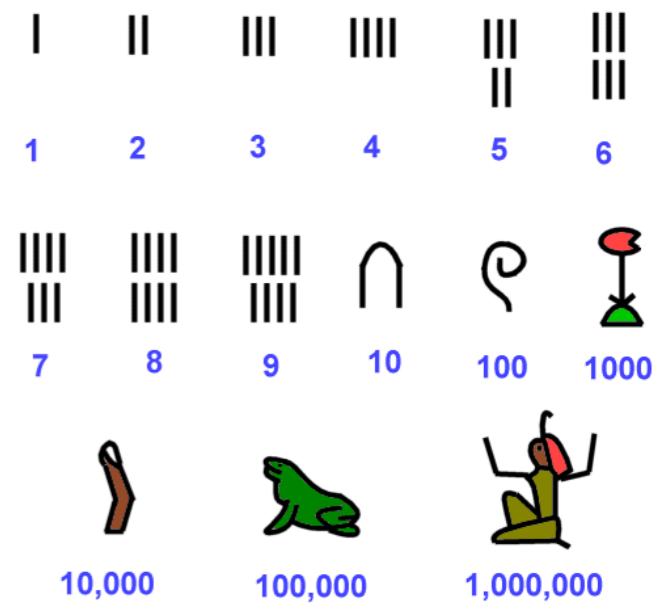
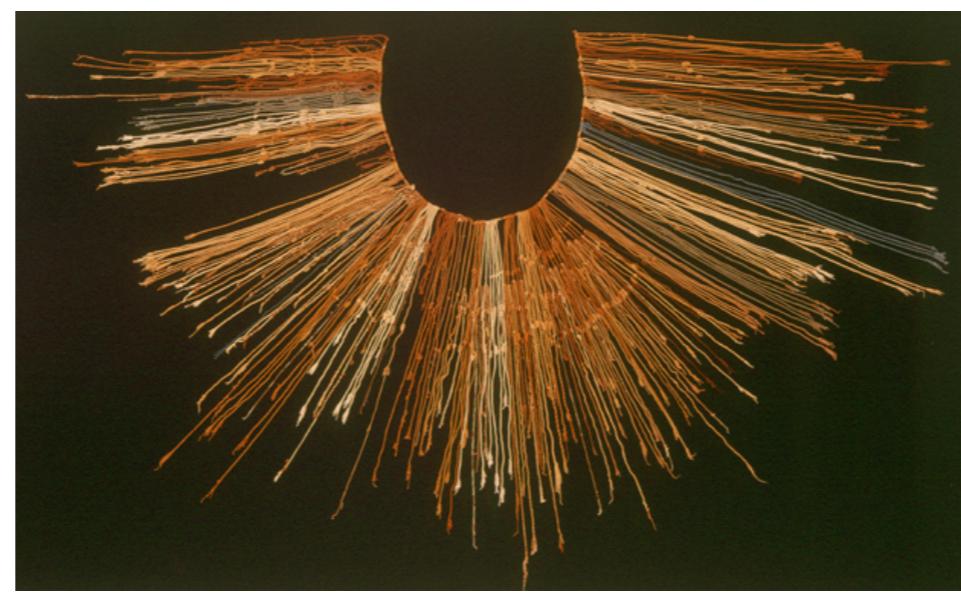


The History of Mathematics

Math 190, Professor Jay Cummings



Syllabus Key Points

- Name: Jay Cummings
- Email: Jay.Cummings@csus.edu
- Office: Shasta 253
- Office Hours: Monday 10–11:30, Thursday 12-1:20

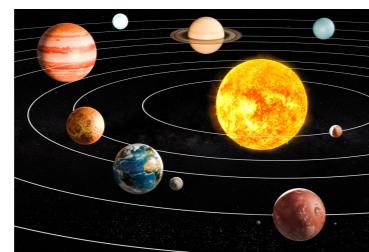
Syllabus Key Points

- Homework:
 - Written Homeworks (25%). Due on Thursdays about every other week
 - Perusall (15%). Discuss notes online (1 per chapter)
- Exams:
 - Midterms (30%) on Thursday of Week 6 & Week 12
 - Final Exam (20%), comprehensive final exam
 - Essay-based, get questions 1 week ahead of time
- Attendance/Participation (10%). Mostly graded on showing up and paying attention.

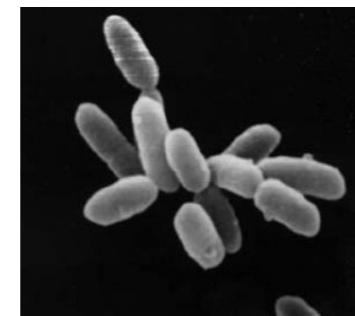
A Brief History of Time



Universe born

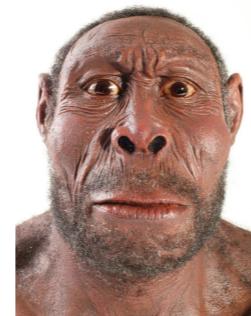


Solar system born



Life on Earth

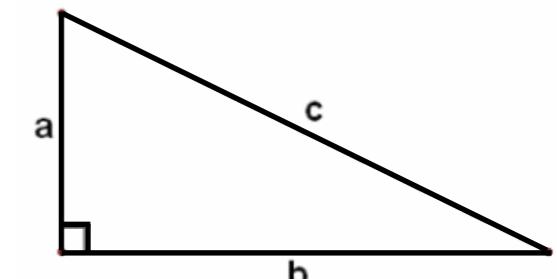
Homo Habitus tools



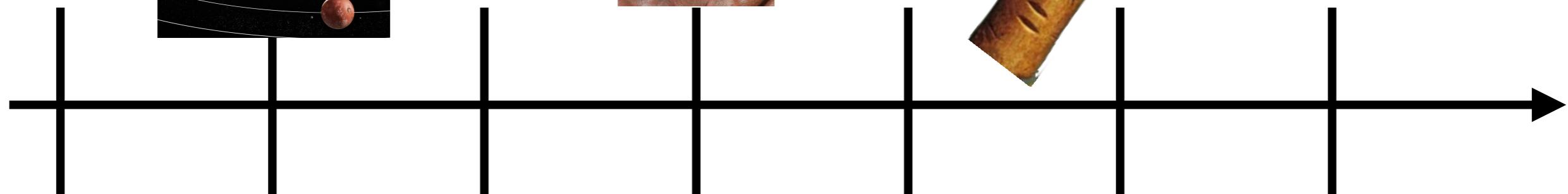
Homo Sapiens, fire



Tally marks on bone



Pythag theorem proved



14 billion years ago

4.5 billion years ago

4 billion years ago

3 million years ago

200,000 years ago

43,000 years ago

2,500 years ago

Lebombo Bone

- From ~41,000 BC.
- 29 clear notches.



Lebombo Bone



Ishango Bone

- From ~18,000 BC.
 - Has 3 columns, with 48, 60 and 60 notches.



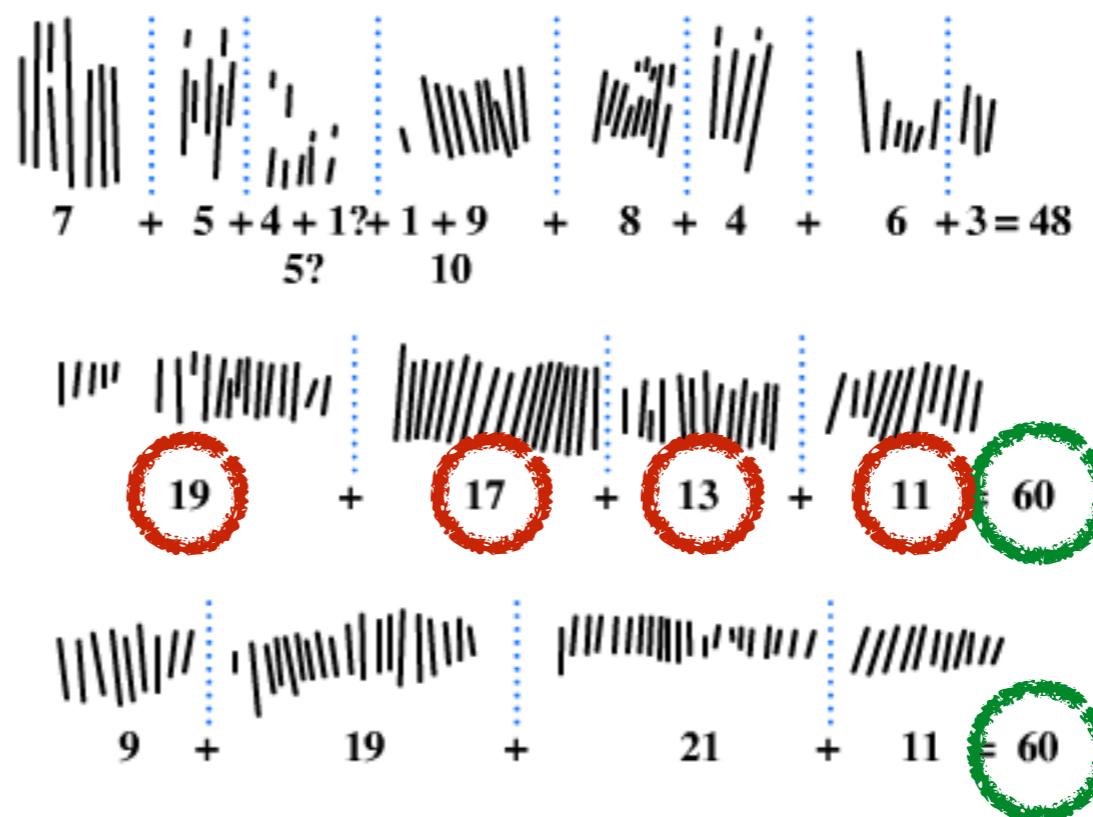
$$\begin{array}{r} \text{|||} \quad \text{|||} \quad \text{'} \quad \text{'|} \quad \text{|} \quad \text{\backslash} \quad \text{|||||} \quad \text{|||||} \quad \text{|} \\ 7 \quad + \quad 5 + 4 + 1? + 1 + 9 \quad + \quad 8 \quad + \quad 4 \quad + \quad 6 \quad + 3 = 48 \\ \hline 5? \quad \quad \quad 10 \end{array}$$

$$\begin{array}{r} \text{|||||} \text{ |||||} \text{ |||||} \\ 19 \\ + \end{array} \quad \begin{array}{r} \text{|||||} \text{ |||||} \text{ |||||} \\ 17 \\ + \end{array} \quad \begin{array}{r} \text{|||||} \text{ |||||} \text{ |||||} \\ 13 \\ + \end{array} \quad \begin{array}{r} \text{|||||} \text{ |||||} \text{ |||||} \\ 11 \\ = 60 \end{array}$$

$$\begin{array}{r} \boxed{\text{V V V V}} + \boxed{\text{V V V V V V V V}} + \boxed{\text{V V V V V V V V V V}} + \boxed{\text{V V V V V V}} = 60 \\ 9 + 19 + 21 + 11 = 60 \end{array}$$

Ishango Bone

- From ~18,000 BC.
 - Has 3 columns, with 48, 60 and 60 notches.



Both bones are the fibula of baboons!

Ishango Bone



How Humans Count

Objects:

- Fingers. (Many methods)
- Tally marks. (Congo, Czech, etc in Stone Age.)
- Knots in rope. (Peru, China)
- Shells, beads. (Native Americans)

Question: If counting is the start of math history, does it have to be humans counting?

Number Bases

- Base 10 (decimal system): Now, almost everyone
- Base 20 (vegesimal system): Mayans, Nigeria
- Base 5: Cherokee, Northern Australia, Siberia, etc.
- Base 60 (sexagesimal system): Babylonians
- Base 4: Papua New Guinea, my son at 22 months
- Base 2 (binary system): Computers

Number Base Notation

Notation. When we write a number like 3,835, that is notation for

$$\begin{aligned} & 3 \cdot 1,000 + 8 \cdot 100 + 3 \cdot 10 + 5 \\ &= 3 \cdot 10^3 + 8 \cdot 10^2 + 3 \cdot 10^1 + 5 \cdot 10^0. \end{aligned}$$

Notation. A number in base b is expressed as

$$(a_k a_{k-1} \dots a_2 a_1 a_0)_b,$$

where each a_i is an integer between 0 and $b - 1$.

Converting the number $(a_k a_{k-1} \dots a_2 a_1 a_0)_b$ to base 10 gives

$$a_k b^k + a_{k-1} b^{k-1} + \dots + a_2 b^2 + a_1 b^1 + a_0 \cdot b^0.$$

Practice

Example. $(123)_4$ is a base 4 number.

$(11010)_2$ is a base 2 number.

$(11010)_3$ is a base 3 number.

Practice

Example. Write $(123)_4$ in base 10.

Solution.

$$1 \cdot 4^2 + 2 \cdot 4^1 + 3 \cdot 4^0$$

$$= 16 + 8 + 3$$

$$= 27$$

Practice

Example. Write the (base 10) number 34 in base 3.

Solution. The powers of 3 that are less than 34 are 27, 9, 3 and 1. After 1 copy of 27, we are left with

$$34 - 27 = 7.$$

We don't need any 9s. After 2 copies of 3, we are left with

$$7 - 2 \cdot 3 = 1.$$

Then, we need 1 copy of 1. In summary:

$$34 = 1 \cdot 3^3 + 0 \cdot 3^2 + 2 \cdot 3^1 + 1 \cdot 3^0$$

So,

$$(34)_{10} = (1021)_3.$$

4 Ways People Write Numbers

- Positional System: Each *position* is a power of 10.

Example: $3,854 = 3 \cdot 1,000 + 8 \cdot 100 + 5 \cdot 10 + 4$

$$= 3 \cdot 10^3 + 8 \cdot 10^2 + 5 \cdot 10^1 + 4$$

- Multiplicative Grouping System: Have a character for each power of 10

Example: Chinese-Japanese numeral system

4 Ways People Write Numbers

1 = 一	10 = 十	100 = 一百	1,000 = 一千
2 = 二	20 = 二十	200 = 二百	2,000 = 二千
3 = 三	30 = 三十	300 = 三百	3,000 = 三千
4 = 四	40 = 四十	400 = 四百	4,000 = 四千
5 = 五	50 = 五十	500 = 五百	5,000 = 五千
6 = 六	60 = 六十	600 = 六百	6,000 = 六千
7 = 七	70 = 七十	700 = 七百	7,000 = 七千
8 = 八	80 = 八十	800 = 八百	8,000 = 八千
9 = 九	90 = 九十	900 = 九百	9,000 = 九千
10 = 十	100 = 一百	1,000 = 一千	10,000 = 一万

Example: Chinese-Japanese numeral system

$$5,062 = 5 \cdot 1,000 + 6 \cdot 10 + 2$$

= 五千六十二

4 Ways People Write Numbers

- Simple Grouping System: Add everything up.

Example: Egyptian hieroglyphics

4 Ways People Write Numbers

- Simple Grouping System: Add everything up.

$1 = |$, $10 = \cap$, $100 = \text{ꝛ}$, $1,000 = \text{ҝ}$

$10,000 = \text{Ѡ}$, $100,000 = \text{ѿ}$, $1,000,000 = \text{ѿѿѿ}$

$$\begin{aligned}321 &= \text{ꝛꝛꝛ} \cap\cap | \\&= \text{ꝛꝛꝛ} | \cap\cap \\&= \text{ꝛ}\cap\text{ꝛ} | \text{ꝛ}\cap\end{aligned}$$

4 Ways People Write Numbers

- Ciphered systems: It's... complicated. For a base b , we need symbols for

$$1, 2, \dots, b - 1$$
$$b, 2b, \dots, (b - 1)b$$
$$b^2, 2b^2, \dots, (b - 1)b^2$$
$$b^3, 2b^3, \dots, (b - 1)b^3$$
$$\vdots$$

Example: Ionic Greek system

4 Ways People Write Numbers

$1 = \alpha$ (alpha)	$10 = \iota$ (iota)	$100 = \rho$ (rho)
$2 = \beta$ (beta)	$20 = \kappa$ (kappa)	$200 = \sigma$ (sigma)
$3 = \gamma$ (gamma)	$30 = \lambda$ (lambda)	$300 = \tau$ (tau)
$4 = \delta$ (delta)	$40 = \mu$ (mu)	$400 = \upsilon$ (upsilon)
$5 = \varepsilon$ (epsilon)	$50 = \nu$ (nu)	$500 = \phi$ (phi)
$6 = \varsigma$ (vau)	$60 = \xi$ (xi)	$600 = \chi$ (chi)
$7 = \zeta$ (zeta)	$70 = \omicron$ (omicron)	$700 = \psi$ (psi)
$8 = \eta$ (eta)	$80 = \pi$ (pi)	$800 = \omega$ (omega)
$9 = \theta$ (theta)	$90 = \kappa$ (koppa)	$900 = \lambda$ (sampi)

Example: Ionic Greek system

Question: Which
system is best?



Zero



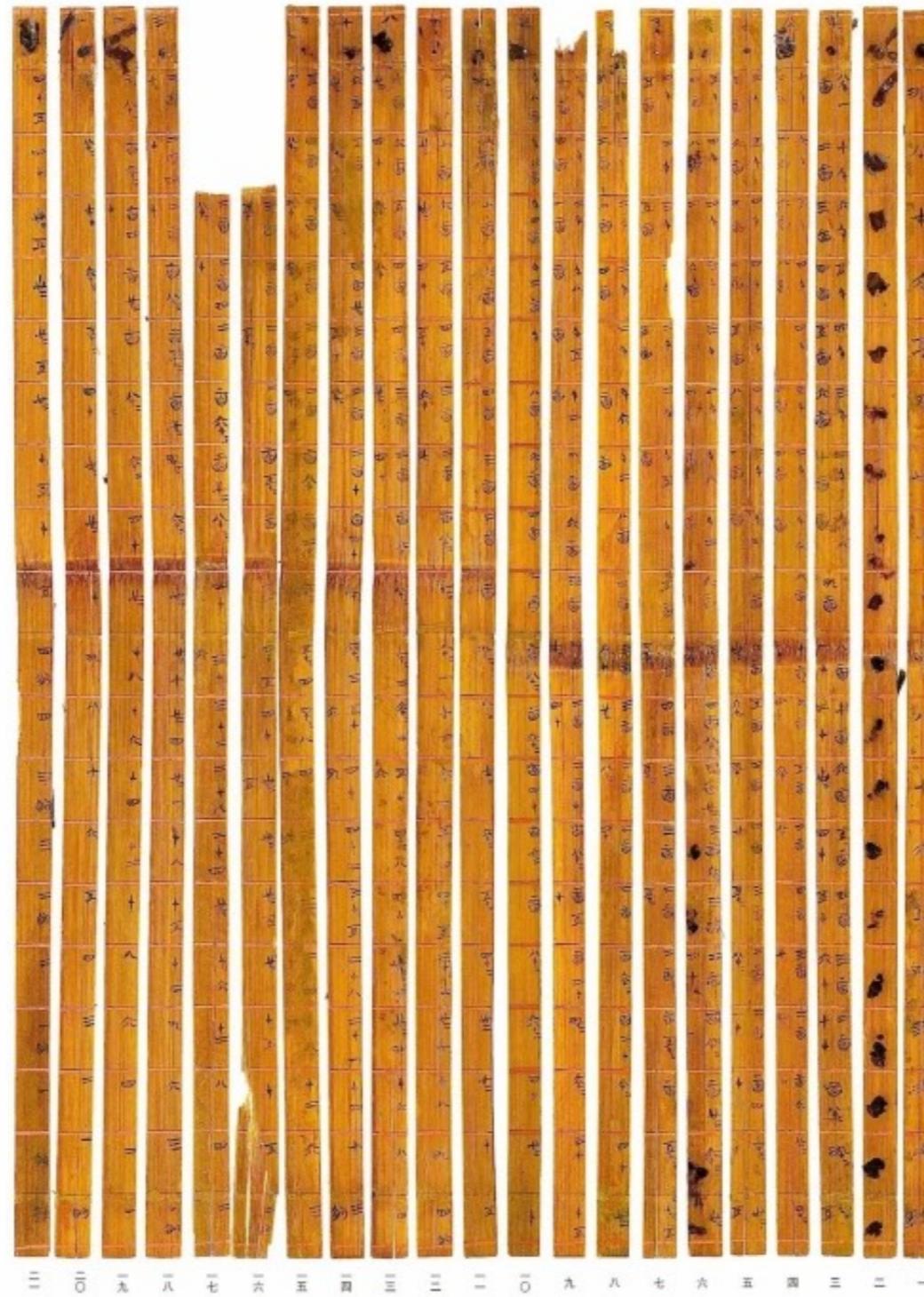
The Indians and the Mayans were the two civilizations to discover zero and recognize its deep potential as:

- A symbol
- A number
- A magnitude
- A direction separator
- A place-holder
- An idea

This was a big deal.

Think Like A
Math Historian

Think Like A Math Historian



Think Like A Math Historian

8	5	90	20	9	10	40	60	5400	2	90
	450	80	1800	810	900	3600	5400	4800	180	8100
	400	70	1600	720	800	3200	4800	4200	160	7200
	350	60	1400	630	700	2800	4200	3600	140	6300
	300	50	1200	540	600	2400	3600	3000	120	5400
	250	40	1000	450	500	2000	4500	4000	100	4500
	200	320	800	360	400	1600	540	480	1800	3600
	150	7200	600	270	300	1200	3500	3000	60	2700
	100	6400	280	30	240	2000	4200	3600	90	5600
	560	280	400	560	400	1600	4800	4000	1800	4900
	480	240	200	180	200	800	2700	2400	1200	4200
	400	200	10	160	100	400	2400	2000	400	3500
	320	160	180	160	100	360	2100	1800	20	720
	240	120	150	140	90	320	1800	1500	18	630
	160	80	120	120	80	280	1500	1200	60	2100
	80	40	120	120	100	280	1200	1000	16	1400
	30	80	8	120	100	240	900	120	50	450
	72	36	7	90	80	200	600	500	40	700
	25	36	6	60	60	160	300	240	12	630
	64	30	5	30	40	120	270	200	30	360
	20	32	27	20	36	80	270	240	10	560
	56	720	28	24	20	120	54	400	120	490
	15	28	4	24	27	80	240	200	8	180
	48	640	3	21	18	10	48	350	6	420
	10	24	21	18	15	2	210	300	8	350
	40	560	2	18	15	20	42	300	4	280
	5	20	18	15	15	180	150	120	7	210
	32	480	1	15	15	36	36	200	2	140
	21/2	400	16	15	35	120	120	30	6	70
	24	320	12	12	28	90	90	150	5	35
	16	240	8	9	21	60	60	100	4	42
	8	240	4	6	14	30	30	18	3	30
	4	160	4	3	14	12	12	50	2	22
	2	80	2	3	7	6	6	25	1	11
	40	40	11/2	11/2	31/2	31/2	3	1/2	1/2	

Think Like A Math Historian

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
4 ^{1/2}	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
3 ^{1/2}	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
2 ^{1/2}	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
1 ^{1/2}	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
1 ^{1/2}	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
1/4	1/2	1	1 ^{1/2}	2	2 ^{1/2}	3	3 ^{1/2}	4	4 ^{1/2}	5	10	15	20	25	30	35	40	45	1/2

We will come back to this soon.

Arithmetic

Multiplication Tables

- After counting comes arithmetic. Simple addition and subtraction could be done in head, on fingers, or with simple tools. Multiplication was probably much harder.
- First known multiplication table is a 4,000 year old Babylonian (base 60) multiplication table.

Multiplication Tables

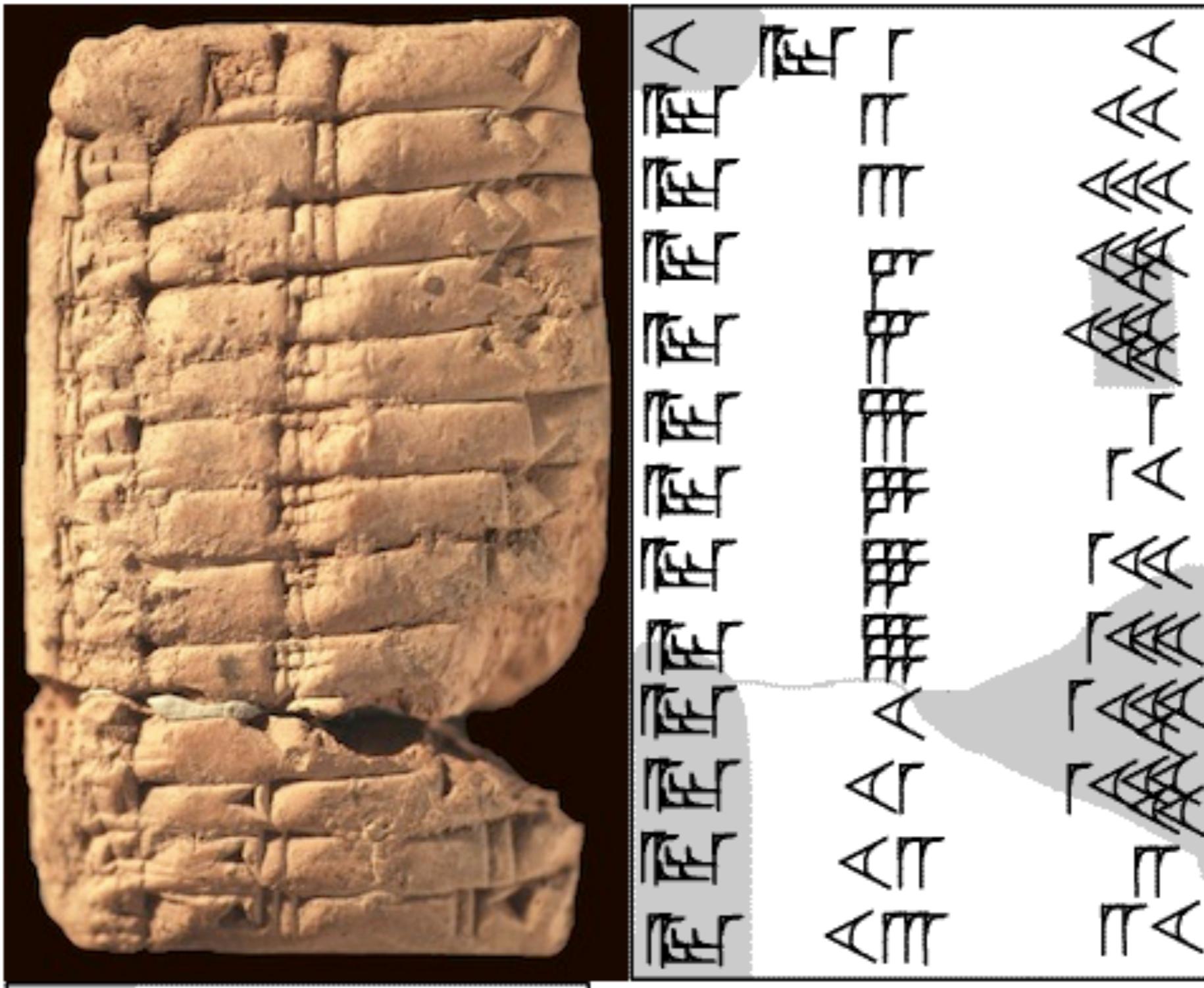
- After counting and subtracting or with simple multiplication was much harder.
- First known multiplication table from Babylonian (but not the first multiplication table)
- c. Simple addition
→ in head, on fingers, on fingers
ion was probably
is a 4,000 year old
ation table.



MS 3866
Multiplication table for $1.12(=72)$.
Babylonia, 19th c. BC

Multiplication Tables

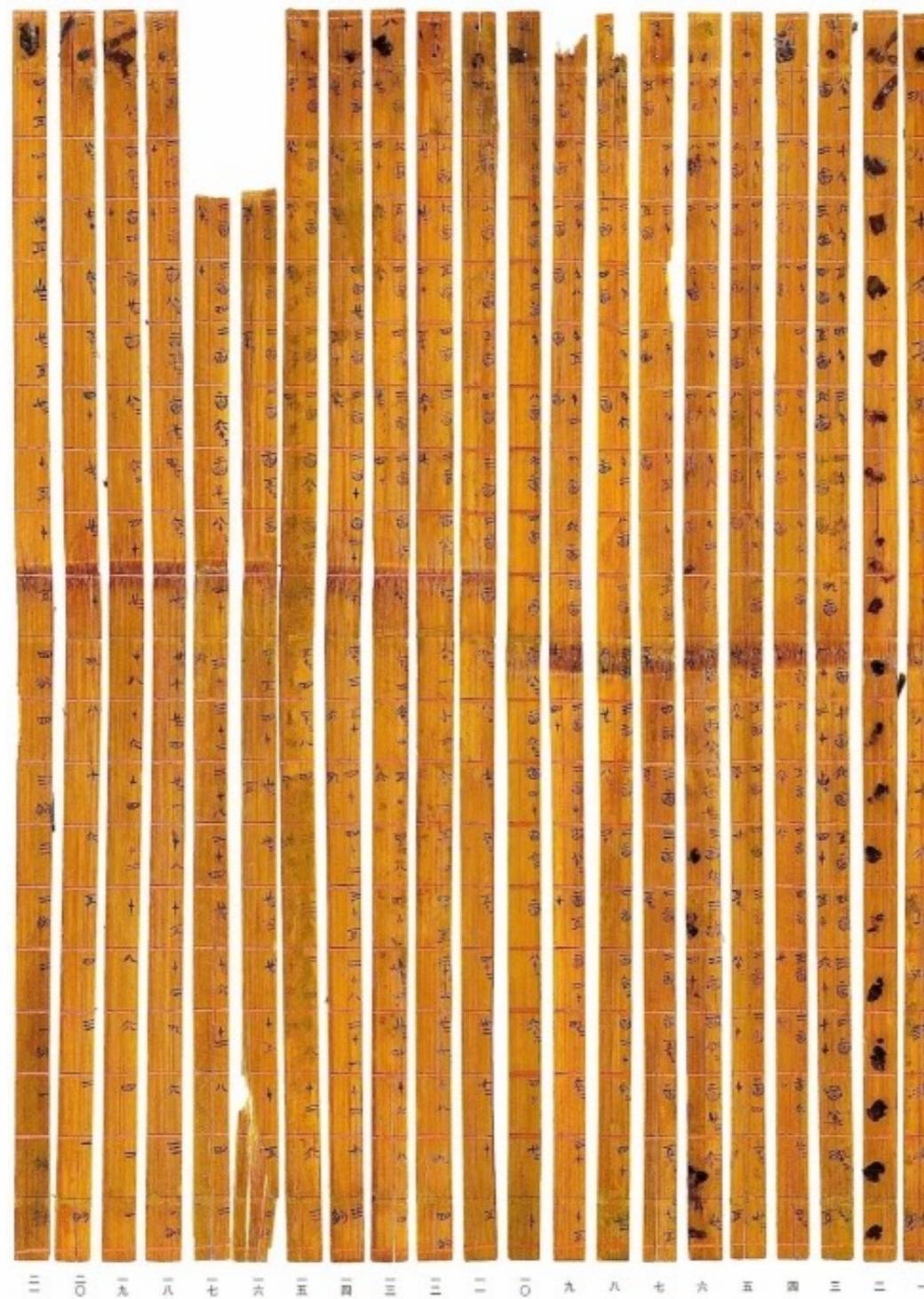
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Multiplication Tables

- After counting comes arithmetic. Simple addition and subtraction could be done in head, on fingers, or with simple tools. Multiplication was probably much harder.
- First known multiplication table is a 4,000 year old Babylonian (base 60) multiplication table.
- First known *decimal* multiplication table is a 2,300 year old Chinese multiplication table.
- It was discovered in 2009!

Tsinghua Bamboo Strips



Tsinghua Bamboo Slips

- An ancient table, written on bamboo strips and found in China, contains what is essentially a multiplication table.
- With a little extra work, one can use it to compute the product of any two integers or half integers from 0.5 to 99.5.
That is, you can multiply together any two numbers from this list: 0.5, 1, 1.5, 2, 2.5, ..., 99.5.
- This is the first known decimal multiplication table.

Tsinghua Bamboo Slips

- This multiplication table can compute the product of any two numbers from the set $\{0.5, 1, 1.5, 2, 2.5, 3, 3.5, \dots, 99.5\}$. How?

$1/2$	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
$4\frac{1}{2}$	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
$3\frac{1}{2}$	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
$2\frac{1}{2}$	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
$1\frac{1}{2}$	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
$\frac{1}{2}$	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	10	15	20	25	30	35	40	45	$\frac{1}{2}$

Think Like A
Math Historian

Tsinghua Bamboo Slips

- This multiplication table can compute the product of any two numbers from the set $\{0.5, 1, 1.5, 2, 2.5, 3, 3.5, \dots, 99.5\}$. How?

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
$4\frac{1}{2}$	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
$3\frac{1}{2}$	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
$2\frac{1}{2}$	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
$1\frac{1}{2}$	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
$\frac{1}{2}$	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	10	15	20	25	30	35	40	45	$\frac{1}{2}$

Tsinghua Bamboo Slips

- Example: $24 \cdot 36.5 = ?$

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
$4\frac{1}{2}$	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
$3\frac{1}{2}$	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
$2\frac{1}{2}$	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
$1\frac{1}{2}$	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
$\frac{1}{2}$	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	10	15	20	25	30	35	40	45	$\frac{1}{2}$

Tsinghua Bamboo Slips

- Example: $24 \cdot 36.5 = ?$

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
$4\frac{1}{2}$	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
$3\frac{1}{2}$	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
$2\frac{1}{2}$	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
$1\frac{1}{2}$	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
$1\frac{1}{2}$	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
$1\frac{1}{4}$	$1\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	10	15	20	25	30	35	40	45	$1\frac{1}{2}$

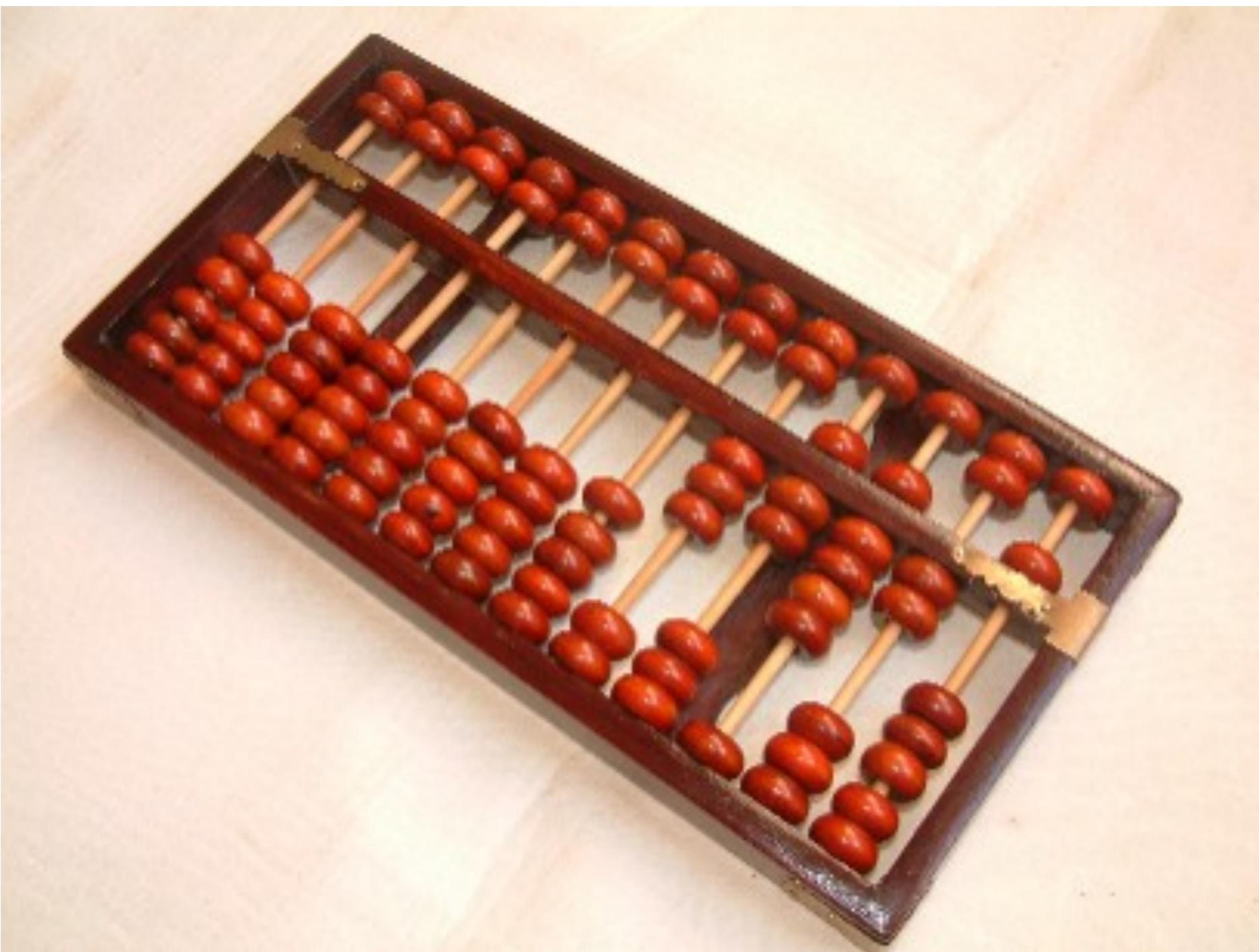
$$24 \cdot 36.5 = (20 + 4) \cdot (30 + 6 + 0.5)$$

$$= 20 \cdot 30 + 20 \cdot 6 + 20 \cdot 0.5 + 4 \cdot 30 + 4 \cdot 6 + 4 \cdot 0.5$$

= Look up on table, then add together

Ancient Calculators

Abacus



The Incan Empire



The Incan Empire



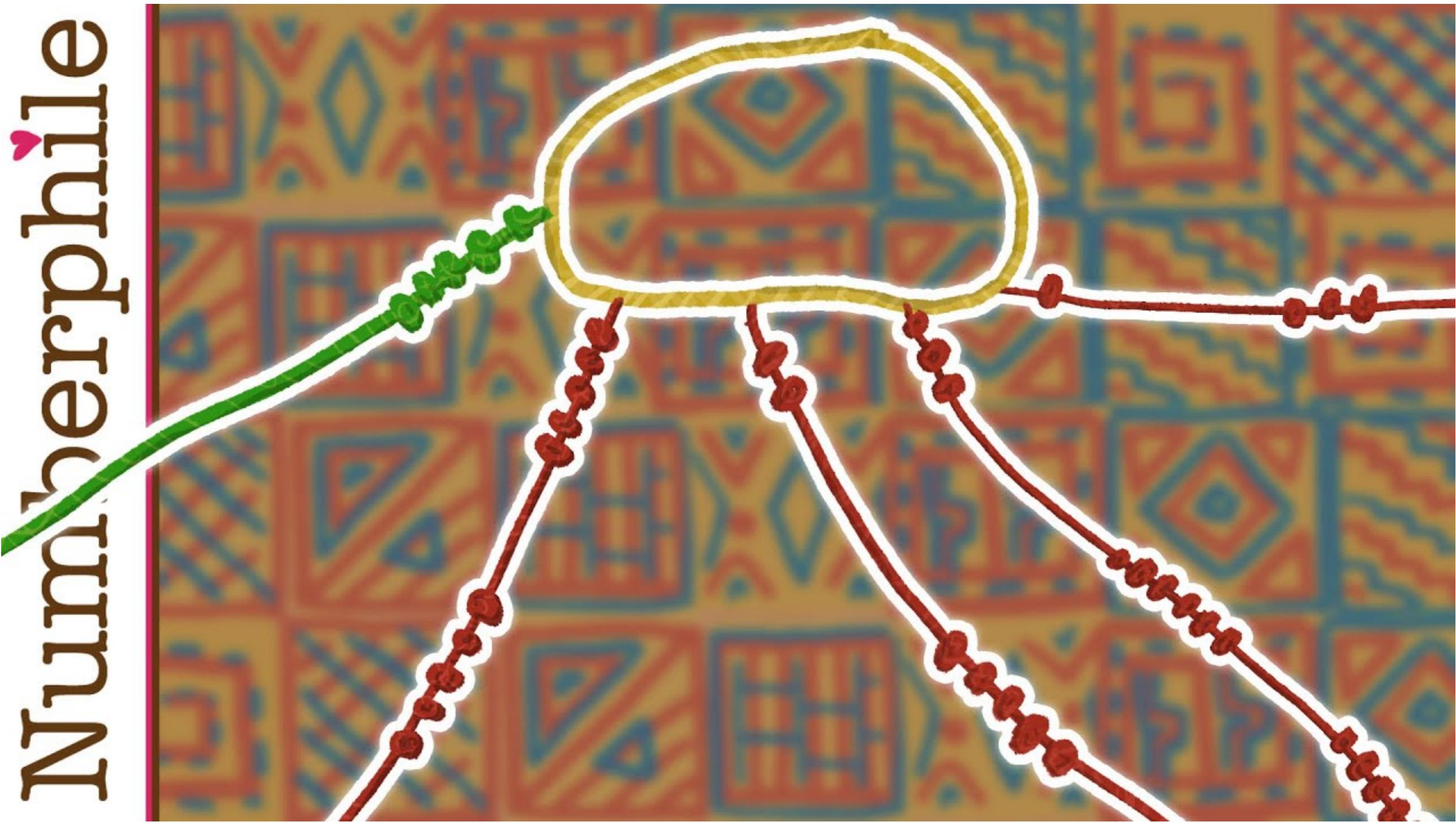
The Incan Empire

- The Incans were a great but relatively short-lived empire.
- They did not have a written language.
- How do you record numbers and do math without writing?
- The Incans used what is called a *quipu*.

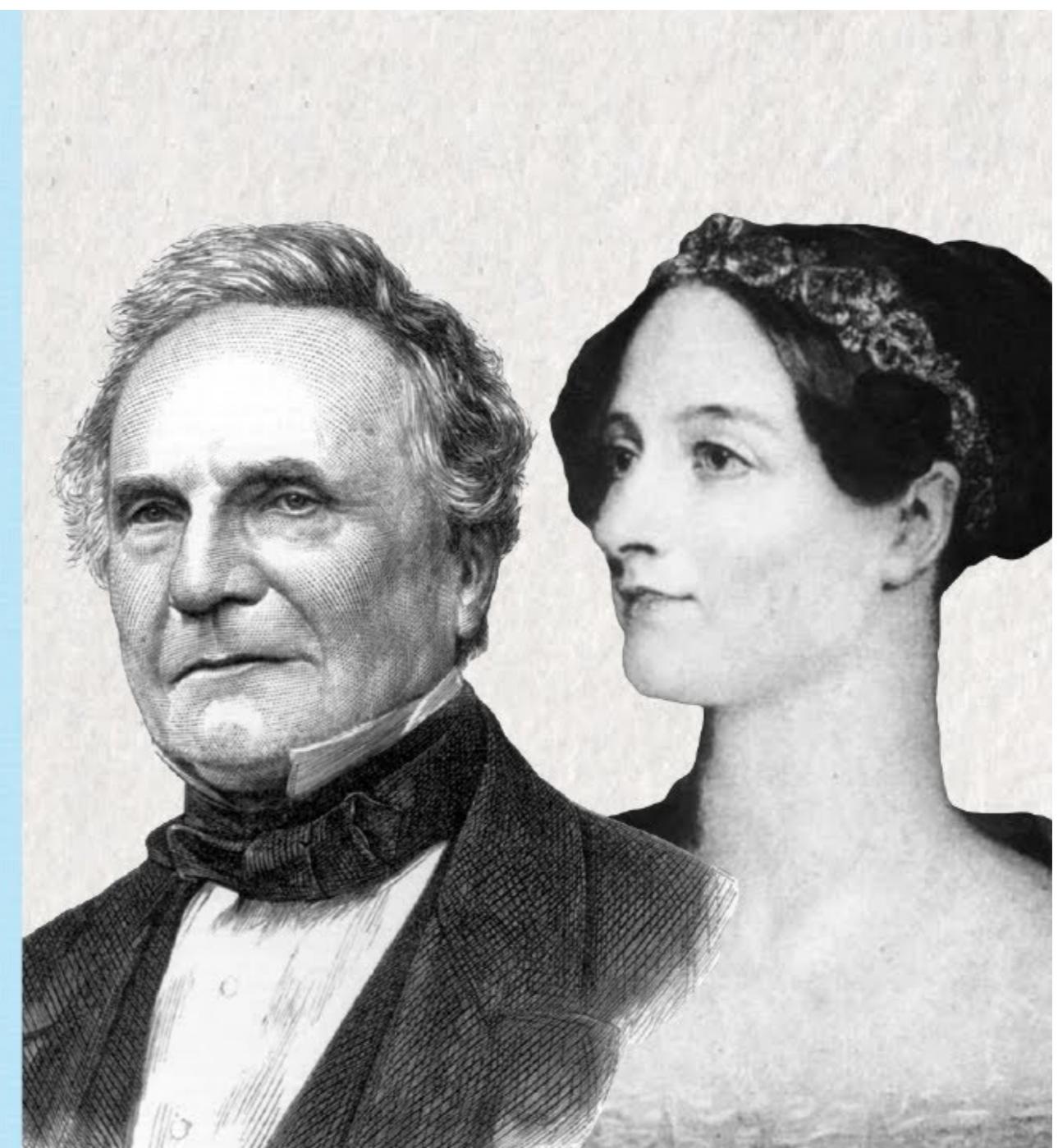
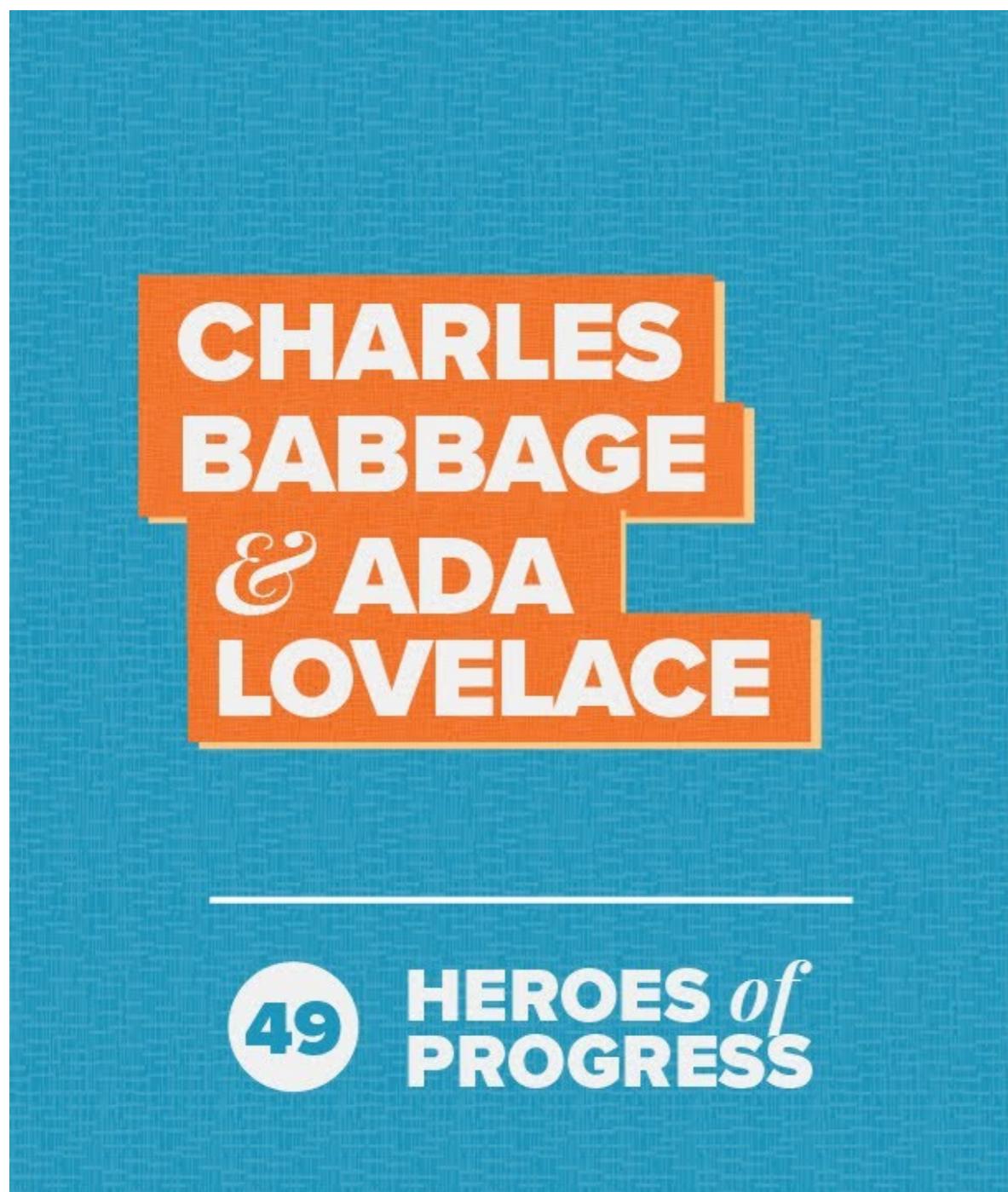
Devices



Devices



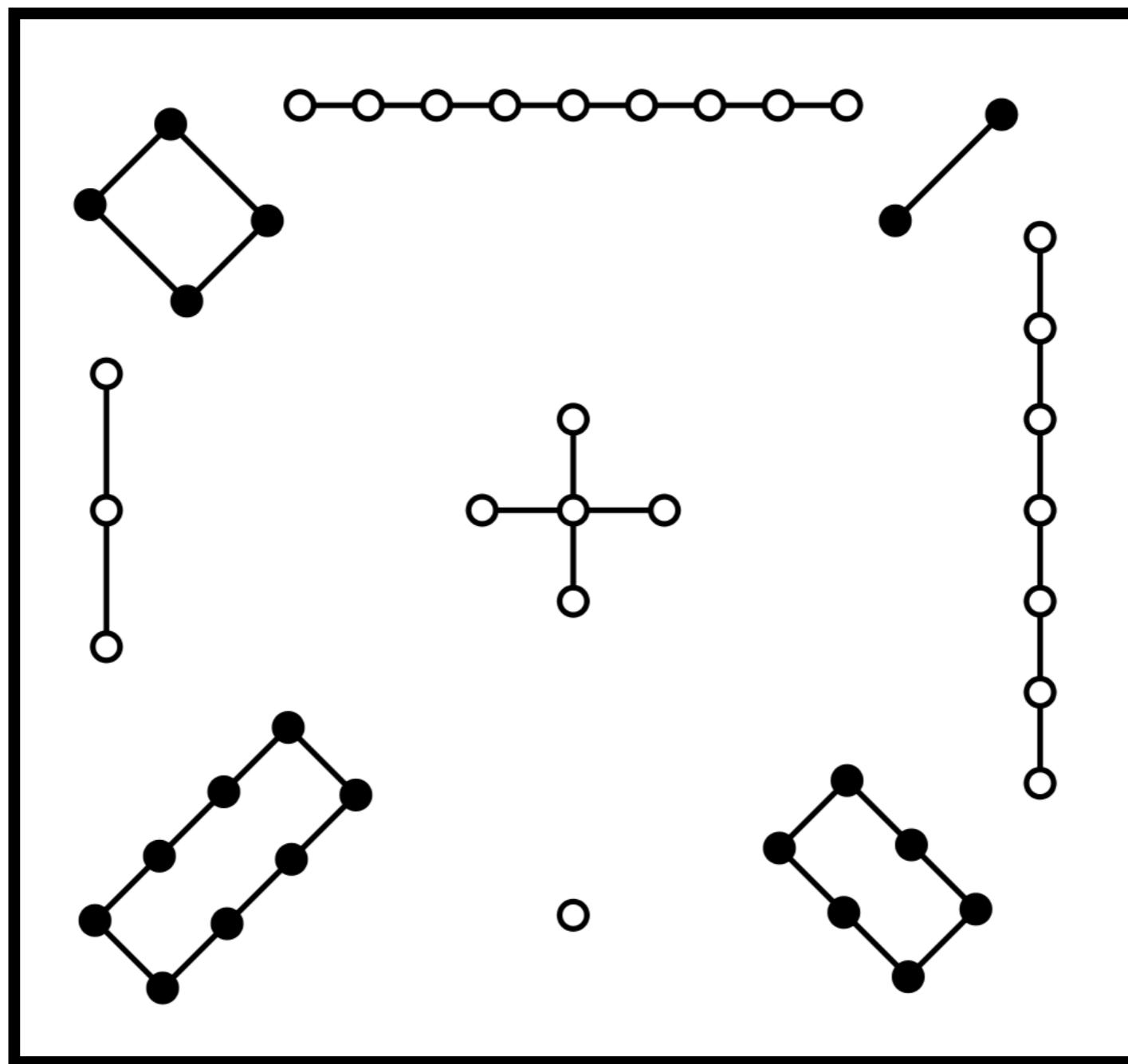
Aftermath: Computers



Think Like A
Math Historian

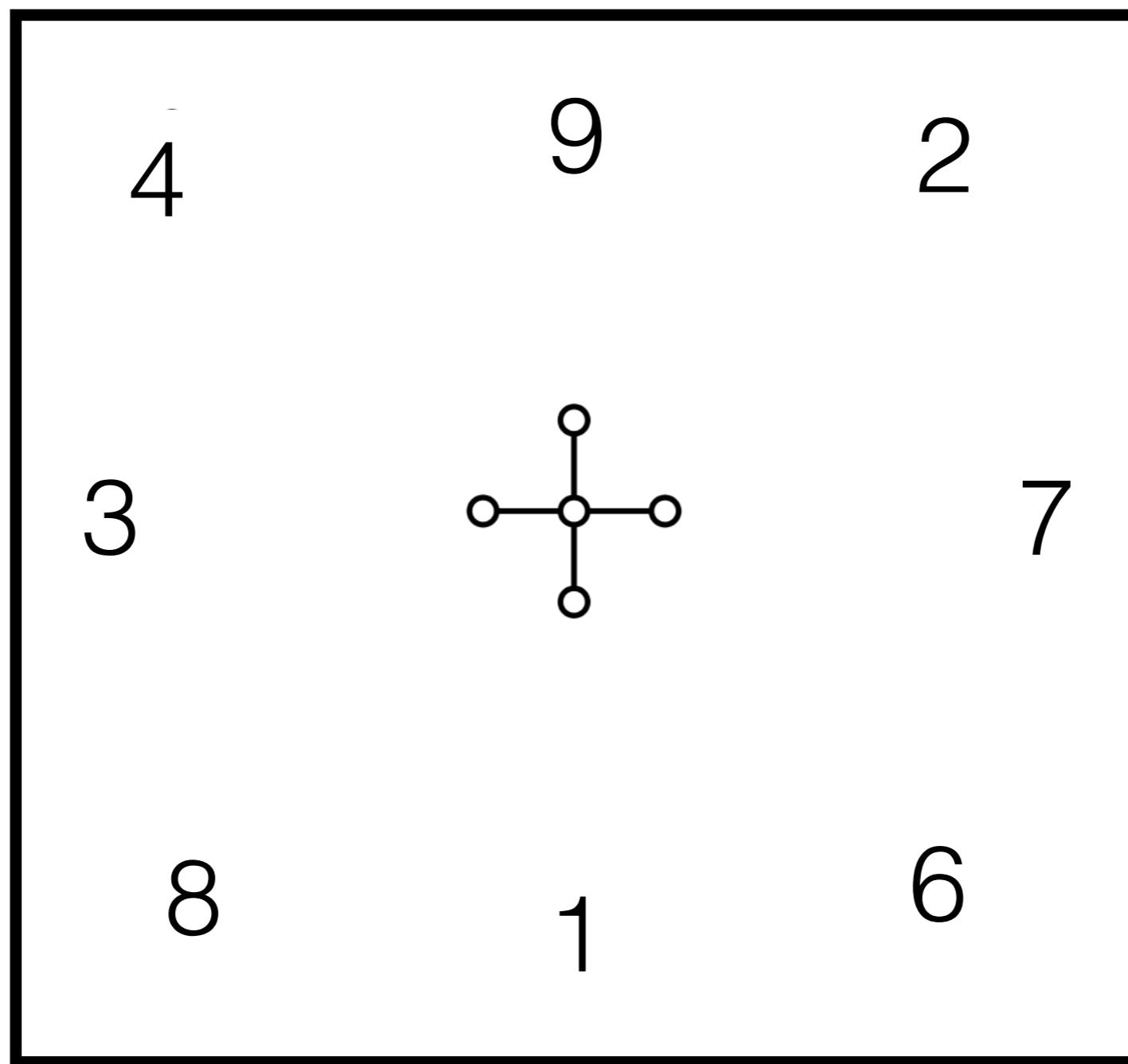
Think Like a Math Historian

- Hint: Focus on how many dots are in each group.



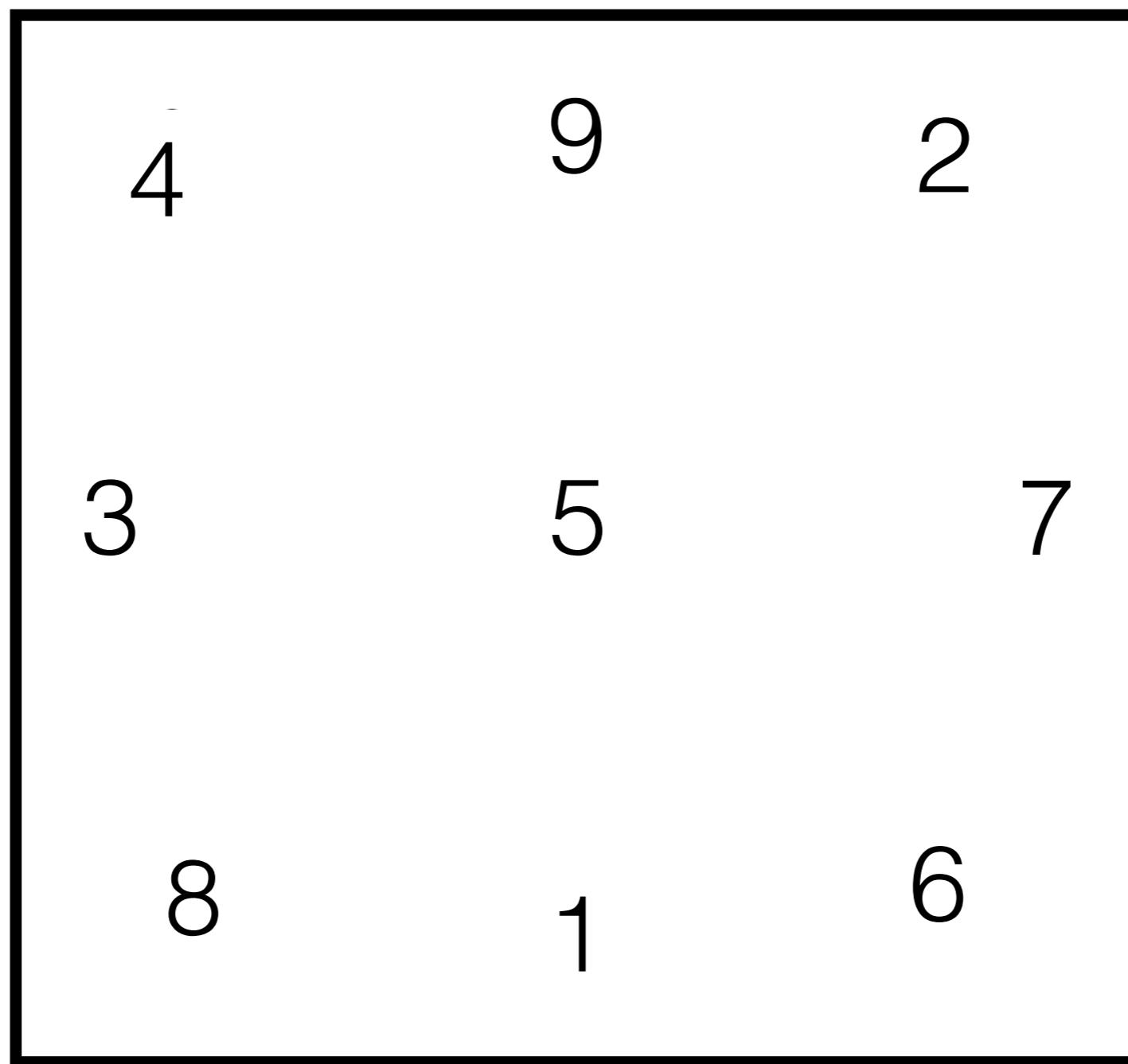
Think Like a Math Historian

- Hint: Focus on how many dots are in each group.



Think Like a Math Historian

- Hint: Focus on how many dots are in each group.



Think Like a Math Historian

- Magic Square

4	9	2
3	5	7
8	1	6

Think Like a Math Historian

- Magic Square

4	9	2
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Think Like a Math Historian

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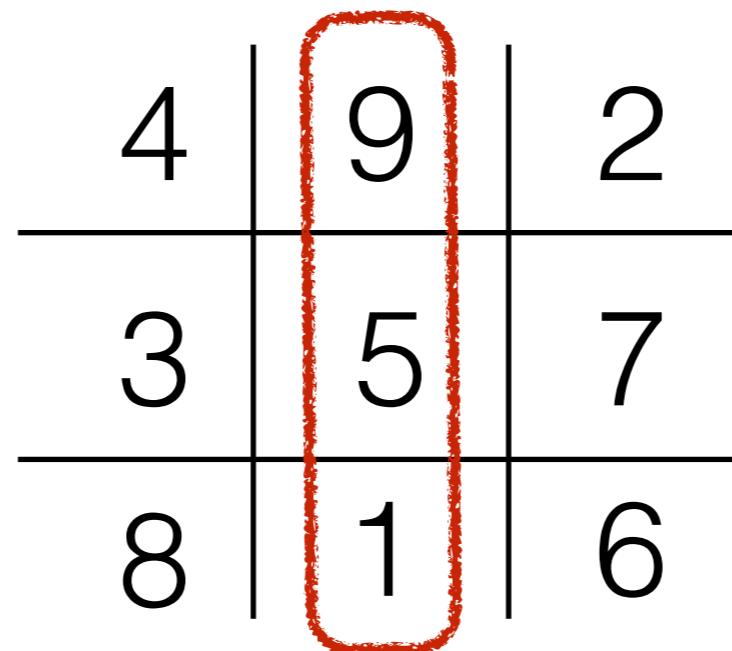
Think Like a Math Historian

- Magic Square

4	9	2
3	5	7
8	1	6

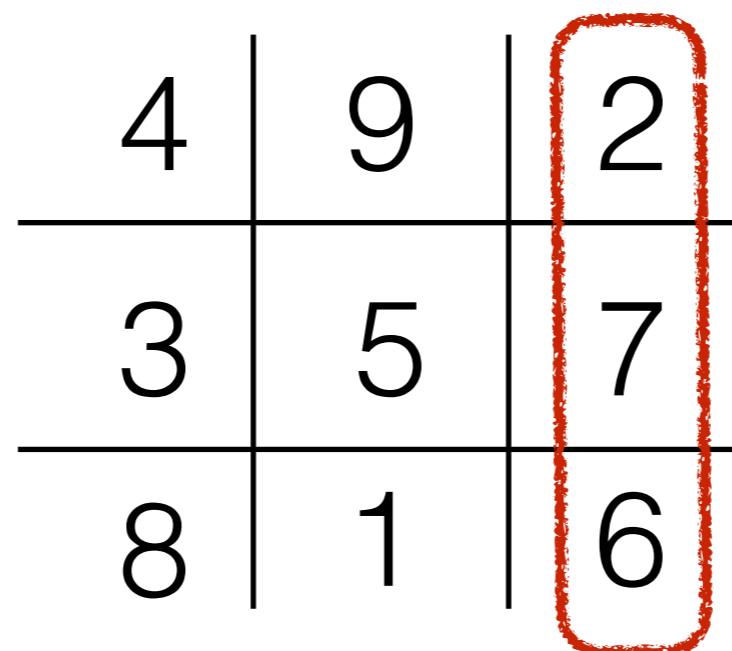
Think Like a Math Historian

- Magic Square



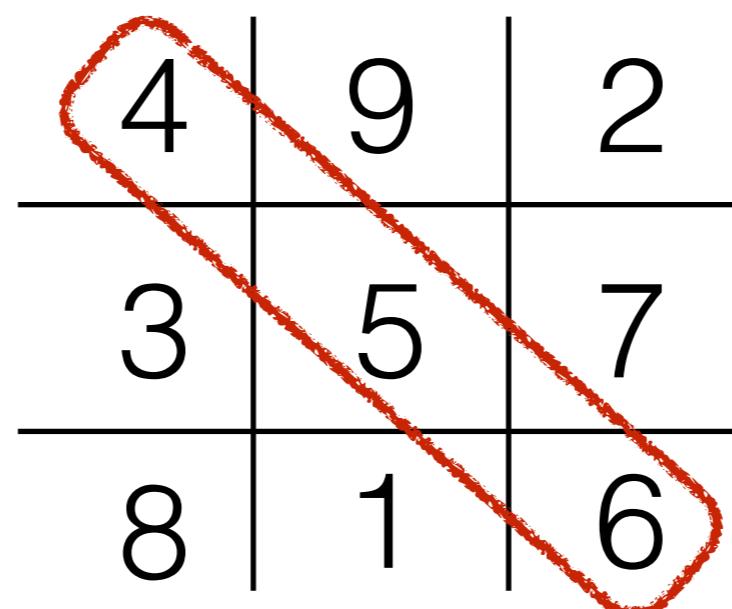
Think Like a Math Historian

- Magic Square



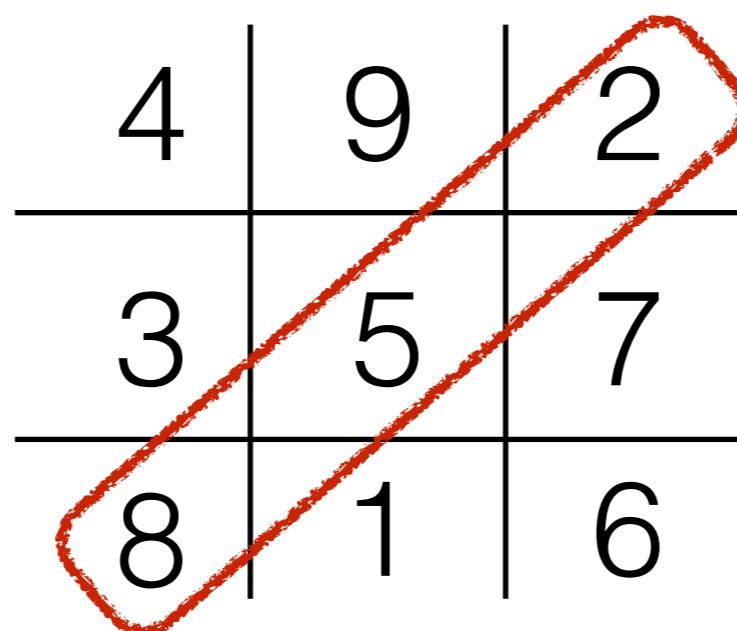
Think Like a Math Historian

- Magic Square



Think Like a Math Historian

- Magic Square



Think Like a Math Historian

- Magic Square



The Aftermath

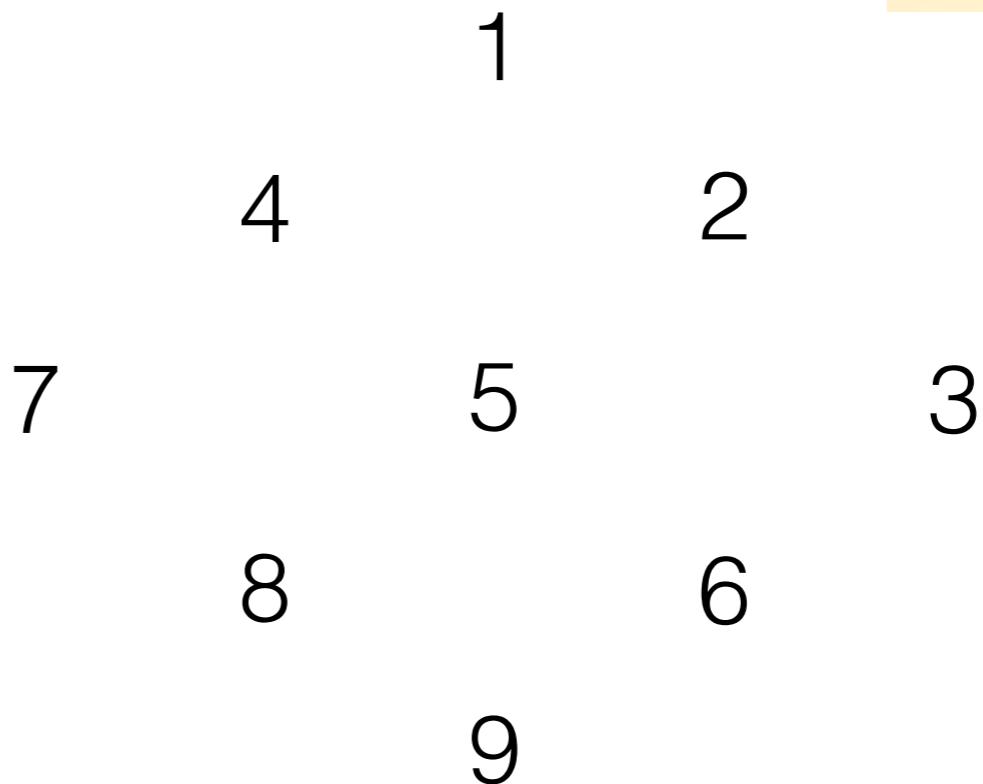
- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.



The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

Step 1: Write numbers
in a diamond.



The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

Step 2: Swap corners.

		1
	4	2
7		5
	8	6
		9

The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

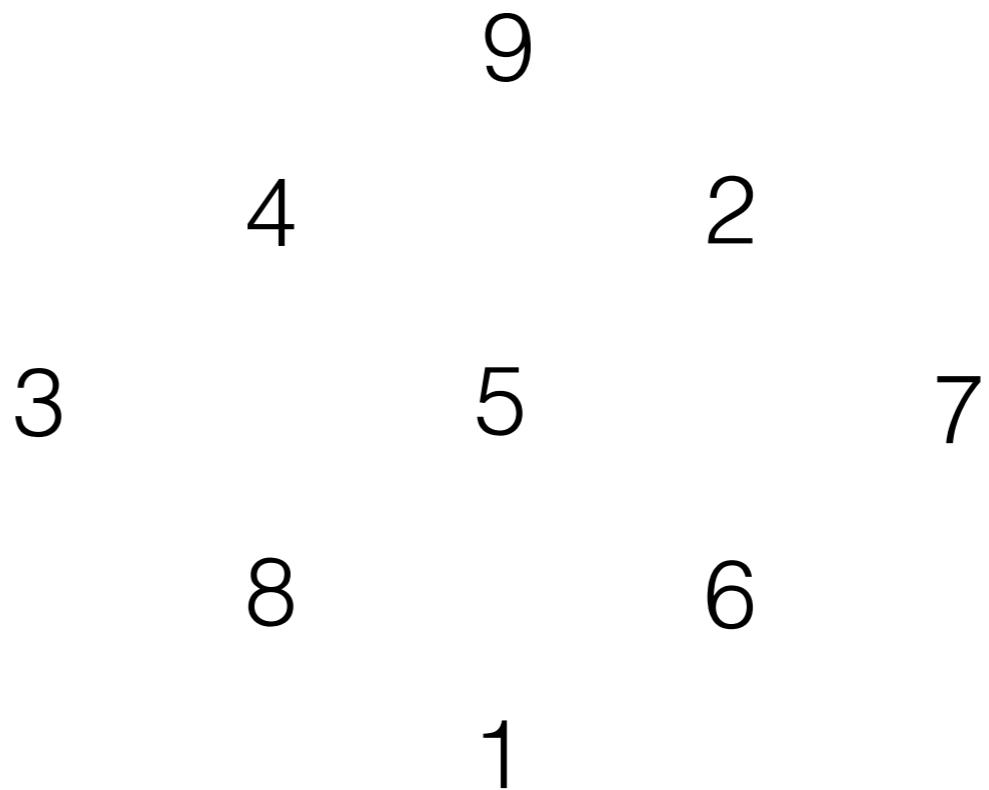
Step 2: Swap corners.

		9	
	4		2
7		5	3
	8		6
		1	

The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

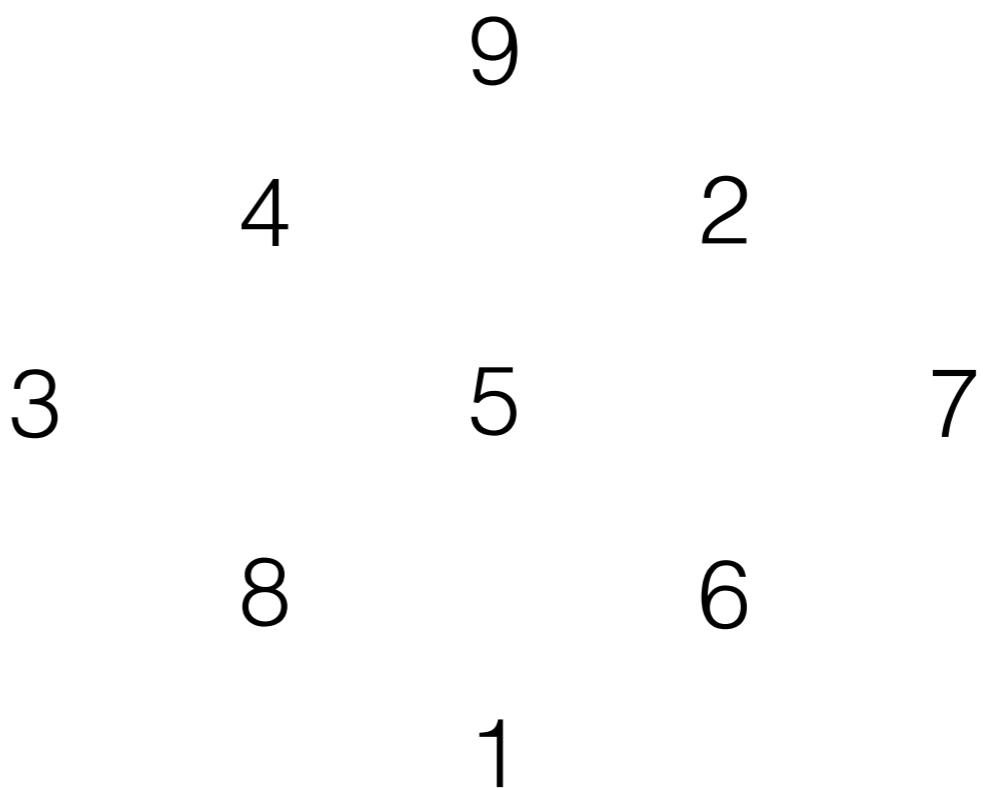
Step 2: Swap corners.



The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

Step 3: Scrunch together.



The Aftermath

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3	5	7
8	1	6

The Aftermath

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Step 3: Scrunch together.

4	9	2
3	5	7
8	1	6

The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

Step 3: Scrunch together.

4	9	2
3	5	7
8	1	6

Hui also came up with a similar algorithm to construct a 4x4 magic square.

The Aftermath

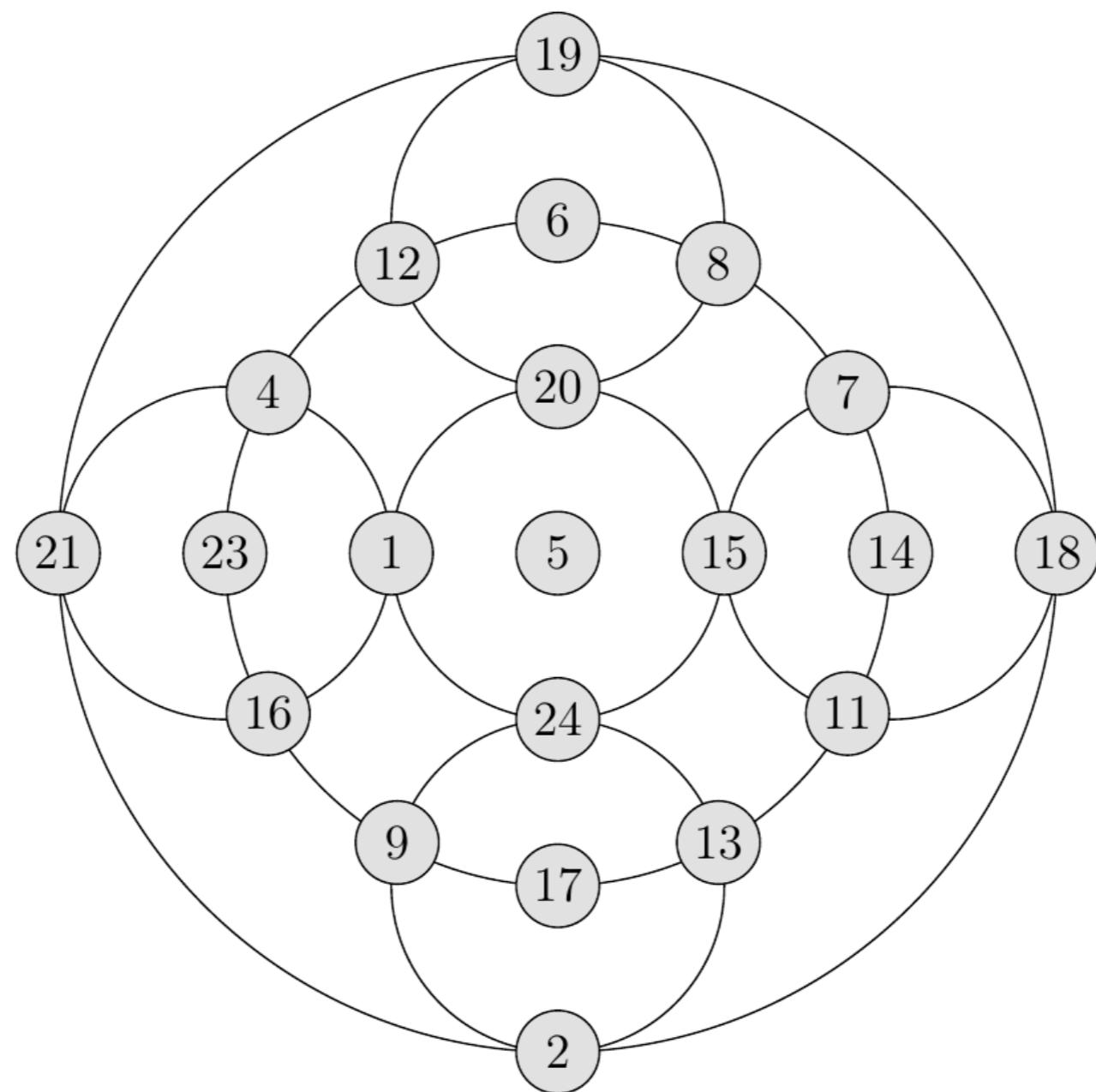
- Yang Hui constructed a magic square of every size up to 10x10.

一	二	二	四	四	六	六	八	八	一百
九	八	七	六	五	四	三	二	一	二
九	九	九	二	九	二	九	二	九	二
三	十	二	三	八	四	五	六	七	八
九	八	七	六	五	四	三	七	二	四
七	四	七	四	七	四	七	二	四	七
五	十	二	三	四	五	六	七	八	九
六	六	五	六	五	六	五	六	五	六
九	八	七	六	五	四	三	五	二	十
五	六	五	六	五	六	五	六	五	六
十四	七	三	二	五	四	七	四	六	九
八	九	六	七	四	五	三	二	三	八
八	三	八	三	八	三	八	二	三	十三
十二	九	三	二	五	二	九	二	六	九
九	九	七	七	五	五	三	三	三	八
一	十	一	七十	一	五十	三	三十	十一	十

1	20	21	40	41	60	61	80	81	100
99	82	79	62	59	42	39	22	19	2
3	18	23	38	43	58	63	78	83	98
97	84	77	64	57	44	37	24	17	4
5	16	25	36	45	56	65	76	85	96
95	86	75	66	55	46	35	26	15	6
14	7	34	27	54	47	74	67	94	87
88	93	68	73	48	53	28	33	8	13
12	9	32	29	52	49	72	69	92	89
91	90	71	70	51	50	31	30	11	10

The Aftermath

- Yang Hui also constructed six magic circles.



Algorithm to Construct Magic Squares

Shout-Outs!

- Crows

Counting Crows

- Other animals may count too, to some extent. For example, crows.

Counting Crows

- Other animals may count too, to some extent. For example, crows.
- Old story of a hunter hiding in a shelter.



Counting Crows

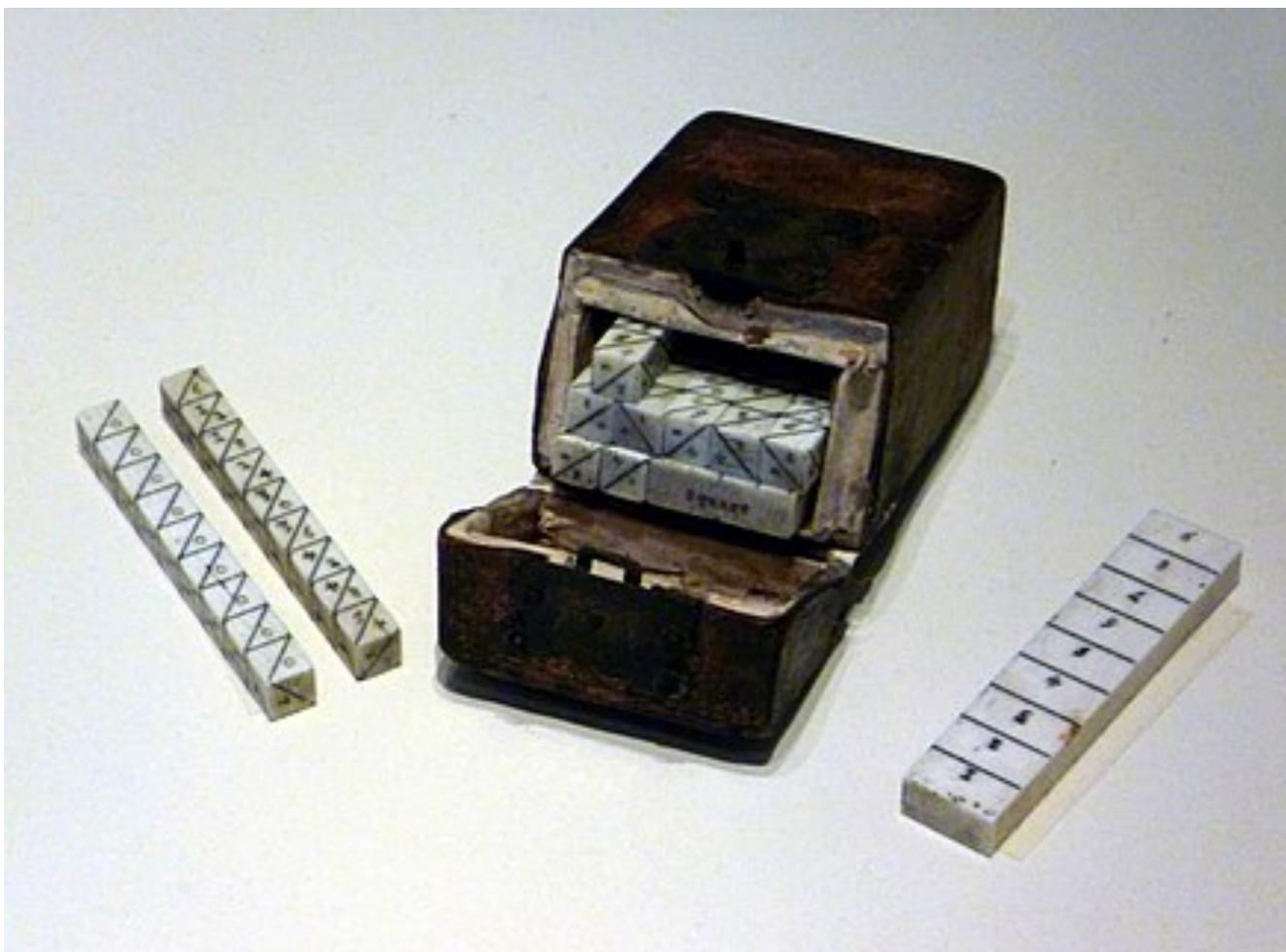


Shout-Outs!

- Napier's Bones

Shout-Outs!

- Napier's Bones



Napier's Bones										
	0	1	2	3	4	5	6	7	8	9
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	0	1	2
3	0	0	0	0	0	1	2	1	3	4
4	0	0	0	0	1	2	1	2	3	4
5	0	0	0	1	2	3	2	3	4	5
6	0	0	1	2	3	4	3	4	5	6
7	0	0	1	2	3	4	2	3	5	6
8	0	0	1	2	3	4	2	3	6	7
9	0	0	1	2	3	4	5	4	6	8

Invented by John Napier 1550-1617 Made in USA by Creative Crafthouse

People's History

People's History of Numbers

- A “people’s history” studies the history of non-elites.
- Much of the progression to create number systems is a people’s history. Merchants lead the way.
- Hindu-Arabic numerals “democratized” math.
- Some Roman elites fought back in an effort to keep math opaque.