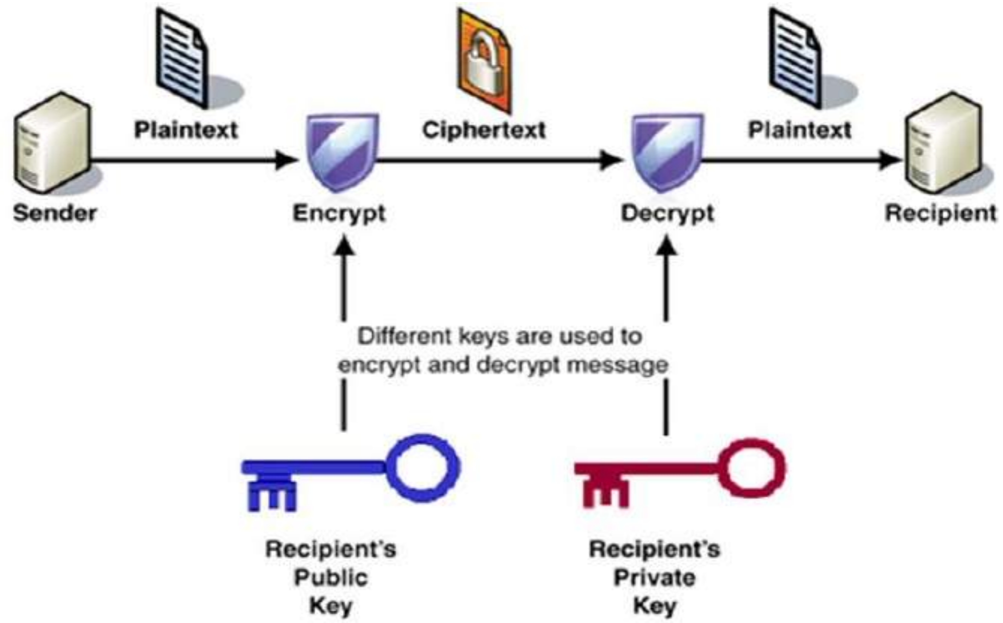

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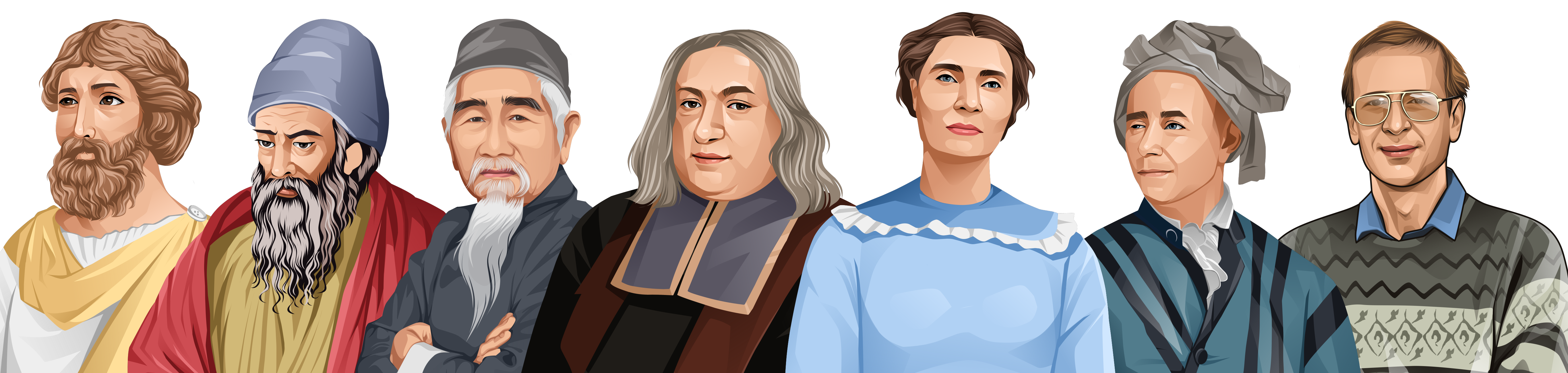
Chapter 5: Number Theory



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- It is said that Pythagoreans believed that numbers have mystical powers—especially integers.

- Their beliefs held that all lengths are *rational numbers.* That is, a ratio of integers.

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- They failed. We know the secret. And I can prove it.

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QED

Poem on Hippasus

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- Hippasus showed 1 and $\sqrt{2}$ are incommensurable.

Perfect Numbers

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- **Definition.** A number N to be *perfect* if N's proper divisors sum to N.

Challenge: Try to find a perfect number

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- Perfect numbers may have originated with the Pythagoreans.

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- And the only proper divisors of $2^{n-1}(2^n - 1)$ are
$1 \ , \ 2 \ , \ 2^2 \ , \ 2^3 \ , \ \ldots \ , \ 2^{n-1}$
and
$2^n - 1 \ , \ 2(2^n - 1) \ , \ 2^2(2^n - 1) \ , \ \ldots \ , \ 2^{n-2}(2^n - 1) \ .$

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- Largest known prime (Feb 2023) is $2^{82,589,933} - 1$.

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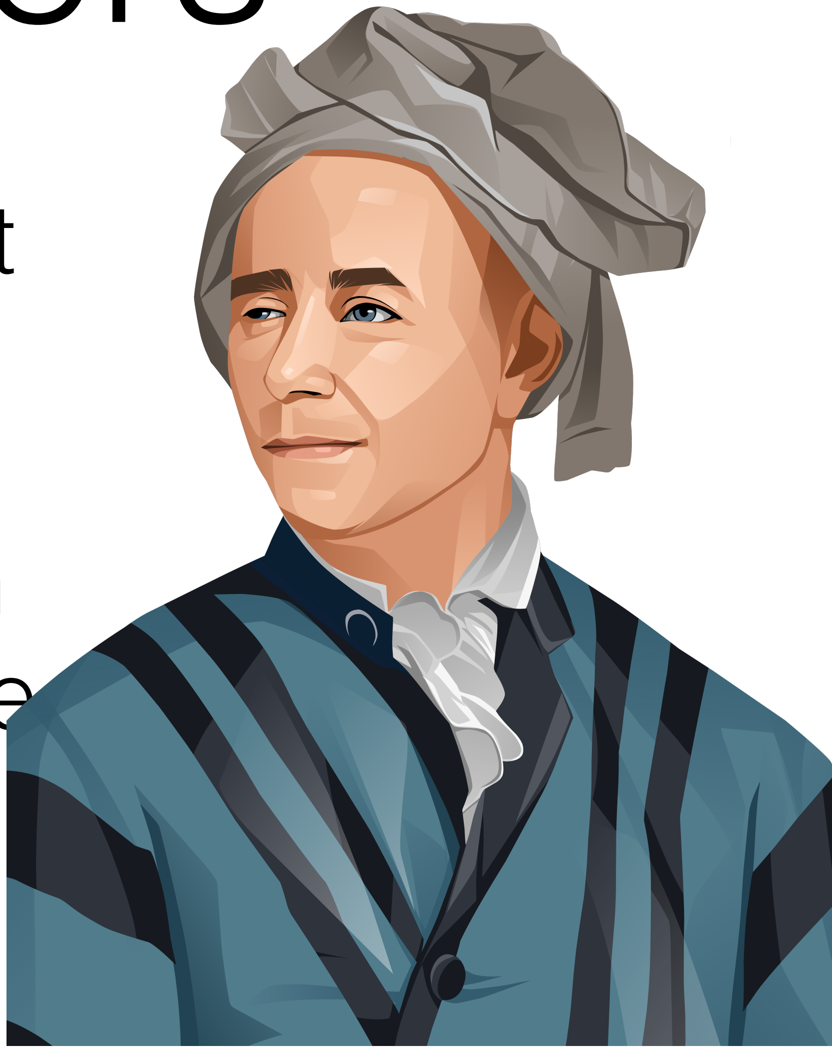

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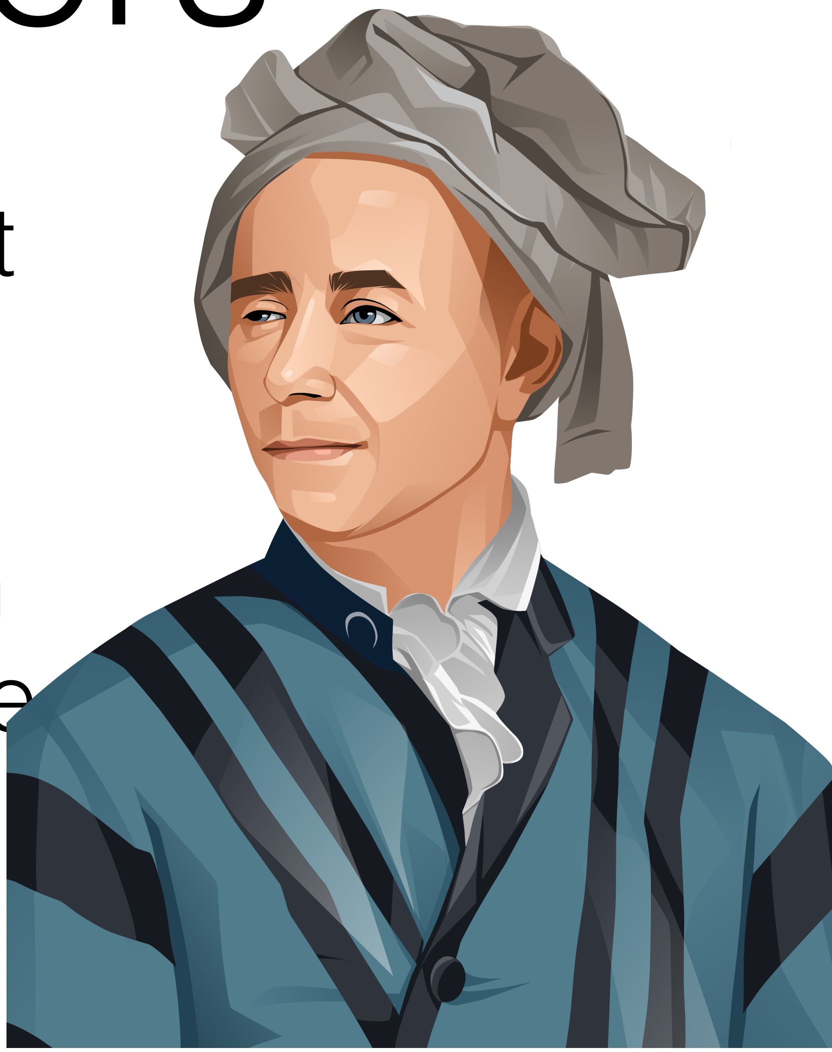

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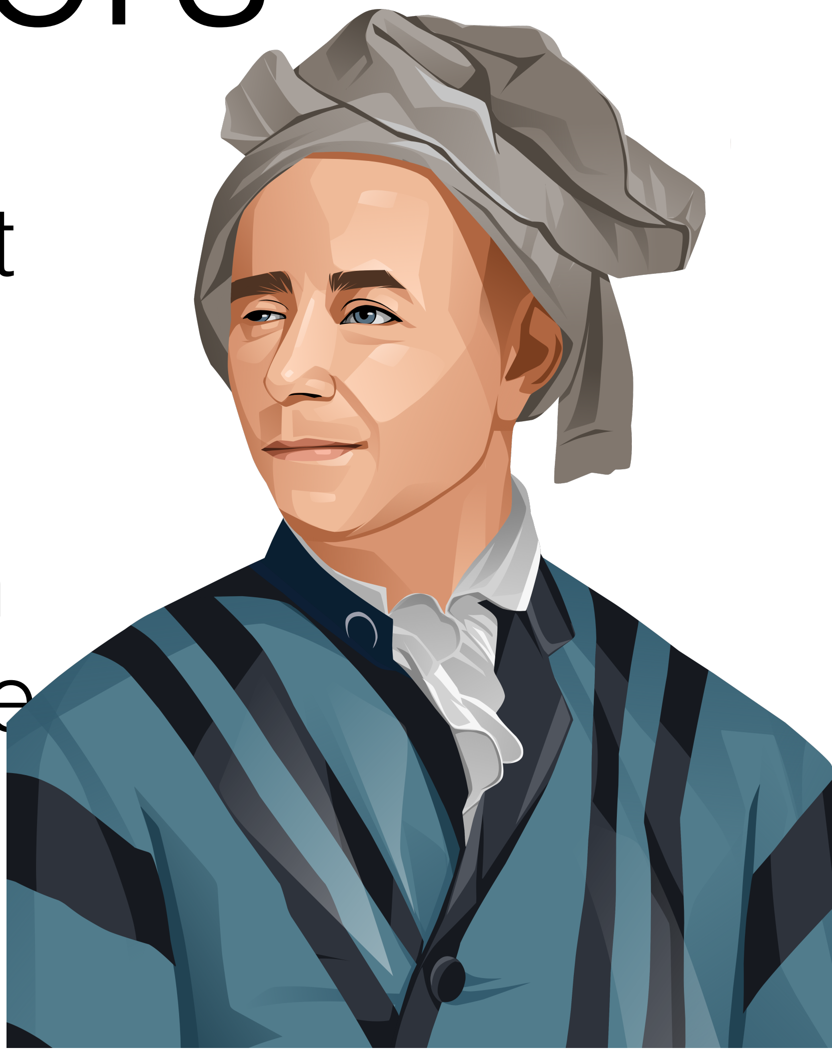

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- Today, 51 are known.

Perfect Numbers

<u>**Open questions:**</u>

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- Are there infinitely many perfect numbers?
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- Are there infinitely many Mersenne primes?

The Aftermath: Fermat’s Little Theorem

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- While studying perfect numbers, Fermat proved what today is called *Fermat's little theorem*. It says:

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- This is very important in number theory, including in public key cryptography.

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- He rarely published but was well-known in math circles. He communicated via letter with top mathematicians. He often withheld his proofs.

RSA Cryptosystem

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Primes

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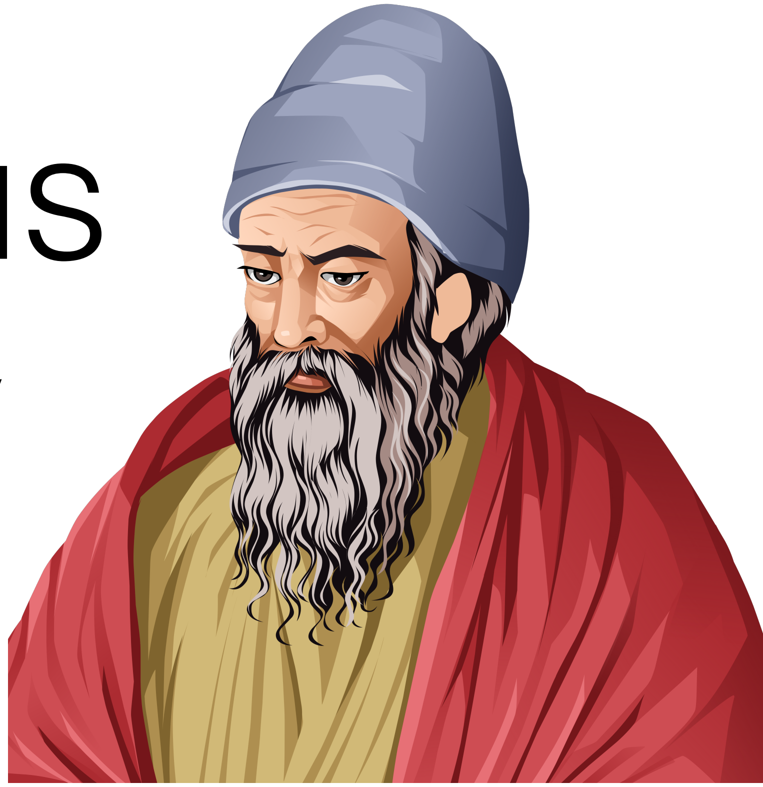
The Most Famous Proof in History



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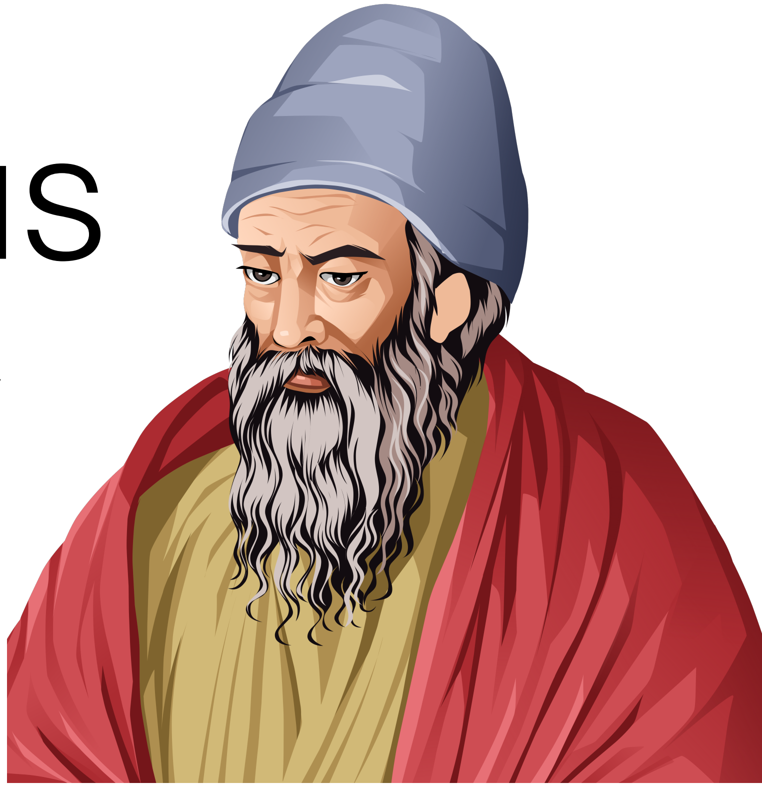

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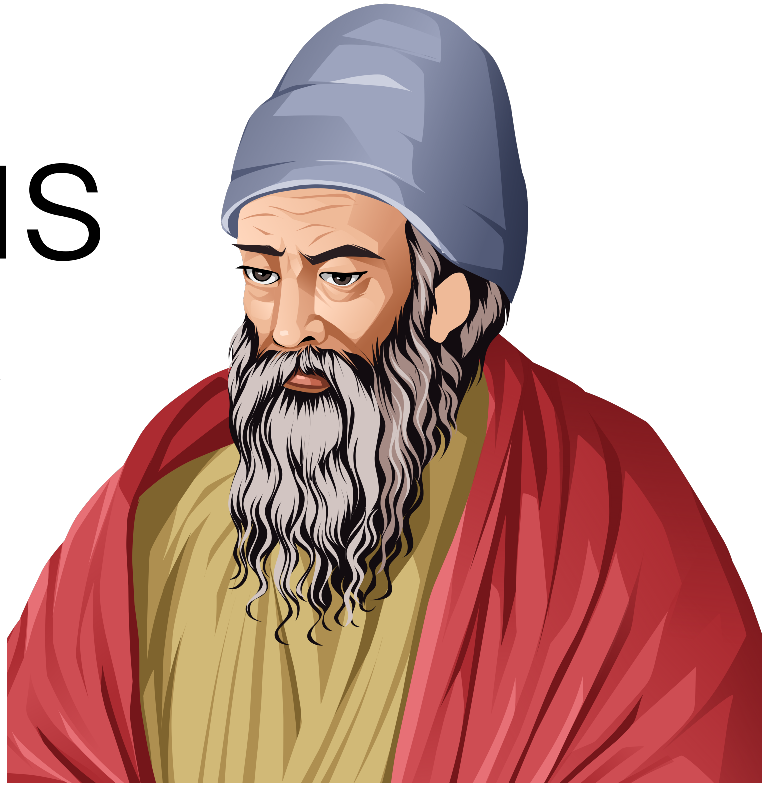

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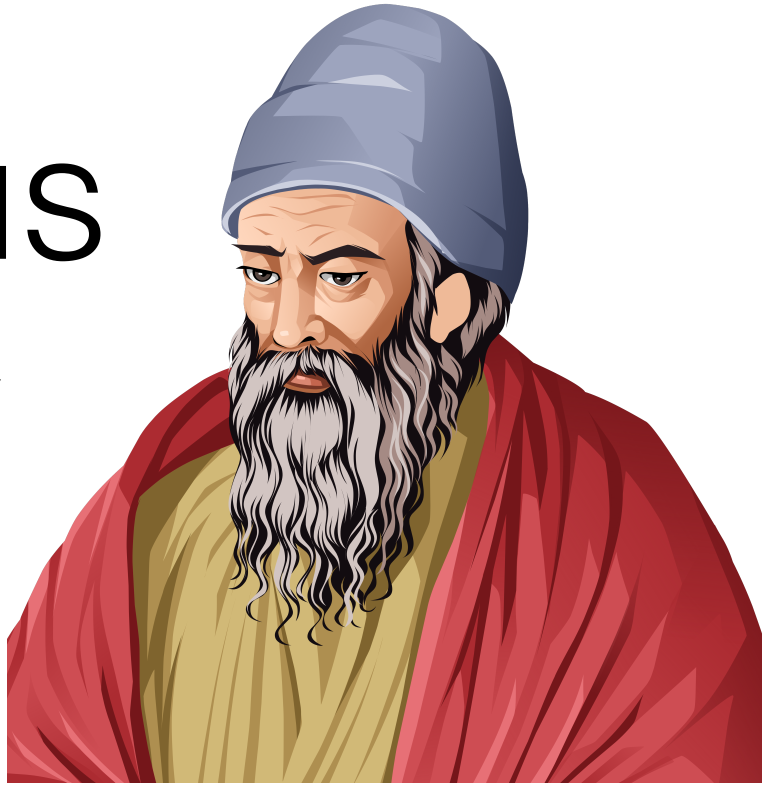

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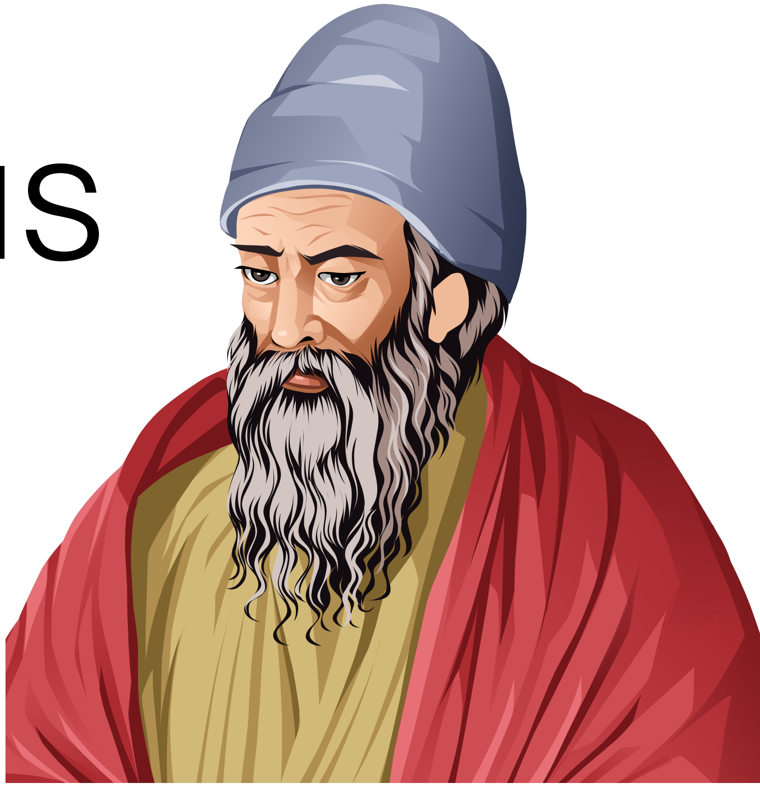

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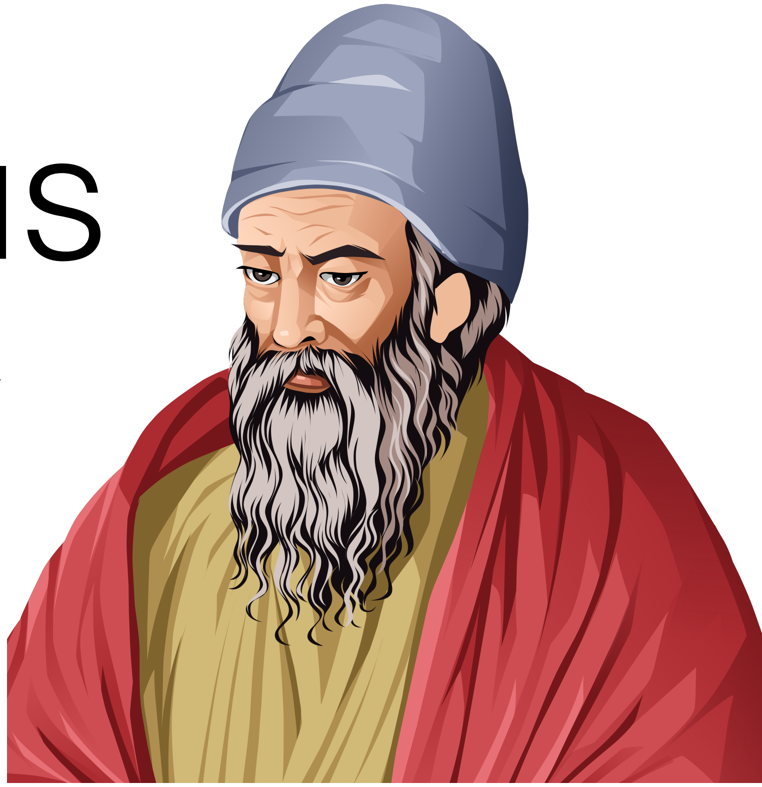

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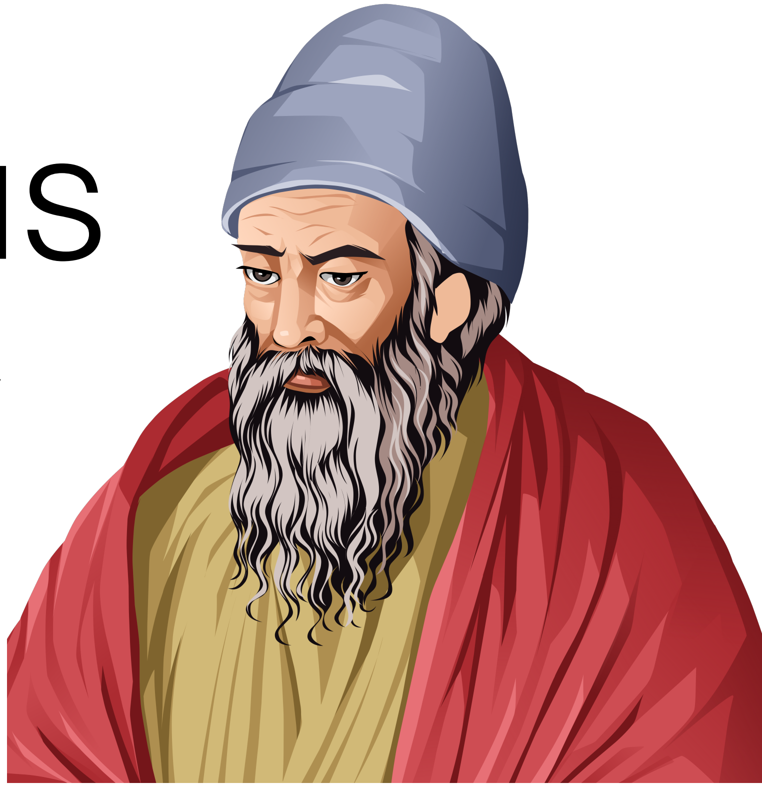

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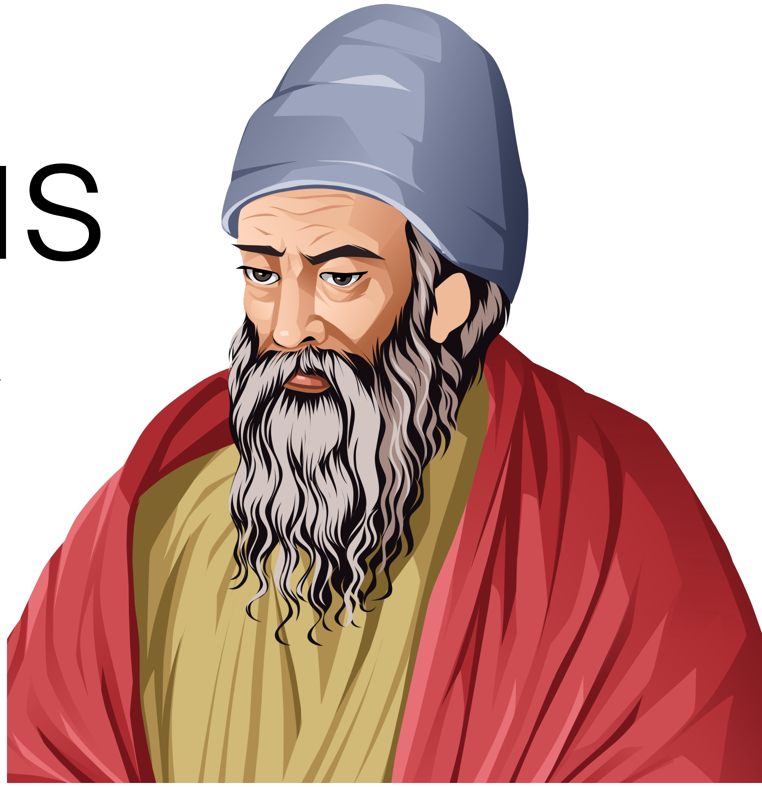

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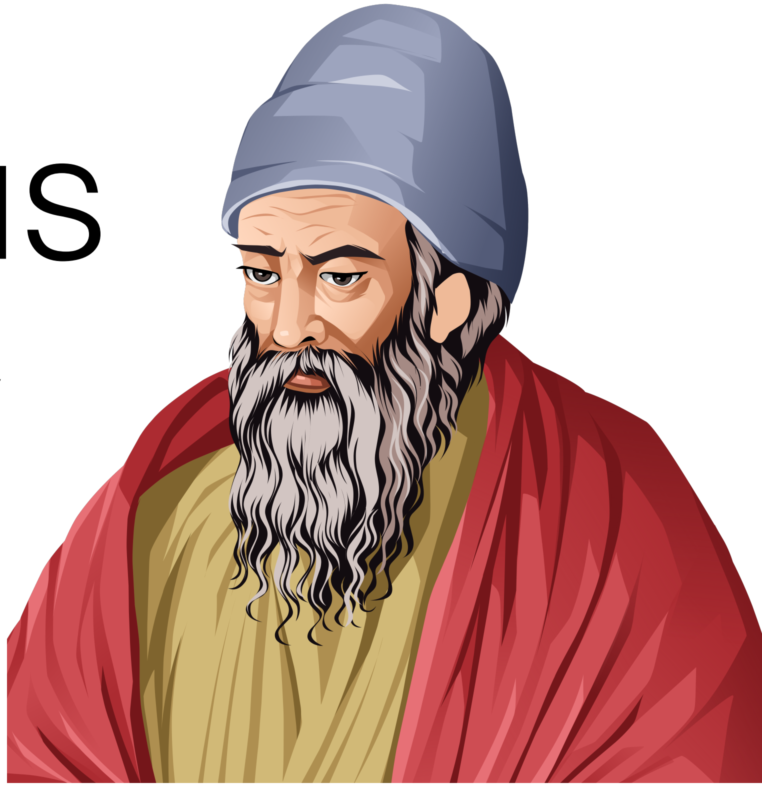

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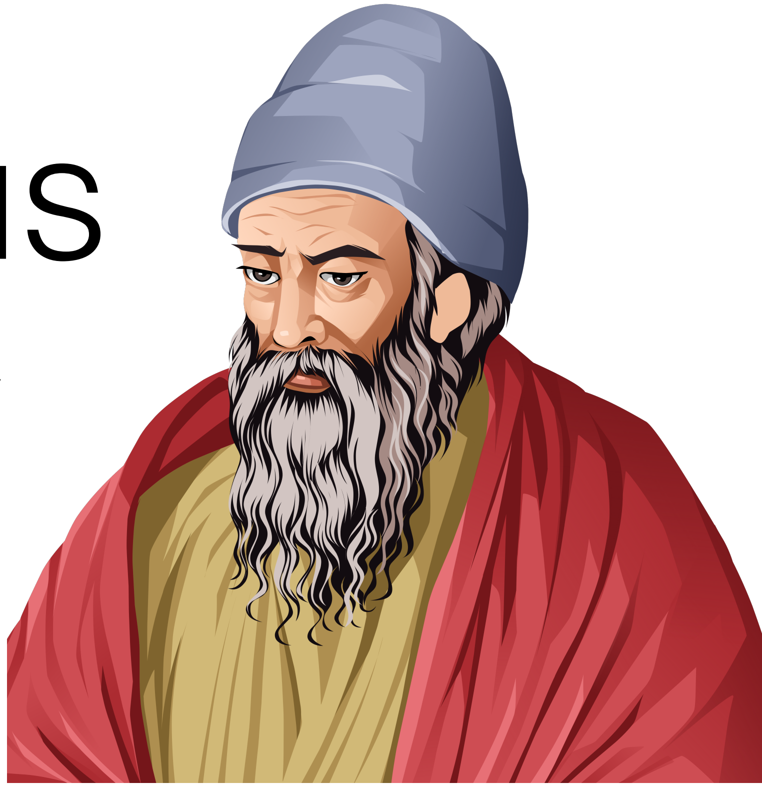

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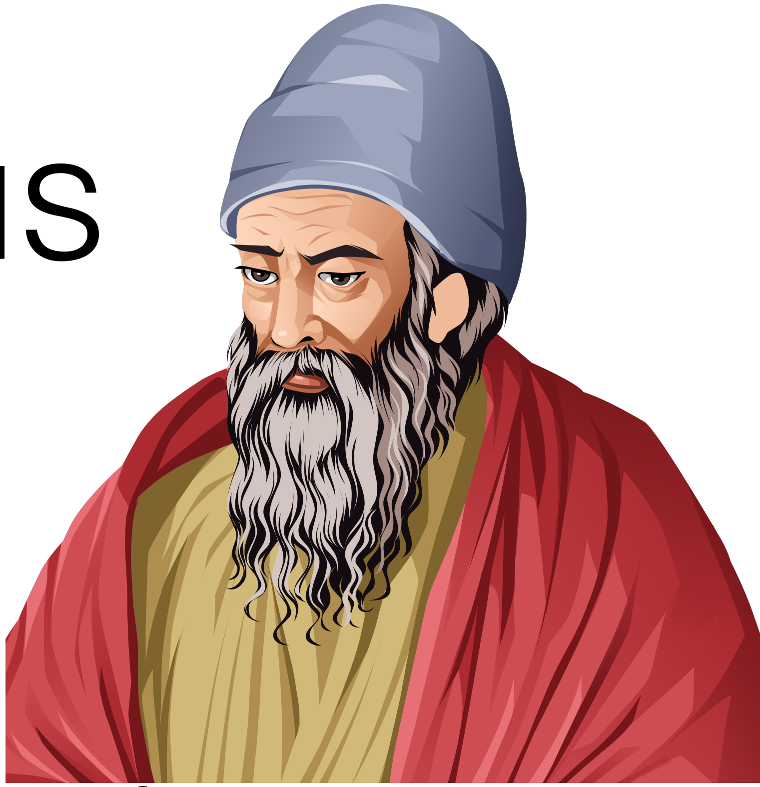

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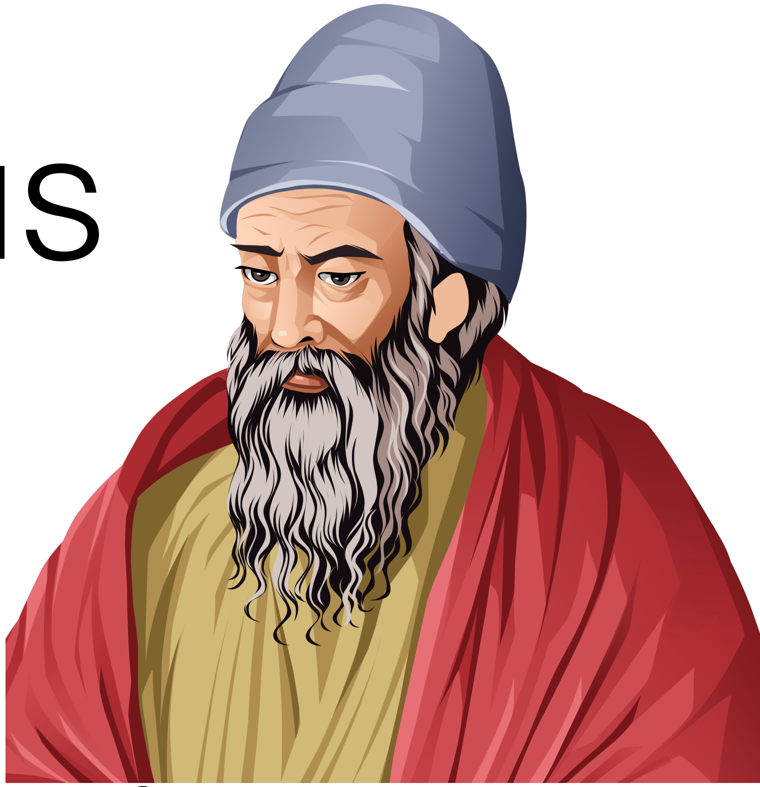

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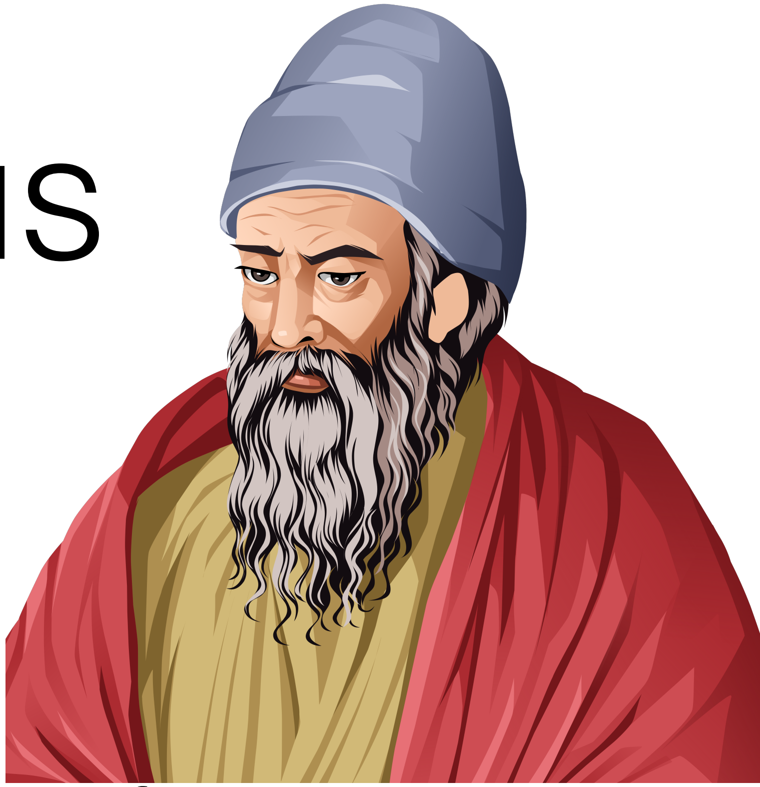

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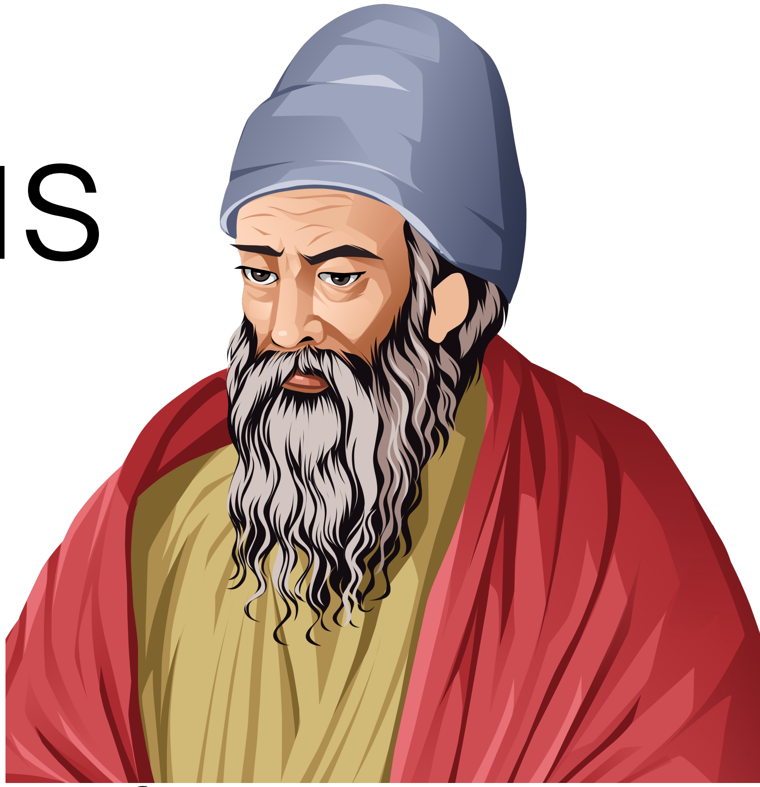

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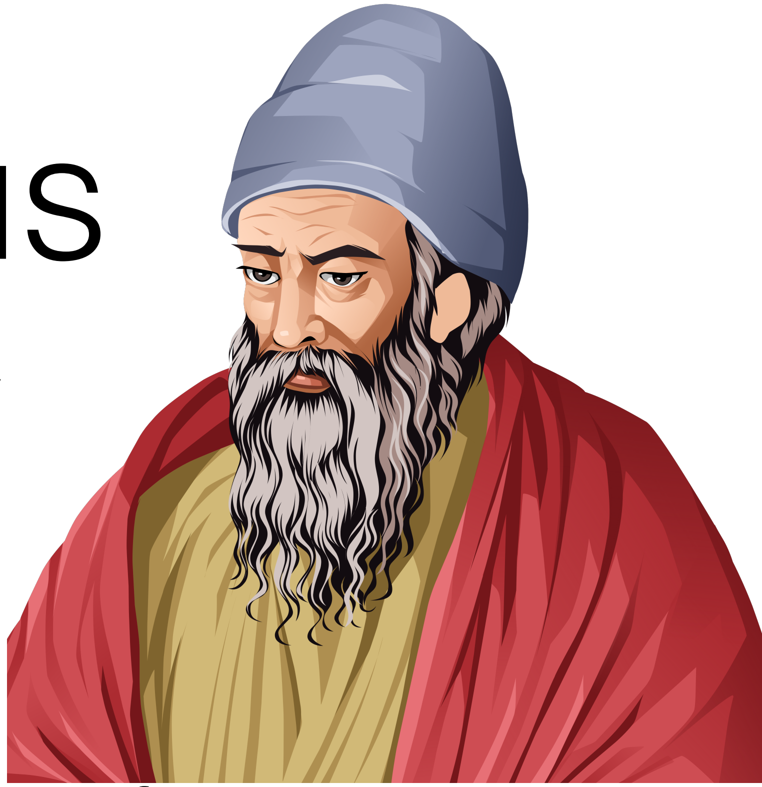

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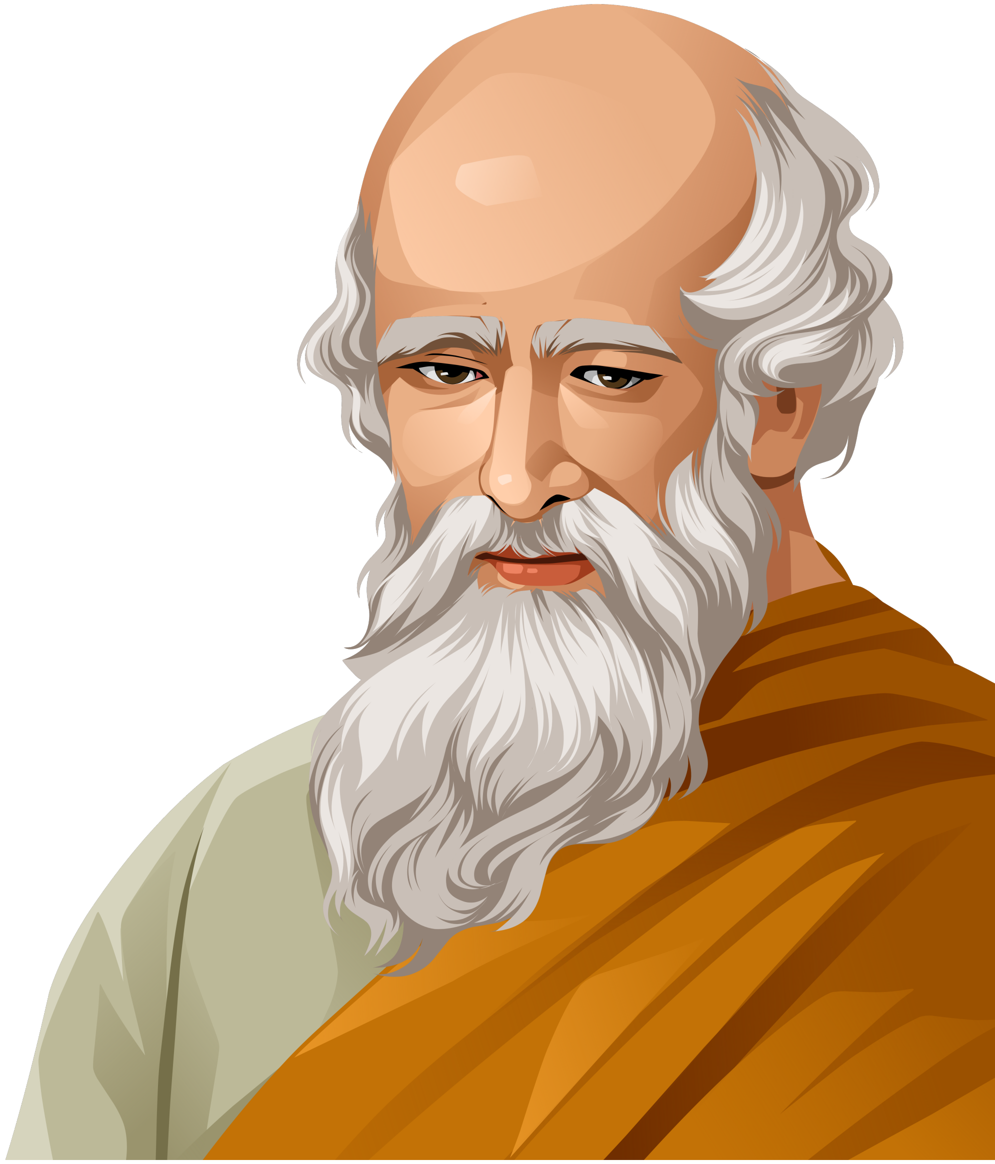
Euclid Poem

Where’s Archimedes?



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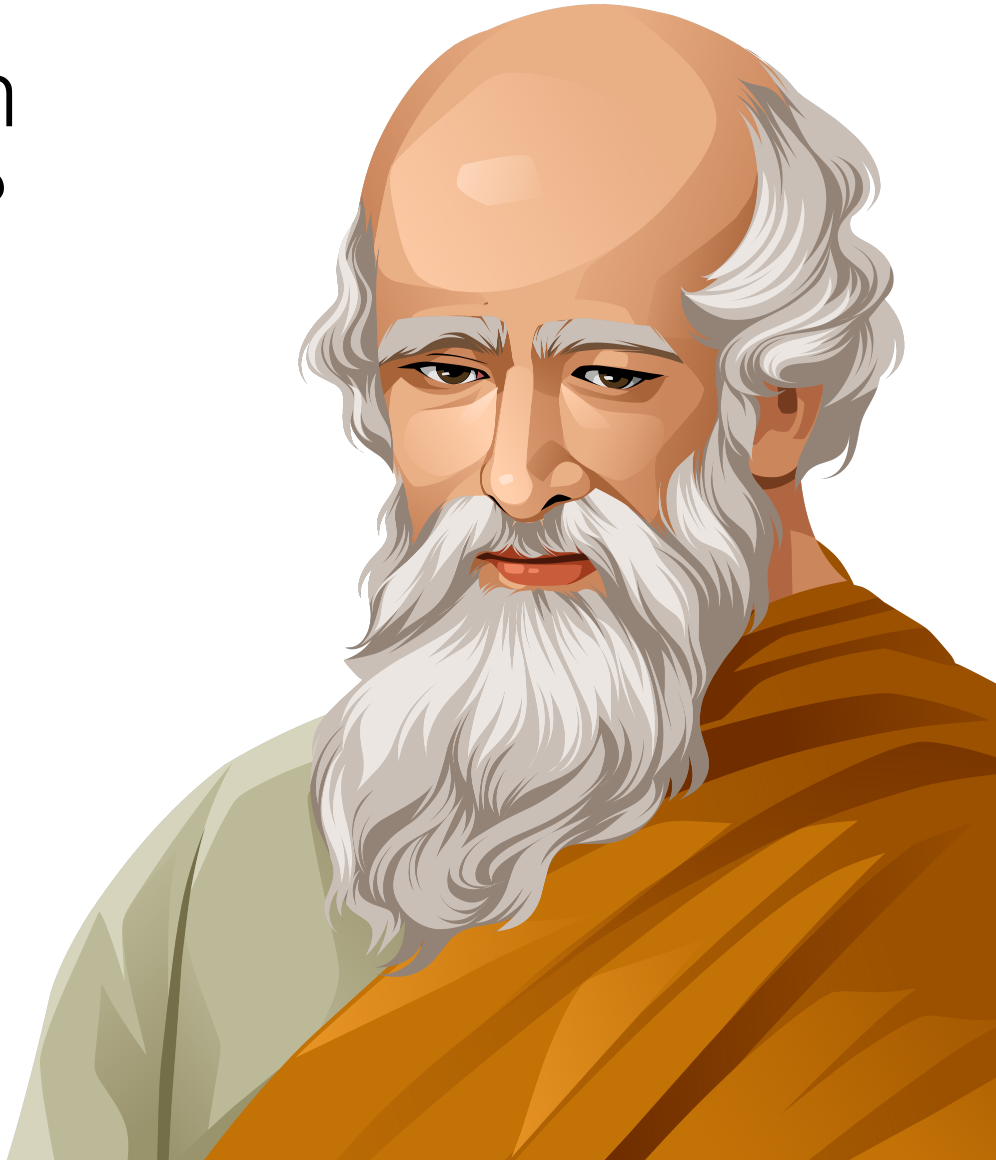

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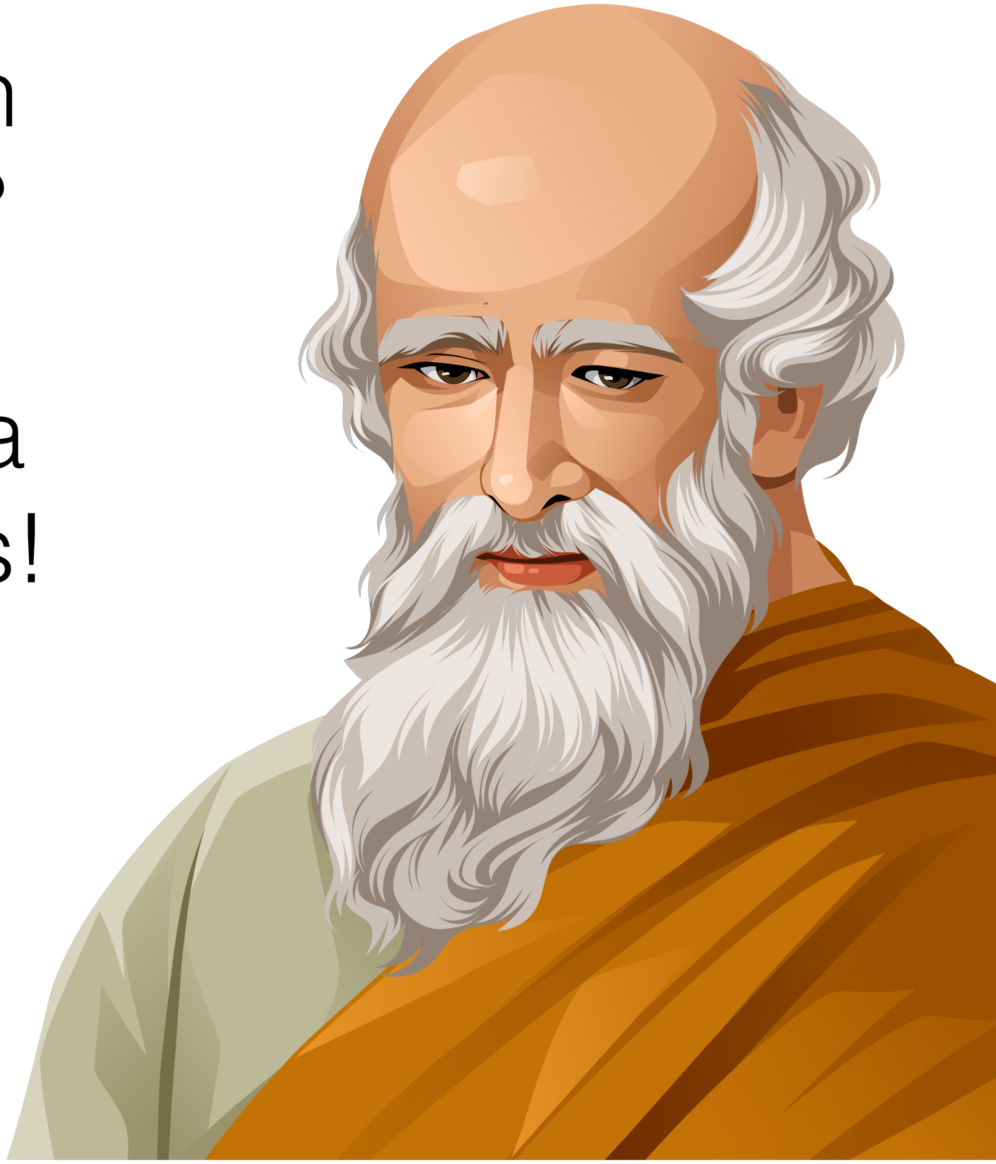

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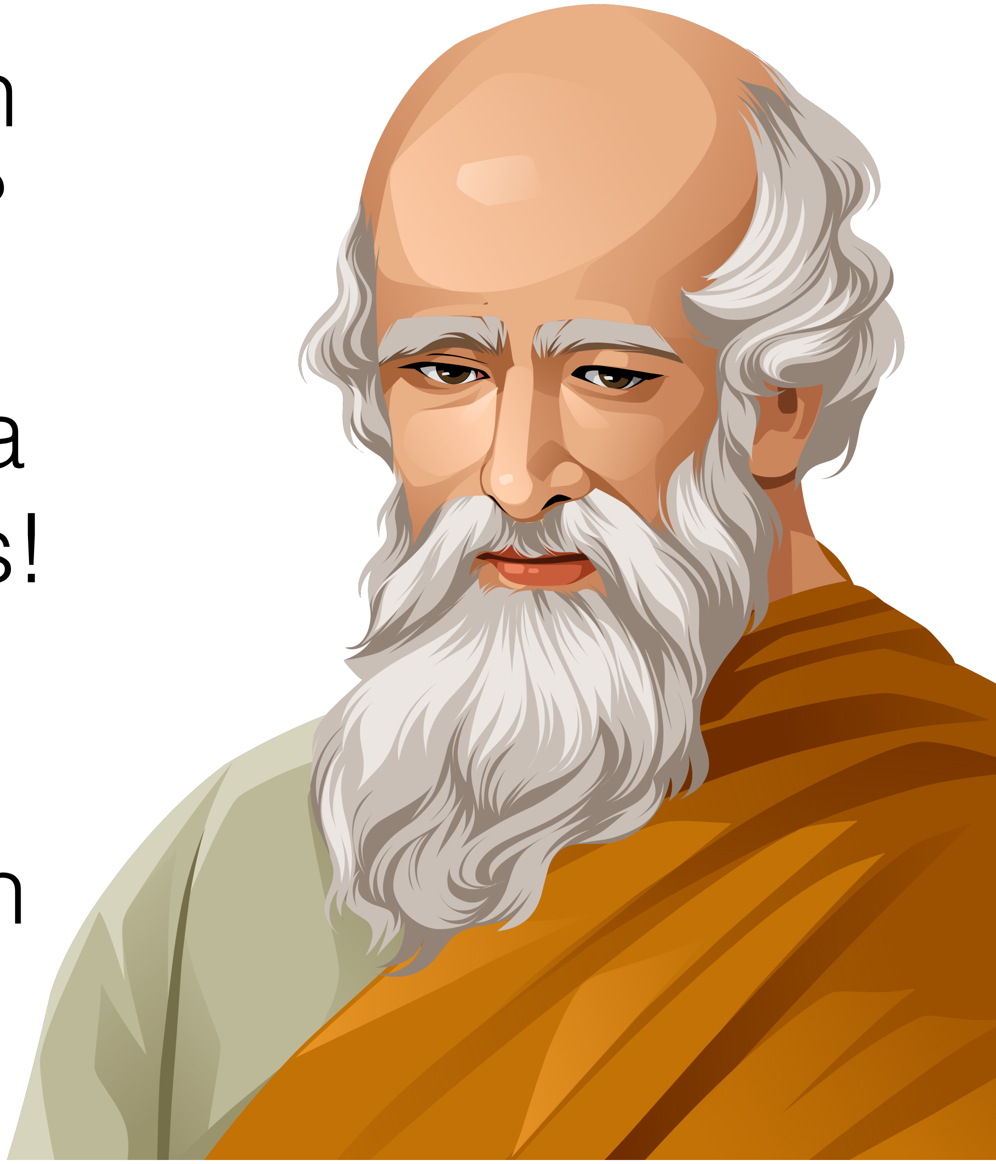

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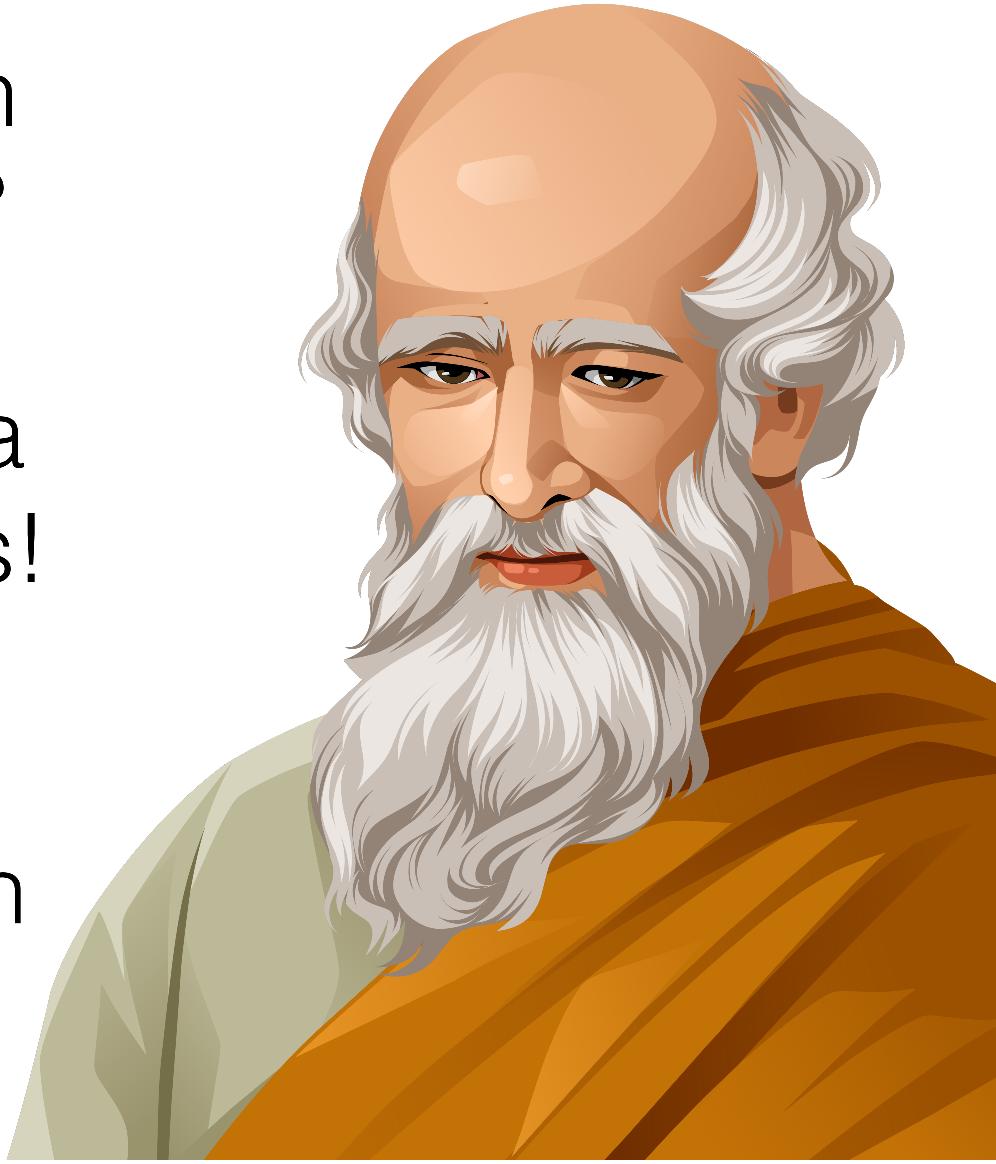
- We will discuss him in Chapter 7.



Three Notable Problems About Primes

Problem 1

Problem 1

- What do you notice:

$$
\begin{aligned}
4 &= 2 + 2 \\
6 &= 3 + 3 \\
8 &= 5 + 3 \\
10 &= 7 + 3 \\
12 &= 7 + 5 \\
14 &= 7 + 7 \\
16 &= 13 + 3 \\
18 &= 13 + 5 \\
20 &= 17 + 3 \\
&\vdots
\end{aligned}
$$

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  Every positive even integer can be written as the sum of two primes.

Problem 2

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- What do you notice:

3 and 5
5 and 7
11 and 13
17 and 19
29 and 31
41 and 43
59 and 61
71 and 73
$\vdots$

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- Over the next year, a riveting polymath project brought the gap down to 246.

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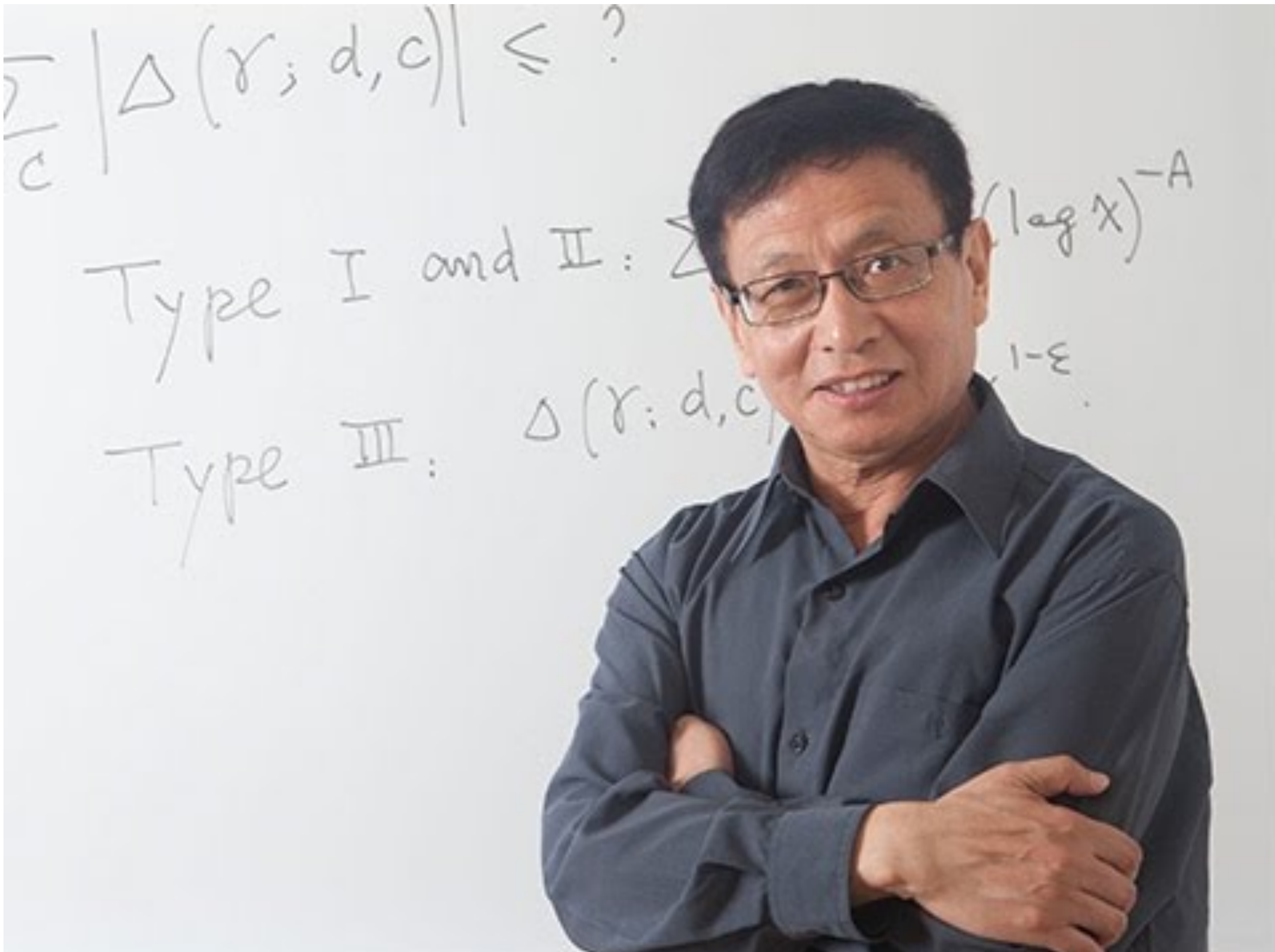

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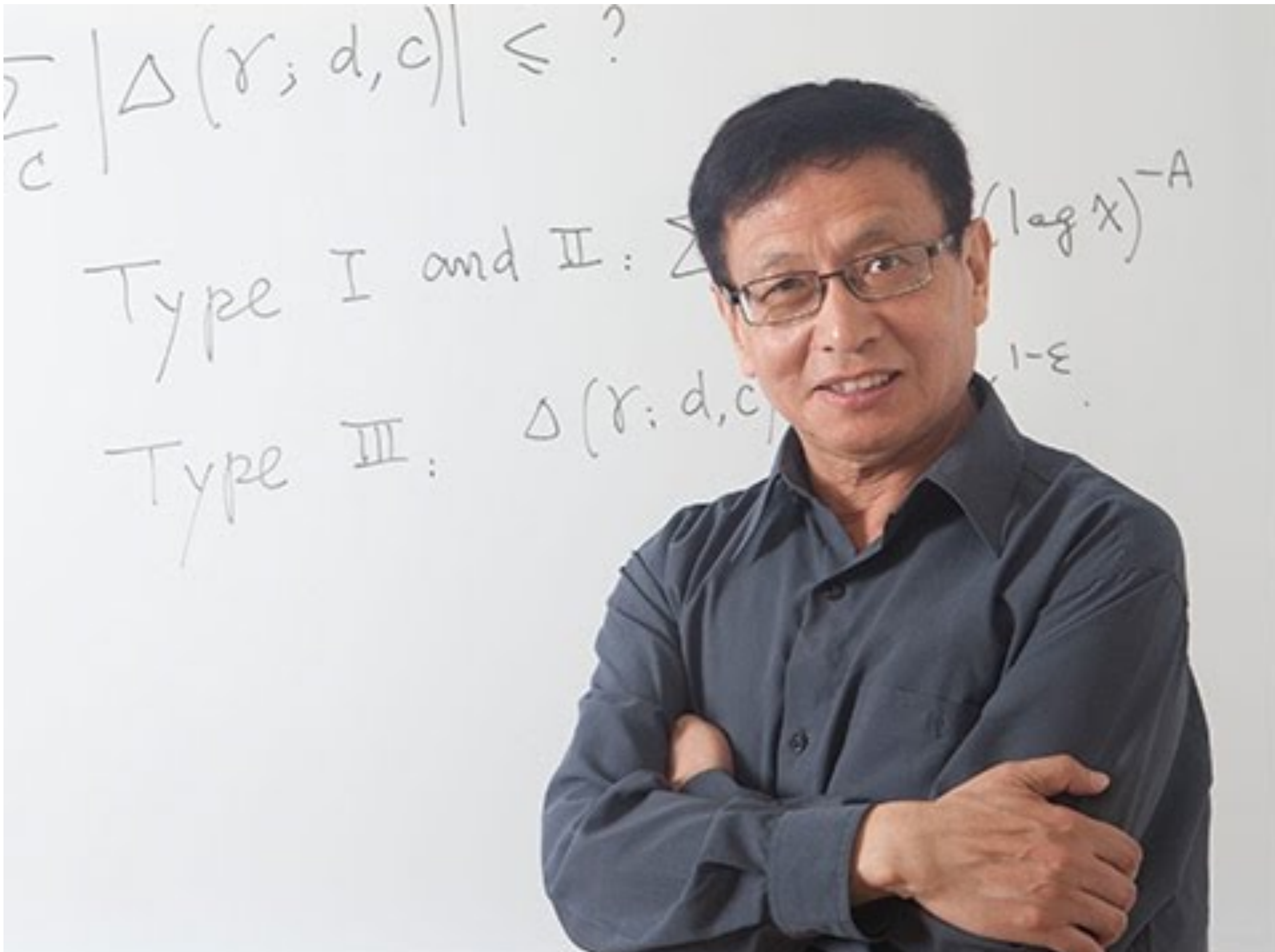

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- Let $\pi(N)$ be the number of primes between 1 and N.
- The prime number theorem says

$$\lim_{N \to \infty} \frac{\pi(N)}{N/\log(N)} = 1.$$

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- Example: “If my troops march in rows of 5, there are 4 soldiers left over. If they march in rows of 8 there are 2 left over. In rows of 9, there are 7 left over.”

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- The method to do so is called the Chinese remainder theorem.
- It first appeared in Sun Zi's book *Mathematical Manual*, written in the 3rd century AD.

Chinese Remainder Theorem

Chinese Remainder Theorem

- If you feel comfortable with modular arithmetic, check it out in the notes.

Theorem.

Theorem 1.4 (Sun Zi)**.** Suppose $n_1, n_2, \ldots, n_k$ are integers larger than 1. If the n_i are pairwise relatively prime, then for any integers $a_1, a_2, \ldots, a_k$, the system of modular congruences

$$\begin{aligned} x &\equiv a_1 \pmod{n_1} \\ x &\equiv a_2 \pmod{n_2} \\ &\vdots \\ x &\equiv a_k \pmod{n_k} \end{aligned}$$

has a solution. Moreover, if x_1 and x_2 are both solutions to this system, then $x_1 \equiv x_2 \pmod{n_1 n_2 \cdots n_k}$.

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- Question: Can you think of a triple of positive integers that satisfy
$$x^3 + y^3 = z^3 \,?$$

Fermat's Last Theorem

"Theorem." Suppose n is an integer larger than 2. There are no positive integers x, y and z such that

$$x^n + y^n = z^n \,.$$

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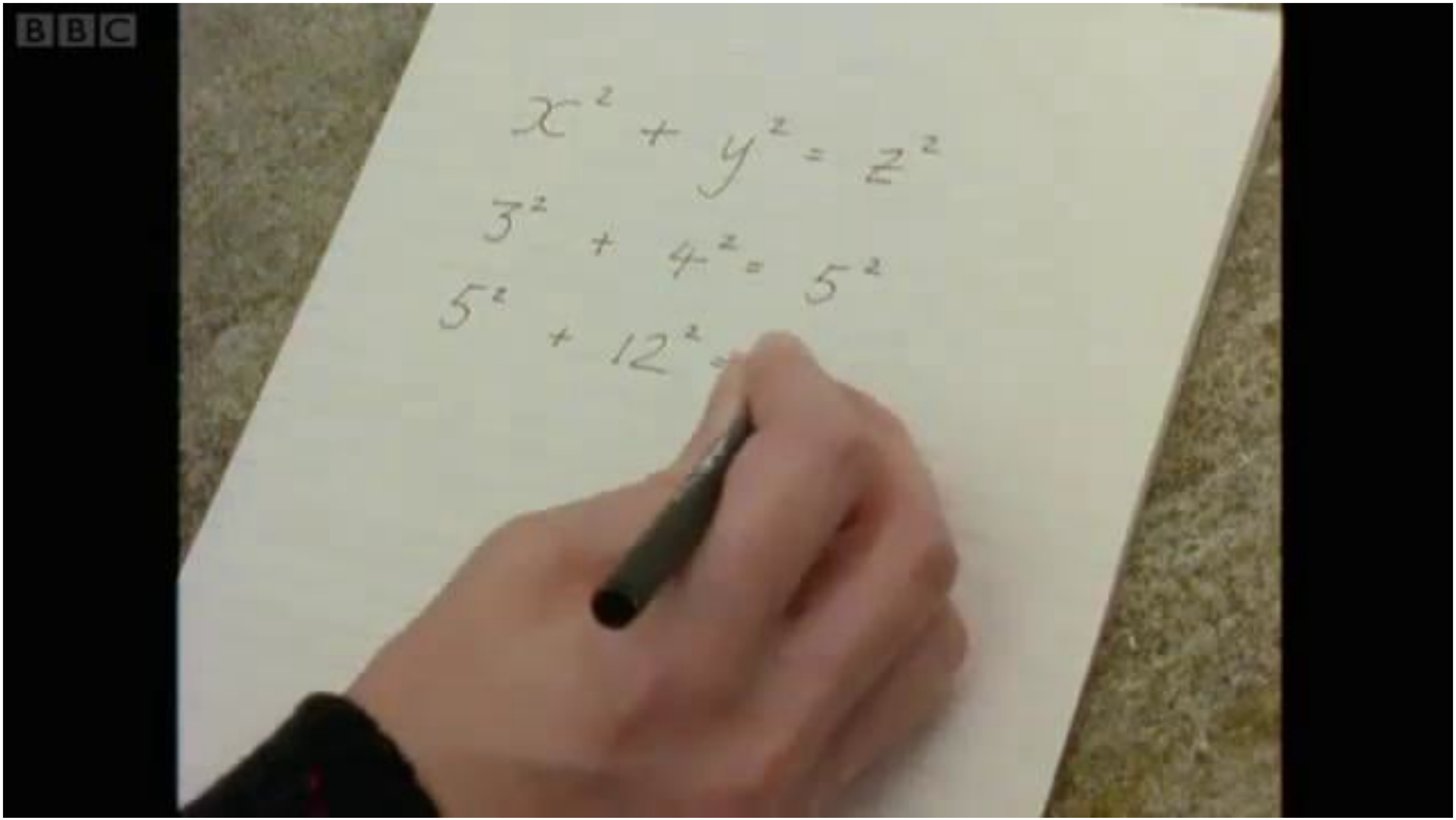

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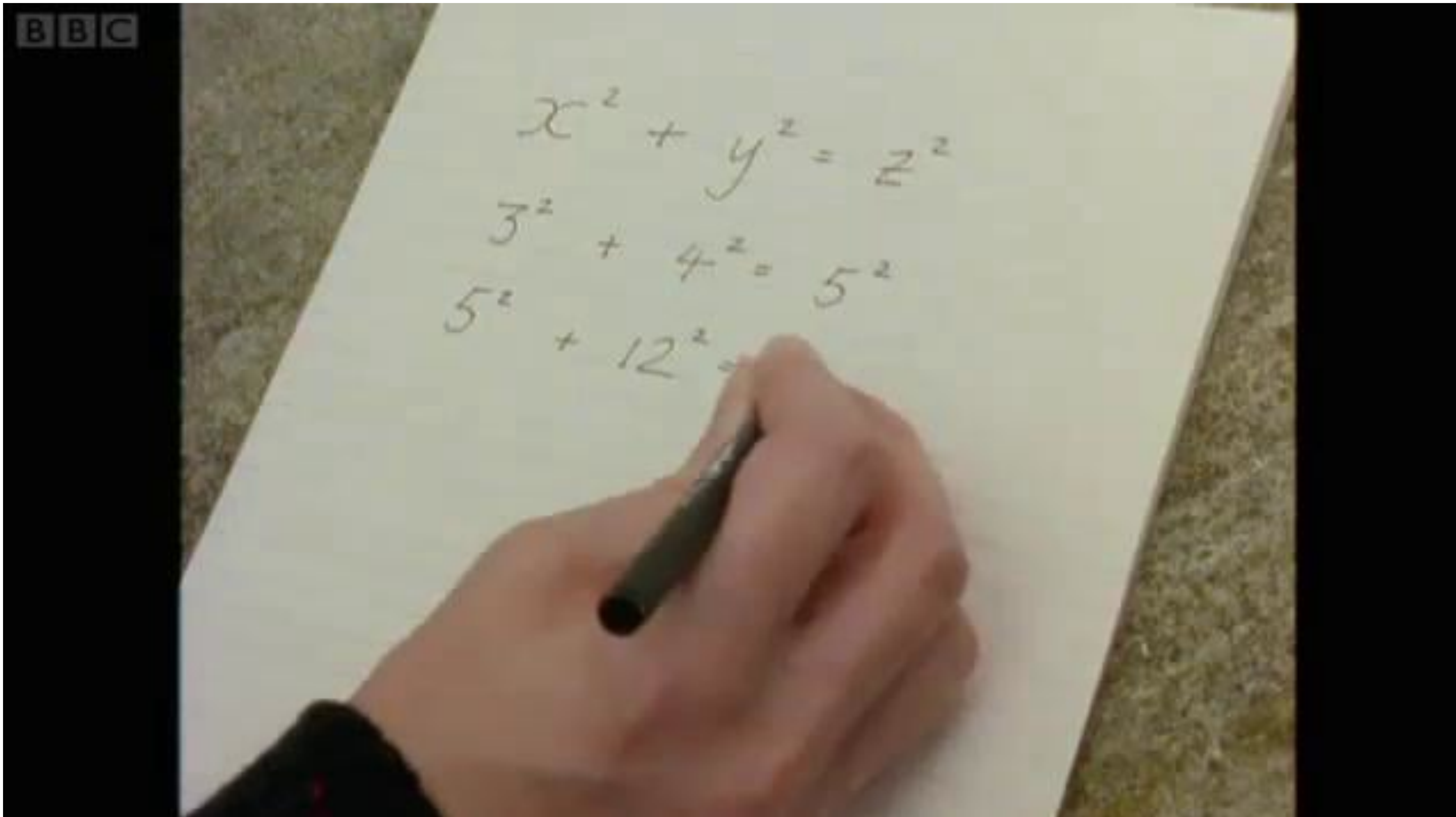

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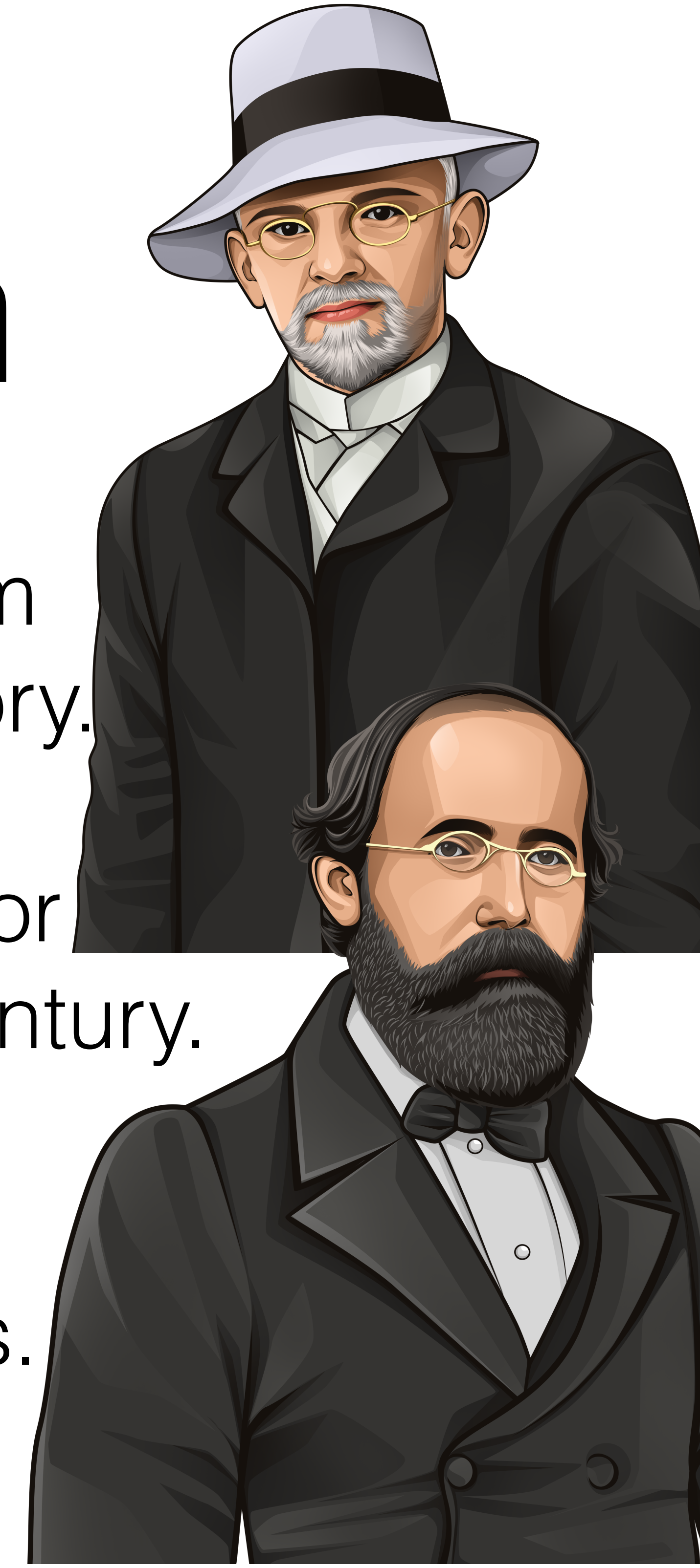

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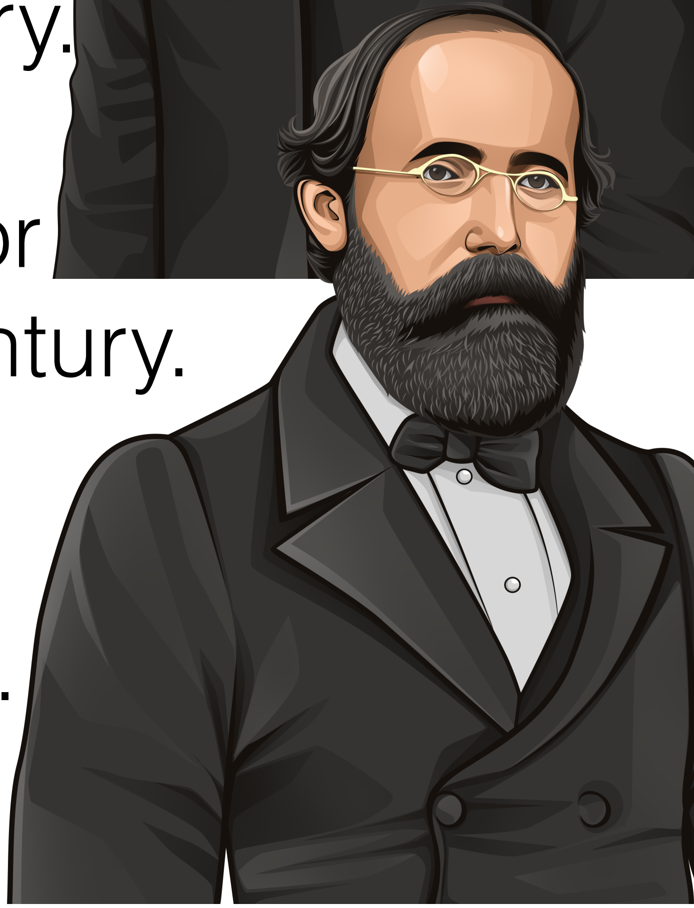

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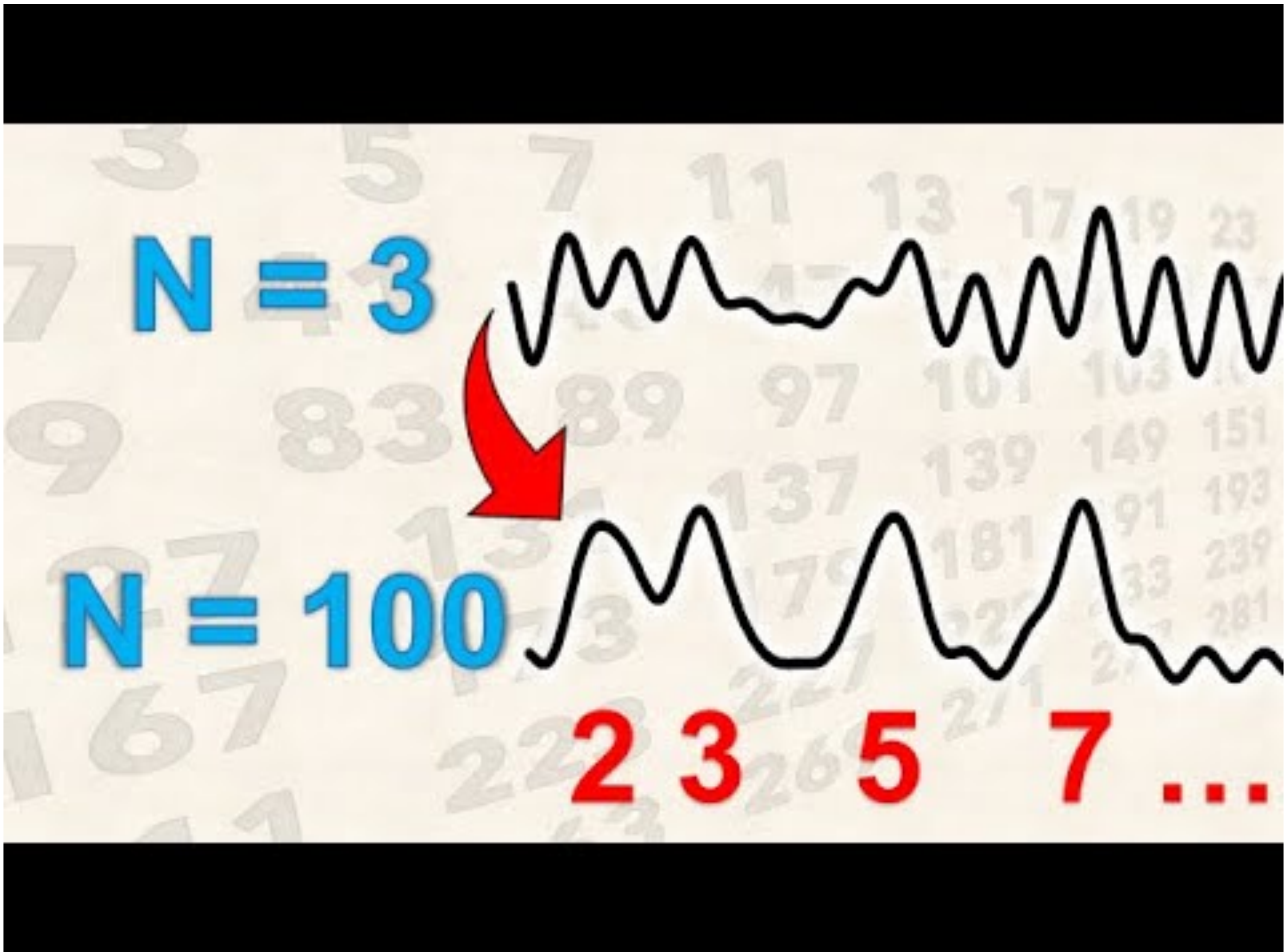
- In 2000, the Clay Institute published 7 problems for the next century and offered $1 million for any solution.

The Riemann Hypothesis



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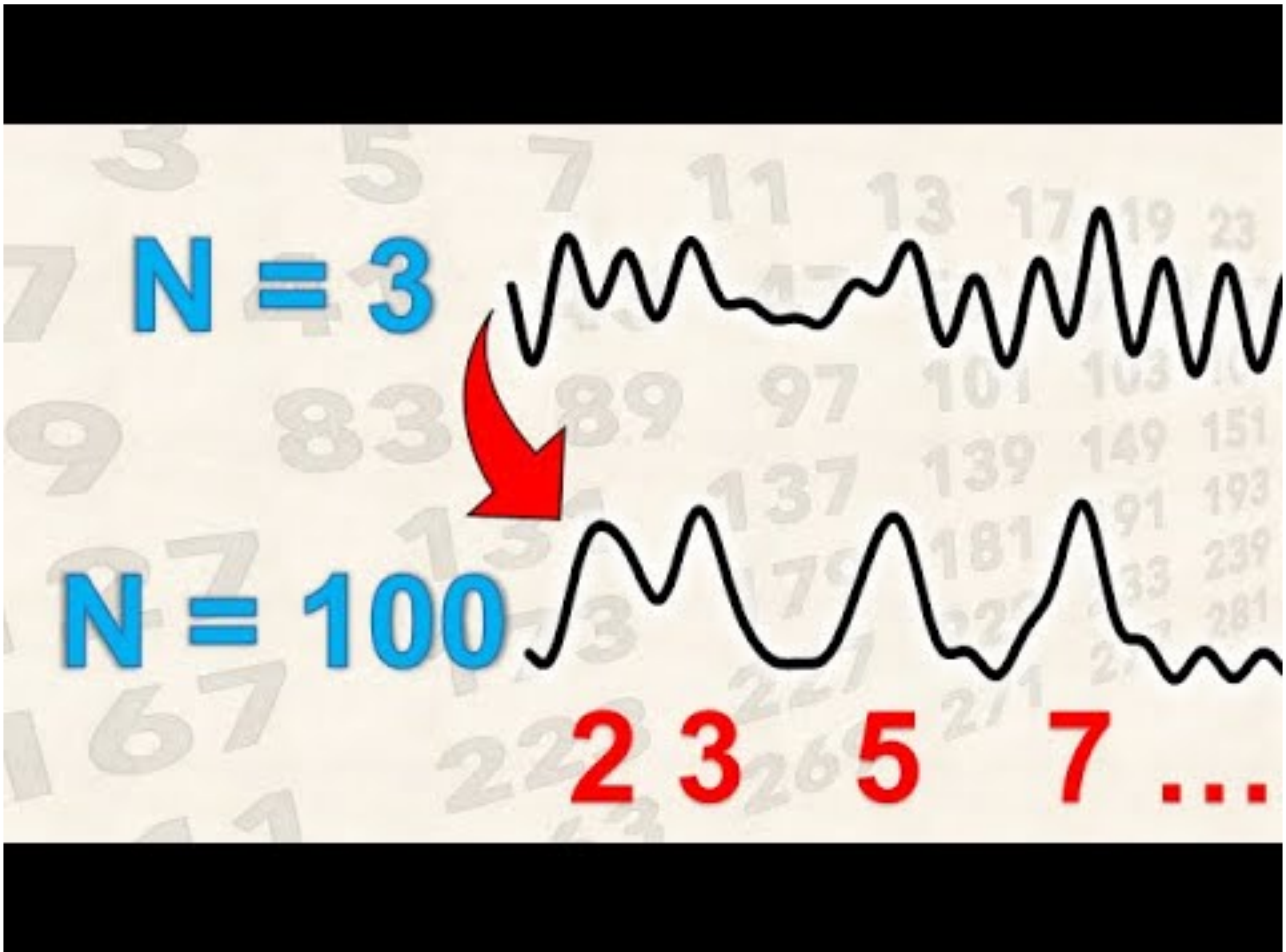
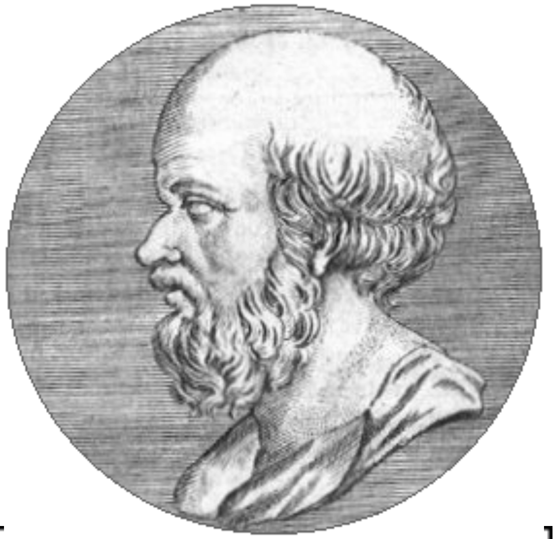


Shout-outs

Eratosthenes



	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
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Prime numbers

Eratosthenes

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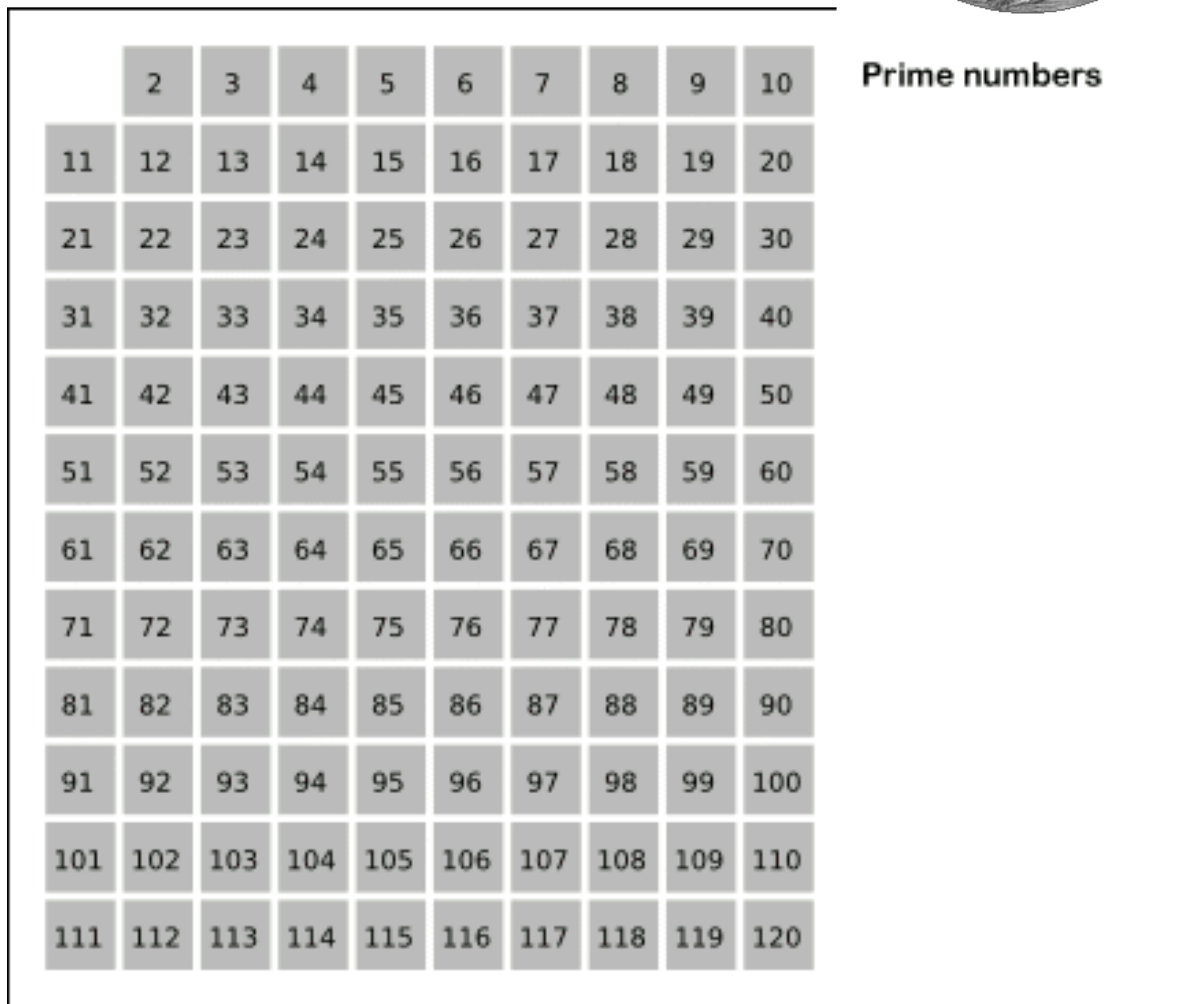

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- Sieve of Eratosthenes: Method for finding primes.
- First to calculate the circumference of the Earth.

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Prime numbers

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- Solved the problem of vibrating plates, winning a prestigious prize for her work.

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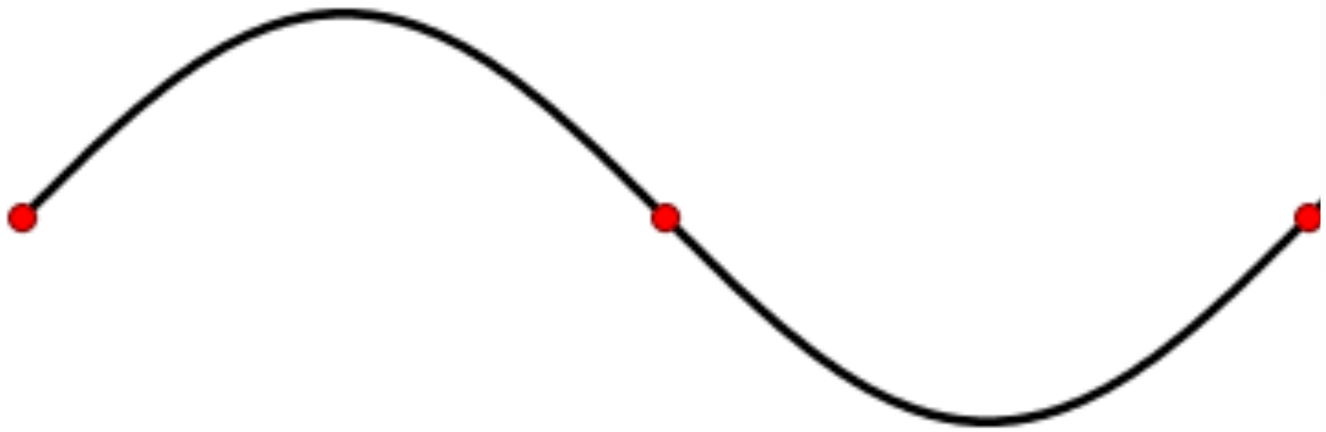
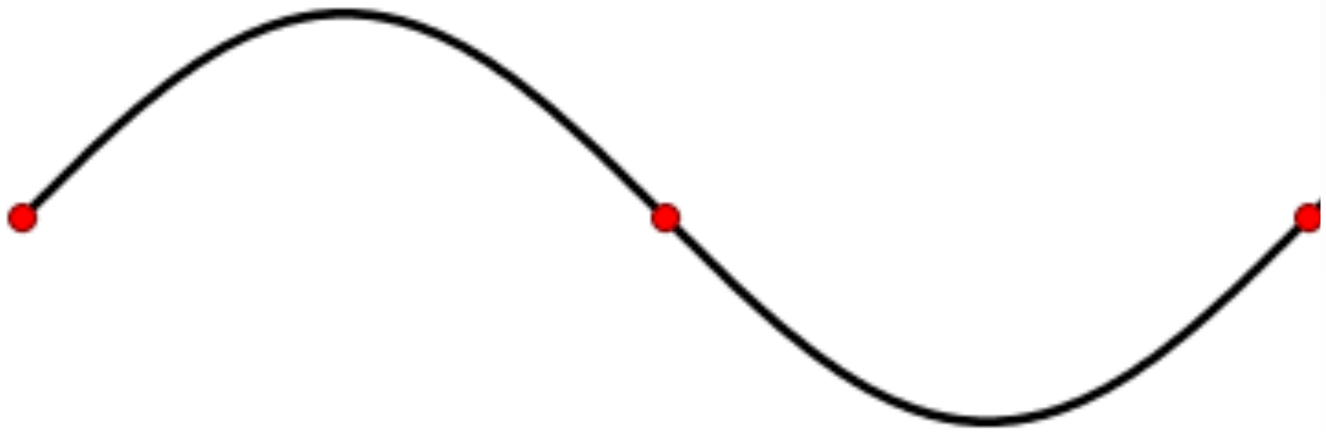


Sofie Germain

- Standing wave:



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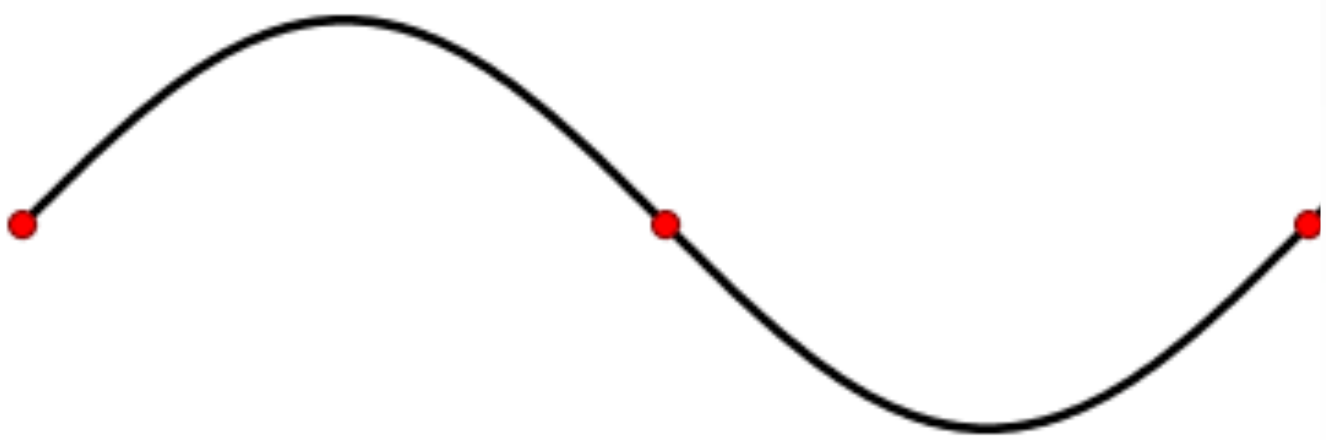

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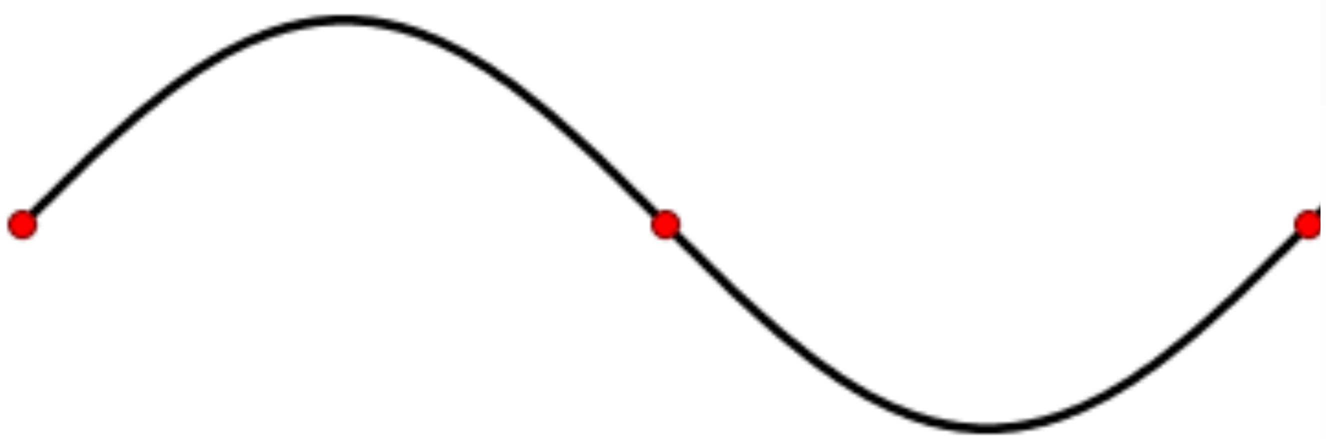

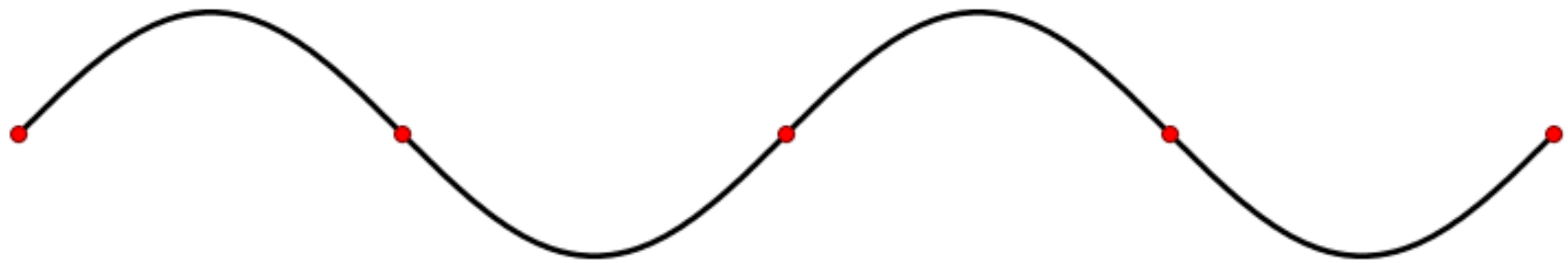

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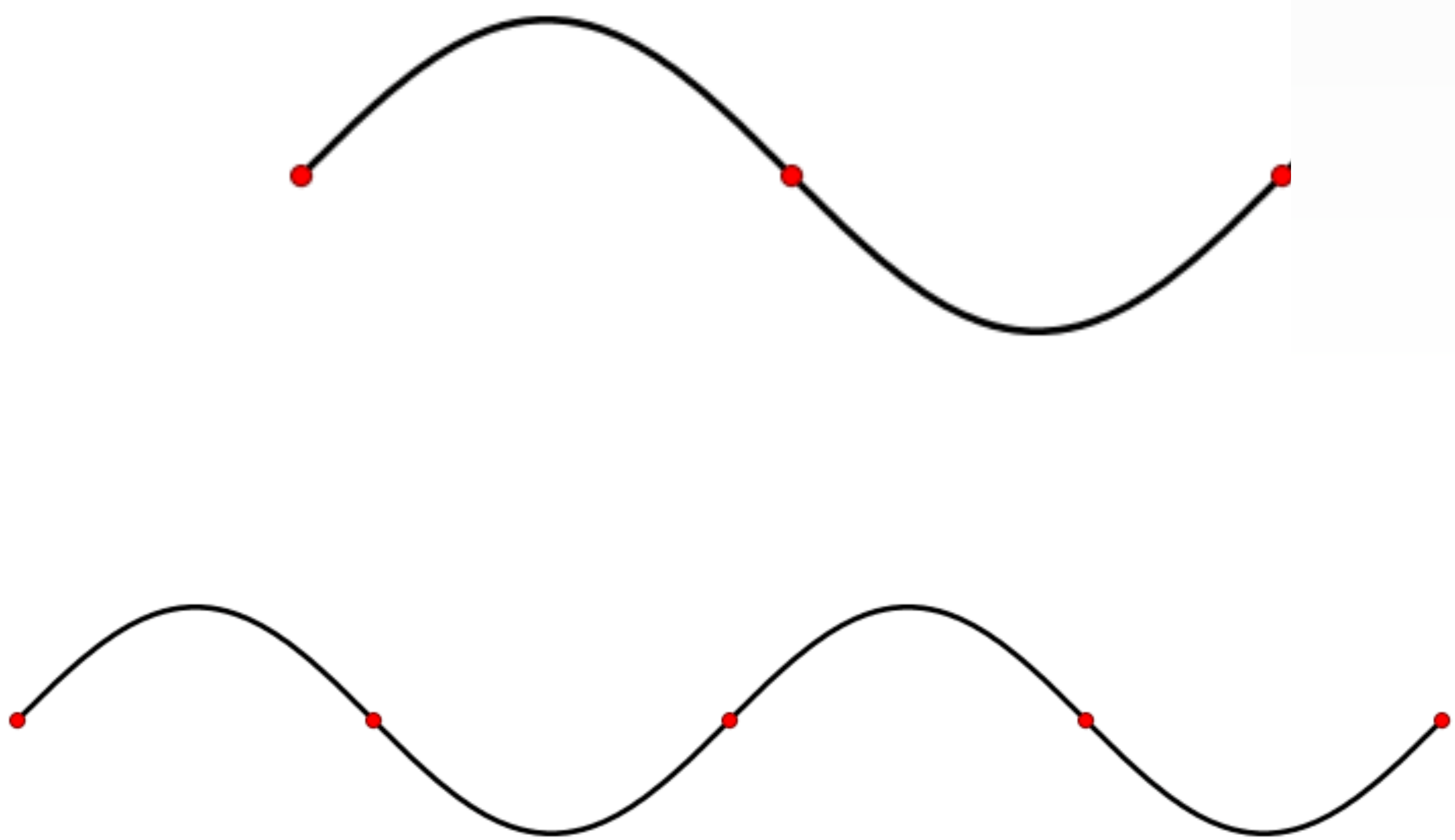

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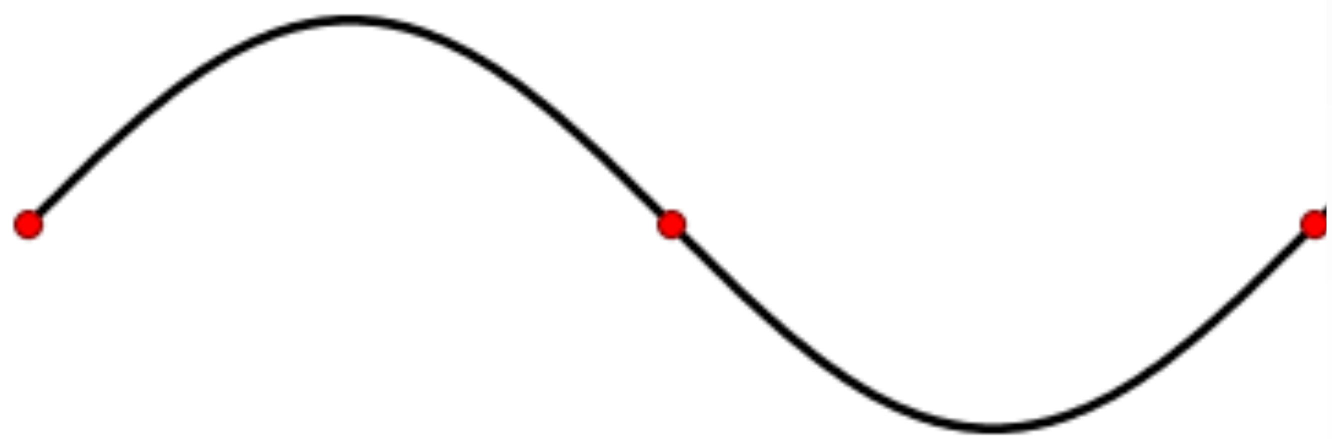

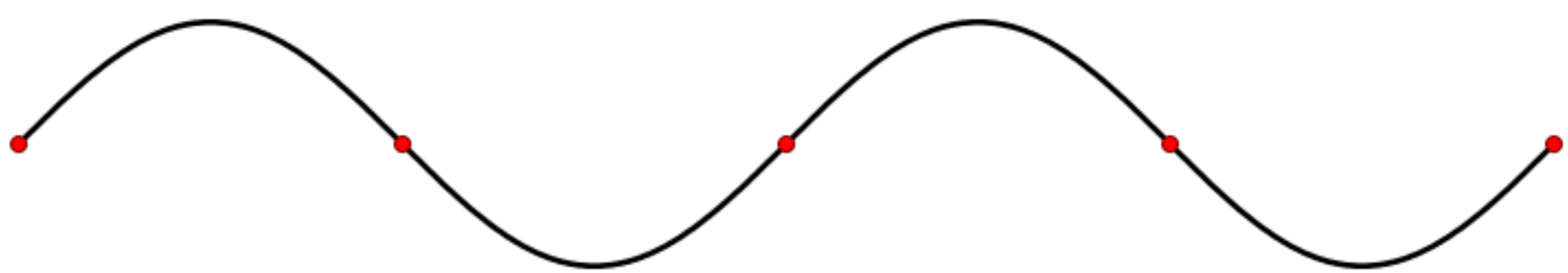

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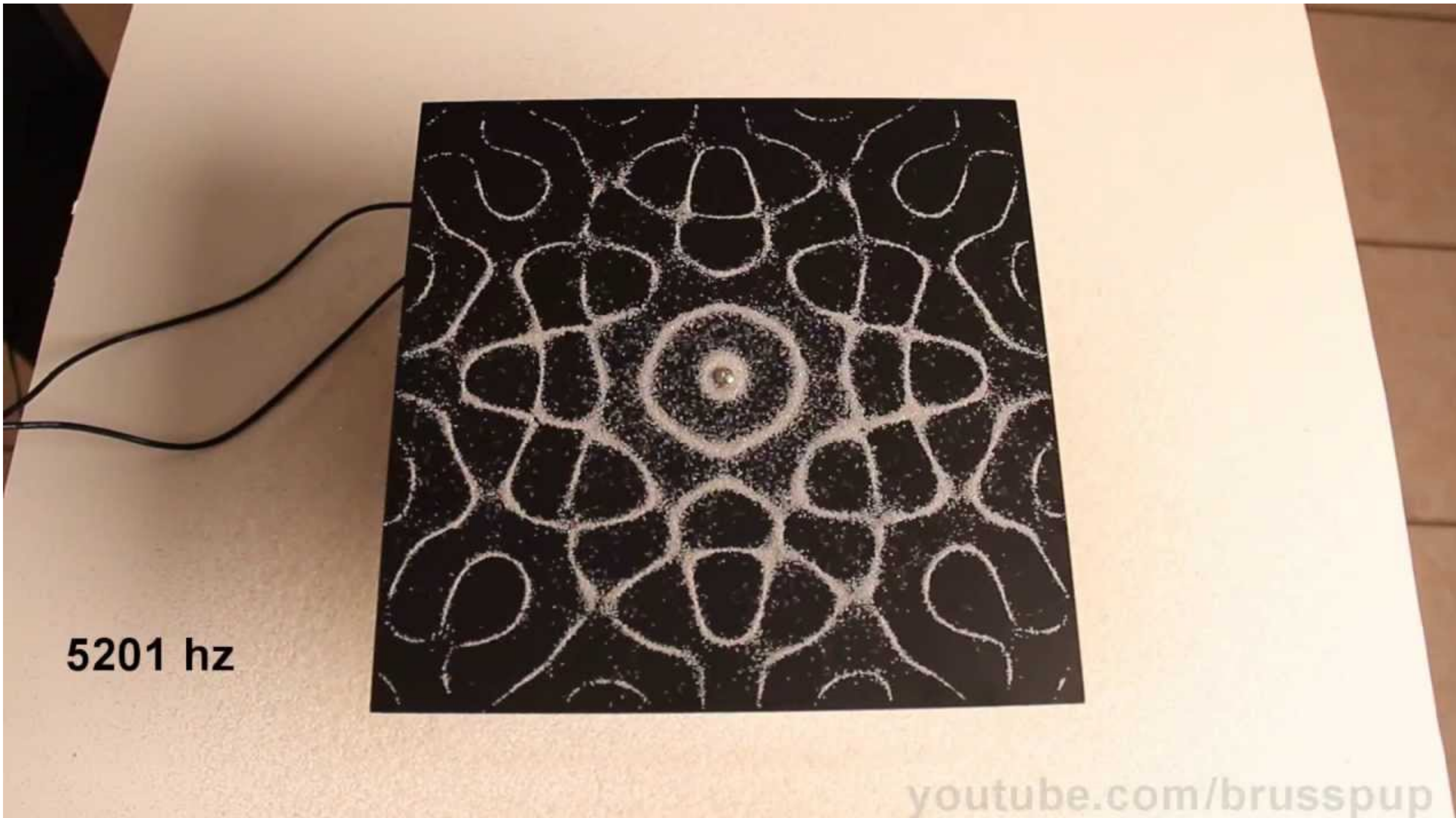

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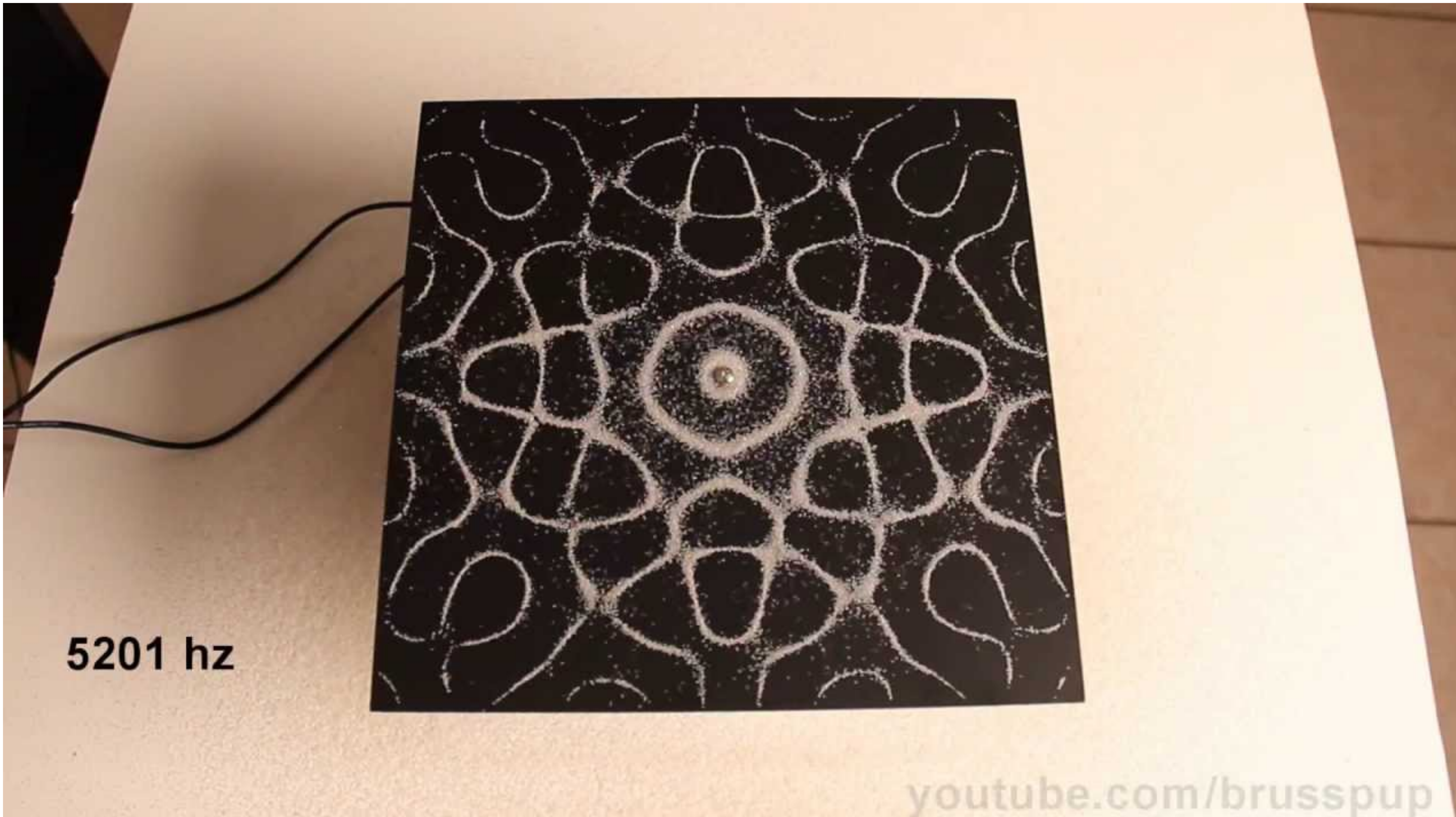

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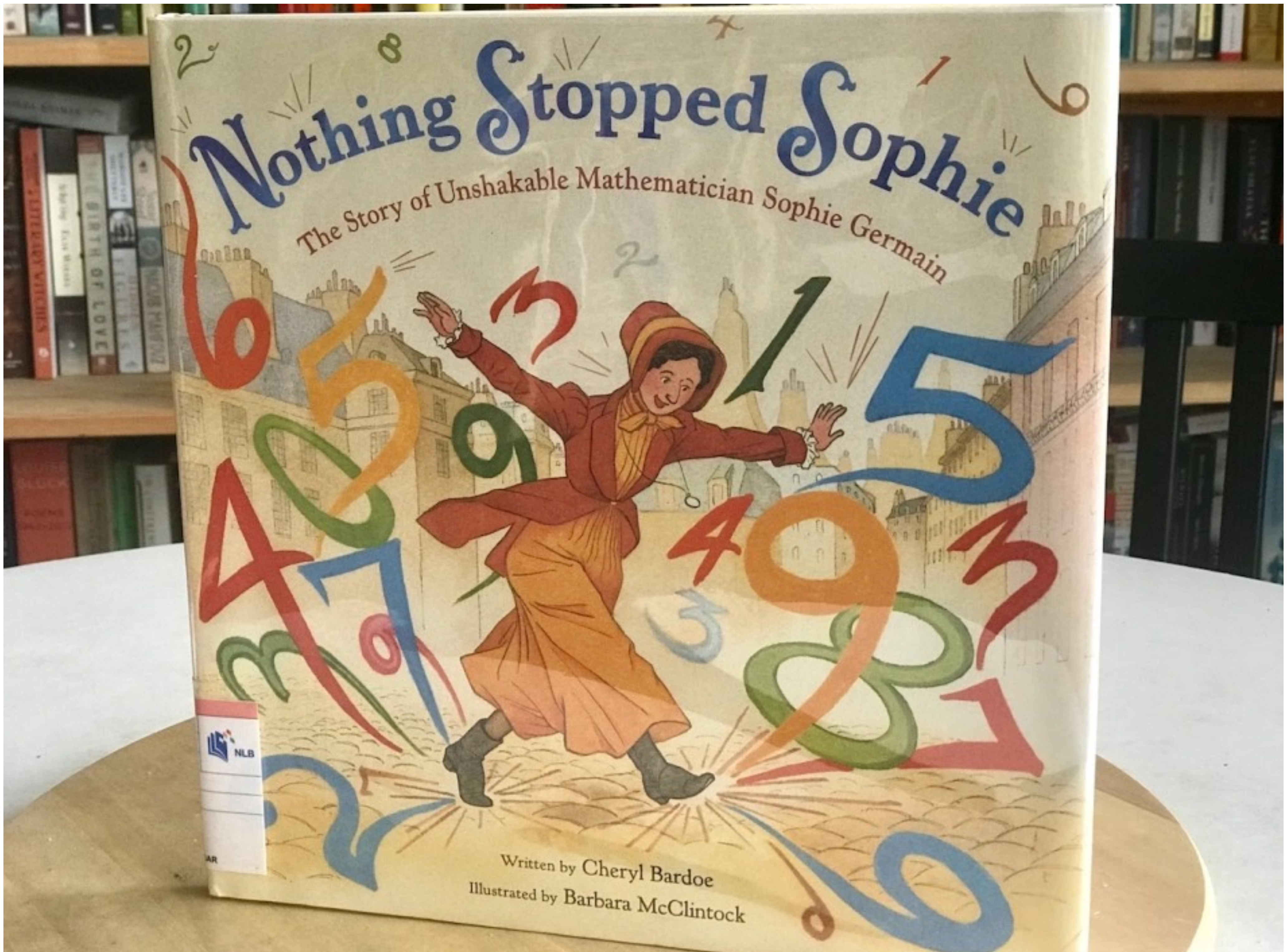

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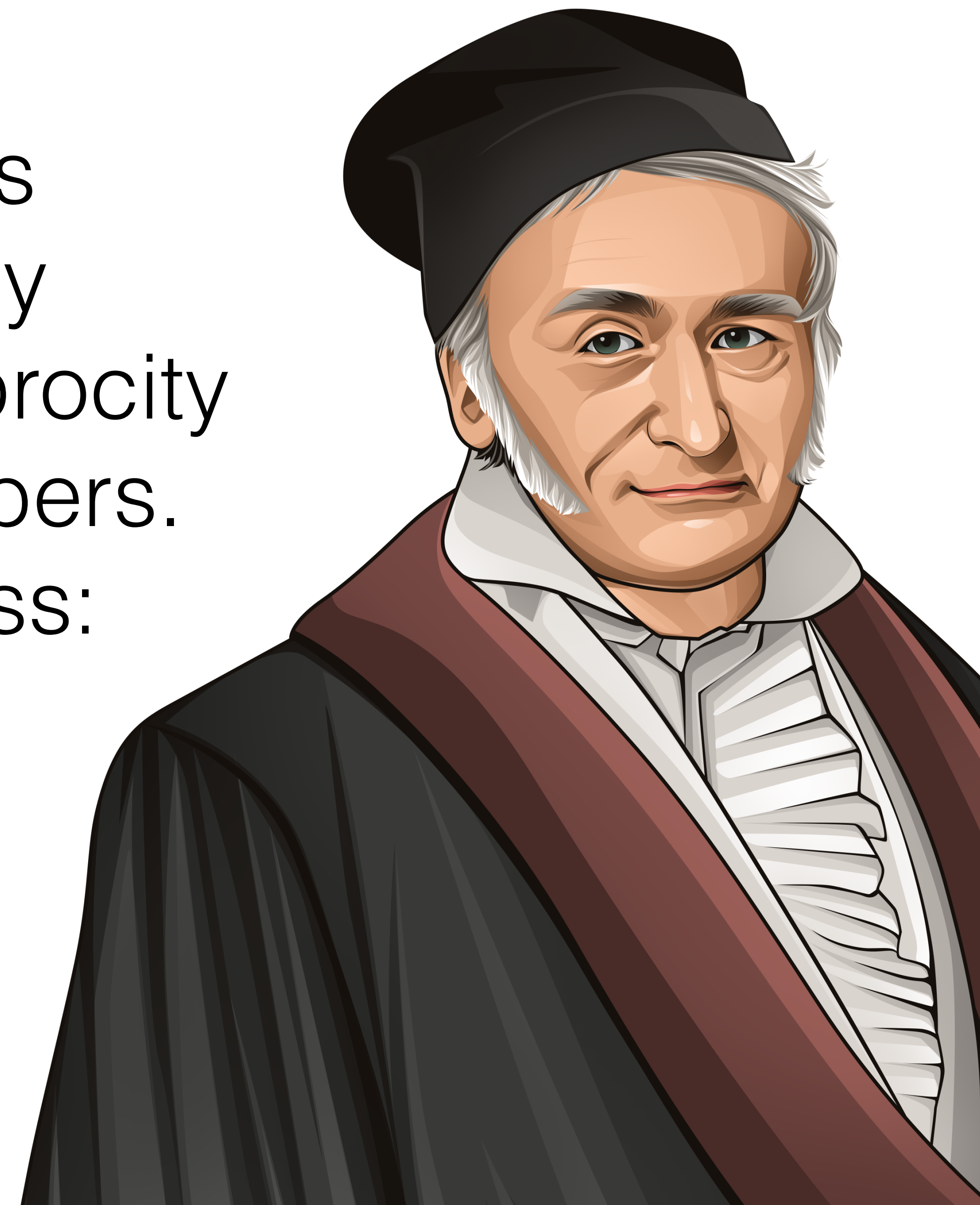

Gauss

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Gauss' braid, Gauss's constant, Gaussian curvature, Gaussian distribution, Gaussian filter, Gaussian fixed point, Gauss's formula, Gaussian function, Gauss's inequality, Gaussian integer, Gauss line, Gauss map, Gaussian measure, Gaussian quadrature, Gauss sum, Gaussian surface, Gauss transformation, several Gauss's lemmas, and at least a dozen Gauss's theorems.

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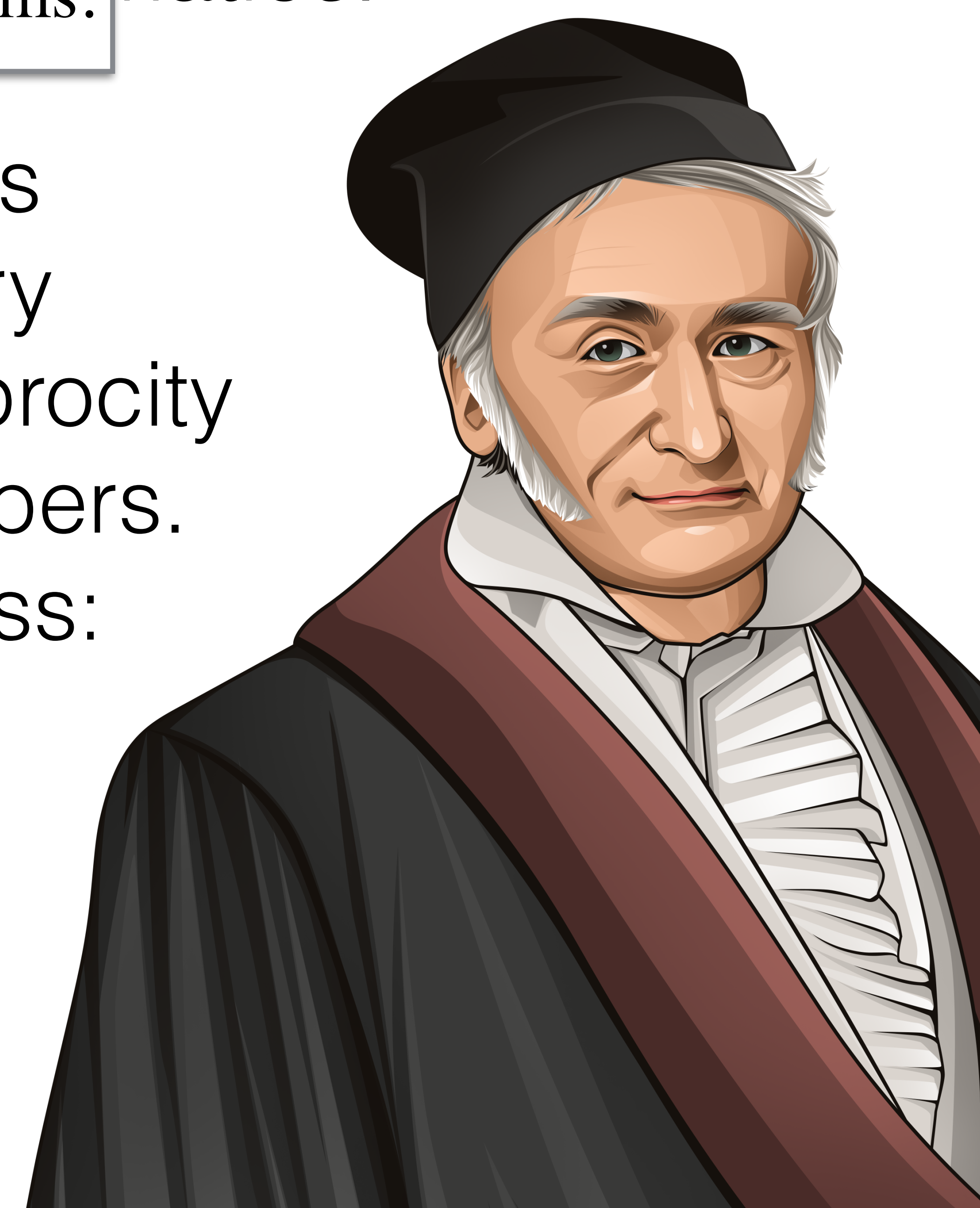

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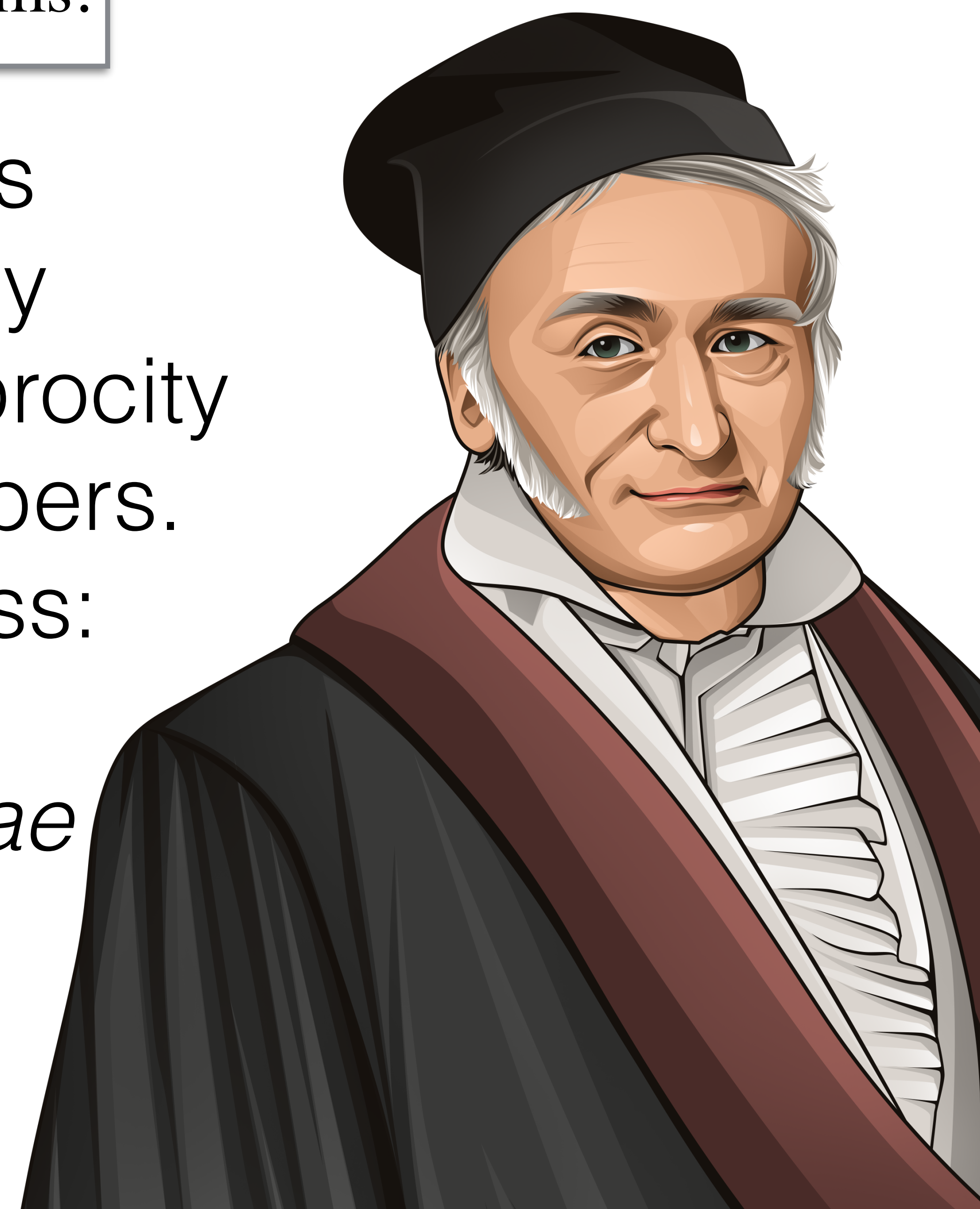
- His book *Disquisitiones Arithmeticae* is probably the most important number theory book in history.



People’s History

People’s History of Equality

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- These difficulties kept countless women from doing math or for being remembered for the math they did.
- As gender inequality has decreased, the number of women in math has increased.

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- And many discoveries outside of Europe are named after the first European to discover them.
- Things are way better than they were.
- But the demographics of math PhD-holders indicate that there is a lot more work to go.