (BP)

Q7

41. Let $X = \{1, 2, ..., n\}$, $\mathcal{A} = \{A \subseteq X \mid n \notin A\}$, and $\mathcal{B} = \{A \subseteq X \mid n \in A\}$. Show that $|\mathcal{A}| = |\mathcal{B}|$ by (BP).