— Exercises —

Exercise 2.1. Express $\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \ldots, \frac{11}{12}$ as Egyptian fractions. Use as few parts as you can, and do not use the same part more than once (for example, you should not write $\frac{5}{12}$ as $\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}$). Write your results in both Egyptian and modern notation.

Recall

$1 = |$, $10 = \cap$, $100 =$ 𓍢 , $1{,}000 =$ [lotus]

$10{,}000 =$ [finger] , $100{,}000 =$ [tadpole] , $1{,}000{,}000 =$ [kneeling man]

Of course, Egyptians didn't write things using our modern "$\frac{1}{n}$" notation. They wrote $\frac{1}{n}$ by either placing a dot or an oval above the number n (the dot and oval symbols mean "part" or "mouth"). For instance:

$10 = \cap$ $\qquad \frac{1}{10} = \dot{\cap}$

$3 = |||$ $\qquad \frac{1}{3} = \overline{|||}$

This gave them a symbol for each unit fraction, but except for two exceptions, these were the only fractions they could write directly.[5] They then expressed all other fractions as a sum of distinct[6] unit fractions. For example, to write $\frac{2}{5}$, they would use the fact that

$$\frac{2}{5} = \frac{1}{3} + \frac{1}{15}.$$

And so, $\frac{2}{5}$ would be written as

$$\overline{|||}\ \overline{\cap|||||}\,.$$

A second drawback is the fact that each representation is not unique. One way to see this is to simply note that

$$* \quad \frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}.$$

Egyptian, Modern

$* \ \frac{1}{12} = \overline{\cap||}\ \overline{𓍢\cap\cap\cap\cap\cap||||||}\ , \ \frac{1}{13} + \frac{1}{156}$

equivalent

$\frac{1}{6}\ \ \frac{2}{12} = \frac{1}{6} = \frac{1}{6+1} + \frac{1}{6(6+1)}$

$\overline{|||||||}\ \overline{\cap\cap\cap\cap||}\ , \ \frac{1}{7} + \frac{1}{42}$

$\frac{1}{4}\ \ \frac{3}{12} = \frac{1}{4} = \frac{1}{4+1} + \frac{1}{4(4+1)}$

$\overline{|||||}\ \overline{\cap\cap}\ , \ \frac{1}{5} + \frac{1}{20}$

$\frac{1}{3}\ \ \frac{4}{12} = \frac{1}{3} = \frac{1}{12+1} + \frac{1}{12(12+1)}$

$\overline{\cap|||}\ \overline{𓍢\cap\cap\cap\cap\cap||||||}\ , \ \frac{1}{13} + \frac{1}{156}$

$\frac{5}{12} = \underset{\text{can reduce}}{\frac{4}{12}} + \frac{1}{12}$

$\overline{||||}\ \overline{\cap||}\ , \ \frac{1}{4} + \frac{1}{12}$

$\frac{1}{2}\ \ \frac{6}{12} = \frac{1}{12} + \frac{4}{12}\}\frac{1}{3}$

$\overline{\cap||}\ \overline{|||}\ , \ \frac{1}{12} + \frac{1}{3}$

$\frac{7}{12} = \frac{1}{12} + \frac{6}{12}\}\frac{1}{2}$

$\overline{\cap||}\ \overline{||}\ , \ \frac{1}{12} + \frac{1}{2}$

$\frac{2}{3}\ \ \frac{8}{12} = \frac{2}{12}\}\frac{1}{6}\ \ \frac{6}{12}\{\frac{1}{2}$

$\overline{||||||}\ \overline{||}\ , \ \frac{1}{6} + \frac{1}{2}$

$\frac{3}{4}\ \ \frac{9}{12} = \frac{3}{12}\{\frac{1}{4}\}\ \ \frac{6}{12}\{\frac{1}{2}\}$

$\overline{||||}\ \overline{||}\ , \ \frac{1}{4} + \frac{1}{2}$

$\frac{5}{6}\ \ \frac{10}{12} =$ Better to just reduce $\rightarrow \frac{2}{6}\{\frac{1}{3}\}\ \ \frac{3}{6}\{\frac{1}{2}\}$

$\overline{|||}\ \overline{||}\ , \ \frac{1}{3} + \frac{1}{2}$

$\frac{11}{12} = \frac{1}{12}\quad \frac{10}{12}\{\frac{1}{3}+\frac{1}{2}\}$

$\overline{\cap||}\ \overline{|||}\ \overline{||}\ , \ \frac{1}{12} + \frac{1}{3} + \frac{1}{2}$

Exercise 2.2. Read Sketch 4. Parts (a)-(d): Write the following numbers in expanded sexagesimal notation (eg., $71\frac{1}{4} = 1, 11; 15$), then translate into Babylonian notation.

(a) $120\ \frac{1}{3}$

60^1 60^0 Fractionals $\quad \frac{1}{3} = \frac{20}{60}$

$$\underbrace{(2 \times 60) + 0 + (20 \times \tfrac{1}{60})}_{\text{Expanded sexagesimal Notation}}$$

$$\underbrace{2.0;20}_{\text{Babylonian Notation (Cuneiform)}}$$

How to write the number zero 0?

Babylonians did not use the zero (this concept had not been invented), but from the 3rd century they used the symbol

(b) $\frac{1}{100}$

$$\frac{1}{100} = \frac{x}{60} \Rightarrow \frac{60}{100} = \frac{100x}{100}$$

$$\underbrace{x = 0.6 \ldots\ 100 > 60, \text{ so}}_{\text{Expanded sexagesimal Notation}}$$

$$\underbrace{0;0,36}_{\text{Babylonian Notation (Cuneiform)}}$$

The Babylons did not use a symbol to indicate where the fractional part begin. It was based on the context.

(c) $12\frac{1}{5}$

60^0 Fractionals

$$\underbrace{12 + (\tfrac{1}{5} \times 60)}_{\text{Expanded sexagesimal Notation}}$$

$0.2 \cdot 60 = 12$

$$\underbrace{12;12}_{\text{Babylonian Notation (Cuneiform)}}$$

(d) 81.23

60^1 60^0 Fractionals

$$(1 \times 60) + 21 + (0.23 \times 60)$$

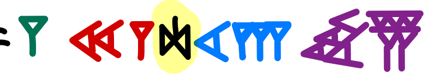
13.8

$0.8 \times 60 = 48$

Expanded sexagesimal Notation

$$\underbrace{1.21;13.48}_{\text{Babylonian Notation (Cuneiform)}}$$

Part (e): What notation would you add to the Babylonian system to make their numerals easier to decipher?

I would add a butterfly (⋈), so it would serve as my "floating point" as the separator between whole numbers and fractional values (eg 81.23 = )

Babylonian Notation

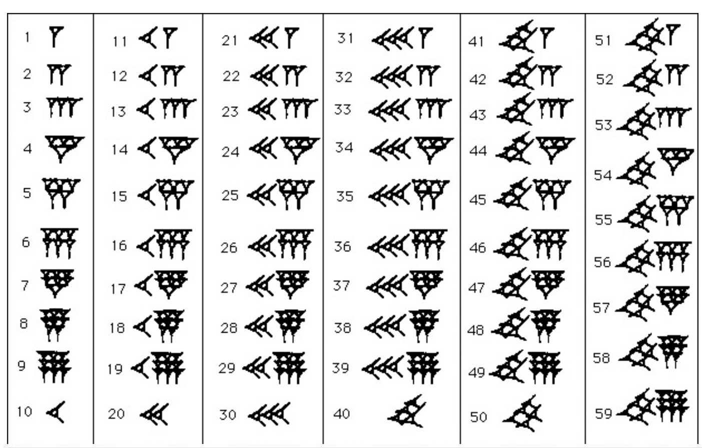
How to write the number zero 0?

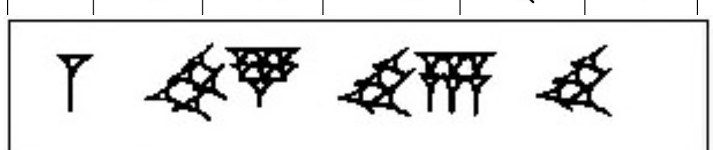
Babylonians did not use the zero (this concept had not been invented), but from the 3rd century they used the symbol

How to count using Babylonian numerals?

Babylonian numbers chart (base60)

0 (zero)		1		2		3		4	
5		6		7		8		9	
10		11		12		13		14	
15		16		17		18		19	
20		21		22		23		24	
25		26		27		28		29	
30		31		32		33		34	
35		36		37		38		39	
40		41		42		43		44	
45		46		47		48		49	
50		51		52		53		54	
55		56		57		58		59	





1,57,46,40 = 424000

Exercise 2.3. The prevalence of computers has led to increasingly common usage of decimals to express fractional parts. Give two advantages and two disadvantages to writing fractions in decimal form.[24]

Disadvantages

Loss of precision

edit fork download

```
result = 1.0 / 3.0
print("Result in Python:", result)
```

Success #stdin #stdout 0.03s 9528KB

stdin

Standard input is empty

Sometimes will produce a four

stdout

Result in Python: 0.3333333333333333

Precision to the 16th place

can't express in its true decimal form and may have rounding errors. It may also cause promotion/demotion issues in data

by extension →

Complex fractions

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```
result = 1.0 / 7.0
print("Result in Python:", result)
```

Success #stdin #stdout 0.03s 9512KB

stdin

Standard input is empty

stdout

Result in Python: 0.14285714285714285

Fractions converted to decimal may result into long, non-repeating, non-terminating decimals

Advantages

Ease of comparison: Decimals allow one to make comparison; for as fractions $\frac{1}{2}$, $\frac{4}{8}$, $\frac{8}{16}$ are the same, so representing as decimal it's easy to validate if $0.5 \equiv 1.5$

Compatibility: When it comes to calculations or finances the decimal system allows "data flow", for example, from one calculation software into a financial application (ie numbers to dollars)

Exercise 2.4. Some Babylonian texts say "seven has no inverse." What do you think they meant by this?

It is to denote that there is no whole number or fraction that can be multiplied by 7 to produce a whole number.

For example: $7 \times x = 1$

7 is a prime number, and so there was no way to divide 7 into equal parts. They can represent denominators of 7 as approximations in their sexagesimal notation, but not accurately

This holds true for other fractions like

$\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{11}$