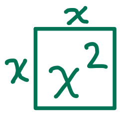
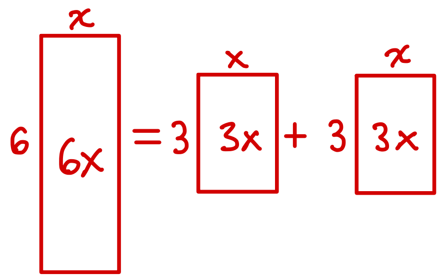
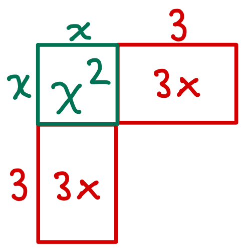
Exercise 6.1. Using al-Khwārizmī's approach that was discussed in this chapter, find the positive solution to $x^2 + 6x = 112$. Draw all of the needed diagrams.



rooted in geometry "x^2" represents the area of a square with side length x



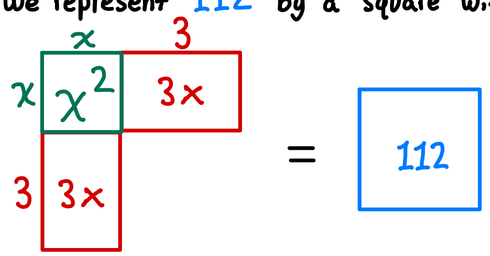
the "$6x$" term is the area of a rectangle with side lengths 6 and x.
Or the combination of two rectangles, each with side lengths 3 and x



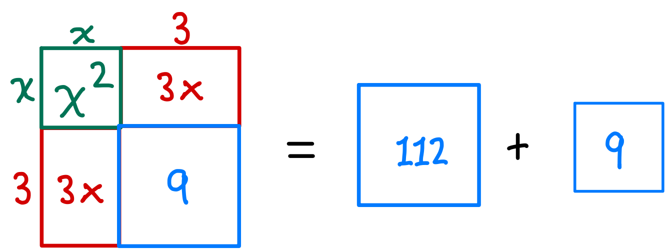
Combine the square with these two rectangles. We get a shape with area $x^2 + 6x$

* This is one side of the equation that we are trying to solve.

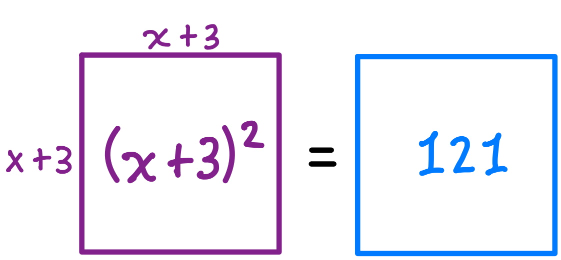
The equation in full is $x^2 + 6x = 112$
We represent "112" by a square with that area.



Now we complete the square...
Balance the equation/drawing



Completing the square allows us to rewrite as $x^2 + 6x$ as a partial square, and by completing the square we then gather all x-terms into a single squared equation.



With Al-Khwarizmi's method, by completing the square, we can demonstrate via geometry to transform the original equation $x^2 + 6x = 112$ into an equivalent equation $(x+3)^2 = 121$. Now we can simply take the square root to solve...

$$(x+3)^2 = 121$$

$$x + 3 = 11$$

$$x = 8$$

Exercise 6.9. In his math duel with Tartaglia, Antonio Fior posed the following problems.

(a) His first problem was this:

> Find the number which added to its cube root gives 6. $\rightarrow$ $x + \sqrt[3]{x} = 6$

Solve Fior's problem.

$$x + \sqrt[3]{x} = 6$$
$$\Rightarrow x = 6 - \sqrt[3]{x}$$
$$\Rightarrow (x)^3 = (6 - \sqrt[3]{x})^3$$
$$\Rightarrow x^3 = 6^3 - 3\cdot 6^2 \sqrt[3]{x} + 3\cdot 6(\sqrt[3]{x})^2 - (\sqrt[3]{x})^3$$
$$\Rightarrow x^3 = 216 - 108\sqrt[3]{x} + 18x - x$$
$$\Rightarrow x^3 + x = 216 - 108\sqrt[3]{x} + 18$$
$$\Rightarrow x^3 + x - 18x - 216 = -108\sqrt[3]{x}$$
$$\Rightarrow \frac{x^3 + x - 18x - 216}{-108} = \sqrt[3]{x} \Rightarrow \left(\frac{x^3 + x - 18x - 216}{-108}\right)^3 = x$$

(b) His second problem was this:

> Find two numbers in double proportion [x and $2x$] such that if the square of the larger number is multiplied by the lesser, and this product is added to the sum of the two original two numbers, the result is 40.

Solve Fior's problem. $(x^2)(2x) + (x + 2x) = 40$

(c) His ninth problem was this:

> Find a number which added to four times its cube root gives 17.

Solve Fior's problem. $x + 4\sqrt[3]{x} = 17$

(d) His eleventh problem was this:

> Divide twenty into two parts such that one is the cube root of the other.

Solve Fior's problem.