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Solar Energy Materials & Solar Cells 81 (2004) 269–277
Solar Energy Materials & Solar Cells
www.elsevier.com/locate/solmat

Exact analytical solutions of the parameters of real solar cells using Lambert W-function

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Received 11 August 2003; accepted 4 November 2003

Abstract

Exact closed-form solution based on Lambert W-function are presented to express the transcendental current–voltage characteristic containing parasitic power consuming parameters like series and shunt resistances. The W-function expressions are derived using Maple software. Different parameters of solar cell are calculated using W-function method and compared with experimental data of Charles et al. for two solar cells (blue and grey). Percentage accuracy of W-function method is also calculated to prove the significance of the method.

Keywords: Lambert W-function; Current–voltage characteristics; Solar cell

1. Introduction

Expressions where linear and exponential responses are combined appear in many problems of physics and engineering. Some examples are current–voltage relationships of solar cells, photo detectors and diodes used as circuit elements. It is always preferable to express current as an explicit analytical function of the terminal voltage and vice versa. Such exercise would be computationally advantageous in device models that are to be used repeatedly in circuit simulator programs, in problems of device parameter extraction, etc. Several attempts have been approached traditionally using iterative or analytical approximations [1–3], Lagrange's method of undetermined multipliers [4], approximation methods, least-squares numerical

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0927-0248/$ - see front matter
doi:10.1016/j.solmat.2003.11.018

techniques [5] to achieve the explicit solutions containing only common elementary functions. A careful search of literature reveals that use of a function known as Lambert W-function [6,7] commonly as "W-function" which is not frequently used in electronics problem is extremely important for solving such kind of problems. Solutions based on this function are exact and explicit and are easily differentiable. W-function originated from work of J.H. Lambert [8] on trinomial equation that he published in 1758 and was discussed by Euler [9] in 1779. W-function is defined by the solution of equation $W\exp(W) = x$. Although rarely used, its properties are well documented [10–13] and several algorithms were published for calculating W-function. Some recent work includes exact analytical solution based on W-function for the case of a non-ideal diode model comprised of a single exponential and a series parasitic resistance, bipolar transistor circuit analysis using W-function [14], and photorefractive two-wave mixing [15].

To the best of our knowledge no analysis of solar cells using W-function is available in literature. The current–voltage relation of solar cell is transcendental in nature, hence it is not possible to solve it for voltage in terms of current explicitly and vice versa. This paper describes the use of W-function to find the explicit solution for the current and voltage and use them to extract different parameters of solar cells. Comparisons are also made with the experimental data.

2. Theory

The single diode model assumes that the dark current can be described by a single exponential dependence modified by the diode ideality factor n. The current–voltage relationship is given by

$$i = I_{\mathrm{ph}} - \frac{V + iR_{\mathrm{s}}}{R_{\mathrm{sh}}} - I_{\mathrm{o}}\left(\mathrm{e}^{\left(\frac{V+iR_{\mathrm{s}}}{nV_{\mathrm{th}}}\right)} - 1\right), \tag{1}$$

where i and V are terminal current and voltage in amperes and volts respectively, I_{o} the junction reverse current (A), n the junction ideality factor and V_{th} the thermal voltage (kT/q), and R_{s} and R_{sh} are series and shunt resistance, respectively. Eq. (1) is transcendental in nature hence it is not possible to solve it for V in terms of i and vice versa. However, explicit solution for current and voltage can be expressed using W-function as follows:

$$i = -\frac{V}{R_{\mathrm{s}} + R_{\mathrm{sh}}} - \frac{\mathrm{Lambert}W\left(\frac{R_{\mathrm{s}}I_{\mathrm{o}}R_{\mathrm{sh}}\mathrm{e}^{\left(\frac{R_{\mathrm{sh}}(R_{\mathrm{s}}I_{\mathrm{ph}}+R_{\mathrm{s}}I_{\mathrm{o}}+V)}{nV_{\mathrm{th}}(R_{\mathrm{s}}+R_{\mathrm{sh}})}\right)}}{nV_{\mathrm{th}}(R_{\mathrm{s}}+R_{\mathrm{sh}})}\right)nV_{\mathrm{th}}}{R_{\mathrm{s}}} + \frac{R_{\mathrm{sh}}(I_{\mathrm{o}} + I_{\mathrm{ph}})}{R_{\mathrm{s}} + R_{\mathrm{sh}}}, \tag{2}$$

$$V = -iR_s - iR_{sh} + I_{ph}R_{sh} - nV_{th}\,\mathrm{Lambert}W\left(\frac{I_oR_{sh}e^{\left(\frac{R_{sh}(-i+I_{ph}+I_o)}{nV_{th}}\right)}}{nV_{th}}\right) + I_oR_{sh}. \quad (3)$$

The arguments of the W-function in Eqs. (2) and (3) only contains corresponding variable and the model's parameters.

To obtain short-circuit current, I_{sc}, substitute $V=0$ in Eq. (2), explicit solution of I_{sc} using W-function is

$$I_{sc} = -\frac{\mathrm{Lambert}W\left(\frac{R_sI_oR_{sh}e^{\left(\frac{R_{sh}(R_sI_{ph}+R_sI_o)}{nV_{th}(R_s+R_{sh})}\right)}}{nV_{th}(R_s+R_{sh})}\right)nV_{th}}{R_s} + \frac{R_{sh}(I_{ph}+I_o)}{R_s+R_{sh}}. \quad (4)$$

Similarly explicit solution of open circuit voltage V_{oc} in terms of W-function can be evaluated by substituting $i=0$ in Eq. (3)

$$V_{oc} = I_{ph}R_{sh} - nV_{th}\,\mathrm{Lambert}W\left(\frac{I_oR_{sh}e^{\left(-\frac{R_{sh}(-I_{ph}-I_o)}{nV_{th}}\right)}}{nV_{th}}\right) + I_oR_{sh}. \quad (5)$$

The dynamic resistance R_{so} and R_{sho} at the open-circuit voltage and short-circuit current are given by

$$R_{so} = -(\partial V/\partial i)_{V=V_{oc}},$$

$$R_{so} := R_s + R_{sh} - \frac{\mathrm{Lambert}W\left(\frac{I_oR_{sh}e^{\left(-\frac{R_{sh}(-I_{ph}-I_o)}{nV_{th}}\right)}}{nV_{th}}\right)R_{sh}}{1+\mathrm{Lambert}W\left(\frac{I_oR_{sh}e^{\left(-\frac{R_{sh}(-I_{ph}-I_o)}{nV_{th}}\right)}}{nV_{th}}\right)}, \quad (6)$$

$$R_{sho} = -(\partial V/\partial i)_{i=I_{sc}},$$

$$R_{sho} := R_s + R_{sh} - \frac{\mathrm{Lambert}W\left(\frac{I_oR_{sh}e^{\left(-\frac{R_{sh}(I_{sc}-I_{ph}-I_o)}{nV_{th}}\right)}}{nV_{th}}\right)R_{sh}}{1+\mathrm{Lambert}W\left(\frac{I_oR_{sh}e^{\left(-\frac{R_{sh}(-I_{sc}-I_{ph}-I_o)}{nV_{th}}\right)}}{nV_{th}}\right)}. \quad (7)$$

R_{so} and R_{sho} are also the slopes of I–V curve at open- and short-circuit conditions.

The output power is given by

$$P = Vi.$$

Power can be expressed explicitly in terms of i and V as follows:

$$P(i) = i\left(-iR_s - iR_{sh} + I_{ph}R_{sh} - nV_{th}\,\mathrm{Lambert}W\left(\frac{I_oR_{sh}\mathrm{e}^{\left(\frac{R_{sh}(-i+I_{ph}+I_o)}{nV_{th}}\right)}}{nV_{th}}\right) + I_oR_{sh}\right), \tag{8}$$

$$P(V) = V\left(-VR_s - \mathrm{Lambert}W\left(\frac{R_sI_oR_{sh}\mathrm{e}^{\left(\frac{R_{sh}(R_sI_{ph}+R_sI_o+V)}{nV_{th}(R_{sh}+R_s)}\right)}}{nV_{th}(R_{sh}+R_s)}\right)nV_{th}R_{sh} - \mathrm{Lambert}W\left(\frac{R_sI_oR_{sh}\mathrm{e}^{\left(\frac{R_{sh}(R_sI_{ph}+R_sI_o+V)}{nV_{th}(R_{sh}+R_s)}\right)}}{nV_{th}(R_{sh}+R_s)}\right) \times nV_{th}R_s + R_sI_{ph}R_{sh} + R_sI_oR_{sh}\right)/(R_s(R_{sh}+R_s)). \tag{9}$$

To obtain maximum-power output condition we have to optimize $P(i)$ and $P(V)$ as follows:

Differentiate $P(i)$ and $P(V)$ w.r.t. i and V, respectively, and then solve them to obtain optimum current, i_m and optimum voltage, V_m, corresponding to maximum power output condition:

$$\begin{aligned}(\partial P/\partial i)_{i=i_m} &= -iR_s - iR_{sh} + I_{ph}R_{sh} - nV_{th}\,\mathrm{Lambert}W\left(\frac{I_oR_{sh}\mathrm{e}^{\left(-\frac{R_{sh}(-i+I_{ph}+I_o)}{nV_{th}}\right)}}{nV_{th}}\right) \\ &\quad + I_oR_{sh} \\ &\quad + i\left(-R_s - R_{sh} + \frac{\mathrm{Lambert}W\left(\frac{I_oR_{sh}\mathrm{e}^{\left(\frac{R_{sh}(-i+I_{ph}+I_o)}{nV_{th}}\right)}}{nV_{th}}\right)R_{sh}}{1 + \mathrm{Lambert}W\left(\frac{I_oR_{sh}\mathrm{e}^{\left(\frac{R_{sh}(-i+I_{ph}+I_o)}{nV_{th}}\right)}}{nV_{th}}\right)}\right),\end{aligned} \tag{10}$$

$$
\begin{aligned}
&-(\partial P/\partial V)_{V=V_\mathrm{m}}\\
&= -\frac{V}{R_\mathrm{s}+R_\mathrm{sh}} - \frac{\mathrm{LambertW}\left(\dfrac{R_\mathrm{s}I_\mathrm{o}R_\mathrm{sh}\mathrm{e}^{\left(\frac{R_\mathrm{sh}(I_\mathrm{ph}R_\mathrm{s}+I_\mathrm{o}R_\mathrm{s}+V)}{nV_\mathrm{th}(R_\mathrm{s}+R_\mathrm{sh})}\right)}}{nV_\mathrm{th}(R_\mathrm{s}+R_\mathrm{sh})}\right)nV_\mathrm{th}}{R_\mathrm{s}}\\
&\quad + \frac{R_\mathrm{sh}(I_\mathrm{ph}+I_\mathrm{o})}{R_\mathrm{s}+R_\mathrm{sh}} + V\left(-\frac{1}{R_\mathrm{s}+R_\mathrm{sh}}\right.\\
&\quad \left. - \frac{\mathrm{LambertW}\left(\dfrac{R_\mathrm{s}I_\mathrm{o}R_\mathrm{sh}\mathrm{e}^{\left(\frac{R_\mathrm{sh}(I_\mathrm{ph}R_\mathrm{s}+I_\mathrm{o}R_\mathrm{s}+V)}{nV_\mathrm{th}(R_\mathrm{s}+R_\mathrm{sh})}\right)}}{nV_\mathrm{th}(R_\mathrm{s}+R_\mathrm{sh})}\right)R_\mathrm{sh}}{\left(1+\mathrm{LambertW}\left(\dfrac{R_\mathrm{s}I_\mathrm{o}R_\mathrm{sh}\mathrm{e}^{\left(\frac{R_\mathrm{sh}(I_\mathrm{ph}R_\mathrm{s}+I_\mathrm{o}R_\mathrm{s}+V)}{nV_\mathrm{th}(R_\mathrm{s}+R_\mathrm{sh})}\right)}}{nV_\mathrm{th}(R_\mathrm{s}+R_\mathrm{sh})}\right)\right)(R_\mathrm{s}+R_\mathrm{sh})R_\mathrm{s}}\right). \qquad (11)
\end{aligned}
$$

Solving the above equations we can obtain i_m and V_m and hence maximum output power $P_\mathrm{m}(V_\mathrm{m}i_\mathrm{m})$.

Fill factor (FF), which is a measure of squareness of I–V curve, is found to be

$$\mathrm{FF} = (P_\mathrm{m})/(V_\mathrm{oc}I_\mathrm{sc}).$$

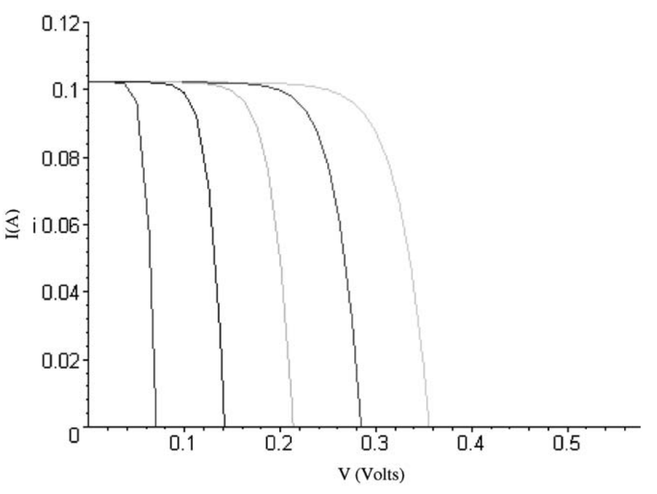

Fig. 1. Current–voltage characteristics of solar cell for different values of ideality factors.

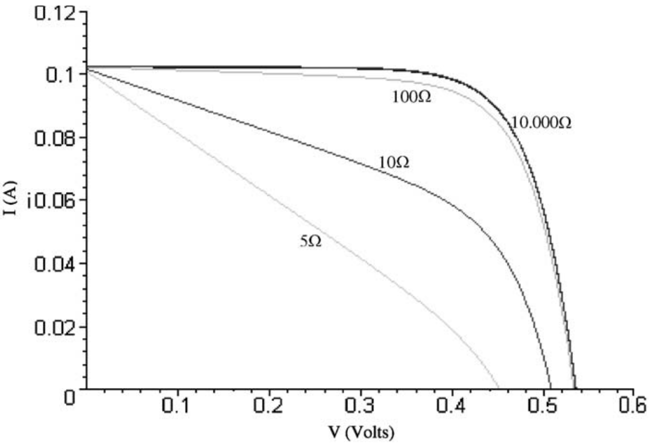

Fig. 2. Current–voltage characteristics of solar cell for different values of shunt resistance.

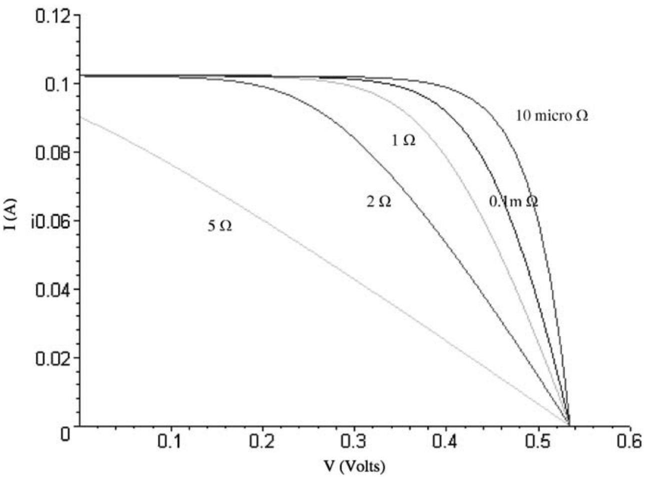

Fig. 3. Current–voltage characteristics of solar cell for different values of series resistance.

The two ratios V_m/V_{oc} and I_m/I_{sc} and the FF all improve with increasing value of V_{oc} and decreasing value of n the ideality factor. The nearer the value of n is to unity, the better the device performance is, other parameters being equal (Fig. 1). FF degrades with increasing value of R_s and increases with increasing value of R_{sh} (Figs. 2 and 3).

3. Calculations

We evaluated different parameters for two solar cells (namely blue solar cell and grey solar cells) using data of Phang et al. [16] and Charles et al. [17] (Table 1) and

Table 1
Comparison between experimental and theoretical data and relative accuracy

Parameters	Experimental data of Charles et al.		Data using W-function		Accuracy (%)	
	Blue solar cell	Grey solar cell	Blue solar cell	Grey solar cell	Blue solar cell	Grey solar cell
V_{oc} (V)	0.536	0.524	0.53465	0.52093	0.251	0.585
I_{sc} (A)	0.1023	0.561	0.10229	0.55931	0.009	0.301
R_{so} (Ω)	0.45	0.162	0.44298	0.16121	1.56	0.487
R_{sho} (Ω)	1000	25.9	997.4018	25.896	0.259	0.015
V_m (V)	0.437	0.390	0.43191	0.38473	1.16	1.35
I_m (A)	0.0925	0.481	0.093396	0.48335	0.968	0.488
T (K)	300	307				

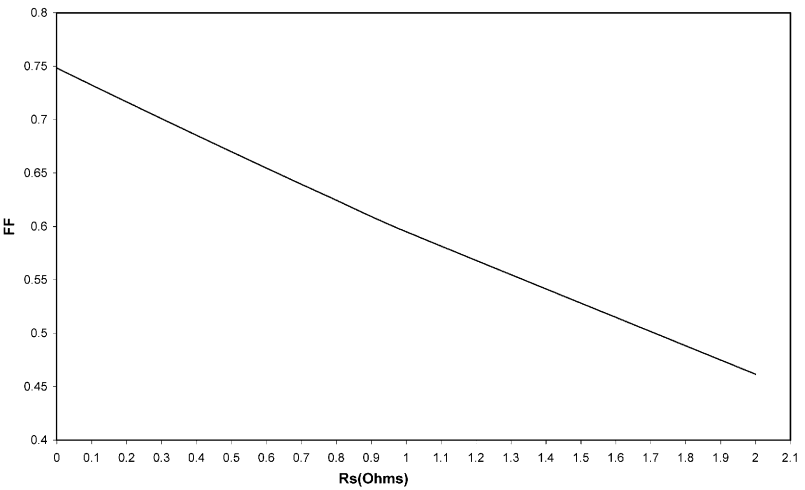

Fig. 4. Fill factor Vs Series Resistance.

equations derived above. Calculated parameters using W functions are compared with experimental data by Charles et al. (Table 1). Relative accuracy has also been calculated (Table 1).

4. Results and discussion

The approach for extracting solar cell parameters using W-function is made first time.

Various other methods suggested earlier have either larger computational time or are less accurate due to various approximations. Exact explicit analytical solution for current, voltage, short-circuit current, open-circuit voltage, output power are

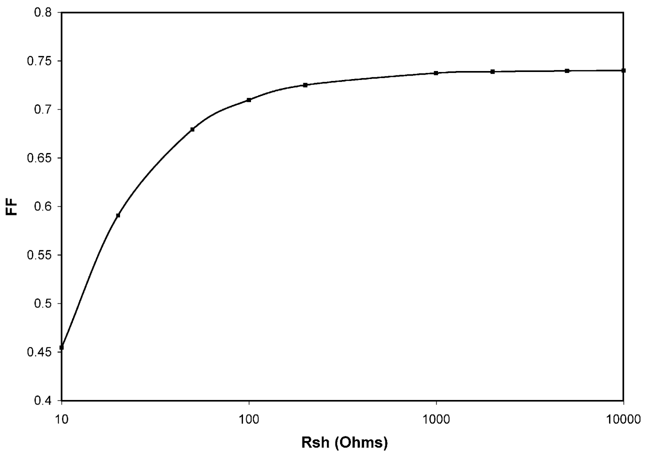

Fig. 5. Fill factor Vs Shunt Resistance.

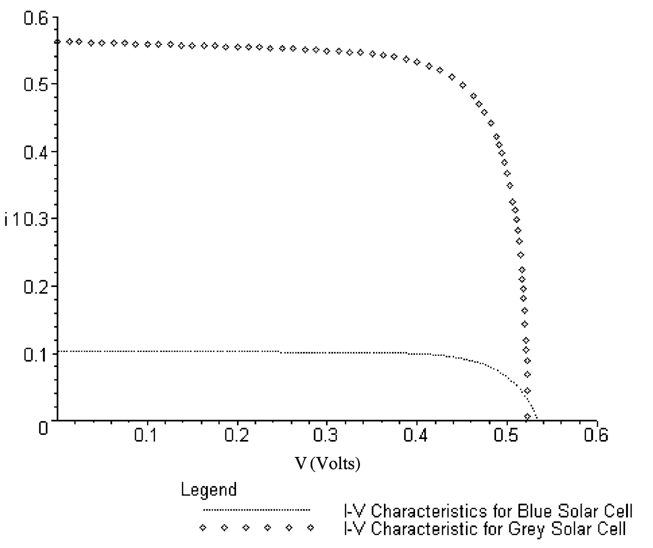

Fig. 6. Current–voltage characteristics of blue and grey solar cell, curve 1 (solid line) for blue solar cell, curve 2 (dotted line) for grey solar cell.

presented. For both solar cells calculated parameters using W-function are in better agreement with experimental data by Charles et al. Accuracy is more as no approximations are made for solving the equations. Various curves of FF versus R_s (Fig. 4), FF versus R_{sh} (Fig. 5) are plotted. I–V curves for various R_s and R_{sh} (Figs. 2 and 3) are also plotted. It was found that for $R_s > 1\ \Omega$ and $R_{sh} < 10\ \Omega$ I–V plot is a triangle which is worst case for solar cell. Current-voltage characteristics of blue and grey solar cell for an experimental case are shown in Fig. 6.

It can be concluded that W-function-type solutions are attractive alternatives to extract and study solar cell parameters.

References

[1] T.A. Fjeldly, B.J. Moon, M. Shur, IEEE Trans. Electron. Dev. 38 (1991) 1976–1977.
[2] M.T. Abuelma'atti, Electron. Lett. 28 (1992) 594–595.
[3] A. Oritz-Conde, F.J. Garcia, Electron. Lett. 28 (1992) 1964–1965.
[4] L.S. Kothari, A. Kapoor, J. Appl. Phys. 53 (1982) 5982–5984.
[5] S.K. Datta, Solid State Electron. 35 (1992) 1667–1673.
[6] T.C. Banwell, A. Jayakumar, Electron. Lett. 36 (2000) 291–292.
[7] A. Oritz-Conde, F.J. Garcia, Solid State Electron. 440 (2000) 1861–1864.
[8] J.H. Lambert, Acta Helv. Phys. Math. Anat. Bot-Med. 3 (1758) 3, 128.
[9] L. Euler, De Serie Lambertina Plurimisque eius ingignibus properitatibus, Lenolardi Euleri opera Omania, Series Prima, Vol. VI, 1921, p. 350.
[10] D.A. Barry, P.J. Culligan-Hensley, S.J. Barry, ACM Trans. Math. Software 21 (1995) 161–171.
[11] D.A. Barry, P.J. Culligan-Hensley, S.J. Barry, ACM Trans. Math. Software 21 (1995) 172–181.
[12] R.M. Corless, G.H. Gonnet, D.E.G. Hare, D.J. Jeffry, D.E. Kunth, ACM Trans. Math. Software 5 (1996) 329–359.
[13] F.N. Fritsch, R.E. Shafer, W.P. Crowley, Commun. ACM 16 (1973) 123–124.
[14] Banwell, T.C., IEEE Trans. Circuits Systems—I Fundamental Theory and Appl. 47 (2000).
[15] H. Rehn, J. Mod. Phys. 45 (1998) 2197–2199.
[16] J.C.J. Phang, D.S.H. Chan, J.R. Philips, Electron. Lett. 20 (1984) 406.
[17] J.P. Charles, M. Abdelkrim, Y.H. Muoy, P. Mialha, Solar Cells 4 (1981) 169.