California State University Sacramento - Math 101

Homework Assignment 3

1) Example 1.2.1 on page 6. **Example 1.2.1.** Let $A = \{a, b, c, d\}$. All the 3-permutations of A are

2) Example 1.2.2 on page 7.

3) Example 1.2.3 on page 8.

4) Example 1.2.4 on page 9.

Example 1.2.2. Let $E = \{a, b, c, ..., x, y, z\}$ be the set of the 26 English alphabets. Find the number of 5-letter words that can be formed from E such that the first and last letters are distinct vowels and the remaining three are distinct consonants.

5) Problem 4 on page 50.

6) Problem 2(i) and 2(ii) on page 50.

7) Problem 14(i), 14(ii), 14(iii), and 14(iv) on page 51.

Example 1.2.3. There are 7 boys and 3 girls in a gathering. In how many ways can they be arranged in a row so that

(i) the 3 girls form a single block (i.e. there is no boy between any two of the girls)?

(ii) the two end-positions are occupied by boys and no girls are adjacent?

Example 1.2.4. Between 20000 and 70000, find the number of even integers in which no digit is repeated.

4. How many 5-letter words can be formed using A, B, C, D, E, F, G, H, I, J,
   (i) if the letters in each word must be distinct?
   (ii) if, in addition, A, B, C, D, E, F can only occur as the first, third or fifth letters while the rest as the second or fourth letters?

2. There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if
   (i) there are no restrictions?
   (ii) the 5 girls must be together (forming a block)?

14. Let $n, r \in \mathbf{N}$ with $r \leq n$. Prove each of the following identities:
   (i) $P_r^n = nP_{r-1}^{n-1}$,
   (ii) $P_r^n = (n-r+1)P_{r-1}^n$,
   (iii) $P_r^n = \frac{n}{n-r}P_r^{n-1}$, where $r < n$,
   (iv) $P_r^{n+1} = P_r^n + rP_{r-1}^n$,
   (v) $P_r^{n+1} = r! + r(P_{r-1}^n + P_{r-1}^{n-1} + \cdots + P_{r-1}^r)$.