California State University Sacramento - Math 101

Quiz #12

Name: ____________________

1) Solve the recurrence $a_{n+2} = a_{n+1} + 6a_n$ given that $a_0 = 0$ and $a_1 = 1$.

2) Solve the recurrence $a_{n+2} = 6a_{n+1} - 9a_n$ given that $a_0 = 1$ and $a_1 = 2$.

3) State the Inclusion-Exclusion Principle in the case of three finite sets A, B, and C.

4) Find the number of integers in the set $\{1, 2, \ldots, 180\}$ that are divisible by 4 or 9.

5) Find the number of integers in the set $\{1, 2, \ldots, 180\}$ that are divisible by 2, 3, or 5.

6) Suppose that 80 students played three sports; basketball, soccer, or volleyball. Each student may play one, two or all three sports. If 20 students played basketball, 30 played soccer, 45 played volleyball, and 5 played all three sports, how many students played at least two sports?

7) Suppose that 80 students played three sports; basketball, soccer, or volleyball. Each student may play one, two or all three sports. Assume that 45 students played basketball, 30 played soccer, and 45 played volleyball. Also, suppose that at most 15 students played both basketball and soccer, at most 10 students played both basketball and volleyball, and at most 20 students played both soccer and volleyball. Find the largest possible number of students that could have played all three sports.

8) (a) Solve the recurrence relation $a_{n+3} = a_{n+2} + a_{n+1} - a_n$ with initial conditions $a_0 = 1$, $a_1 = 1$, and $a_2 = 2$.

(b) Let a_n be the number of ways of walking up n stairs where you may take one, two, or three stairs at a time. Find a recurrence relation for a_n and initial conditions for a_1 and a_2 (we define $a_0 = 1$).