Euclid Handout

Definitions

- **Definition 1.** A *point* is that which has no part.
- **Definition 2.** A *line* is breadthless length.
- **Definition 4.** A *straight line* is a line which lies evenly with the points on itself.
- **Definition 10.** When a striaght line standing on another straight line makes the adjacent angles equal to one another, each of the equal angles is *right* and the straight line standing on the other is called *perpendicular* to that on which it stands.

Today we say a right angle is one which measures $90°$, and every other degree measurement is possible too. But in *Elements*, only a right angle ever gets mentioned or used, and it was defined using perpendicular lines.

The next definition is that of a circle, where the "one point" is the circle's center, and the "straight lines" are the radii. Basically, it says that when you draw a straight line segment that starts at the center of the circle and ends at the circle itself, the length of this segment is always the same — which today we call its *radius.*

- **Definition 15.** A *circle* is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to each other

Definitions 19 to 22 were of *triangles* (plane figures contained by three straight lines), *quadrilaterals* (contained by four straight lines), *equilateral triangles* (all sides are equal), and *isosceles triangles* (precisely two of its sides are equal).

- **Definition 23.** *Parallel* straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

Common Notions

- **Common Notion 1.** Things which are equal to the same thing are also equal to one another.
- **Common Notion 2.** If equals be added to equals, the wholes are equal.
- **Common Notion 3.** If equals be subtracted from equals, the remainders are equal.
- **Common Notion 4.** Things which coincide with one another are equal to one another.
- **Common Notion 5.** The whole is greater than the part.

Postulates

- **Postulate 1.** [It is possible] to draw a straight line from any point to any [other] point.
- **Postulate 2.** [It is possible] to produce a finite straight line continuously in a straight line.

These are the two axioms that allow you to use a straightedge in your calculuations. The first says you can use it to connect two points, while the second says that you can cut a line off at some points (for example, this allows you to chop the ends off three intersecting lines in order to form a triangle).

We have a straightedge, now we need a compass.

- **Postulate 3.** [It is possible] to describe a circle with any center and distance.

The "distance" here is the radius of the circle. A common technique when doing straightedge-and-compass constructions is to measure a distance with your compass, lock the compass to store this distance, and to transfer that distance to another part of your construction. Euclid didn't assert in his postulate that this is allowed. Why not? Because he didn't need to. His third proposition of Book I was a proof that that one can do this. This highlights the brilliance of his choice of postulates — he never assumed something that could be proven from the other postulates.

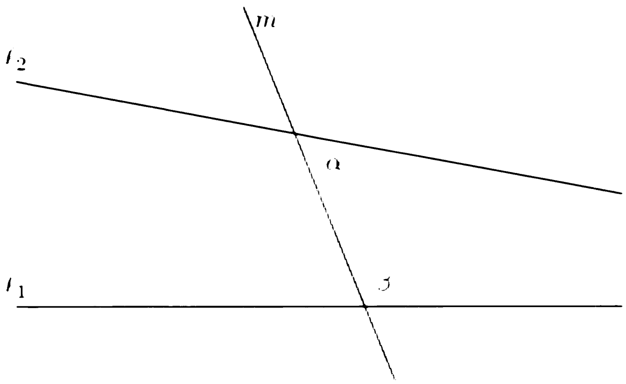
We have lines and circles, what about angles? Euclid kept his lone assumption about angles remarkably simple:

- **Postulate 4.** All right angles are equal to one another.

That's it. It's so simple you would be forgiven for not realizing that this should be explicitly assumed. All it says is that a right angle in one part of the plane is equal to another at a different location in the plane.

But since every good story needs some conflict, his final postulate provided just that.

- **Postulate 5** (*The parallel postulate*)**.** If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, then two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.



If $\alpha + \beta$ is smaller than 180° (or, in Euclid's language, is smaller than two right angles), then the lines ℓ_1 and ℓ_2 will intersect; moreover, they will intersect on the right-hand side of the picture, since this is the side containing the α and β angles.

Basically, if $\alpha + \beta = 180^\circ$ (i.e. "equals two right angles"), then the two lines would be parallel. Otherwise, on one of the two sides the sum is less than 180°, and on that side you will be getting an intersection.