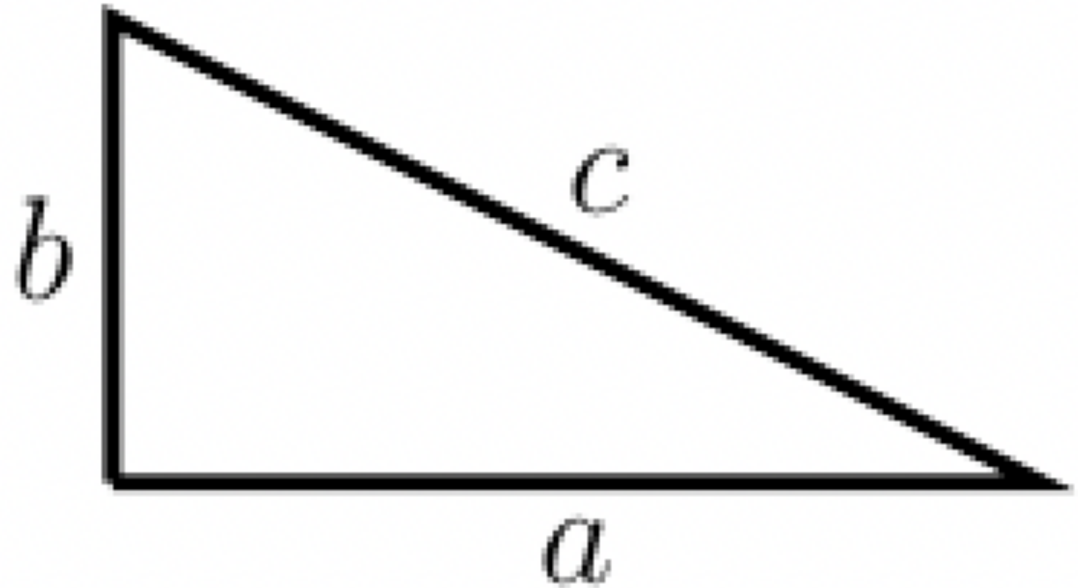
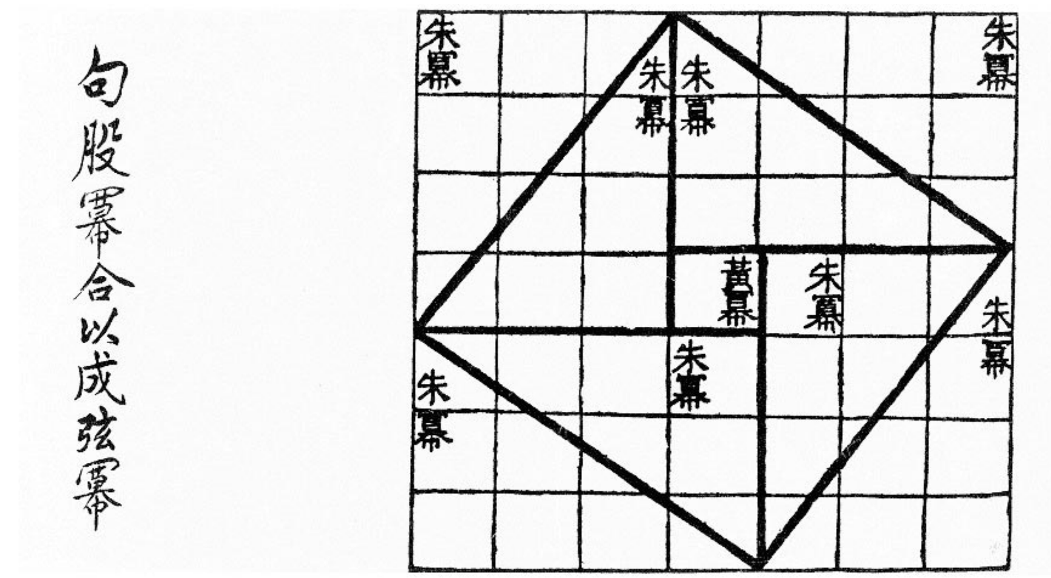


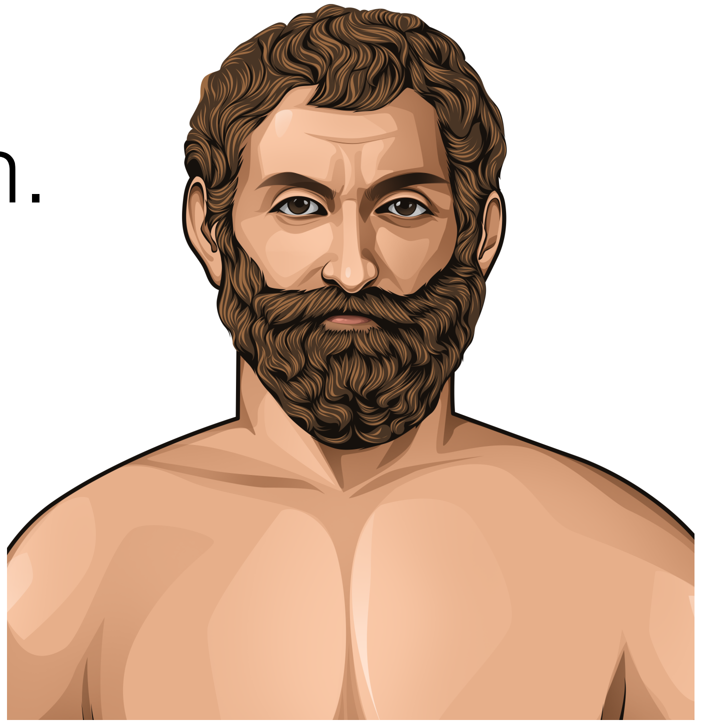


Chapter 3: Demonstrative Mathematics

Demonstrative Math

- Demonstrative math = Proof-based math. Before: *What* is true. After: *Why* is it true.

- Thales of Miletus (~624-545 BC, from Miletus) was a Greek mathematician.



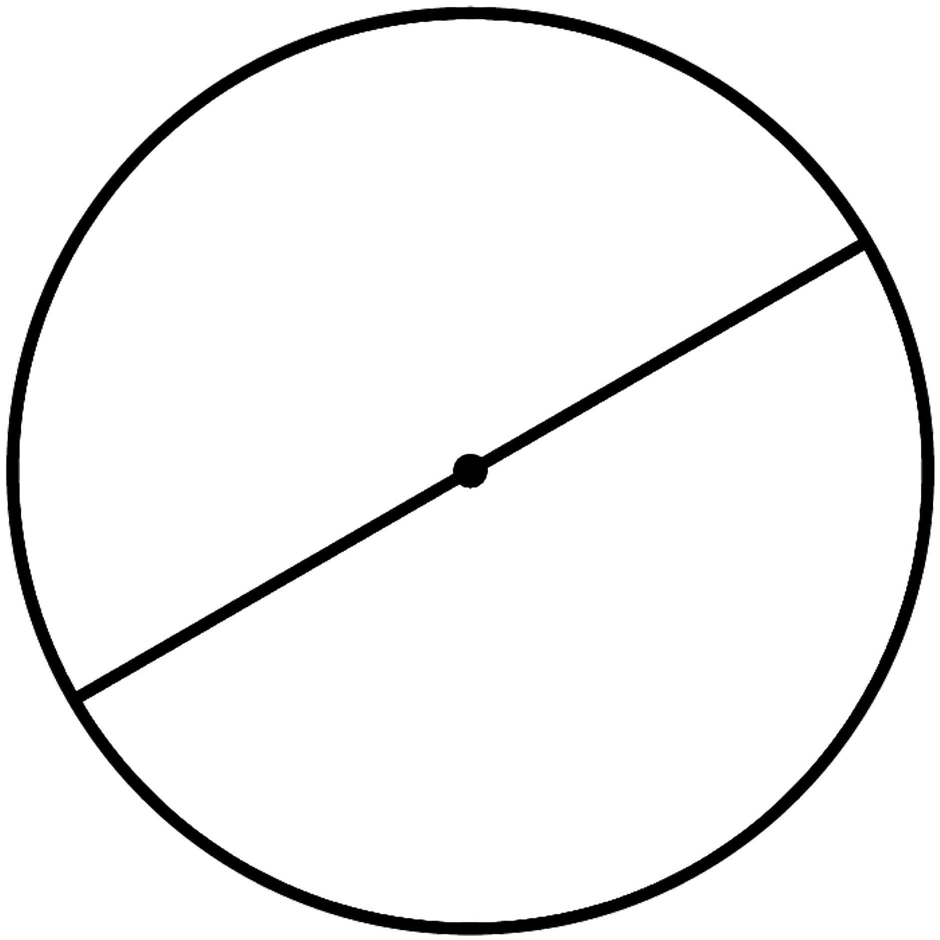
The First Proof

- Thales is the first person to whom a mathematical proof is attributed.
- He’s also called the Father of Science because he used evidence and theories, not mythology, to study and explain the world.
- Aesop had two stories about him.
- First absent-minded prof in history.

The First Proof

Theorem. The diameter of any circle divides that circle into two equal halves.

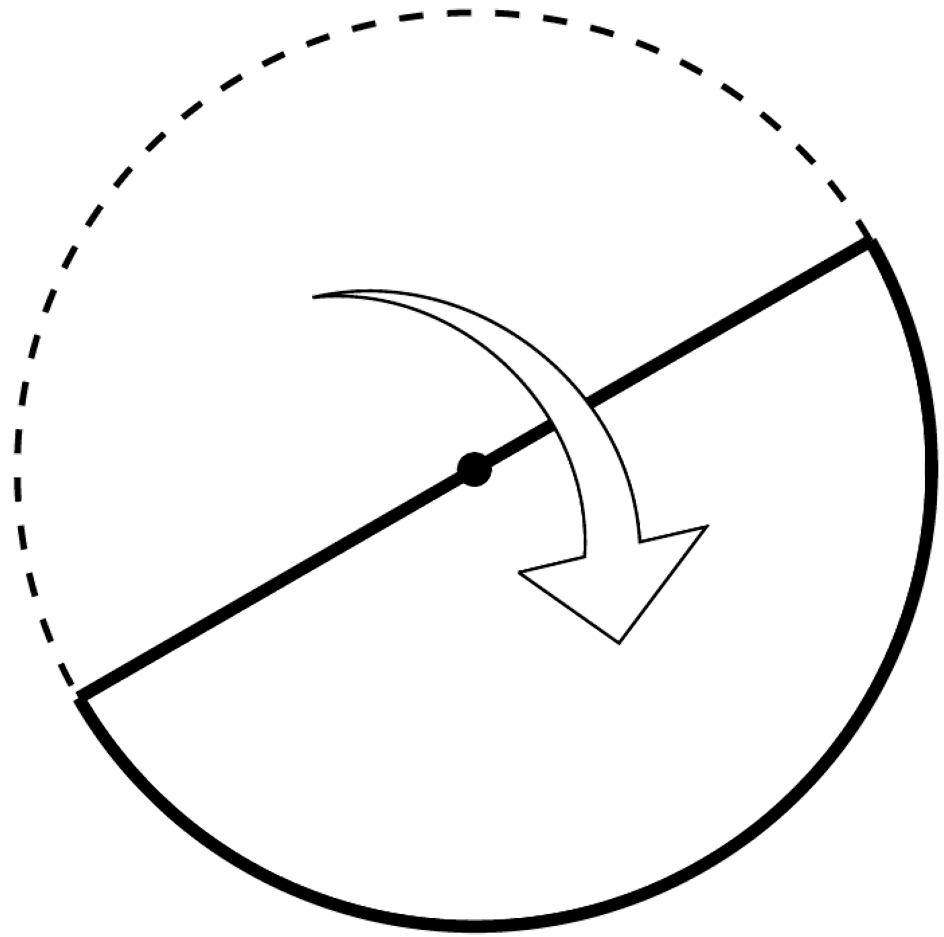
Proof. Suppose you have a circle and diameter



The First Proof

Theorem. The diameter of any circle divides that circle into two equal halves.

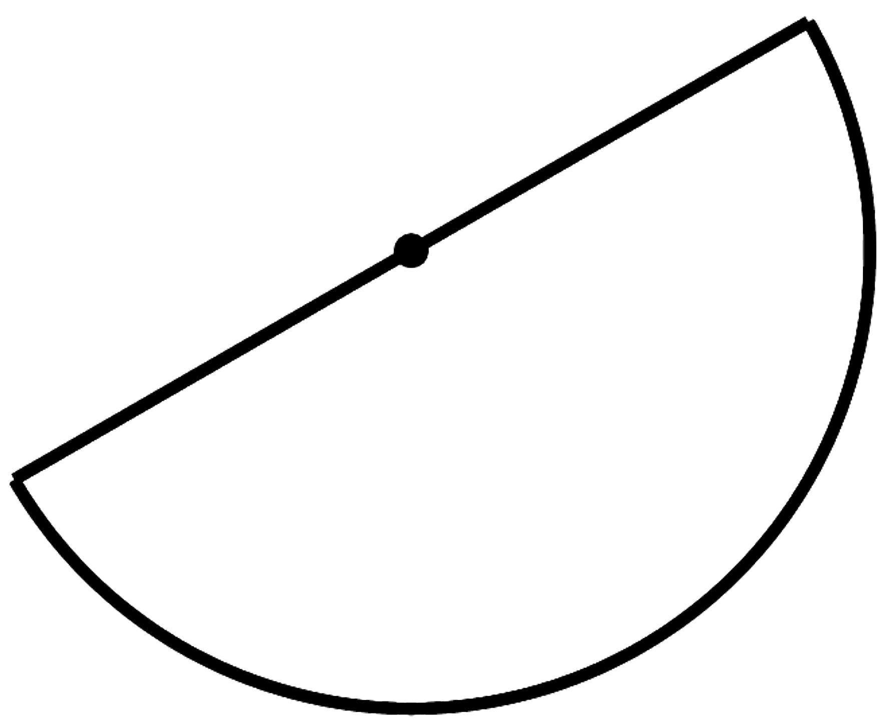
Proof. Take half the circle and flip it over the other half.



The First Proof

Theorem. The diameter of any circle divides that circle into two equal halves.

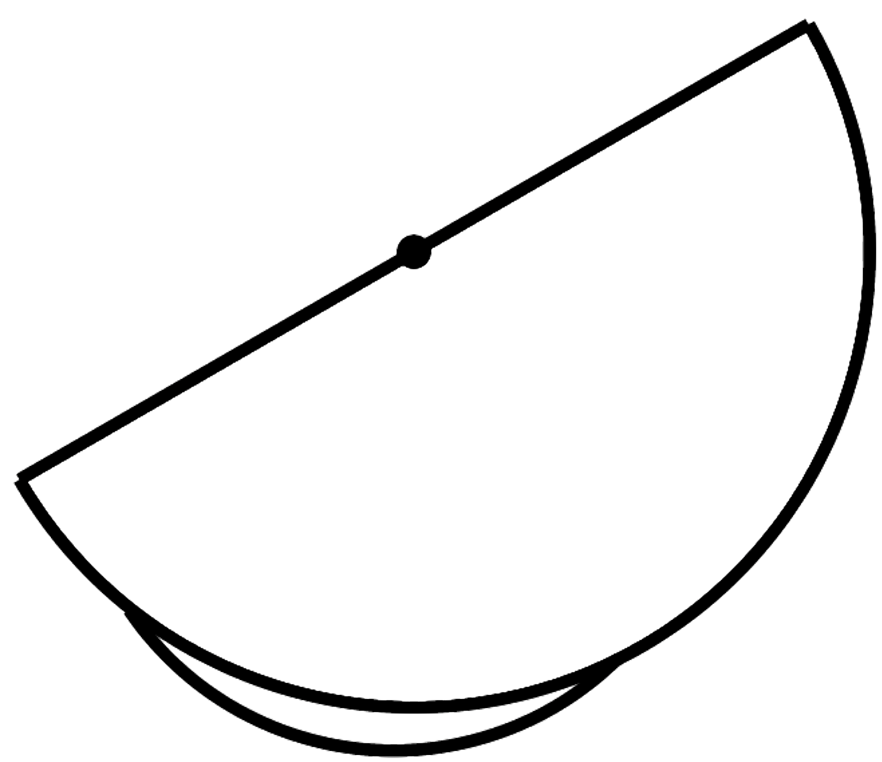
Proof. We want to show that the picture looks like this:



The First Proof

Theorem. The diameter of any circle divides that circle into two equal halves.

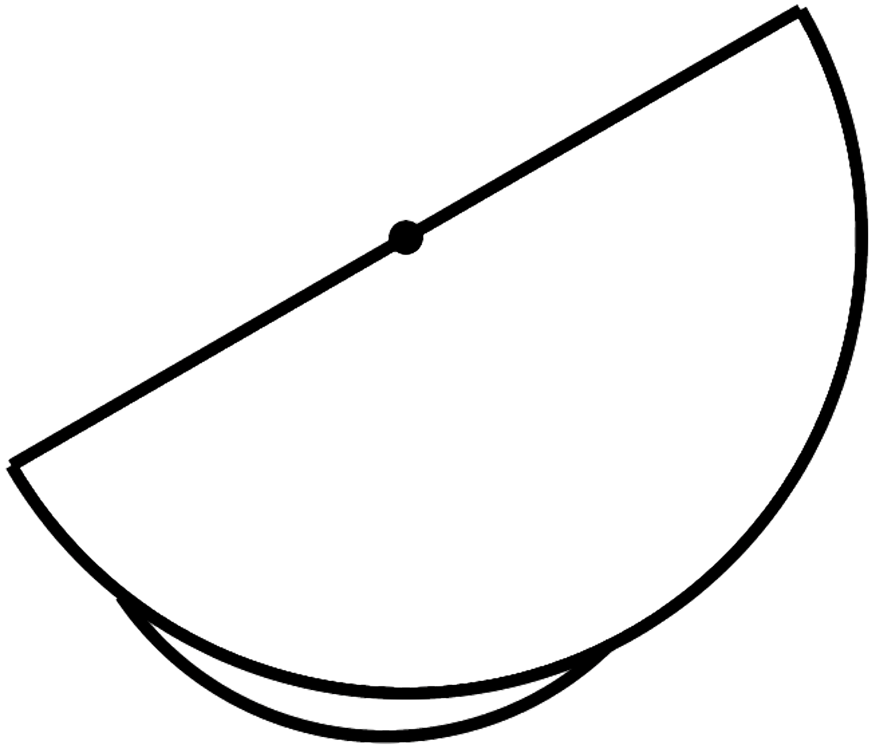
Proof. If it doesn't, then it looks something like this:



The First Proof

Theorem. The diameter of any circle divides that circle into two equal halves.

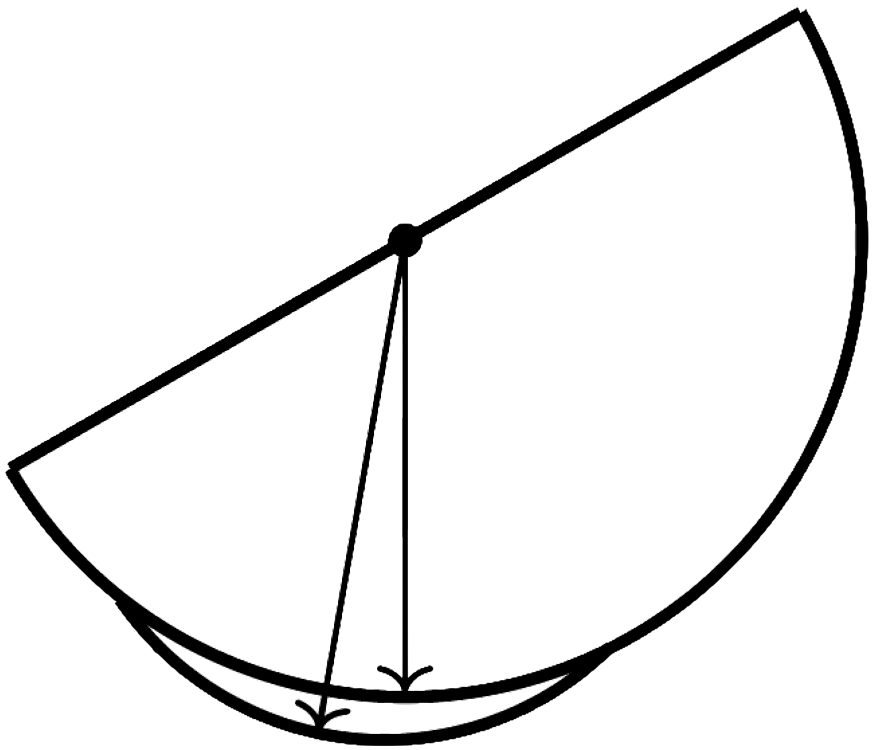
Proof. This gives us a contradiction. Why?



The First Proof

Theorem. The diameter of any circle divides that circle into two equal halves.

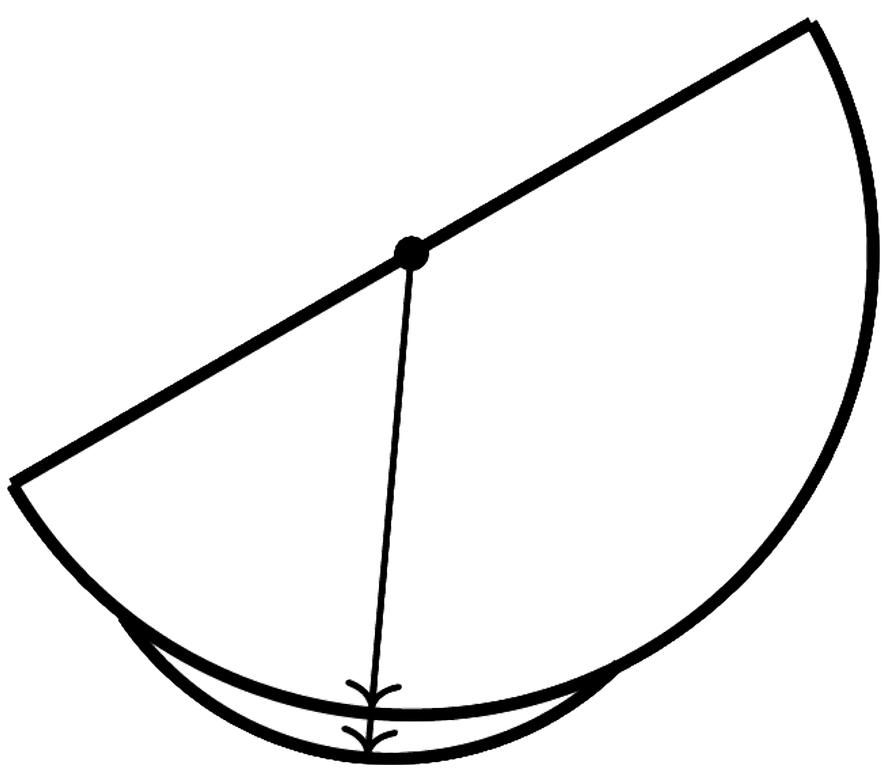
Proof. It's a contradiction because it produced two separate radii! But a circle has just one radius.



The First Proof

Theorem. The diameter of any circle divides that circle into two equal halves.

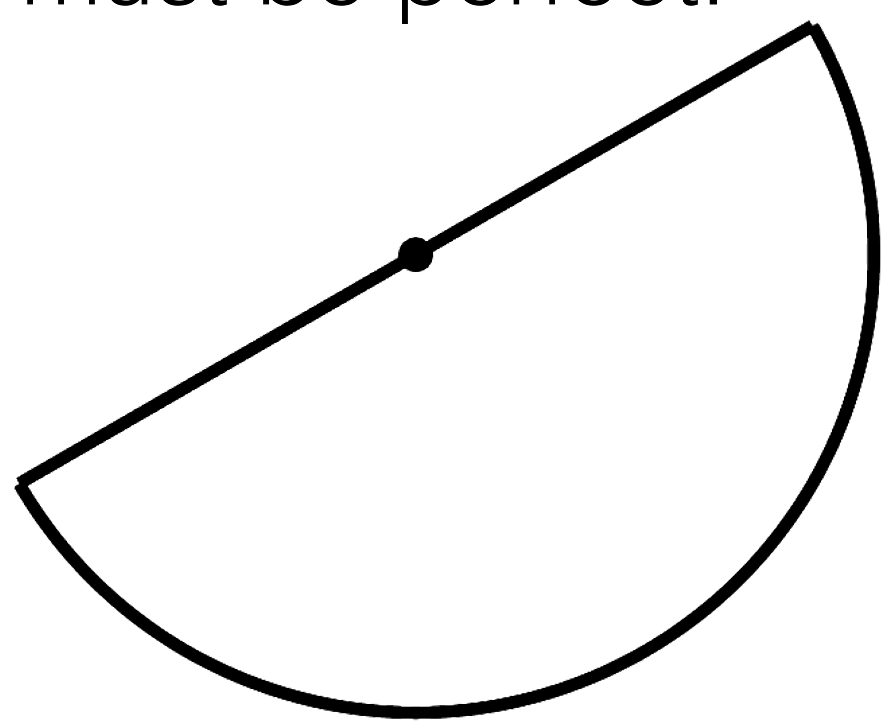
Proof. This fact is even clearer if you line up the two radii that appear.



The First Proof

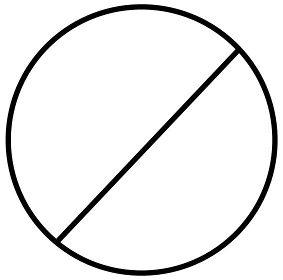
Theorem. The diameter of any circle divides that circle into two equal halves.

Proof. Either way, the conclusion is the same. If the two parts did not perfectly overlap then we would get two radii, which is impossible. So the overlap must be perfect.



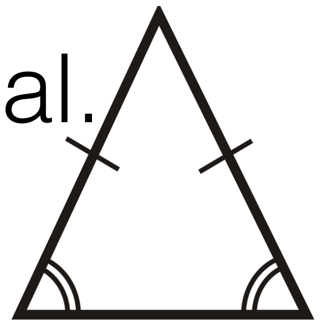
The First Proof

Other things Thales proved:

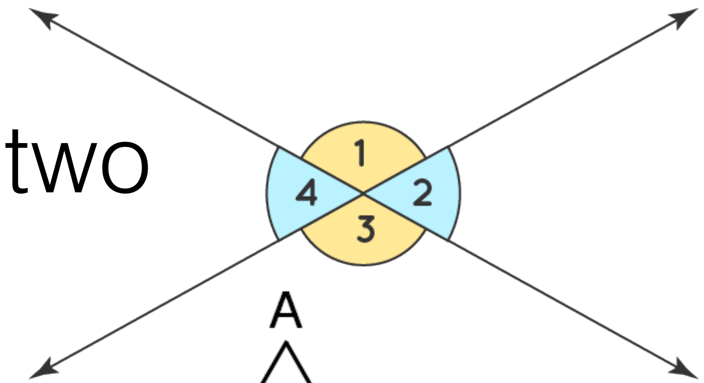


1. A circle is bisected by any diameter.

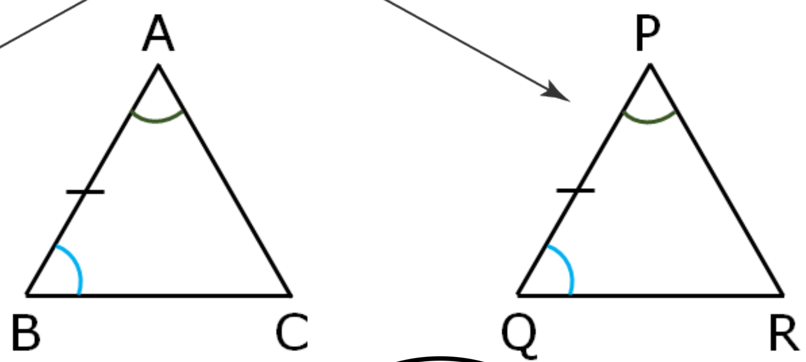
2. The base angles of an isosceles triangle are equal.



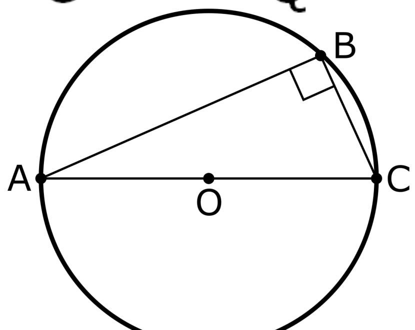
3. The vertical angles formed by two intersecting lines are equal.



4. The ASA triangle congruence.

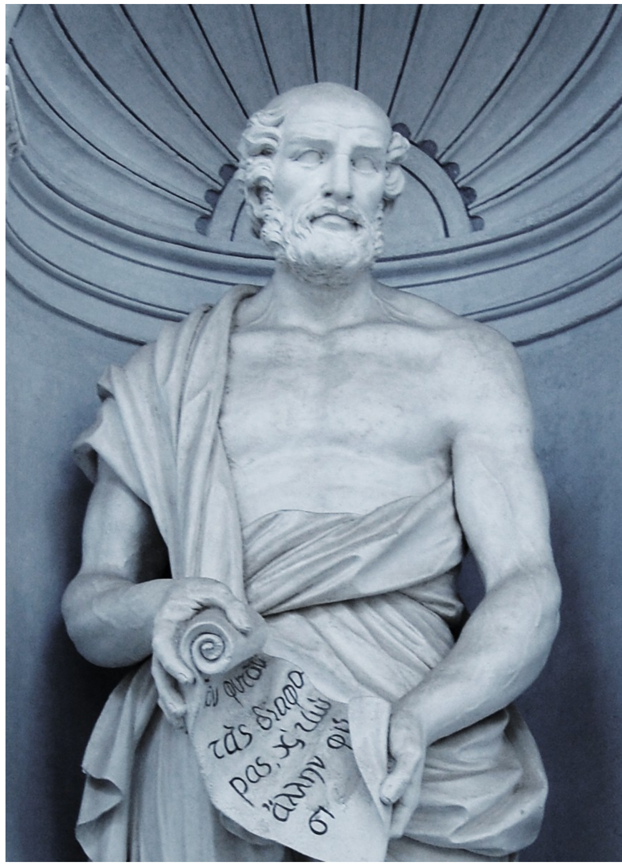


5. The angle inscribed in a semicircle is a right angle.



Historial Uncertainty

- Eudemus of Rhodes (~370 BC — ~300 BC) is considered the first historian of science. He also worked closely with Aristotle.
- He wrote *History of Geometry*, a book about all the geometry known to the Greeks. Sadly, no copy of this book exists today.



Historial Uncertainty

- Proclus (412 AD — 485 AD) gave a short sketch of the history of geometry which seems to be based on Eudemus' book *History of Geometry*. He discusses Thales, Pythagoras and others.

- Stories about Pythagoras are often contradictory or clearly false. This casts doubt on the rest.

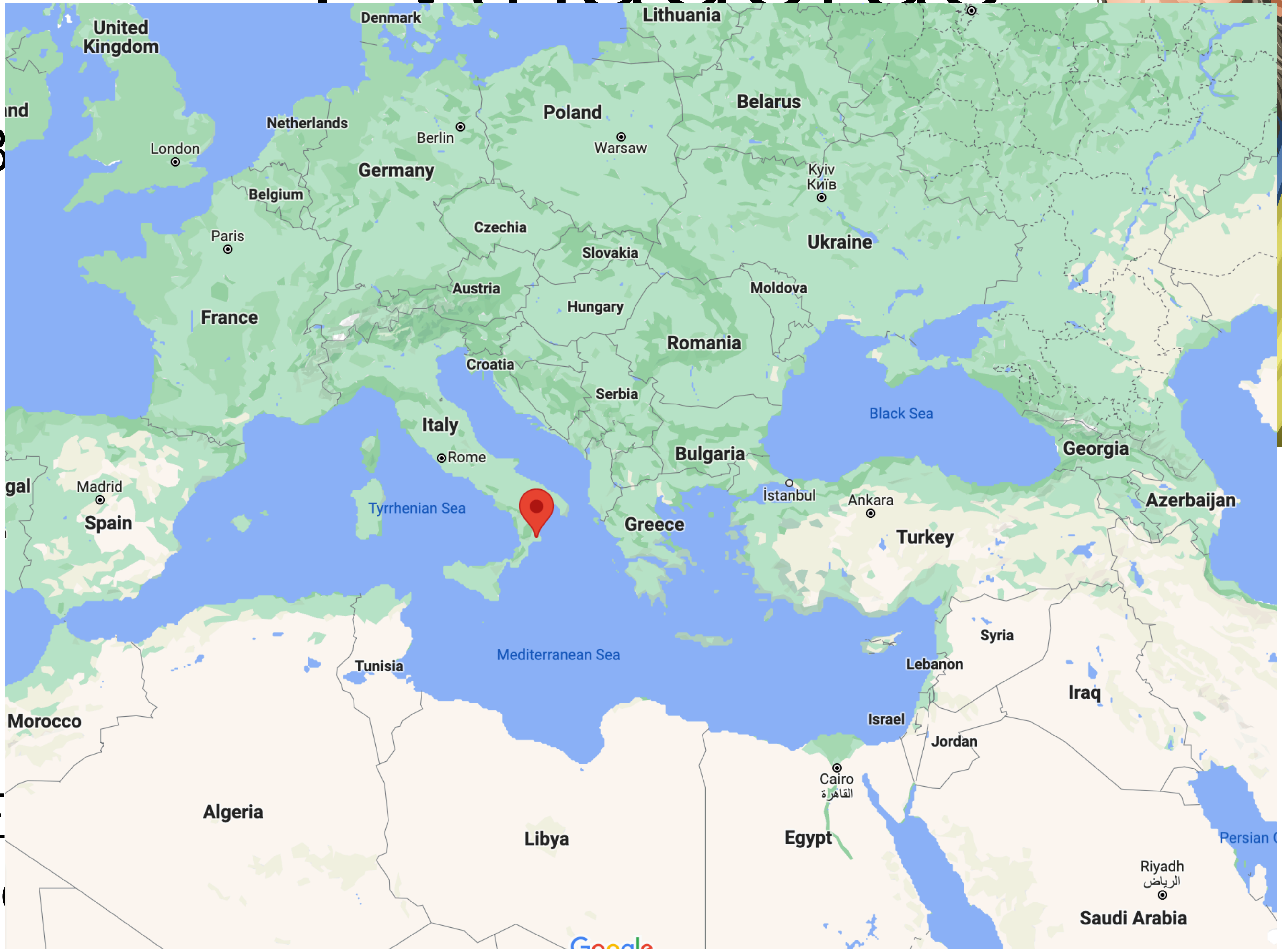
- Yet, from Proclus and a few others, there is (non-conclusive) evidence that Pythagoras made important contributions to math.

Pythagoras

- Born in Samos around 572 BC

- Established his “Brotherhood” in Crotona, in southern Italy.

Pythagoras



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Pythagorean Brotherhood

- It was basically a cult, with Pythagoras at the helm. Special diet, exercise, activities, rituals. May have studied numerology and mathematics.

Pythagorean Brotherhood

- Pythagoras (or one of his followers) *may* have proved the Pythagorean theorem.
- If they did, then it was probably a "dissection proof."
- If it was, then it may have been the proof that I will show you in a moment.

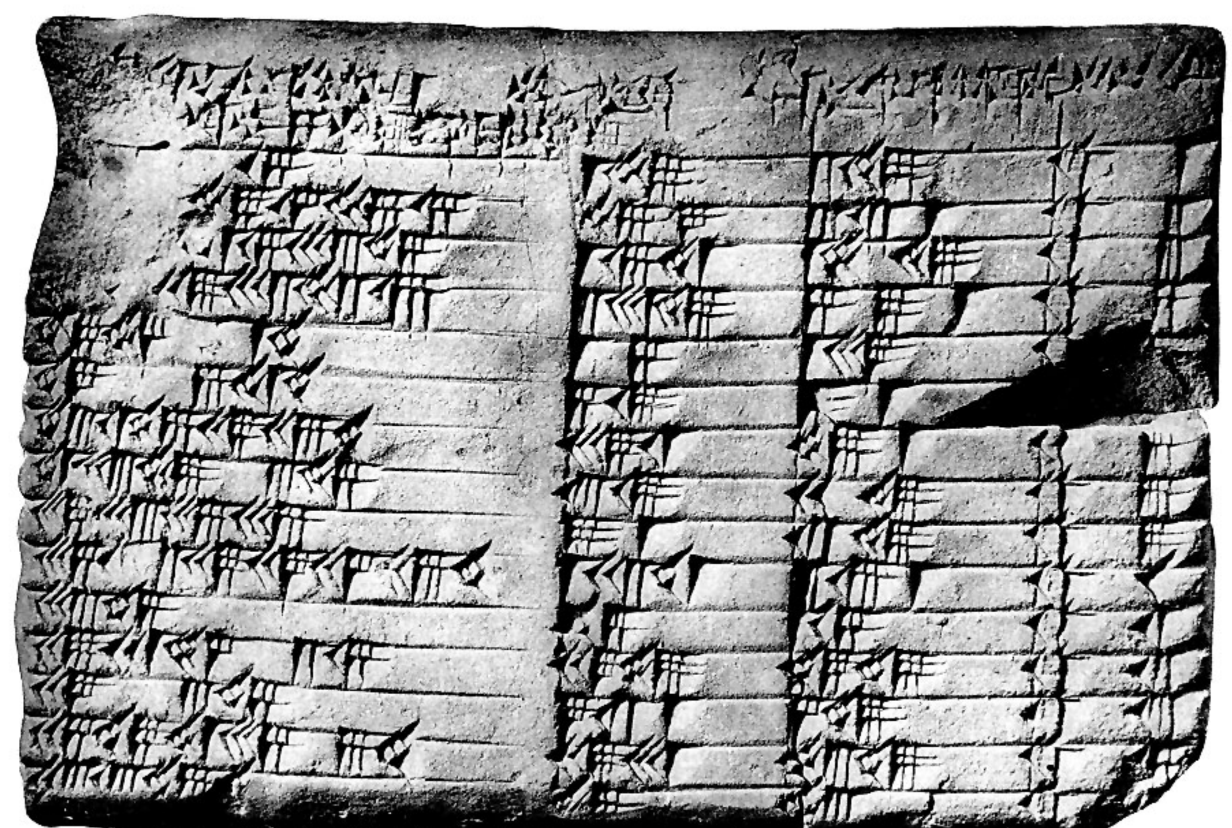
Pythagorean Triples

- Definition: A *Pythagorean triple* is a triple of positive integers (a, b, c) satisfying $a^2 + b^2 = c^2$.

- Example: $3^2 + 4^2 = 5^2$, so $(3,4,5)$ is a Pythagorean triple.

- Recall: 15 PTs appeared in Plimpton 322 (~1800 BC).

- Nearly all ancient cultures found some Pythagorean triples.

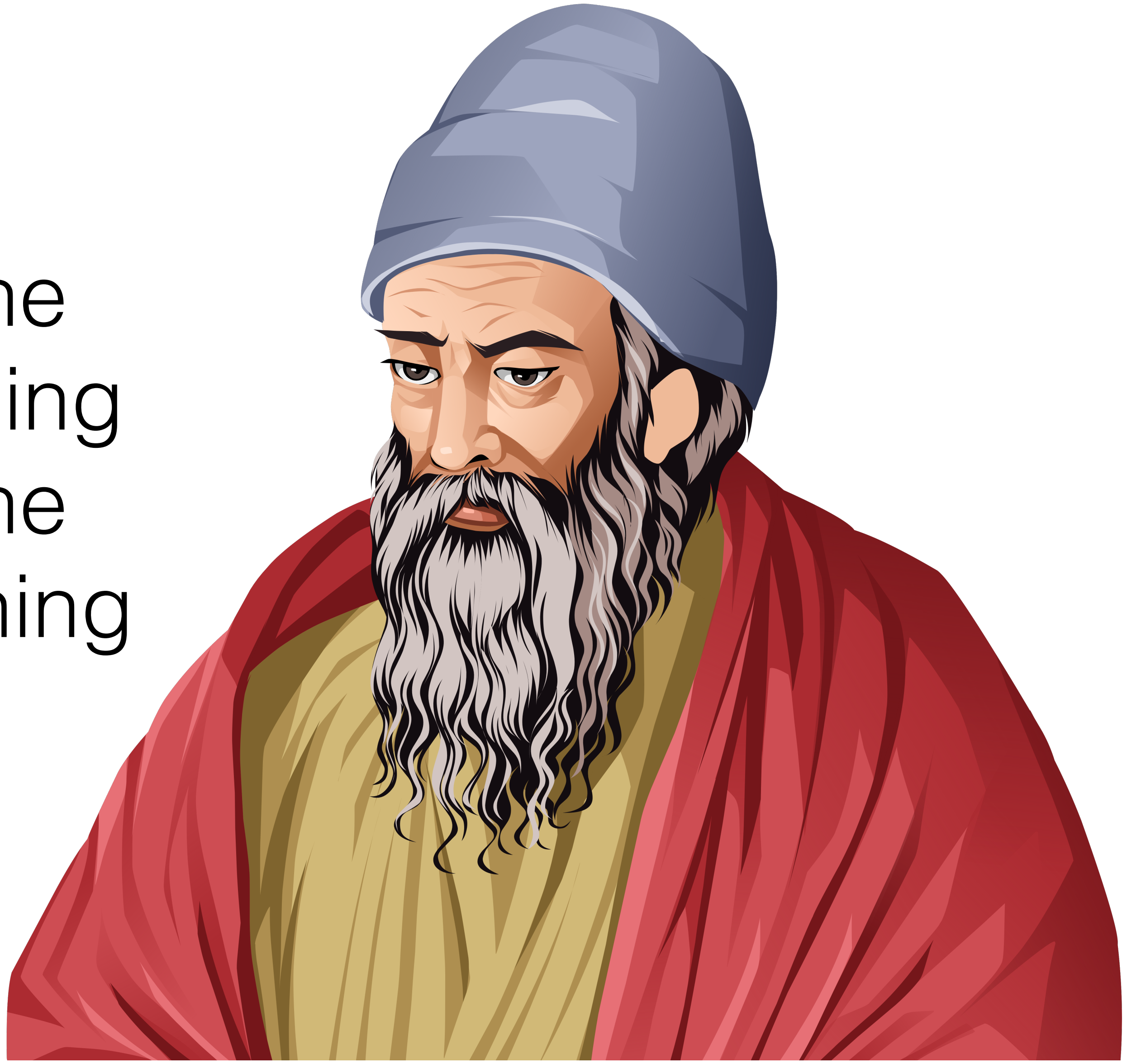


Pythagorean Theorem

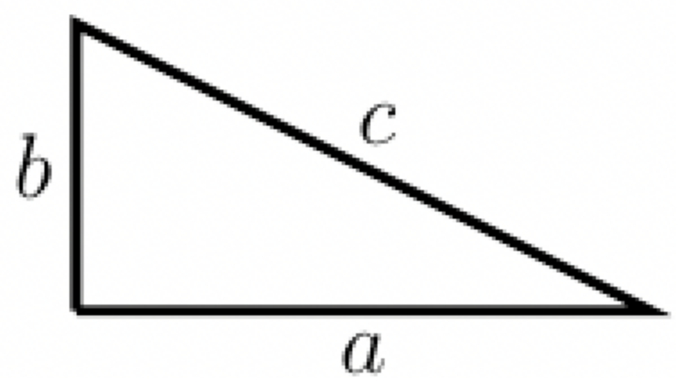
- Warm-up: State the Pythagorean theorem.

- Euclid:

  "In right-angled triangles, the square on the side subtending the right angle is equal to the squares on the side containing the right angle."

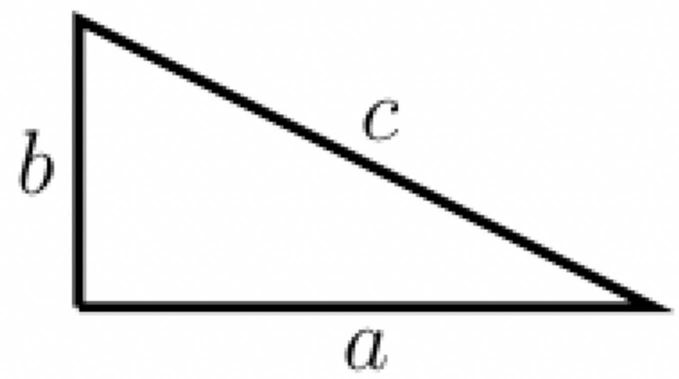


Pythagorean Theorem

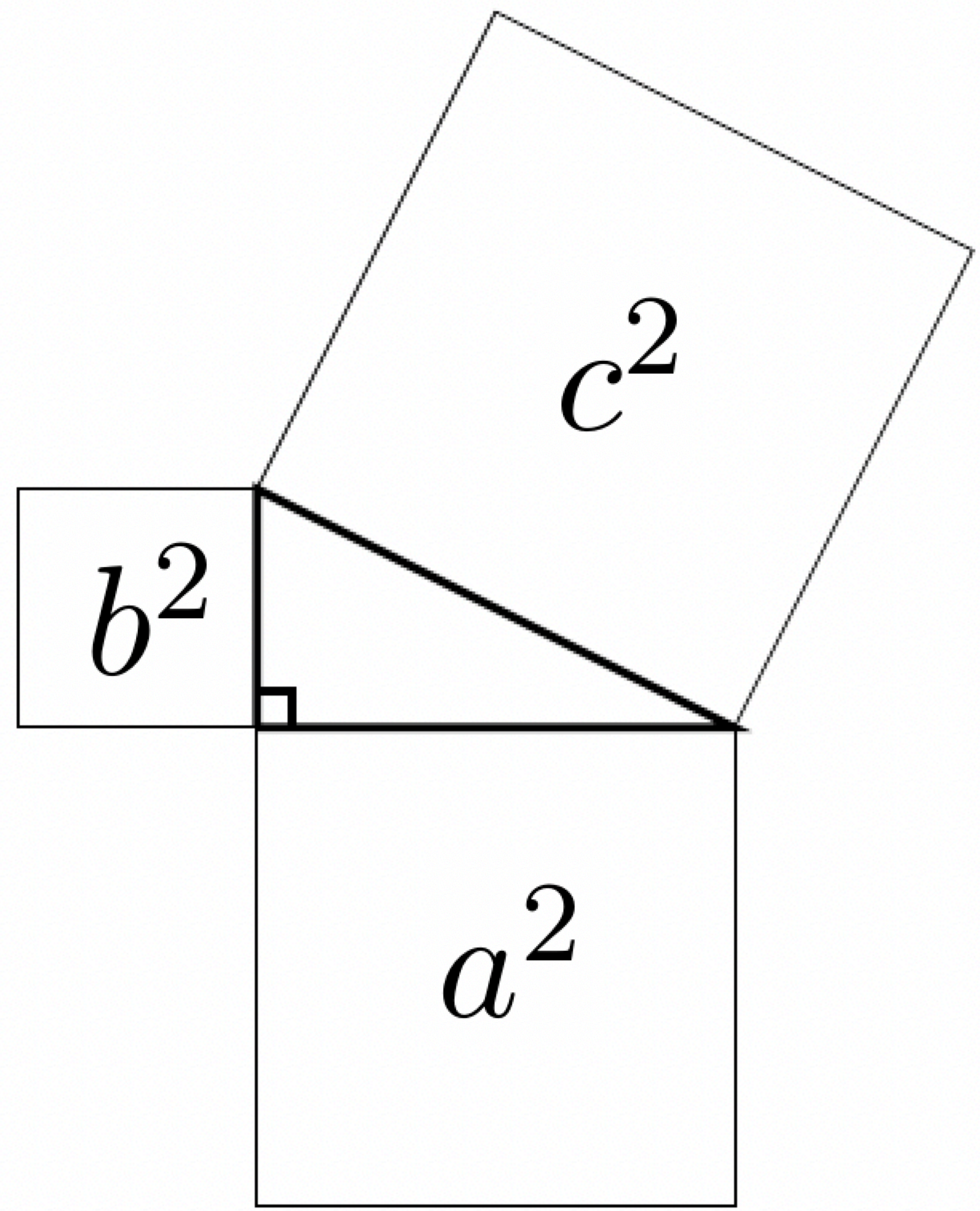


$$a^2 + b^2 = c^2$$

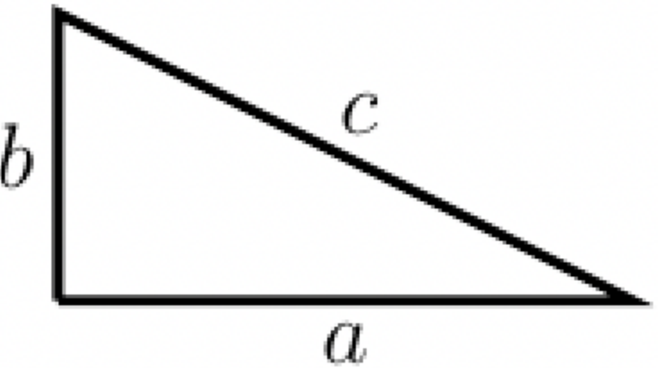
Pythagorean Theorem

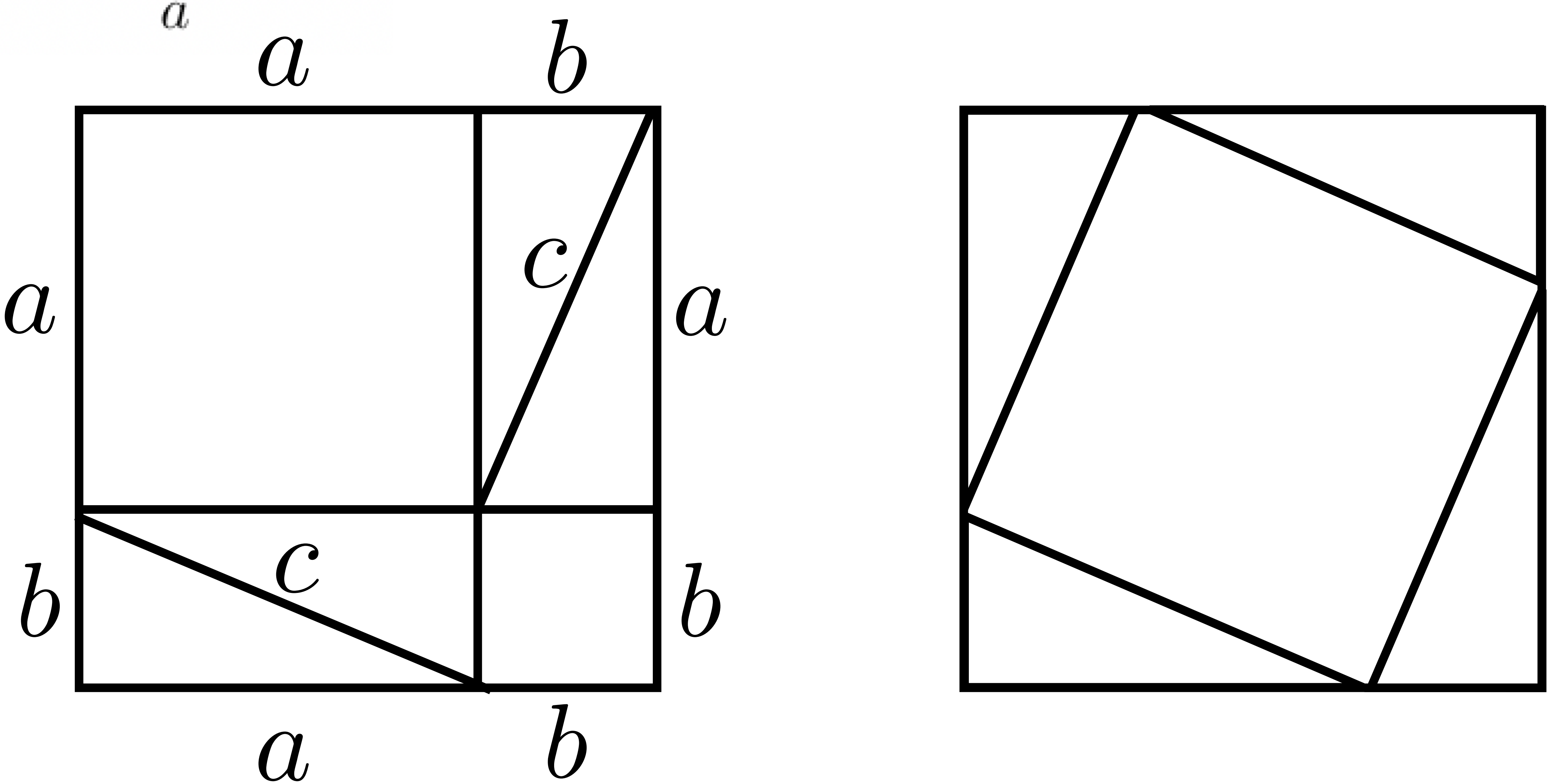


$$a^2 + b^2 = c^2$$

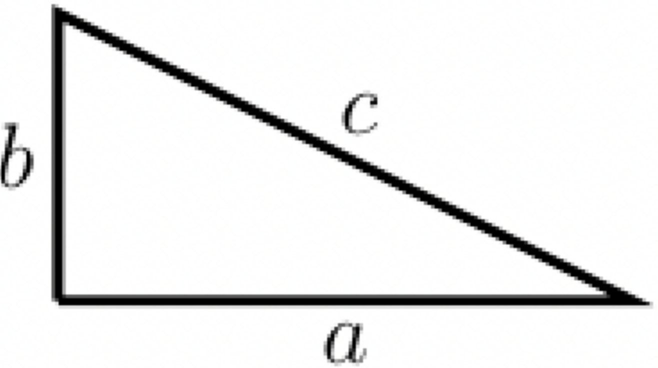


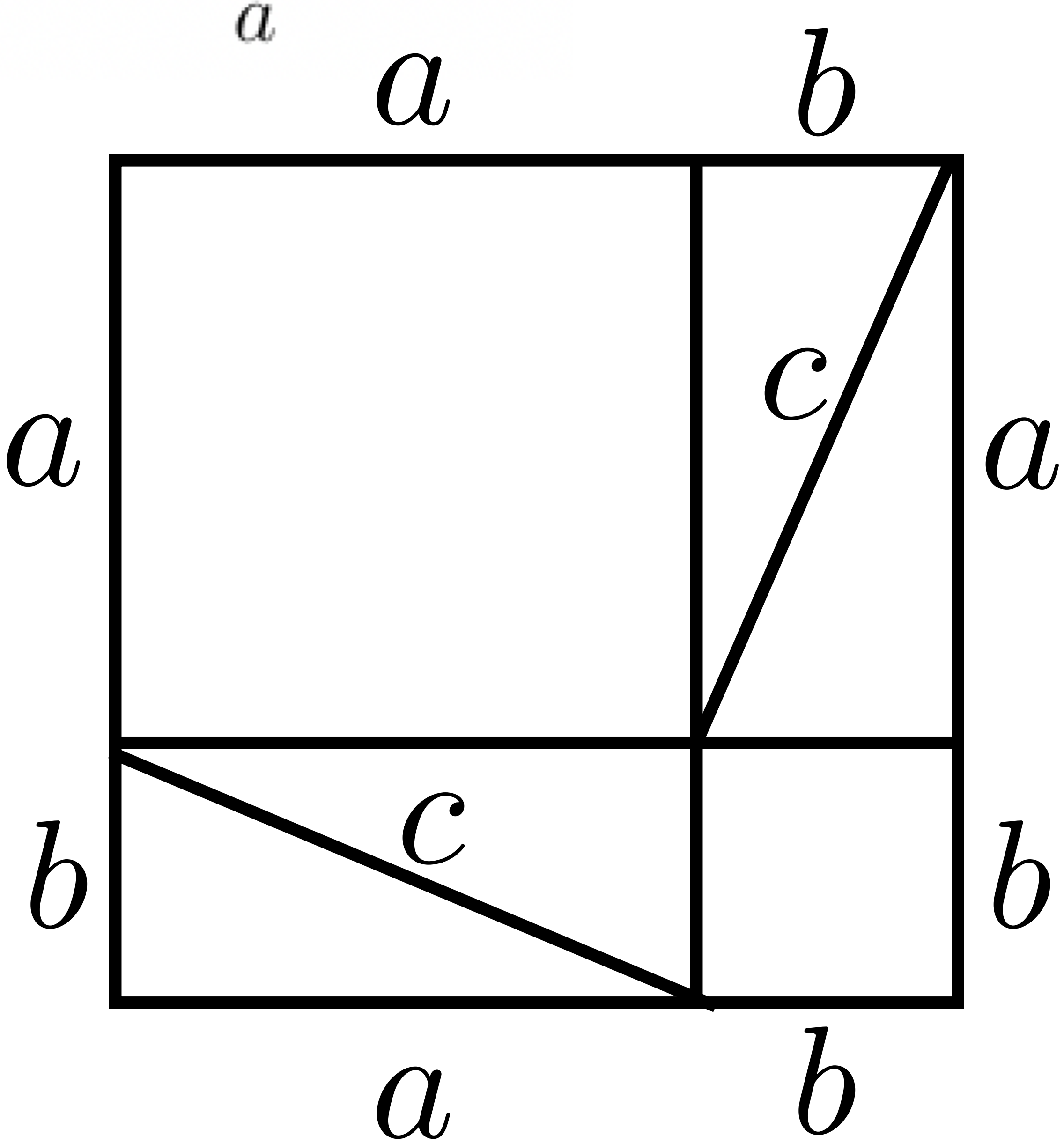
How They *Might* Have Proved It

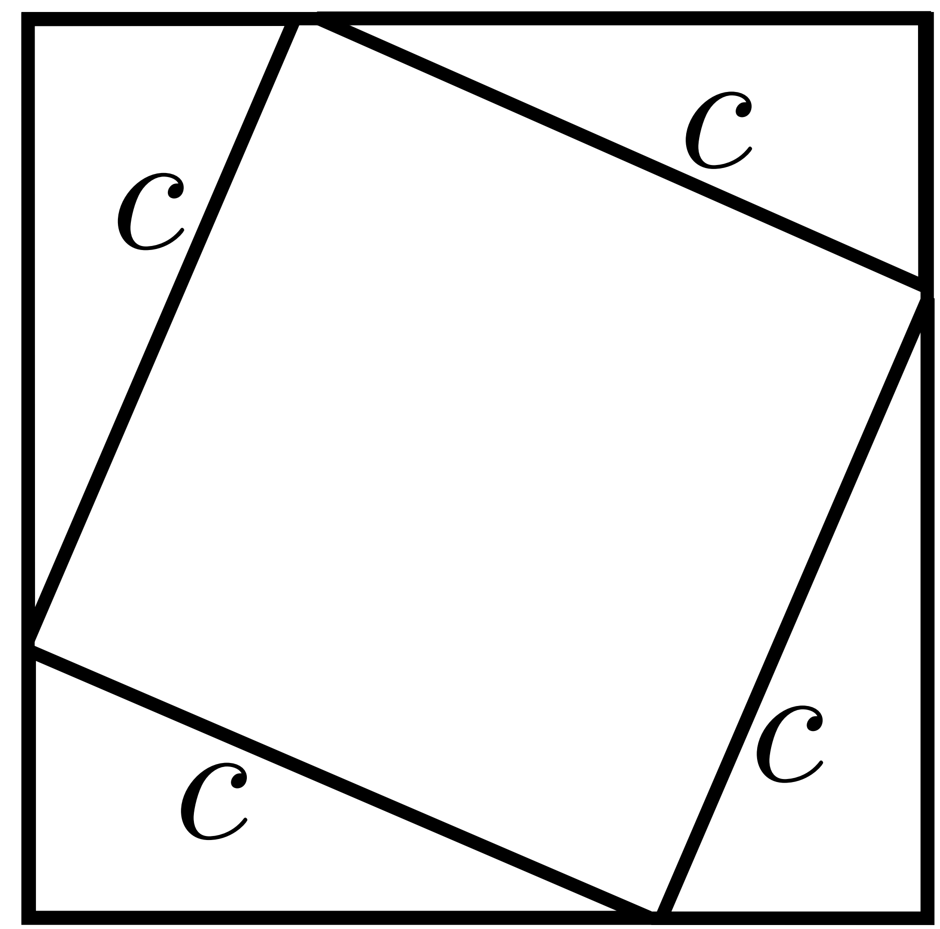




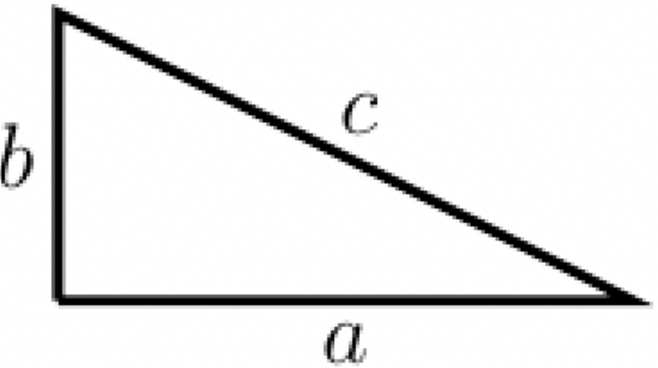
How They *Might* Have Proved It

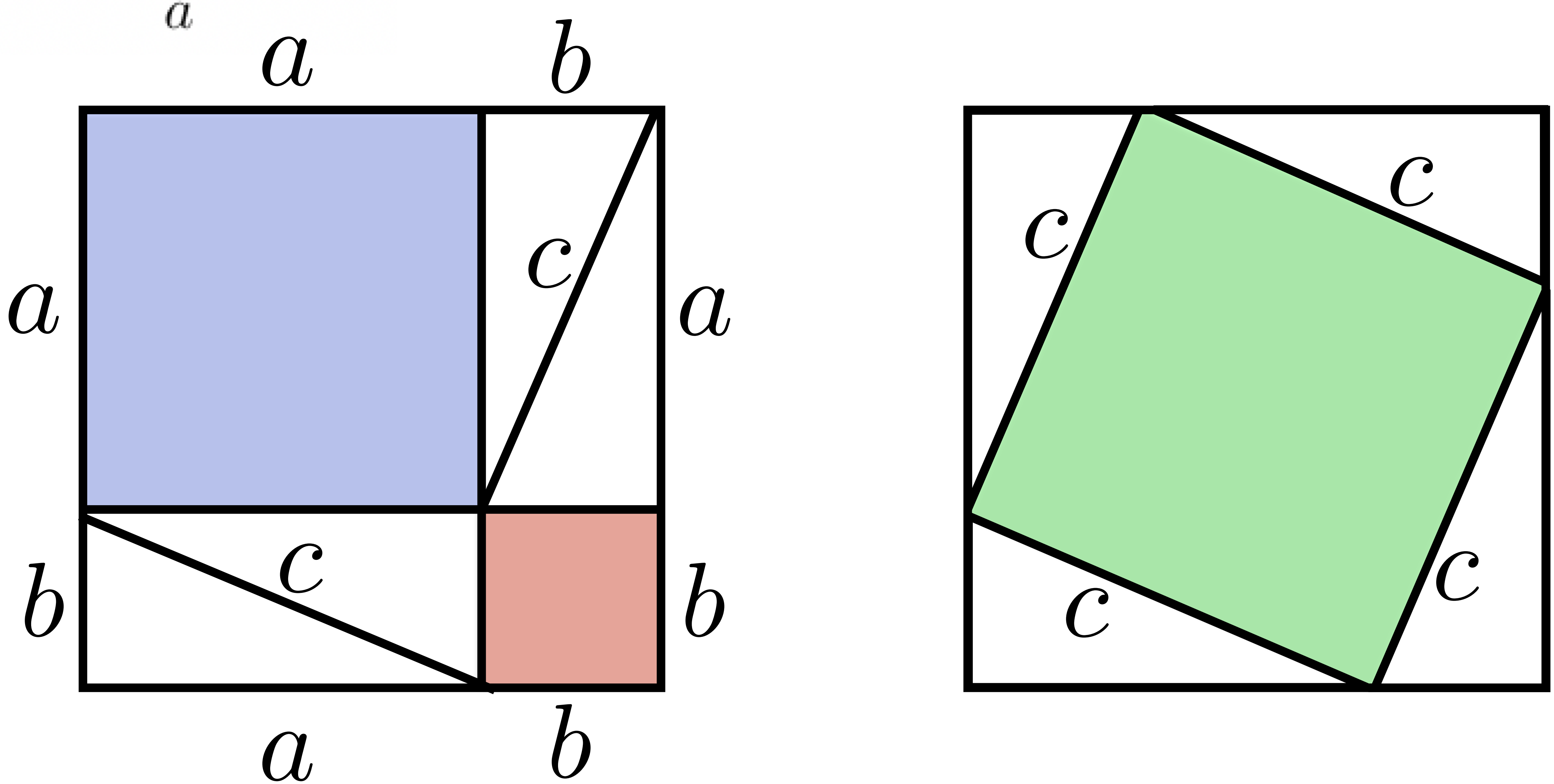






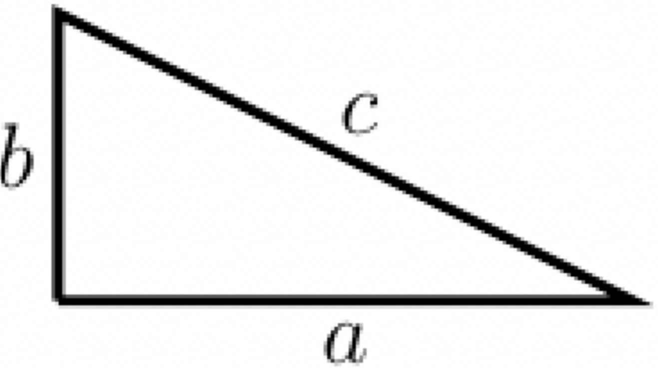
How They *Might* Have Proved It

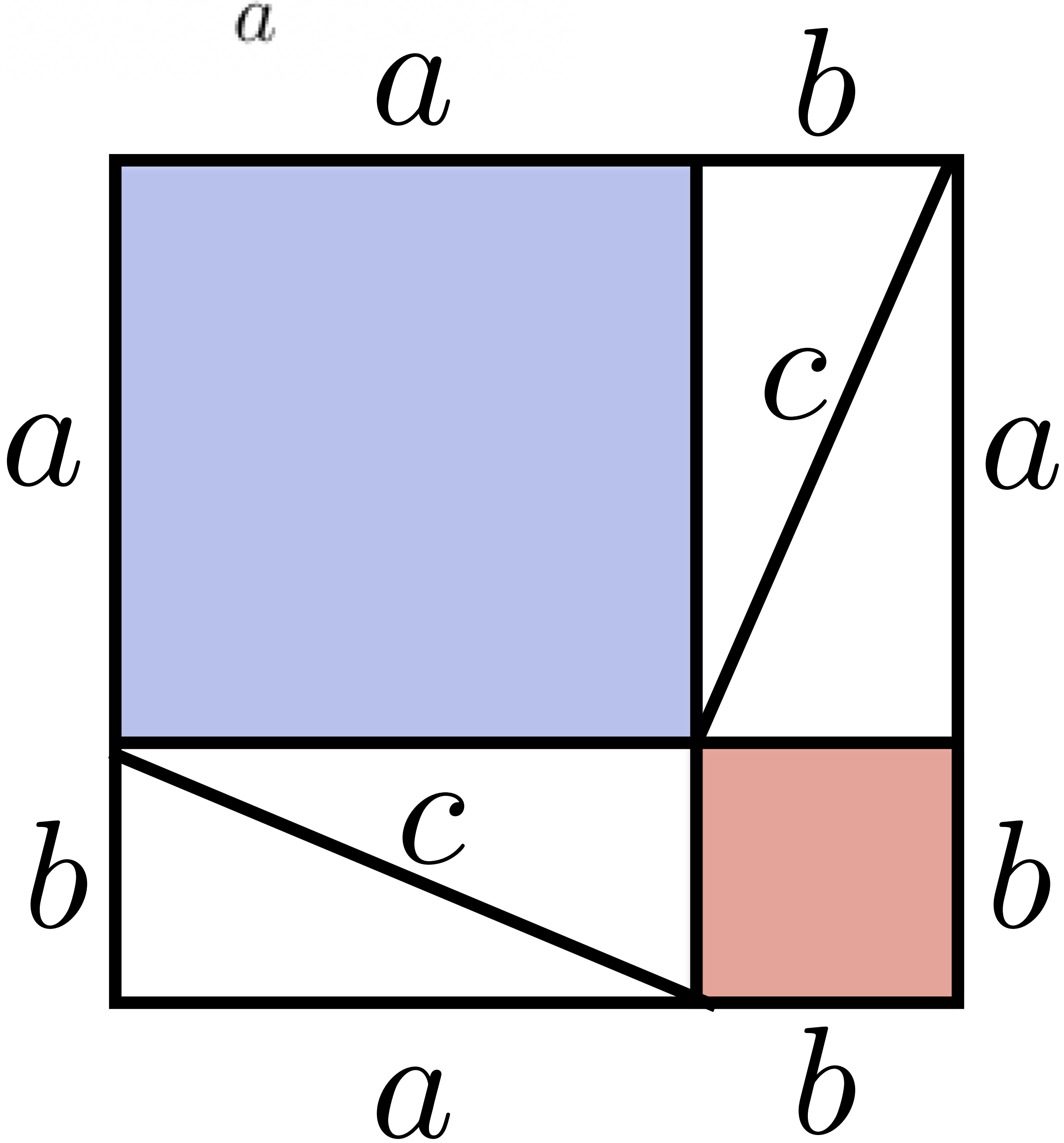


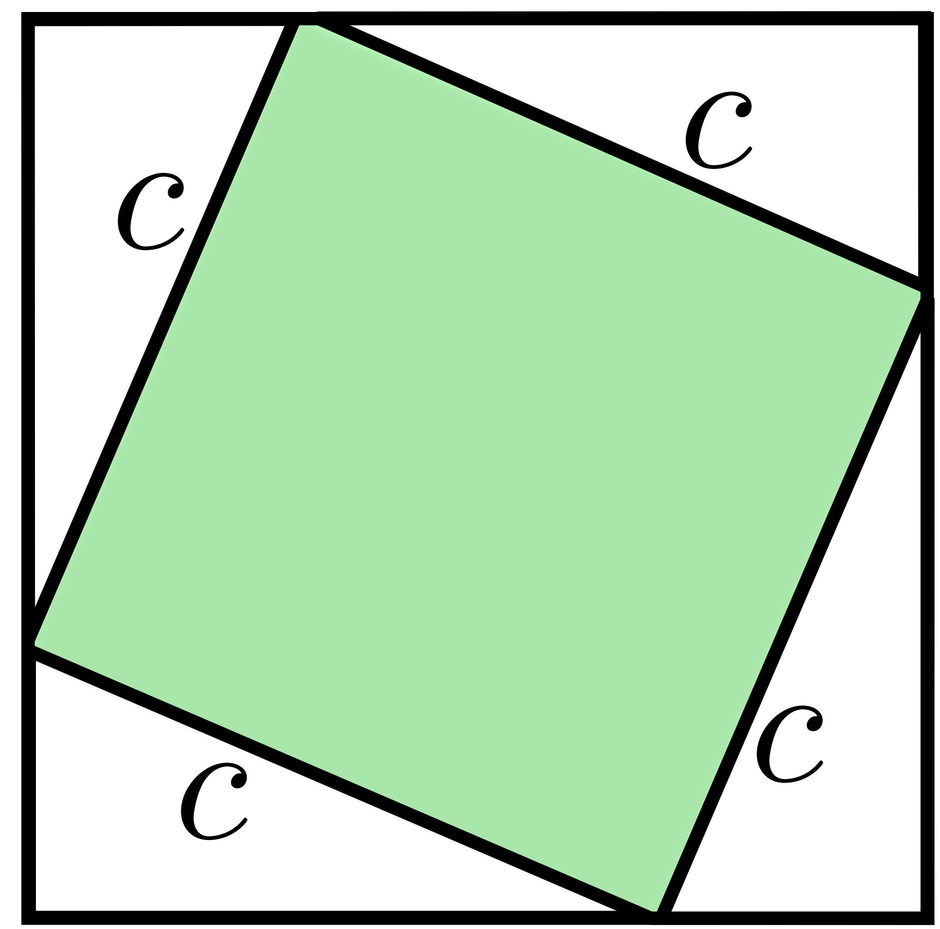


$a^2 + b^2$ + 4 triangles

How They *Might* Have Proved It

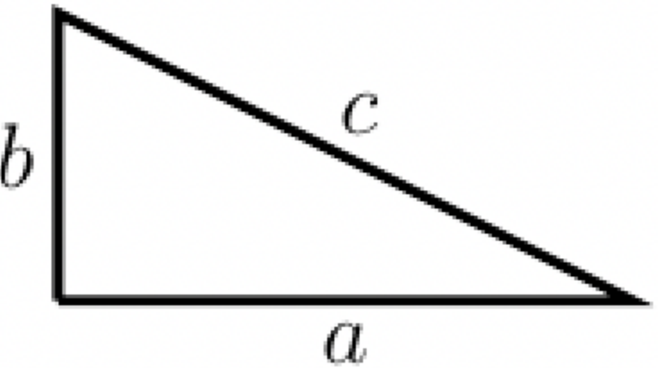


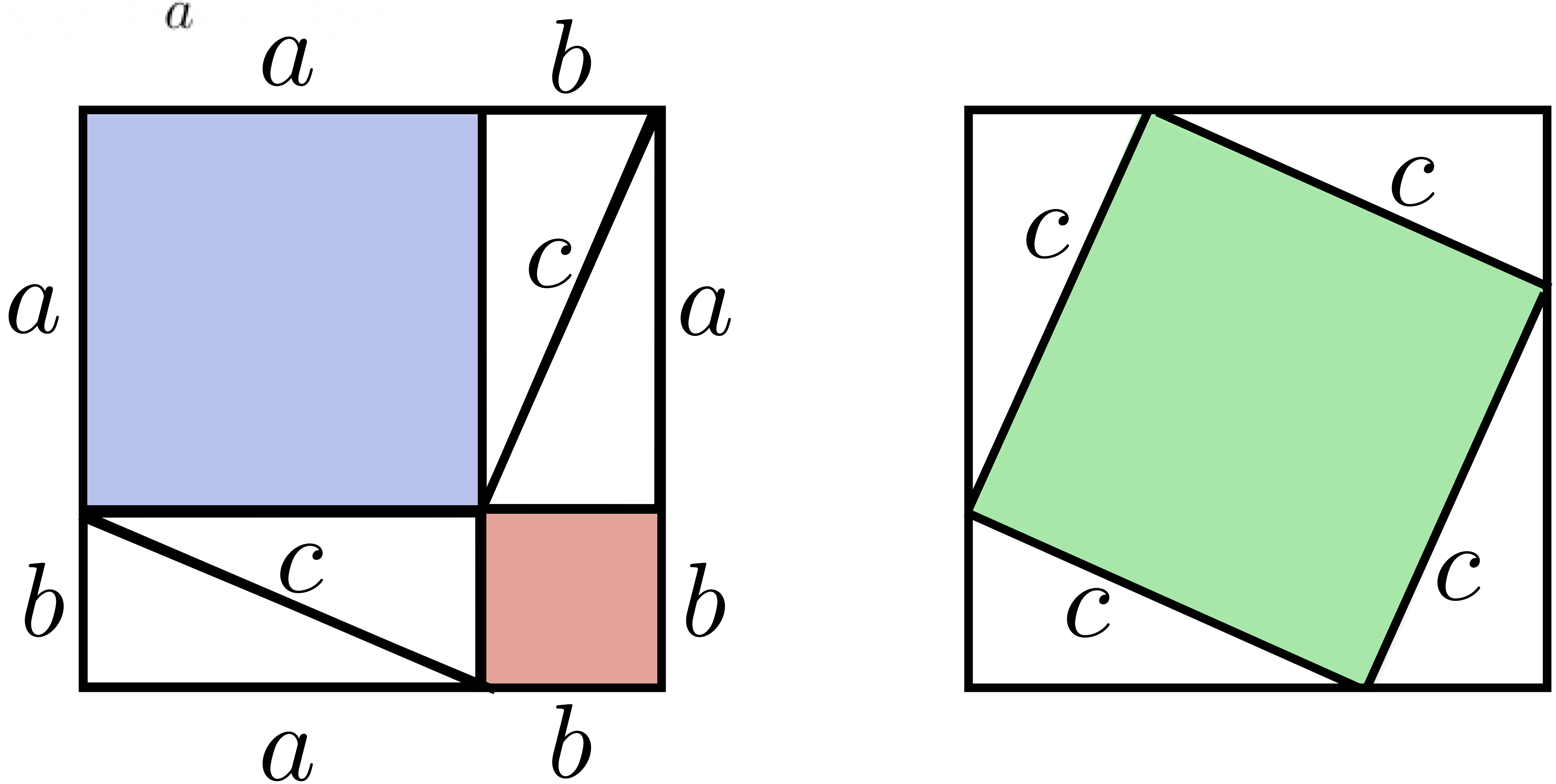




$a^2 + b^2$ + ~~4 triangles~~ $= c^2$ + ~~4 triangles~~

How They *Might* Have Proved It





$$a^2 + b^2 = c^2$$

A *Possible* Proof

- One of the oldest math texts in China is *Zhoubi Suanjing*. It was written around the time of Pythagoras.

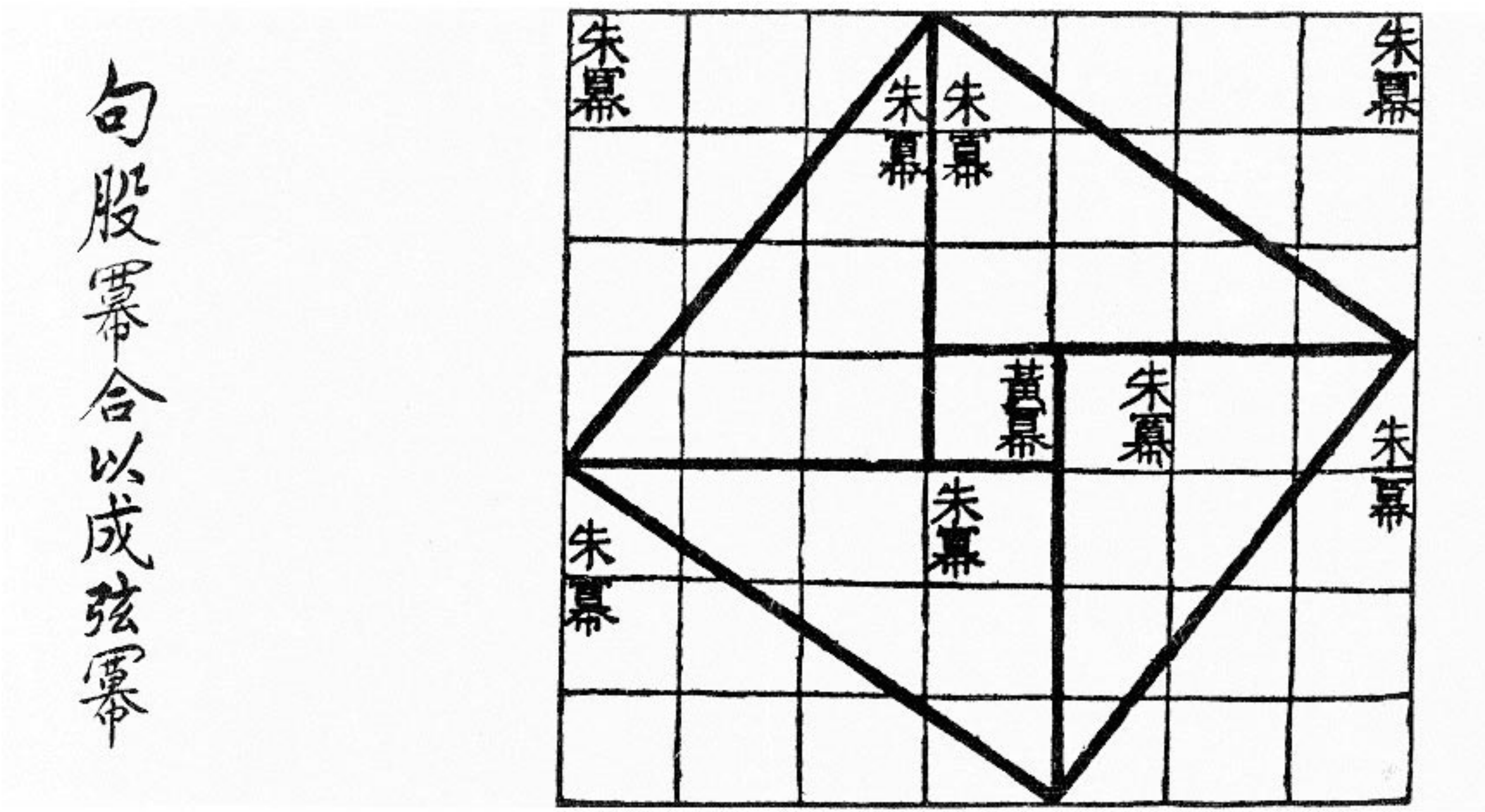
- The book uses the Pythagorean theorem many times and alludes to a diagram.

- A diagram was added to the text in the 3rd century. Unclear if it was the one intended by the original author.

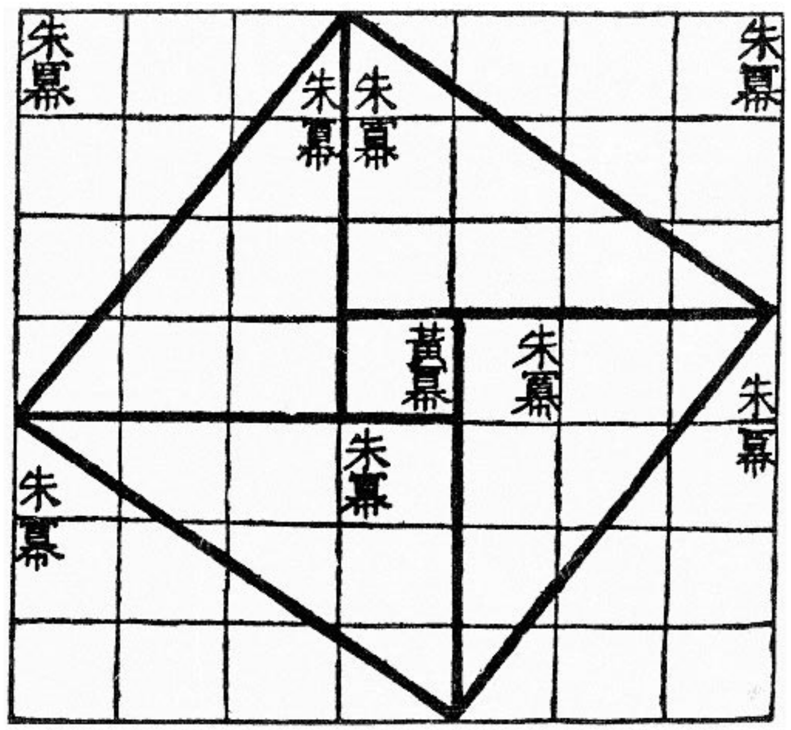
- Time to Think Like A Math Historian™!

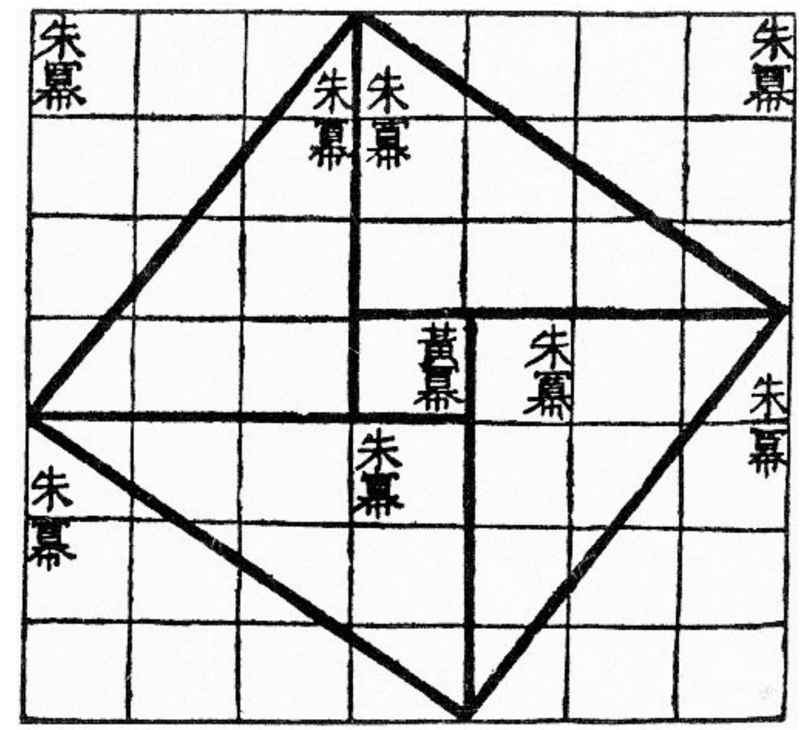
Think Like A
Math Historian

Think Like A Math Historian

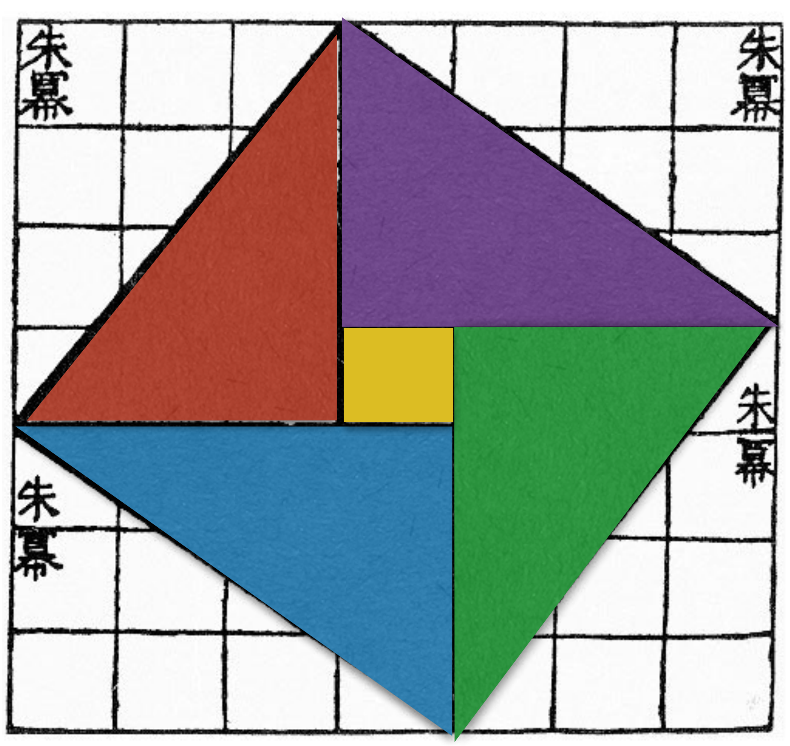


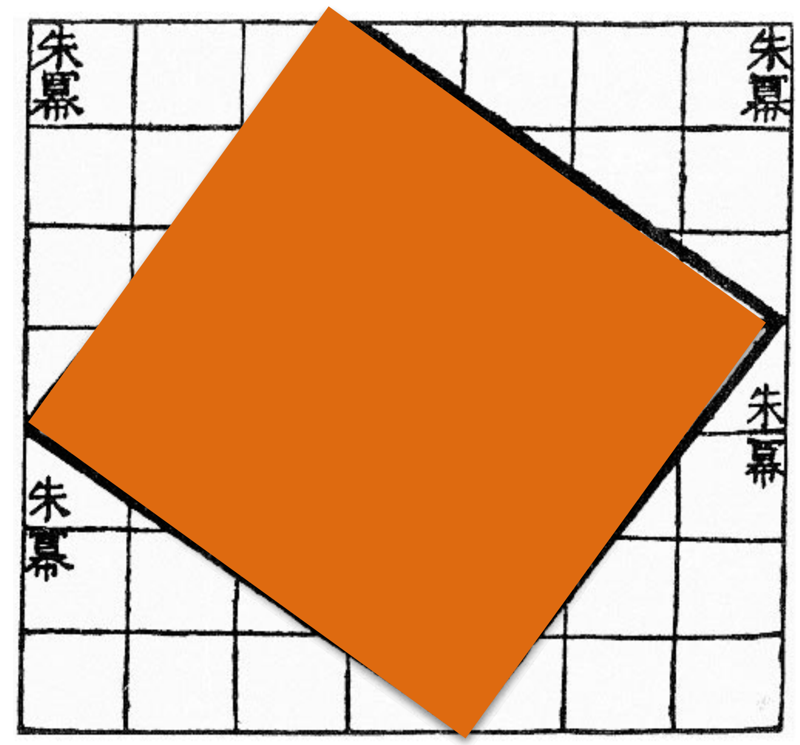
First *Possible* Proof





First *Possible* Proof

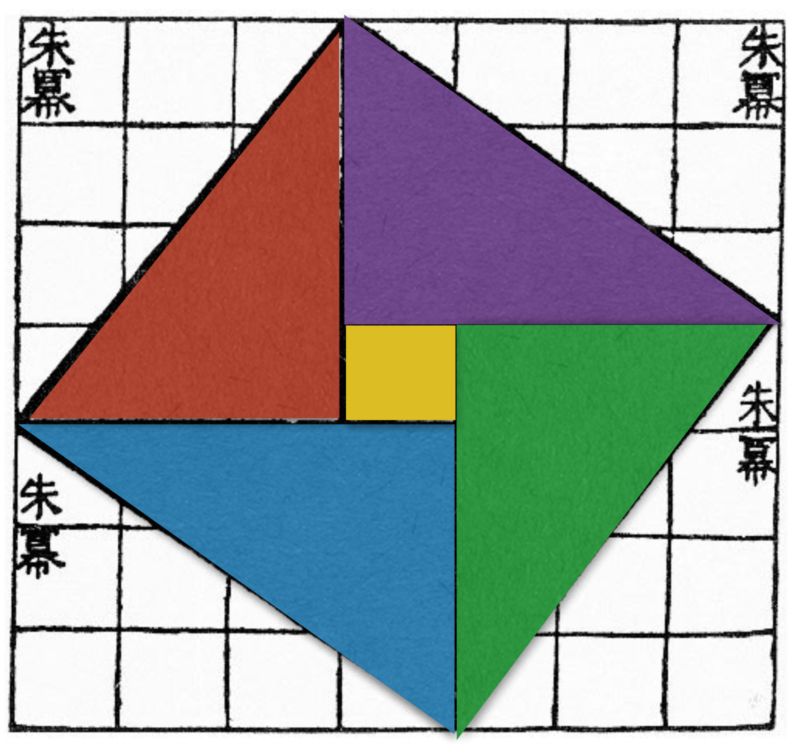


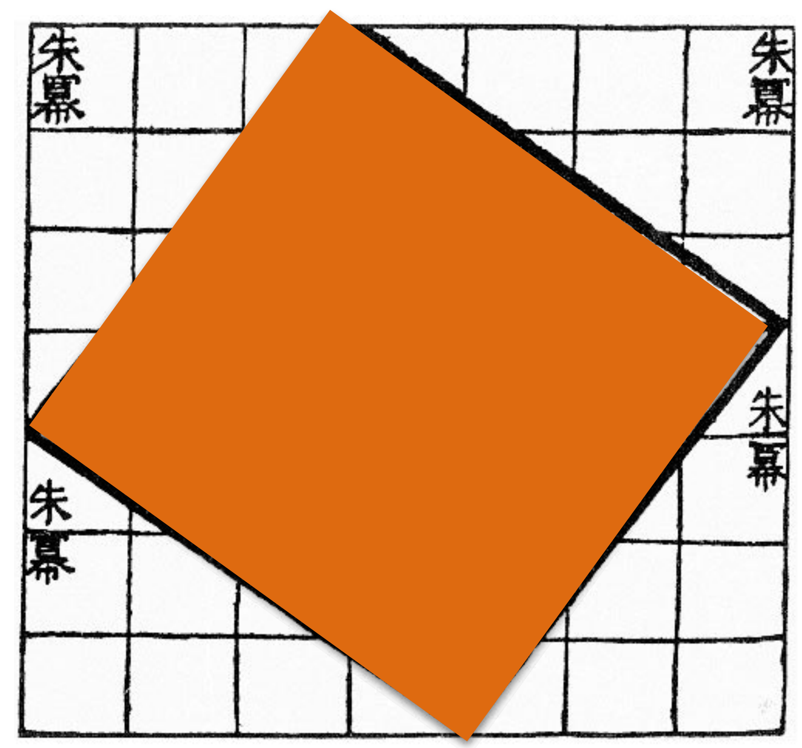


4 triangles + little square = Big square

$$4 \cdot \frac{1}{2}(3 \cdot 4)$$

First *Possible* Proof

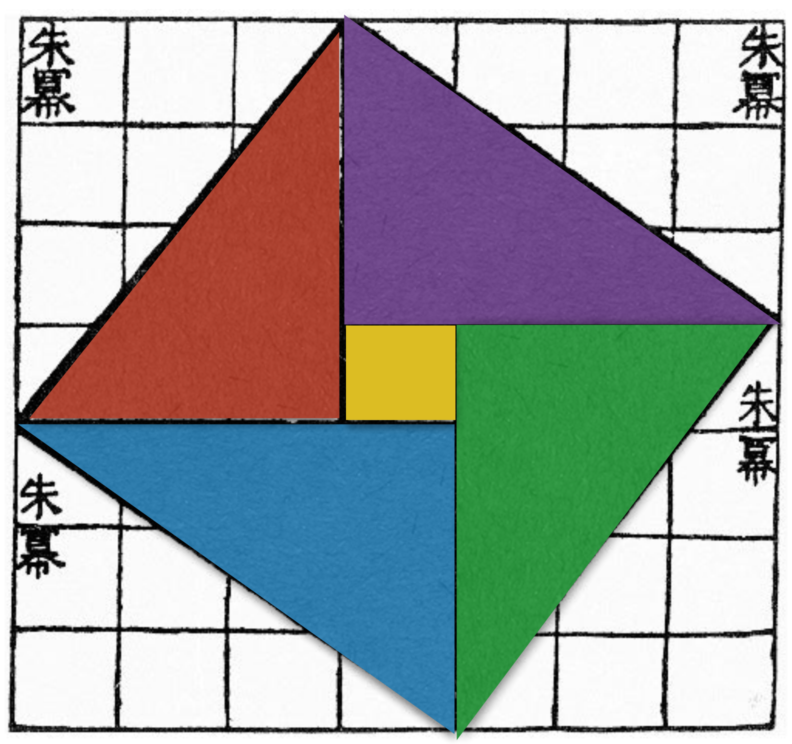


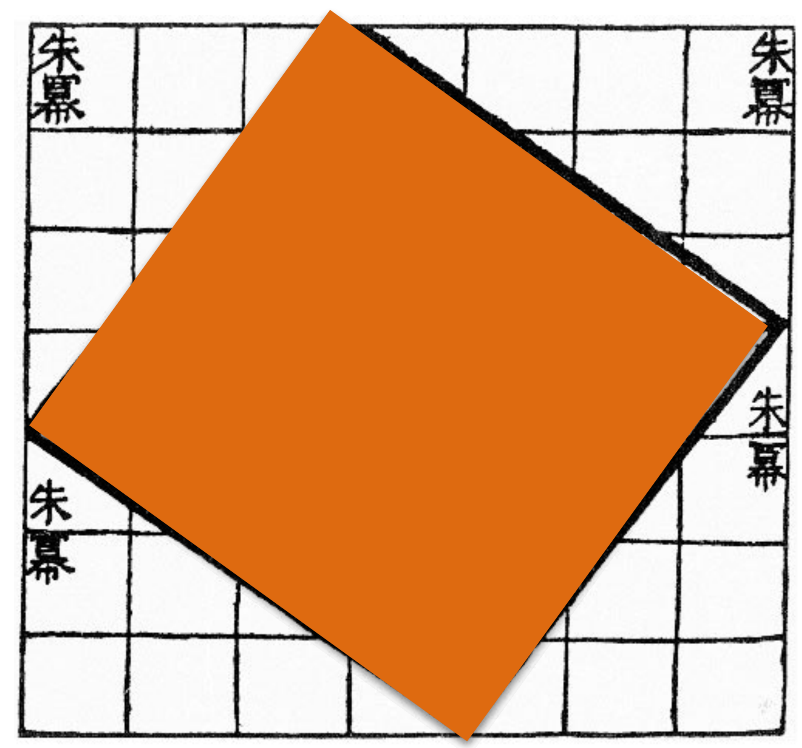


4 triangles + little square = Big square

$$4 \cdot \frac{1}{2}(3 \cdot 4) \quad + \quad 1 \cdot 1$$

First *Possible* Proof

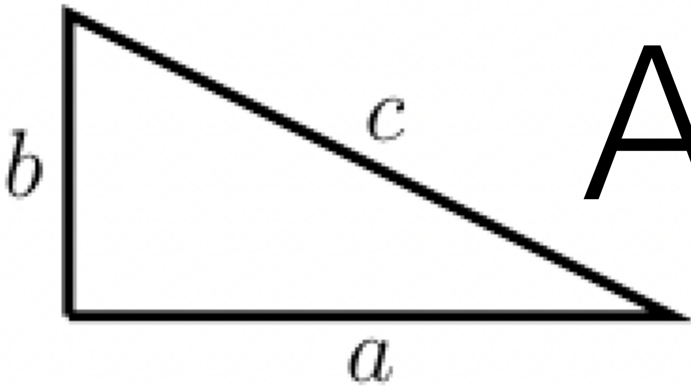


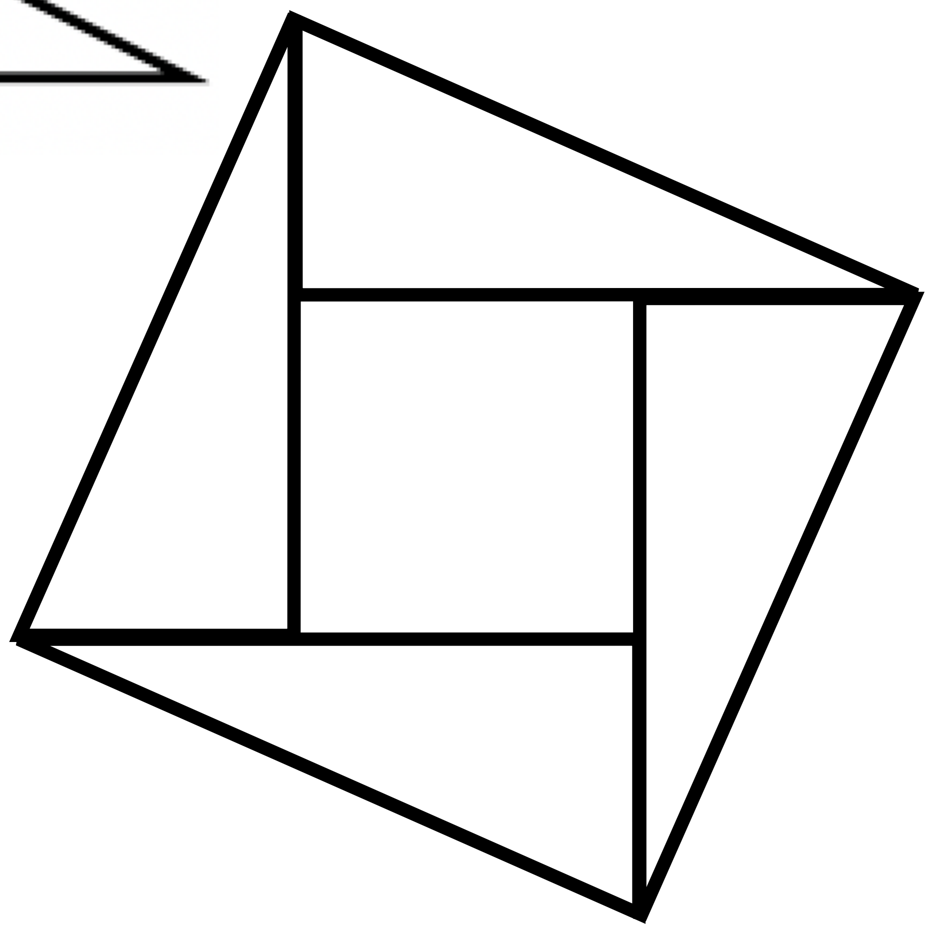


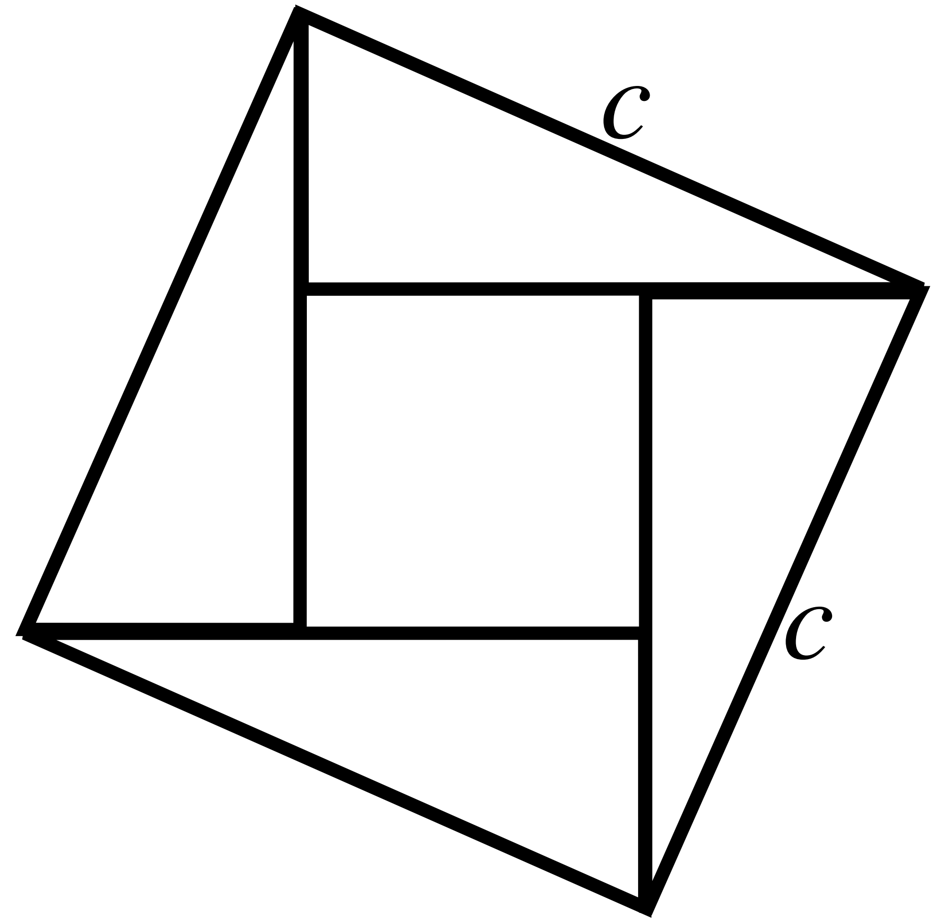
4 triangles + little square = Big square

$$4 \cdot \frac{1}{2}(3 \cdot 4) \quad + \quad 1 \cdot 1 \quad = \quad 5^2$$

A *Possible* Proof

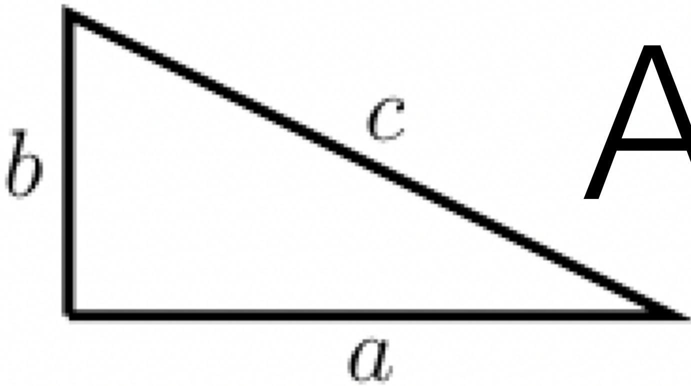




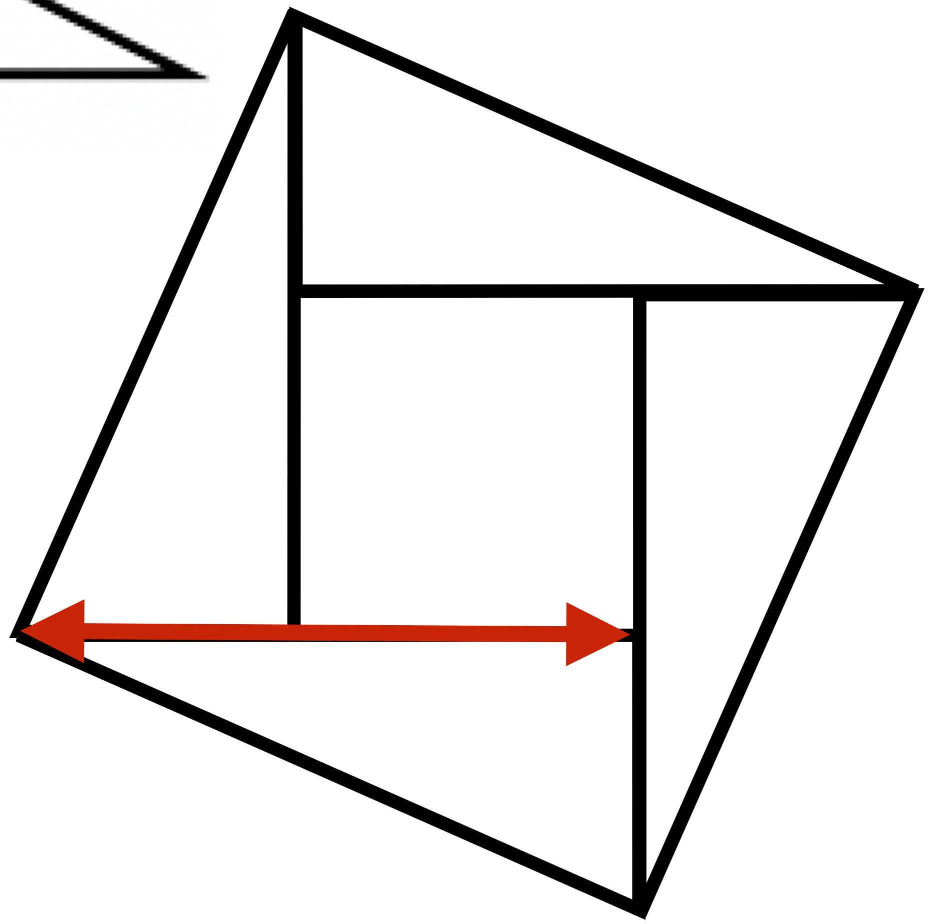


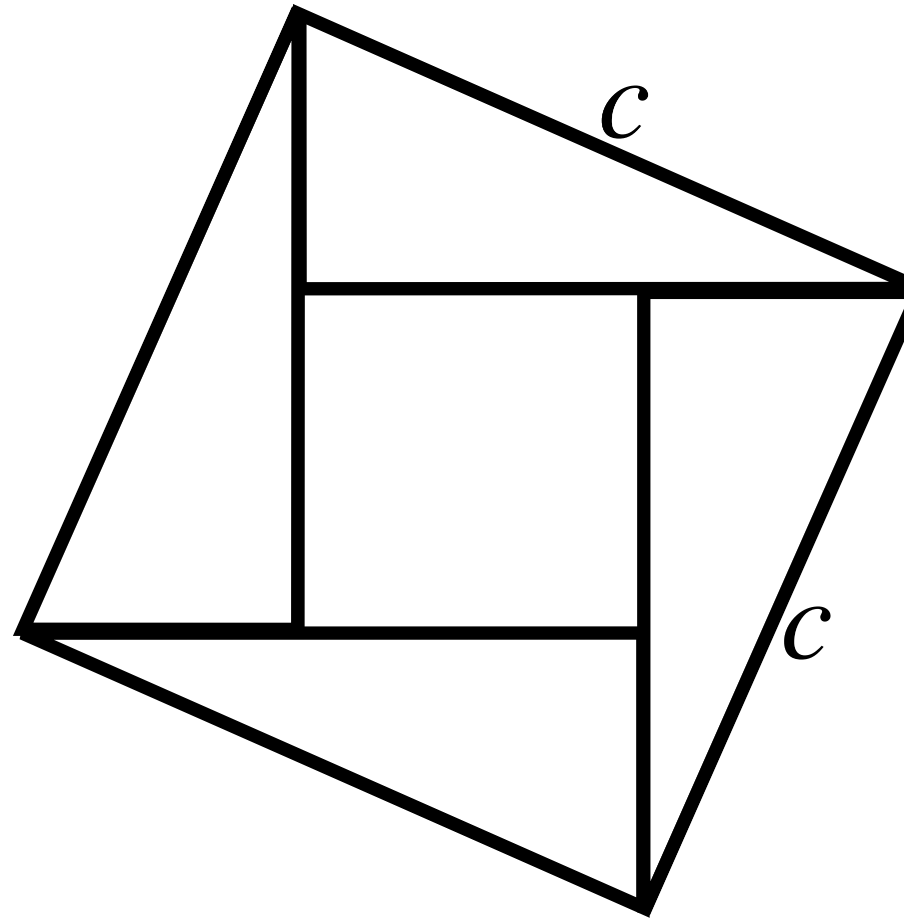
4 triangles + little square = Big square

$= c^2$



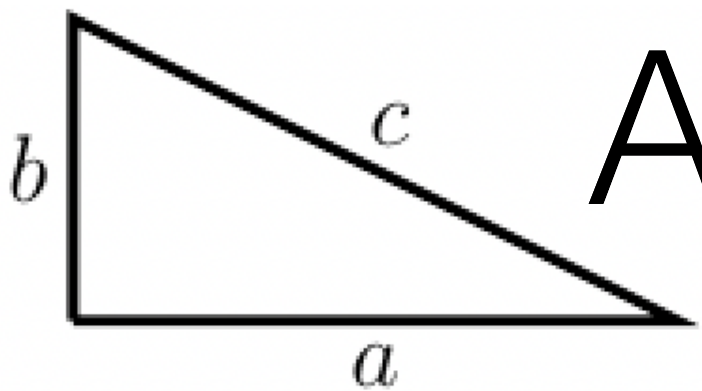
A *Possible* Proof



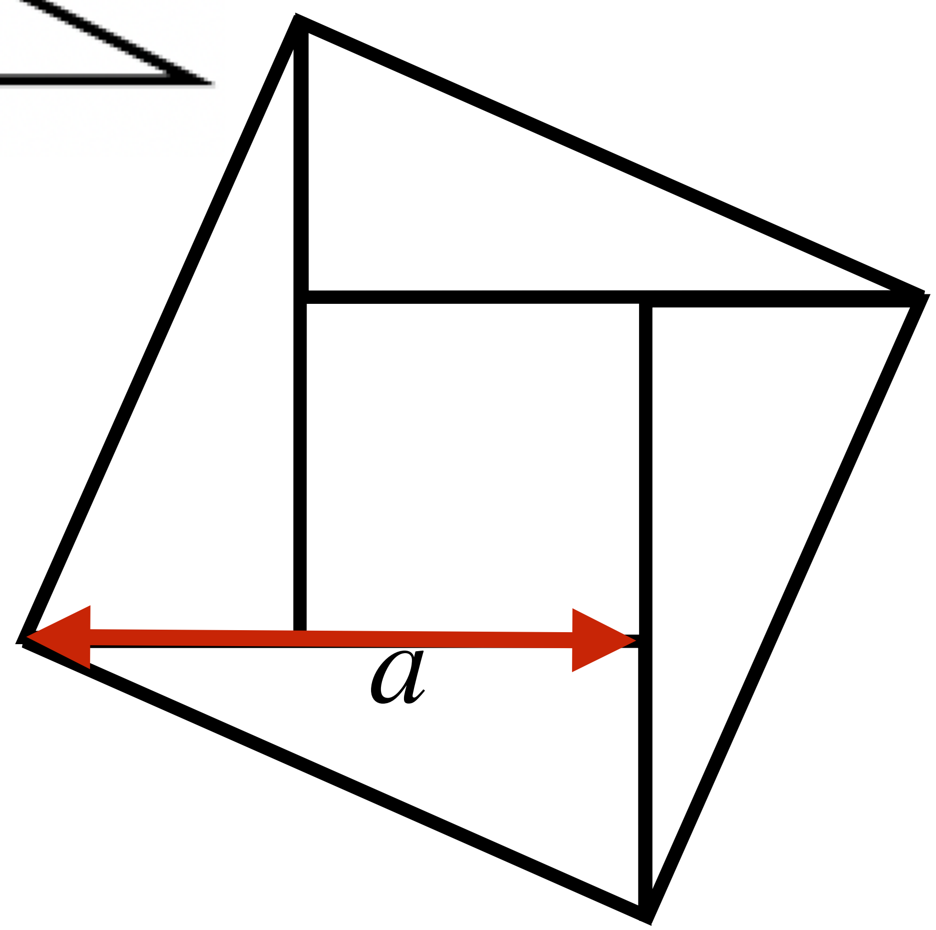


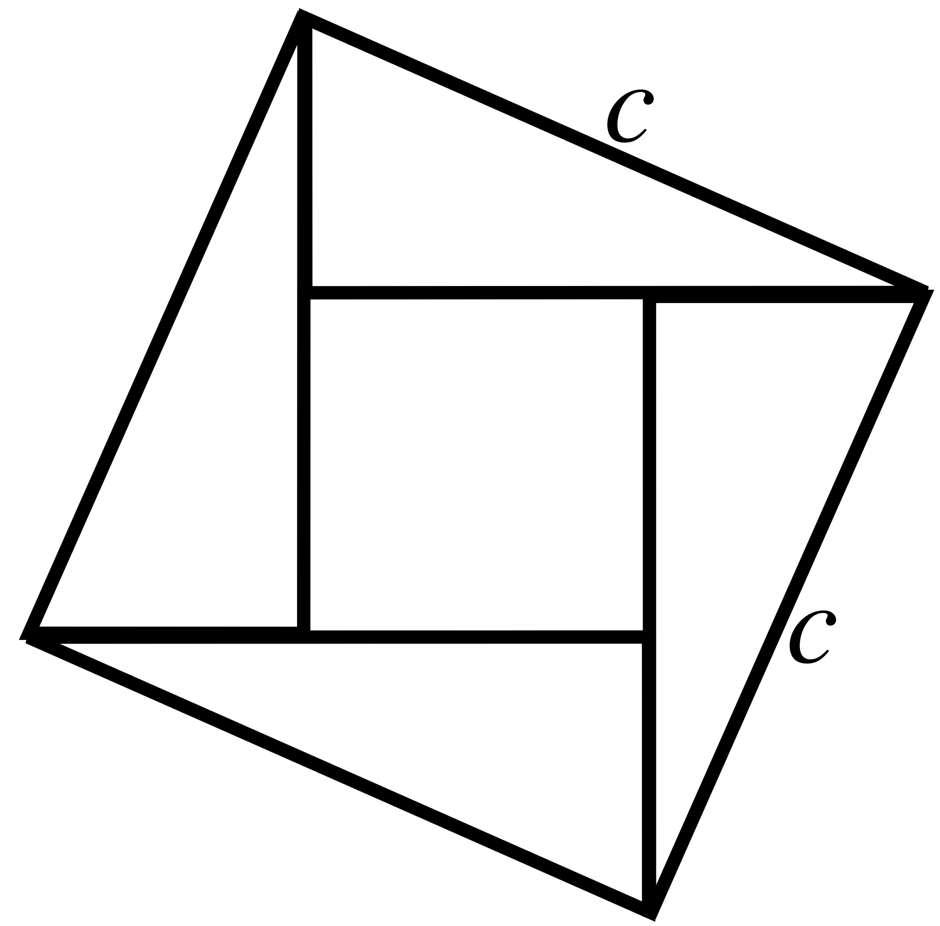
4 triangles + little square = Big square

= c^2



A *Possible* Proof

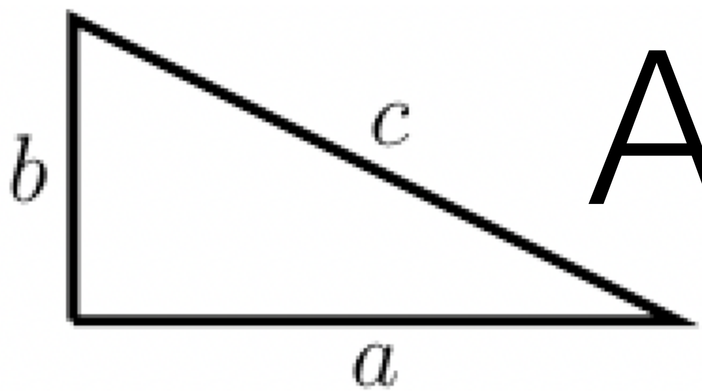


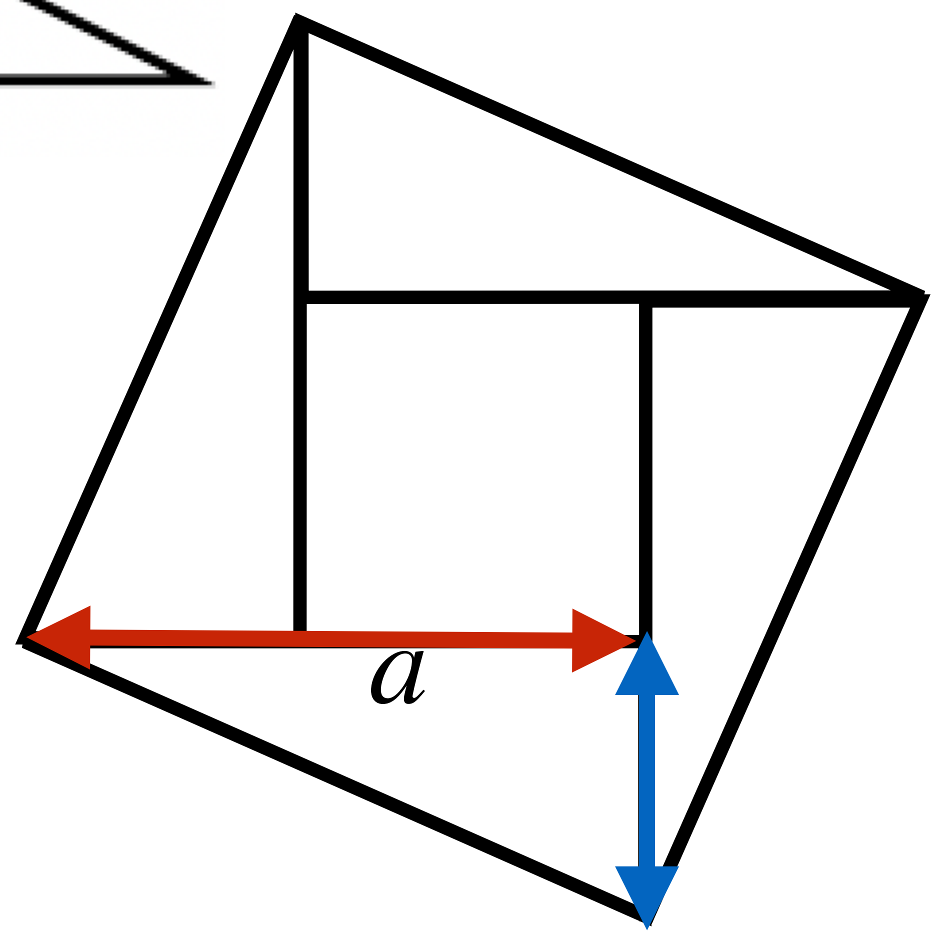


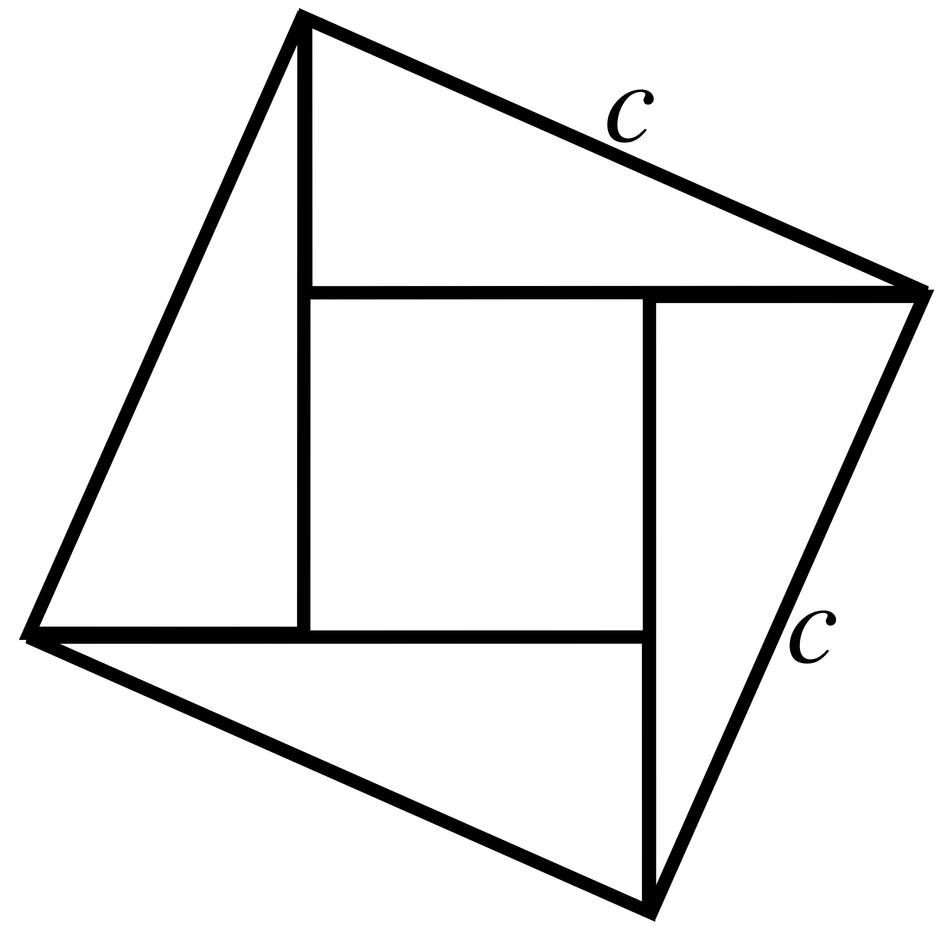
4 triangles + little square = Big square

$= c^2$

A *Possible* Proof



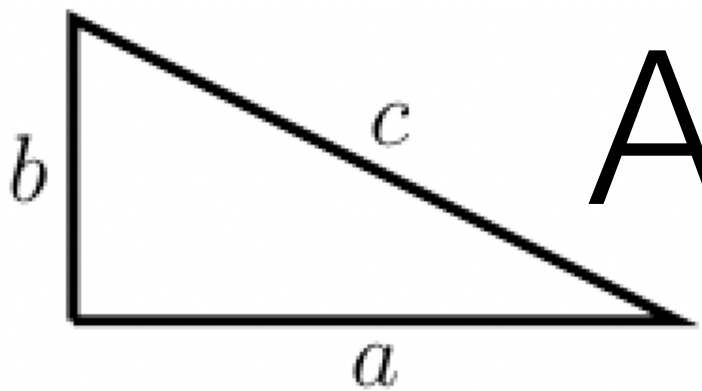


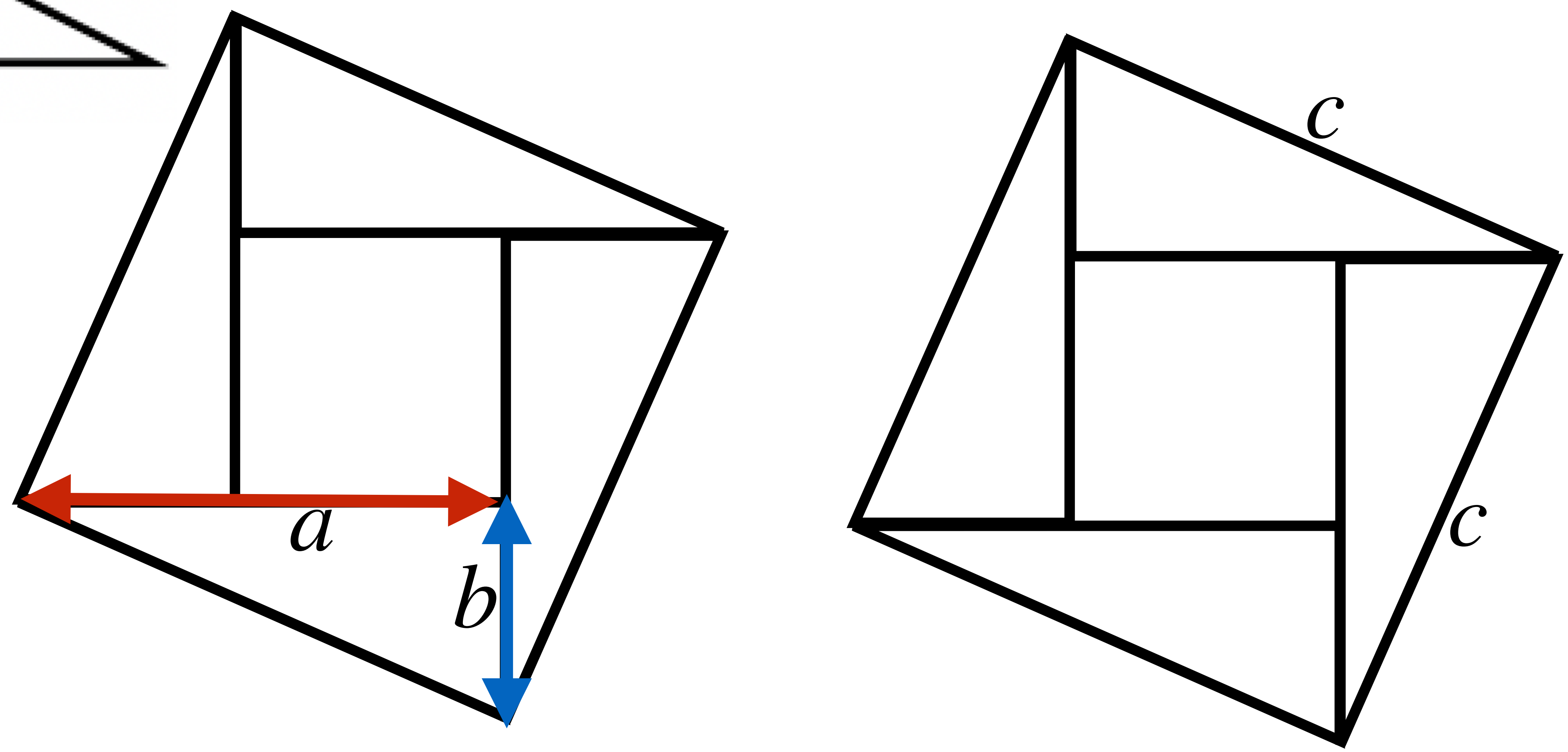


4 triangles + little square $=$ Big square

$= \quad c^2$

A *Possible* Proof

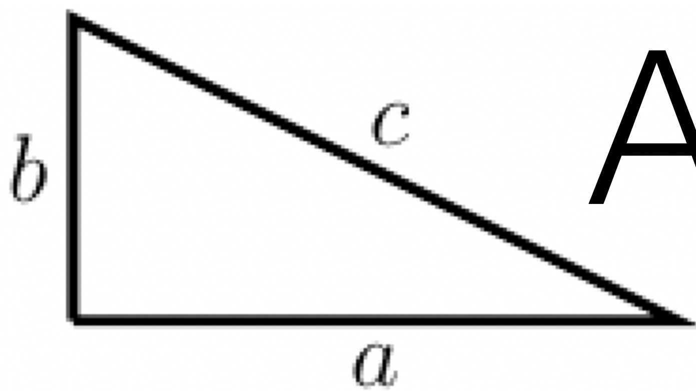


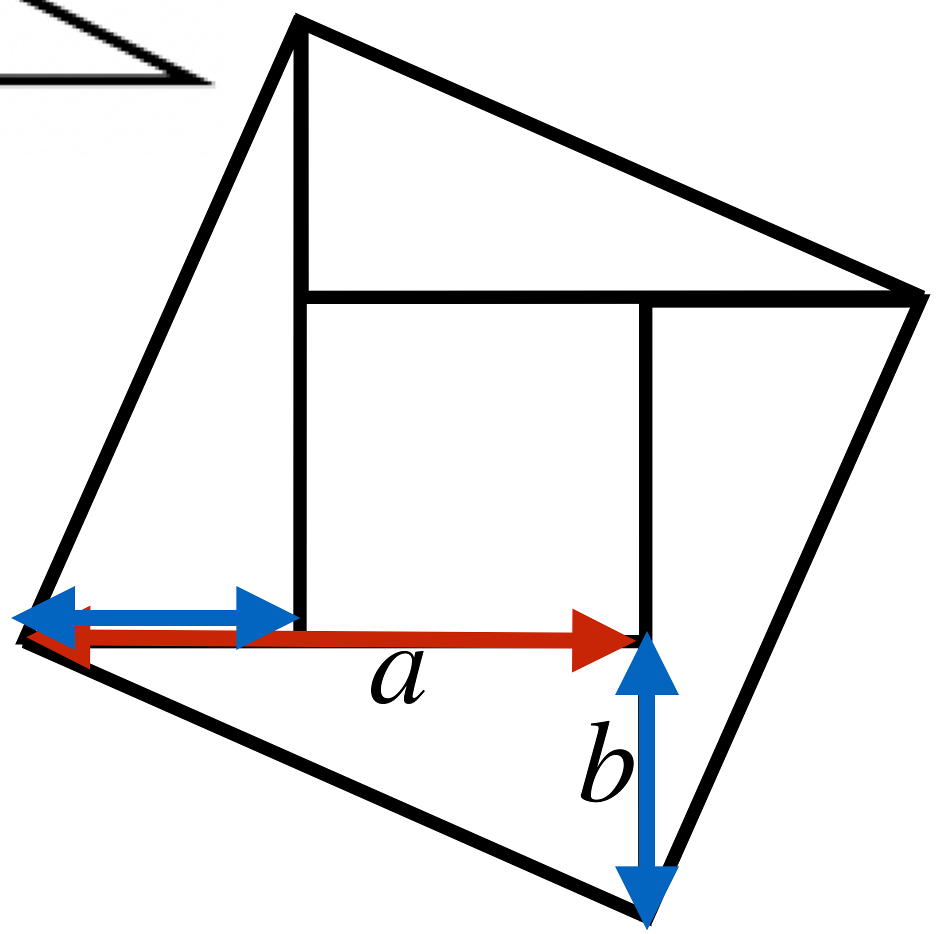


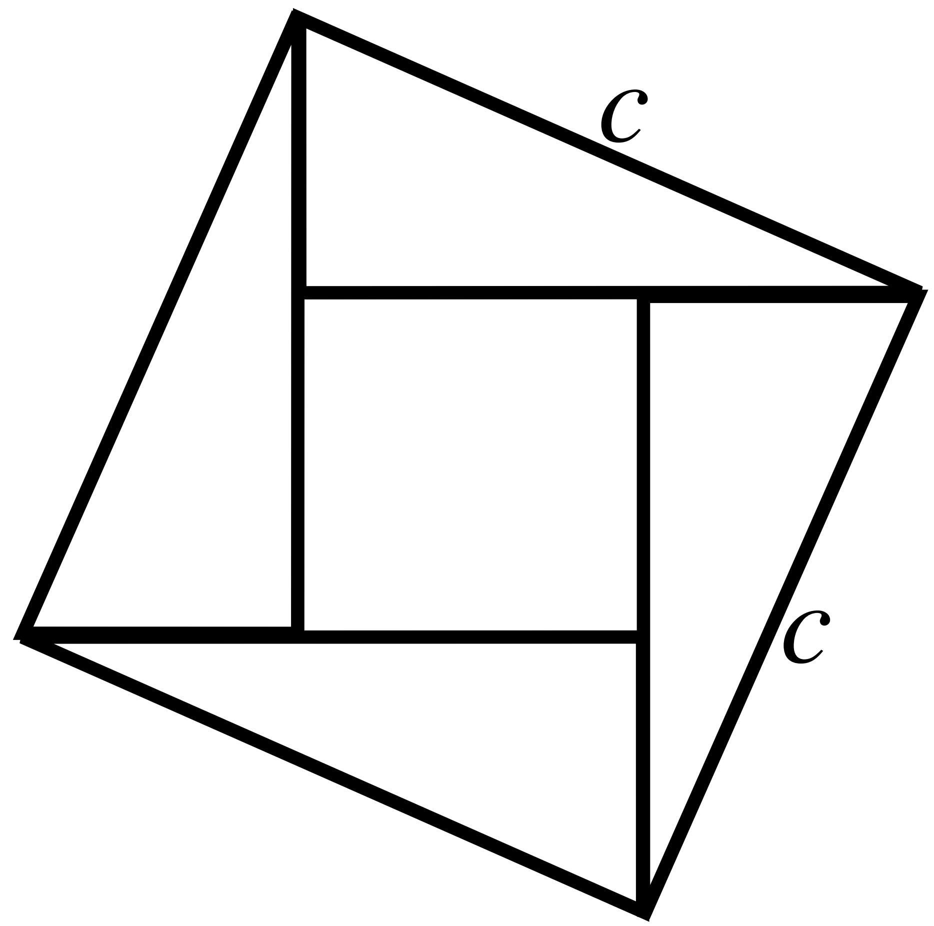
4 triangles + little square = Big square

$$4\cdot\left(\frac{1}{2}a\cdot b\right) + \qquad = c^2$$

A *Possible* Proof

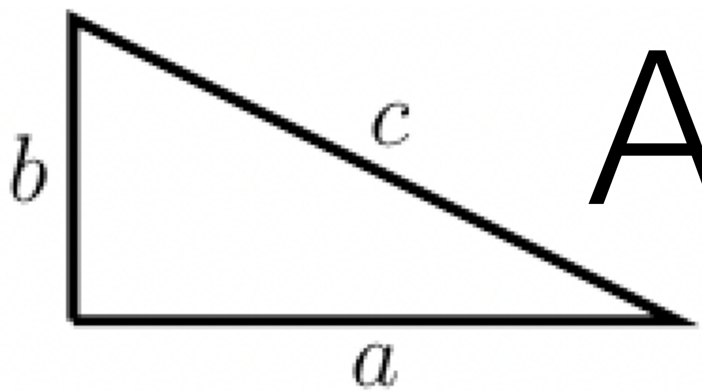




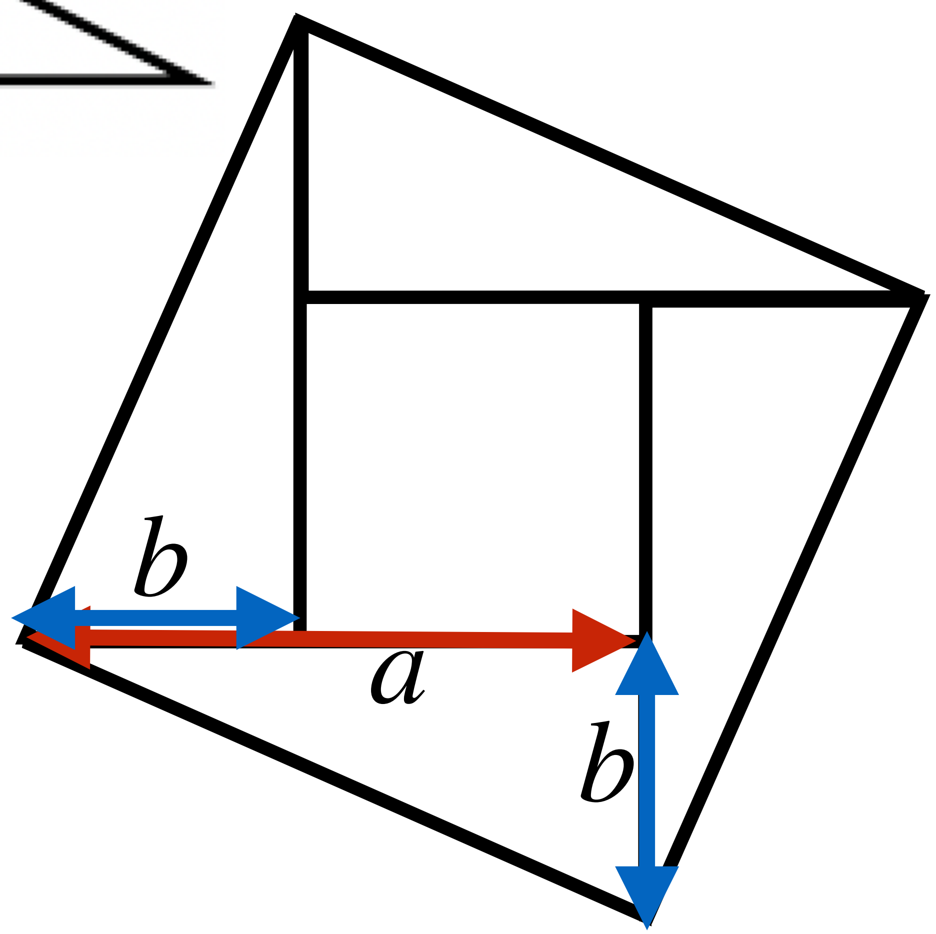


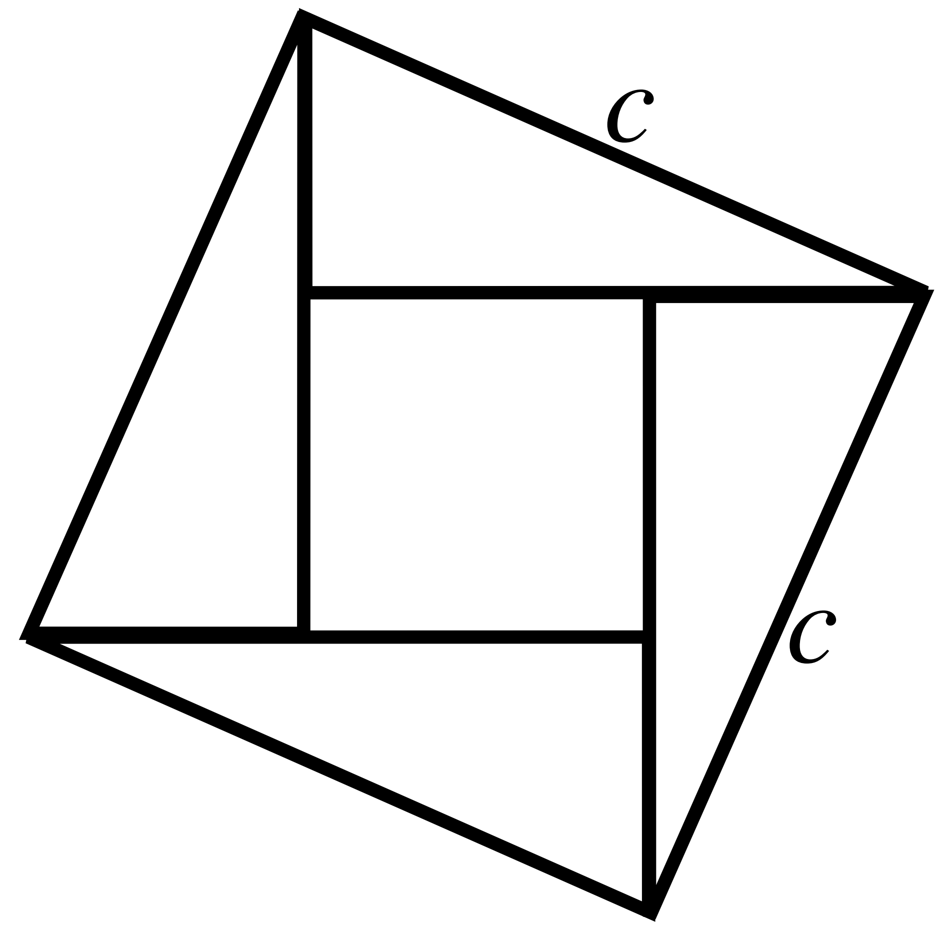
4 triangles + little square = Big square

$$4 \cdot \left(\frac{1}{2}a \cdot b\right) \quad + \qquad = \quad c^2$$



A *Possible* Proof





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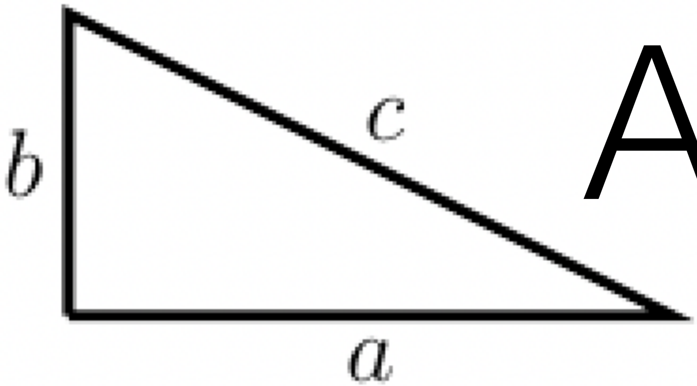

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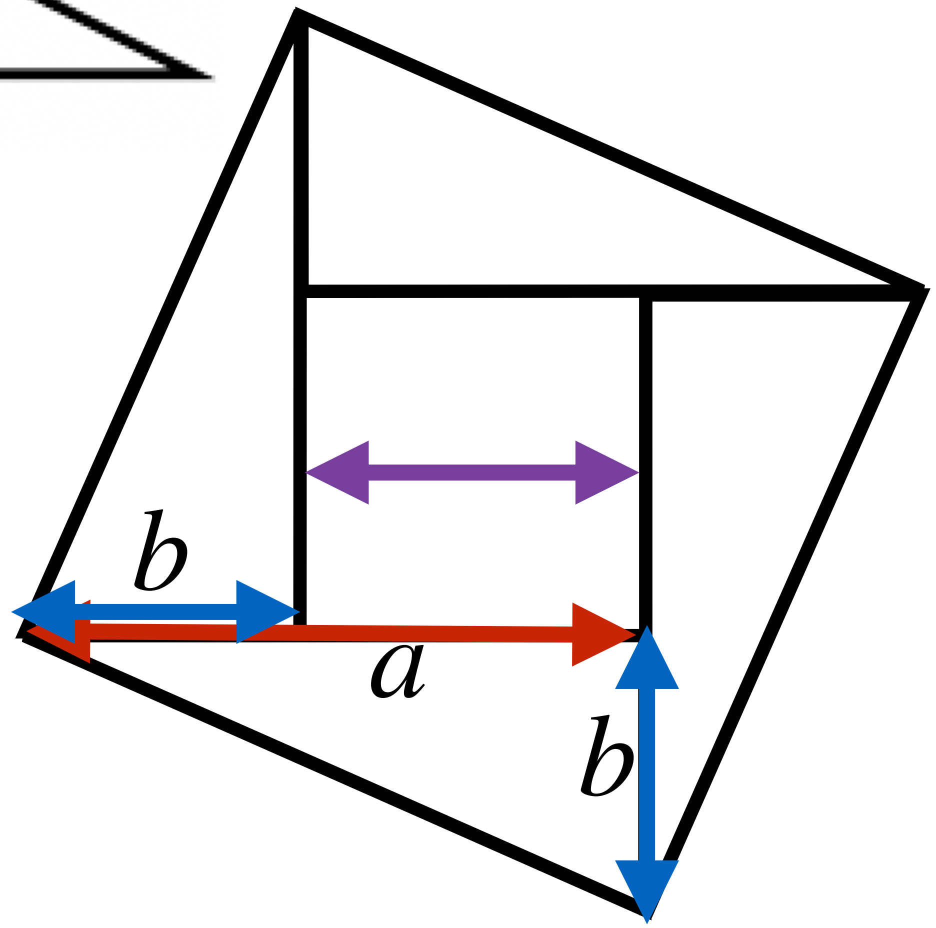

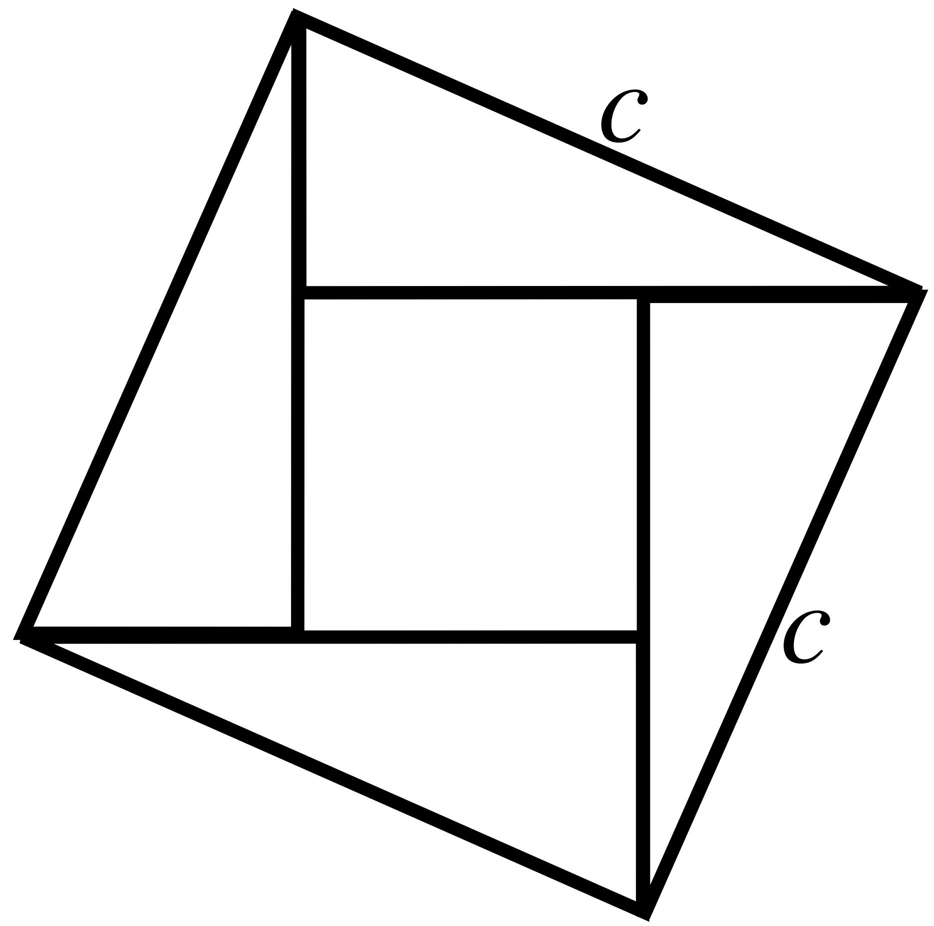

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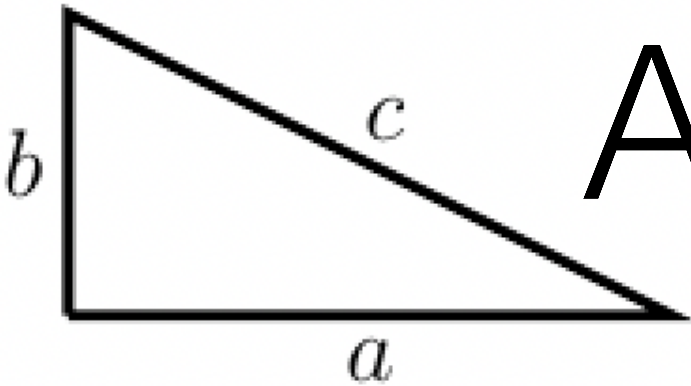
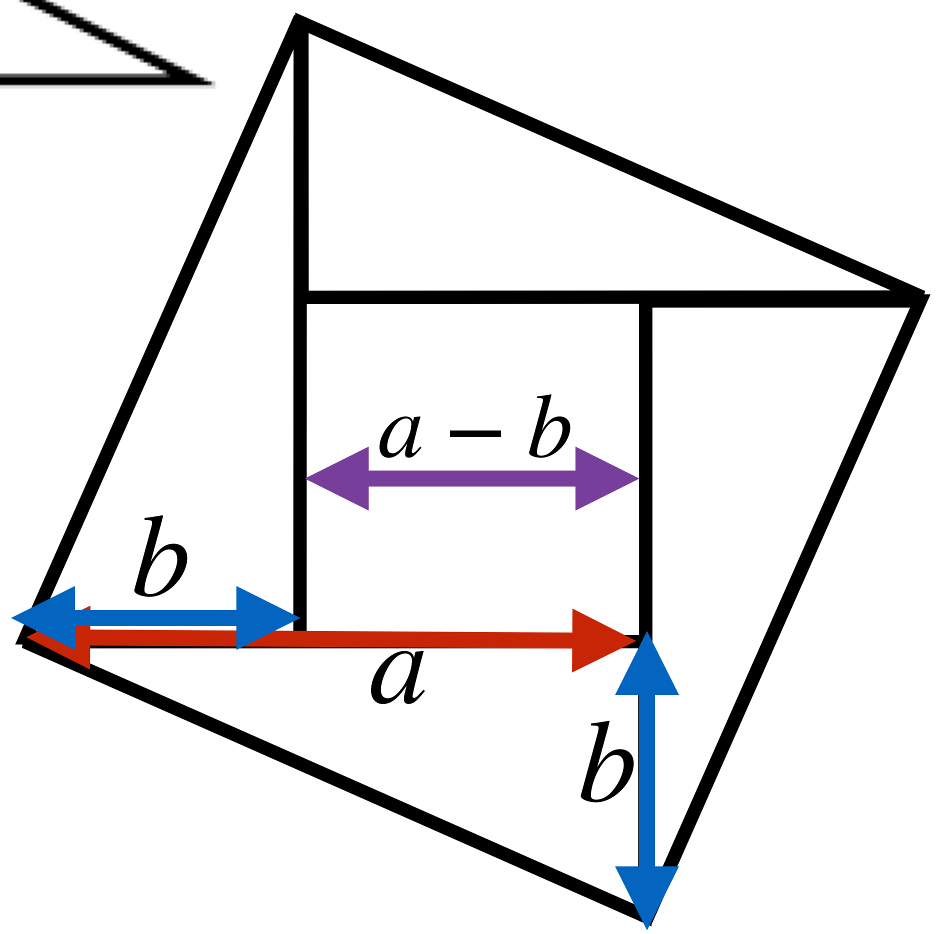
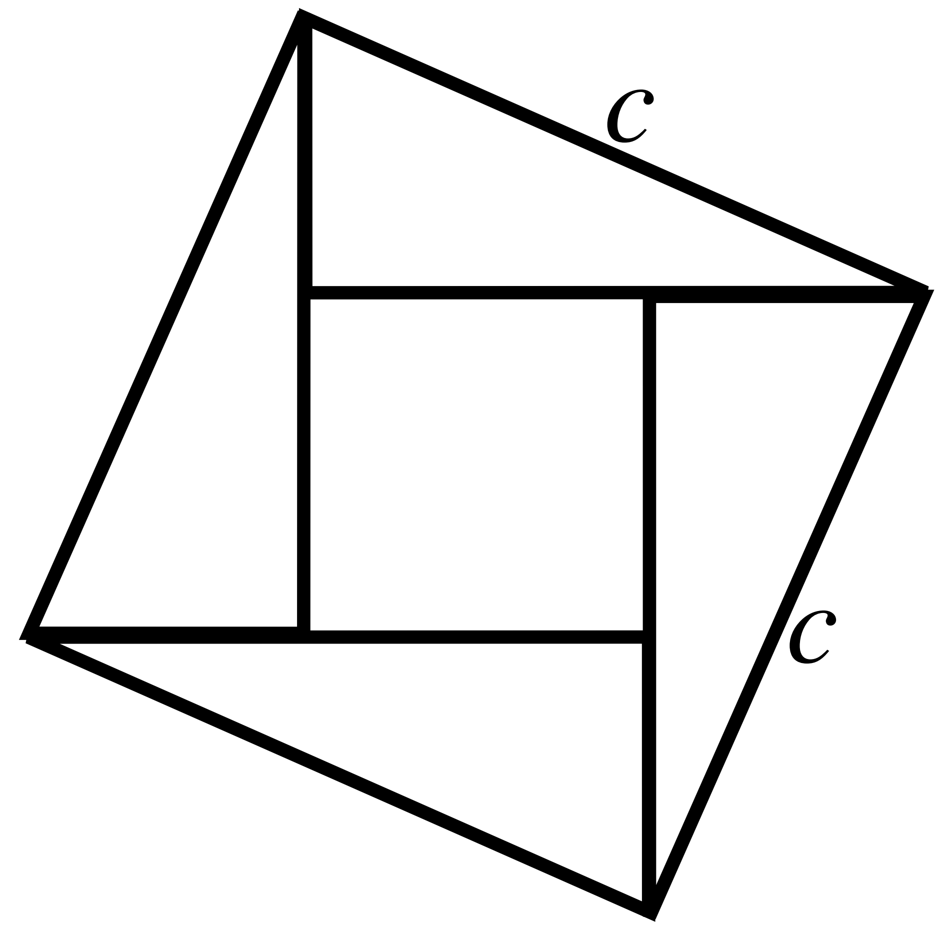
$$4 \cdot \left(\frac{1}{2} a \cdot b\right) \quad + \qquad\qquad = \quad c^2$$

A *Possible* Proof







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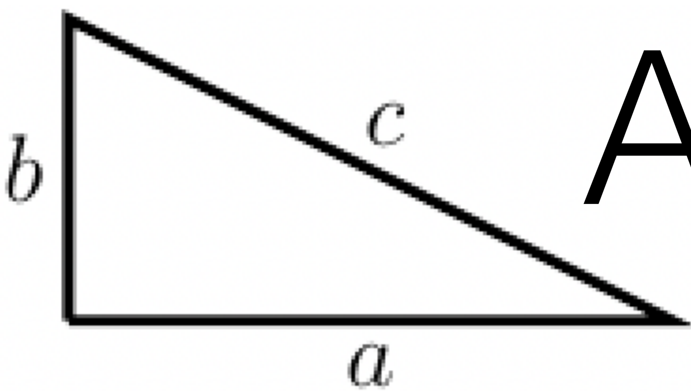

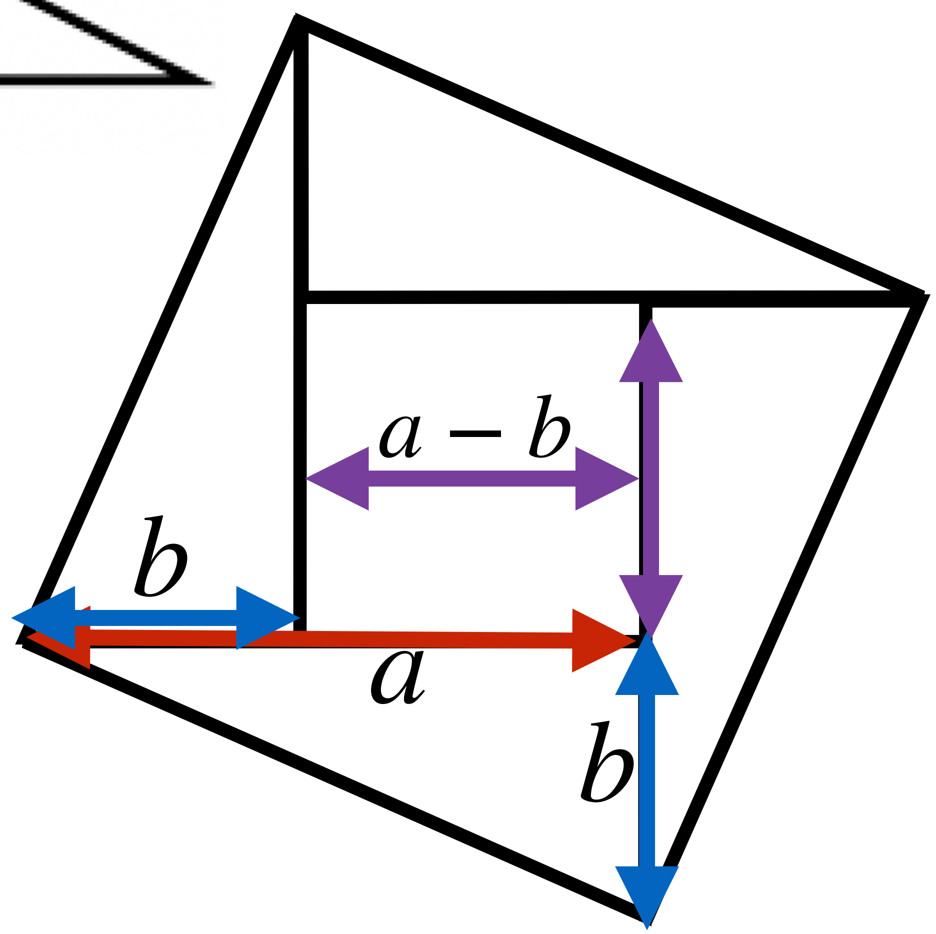

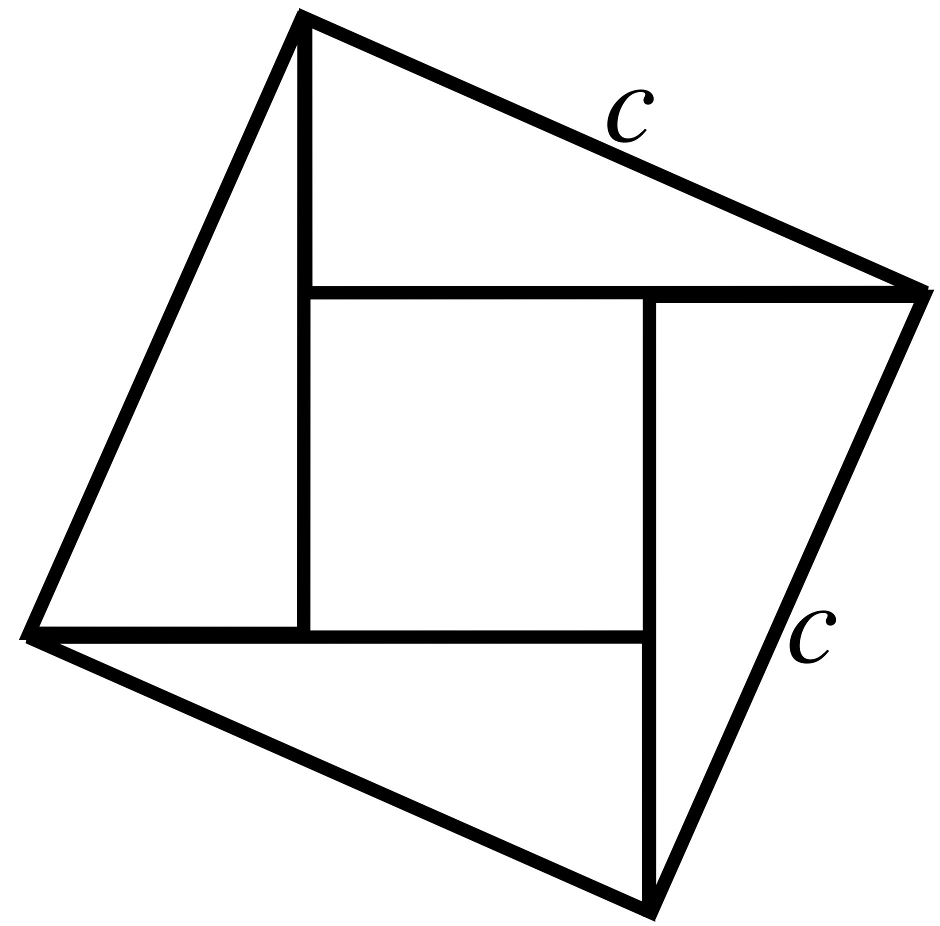

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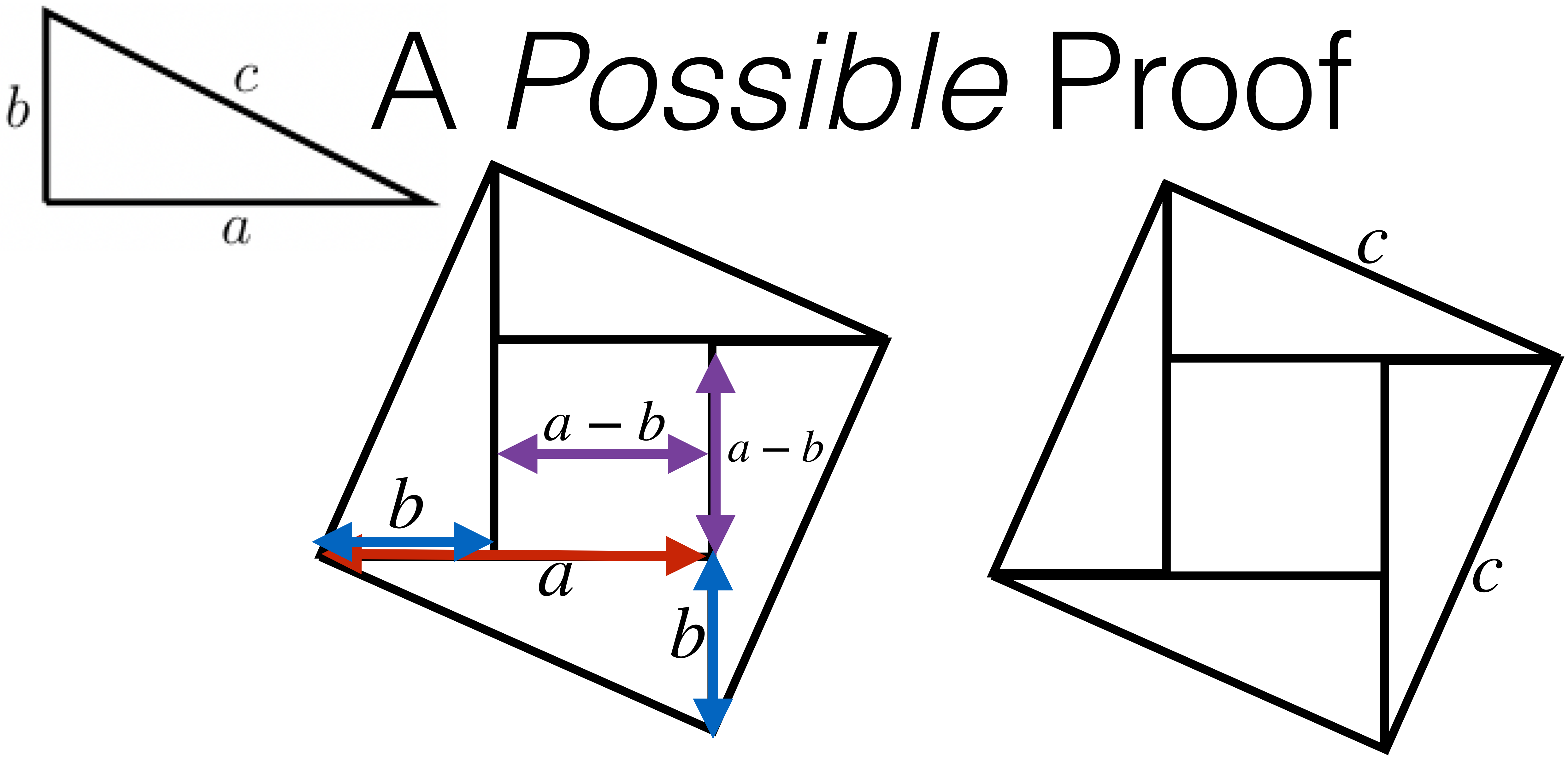

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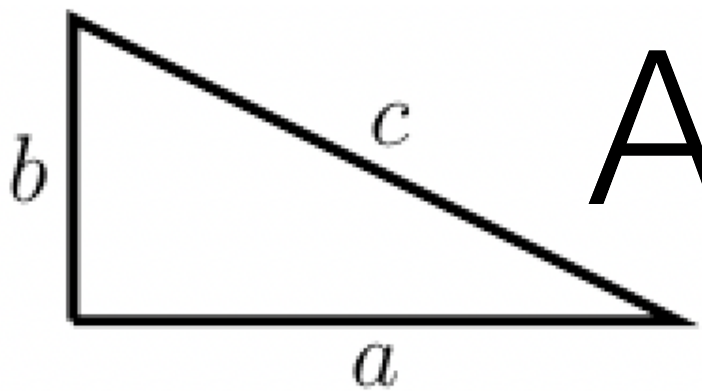
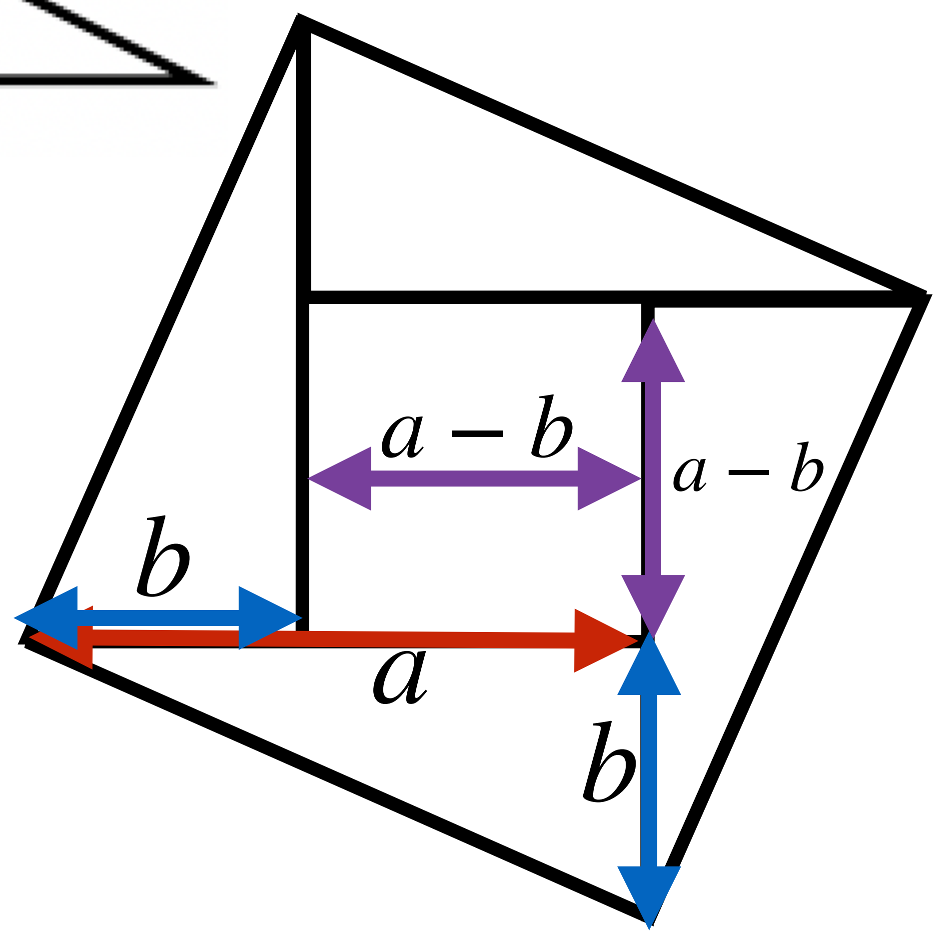
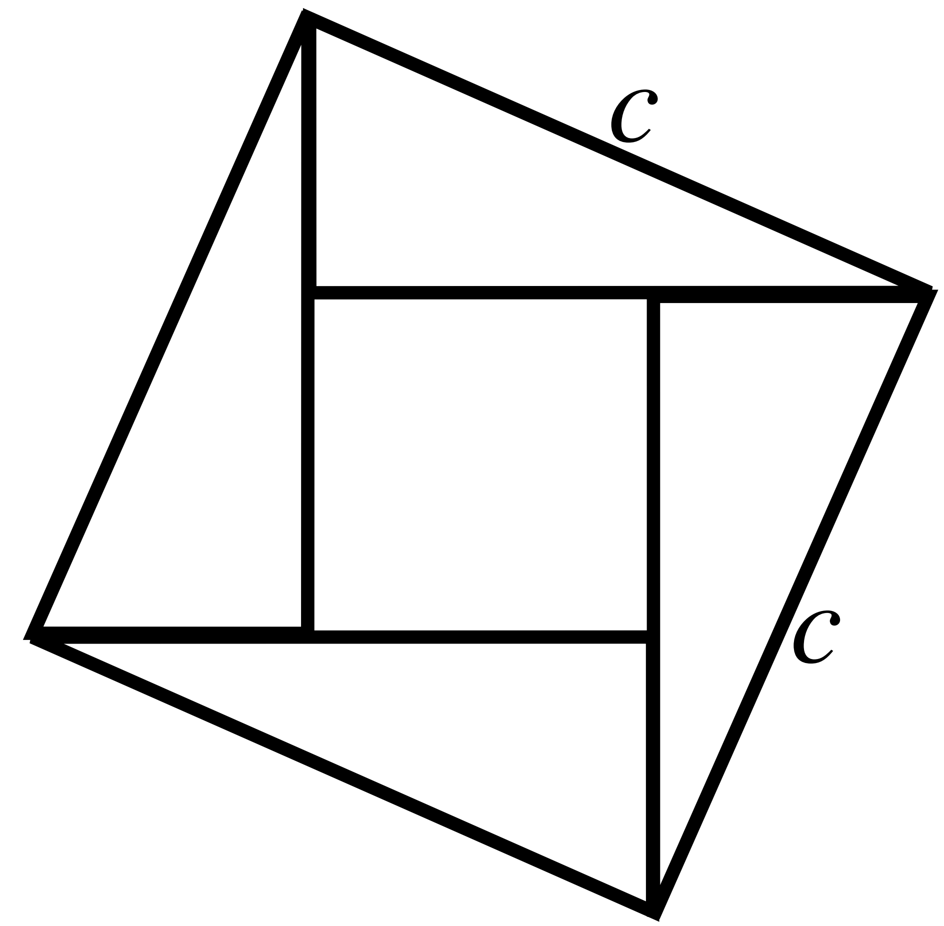
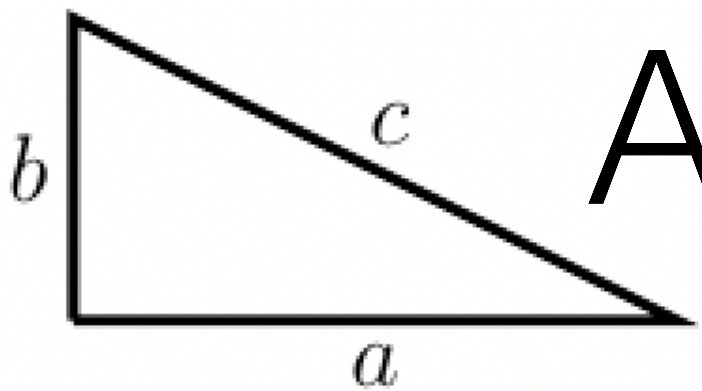






4 triangles + little square = Big square

$$4 \cdot \left(\frac{1}{2} a \cdot b\right) \quad + \quad (a-b)^2 \quad = \quad c^2$$



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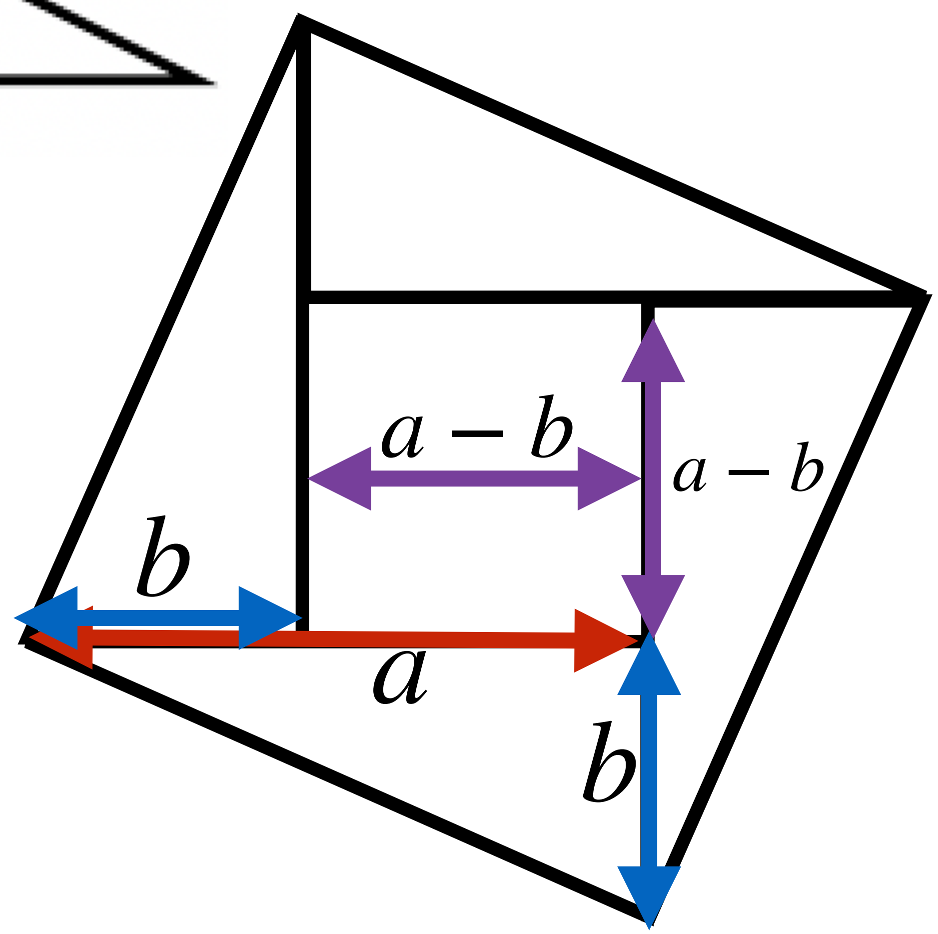
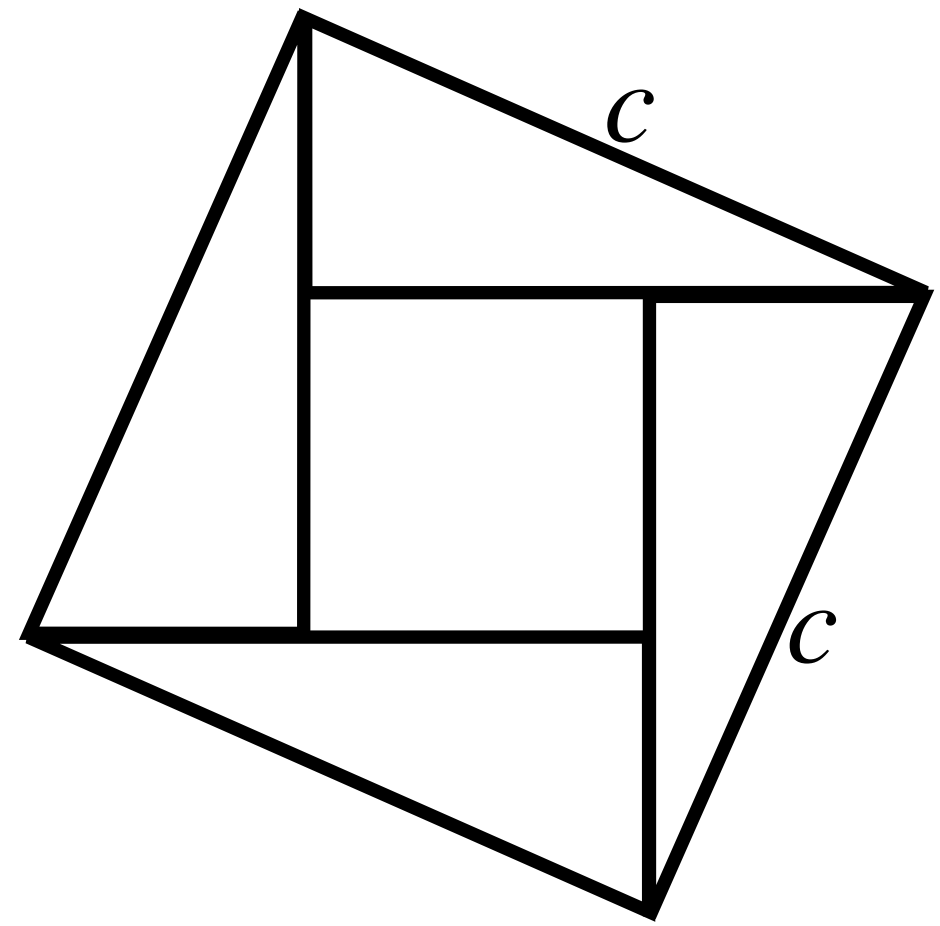
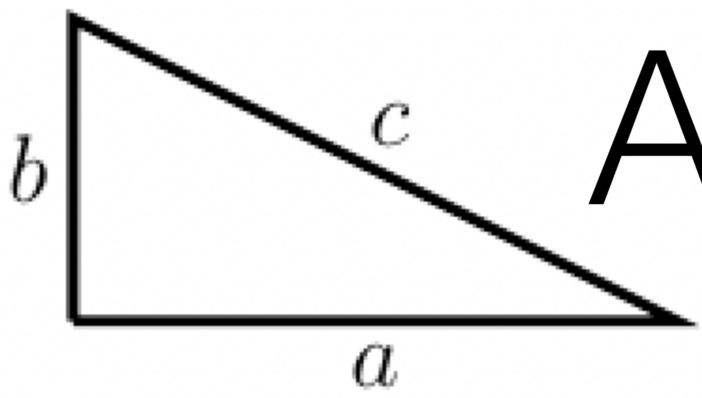




4 triangles + little square = Big square

$$4 \cdot \left(\frac{1}{2} a \cdot b\right) + (a-b)^2 = c^2$$

$$2ab + a^2 + b^2 - 2ab = c^2$$



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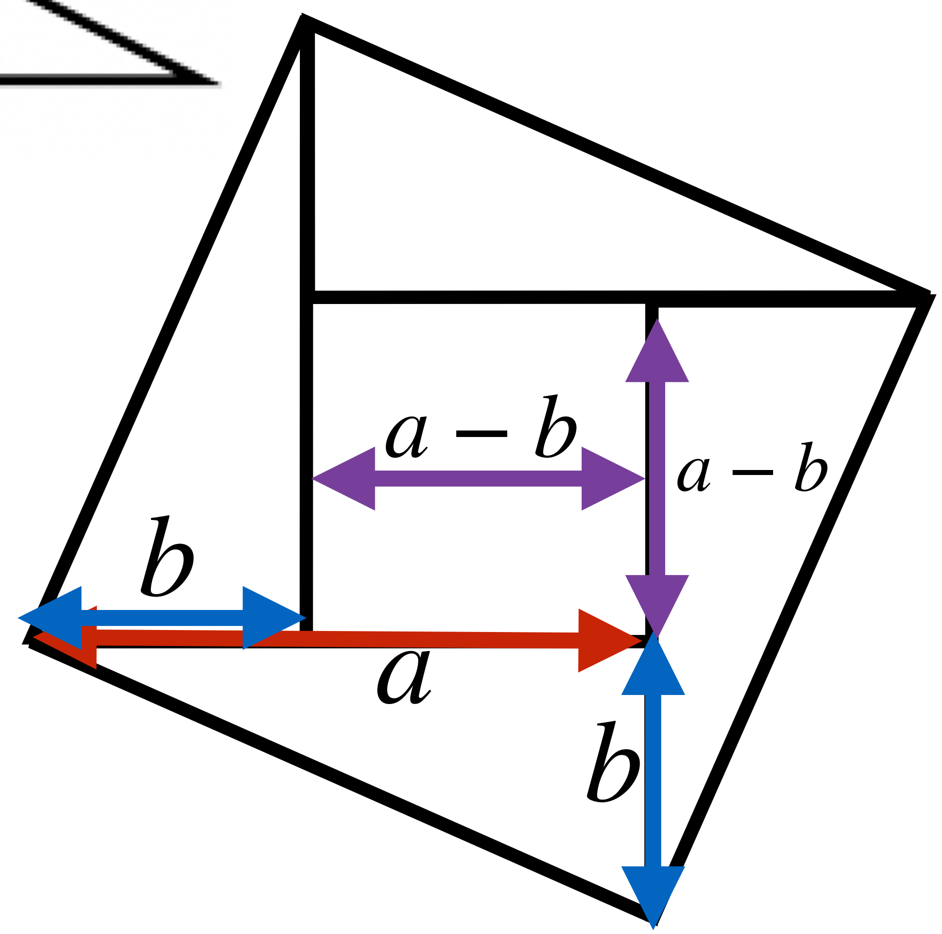
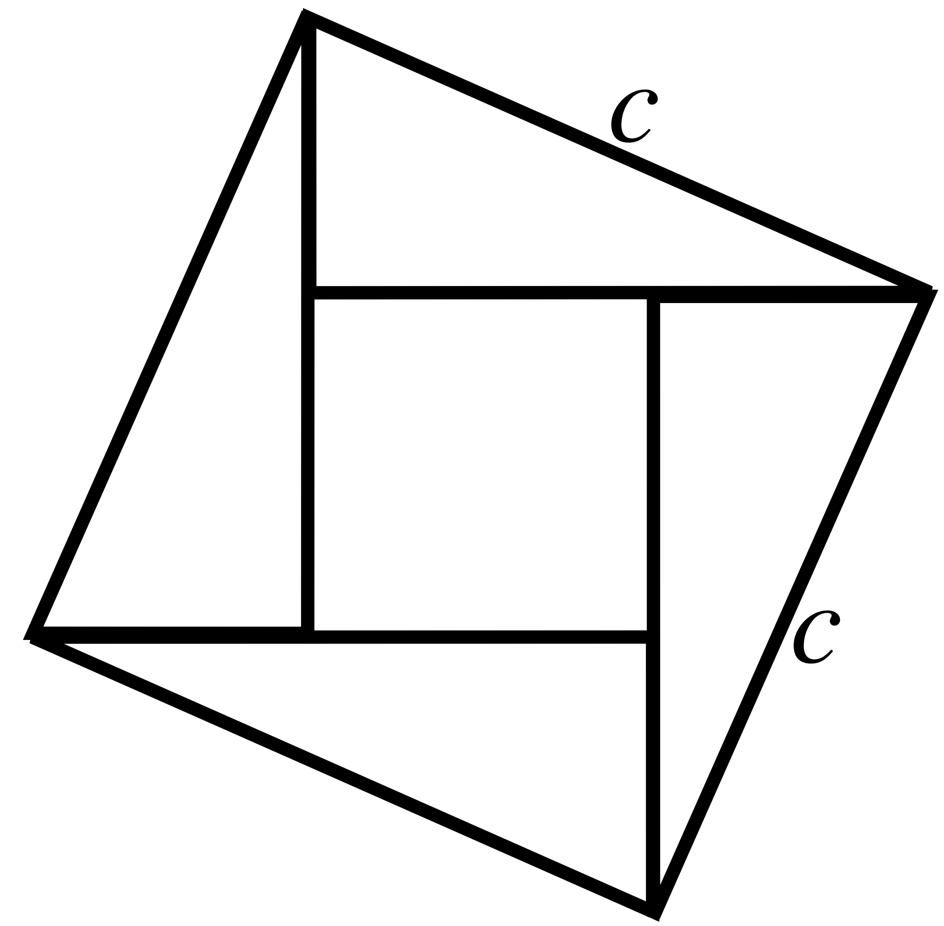
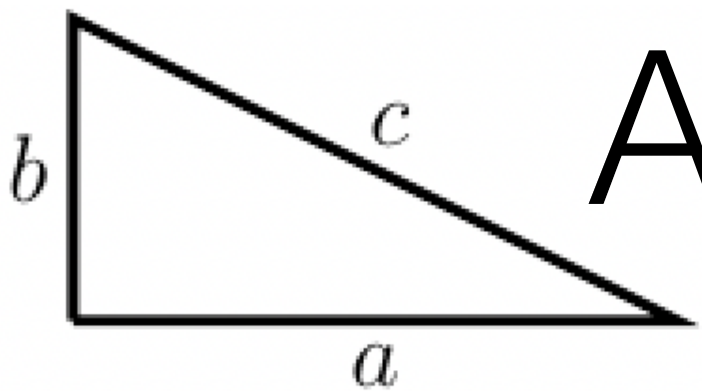




4 triangles + little square = Big square

$$4 \cdot \left(\frac{1}{2} a \cdot b\right) \quad + \quad (a-b)^2 \quad = \quad c^2$$

$$\cancel{2ab} + a^2 + b^2 - \cancel{2ab} \quad = \quad c^2$$



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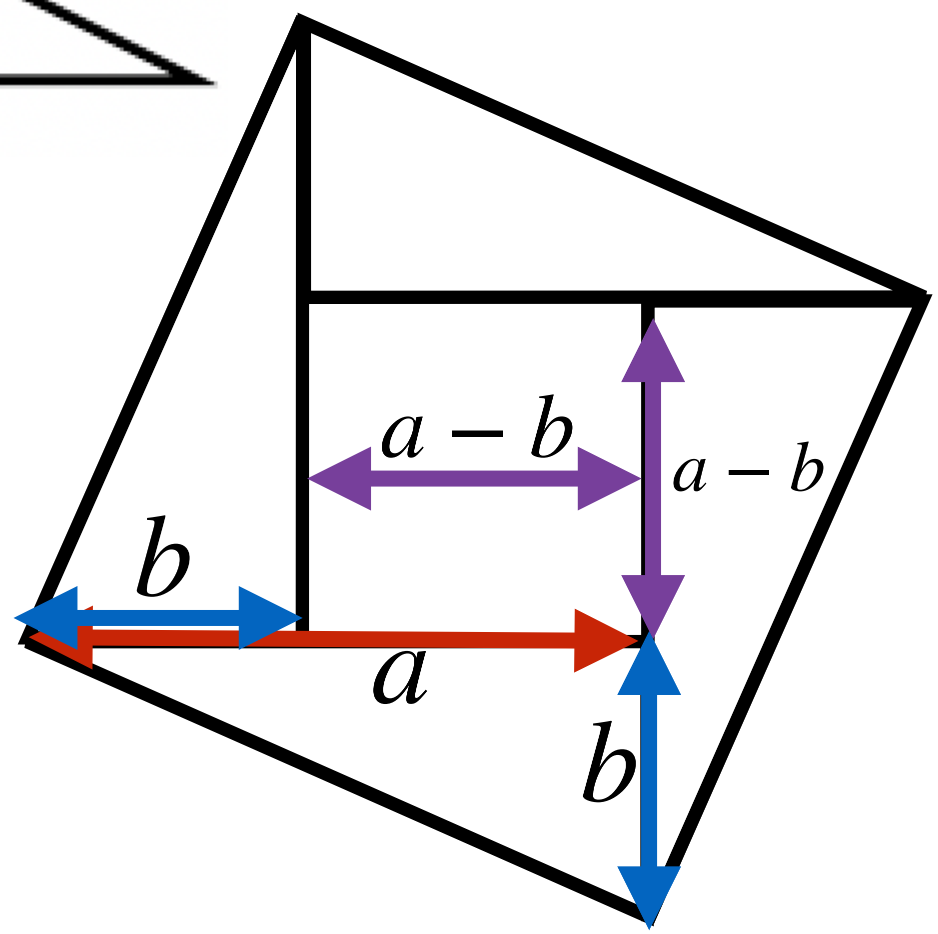

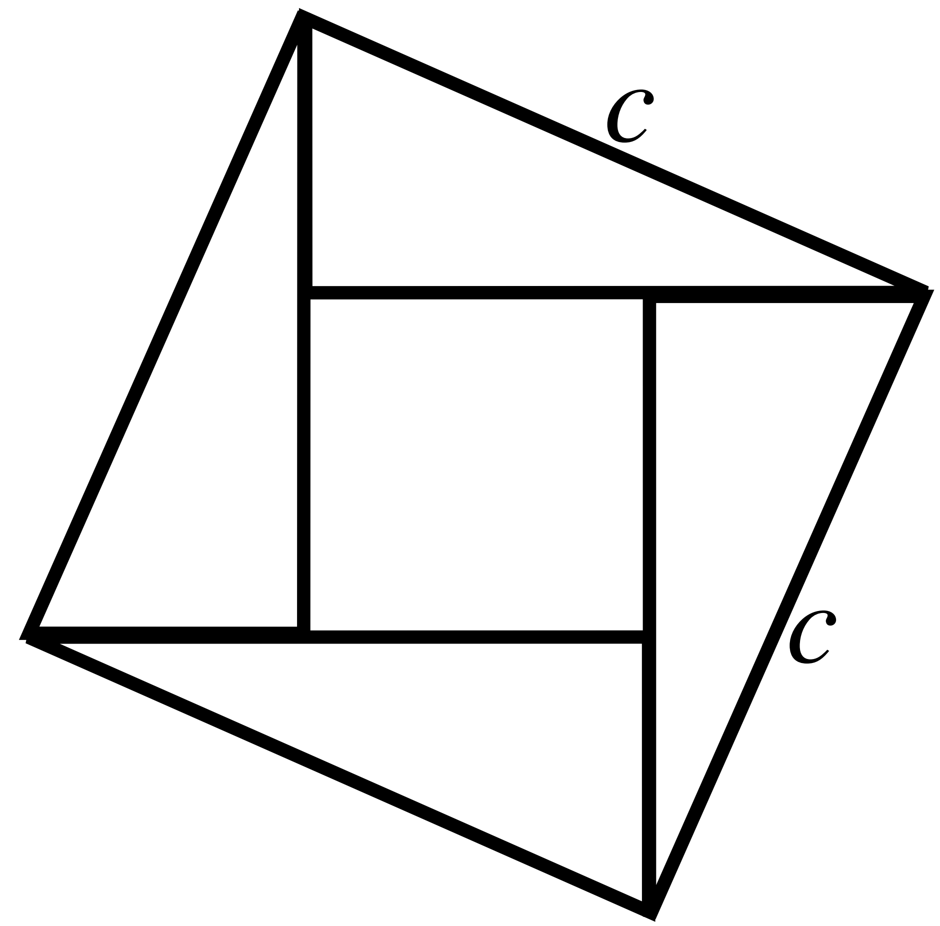

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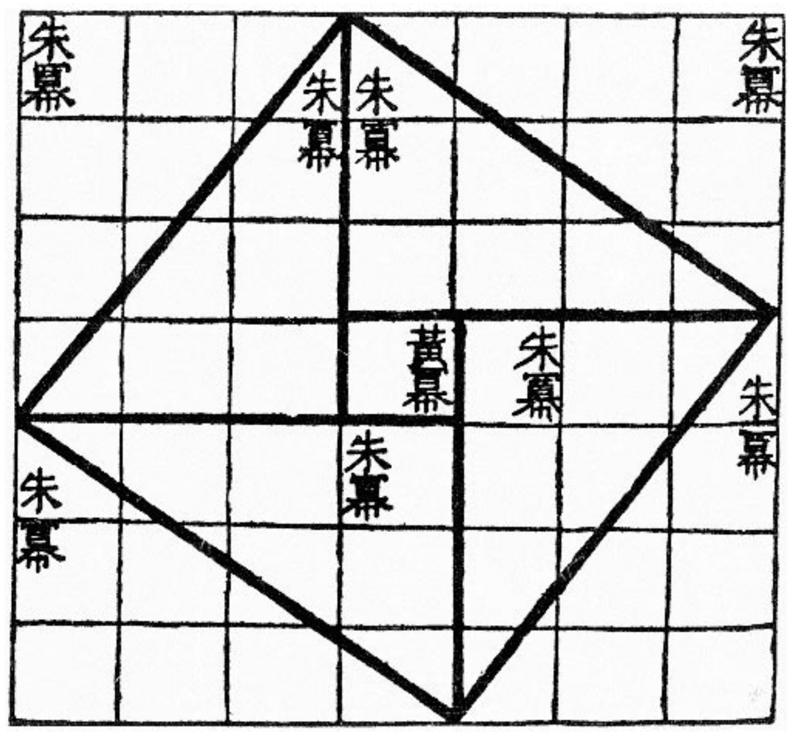
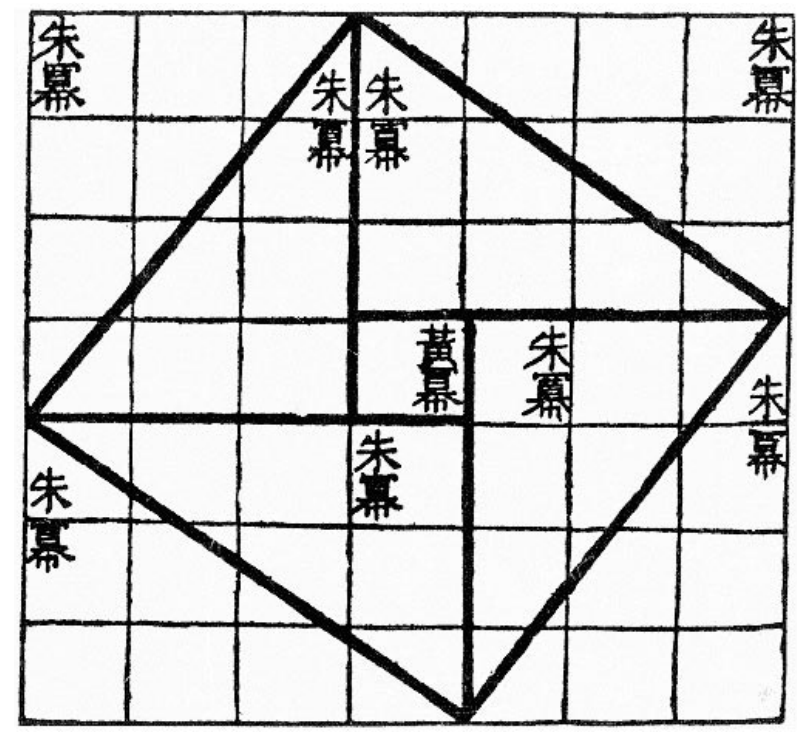
$$\cancel{2ab} + a^2 + b^2 - \cancel{2ab} \quad = \quad c^2$$

$$\boxed{a^2 + b^2 = c^2}$$

Second *Possible* Proof





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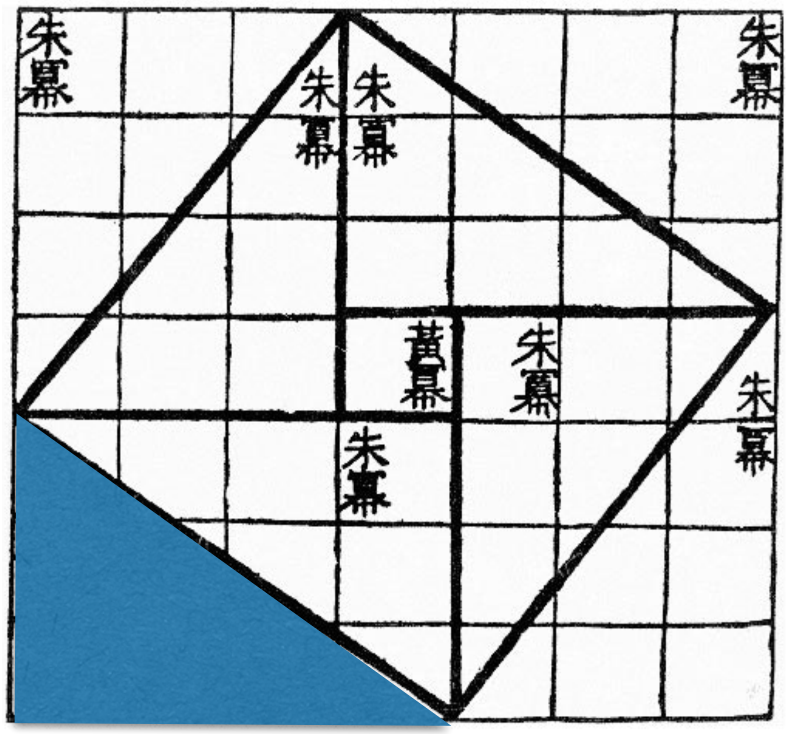

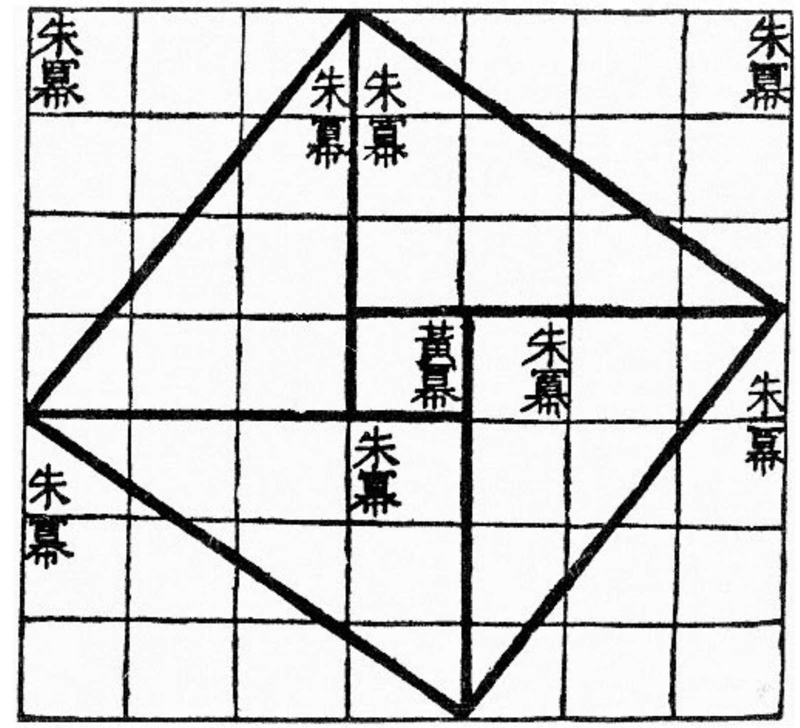

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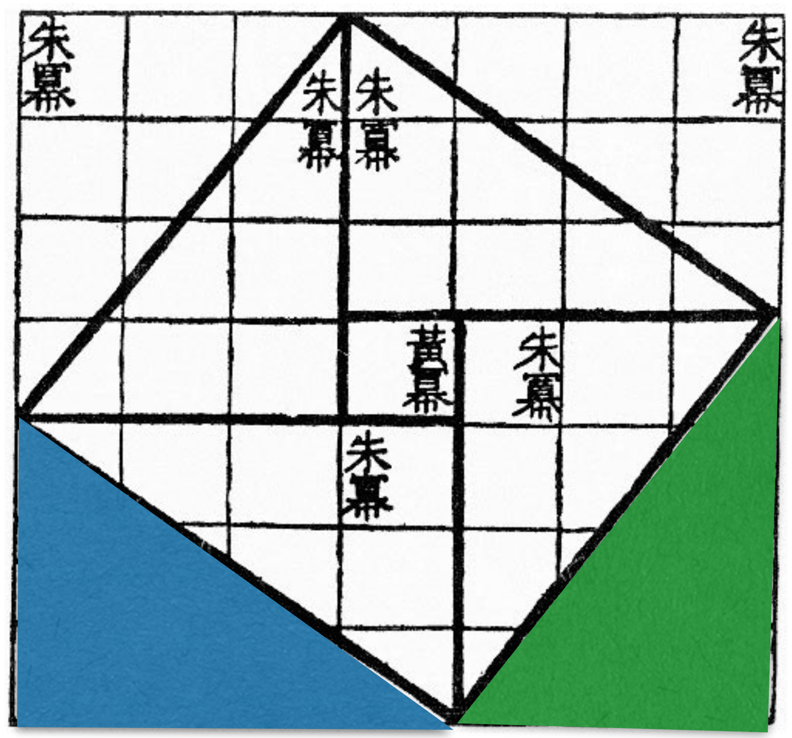

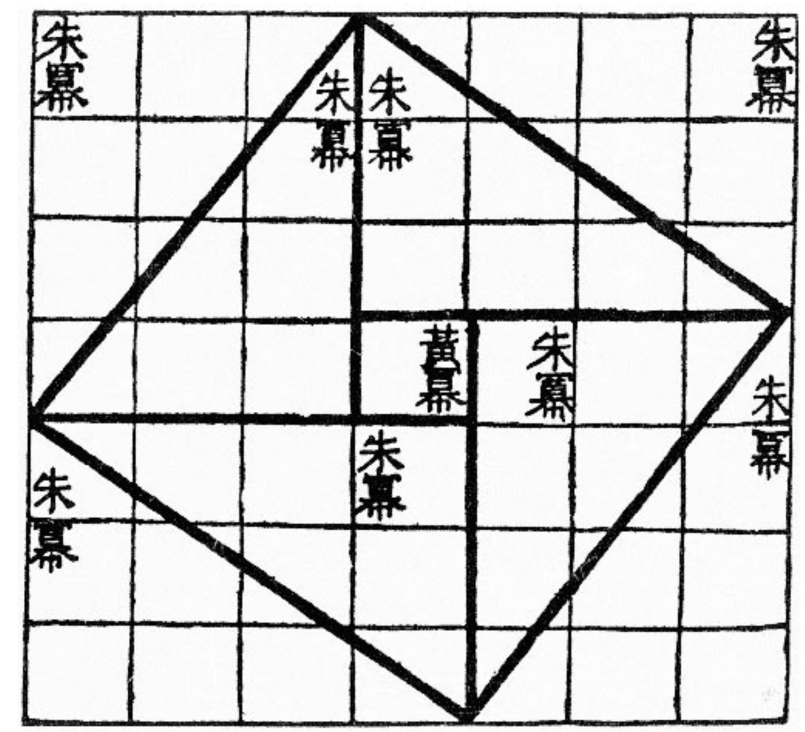

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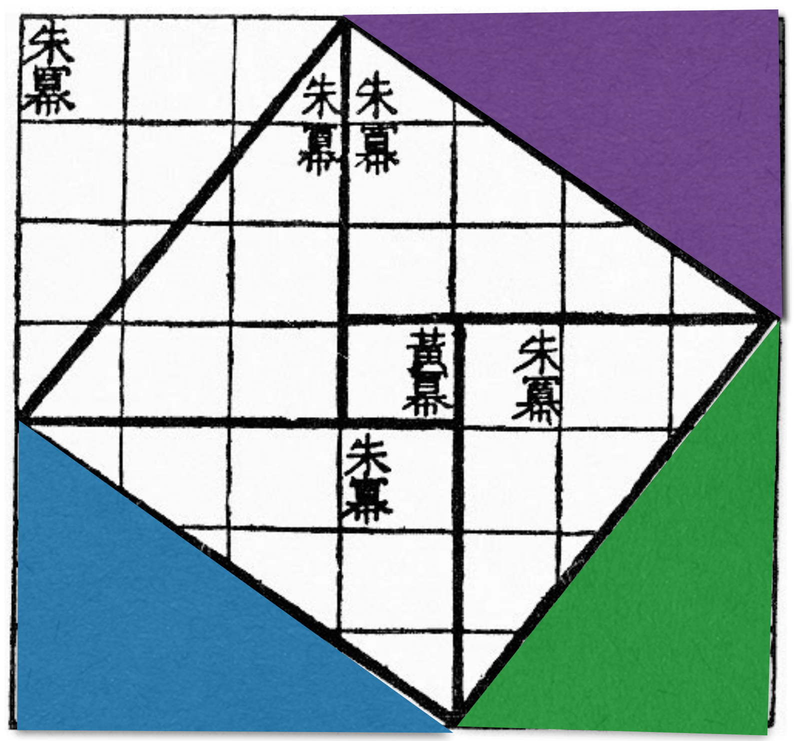

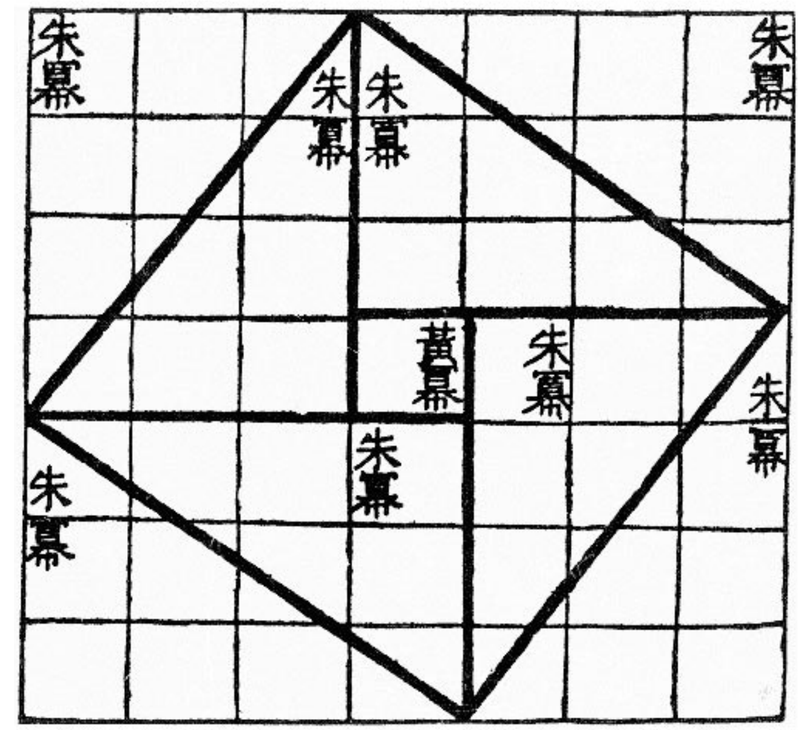

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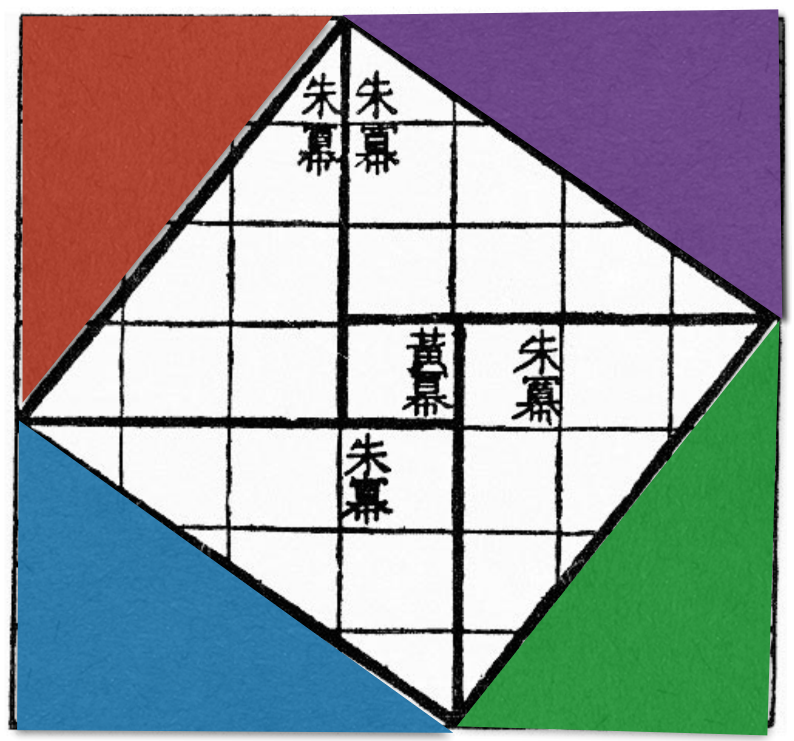

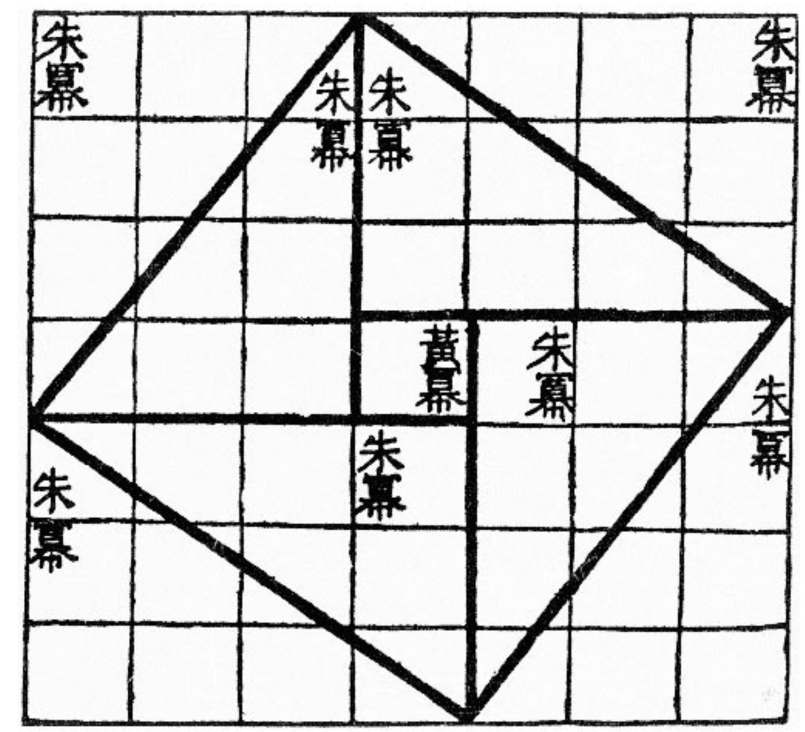

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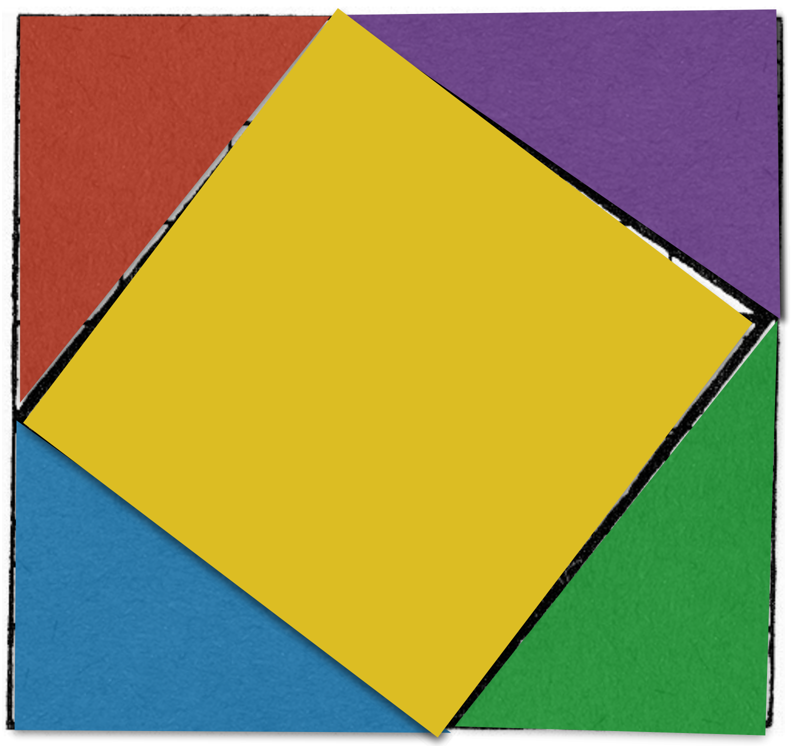

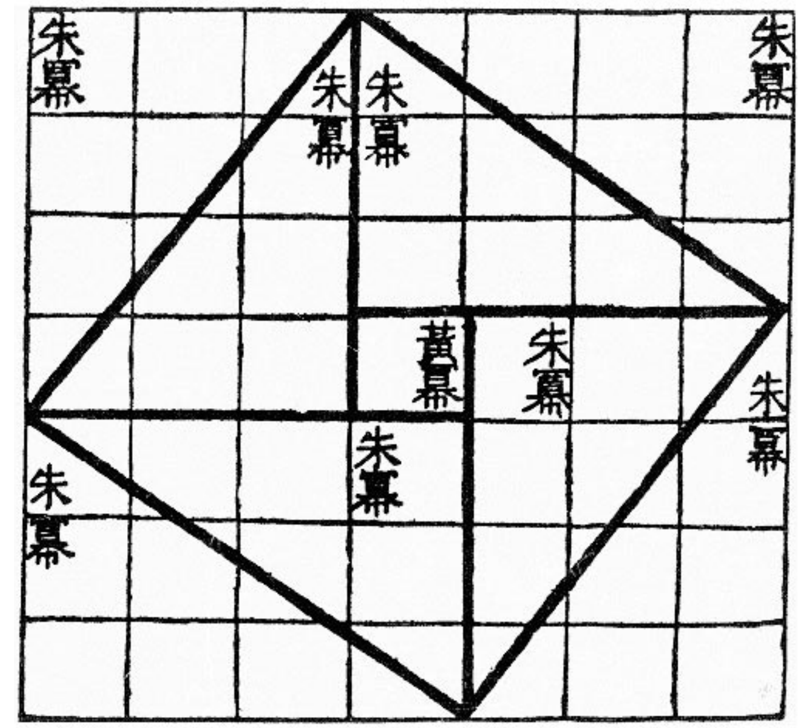

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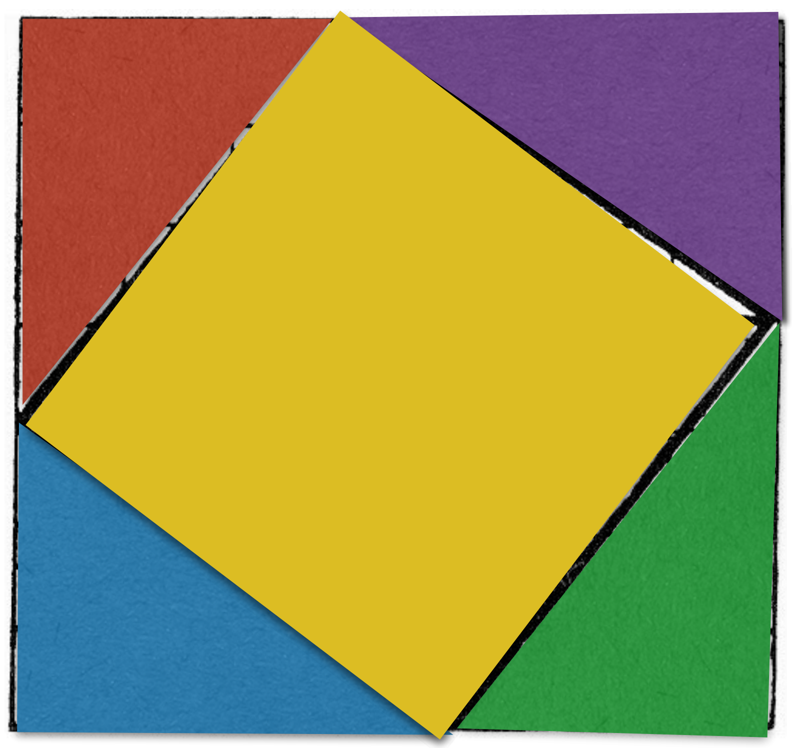

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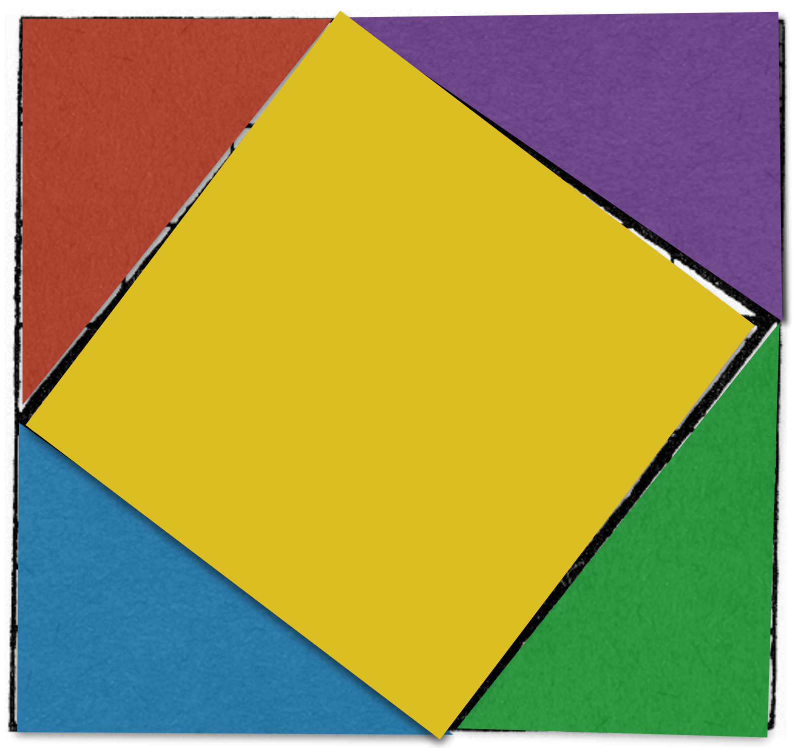
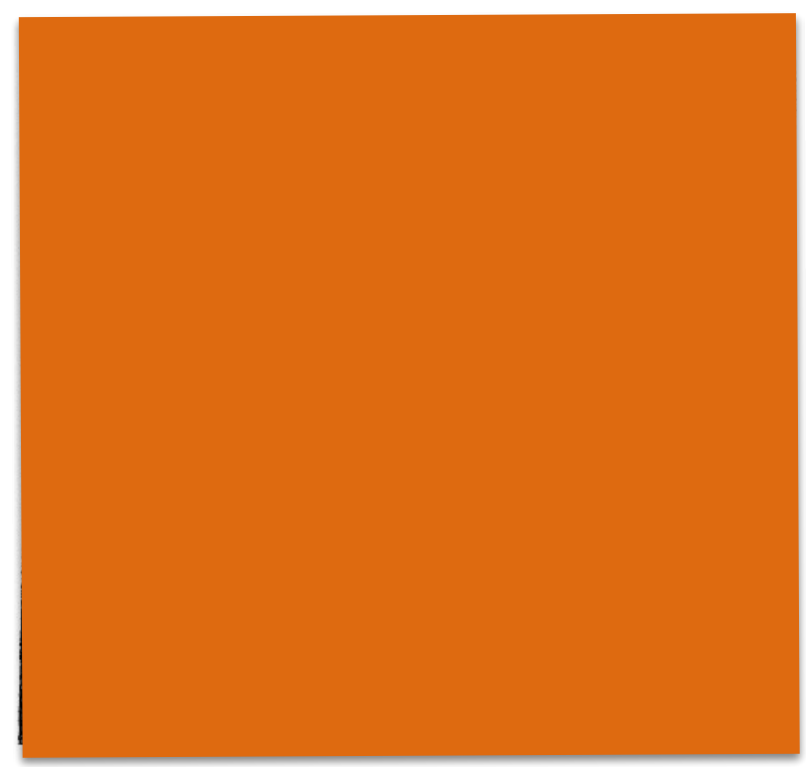
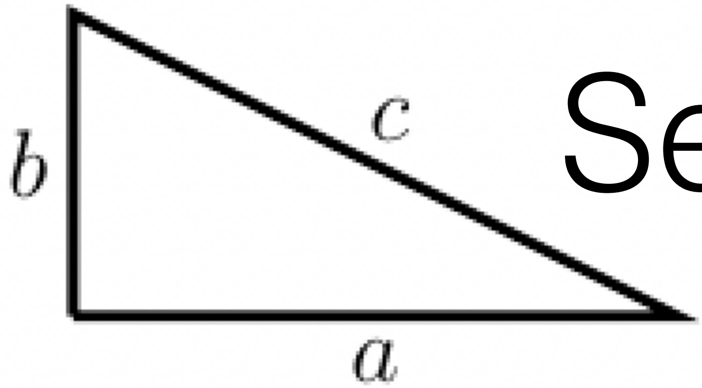




4 triangles + middle square = Whole square

Second *Possible* Proof



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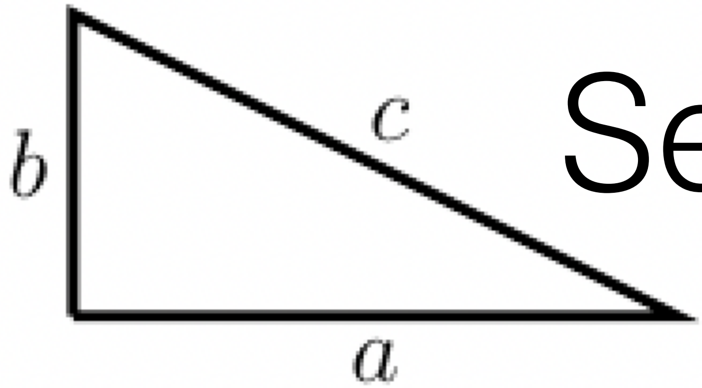

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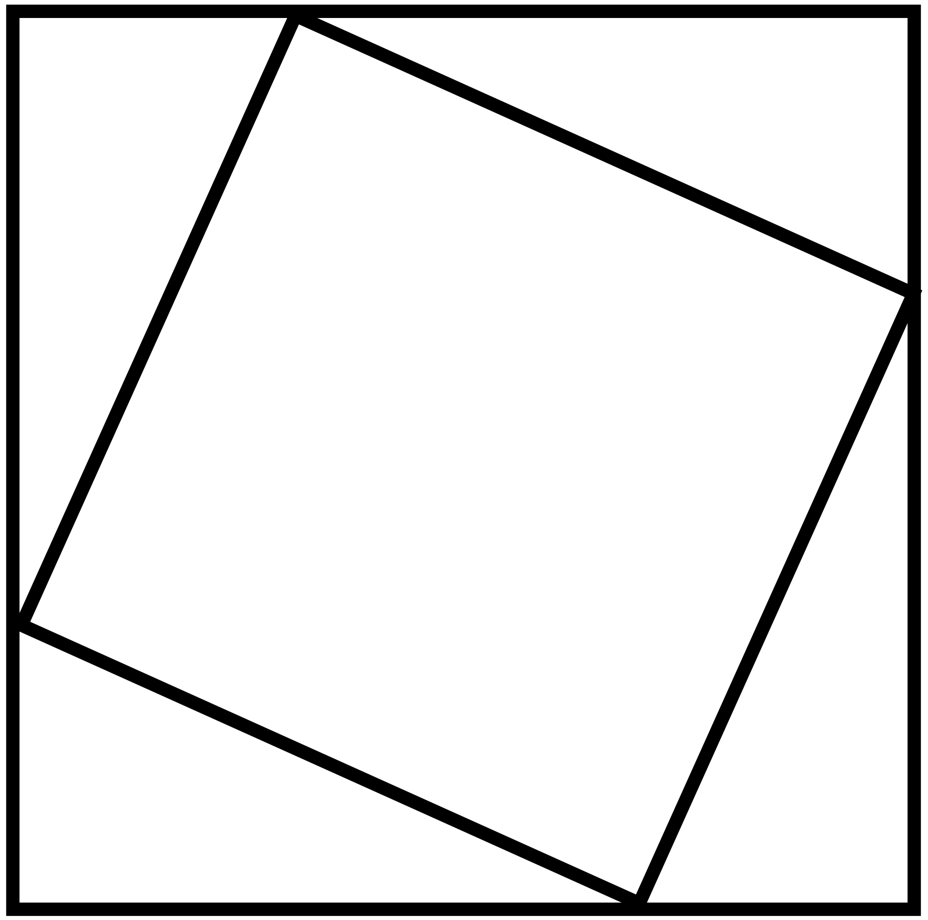

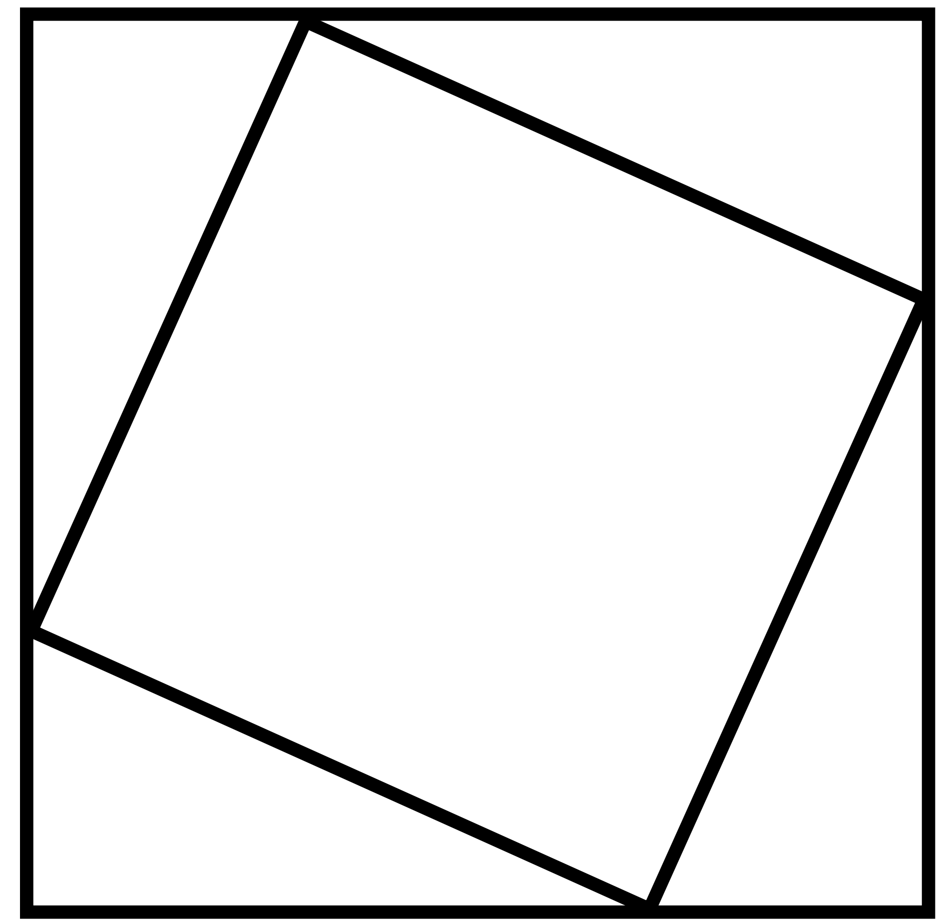

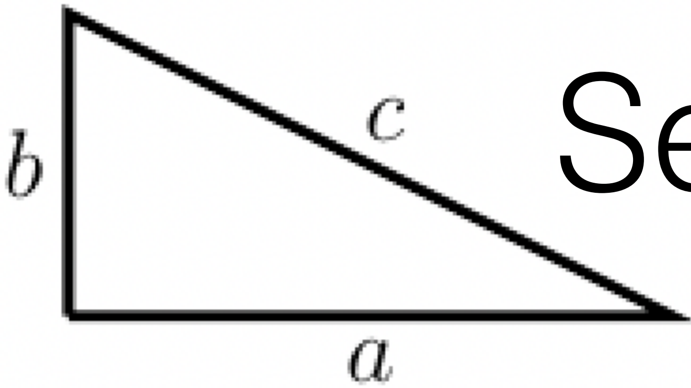

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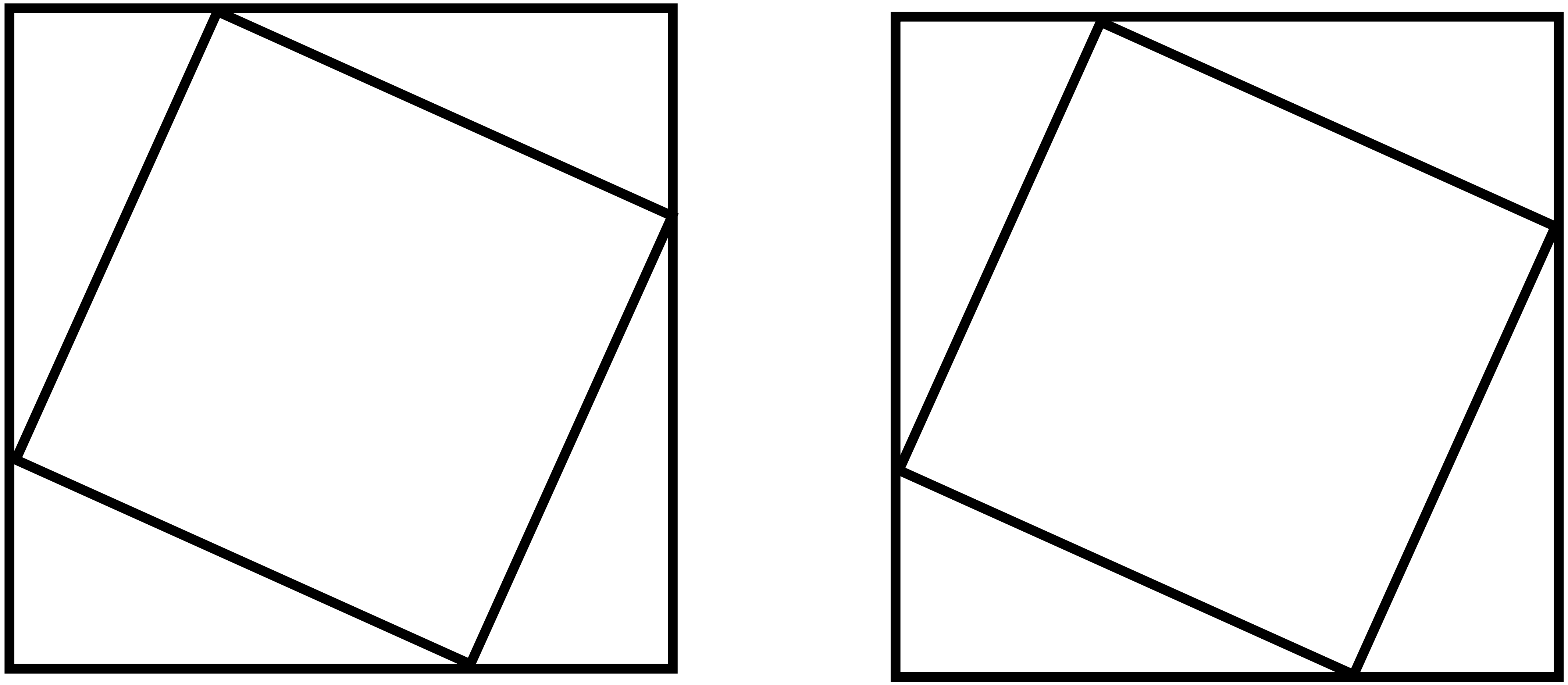

4 triangles + middle square = Whole square

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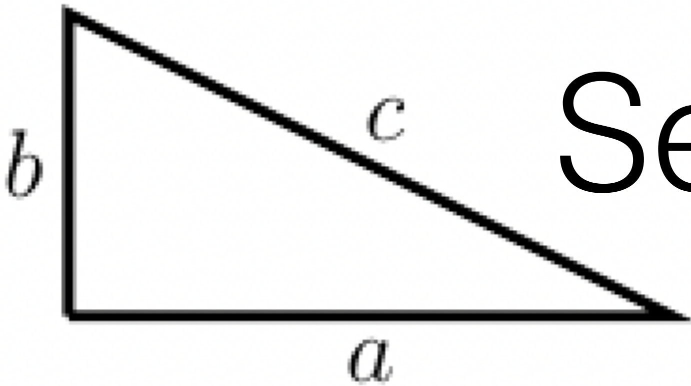

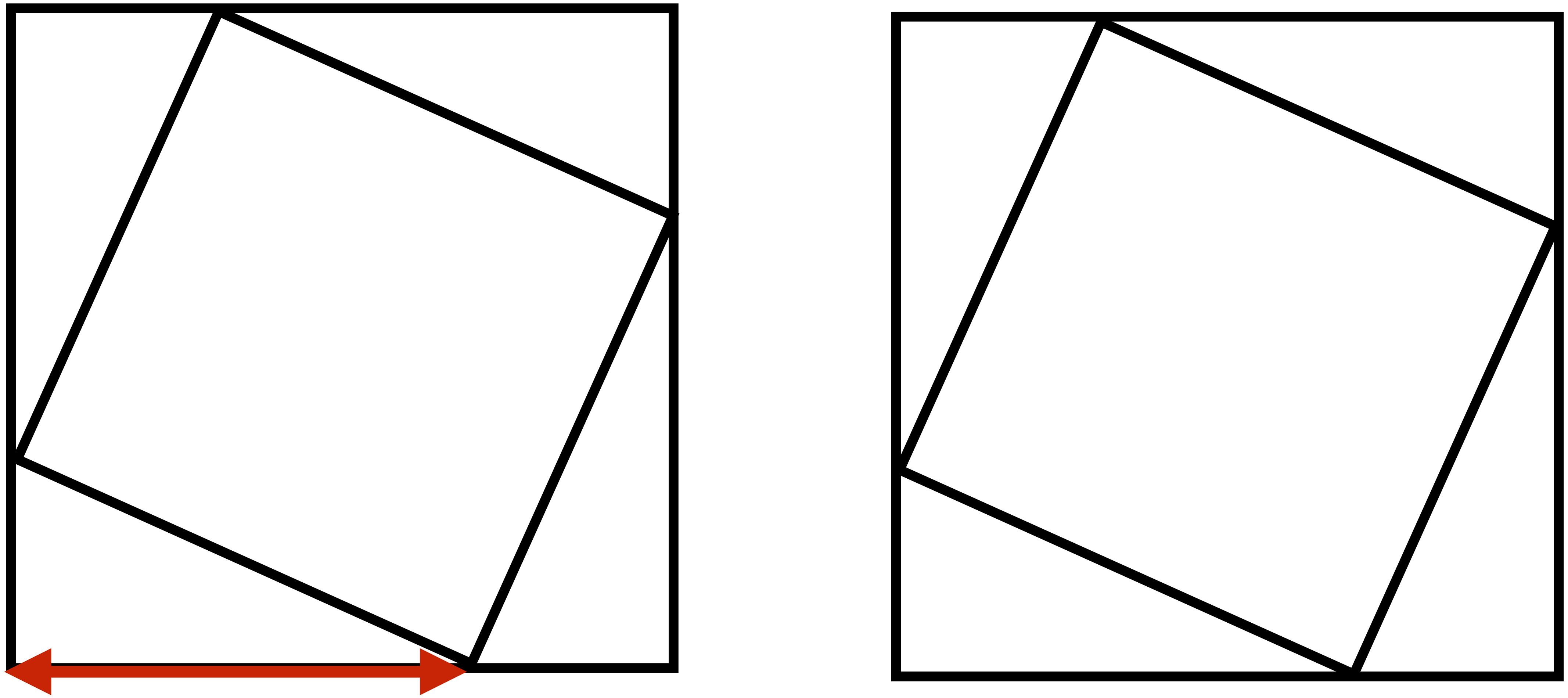

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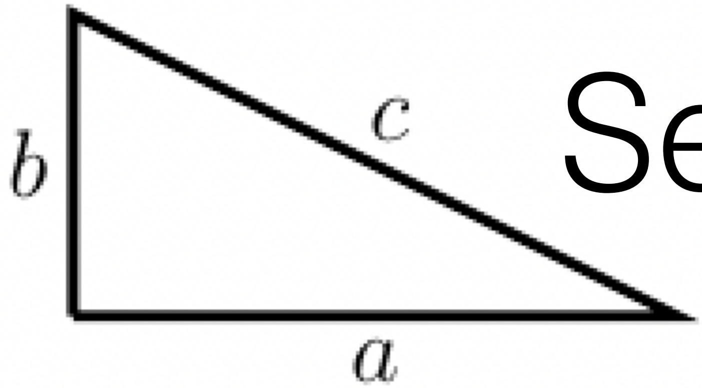

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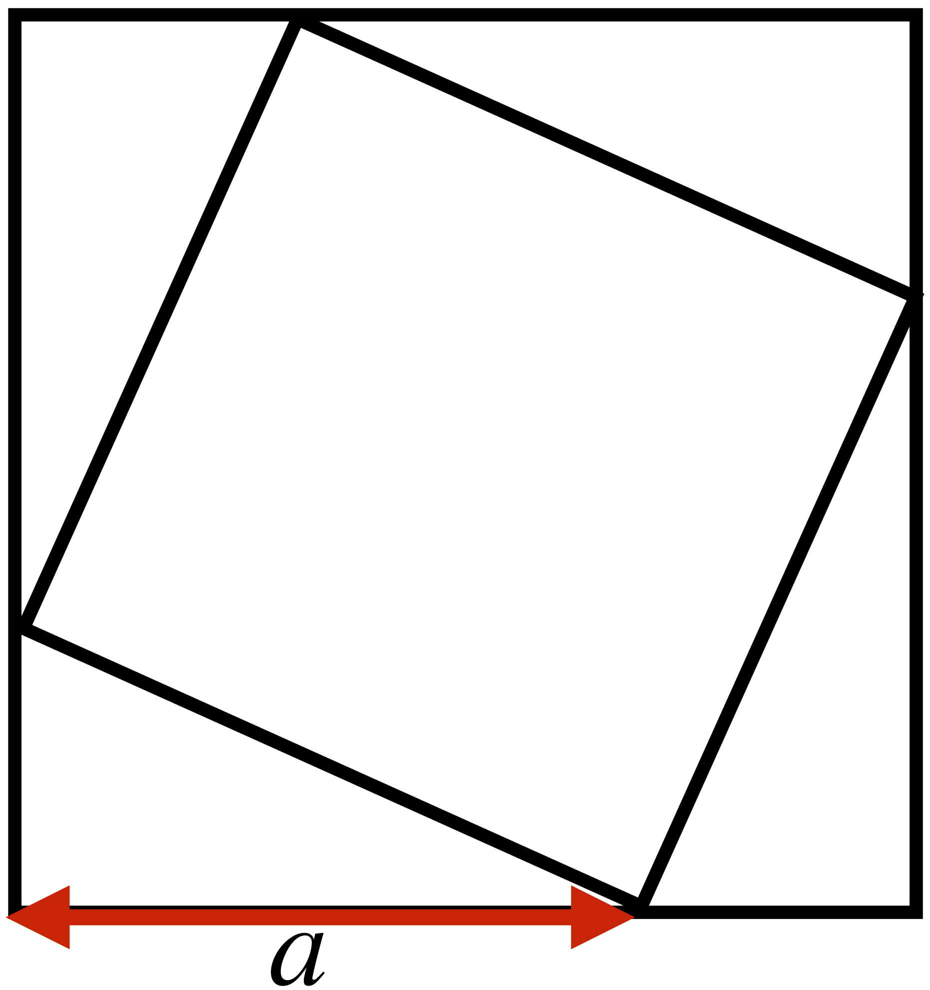

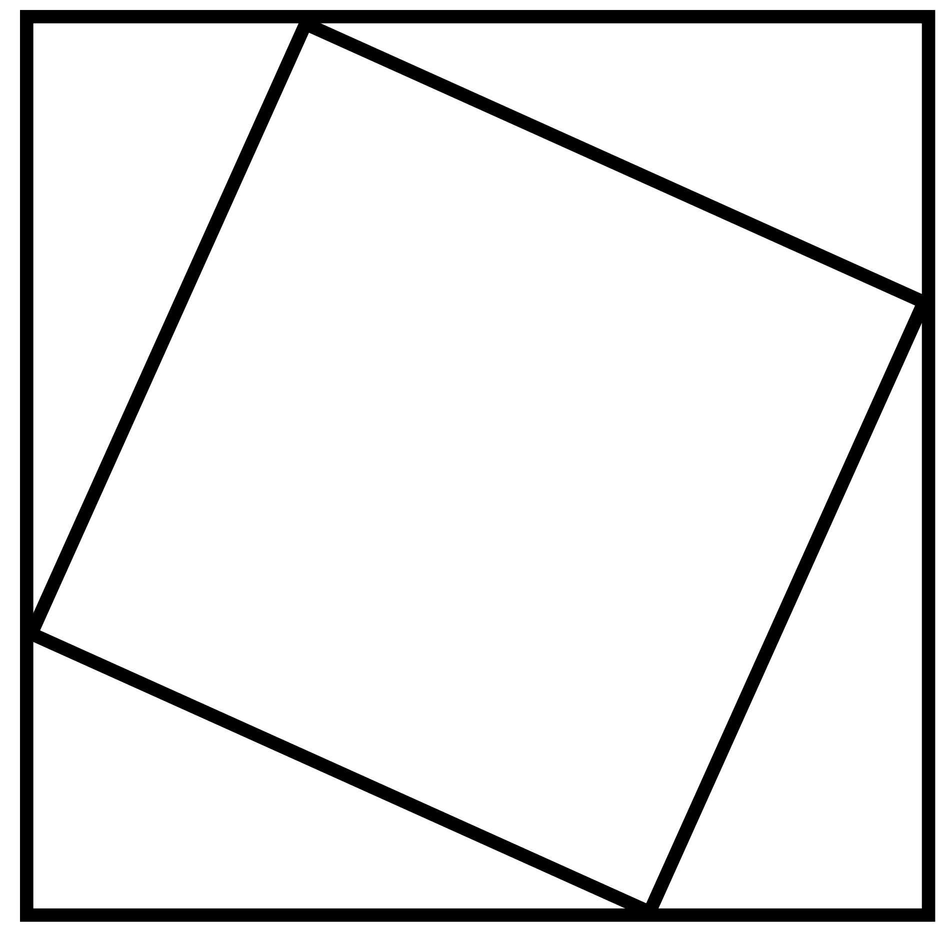

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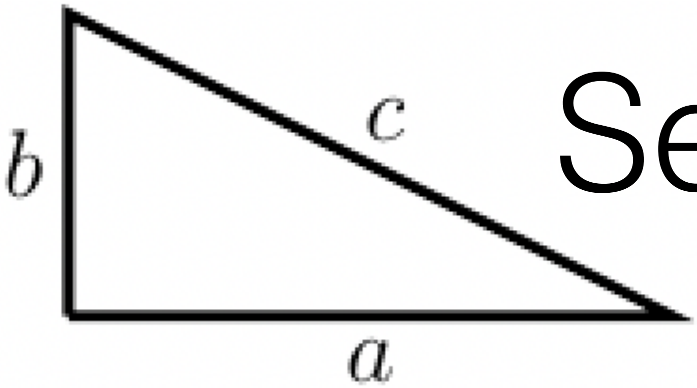

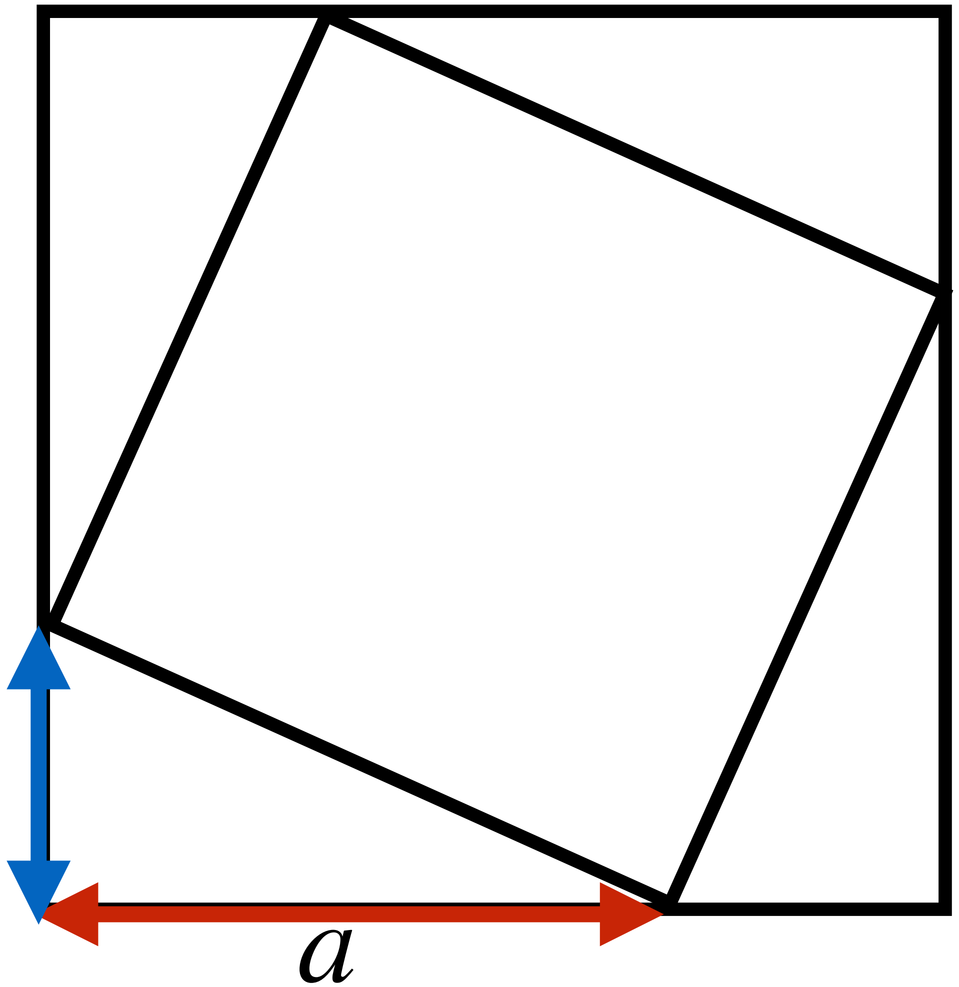

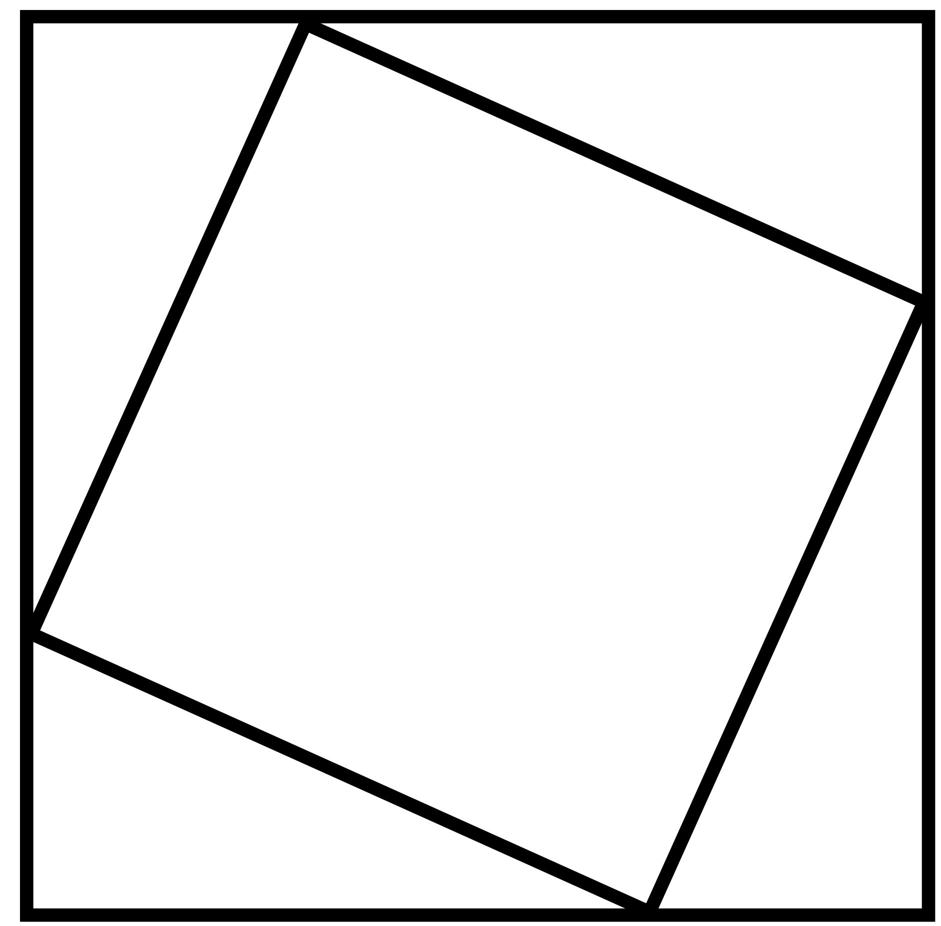

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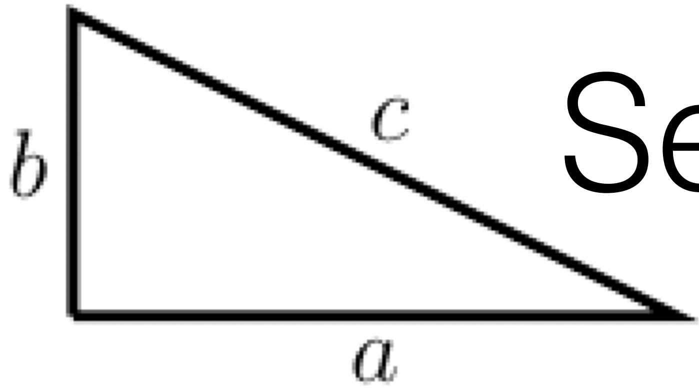

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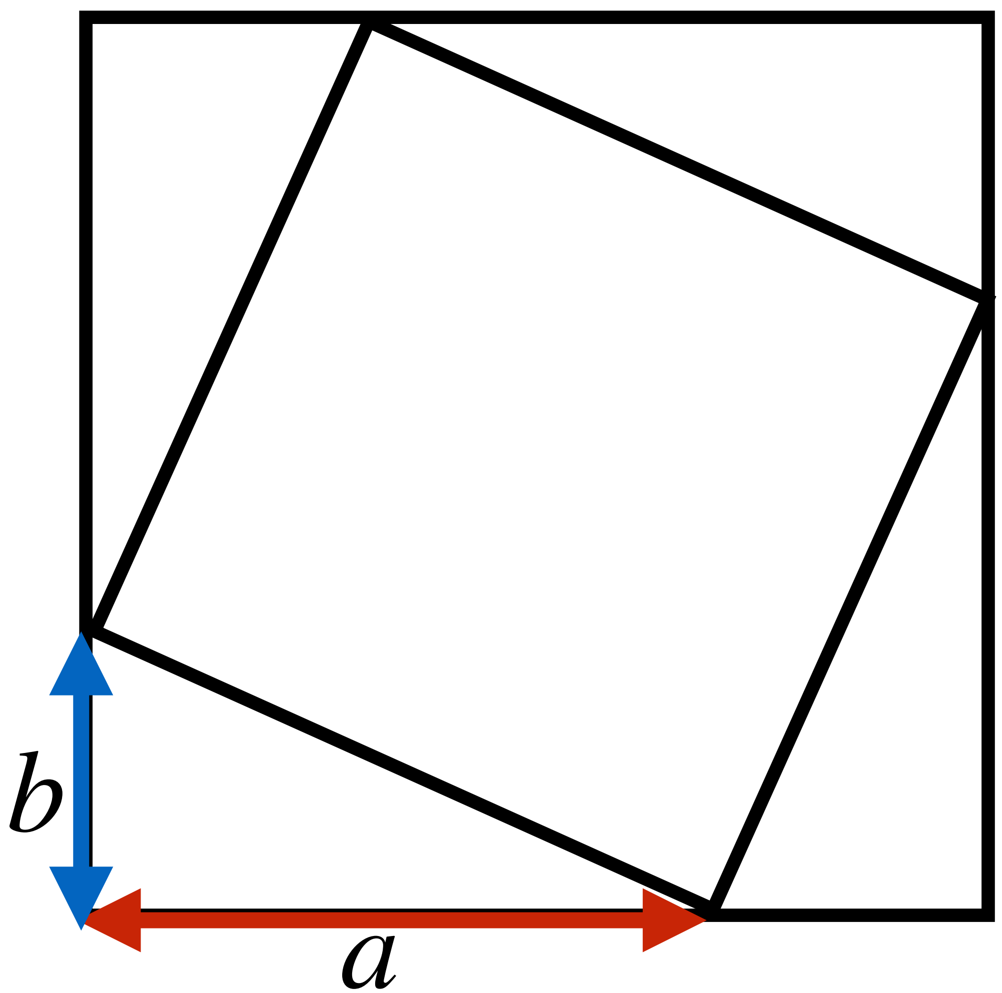

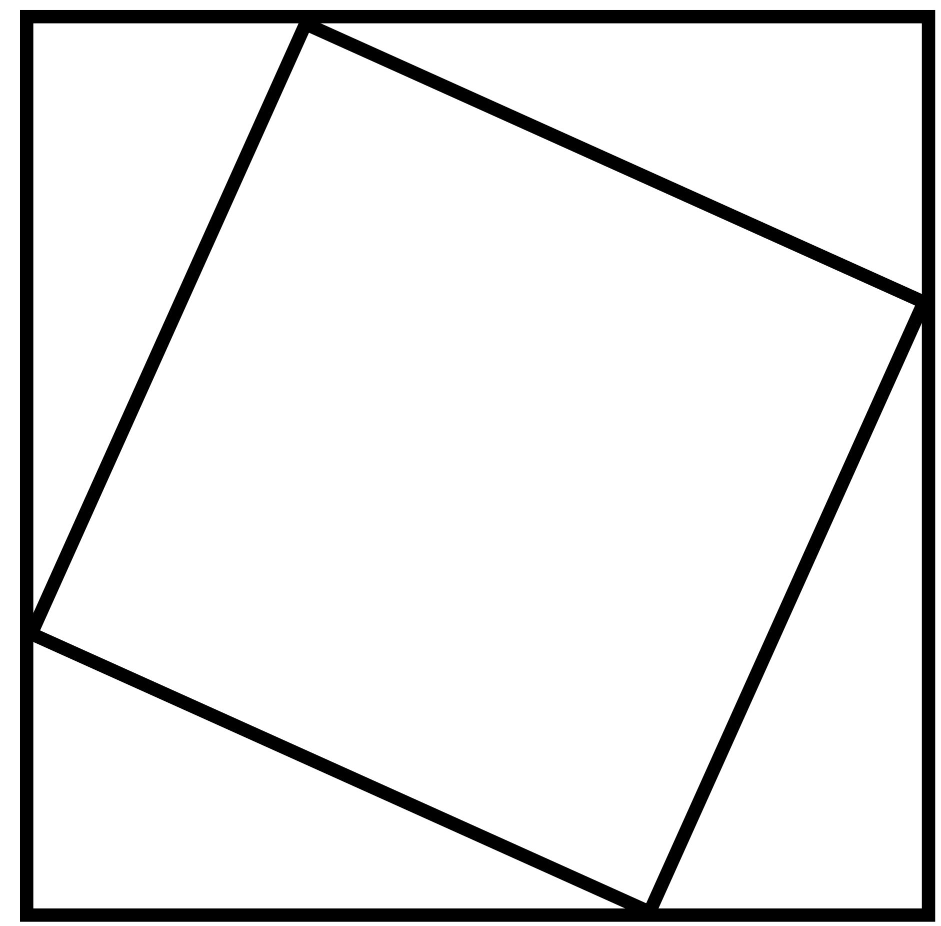

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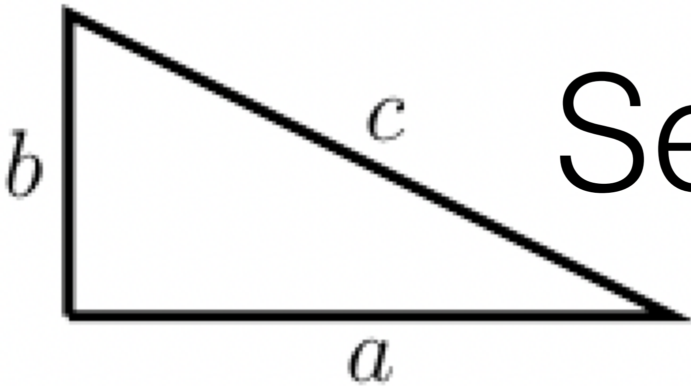
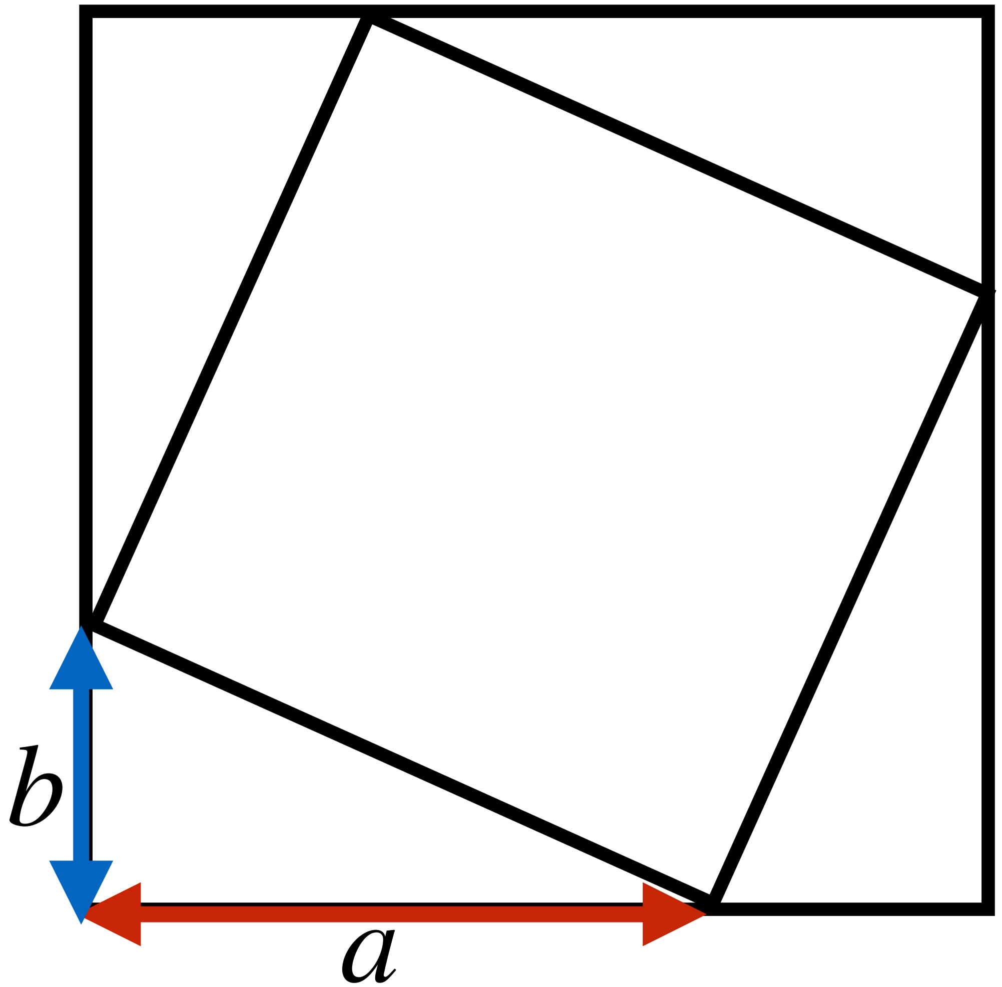
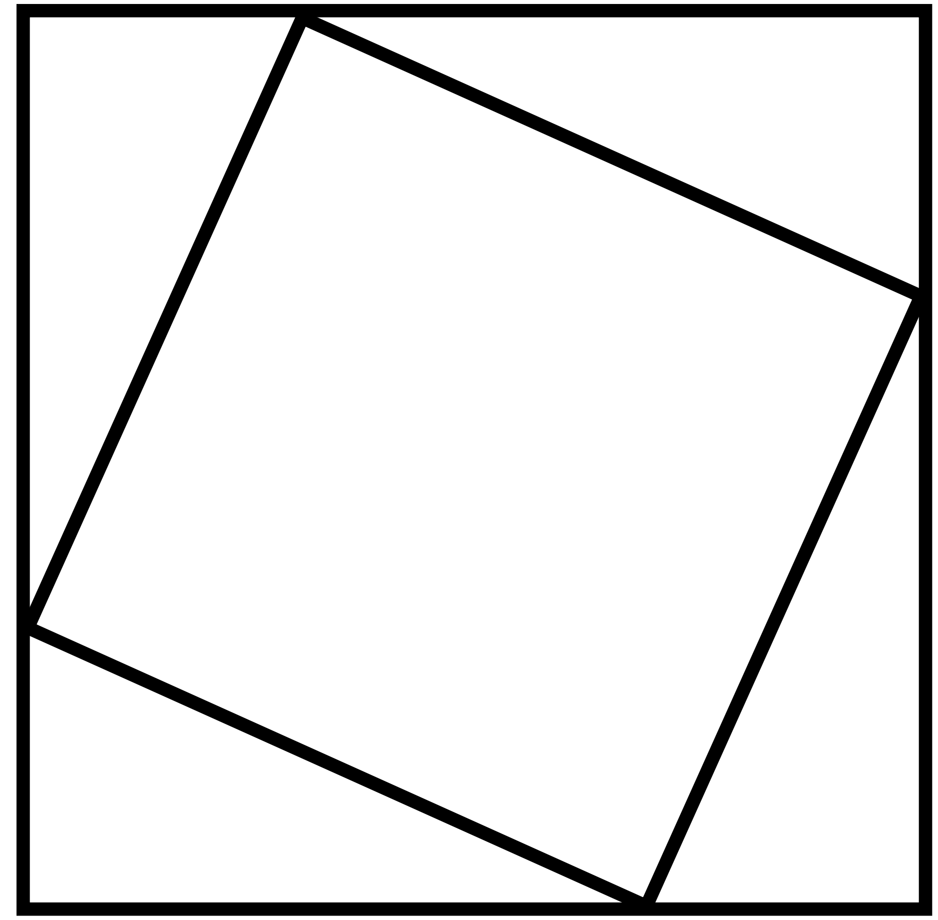


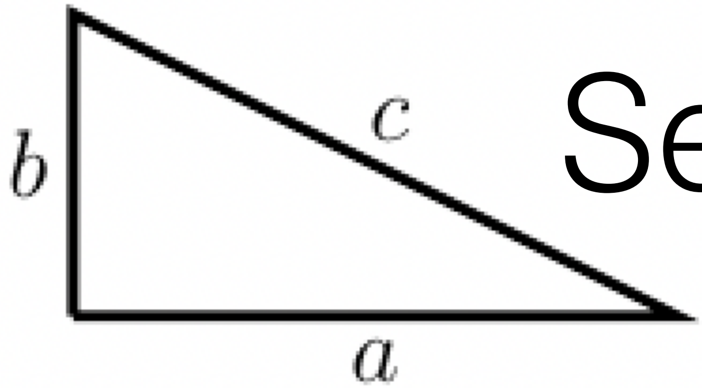


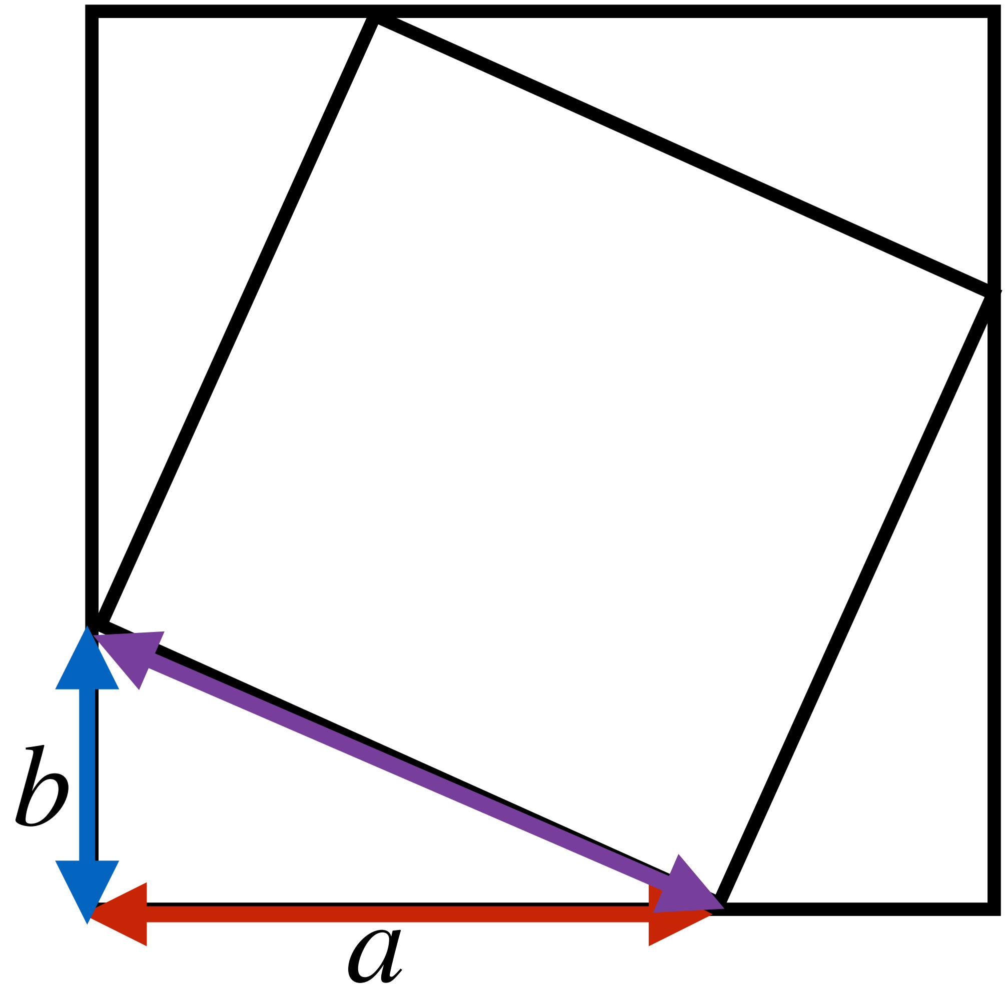


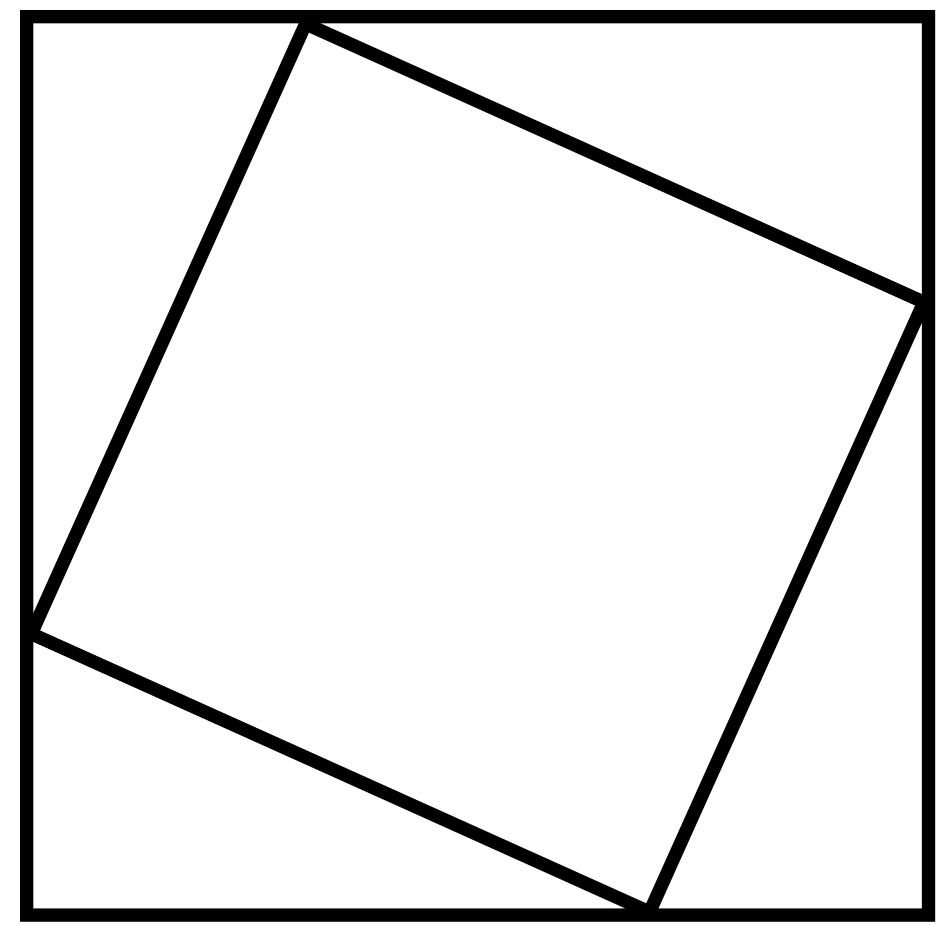
4 triangles + middle square = Whole square

$$4 \cdot \left(\frac{1}{2} a \cdot b \right) \quad +$$

Second *Possible* Proof







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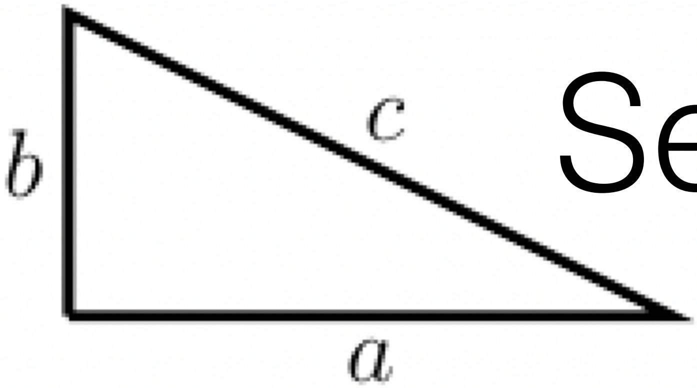

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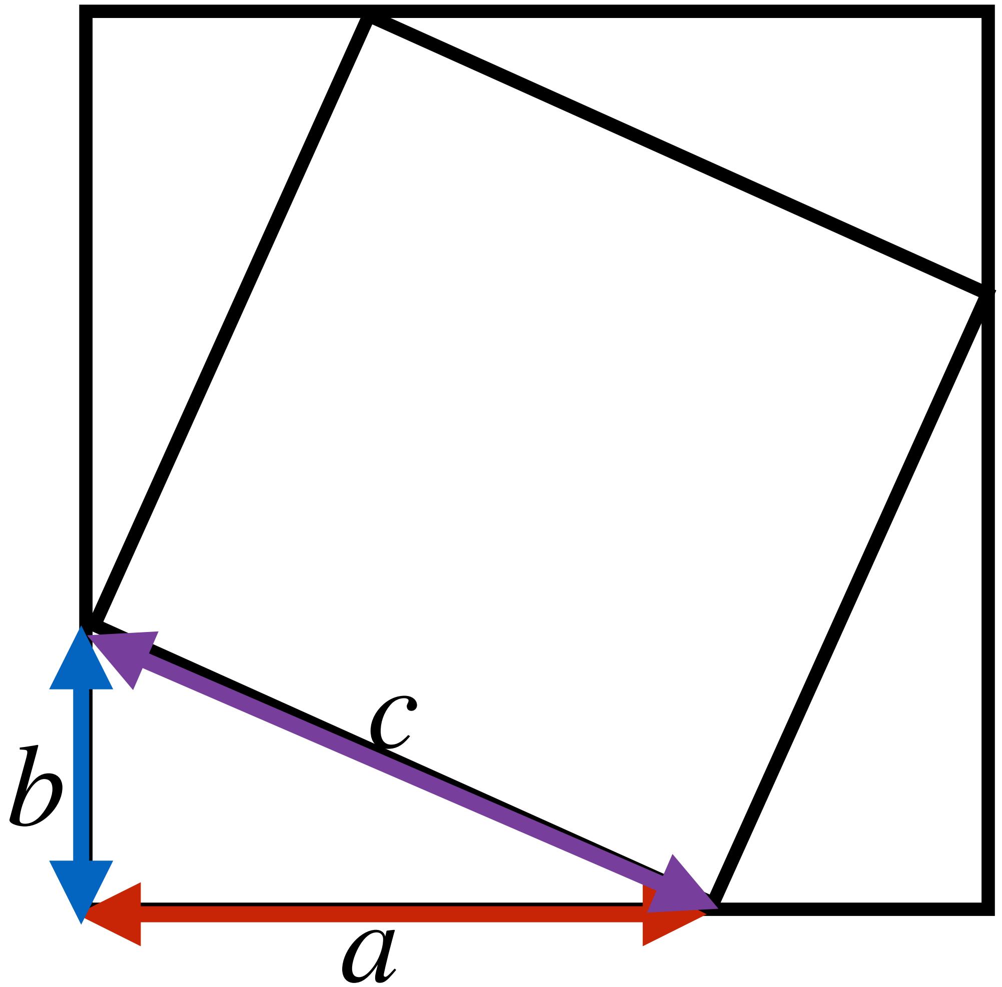

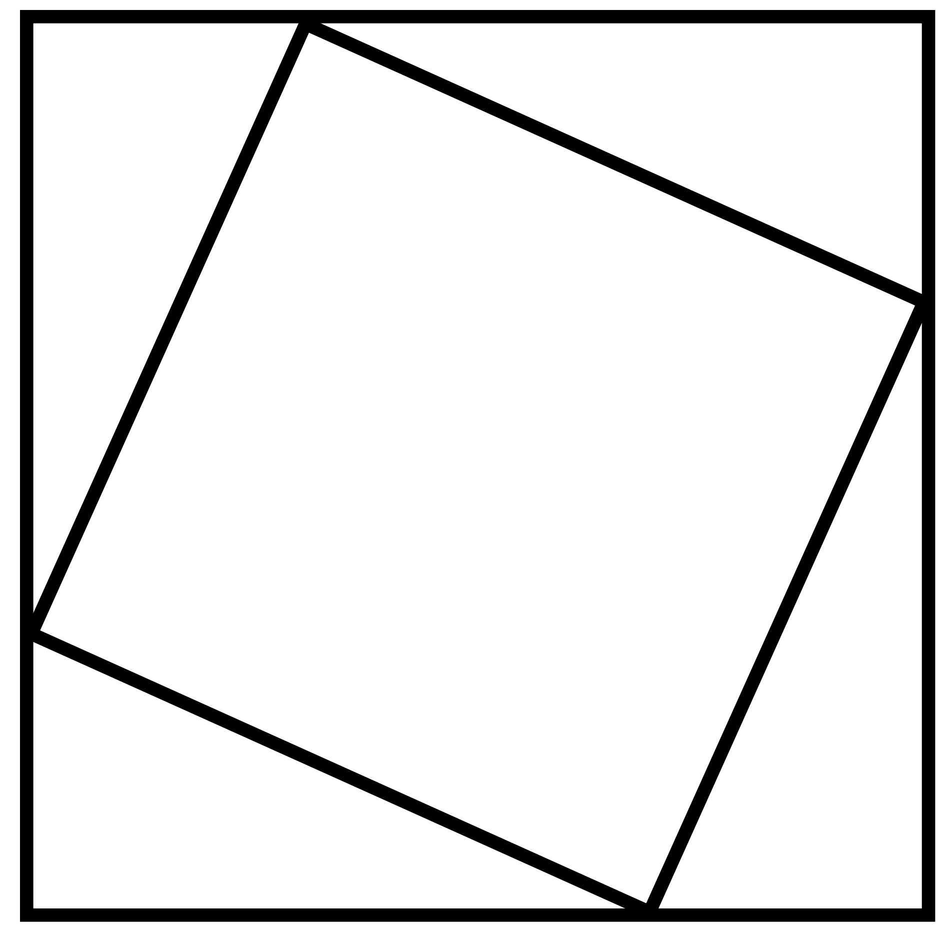

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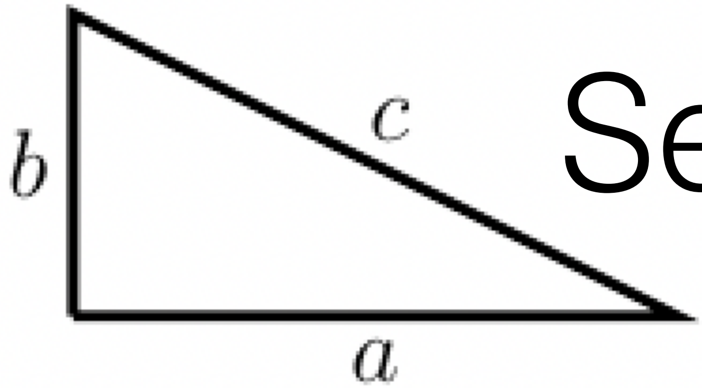

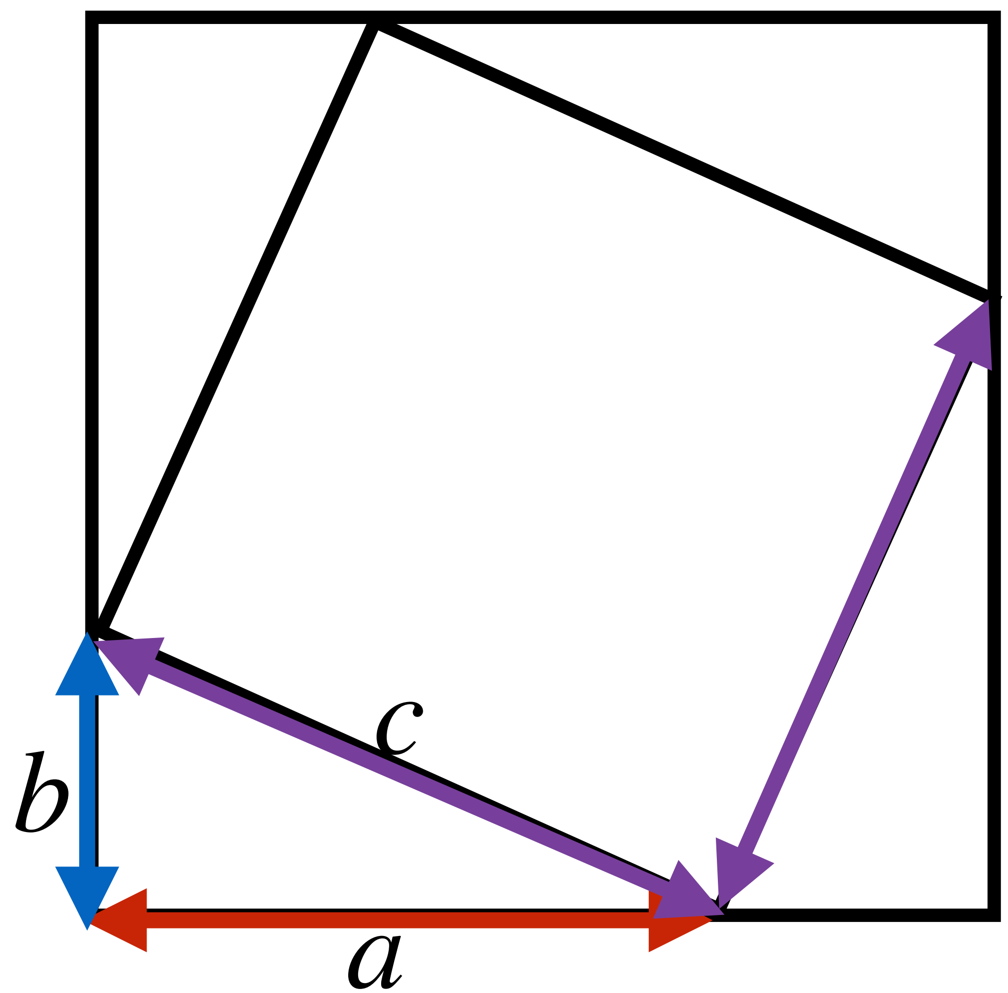

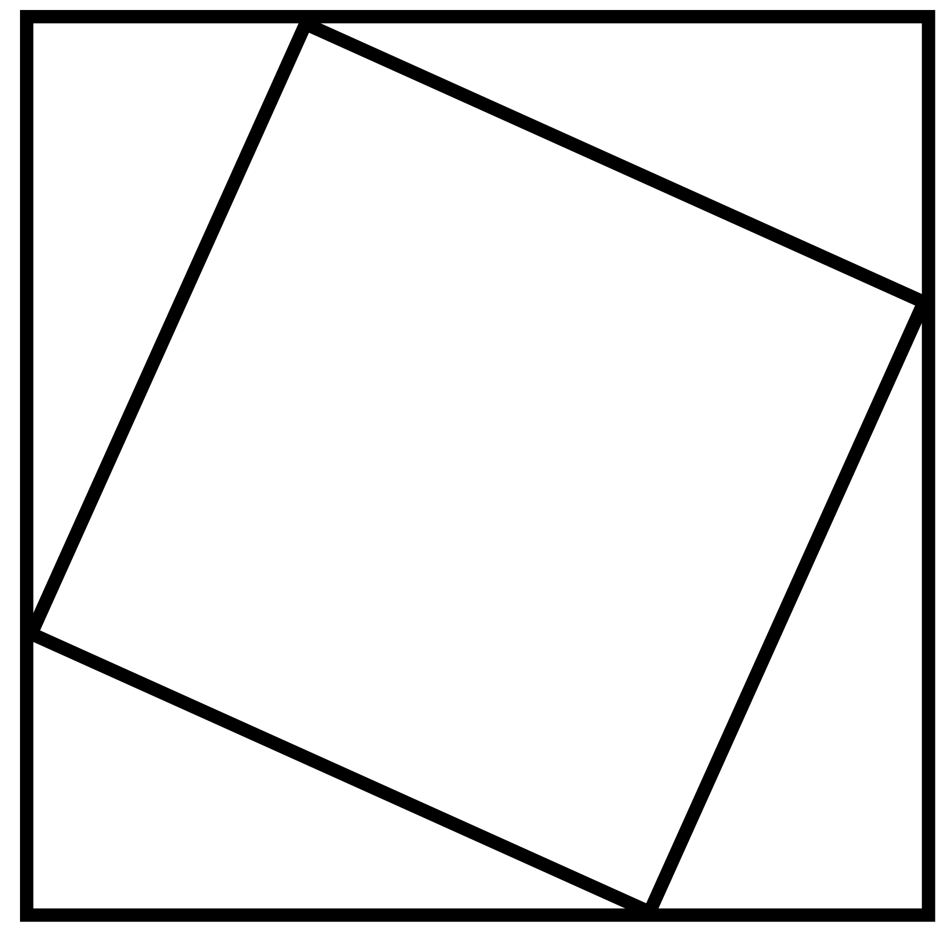

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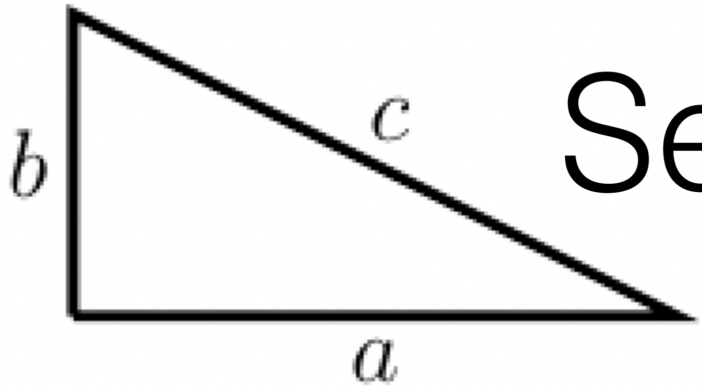
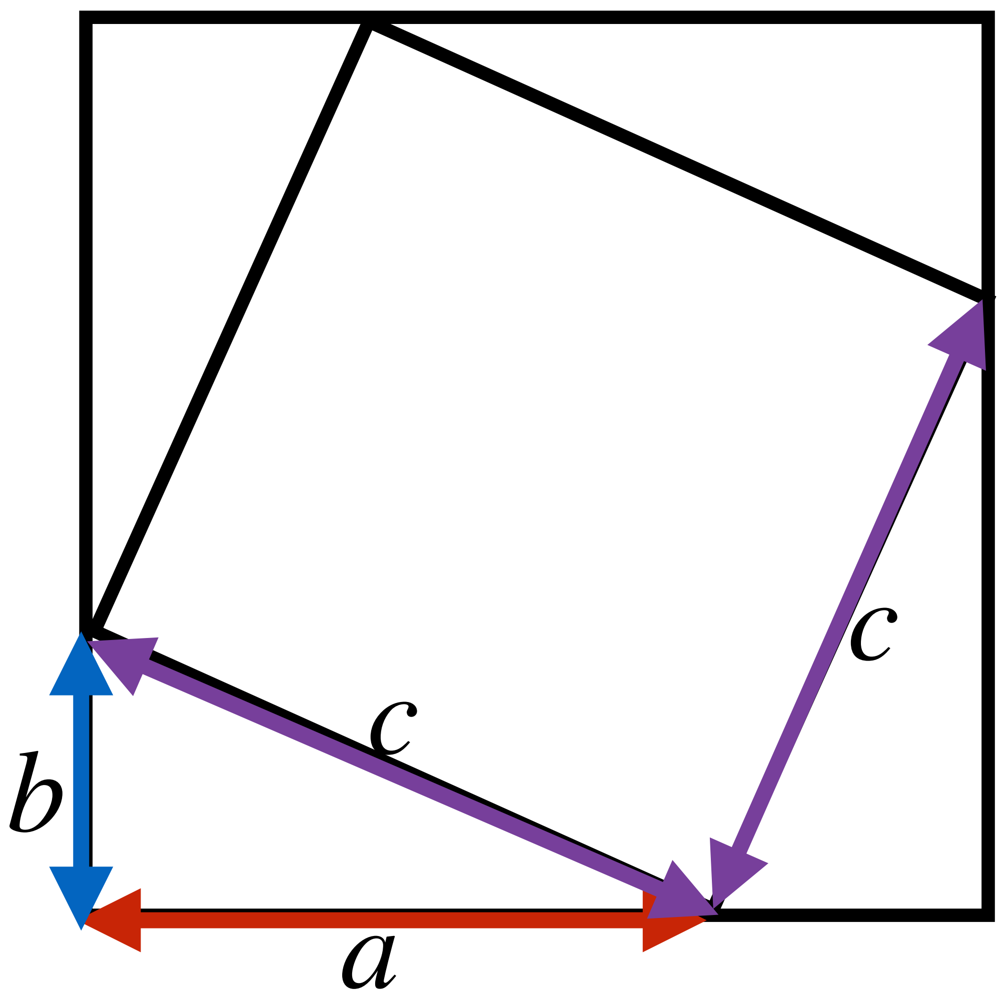
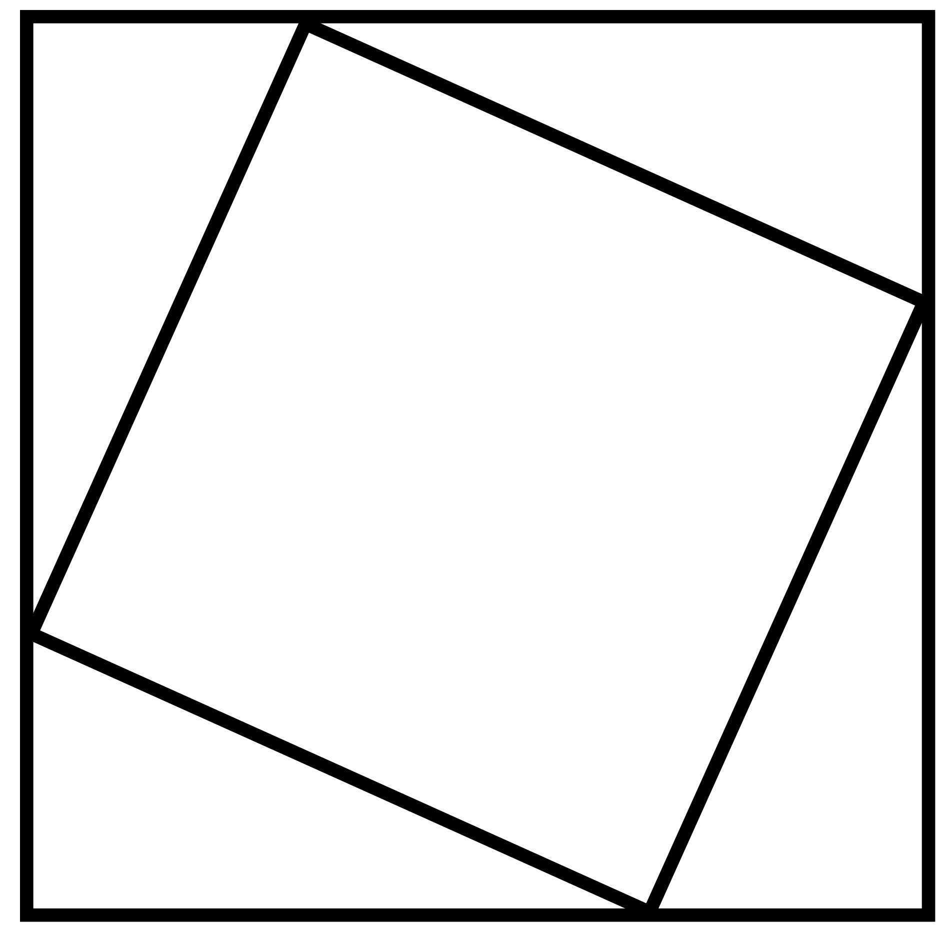


Second *Possible* Proof





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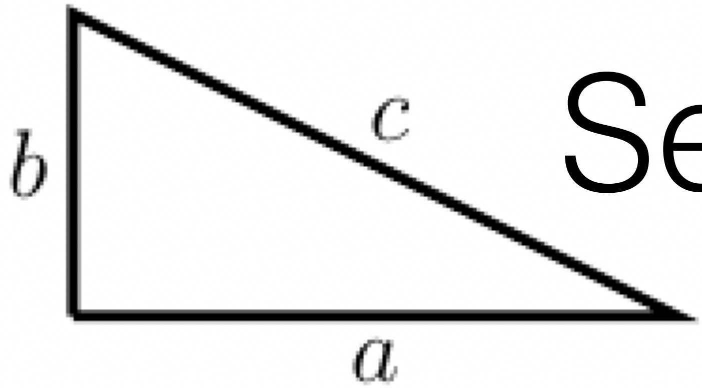

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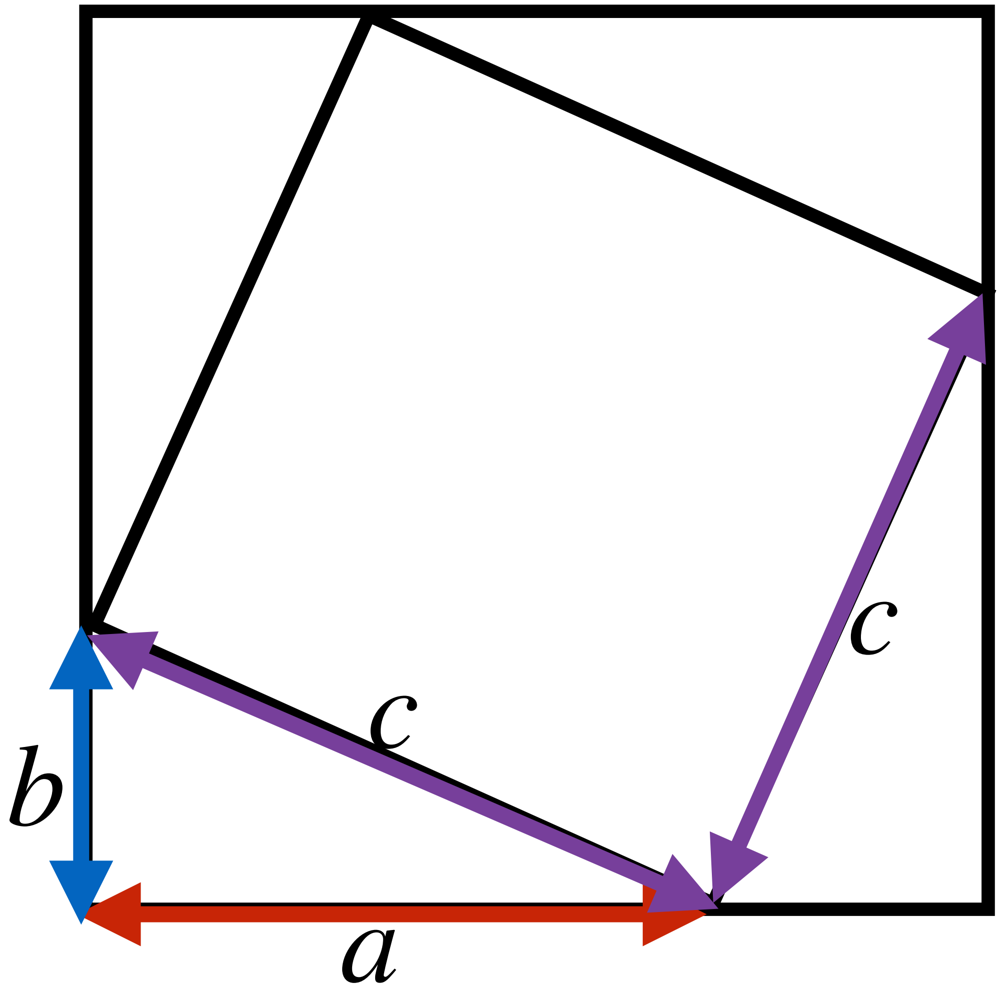

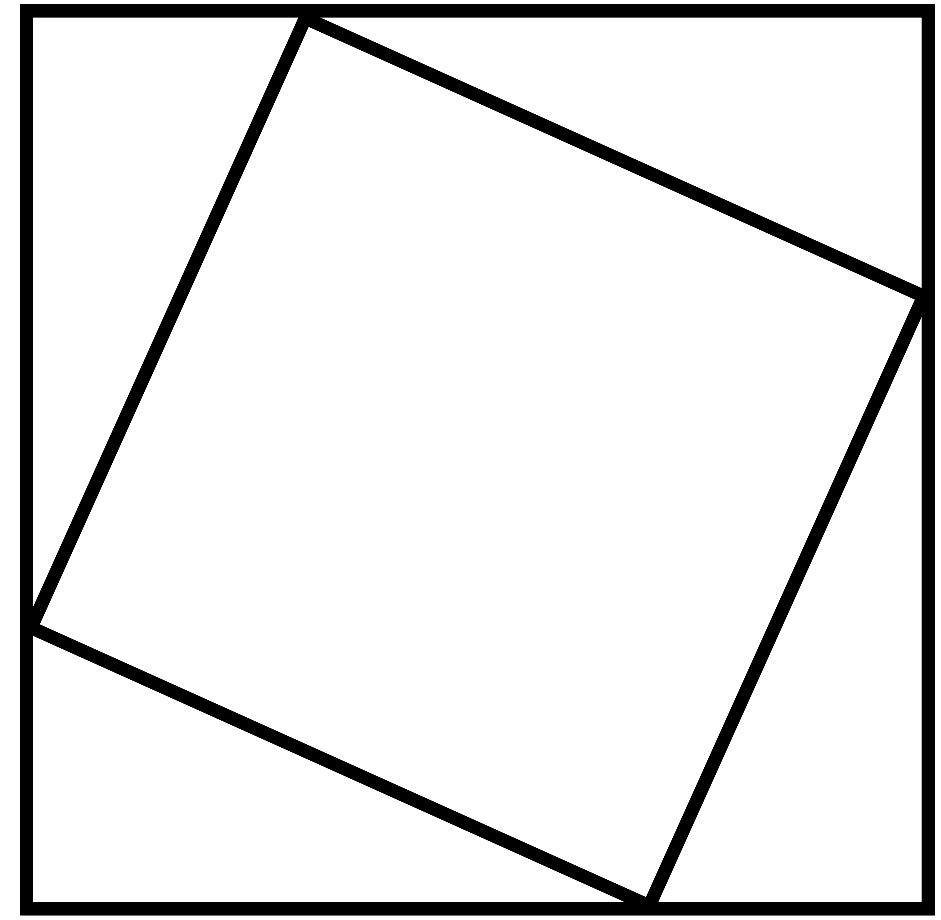

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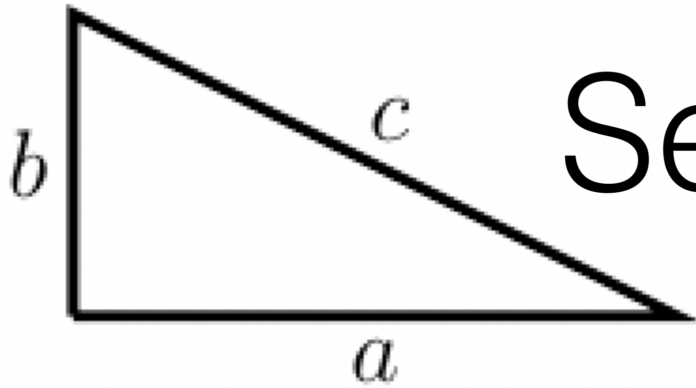

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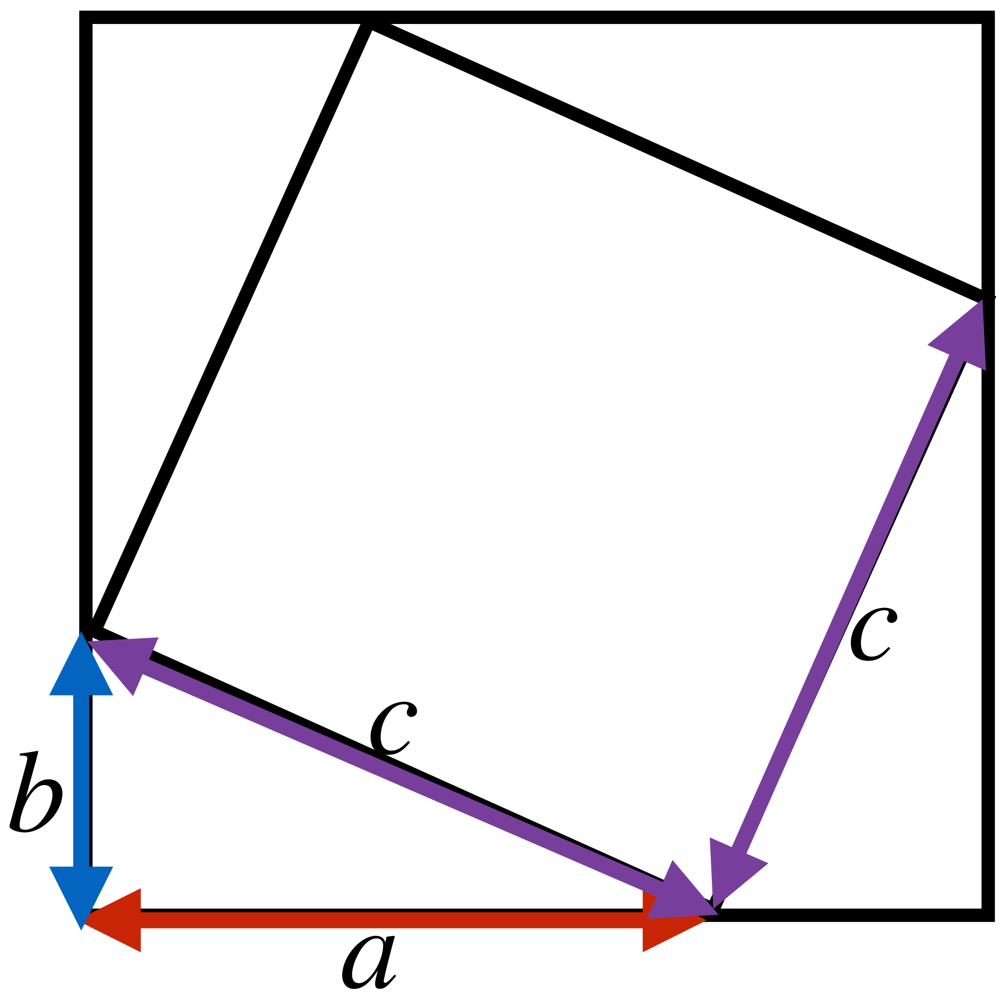

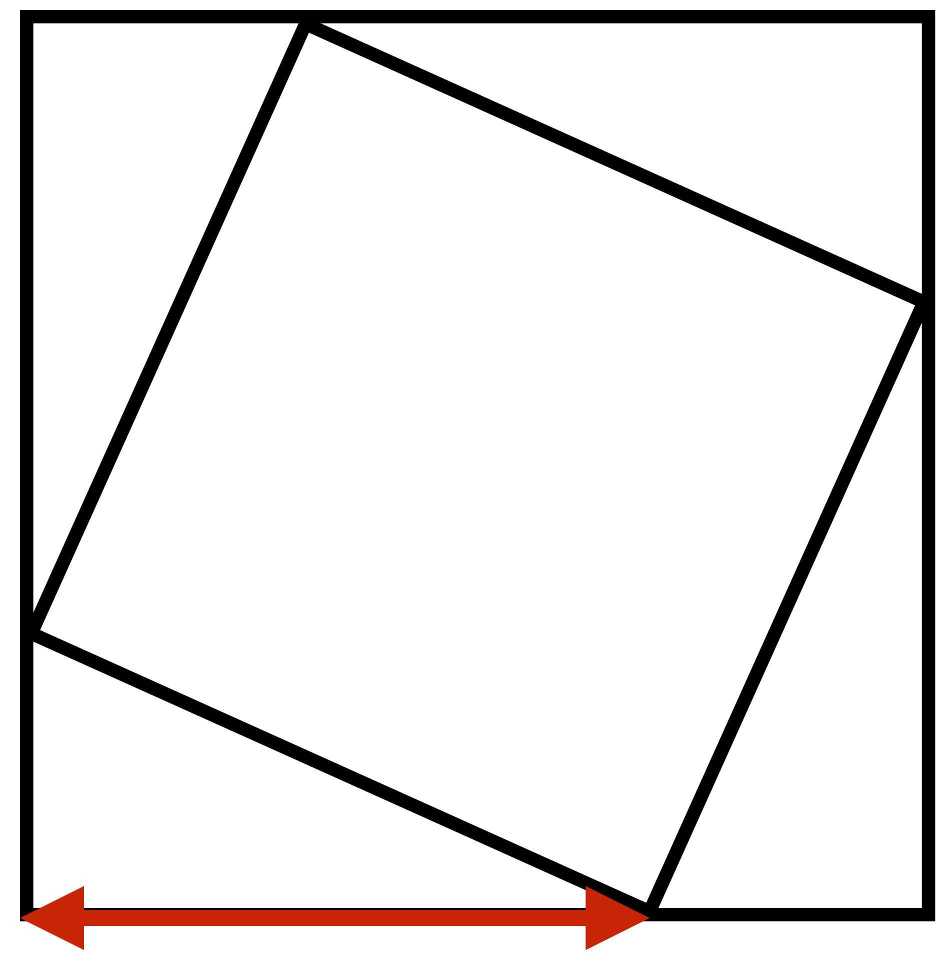

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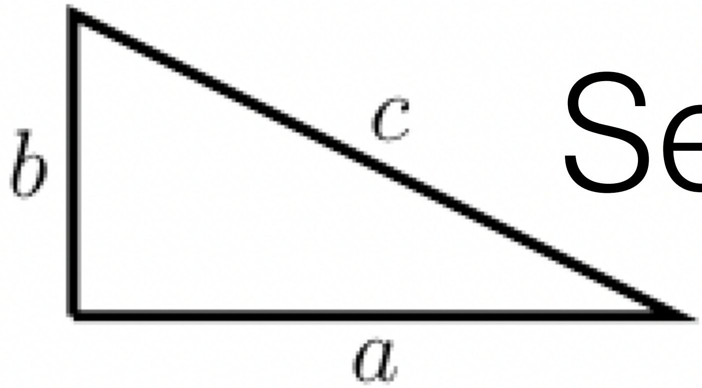

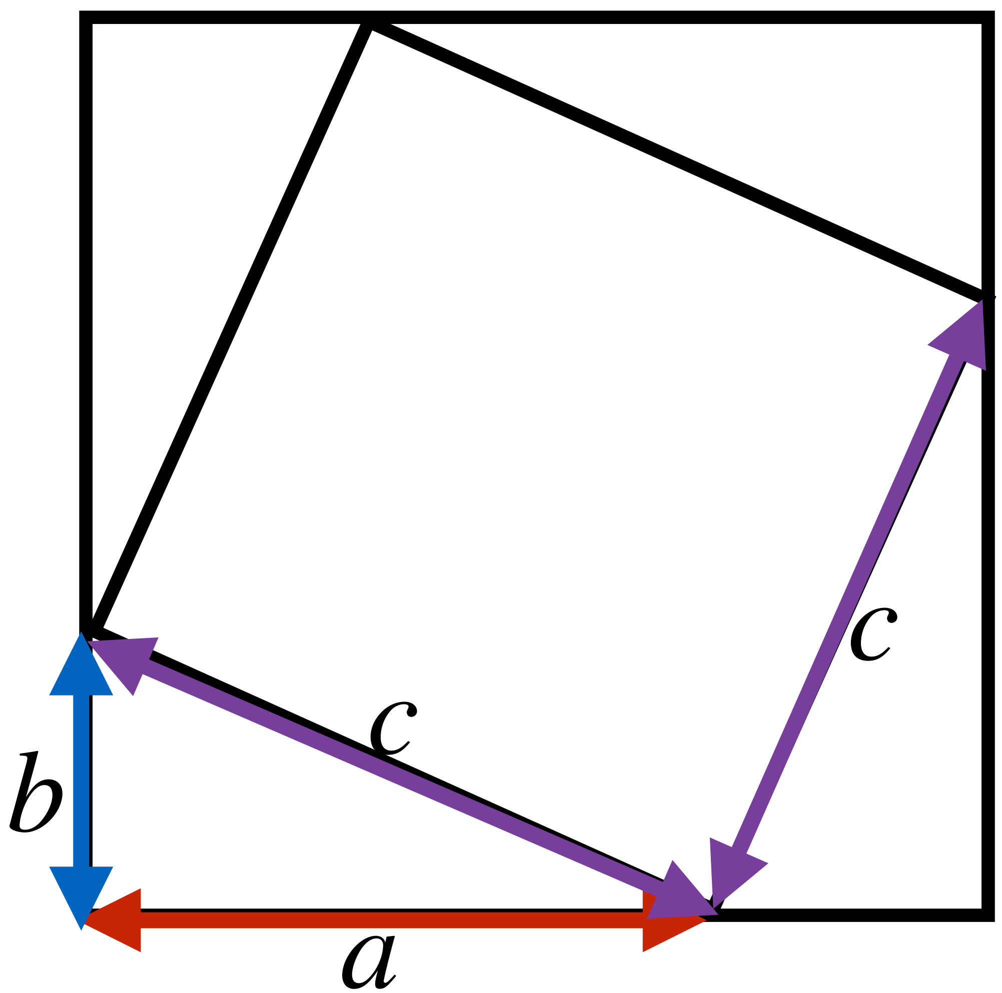

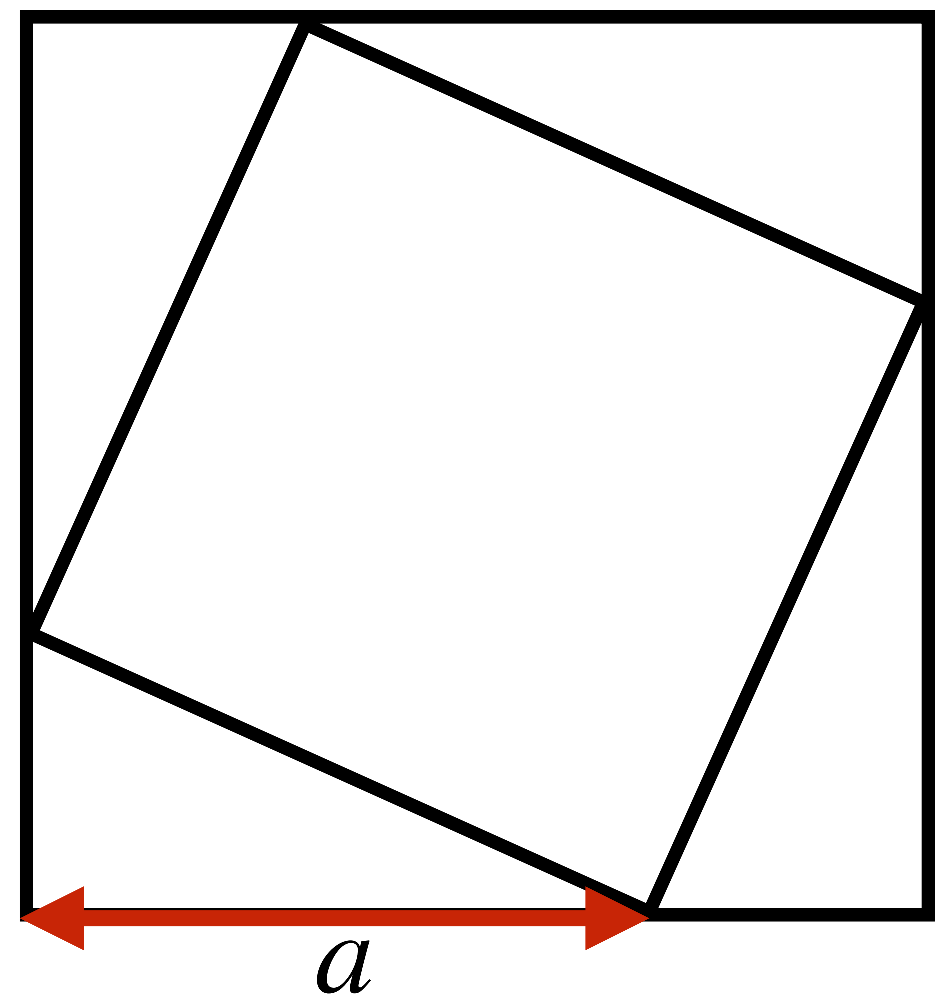

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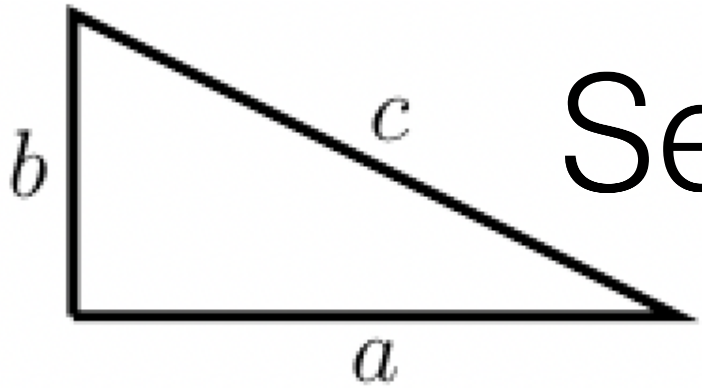

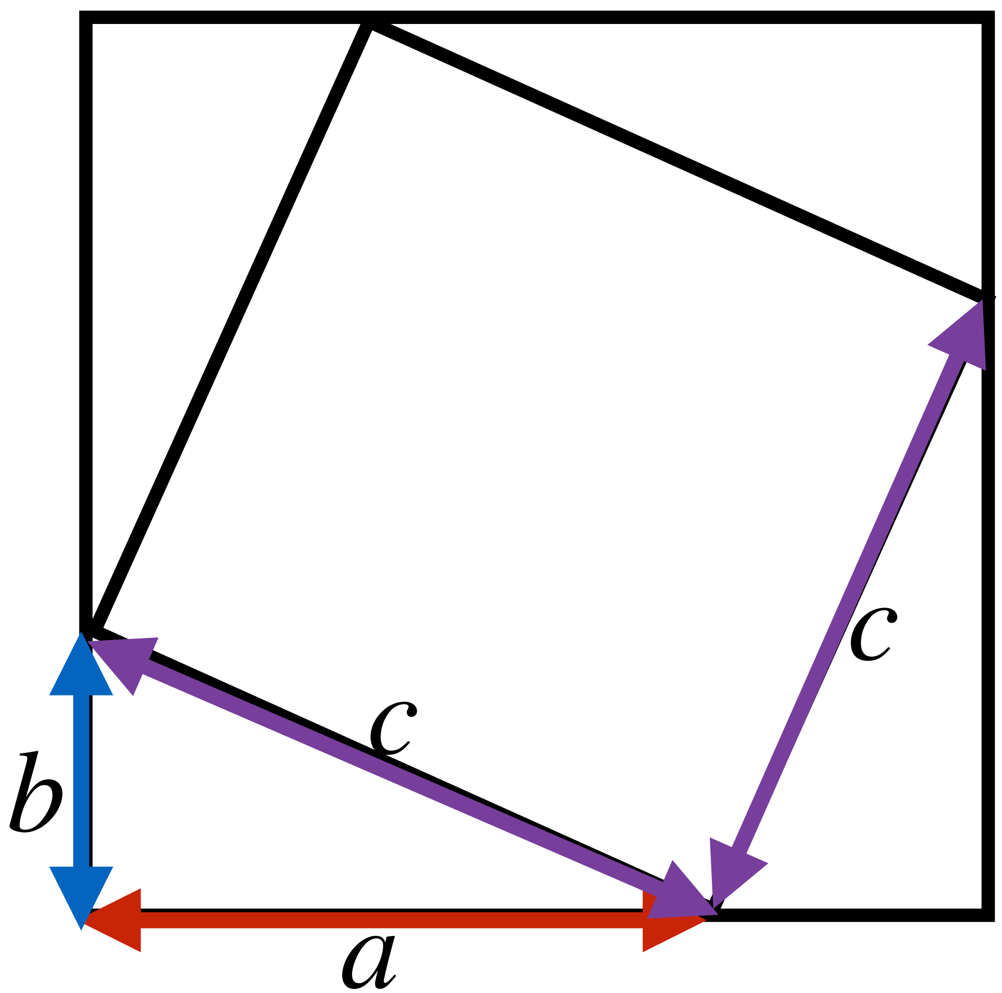

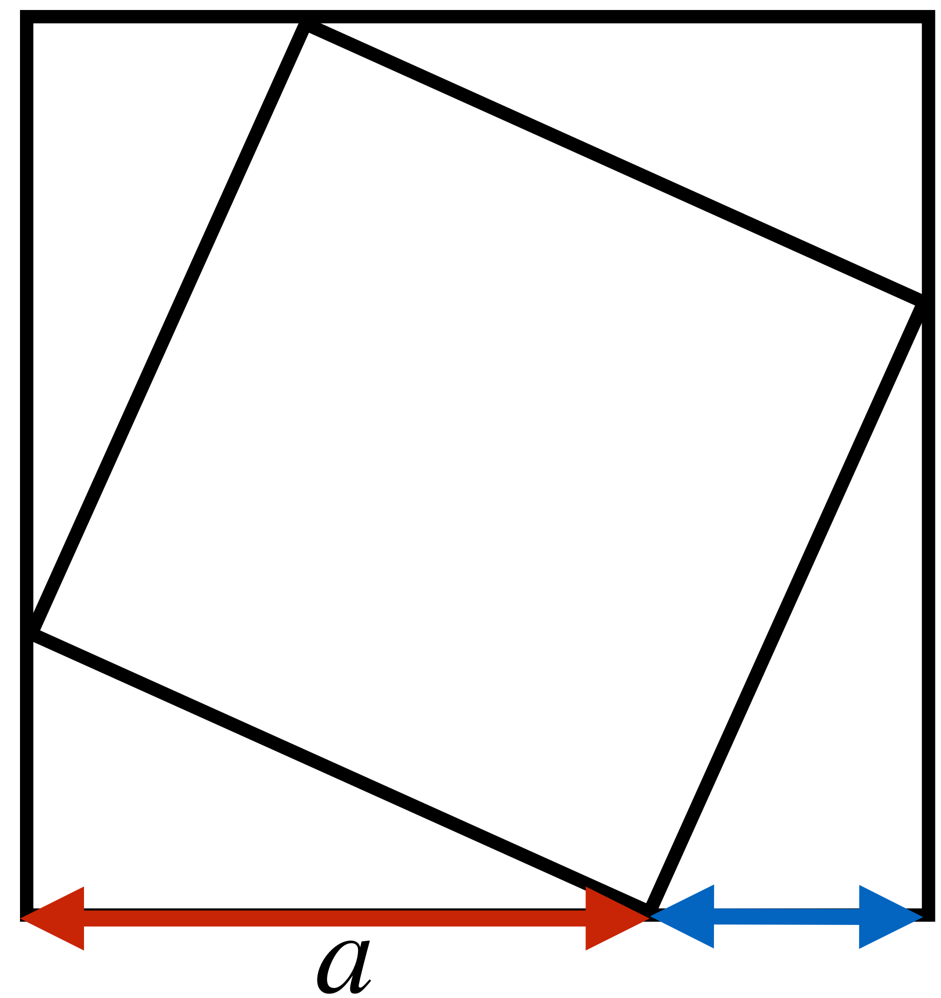

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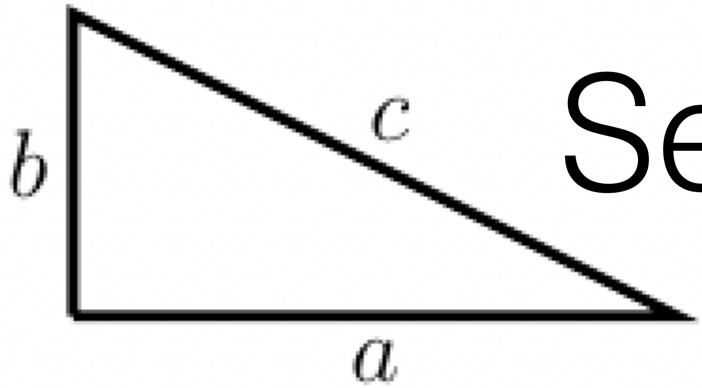

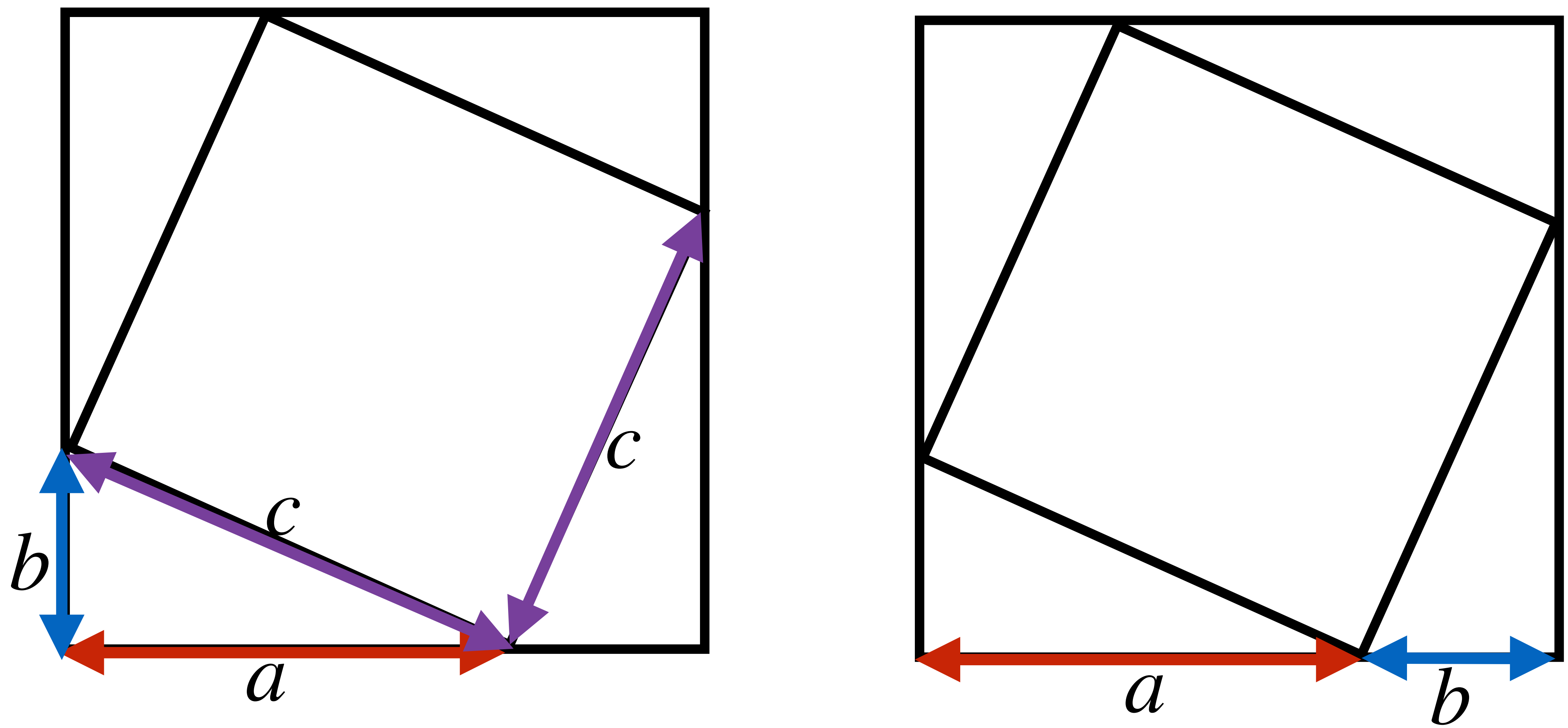

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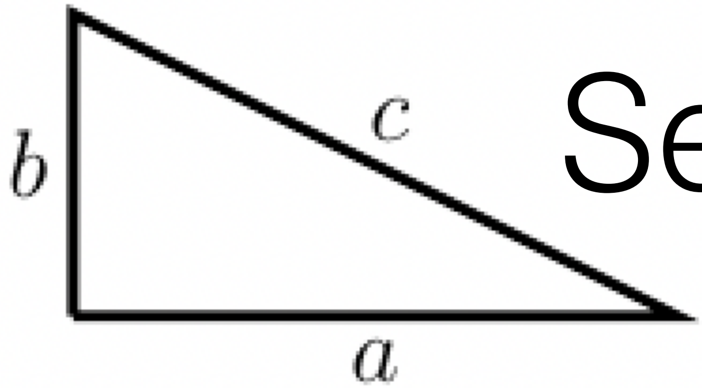
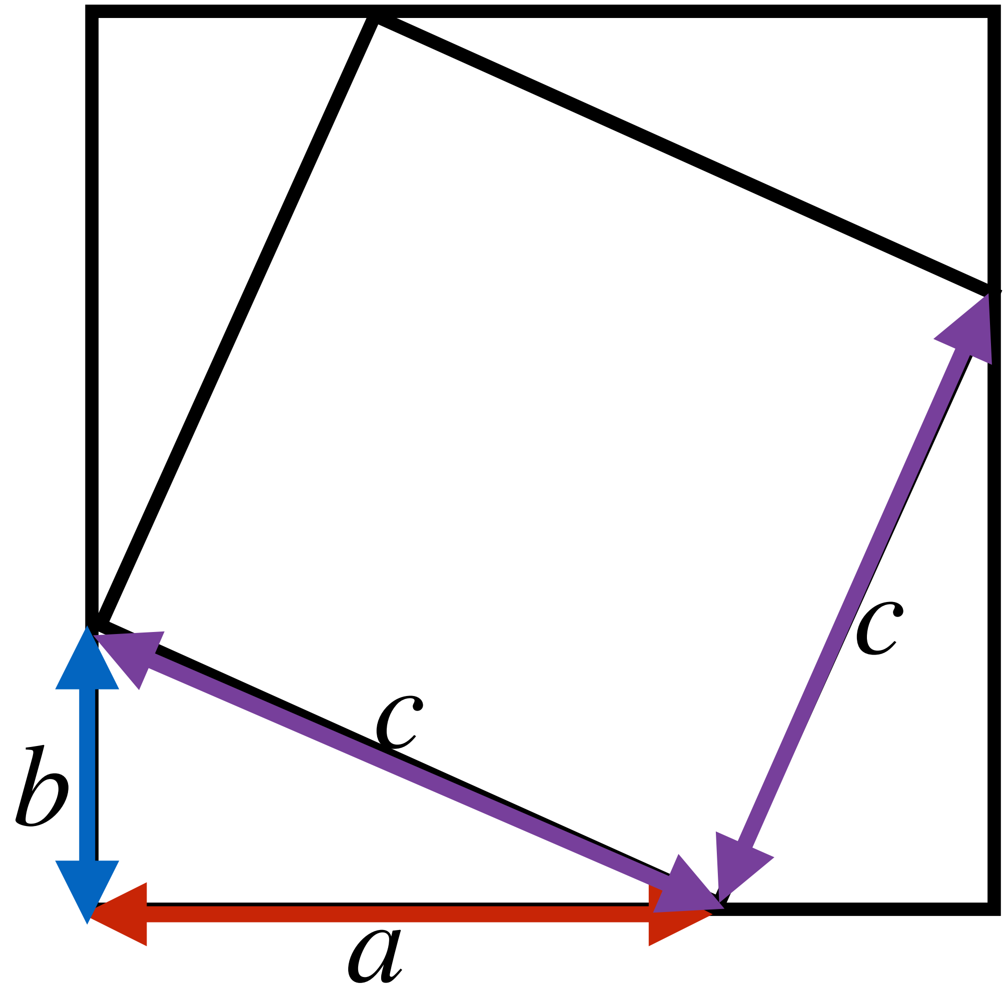
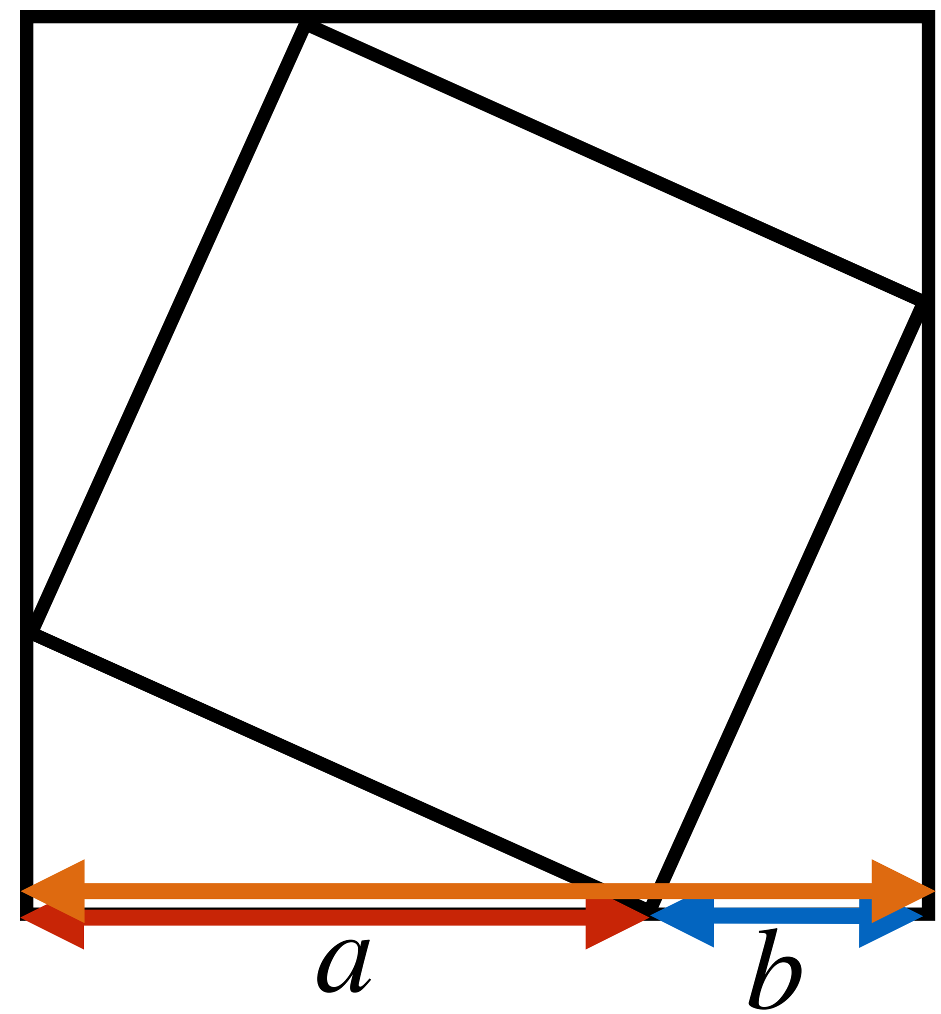
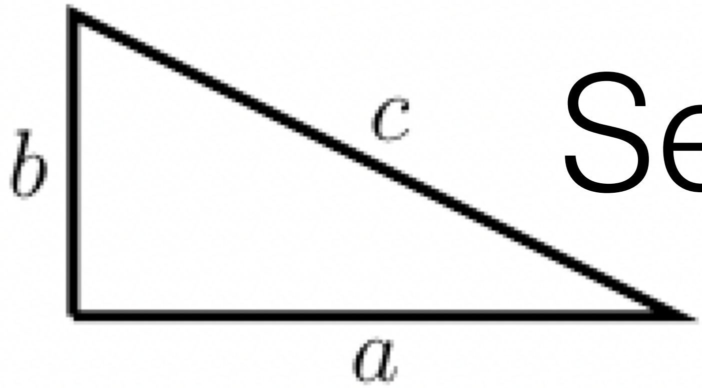






4 triangles + middle square = Whole square

$$4 \cdot \left(\frac{1}{2} a \cdot b\right) + c^2$$



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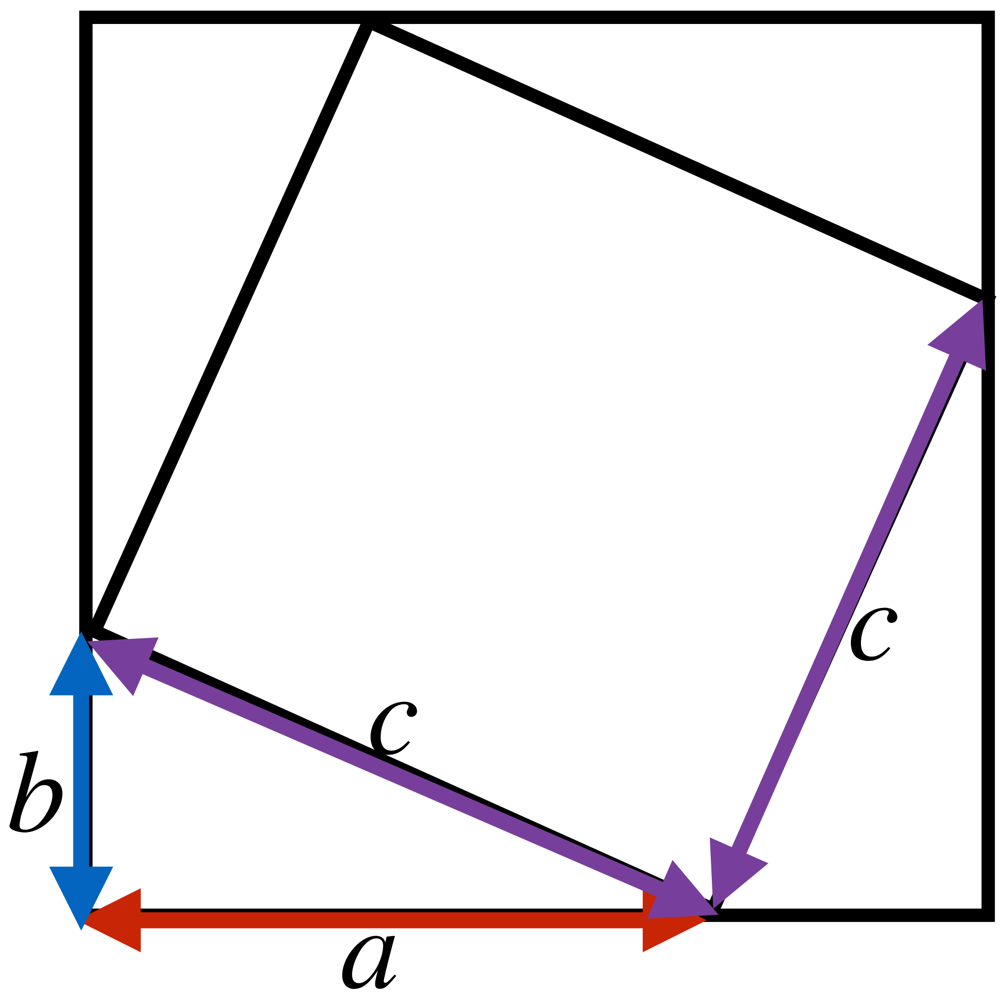

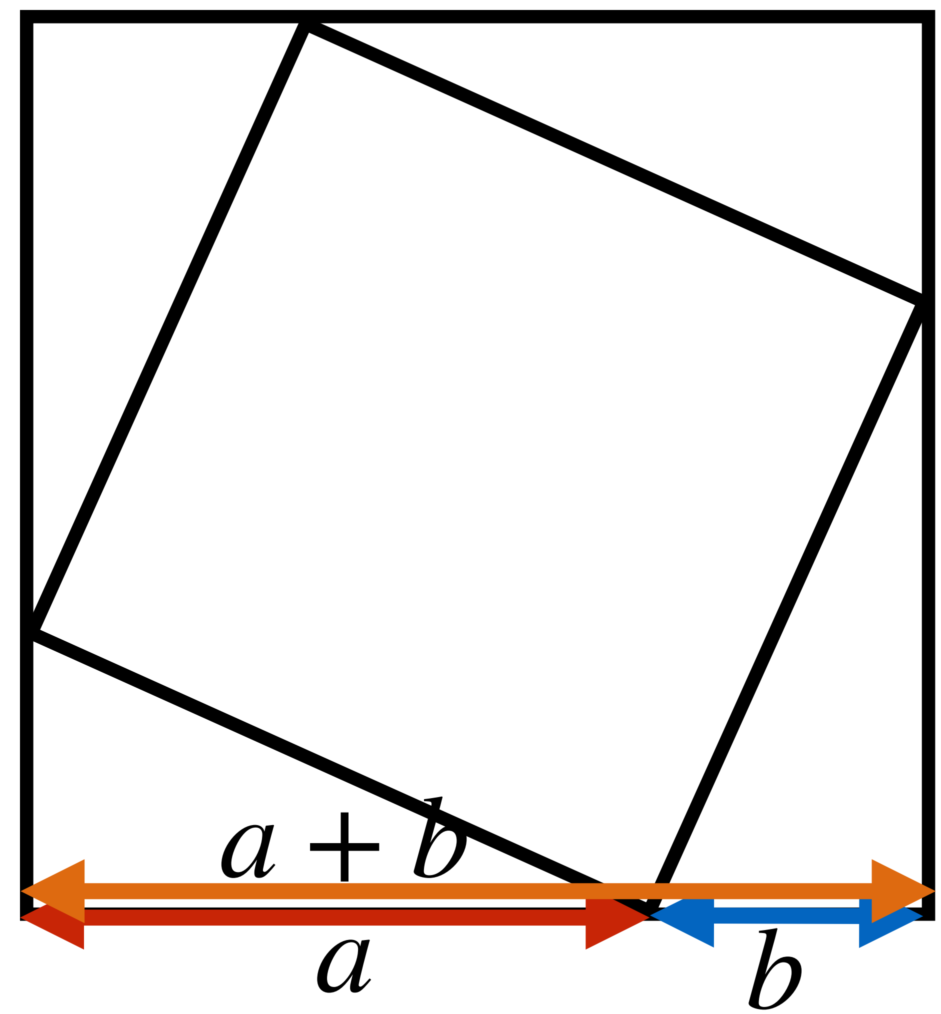

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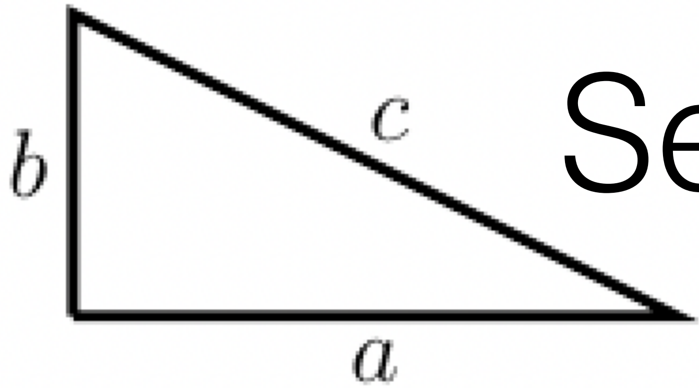

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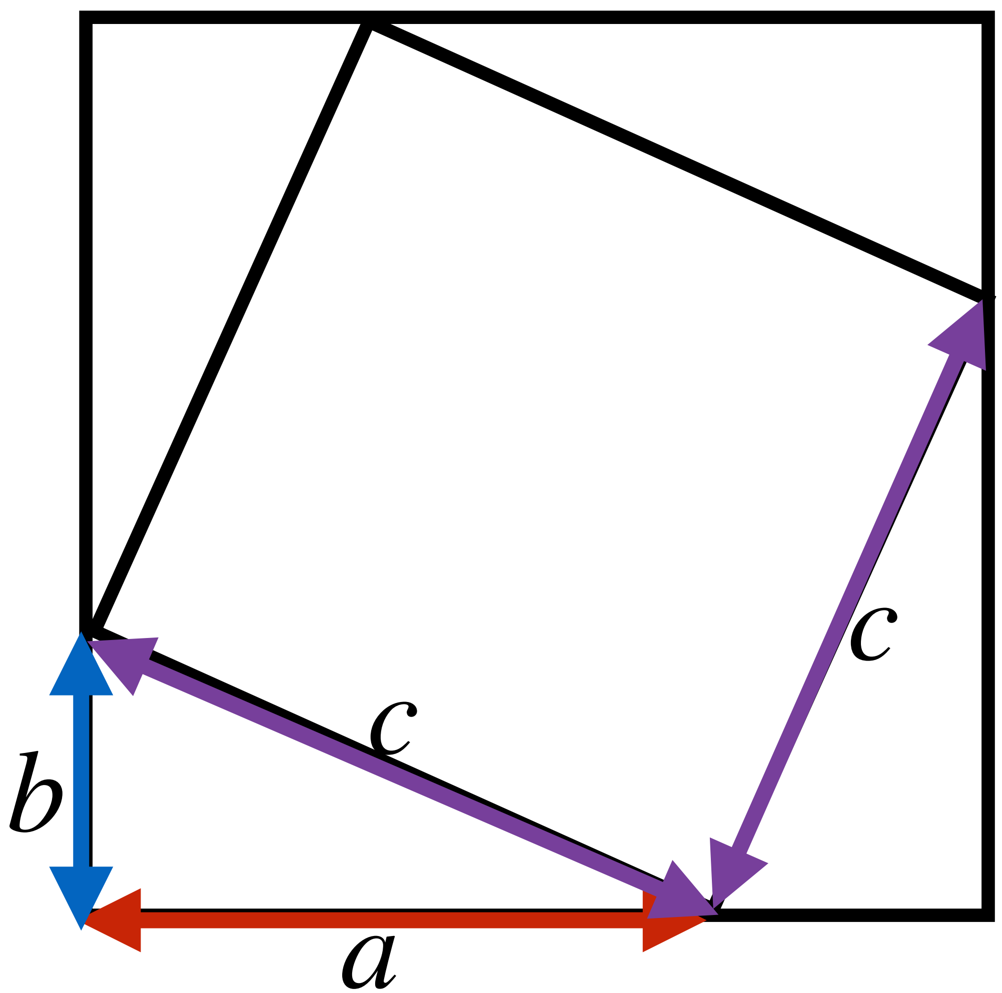

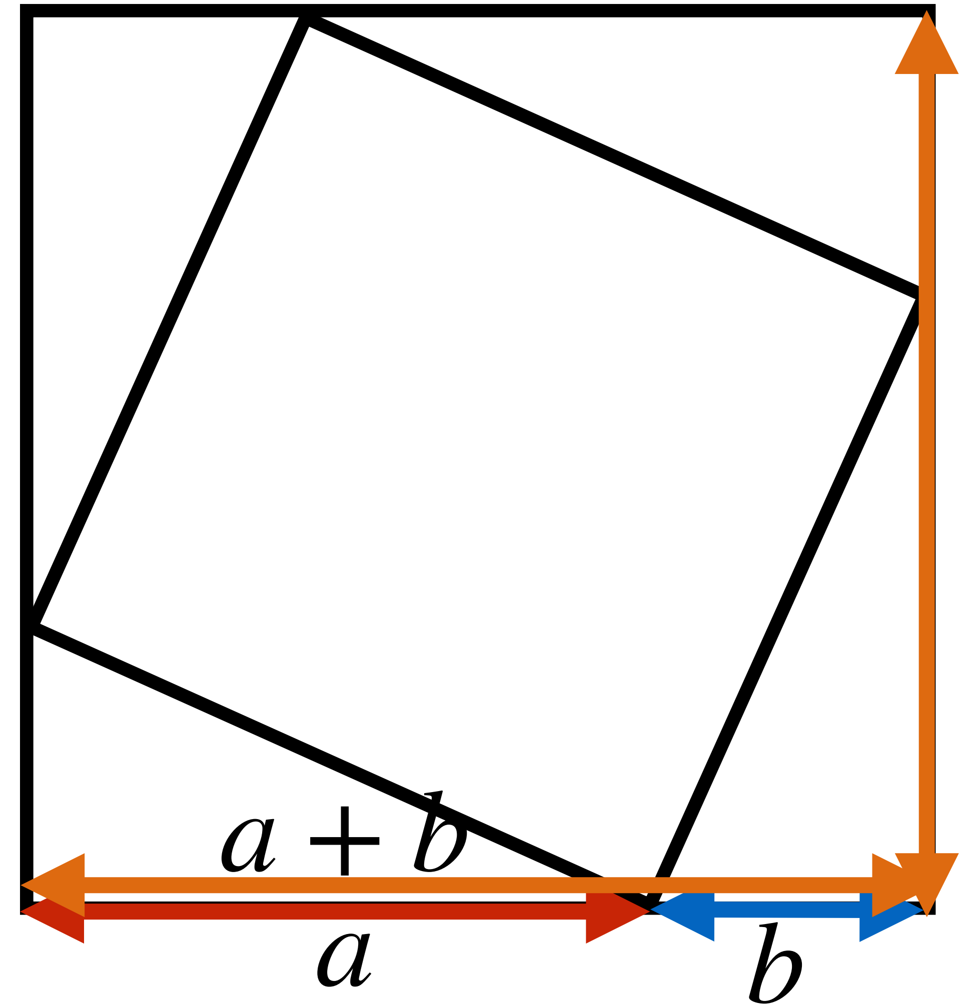

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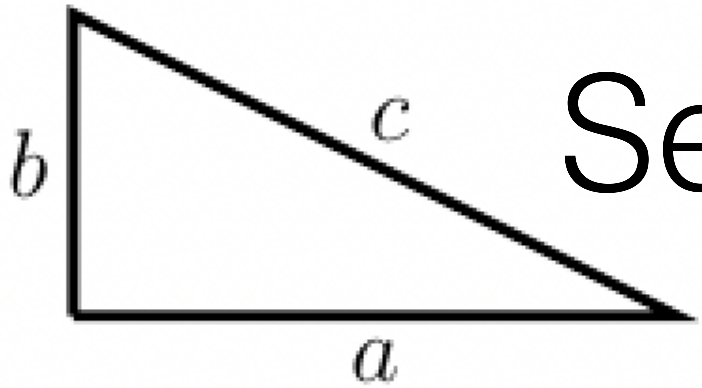

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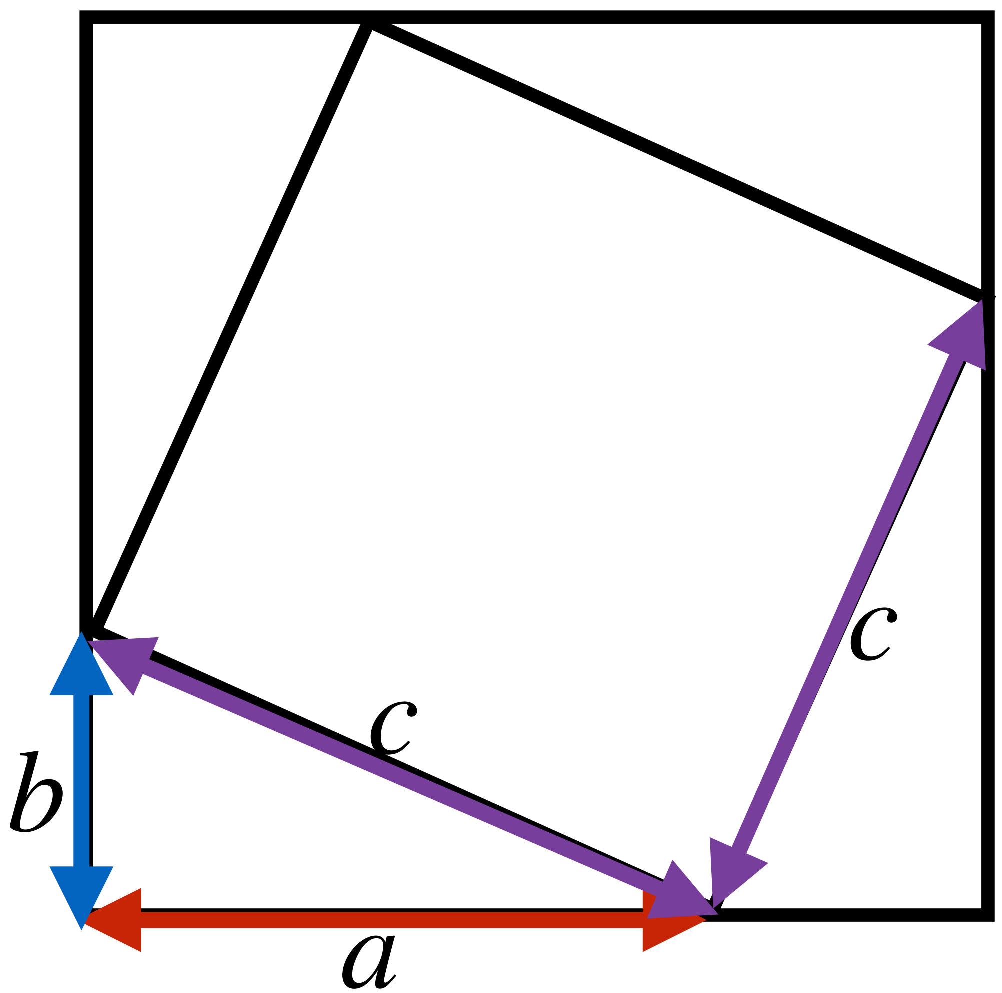

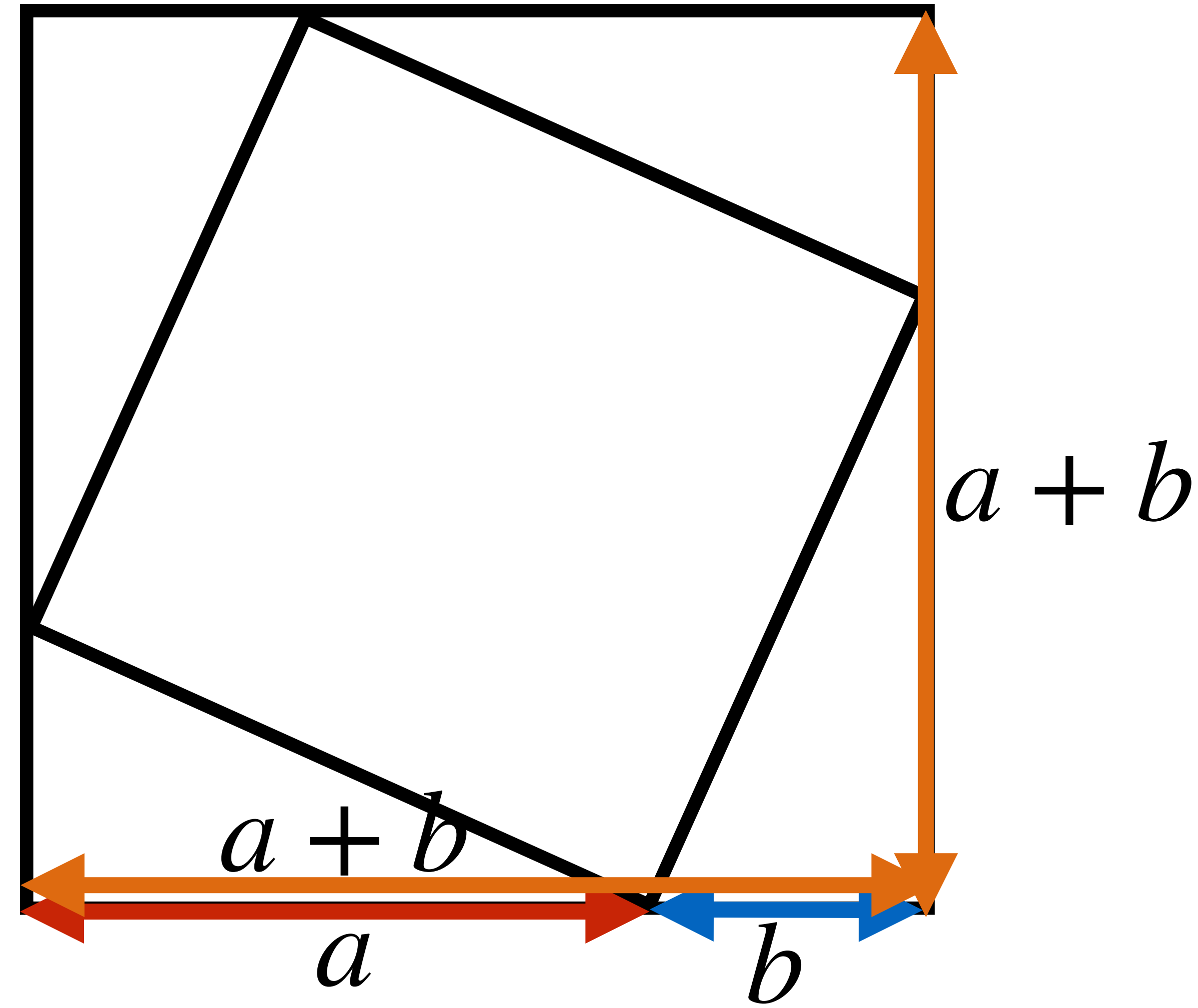

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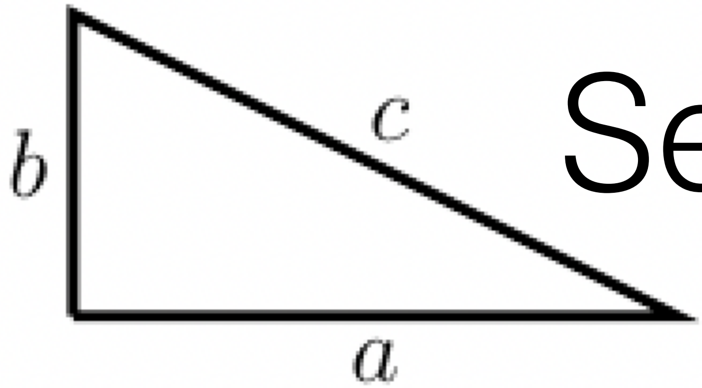

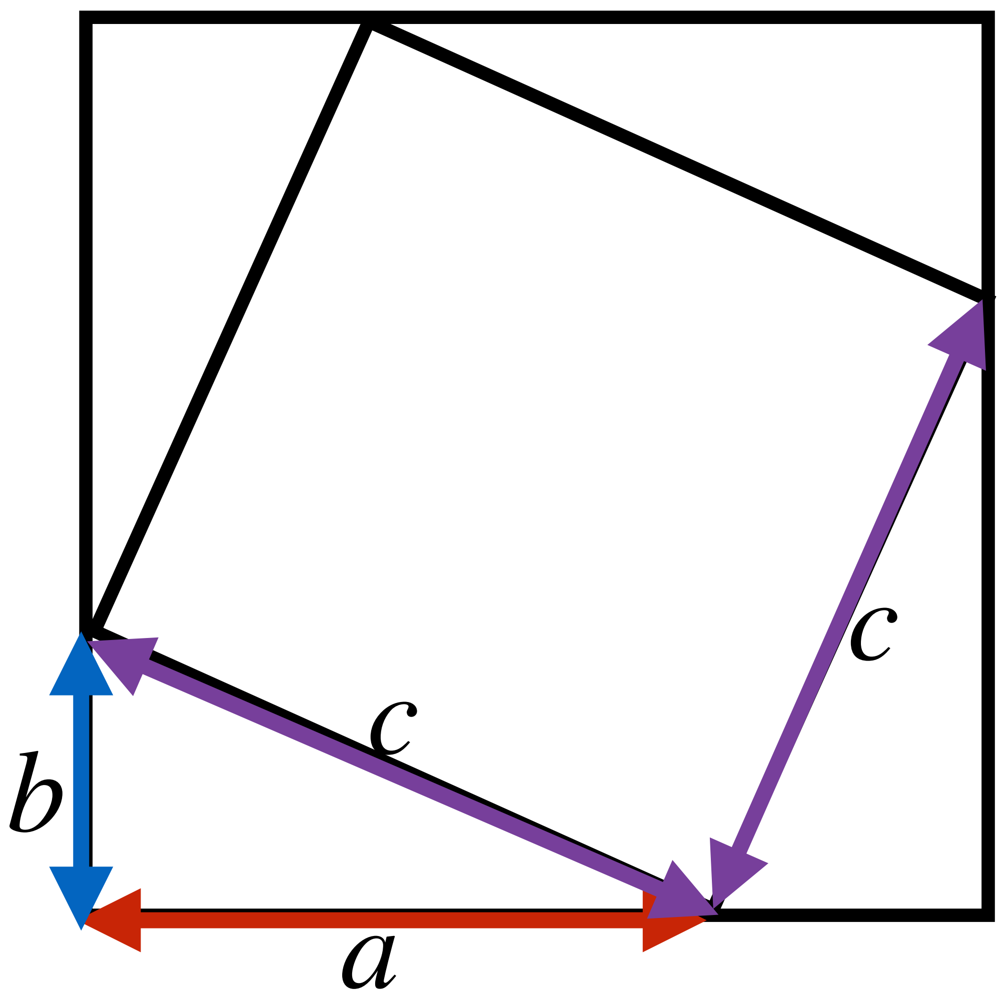

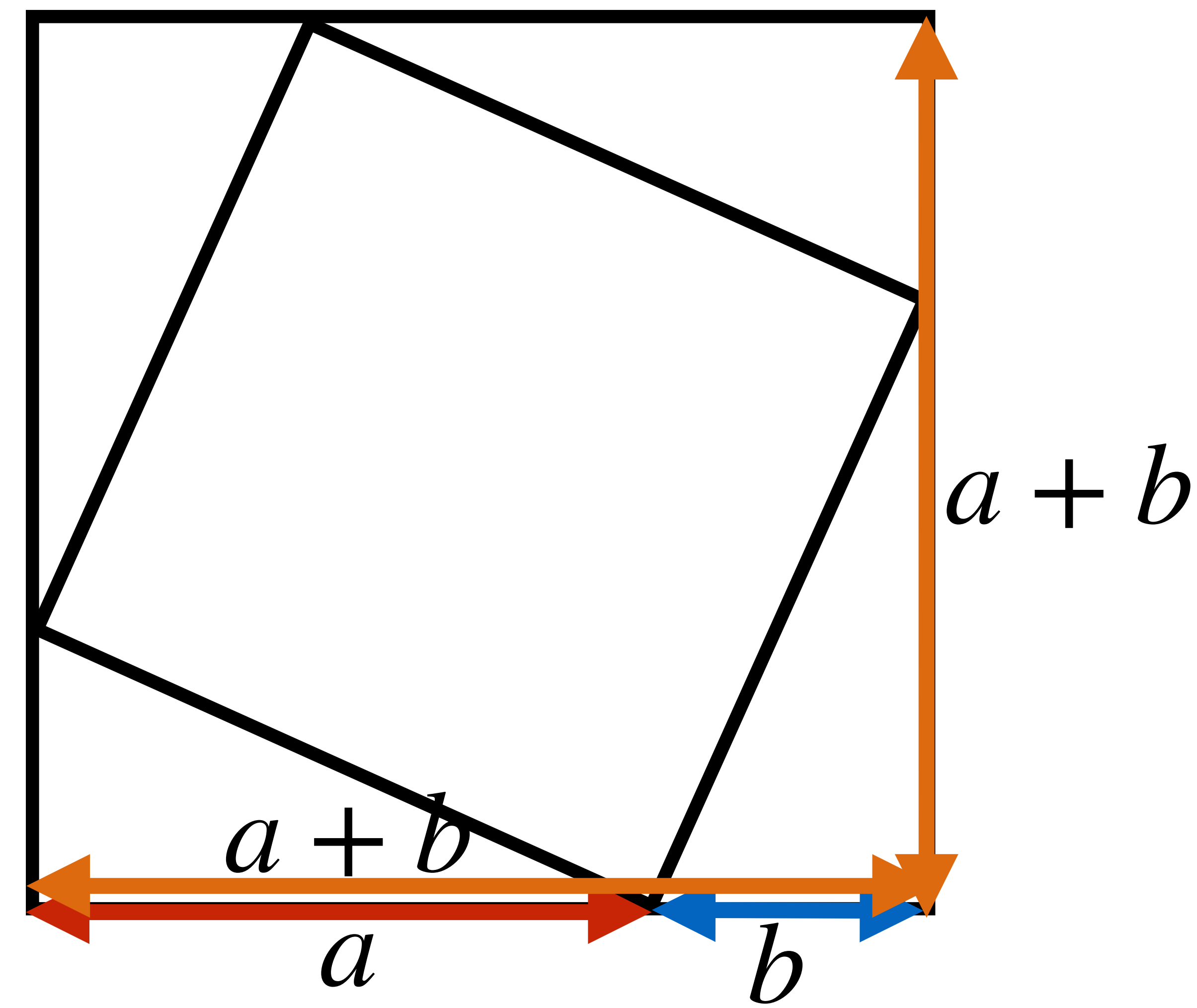

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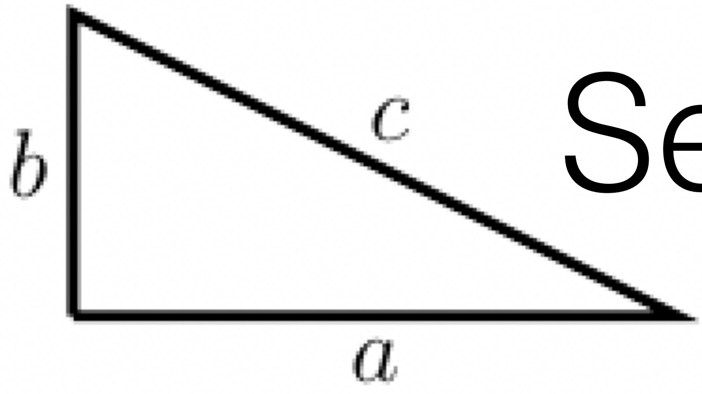
$$4 \cdot \left(\frac{1}{2} a \cdot b\right) \quad + \quad c^2 \quad = \quad (a+b)^2$$



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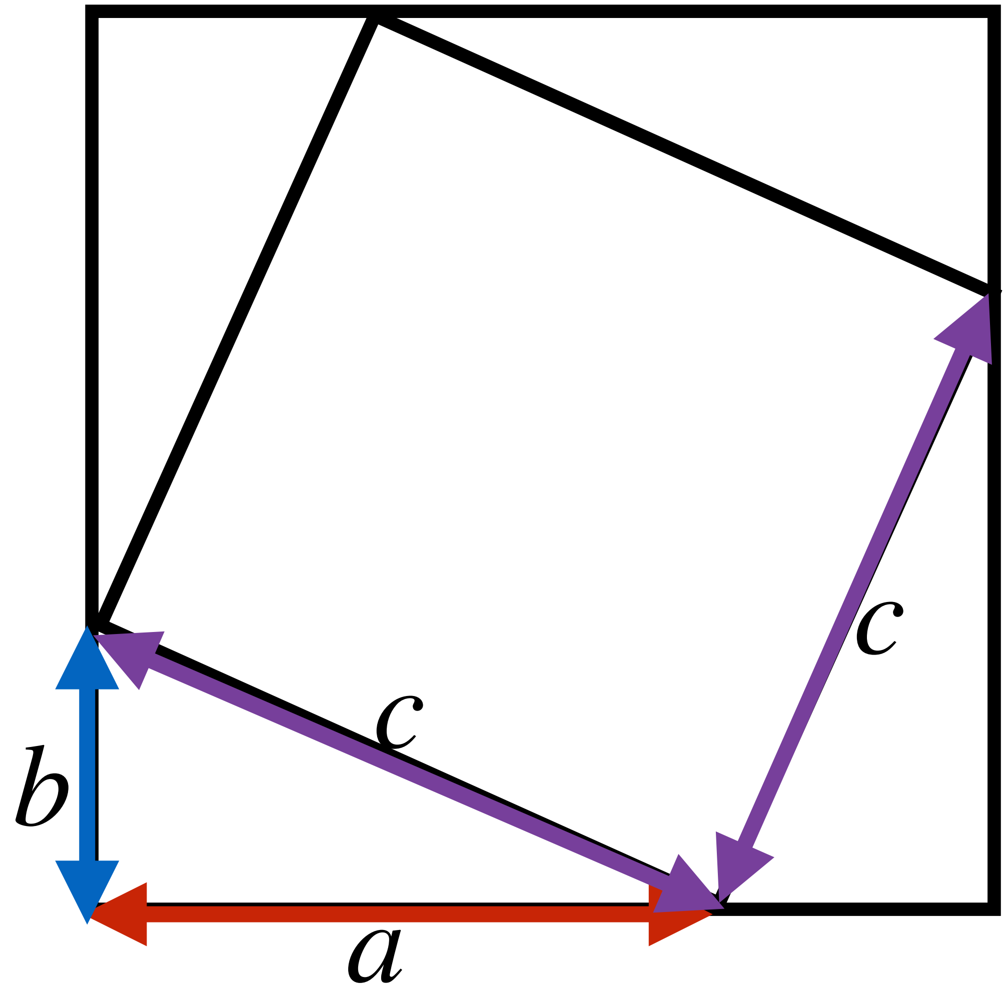
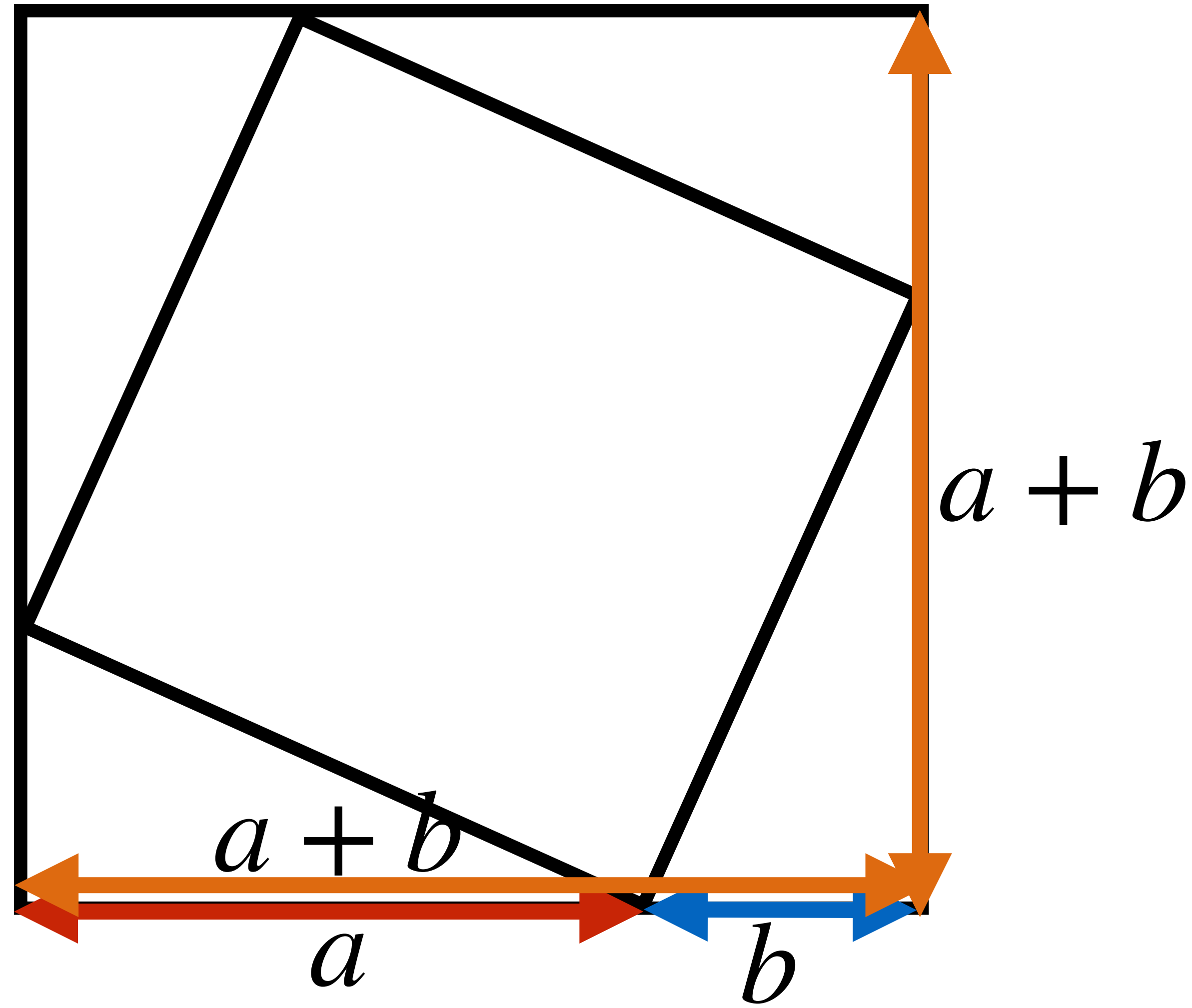


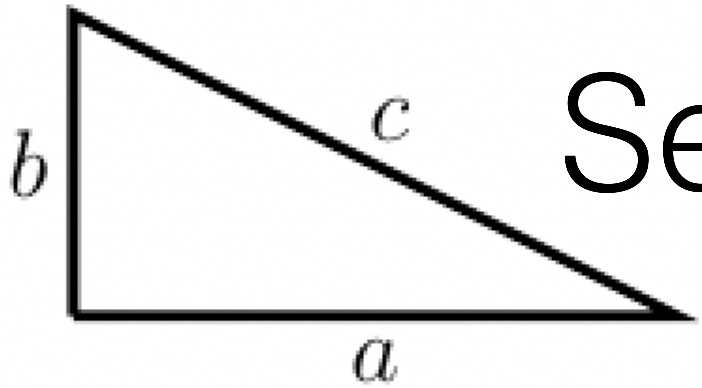


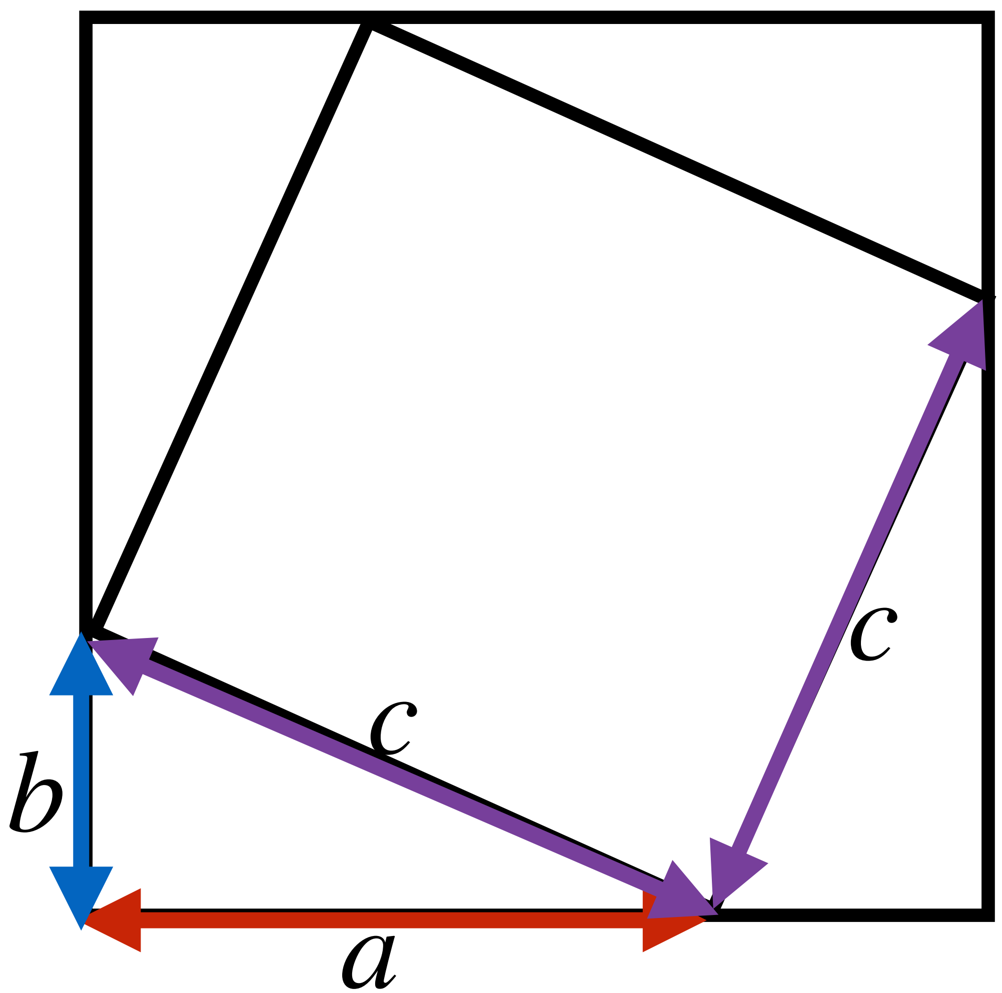
4 triangles + middle square = Whole square

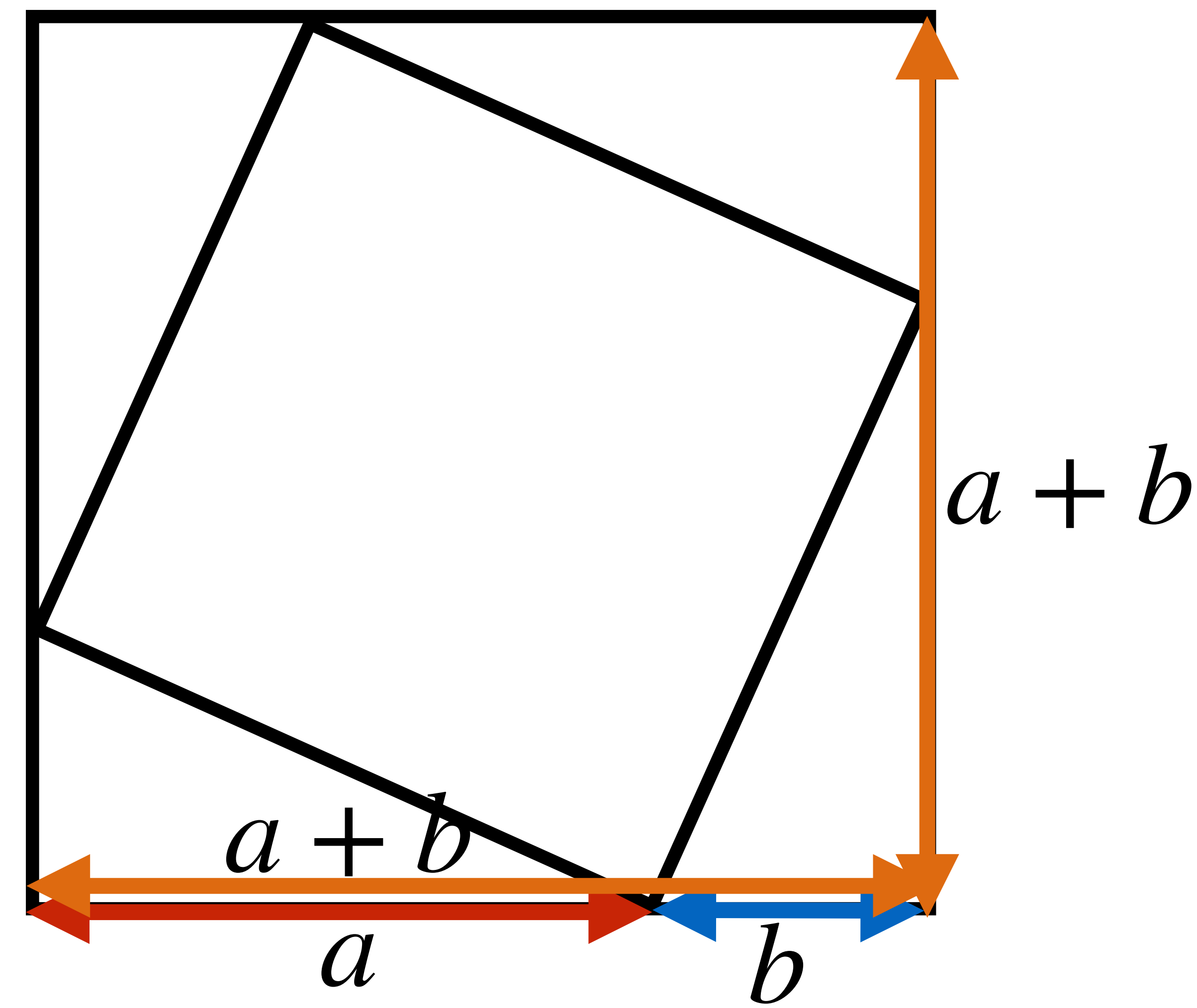
$$4 \cdot \left(\frac{1}{2} a \cdot b\right) + c^2 = (a+b)^2$$

$$2ab + c^2 = a^2 + b^2 + 2ab$$

Second *Possible* Proof







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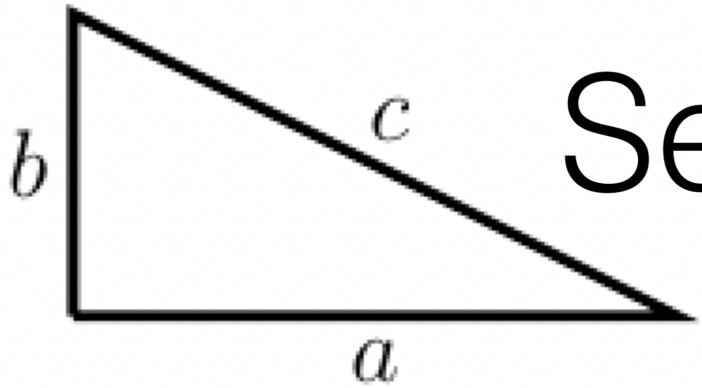
$$4 \cdot \left(\frac{1}{2} a \cdot b\right) \quad + \quad c^2 \quad = \quad (a+b)^2$$

$$\cancel{2ab} \quad + \quad c^2 \quad = \quad a^2 + b^2 + \cancel{2ab}$$



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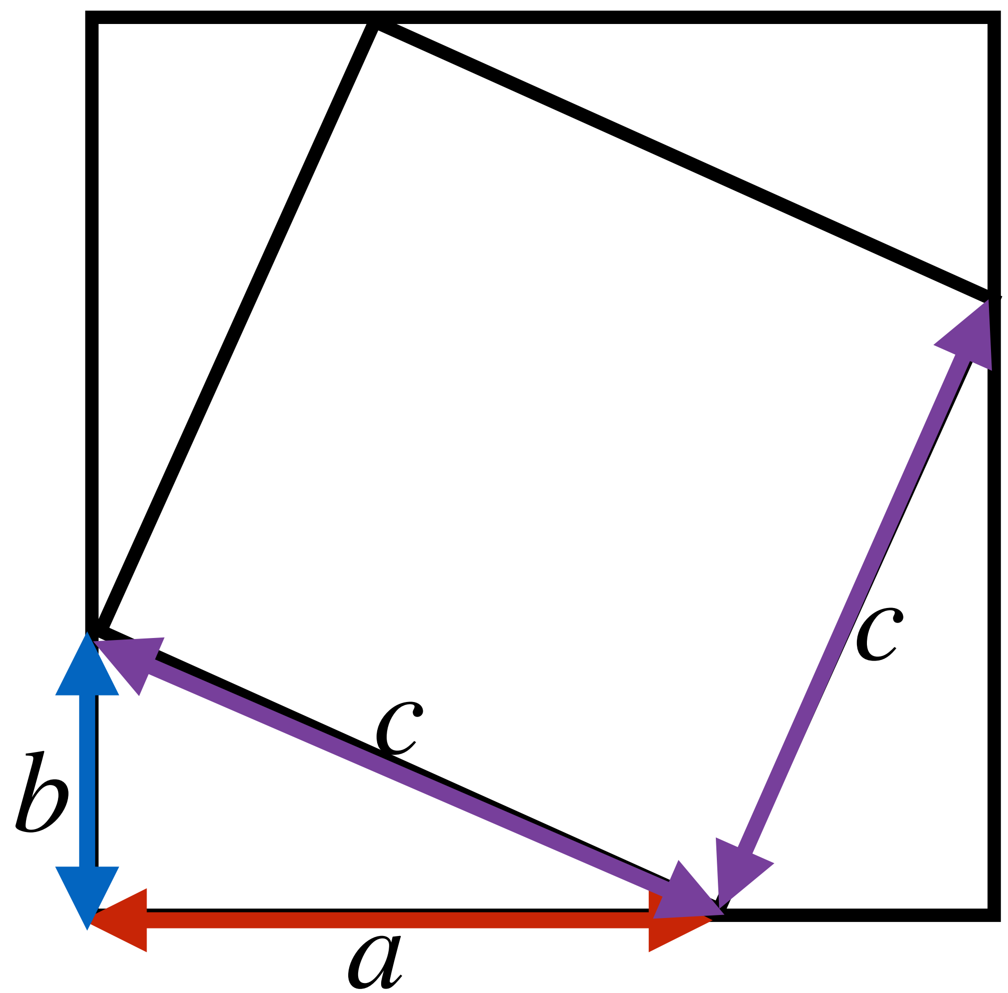

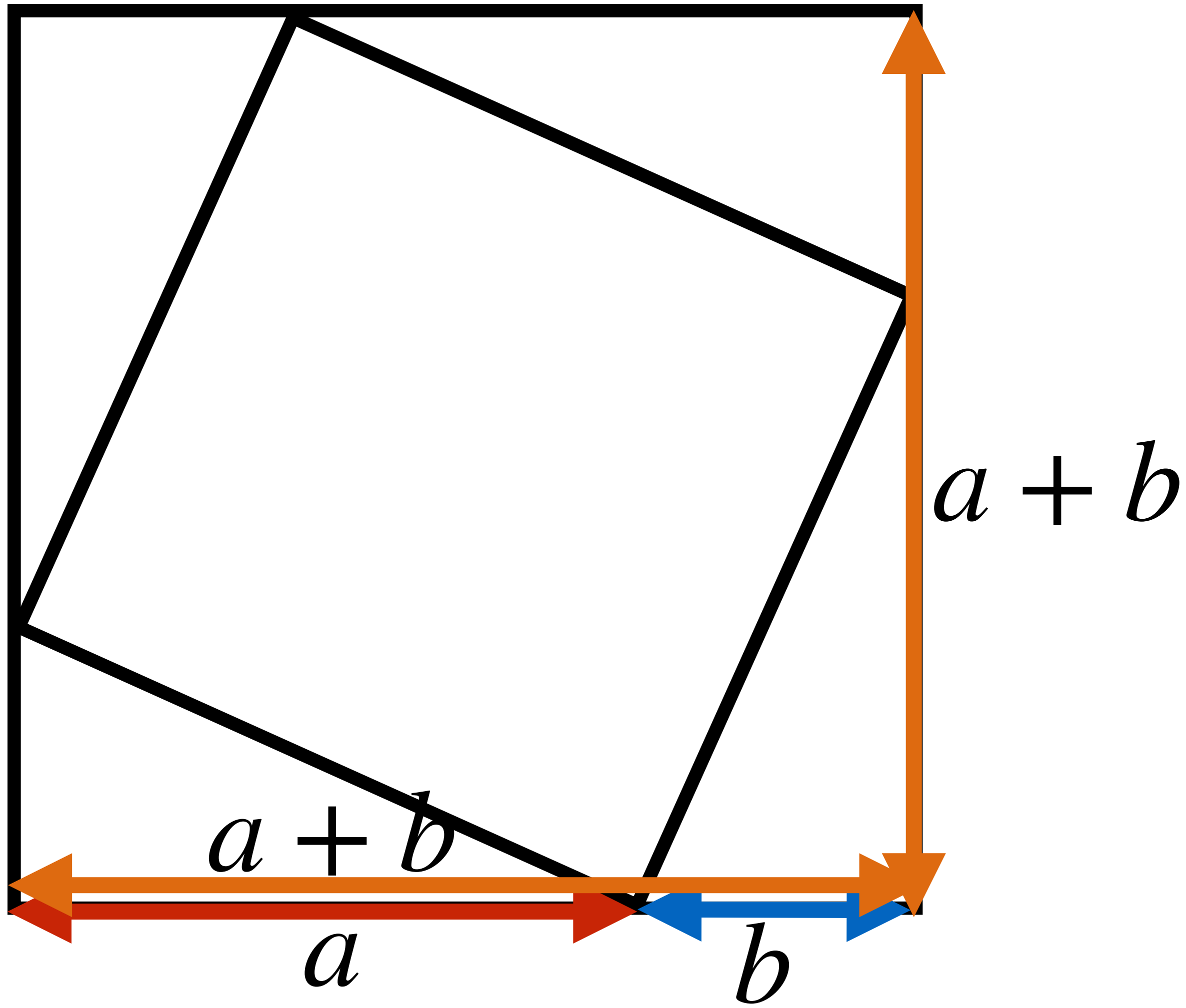

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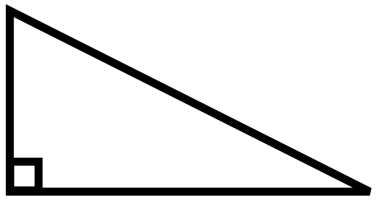
Generalized Pythagorean



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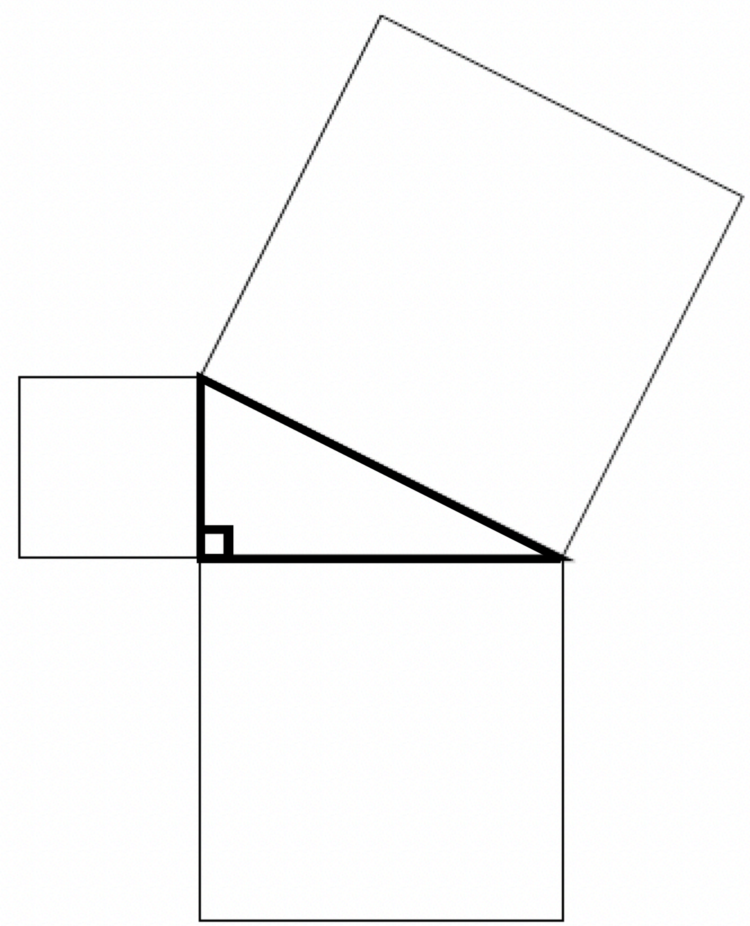
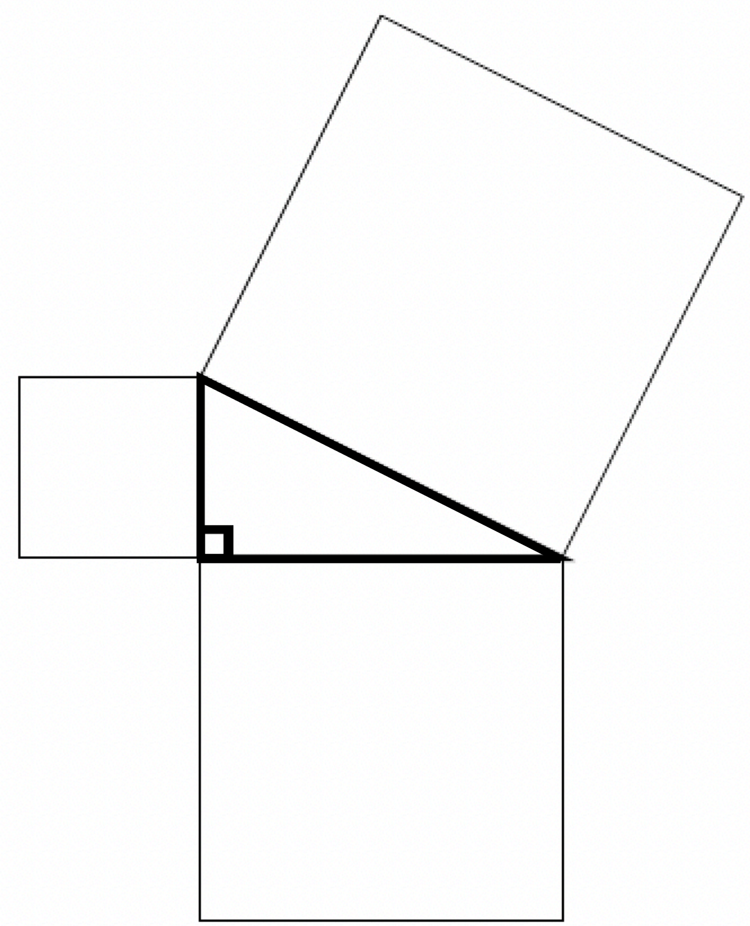


Generalized Pythagorean

If you attach *any* shape to the sides of a right triangle, the area inside the largest one equals the sum of the areas inside the smaller two.



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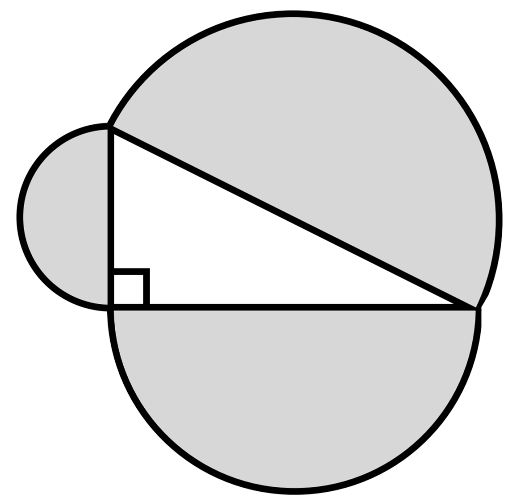

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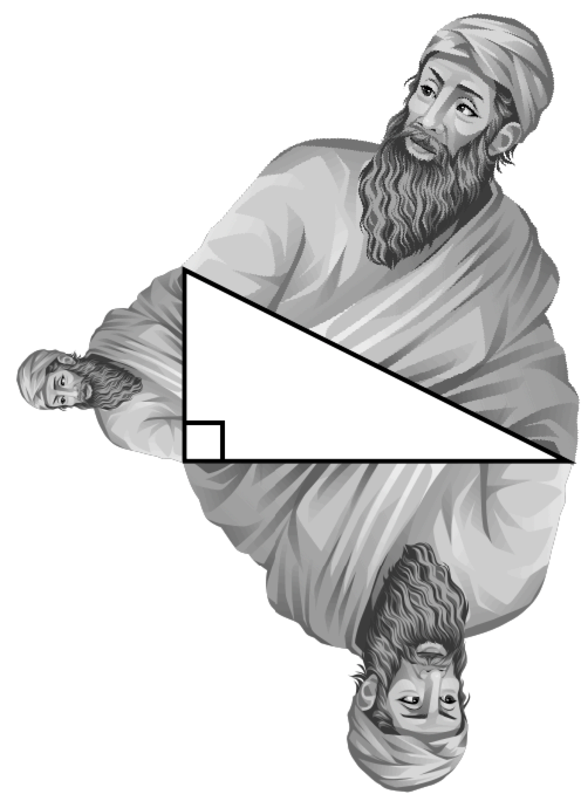

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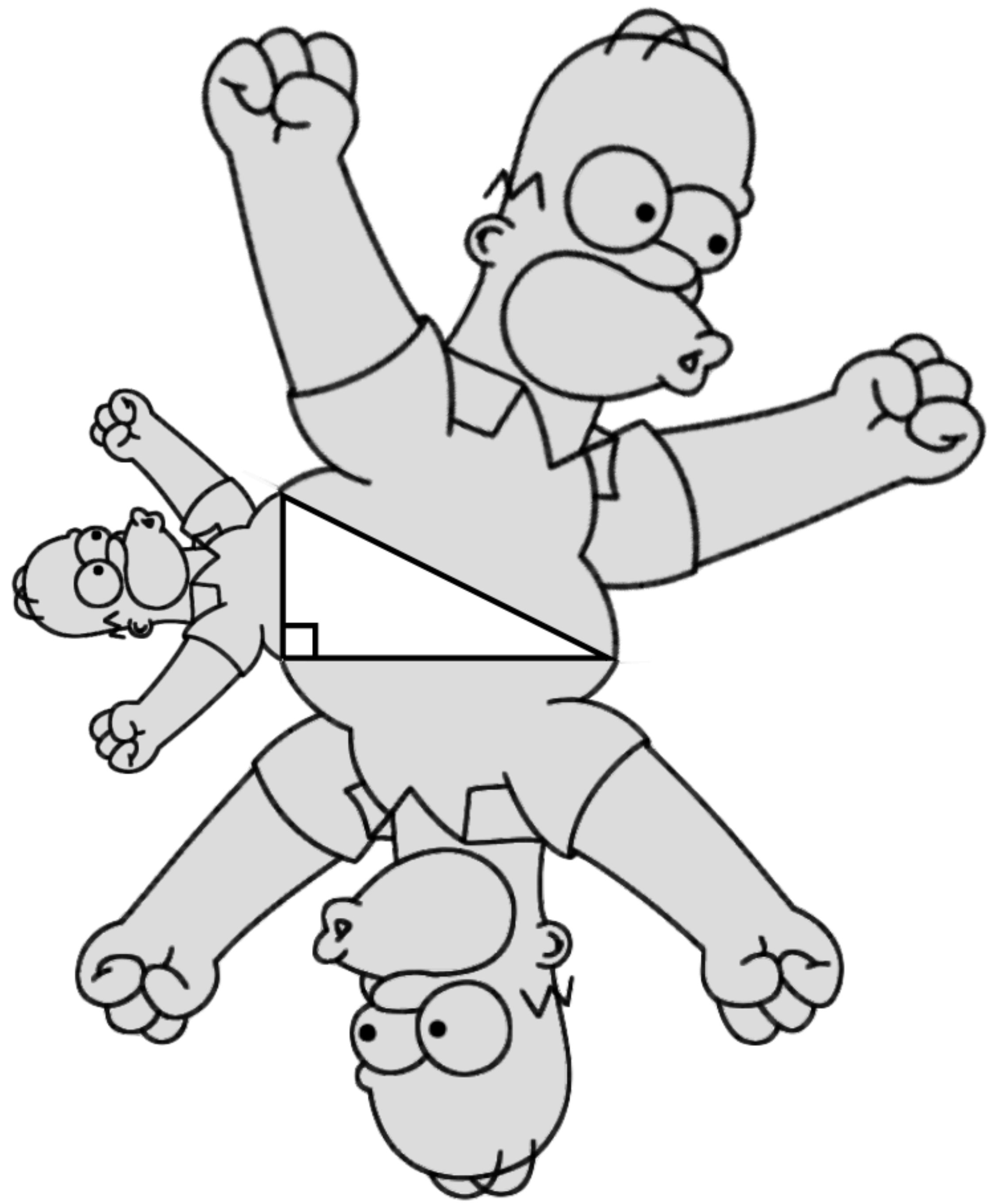
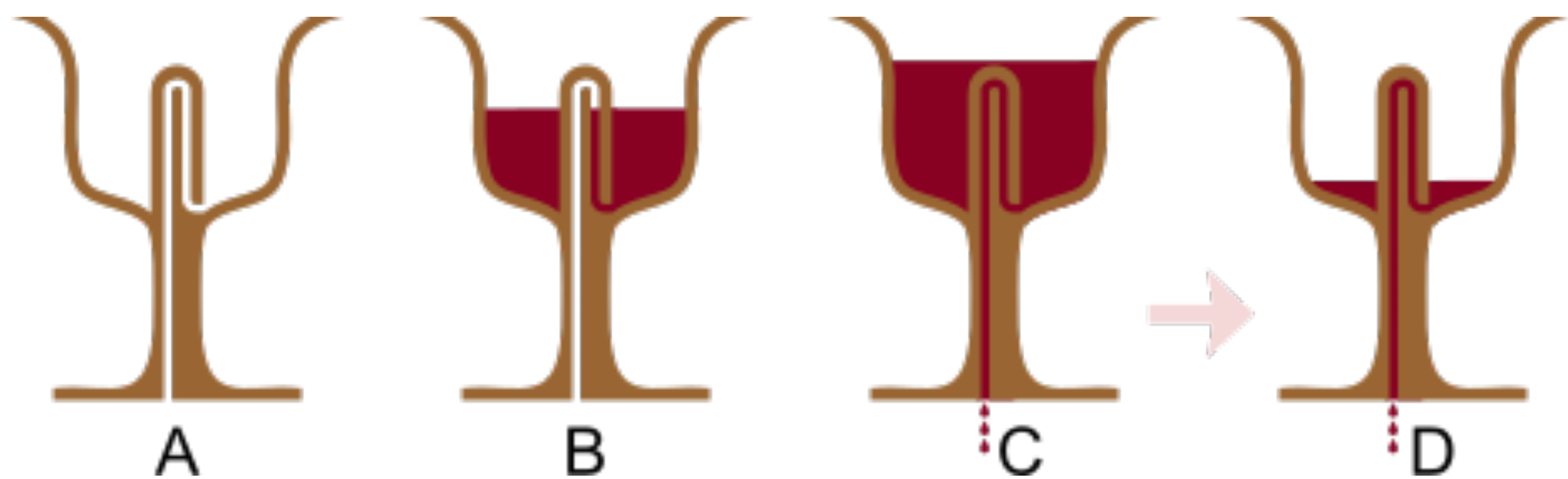
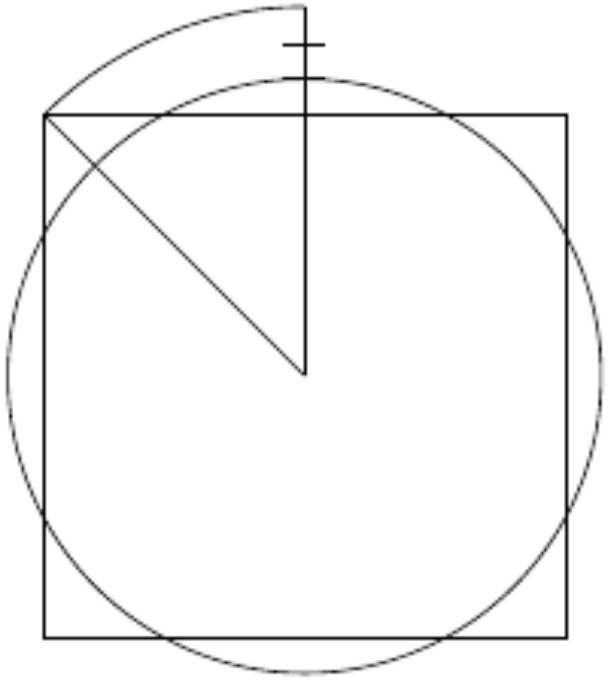
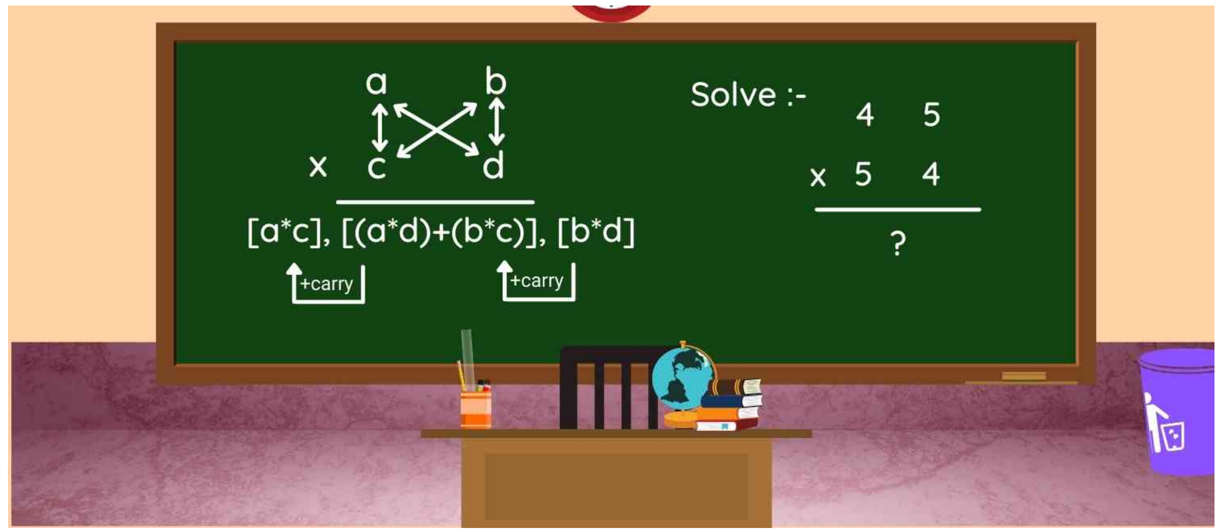


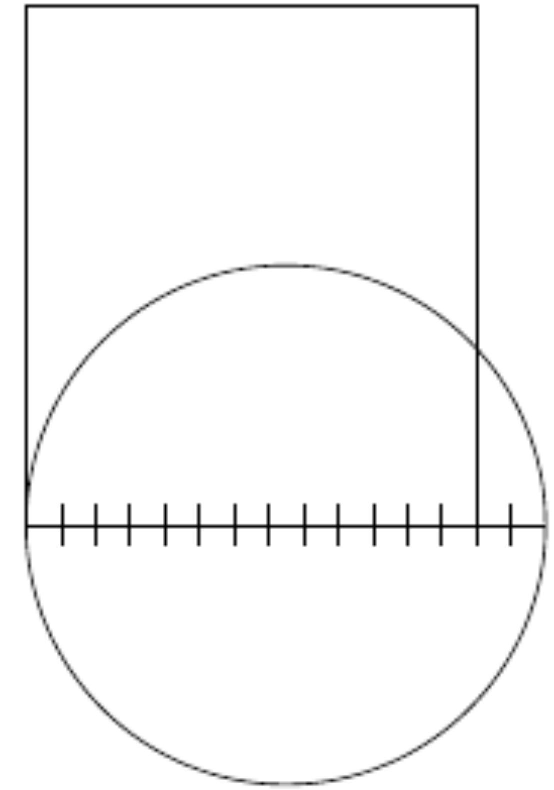
Pythagorean Cup

- A practical joke device whose invention is credited to Pythagoras.
- Also called a “justice cup”

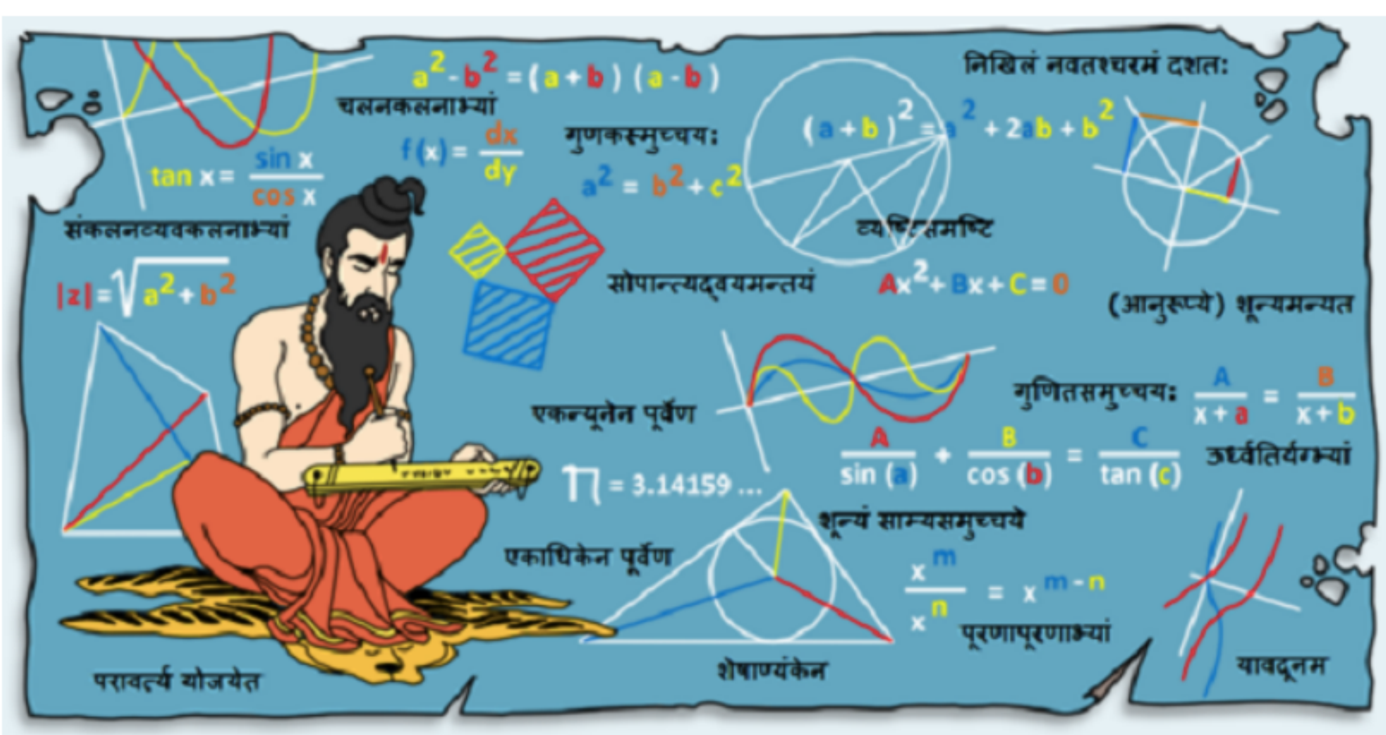


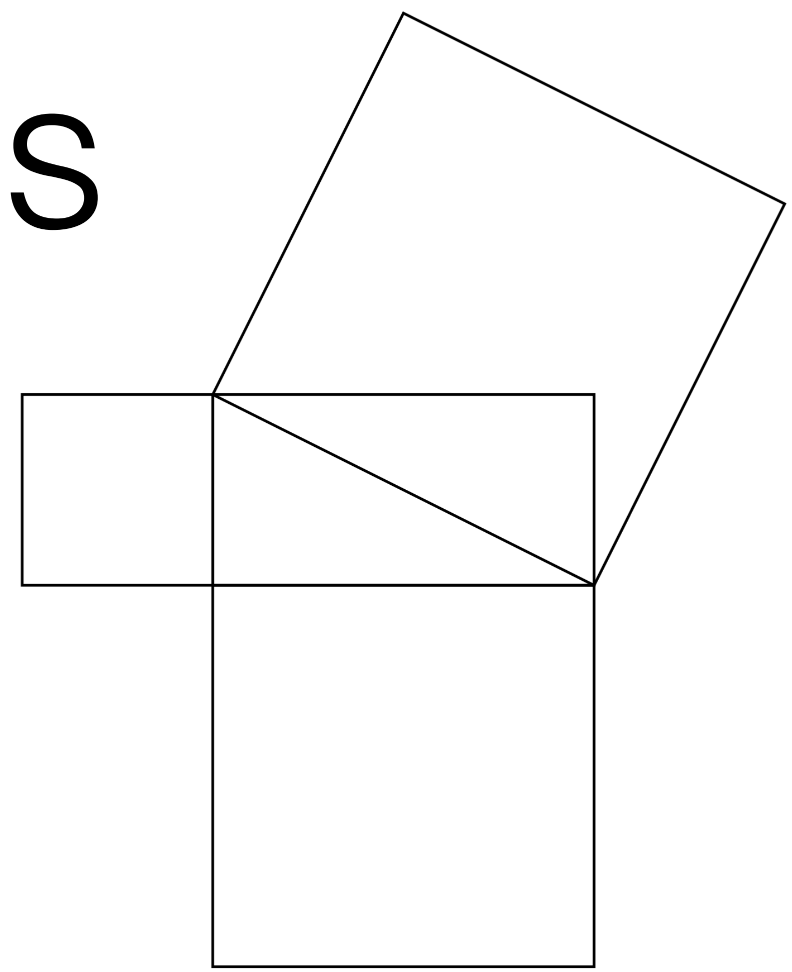






Ancient Indian Mathematics





Ancient India

- The ancient Indians used the Pythagorean theorem well.

- There is no evidence they proved it, but they used it so well that they may have. They certainly understood it extremely well.

Ancient India

- The Vedic people migrated from modern-day Iran to the Indian subcontinent around 1500 BC.

Ancient India

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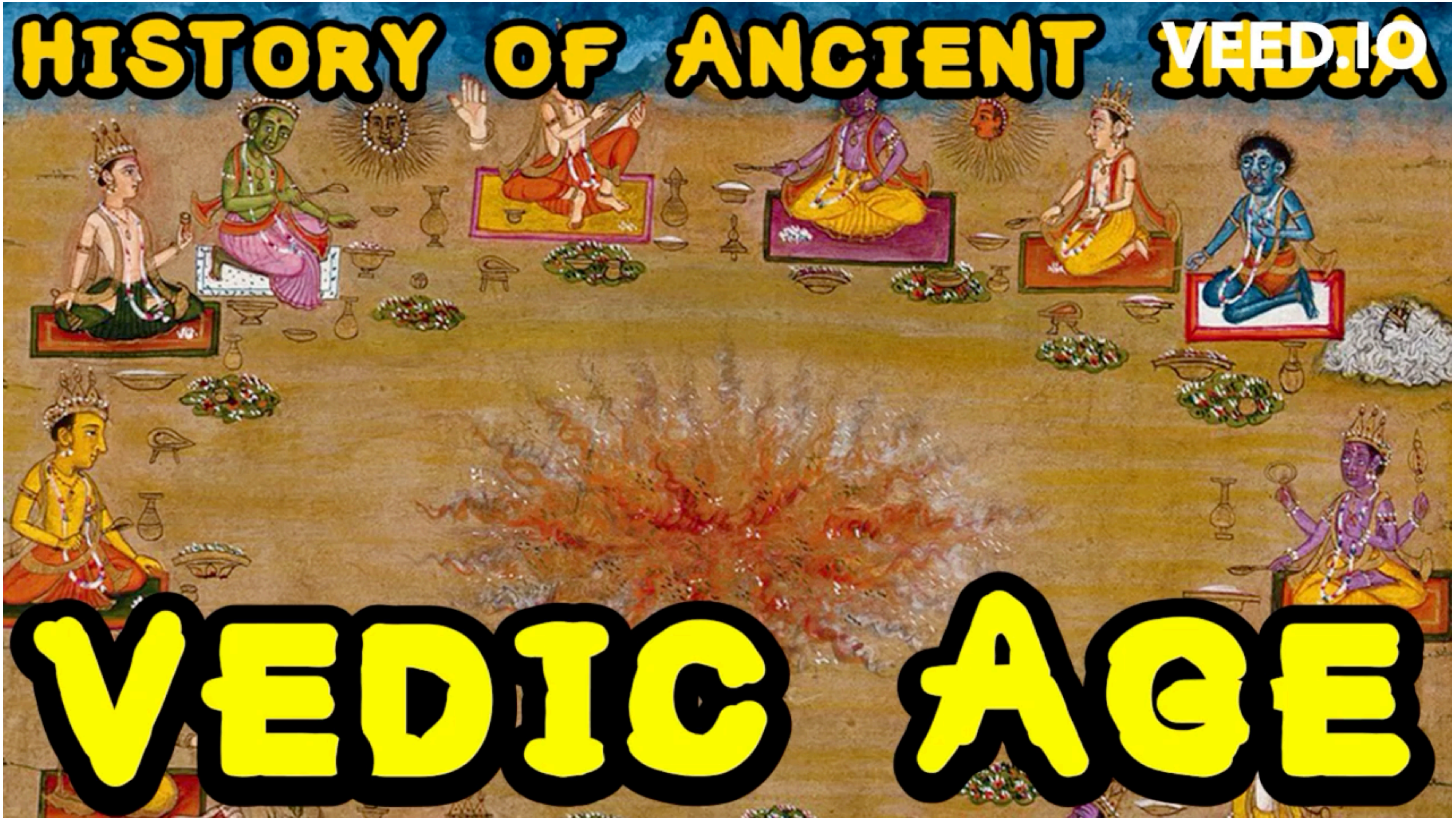
- “Vedic” refers to the Vedas—the oldest religious texts of Hinduism.

- The Vedas contain appendices called *sulbasutras*, which contain geometric instructions for how to build fire-altars.

- These tell us some of the geometry they knew.

Ancient India



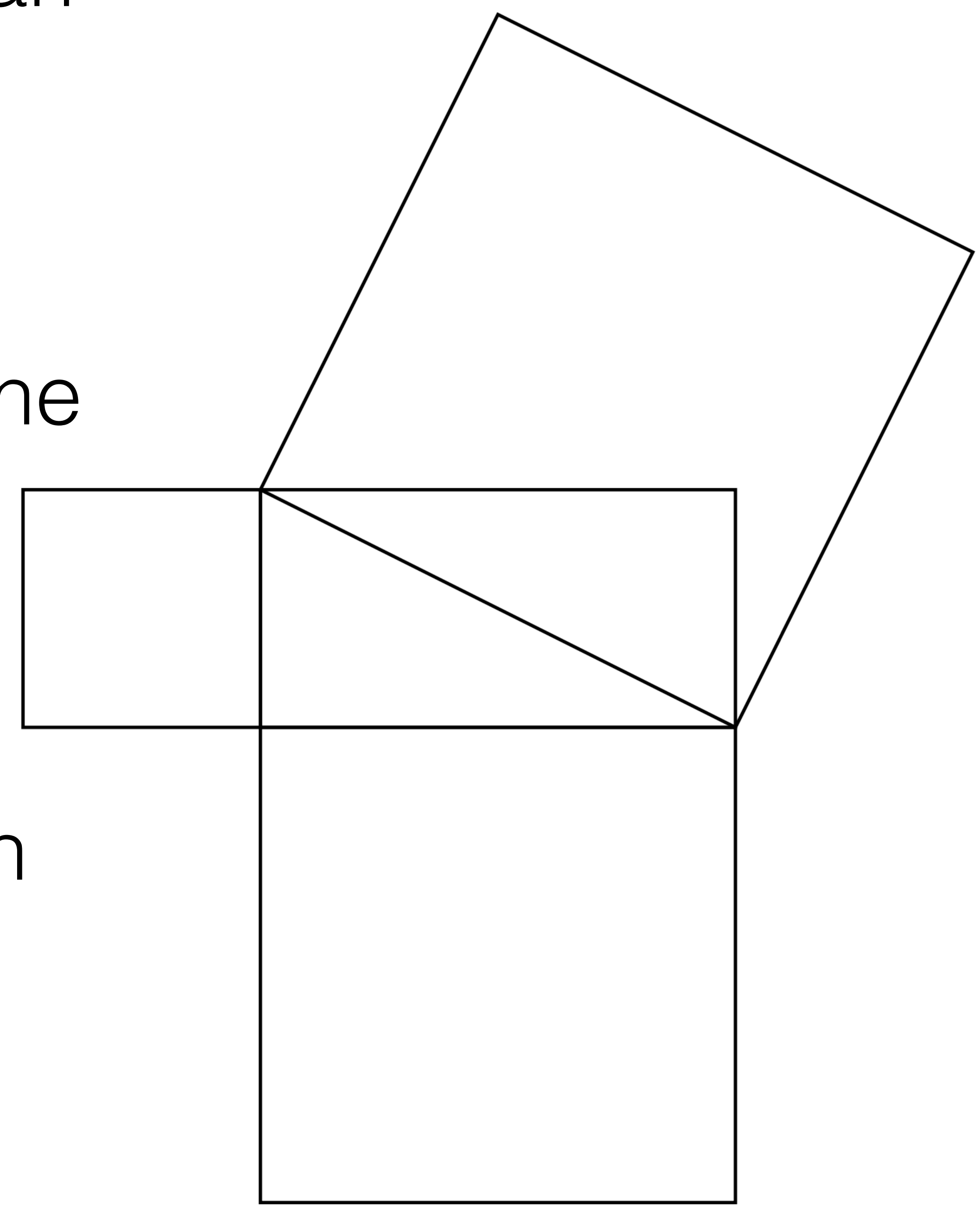
Ancient India

- Reminder:
- The Vedas contain appendices called *sulbasutras*, which contain geometric instructions for how to build fire-altars.
- These tell us some of the geometry they knew.

Ancient India

- Their version of the Pythagorean theorem:

"The rope of the diagonal of a rectangle makes an [area] which the vertical and horizontal sides make together."



- Three examples of using this in their *sulbasutras*:

Ancient India

- **1. Squaring a pair of squares.** Meaning: Given a pair of squares, create a third square whose area is equal to the sum of the other two.

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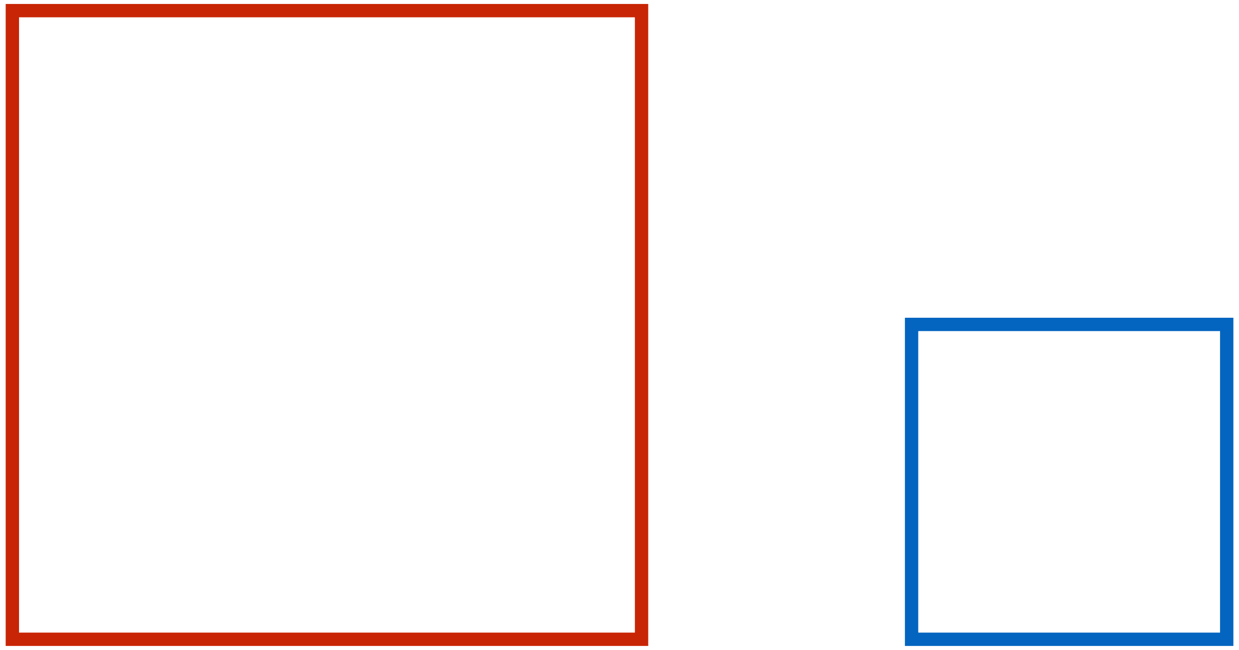

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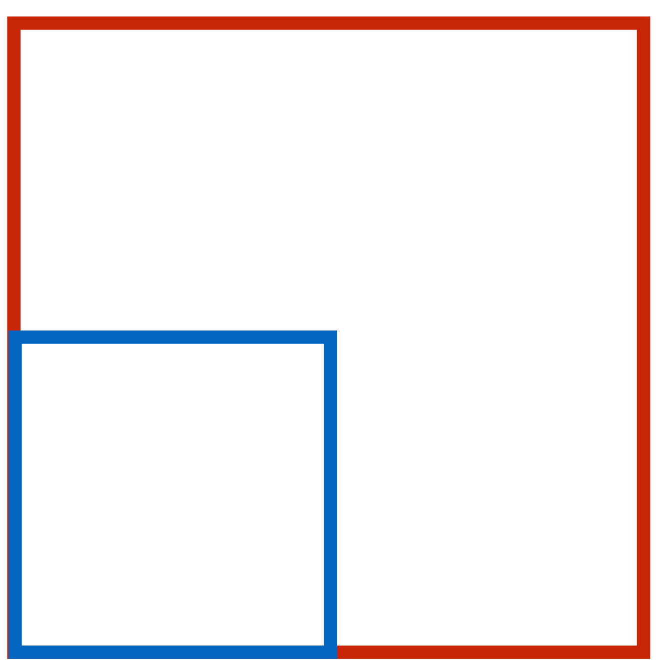

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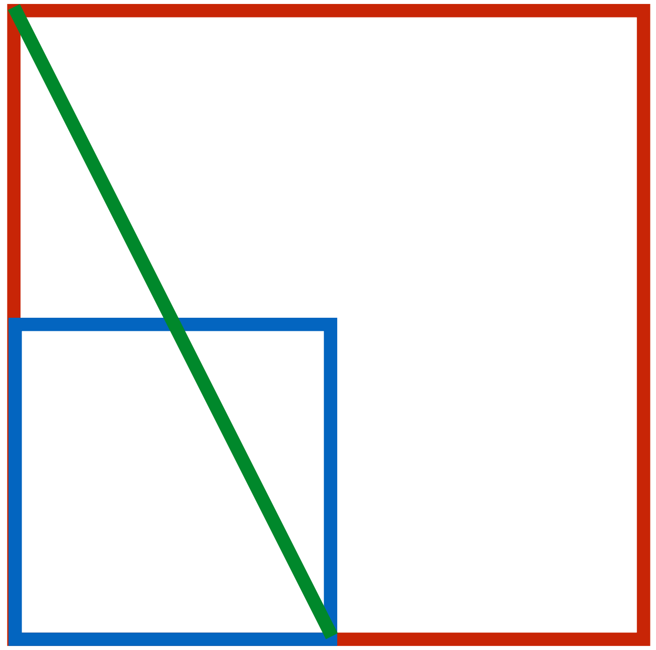

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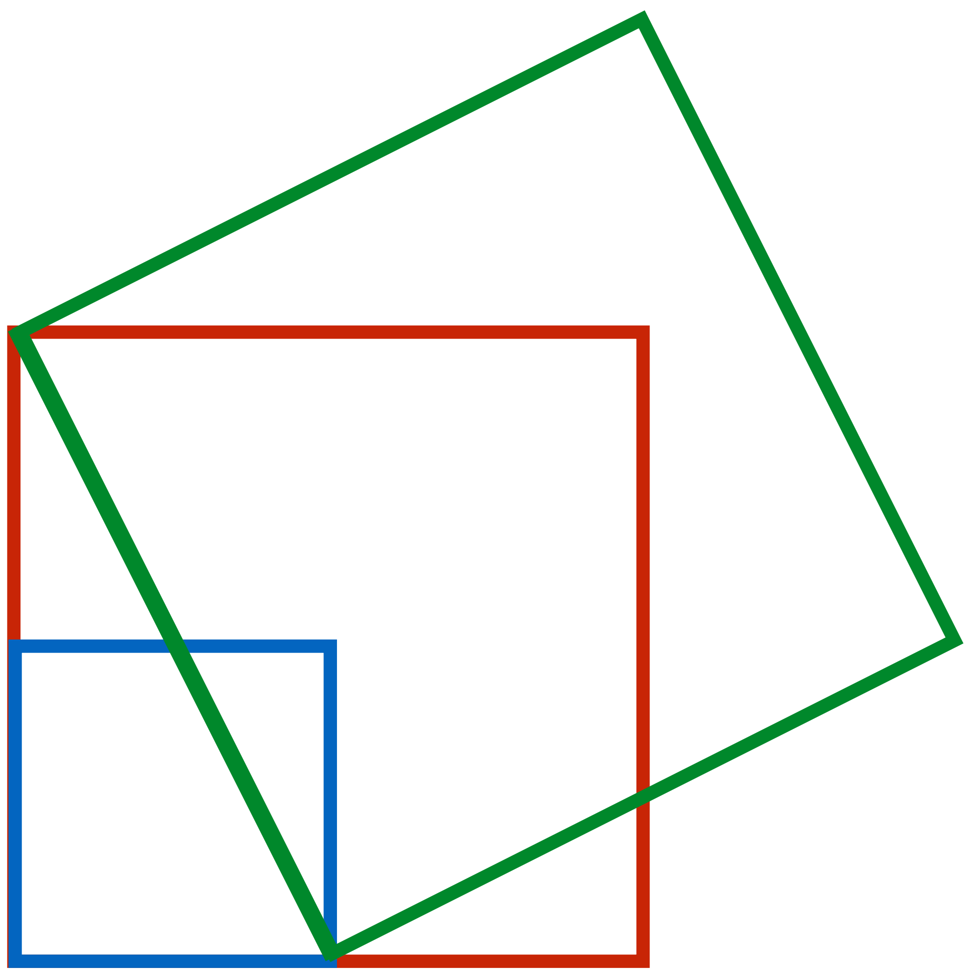

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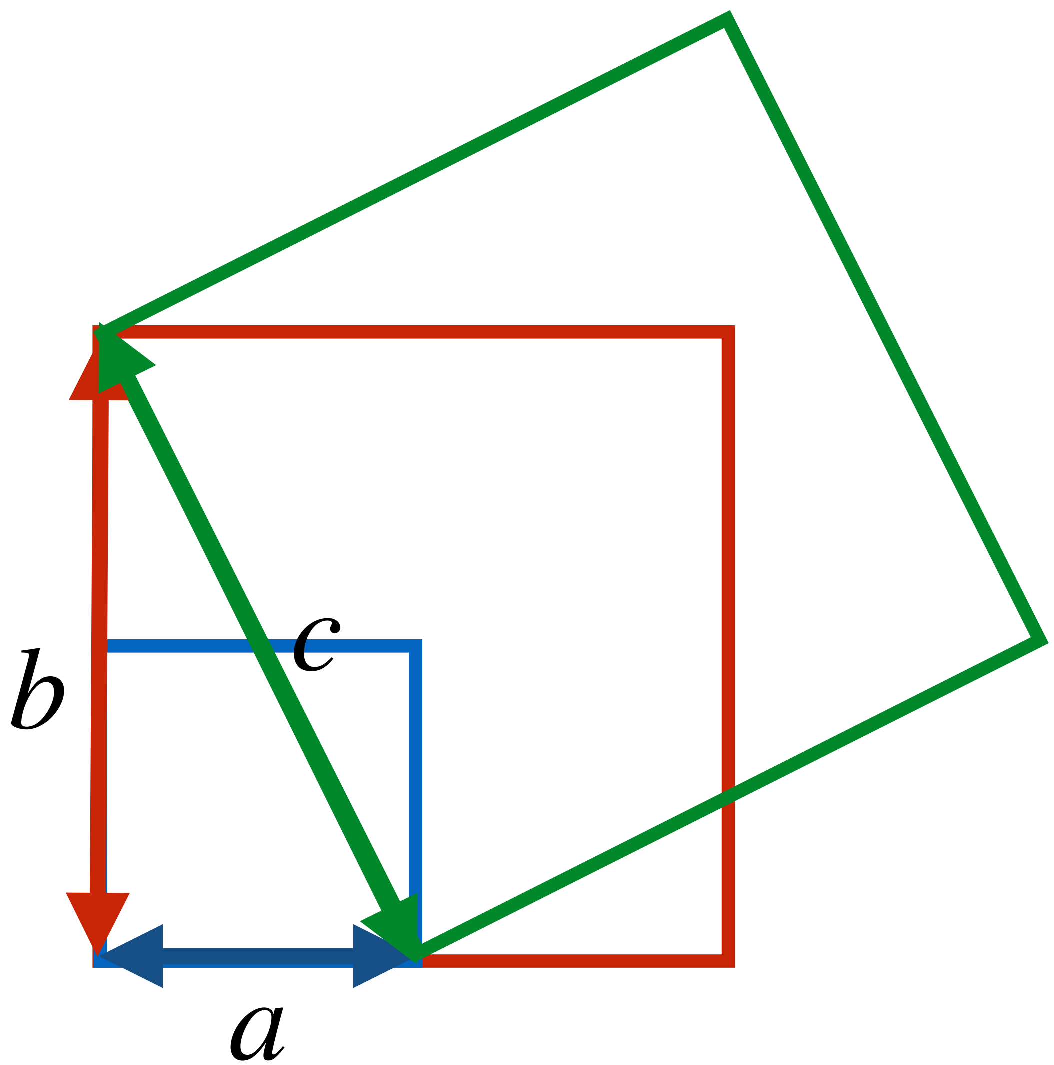

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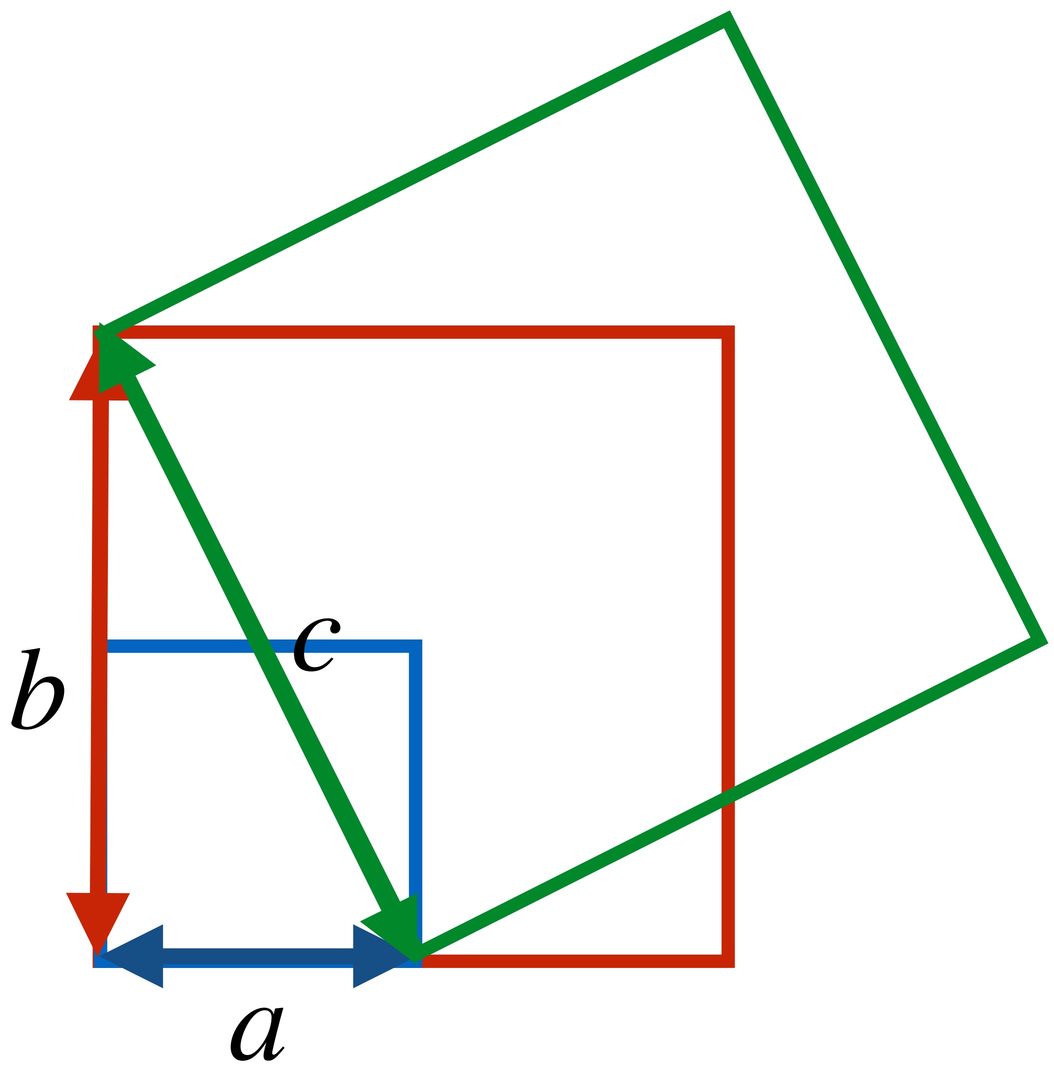

$$\implies a^2 + b^2 = c^2$$

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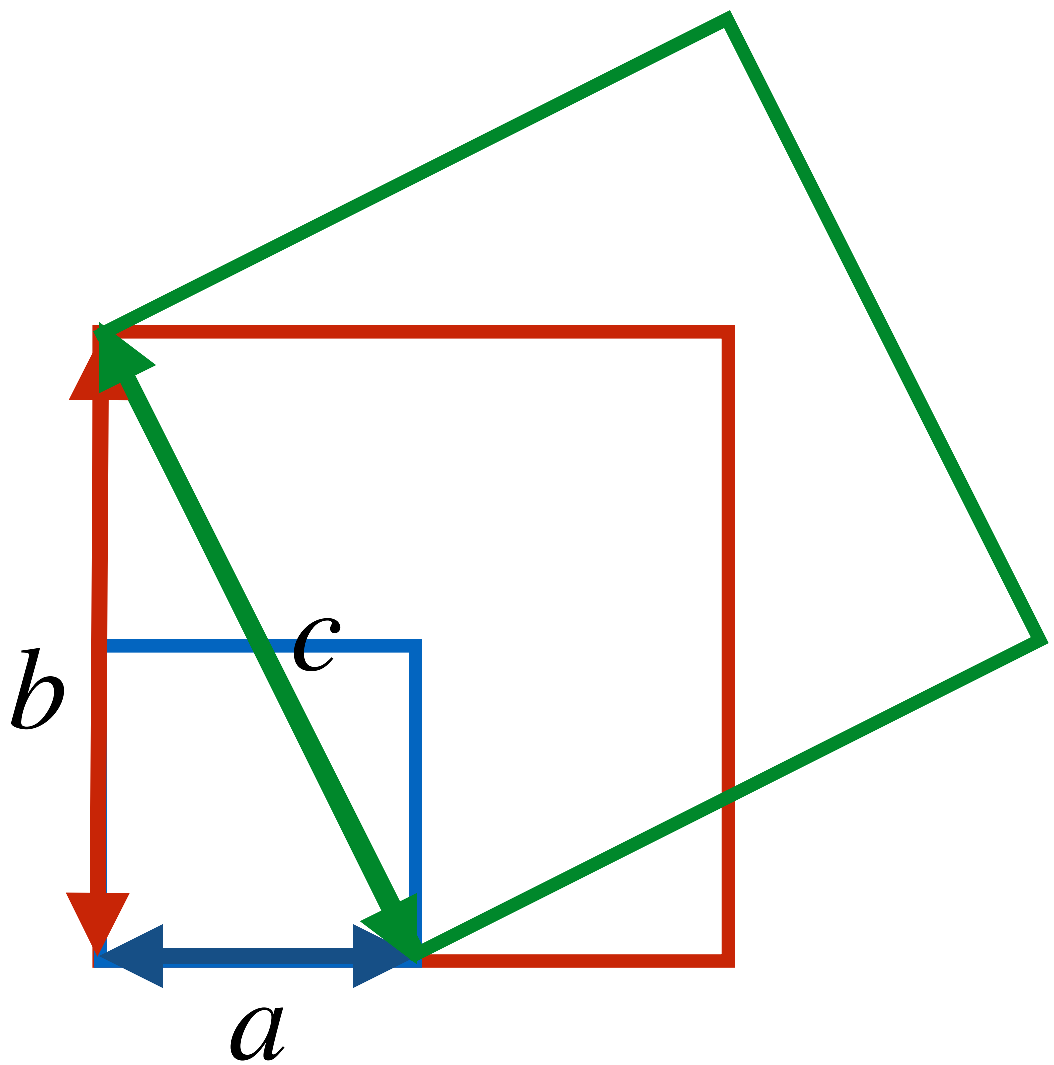
- **1. Squaring a pair of squares.** Meaning: Given a pair of squares, create a third square whose area is equal to the sum of the other two.



$$\Longrightarrow a^2 + b^2 = c^2$$

$$\Longrightarrow \square + \square = \square$$

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the area of the rectangle.

Ancient India

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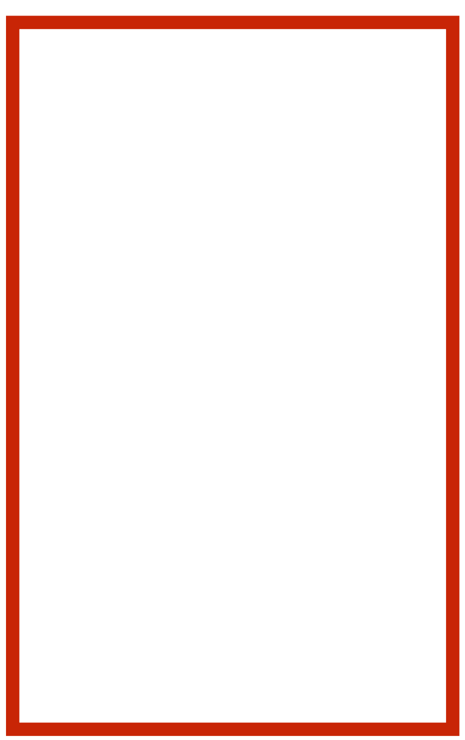
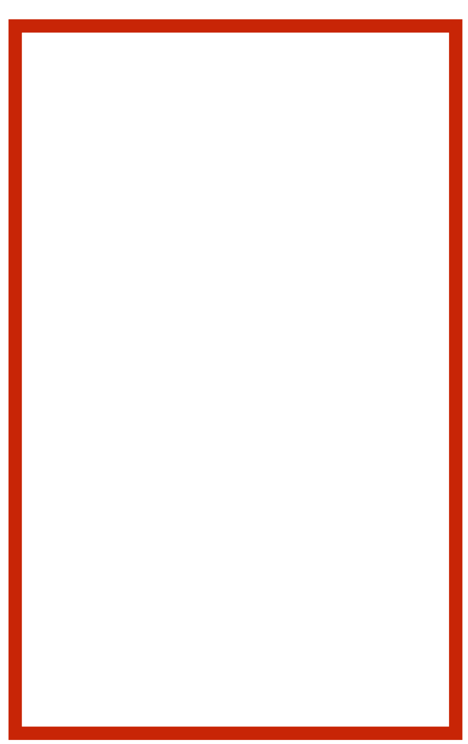


Ancient India

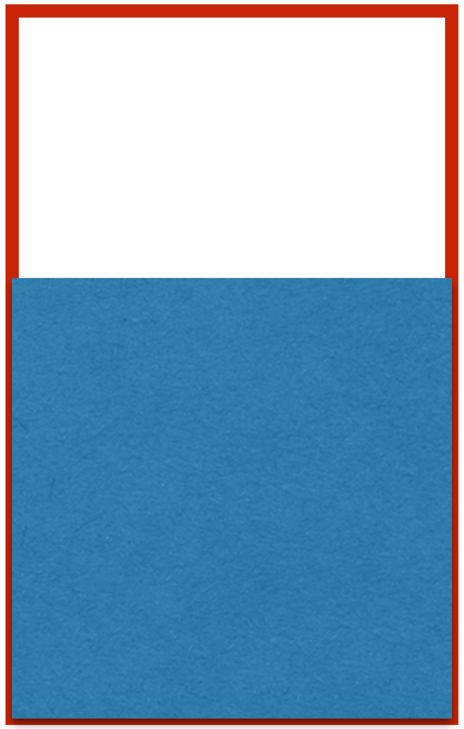
- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the area of the rectangle.

1. Create Square in the bottom of the rectangle.



Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the area of the rectangle.



1. Create Square in the bottom of the rectangle.

Ancient India

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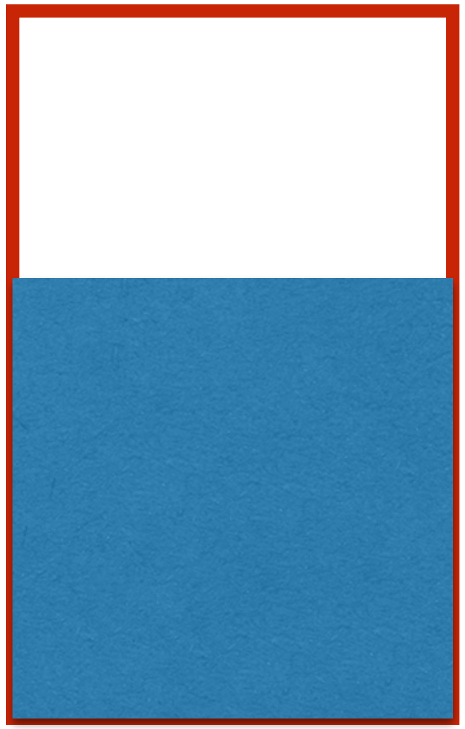

1. Create Square in the bottom of the rectangle.

Total Area =

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the area of the rectangle.

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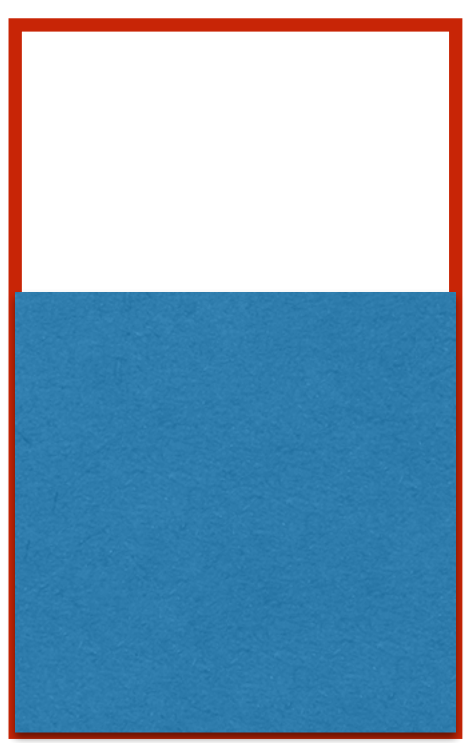
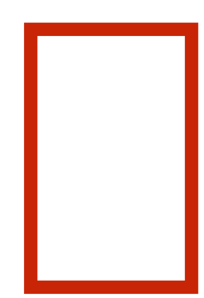
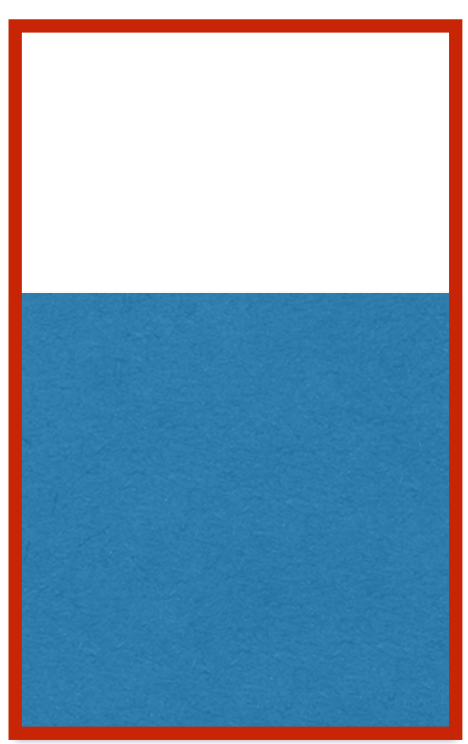


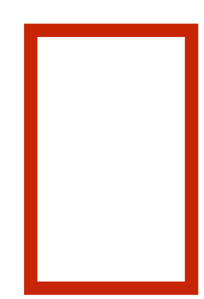
Total Area = 

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

2. Cut the top region in half



Total Area = 

Ancient India

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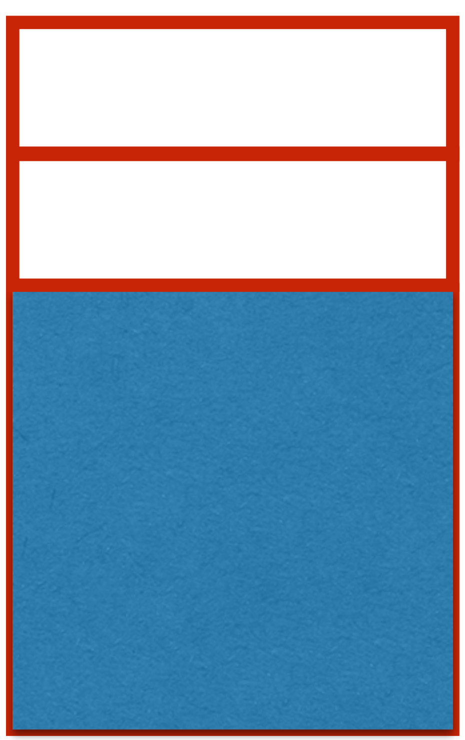
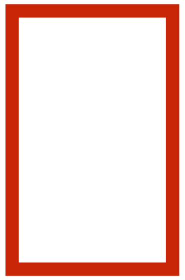
2. Cut the top region in half

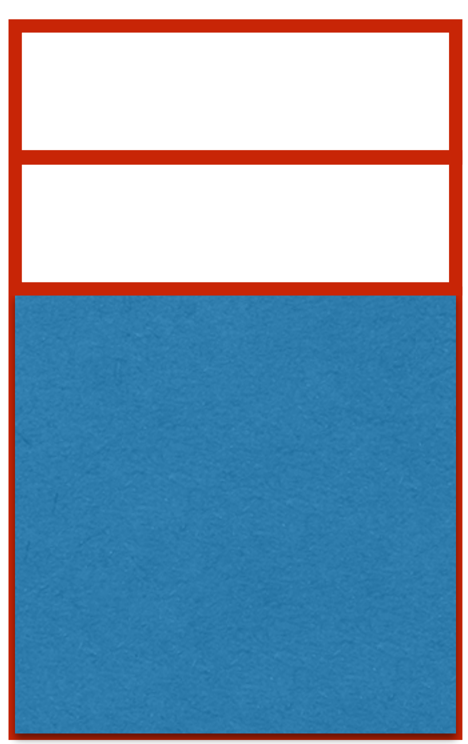


Total Area =

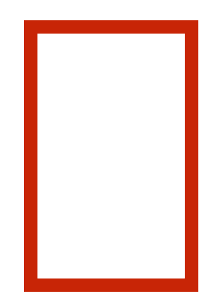


Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.



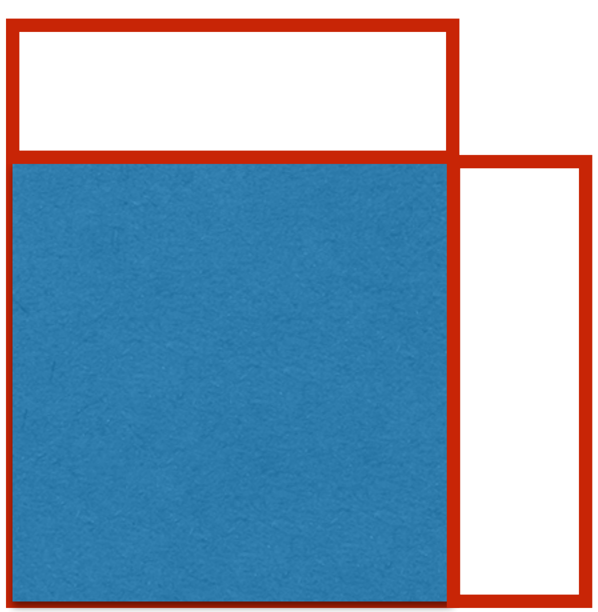
3. Move the top-most rectangle to the right side of the square

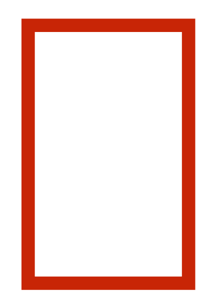
Total Area = 

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

3. Move the top-most rectangle to the right side of the square

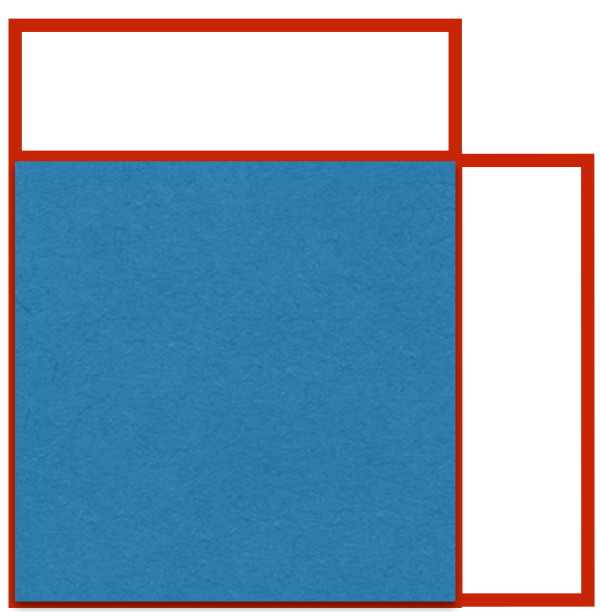


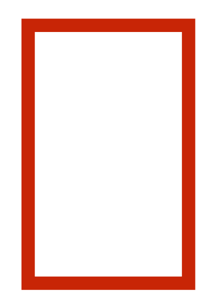
Total Area = 

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

4. Fill in the gap to form a square.



Total Area = 

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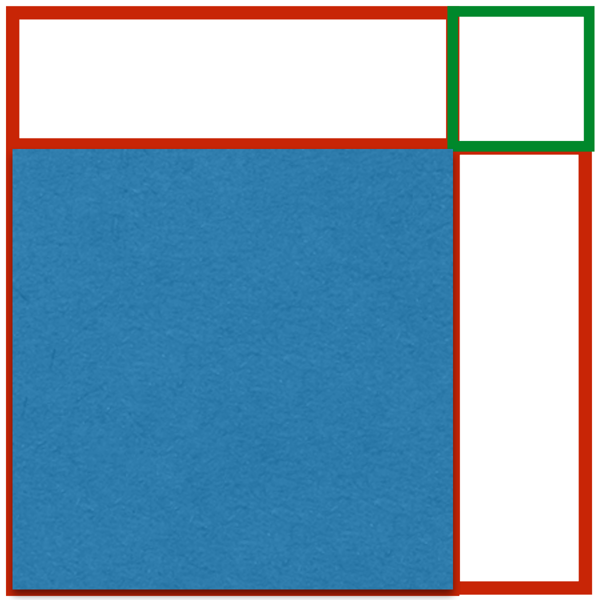
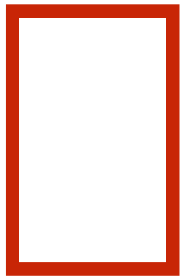


Total Area = 

Ancient India

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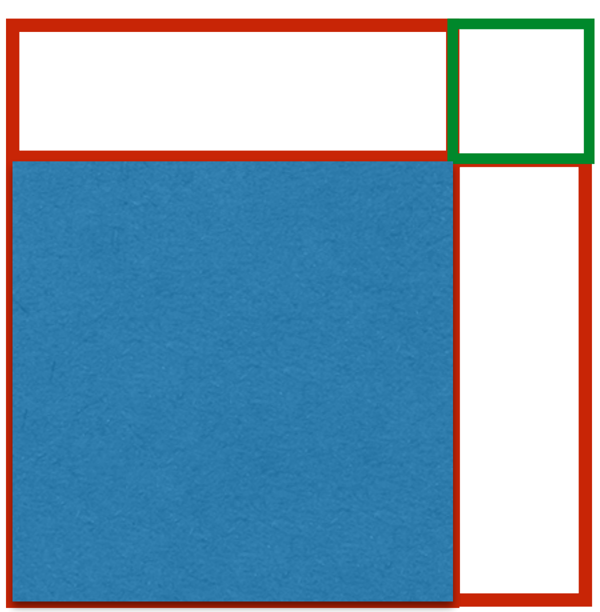
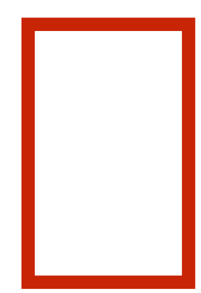
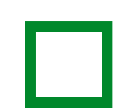


Total Area =  + 

Ancient India

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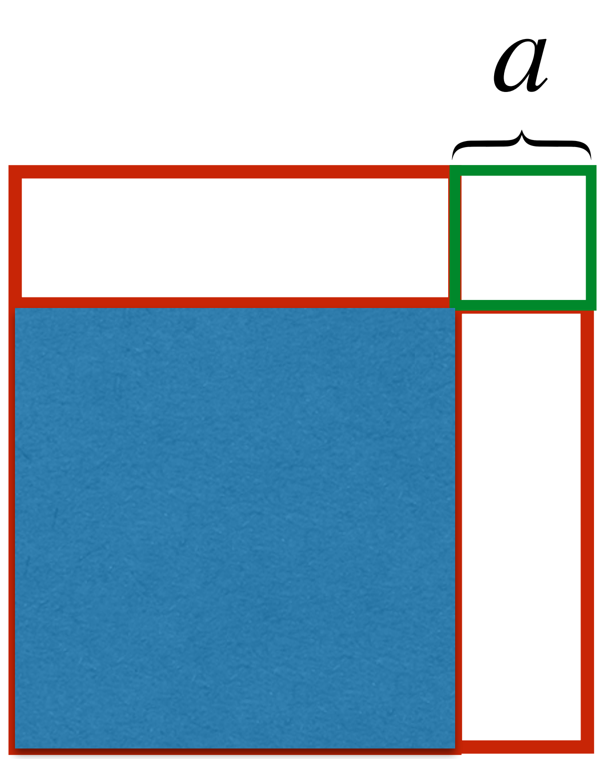
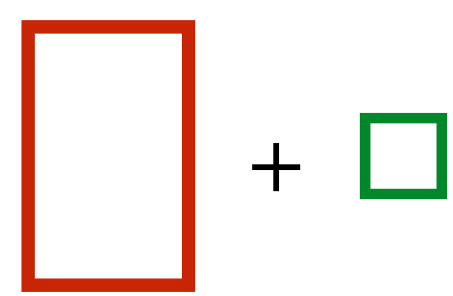


Total Area = 

Ancient India

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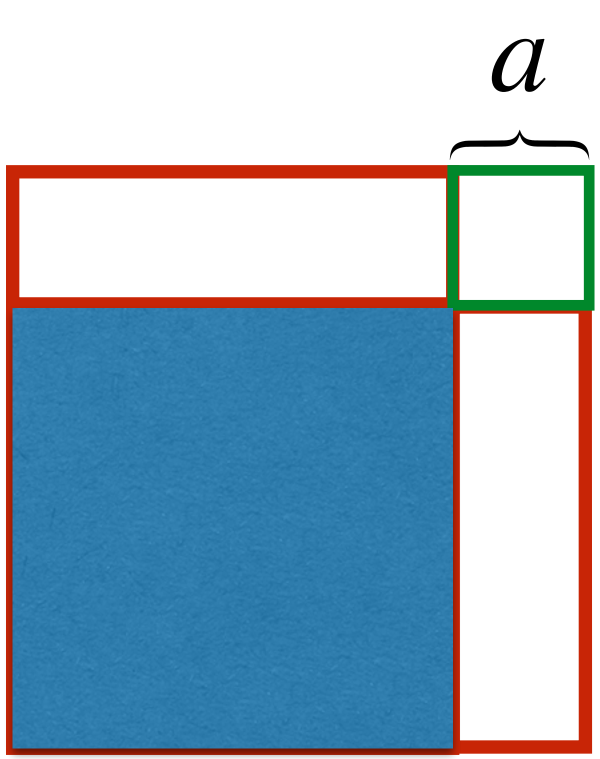
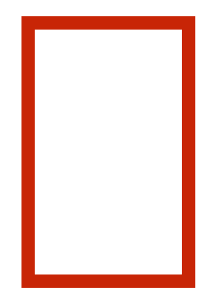
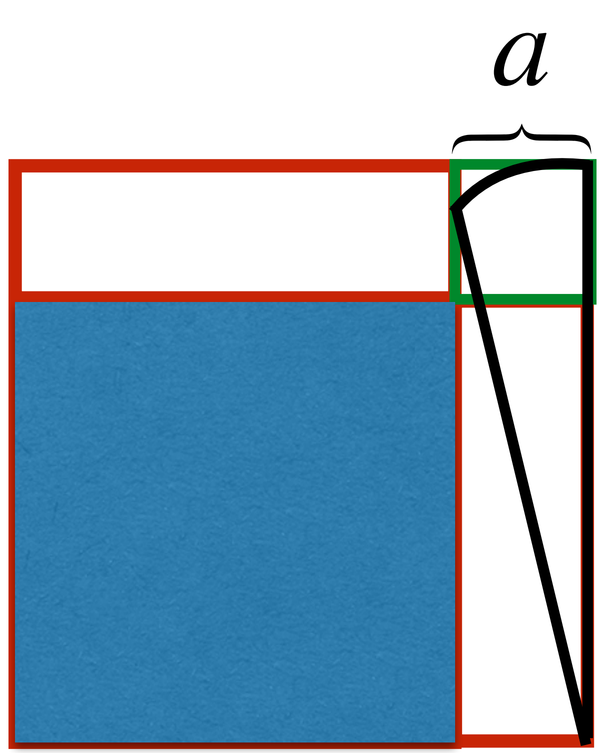


Total Area =  + a^2

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

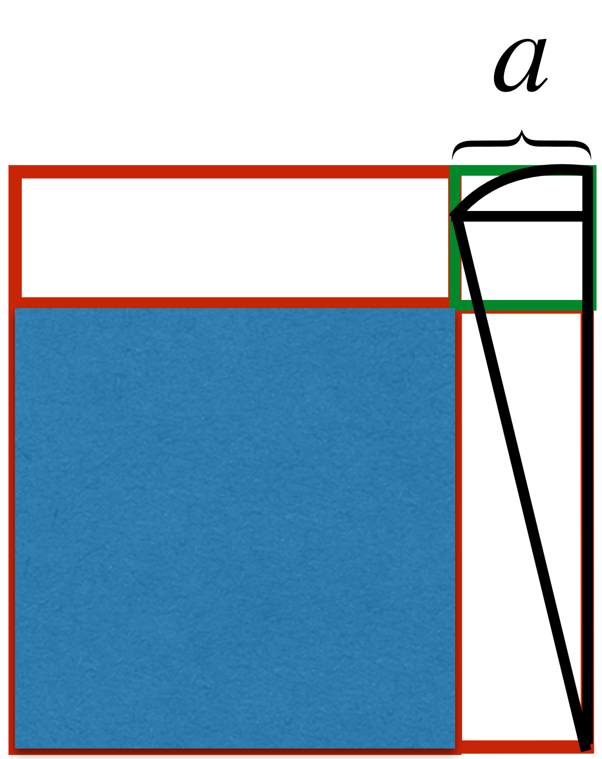
5. Using a compass, make the arc shown.



Total Area = $\boxed{} + a^2$

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.



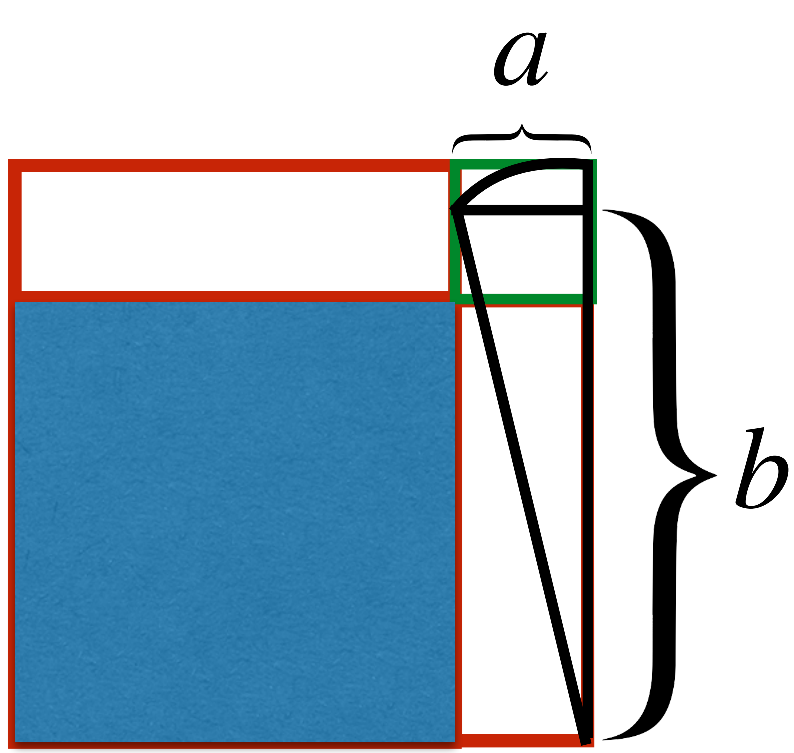
6. Draw the following horizontal line

Total Area = $\square$ + a^2

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

7. Label this length b, and let the hypotenuse be c:



Total Area = $\boxed{}$ + a^2

Ancient India

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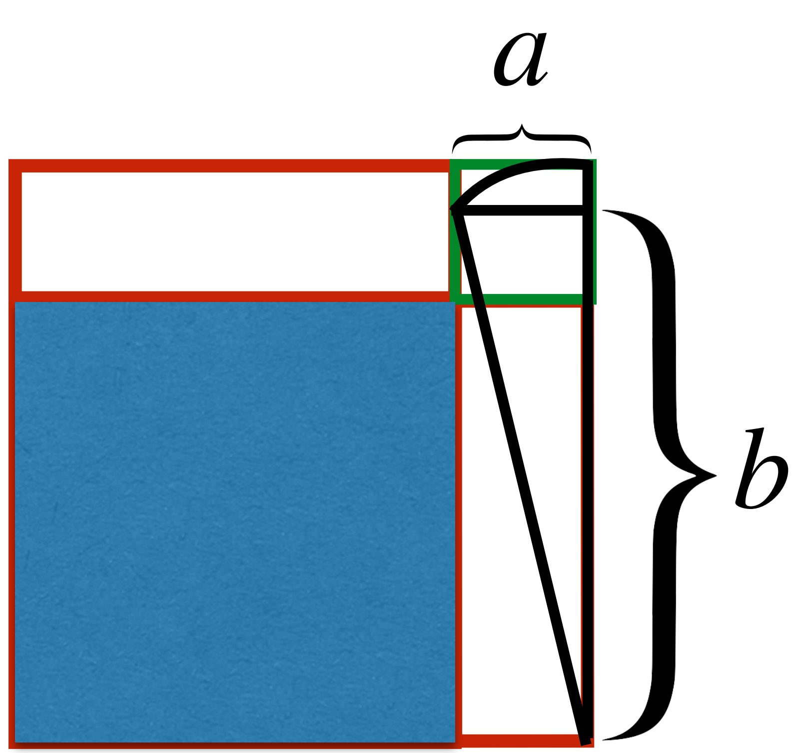

$$c^2 = \boxed{} + a^2$$

Ancient India

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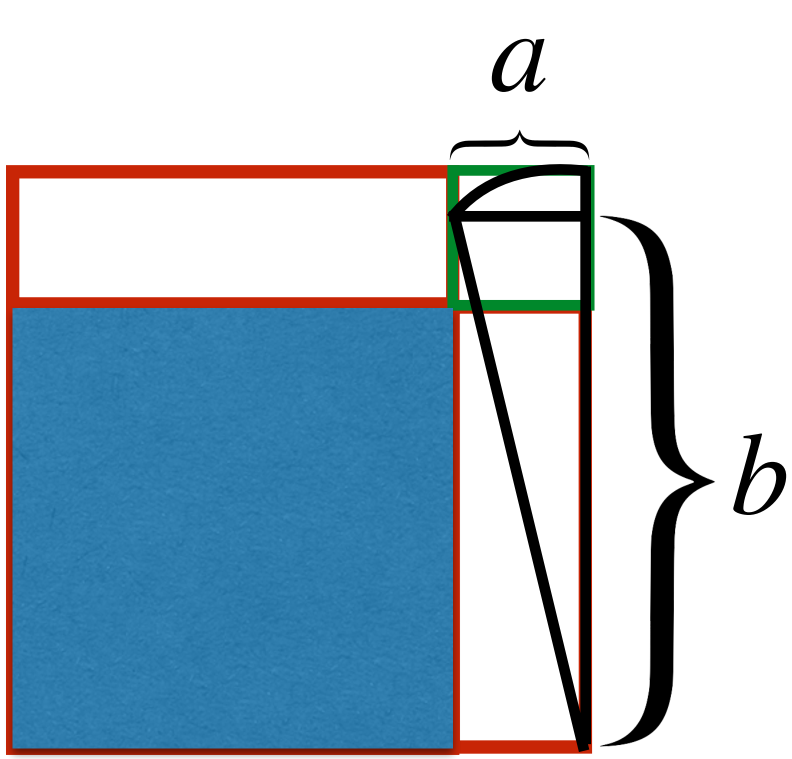

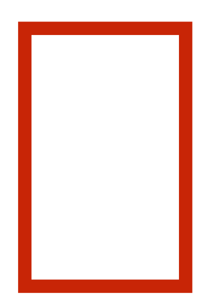

$= c^2 - a^2$

Ancient India

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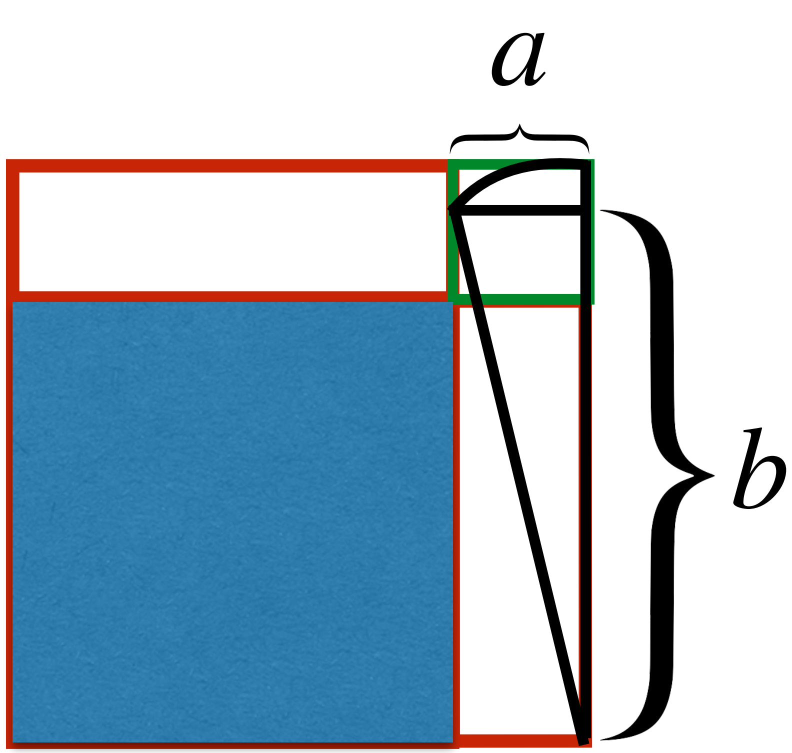
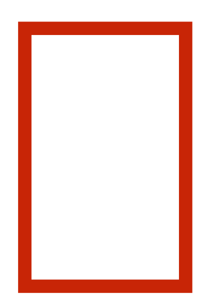


By the Pythagorean theorem,

$$a^2 + b^2 = c^2.$$



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Ancient India

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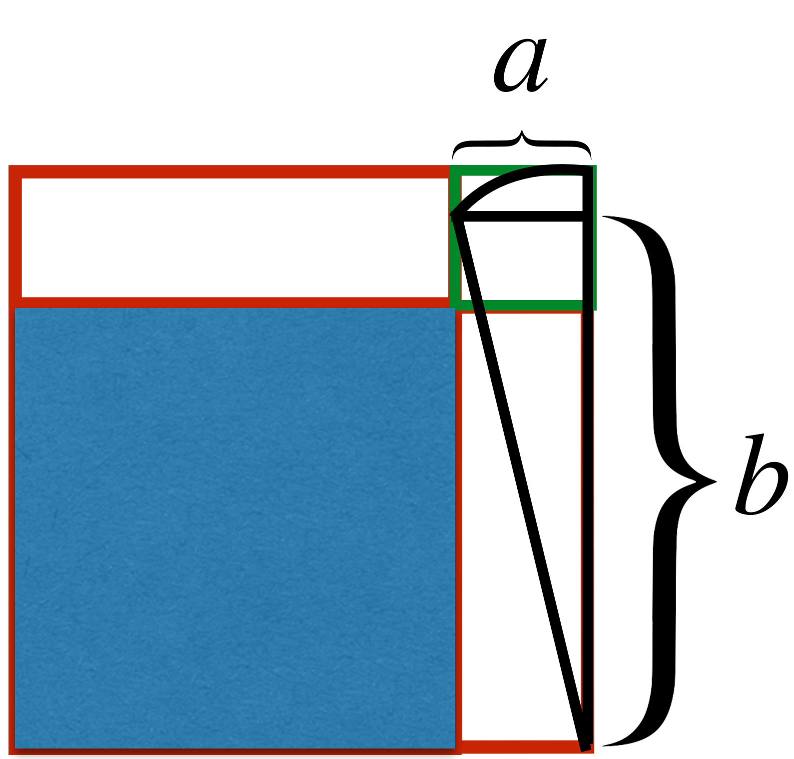
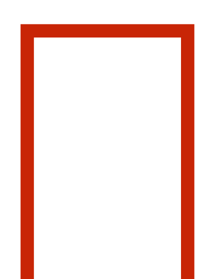


7. Label this length b, and let the hypotenuse be c:

By the Pythagorean theorem,

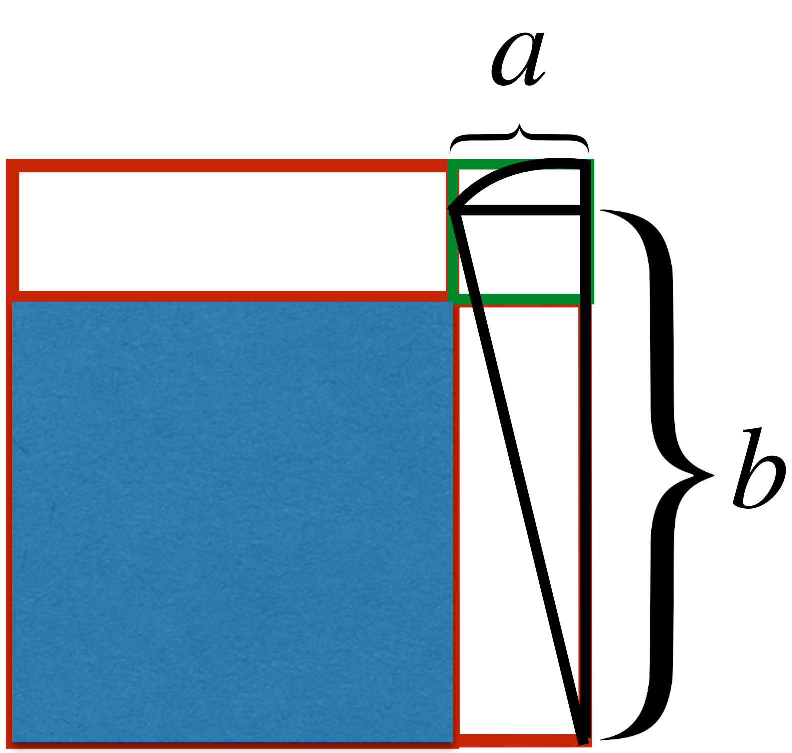
$$a^2 + b^2 = c^2.$$

$$\implies b^2 = c^2 - a^2.$$

 $= c^2 - a^2$

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.



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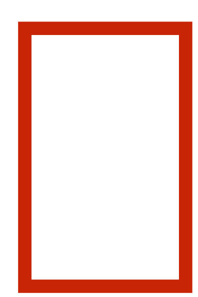
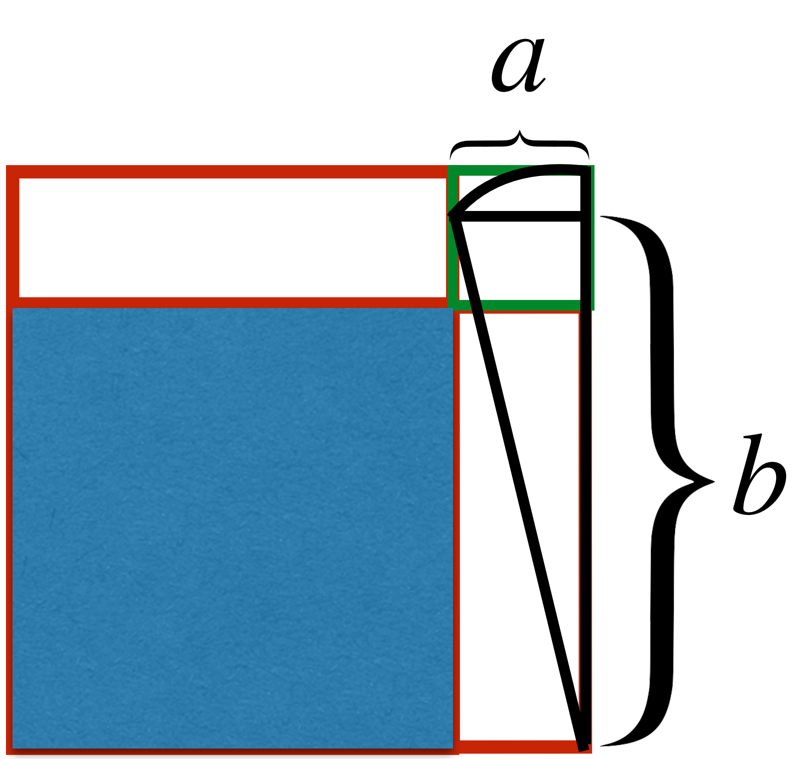
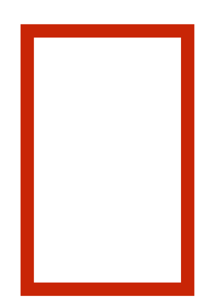
 $= \; b^2$

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

8. Build a square on this line segment.





$= b^2$

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

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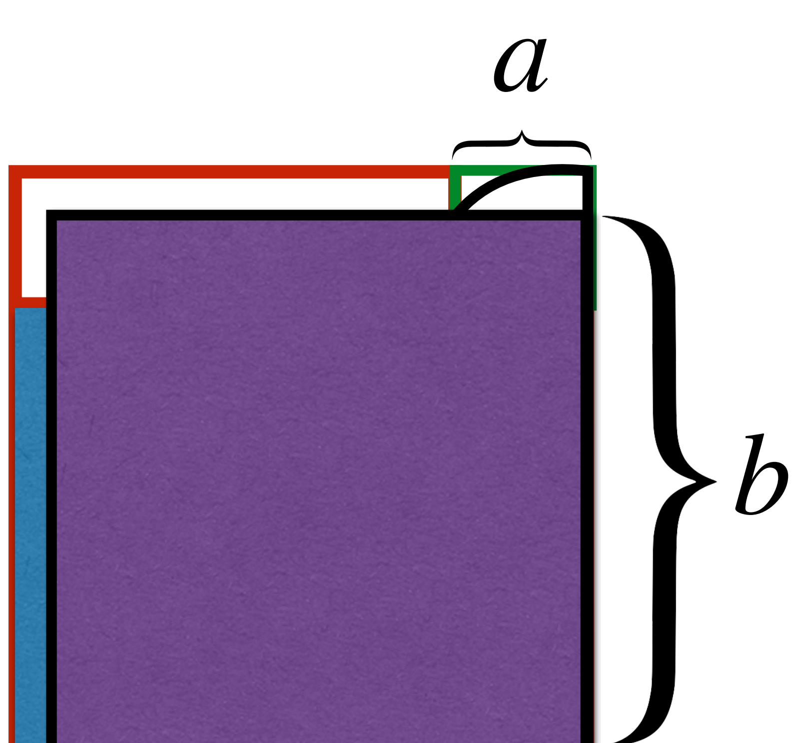
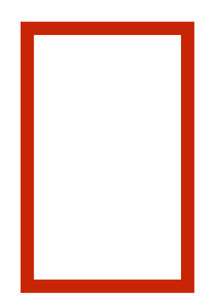
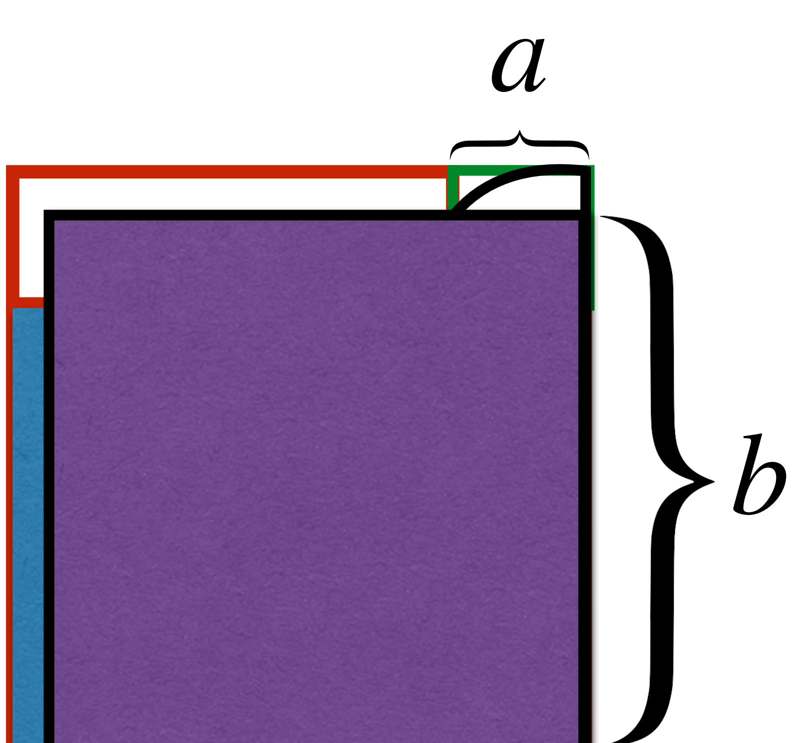


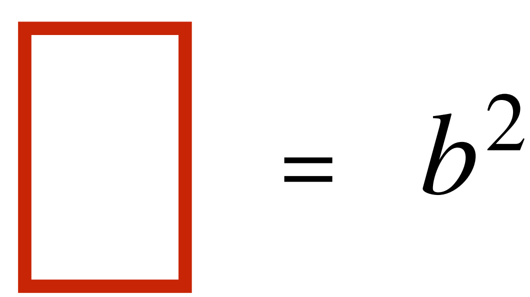
 $= b^2$

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

8. Build a square on this line segment. The area of this square equals b^2.





Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

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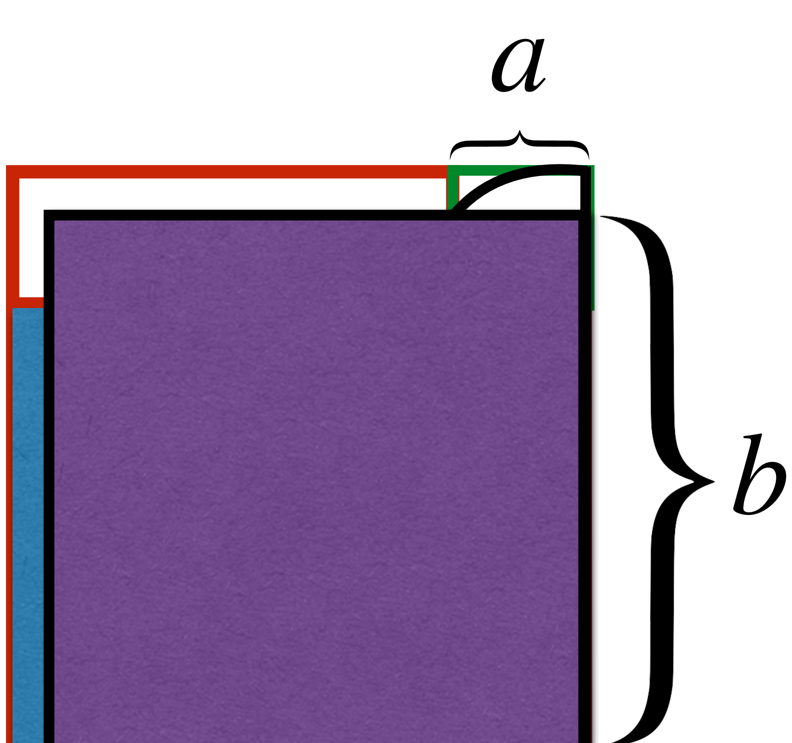
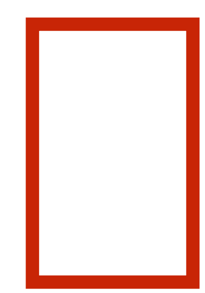
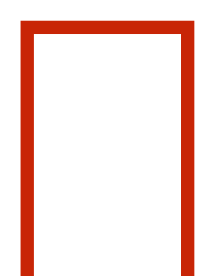


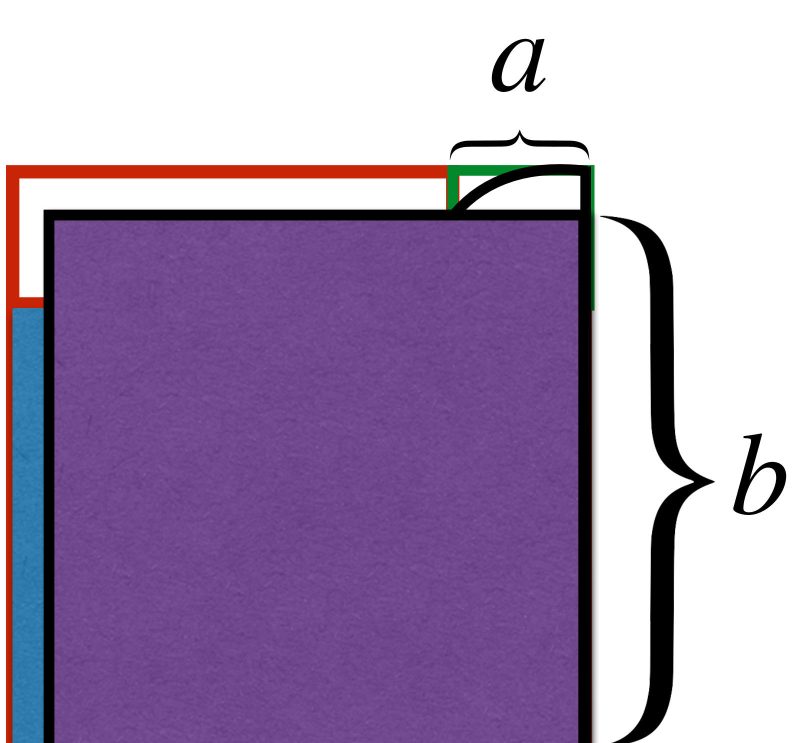
So,  = 

 $= b^2$

Ancient India

- **2. Squaring a rectangle.** Meaning: Given a rectangle, create a square whose area is equal to the sum of the other two.

9. So, this square has the same area as the original rectangle.



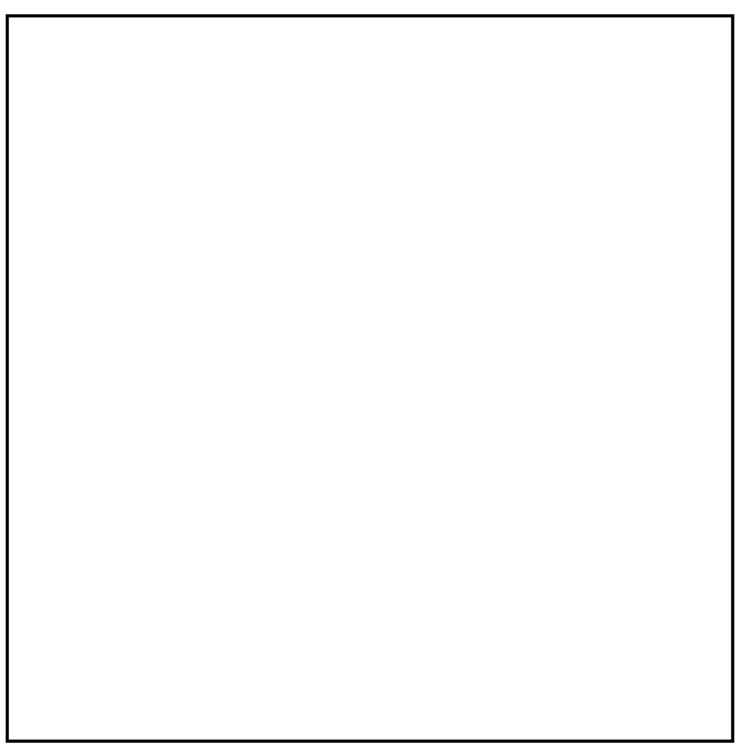
Ancient India

- **3. Circling a square or squaring a circle.** Meaning: Given a square, create a circle whose area is equal to that of the square. Or, given a circle, create a square of the same area.

- Note: While the last two gave perfect constructions, these ones do not. Also, they did not note in their work that these two were imperfect.

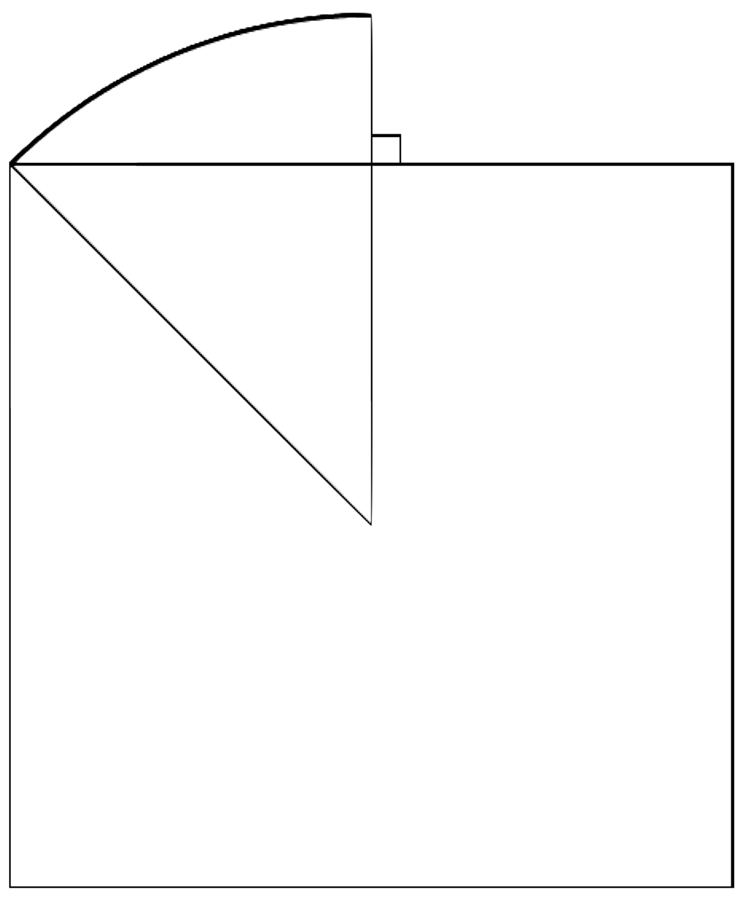
Ancient India

- **3. Circling a square or squaring a circle.**



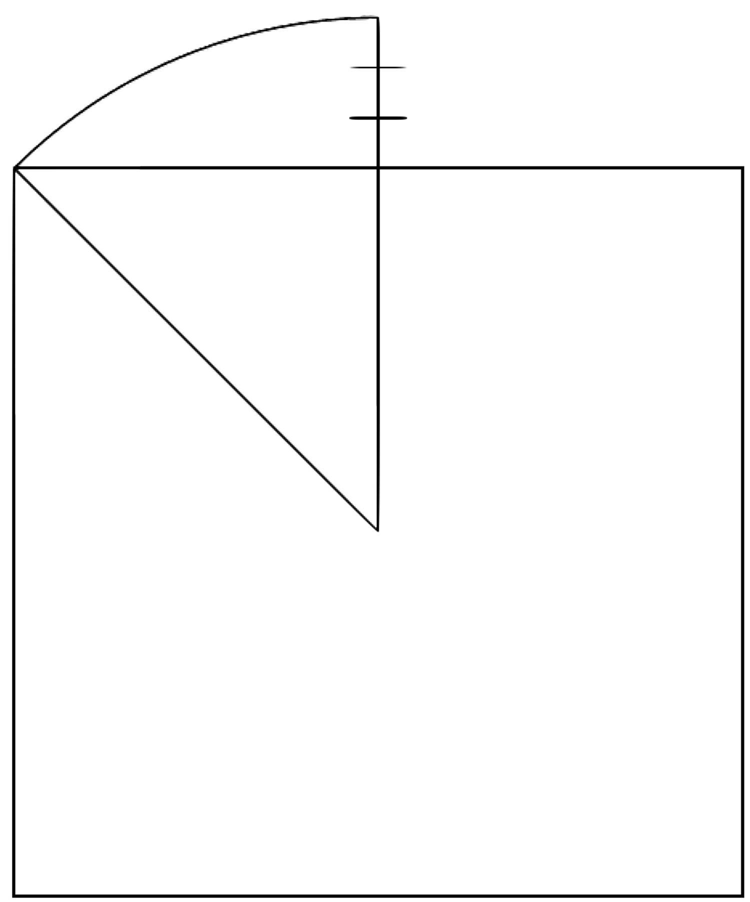
Ancient India

- **3. Circling a square or squaring a circle.**



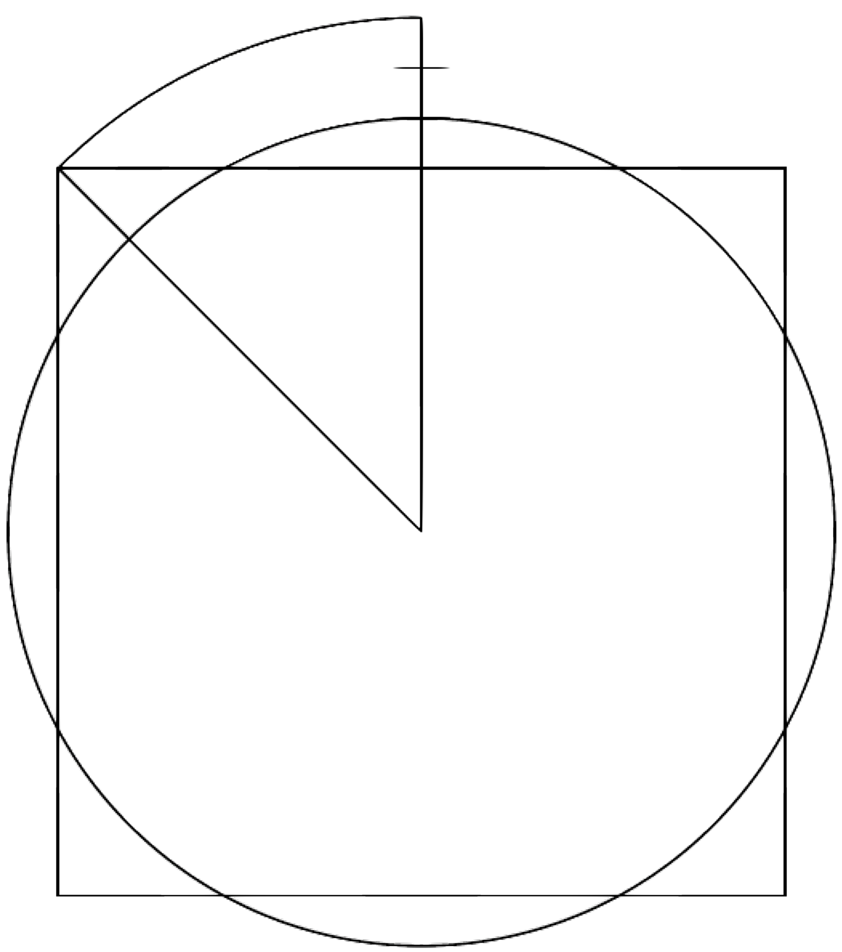
Ancient India

- **3. Circling a square or squaring a circle.**



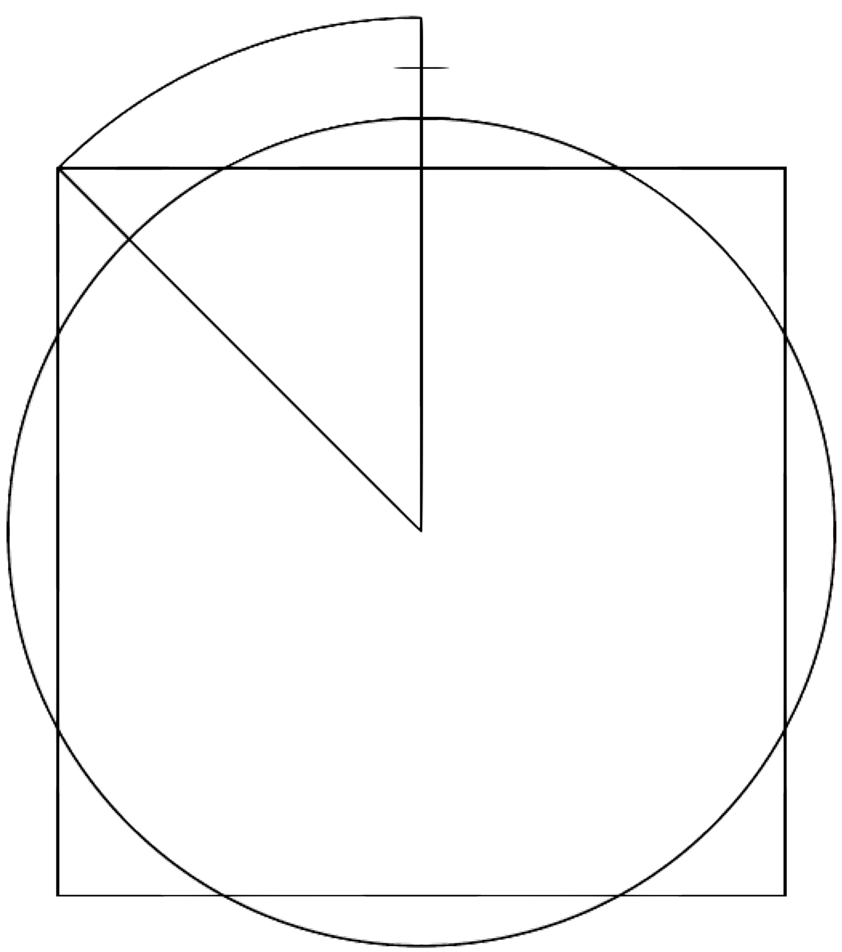
Ancient India

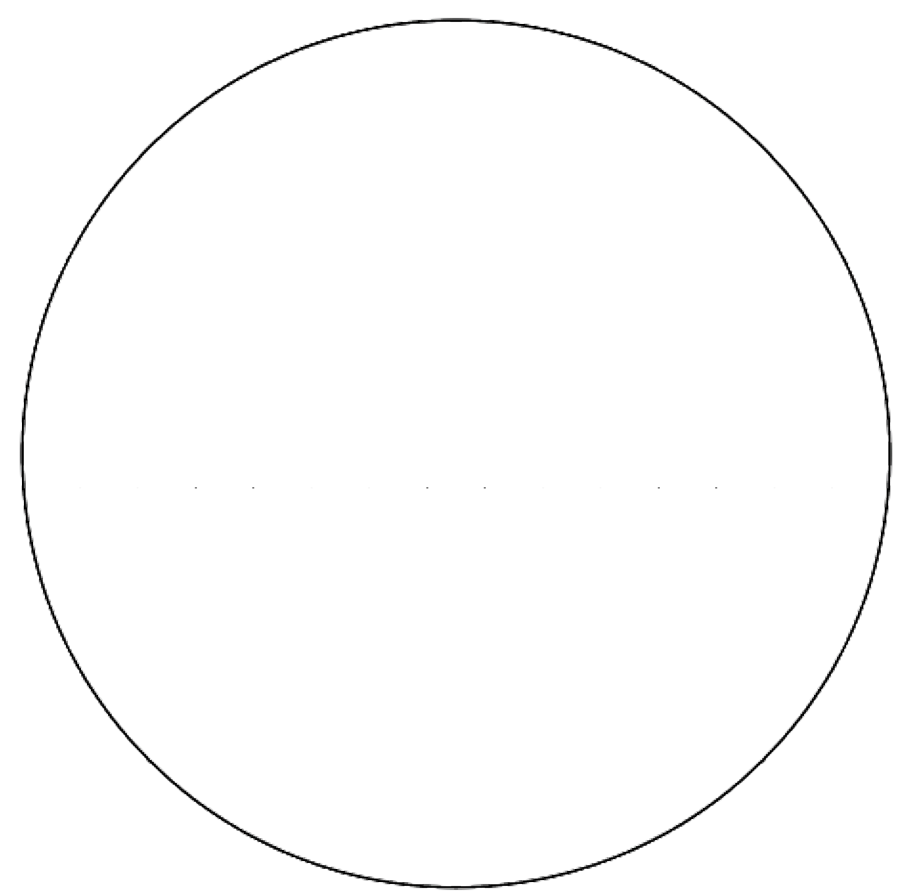
- **3. Circling a square or squaring a circle.**



Ancient India

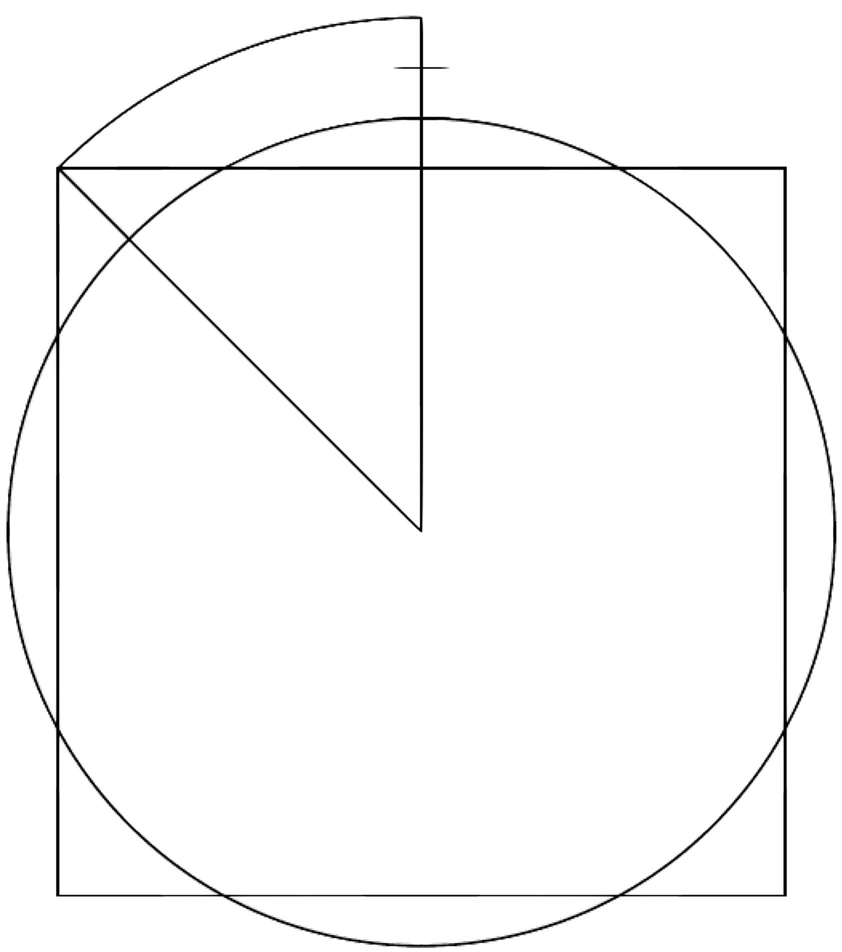
- **3. Circling a square or squaring a circle.**

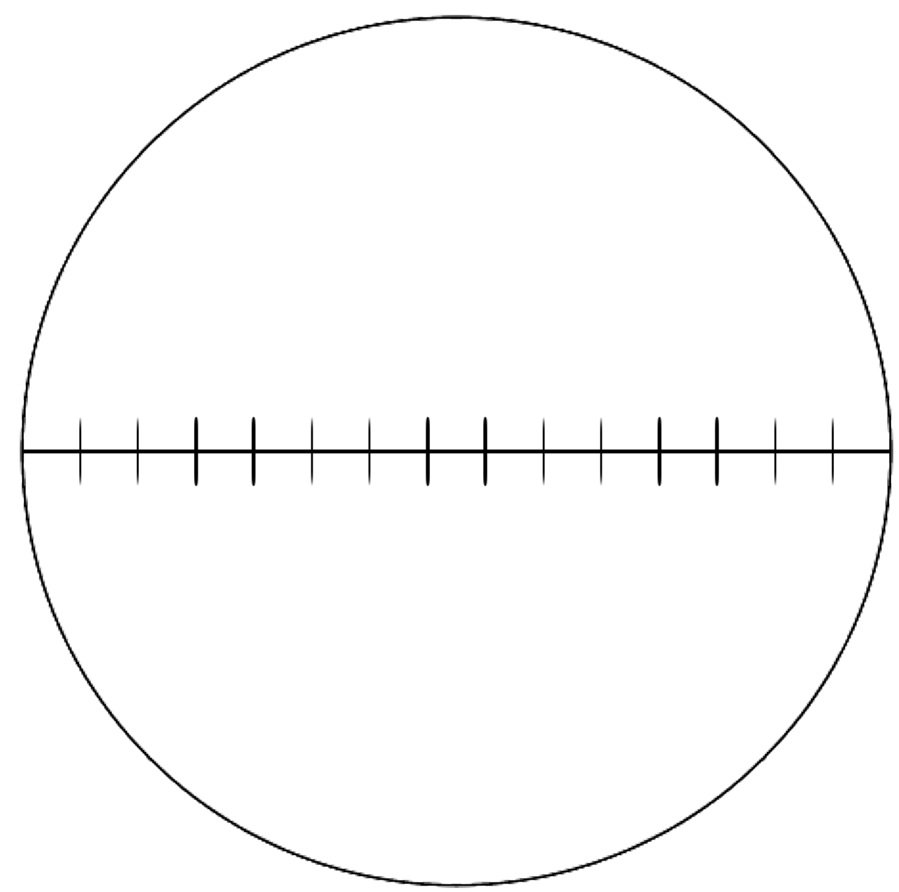




Ancient India

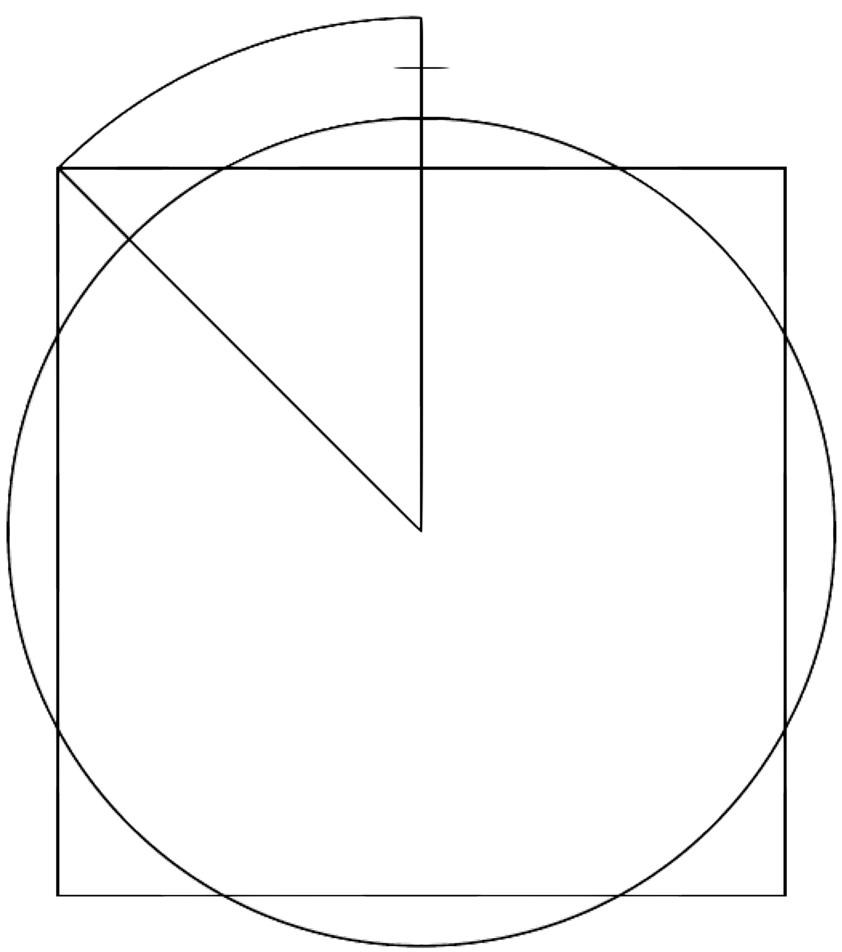
- **3. Circling a square or squaring a circle.**

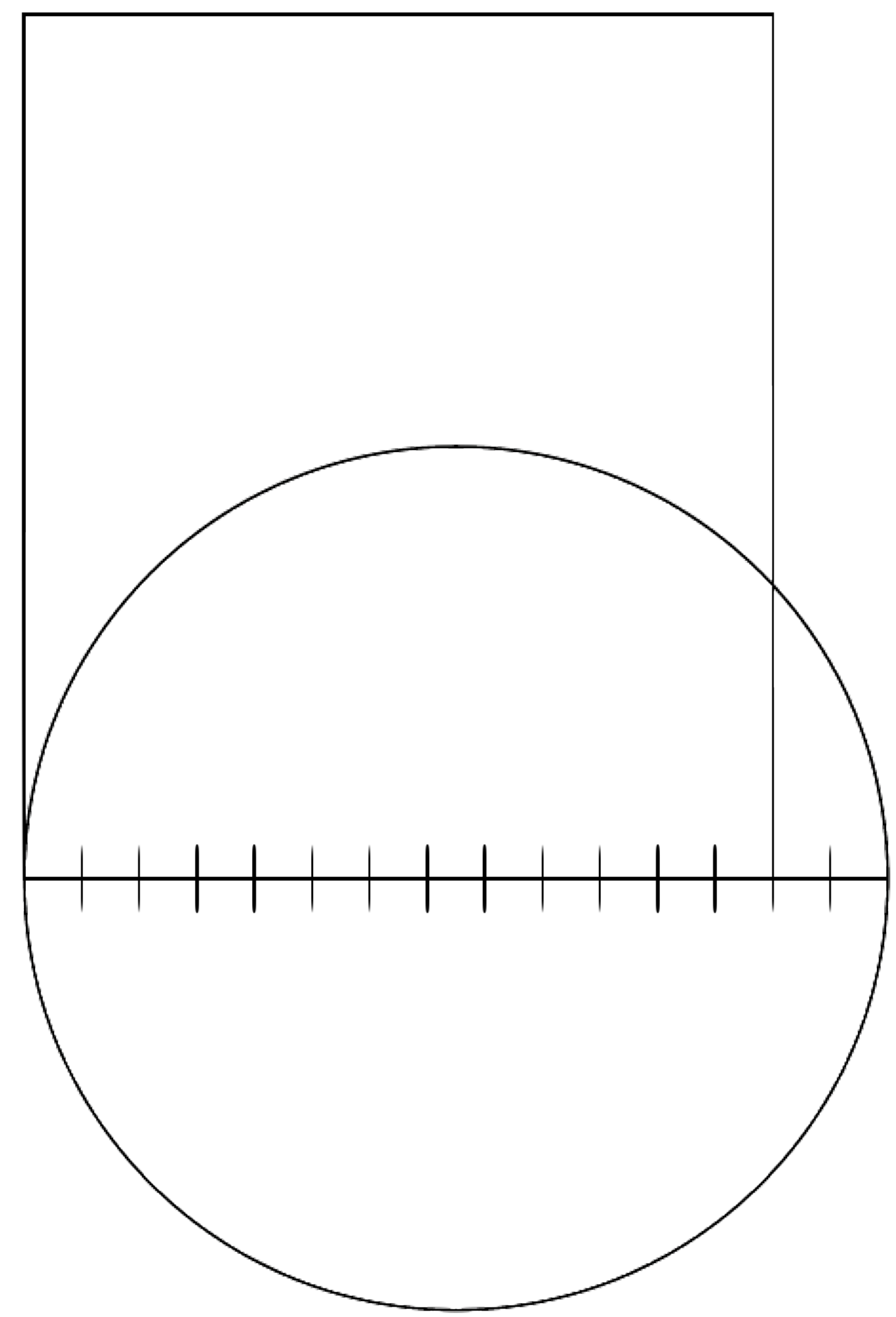




Ancient India

- **3. Circling a square or squaring a circle.**





The Aftermath

- The problem of “squaring a circle” is this: Given a circle, use a straightedge and compass to create a square of the same area.

- This was proved to be impossible in 1882. To do so, Ferdinand von Lindemann proved that π is transcendental (i.e., not the root of a polynomial with rational coefficients).

- Proof sketch given in the notes. It’s a neat argument—but is subtle and better read than heard.

Shout-outs!

Shout-outs!

m & n are integers
'm' is greater than 'n'

- Euclid characterized all *primitive Pythagorean triples.* They are all of this form:
$$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2$$

- Pythagoreans were also said to be pioneers in the math of music.

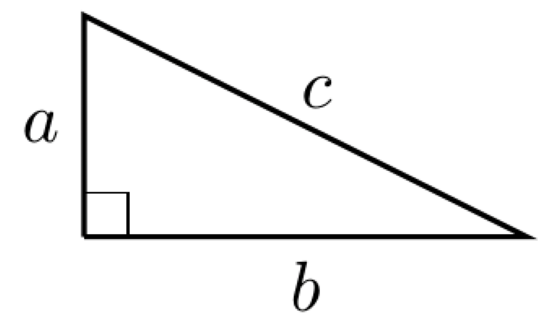
Shout-outs!

- A more modern figure who prominently weaved together math and mysticism is Englishman John Dee. In fact, during his life in England, math was considered by many to be pseudo-magical, and in 1555 Dee was arrested for the crime of "calculating."

- Lastly: My favorite proof of the Pythagorean theorem.

Shout-outs!

Take any right triangle:



$\underline{\text{Goal:}}$ $a^2 + b^2 = c^2$

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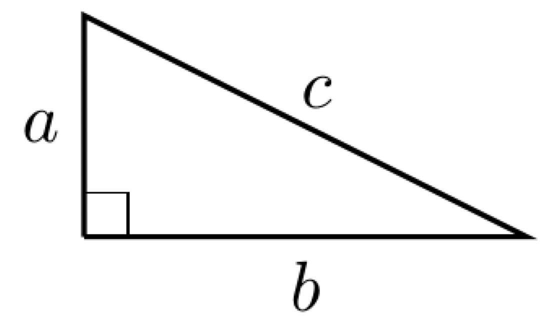

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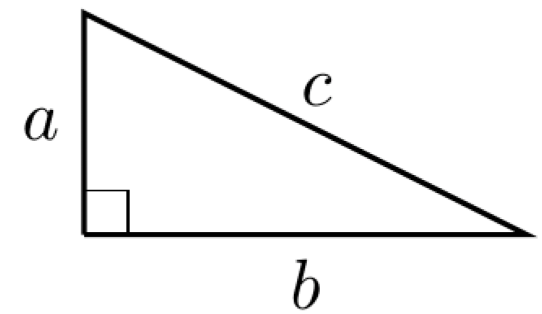
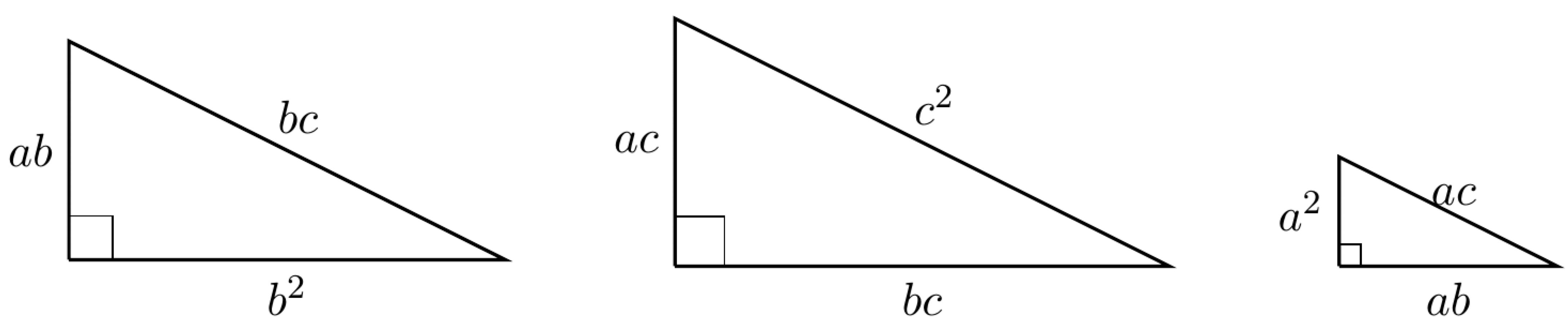


Now, scale up this triangle three times, first by a factor of b, next by a factor of c, and last by a factor of a. This produces these three similar triangles:



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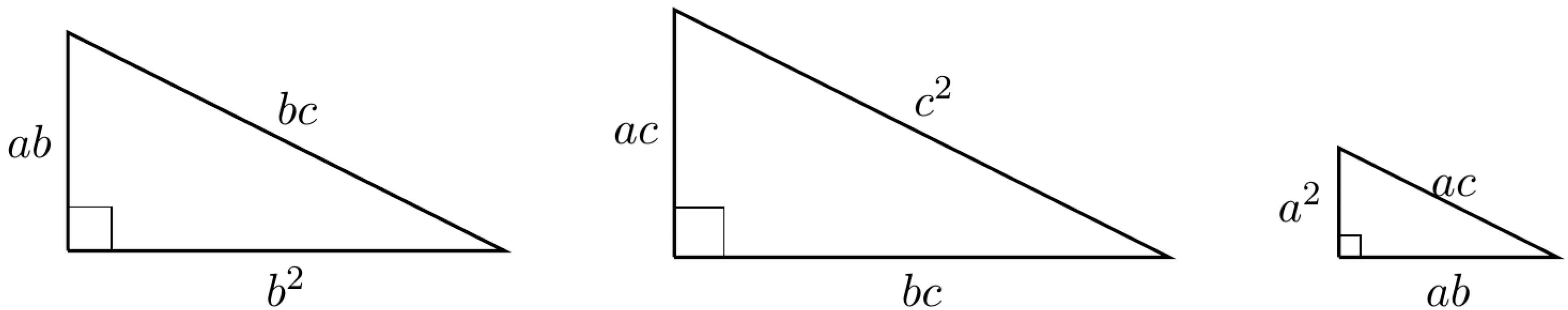

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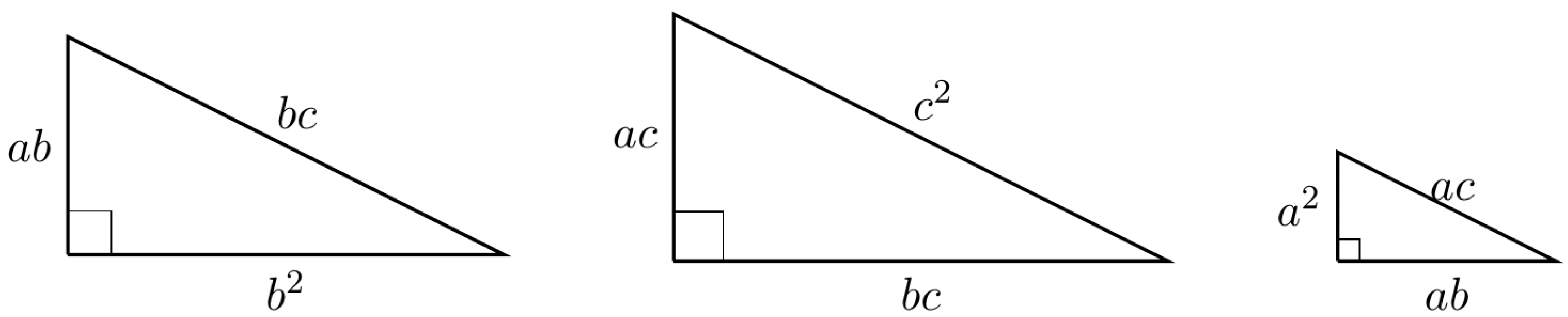
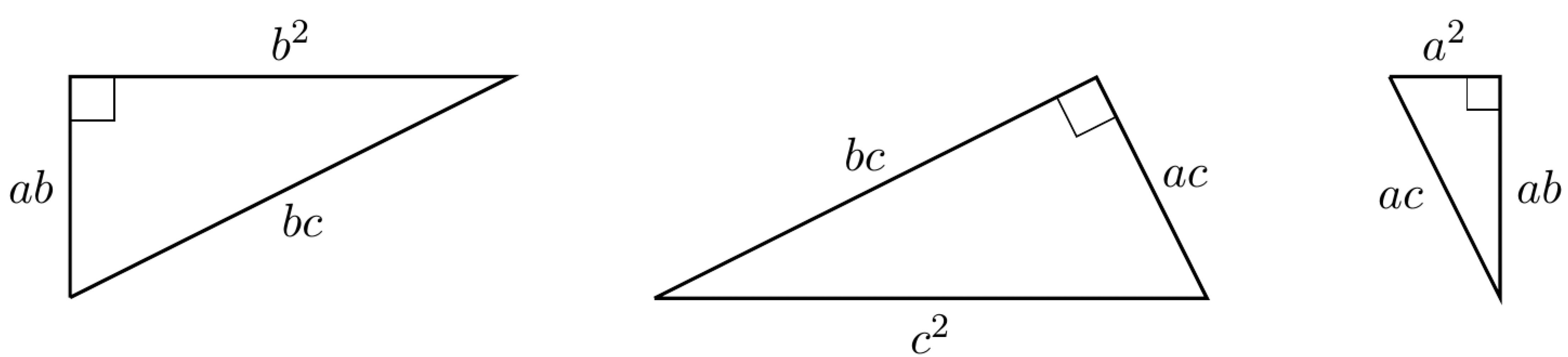


Next, by simply rotating or mirroring them, we arrive here:



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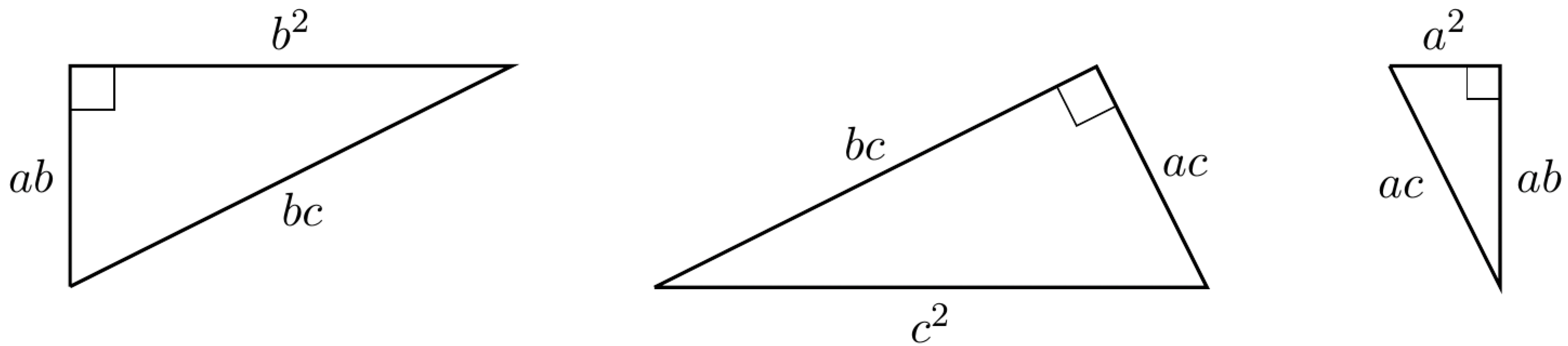

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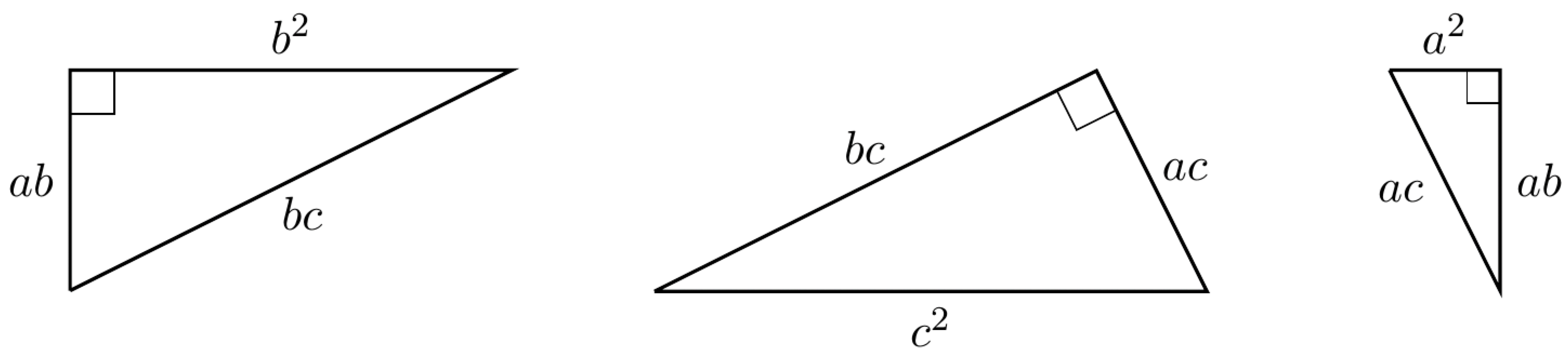

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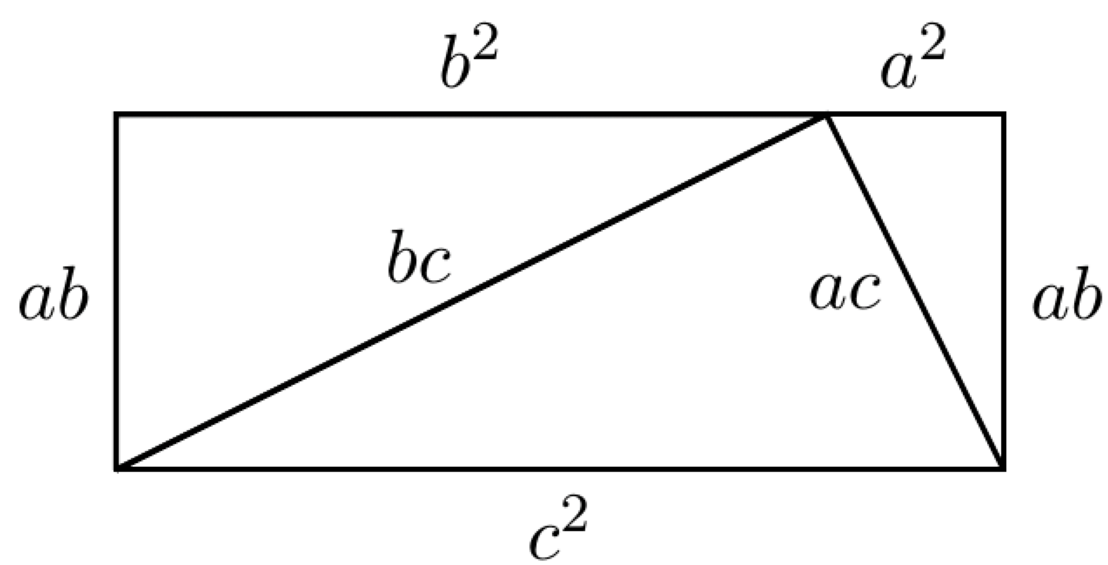

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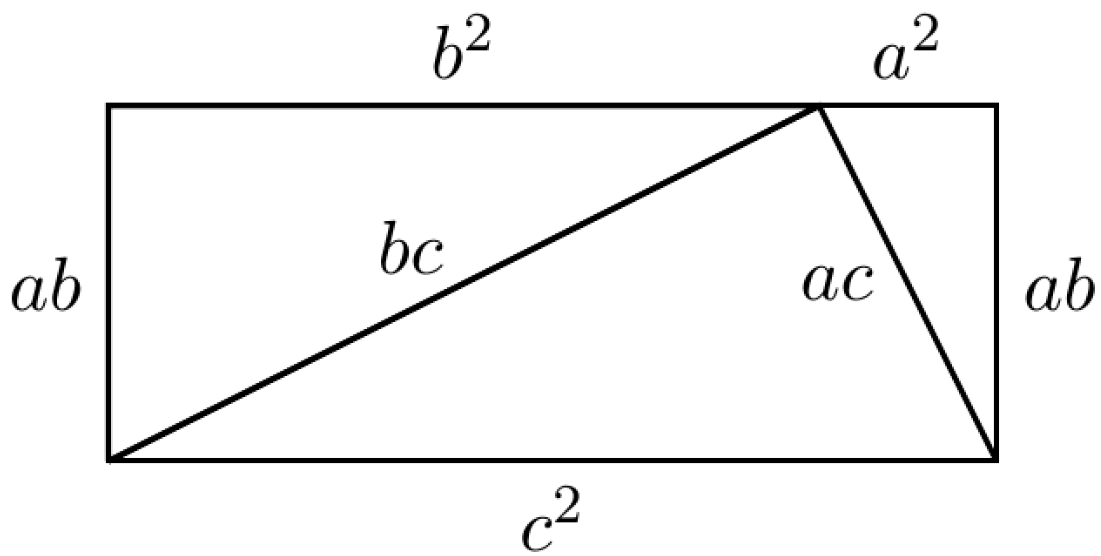

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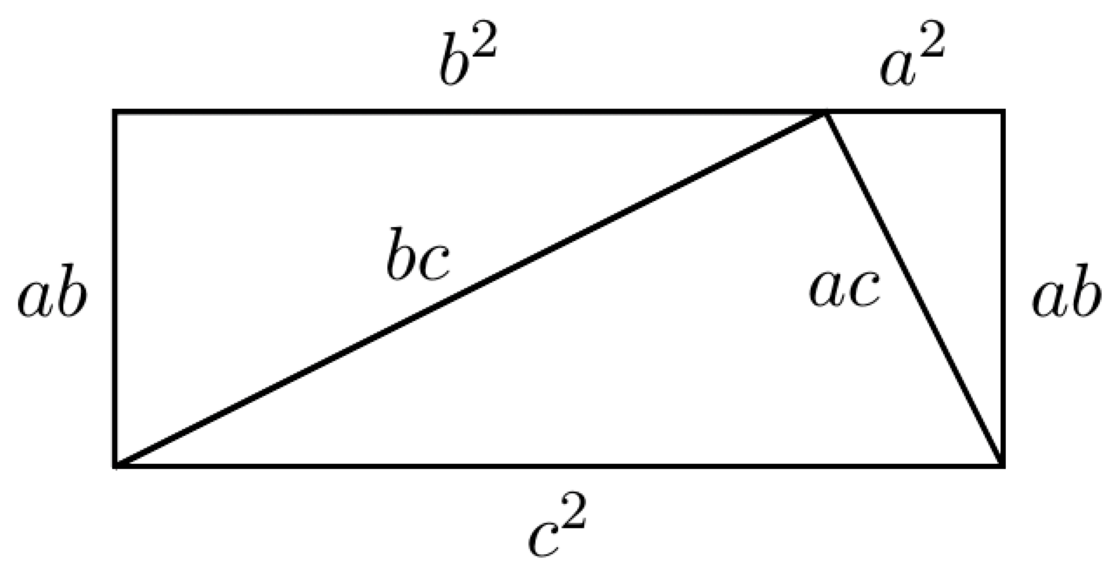

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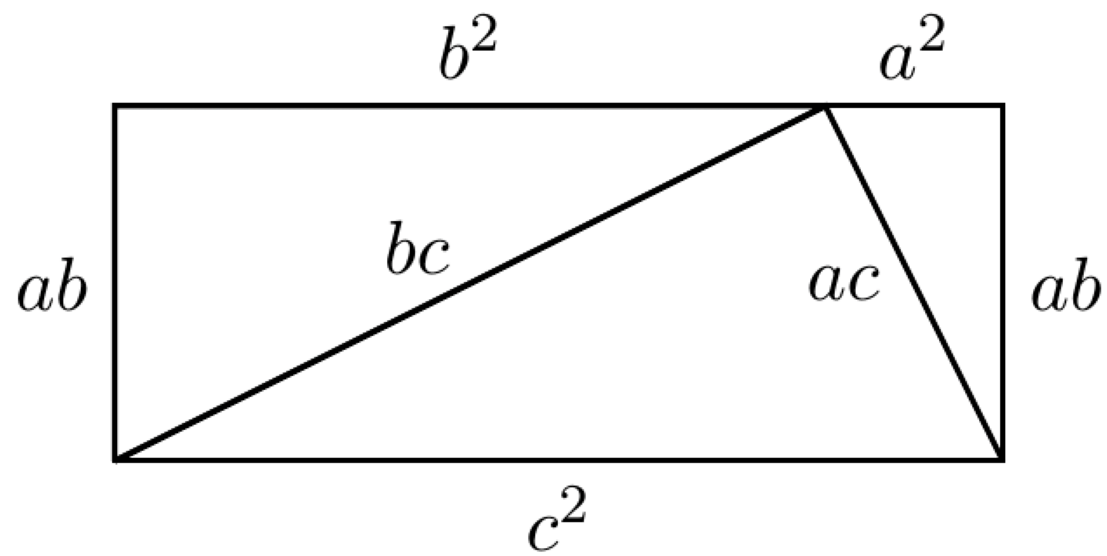

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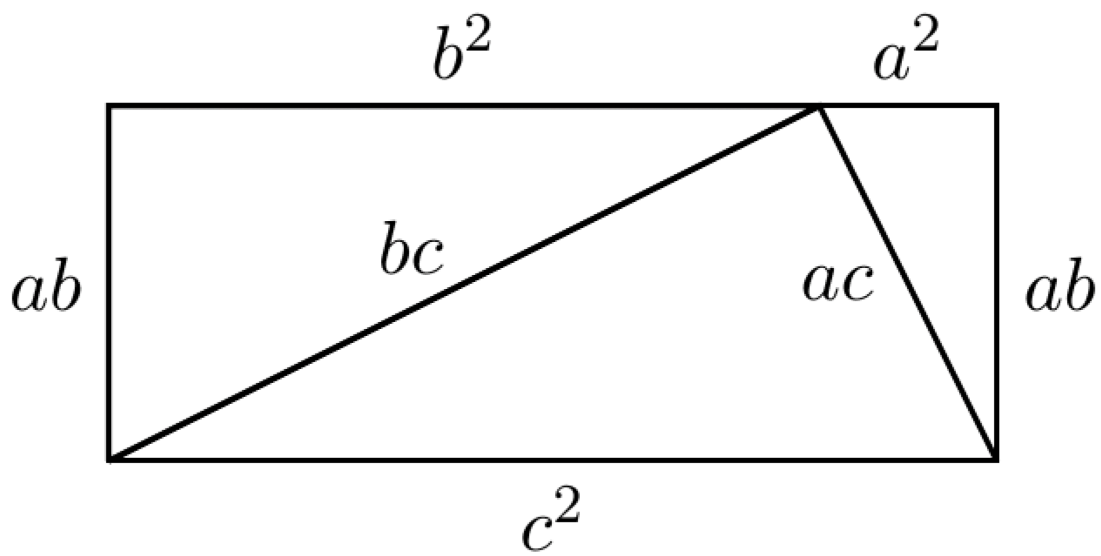

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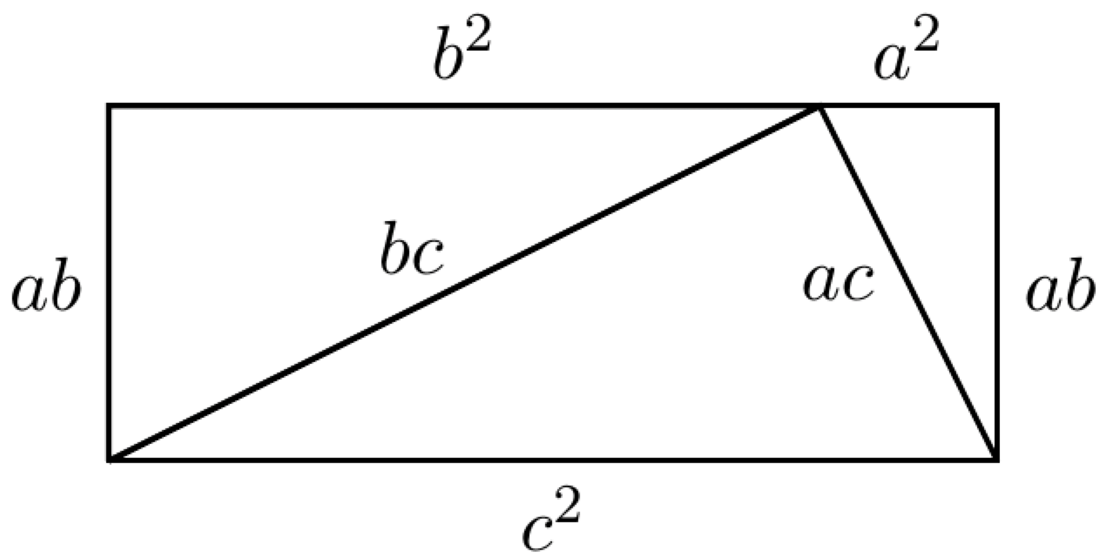

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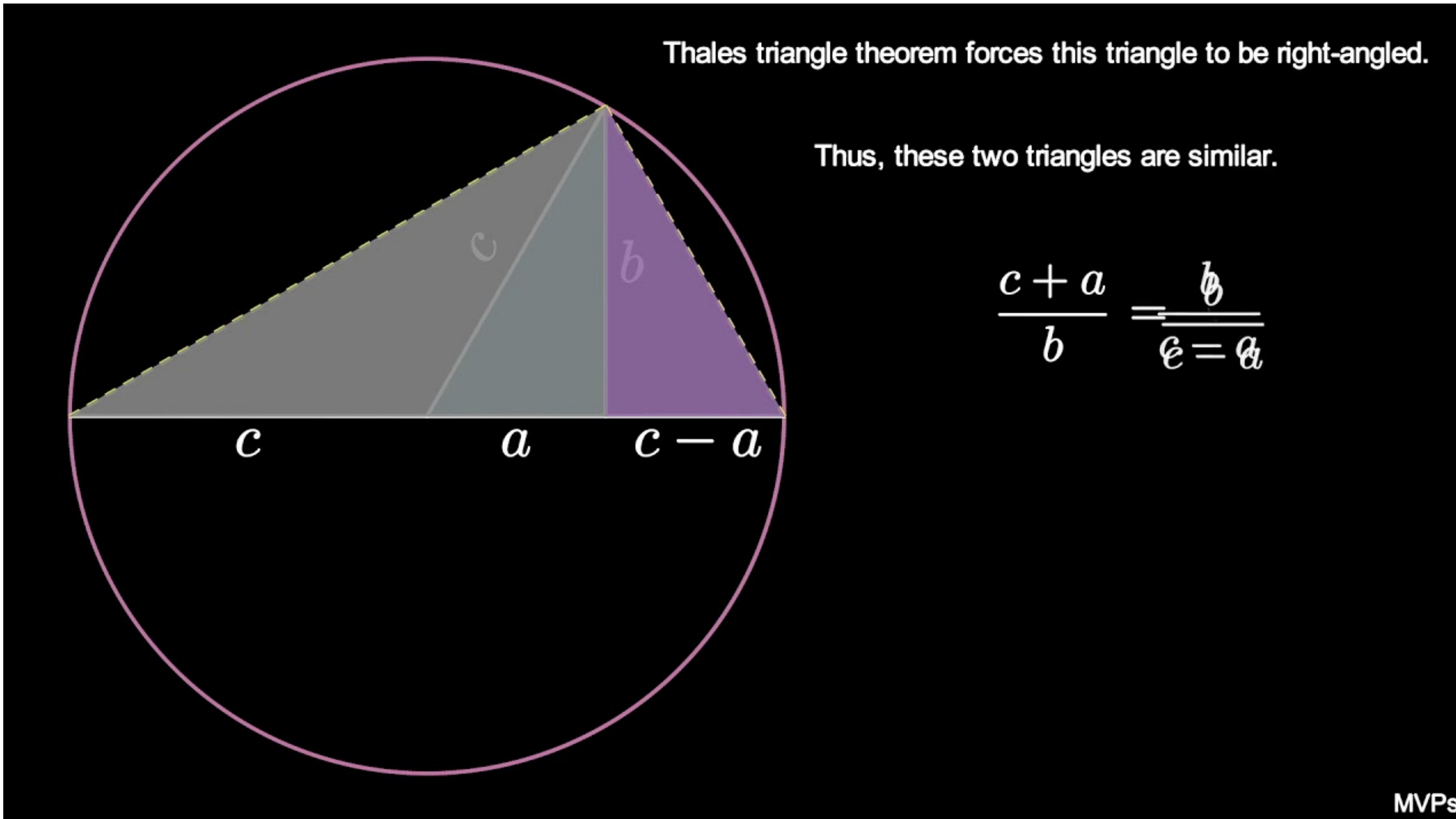
More Proofs of the Pythagorean Theorem

More Pythagorean Proofs



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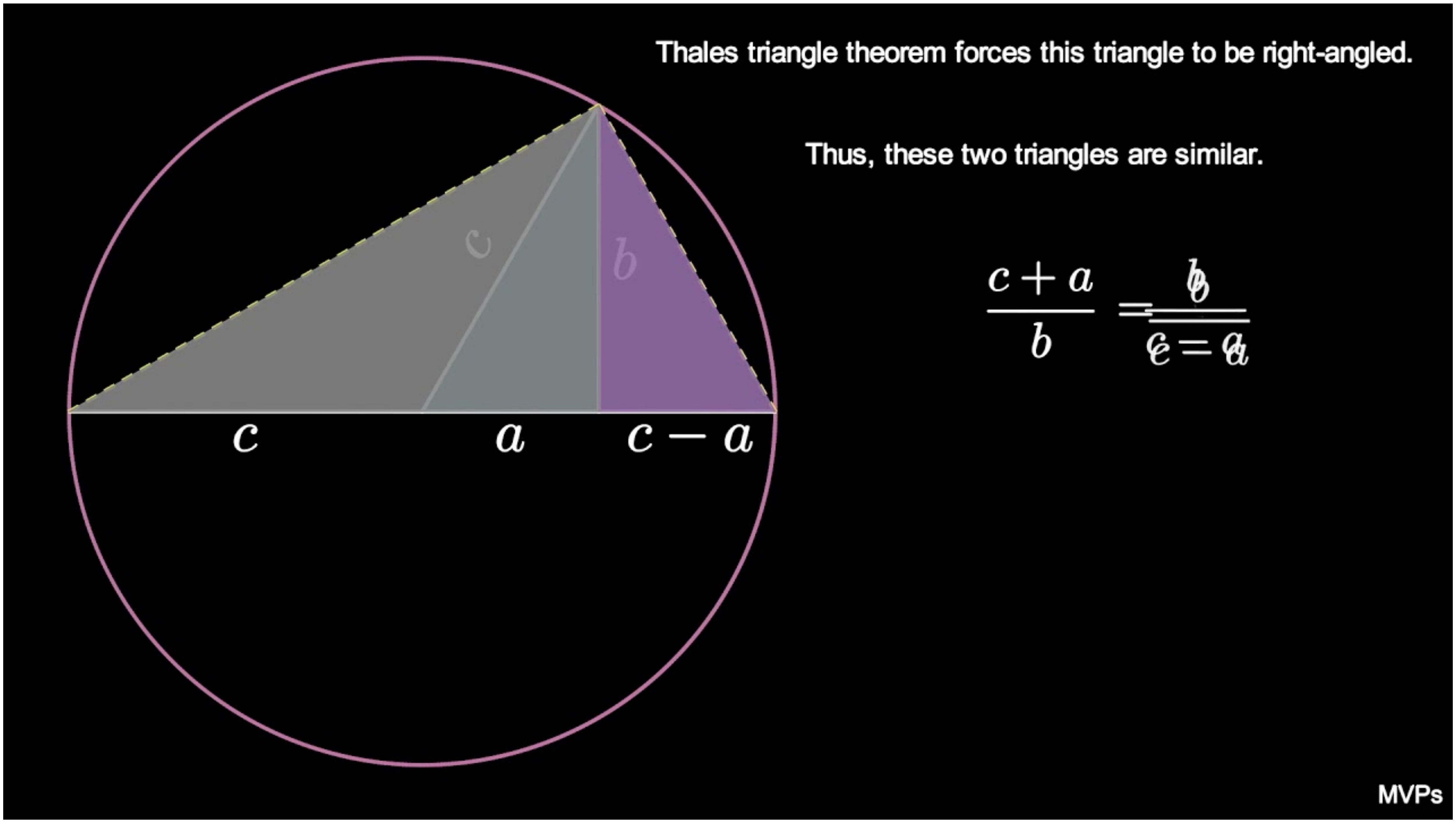

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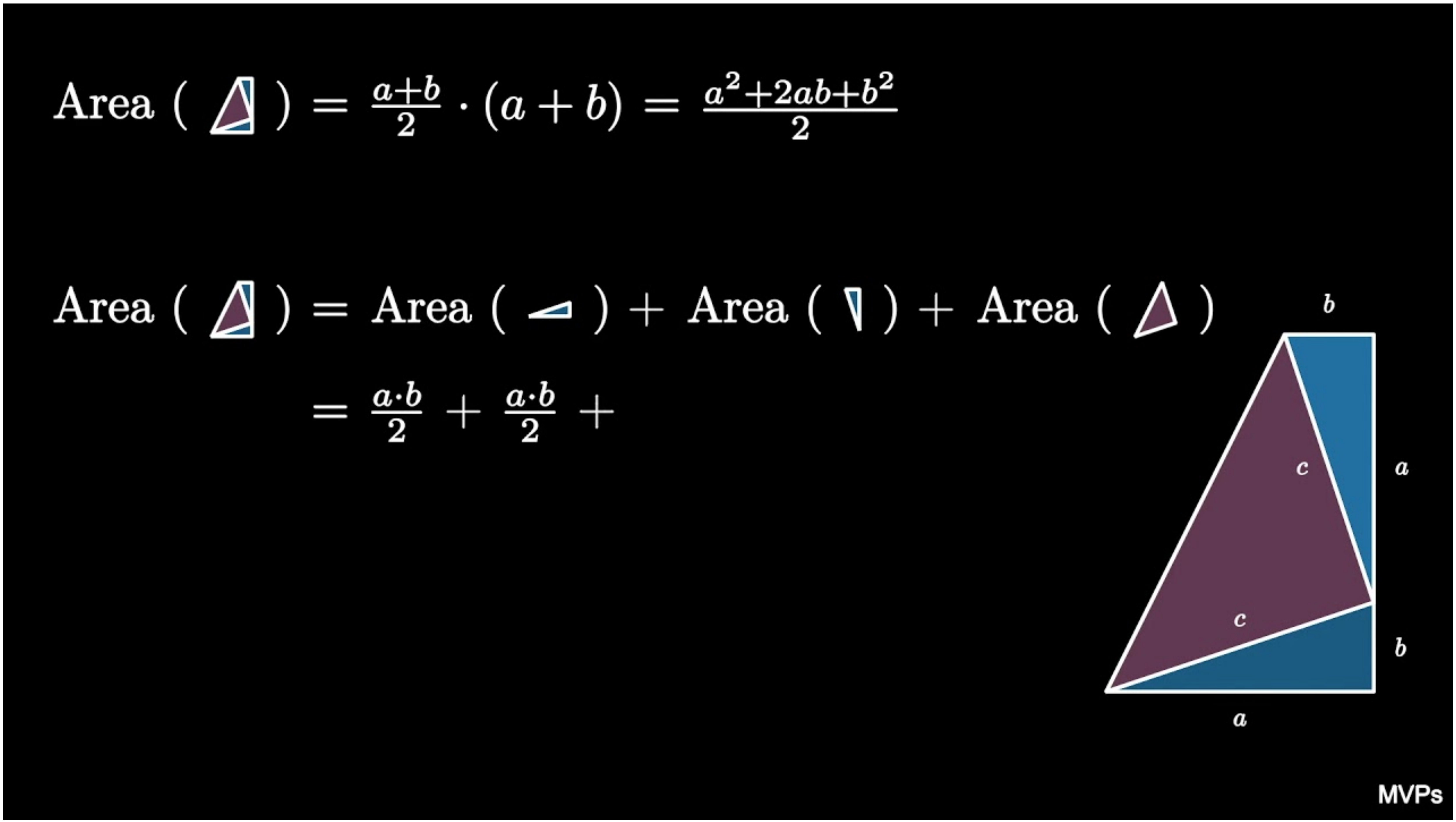

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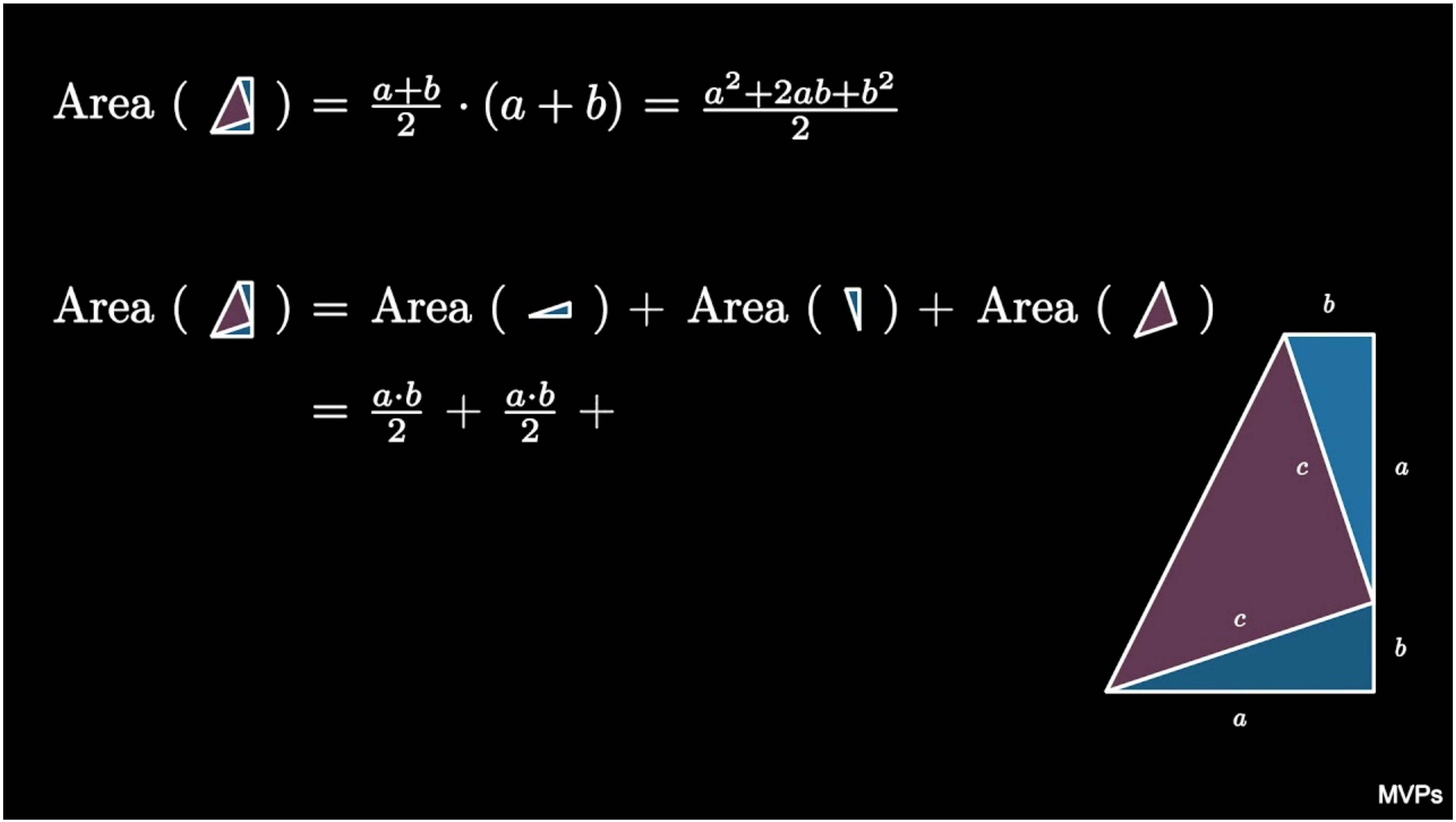

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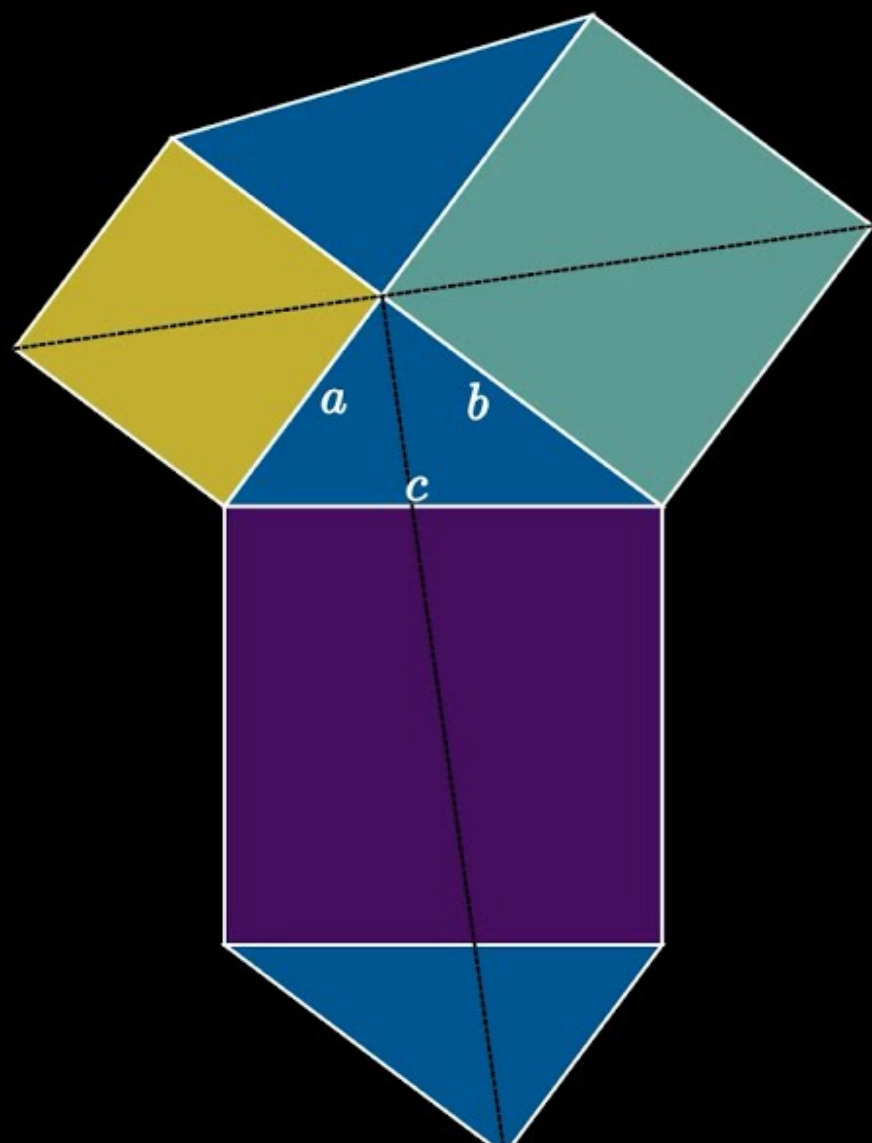

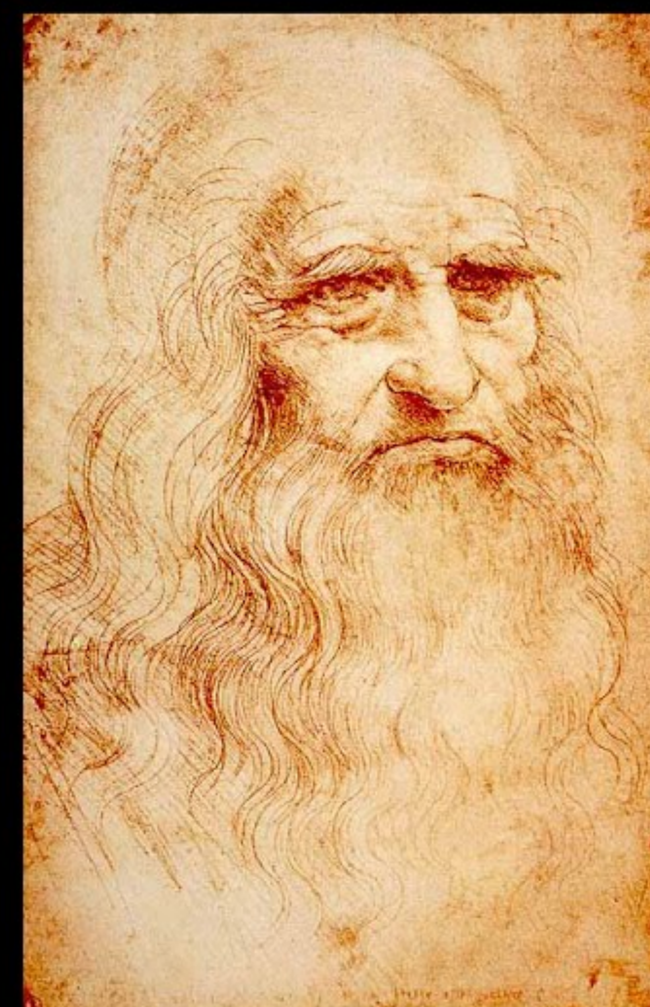

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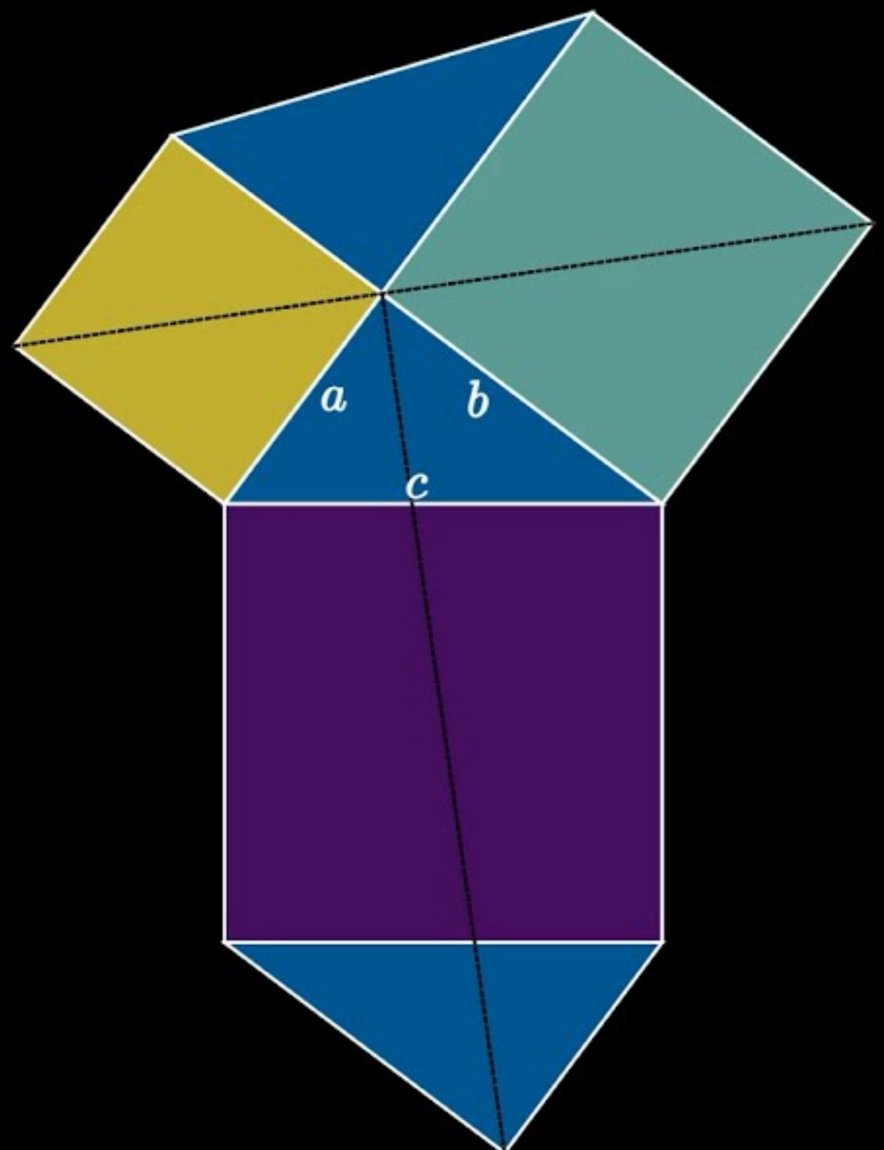

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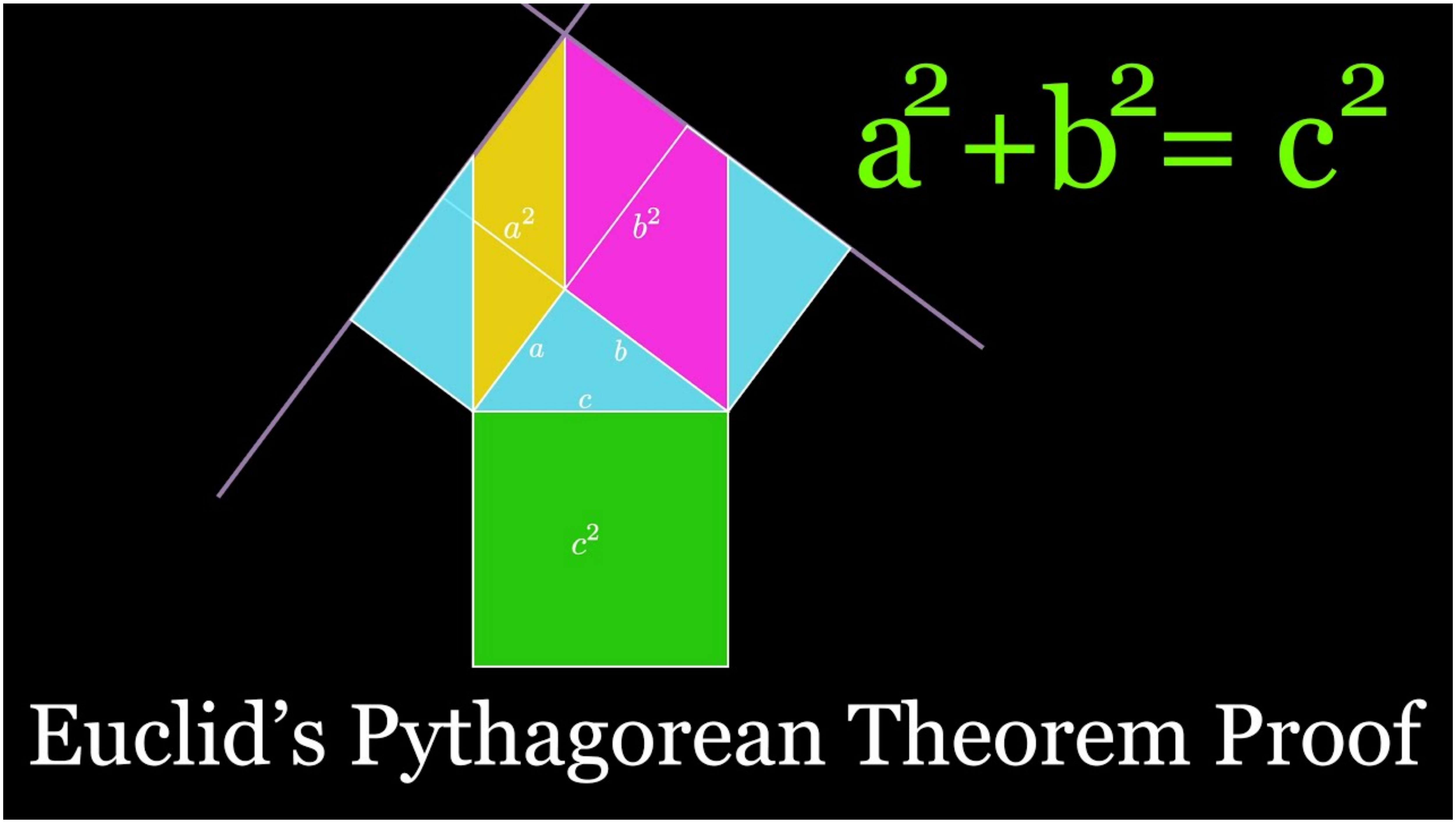

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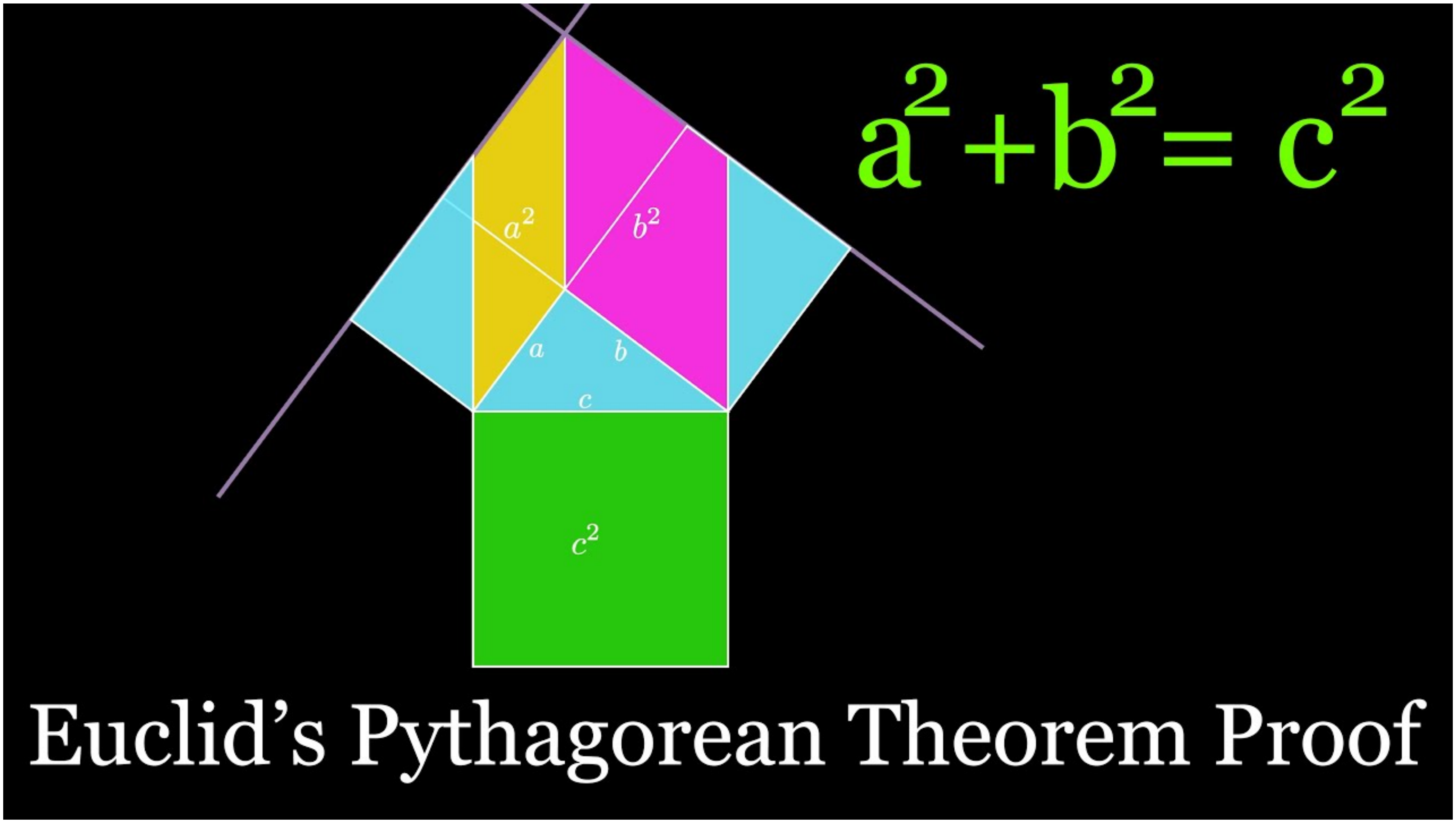

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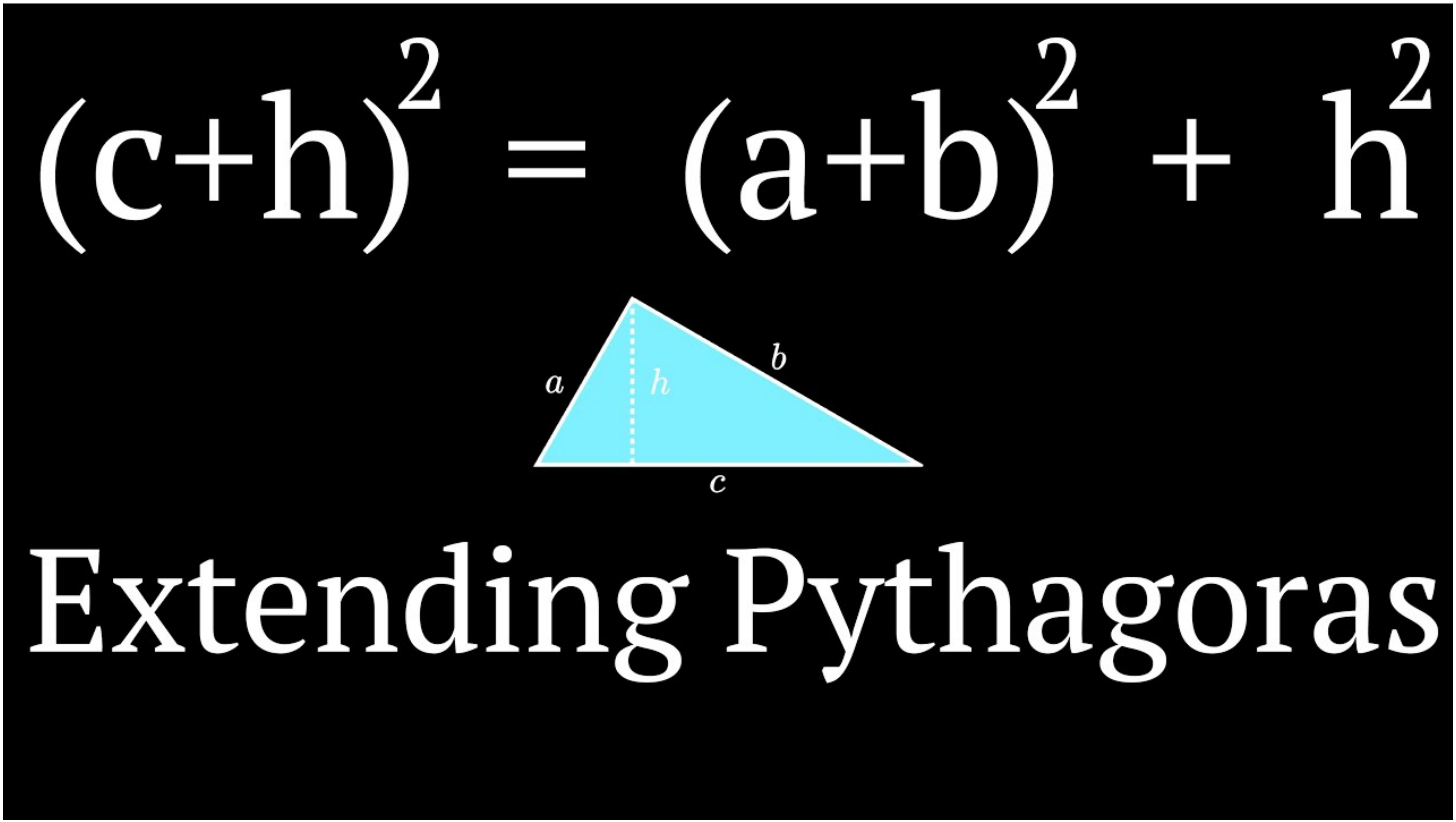

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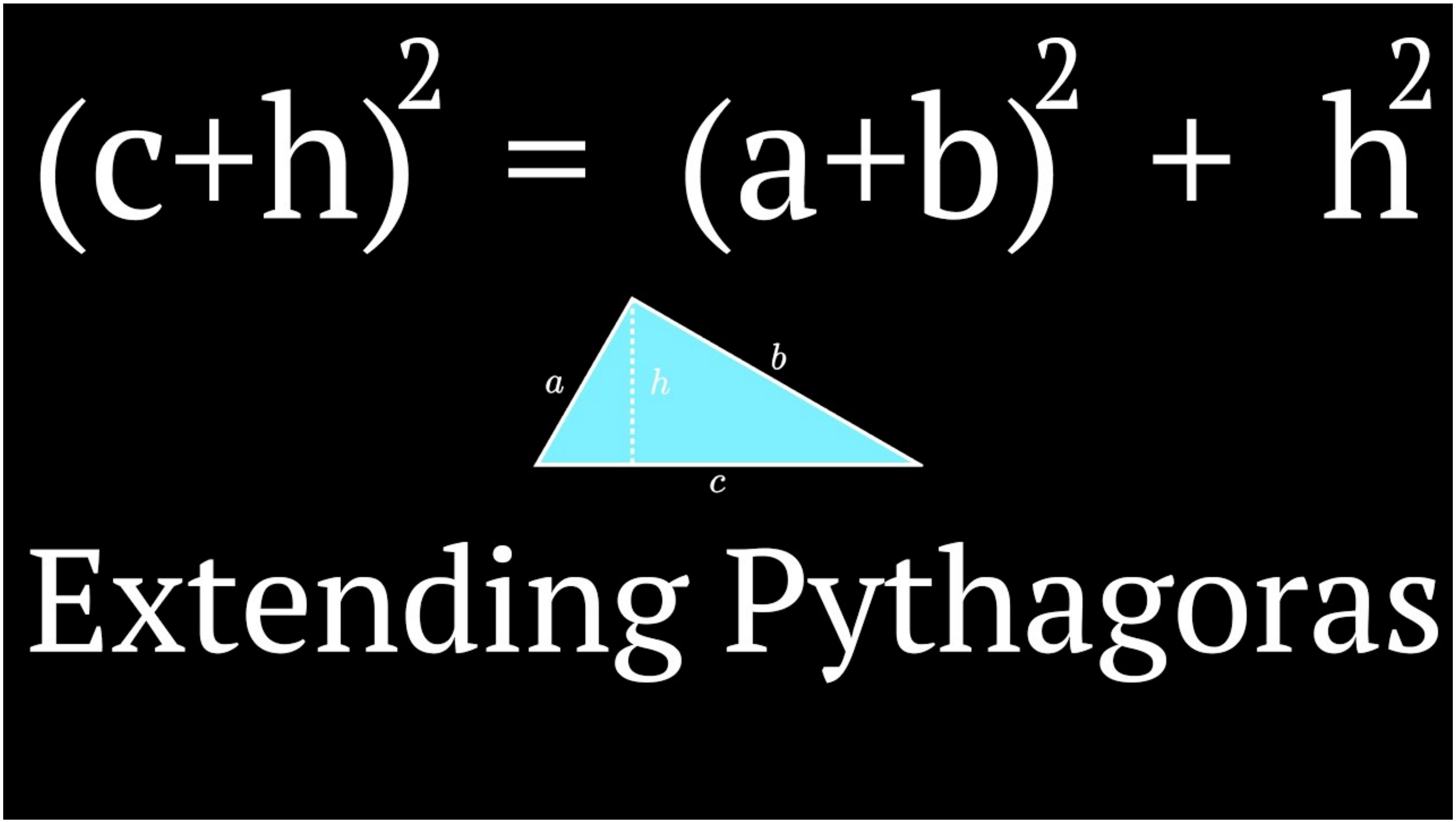

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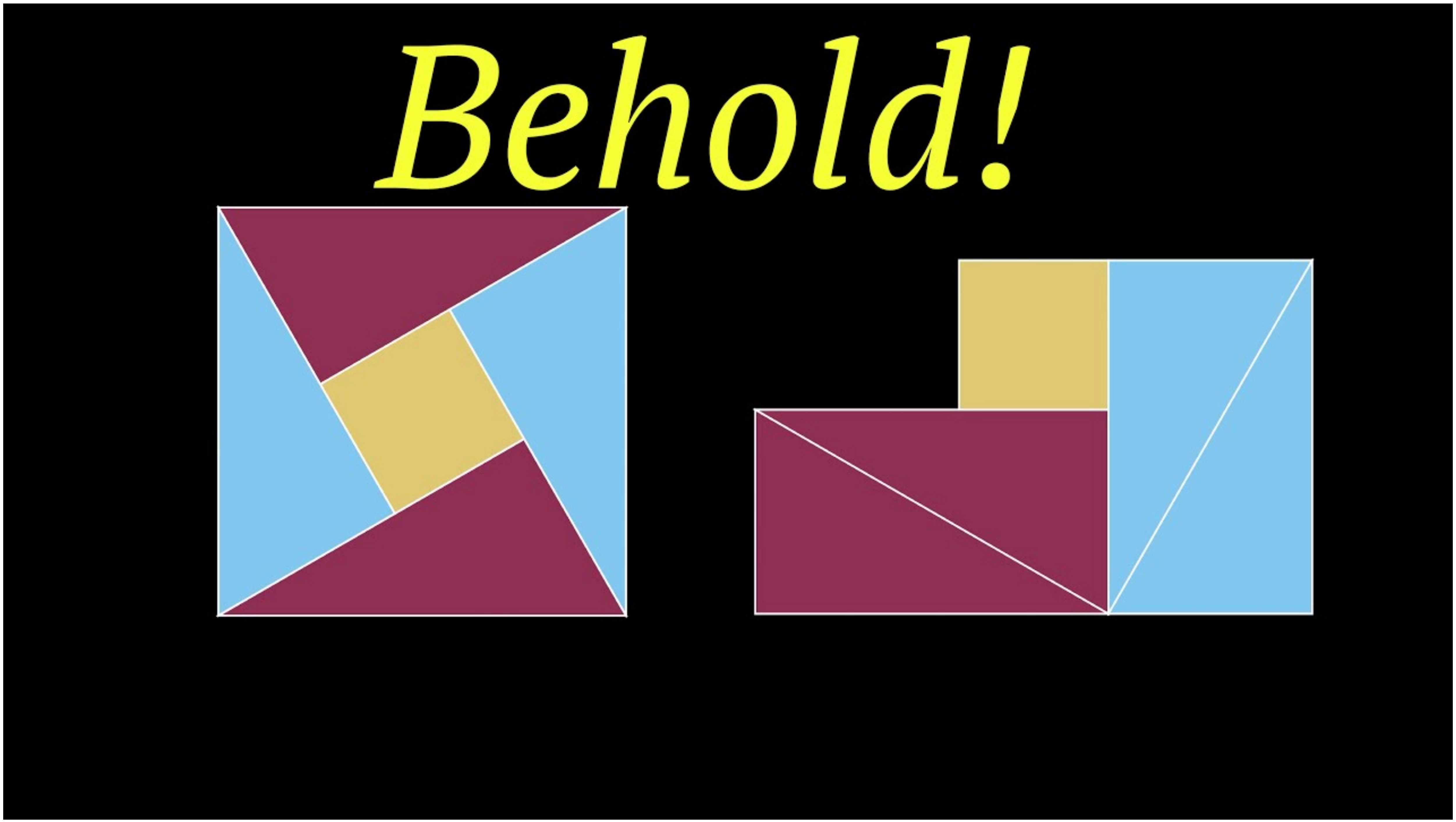

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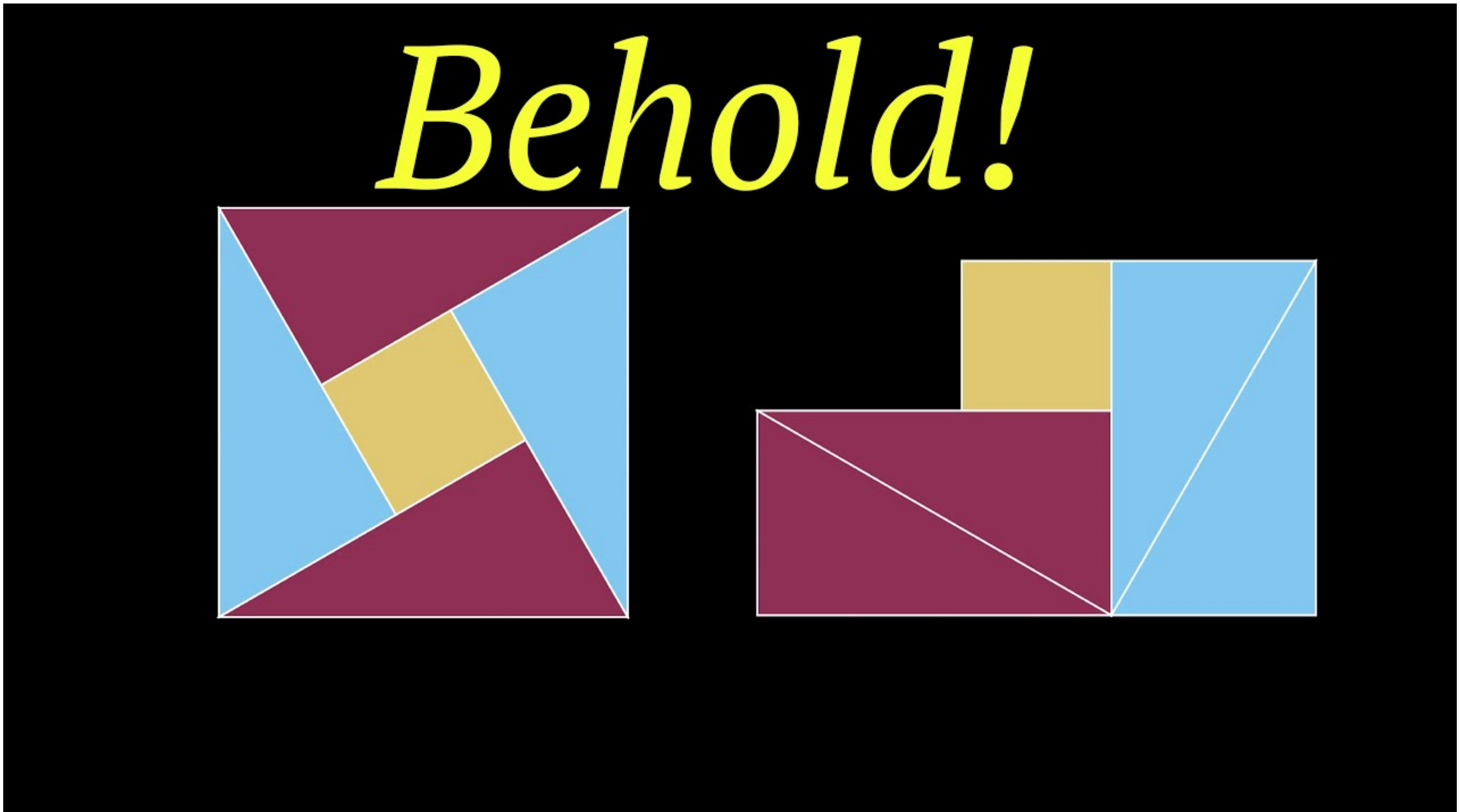

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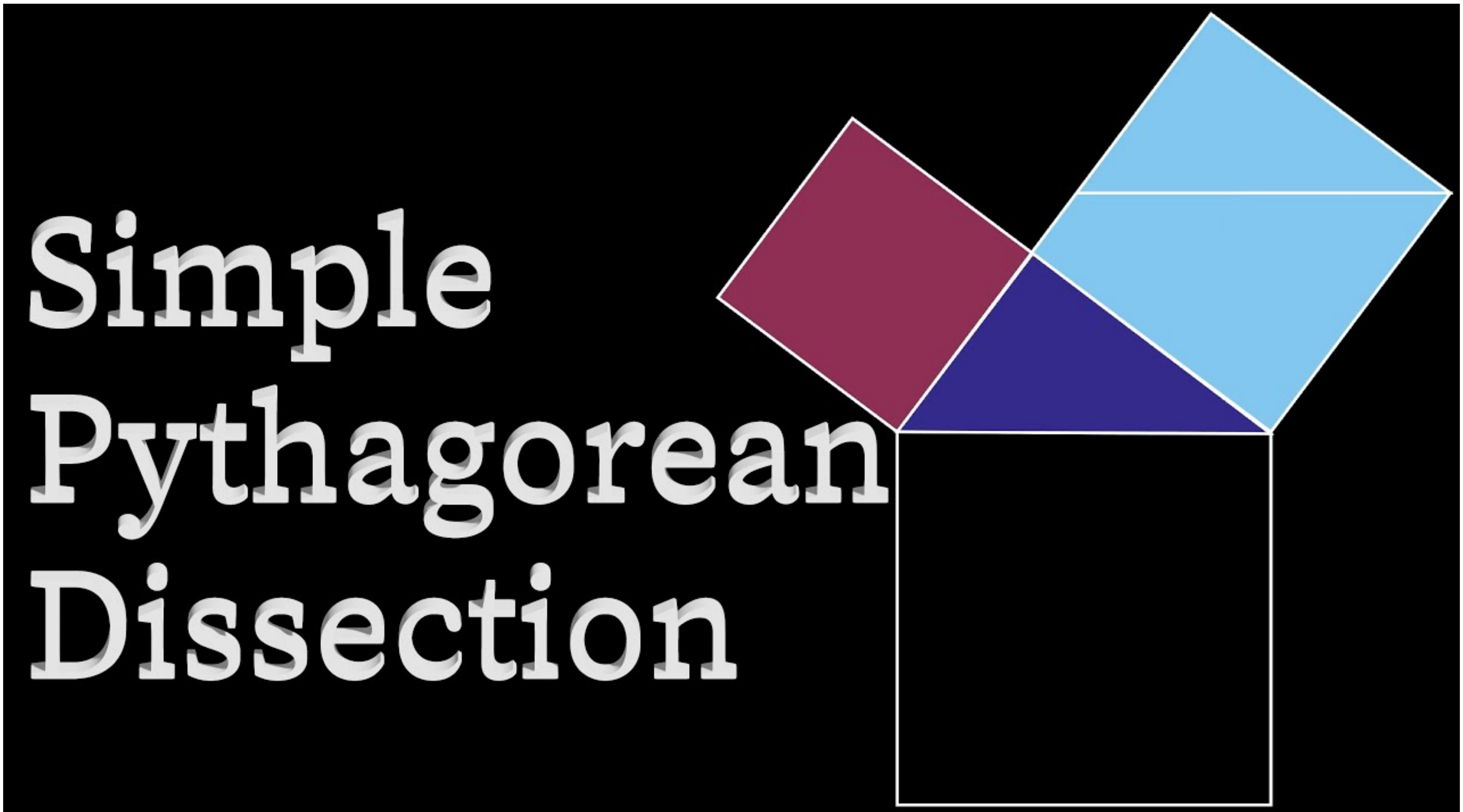

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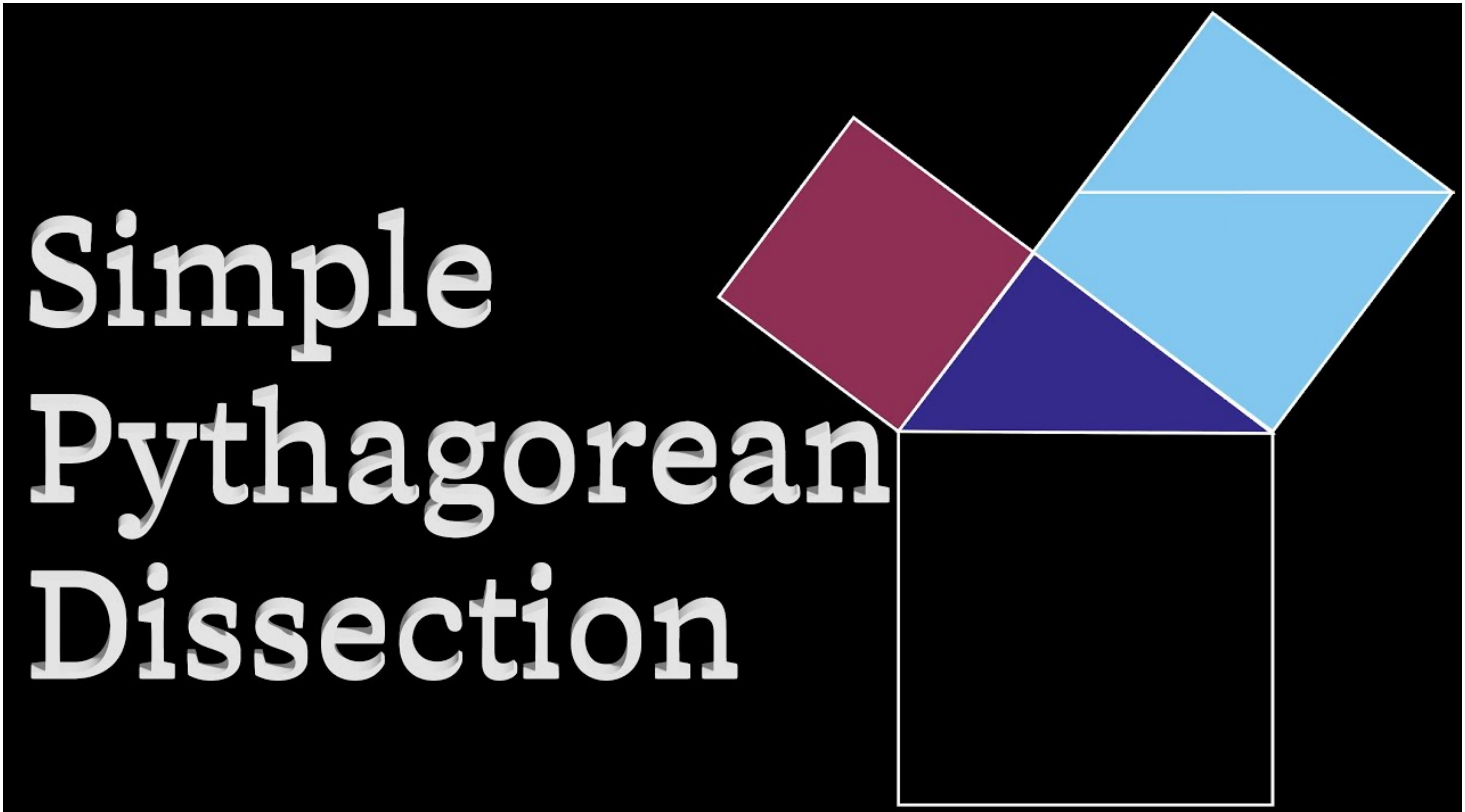

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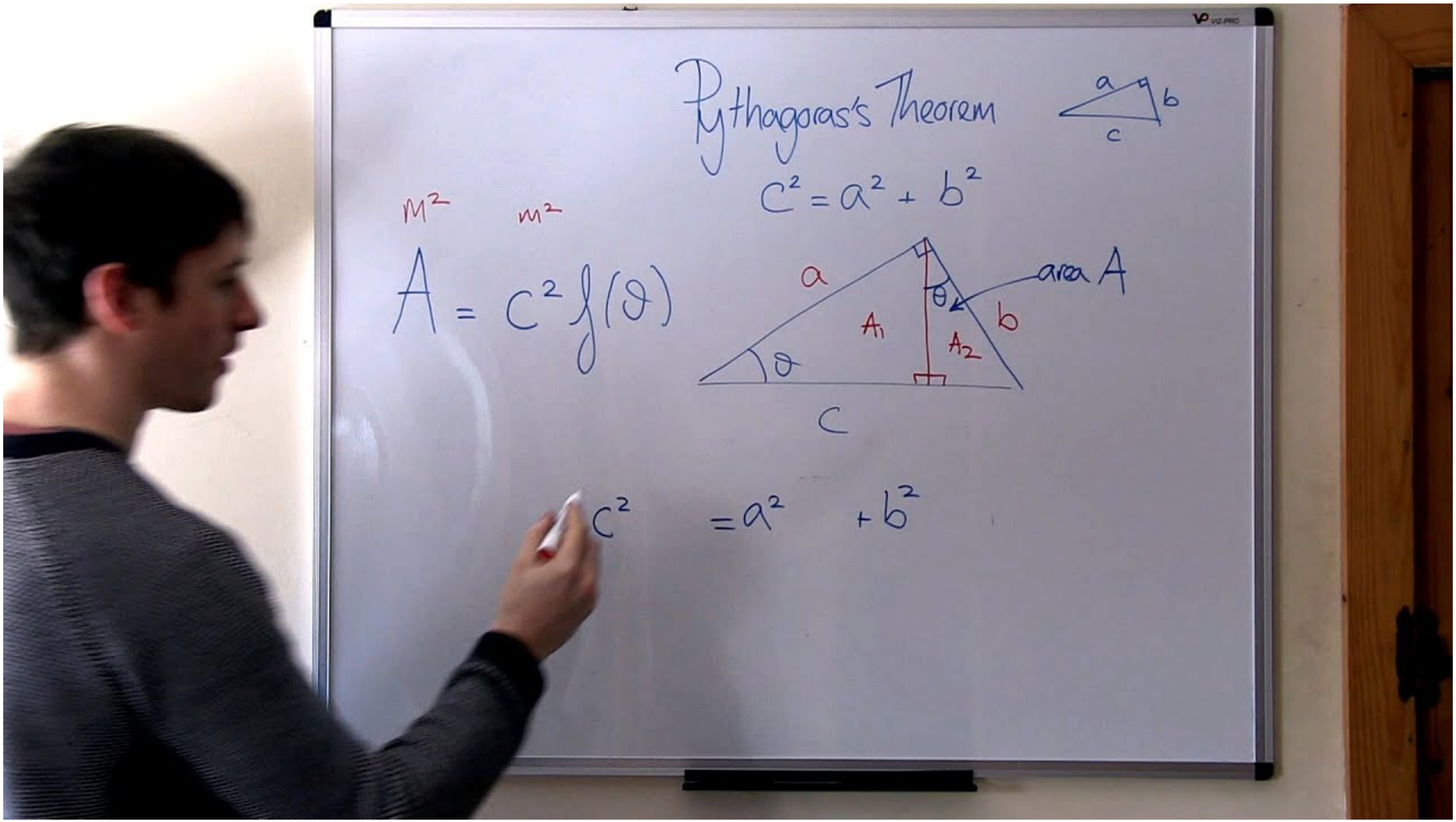

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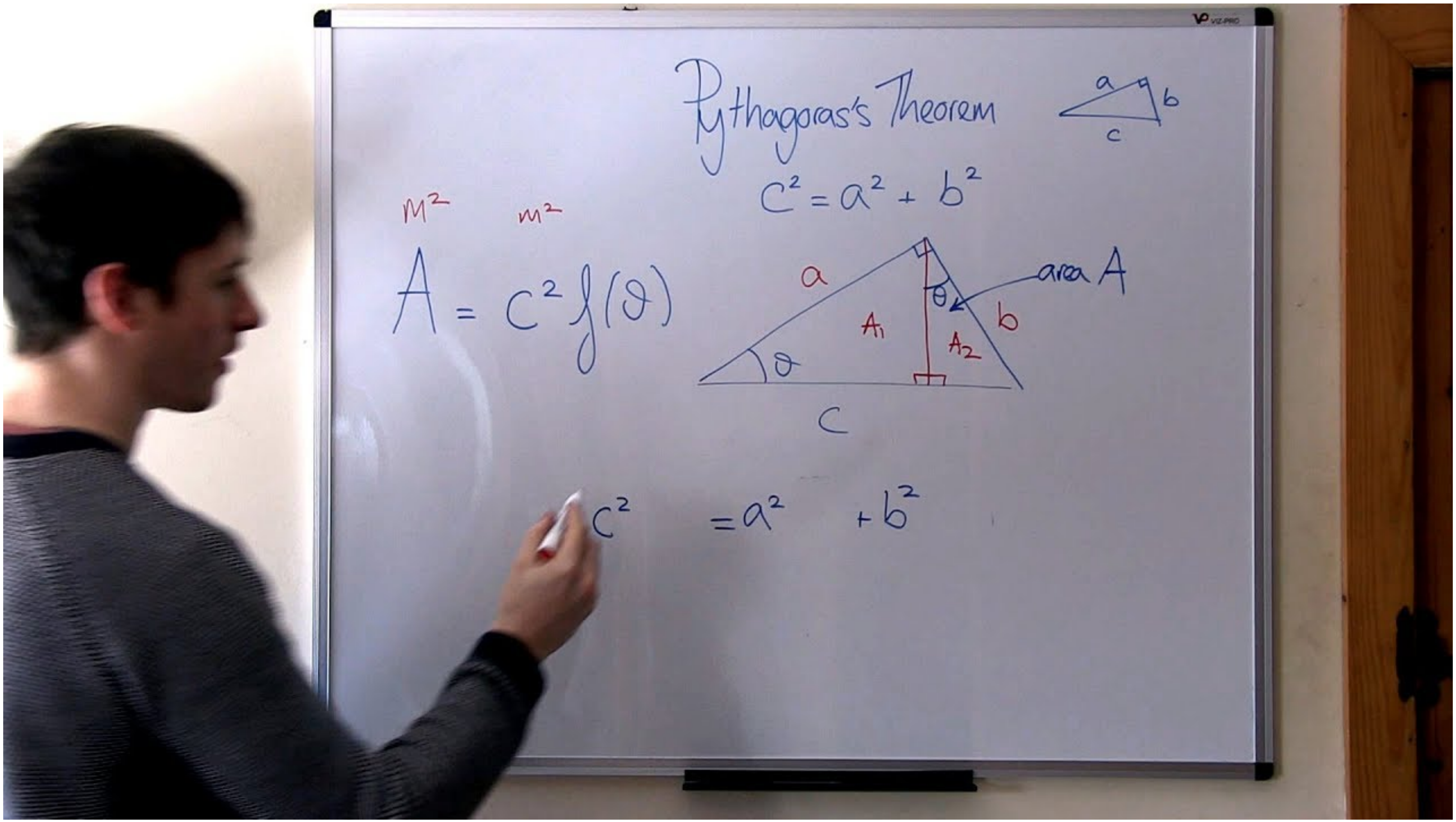

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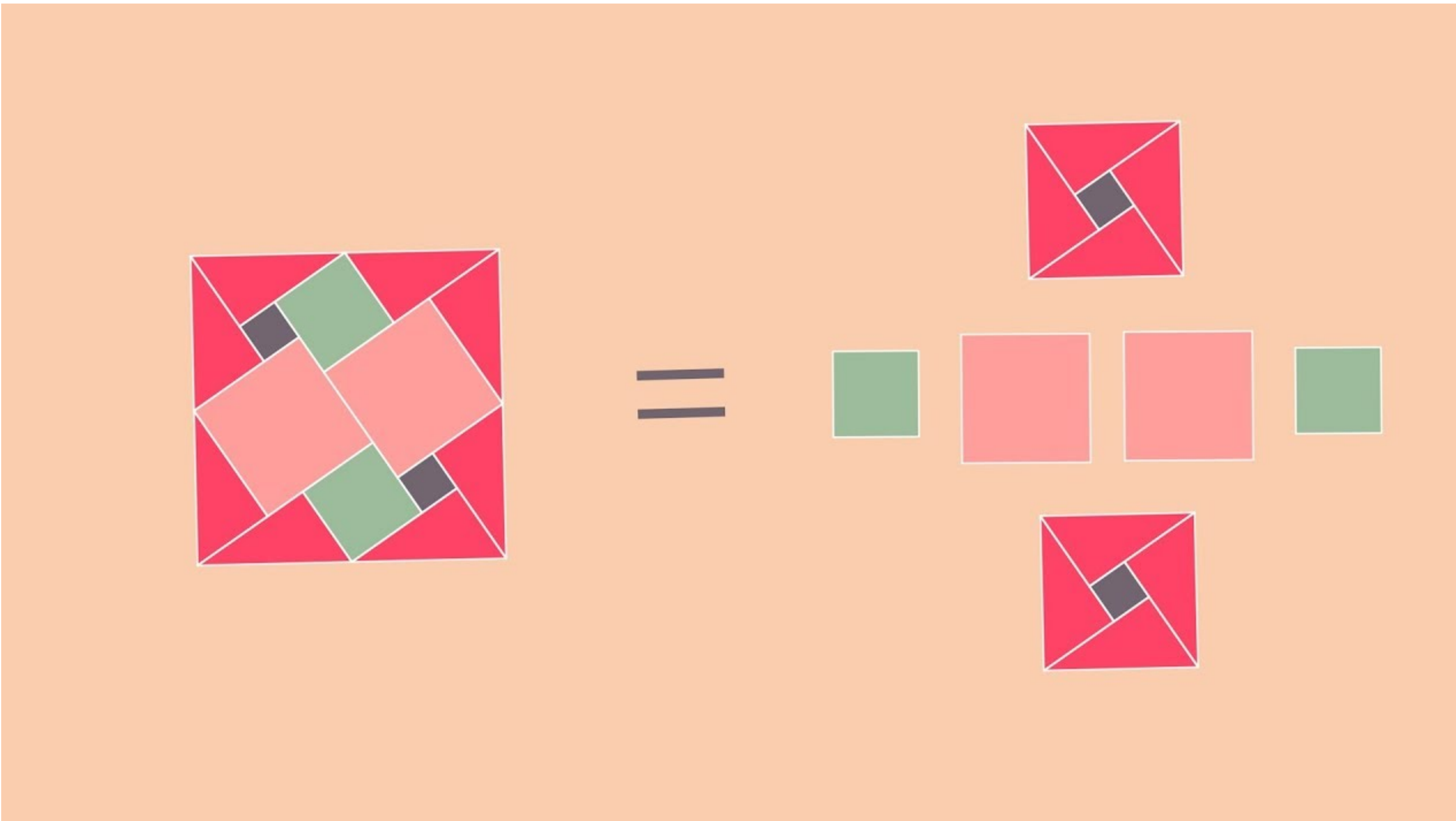

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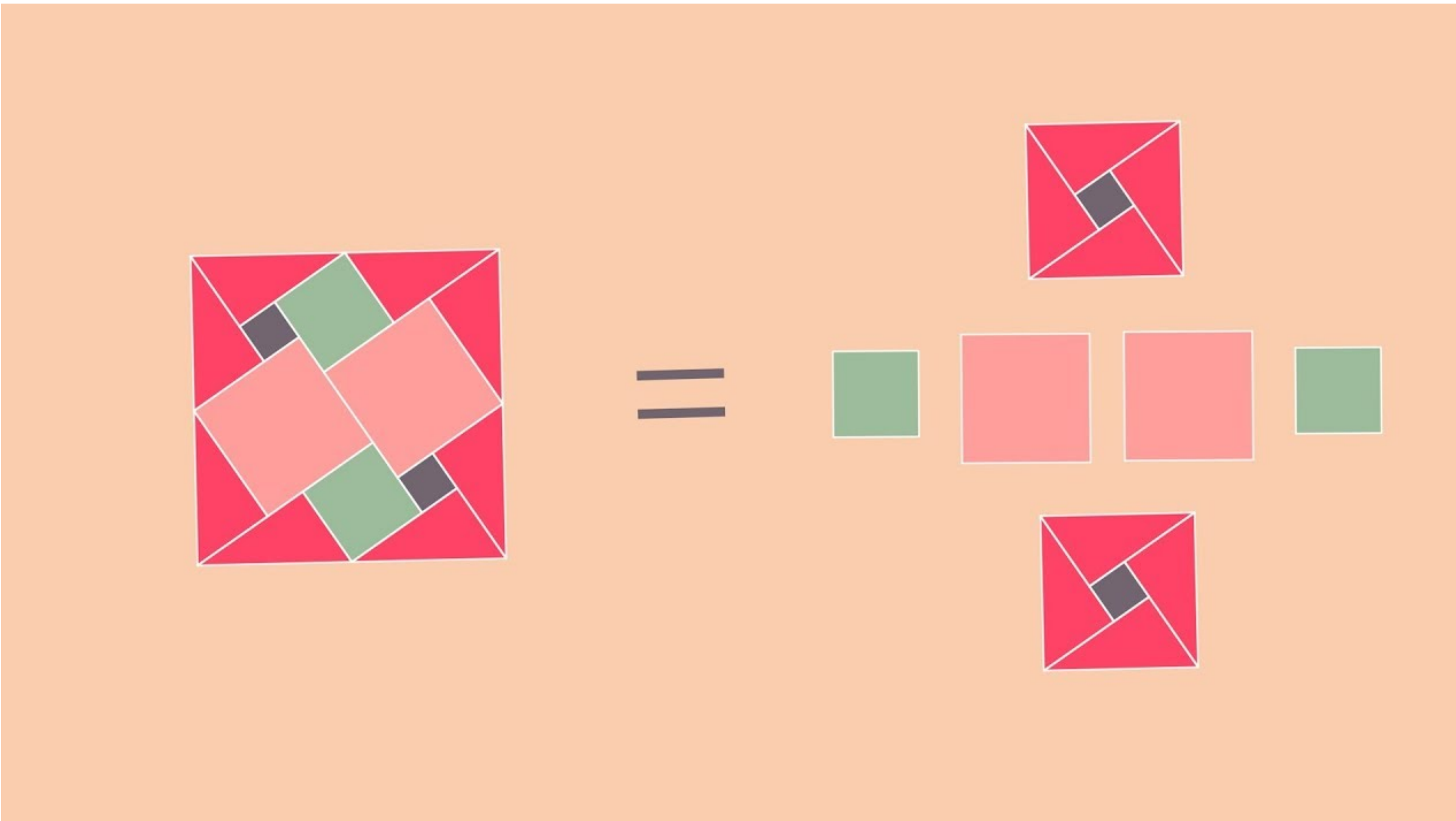

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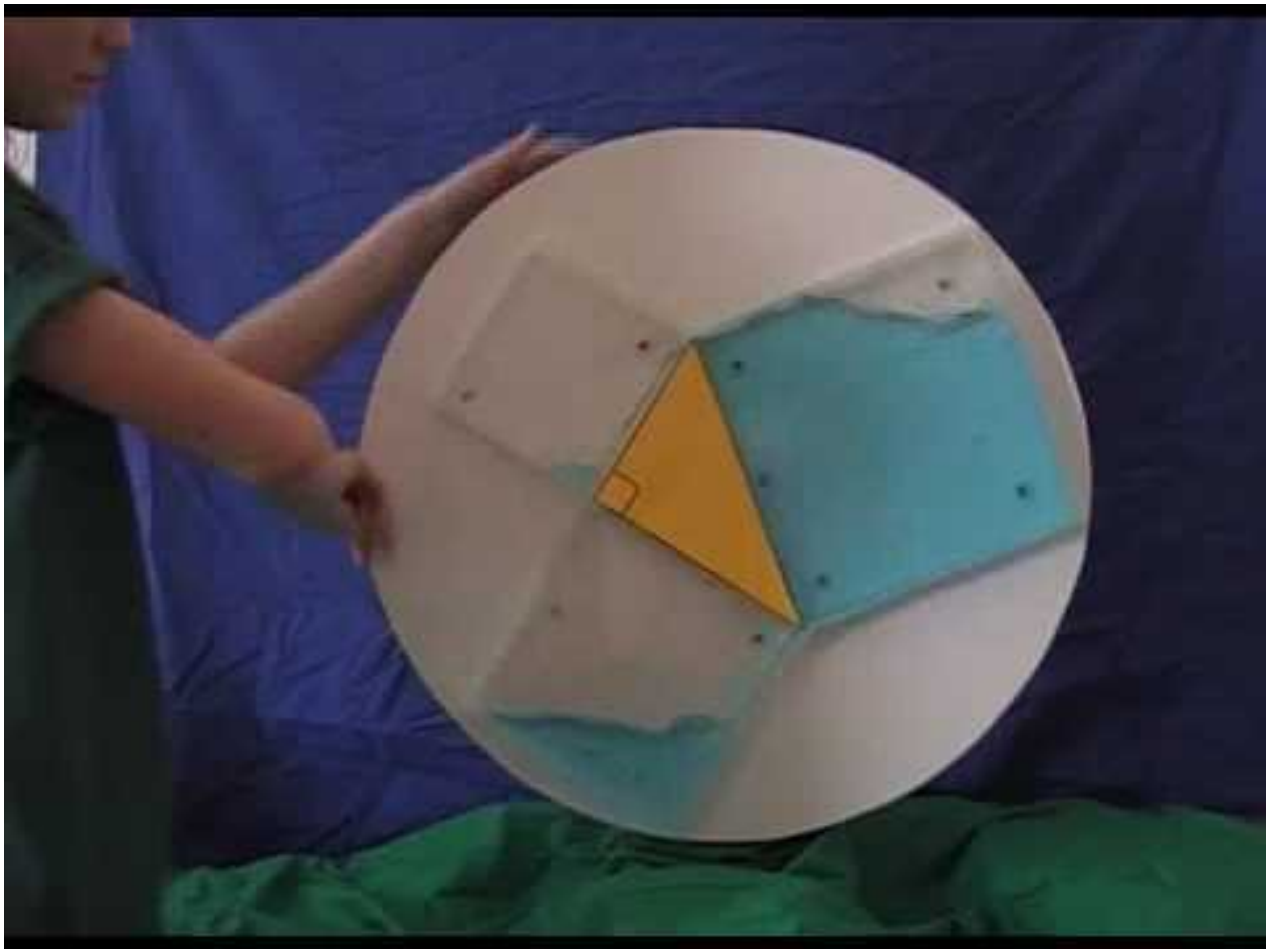

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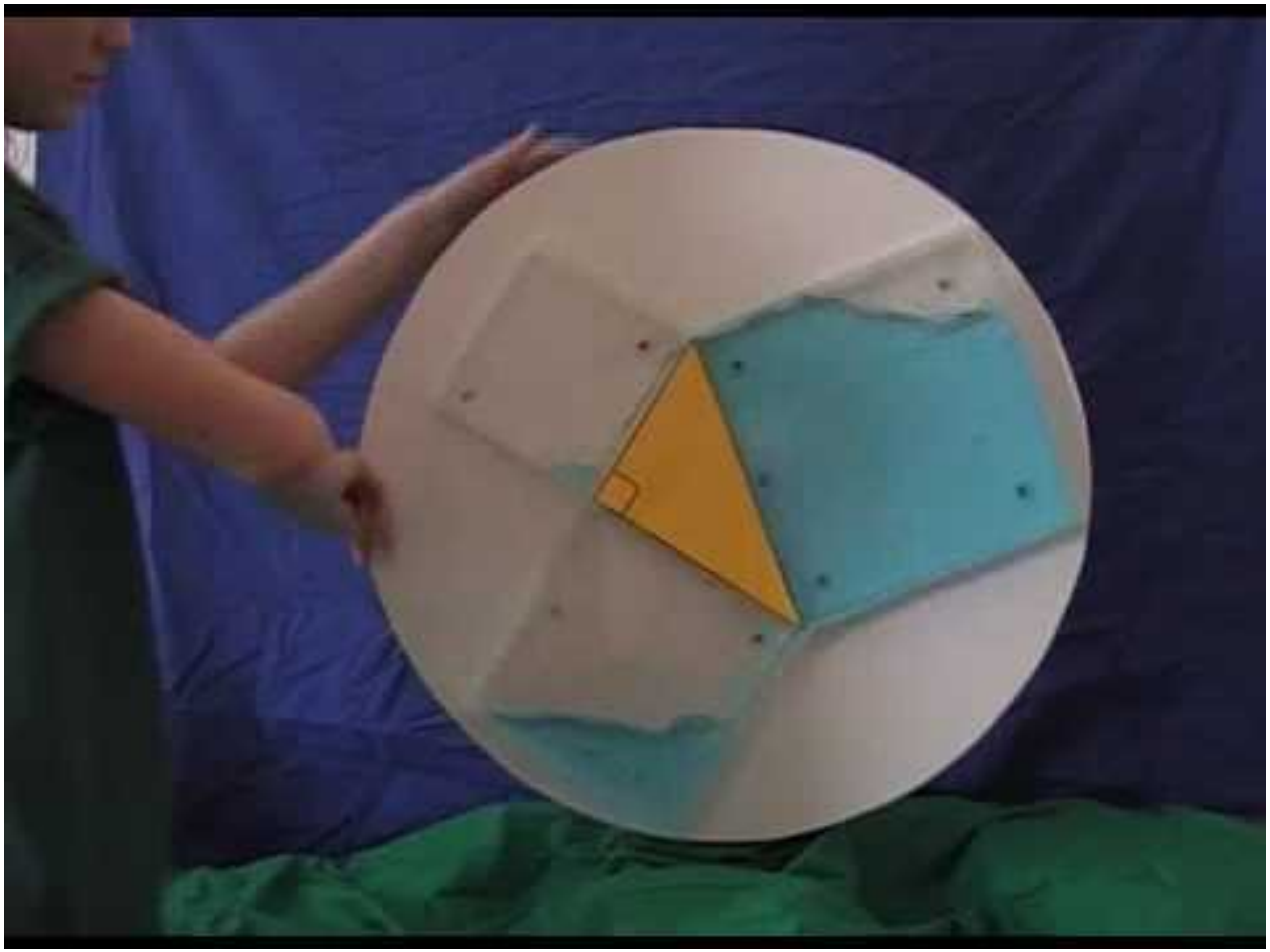

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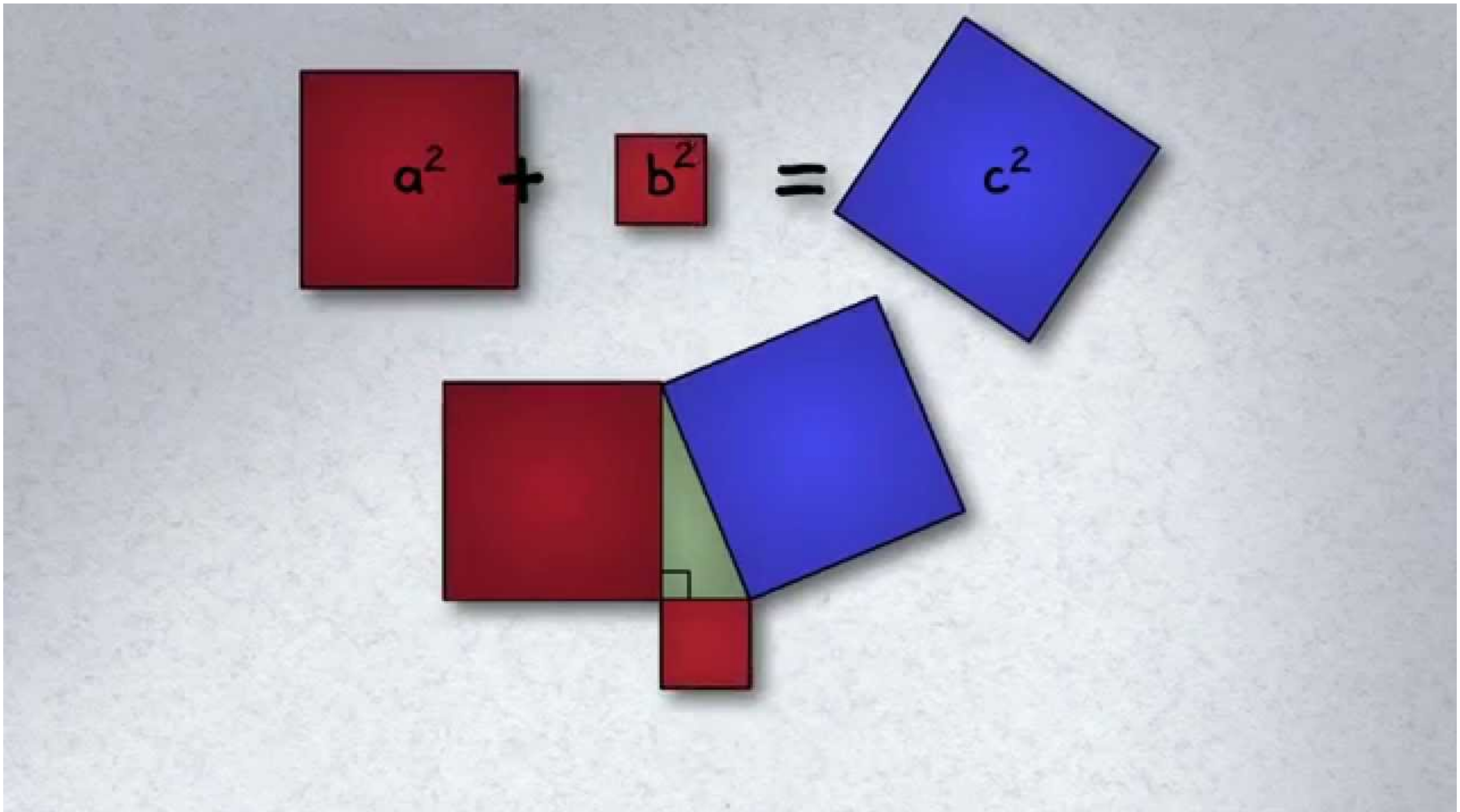

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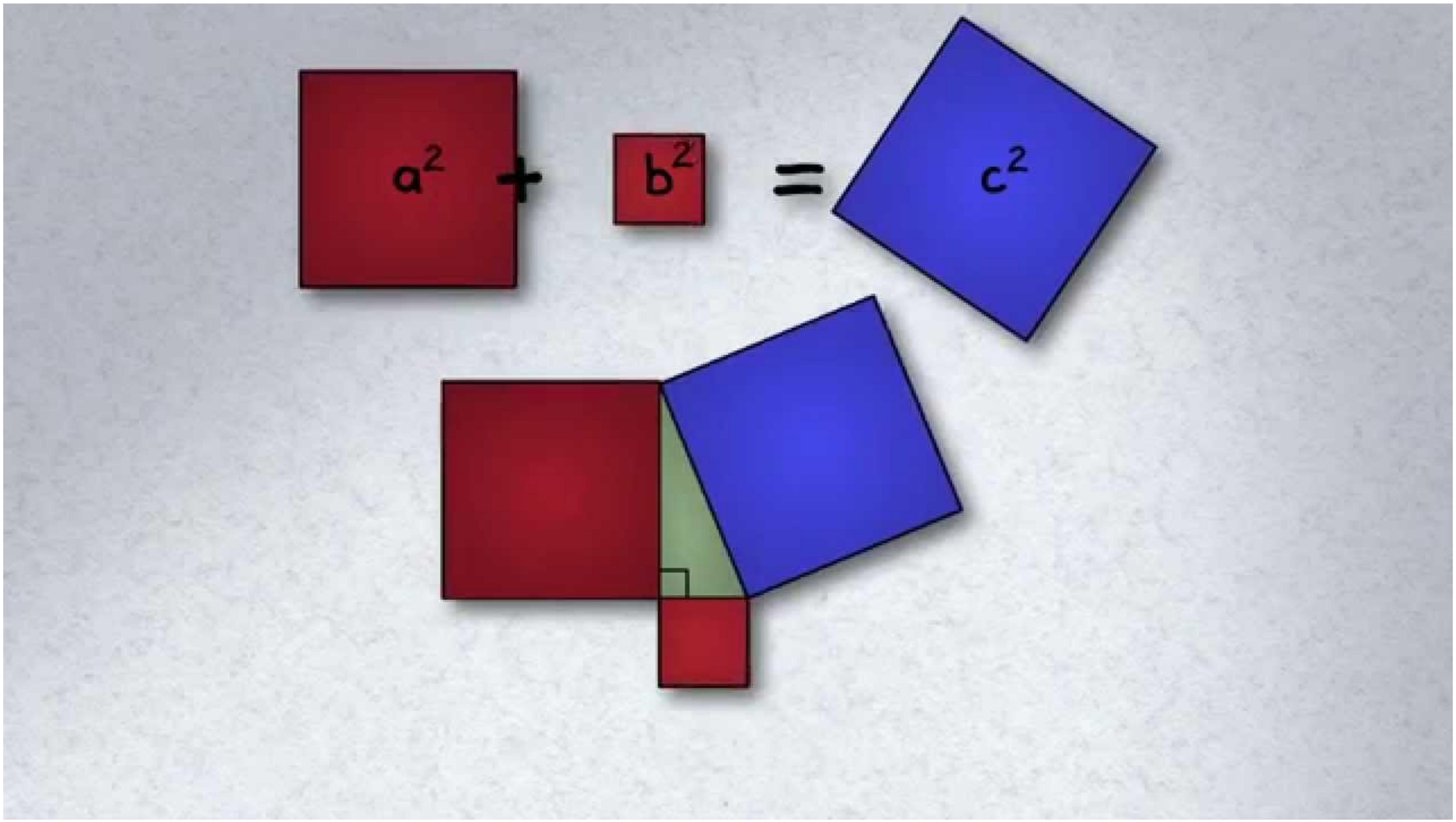

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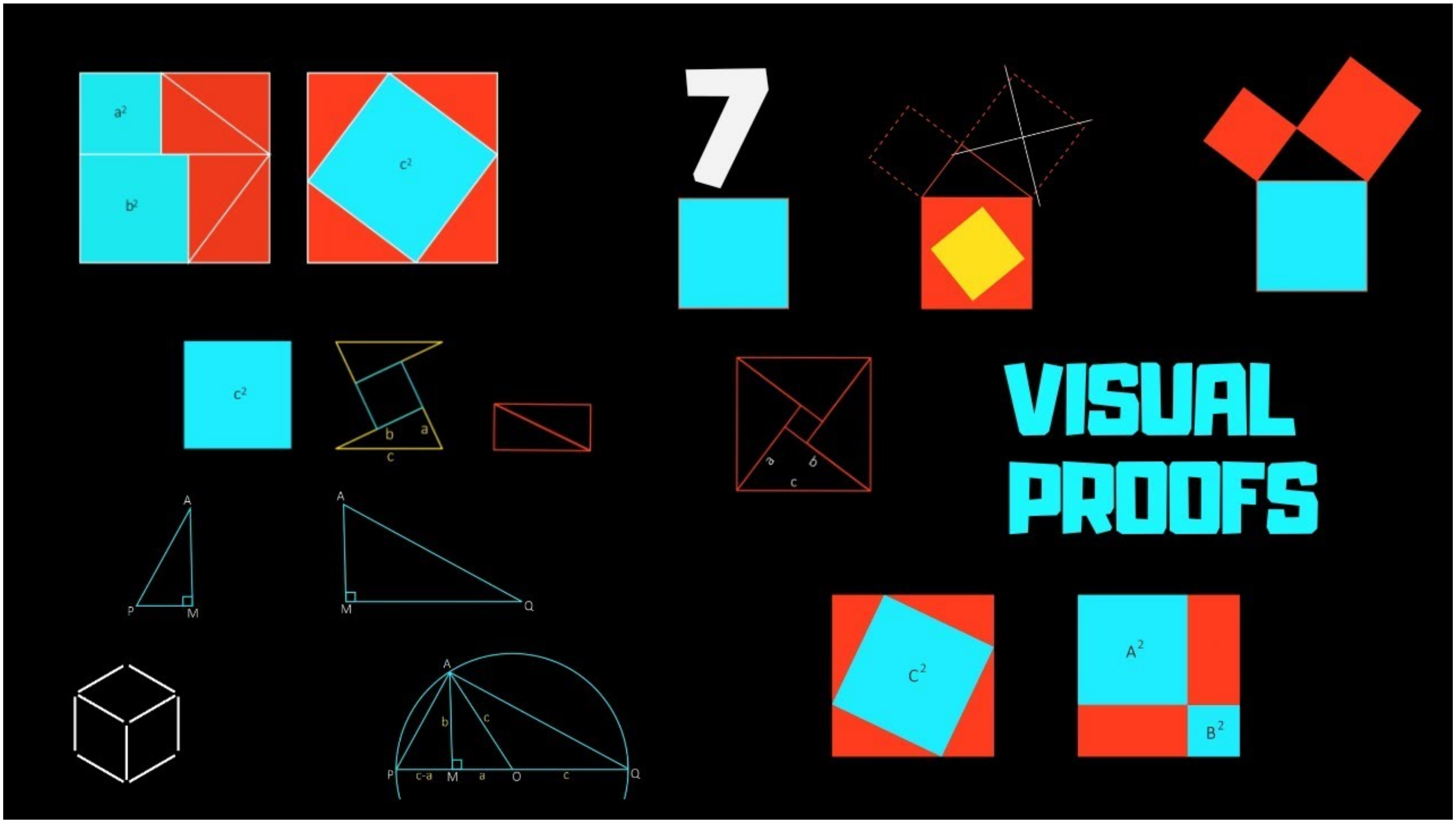

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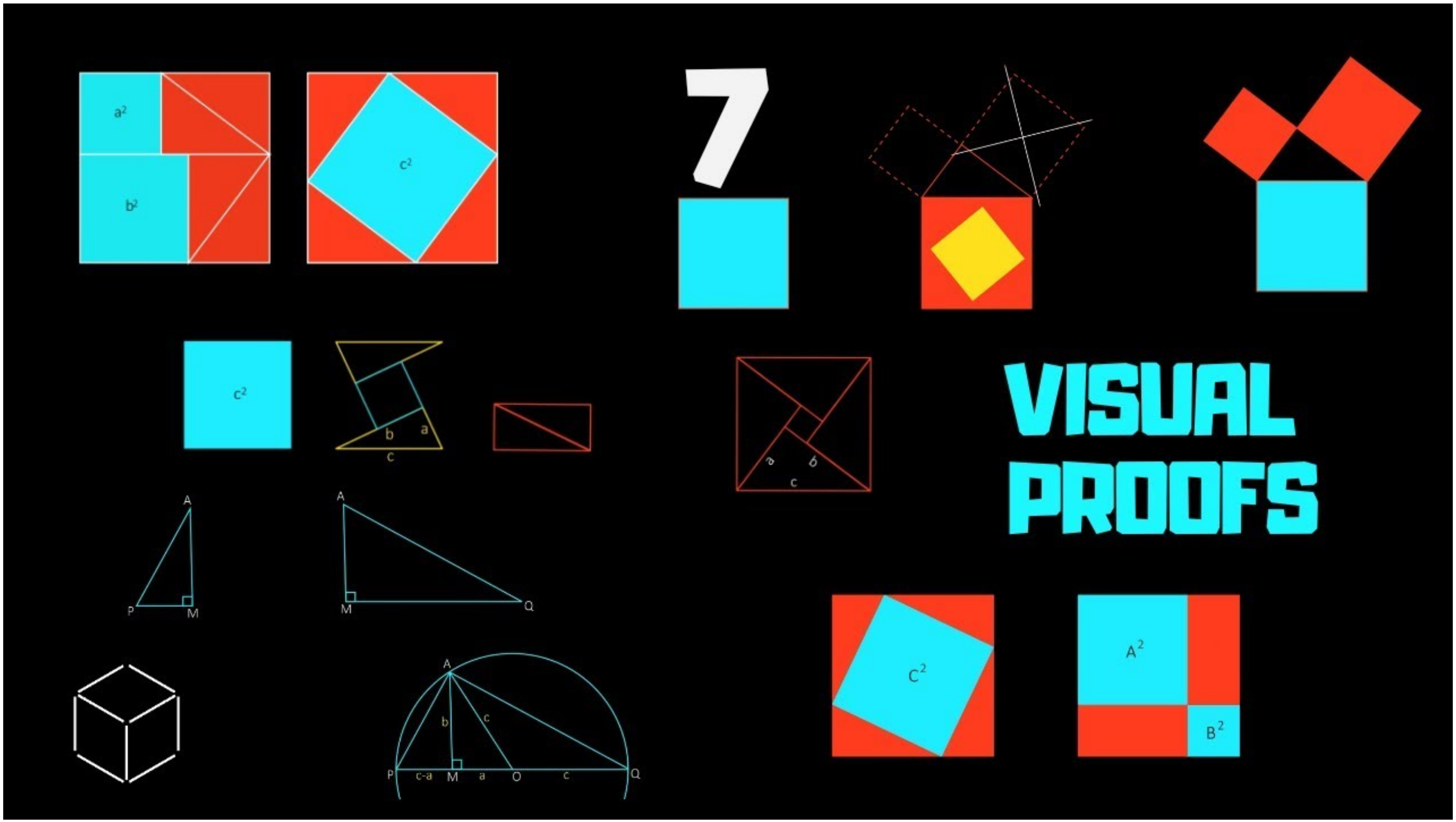

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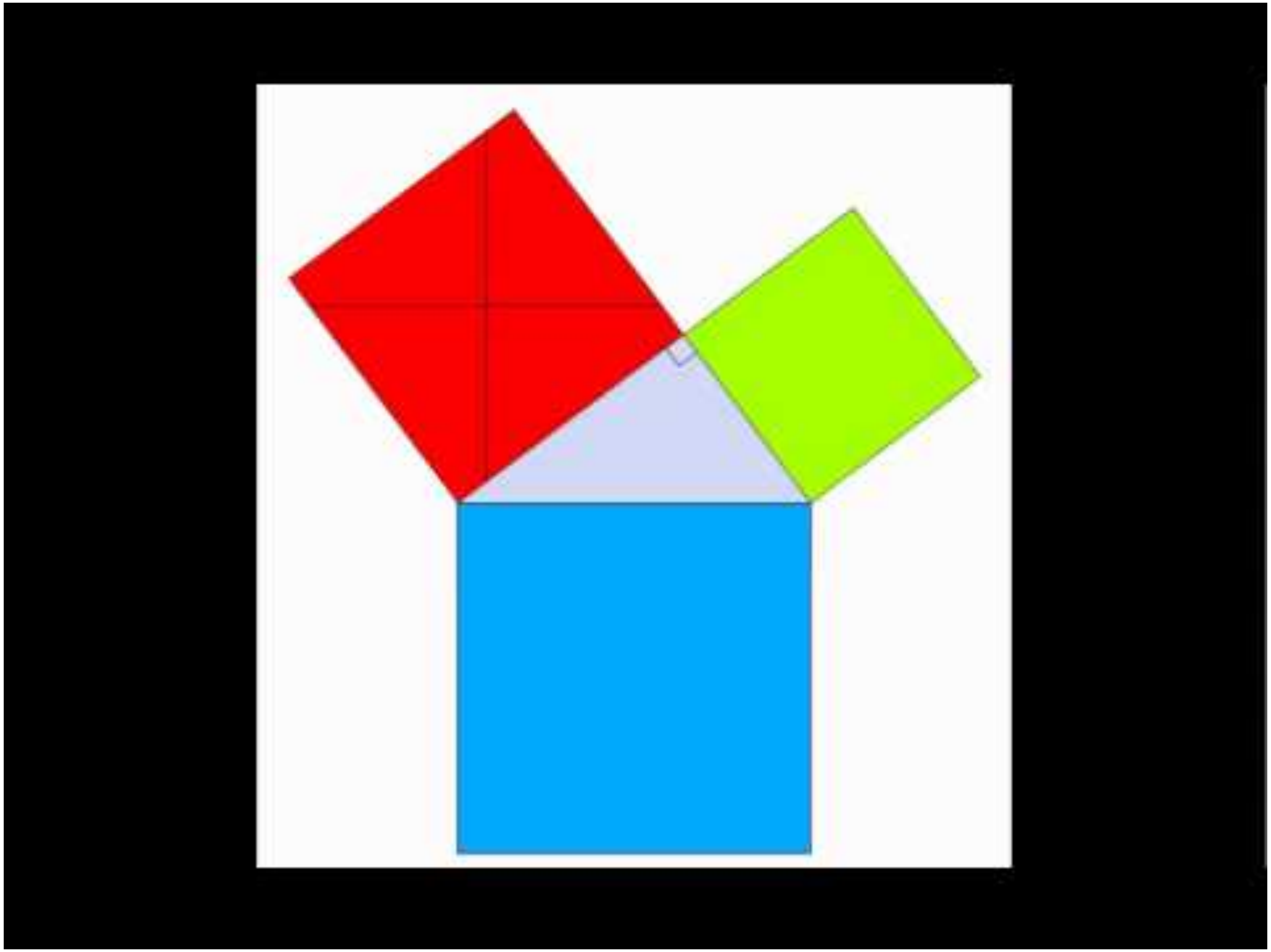

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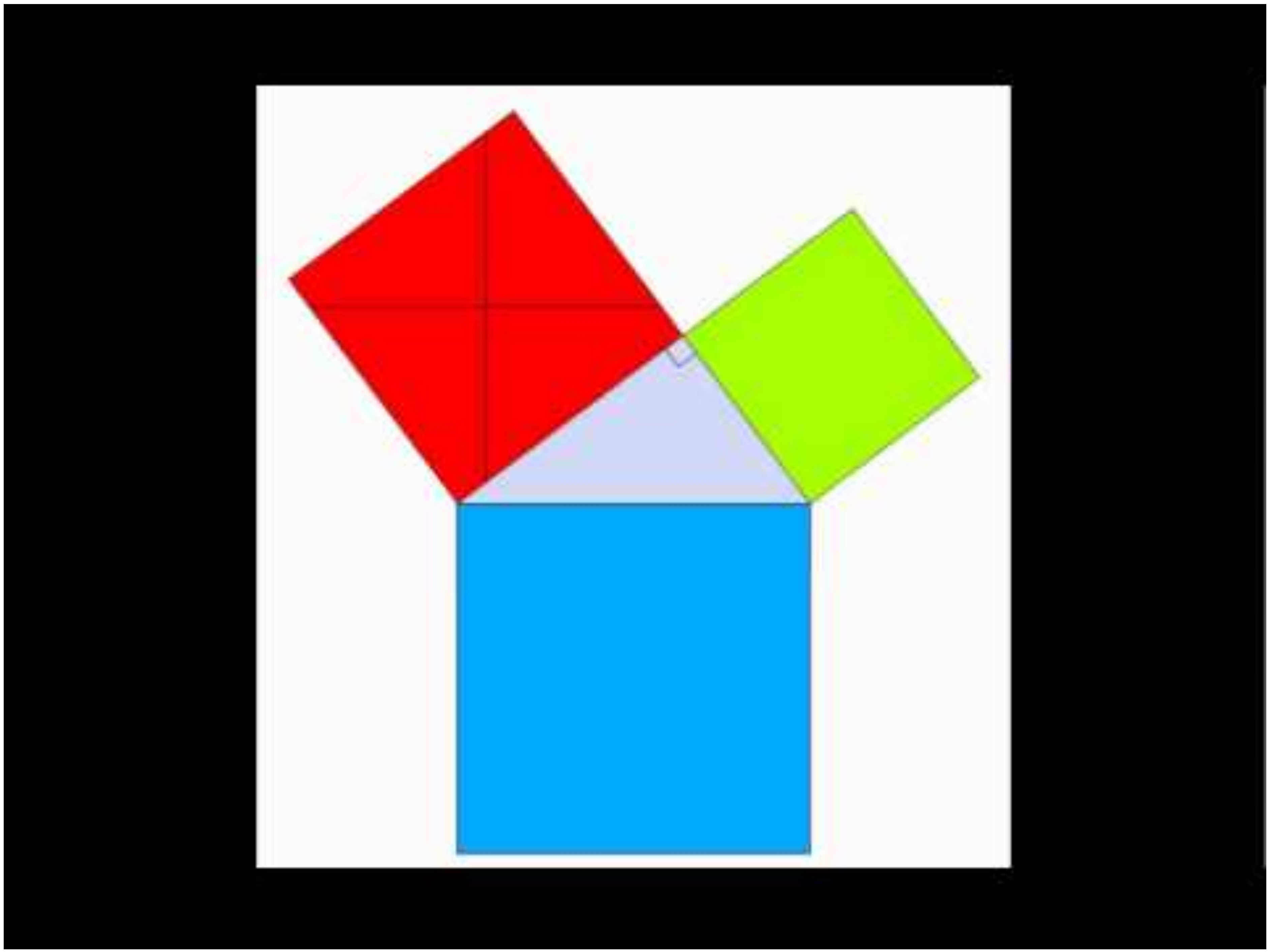
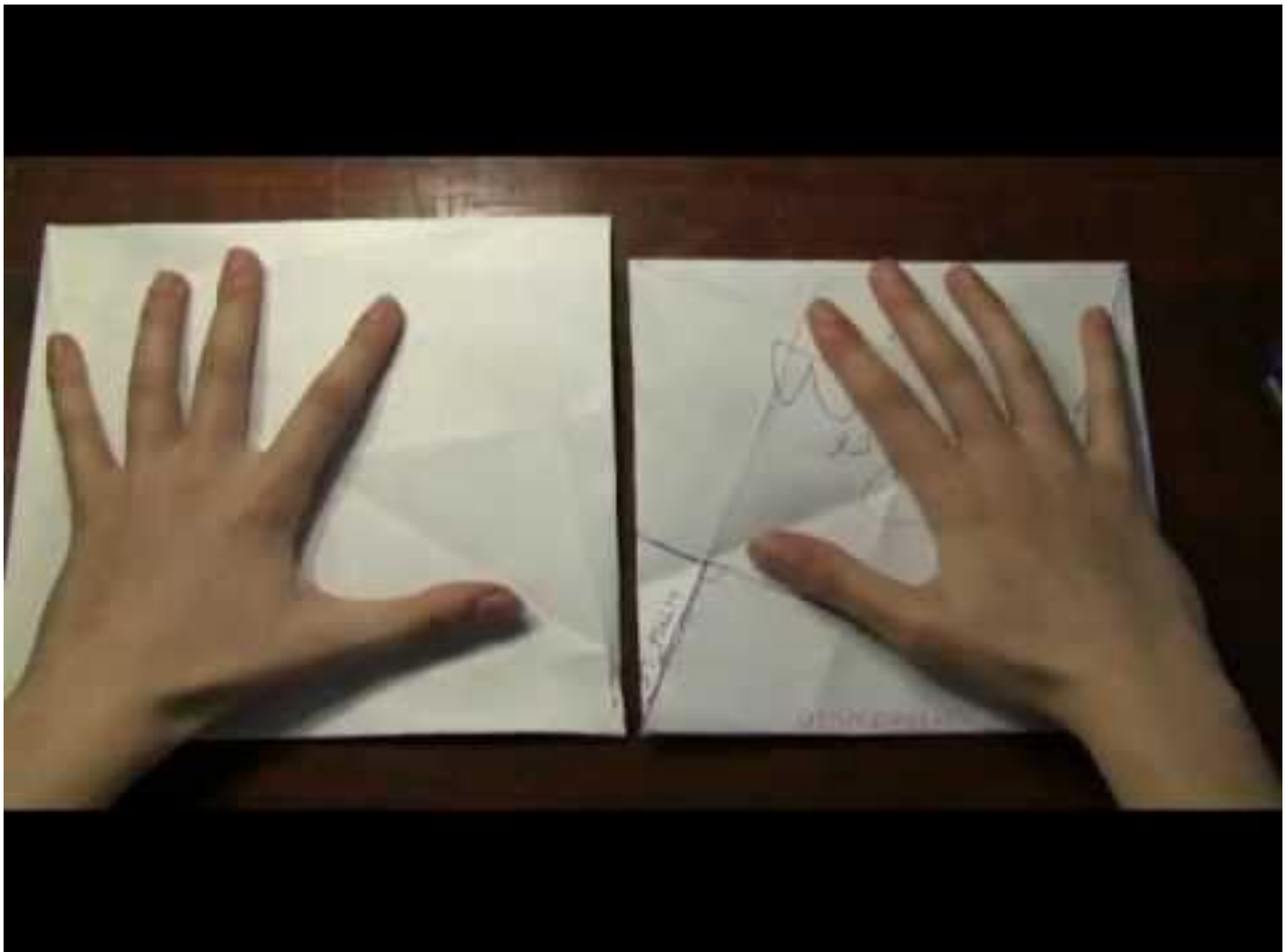


Pythagorean Theorem



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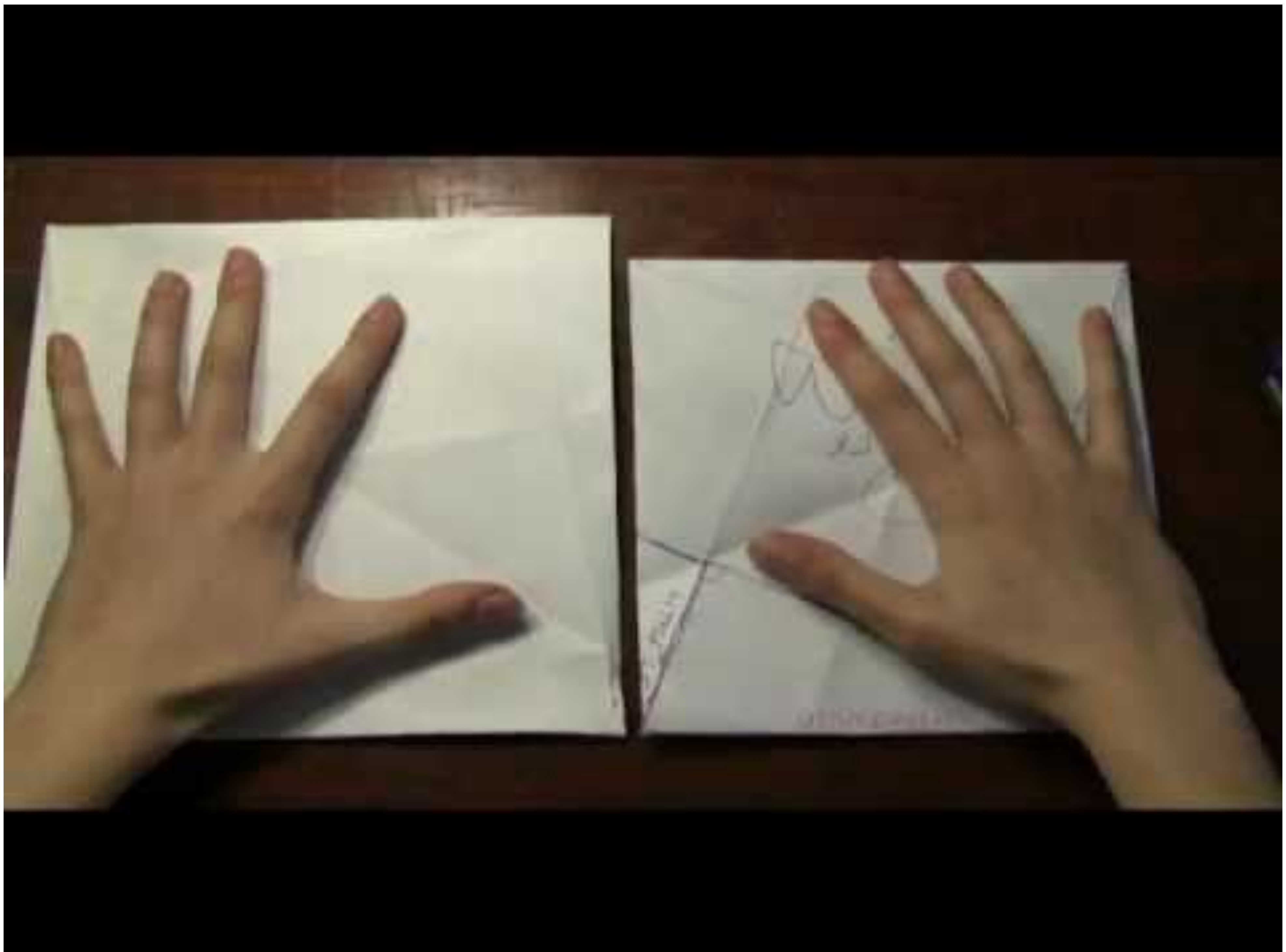


4WWL


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> My brother … taught me what he had been learning in those 3 months; that is, the *Practical* part of *Common Arithmetick* … This was my first entry into *Mathematicks*, and all the *Teaching* I had. … I did thenceforth prosecute it, (at School and in the University) not as a formal Study, but as a pleasing Diversion, at spare hours; as books of *Arithmetick*, or others *Mathematical* fel occasionally in my way. … For Mathematicks, (at that time, with us) was scarce looked upon as *Academical* Studies, but rather *Mechanical*; as the business of *Traders*, *Merchants*, *Seamen*, *Carpenters*, *Suveyors of Lands*, or the like; and perhaps *Almanack-makers in London*. And amongst more than Two hundred Students (at that time) in our College, I do not know of any Two (perhaps not any) who had more of *Mathematicks* than I, (if so much) which was then but little; And but very few, in that whole University. For the Study of *Mathematicks* was at that time more cultivated in London than in the Universities.

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- Instead, an example from Roman land surveyor Marcus Junius Nipsius.

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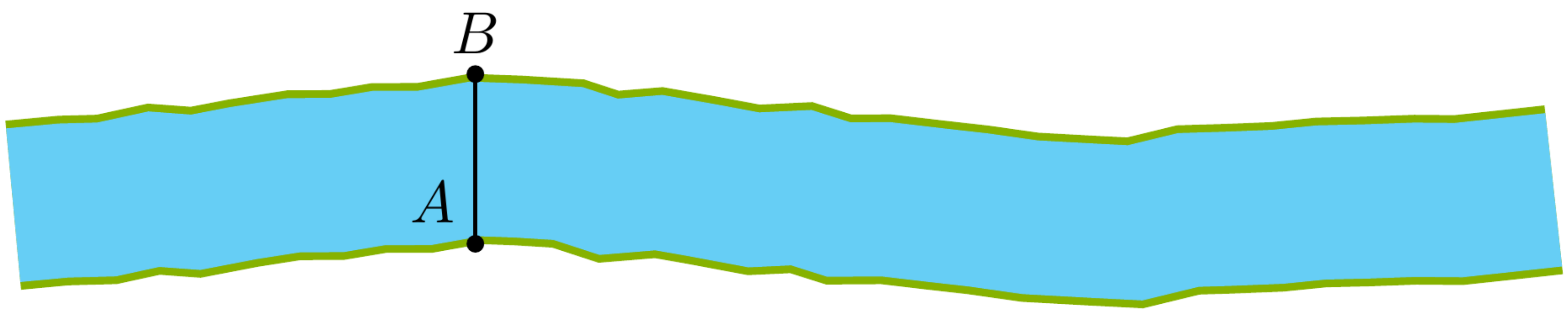

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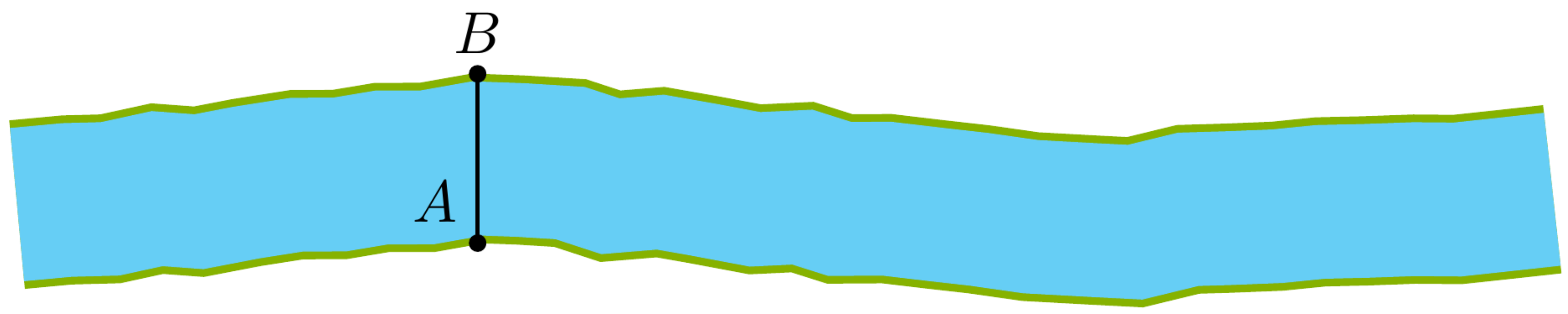

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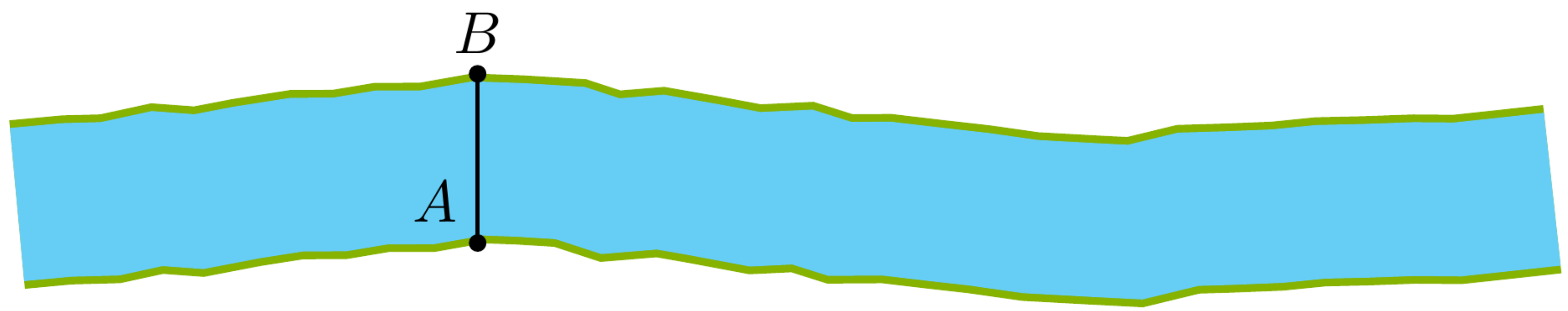
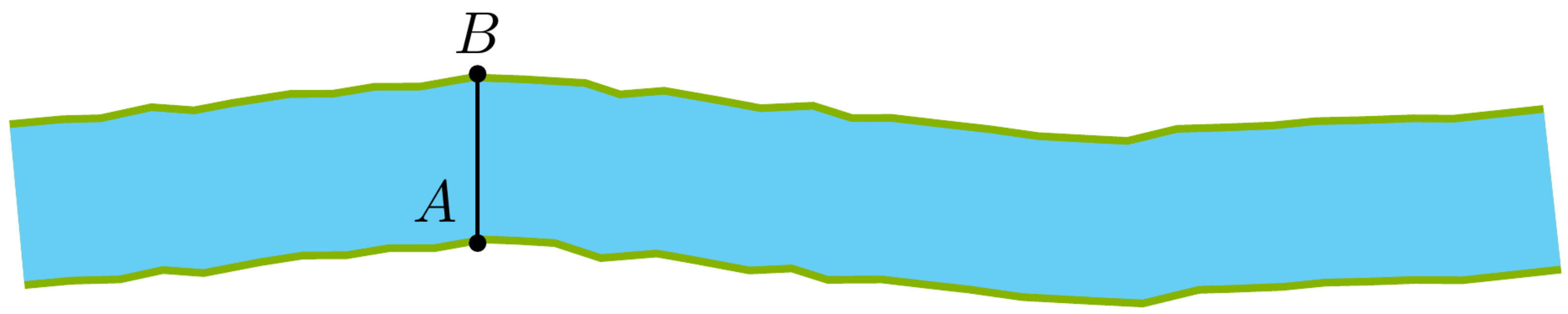


- Maybe, $\boldsymbol{B}$ is some tree, or something else in sight.

- Rules: You must stay on your side of the river.

People's History of Surveying



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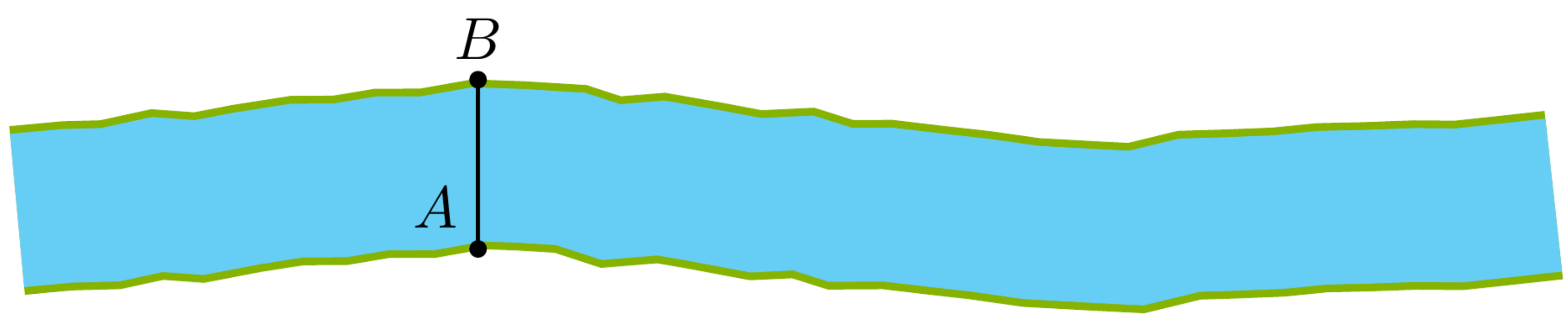

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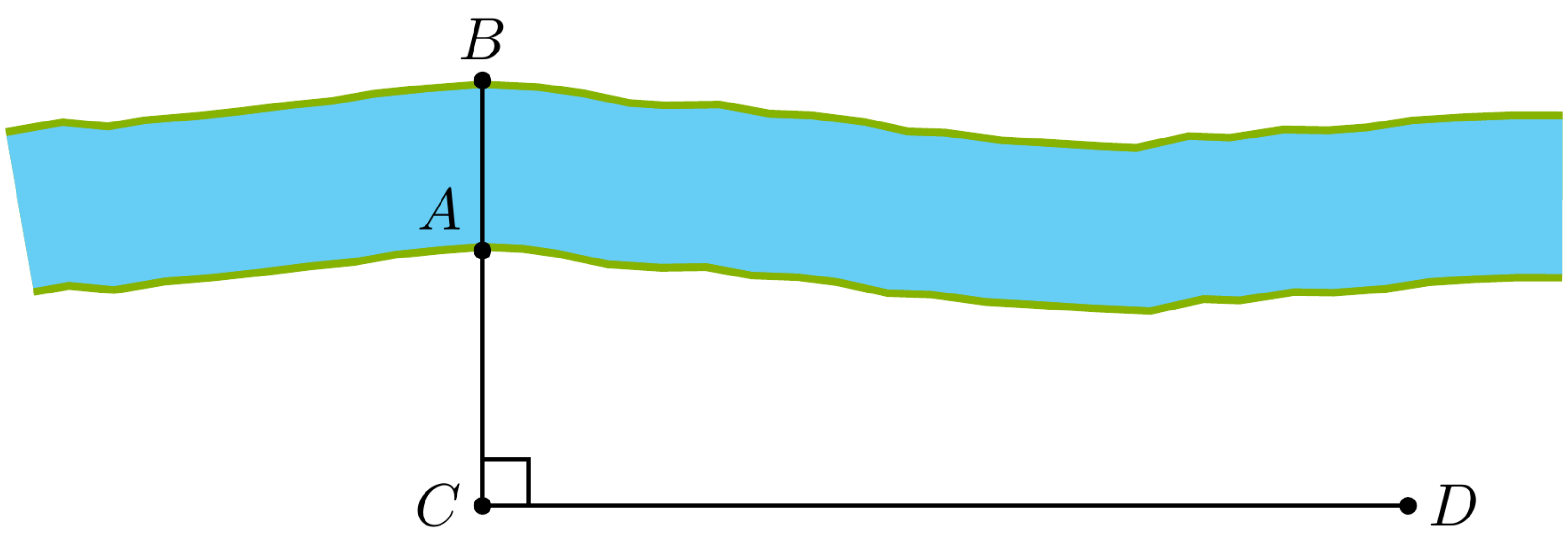

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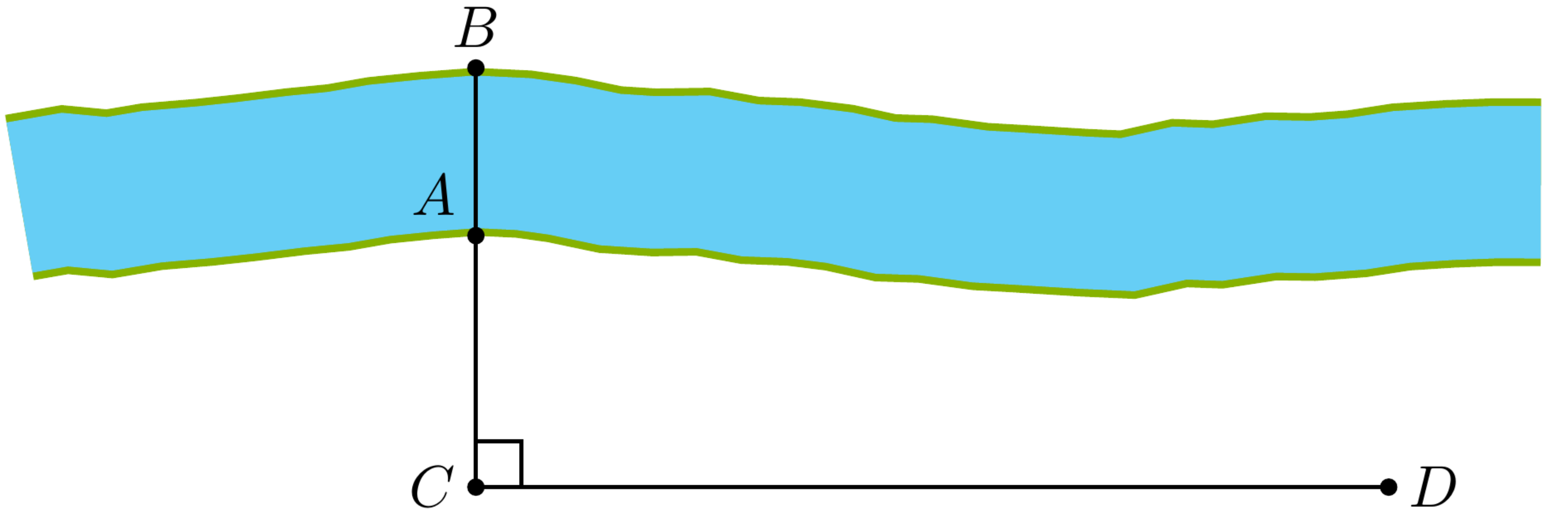
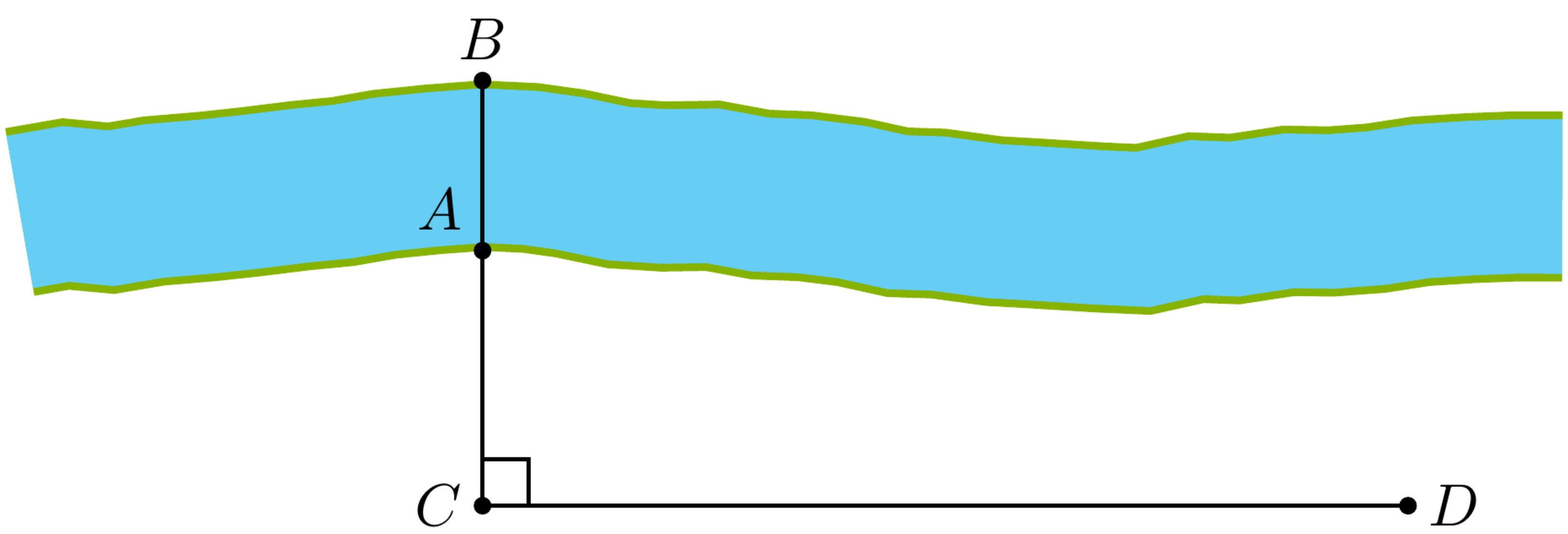


People's History of Surveying

- Make another right angle, this time at D, drawing a ray downward



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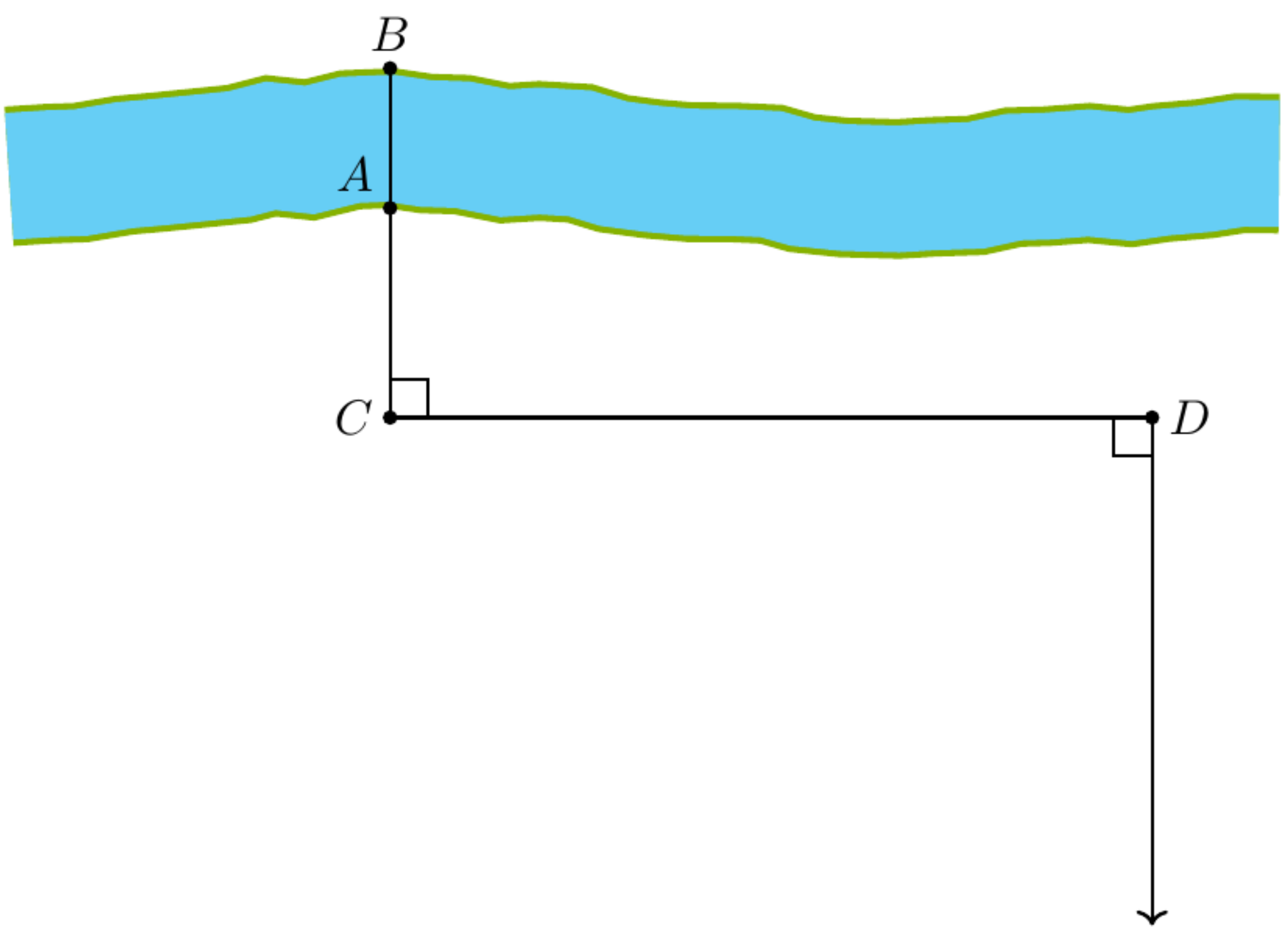

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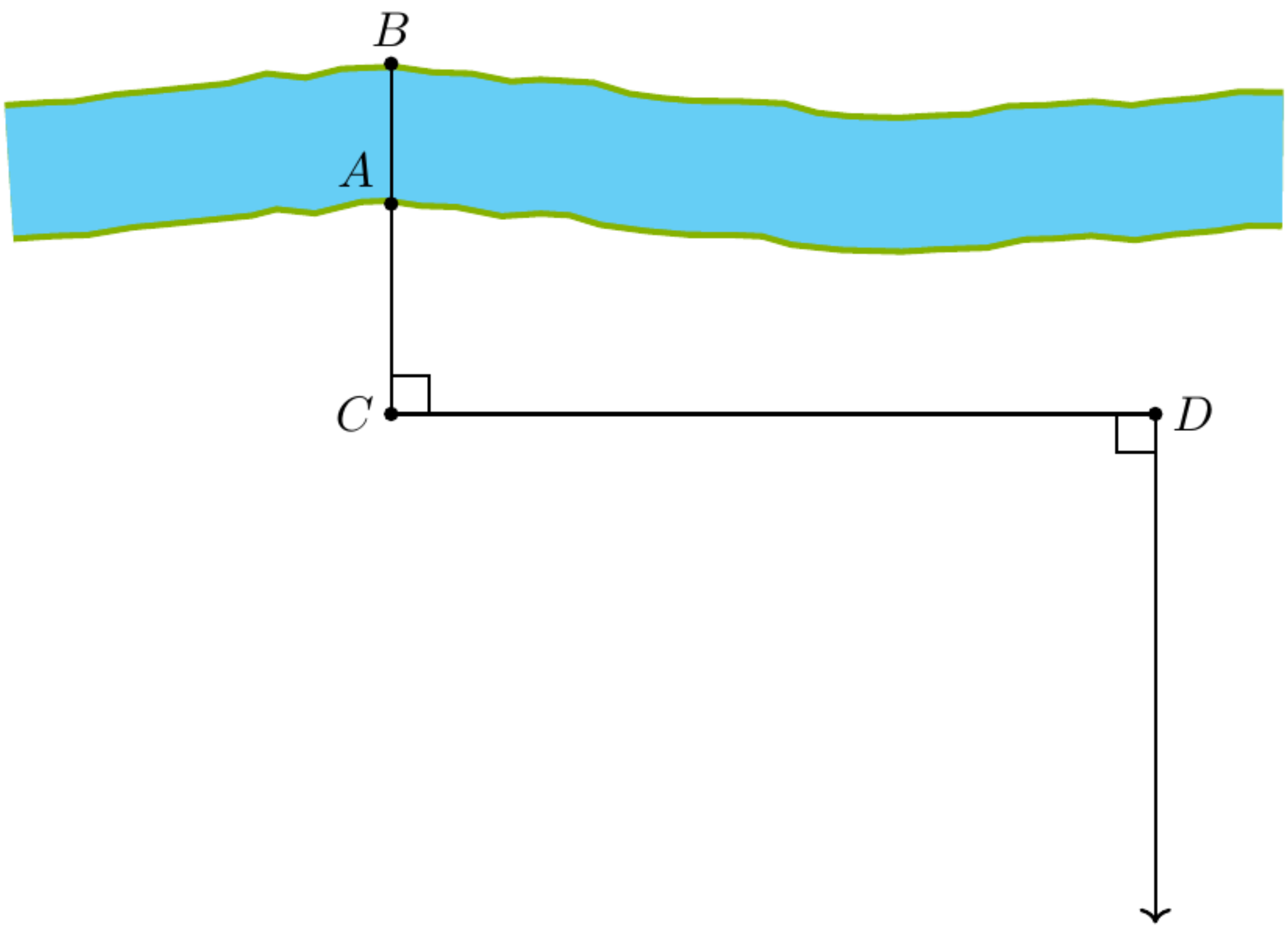

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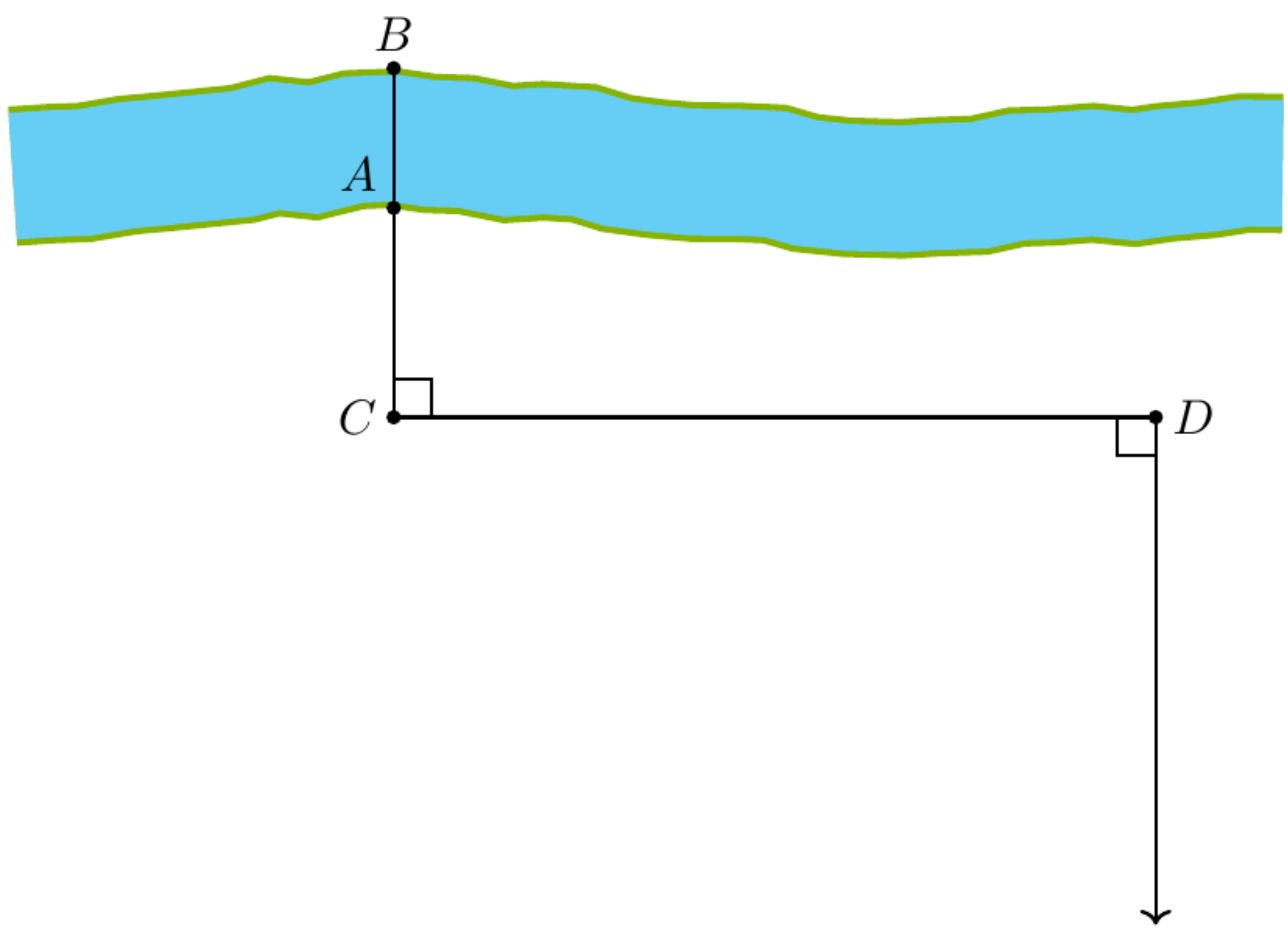
- Bisect $\overline{CD}$, call the midpoint E. Then, draw $\overline{BE}$, and continue this line until it meets the ray.



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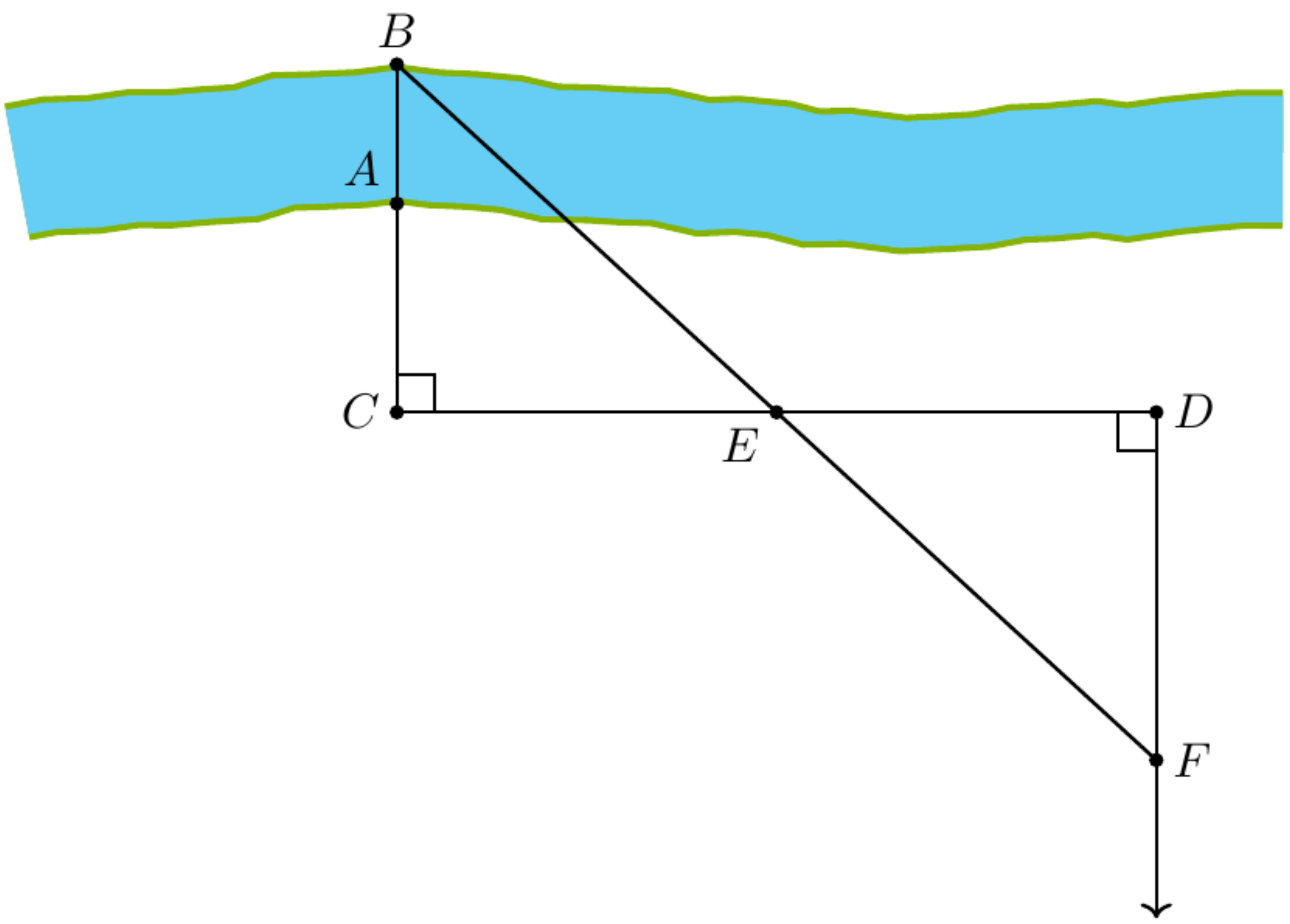
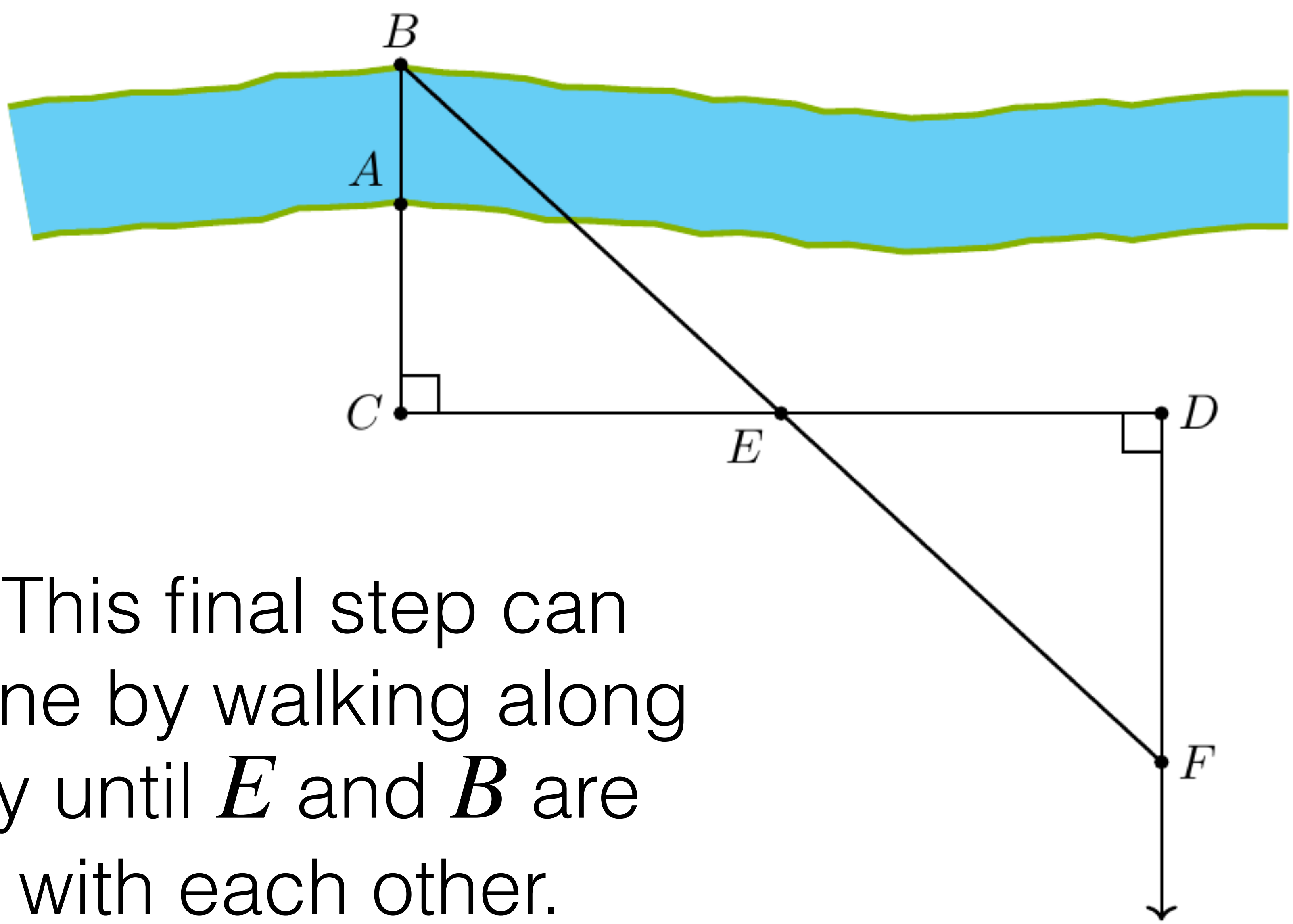


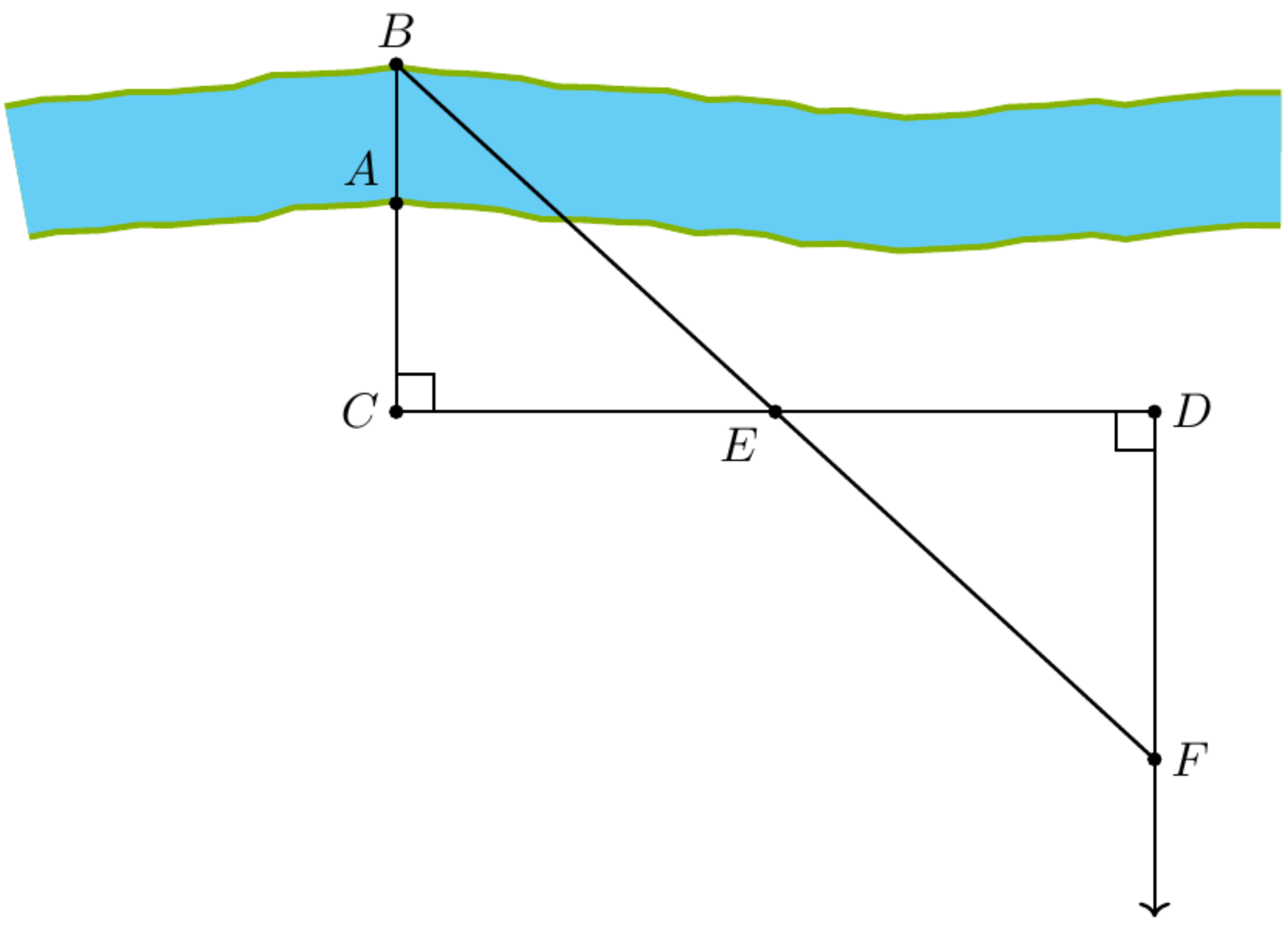
People's History of Surveying

- Bisect $\overline{CD}$, call the midpoint E. Then, draw $\overline{BE}$, and continue this line until it meets the ray.



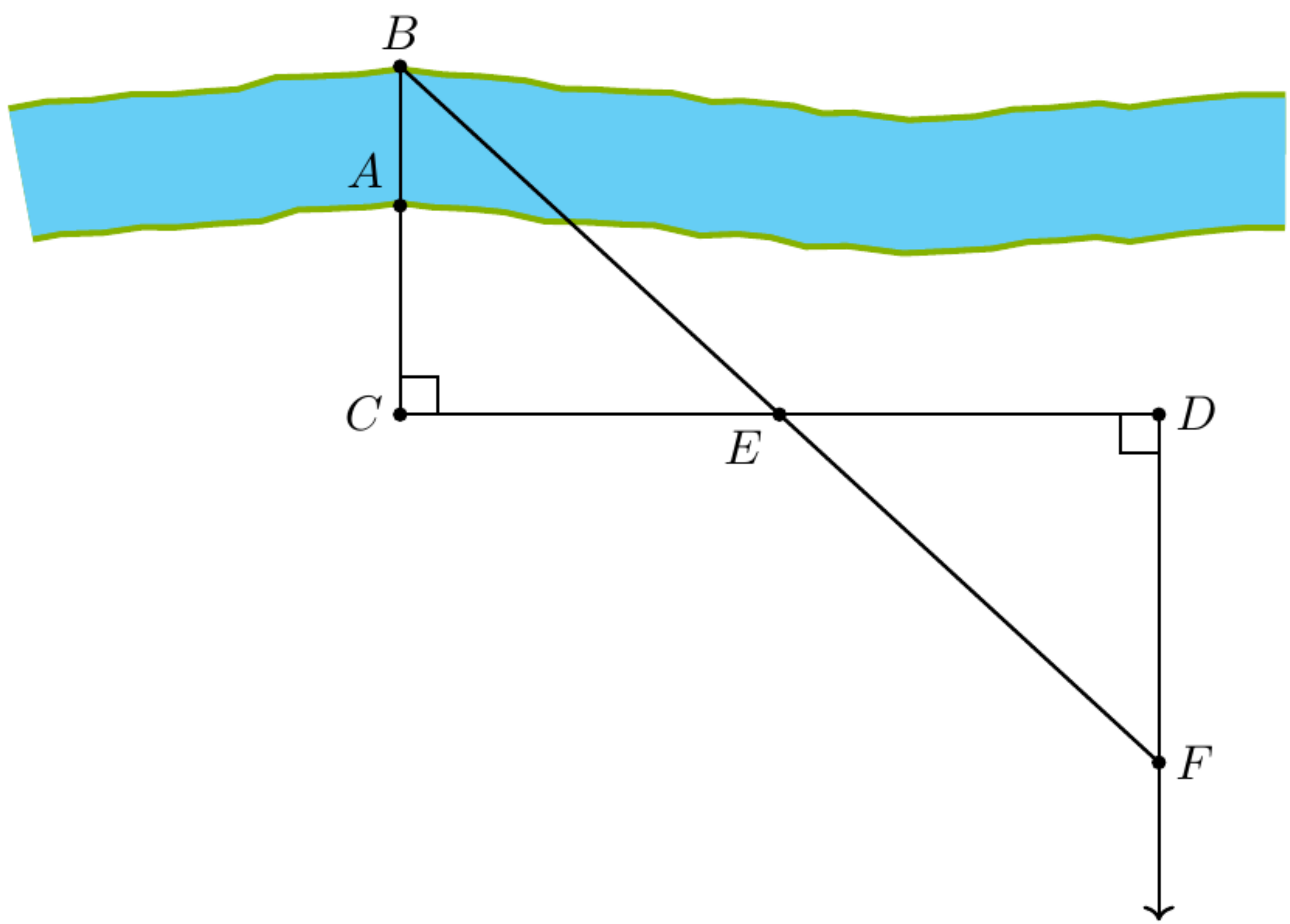
- Note: This final step can be done by walking along the ray until E and B are in line with each other.

People's History of Surveying



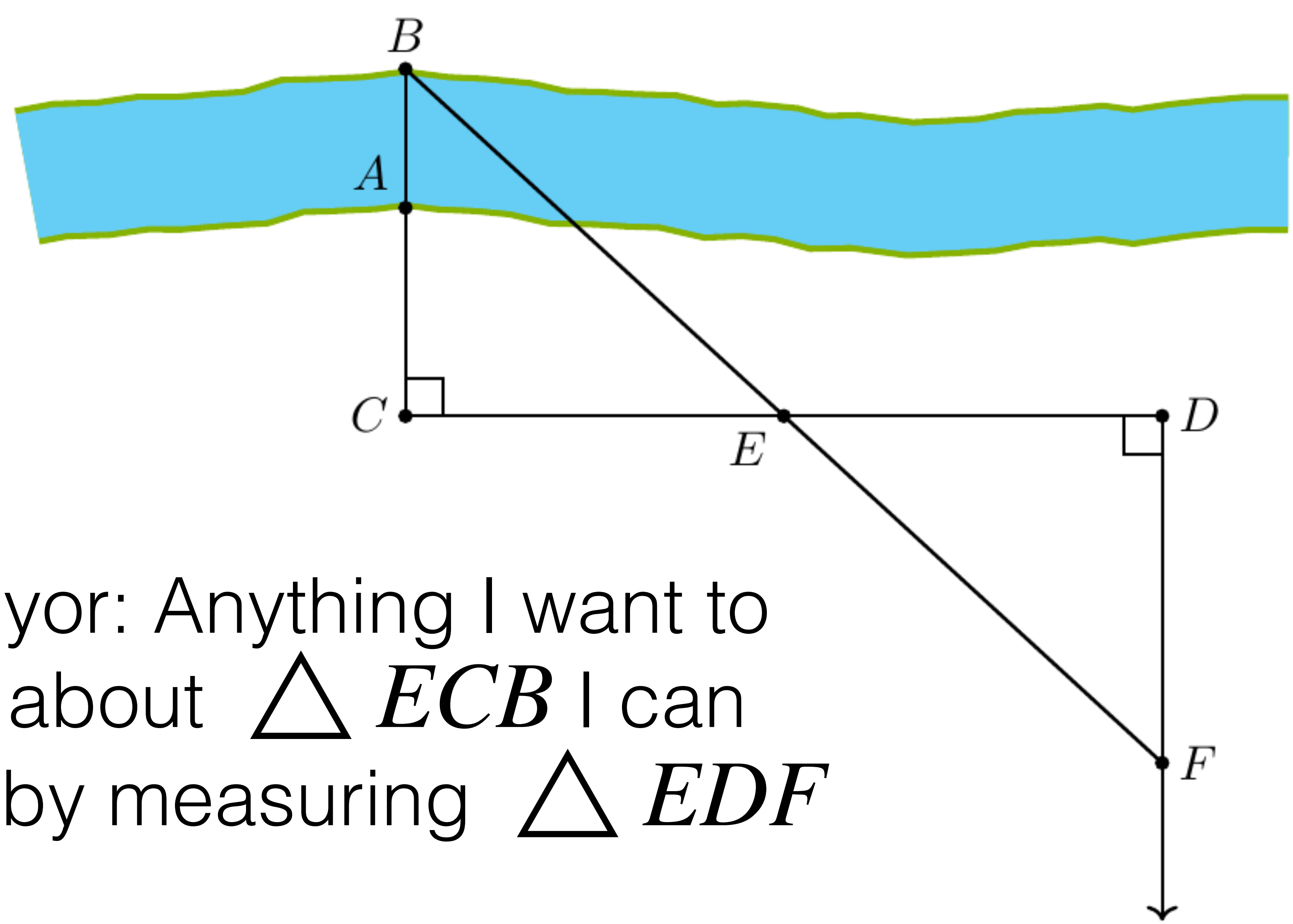
People's History of Surveying

- Mathematician: $\triangle ECB$ and $\triangle EDF$ are congruent.



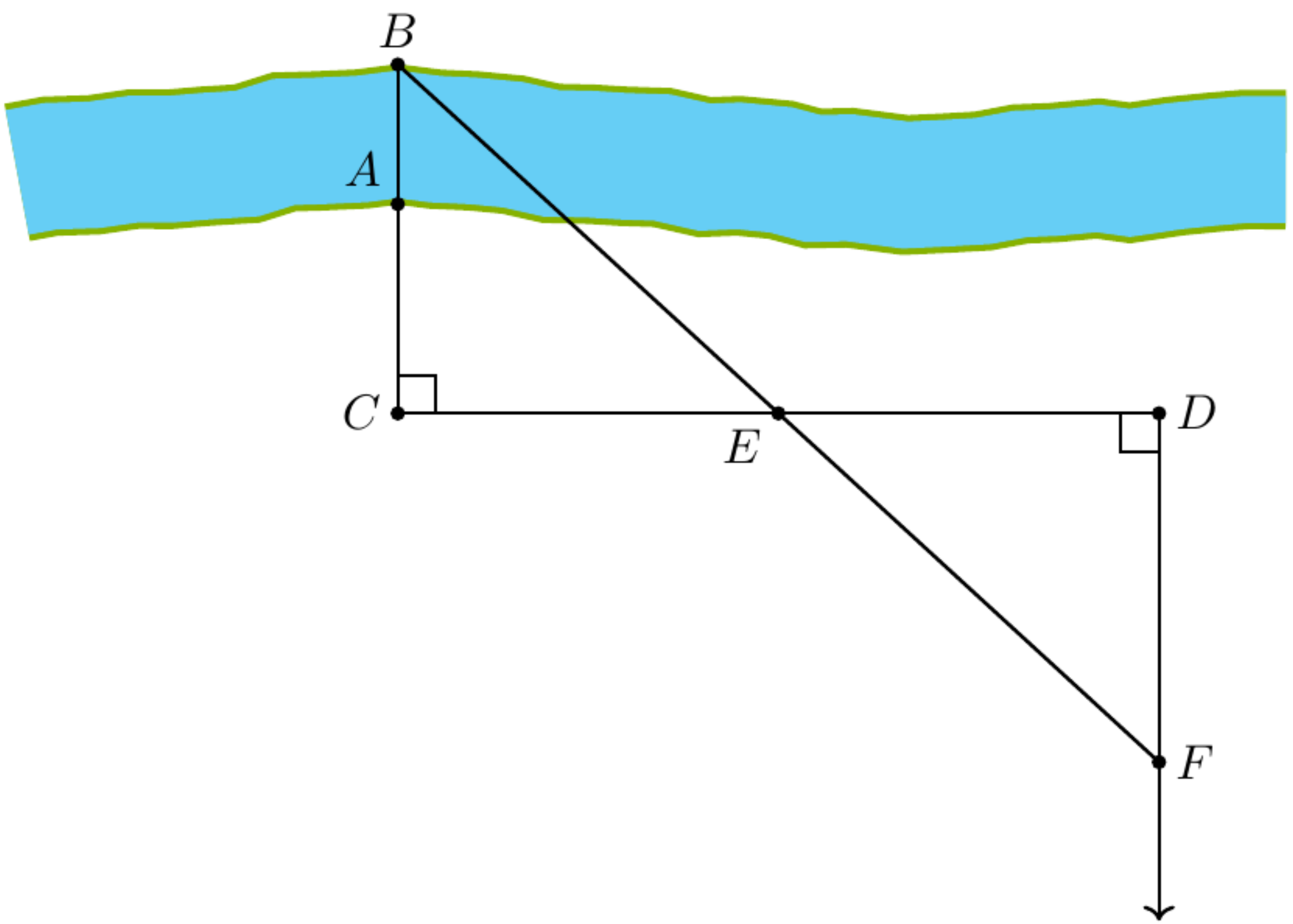
People's History of Surveying

- Mathematician: $\triangle\, ECB$ and $\triangle\, EDF$ are congruent.



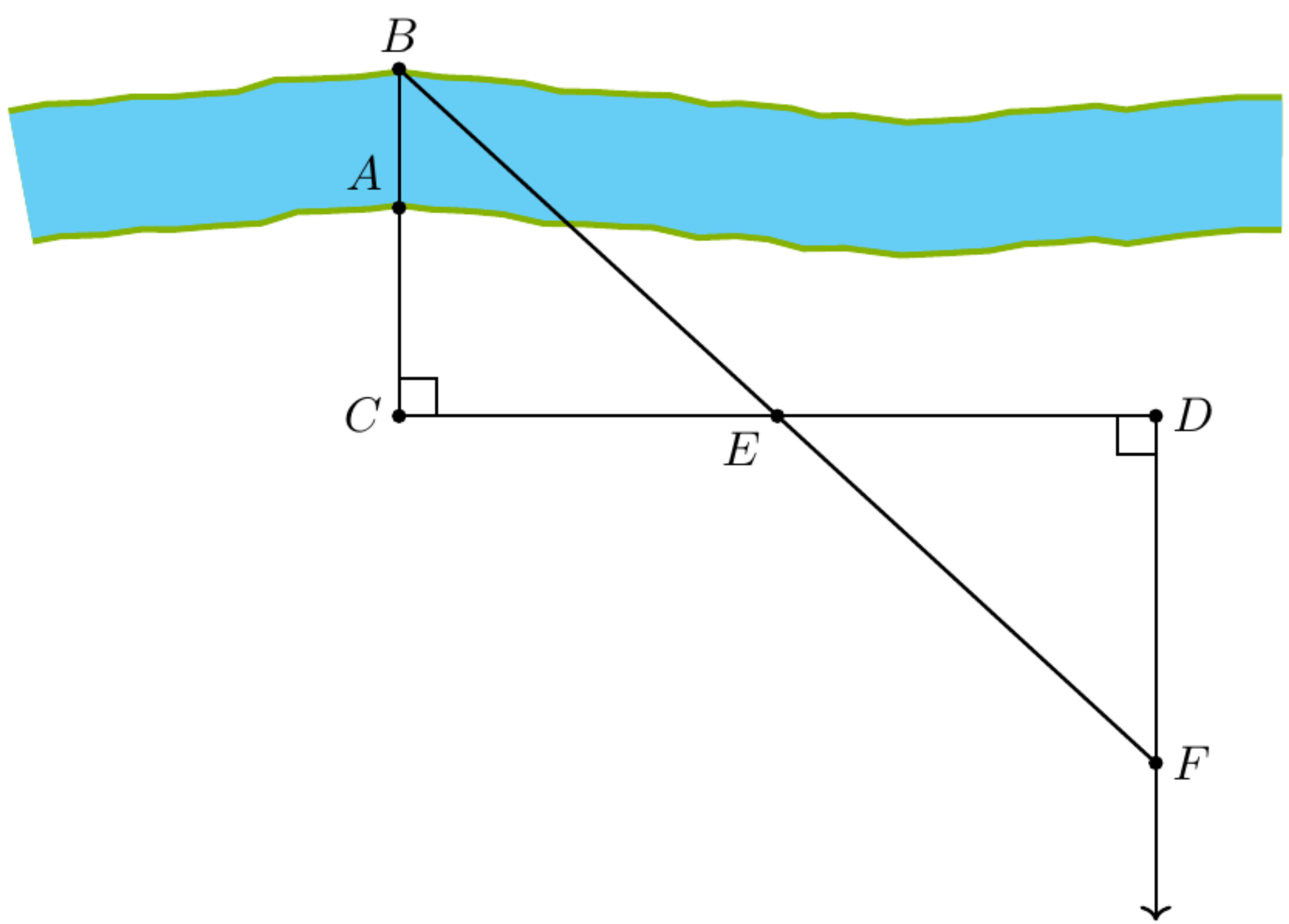
- Surveyor: Anything I want to know about $\triangle\, ECB$ I can learn by measuring $\triangle\, EDF$

People's History of Surveying



People's History of Surveying

- To find the distance $\overline{AB}$, add the point G.



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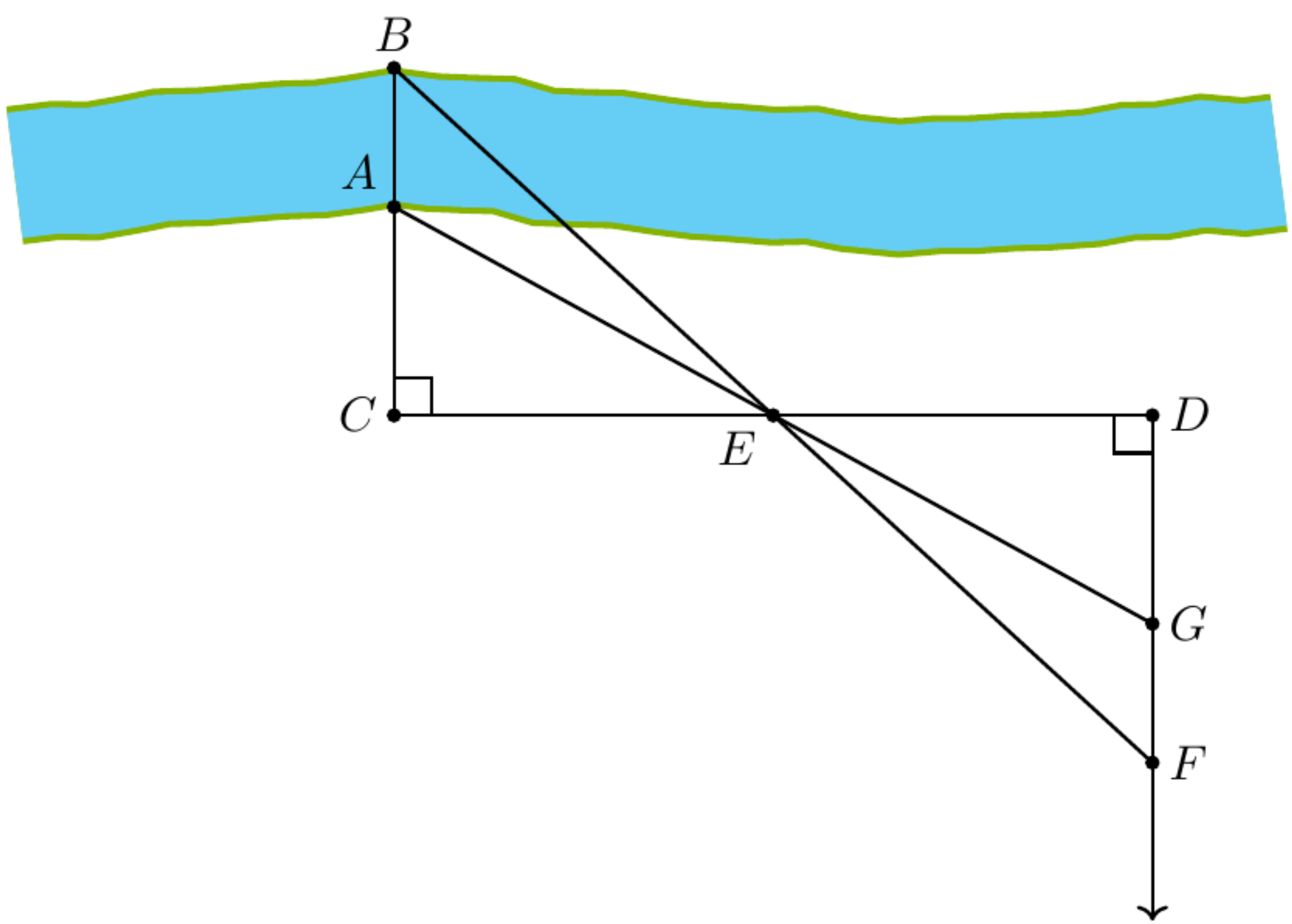

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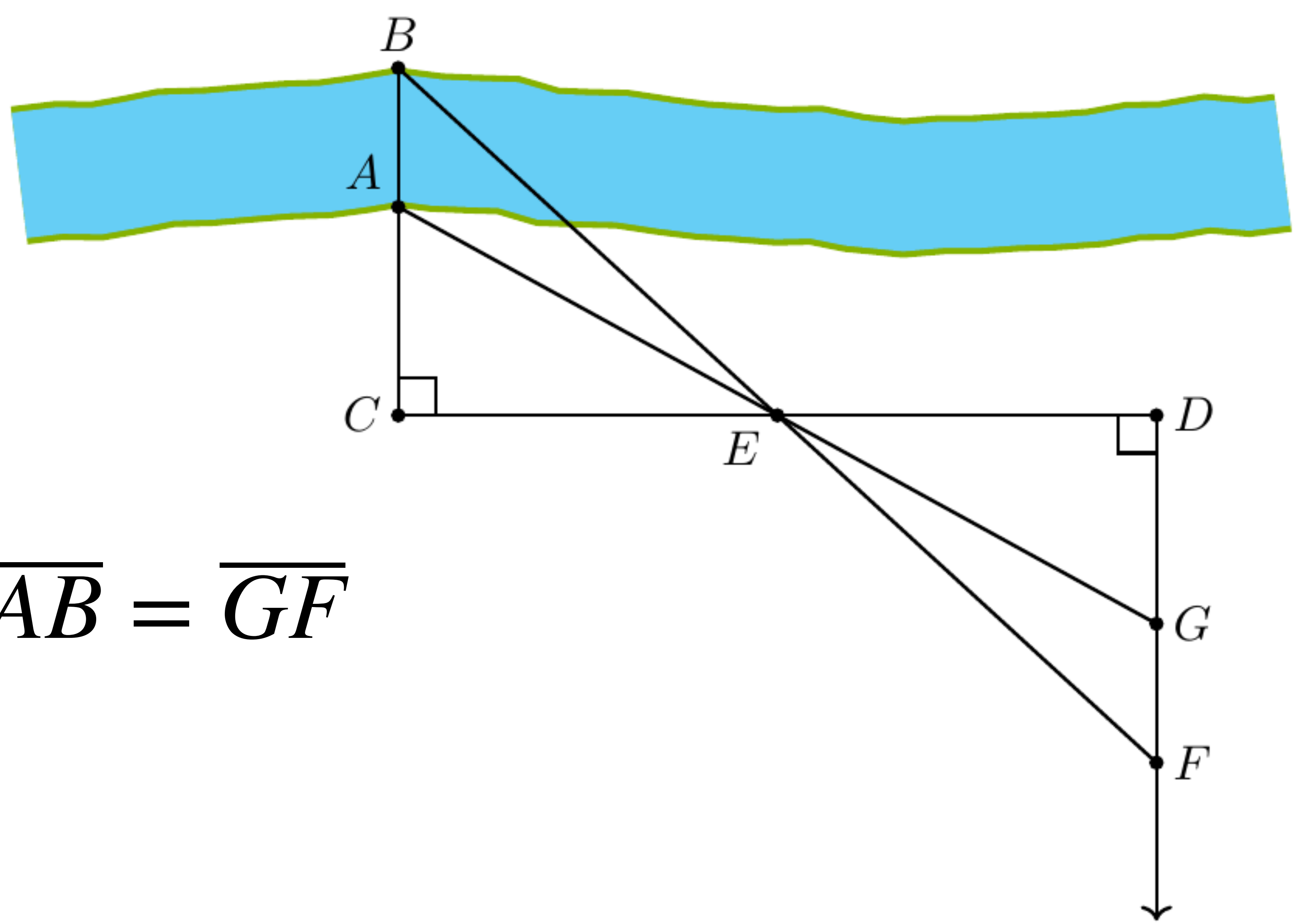

- Now, $\overline{AB} = \overline{GF}$

People's History of Surveying

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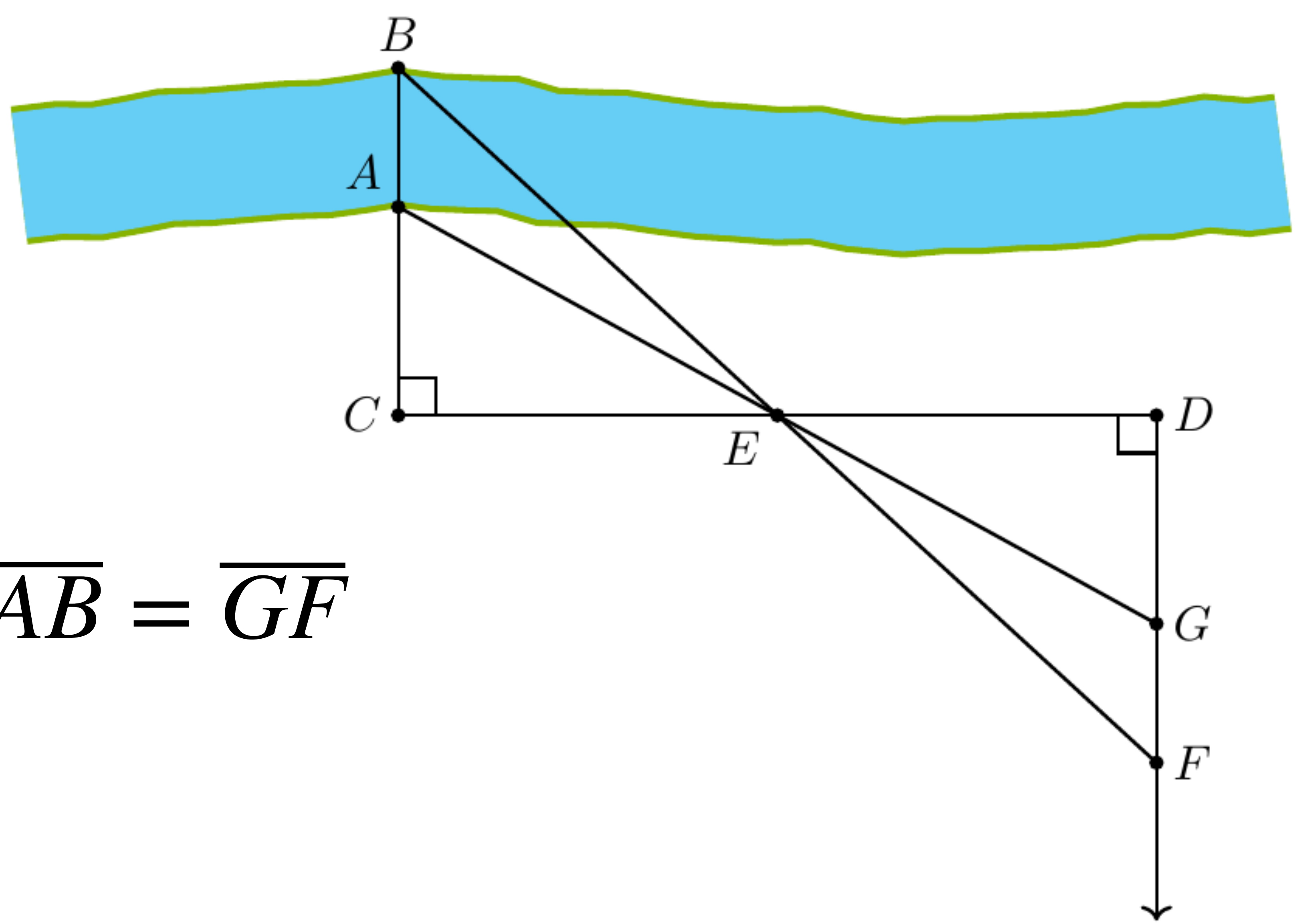
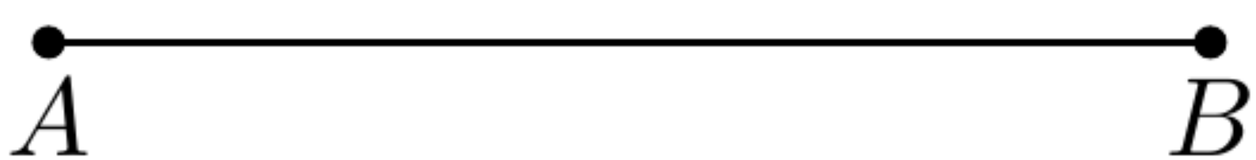


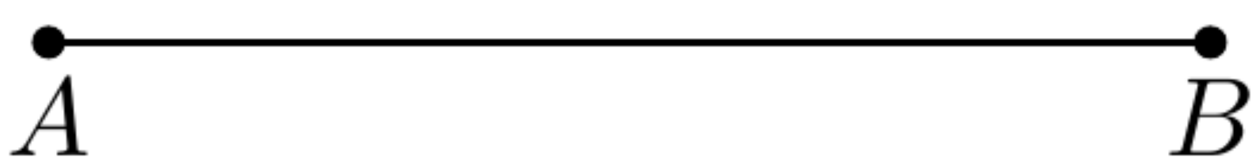
- Now, $\overline{AB} = \overline{GF}$
- Cool!

People's History of Surveying

- A related problem: An object is at an inaccessible point A. You are at B. How do you find this distance?

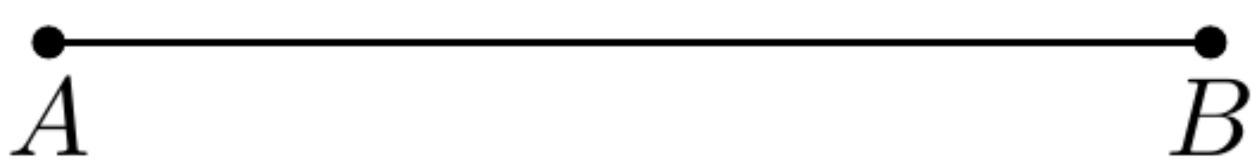


People's History of Surveying



People's History of Surveying

- Extend $\overline{AB}$ to a point C, draw a perpendicular at C to a point D.



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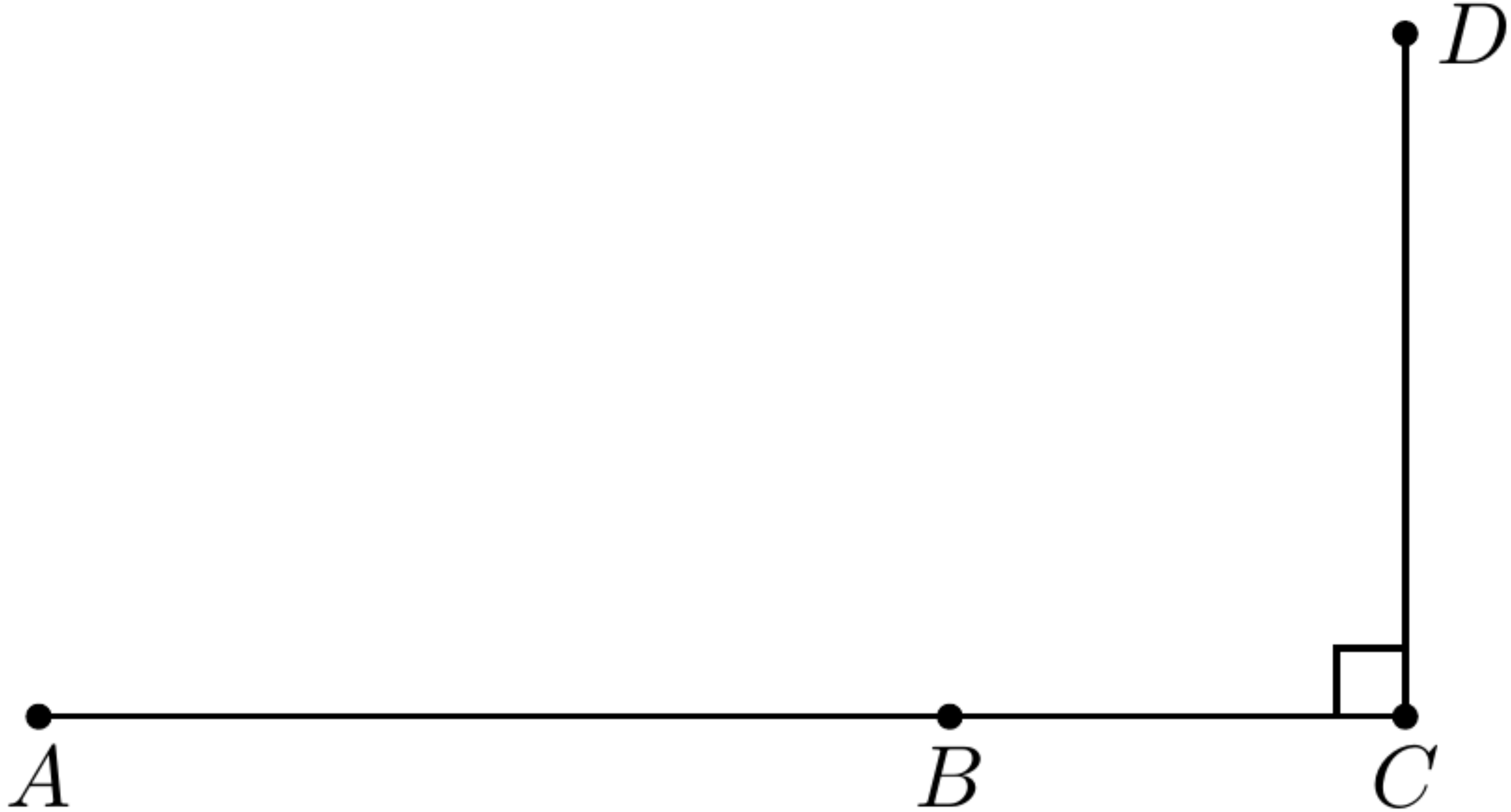

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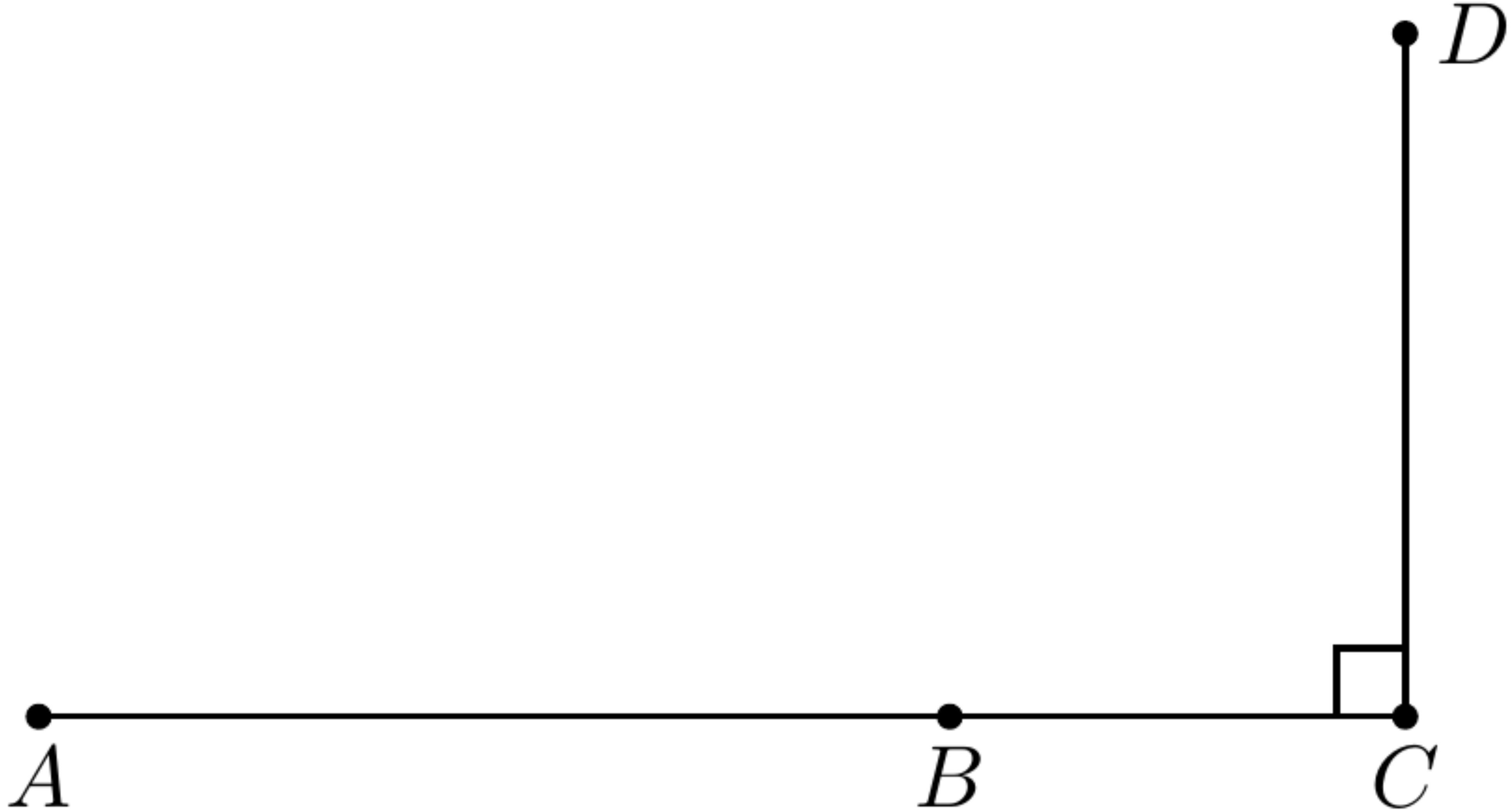
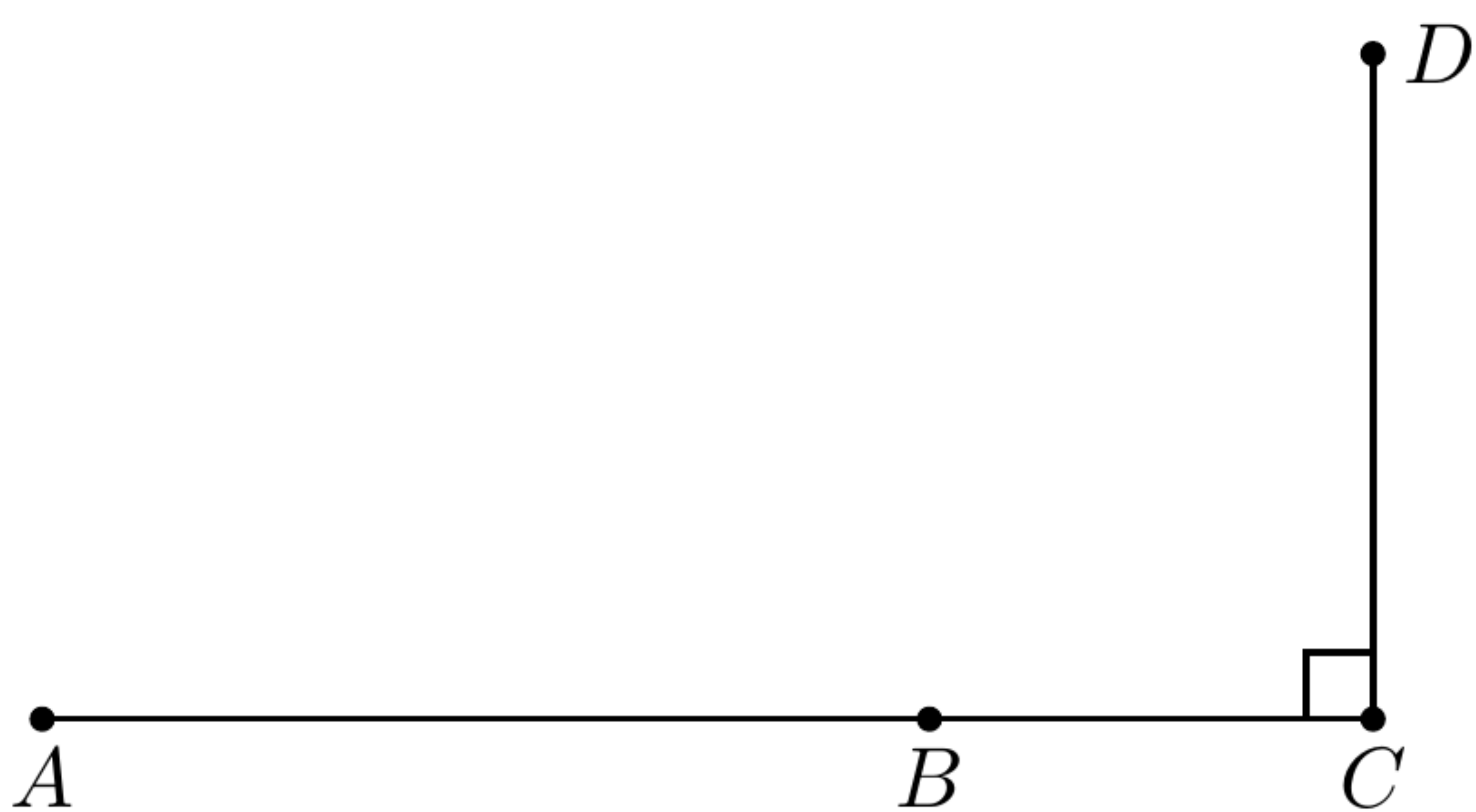


People's History of Surveying

- Walk along the perpendicular to B until you are in line with A and D.



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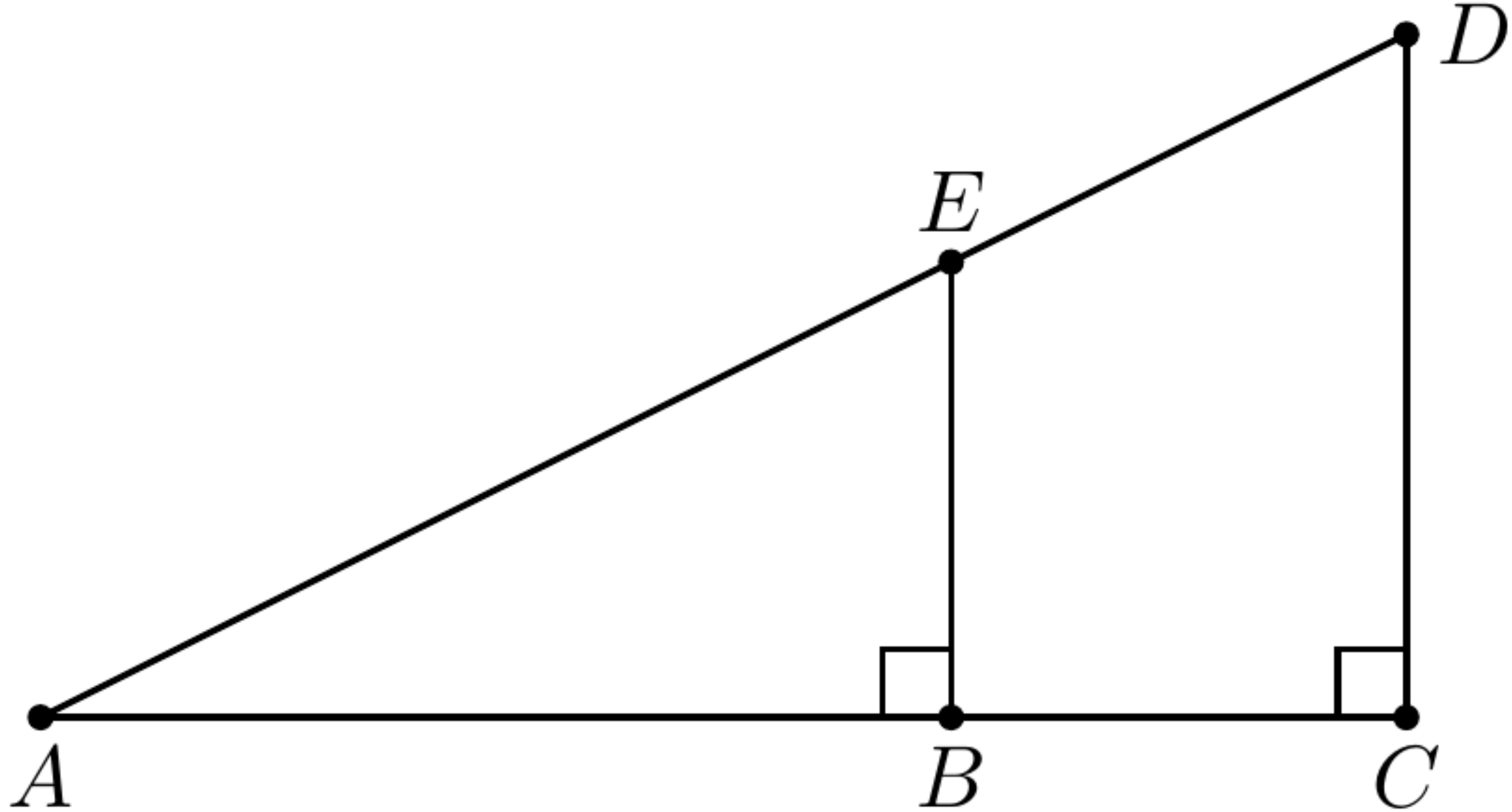
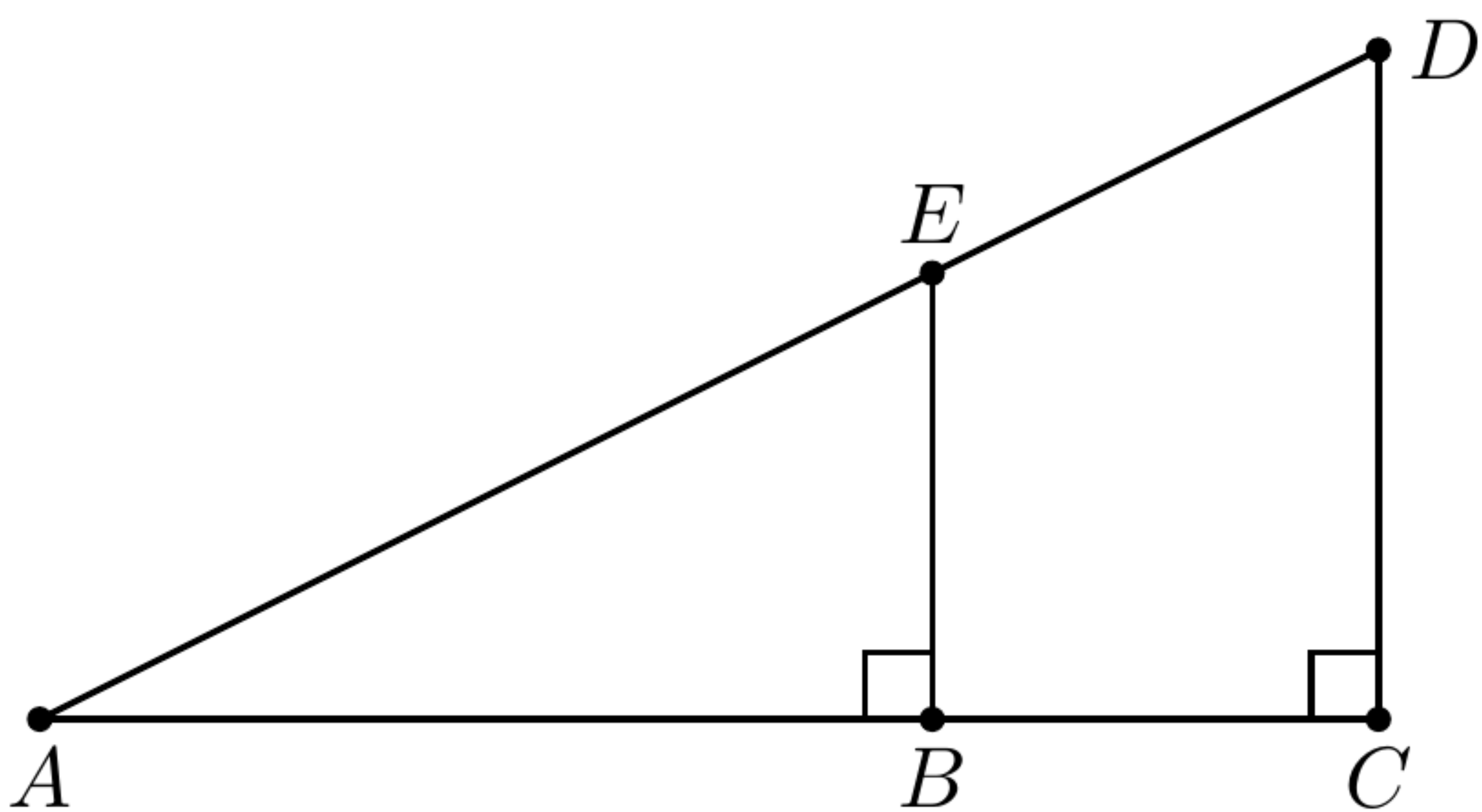
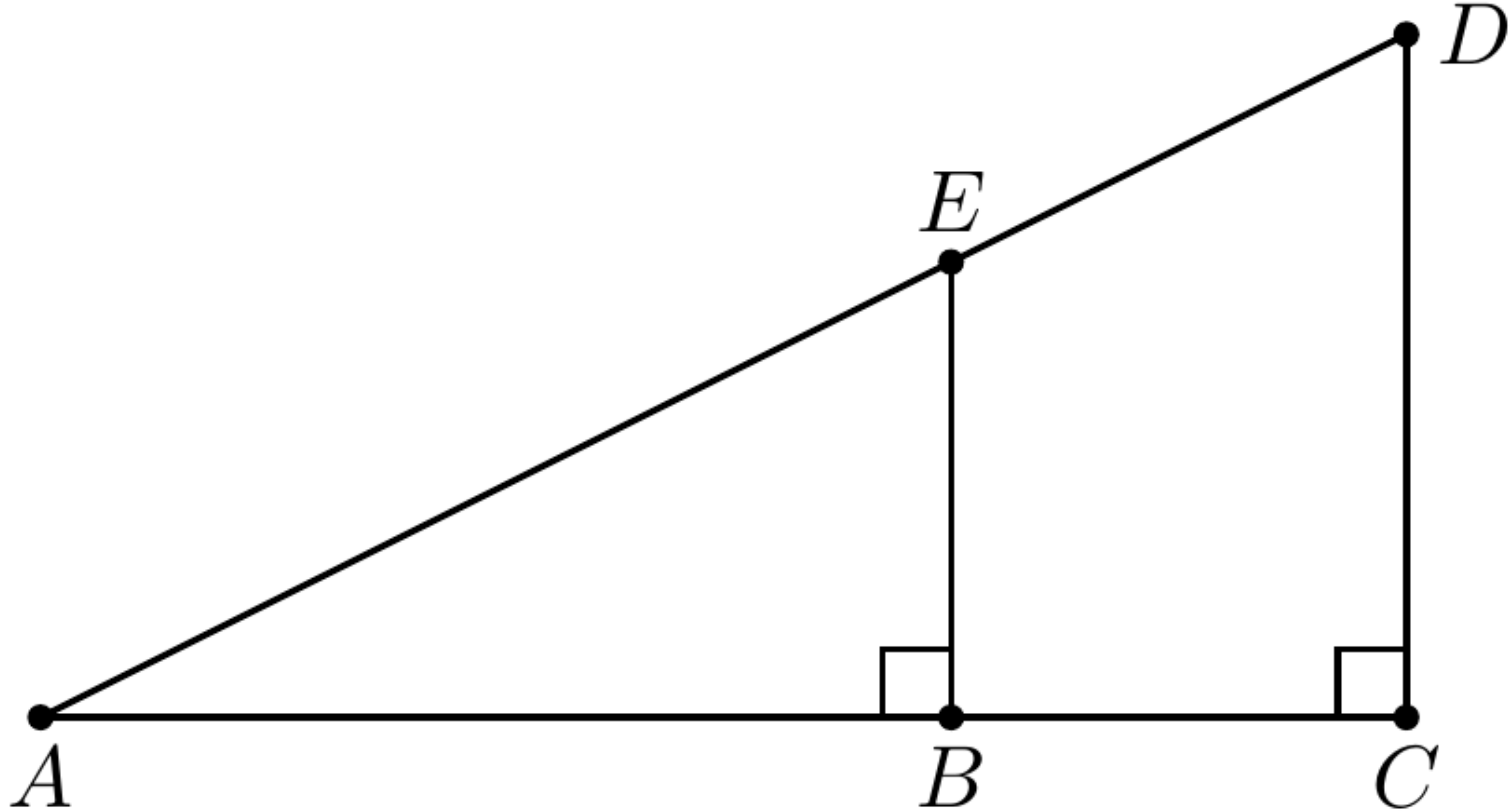


People's History of Surveying



People's History of Surveying

- By similar triangles, $$\frac{\overline{AB}}{\overline{BE}} = \frac{\overline{AC}}{\overline{CD}}$$



People's History of Surveying

- By similar triangles,
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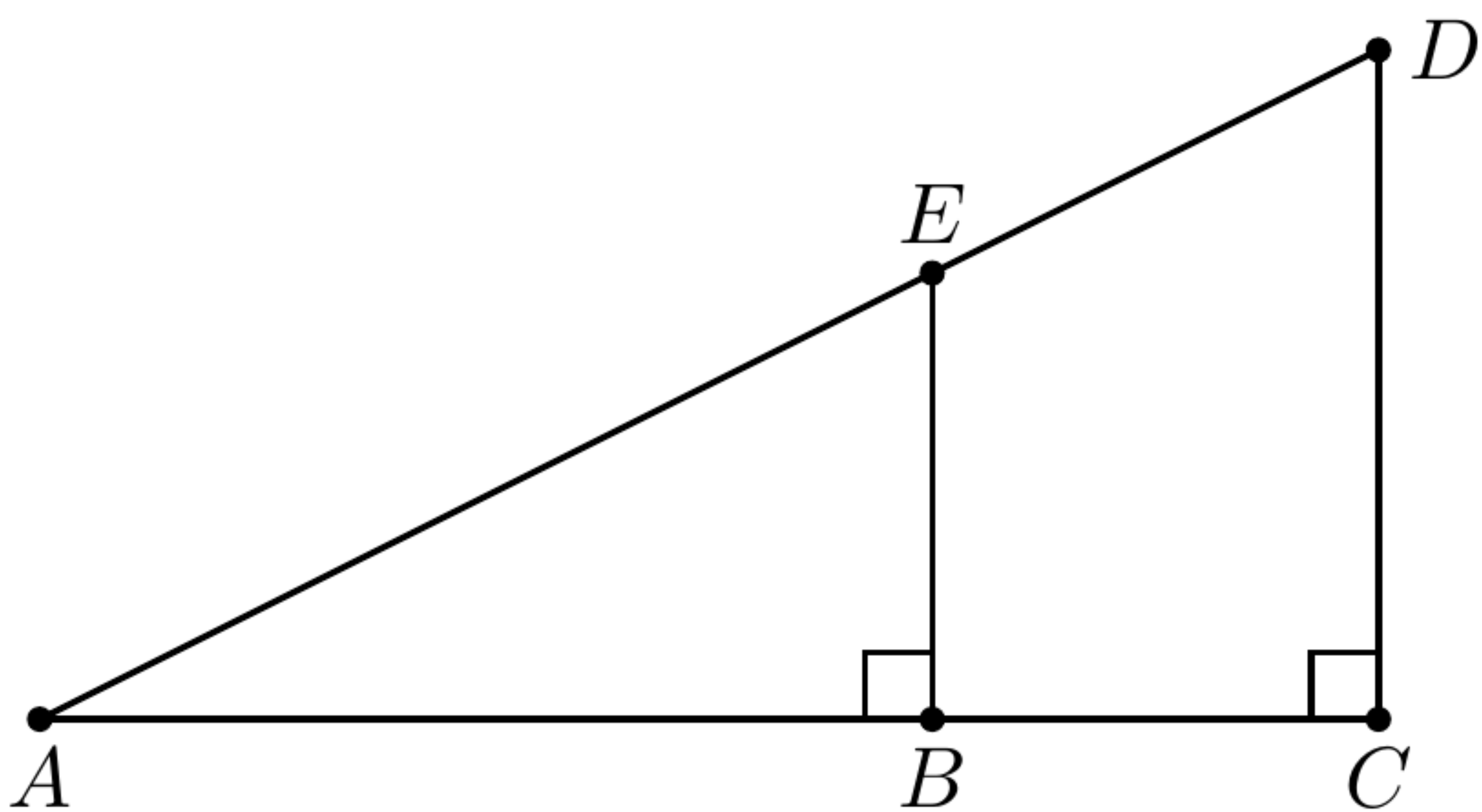
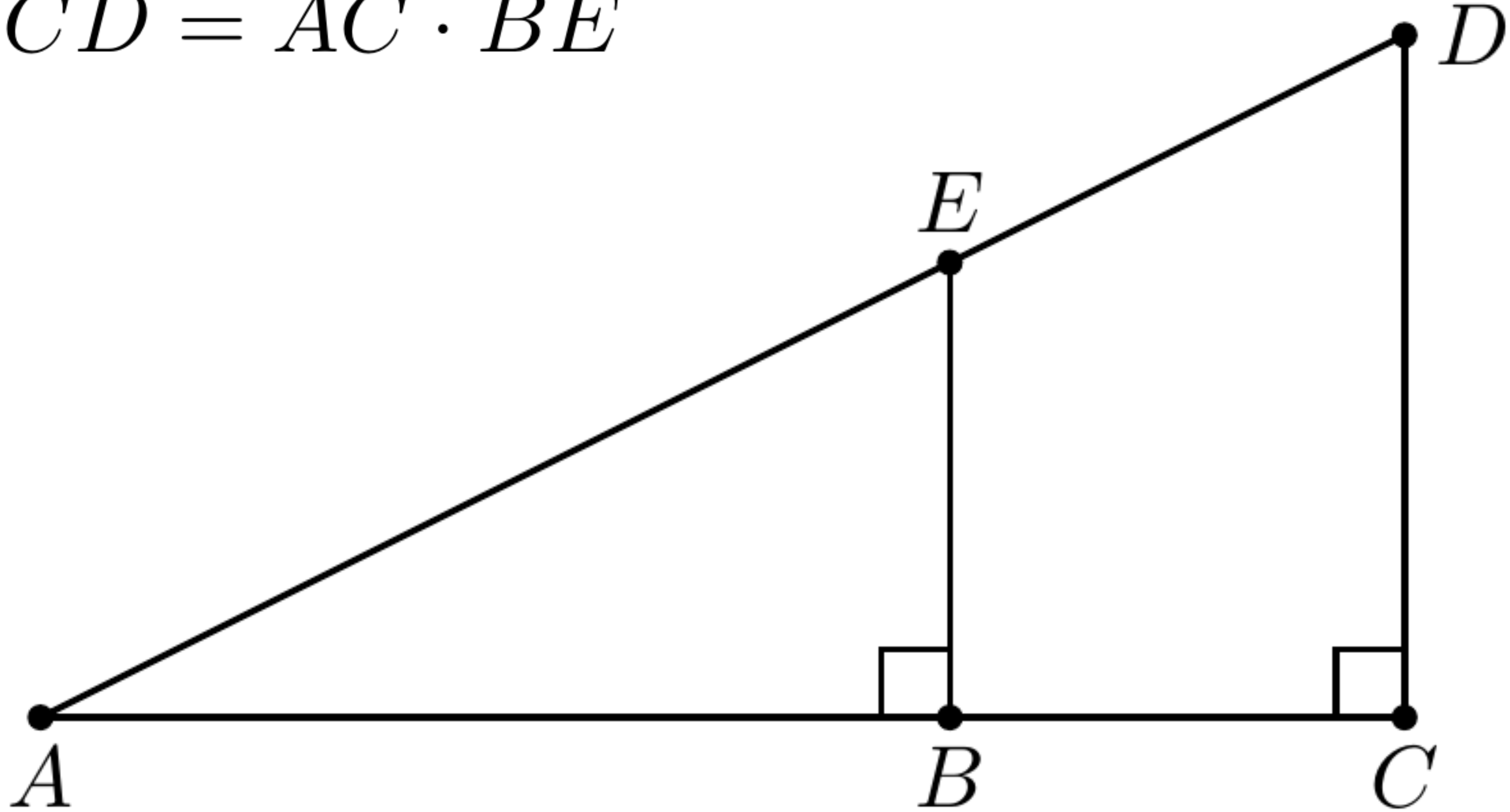


People's History of Surveying

- By similar triangles, $$\frac{\overline{AB}}{\overline{BE}} = \frac{\overline{AC}}{\overline{CD}}$$
- Thus,

$$\overline{AB} \cdot \overline{CD} = \overline{AC} \cdot \overline{BE}$$



People's History of Surveying

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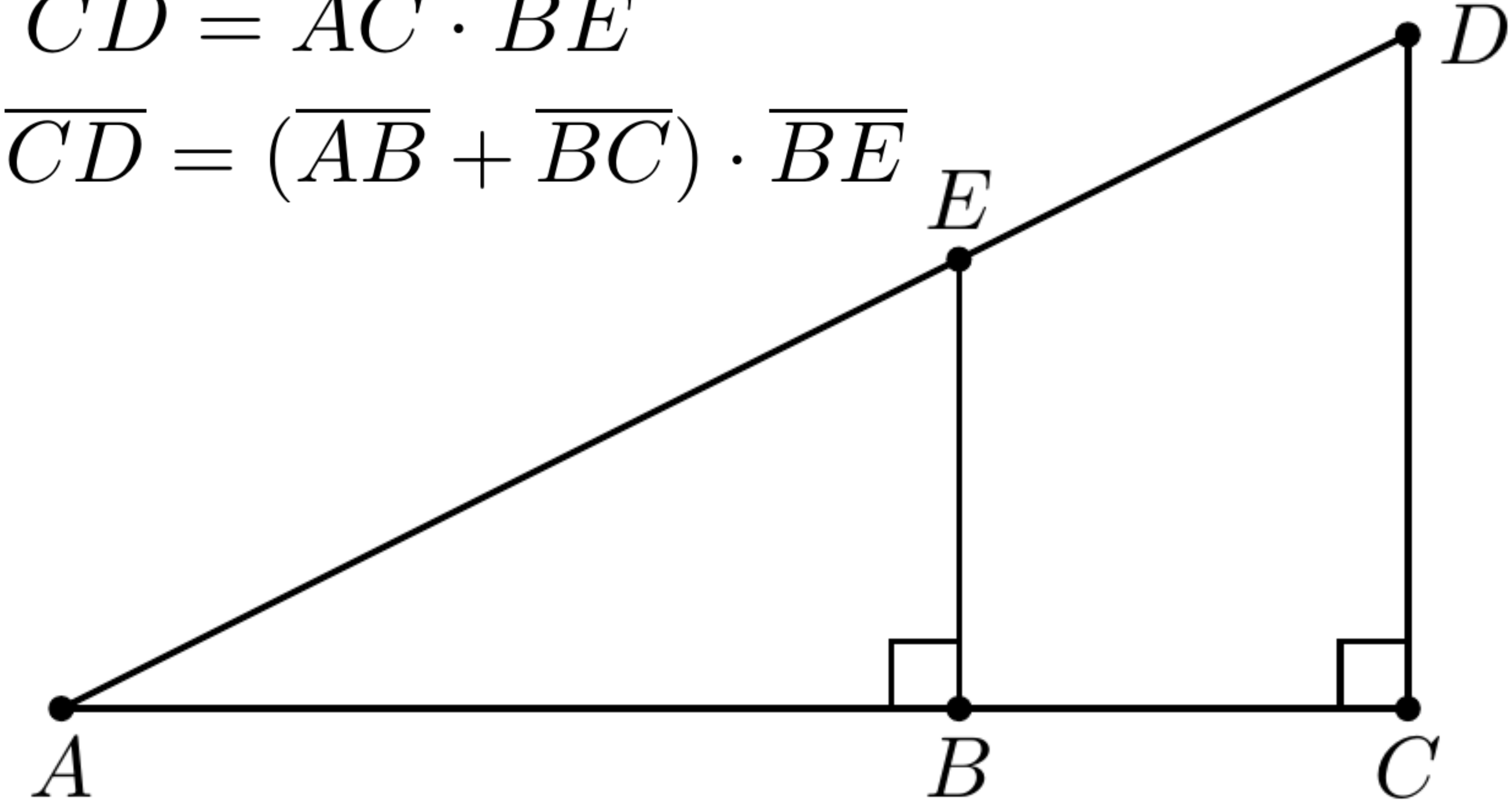
$$\overline{AB} \cdot \overline{CD} = (\overline{AB} + \overline{BC}) \cdot \overline{BE}$$



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- Everything besides $\overline{AB}$ is known. So we can find $\overline{AB}$.