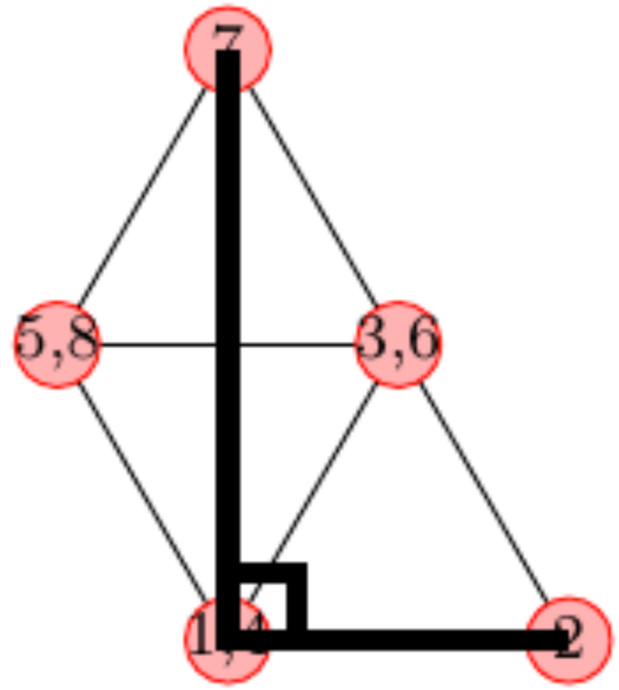
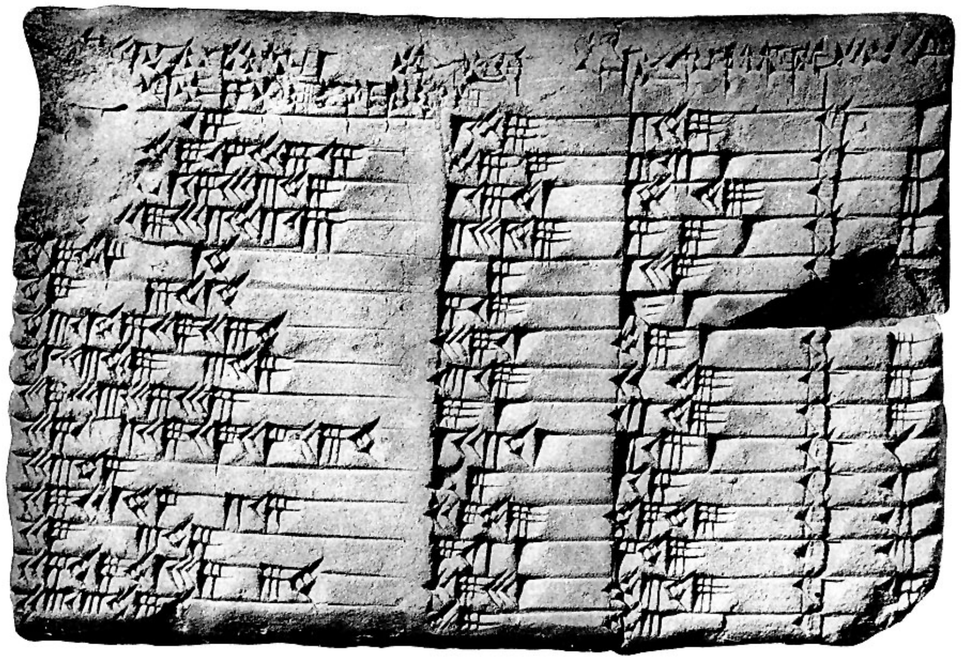
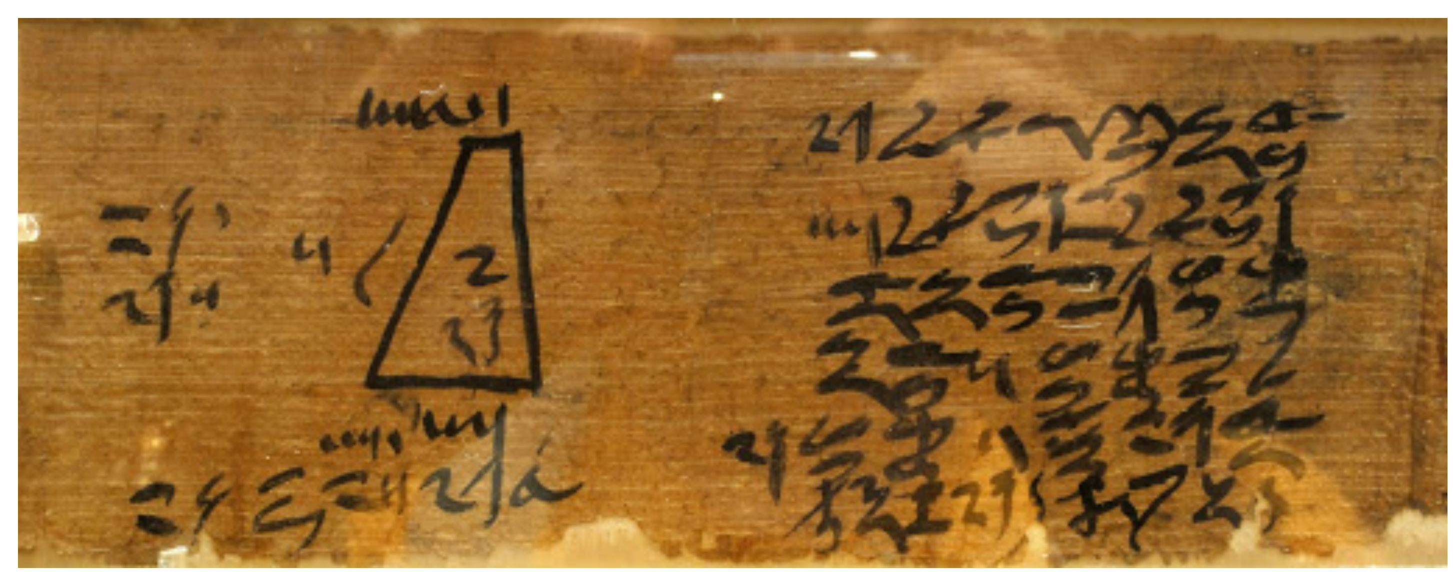


Chapter 2: Ancient Methods



$$\begin{array}{l} 26 \to 31 \\ 13 \to 62 \\ 6 \to 124 \\ 3 \to 248 \\ 1 \to 496 \end{array} \qquad \begin{array}{r} 62 \\ +\ 248 \\ 496 \\ \hline 806 \end{array}$$

Egyptian and Babylonian Mathematics

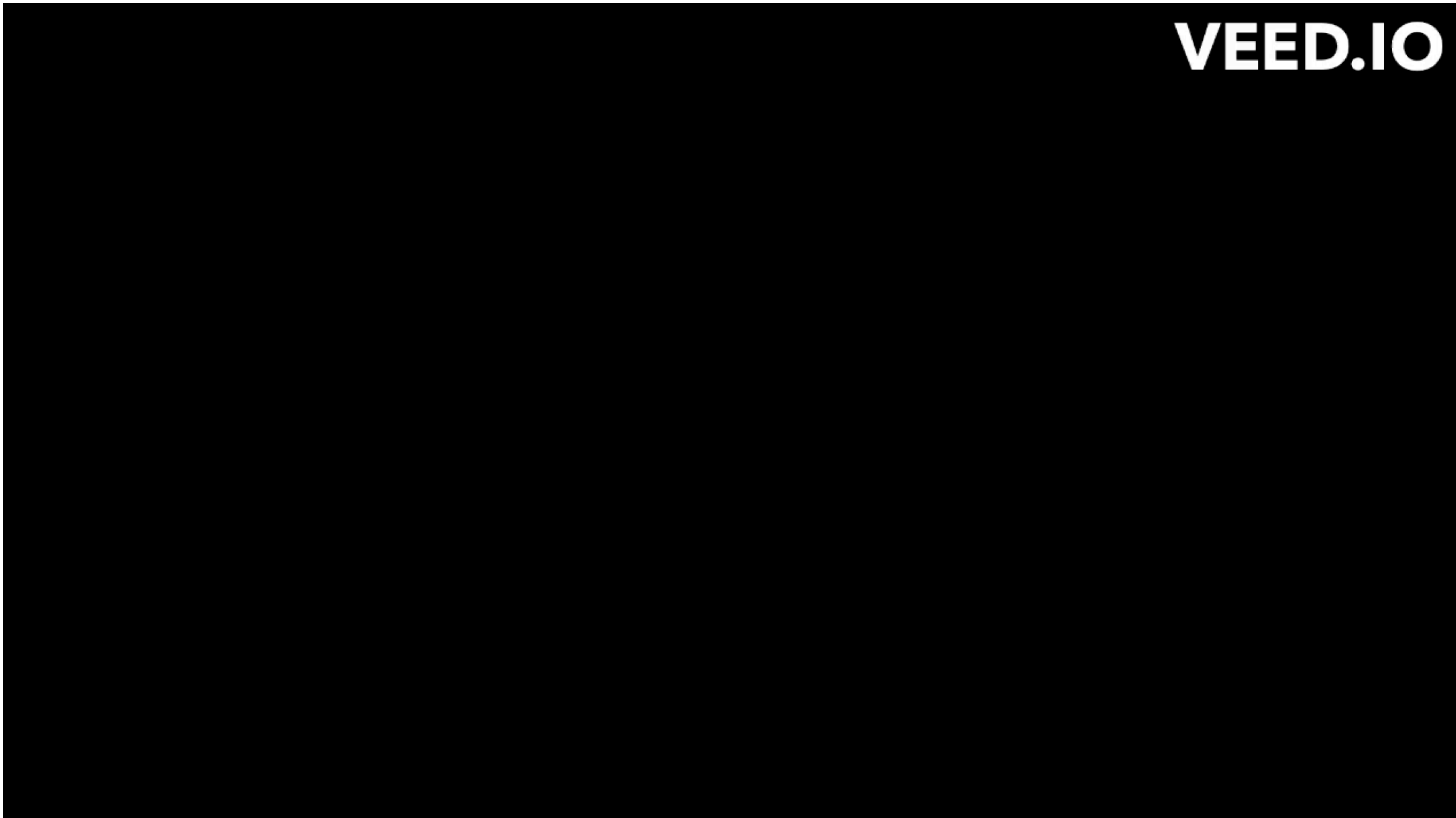


Ancient Egyptian Mathematics

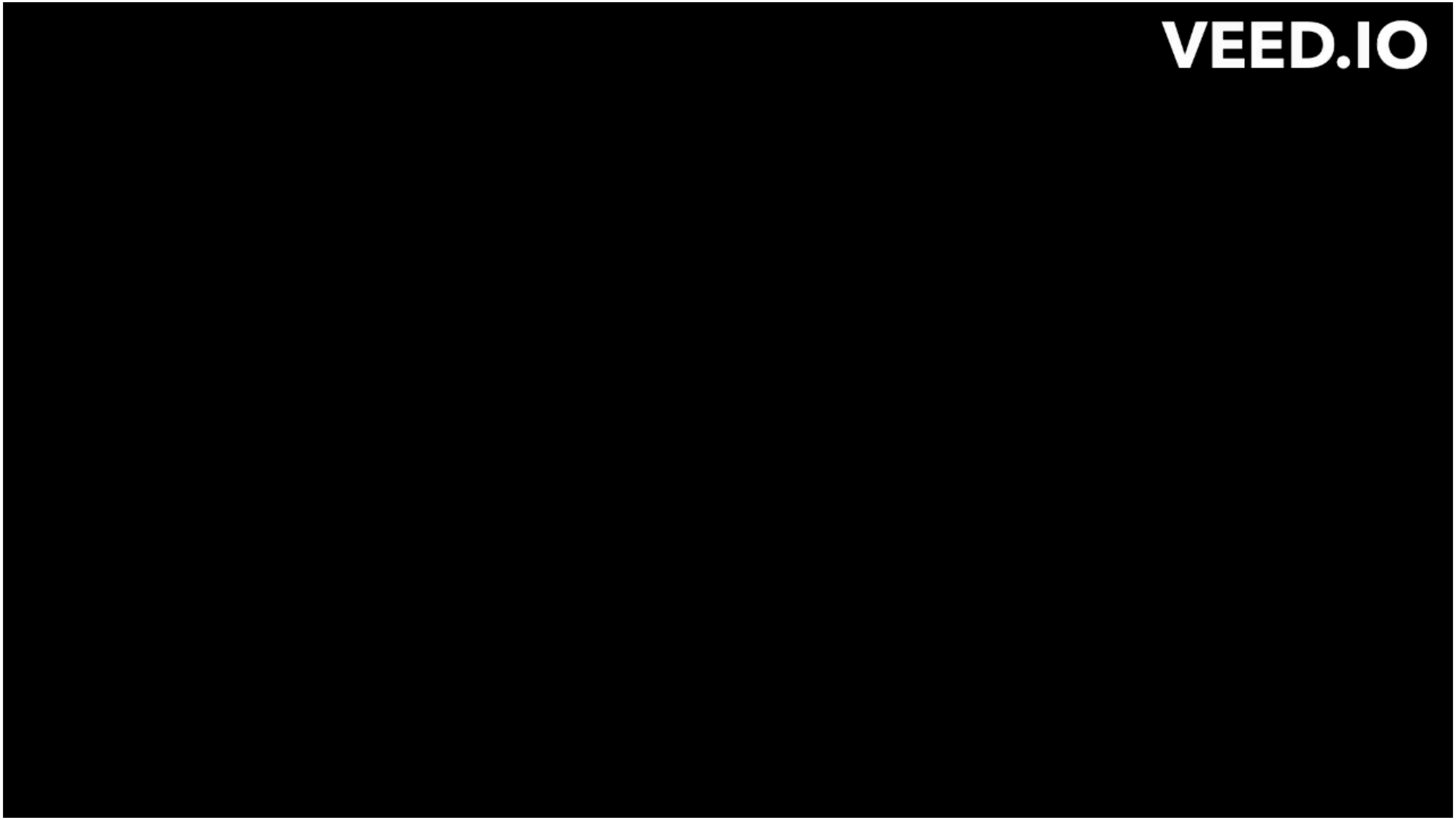
Egypt


Romania
Croatia
Serbia
Italy
Rome
Bulgaria
Black Sea
Georgia
Uzbekistan
drid
pain
Tyrrhenian Sea
Istanbul
Ankara
Azerbaijan
Greece
Turkey
Turkmenistan
Ta
Syria
Tunisia
Mediterranean Sea
Lebanon
Afghanistan
Iraq
Israel
Iran
Cairo
القاهرة
Paki
Algeria
Libya
Egypt
Persian Gulf
Riyadh
الرياض
United Arab
Emirates
Saudi Arabia
Jeddah
جدة
Red Sea
Oman
Mali
Niger
Sudan
Chad
Yemen
Eritrea
Burkina
Faso
Gulf of Aden
Arabian Sea
Djibouti
Benin
Nigeria

Egypt



Egypt



Utilitarian “Theorems”

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- Important example: Constructing right angles.

Think Like A Math Historian

Utilitarian “Theorems”

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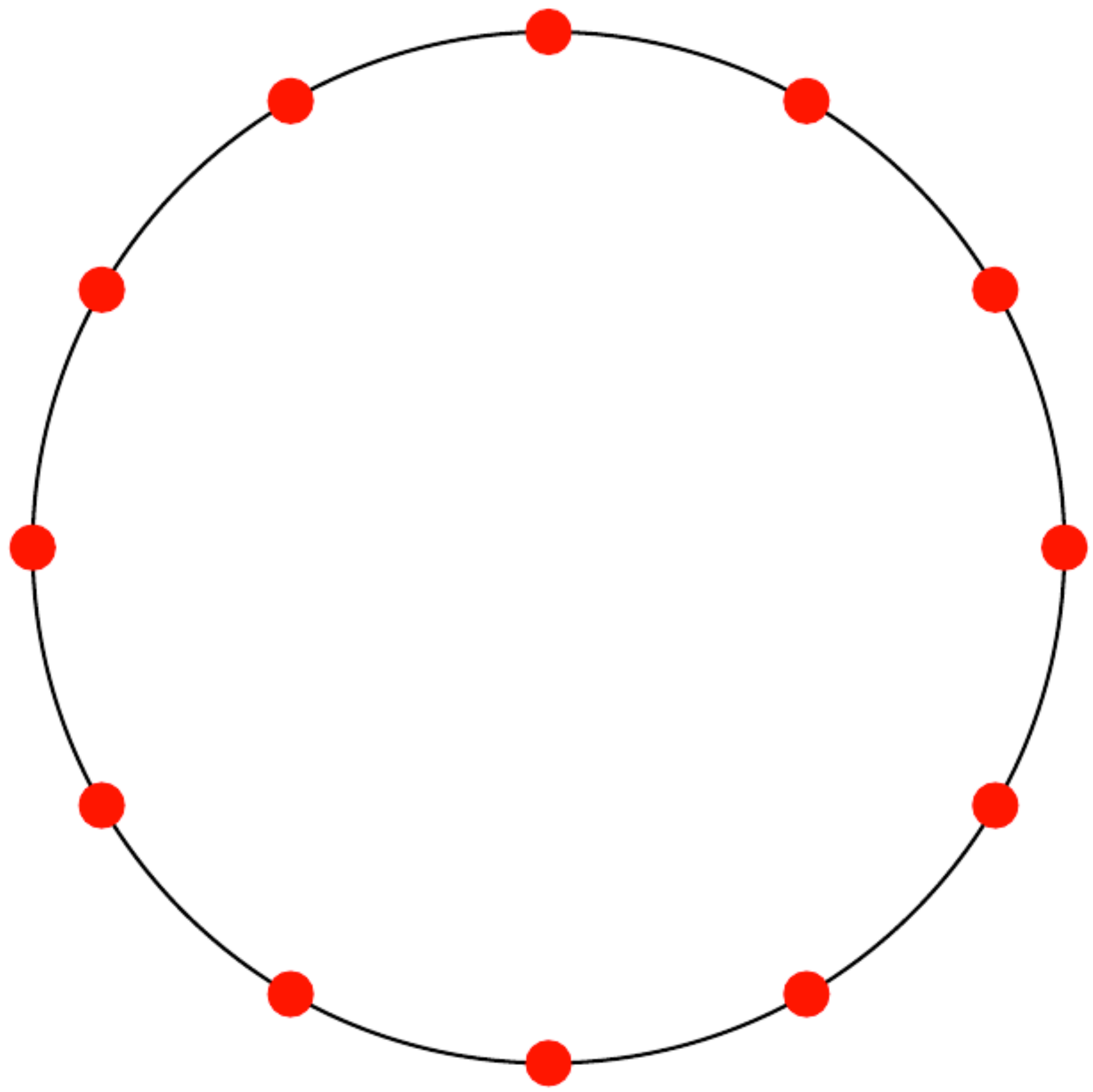

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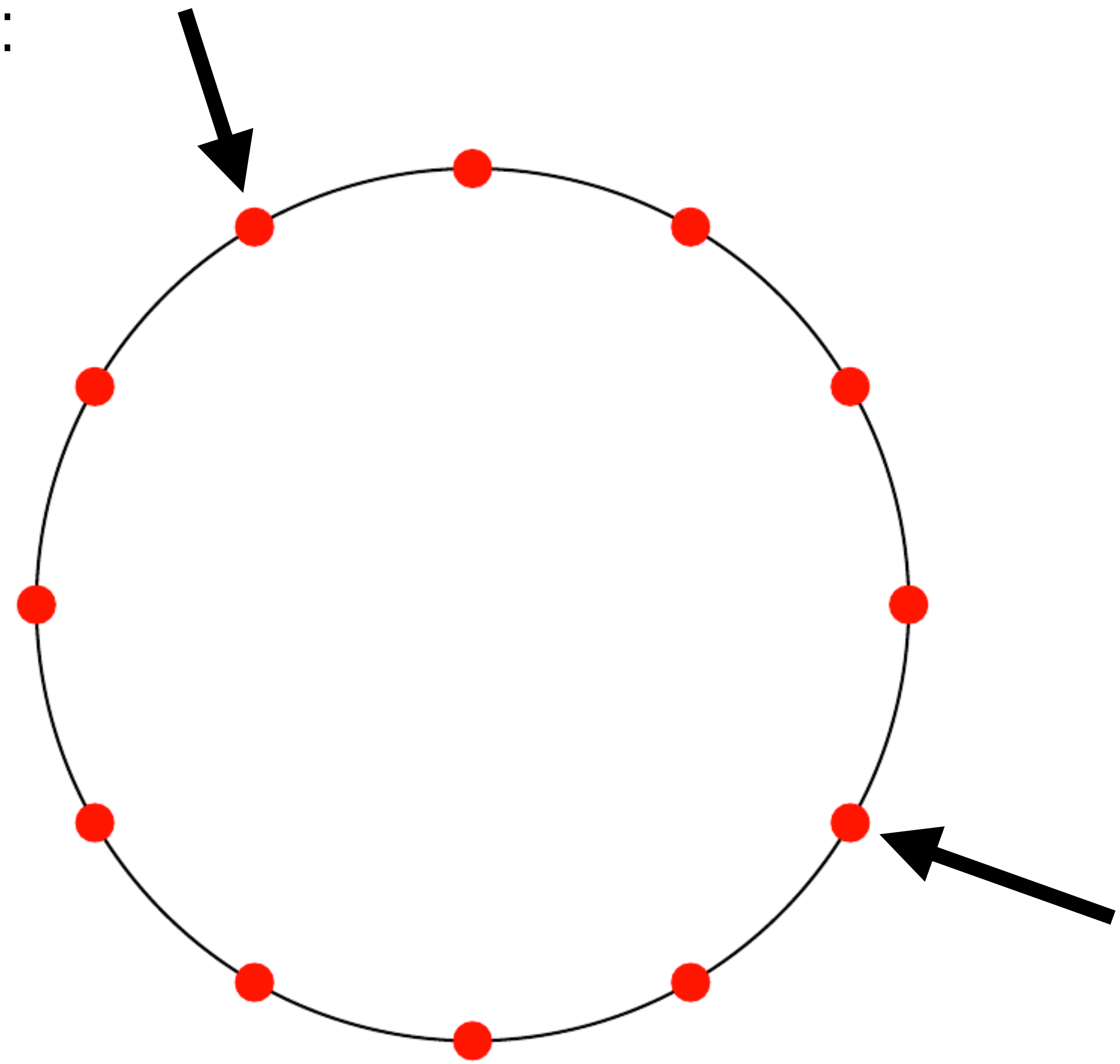

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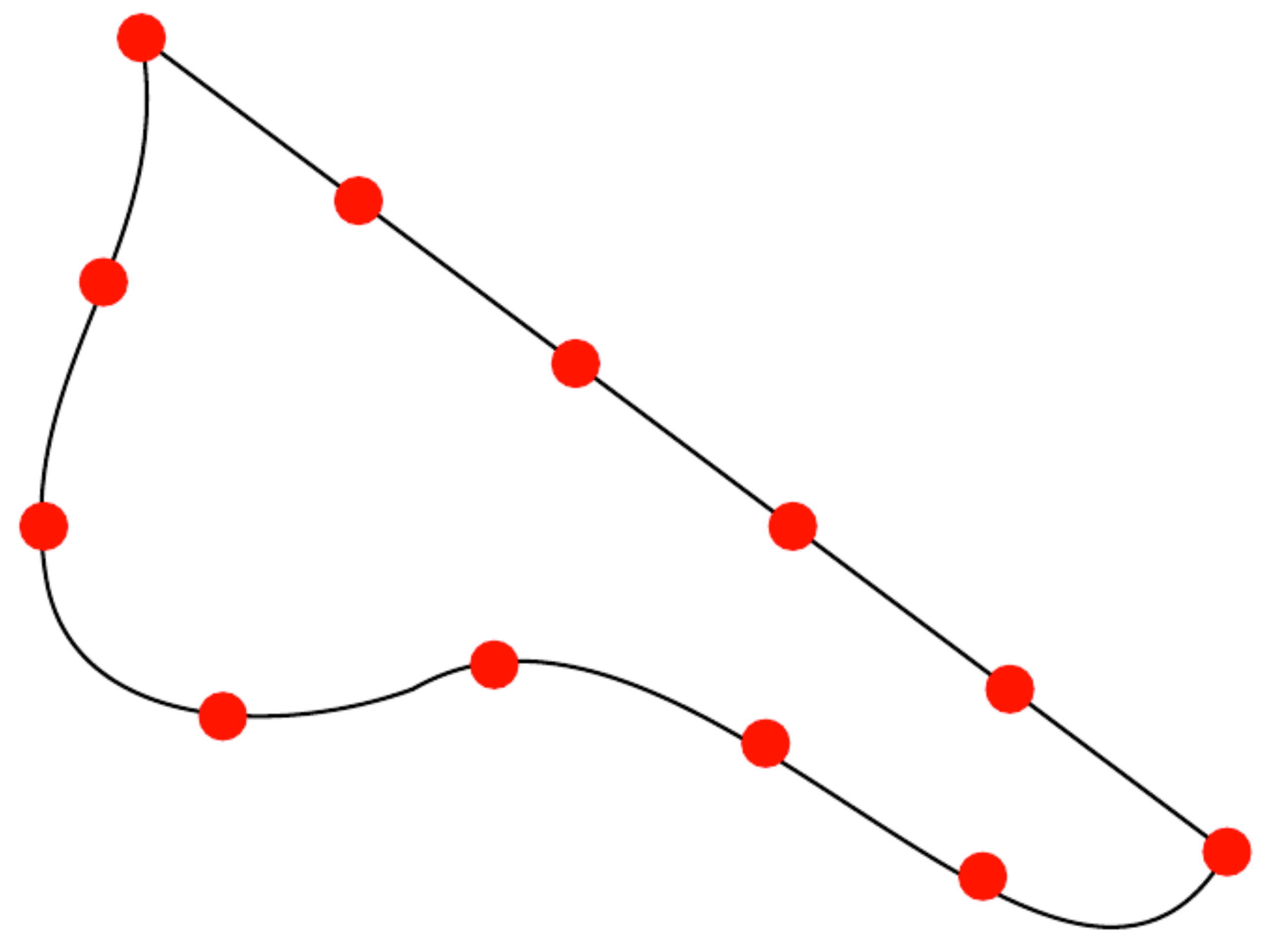

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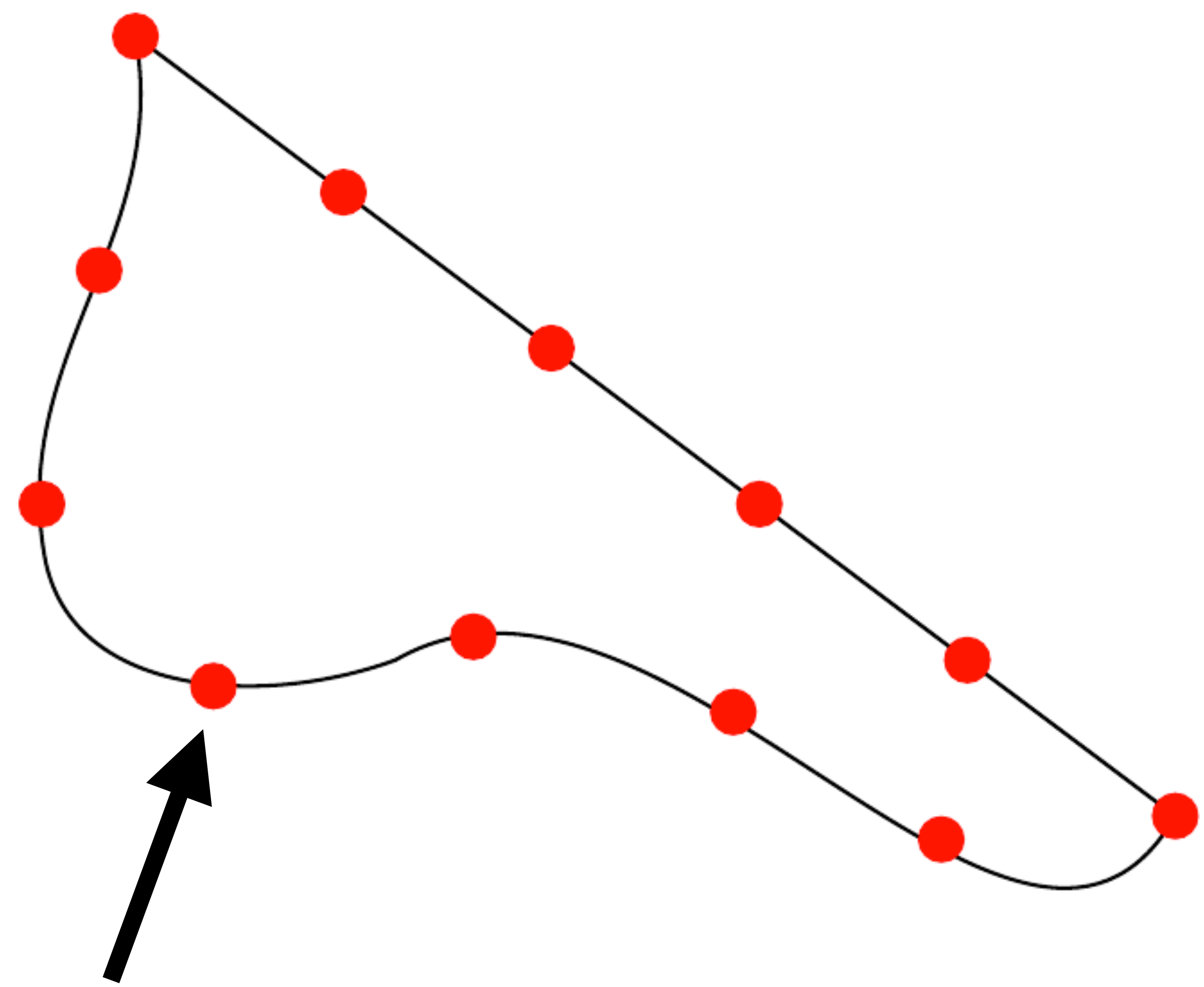

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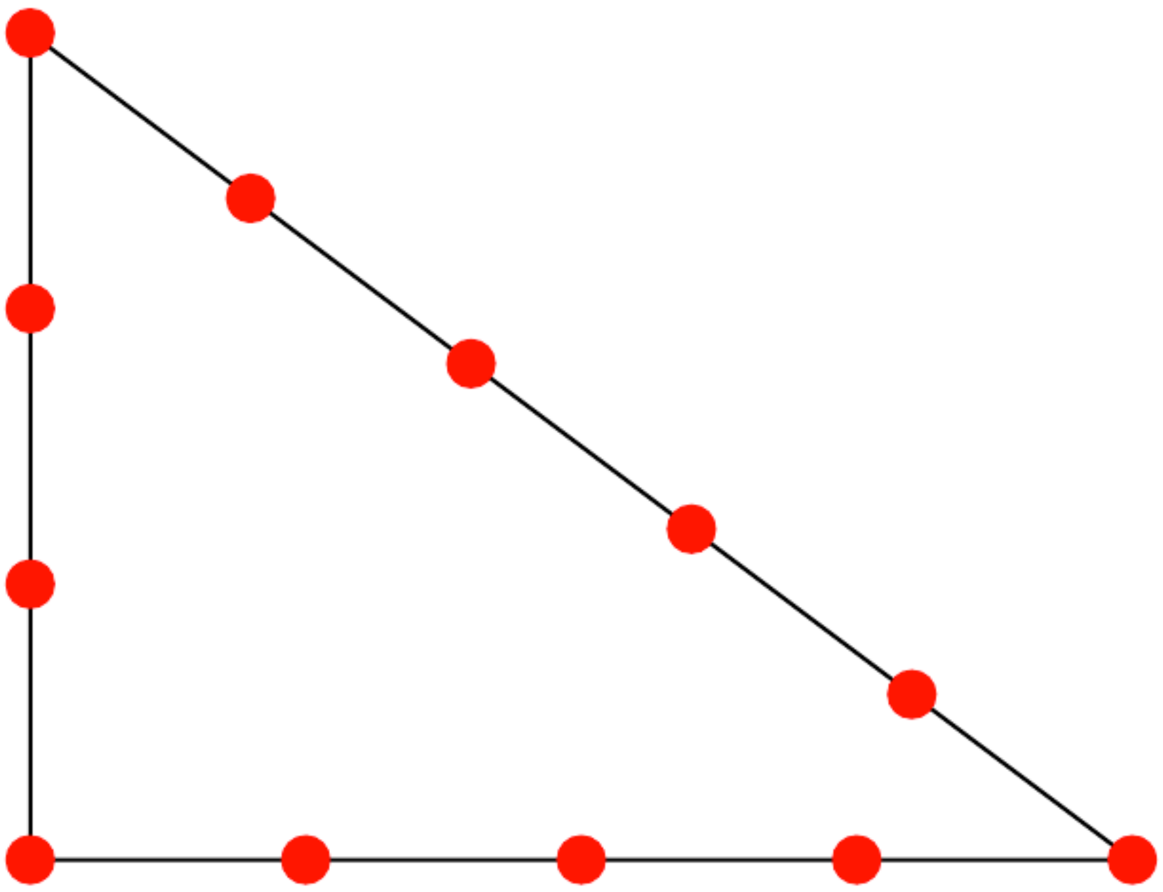

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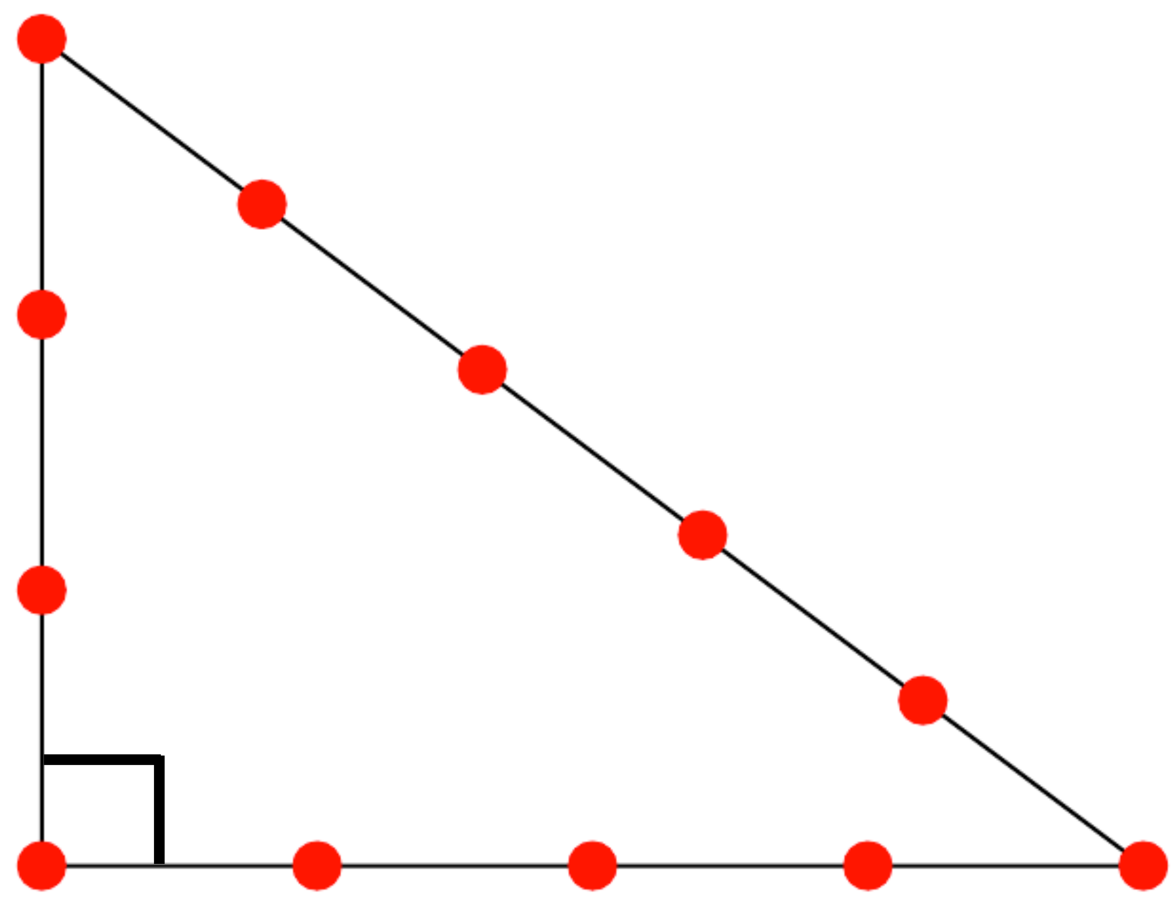

Egyptian Numerical Methods

Egyptian Methods

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- And both are perishable, but the Egyptian climate preserved many of them well.

Egyptian Methods

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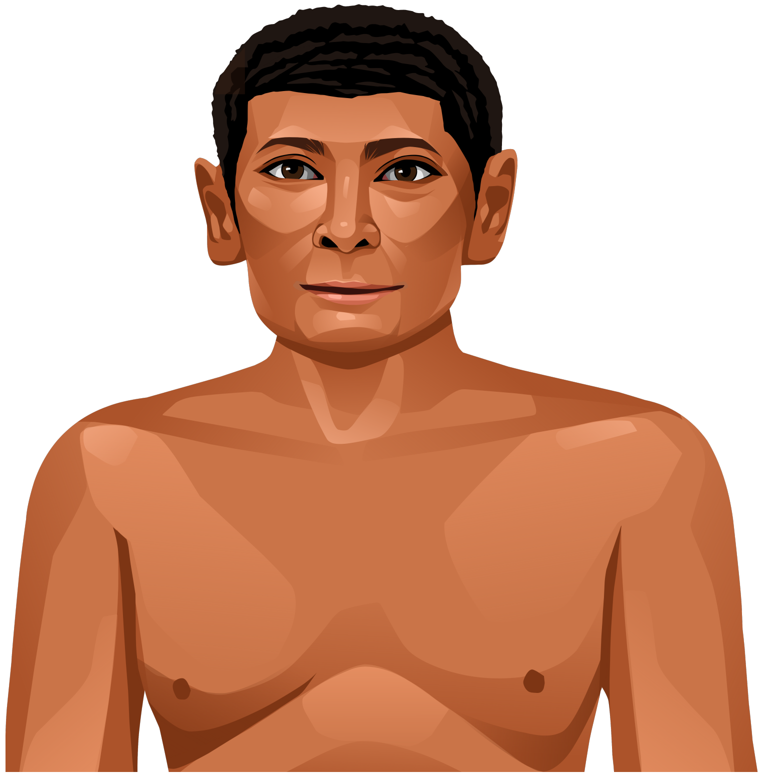
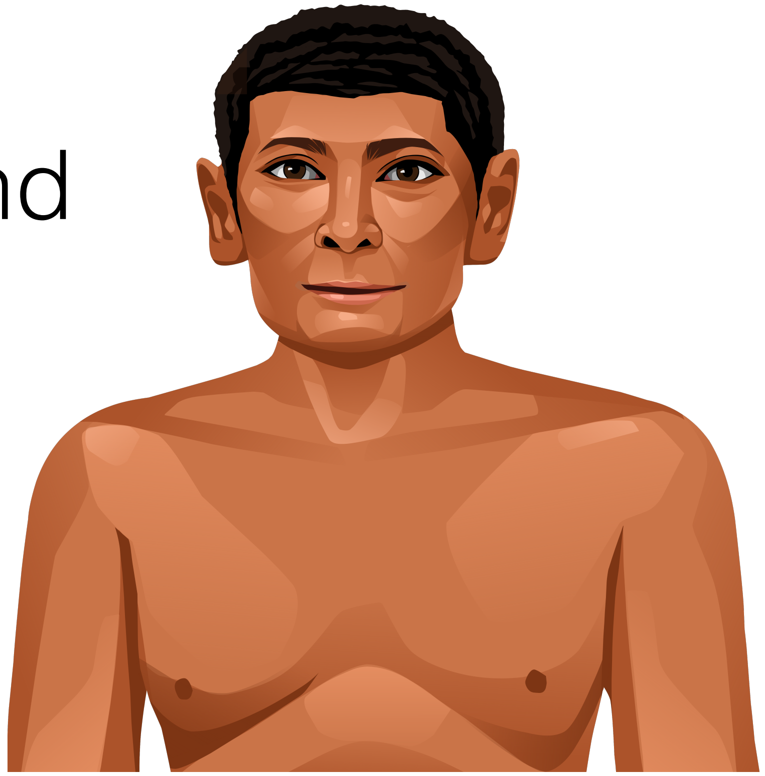


Egyptian Methods



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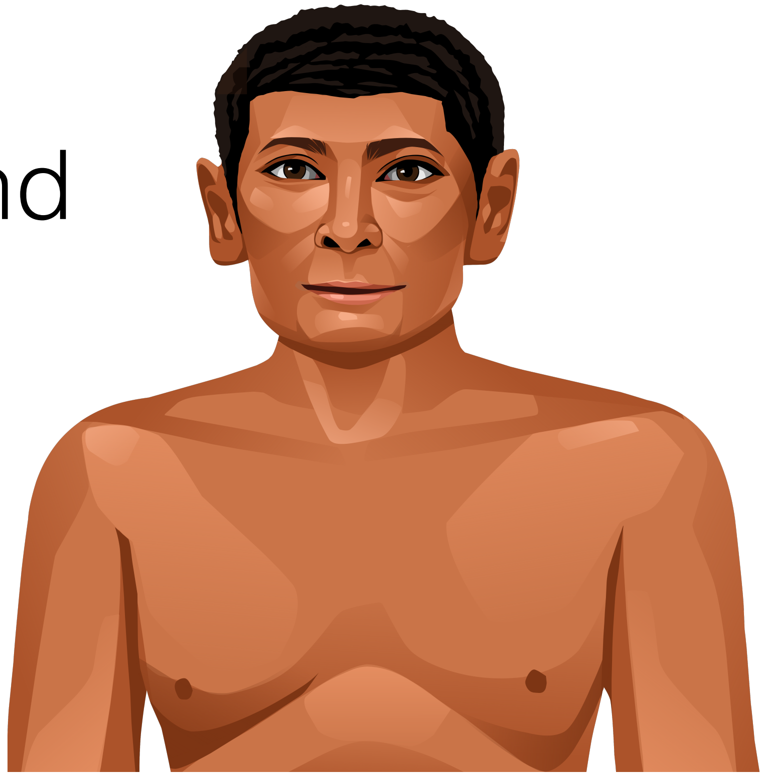
Egyptian Methods



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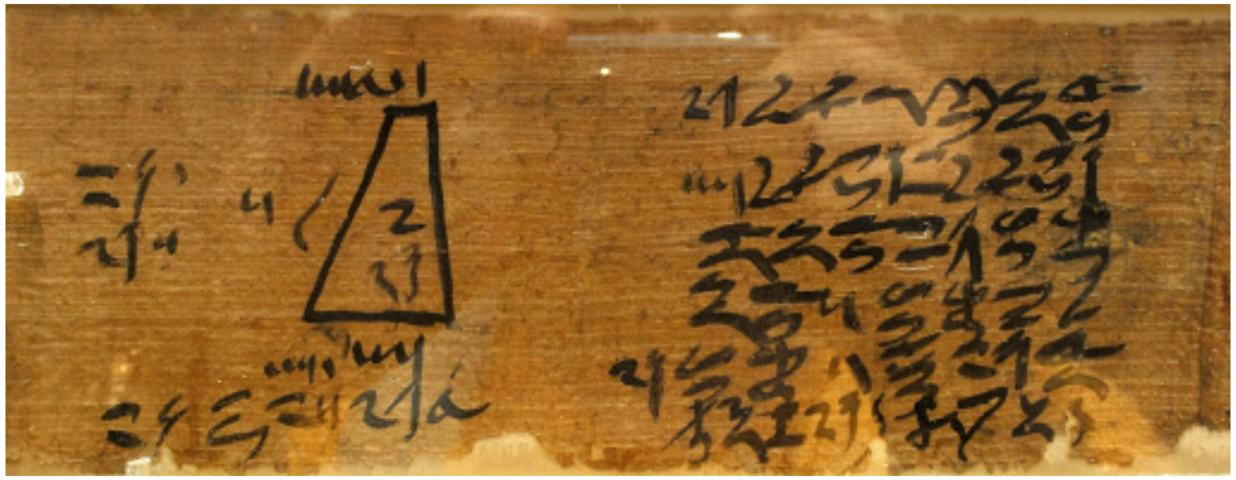
- Combined: 85 problems and solutions (all numerical) on arithmetic, algebra and geometry.



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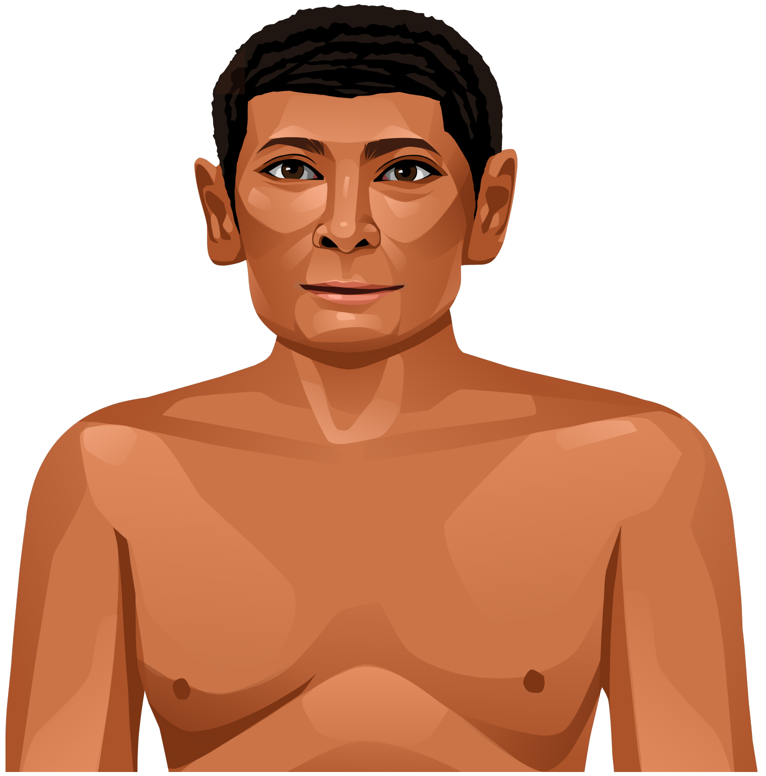

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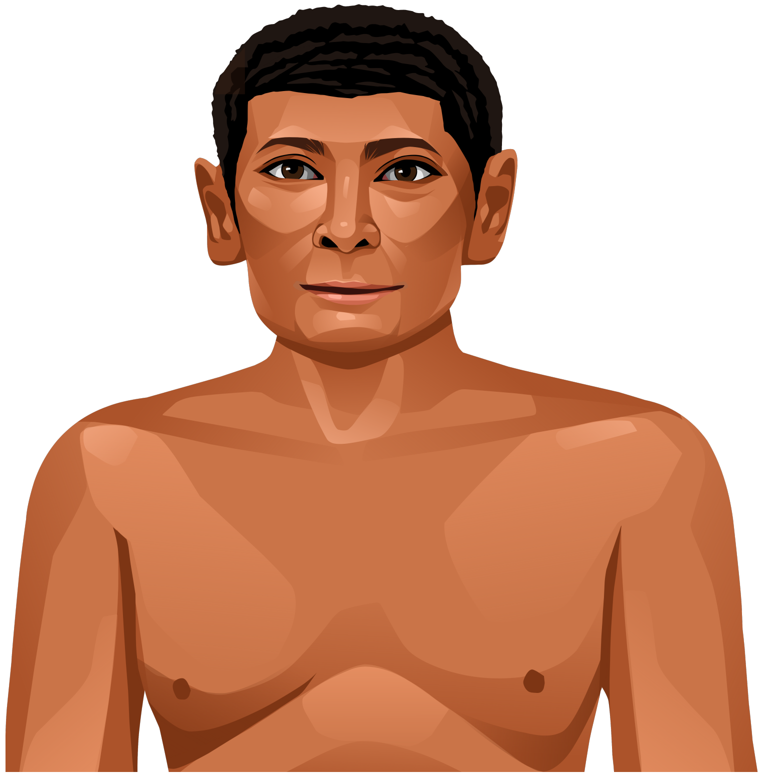

- Rind pictures:

  https://www.britishmuseum.org/collection/object/Y_EA10058

Think Like A
Math Historian

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$$
\begin{array}{l}
1 \to 21 \\
2 \to 42 \\
4 \to 84 \\
8 \to 168 \\
16 \to 336 \\
32 \to 672
\end{array}
\qquad
\begin{array}{r}
21 \\
84 \\
+\ 168 \\
672 \\
\hline
945
\end{array}
$$

Think Like A Math Historian

Why does it work?

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$$\begin{aligned} 45 \bullet 21 &= (32 + 8 + 4 + 1) \cdot 21 \\ &= 32 \cdot 21 + 8 \cdot 21 + 4 \cdot 21 + 1 \cdot 21 \\ &= 672 + 168 + 84 + 21 \\ &= 945. \end{aligned}$$

Think Like A Math Historian

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$$6 \to 124$$

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$$1 \to 496$$

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Why does it work? You'll tell me on your homework!

Egyptian Fractions
(On Board)

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$$\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}.$$

- Second, arithmetic is tough. Even simply multiplying by 2 can make a fraction unrecognizable.

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1. Find the largest unit fraction less than $\frac{m_1}{n_1}$; suppose it is $\frac{1}{k_1}$.
2. Subtract: $\frac{m_1}{n_1} - \frac{1}{k_1}$, and simplify this to a single new fraction, $\frac{m_2}{n_2}$.
3. Now find the largest unit fraction less than $\frac{m_2}{n_2}$; suppose it is $\frac{1}{k_2}$.
4. Subtract: $\frac{m_2}{n_2} - \frac{1}{k_2}$, and simplify this to a single new fraction, $\frac{m_3}{n_3}$.
5. Continue in this way.

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One can prove that this algorithm at some point terminates, producing a *finite* list sum of unit fractions equaling $\frac{m_1}{n_1}$:

$$\frac{m_1}{n_1} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_t}.$$

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- Suppose you have, say, 5 cookies and 6 people. How do you cut them to give everyone the same amount?
- Could try to cut a sliver off each cookie and give those slivers to one person, but that wouldn't go well.
- Writing it with Egyptian fractions, $\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$. So one way is to give everyone a half and a third!

Geometry

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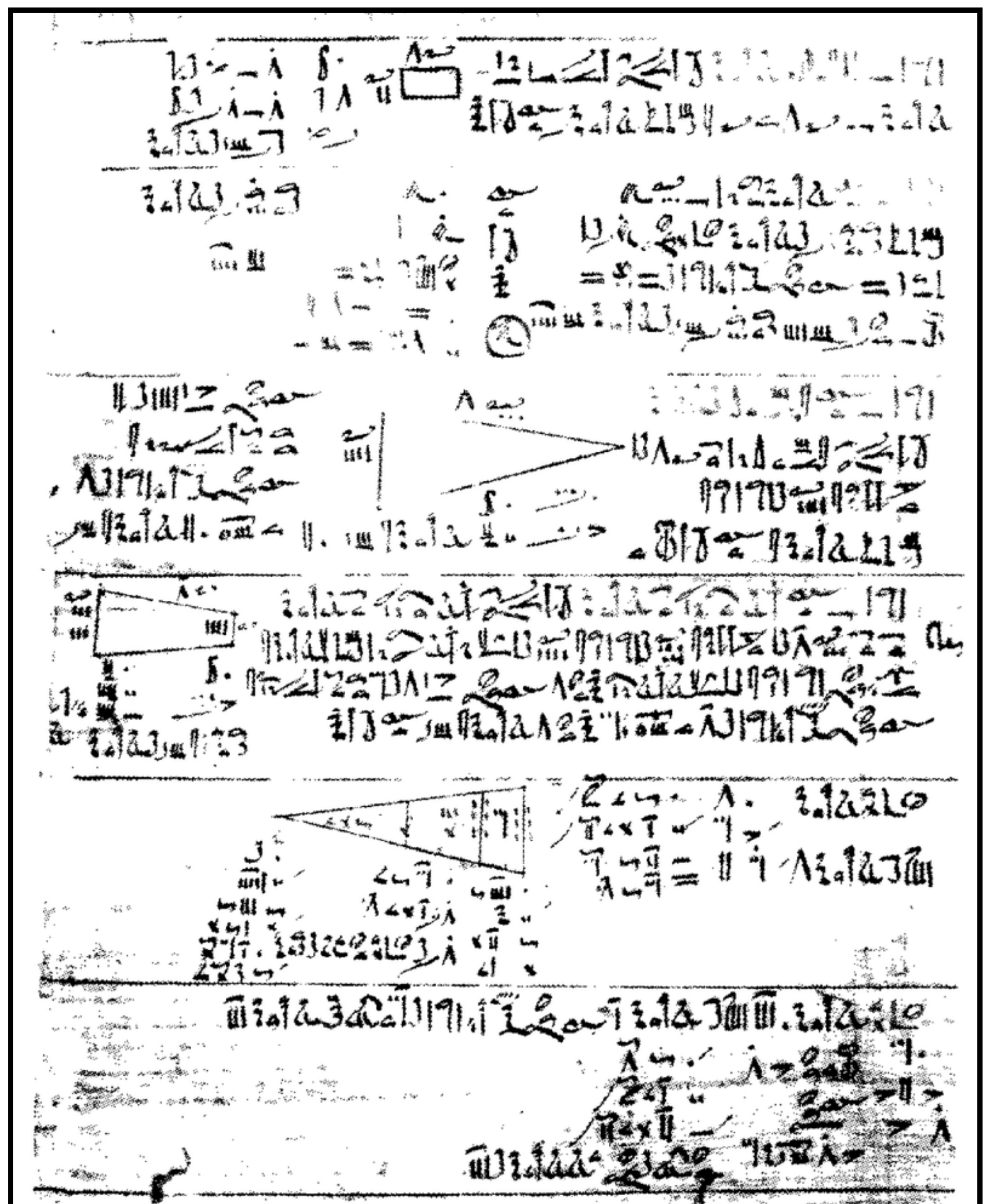

Geometry

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- This essentially means they were using $\pi = \dfrac{256}{81} = 3.1604\ldots$ Which isn't too bad.

Babylonian Mathematics

Mesopotamia

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Babylonia

Romania
Croatia
Serbia
Italy
Rome
Bulgaria
Black Sea
Georgia
Uzbekistan
drid
pain
Tyrrhenian Sea
Istanbul
Ankara
Azerbaijan
Greece
Turkey
Turkmenistan
Syria
Tunisia
Mediterranean Sea
Lebanon
Afghanistan
Iraq
Israel
Iran
Paki
Cairo
القاهرة
Algeria
Libya
Egypt
Persian Gulf
Riyadh
الرياض
United Arab Emirates
Saudi Arabia
Jeddah
جدة
Red Sea
Oman
Mali
Niger
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- Babylonian mathematics was generally more advanced than Ancient Egyptian mathematics.

Think Like A
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Babylonian Methods

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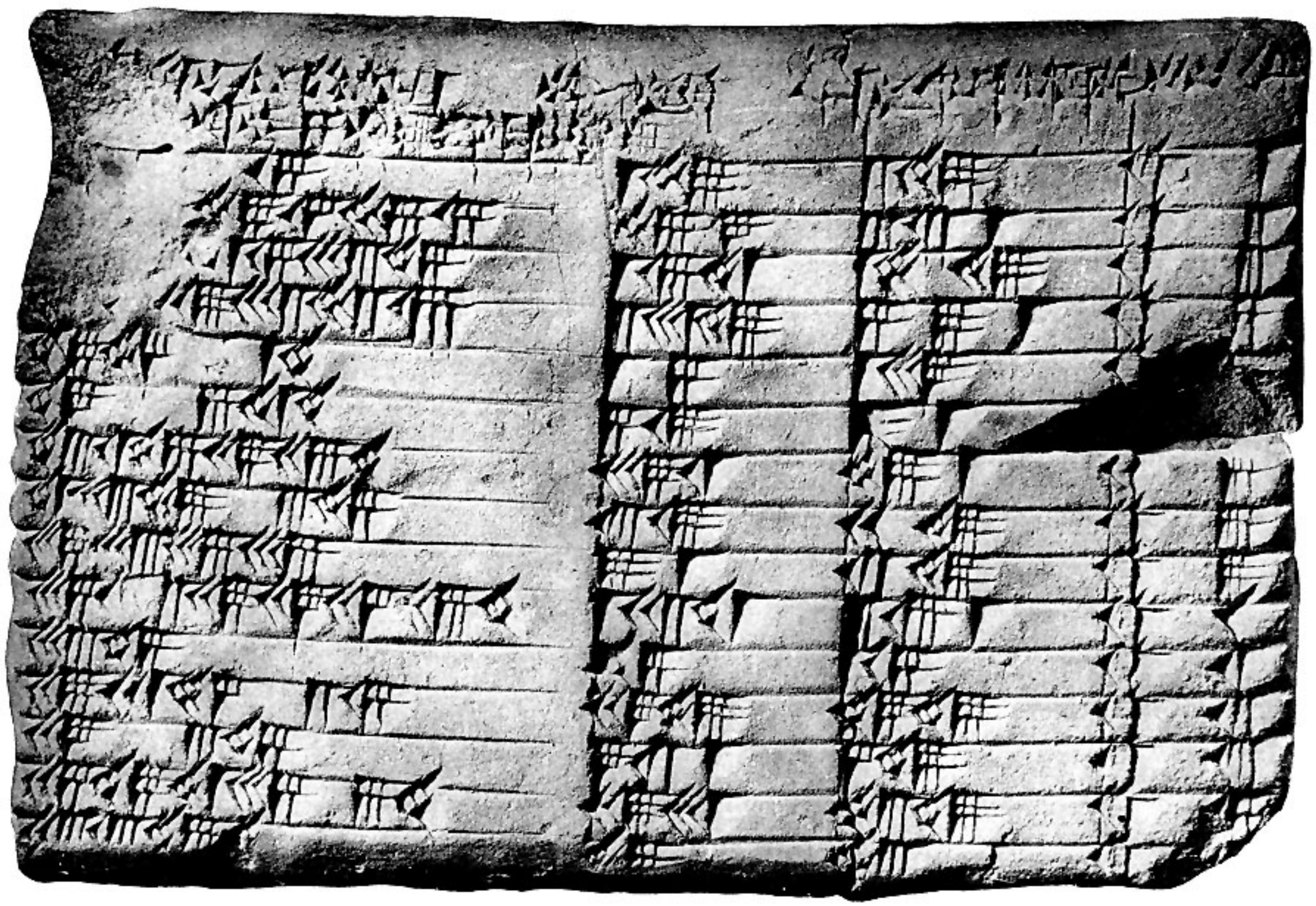

Babylonian Methods

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Ratio	Width	Diagonal	Row number
1; 59 0 15	1 59	2 49	1
1; 56 56 58 14 50 6 15	56 7	1 20 25*	2
1; 55 7 41 15 33 45	1 16 41	1 50 49	3
1; 53 10 29 32 52 16	3 31 49	5 9 1	4
1; 48 54 1 40	1 5	1 37	5
1; 47 6 41 40	5 19	8 1	6
1; 43 11 56 28 26 40	38 11	59 1	7
1; 41 33 45 14 3 45	13 19	20 49	8
1; 38 33 36 36	8 1*	12 49 9	9
1; 35 10 2 28 27 24 26 40	1 22 41	2 16 1	10
1; 33 45	45	1 15	11
1; 29 21 54 2 15	27 59	48 49	12
1; 27 0 3 45	2 41*	4 49	13
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1; 35 10 2 28 27 24 26 40	1 22 41	2 16 1	10
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$$= 12{,}709$$

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Ratio	Width	Diagonal	Row number
$\frac{28,561}{14,400}$	119	169	1
$\frac{23,280,625}{11,943,936}$	3,367	4,825	2
$\frac{44,209,201}{23,040,000}$	4,601	6,649	3
$\frac{343,768,681}{182,250,000}$	12,709	18,541	4
$\frac{9,409}{5,184}$	65	97	5
$\frac{231,361}{129,600}$	319	481	6
$\frac{12,538,681}{7,290,000}$	2,291	3,541	7
$\frac{1,560,001}{921,600}$	799	1,249	8
$\frac{591,361}{360,000}$	481	46,149	9
$\frac{66,601,921}{41,990,400}$	4,961	8,161	10
$\frac{25}{16}$	45	75	11
$\frac{8,579,041}{5,760,000}$	1,679	2,929	12
$\frac{5,034,241}{34,560,00}$	161	289	13
$\frac{10,426,441}{7,290,000}$	1,771	3,229	14
$\frac{678,811}{486,000}$	56	106	15

Babylonian Methods

- Plimpton 322 is a tablet from around 1800 BC.

Ratio	Width	Diagonal	Row number	Height
$\frac{28,561}{14,400}$	119	169	1	120
$\frac{23,280,625}{11,943,936}$	3,367	4,825	2	3,456
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$$45^2 + 60^2 = 75^2$$

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$$\text{Ratio} = \frac{(\text{Diagonal})^2}{(\text{Height})^2}$$

$$\frac{(\text{Width})^2}{(\text{Height})^2} + 1 = \frac{(\text{Diagonal})^2}{(\text{Height})^2}$$

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$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

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Babylonian Methods

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- Discuss Hero’s method on the board.

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- It's so good that computers use it today. It's quadratically convergent. It has been called "the greatest known computation of the ancient world."

Babylonian Methods

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- Fun Fact: Hero of Alexandria was also a great inventor. He invented:

Babylonian Methods

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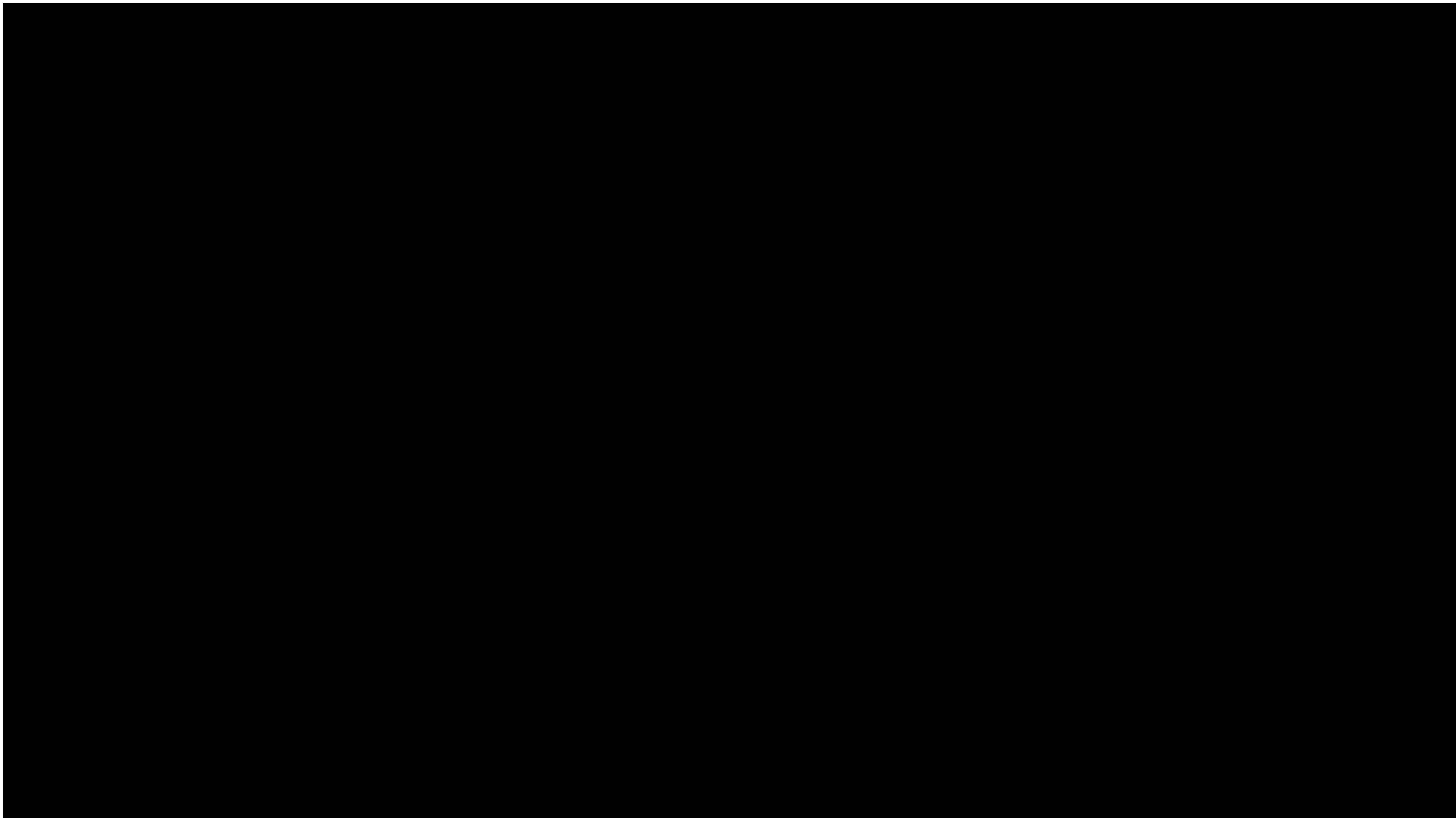
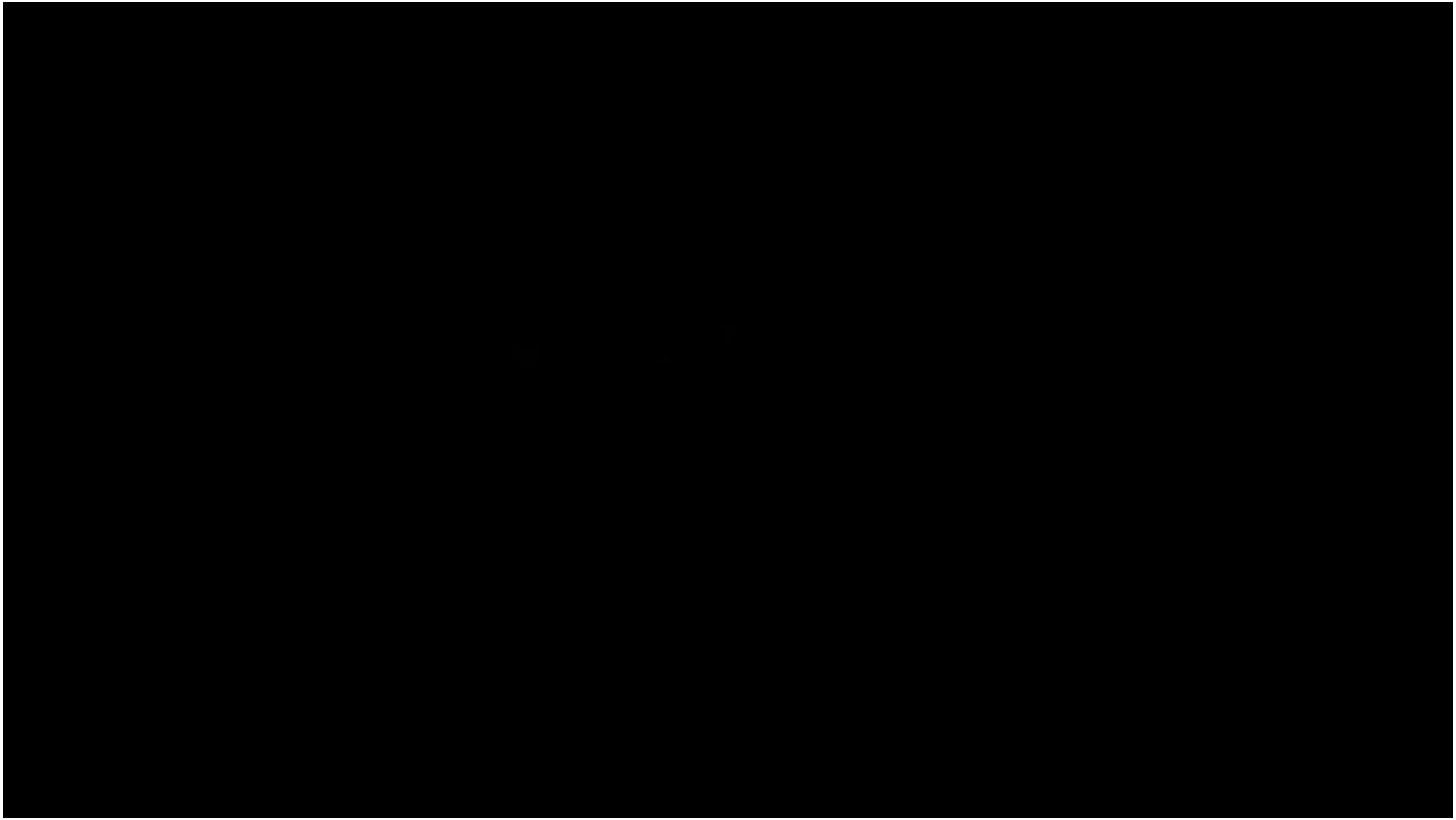
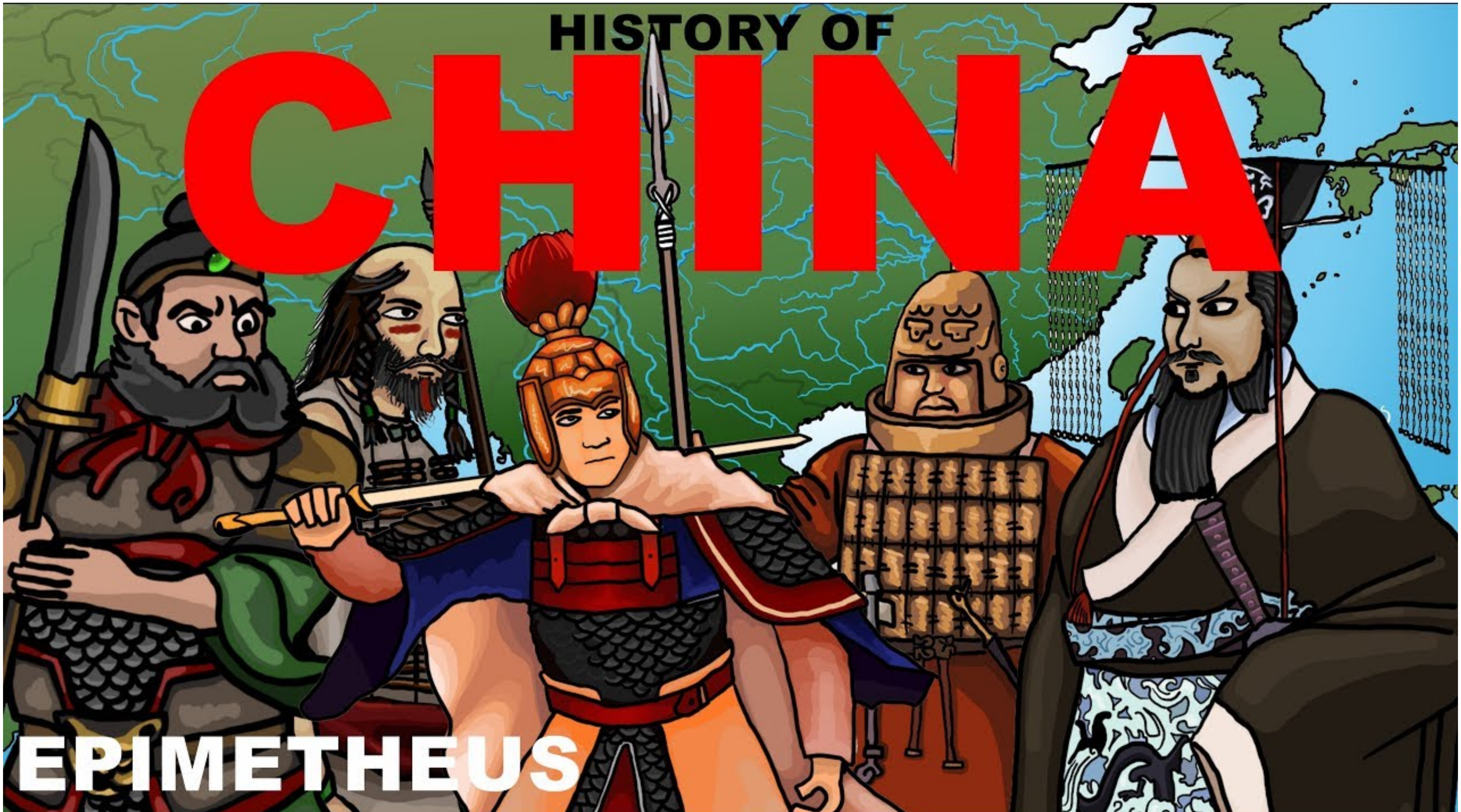


Vending Machine



Ancient Chinese Mathematics

Chinese History



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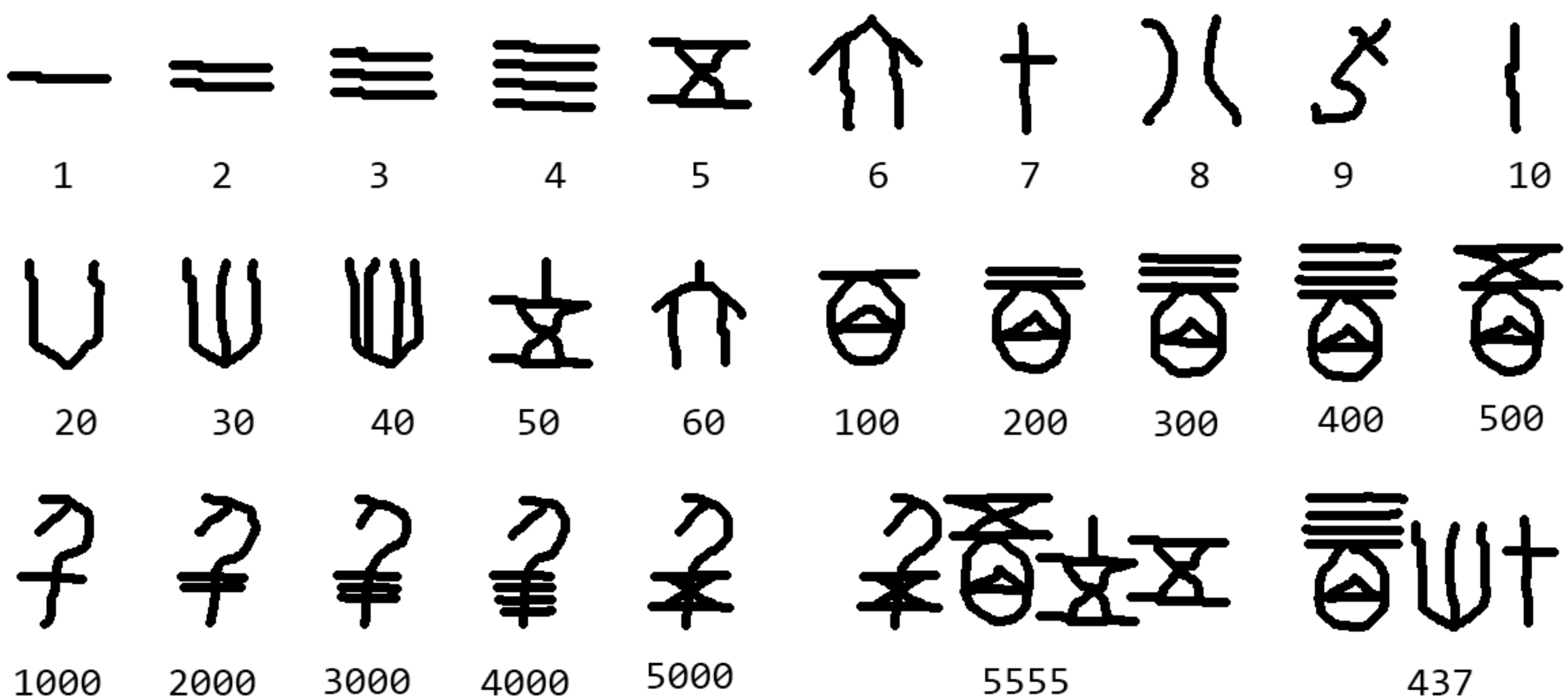

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- Qin joined remote stretches of wall into The Great Wall of China. Required many math techniques for materials/manpower needed, and its design.

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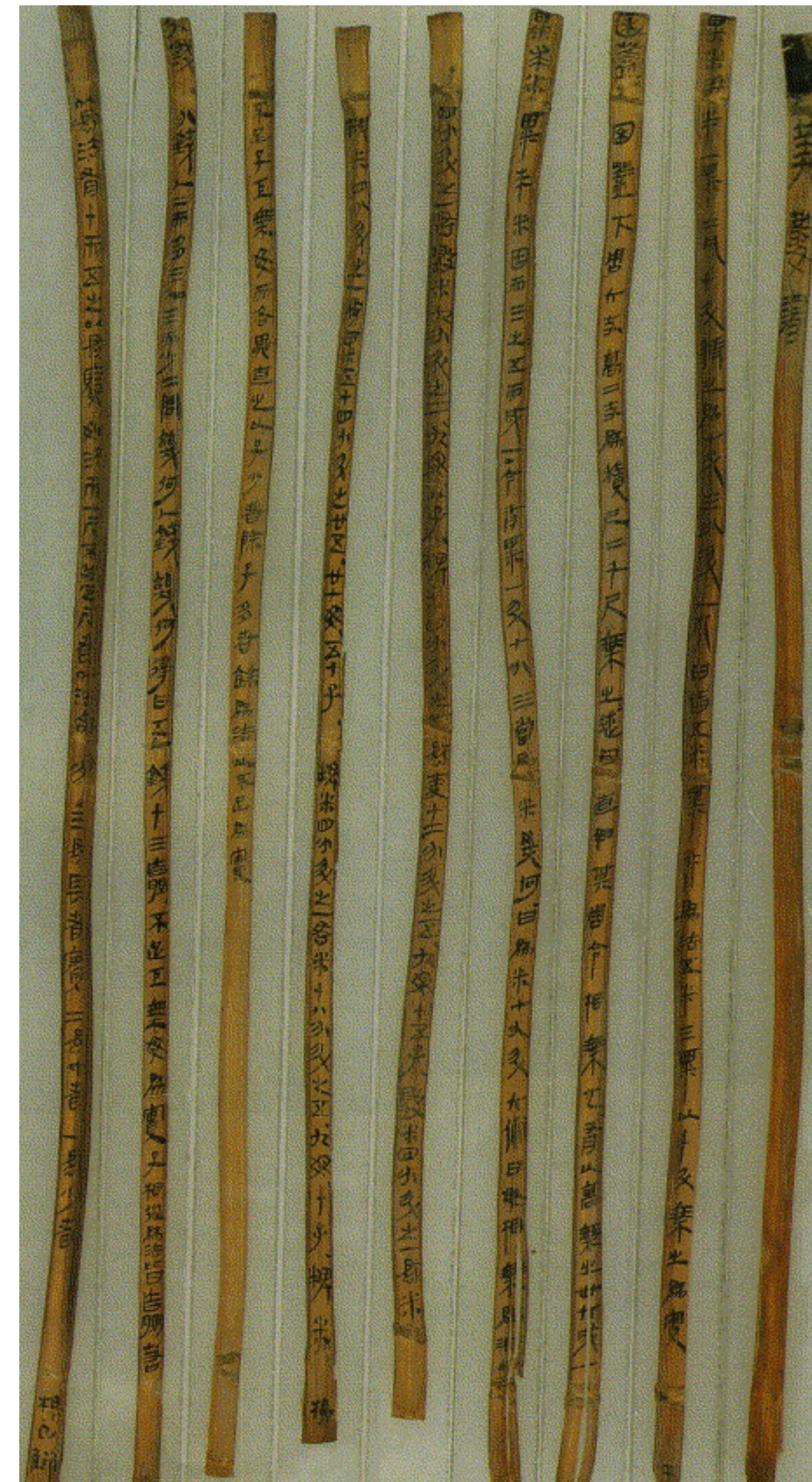

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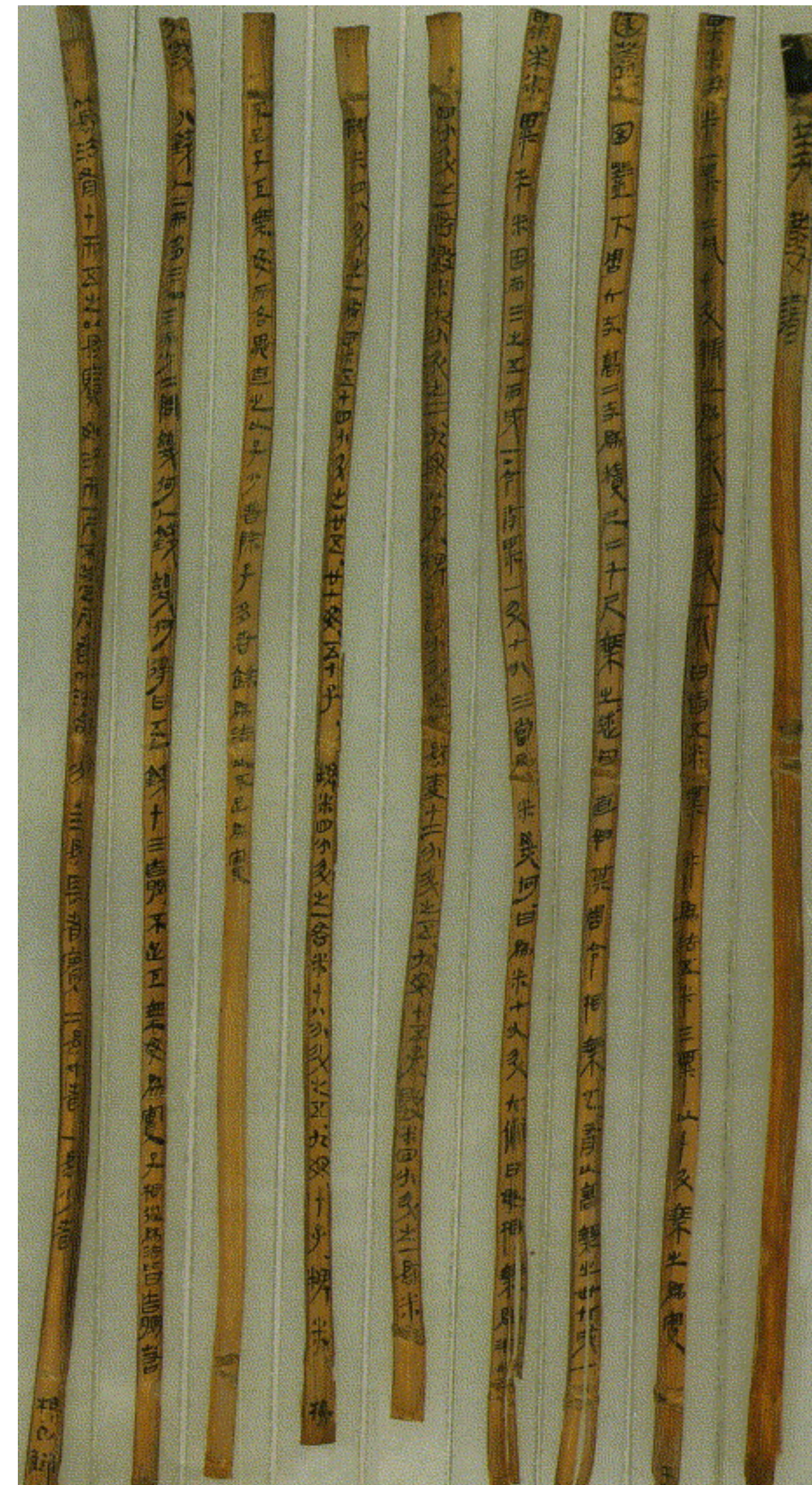

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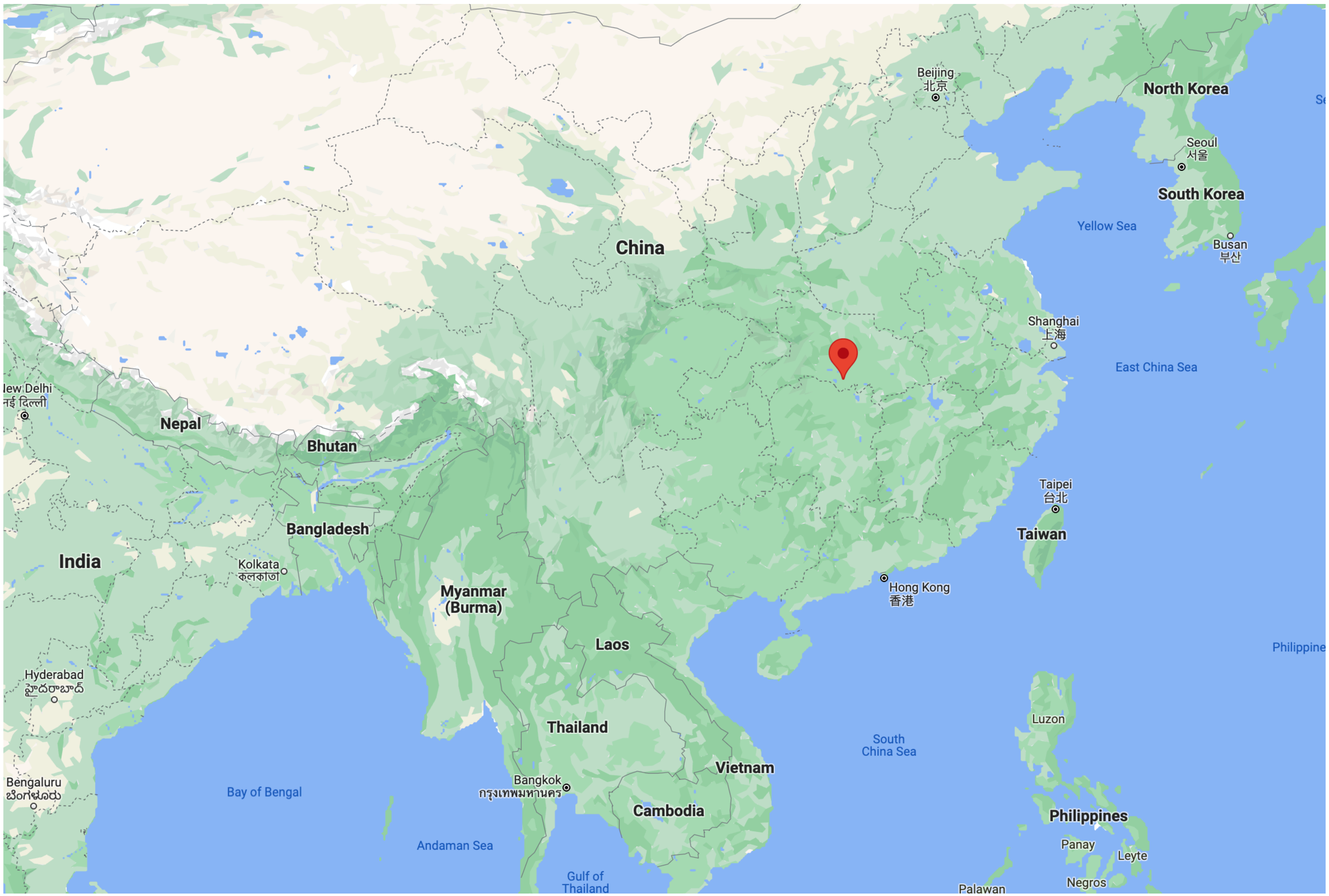

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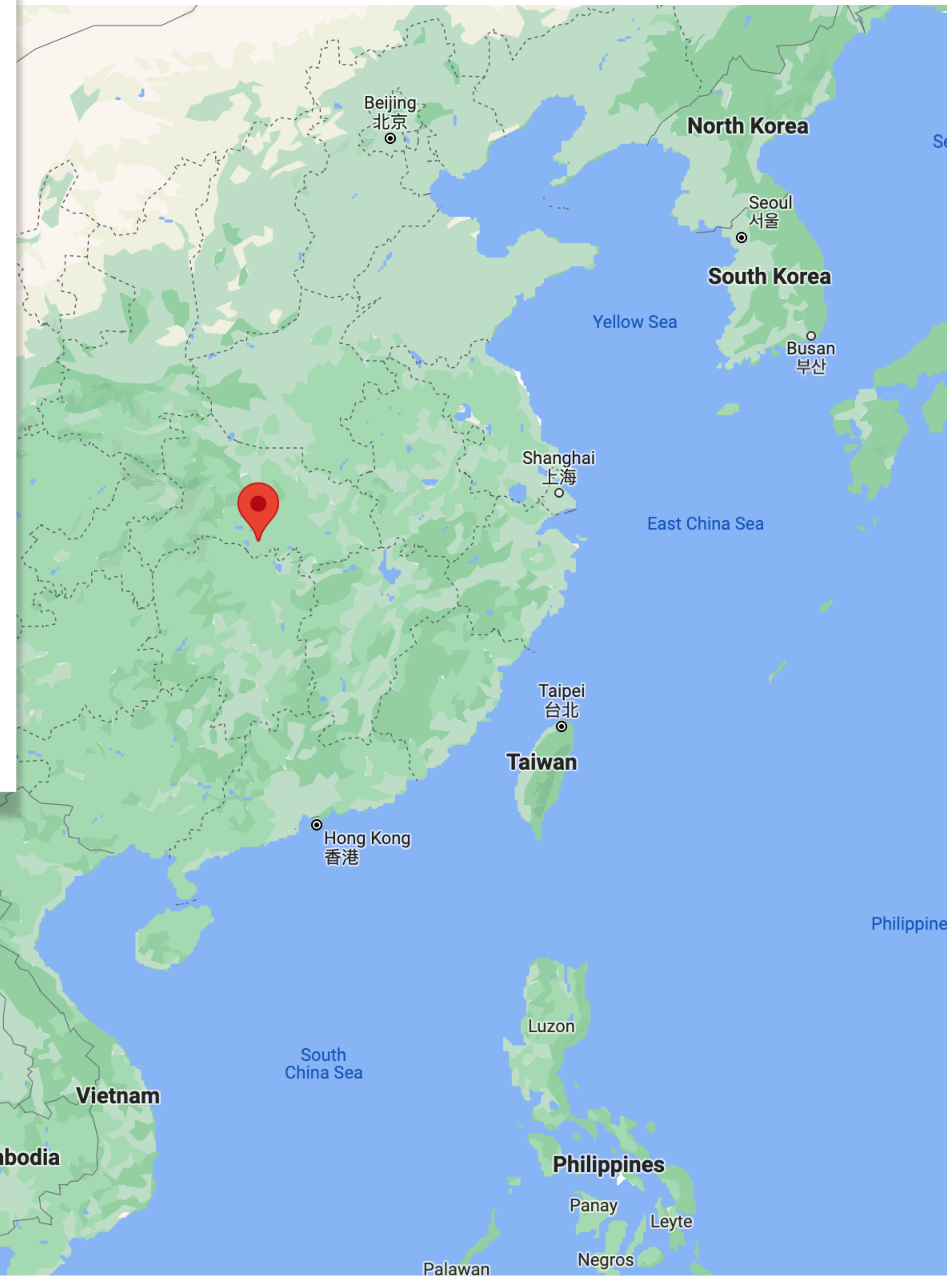

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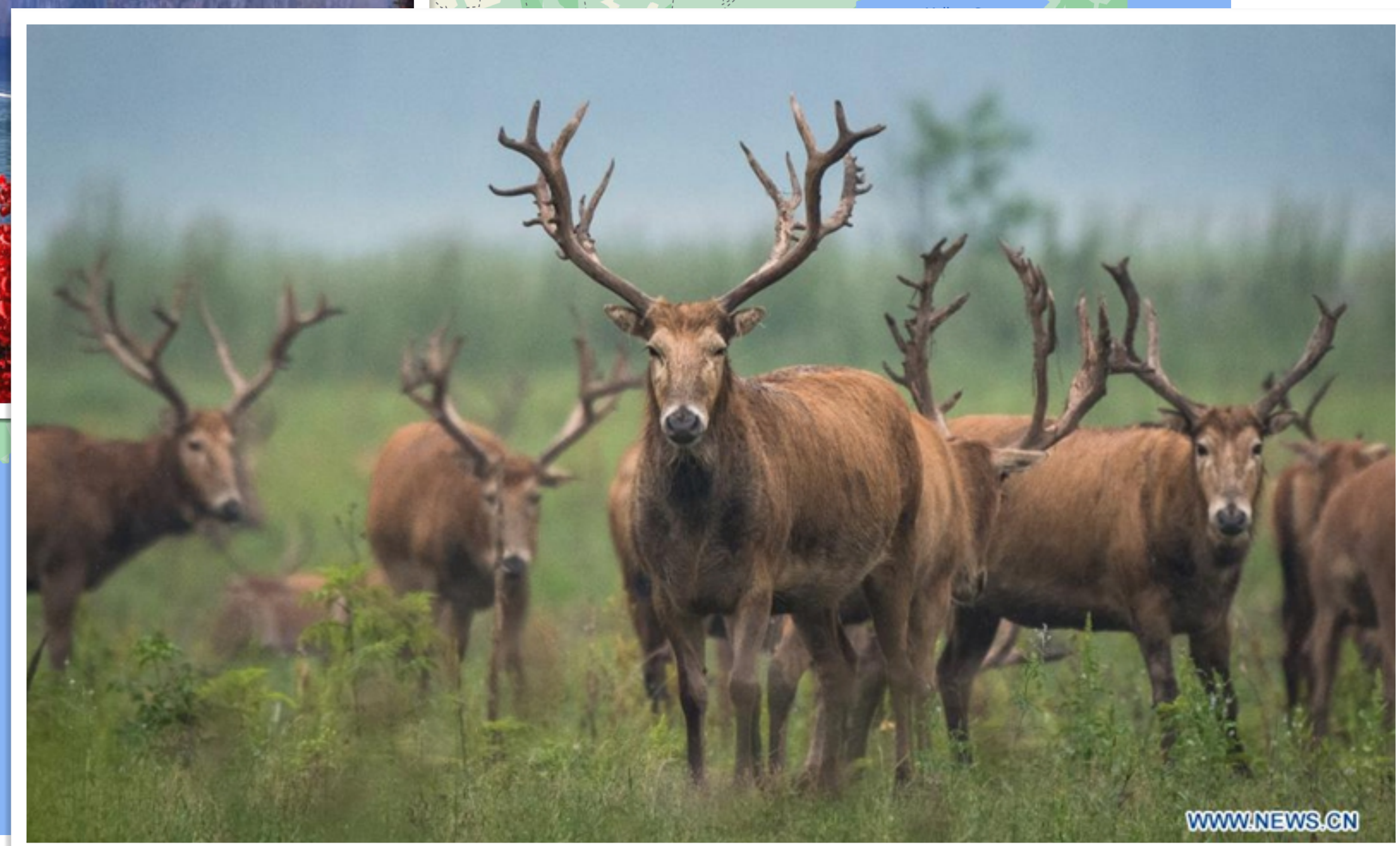

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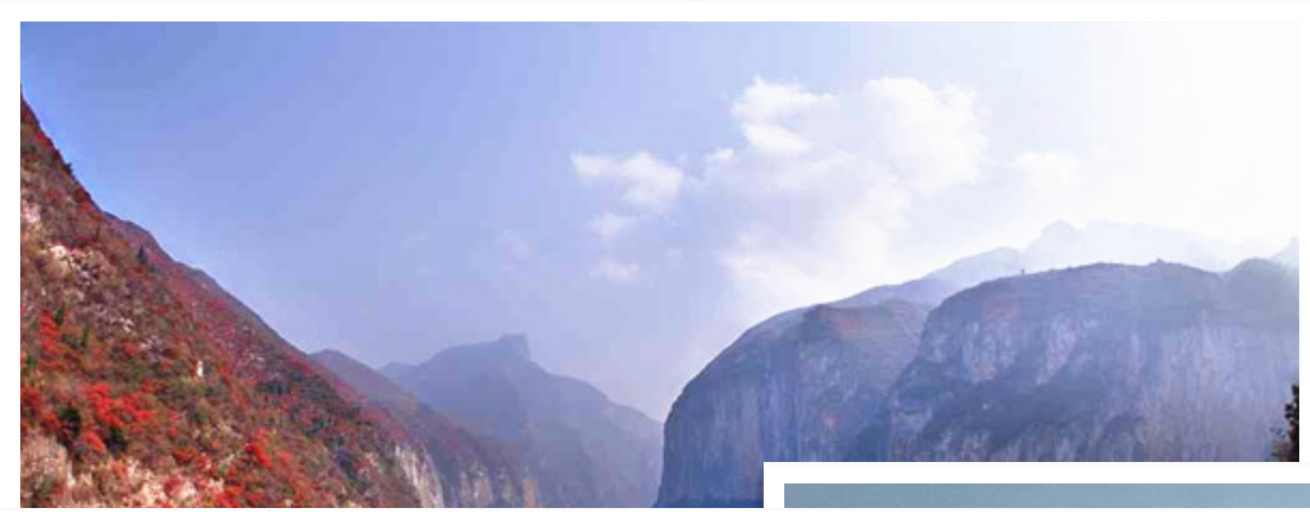

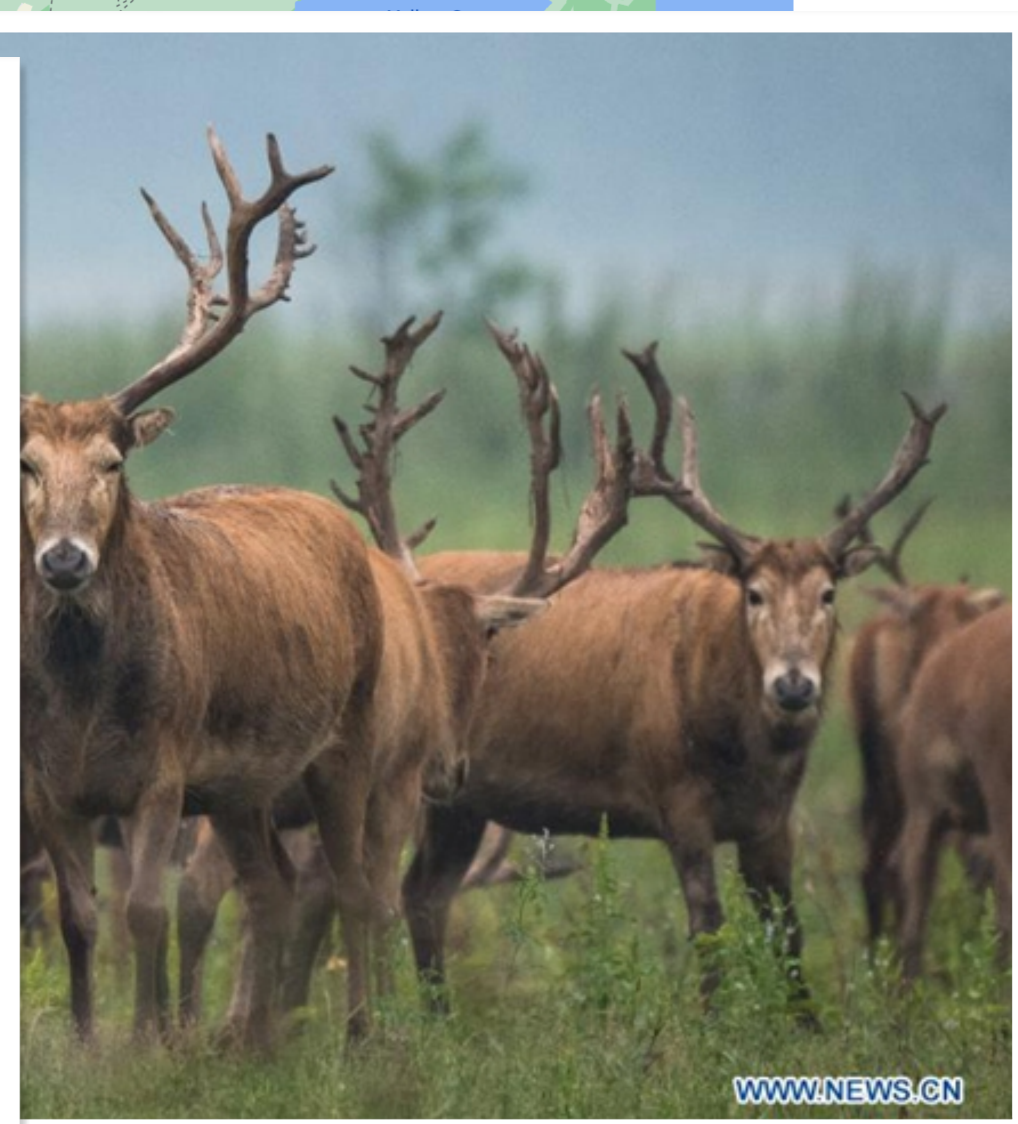

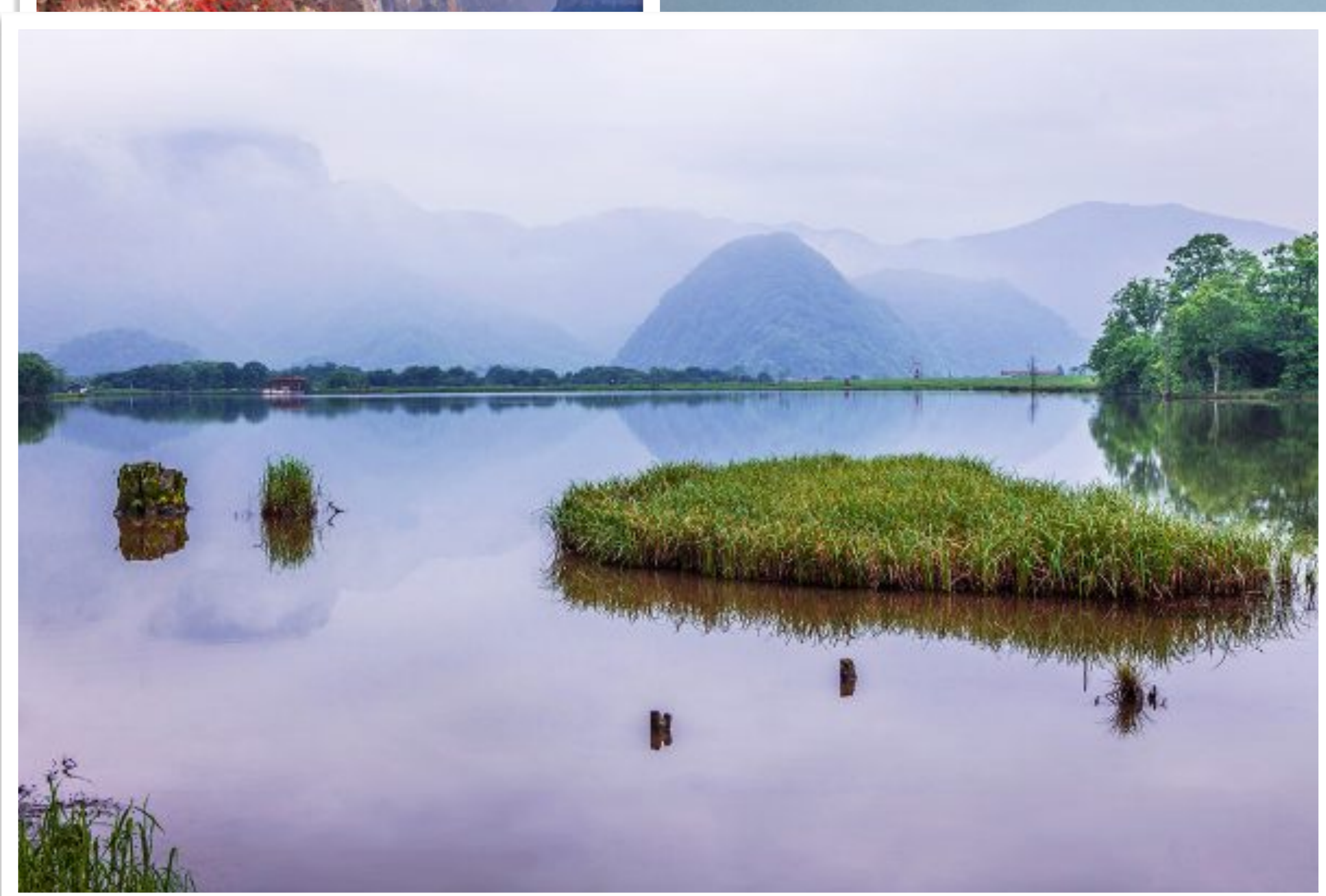

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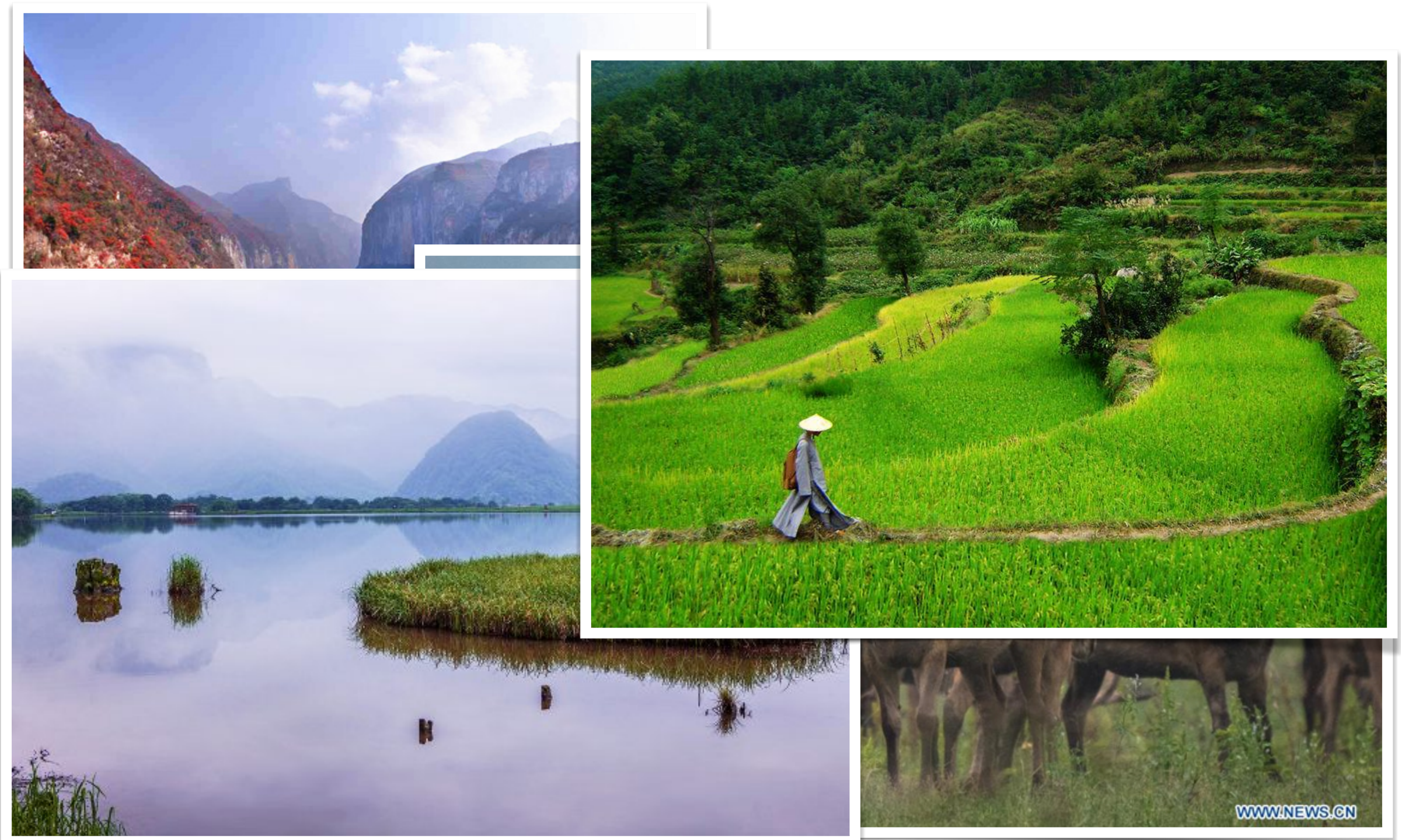

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Think Like A Math Historian

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- This is the first example in history of an important area of math. What area of math is it?

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- In their particular example, $c - a = 1$. So if their example was supposed to only represent those cases, it was correct. Otherwise it had an error.

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- If you guess the number of people and coins, and once you are too high and once you are too low, here is a procedure to tell you how to combine your wrong guesses into a correct solution.

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- Modern notation: $x + \frac{1}{4}x = 15$. Solve for x. They give a method to solve this problem.

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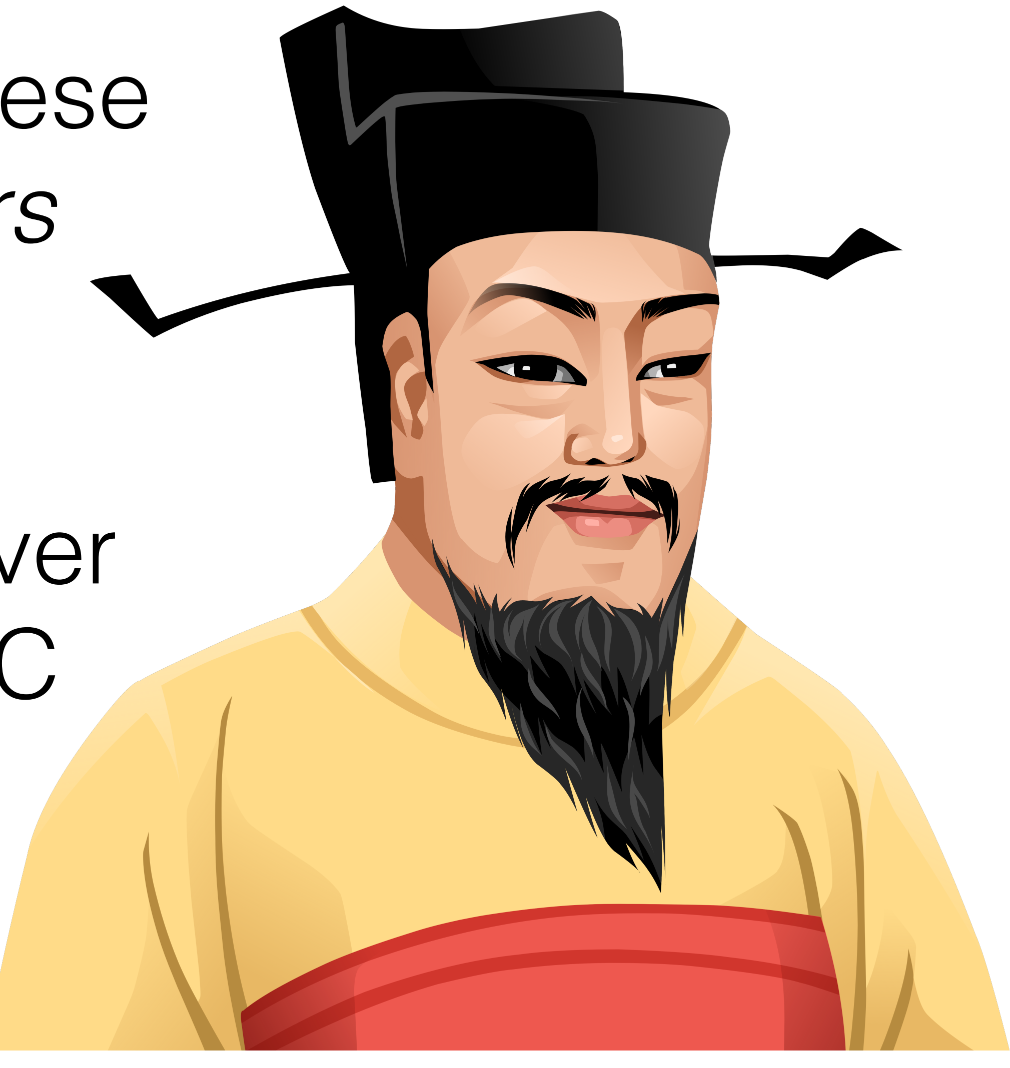

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- It is organized like the *Book on Numbers and Computation*. That is, a problem is stated, an answer is given, and a procedure is presented for how to solve the problem and similar problems.

Show a copy of
Nine Chapters

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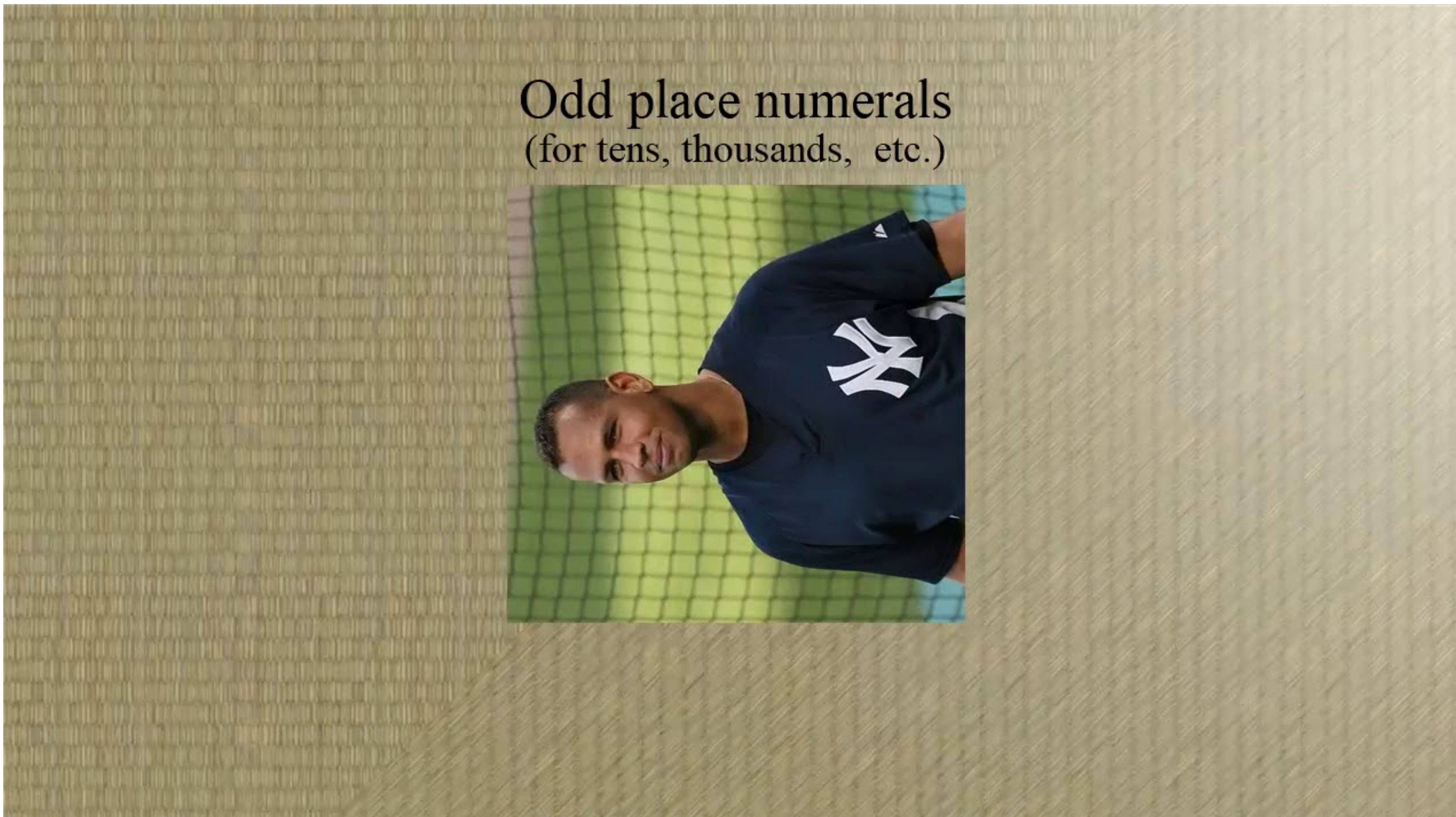
- *Fang cheng* is essentially Gaussian elimination (discovered 2,000 years before Gauss!)

Chinese Mathematics

- First example on *fang cheng* in the *Nine Chapters*:

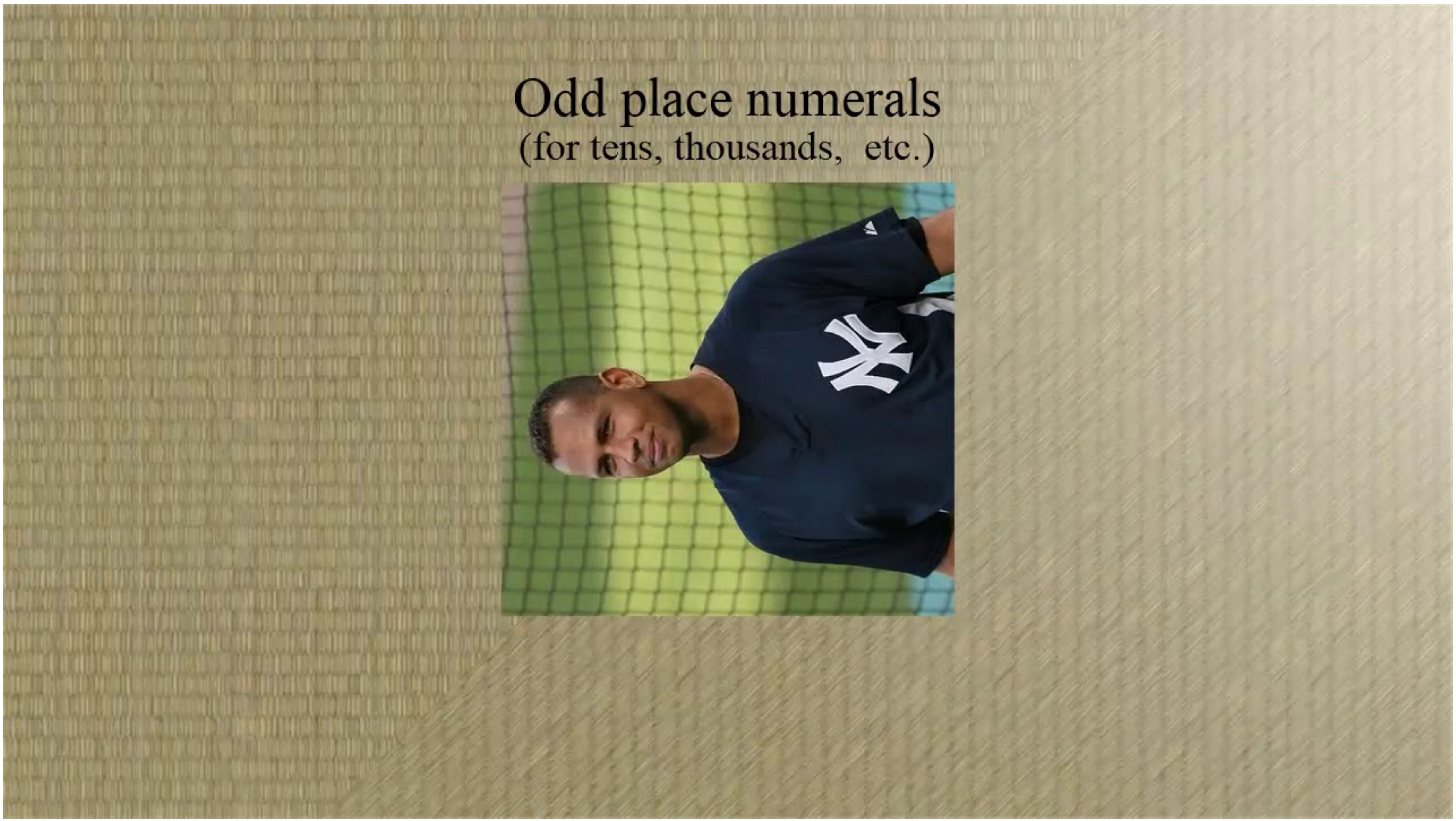
Given 3 bundles of top grade paddy, 2 bundles of medium grade paddy, and 1 bundle of low grade paddy, together they yield 39 *dou* of grain. Given 2 bundles of top grade paddy, 3 bundles of medium grade paddy, and 1 bundle of low grade paddy, together they yield 34 *dou* of grain. Given 1 bundle of top grade paddy, 2 bundles of medium grade paddy, and 3 bundles of low grade paddy, together they yield 26 *dou* of grain. Problem: how much grain does one bundle of high, medium and low grade paddy together yield? Answer: Top grade paddy yields $9\frac{1}{4}$ *dou* per bundle; medium grade paddy yields $4\frac{1}{4}$ *dou* per bundle; low grade paddy yields $2\frac{3}{4}$ *dou* per bundle.

Chinese Mathematics



Chinese Mathematics

- Solving on a counting board.



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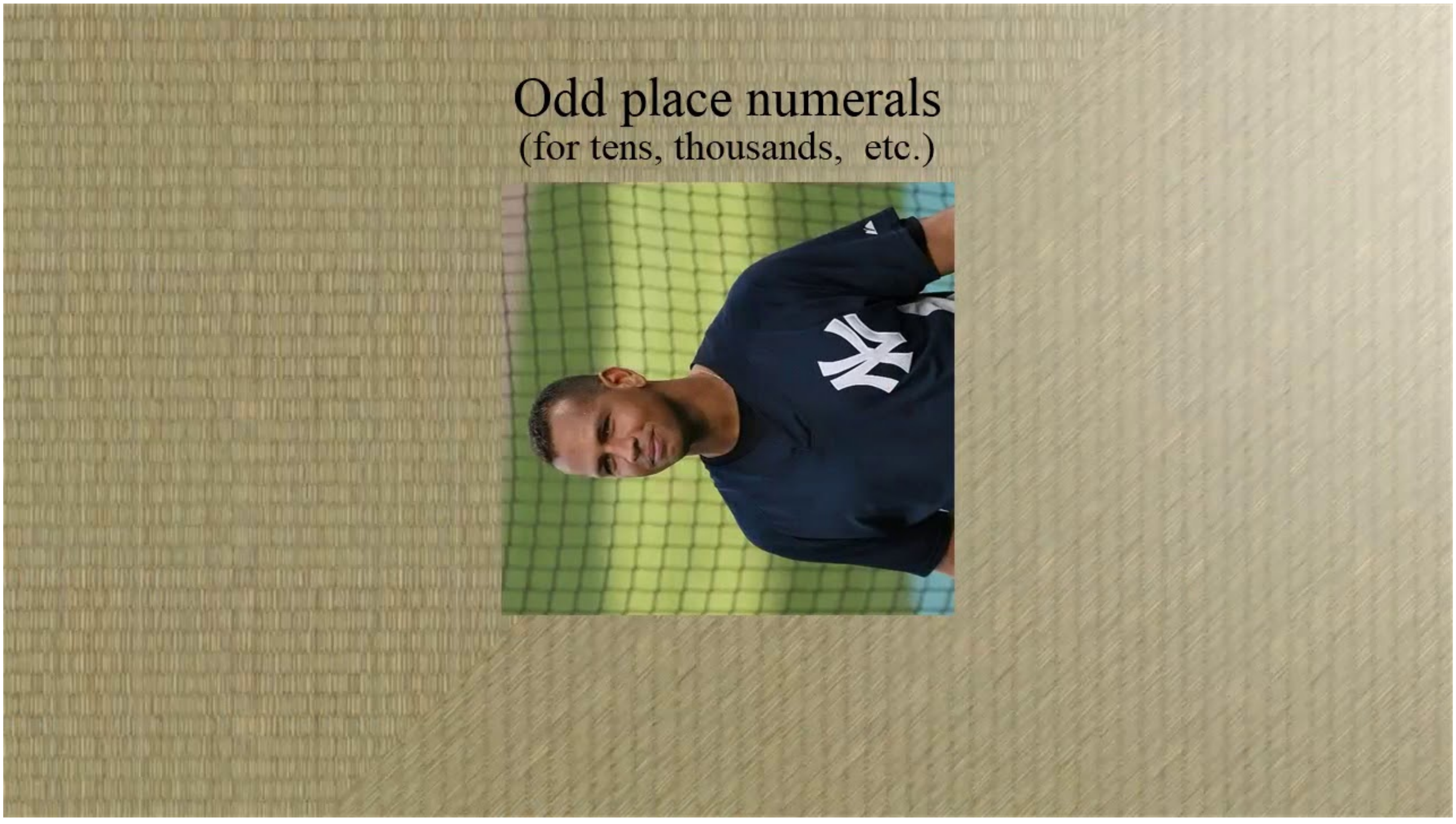

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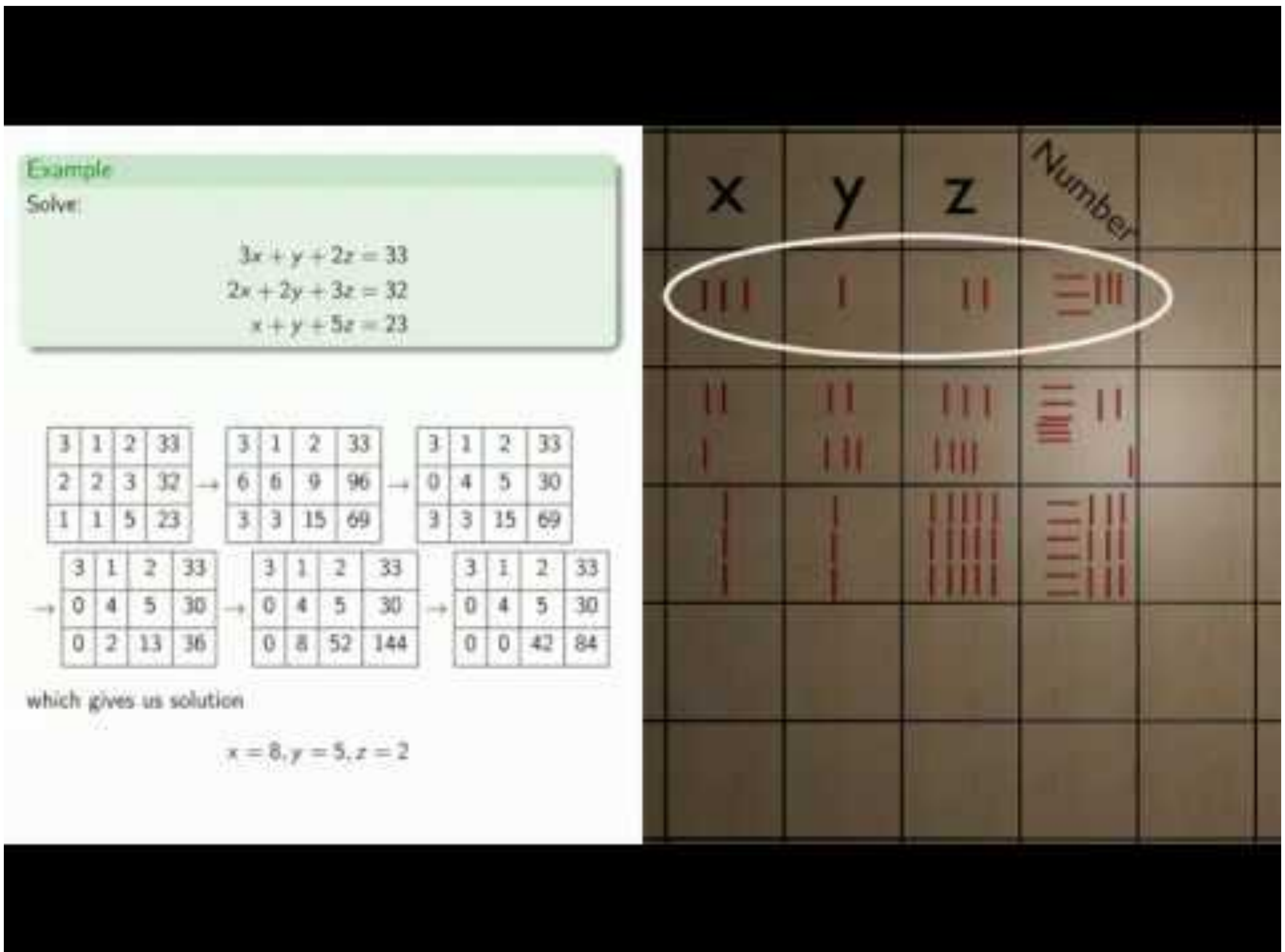

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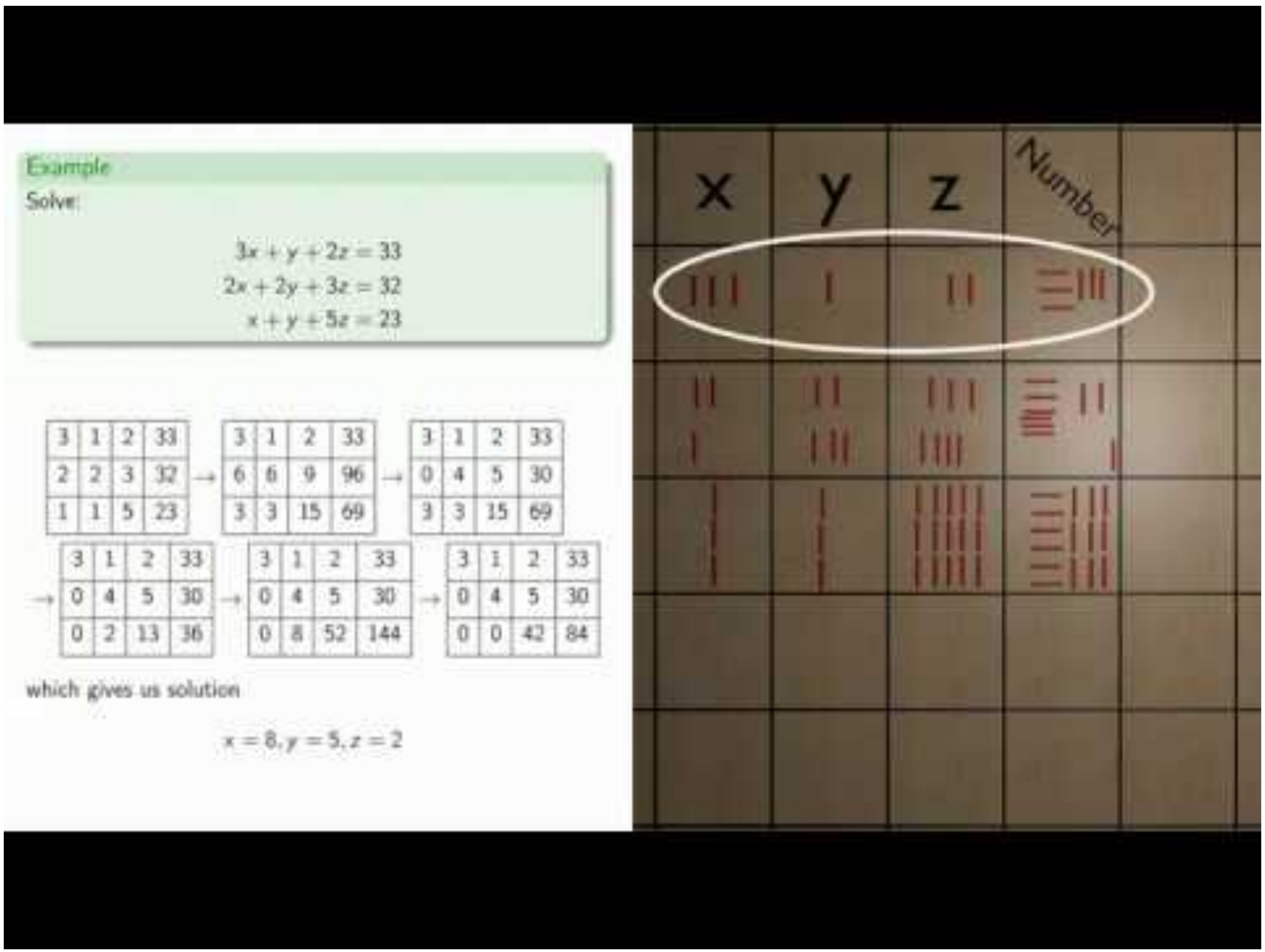

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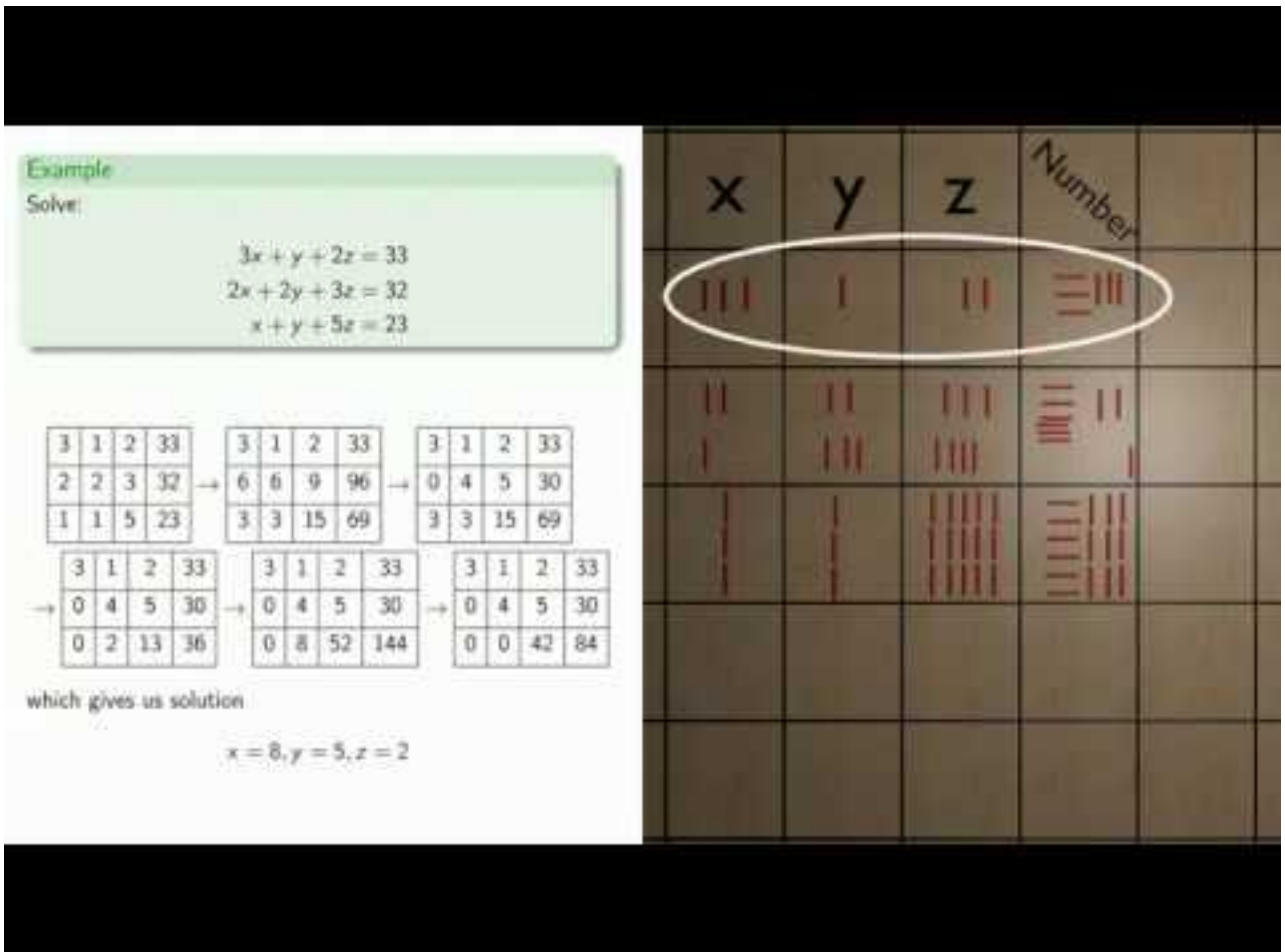

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- Gaussian elimination is happy to introduce fractions as soon as back-substitution begins.

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Chapter 9: Problems using the Pythagorean theorem.

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- Issac Newton was the first in the West. He noticed that no Algebra book he knew of solved systems of linear equations, so he created Gaussian elimination.

> "And you are to know, that by each Æquation one unknown Quantity may be taken away, and consequently, when there are as many Æquations and unknown Quantities, all at length may be reduc'd into one, in which there shall be only one Quantity unknown."

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- Calling it Gaussian elimination started in the 1950s based on a misreading of history.

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- Before universal education, it was really important to clearly communicate how to do arithmetic.
- One pioneer in this was Wang Zhenyi, from 18th century China. She wrote a five-volume text doing so.

People’s History

People’s History of Trigonometry

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- The Mayans had a similar method.

Utilitarian “Theorems”

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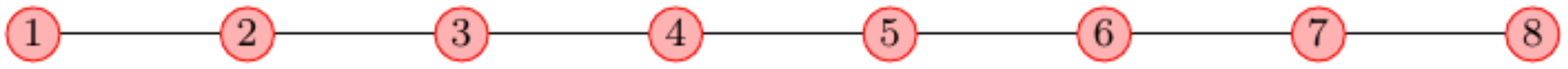

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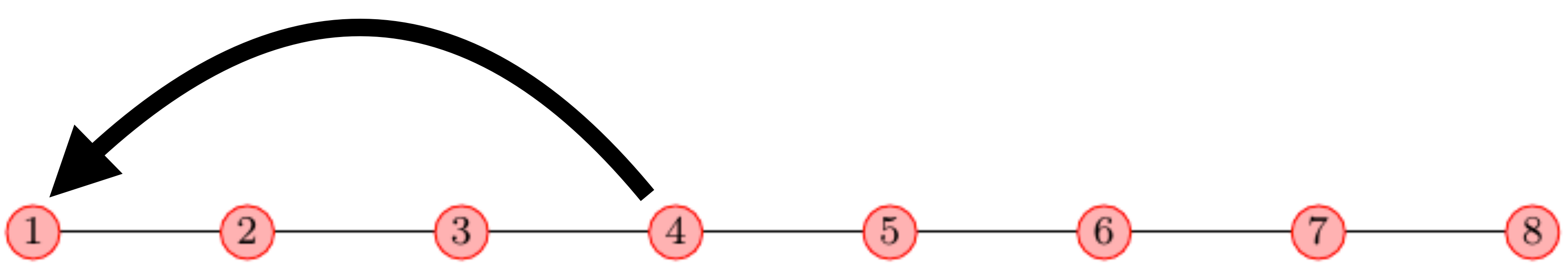

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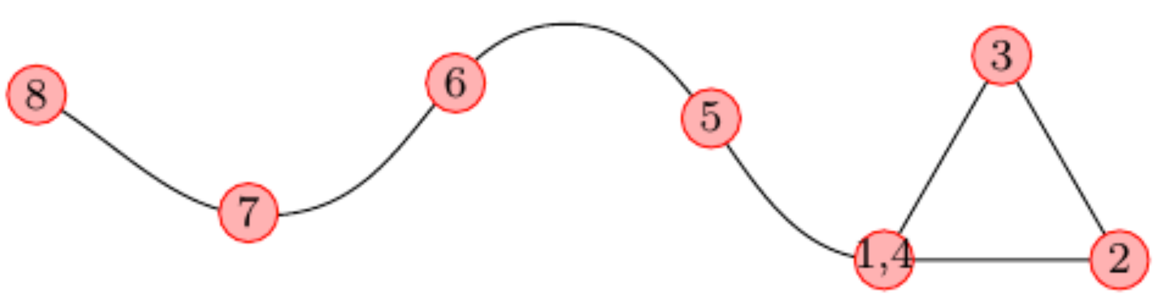

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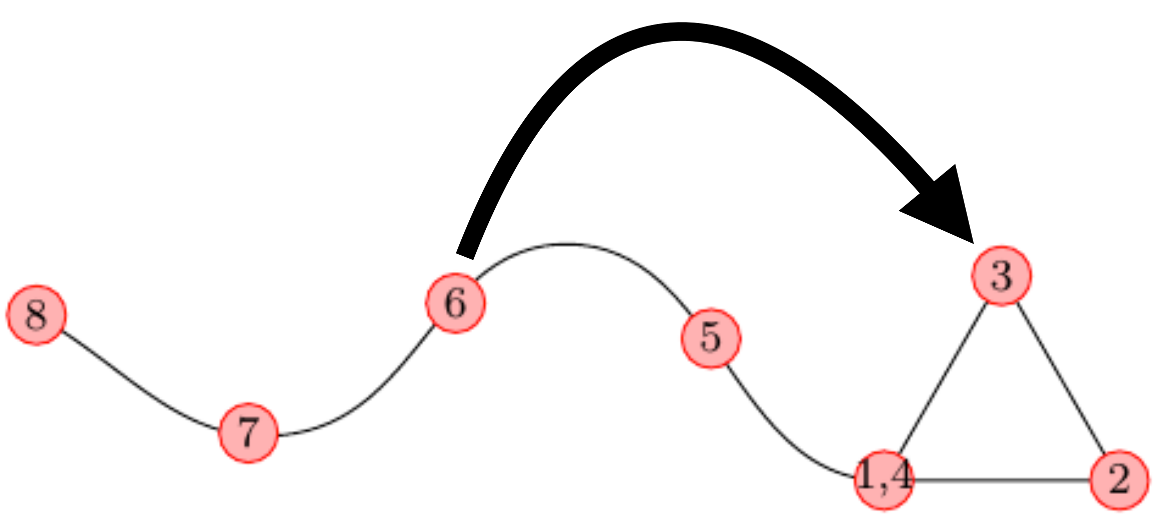

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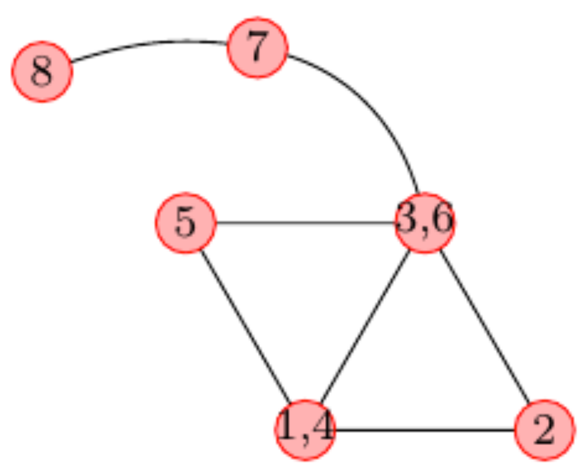

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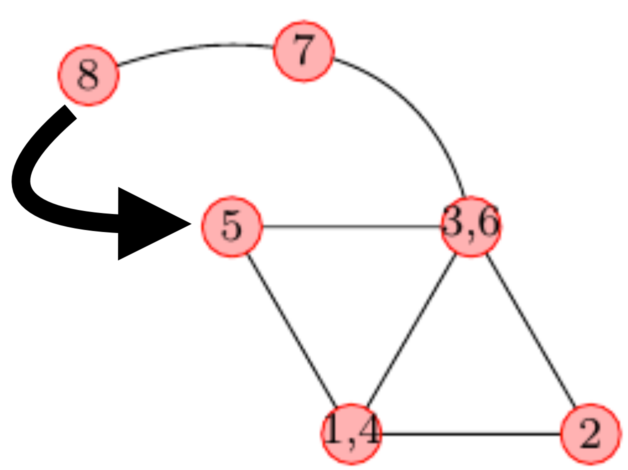

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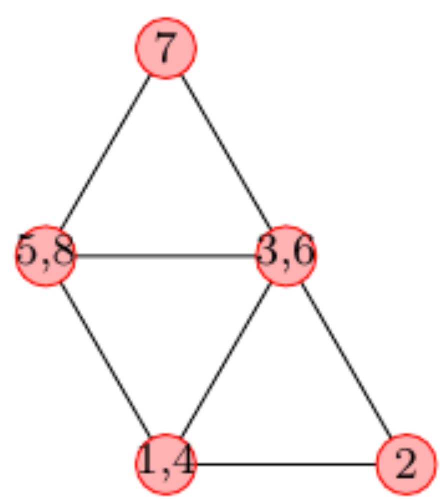

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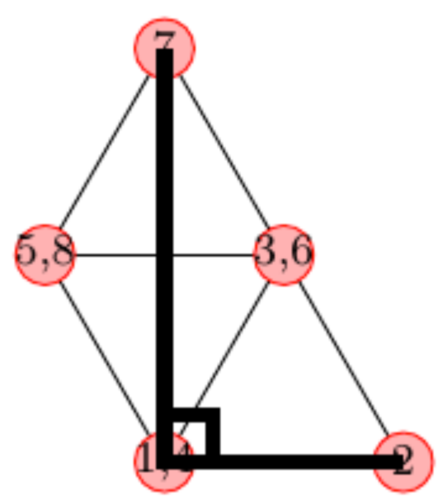

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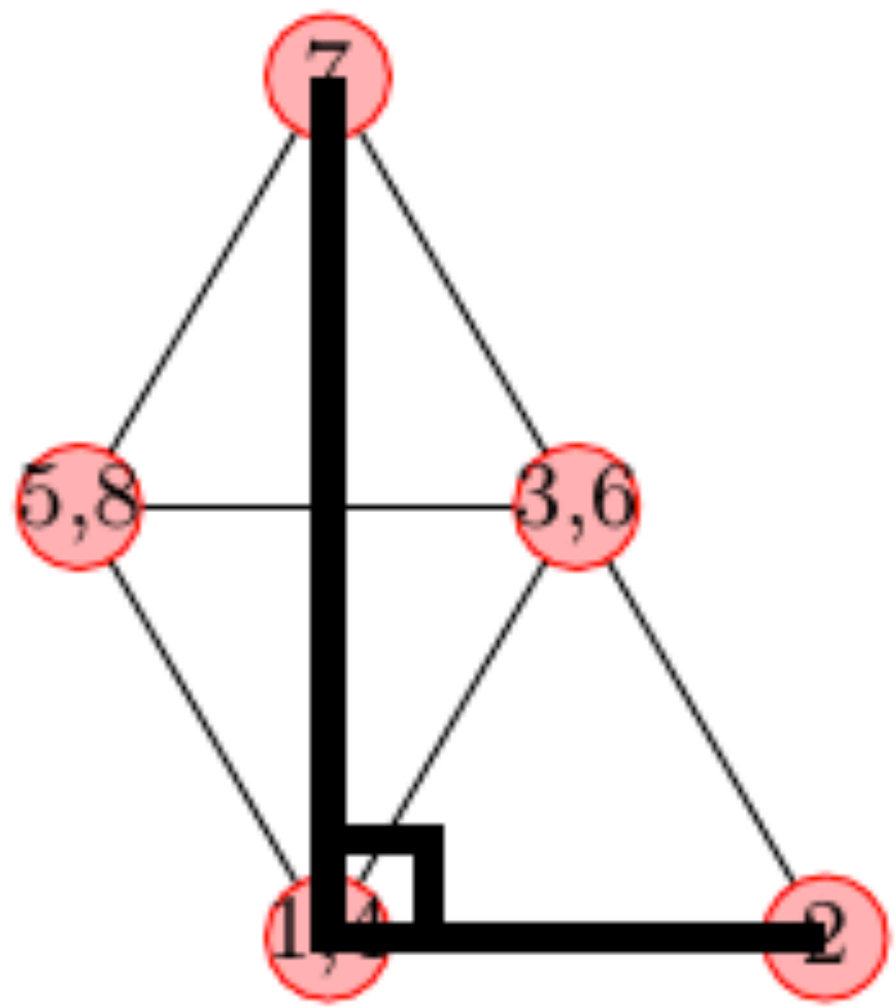

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- Roman architect Vitruvius recorded a clever approach using the Sun.

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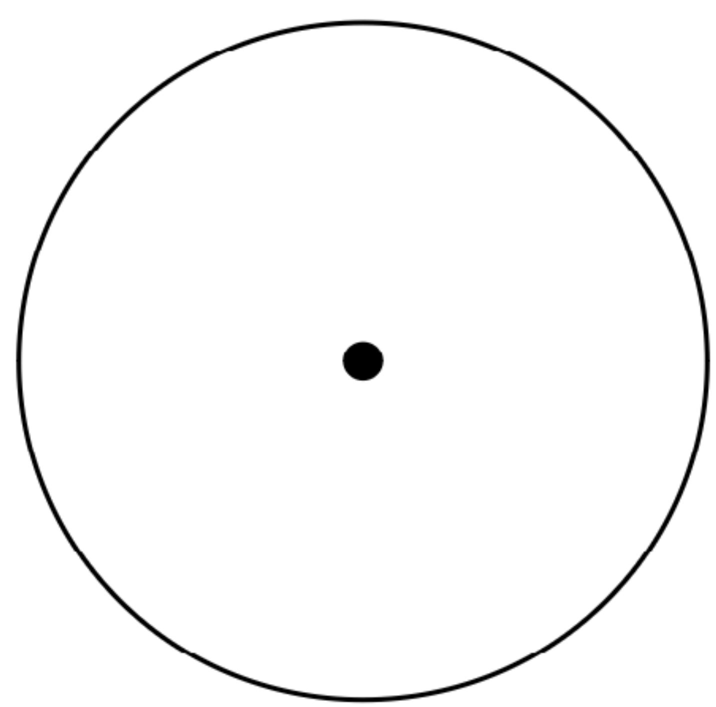

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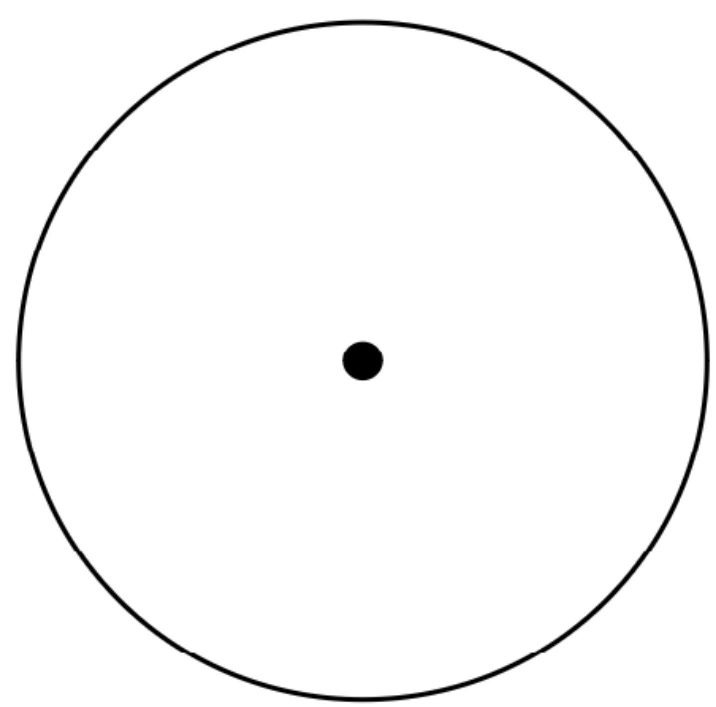
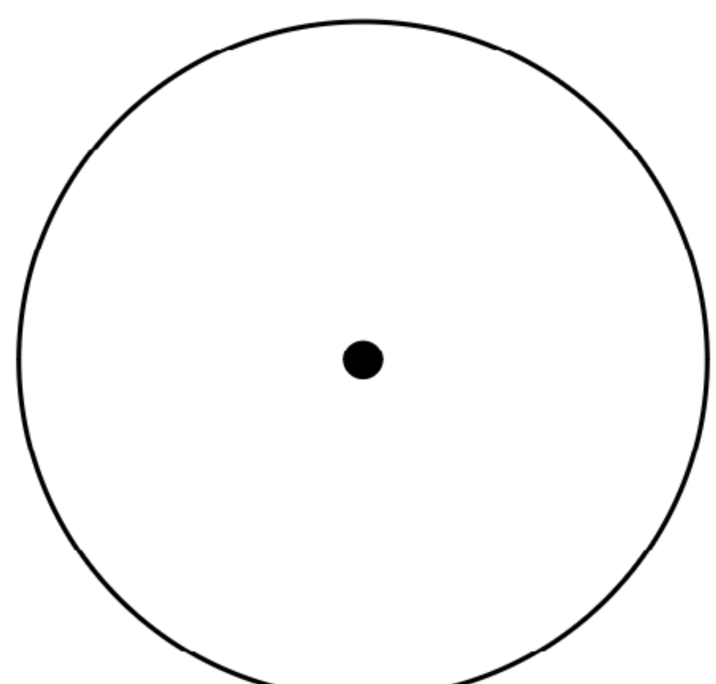


People's History of Trigonometry

- Step 2: At the start of the day, the shadow cast by the stick will be outside the circle.



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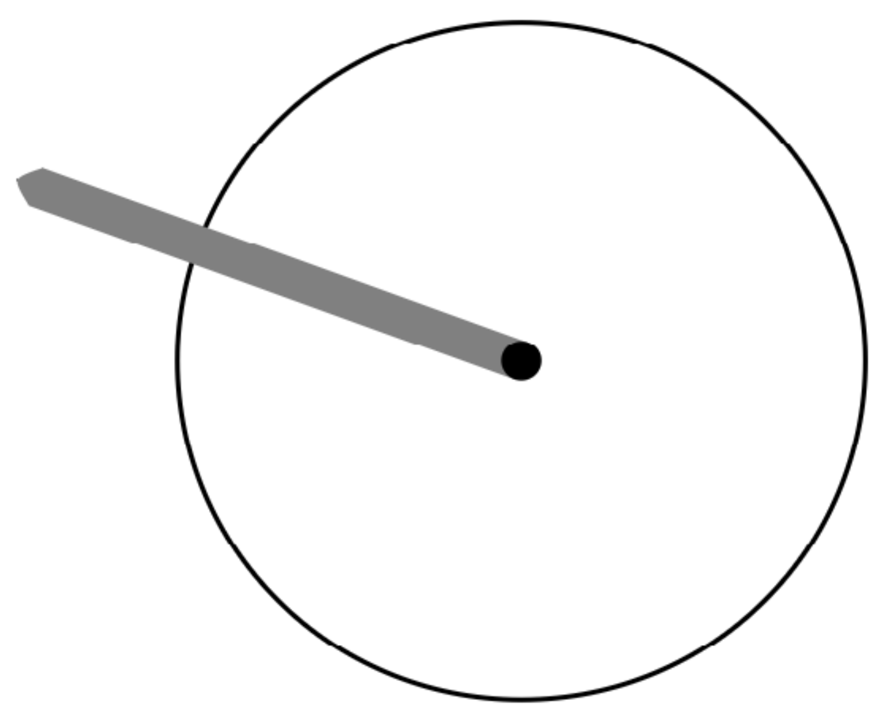

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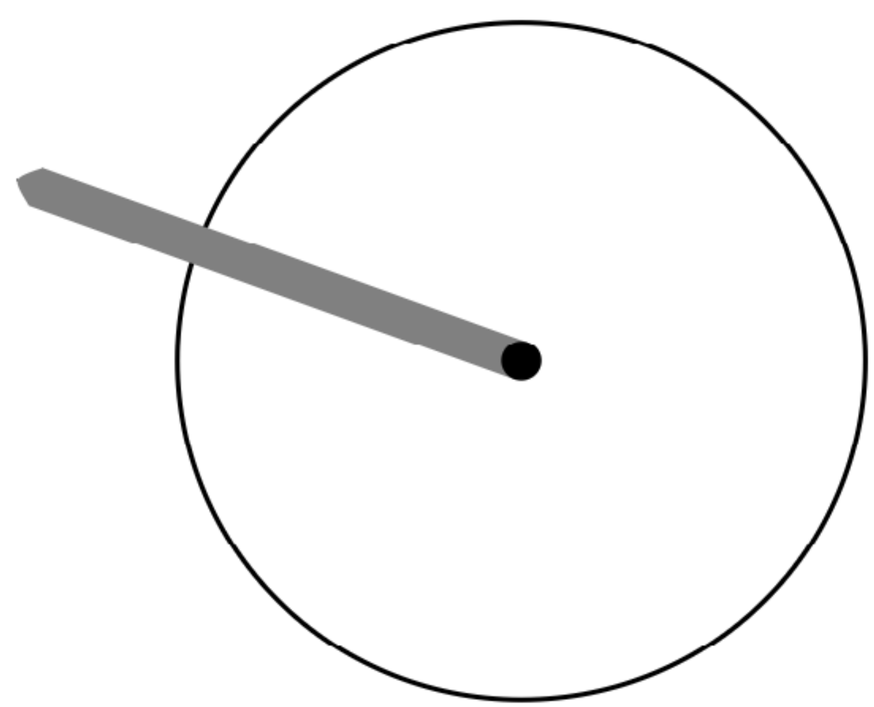

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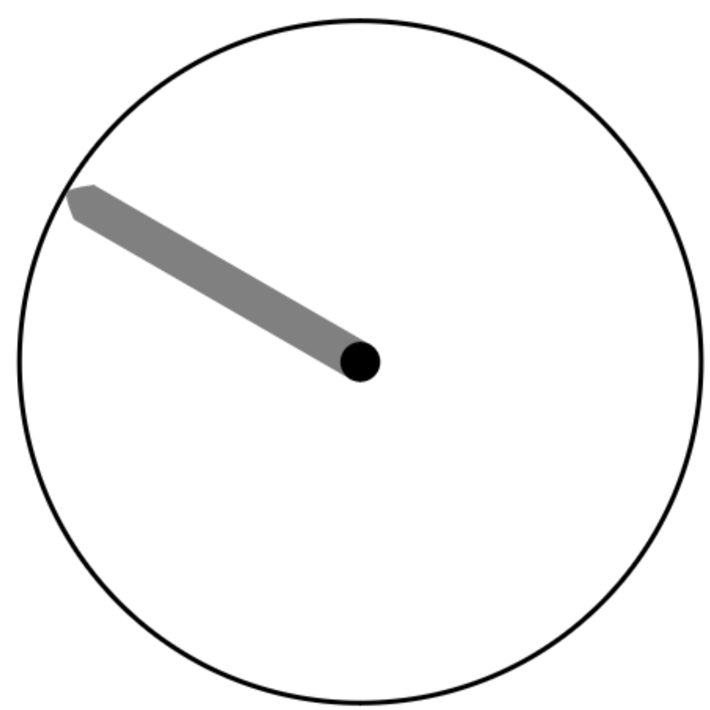

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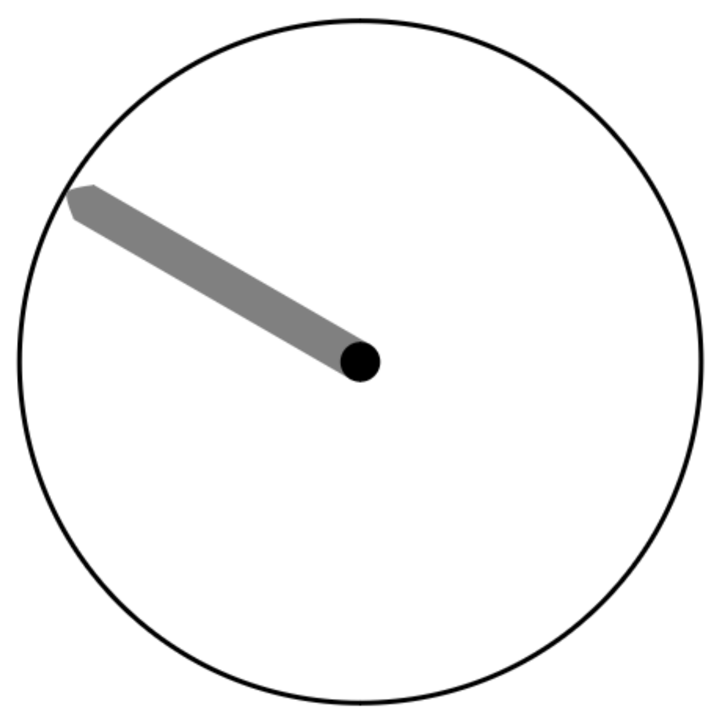

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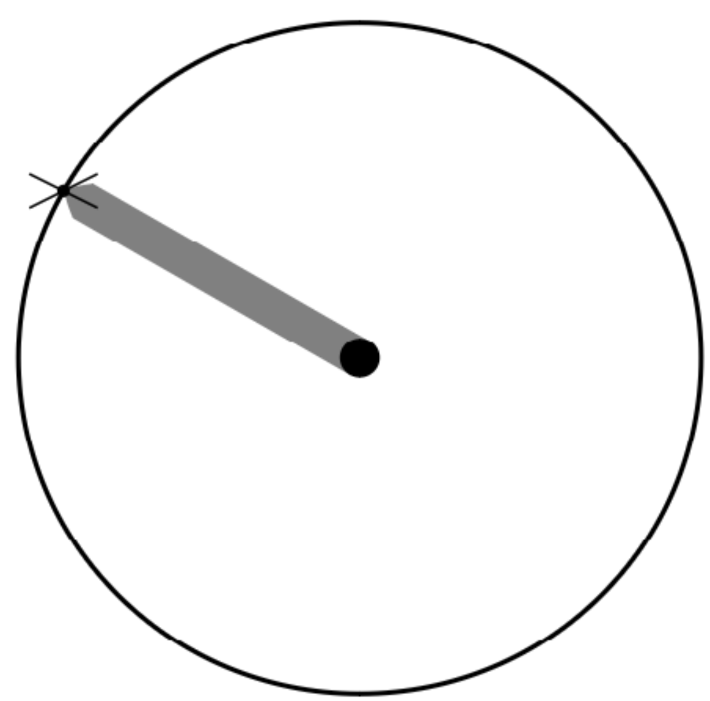

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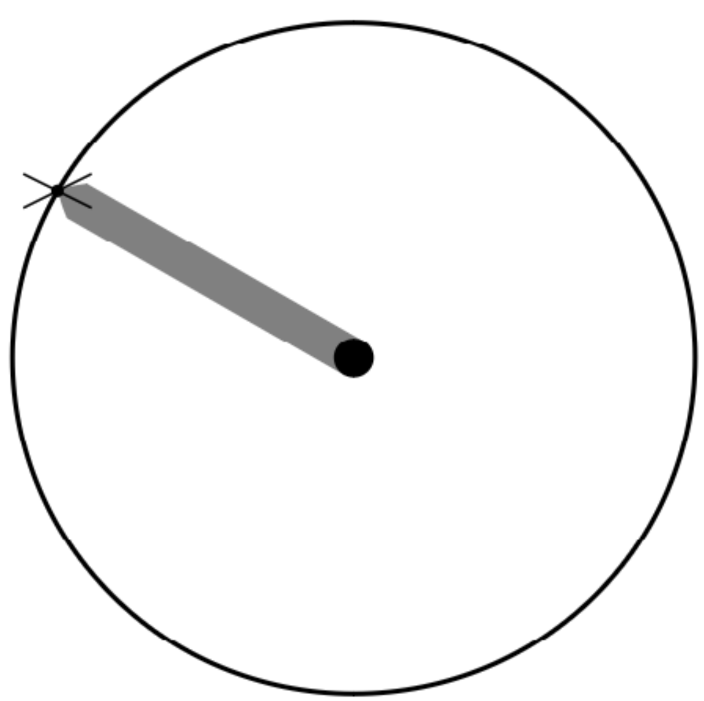

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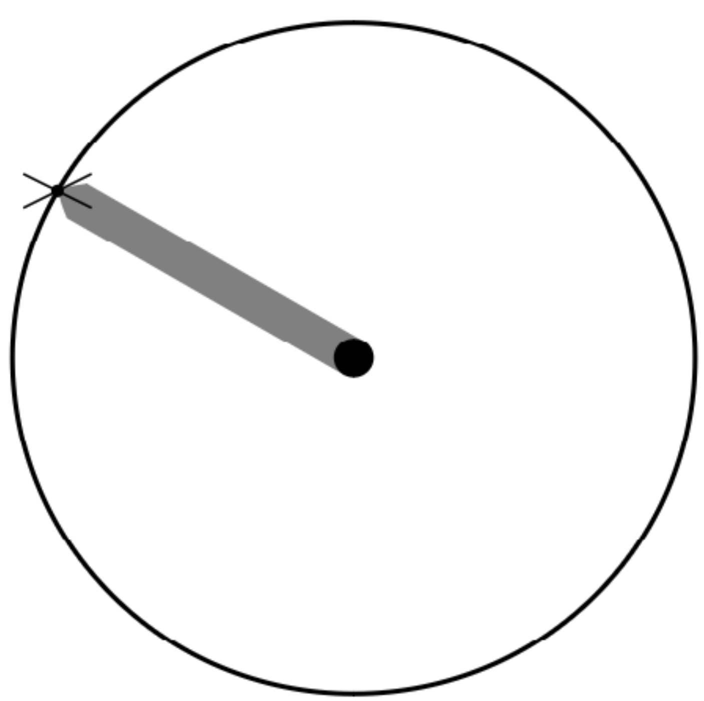

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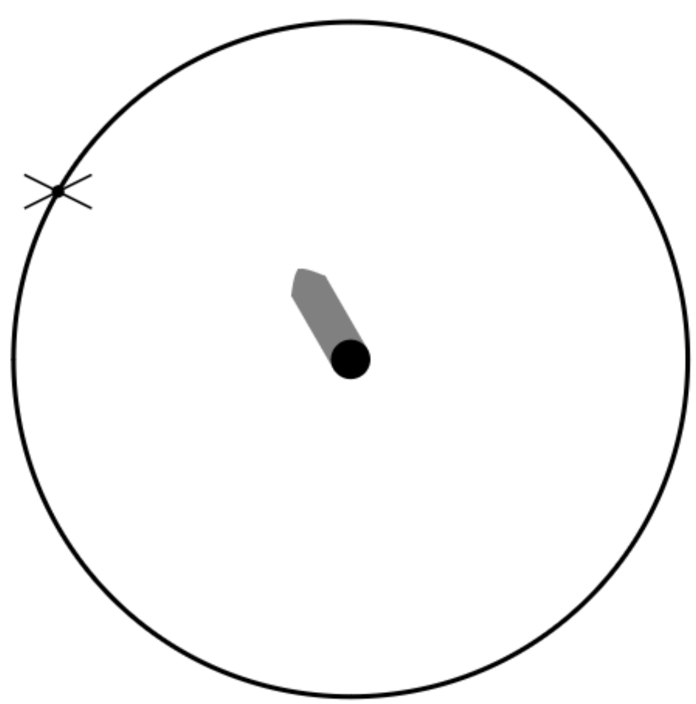

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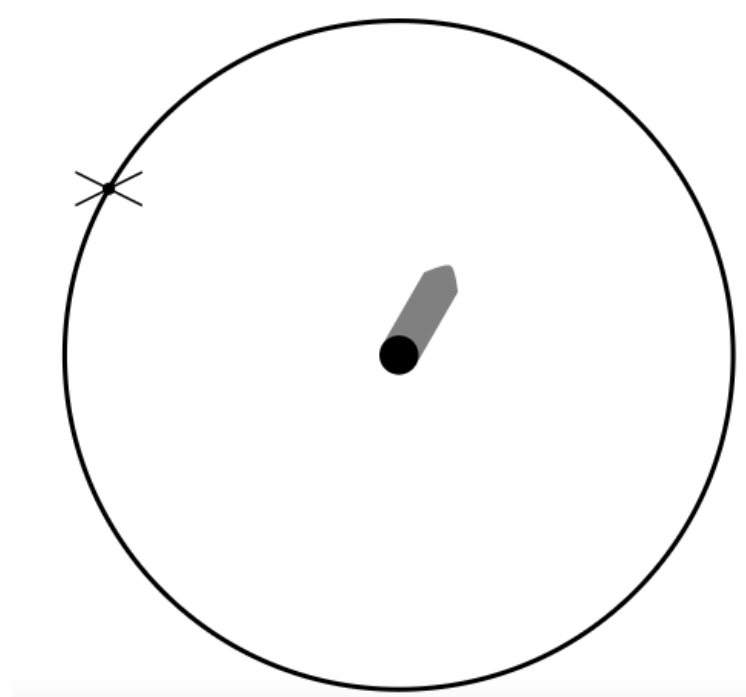

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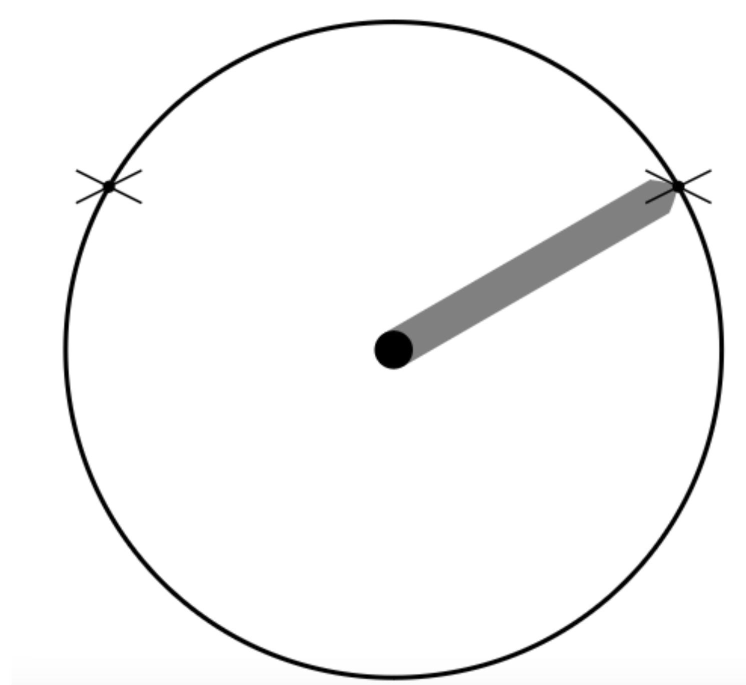

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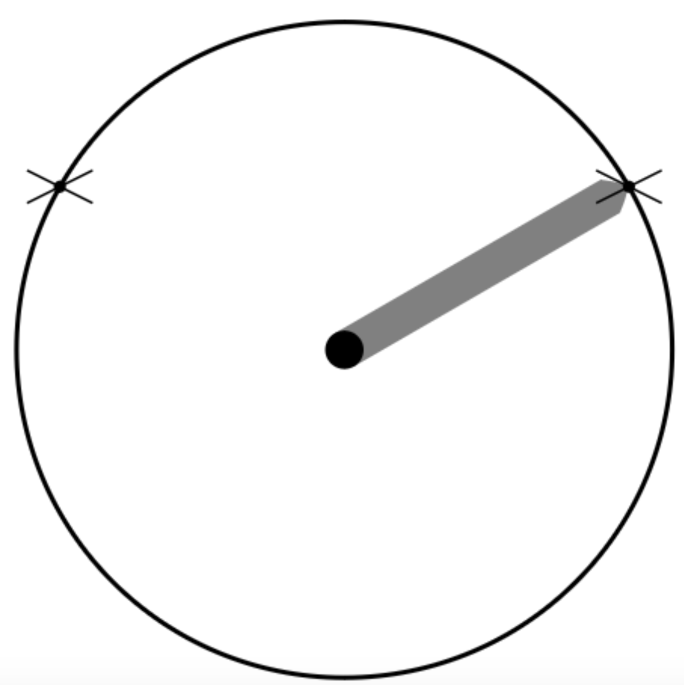

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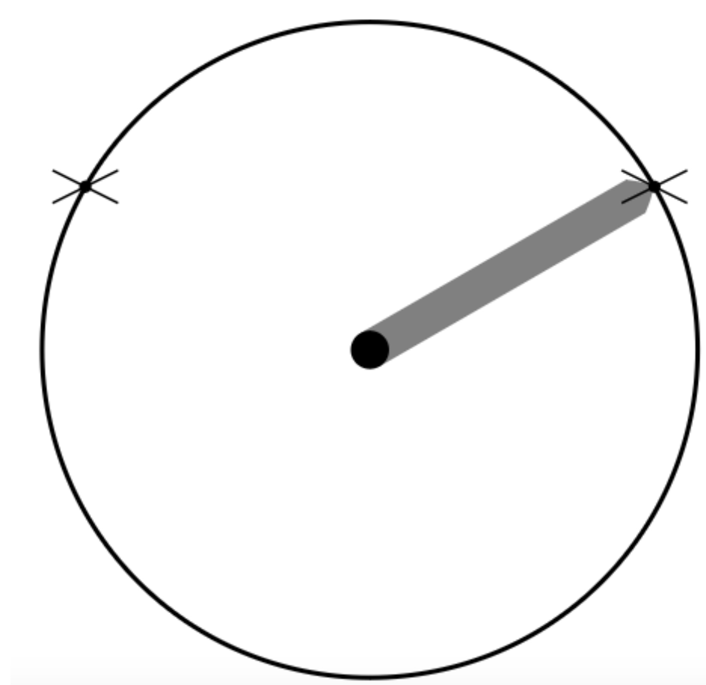

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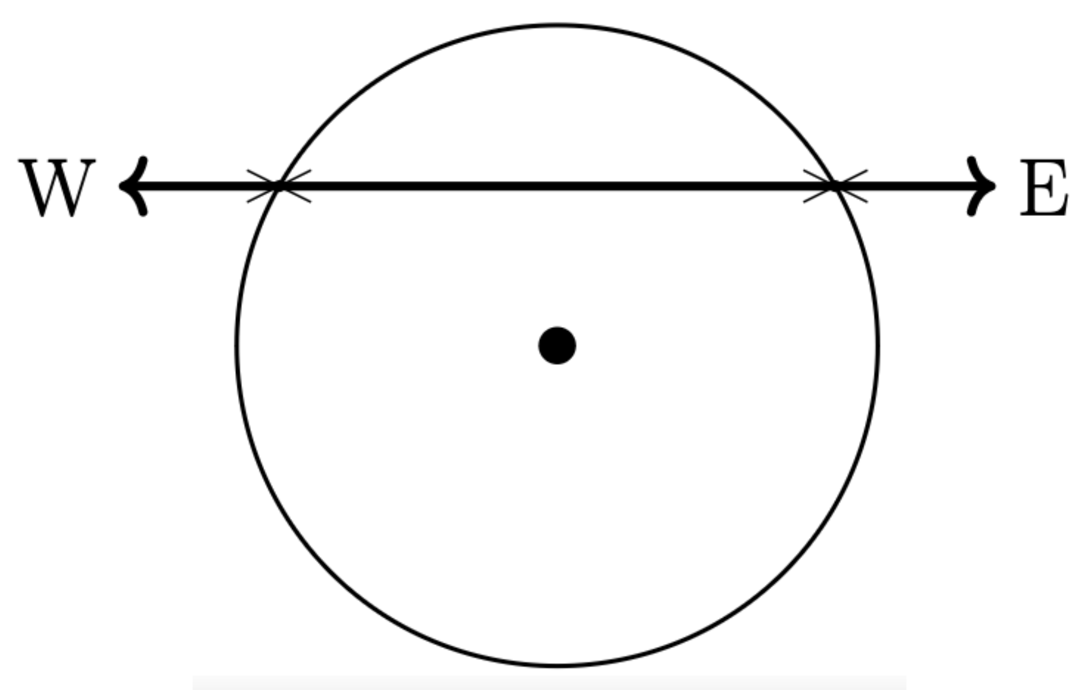

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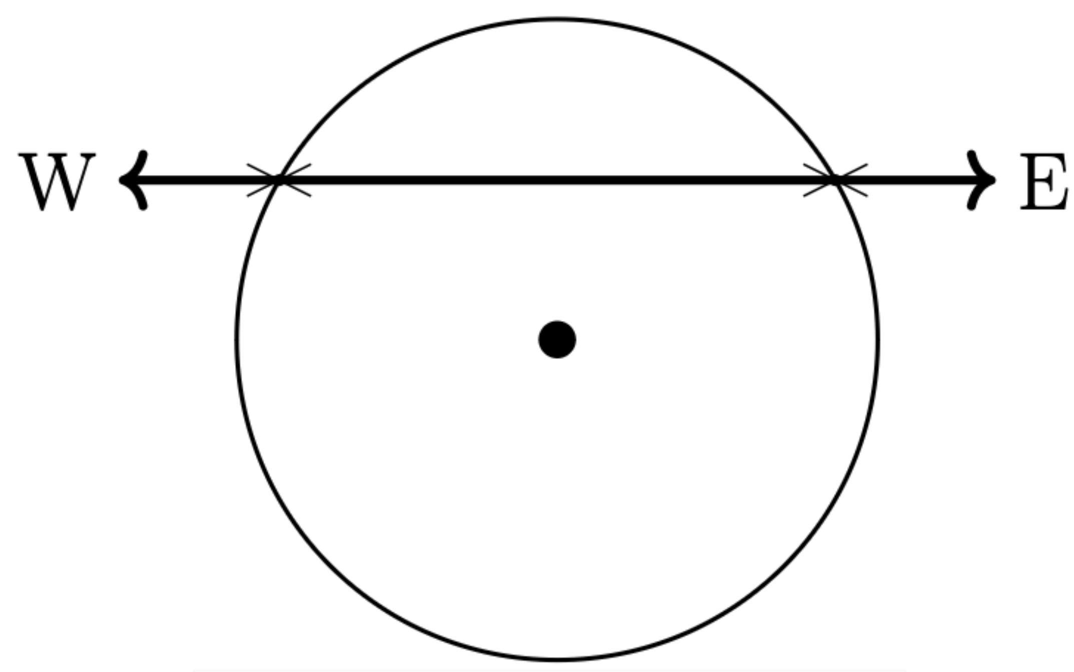
- Step 4: Connect the marks. Draw a perpendicular.



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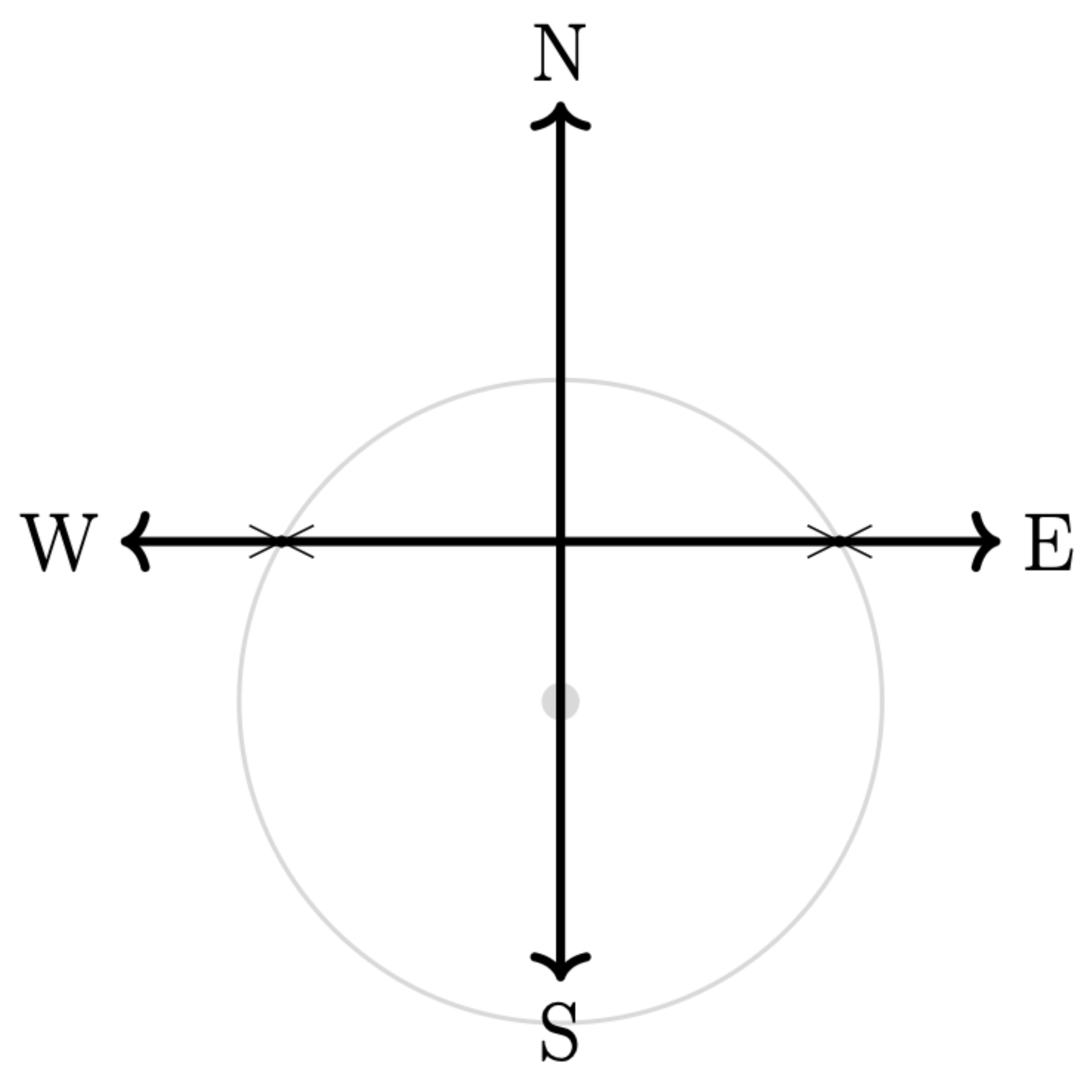

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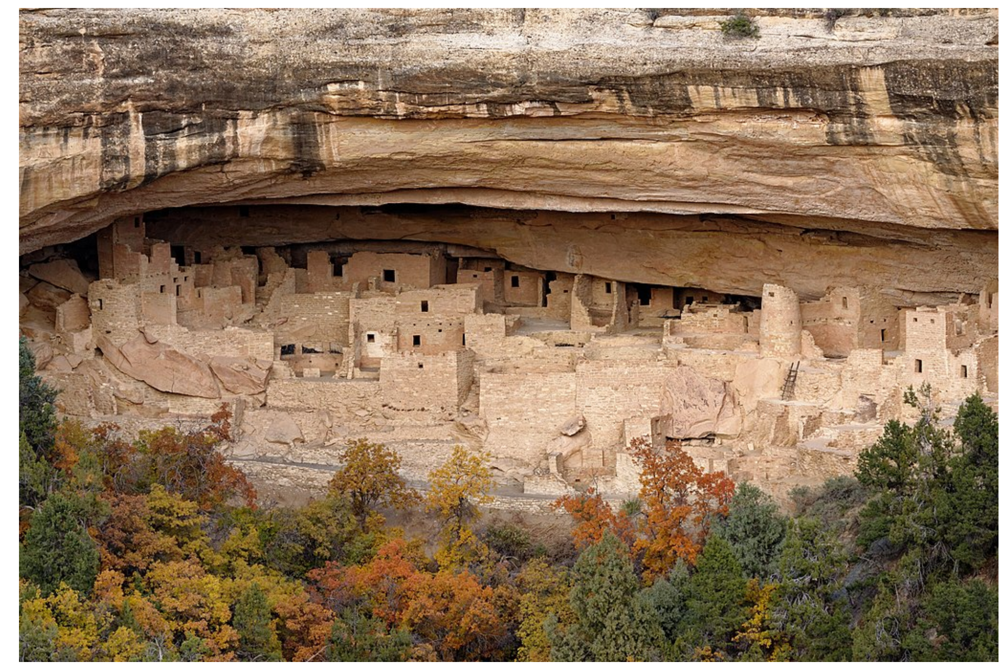

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- They constructed elaborate buildings that could be used to identify the solstices and the 18.6-year lunar cycle.
- But they left no records, so some speculation is required.