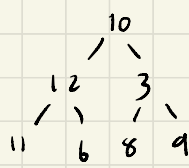4a) [10, 12, 3, 11, 6, 8, 9]
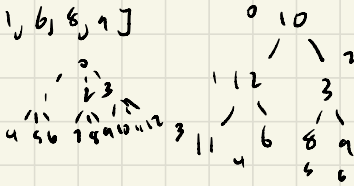

heapify(Item)
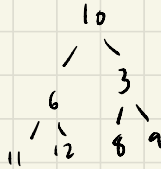start with rightmost non leaf node
"3"

[10, 12, 3, 11, 6, 8, 9]

heapify (3)
3 is less than leaf nodes (8,9) so
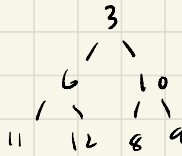no swap needed

[10, 6, 3, 11, 12, 4, 9]

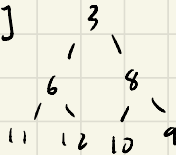heapify (12)
swaps 12 and 6

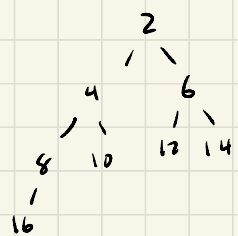[3, 6, 10, 11, 12, 8, 9]

heapify (10)
swaps 10 and 3

[3, 6, 8, 11, 12, 10, 9]

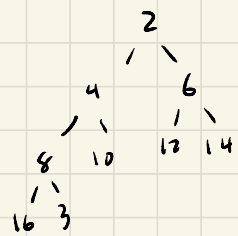swaps 10 and 8
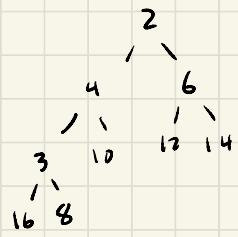
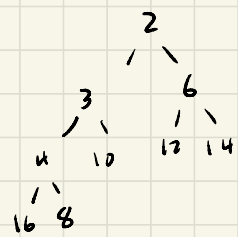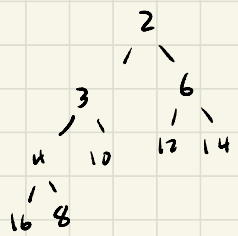4b) [2, 4, 6, 8, 10, 12, 14, 16]   original

```
        2
       / \
      4   6
     / \  / \
    8 10 12 14
   /
  16
```

insert (3)

```
        2
       / \
      4   6
     / \  / \
    8 10 12 14
   / \
  16  3
```

trickle up (3)

```
        2
       / \
      4   6
     / \  / \
    3 10 12 14
   / \
  16  8
```

swap 3 and 8

```
        2
       / \
      3   6
     / \  / \
    4 10 12 14
   / \
  16  8
```
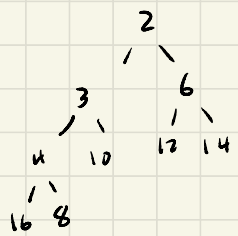
swap 3 and 4

pop (top element: 2)   (original)

```
        2
       / \
      3   6
     / \  / \
    4 10 12 14
   / \
  16  8
```

2

8
3   6
4 10 12 14
16
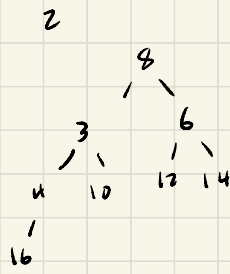
top element(2) disappears,
replace with last node.(8)

8
3   6
4 10 12 14
16

heapify (8)

3
4   6
4 10 12 14
16

swap 8 and 3

3
4   6
8 10 12 14
16
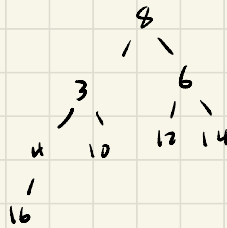
swap 8 and 4
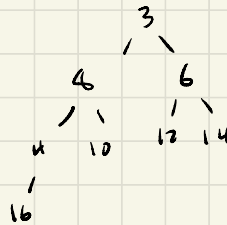
pop(top element : 3)
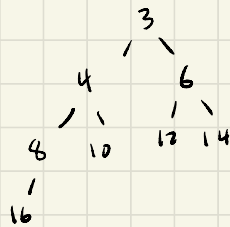
3   16
4   6
8 10 12 14
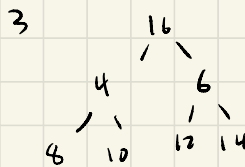
top element(3) disappears
last node (16) becomes
new top.

heapify (16)

4
16   6
8 10 12 14

swaps 16 and 4

4
8   6
16 10 12 14

swaps 16 and 8

insert(5)

```
        4          (original)
      /   \
     8      6
    / \    / \
  16  10 12  14
```

```
        4          add 5 to last node
      /   \
     8      6
    / \    / \
  16  10 12  14
  /
 5
```

```
        4          trickle up(5)
      /   \         swap 5 and 16
     8      6
    / \    / \
   5  10 12  14
  /
16
```

```
        4          swap 5 and 8.
      /   \
     5      6
    / \    / \
   8  10 12  14
  /
16
```