Exam 1 Essay Questions

On the day of your exam, I will choose 4 of these 6 problems. You will have to write on 3 of the 4 chosen problems.

1. **Discuss the positional number system, the multiplicative grouping system, and the simple grouping system. Give an example of each system (using proper symbols) and write about their merits and drawbacks.**

2. **Discuss how ancient Egyptians wrote their integers and their fractions. Use their symbols and include examples. We also talked about tow methods the Egyptians used to multiply two numbers. Using one of these methods (your choice), show how many they would have multiplied 25 • 36. (Note: These numbers will be changed on exam day.)**

3. **Discuss math done in ancient China. Include a discussion of the Nine Chapters and the fang cheng method**

4. **State and prove the Pythagorean theorem. You may give any of the 300+ valid proofs**

5. **Ancient Indians found a way to "square a pair of squares" and to "square a rectangle." Describe what is meant by this, and show the procedure they used.**

6. **Discuss Euclid's Elements. Include a discussion on how it was structured and its historical impact on math. Also, show how Euclid bisected an angle using a straightedge and compass.**

1. Discuss the positional number system, the multiplicative grouping system, and the simple grouping system. Give an example of each system (using proper symbols) and write about their merits and drawbacks.

Multiplicative-Grouping Chinese	Simple-Grouping Egyptian	Positional-Grouping Hindu-Arabic
	𓆼𓍢𓍢𓍢𓎆𓏺	1311
七千三百九		7309
		310
		625
		1275

- Positional System: Each *position* is a power of 10.

Example:

$$\boxed{\begin{aligned} 3{,}854 &= 3 \cdot 1{,}000 + 8 \cdot 100 + 5 \cdot 10 + 4 \\ &= 3 \cdot 10^3 + 8 \cdot 10^2 + 5 \cdot 10^1 + 4 \end{aligned}}$$

Merits

Versatility
widely used today in everyday numbering decimal base 10 to computer binary base 2

Drawbacks

Notation Complexity:
Large numbers can be challenging to read & write; and, require notation to separate groupings 1,000

- Simple Grouping System: Add everything up.

$1 =$ 𓏺 , $10 =$ 𓎆 , $100 =$ 𓍢 , $1{,}000 =$ 𓆼

$10{,}000 =$ 𓂭 , $100{,}000 =$ 𓆐 , $1{,}000{,}000 =$ 𓁨

321 = 𓍢𓍢𓍢𓎆𓎆𓏺
= 𓍢𓍢𓍢𓏺𓎆𓎆
= 𓍢𓎆𓍢𓏺𓍢𓎆

Merits

Simple just add everything up

Drawbacks

Multiple ways to represent one number

- Multiplicative Grouping System: Have a character for each power of 10

Example: Chinese-Japanese numeral system

1 = 一	10 = 十	100 = 一百	1,000 = 一千
2 = 二	20 = 二十	200 = 二百	2,000 = 二千
3 = 三	30 = 三十	300 = 三百	3,000 = 三千
4 = 四	40 = 四十	400 = 四百	4,000 = 四千
5 = 五	50 = 五十	500 = 五百	5,000 = 五千
6 = 六	60 = 六十	600 = 六百	6,000 = 六千
7 = 七	70 = 七十	700 = 七百	7,000 = 七千
8 = 八	80 = 八十	800 = 八百	8,000 = 八千
9 = 九	90 = 九十	900 = 九百	9,000 = 九千
10 = 十	100 = 一百	1,000 = 一千	10,000 = 一万

Example: Chinese-Japanese numeral system

$$5{,}062 = 5 \cdot 1{,}000 + 6 \cdot 10 + 2$$
= 五千六十二

Merits

Intuitive for Multiples:
this system is based on grouping objects into equal sized sets. Making it highly intuitive for counting and representing multiples. Simplifies calculation with repeated additions

Drawbacks

Inefficient for Complex Calculations:
complex arithmetic operations like division or non-integer calculations can be cumbersome and less efficient than the Positional System

2. Discuss how ancient Egyptians wrote their integers and their fractions. Use their symbols and include examples. We also talked about tow methods the Egyptians used to multiply two numbers. Using one of these methods (your choice), show how many they would have multiplied 25 • 36. (Note: These numbers will be changed on exam day.)

Exercise 2.1. Express $\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \ldots, \frac{11}{12}$ as Egyptian fractions. Use as few parts as you can, and do not use the same part more than once (for example, you should not write $\frac{5}{12}$ as $\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}$). Write your results in both Egyptian and modern notation.

$$\frac{1}{12} =$$

$$\frac{2}{12} = (\frac{1}{6}) =$$

$$\frac{3}{12} = (\frac{1}{4}) =$$

$$\frac{4}{12} = (\frac{1}{3}) =$$

$$\frac{5}{12} =$$

$$\frac{6}{12} = (\frac{1}{2}) =$$

$$\frac{7}{12} =$$

$$\frac{8}{12} = (\frac{2}{3}) =$$

$$\frac{9}{12} = (\frac{3}{4}) =$$

$$\frac{10}{12} = (\frac{5}{6}) =$$

$$\frac{11}{12} =$$

Recall

1 = | , 10 = ∩ , 100 = 𓍢 , 1,000 = 𓆼

10,000 = 𓂭 , 100,000 = 𓆐 , 1,000,000 = 𓁨

Of course, Egyptians didn't write things using our modern "$\frac{1}{n}$" notation. They wrote $\frac{1}{n}$ by either placing a dot or an oval above the number n (the dot and oval symbols mean "part" or "mouth"). For instance:

$10 = \cap \qquad \frac{1}{10} = \dot{\cap}$

$3 = ||| \qquad \frac{1}{3} = \overset{\frown}{|||}$

This gave them a symbol for each unit fraction, but except for two exceptions, these were the only fractions they could write directly.[5] They then expressed all other fractions as a sum of distinct[6] unit fractions. For example, to write $\frac{2}{5}$, they would use the fact that

$$\frac{2}{5} = \frac{1}{3} + \frac{1}{15}.$$

And so, $\frac{2}{5}$ would be written as

$$\overset{\frown}{|||} \ \ \overset{\frown}{\cap|||||}\ .$$

A second drawback is the fact that each representation is not unique. One way to see this is to simply note that

$$\ast \quad \frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}.$$

$\underbrace{25}_{\text{1st column}} \cdot \underbrace{36}_{\text{2nd column}}$

Find a combination of numbers in the 1st column that adds up to the header (25)

1st column | 2nd column

1 → 36

2   72

4   144

8 → 288

→ 16 → 576

match the cardinality of the 1st column (5 doublings)

Next double would exceed 1st column #25

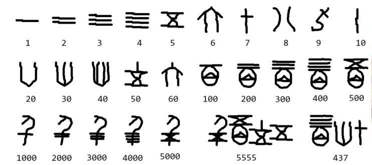
(Plus) +

(equals) = 900

3. Discuss math done in ancient China. Include a discussion of the Nine Chapters and the fang cheng method

Chinese Mathematics

- These bones contain numerals.
- Used a simple grouping system.



- We have learned about Chinese characters, multiplication table, and magic squares.
- Older than all these are *oracle bones*. These date back to the Shang Dynasty (1600–1050 BC).
- Bones from turtles and ox with writing on them, used to predict the future.
- They contain thousands characters, older than modern Chinese characters.

By the Shang dynasty (1600–1050 BC), the Chinese had developed:

1. A real number system that includes significantly large and negative numbers
2. More than one numeral system (base 2 and base 10)
3. Basic algebra
4. Basic geometry
5. Basic trigonometry

- The most important book in Chinese math history is *The Nine Chapters on the Mathematical Art*.
- It was written by many authors over hundreds of years, from ~1000BC to ~100 BC. In 263 AD, Liu Hui published an annotated edition.
- It is organized like the *Book on Numbers and Computation*. That is, a problem is stated, an answer is given, and a procedure is presented for how to solve the problem and similar problems.

- The *Book on Numbers and Computation* only considered systems of 2 linear equations with 2 unknowns.

chapter 8

- In the *Nine Chapters*, a procedure called *fang cheng* is presented which can solve larger systems. The examples given solved systems with 3 equations and 3 unknowns, 4 equations and 4 unknowns, and 5 equations and 5 unknowns.

via use of the counting board

- *Fang cheng* is essentially Gaussian elimination (discovered 2,000 years before Gauss!)

- The only difference between *fang cheng* and Gaussian elimination is in the final step (back substitution).
- *Fang cheng* uses a more complicated approach so that there are only integers until the very end. This is advantageous when solving the problem using a counting board.
- Gaussian elimination is happy to introduce fractions as soon as back-substitution begins.

In dividing coins, if each person receives 2, there is a surplus of 3; if each person receives 3, there is a shortage of 2. It is asked how many persons and coins are there? The answer: 5 persons and 13 coins. Excess and deficit: cross multiply the denominators to become the dividend; the numerators are added to become the divisor. Both excess or deficit: the numerators are cross multiplied by the denominators and each is set aside. The lesser of the numerators is subtracted from the greater of the numerators and the remainder is the divisor. Take the deficit as the dividend.

- Hint: Use variables. If x_1 is the number of people, and x_2 is the number of coins, then:

$$2x_1 + 3 = x_2$$
$$3x_1 - 2 = x_2$$

- Rewriting,

$$2x_1 - x_2 = -3$$
$$3x_1 - x_2 = 2$$

- This was the first (notable) linear algebra problem in history.

Chapter 7

Excess and deficit

- Today it can be written with matrices:

$$\begin{pmatrix} a & -1 \\ c & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -b \\ d \end{pmatrix}$$

- Finding the augmented matrix and row reducing gives the solutions:

$$x_1 = \frac{b+d}{c-a} \quad \text{and} \quad x_2 = \frac{bc+da}{c-a}$$

Example

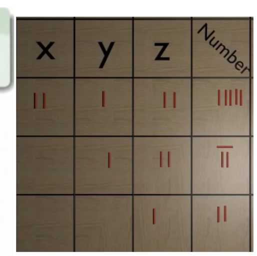
Assume the first, second, and third columns correspond to x, y, z. Interpret the equations shown.

The first row shows $2x$, $1y$, $2z$, and 5, so the equation is

$$2x + 1y + 2z = 5$$

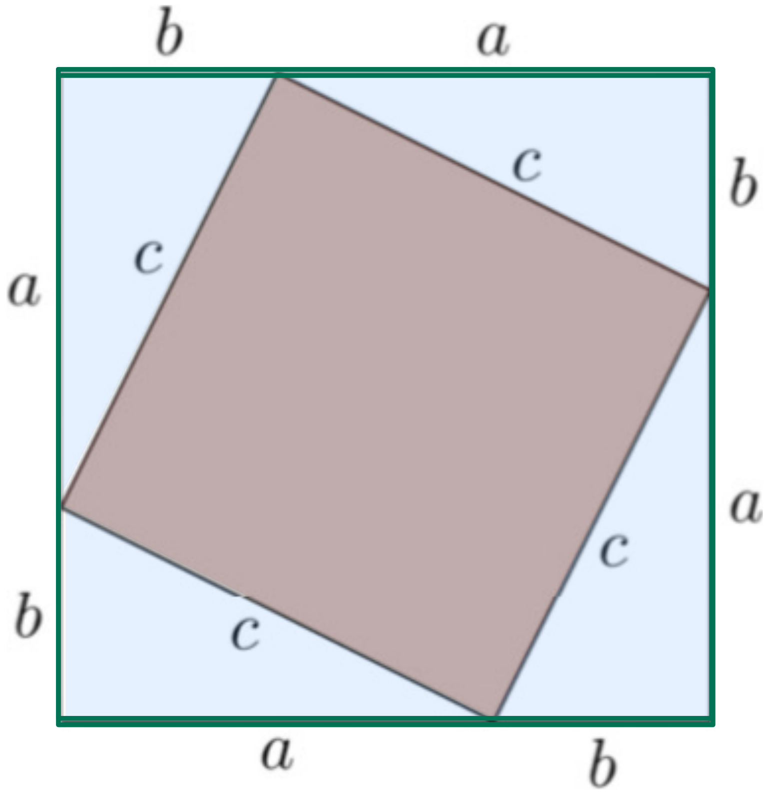
The second row shows no xs, but $1y$, $2z$, and 7, so the equation is

$$y + 2z = 7$$



4. State and prove the Pythagorean theorem. You may give any of the 300+ valid proofs

Pythagorean theorem, which says that if "a" and "b" are the lengths of the legs of a right triangle, and "c" the length of the hypotenuse, then $a^2 + b^2 = c^2$



1) Draw a square
2) Draw a smaller square inside the Larger square
3) Label all the sides with 'a', 'b', and 'c'

4) Law of Squares 2 legs of the outer square multiplied together gives the area of the larger square

5) area of the inner square and the area of all 4 right triangles bounded by the outer square

$$\underbrace{(a+b)^2}_{\text{Law of squares}} = \underbrace{c^2}_{\text{inner - square}} + \underbrace{\left(4 \cdot \frac{1}{2} a \cdot b\right)}_{\text{4 right triangles bounded}}$$

$$a^2 + 2ab + b^2 = c^2 + 2ab$$

$$a^2 \cancel{+ 2ab} + b^2 = c^2 \cancel{+ 2ab}$$

$$\therefore\ a^2 + b^2 = c^2$$

5. Ancient Indians found a way to "square a pair of squares" and to "square a rectangle." Describe what is meant by this, and show the procedure they used.

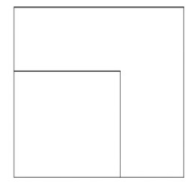
– 1. Squaring a pair of squares –

Task 1: Given a pair of squares, create a third square whose area is equal to the sum of the other two.

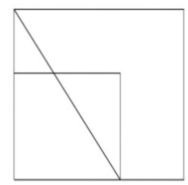
If this already made you think about the geometric form of the Pythagorean theorem, then good job! In essence, what you want to do is create a triangle whose legs are exactly as long as the sides of your two squares. Once you have such a triangle, you simply build a square on the hypotenuse, as we discussed with Pythagoras and Euclid.

The Sulbasutras' instructions for how to accomplish Task 1 is very similar to the idea we just discussed. Indeed, let's suppose you had two squares.

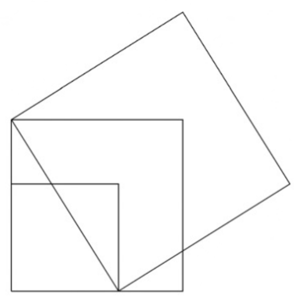
Imagine you placed the smaller square inside the larger one, so their bottom-left corners aligned.



Next, connect the two vertices shown below.

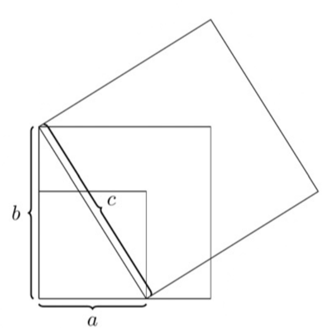


And finally, construct a square on that line segment.[10]



Do you see the triangle? Can you already see why the Pythagorean theorem proves that this new square satisfies Task 1?

This constructed square does indeed have an area that is equal to the sum of the areas of the two original squares. And while the Sulbasutras do not prove that this is true, its proof boils down to an application of the Pythagorean theorem. Indeed, consider labeling the triangle like this:



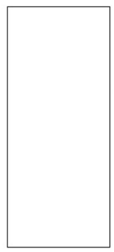
Then by the Pythagorean theorem, $a^2 + b^2 = c^2$, and since this diagram shows that a^2 and b^2 are the areas of the original two squares, and c^2 is the area of the new square, the construction did indeed work.

1. Ancient Indians found a way to “square a pair of squares” and to “square a rectangle.” Describe what is meant by this, and show the procedure they used.

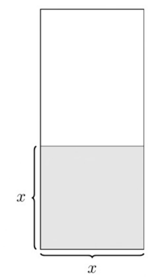
– 2. Squaring[11] a rectangle. –

Task 2: Given a rectangle, create a square of the same area.

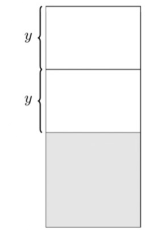
Baudhayana’s Sulbasutra solved this in the following way.[12] Suppose you were given a rectangle:



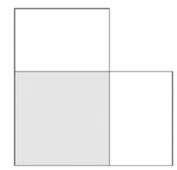
Form a square on the bottom of this rectangle.[13]



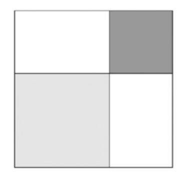
Next, chop the top (non-shaded) rectangle in two.



This created two copies of a rectangle. Rotate the top of these rectangles 90° and move it onto the right side of the shaded square.

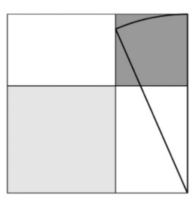


Then fill in the gap to form a square.

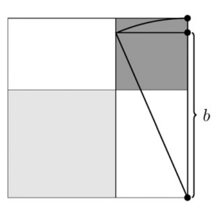


Notice that until we added the darker square, the total area had been preserved at each stage. So this new square does not have the same area as the original rectangle—it overshoots that answer by the area of that dark square.

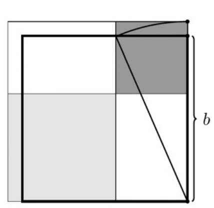
To create a square of the correct area, there is one last thing to do. Using your compass, draw an arc that is centered on the lower-right corner, begins at the upper-right corner, and sweeps left until it meets the vertical line to its left.



Look where this arc meets the vertical line—this is an important point. Draw a horizontal line segment from this point until it meets the line on its right.

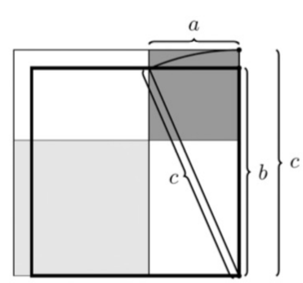


The distance b labeled above will be the height of our final square. Now we simple create a $b \times b$ square off of it.



This square is the one we are after. Indeed, this square has the same area as the original rectangle, and the reason comes down to the Pythagorean theorem. Let a be the side-length of the small dark square in the upper-right corner. And let c be the length of the hypotenuse of the triangle (and by following the arc, this also means c is the height of the entire big square).

Here is that picture:



Since this trianle is a right triangle, we see that $a^2 + b^2 = c^2$. Or, $b^2 = c^2 - a^2$ To summarize, the whole square above (area $= c^2$) overshoots our desired area by the area in that dark square (area $= a^2$). So our goal was to construct a square of area $c^2 - a^2$. And how about that—we just constructed a $b \times b$ square and showed that $b^2 = c^2 - a^2$. Thus, this constructed square does indeed have the area of the original rectangle.

While the Sulbasutras contained none of this justification, it is a reasonable guess that some Vedic mathematicians had a justification in mind. That said, these mathematicians did err in our third and final example. Again, the goal was to create a new shape with the same area as a shape you already have.

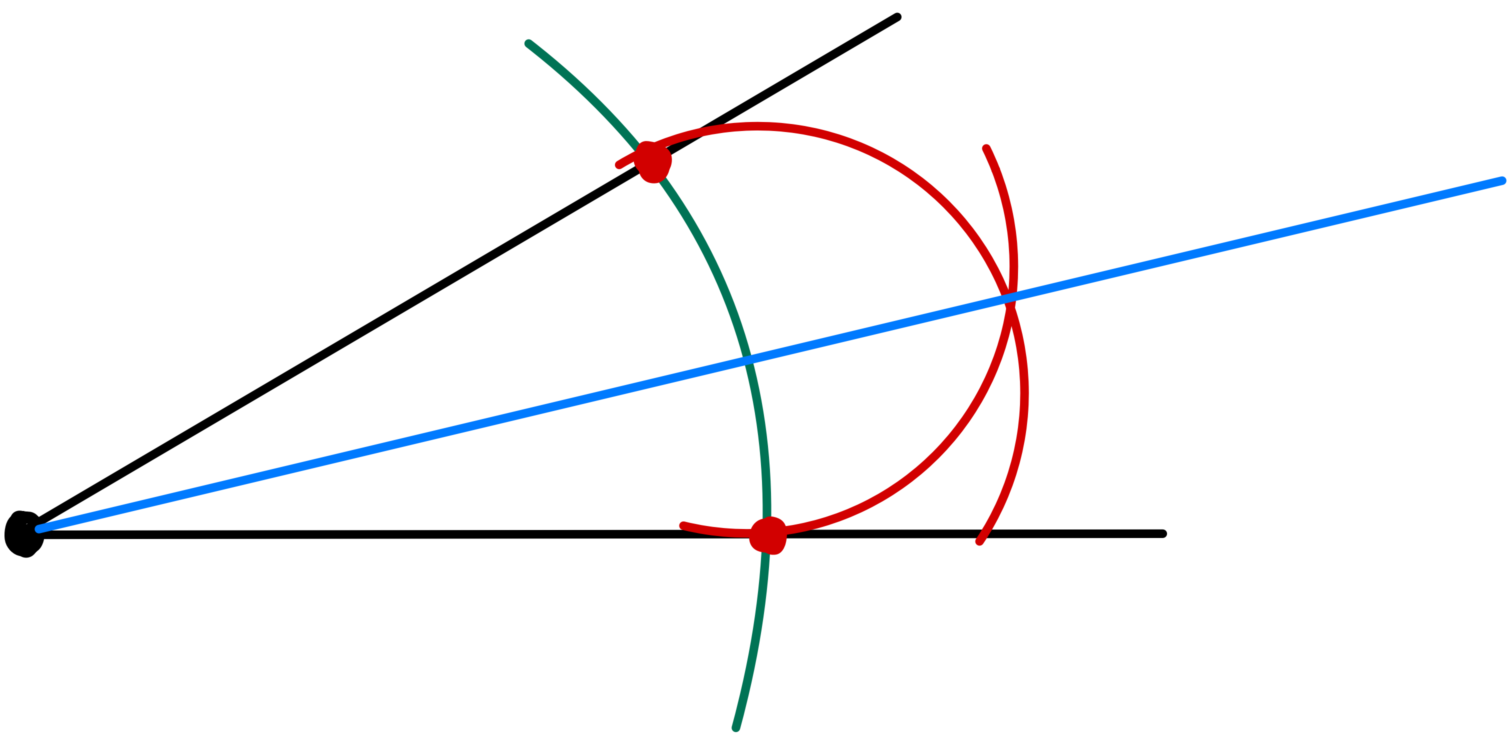
6. **Discuss Euclid's Elements. Include a discussion on how it was structured and its historical impact on math. Also, show how Euclid bisected an angle using a straightedge and compass.**

The Elements

- Euclid wrote the *Elements*, which is the most important book in math history. Much of it is not his original work—it is compiled from other sources.
- The major sources were likely the Pythagoreans, Hippocrates and Eudoxus. Many of the proofs are believed to be original to Euclid.
- Its success is due to its logical and axiomatic presentation. This deductive style is the central approach to mathematics today.

- The text was the primary math textbook for *two thousand years*.
- It is divided into 13 "books" on topics from geometry, algebra and number theory.
- Book I is devoted to propositions from planar geometry.
- Euclid's postulates allow him to use a straightedge and compass, which play a central role.

Book I.9



1. **Discuss the positional number system, the multiplicative grouping system, and the simple grouping system. Give an example of each system (using proper symbols) and write about their merits and drawbacks.**

2. Discuss how ancient Egyptians wrote their integers and their fractions. Use their symbols and include examples. We also talked about tow methods the Egyptians used to multiply two numbers. Using one of these methods (your choice), show how many they would have multiplied 25 • 36. (Note: These numbers will be changed on exam day.)

3. Discuss math done in ancient China. Include a discussion of the Nine Chapters and the fang cheng method

4. **State and prove the Pythagorean theorem. You may give any of the 300+ valid proofs**

5. Ancient Indians found a way to "square a pair of squares" and to "square a rectangle." Describe what is meant by this, and show the procedure they used.

6. **Discuss Euclid's Elements. Include a discussion on how it was structured and its historical impact on math. Also, show how Euclid bisected an angle using a straightedge and compass.**

1. Discuss the positional number system, the multiplicative grouping system, and the simple grouping system. Give an example of each system (using proper symbols) and write about their merits and drawbacks.

Multiplicative-Grouping	Simple-Grouping	Positional-Grouping
Chinese	Egyptian	Hindu-Arabic
一千三百十一 ($\underbrace{一千}_{1000}\underbrace{三百}_{300}\underbrace{十}_{10}\underbrace{一}_{1}$)	1311: $\underbrace{𓆼}_{1000}\underbrace{𓍢𓍢𓍢}_{300}\underbrace{𓎆}_{10}\underbrace{𓏺}_{1}$	1311
7309: 七千三百九 ($\underbrace{七千}_{7000}\underbrace{三百}_{300}\underbrace{九}_{9}$)	𓆼𓆼𓆼𓆼𓆼𓆼𓆼𓍢𓍢𓍢𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺	7309
三百十 ($\underbrace{三百}_{300}\underbrace{十}_{10}$)	𓍢𓍢𓍢𓎆	310
六百二十五 ($\underbrace{六百}_{600}\underbrace{二十}_{20}\underbrace{五}_{5}$)	𓍢𓍢𓍢𓍢𓍢𓍢𓎆𓎆𓏺𓏺𓏺𓏺𓏺	625
一千二百七十五 ($\underbrace{一千}_{1000}\underbrace{二百}_{200}\underbrace{七十}_{70}\underbrace{五}_{5}$)	𓆼𓍢𓍢𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺	1275

- Positional System: Each *position* is a power of 10.

Example: $$\boxed{\begin{aligned}3{,}854 &= 3\cdot 1{,}000 + 8\cdot 100 + 5\cdot 10 + 4\\ &= 3\cdot 10^3 + 8\cdot 10^2 + 5\cdot 10^1 + 4\end{aligned}}$$

Merits

Versatility

Widely used today in everyday numbering decimal base 10 to computer binary base 2

Drawbacks

Notation Complexity:

Large numbers can be challenging to read & write; and, require notation to separate groupings 1,000

- Simple Grouping System: Add everything up.

1 = 𓏺 , 10 = 𓎆 , 100 = 𓍢 , 1,000 = 𓆼

10,000 = 𓂭 , 100,000 = 𓆐 , 1,000,000 = 𓁨

$$\begin{aligned}321 &= 𓍢𓍢𓍢𓎆𓎆𓏺\\ &= 𓍢𓍢𓍢𓏺𓎆𓎆\\ &= 𓍢𓎆𓍢𓏺𓍢𓎆\end{aligned}$$

Merits

Simple just add everything up

Drawbacks

Multiple ways to represent one number

- Multiplicative Grouping System: Have a character for each power of 10

Example: Chinese-Japanese numeral system

1 = 一	10 = 十	100 = 一百	1,000 = 一千
2 = 二	20 = 二十	200 = 二百	2,000 = 二千
3 = 三	30 = 三十	300 = 三百	3,000 = 三千
4 = 四	40 = 四十	400 = 四百	4,000 = 四千
5 = 五	50 = 五十	500 = 五百	5,000 = 五千
6 = 六	60 = 六十	600 = 六百	6,000 = 六千
7 = 七	70 = 七十	700 = 七百	7,000 = 七千
8 = 八	80 = 八十	800 = 八百	8,000 = 八千
9 = 九	90 = 九十	900 = 九百	9,000 = 九千
10 = 十	100 = 一百	1,000 = 一千	10,000 = 一万

Example: Chinese-Japanese numeral system

$$\boxed{\begin{aligned}5{,}062 &= 5\cdot 1{,}000 + 6\cdot 10 + 2\\ &= 五千六十二\end{aligned}}$$

Merits

Intuitive for Multiples:

this system is based on grouping objects into equal sized sets. Making it highly intuitive for counting and representing multiples. Simplifies calculation with repeated additions

Drawbacks

Inefficient for Complex Calculations:

complex arithmetic operations like division or non-integer calculations can be cumbersome and less efficient than the Positional System

2. Discuss how ancient Egyptians wrote their integers and their fractions. Use their symbols and include examples. We also talked about tow methods the Egyptians used to multiply two numbers. Using one of these methods (your choice), show how many they would have multiplied 25 • 36. (Note: These numbers will be changed on exam day.)

Exercise 2.1. Express $\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \dots, \frac{11}{12}$ as Egyptian fractions. Use as few parts as you can, and do not use the same part more than once (for example, you should not write $\frac{5}{12}$ as $\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}$). Write your results in both Egyptian and modern notation. Egyptian, Modern

$\frac{1}{12} = \overline{\cap||}$, $\frac{1}{12}$

$\frac{2}{12} = (\frac{1}{6}) = \frac{1}{6+1} + \frac{1}{6(6+1)}$

$\overline{|||||||}\ \overline{\cap\cap\cap\cap||}$, $\frac{1}{7}+\frac{1}{42}$

$\frac{3}{12} = (\frac{1}{4}) = \frac{1}{4+1} + \frac{1}{4(4+1)}$

$\overline{|||||}\ \overline{\cap\cap}$, $\frac{1}{5}+\frac{1}{20}$

$\frac{4}{12} = (\frac{1}{3}) = \frac{1}{12+1} + \frac{1}{12(12+1)}$

$\overline{\cap|||}$ 𓍢$\overline{\cap\cap\cap\cap\cap||||||}$, $\frac{1}{13}+\frac{1}{156}$

can reduce

$\frac{5}{12} = \frac{4}{12} + \frac{1}{12}$

$\overline{||||}\ \overline{\cap||}$, $\frac{1}{4}+\frac{1}{12}$

$\frac{6}{12} = (\frac{1}{2}) = \frac{1}{12} + \frac{4}{12}\}\frac{1}{3}$

$\overline{\cap||}\ \overline{|||}$, $\frac{1}{12}+\frac{1}{3}$

$\frac{7}{12} = \frac{1}{12} + \frac{6}{12}\}\frac{1}{2}$

$\overline{\cap||}\ \overline{||}$, $\frac{1}{12}+\frac{1}{2}$

$\frac{8}{12} = (\frac{2}{3}) = \frac{2}{12}\to\frac{1}{6} + \frac{6}{12}\to\frac{1}{2}$ $\overline{||||||}\ \overline{||}$, $\frac{1}{6}+\frac{1}{2}$

$\frac{9}{12} = (\frac{3}{4}) = \frac{3}{12}\{\frac{1}{4}\}\ \frac{6}{12}\{\frac{1}{2}\}$ $\overline{||||}\ \overline{||}$, $\frac{1}{4}+\frac{1}{2}$

$\frac{10}{12} = (\frac{5}{6}) =$ Better to just reduce $\to \frac{2}{6}\{\frac{1}{3}\}\ \frac{3}{6}\{\frac{1}{2}\}$

$\overline{|||}\ \overline{||}$, $\frac{1}{3}+\frac{1}{2}$

$\frac{11}{12} = \frac{1}{12}\quad \frac{10}{12}\{\frac{1}{3}+\frac{1}{2}\}$ Better to just reduce $\to \frac{2}{6}\{\frac{1}{3}\}\ \frac{3}{6}\{\frac{1}{2}\}$

$\overline{\cap||}\ \overline{|||}\ \overline{||}$, $\frac{1}{12}+\frac{1}{3}+\frac{1}{2}$

Recall

1 = 𓏺 , 10 = 𓎆 , 100 = 𓍢 , 1,000 = 𓆼

10,000 = 𓂭 , 100,000 = 𓆐 , 1,000,000 = 𓁨

Of course, Egyptians didn't write things using our modern "$\frac{1}{n}$" notation. They wrote $\frac{1}{n}$ by either placing a dot or an oval above the number n (the dot and oval symbols mean "part" or "mouth"). For instance:

10 = ∩ $\qquad \frac{1}{10} = \dot{\cap}$

3 = ||| $\qquad \frac{1}{3} = \overline{|||}$

This gave them a symbol for each unit fraction, but except for two exceptions, these were the only fractions they could write directly.[5] They then expressed all other fractions as a sum of distinct[6] unit fractions. For example, to write $\frac{2}{5}$, they would use the fact that

$$\frac{2}{5} = \frac{1}{3} + \frac{1}{15}.$$

And so, $\frac{2}{5}$ would be written as

$$\overline{|||}\ \overline{\cap|||||}\ .$$

A second drawback is the fact that each representation is not unique. One way to see this is to simply note that

$$\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}.$$

$25 \cdot 36$ — Find a combination of numbers in the 1st column that adds up to the header (25)

1st column | 2nd column

match the cardinality of the 1st column (5 doublings)

1st column	2nd column
1	36
2	72
4	144
8	288
16	576

Next double would exceed 1st column #25

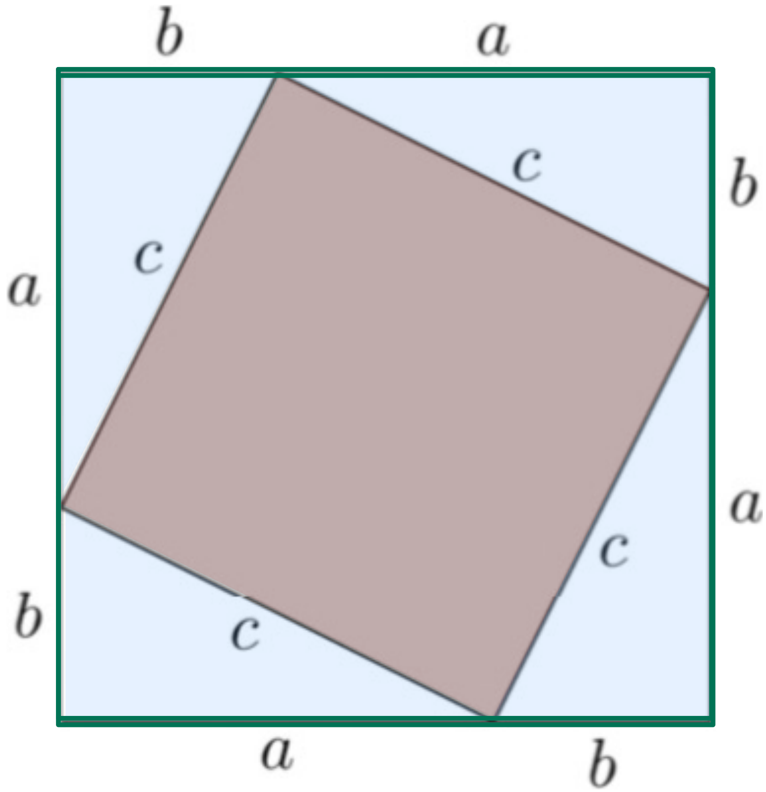
(Plus) +

(equals) = 900

3. Discuss math done in ancient China. Include a discussion of the Nine Chapters and the fang cheng method

4. State and prove the Pythagorean theorem. You may give any of the 300+ valid proofs

Pythagorean theorem, which says that if "a" and "b" are the lengths of the legs of a right triangle, and "c" the length of the hypotenuse, then $a^2 + b^2 = c^2$



1) Draw a square
2) Draw a smaller square inside the Larger square
3) Label all the sides with 'a', 'b', and 'c'
4) Law of Squares 2 legs of the outer square multiplied together gives the area of the larger square
5) area of the inner square and the area of all 4 right triangles bounded by the outer square

$$\underbrace{(a+b)^2}_{\text{Law of squares}} = \underbrace{c^2}_{\text{inner - square}} + \underbrace{\left(4 \cdot \frac{1}{2} a \cdot b\right)}_{\text{4 right triangles bounded}}$$

$$a^2 + 2ab + b^2 = c^2 + 2ab$$

$$a^2 \cancel{+ 2ab} + b^2 = c^2 \cancel{+ 2ab}$$

$$\therefore\ a^2 + b^2 = c^2$$

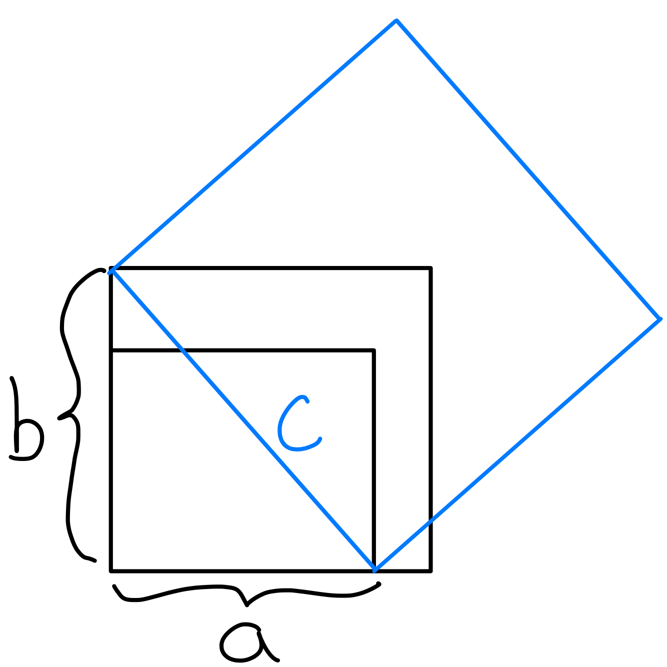
5. Ancient Indians found a way to "square a pair of squares" and to "square a rectangle." Describe what is meant by this, and show the procedure they used.

The concept of "squaring a pair of squares" and to "square a rectangle" are related to Vedic mathematics where both concepts are to generate another Shape that has an equal area equal to that of anothe or is the sum of two others

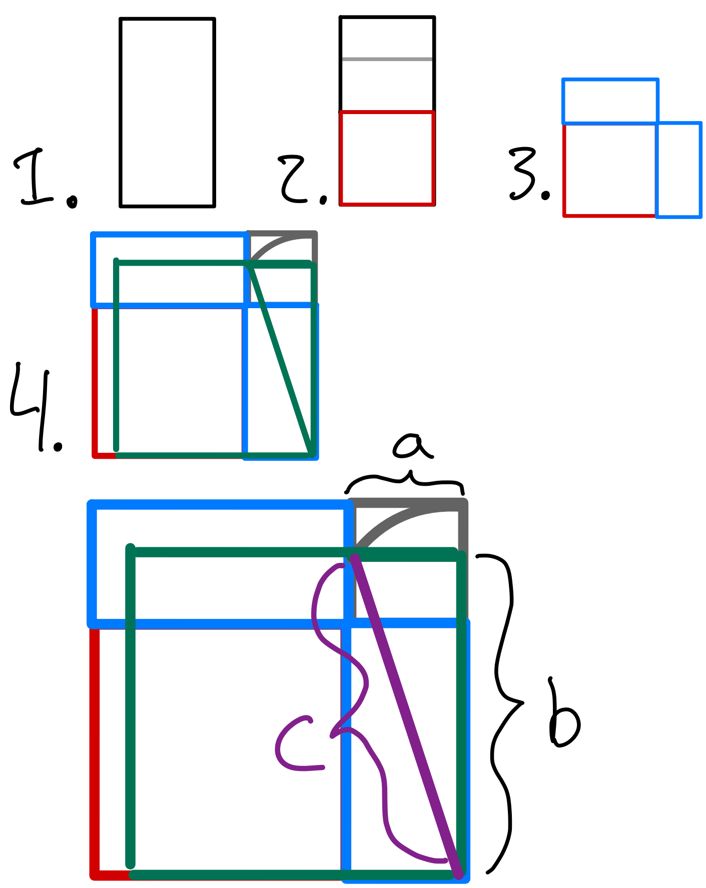
In the case of Squaring a pair of Squares Mathematically if you have two squares with side lengths 'a' and 'b', the goal is to find a square with a side length 'c' such that: $a^2+b^2=c^2$

In the case of "Squaring a Rectangle" involves finding a square whose area is equal to that of a given rectangle Mathematically, if you have a rectangle with sides of length 'a' and 'b', the goal is to find a square with a side length 'c' such that: $c^2=a*b$

"Squaring a pair of squares"



"square a rectangle"



6. **Discuss Euclid's Elements. Include a discussion on how it was structured and its historical impact on math. Also, show how Euclid bisected an angle using a straightedge and compass.**

Book I.9