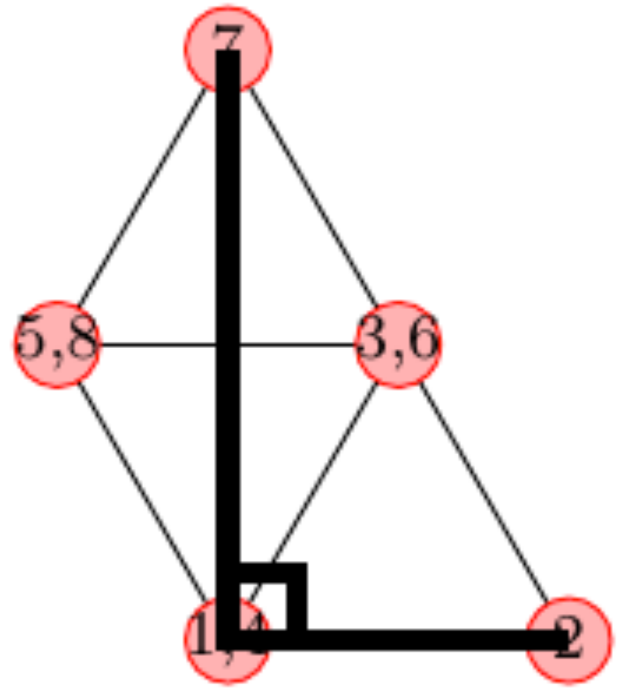
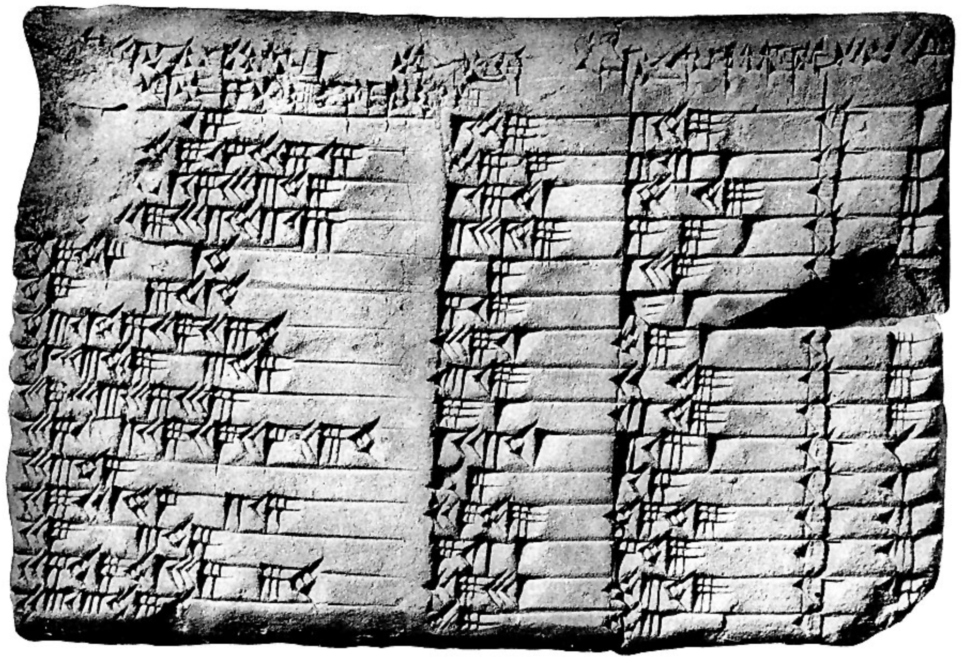


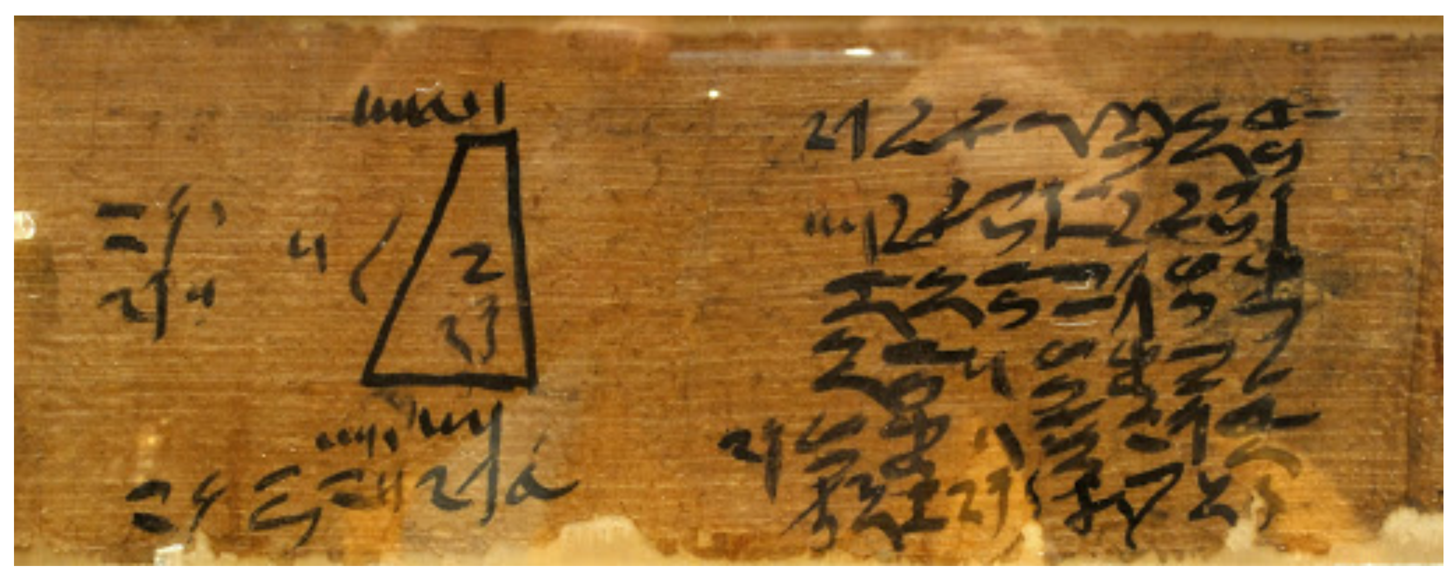
Chapter 2: Ancient Methods



$$\begin{array}{l} 26 \to 31 \\ 13 \to 62 \\ 6 \to 124 \\ 3 \to 248 \\ 1 \to 496 \end{array}$$

$$\begin{array}{r} 62 \\ +\ 248 \\ 496 \\ \hline 806 \end{array}$$

Egyptian and Babylonian Mathematics



Ancient Egyptian Mathematics

Egypt

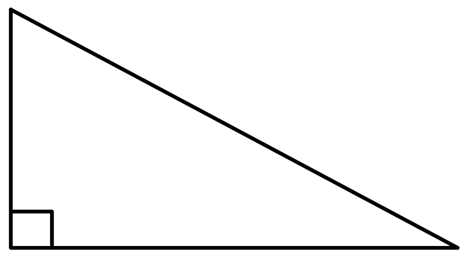

Romania
Croatia
Serbia
Italy
Rome
Black Sea
Georgia
Uzbekistan
Bulgaria
Istanbul
Ankara
Azerbaijan
Tyrrhenian Sea
Greece
Turkey
Turkmenistan
Syria
Lebanon
Tunisia
Mediterranean Sea
Iraq
Afghanistan
Israel
Iran
Cairo
القاهرة
Algeria
Libya
Egypt
Persian Gulf
Riyadh
الرياض
United Arab Emirates
Saudi Arabia
Jeddah
جدة
Red Sea
Oman
Mali
Niger
Sudan
Yemen
Chad
Eritrea
Burkina Faso
Gulf of Aden
Arabian Sea
Djibouti
Benin

Egypt

Pragmatic or Piratical?

Utilitarian "Theorems"

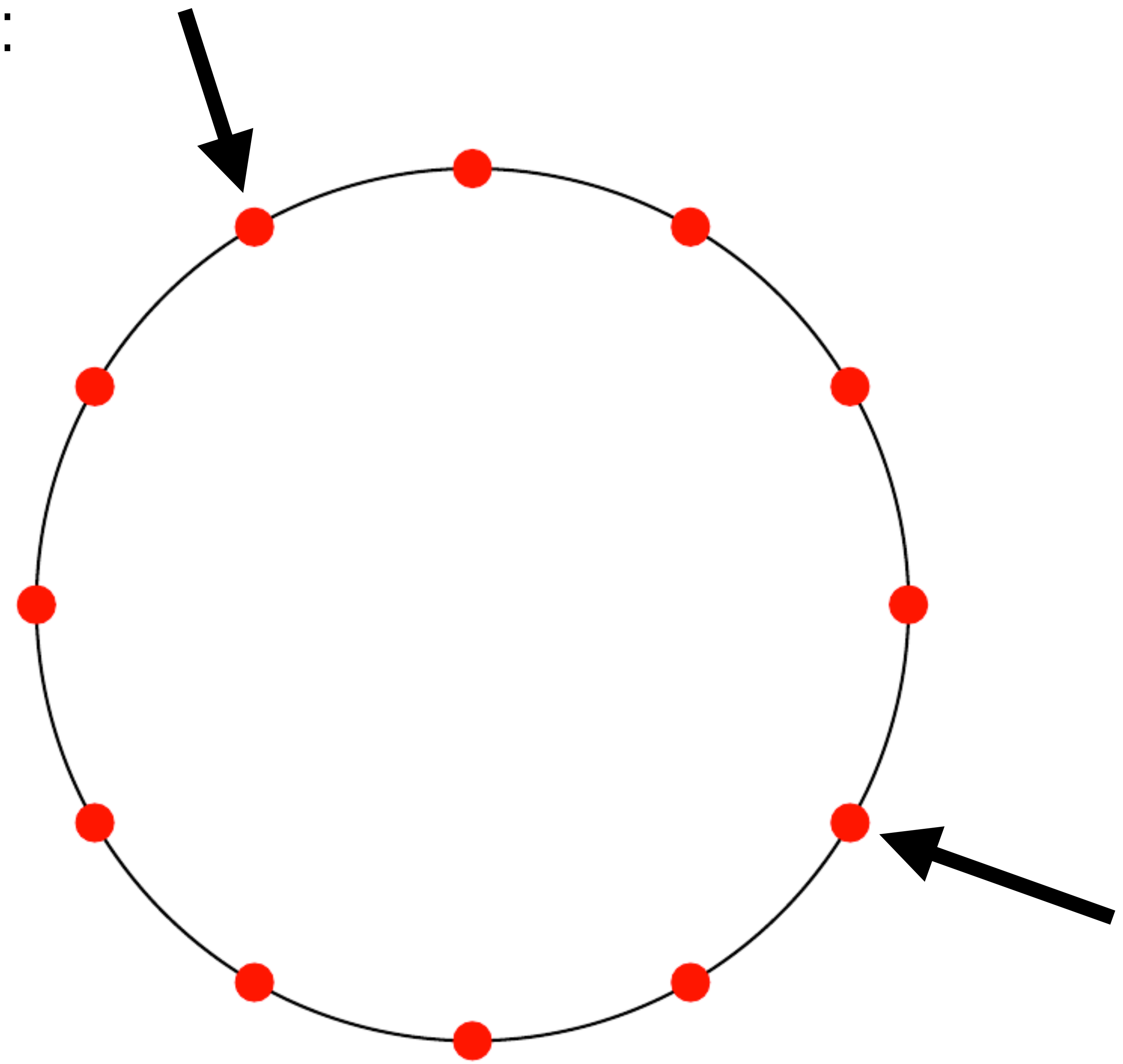
- Today, we demand proof of all of our theorems.
- Before this formality, scholars still discovered many true results.
- Often the goal was to use the mathematical property to some practical end. They were utilitarian.
- Important example: Constructing right angles.



Think Like A Math Historian

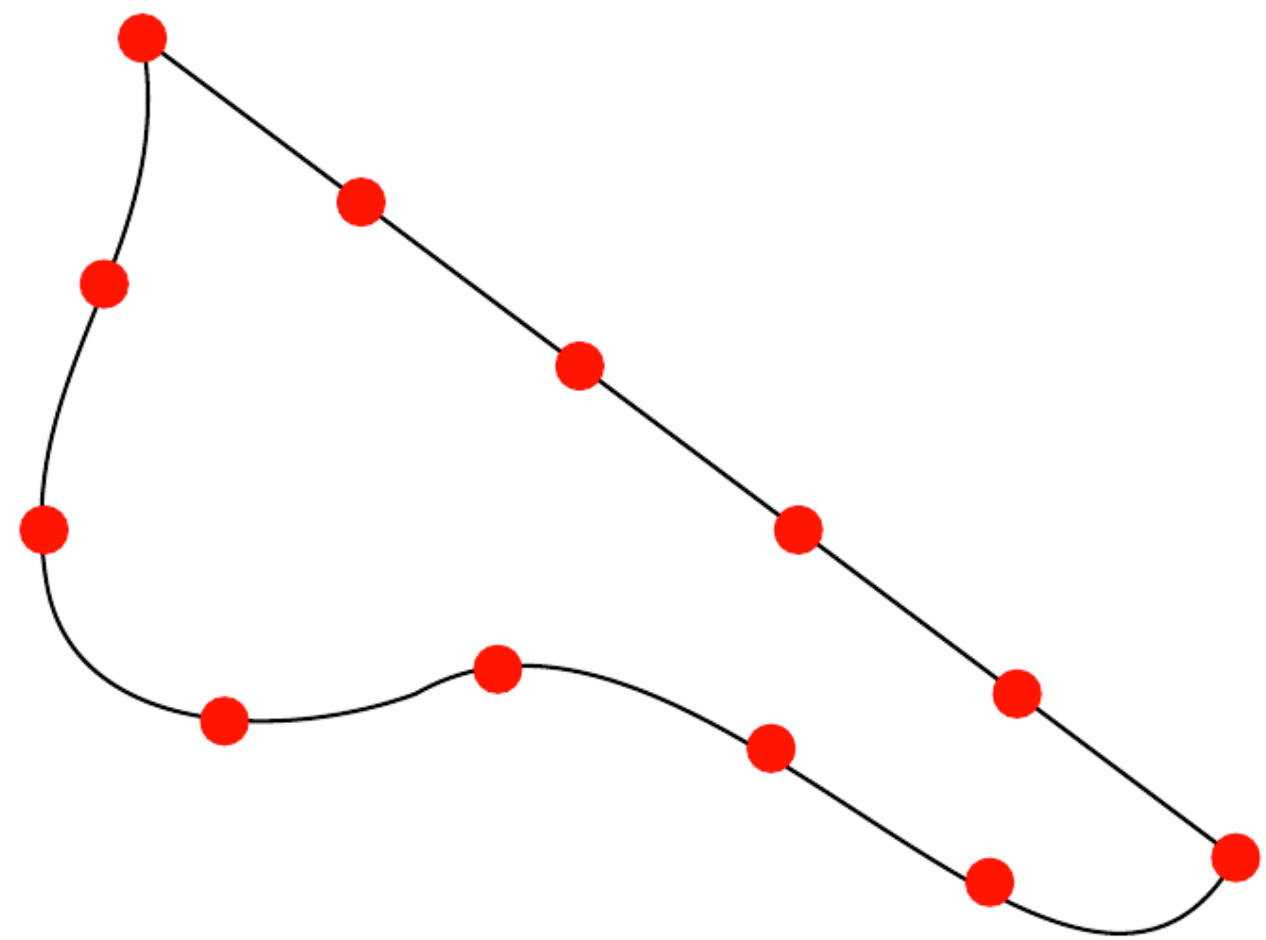
Utilitarian "Theorems"

- Egyptian method:



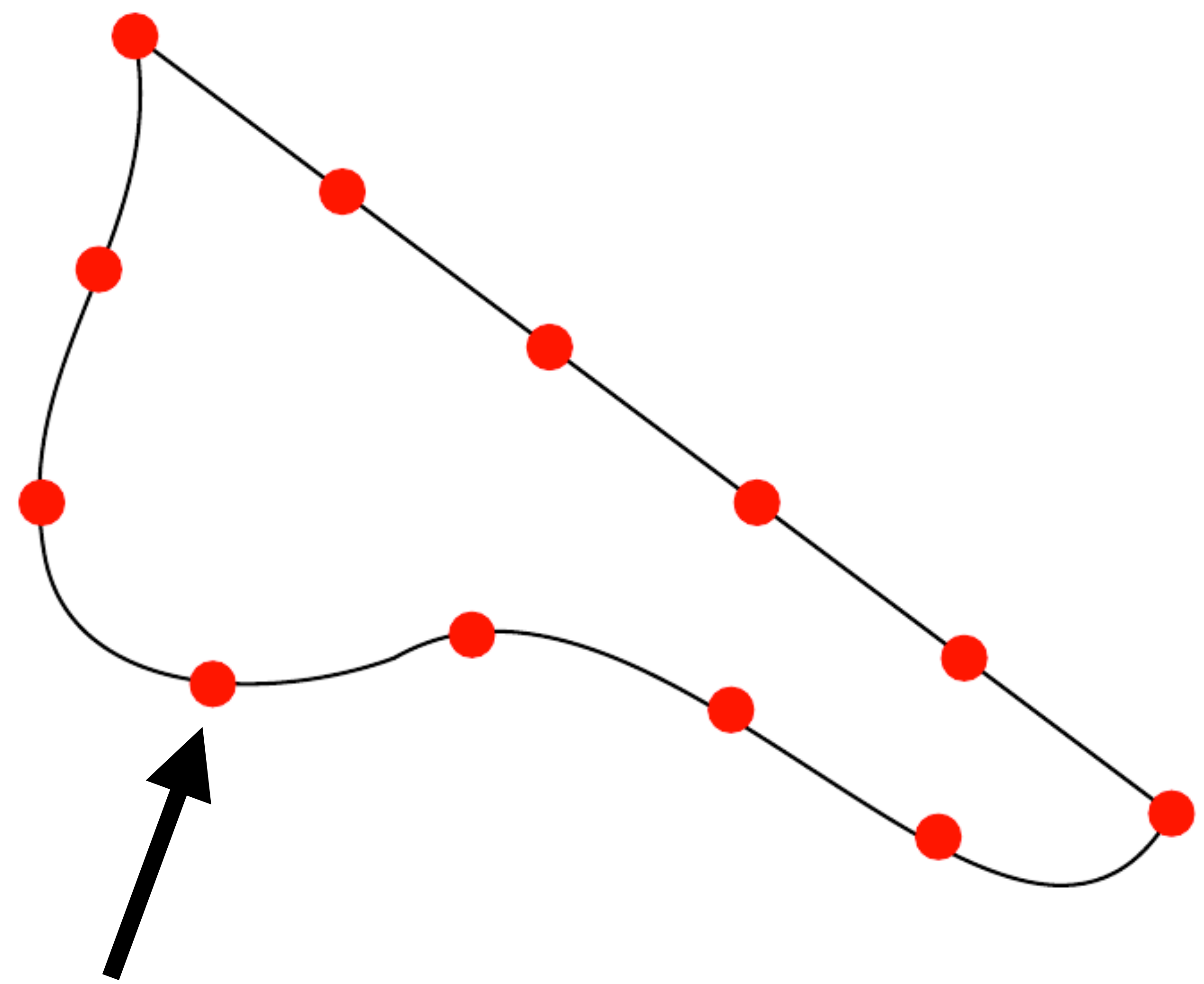
Utilitarian “Theorems”

- Egyptian method:



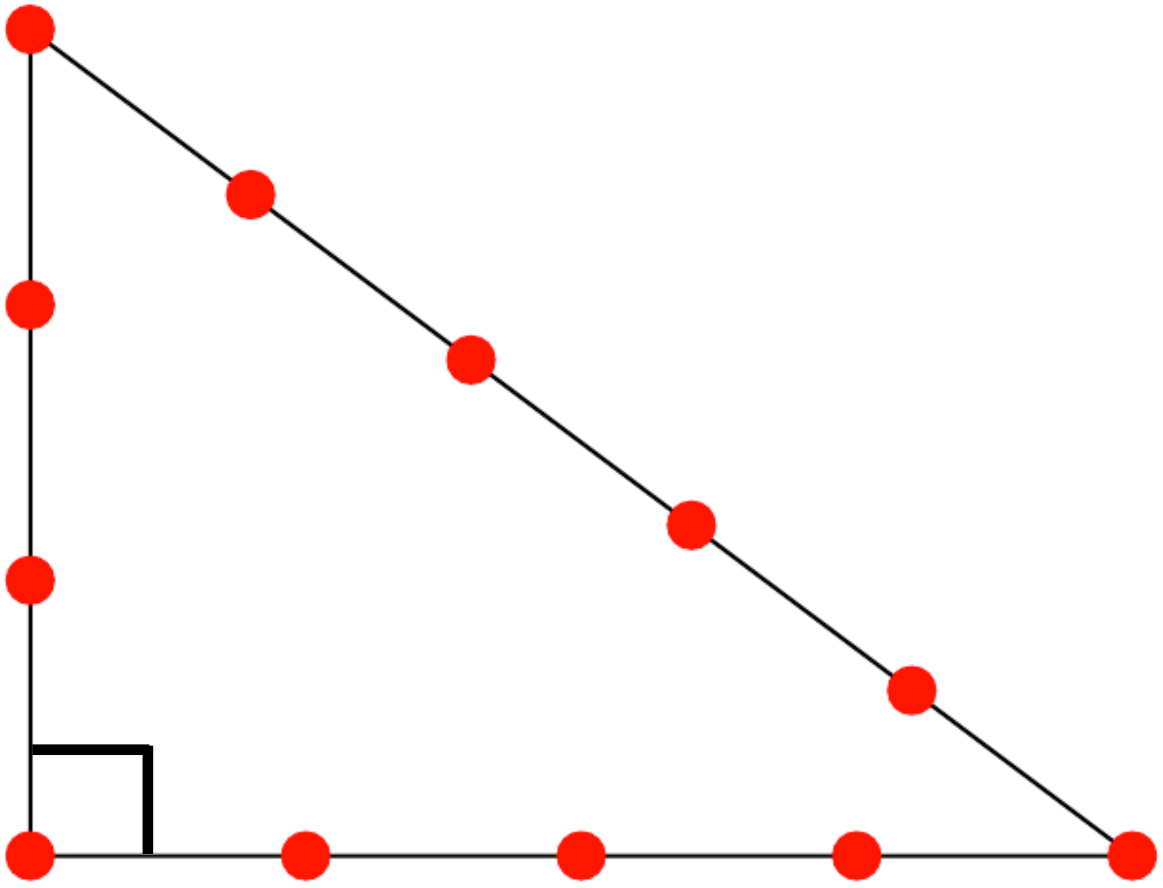
Utilitarian “Theorems”

- Egyptian method:



Utilitarian “Theorems”

- Egyptian method:



Egyptian Numerical Methods

Egyptian Methods

- Most of what we know are due to some old documents written on *papyrus* or *parchment*.
- Papyrus is made from a water reed called *papu*. Parchment is made from animal skins.
- Both were difficult to make and expensive.
- And both are perishable, but the Egyptian climate preserved many of them well.

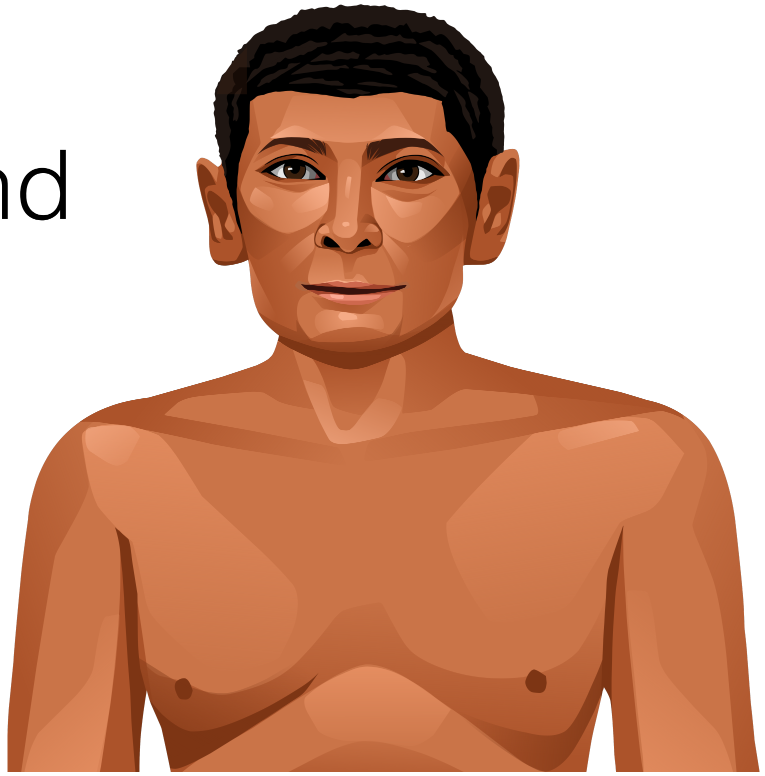
Making Papyrus

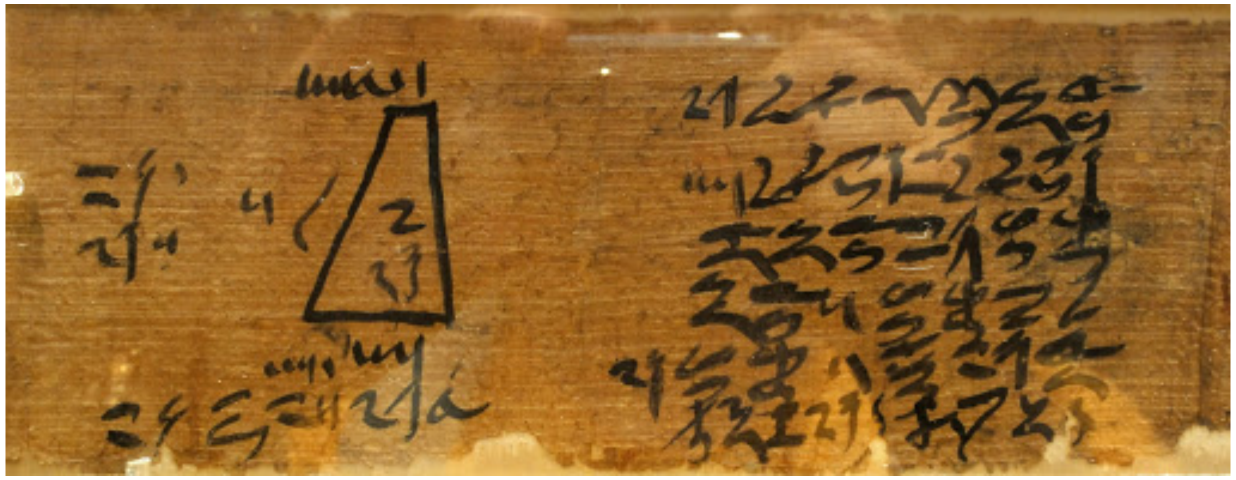
Egyptian Methods

Egyptian Methods

- Rhind Papyrus (~1650 BC; 18 ft x 13 in) and Moscow Papyrus (~1850 BC; 16 ft x 3 in). Written by a scribe named Ahmes.

- Combined: 85 problems and solutions (all numerical) on arithmetic, algebra and geometry.

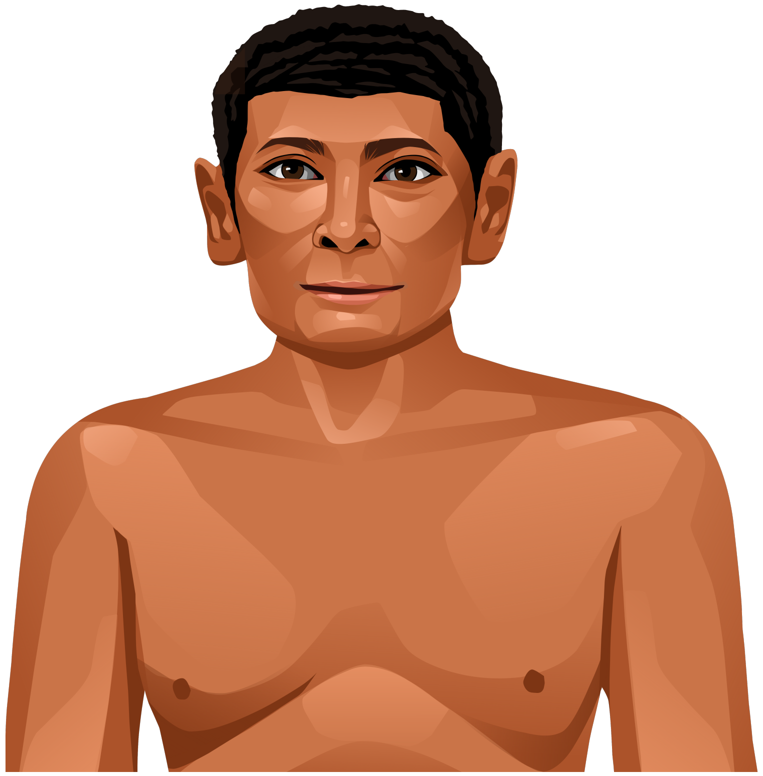




Egyptian Methods

- Rind pictures:

  https://www.britishmuseum.org/collection/object/Y_EA10058



Think Like A
Math Historian

Think Like A Math Historian

- Here is a method that the ancient Egyptians used to compute a product of integers.

- Example: To compute 45•21, here is what they'd do:

Binary?

$$
\begin{array}{rcl}
1 & \to & 21 \\
2 & \to & 42 \\
4 & \to & 84 \\
8 & \to & 168 \\
16 & \to & 336 \\
32 & \to & 672
\end{array}
\qquad
\begin{array}{r}
21 \\
84 \\
+\ 168 \\
672 \\
\hline
945
\end{array}
$$

Think Like A Math Historian

Why does it work?

$$1 \to 21$$
$$2 \to 42$$
$$4 \to 84$$
$$8 \to 168$$
$$16 \to 336$$
$$32 \to 672$$

$$\begin{array}{r} 21 \\ 84 \\ + \quad 168 \\ 672 \\ \hline 945 \end{array}$$

$$45 \bullet 21 = (\overbrace{32 + 8 + 4 + 1}^{\text{Binary!}}) \cdot 21$$

$$= 32 \cdot 21 + 8 \cdot 21 + 4 \cdot 21 + 1 \cdot 21$$

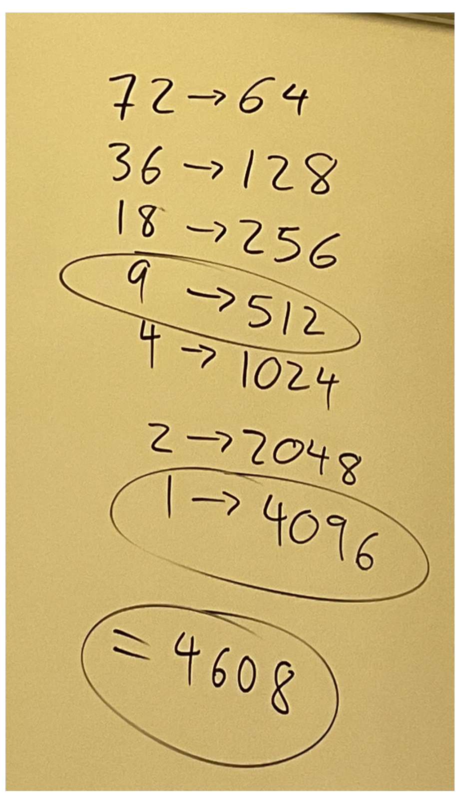
$$= 672 + 168 + 84 + 21$$

$$= 945.$$

Think Like A Math Historian

- Here is another method that the ancient Egyptians used to compute a product of integers.

- Example: To compute 26•31, here is what they'd do:



$$
\begin{array}{l}
26 \to 31 \\
13 \to 62 \\
6 \to 124 \\
3 \to 248 \\
1 \to 496
\end{array}
\qquad
\begin{array}{r}
62 \\
+\ 248 \\
496 \\
\hline
806
\end{array}
$$

Why does it work? You (might) tell me on your homework!

$$72 \cdot 64$$

$$72 \to 64$$

$$36 \to 128$$

$$18 \to 256$$

$$9 \to 512$$

$$4 \to 1024$$

$$2 \to 2048$$

$$1 \to 4096$$

$$= 4608$$

Egyptian Fractions

- **Definition.** A *unit fraction* is one of the form $\dfrac{1}{n}$, where n is a positive integer.

- In Ancient Egypt, putting a dot or circle above a hieroglyph means to invert it. For example,

$$10 = \cap \qquad \frac{1}{10} = \dot{\cap}$$

$$3 = ||| \qquad \frac{1}{3} = \overset{\frown}{|||}$$

Egyptian Fractions

- For all other fractions, the goal is to write them as a sum of distinct unit fractions.

- For example, our $\dfrac{2}{5}$ was viewed as $\dfrac{1}{3}+\dfrac{1}{15}$. That is,

Egyptian Fractions

- The Rhind papyrus noted that $\dfrac{2}{3n} = \dfrac{1}{2n} + \dfrac{1}{6n}$.

- This is a formula for how to take 2/3 of an Egyptian fraction: Apply this rule to each term.

- The Rhind also had tables of values. Example: how to write

$$\frac{1}{10}, \frac{2}{10}, \ \cdots \ , \frac{9}{10}$$

as Egyptian fractions.

Egyptian Fractions

- Question: What are the advantages and disadvantages of writing fractions only in terms of Egyptian fractions?

- Advantage: Maybe intuitive for some applications.

- Disadvantages: First, non-uniqueness:

$$\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}.$$

- Second, arithmetic is tough. Even simply multiplying by 2 can make a fraction unrecognizable.

Egyptian Fractions

- The Aftermath: Can every fraction be written in this way? Fibonacci, in 1202: Yes!

1. Find the largest unit fraction less than $\frac{m_1}{n_1}$; suppose it is $\frac{1}{k_1}$.
2. Subtract: $\frac{m_1}{n_1} - \frac{1}{k_1}$, and simplify this to a single new fraction, $\frac{m_2}{n_2}$.
3. Now find the largest unit fraction less than $\frac{m_2}{n_2}$; suppose it is $\frac{1}{k_2}$.
4. Subtract: $\frac{m_2}{n_2} - \frac{1}{k_2}$, and simplify this to a single new fraction, $\frac{m_3}{n_3}$.
5. Continue in this way.

One can prove that this algorithm at some point terminates, producing a *finite* list sum of unit fractions equaling $\frac{m_1}{n_1}$:

$$\frac{m_1}{n_1} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_t}.$$

Egyptian Fractions

- Here's a fun application.

- Suppose you have, say, 5 cookies and 6 people. How do you cut them to give everyone the same amount?

- Could try to cut a 1/6 sliver off each cookie and give those slivers to one person, but that wouldn't go well.

Egyptian Fractions

- Here’s a fun application.

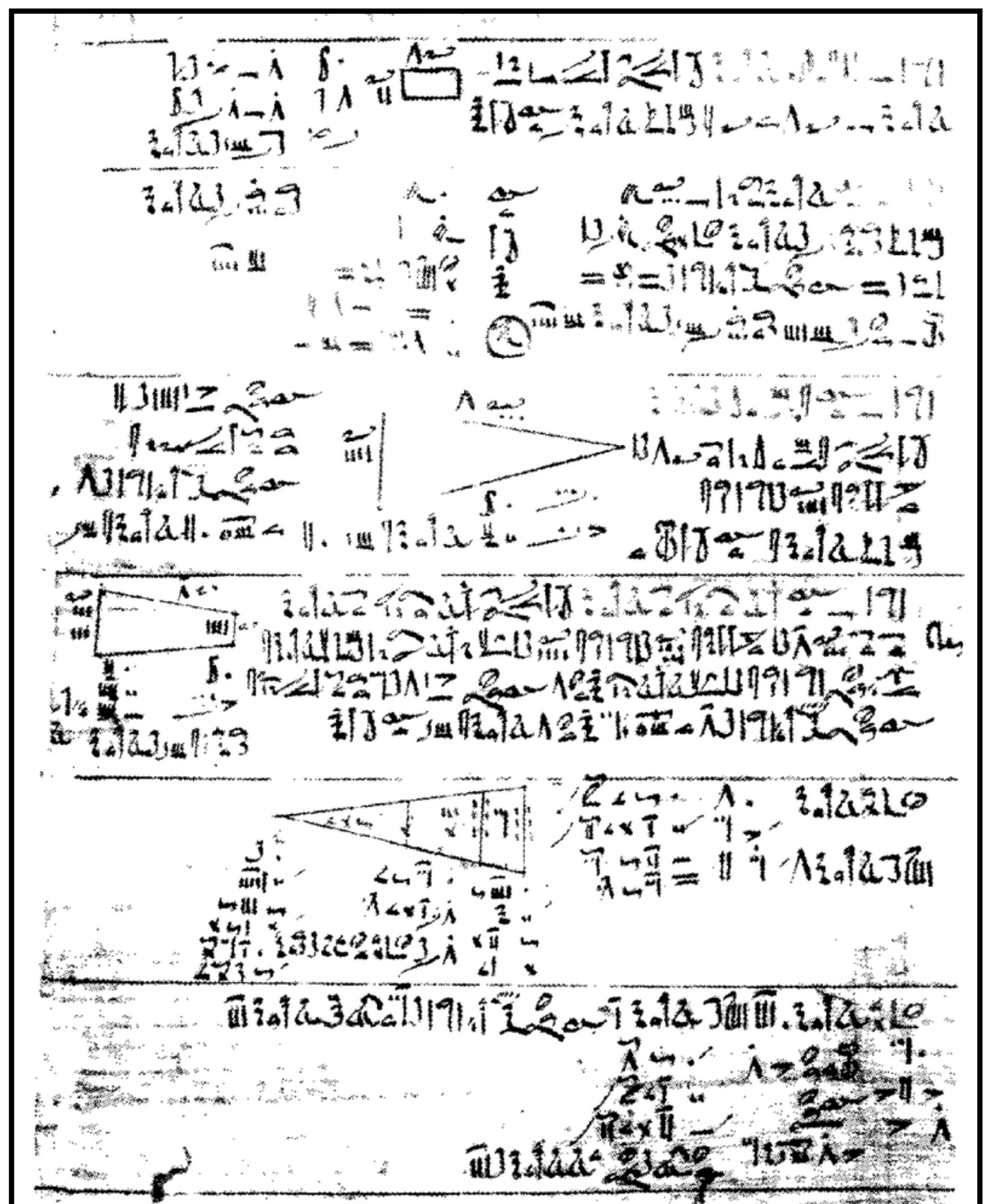
- Suppose you have, say, 5 cookies and 6 people. How do you cut them to give everyone the same amount?

- Could try to cut a 1/6 sliver off each cookie and give those slivers to one person, but that wouldn’t go well.

- Writing it with Egyptian fractions, $\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$. So one way is to give everyone a half and a third!

Geometry

- The Rhind papyrus also has a number of geometry problems, including areas, volumes and slopes.



Geometry

- In Problem 41, Ahmes computes the volume of a cylindrical granary. Given a diameter of d and height of h, he says its volume is

$$[(1 - 1/9)d]^2 h = \frac{256}{81} r^2 h\,.$$

- This essentially means they were using $\pi = \dfrac{256}{81} = 3.1604\ldots$ Which is wrong, but not too bad.

Babylonian Mathematics

Mesopotamia

Babylonia


Romania
Croatia
Serbia
Italy
Rome
Bulgaria
Black Sea
Georgia
Uzbekistan
Istanbul
Ankara
Azerbaijan
Tyrrhenian Sea
Greece
Turkey
Turkmenistan
Syria
Tunisia
Mediterranean Sea
Lebanon
Afghanistan
Iraq
Israel
Iran
Cairo
القاهرة
Algeria
Libya
Egypt
Persian Gulf
Riyadh
الرياض
United Arab Emirates
Saudi Arabia
Jeddah
جدة
Red Sea
Oman
Mali
Niger
Sudan
Yemen
Chad
Eritrea
Burkina Faso
Gulf of Aden
Arabian Sea
Djibouti
Benin
Nigeria

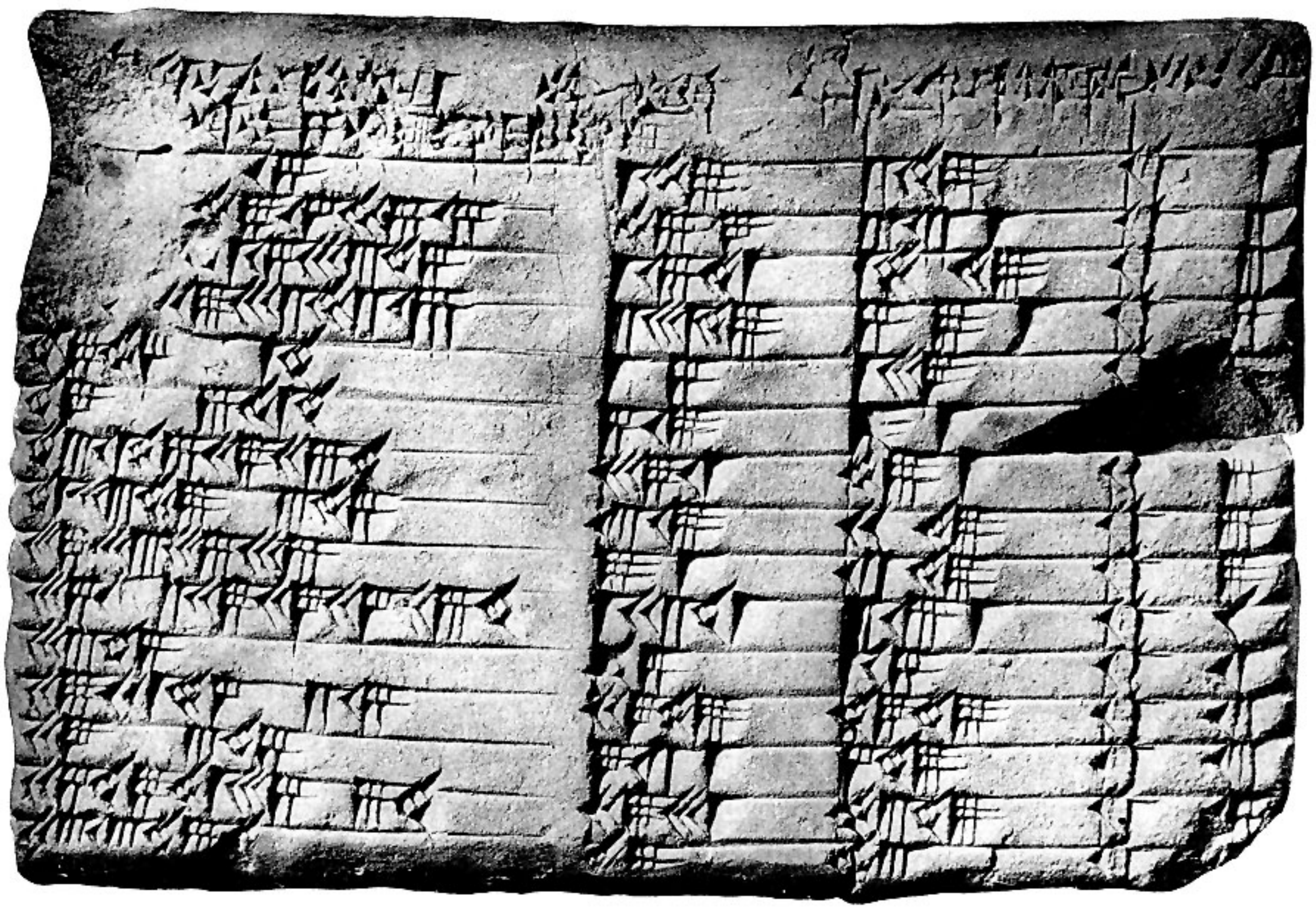
Babylonian Methods

- The earliest written records were all accounting records.
- Recall that the Babylons used base 60.
- Babylonian mathematics was generally more advanced than Ancient Egyptian mathematics.

Think Like A
Math Historian

Babylonian Methods

- Plimpton 322 is a tablet from around 1800 BC.



Babylonian Methods

- Plimpton 322 is a tablet from around 1800 BC.

Ratio	Width	Diagonal	Row number
1; 59 0 15	1 59	2 49	1
1; 56 56 58 14 50 6 15	56 7	1 20 25*	2
1; 55 7 41 15 33 45	1 16 41	1 50 49	3
1; 53 10 29 32 52 16	3 31 49	5 9 1	4
1; 48 54 1 40	1 5	1 37	5
1; 47 6 41 40	5 19	8 1	6
1; 43 11 56 28 26 40	38 11	59 1	7
1; 41 33 45 14 3 45	13 19	20 49	8
1; 38 33 36 36	8 1*	12 49 9	9
1; 35 10 2 28 27 24 26 40	1 22 41	2 16 1	10
1; 33 45	45	1 15	11
1; 29 21 54 2 15	27 59	48 49	12
1; 27 0 3 45	2 41*	4 49	13
1; 25 48 51 35 6 40	29 31	53 49	14
1; 23 13 46 40	56	1 46	15

Babylonian Methods

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1; 53 10 29 32 52 16	3 31 49	5 9 1	4
1; 48 54 1 40	1 5	1 37	5
1; 47 6 41 40	5 19	8 1	6
1; 43 11 56 28 26 40	38 11	59 1	7
1; 41 33 45 14 3 45	13 19	20 49	8
1; 38 33 36 36	8 1*	12 49 9	9
1; 35 10 2 28 27 24 26 40	1 22 41	2 16 1	10
1; 33 45	45	1 15	11
1; 29 21 54 2 15	27 59	48 49	12
1; 27 0 3 45	2 41*	4 49	13
1; 25 48 51 35 6 40	29 31	53 49	14
1; 23 13 46 40	56	1 46	15

$$3 \cdot 60^2 + 31 \cdot 60 + 49$$
$$= 12{,}709$$

Babylonian Methods

- Plimpton 322 is a tablet from around 1800 BC.

Ratio	Width	Diagonal	Row number	Height
$\frac{28{,}561}{14{,}400}$	119	169	1	120
$\frac{23{,}280{,}625}{11{,}943{,}936}$	3,367	4,825	2	3,456
$\frac{44{,}209{,}201}{23{,}040{,}000}$	4,601	6,649	3	4,800
$\frac{343{,}768{,}681}{182{,}250{,}000}$	12,709	18,541	4	13,500
$\frac{9{,}409}{5{,}184}$	65	97	5	72
$\frac{231{,}361}{129{,}600}$	319	481	6	360
$\frac{12{,}538{,}681}{7{,}290{,}000}$	2,291	3,541	7	2,700
$\frac{1{,}560{,}001}{921{,}600}$	799	1,249	8	960
$\frac{591{,}361}{360{,}000}$	481	46,149	9	600
$\frac{66{,}601{,}921}{41{,}990{,}400}$	4,961	8,161	10	6,480
$\frac{25}{16}$	45	75	11	60
$\frac{8{,}579{,}041}{5{,}760{,}000}$	1,679	2,929	12	2,400
$\frac{5{,}034{,}241}{34{,}560{,}00}$	161	289	13	240
$\frac{10{,}426{,}441}{7{,}290{,}000}$	1,771	3,229	14	2,700
$\frac{678{,}811}{486{,}000}$	56	106	15	90

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$\frac{678{,}811}{486{,}000}$	56	106	15	90

3-4-5 Triangle

$$45^2 + 60^2 = 75^2$$

$$\frac{45}{15} \downarrow 3 \qquad \frac{60}{15} \downarrow 4 \qquad \frac{75}{15} \downarrow 5$$

Babylonian Methods

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Ratio	Width	Diagonal	Row number	Height
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$\frac{678{,}811}{486{,}000}$	56	106	15	90

$$(\text{Width})^2 + (\text{Height})^2 = (\text{Diagonal})^2$$

Questions:

1. Why these (large) Pythagorean triples?

2. What's happening in the first column?

Babylonian Methods

- Plimpton 322 is a tablet from around 1800 BC.

Ratio	Width	Diagonal	Row number	Height
$\frac{28{,}561}{14{,}400}$	119	169	1	120
$\frac{23{,}280{,}625}{11{,}943{,}936}$	3,367	4,825	2	3,456
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$\frac{10{,}426{,}441}{7{,}290{,}000}$	1,771	3,229	14	2,700
$\frac{678{,}811}{486{,}000}$	56	106	15	90

2. What's happening in the first column?

$$\text{Ratio} = \frac{(\text{Diagonal})^2}{(\text{Height})^2}$$

$$\frac{(\text{Width})^2}{(\text{Height})^2} + 1 = \frac{(\text{Diagonal})^2}{(\text{Height})^2}$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

Babylonian Methods

- YBC 7289:

Babylonian Methods

- Consider the sequence with

$$a_1 = 1 \qquad \text{and} \qquad \underbrace{a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right)}_{\text{Taking an average of both over and underestimate}}.$$

- This gives:

$$a_1 = 1$$

$$a_2 = \frac{1}{2}\left(1 + \frac{2}{1}\right) = \frac{3}{2} = 1.5$$

$$a_3 = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3/2}\right) = \frac{17}{12} = 1.41666\ldots$$

Babylonian Methods

- Continuing: $$a_1 = 1$$

$$a_2 = \frac{1}{2}\left(1 + \frac{2}{1}\right) = \frac{3}{2} = 1.5$$

$$a_3 = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3/2}\right) = \frac{17}{12} = 1.41666\ldots$$

$$a_4 = \frac{1}{2}\left(\frac{17}{12} + \frac{2}{17/12}\right) = \frac{577}{408} = 1.4142156\ldots$$

- Looks a lot like $$\sqrt{2} = 1.4142135\ldots$$

- Indeed, $a_1, a_2, a_3, \ldots$ *converges* to $\sqrt{2}$.

Babylonian Methods

- If you continue the above sequence a couple more steps, you get the fraction on YBC 7289. That is, YBC 7289 shows a triangle (inside of a square), whose hypotenuse gives an estimate of $\sqrt{2}$ accurate to 5 decimal places.

- They figured this out 4,000 years ago!!!!!!!

- !!

Babylonian Methods

- Hero of Alexandria (10-70AD) described this method.

- It’s so good that computers use it today. It’s quadratically convergent. It has been called “the greatest known computation of the ancient world.”

Babylonian Methods

- Fun Fact: Hero of Alexandria was also a great inventor. He invented:

  -First steam-powered engine

  -First wind-power machine

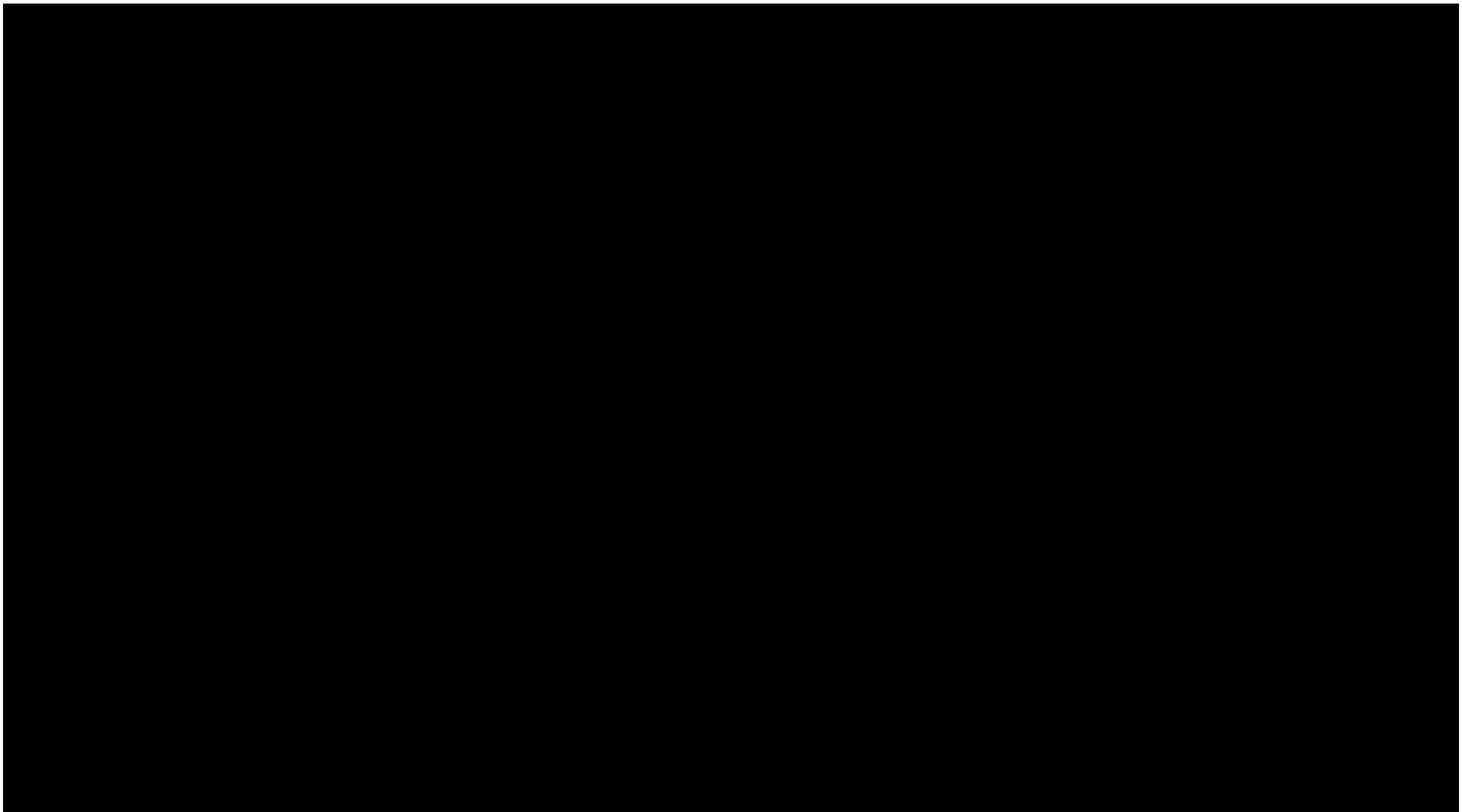
  -Syringes

  -Self-powered fountain

  -Mechanical play

  -Vending machine

Vending Machine



Ancient Chinese Mathematics

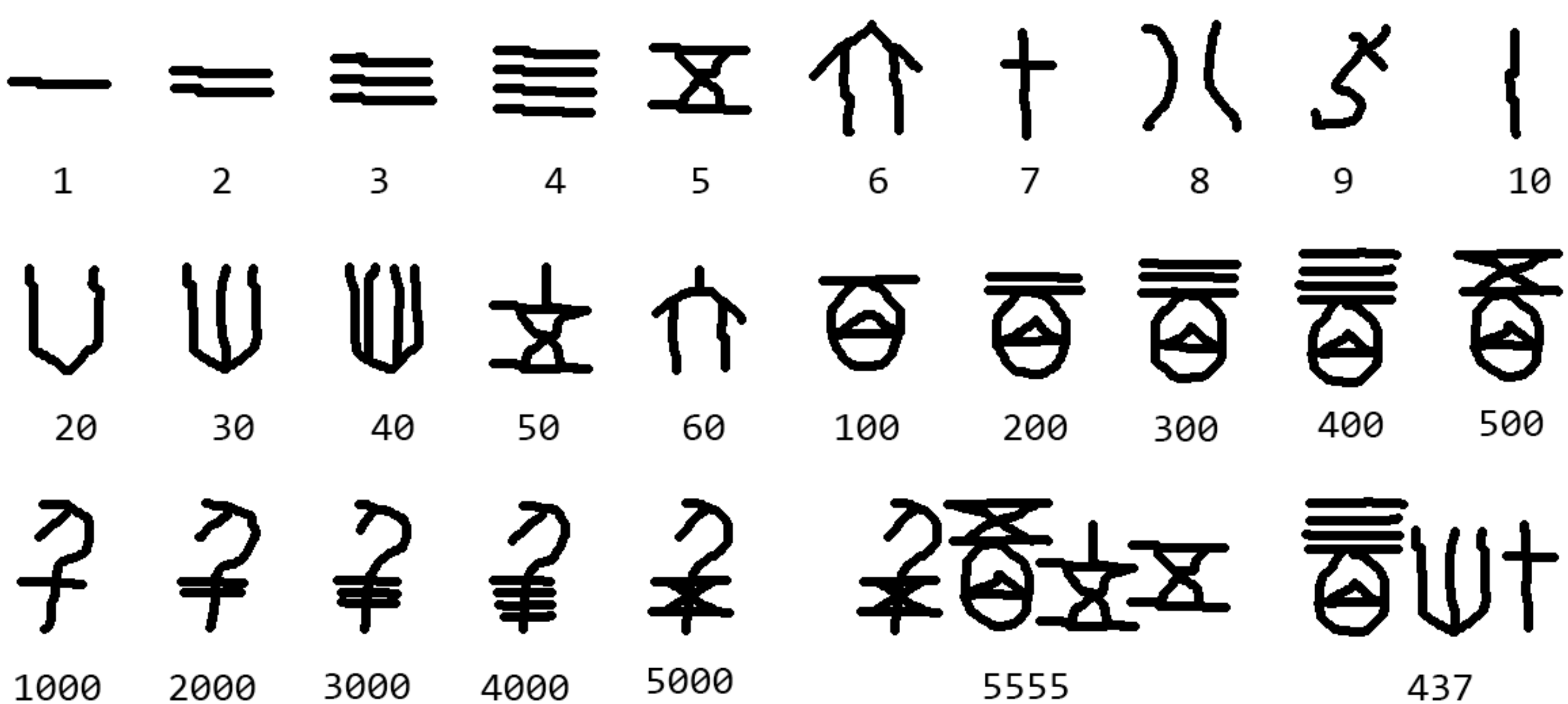
Chinese History

Chinese Mathematics

- We have learned about Chinese characters, multiplication table, and magic squares.
- Older than all these are *oracle bones*. These date back to the Shang Dynasty (1600–1050 BC).
- Bones from turtles and ox with writing on them, used to predict the future.
- They contain thousands characters, older than modern Chinese characters.

Chinese Mathematics

- These bones contain numerals.
- Used a simple grouping system.



Chinese Mathematics

- By the Shang dynasty (1600–1050 BC), the Chinese had developed:

1. A real number system that includes significantly large and negative numbers

2. More than one numeral system (base 2 and base 10)

3. Basic algebra

4. Basic geometry

5. Basic trigonometry

Chinese Mathematics

- Qin Dynasty (221–210 BC), under Emperor Qin, the first emperor of a unified China.

- Qin burned or banned many books, and executed many scholars. We don’t know what math knowledge was lost.

- Qin joined remote stretches of wall into The Great Wall of China. Required many math techniques for materials/manpower needed, and its design.

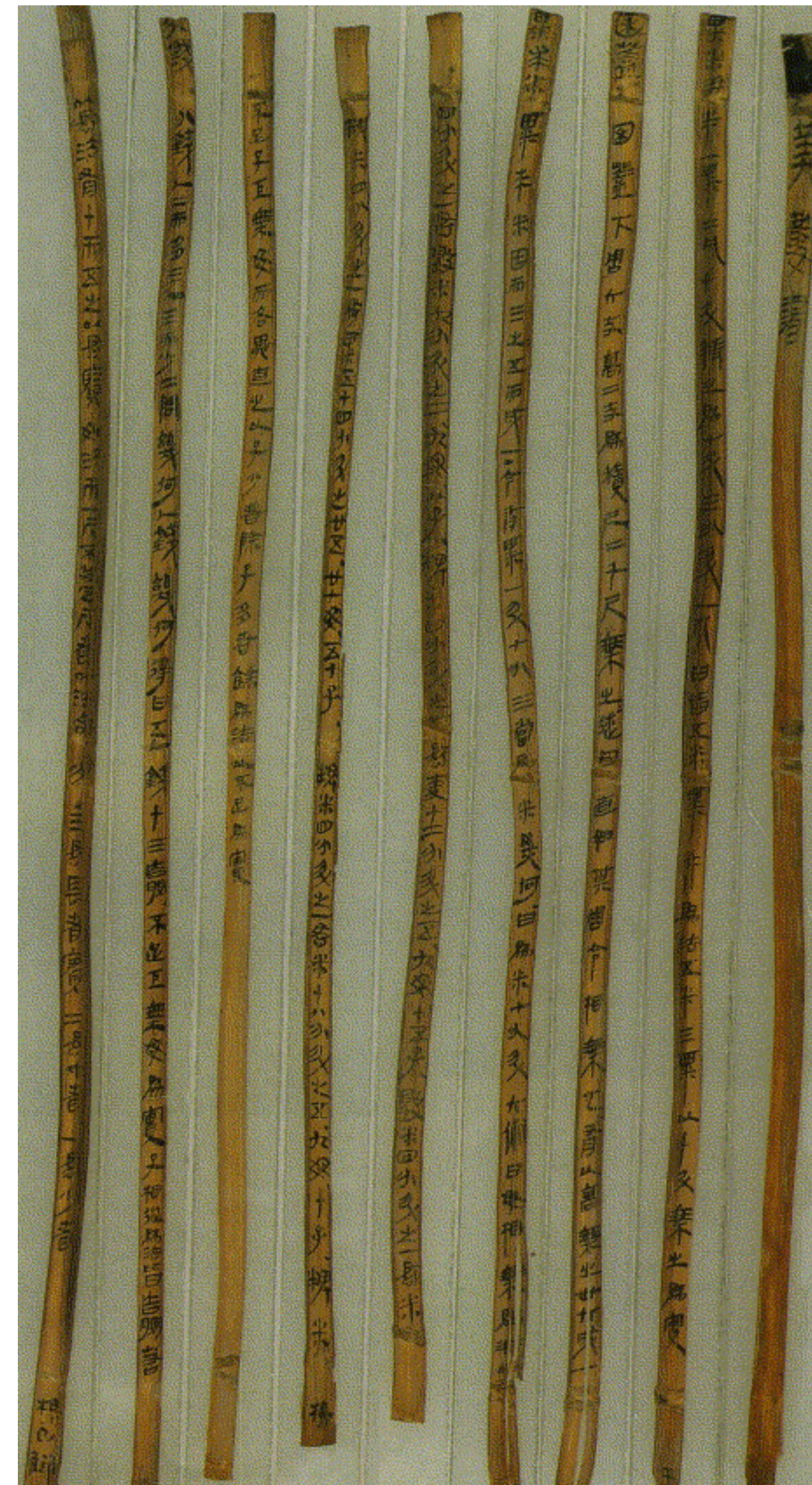
Chinese Mathematics

- The first book: Book on Numbers and Computation (~186 BC).

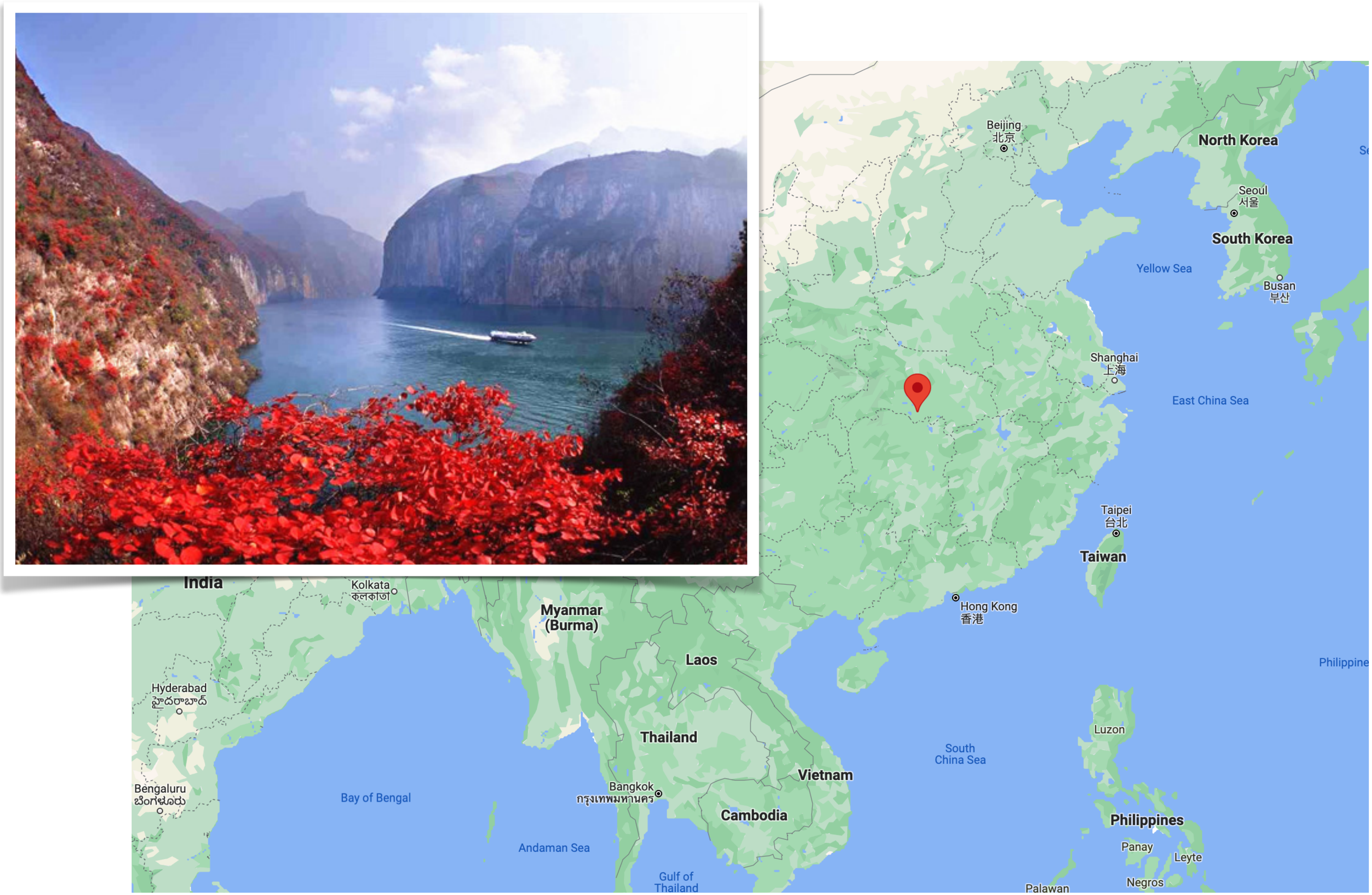
- Discovered in 1984 in a tomb. Written in ink on bamboo strips. 1200 bamboo strips comprising texts on math, government, law, and therapeutic gymnastics.

Nice

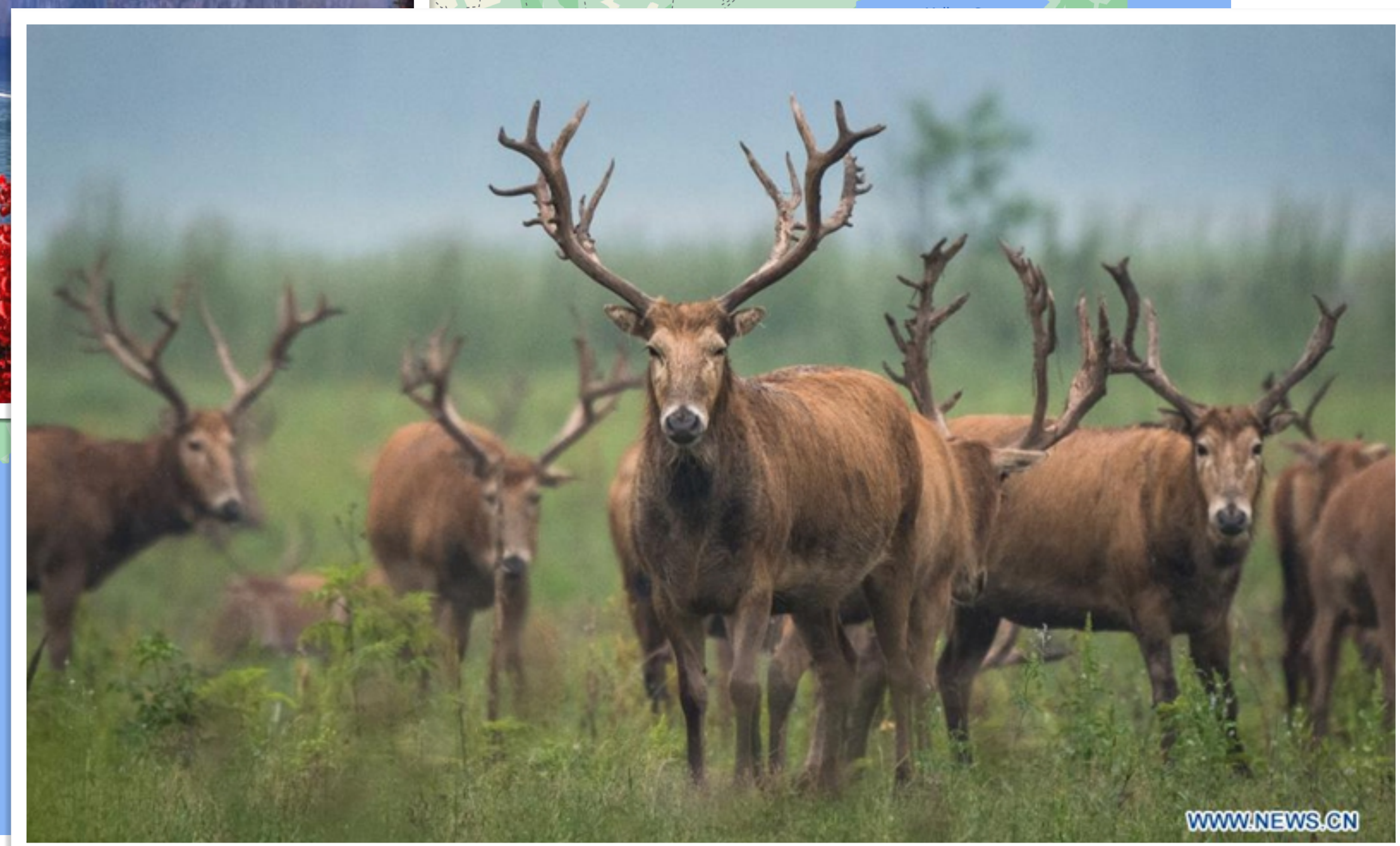
- Book contained 69 math questions, each followed by an answer and a method to solve it.



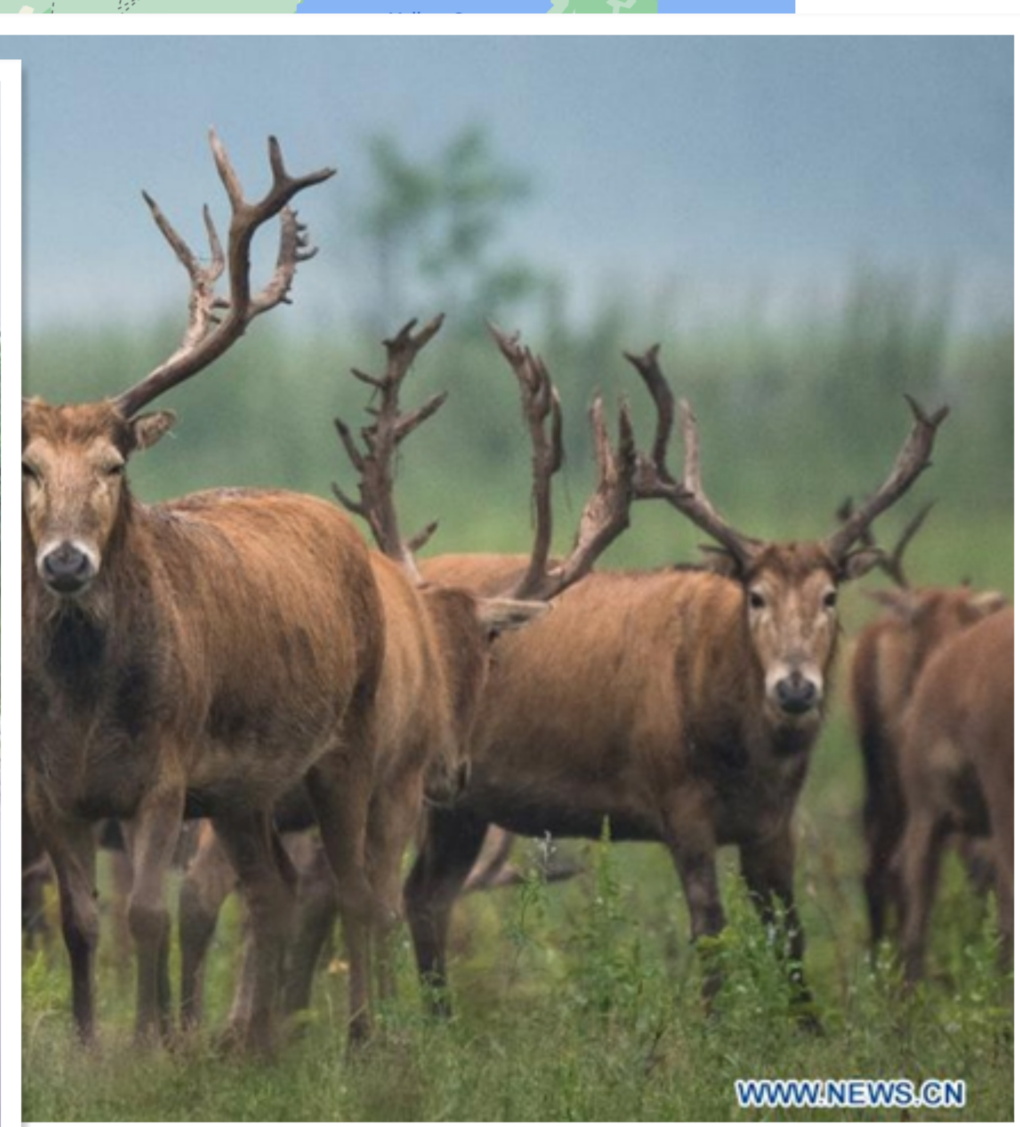
Chinese Mathematics



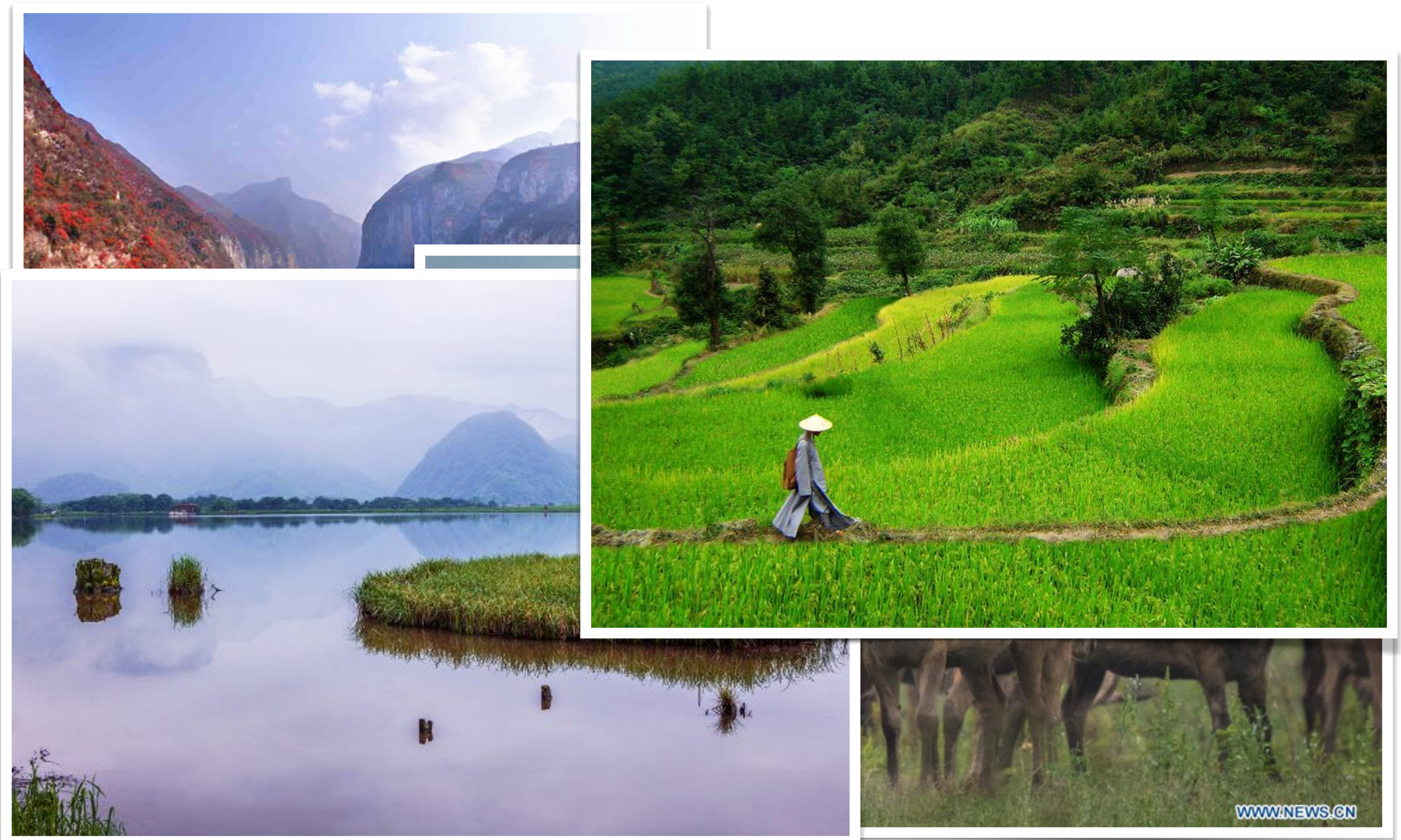
Chinese Mathematics



Chinese Mathematics



Chinese Mathematics



Chinese Mathematics

Chinese Mathematics

- Important topic in the book: excess and deficit problems. The book's first example on this:

> In dividing coins, if each person receives 2, there is a surplus of 3; if each person receives 3, there is a shortage of 2. It is asked how many persons and coins are there? The answer: 5 persons and 13 coins. Excess and deficit: cross multiply the denominators to become the dividend; the numerators are added to become the divisor. Both excess or deficit: the numerators are cross multiplied by the denominators and each is set aside. The lesser of the numerators is subtracted from the greater of the numerators and the remainder is the divisor. Take the deficit as the dividend.

Think Like A Math Historian

Chinese Mathematics

- This is the first example in history of an important area of math. What area of math is it?

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Chinese Mathematics

- Hint: Use variables. If x_1 is the number of people, and x_2 is the number of coins, then:

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Chinese Mathematics

- Hint: Use variables. If x_1 is the number of people, and x_2 is the number of coins, then:

$$2x_1 + 3 = x_2$$
$$3x_1 - 2 = x_2$$

- Rewriting,

$$2x_1 - x_2 = -3$$
$$3x_1 - x_2 = 2$$

In dividing coins, if each person receives 2, there is a surplus of 3; if each person receives 3, there is a shortage of 2. It is asked how many persons and coins are there? The answer: 5 persons and 13 coins. Excess and deficit: cross multiply the denominators to become the dividend; the numerators are added to become the divisor. Both excess or deficit: the numerators are cross multiplied by the denominators and each is set aside. The lesser of the numerators is subtracted from the greater of the numerators and the remainder is the divisor. Take the deficit as the dividend.

Chinese Mathematics

- This was the first (notable) linear algebra problem in history.

- Today it can be written with matrices:

$$\begin{pmatrix} a & -1 \\ c & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -b \\ d \end{pmatrix}$$

- Finding the augmented matrix and row reducing gives the solutions:

$$x_1 = \frac{b+d}{c-a} \qquad \text{and} \qquad x_2 = \frac{bc+da}{c-a}$$

Chinese Mathematics

- Finding the augmented matrix and row reducing gives the solutions:

$$x_1 = \frac{b+d}{c-a} \qquad \text{and} \qquad x_2 = \frac{bc+da}{c-a}$$

- In the book, they gave the answers

$$x_1 = b+d \qquad \text{and} \qquad x_2 = bc+da$$

- In their particular example, $c - a = 1$. So if their example was supposed to only represent those cases, it was correct. Otherwise it had an error.

Chinese Mathematics

- Another way to think about these problems is as a guess-and-check procedure.

> In dividing coins, if each person receives 2, there is a surplus of 3; if each person receives 3, there is a shortage of 2. It is asked how many persons and coins are there?

- If you guess the number of people and coins, and once you are too high and once you are too low, here is a procedure to tell you how to combine your wrong guesses into a correct solution.

Chinese Mathematics

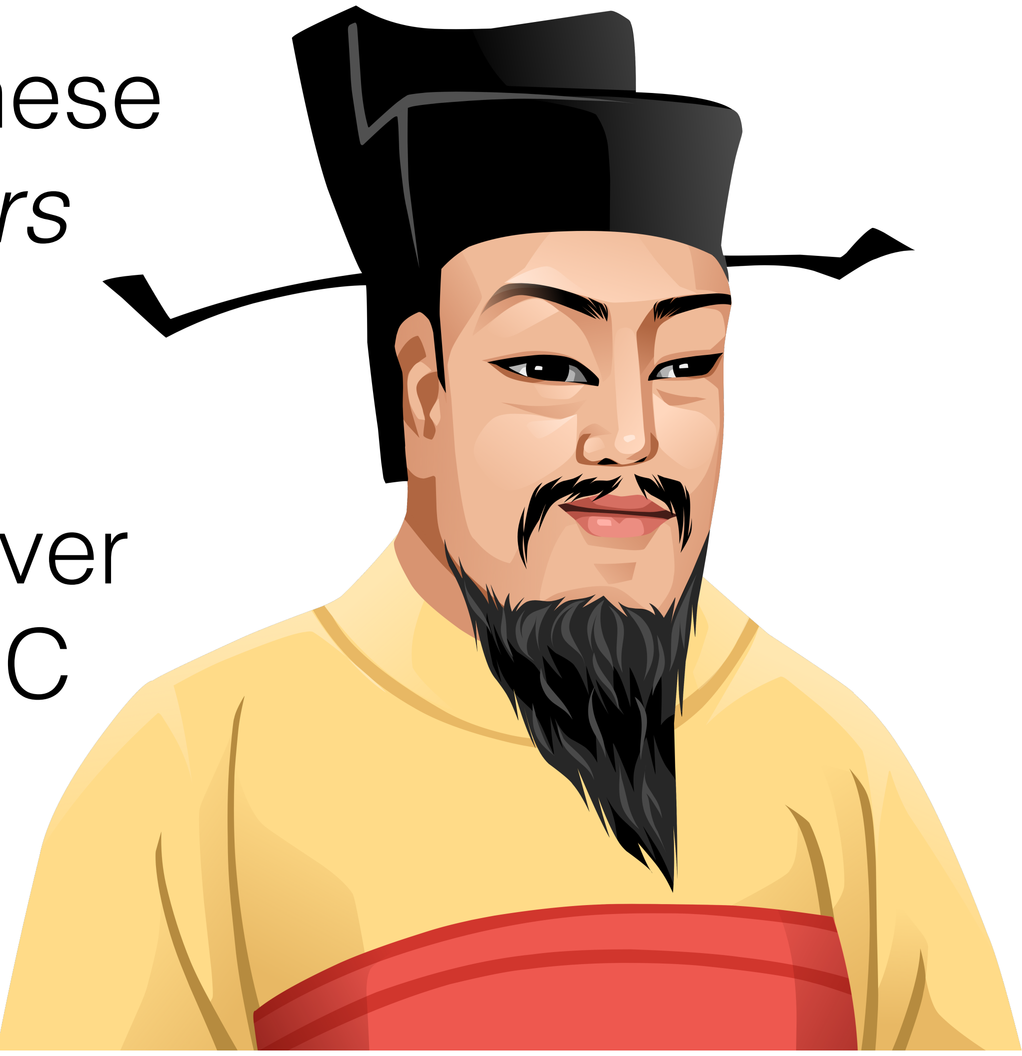
- With two unknowns and two guesses, this is called *double false position*.

- With a single guess and a single unknown, it is called *simple false position*. These problems occurred earlier, in both Egyptian and Babylonian mathematics. Example from the Rhind papyrus:

> Find a quantity such that when it is added to 1/4 of itself, the result is 15.

- Modern notation: $x + \frac{1}{4}x = 15$. Solve for x. They give a method to solve this problem.

Chinese Mathematics



- The most important book in Chinese math history is *The Nine Chapters on the Mathematical Art*.
- It was written by many authors over hundreds of years, from ~1000BC to ~100 BC. In 263 AD, Liu Hui published an annotated edition.
- It is organized like the *Book on Numbers and Computation*. That is, a problem is stated, an answer is given, and a procedure is presented for how to solve the problem and similar problems.

Show a copy of

Nine Chapters

Chinese Mathematics

- The *Book on Numbers and Computation* only considered systems of 2 linear equations with 2 unknowns.
- In the *Nine Chapters*, a procedure called *fang cheng* is presented which can solve larger systems. The examples given solved systems with 3 equations and 3 unknowns, 4 equations and 4 unknowns, and 5 equations and 5 unknowns.
- *Fang cheng* is essentially Gaussian elimination (discovered 2,000 years before Gauss!)

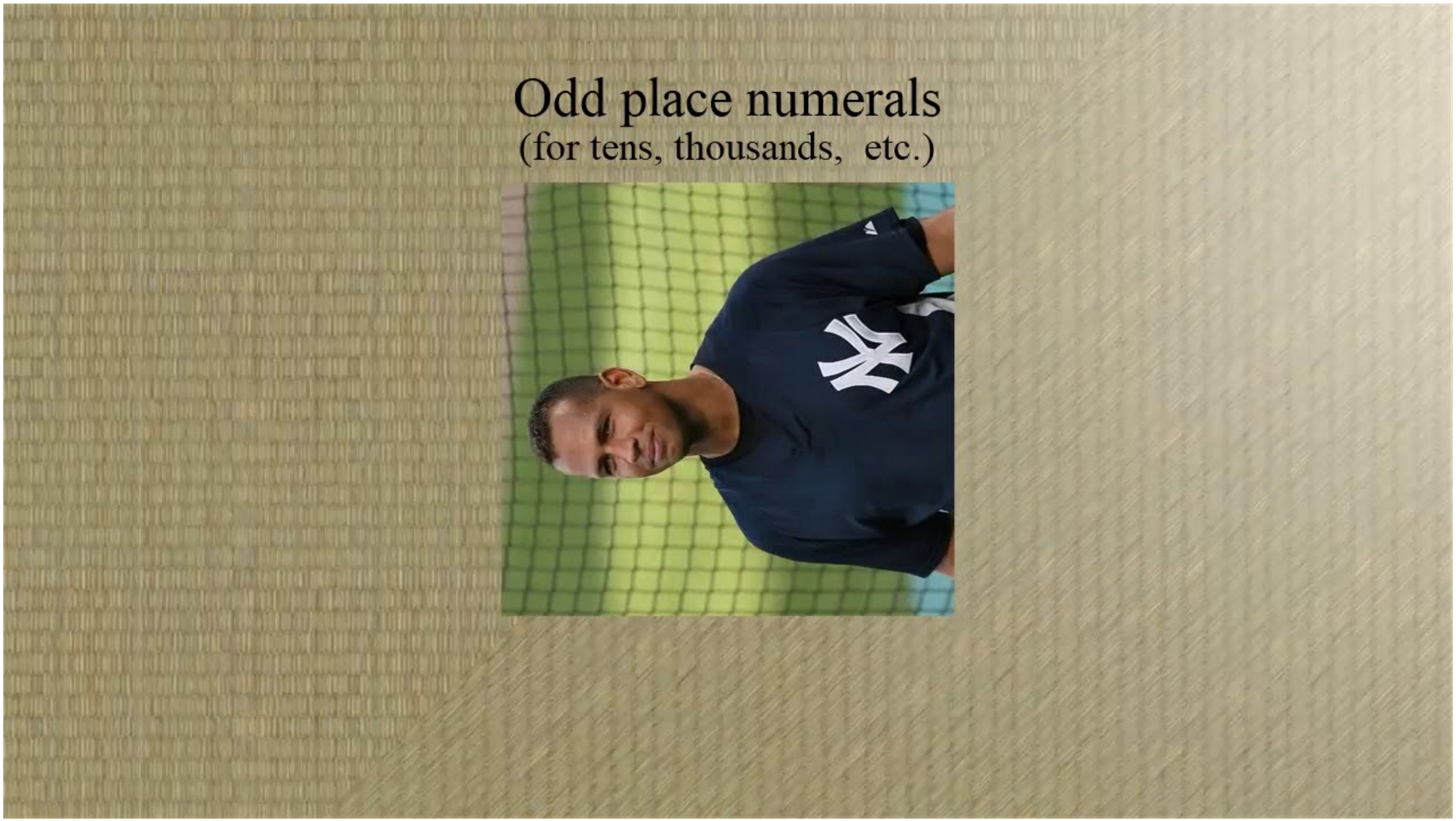
Chinese Mathematics

- First example on *fang cheng* in the *Nine Chapters*:

> Given 3 bundles of top grade paddy, 2 bundles of medium grade paddy, and 1 bundle of low grade paddy, together they yield 39 *dou* of grain. Given 2 bundles of top grade paddy, 3 bundles of medium grade paddy, and 1 bundle of low grade paddy, together they yield 34 *dou* of grain. Given 1 bundle of top grade paddy, 2 bundles of medium grade paddy, and 3 bundles of low grade paddy, together they yield 26 *dou* of grain. Problem: how much grain does one bundle of high, medium and low grade paddy together yield? Answer: Top grade paddy yields $9\frac{1}{4}$ *dou* per bundle; medium grade paddy yields $4\frac{1}{4}$ *dou* per bundle; low grade paddy yields $2\frac{3}{4}$ *dou* per bundle.

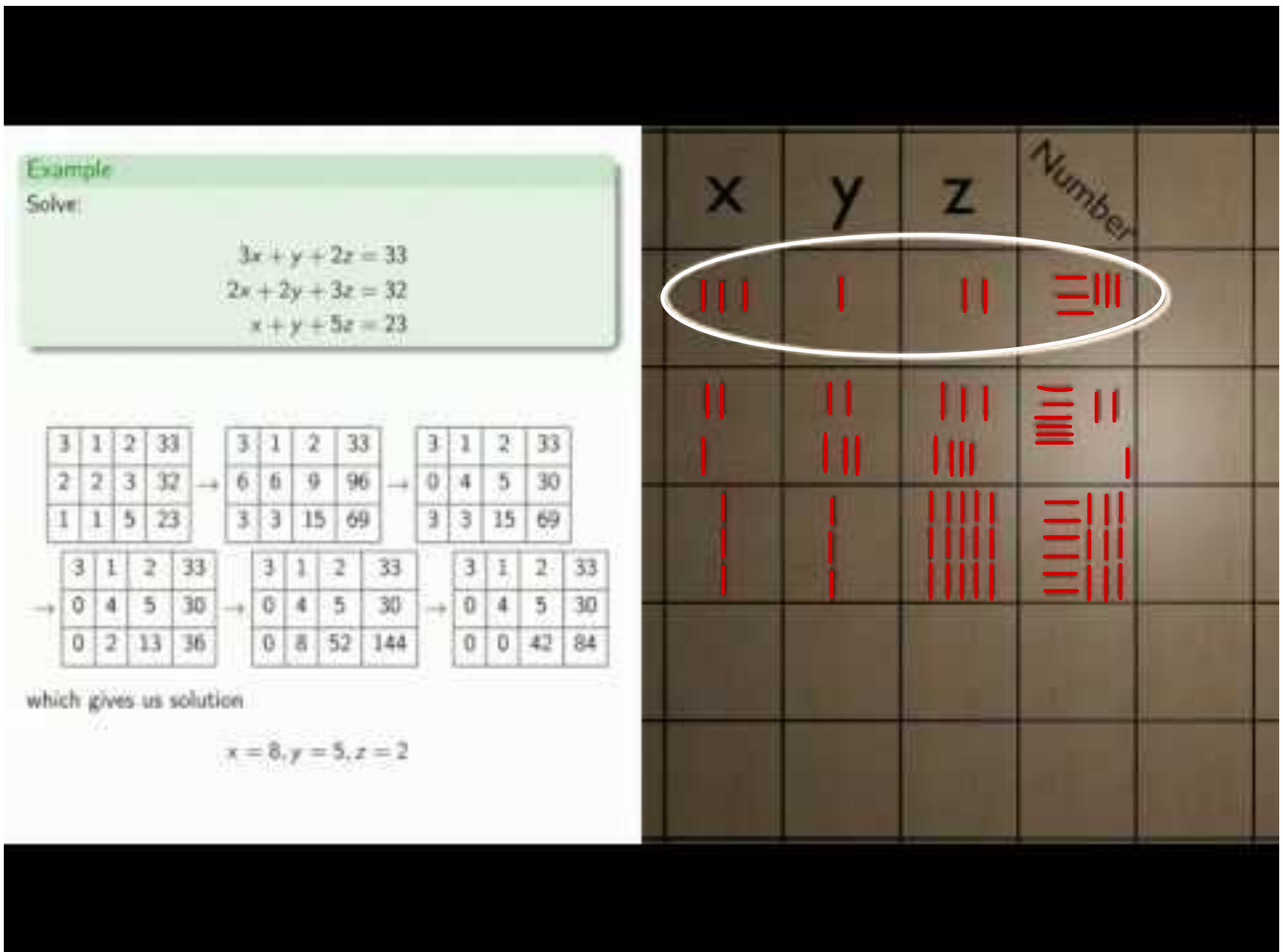
Chinese Mathematics

- Solving on a counting board.



Chinese Mathematics

- The *feng cheng* procedure



Chinese Mathematics

- The only difference between *fang cheng* and Gaussian elimination is in the final step (back substitution).
- *Fang cheng* uses a more complicated approach so that there are only integers until the very end. This is advantageous when solving the problem using a counting board.
- Gaussian elimination is happy to introduce fractions as soon as back-substitution begins.

Chinese Mathematics

- The *Nine Chapters* also covered:

Chapter 1: Finding areas of fields (squares, rectangles, trapezoids, circles). Liu Hui's calculated π to 3.14159.

Chapter 2: Exchange of commodities at different rates; unit pricing. Proportions and fractions.

Chapter 3: Distributing commodities and money at proportional rates. Arithmetic and geometric sums.

Linear growth

exponential growth

Chapter 4: Finding dimensions of a shape given its area or volume. Finding the square and cube roots.

Chinese Mathematics

- The *Nine Chapters* also covered:

Chapter 5: Finding the volume of various shapes.

Chapter 6: More advanced word problems on proportion, involving work, distances and rates.

Chapter 7: Excess and deficit.

Chapter 8: *Fang cheng*.

Chapter 9: Problems using the Pythagorean theorem.

The Aftermath

- Systems of linear equations appeared in the West from Greek mathematician Diophantus and Hindu mathematician Aryabhata. But their work did not go as far as the Chinese did.

- Issac Newton was the first in the West. He noticed that no Algebra book he knew of solved systems of linear equations, so he created Gaussian elimination.

> “And you are to know, that by each Æquation one unknown Quantity may be taken away, and consequently, when there are as many Æquations and unknown Quantities, all at length may be reduc’d into one, in which there shall be only one Quantity unknown.”

The Aftermath

- Gauss wrote about the procedure, worked on a related problem (least squares), and invented some related notation, but played no role in creating the algorithm itself.

- Calling it Gaussian elimination started in the 1950s based on a misreading of history.

Shout-outs!

- Math is not just about figuring things out, it is also about communication.
- Before universal education, it was really important to clearly communicate how to do arithmetic.
- One pioneer in this was Wang Zhenyi, from 18th century China. She wrote a five-volume text doing so.

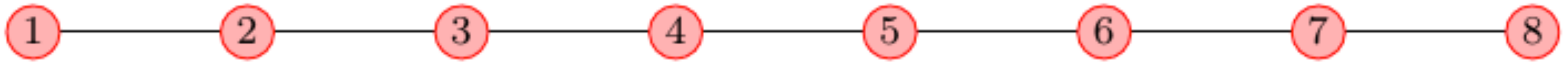
People’s History

People's History of Trigonometry

- The Egyptian artichitect method of creating a right angle is a good bit of people's history.
- The Mayans had a similar method.

People's History of Trigonometry

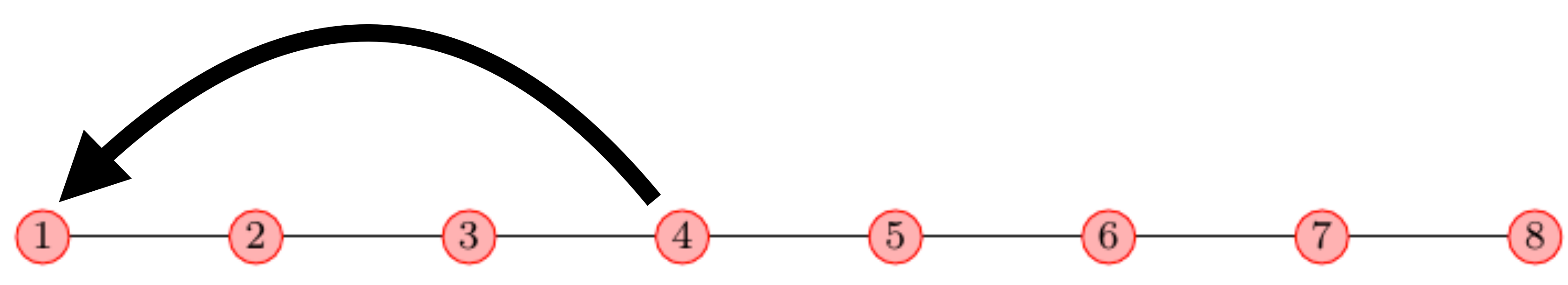
- Mayan method:



People's History of Trigonometry

- Mayan method:

Pinch on over



People's History of Trigonometry

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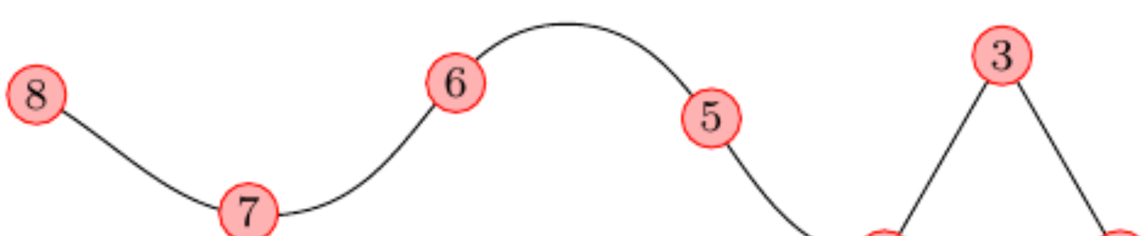

People's History of Trigonometry

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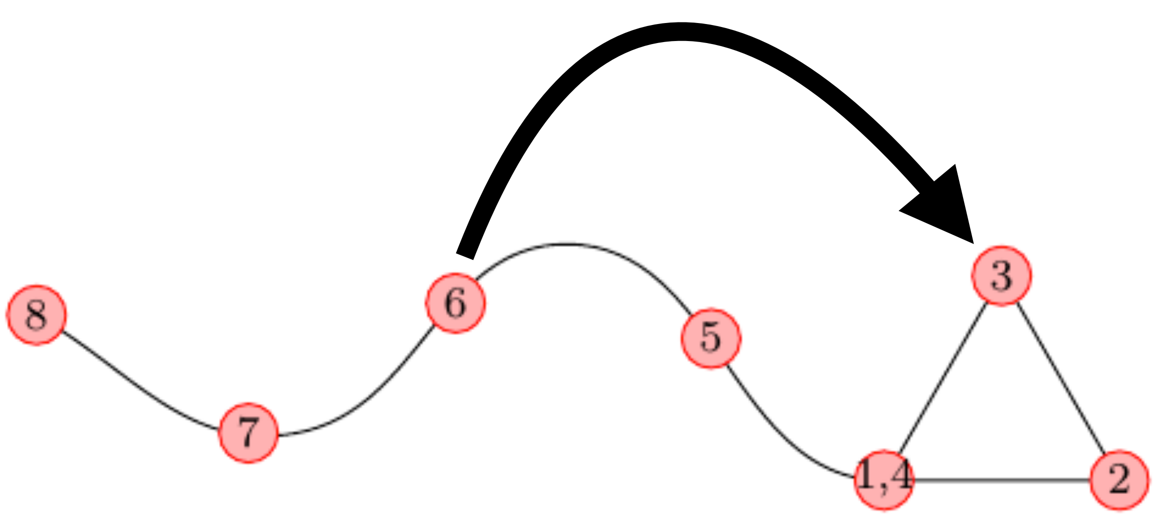

People's History of Trigonometry

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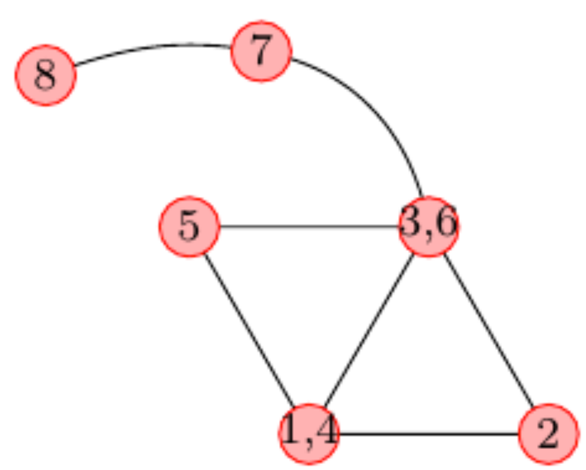

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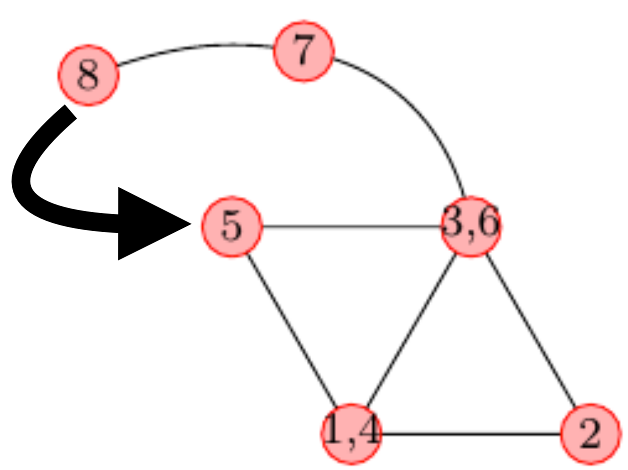

People's History of Trigonometry

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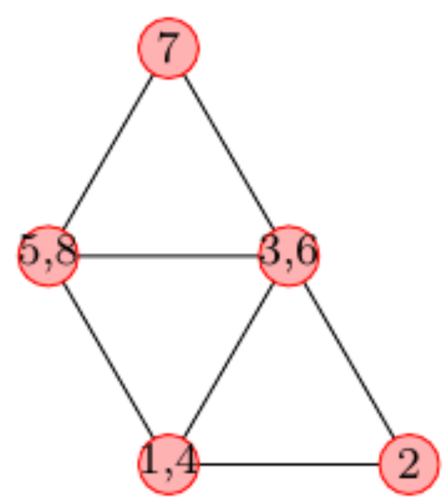

People's History of Trigonometry

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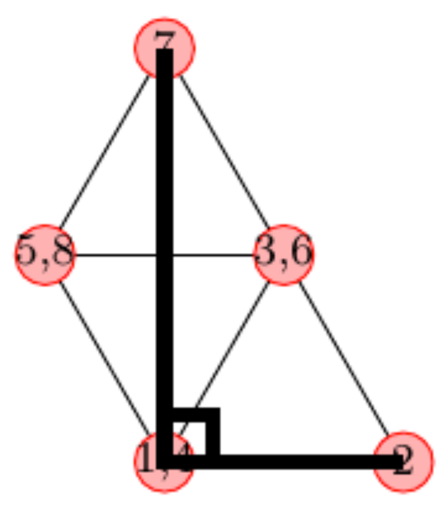

People's History of Trigonometry

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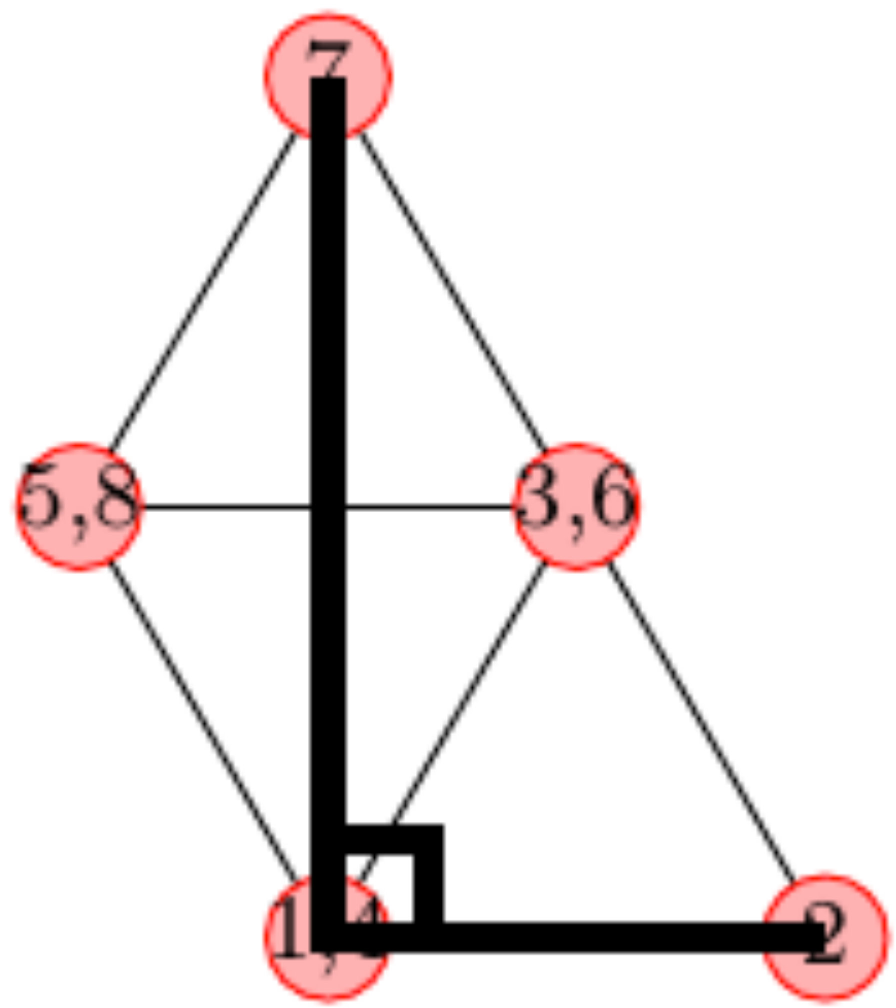


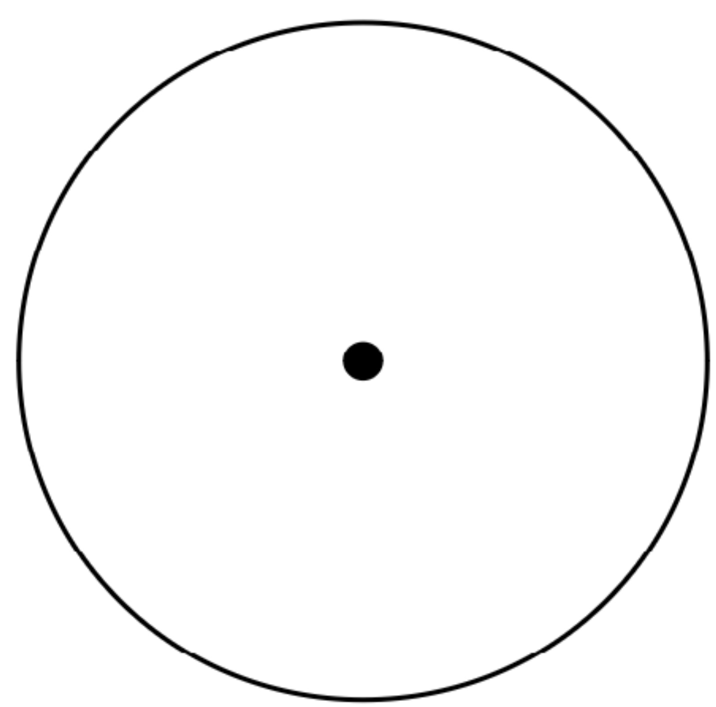
People's History of Trigonometry

- Another development from architects is finding true North/South/East/West.

- Roman architect Vitruvius recorded a clever approach using the Sun.

People's History of Trigonometry

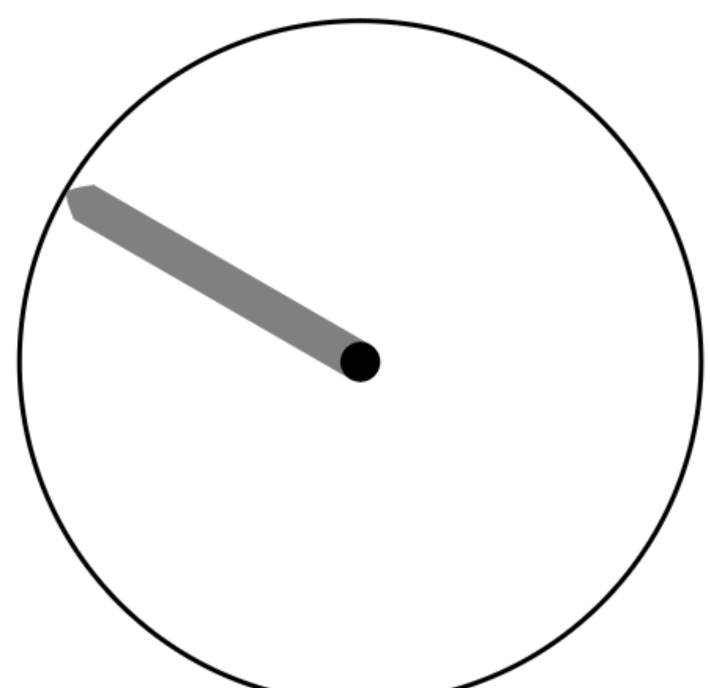
- Step 1: Put a stick in the ground, perpendicular to the ground, and use a compass to draw a circle around it. (Make the circle small enough so that Step 2 works.)



People's History of Trigonometry

- Step 2: At the start of the day, the shadow cast by the stick will be outside the circle.

  Wait until the shadow crosses the circle.
  Mark this spot.



People's History of Trigonometry

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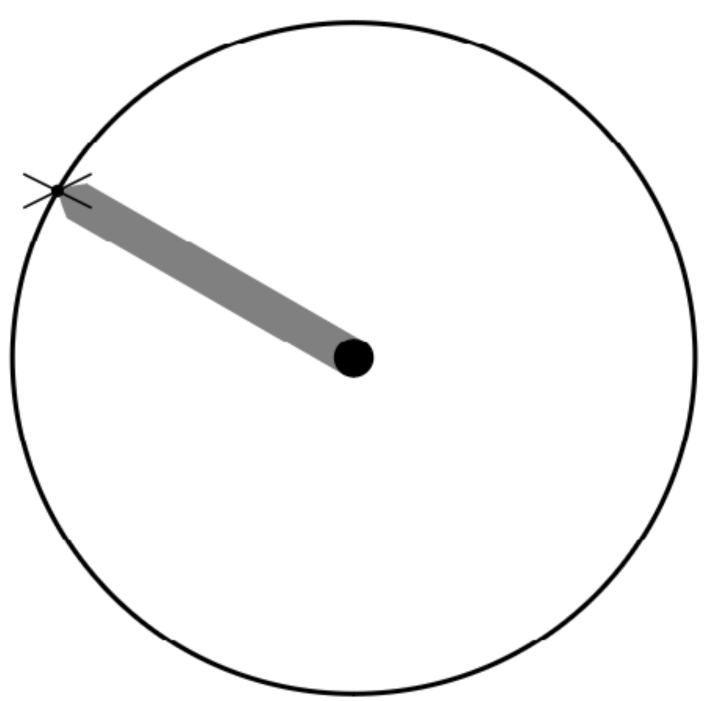
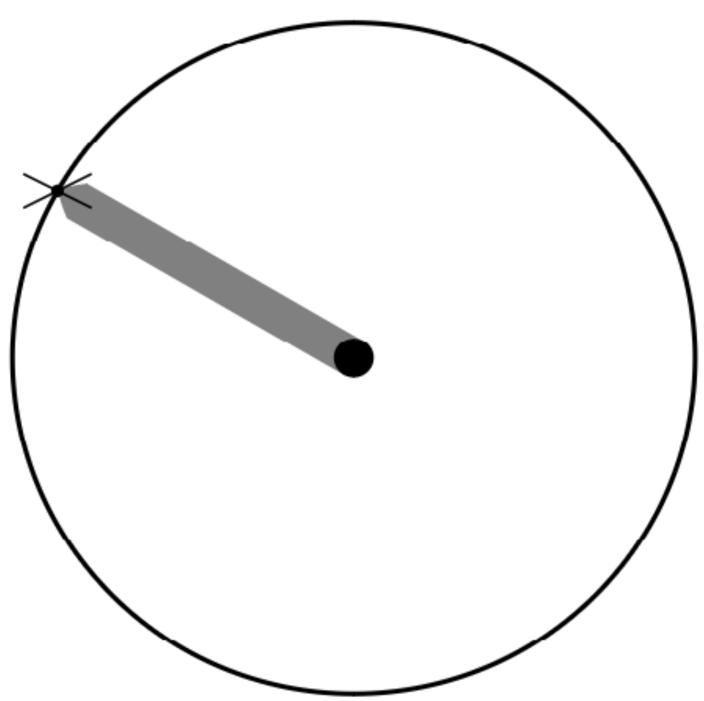


People's History of Trigonometry

- Step 3: Wait until the shadow crosses the circle again. Mark this spot.



People's History of Trigonometry

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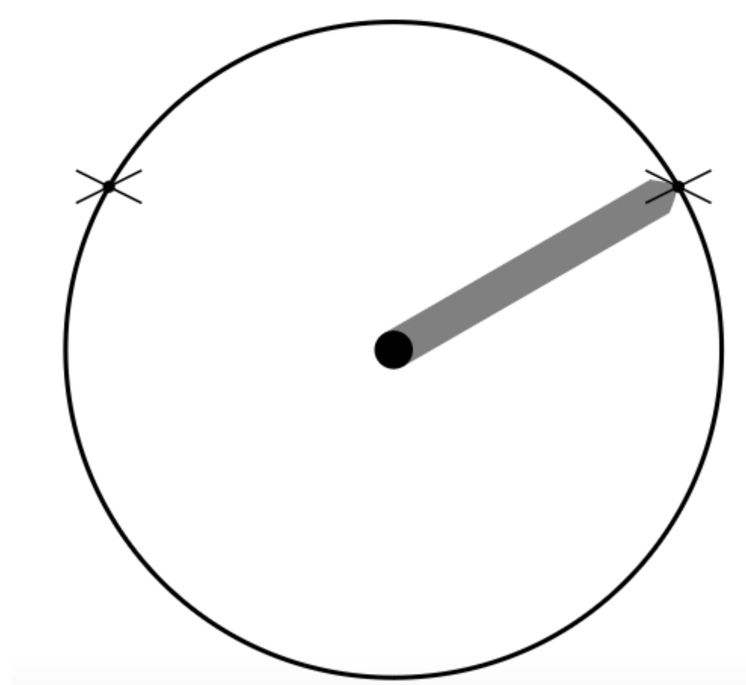
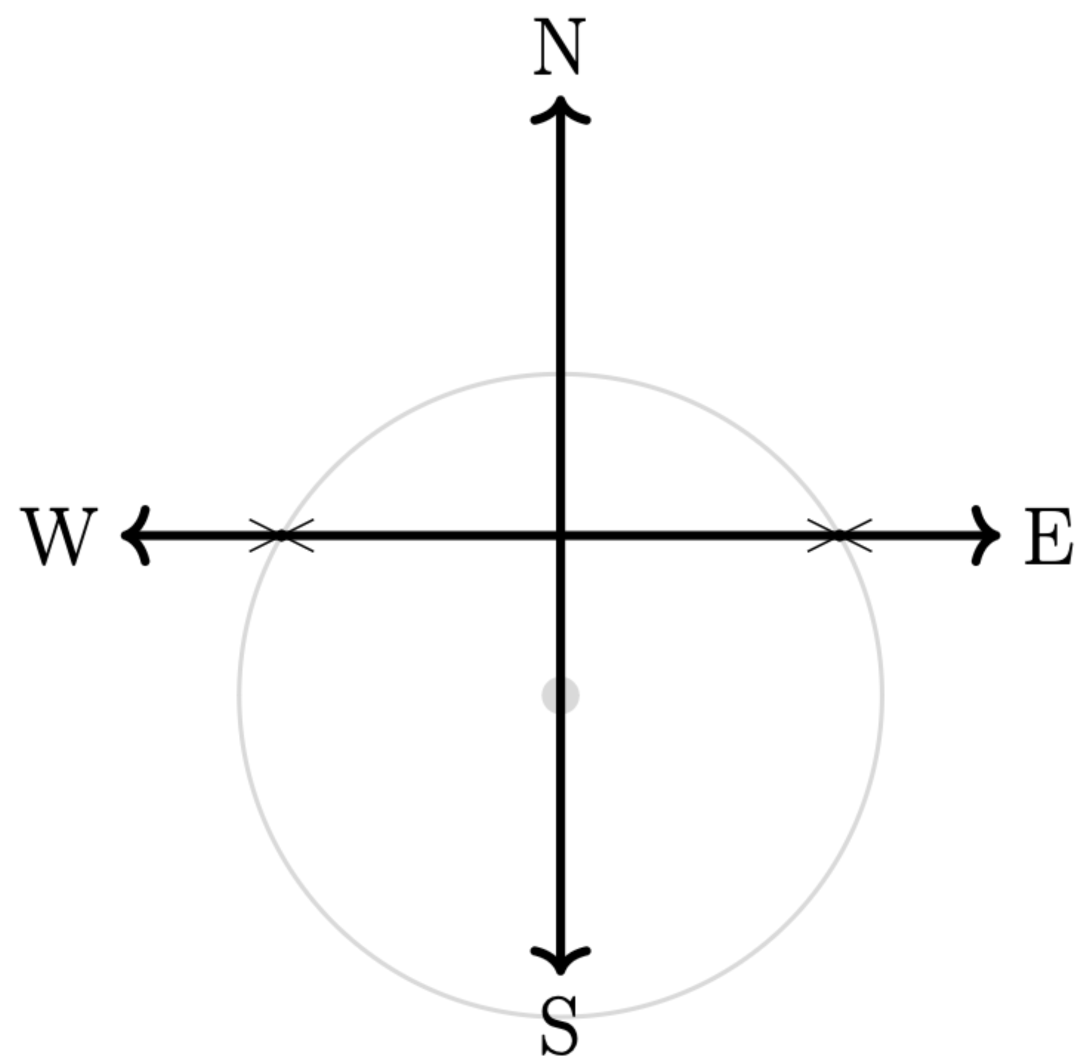


People’s History of Trigonometry

- Step 4: Connect the marks. Draw a perpendicular.



People's History of Trigonometry

- Note: This same technique may have been known to the Ancestral Puebloans.
- They constructed elaborate buildings that could be used to identify the solstices and the 18.6-year lunar cycle.
- But they left no records, so some speculation is required.