3a) void f1(int n) {
int i = 2;
while(i < n) {
//O(1)
i = i * i;
}
}

$\log\lceil \log_2(n-1) \rceil$

$T(n) = \sum_{i=2}^{\log\lceil \log_2(n-1) \rceil} \Theta(1) = \Theta(\log(\log_2(n)))$

$2 = 2^1 \quad 4 = 2^2 \quad 16 = 2^4 \quad 16^2 = 2^8$

n	1	2	3	4	5	6	...	17	...	2^8
k/n	0	0	1	1	2	2		3		4

i = 2
i = 4
i = 16

3b) void f2(int n) {
for(int i = 1; i <= n; i++) {
if((i % (int)sqrt(n)) == 0) {
for(int k = 0; k < pow(i, 3); k++) {
//O(1)
}
}
}
}

j	1	2	3	...	arbitrary j
i	$1\sqrt{n}$	$2\sqrt{n}$	$3\sqrt{n}$		$j\sqrt{n}$

stop when $i = n$

$i = j\sqrt{n}$ so stop when $j\sqrt{n} = n$

$j = \frac{n}{\sqrt{n}}$

$j = \frac{n^{1/2}}{n^{1/2}}$

$j = \sqrt{n}$

$\sum_{i=1}^{n} \Theta(1) + \sum_{j=1}^{\sqrt{n}} \sum_{k=0}^{j^3 n^{3/2}} \Theta(1)$

$i = j\sqrt{n}$

$i^3 = j^3 n^{3/2}$

$\Theta(n) + \sum_{j=1}^{\sqrt{n}} \Theta(j^3 n^{3/2})$

$\sum_{i=1}^{n} \Theta(i^p) = \Theta(n^{p+1})$

$\Theta(n) + \Theta((\sqrt{n})^{3+1} \cdot n^{3/2})$

$\Theta(n) + \Theta(n^2 \cdot n^{3/2}) = \Theta(n^{7/2})$

c)
```
for(int i=1; i<=n; i++){
  for(int k=1; k<=n; k++){
    if(A[k]==i){
      for(int m=1; m<=n; m=m+m){
        // O(1)
      }
    }
  }
}
```

when $m=1$: 1
$m=2$: 1, 2
$m=3$: 1, 2
$m=4$: 1, 2, 4
$m=5$: 1, 2, 4
⋮
$(\log_2 n)+1$

$$T(n): \sum_{i=1}^{n}\left[\sum_{k=1}^{n}\theta(1)+\theta\left[\sum_{m=1}^{(\log_2 n)+1}\theta(1)\right]\right]$$

$$\sum_{i=1}^{n}\left(\theta(n)+\theta(\log_2 n)\right)$$

$$\theta(n^2)+\theta(n\log_2 n)=\underline{\theta(n^2)}$$

d)

size	10		10	15		15	22	22
i	0	...	10	11	...	15	16	... 22

$$T(n)=\theta(1)+\sum_{i=0}^{n-1}\theta(1)+\theta\left(\sum_{j=0}^{size-1}\theta(1)+\theta(1)\right)$$

if we say, u = number of times the if statement has passed

size will be $\left(\frac{3}{2}\right)^u \times 10$

$$\sum_{j=0}^{size-1}\theta(1)=\sum_{j=0}^{\left(\frac{3}{2}\right)^u\times 10-1}\theta(1)=\theta\left(\frac{3}{2}u\right)$$

worst case $\Rightarrow$ $n-1=\left(\frac{3}{2}\right)^u\times 10$

$$\frac{n-1}{10}=\left(\frac{3}{2}\right)^u \qquad u=\log_{\frac{3}{2}}\left(\frac{n-1}{10}\right)$$

$\sum_{u=0}^{\log_{3/2}\left(\frac{n-1}{c_0}\right)} \Theta\left(\frac{3}{2}^u\right)$ $\Rightarrow$ geometric series $= \Theta\left(\frac{3}{2}^{\log_{3/2}\left(\frac{n-1}{c_0}\right)}\right)$

$= \Theta\left(\frac{n-1}{c_0}\right) = \Theta(n)$

$T(n) = \Theta(1) + \Theta(n) + \Theta(n) = \Theta(n)$