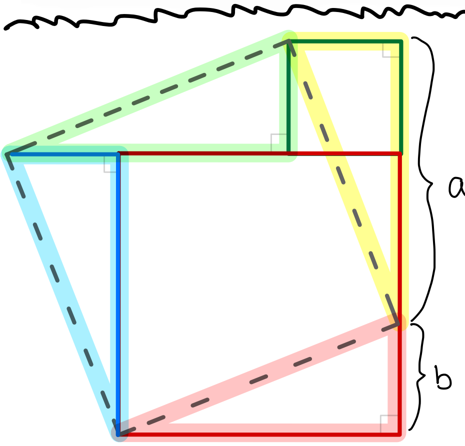
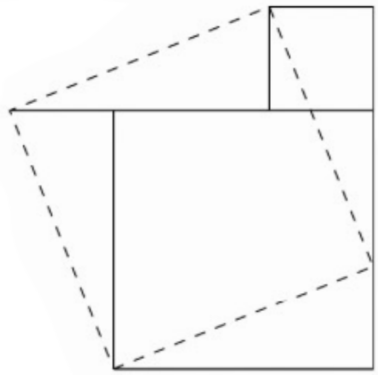
— Exercises —

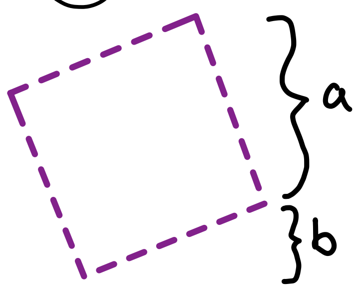
Exercise 3.1. One proof of the Pythagorean theorem, given by 16th-century Indian mathematician Jyesthadeva, is summarized in the following diagram. Write out a complete proof of the Pythagorean theorem based on this diagram.



We know that an area of a triangle: $\frac{1}{2}b \cdot h$, square: a^2

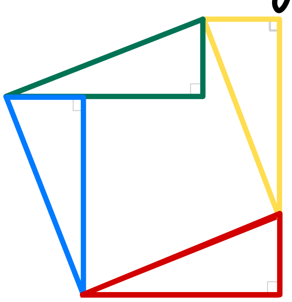


① Total area of the dotted square



$$\rightarrow (a+b)^2$$

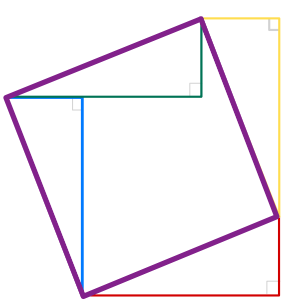
② In the diagram we have 4 triangles, so its fair to say



$$\rightarrow 4\left(\frac{1}{2}ab\right) = 2ab$$

a & b are essentially legs of the right triangle

③ We can equate the two equations by like so...



All the hyp. of all 4 triangles

$$(a+b)^2 = c^2 + 2ab$$

Now with some algebra...

$(a+b)^2 = c^2 + 2ab$

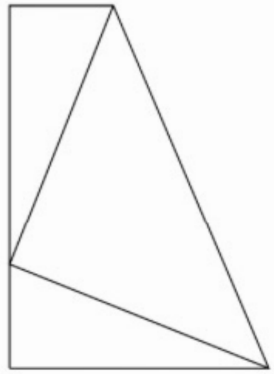
$= a^2 + 2ab + b^2 = c^2 + 2ab$

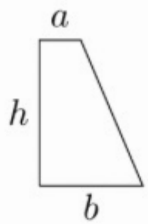
$-2ab$ $-2ab$

and now we are left with

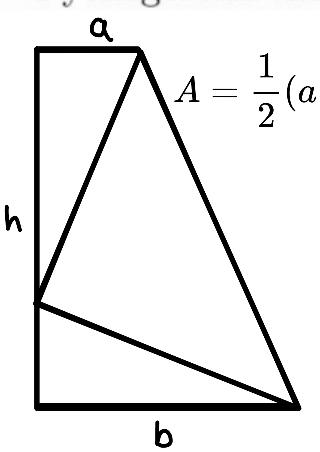
$$a^2 + b^2 = c^2$$

Exercise 3.2. In 1876, U.S. Congressman (and future president) James Garfield discovered a proof of the Pythagorean theorem using the following diagram.

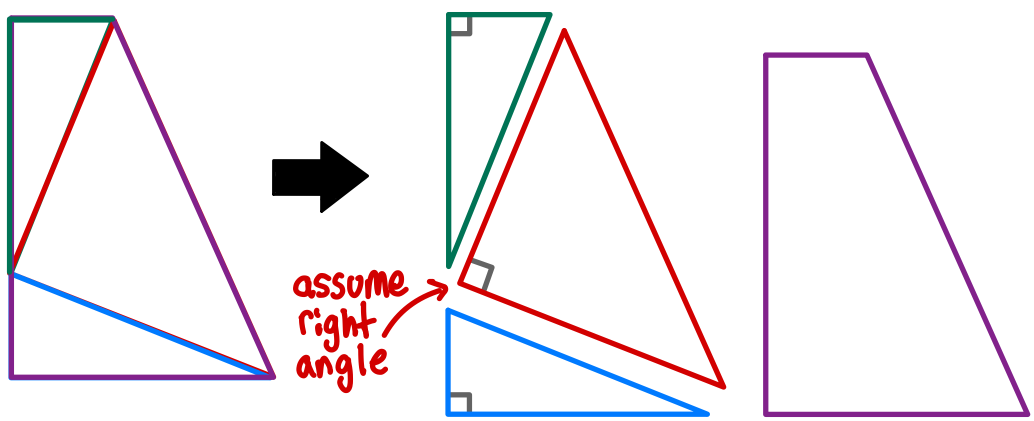


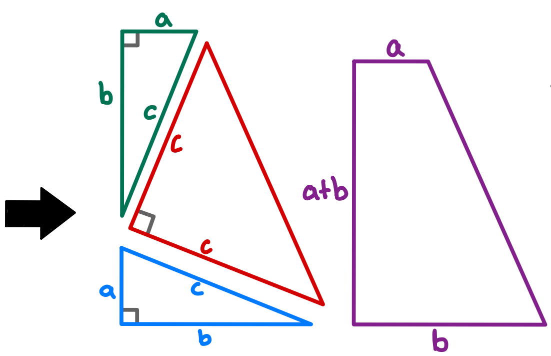
Recall that the area of a trapezoid 

is $\frac{1}{2}(a+b)h$. By writing the area of a Garfield's diagram in two different ways, write out a complete proof of the Pythagorean theorem.

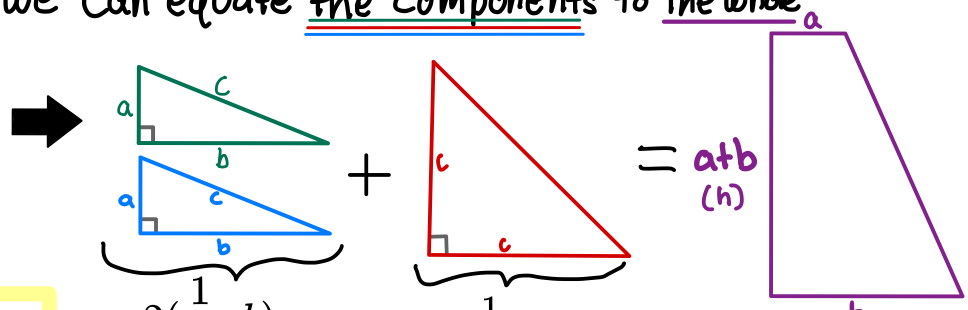


$A = \frac{1}{2}(a+b)h$





With all the components laid out we can equate the components to the whole



$2(\frac{1}{2}ab)$ + $\frac{1}{2}c^2$ = a+b (h)

b

$\frac{1}{2}(a+b)(h)$

↓

$\frac{1}{2}(a+b)(a+b)$

So all together...

$\rightarrow 2(\frac{1}{2}ab) + \frac{1}{2}c^2 = \frac{1}{2}(a+b)(a+b)$

Multiply both sides by 2

$\rightarrow 2[2(\frac{1}{2}ab) + \frac{1}{2}c^2] = 2[\frac{1}{2}(a+b)(a+b)]$

$\rightarrow 2ab + c^2 = a^2 + 2ab + b^2$

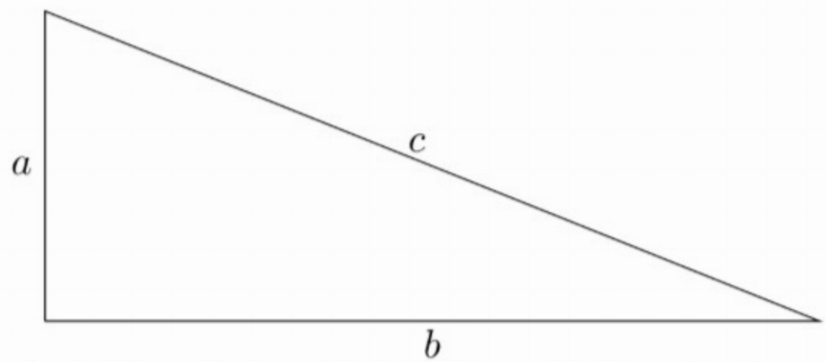
$-2ab \qquad -2ab$

Therefore...

$$c^2 = a^2 + b^2$$

Exercise 3.3. Many proofs of the Pythagorean theorem make use of similar triangles. One of the simplest of these is the following.

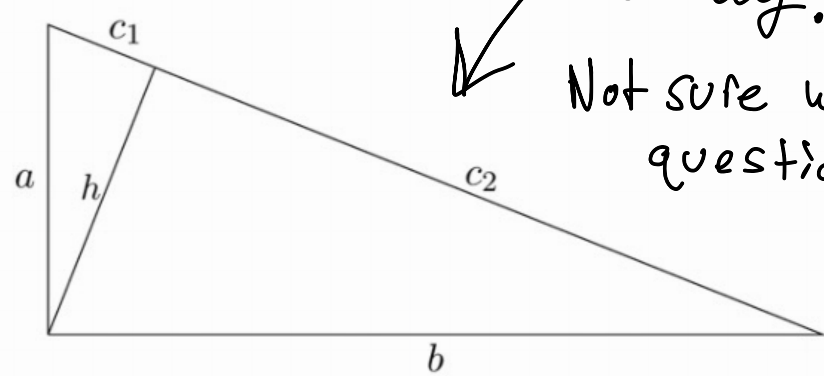
(a) Begin with a right triangle with legs a and b and hypotenuse c.



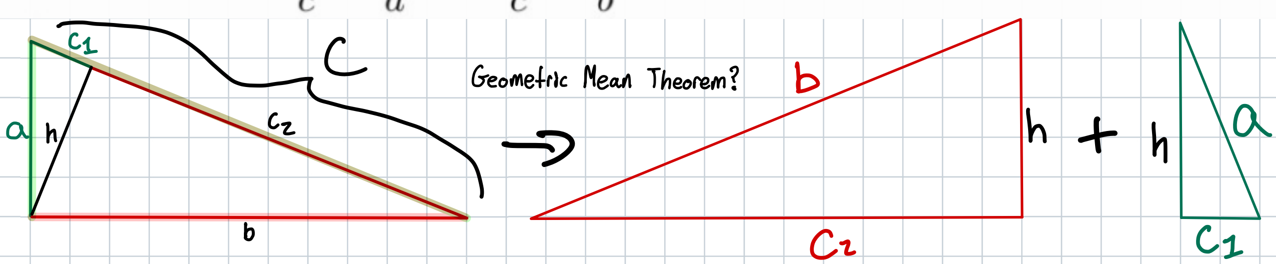
Adding an altitude (of length h, which in turn divides the hypotenuse into two line segments of lengths c_1 and c_2) gives this:

Okay...

Not sure what's the question



(b) Prove that $\dfrac{a}{c} = \dfrac{c_1}{a}$ and $\dfrac{b}{c} = \dfrac{c_2}{b}$.



(c) Cross-multiply each equation and add these equations together. Show that $a^2 + b^2 = c^2$, concluding the proof.

$\frac{a}{c} = \frac{c_1}{a}$ $\quad$ $\frac{b}{c} = \frac{c_2}{b}$

$a^2 = c_1^2$ $\quad$ $b^2 = c_2^2$

$a^2 + b^2 = c_1^2 + c_2^2$

c_1 & c_2 are the two line segments formed by the altitude (h)... Using pythag.

$c_1^2 = a^2 - h^2$ and $c_2^2 = b^2 - h^2$

So... $a^2 + b^2 = (a^2 - h^2) + (b^2 - h^2)$

Simplify & combine like terms

$\cancel{a^2+b^2} = \cancel{a^2+b^2} - 2h^2$

$h^2 = 0$ implies $h = 0$?

c_1 & c_2 are legs a & b respective.

So $c_1 = a$ & $c_2 = b$

Subsitute back

$\frac{a}{c} = \frac{a}{a} \Rightarrow 1 = 1$

$\frac{b}{c} = \frac{b}{b} \Rightarrow 1 = 1$

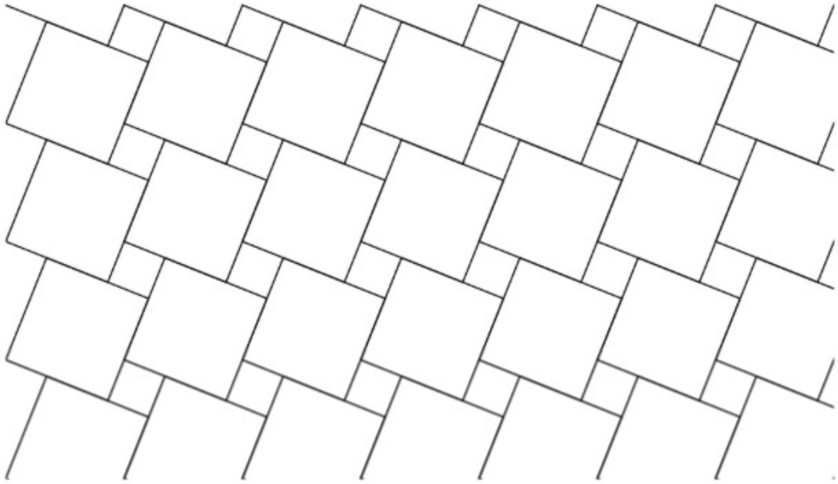
since $c_1 = a$ & $c_2 = b$

$a^2 + b^2 = c_1^2 + c_2^2 = a^2 + b^2$

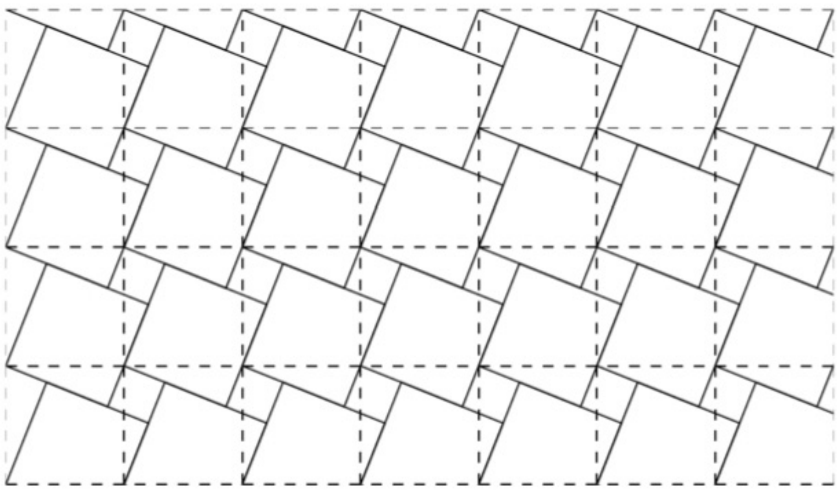
$\therefore a^2 + b^2 = c^2$

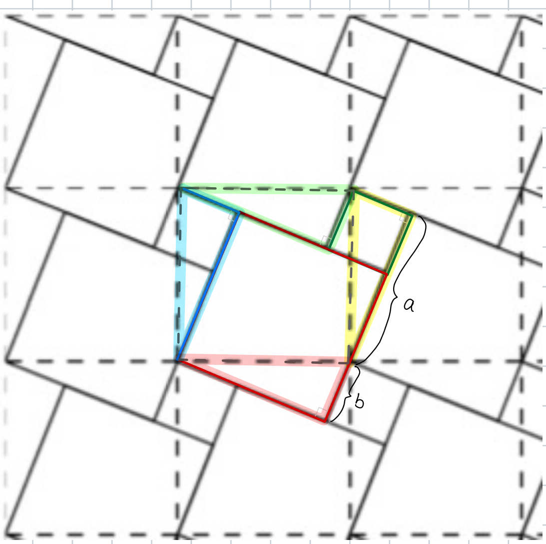
where c is the hypotenuse

Exercise 3.5. In 1974, Wilhelm Magnus proved the Pythagorean theorem using the following tiling pattern. Find a proof based on this tilling.



Hint: It might be helpful to consider these dashed lines:





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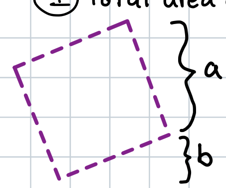

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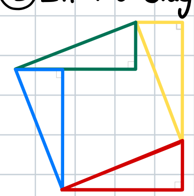

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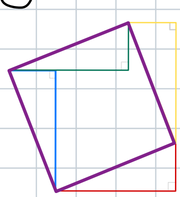

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