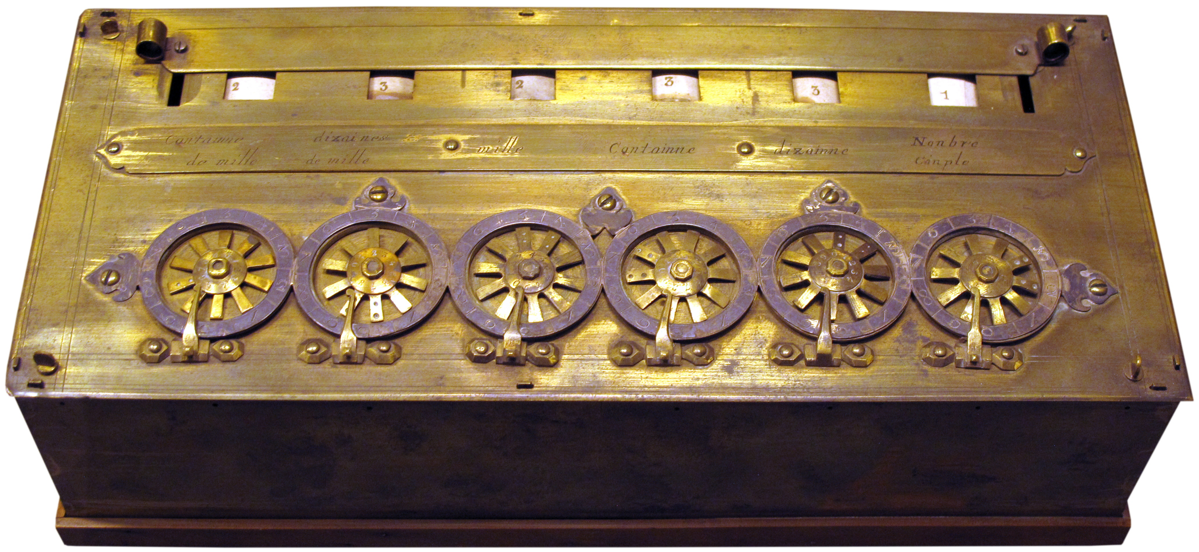
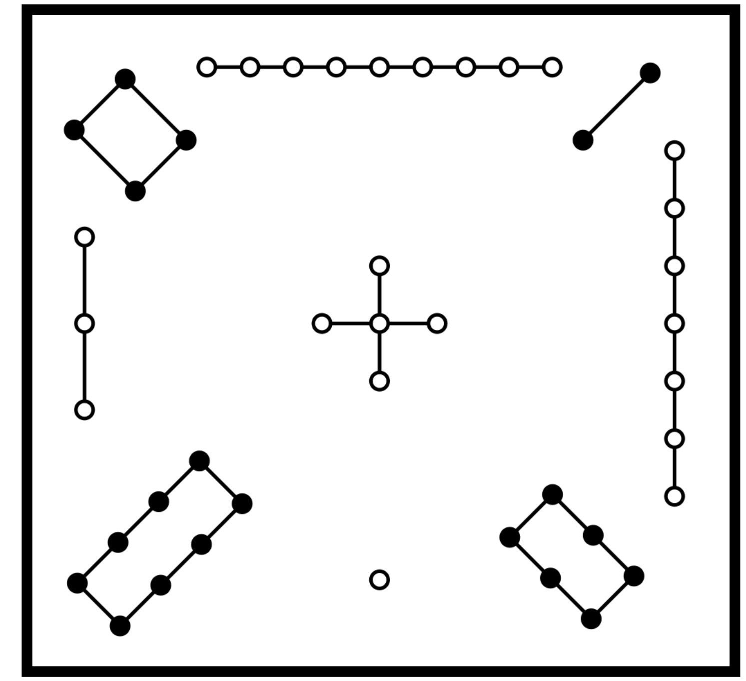
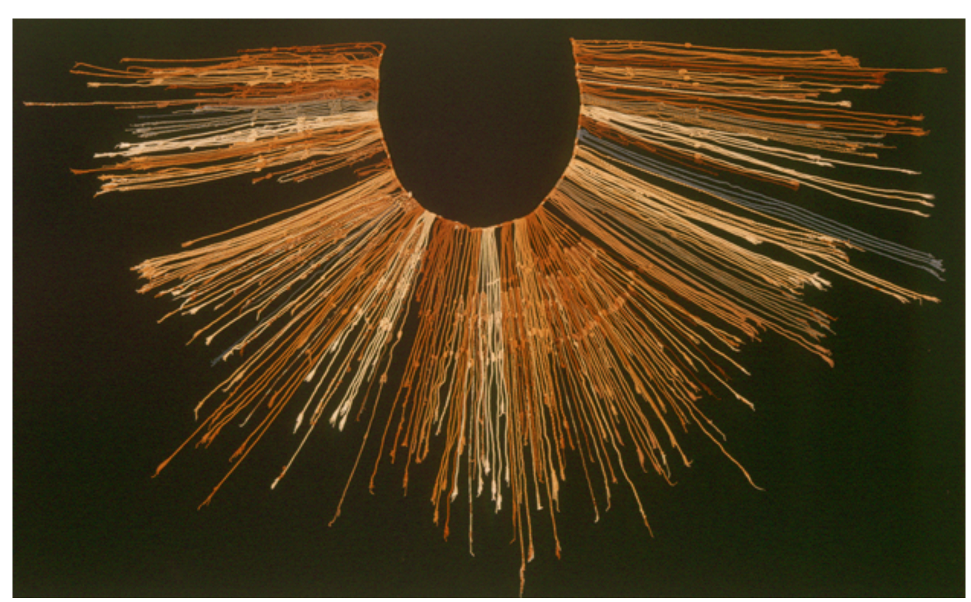
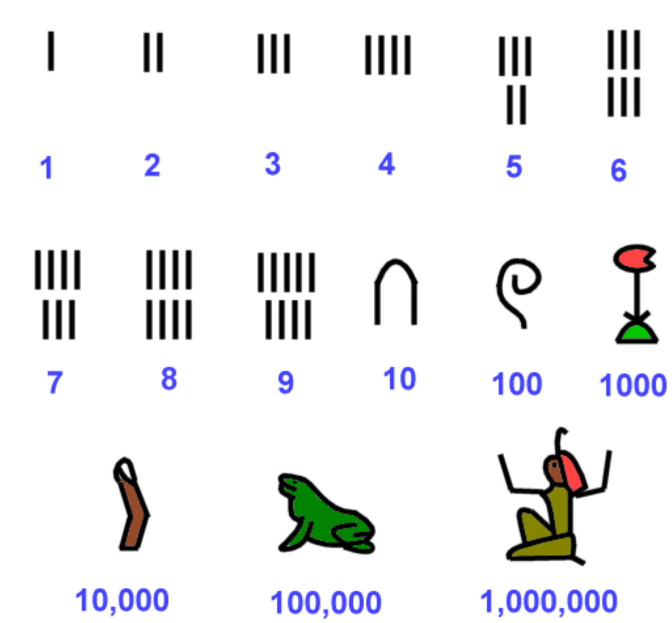




The History of Mathematics

Math 190, Professor Jay Cummings





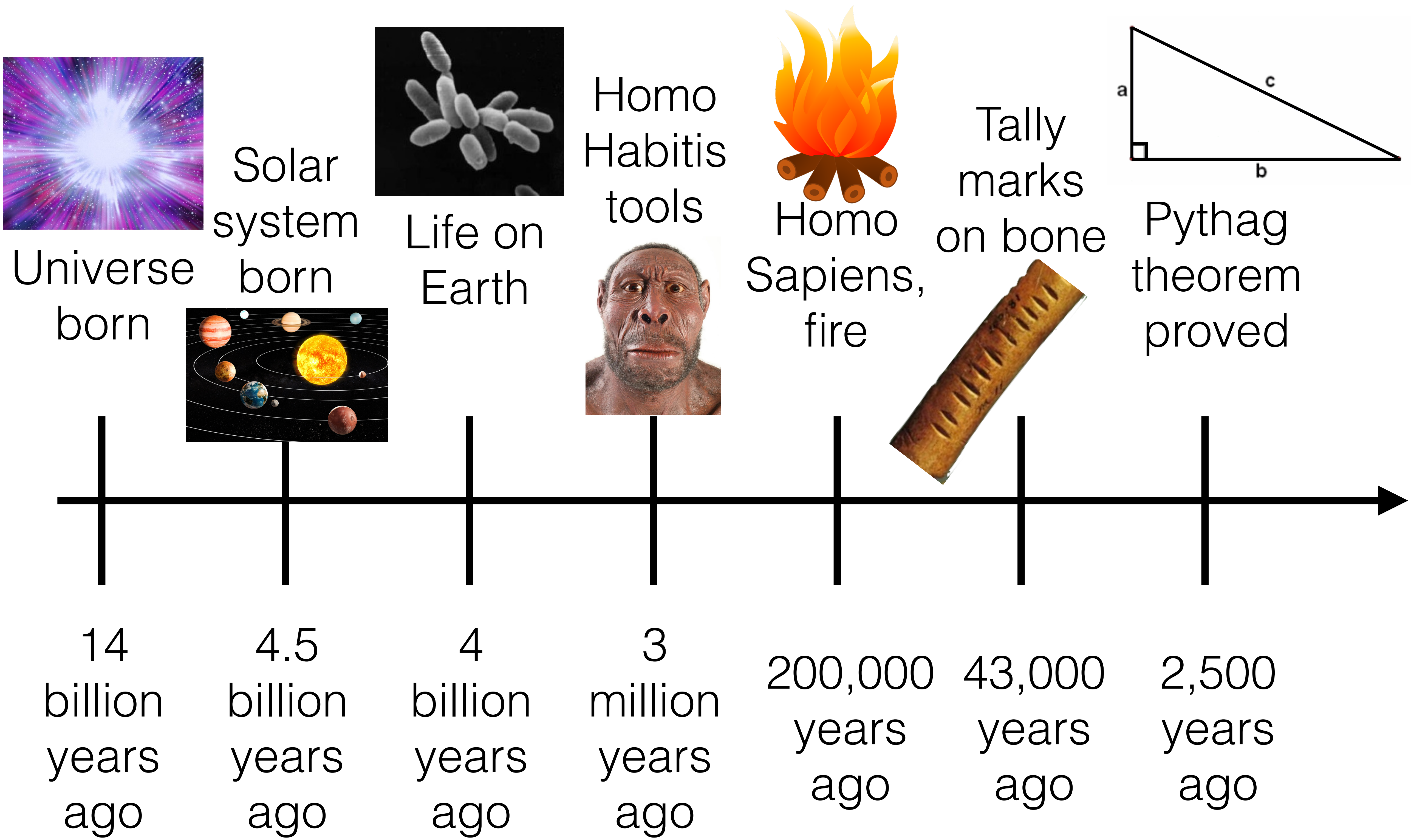
Syllabus Key Points

- Name: Jay Cummings
- Email: Jay.Cummings@csus.edu
- Office: Shasta 253
- Office Hours: Monday 10–11:30, Thursday 12-1:20

Syllabus Key Points

- Homework:
  - Written Homeworks (25%). Due on Thursdays about every other week
  - Perusall (15%). Discuss notes online (1 per chapter)
- Exams:
  - Midterms (30%) on Thursday of Week 6 & Week 12
  - Final Exam (20%), comprehensive final exam
  - Essay-based, get questions 1 week ahead of time
- Attendance/Participation (10%). Mostly graded on showing up and paying attention.

A Brief History of Time



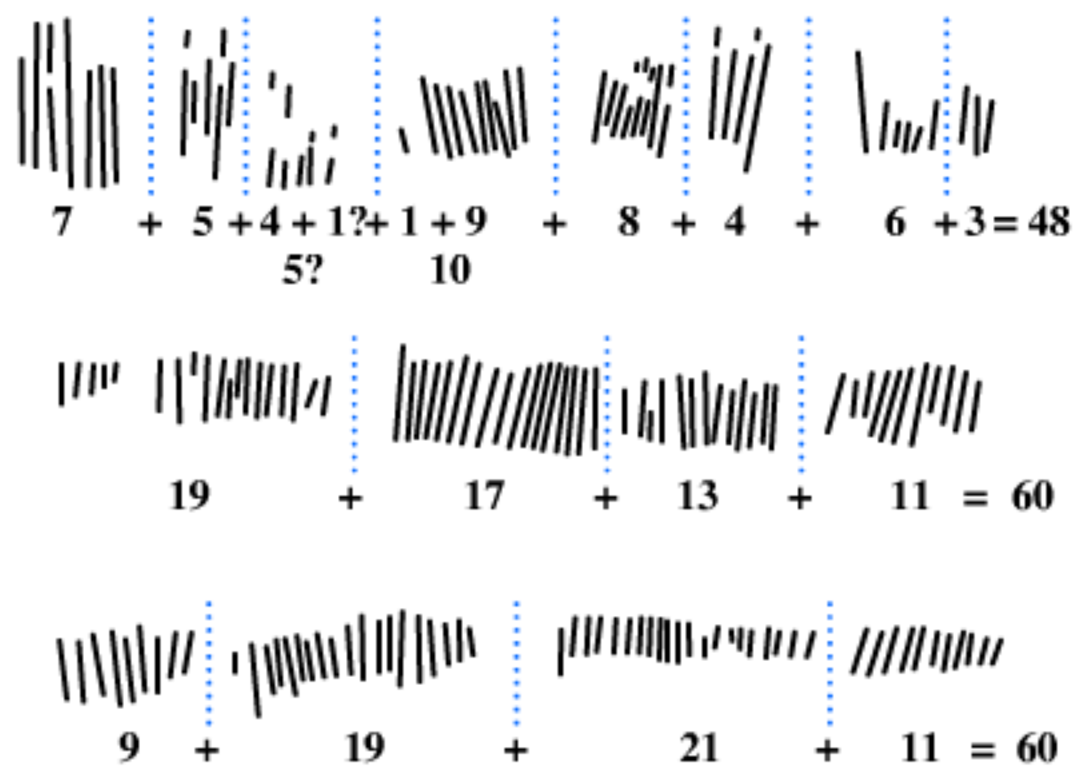
Lebombo Bone

- From ~41,000 BC.
- 29 clear notches.

Lebombo Bone

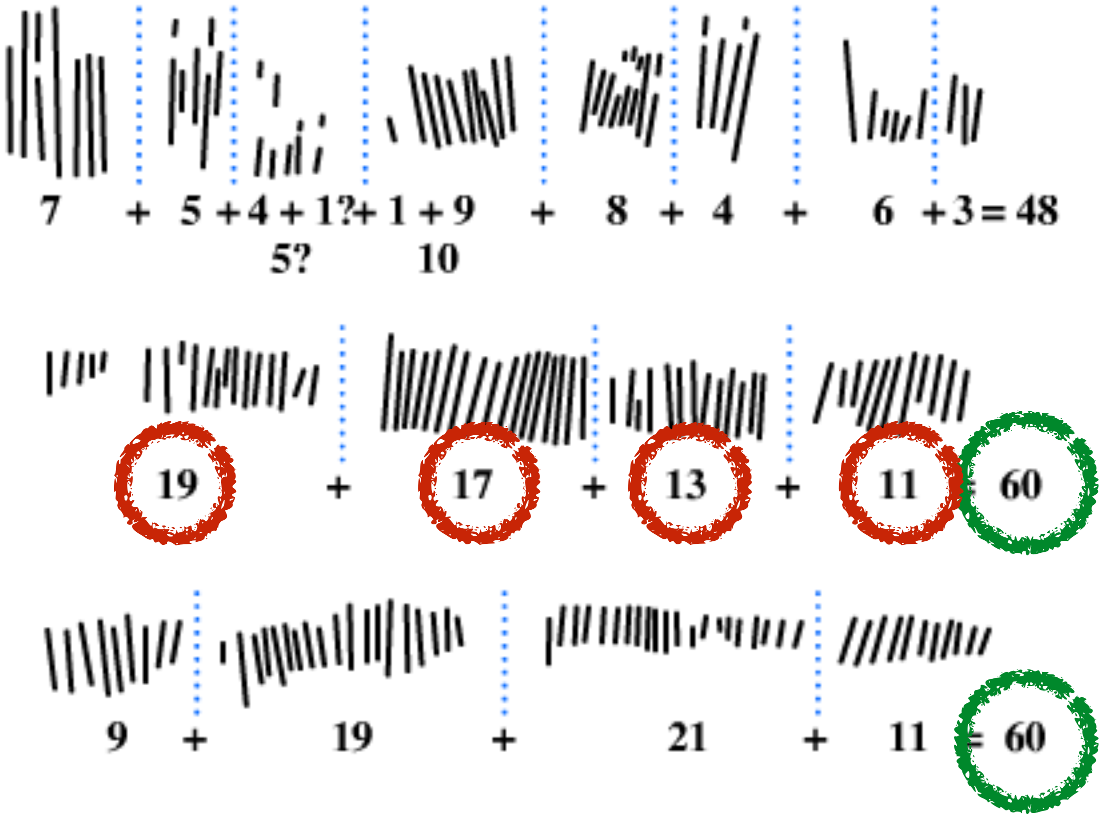
Ishango Bone

- From ~18,000 BC.
- Has 3 columns, with 48, 60 and 60 notches.



Ishango Bone

- From ~18,000 BC.
- Has 3 columns, with 48, 60 and 60 notches.



Both bones are the fibula of baboons!

Ishango Bone

How Humans Count

Objects:

- Fingers.   (Many methods)
- Tally marks.   (Congo, Czech, etc in Stone Age.)
- Knots in rope.   (Peru, China)
- Shells, beads.   (Native Americans)

<u>Question:</u> If counting is the start of math history, does it have to be humans counting?

Number Bases

- Base 10 (decimal system): Now, almost everyone
- Base 20 (vegesimal system): Mayans, Nigeria
- Base 5: Cherokee, Northern Australia, Siberia, etc.
- Base 60 (sexagesimal system): Babylonians
- Base 4: Papua New Guinea, my son at 22 months
- Base 2 (binary system): Computers

Number Base Notation

Notation. When we write a number like 3,835, that is notation for

$$\begin{aligned} &3 \cdot 1{,}000 + 8 \cdot 100 + 3 \cdot 10 + 5 \\ &= 3 \cdot 10^3 + 8 \cdot 10^2 + 3 \cdot 10^1 + 5 \cdot 10^0 . \end{aligned}$$

Notation. A number in base b is expressed as

$$(a_k a_{k-1} \dots a_2 a_1 a_0)_b,$$

where each a_i is an integer between 0 and $b - 1$.

Converting the number $(a_k a_{k-1} \dots a_2 a_1 a_0)_b$ to base 10 gives

$$a_k b^k + a_{k-1} b^{k-1} + \dots + a_2 b^2 + a_1 b^1 + a_0 \cdot b^0.$$

Practice

Example. $(123)_4$ is a base 4 number.

$(11010)_2$ is a base 2 number.

$(11010)_3$ is a base 3 number.

Practice

Example. Write $(123)_4$ in base 10.

Solution.

$$1 \cdot 4^2 + 2 \cdot 4^1 + 3 \cdot 4^0$$
$$= 16 + 8 + 3$$
$$= 27$$

Practice

Example. Write the (base 10) number 34 in base 3.

Solution. The powers of 3 that are less than 34 are 27, 9, 3 and 1.   After 1 copy of 27, we are left with

$$34 - 27 = 7.$$

We don't need any 9s. After 2 copies of 3, we are left with

$$7 - 2 \cdot 3 = 1.$$

Then, we need 1 copy of 1. In summary:

$$34 = 1 \cdot 3^3 + 0 \cdot 3^2 + 2 \cdot 3^1 + 1 \cdot 3^0$$

So,

$$(34)_{10} = (1021)_3 \,.$$

4 Ways People Write Numbers

- Positional System: Each *position* is a power of 10.

  Example: $$3{,}854 = 3 \cdot 1{,}000 + 8 \cdot 100 + 5 \cdot 10 + 4$$
  $$= 3 \cdot 10^3 + 8 \cdot 10^2 + 5 \cdot 10^1 + 4$$

- Multiplicative Grouping System: Have a character for each power of 10

  Example: Chinese-Japanese numeral system

4 Ways People Write Numbers

1 = 一	10 = 十	100 = 一百	1,000 = 一千
2 = 二	20 = 二十	200 = 二百	2,000 = 二千
3 = 三	30 = 三十	300 = 三百	3,000 = 三千
4 = 四	40 = 四十	400 = 四百	4,000 = 四千
5 = 五	50 = 五十	500 = 五百	5,000 = 五千
6 = 六	60 = 六十	600 = 六百	6,000 = 六千
7 = 七	70 = 七十	700 = 七百	7,000 = 七千
8 = 八	80 = 八十	800 = 八百	8,000 = 八千
9 = 九	90 = 九十	900 = 九百	9,000 = 九千
10 = 十	100 = 一百	1,000 = 一千	10,000 = 一万

Example: Chinese-Japanese numeral system

$$5,062 \;=\; 5 \cdot 1,000 + 6 \cdot 10 + 2$$

= 五千六十二

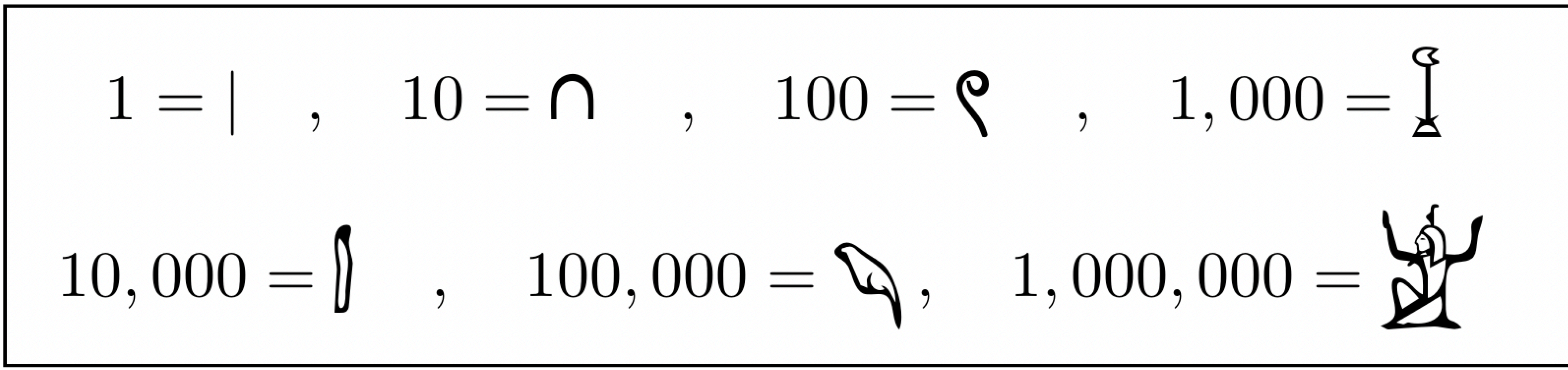
4 Ways People Write Numbers

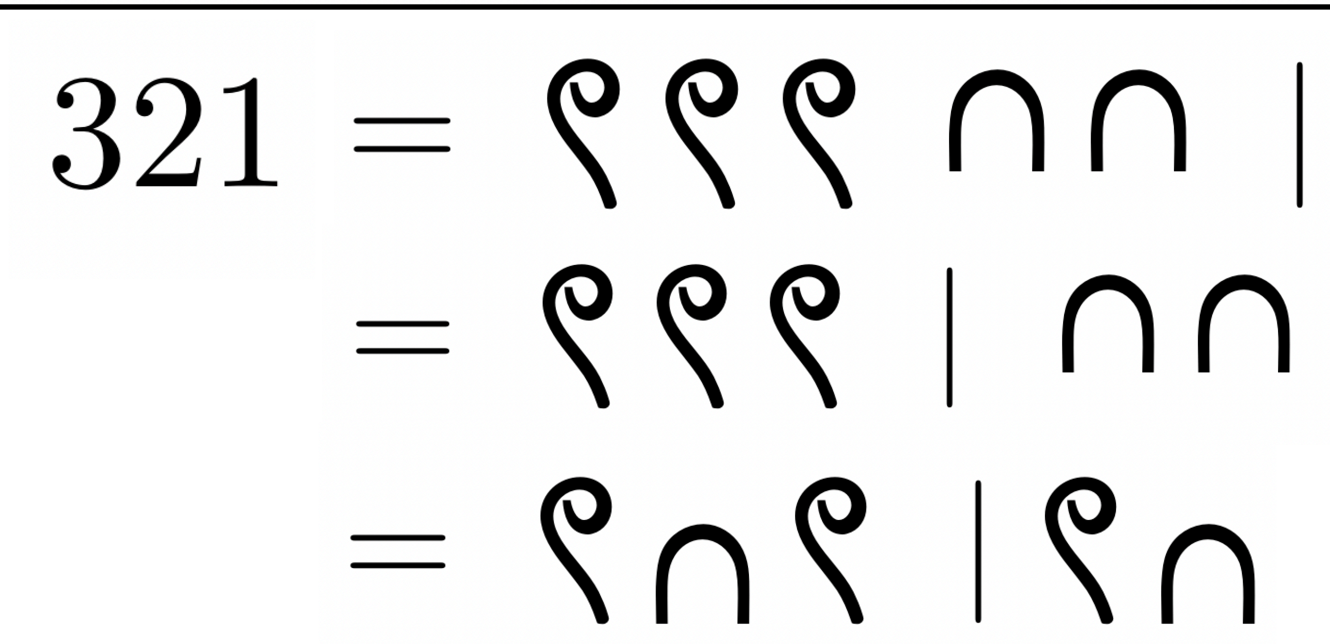
- Simple Grouping System: Add everything up.

  Example: Egyptian hieroglyphics

4 Ways People Write Numbers

- Simple Grouping System: Add everything up.





4 Ways People Write Numbers

- Ciphered systems: It's… complicated. For a base b, we need symbols for

$$
\begin{gathered}
1, 2, \ldots, b - 1 \\
b, 2b, \ldots, (b-1)b \\
b^2, 2b^2, \ldots, (b-1)b^2 \\
b^3, 2b^3, \ldots, (b-1)b^3 \\
\vdots
\end{gathered}
$$

Example: Ionic Greek system

4 Ways People Write Numbers

$1 = \alpha$ (alpha)	$10 = \iota$ (iota)	$100 = \rho$ (rho)
$2 = \beta$ (beta)	$20 = \kappa$ (kappa)	$200 = \sigma$ (sigma)
$3 = \gamma$ (gamma)	$30 = \lambda$ (lambda)	$300 = \tau$ (tau)
$4 = \delta$ (delta)	$40 = \mu$ (mu)	$400 = \upsilon$ (upsilon)
$5 = \varepsilon$ (epsilon)	$50 = \nu$ (nu)	$500 = \phi$ (phi)
$6 = \varsigma$ (vau)	$60 = \xi$ (xi)	$600 = \chi$ (chi)
$7 = \zeta$ (zeta)	$70 = o$ (omicron)	$700 = \psi$ (psi)
$8 = \eta$ (eta)	$80 = \pi$ (pi)	$800 = \omega$ (omega)
$9 = \theta$ (theta)	$90 = \varsigma$ (koppa)	900 = ϡ (sampi)

Example: Ionic Greek system

Question: Which system is best?

Zero

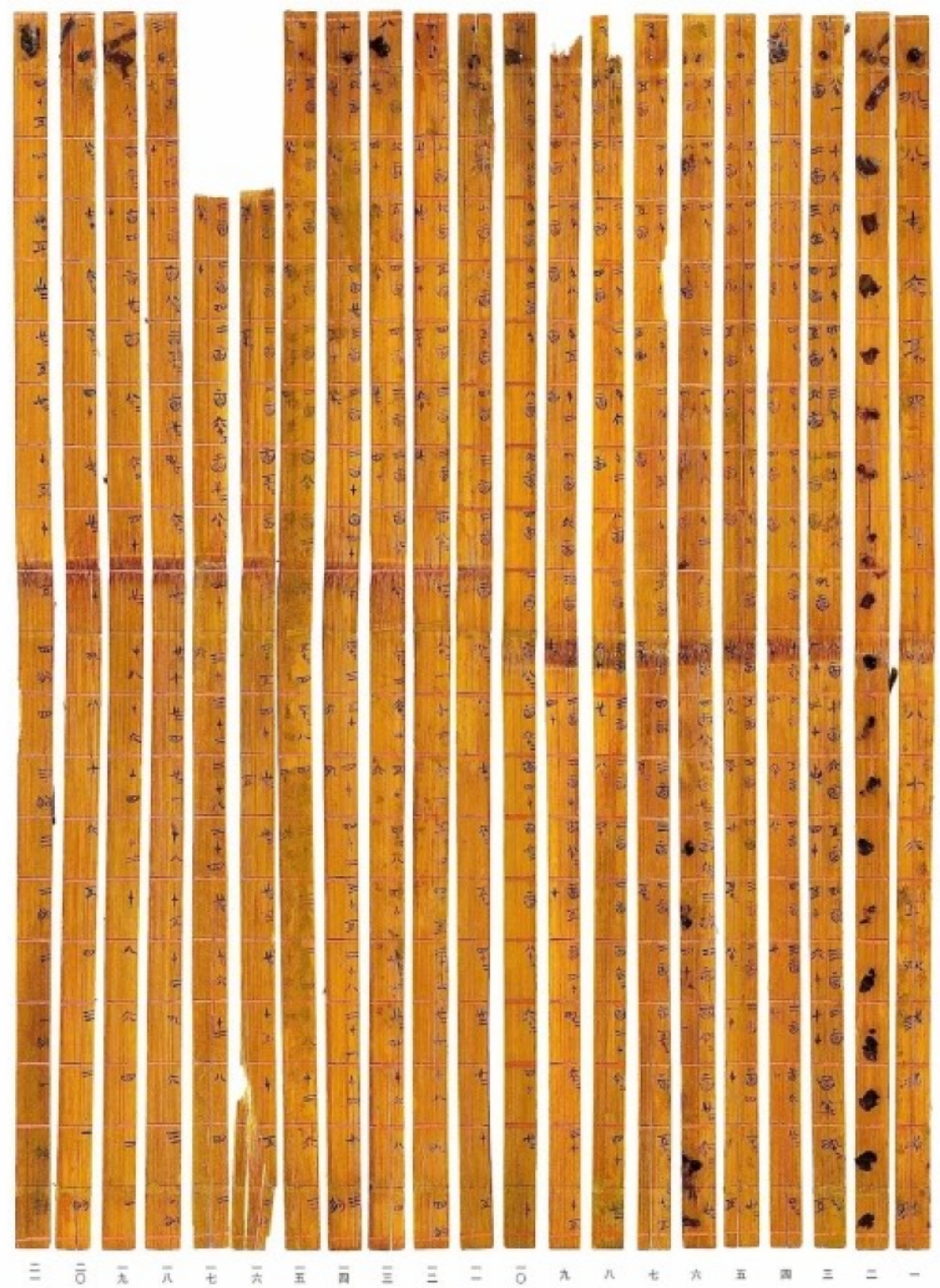
The Indians and the Mayans were the two civilizations to discover zero and recognize its deep potential as:

- A symbol
- A number
- A magnitude
- A direction separator
- A place-holder
- An idea

This was a big deal.

Think Like A Math Historian

Think Like A Math Historian



Think Like A Math Historian

8	5	80	4		3	20	7	9	10	1/2	40	30	6	50	60	2	1	90	70
720	450	7200	360	90	270	1800	630	810	900	45	3600	2700	540	4500	5400	180	90	8100	6300
640	400	6400	320	80	240	1600	560	720	800	40	3200	2400	480	4000	4800	160	80	7200	5600
560	350	5600	280	70	210	1400	490	630	700	35	2800	2100	420	3500	4200	140	70	6300	4900
480	300	4800	240	60	180	1200	420	540	600	30	2400	1800	360	3000	3600	120	60	5400	4200
400	250	4000	200	50	150	1000	350	450	500	25	2000	1500	300	2500	3000	100	50	4500	3500
320	200	3200	160	40	120	800	280	360	400	20	1600	1200	240	2000	2400	80	40	3600	2800
240	150	2400	120	30	90	600	210	270	300	15	1200	900	180	1500	1800	60	30	2700	2100
160	100	1600	80	20	60	400	140	180	200	10	800	600	120	1000	1200	40	20	1800	1400
80	50	800	40	10	30	200	70	90	100	5	400	300	60	500	600	20	10	900	700
72	45	720	36	9	27	180	63	81	90	$4\frac{1}{2}$	360	270	54	450	540	18	9	810	630
64	40	640	32	8	24	160	56	72	80	4	320	240	48	400	480	16	8	720	560
56	35	560	28	7	21	140	49	63	70	$3\frac{1}{2}$	280	210	42	350	420	14	7	630	490
48	30	480	24	6	18	120	42	54	60	3	240	180	36	300	360	12	6	540	420
40	25	400	20	5	15	100	35	45	50	$2\frac{1}{2}$	200	150	30	250	300	10	5	450	350
32	20	320	16	4	12	80	28	36	40	2	160	120	24	200	240	8	4	360	280
24	15	240	12	3	9	60	21	27	30	$1\frac{1}{2}$	120	90	18	150	180	6	3	270	210
16	10	160	8	2	6	40	14	18	20	1	80	60	12	100	120	4	2	180	140
8	5	80	4	1	3	20	7	9	10	$\frac{1}{2}$	40	30	6	50	60	2	1	90	70
4	$2\frac{1}{2}$	40	2	$\frac{1}{2}$	$1\frac{1}{2}$	10	$3\frac{1}{2}$	$4\frac{1}{2}$	5	$\frac{1}{4}$	20	15	3	25	30	1	$\frac{1}{2}$	45	35

Think Like A Math Historian

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
$4\frac{1}{2}$	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
$3\frac{1}{2}$	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
$2\frac{1}{2}$	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
$1\frac{1}{2}$	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
$\frac{1}{2}$	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	10	15	20	25	30	35	40	45	$\frac{1}{2}$

We will come back to this soon.

Arithmetic

Multiplication Tables

- After counting comes arithmetic. Simple addition and subtraction could be done in head, on fingers, or with simple tools. Multiplication was probably much harder.

- First known multiplication table is a 4,000 year old Babylonian (base 60) multiplication table.

Multiplication Tables

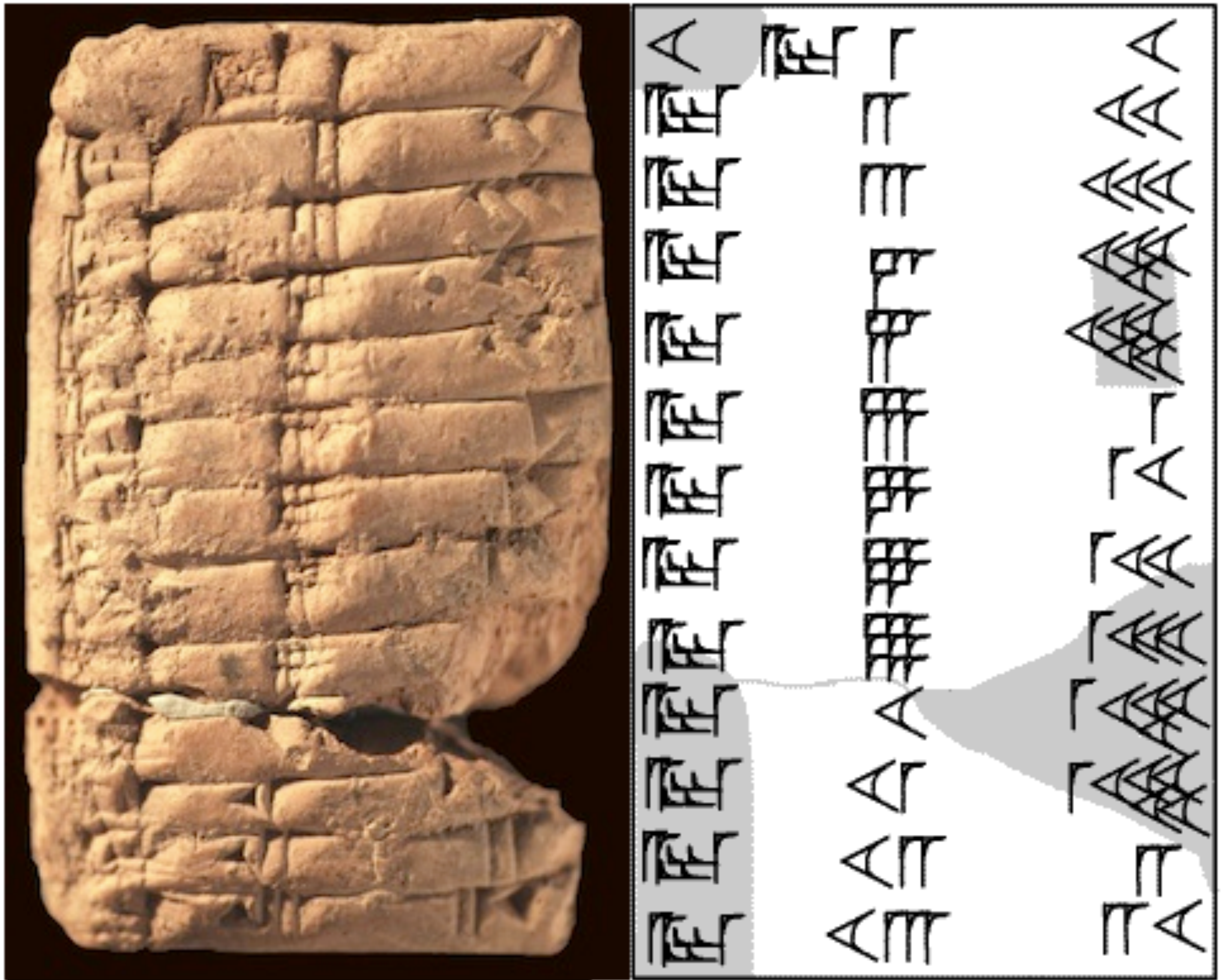

MS 3866
Multiplication table for 1.12(=72).
Babylonia, 19th c. BC

- After counting [illegible] c. Simple addition and subtractio[illegible] in head, on fingers, or with simple [illegible]ion was probably much harder.

- First known mu[illegible] is a 4,000 year old Babylonian (b[illegible]ation table.

Multiplication Tables

- Aft … on and … gers, or … y mu…
- Firs … old Ba…



Multiplication Tables

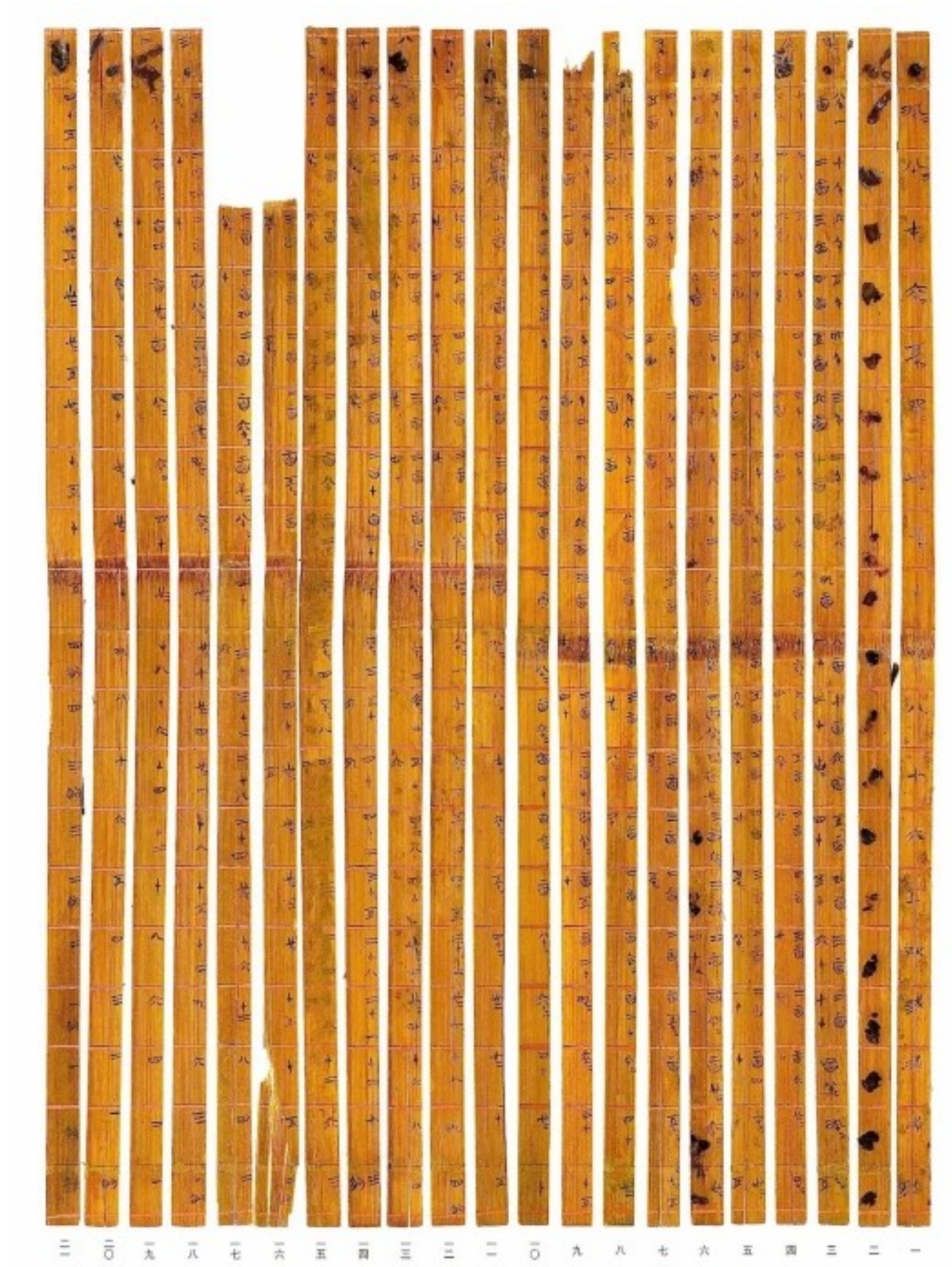
- After counting comes arithmetic. Simple addition and subtraction could be done in head, on fingers, or with simple tools. Multiplication was probably much harder.

- First known multiplication table is a 4,000 year old Babylonian (base 60) multiplication table.

- First known *decimal* multiplication table is a 2,300 year old Chinese multiplication table.

- It was discovered in 2009!

Tsinghua Bamboo Strips



Tsinghua Bamboo Slips

- An ancient table, written on bamboo strips and found in China, contains what is essentially a multiplication table.

- With a little extra work, one can use it to compute the product of any two integers or half integers from 0.5 to 99.5.
  That is, you can multiply together any two numbers from this list: 0.5, 1, 1.5, 2, 2.5, … , 99.5.

- This is the first known decimal multiplication table.

Tsinghua Bamboo Slips

- This multiplication table can compute the product of any two numbers from the set {0.5,1,1.5,2,2.5,3,3.5,…,99.5}. How?

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
$4\frac{1}{2}$	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
$3\frac{1}{2}$	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
$2\frac{1}{2}$	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
$1\frac{1}{2}$	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
$\frac{1}{2}$	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	10	15	20	25	30	35	40	45	$\frac{1}{2}$

Think Like A
Math Historian

Tsinghua Bamboo Slips

- This multiplication table can compute the product of any two numbers from the set {0.5,1,1.5,2,2.5,3,3.5,…,99.5}. How?

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
4½	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
3½	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
2½	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
1½	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
½	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
¼	½	1	1½	2	2½	3	3½	4	4½	5	10	15	20	25	30	35	40	45	½

Tsinghua Bamboo Slips

- Example: $24 \cdot 36.5 = ?$

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
4 1/2	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
3 1/2	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
2 1/2	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
1 1/2	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
1/4	1/2	1	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	10	15	20	25	30	35	40	45	1/2

Tsinghua Bamboo Slips

- Example: $24 \cdot 36.5 = ?$

1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
4 1/2	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
3 1/2	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
2 1/2	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
1 1/2	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
1/4	1/2	1	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	10	15	20	25	30	35	40	45	1/2

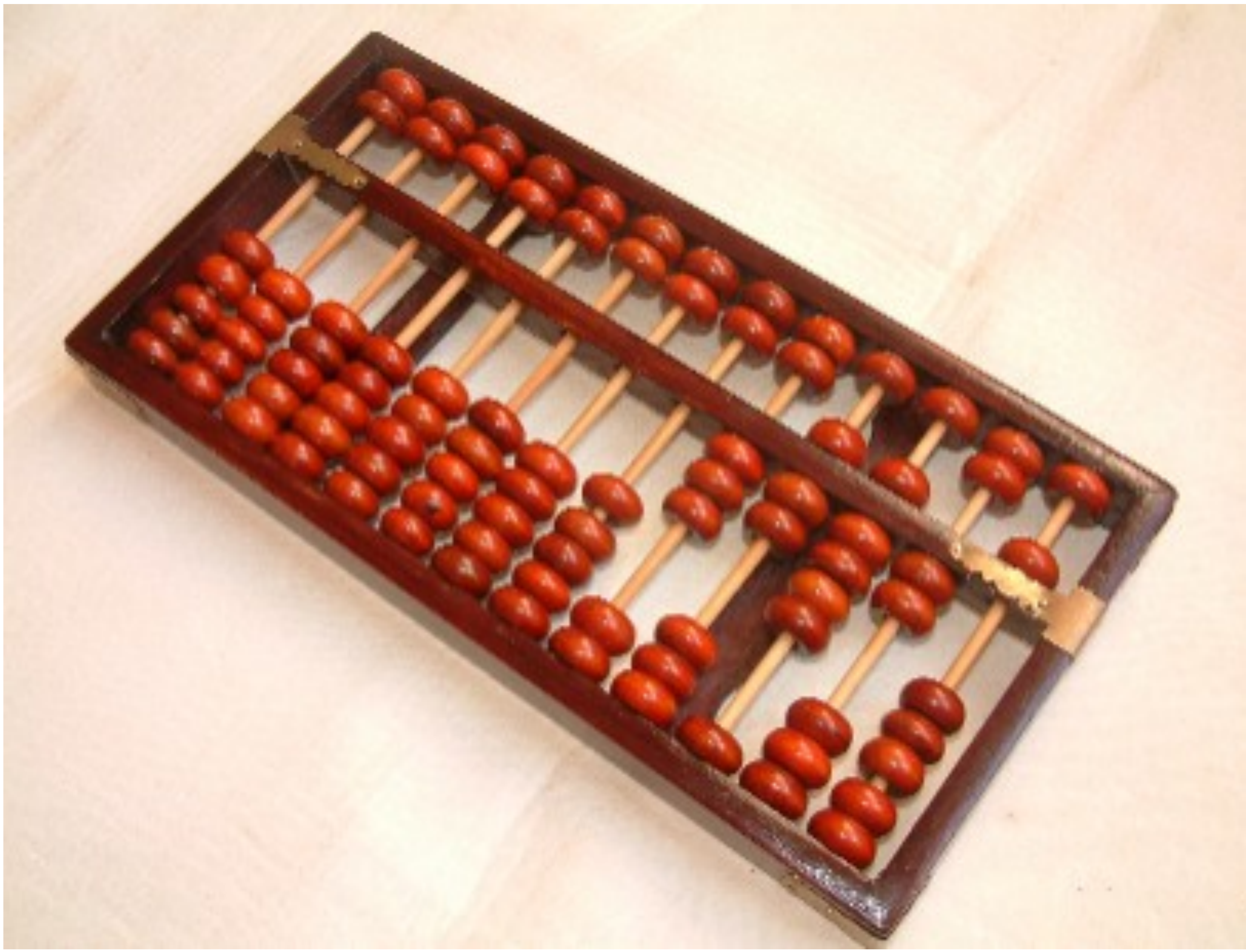
$$24 \cdot 36.5 = (20 + 4) \cdot (30 + 6 + 0.5)$$

$$= 20 \cdot 30 + 20 \cdot 6 + 20 \cdot 0.5 + 4 \cdot 30 + 4 \cdot 6 + 4 \cdot 0.5$$

= Look up on table, then add together

Ancient Calculators

Abacus



The Incan Empire

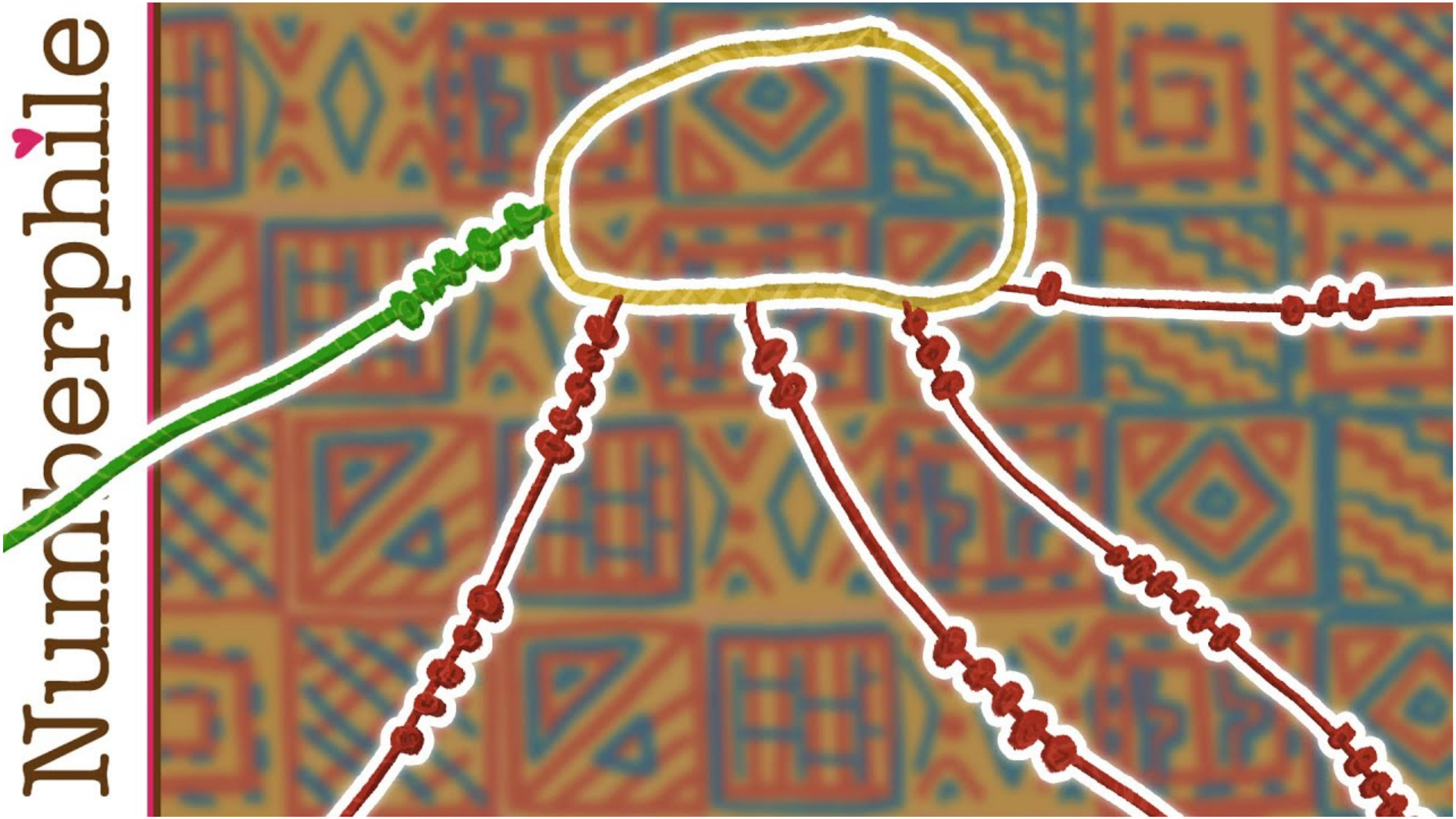
The Incan Empire

The Incan Empire

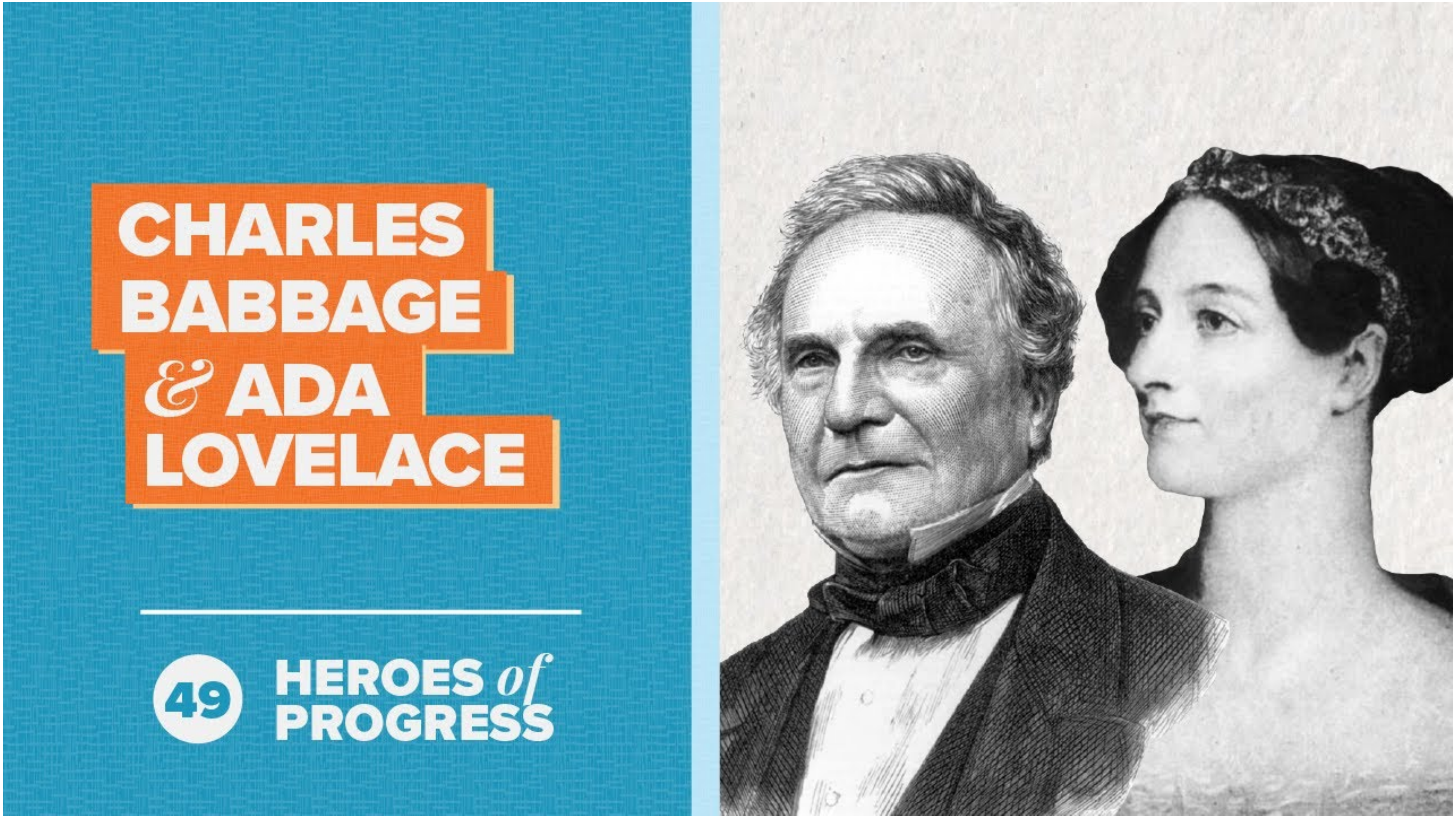
- The Incans were a great but relatively short-lived empire.
- They did not have a written language.
- How do you record numbers and do math without writing?
- The Incans used what is called a *quipu*.

Devices

Devices



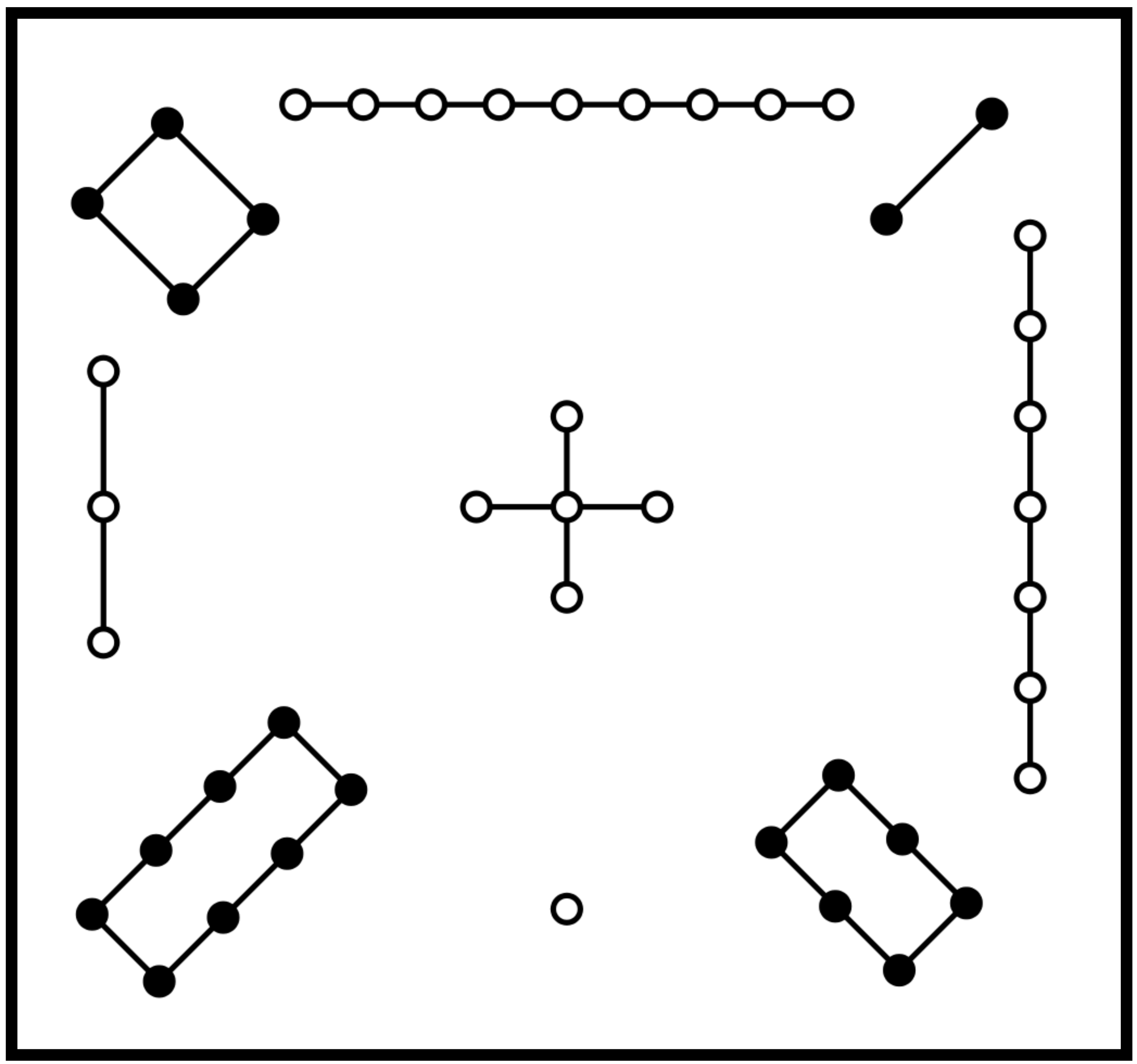
Aftermath: Computers



Think Like A
Math Historian

Think Like a Math Historian

- Hint: Focus on how many dots are in each group.



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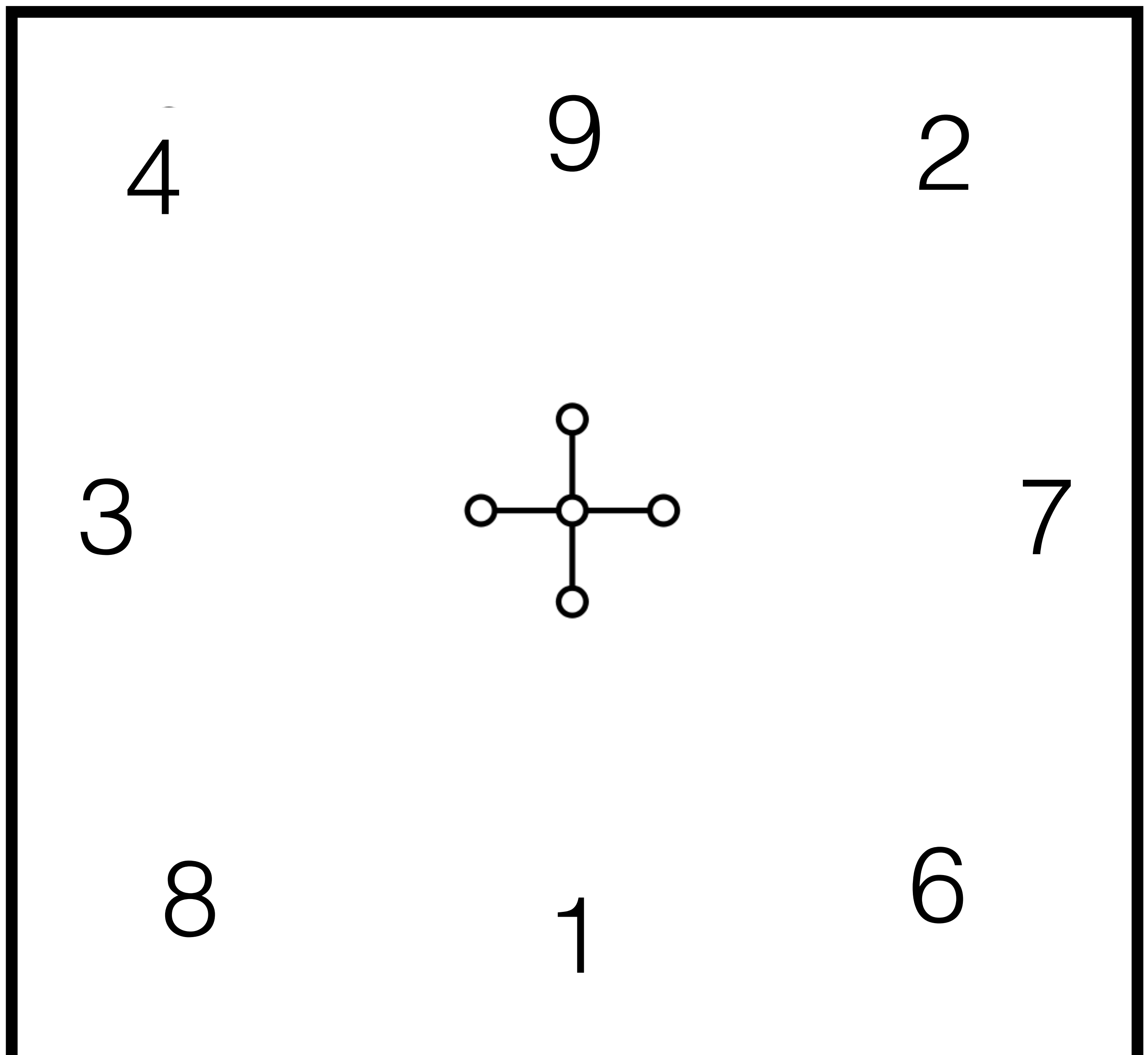

Think Like a Math Historian

- Hint: Focus on how many dots are in each group.

4	9	2
3	5	7
8	1	6

Think Like a Math Historian

- Magic Square

4	9	2
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Think Like a Math Historian

- Magic Square

The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

Step 1: Write numbers in a diamond.

1

4 2

7 5 3

8 6

9

The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

<u>Step 2:</u> Swap corners.

1

4 2

7 5 3

8 6

9

The Aftermath

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<u>Step 2:</u> Swap corners.

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8 6

1

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<u>Step 2:</u> Swap corners.

9

4 2

3 5 7

8 6

1

The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

<u>Step 3:</u> Scrunch together.

		9		
	4		2	
3		5		7
	8		6	
		1		

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8	1	6

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3	5	7
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The Aftermath

- Yang Hui, a much more modern Chinese mathematician (~1238-1298 AD) had a 3-step procedure to create this square.

<u>Step 3:</u> Scrunch together.

4	9	2
3	5	7
8	1	6

Hui also came up with a similar algorithm to construct a 4x4 magic square.

The Aftermath

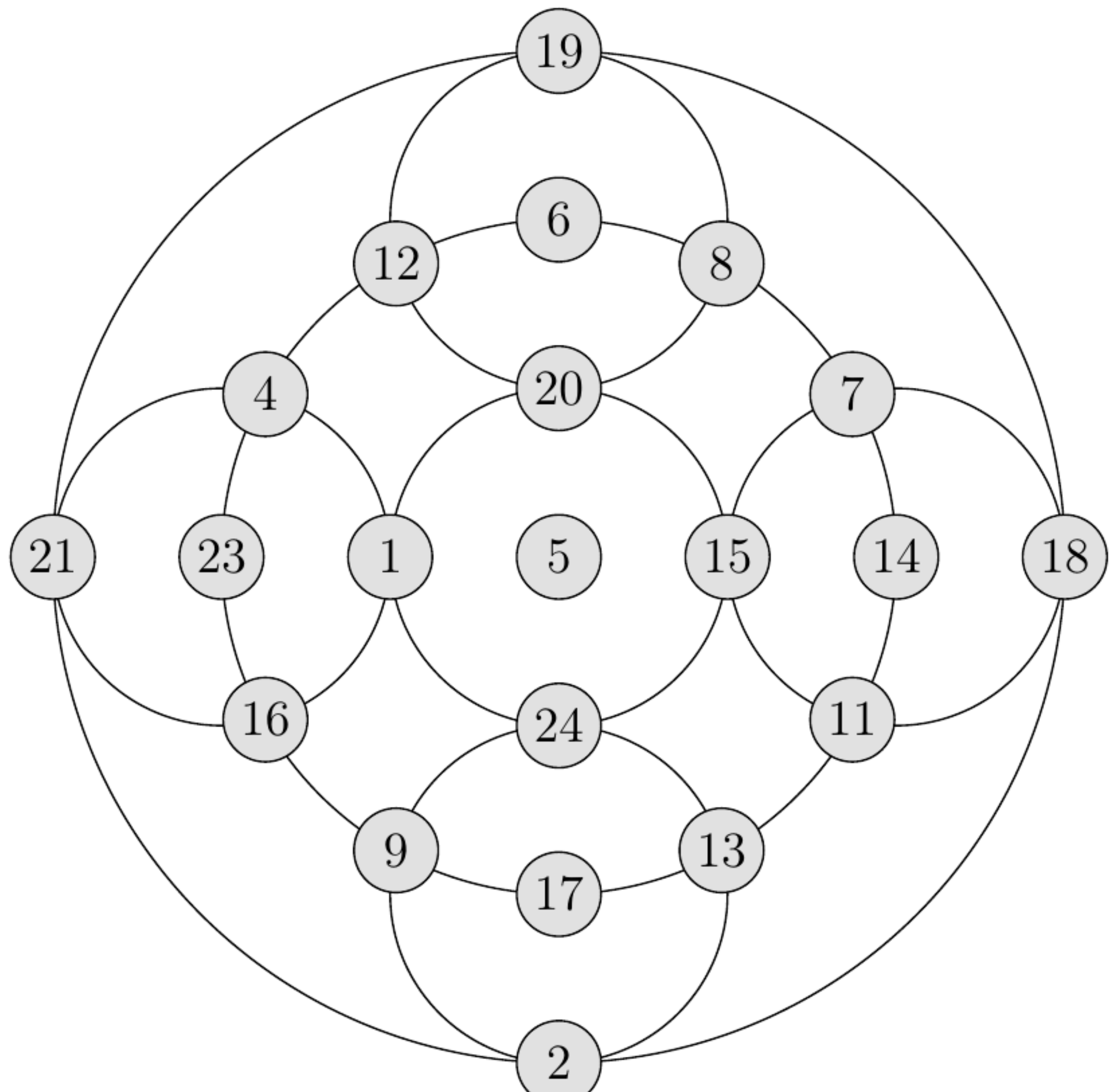
- Yang Hui constructed a magic square of every size up to 10x10.

一	二十	二一	四十	四一	六十	六一	八十	八一	一百
九九	八二	七九	六二	五九	四二	三九	二二	十九	二
三	十八	二三	三八	四三	五八	六三	七八	八三	九八
九七	八四	七七	六四	五七	四四	三七	二四	十七	四
五	十六	二五	三六	四五	五六	六五	七六	八五	九六
九五	八六	七五	六六	五五	四六	三五	二六	十五	六
十四	七	三四	二七	五四	四七	七四	六七	九四	八七
八八	九三	六八	七三	四八	五三	二八	三三	八	十三
十二	九	三二	二九	五二	四九	七二	六九	九二	八九
九一	九十	七一	七十	五一	五十	三一	三十	十一	十

1	20	21	40	41	60	61	80	81	100
99	82	79	62	59	42	39	22	19	2
3	18	23	38	43	58	63	78	83	98
97	84	77	64	57	44	37	24	17	4
5	16	25	36	45	56	65	76	85	96
95	86	75	66	55	46	35	26	15	6
14	7	34	27	54	47	74	67	94	87
88	93	68	73	48	53	28	33	8	13
12	9	32	29	52	49	72	69	92	89
91	90	71	70	51	50	31	30	11	10

The Aftermath

- Yang Hui also constructed six magic circles.



Algorithm to Construct Magic Squares

Shout-Outs!

- Crows

Counting Crows

- Other animals may count too, to some extent. For example, crows.

Counting Crows

- Other animals may count too, to some extent. For example, crows.
- Old story of a hunter hiding in a shelter.

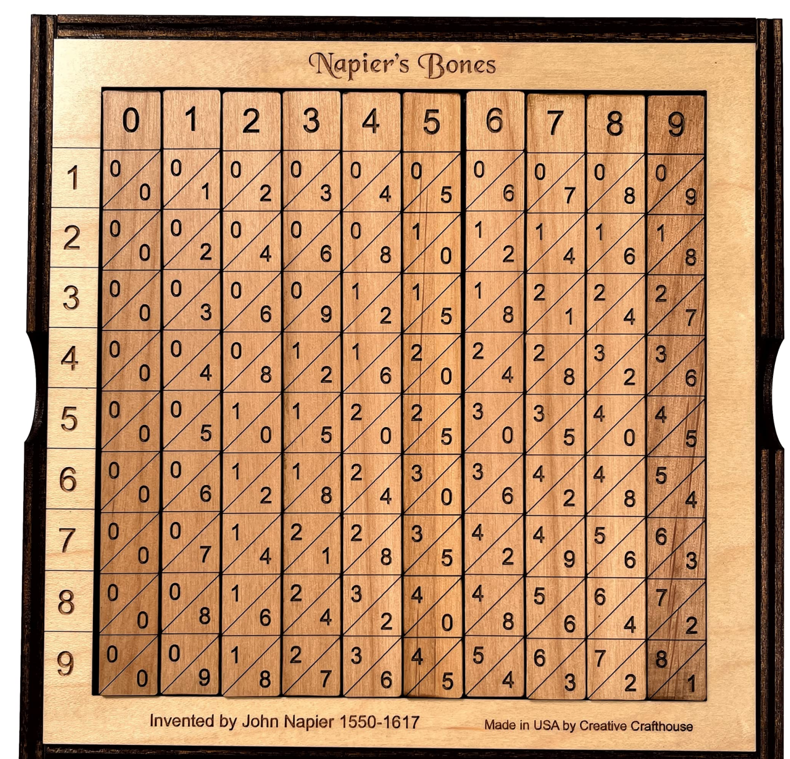
Counting Crows

Shout-Outs!

- Napier’s Bones

Shout-Outs!

- Napier's Bones



People’s History

People’s History of Numbers

- A “people’s history” studies the history of non-elites.
- Much of the progression to create number systems is a people’s history. Merchants lead the way.
- Hindu-Arabic numerals “democratized” math.
- Some Roman elites fought back in an effort to keep math opaque.