1) (a) $\binom{11}{3} = \frac{11!}{3!8!} = \frac{11\cdot10\cdot9}{3\cdot2} = 11\cdot5\cdot3 = 55\cdot3 = 165$

(b) $\binom{8}{0} = 1$

2) $\binom{11}{3} = 165$

3) (a) Letting $x=y=1$ in the Binomial Theorem gives

$$(1+1)^n = \sum_{r=0}^{n}\binom{n}{r}1^r1^{n-r} \quad \text{so} \quad 2^n = \sum_{r=0}^{n}\binom{n}{r}$$

(b) Let $x=2, y=1$ in the Binomial Theorem ← details left to you!

(c) Let $x=-1$ and $y=1$ in

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^ky^{n-k}$$

to get

$$0 = \sum_{k=0}^{n}\binom{n}{k}(-1)^k$$

4) $$\sum_{r=1}^{10}\binom{10}{r} = -\binom{10}{0} + \sum_{r=0}^{10}\binom{10}{r} = -1+2^{10} = -1+1024$$
$$= 1023$$

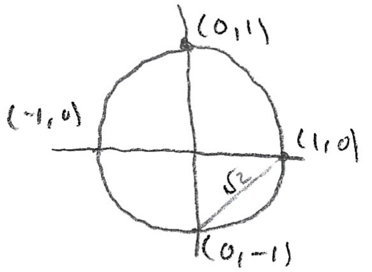
5) (a) $X \setminus A = \{4,5\}$

(b) $\binom{5}{1}+\binom{5}{3}+\binom{5}{5} = 5+10+1 = 16$

(c) $\binom{5}{0}+\binom{5}{2}+\binom{5}{4} = 1+10+5 = 16$

(d) $f(\{5\}) = \{1,2,3,4\}$

6)



By the Pigeonhole Principle, two points must be in the same quadrant and the farthest those two points can be from each other is $\sqrt{2}$.

7) (a) $\frac{100}{12} = 8\frac{4}{12} = 8\frac{1}{3} > 8$ ← $\frac{100}{12}$ is strictly greater than 8

By the Pigeonhole Principle, at least 9 people were born in the same month.

(b) The number of 0-1 sequences of length 6 is $2^6 = 64$ and since there are 100 students, at least two picked the same sequence.

(c) $\binom{15}{2} = \frac{15!}{2!\,13!} = \frac{15 \cdot 14}{2} = 15 \cdot 7 = 95$

If you purchase 95 tickets of all different types, you will have a winning ticket.

8) (a) $\binom{9}{2,3,4}$ (b) $\binom{6}{1,2,3} = \dfrac{6!}{1!2!3!} = \dfrac{6\cdot5\cdot4}{2} = 60$

9) (a) $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a,b,c\}$

(b)

$00 \longrightarrow \emptyset$
$10 \longrightarrow \{x\}$
$01 \longrightarrow \{y\}$
$11 \longrightarrow \{x,y\}$

(c)

$000 \longrightarrow \emptyset$
$100 \longrightarrow \{x\}$
$010 \longrightarrow \{y\}$
$001 \longrightarrow \{z\}$
$110 \longrightarrow \{x,y\}$
$101 \longrightarrow \{x,z\}$
$011 \longrightarrow \{y,z\}$
$111 \longrightarrow \{x,y,z\}$

10) The subsets of X are

$\emptyset$ $\{1\}$ $\{2\}$ $\{3\}$ $\{1,2\}$ $\{1,3\}$ $\{2,3\}$ $\{1,2,3\}$

Only four subsets do not contain 1
so if a five subsets are chosen
one must contain 1.