MATH 190-02 HW3

Matthew Mendoza

TOTAL POINTS

33 / 40

QUESTION 1

1 Question 1 **9 / 10**

- 0 pts Well done!

- 1 pts You should start by assuming you have a right triangle with sides a, b and c, and then build the figure and proceed. Your proof is incomplete without this.

- 6 pts In your solution, you immediately start using the Pythagorean theorem to write $$a^2 + c_1^2 = h^2$$, and proceeding from there. This problem is proving the Pythagorean theorem, so you cannot use it in the proof. Use similar triangles instead.

- 3 pts I am a little confused by your proof. It's a unclear where you are shifting your triangles to. Also, why does it end up in the shape you assert? I think there is probably a proof here, but more explanation and details would be required.

- 3 pts You should start by assuming you have a right triangle with sides a, b and c, and then build the figure and proceed. Your proof is incomplete without this. Also, when you write $$(ab) + \frac12 (c^2)$$, you should explain how you got that.

- 3 pts More explanation is needed. Just writing down equations without saying where they come from does not constitute a proof. Also, you should start by assuming you have a right triangle with sides a, b and c, and then build the figure and proceed. Your proof is incomplete without this.

- 8 pts You were not supposed to look up a proof, you were supposed to figure it out on your own. Either way, what you wrote could perhaps be the basis of a proof, but you would need to make a much more thorough argument.

✓ **- 1 pts** *Say why the purple square is indeed a square.*

- 1 pts Say why that is indeed a trapezoid.

QUESTION 2

2 Question 2 **8 / 10**

- 0 pts Good job!

- 0 pts You should start by assuming you have a right triangle with sides a, b and c, and then build the figure and proceed. Your proof is incomplete without this. Everything else was good!

- 4 pts You are on the right track, but you don't finish it by reaching the conclusion.

- 10 pts It was required that two of your exercises are proofs of the Pythagorean theorem.

- 1 pts State why that is indeed a trapezoid.

- 2 pts Explain what you are doing. A proof needs more than just a list of equations.

- 2 pts Say why that first equation that you wrote down is true.

- **2 pts** Looks like you are on the right track, but more explanation is needed to show your logic.

- **10 pts** Nothing submitted.

✓ **- 2 pts** *State why this is indeed a trapezoid. And don't assume that angle is a right angle, prove it.*

QUESTION 3

3 Question 3 **6 / 10**

- **0 pts** Good job!

- **5 pts** Parts (b), (c) and (d)?

- **5 pts** You show it only in one case. Prove it in general.

- **10 pts** Nothing submitted.

✓ **- 4 pts** *For part (b), use similar triangles.*

QUESTION 4

4 Question 4 **10 / 10**

✓ **- 0 pts** *Good job!*

- **5 pts** You only showed that 2 divides abc.

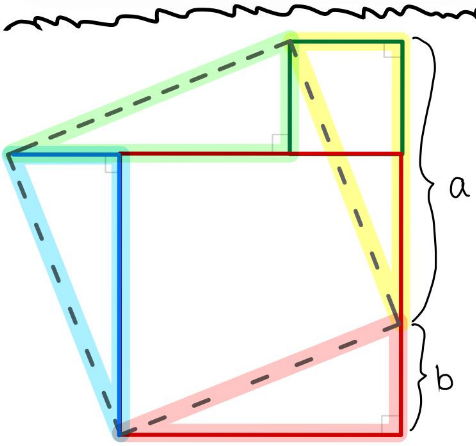
- **5 pts** Your proof only works for 3,4,5 and multiples of it. Why do all the other triples work too?

- **0 pts** Impressive!

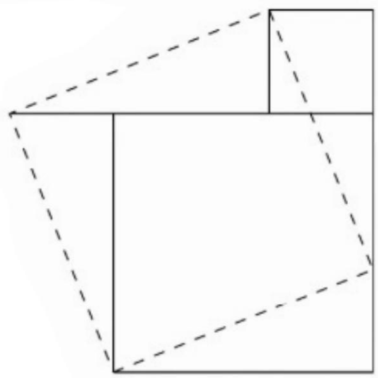
- **10 pts** Nothing submitted.

— Exercises —

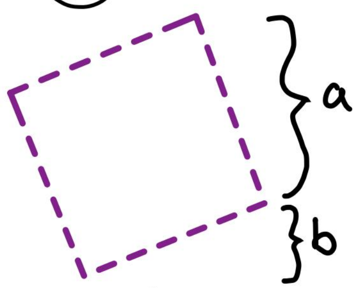
Exercise 3.1. One proof of the Pythagorean theorem, given by 16th-century Indian mathematician Jyesthadeva, is summarized in the following diagram. Write out a complete proof of the Pythagorean theorem based on this diagram.



We know that an area of a triangle: $\frac{1}{2}b \cdot h$, square: a^2

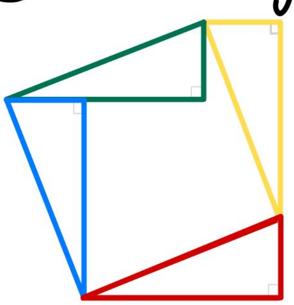


① Total area of the dotted square



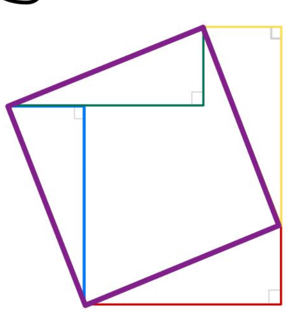
$\rightarrow (a+b)^2$

② In the diagram we have 4 triangles, so its fair to say



$\rightarrow 4(\frac{1}{2}ab) = 2ab$ a & b are essentially legs of the right triangle

③ We can equate the two equations by like so...



All the hyp. of all 4 triangles

$(a+b)^2 = c^2 + 2ab$

Now with some algebra...

$(a+b)^2 = c^2 + 2ab$

$= a^2 + 2ab + b^2 = c^2 + 2ab$

$-2ab \qquad -2ab$

and now we are left with

$a^2 + b^2 = c^2$

1 Question 1 **9 / 10**

- 0 pts Well done!

- 1 pts You should start by assuming you have a right triangle with sides a, b and c, and then build the figure and proceed. Your proof is incomplete without this.

- 6 pts In your solution, you immediately start using the Pythagorean theorem to write $a^2 + c_1^2 = h^2$, and proceeding from there. This problem is proving the Pythagorean theorem, so you cannot use it in the proof. Use similar triangles instead.

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Also, you should start by assuming you have a right triangle with sides a, b and c, and then build the figure and proceed. Your proof is incomplete without this.

- 8 pts You were not supposed to look up a proof, you were supposed to figure it out on your own. Either way, what you wrote could perhaps be the basis of a proof, but you would need to make a much more thorough argument.

✓ **- 1 pts** *Say why the purple square is indeed a square.*

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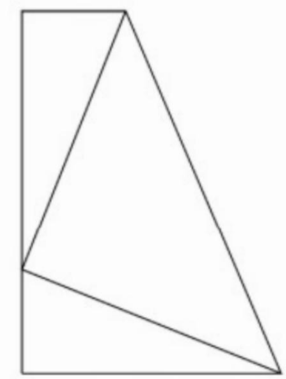
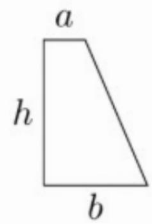
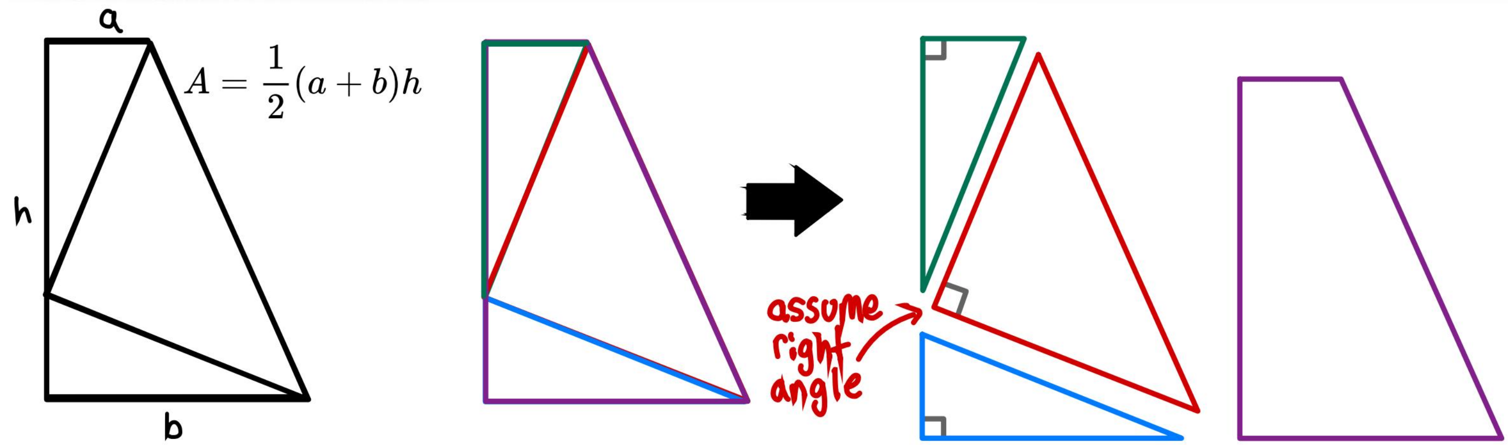
Exercise 3.2. In 1876, U.S. Congressman (and future president) James Garfield discovered a proof of the Pythagorean theorem using the following diagram.

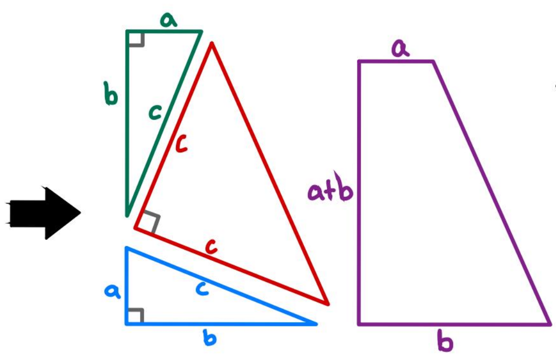


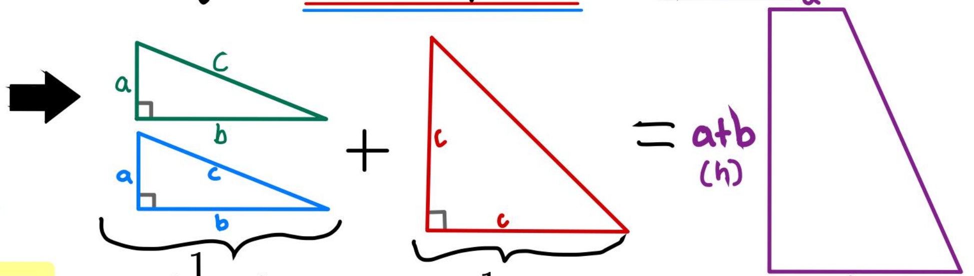
Recall that the area of a trapezoid 

is $\frac{1}{2}(a+b)h$. By writing the area of a Garfield's diagram in two different ways, write out a complete proof of the Pythagorean theorem.



With all the components laid out we can equate the components to the whole





$$2(\frac{1}{2}ab) \qquad \frac{1}{2}c^2$$

$$\downarrow$$

$$\frac{1}{2}(a+b)(h)$$

$$\downarrow$$

$$\frac{1}{2}(a+b)(a+b)$$

So all together...

$$\rightarrow 2(\frac{1}{2}ab) + \frac{1}{2}c^2 = \frac{1}{2}(a+b)(a+b)$$

Multiply both sides by 2

$$\rightarrow 2[2(\frac{1}{2}ab) + \frac{1}{2}c^2] = 2[\frac{1}{2}(a+b)(a+b)]$$

$$\rightarrow 2ab + c^2 = a^2 + 2ab + b^2$$

$-2ab \qquad -2ab$

Therefore...

$$c^2 = a^2 + b^2$$

2 Question 2 **8 / 10**

- 0 pts Good job!

- 0 pts You should start by assuming you have a right triangle with sides a, b and c, and then build the figure and proceed. Your proof is incomplete without this. Everything else was good!

- 4 pts You are on the right track, but you don't finish it by reaching the conclusion.

- 10 pts It was required that two of your exercises are proofs of the Pythagorean theorem.

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- 2 pts Explain what you are doing. A proof needs more than just a list of equations.

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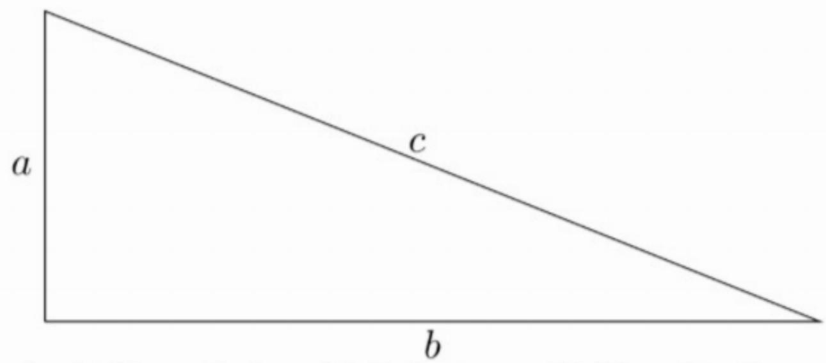
- 2 pts Looks like you are on the right track, but more explanation is needed to show your logic.

- 10 pts Nothing submitted.

✓ **- 2 pts** *State why this is indeed a trapezoid. And don't assume that angle is a right angle, prove it.*

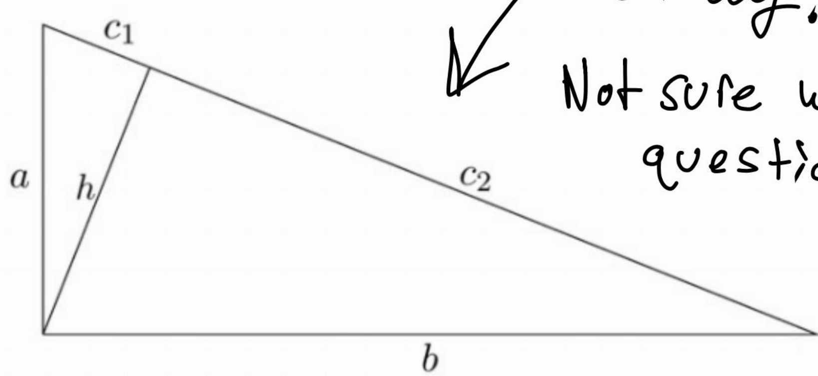
Exercise 3.3. Many proofs of the Pythagorean theorem make use of similar triangles. One of the simplest of these is the following.

(a) Begin with a right triangle with legs a and b and hypotenuse c.

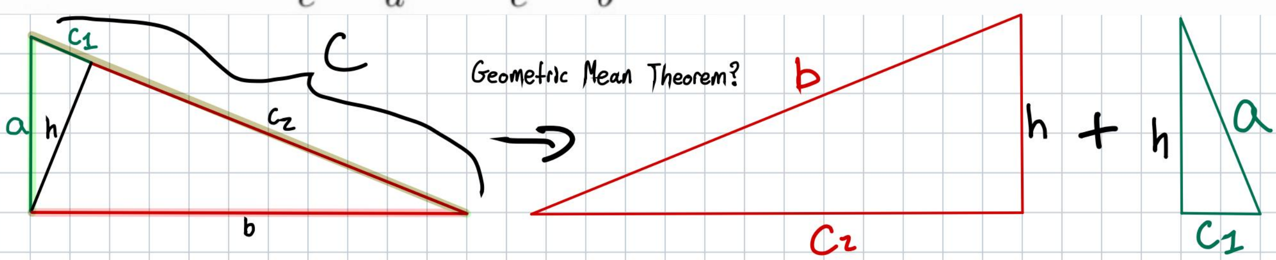


Adding an altitude (of length h, which in turn divides the hypotenuse into two line segments of lengths c_1 and c_2) gives this:

Okay...
Not sure what's the question



(b) Prove that $\frac{a}{c} = \frac{c_1}{a}$ and $\frac{b}{c} = \frac{c_2}{b}$.



(c) Cross-multiply each equation and add these equations together. Show that $a^2 + b^2 = c^2$, concluding the proof.

$\frac{a}{c} = \frac{c_1}{a}$ $\quad$ $\frac{b}{c} = \frac{c_2}{b}$

$a^2 = c_1^2$ $\quad$ $b^2 = c_2^2$

$a^2 + b^2 = c_1^2 + c_2^2$

c_1 & c_2 are the two line segments formed by the altitude (h)... Using pythag.

$c_1^2 = a^2 - h^2$ and $c_2^2 = b^2 - h^2$

So... $a^2 + b^2 = (a^2 - h^2) + (b^2 - h^2)$

Simplify & combine like terms

$\cancel{a^2 + b^2} = \cancel{a^2 + b^2} - 2h^2$

$h^2 = 0$ implies $h = 0$?

c_1 & c_2 are legs a & b respective.

So $c_1 = a$ & $c_2 = b$

Subsitute back

$\frac{a}{c} = \frac{a}{a} \Rightarrow 1 = 1$

$\frac{b}{c} = \frac{b}{b} \Rightarrow 1 = 1$

since $c_1 = a$ & $c_2 = b$

$a^2 + b^2 = c_1^2 + c_2^2 = a^2 + b^2$

$\therefore a^2 + b^2 = c^2$

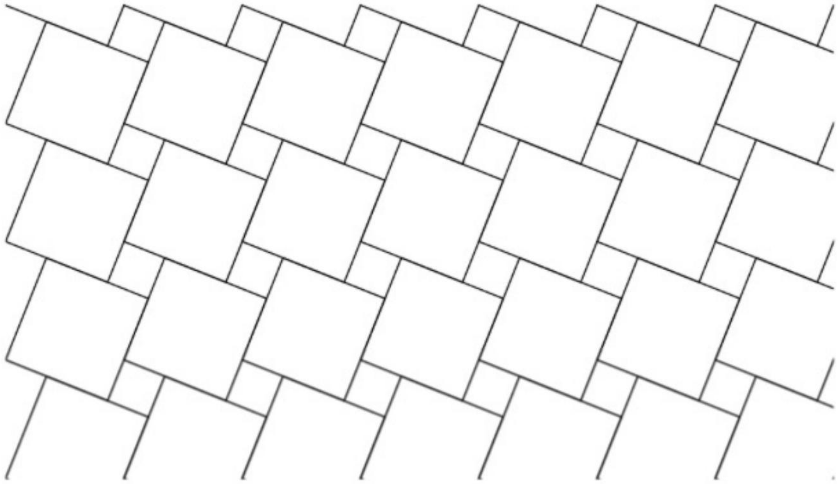
where c is the hypotenuse

3 Question 3 6 / 10

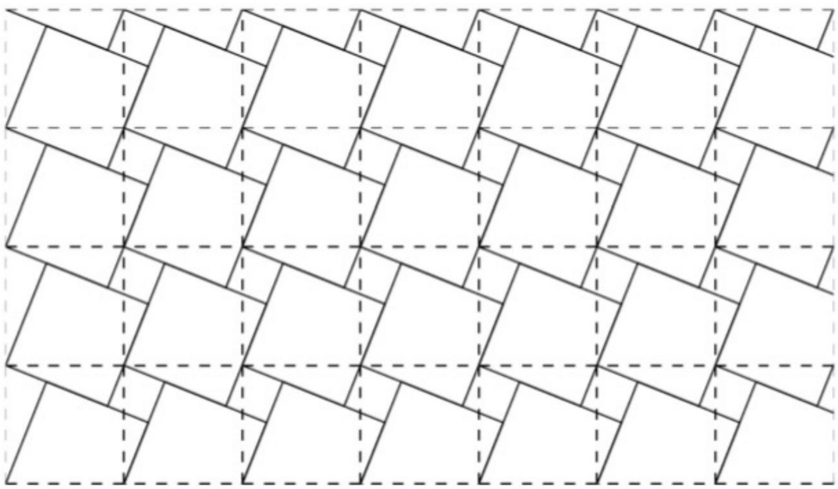
- **- 0 pts** Good job!
- **- 5 pts** Parts (b), (c) and (d)?
- **- 5 pts** You show it only in one case. Prove it in general.
- **- 10 pts** Nothing submitted.

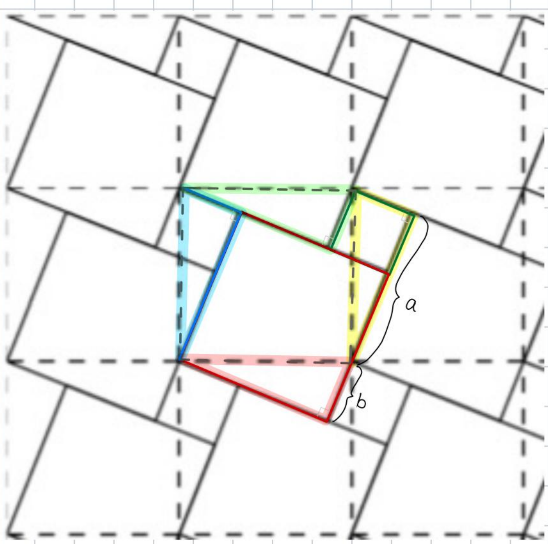
✓ **- 4 pts** *For part (b), use similar triangles.*

Exercise 3.5. In 1974, Wilhelm Magnus proved the Pythagorean theorem using the following tiling pattern. Find a proof based on this tilling.

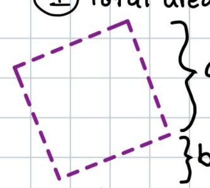


Hint: It might be helpful to consider these dashed lines:





① Total area of the dotted square



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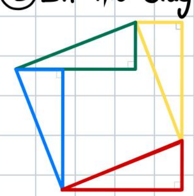
② In the diagram we have 4 triangles, so its fair to say



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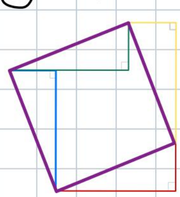

All the hyp. of all 4 triangles

$(a+b)^2 = c^2 + 2ab$ ➡ $(a+b)^2 = c^2 + 2ab$

Now with some algebra...

$= a^2 + 2ab + b^2 = c^2 + 2ab$

$-2ab \qquad -2ab$

and now we are left with

$a^2 + b^2 = c^2$

4 Question 4 **10 / 10**

✓ **- 0 pts** *Good job!*

- 5 pts You only showed that 2 divides abc.

- 5 pts Your proof only works for 3,4,5 and multiples of it. Why do all the other triples work too?

- 0 pts Impressive!

- 10 pts Nothing submitted.